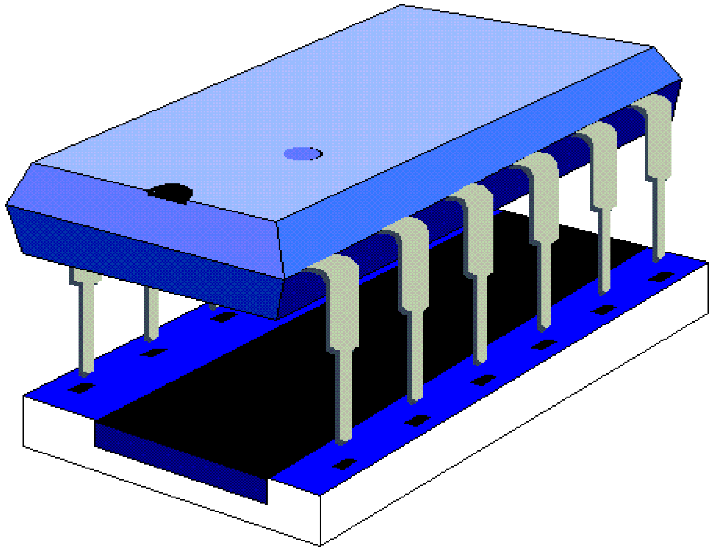
Digital Electronics



Number Systems and Codes

Grayson Himunzowa.

Objectives

- At the end of this chapter the student should be able to
- Understand different number systems in use.
- Convert one number system to another number system.
- Understand codes used in digital systems.
- Carry out binary arithmetic.
- Negate binary numbers

Number Systems

- There are many number systems but we shall only explore the following:
  - Decimal (accepted number system for human operations), has 10 symbols.
  - Binary (number system is used for computer). Has 2 sysmbols.
  - Hexadecimal (has 16 sysmbols),
  - Octal (has eight symbols)
  - Quinary (has five symbols)
  - Duo decimal system (not in use now). It was one of the old fashioned British system. Had 12 symbols.
  - Ternary system (has 3 symbols)

Codes

- Codes
  - BCD (binary cded decimal),
  - Gray code,
  - ASCII (American standard code for information interchange)

Decimal Numbers

- Normal number system used everyday by humans
- Ten symbols used to give the digits (0,1,2,3,4,5,6,7,8,9)
- *Weighted* code - each digit position having an increasing power of 10 as its weight – base 10
- For example a decimal number 123 can be weighted
- Position indicates weights of ten *i.e. 3**10^0, 2*10^1, 1*10^2.
- Which is equal to 123_{10}

General form of weight of decimal number N

A given number N can be represented as

$$N = d_0R^0 + d_1R^1 + d_2R^2 + \ldots + d_nR^n$$

Where d is a digit, R is a radix or base

- The expression can work for all number systems by replacing R.

Binary Numbers

- Similar to the decimal system but operates with a base of two rather than base ten of the decimal system.
- Uses only two possible values or symbols (0 and 1) for each digit.
- Position determines weight in increasing powers of two, *i.e.* 2^0, 2^1, 2^2

Hexadecimal numbers

- The hexadecimal number system has sixteen digits and is of base 16.
- is used primarily for compactness of displaying and writing binary numbers.
- For example: the binary number 01110001 can be written as 71_{16}.
- Long binary numbers are difficult to read and write because it is easy to drop or transpose a bit.
- Most digital systems process data in groups that are multiples of 4 bits, making hexadecimal a convenient way of representation.

Hexadecimal Numbers

- Base 16
- 16 digits - 0 1 2 3 4 5 6 7 8 9 A B C D E F
- A to F represent values 10_{10} to 15_{10}

Convert $A8_{16}$ to decimal

Hexadecimal Numbers

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Octal number system

- Like the hexadecimal number system, the octal number system provides a convenient way to express binary numbers and codes.
- It is used less frequently than hexadecimal.
- Composed of 8 symbols:
- Thus 0,1,2,3,4,5,6,7.
- Its base is 8

Octal Numbers

- Digit positions indicate increasing powers of 8

Exercise

Convert 315_8 to decimal

Octal Numbers

Conversion between binary and octal is simple

Octal Digit	Binary Value
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

- Convert from octal to binary - replace each octal digit with 3 bit binary code

Convert 562_8 to binary

Exercise1

Express in weights:

132.251 (decimal weights)

101101.111_2 (binary weights).

Conversion between number systems

It is easy to convert from decimal to binary and from binary to decimal.

But to convert from other number systems (Hexadeximal, octal) we need first to convert to decimal and then to binary.

We demonstrate converting number systems by way of examples:

What is the decimal value of 1101_2?

- We get the sum of weights of binary number 1101:

  $1*2^3 + 1*2^2 + 0*2^1 + 1*2^0 = 13_{10}$

Convert binary number 101101.111_2

- We again get the sum of weights of binary number 101101.111_2 :

$1*2^3+1*2^2+0*2^1 +1*2^0+1*2^{-1} +1*2^{-2+}1*2^{-3}= 13.875_{10}$

Continue

- Convert 79_{10} to binary.
- Convert 233_{10} to octal and convert it to binary.
- Convert octal number 6072_8 to decimal.
- Convert 233_{10} to hexadecimal.

Convert 79_{10} to binary.

- We devide 79 by the base of binary as we did in primary mathematics:

79/2 = 39 r 1

39/2 = 19 r 1

19/2 = 9 r 1

9/2 = 4 r 1

4/2 = 2 r 0

2/2 = 1 r 0

1/2 = 0 r 1

Hence the binary result is 1001000_2

Exercise

- Convert 233_{10} to octal and convert it to binary.
- Convert octal number 6072_8 to decimal.
- Convert 233_{10} to hexadecimal.

Binary to hexadecimal conversion

- Simply break the number into 4-bit groups,
- Starting at the right-most bit,
- Replace 4-bit group with the equivalent hexadecimal sysmbol.

Example

- Convert the following binary numbers to hexadecimal:

  (a) 1100,1010,0101,0111 = CA57

  (b) 11,1111,0001,0110,1001 = 3F169

Hexadecimal to Binary

- Replace hexadecimal symbols with the appropriate four bits.

Exercise

Determine the binary numbers for the following hexadecimal numbers:

(a) $10A4_{16}$ (b) $CF8E_{16}$ (c) 9742_{16}

Hexadecimal to decimal conversion

- First convert hexadecimal number to binary,
- And then convert from binary to decimal

Exercise

Convert the following hexadecimal numbers to decimal:

(a) $1C_{16}$ (b) $A85_{16}$

Decimal to hexadecimal

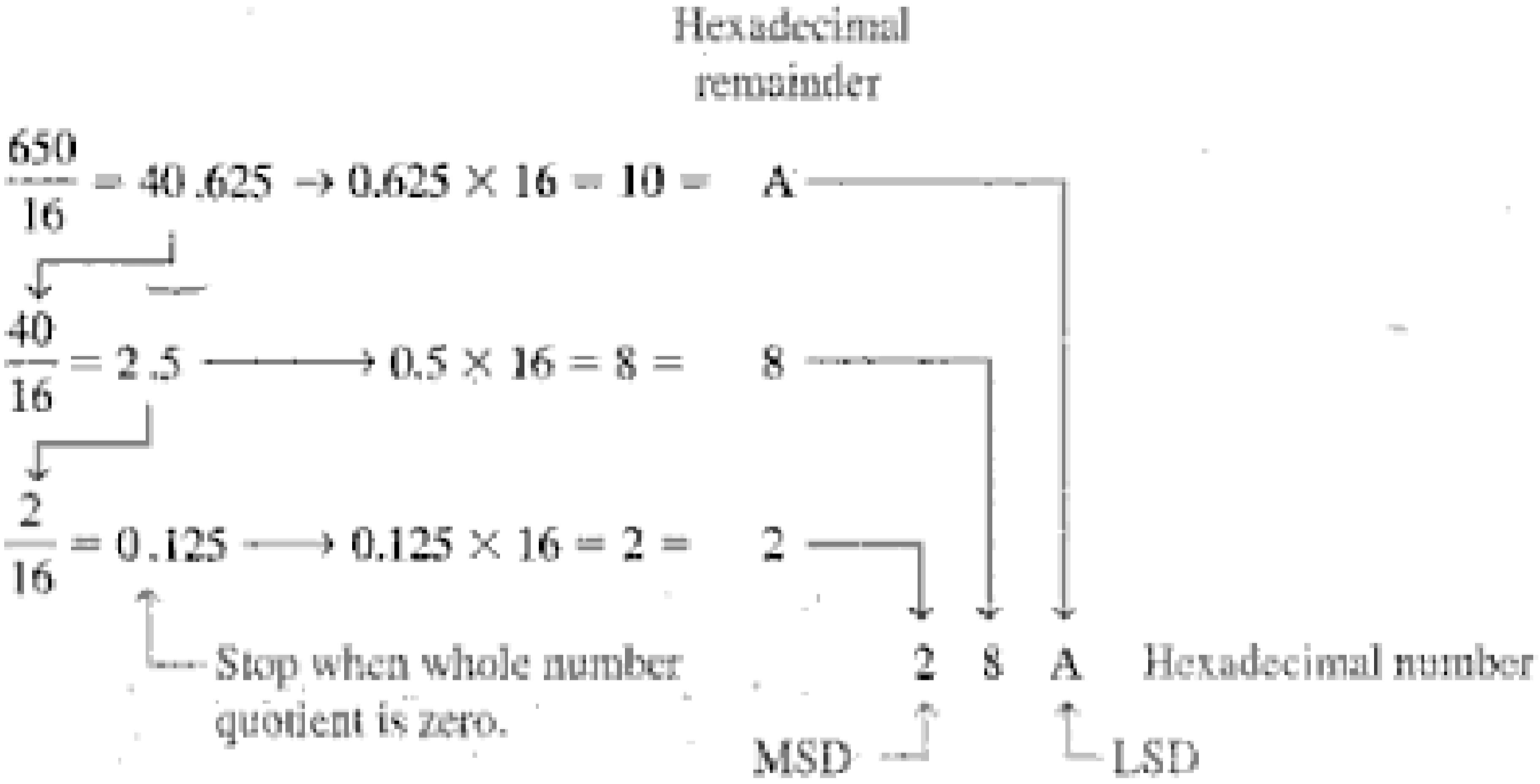
- Repeated division of a decimal number by 16 will produce equivalent hexadecimal number.
- The first remainder produced is lsd and the last remainder produced is msd.

Example

- Convert the decimal number 650 to hexadecimal by repeated division.

soln

- soln


Hexadecimal remainder
650/16 = 40.625 → 0.625 × 16 = 10 = A
40/16 = 2.5 → 0.5 × 16 = 8 = 8
2/16 = 0.125 → 0.125 × 16 = 2 = 2
Stop when whole number quotient is zero.
2 8 A Hexadecimal number
MSD
LSD

Conversion of Decimal fractions to Binary

- To convert a binary fraction to binary, the method is to repeatedly double the decimal fraction.
- If a 1 appears to be on the left hand side of the decimal point after multiplication by 2 is performed, a 1 is added to the right of the binary point to being formed.
- If a 0 remains to the left of the decimal point of the decimal fraction a 0 is added to the right of the binary point.

Example

- Convert 0.875_{10} to binary.

Exercise2

- Convert 0.4375_{10} to binary.
- Convert quinary fraction into octal: fraction is 0.445

Octal to decimal conversion

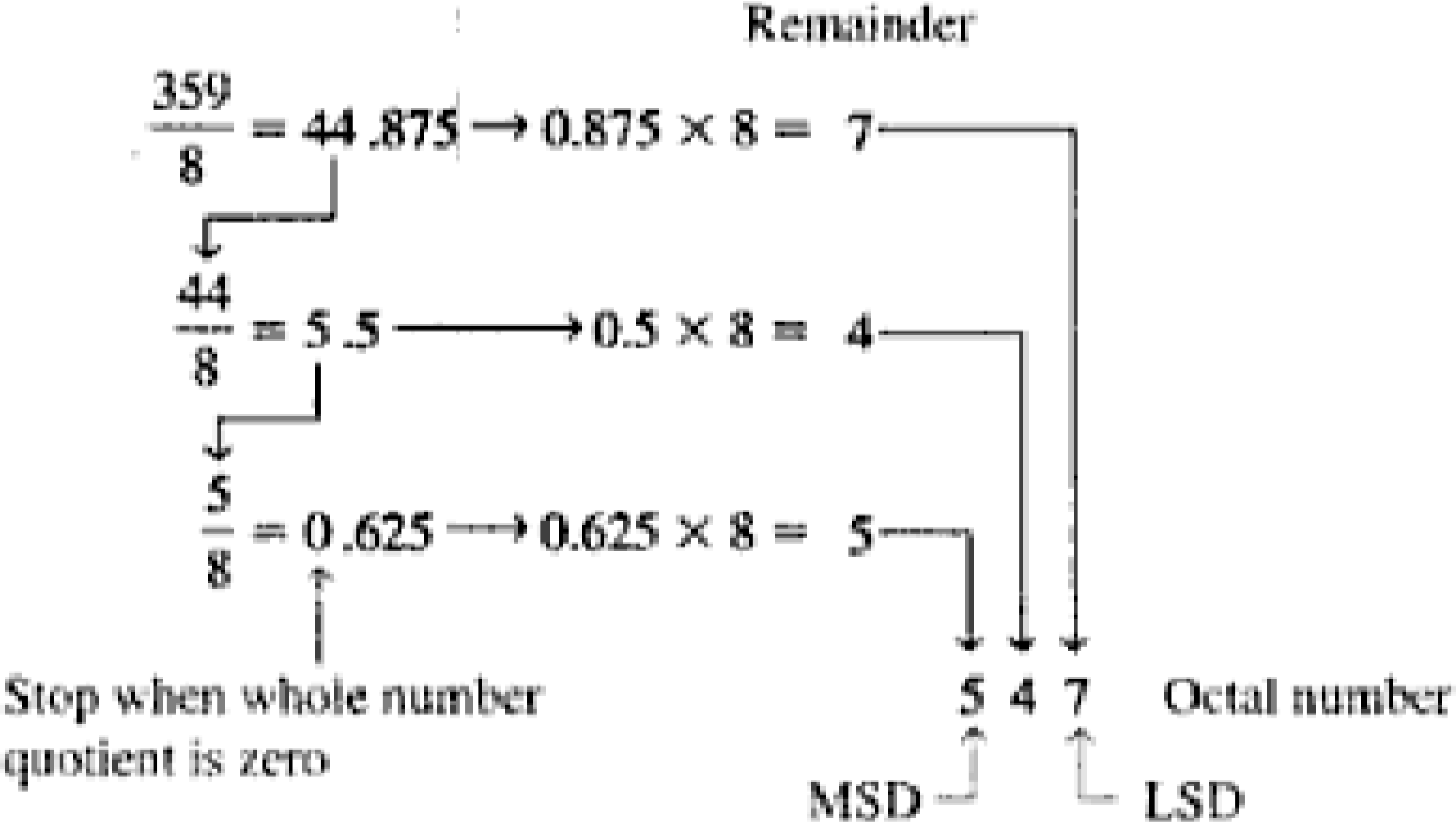
- Convert 2374_8 to decimal.

Weight: 8^3 8^2 8^1 8^0

Octal number: 2 3 7 4

$$\begin{aligned} 2374_8 &= (2 \times 8^3) + (3 \times 8^2) + (7 \times 8^1) + (4 \times 8^0) \\ &= (2 \times 512) + (3 \times 64) + (7 \times 8) + (4 \times 1) \\ &= 1024 + 192 + 56 + 4 = 1276_{10} \end{aligned}$$

Decimal to octal conversion



Octal to binary conversion

- Because each octal digit can be represented by a 3-bit binary number,
- It is very easy to convert from octal to binary,
- Each octal digit is represented by three bits.

OCTAL DIGIT	0	1	2	3	4	5	6	7
BINARY	000	001	010	011	100	101	110	111

Example

- Convert each of the following octal numbers to binary:

  (a) 13_8 (b) 25_8 (c) 140_8 (d) 7526_8

Binary to octal conversion

- Start with right most group of 3-bits,
- Moving to left convert each 3-bit group to equivalent octal digit.
- Example: convert binary 11010000100 to octal number.

End of part one on Number systems and codes

- Please do all exercises as assignment2