

THE UNIVERSITY OF ZAMBIA  
SCHOOL OF ENGINEERING  
DEPARTMENT OF MECHANICAL ENGINEERING

TERM 3 – JULY 2016  
EEE 3112 - ELECTRICAL ENGINEERING PRACTICE

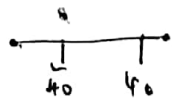
ANSWER ALL QUESTIONS  
CLOSED BOOK

TIME ALLOWED 2 HOURS

**Question 1**

A 4m long beam supported by pin joints at its ends, carries two 40kg motors at  $\frac{1}{4}$  and  $\frac{3}{4}$  of the span of the beam. All motors rotating in the same direction, deliver 60kw at 3000rpm to downstream applications. Find:

- (a) The reactions at the pin joints (6 marks)  
(b) expressions for the slope of the beam at each end in terms of EI. (10 marks)  
(b) expression for the maximum deflection in terms of EI. (9 marks)



**Question 2**

You have been commissioned to design the suspension for a new light truck. The net weight of the light truck up is 3200kg, distributed 35% on the rear axle and 65% on the front axle. When fully loaded an additional 7000kg is added to the light truck, distributed 80% on the rear axle and 20% on the front axle. You are using, on the front axle 2 identical helical close coiled springs in parallel, and on the rear axle 2 identical leaf springs in parallel. Your design constraint is, completely unloaded to fully loaded, the maximum static deflection on either axle should not exceed 15 cm. given for:

60kw  
3000rpm

Helical spring:

Mean coiled diameter	=	30cm
Modulus of rigidity	=	83.5 GN/m <sup>2</sup>
Wire diameter	=	35mm

*Regina T. Njirago*

Leaf spring:

Length	=	1 m
Number of leaves	=	8
Width to thickness ratio	=	8:1
Elastic modulus	=	125 GN/m <sup>2</sup>

Find:

- (a) the number of complete coils required in the helical springs (6 marks)  
(b) the width of the leaf springs (6 marks)  
(c) the deflection at both axles when acted upon the net weight of the pickup only. (6 marks)

11007.36

If the distance between the rear and front axle is 6 m what is the magnitude and direction of the shift in the centre gravity of the pickup when loaded with the additional 5000kg. (7 marks)

**Question 3**

A solid 1m shaft of radius 60 mm and density 4 g/cm<sup>3</sup> carries 4 unbalanced masses spaced as follows:

Masses A, B, C, and D are 13, 12, 11, and 10kg respectively, rotating at radii 10, 11, 12, 13cm respectively, and spaced from one end of the shaft at 30, 45, 60, and 75 cm respectively. the angular spacing from A to B, C, and D are 60, 120, and 180° respectively.

- (a) Balance the shaft using two 15 kg masses at either end (9 marks)
- (b) If before balancing, the shaft rotated at 3000 rpm what will be its rotation after balancing. (7 marks)
- (c) If all masses are removed from the shaft and the power transmitted is 150 kW, determine the maximum shear stress on the shaft. (9 marks)

Note: Uniform disc or cylinder, radius r. I about the central axis is  $\frac{mr^2}{2}$  and  $k = \frac{r}{\sqrt{2}}$

**Question 4**

You are designing the spring damper system for a 1850 kg 4X4 Luxury vehicle. The test road is sinusoidally undulated with a maximum displacement of 36cm. If the maximum allowable displacement of the vehicle relative to the road is 42cm, determine:

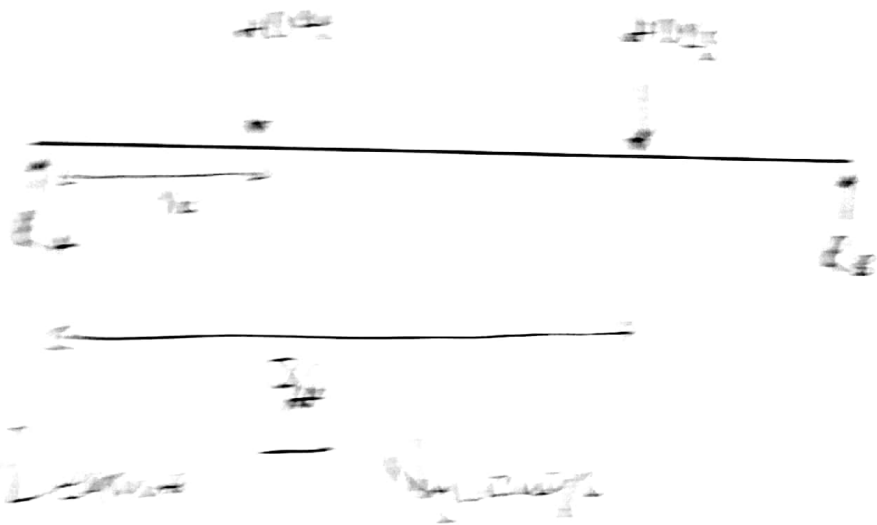
- (a) the damping ratio and state whether it is over, critically or under damped (6 marks)

If the length of one undulation is 2m, and the damping coefficient is 9500kg/s, determine:

- (b) the stiffness of the suspension (5 marks)
- (c) the speed in km/h of the vehicle at maximum displacement (7 marks)
- (d) the maximum displacement of the vehicle if the and the vehicle is moving a 140km/h (7 marks)

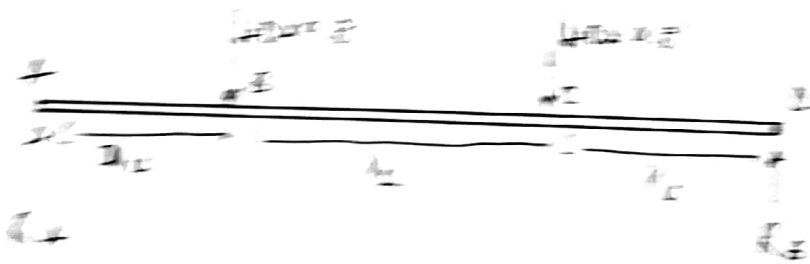
END OF TEST

$$\frac{x}{y} = \sqrt{\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}}$$



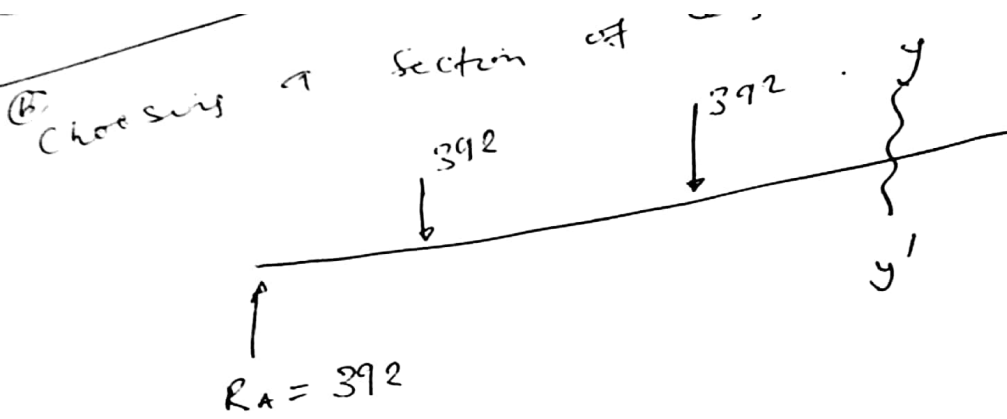
Consider a beam of length  $L$  supported at both ends. A uniformly distributed load  $w$  is applied over the entire length of the beam. The beam is divided into two equal segments of length  $L/2$ . The total length of the beam is  $L$ .

Q. If length of beam =  $L$



$$\sum W = \text{Area} \quad R_1 = wL$$

$$R_2 = wL$$



$$M = 392x - 392(x - \frac{1}{4}l) - 392(x - \frac{3}{4}l)$$

$$EI \frac{d^2y}{dx^2} = M$$

$$EI \frac{d^2y}{dx^2} = 392x - 392(x - \frac{1}{4}l) - 392(x - \frac{3}{4}l)$$

$$EI \frac{dy}{dx} = \frac{392x^2}{2} - \frac{392}{2}(x - \frac{1}{4}l)^2 - \frac{392}{2}(x - \frac{3}{4}l)^2 + C$$

$$EI y = \frac{392}{6}x^3 - \frac{392}{6}(x - \frac{1}{4}l)^3 - \frac{392}{6}(x - \frac{3}{4}l)^3 + Cx + \Delta$$

at ends,  $y = 0$  say at  $x = l$  and  $x = 0$

at  $x = 0$

$x = l$

$$0 = \Delta$$

$$0 = \frac{392l^3}{6} - \frac{392}{6}(l - \frac{1}{4}l)^3$$

$$- \frac{392}{6}(l - \frac{3}{4}l)^3 + Cl$$

(2)

$$-Cl = \frac{392}{6} l^3 - \frac{392}{6} l^3 \left(\frac{27}{64}\right) - \frac{392}{6} l^3 \left(\frac{1}{64}\right)$$

$$-C = \frac{392}{6} l^2 - 27.5625 l^2 - 1.020 l^2$$

$$C = -36.75 l^2$$

Expression for slope of beam

$$EI \frac{dy}{dx} = \frac{392x^2}{2} - \frac{392}{2} \left(x - \frac{1}{4}l\right)^2 - \frac{392}{2} \left(x - \frac{3}{4}l\right)^2 - 36.75l^2$$

end 1 where  $x=0$

$$EI \frac{dy}{dx} = -36.75 l^2$$

$$\frac{dy}{dx} = \frac{-36.75 l^2}{EI}$$

for end 2 where  $x=l$

$$EI \frac{dy}{dx} = \frac{392}{2} l^2 - \frac{392}{2} \left(l - \frac{1}{4}l\right)^2 - \frac{392}{2} \left(l - \frac{3}{4}l\right)^2 - 36.75 l^2$$

$$EI \frac{dy}{dx} = 196 l^2 - \frac{441}{4} l^2 - \frac{49}{4} l^2 - 36.75 l^2$$

$$EI \frac{dy}{dx} = 32.75 l^2$$

(3)

$$\frac{dy}{dx} = \frac{32.75L^2}{EI}$$

③ Expression for Maximum Slope.

for max ~~slope~~ deflection is between  $\frac{1}{4}L$  and  $\frac{3}{4}L$

$$\text{for } \frac{dy}{dx} = 0$$

$$\frac{392}{2}x^2 - \frac{392}{2}\left(x - \frac{1}{4}L\right)^2 - 36.75L^2 = 0$$

$$\frac{392}{2}x^2 - \frac{392}{2}\left(x^2 - \frac{1}{2}xL + \frac{L^2}{16}\right) - 36.75L^2 = 0$$

$$\frac{392}{2}\left[+\frac{1}{2}xL - \frac{L^2}{16}\right] - 36.75L^2 = 0$$

$$98xL - \frac{196L^2}{16} - 36.75L^2 = 0$$

$$\underline{\underline{x = 0.5L}}$$

Max deflection

$$y = \frac{392}{6}(0.5L)^3 - \frac{392}{6}\left(0.5L - \frac{1}{4}L\right)^3 - 36.75L^2$$

$EI$

$$y = \frac{81667c^3 - 102c^3 - 36.75c^2}{EI}$$

---

---

Question 2

net weight = 3200 kg

$\delta = 15 \text{ cm}$

35% on rear axle

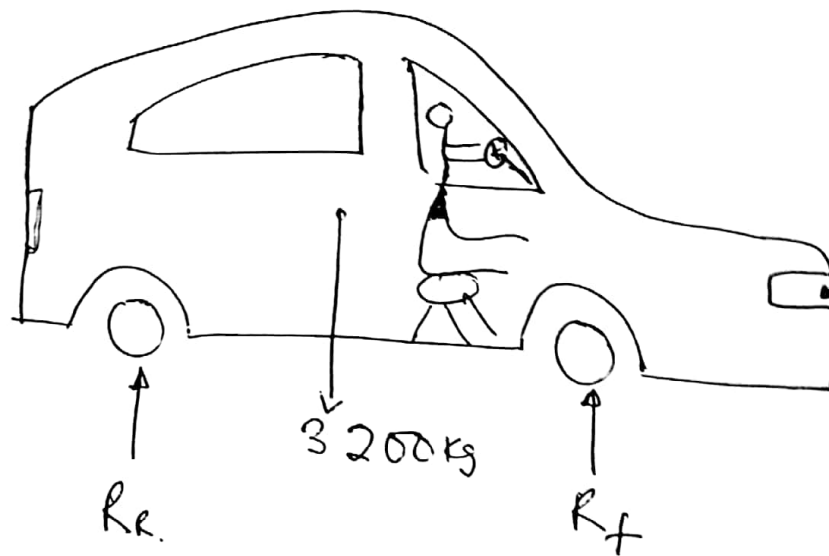
65% on front axle

Loading = 7000 kg

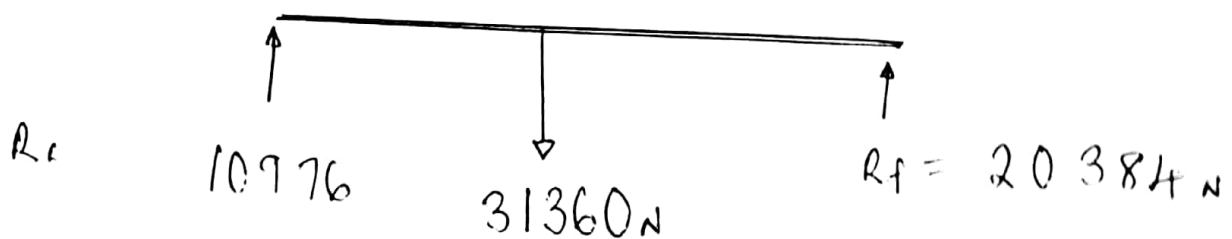
80% on rear axle

20% on front axle

③ diagram



free body diagram



Question 2

net weight = 3200 Kg

$\delta = 15 \text{ cm}$

35% on rear axle

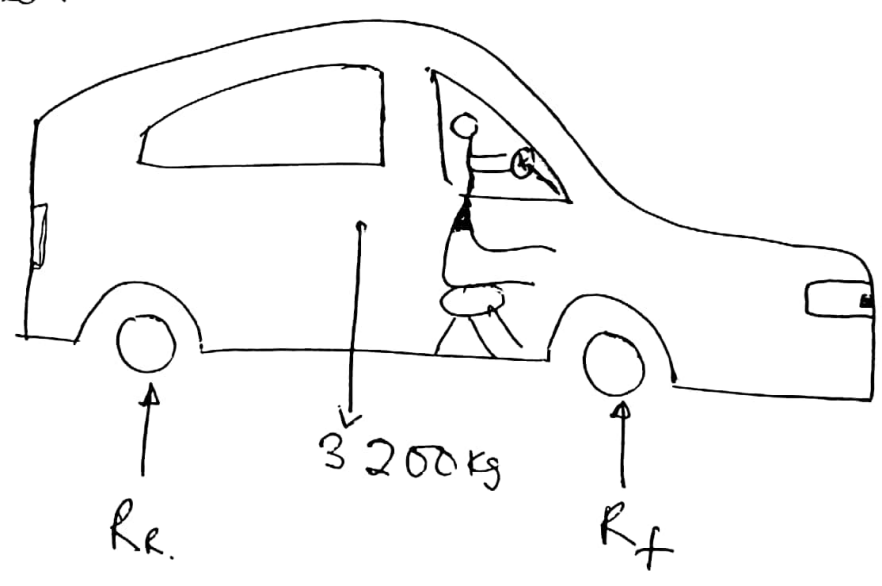
65% on front axle

Loading = 7000 Kg

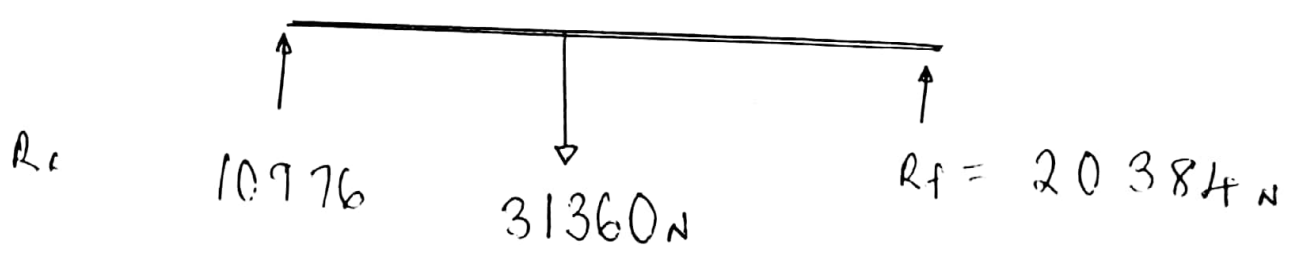
80% on rear axle

20% on front axle

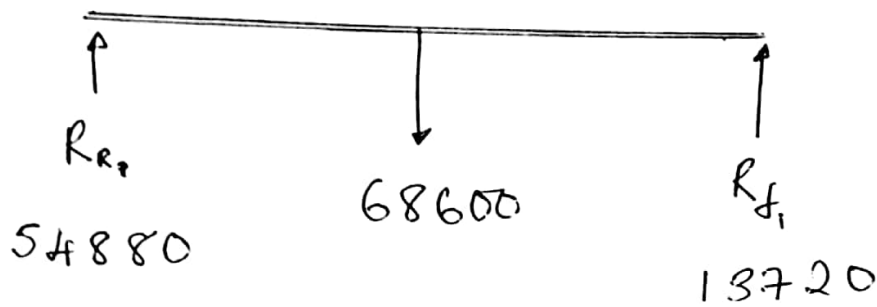
③ diagram



free body diagram



for the loads



$$\begin{aligned} \text{Total force on rear axle} &= 54880 + 10976 \\ &= \underline{\underline{65856 \text{ N}}} \end{aligned}$$

$$\begin{aligned} \text{Total force on front axle} &= 13720 + 26384 \\ &= \underline{\underline{34104 \text{ N}}} \end{aligned}$$

② Number of complete coils in helical springs.

Since springs are in parallel

$$\text{Load/Springs} = \frac{\text{Load}}{2} = 17052 \text{ N}$$

$$\delta = \frac{64 W R^3 n}{C d^4}$$

③

$$64 \times 17052 \times (0.15)^3$$

$$n = \frac{18.795 \times 10^3}{3683.232}$$

$$n = 5.1028$$

number of complete Springs, if we choose 5  
the 0.1028 will cause extra deflection  
∴ Complete coils better be 5 in design

$$n = \underline{5}$$

(5)

(3)

(b)

$$\delta = \frac{3Wl^3}{8ENbt^3}$$

$$\text{Load per spring} = \frac{65856}{2} = 32928$$

$$l = 1\text{m}$$

$$N = 8$$

$$E = 125 \times 10^9$$

$$\frac{b}{t} = \frac{8}{1}$$

$$b = 8t$$

$$t = \left(\frac{b}{8}\right)$$

$$0.15 = \frac{(3)(32928)(1)^3}{(8)(125 \times 10^9)(8)(b)\left(\frac{b}{8}\right)^3}$$

$$\frac{1.2 \times 10^{12} b^4}{512} = 98784$$

$$b^4 = 4.214 \times 10^{-5}$$

$$b = 0.08057\text{m}$$

$$\underline{\underline{b = 8.057\text{cm}}}$$

(c) Deflection when acted upon by net weight

only.

front exle.

$$\delta = \frac{64WR^3n}{9d^4} = \frac{64 * 10192 * (0.15)^3 * 5}{(83.5 \times 10^7) * (0.035)^4}$$

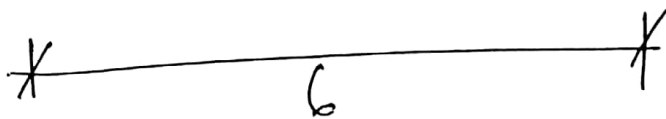
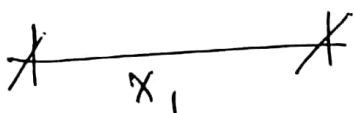
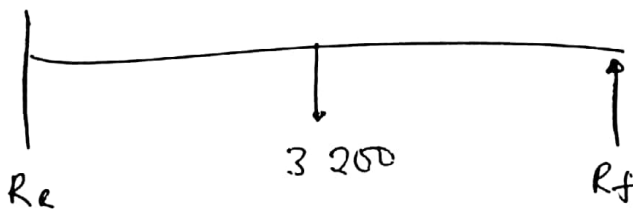
$$\underline{\underline{\delta = 8.784\text{cm}}}$$

deflection on Rear axle.

$$\delta = \frac{3Wl^3}{8ENbt^3} = \frac{3 \times 5488 \times 1^3}{8 \times 125 \times 10^9 \times 8 \times 0.08057 \times (0.01)^3}$$

$$\delta = 2.500 \text{ cm}$$

② Centre of gravity shift

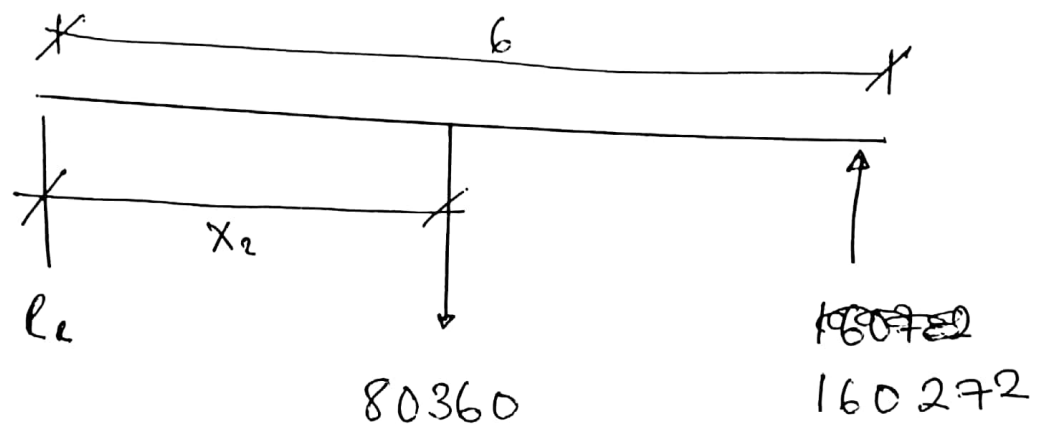


$$\sum M_{R_e} = 0$$

$$(31360 \times x_1) = (20348 \times 6)$$

$$\underline{x_1 = 3.9 \text{ cm}}$$

After SODC loading



provided that after loading the share of weights is as given in question as 7000 kg.

say 80% rear  
20% front

$$\sum M_{R_e} = 0$$

$$80360 x_2 = 160272 \times 6 \times 10$$

$$x_2 = 0.12 \text{ cm} \times 10$$

$$x_2 = 1.2 \text{ cm}$$

from the above calculations  
the centre of gravity shifts towards the  
rear  $2 \times 6$  by  $(3.9 - 1.2) = 2.7 \text{ cm}$

$$3.9 - 1.2 = \underline{\underline{2.7 \text{ cm}}}$$

A	13	0	0.1	0.3	1.3	0.39	0.39	0
B	12	60	0.11	0.45	1.32	0.594	0.297	0.514
C	11	120	0.12	0.6	1.32	0.792	-0.396	0.686
D	10	180	0.13	0.75	1.3	<del>0.975</del> 0.975	<del>-0.475</del> -0.475	0
X	15			0				
Y	15			1				
$\Sigma$							-0.684	1.2

$$\Sigma Mr l \cos \alpha = -0.684$$

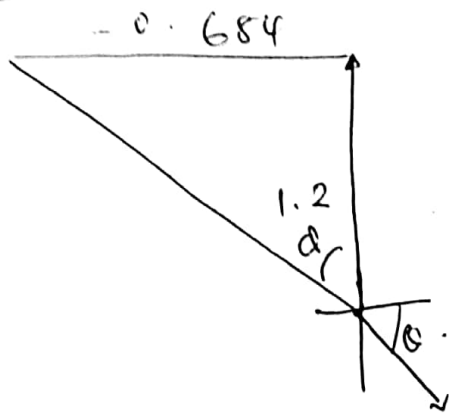
$$\Sigma Mr l \sin \alpha = 1.2$$

$$Mr l = \left[ (1.2)^2 + (0.684)^2 \right]^{1/2} = 1.381$$

$$\text{for } \begin{matrix} y \\ l=1 \\ m=15 \end{matrix} \quad r = \frac{1.381}{15} = \underline{\underline{9.208 \text{ cm}}}$$

$$\alpha = \tan^{-1} \left( \frac{1.2}{-0.684} \right)$$

$$= -60.316^\circ$$



$$\therefore \theta = 300^\circ$$

Solving for mass  $x$  using  $mr$

$\theta$	Mass	$mr$	$mr \cos \theta$	$mr \sin \theta$
0	A	1.3	1.3	0
60	B	1.32	0.66	1.143
120	C	1.32	-0.66	1.143
180	D	1.3	-1.3	0
300	Y	1.3512	0.6906	-1.196
	X			

$$\sum mr \cos \theta = 0.6906$$

$$\sum mr \sin \theta = 1.09$$

$$Mr = \left[ (1.09)^2 + (0.6906)^2 \right]^{1/2}$$

$$Mr = 1.29$$

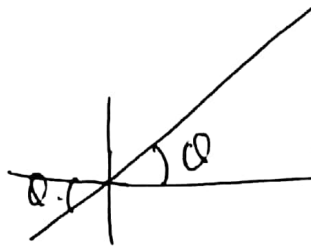
for  $x$   
 $M = 15$

$$\therefore r = \frac{1.29}{15} = \underline{\underline{8.60 \text{ cm}}}$$

(2)

$$\theta = \tan^{-1} \left( \frac{1.09}{0.6908} \right)$$

$$\theta = 57.64^\circ$$



$$\therefore \theta = 180 + 57.64$$

$$= \underline{\underline{237.64^\circ}}$$

Balancing of Masses is

$$X = 15 \text{ Kg, radius} = 8.60 \text{ cm, } l = 0 \text{ m}$$

$$\theta = 237.64^\circ$$

$$Y = \text{Mass} = 15 \text{ Kg, radius} = 9.208 \text{ cm, } l = 1 \text{ m}$$

$$\underline{\underline{\theta = 300^\circ}}$$

Initial rotation = 3000 rpm

using Conservation of Momentum

$$L_i = L_f$$

$$L_i = (I_s + \sum I_m) \omega$$

$$I_s = \frac{Mr^2}{2} = \frac{m \times \left(\frac{60}{1000}\right)^2}{2}$$

calculations for m

$$\rho = 4 \text{ g/cm}^3$$

$$V = \pi r^2 L = 6^2 \cdot \pi \cdot 4 \times 100$$

$$m = \rho V = 45.24 \text{ Kg}$$

$$I_s = \frac{(45.24) \left(\frac{60}{1000}\right)^2}{2} = 0.081432 \text{ Kg m}^2$$

$$\begin{aligned} \sum I_m &= \sum M_i r_i^2 = (13)(0.1)^2 + 12(0.11)^2 + 11(0.12)^2 \\ &\quad + 10 \times (0.13)^2 \\ &= \underline{\underline{0.6026 \text{ Kg m}^2}} \end{aligned}$$

(4)

$$L_i = (0.6026 + 0.081432) 3000$$

$$L_f = \omega_f (0.081432 + \Sigma I_m)$$

$$= \omega_f (0.081432 + 0.6026 + 0.2351)$$

$$L_i = L_f$$

$$\omega_f = \frac{2052.096}{0.9221528}$$

$$\omega_f = \underline{\underline{2225.33 \text{ rpm}}}$$

© again conserving momentum.

$$(2225.33)(0.922132) = 0.081432(\omega_f)$$

$$\omega_f = \underline{\underline{25.19952 \times 10^3 \text{ rpm}}}$$

5

$$P = \frac{2\pi NT}{60}$$

$$T 2\pi N = P \cdot 60$$

$$T = \frac{P \cdot 60}{2\pi N} = \frac{(150000)(60)}{2 * \pi * 25.19952 \times 10^3}$$

$$\underline{T = 56.842 \text{ Nm}}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{TR}{J} = \frac{(56.842)(0.06)}{\frac{\pi}{32} (0.12)^4}$$

$$\boxed{J = \frac{\pi}{32} d^4}$$

$$\tau = \frac{3.41052}{2.035 \times 10^{-5}} = \underline{\underline{167.531 \text{ kN/m}^2}}$$

Question 11

$$m = 1850 \text{ kg}$$

$$y = 36 \text{ cm}$$

$$x = 42 \text{ cm}$$

(a) Since data given is at maximum, this implies that  $\omega = \omega_n$

$$\therefore \frac{\omega}{\omega_n} = 1$$

using 'transmissibility ratio  $T_d$ '

$$T_d = \frac{x}{y} = \left[ \frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

where  $\zeta$  = damping ratio

$$\left( \frac{42}{36} \right)^2 = \frac{1 + 4\zeta^2}{(1-1) + 4\zeta^2} \quad r=1$$

$$\left( \frac{42}{36} \right)^2 = \frac{1}{4\zeta^2} + 1$$

$\Rightarrow \zeta = 0.832$

$$0.3611 = \frac{1}{4\zeta^2}$$

$$\underline{\underline{\zeta = 0.832}}$$

$$\frac{13}{9} \zeta^2 = 1$$

$\therefore$  It is underdamped

length of one modulation is 2 m

$$C = 9500 \text{ kg/s}$$

⑥ Stiffness.

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$K = (\omega_n)^2 m$$

using  $\zeta = \frac{C}{m 2\omega_n}$

$$2\zeta \omega_n m = C$$

$$\omega_n = \frac{C}{m 2\zeta} = \frac{9500}{(1850)(2)(0.832)}$$

$$\omega_n = 3.0858 \text{ rad/s}$$

$$K = (\omega_n)^2 \cdot m = (3.0858)^2 \cdot 1850$$

$$\underline{\underline{K = 17616.37 \text{ N/m}}}$$

② the speed in km/h and Max displacement.

$$\omega_n = 3.086 \text{ rad/s}$$

$$\omega = 2\pi f$$

$$f = \frac{\omega_n}{2\pi} = \frac{3.086}{2\pi} = 0.4911 \text{ Hz}$$

$$\begin{aligned} \text{Speed} &= 2 * 0.4911 \\ &= 0.9823 \text{ m/s} \\ &= \underline{\underline{3.536 \text{ km/h}}} \end{aligned}$$

③ Displacement given speed = 140 km/h

$$\begin{aligned} 140 \text{ km/h} \\ = 38.889 \text{ m/s} \end{aligned}$$

using ratios

$$x - 38.89$$

$$42 - 3.536$$

they are directly

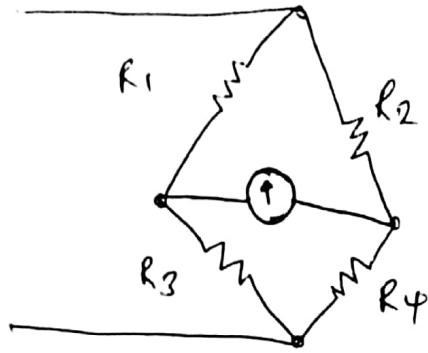
proportional

$$x = \frac{42 * 3.536}{38.89}$$

$$x \approx 3.8188$$

$$\underline{\underline{x = 3.8188 \text{ cm}}}$$

digit 1.  
Jesper's  
Signals



at balance.

$$l_1 = l_3 \quad l_2 = l_4$$

$$l_1 R_1 = l_2 R_2$$

$$l_1 R_3 = l_2 R_4$$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$R_4 R_1 = R_3 R_2$$

$$R_x = \frac{R_3 R_2}{R_1}$$

$$R_x = \frac{4.5 \times 10}{15} = \frac{45}{15}$$

$$= 3$$



$$\log_2 8 = 3$$

$$\log_2 8 = x$$

$$= 2^x = 8$$

$$\log_2 8 = 3$$

**Question 1**

A beam, which is supported through pin joints at its ends, is acted upon by a couple  $M$  in a plane containing the axis of the beam, applied at a point two thirds of the span from one end. Find expressions for the slope of the beam at both ends, and the maximum deflection.

*done* Regina T. Njirero

**Question 2**

Compare the weights of equal lengths of hollow and solid shaft to transmit a given torque for the same maximum shear stress, if inside diameter is  $\frac{1}{4}$  of the outside.

*done*

**Question 3**

You have been commissioned to design the suspension for a new light truck. The net weight of the light truck up is 3000kg, distributed 38% on the rear axle and 62% on the front axle. When fully loaded an additional 5000kg is added to the light truck, distributed 75% on the rear axle and 25% on the front axle. You are using, on the front axle 2 identical helical close coiled springs in parallel, and on the rear axle 2 identical leaf springs in parallel. Your design constraint is, completely unloaded to fully loaded, the maximum static deflection on either axle should not exceed 15 cm, given for:

Helical spring:

- Mean coiled diameter = 25cm
- Modulus of rigidity = 83.5 GN/m<sup>2</sup>
- Wire diameter = 35mm

*done*

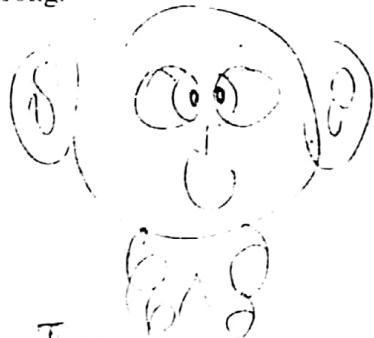
Regina T. Njirero

Leaf spring:

- Length = 1 m
- Number of leaves = 6
- Width to thickness ratio = 8:1
- Elastic modulus = 125 GN/m<sup>2</sup>

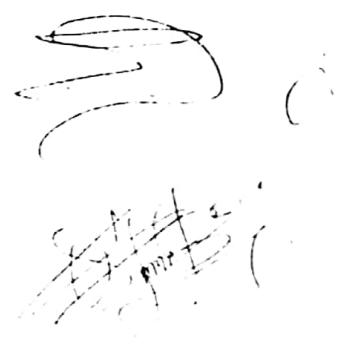
Find:

- (a) the number of coils required in the helical springs (5 marks)
- (b) the width of the leaf springs (5 marks)
- (c) the deflection at both axles when acted upon by the net weight of the pickup only. (4 marks)
- (d) If the distance between the rear and front axle is 6 m what is the magnitude and direction of the shift in the centre gravity of the pickup when loaded with the additional 5000kg. (6 marks)

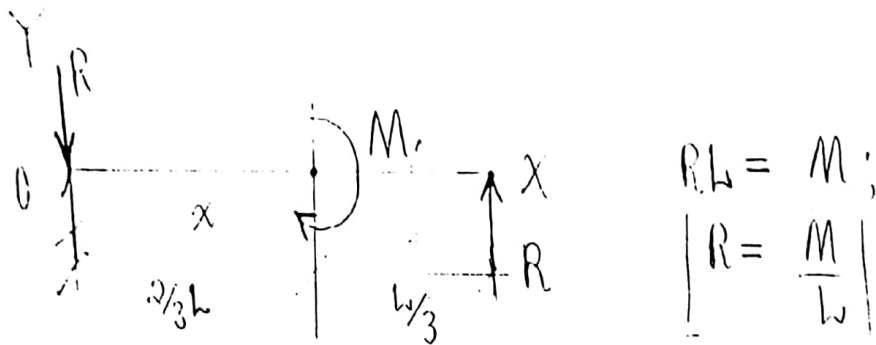


$$e = \frac{L}{C}$$

Student Engineer JERRY ZEE  
 B.Sc., M.Eng., Reg. Eng., M.E.T., Ph.D.



QUESTION 1



$$RL = M;$$

$$R = \frac{M}{L}$$

Moment;  $M_x = ?$

$$EI \frac{d^2 y}{dx^2} = M_x = -Rx + M \left[ x - \frac{2L}{3} \right]^0$$

$$EI \frac{dy}{dx} = -\left(\frac{R}{2}\right)x^2 + M \left(x - \frac{2L}{3}\right) + A$$

$$EI y = -\left(\frac{R}{6}\right)x^3 + \frac{M}{2} \left(x - \frac{2L}{3}\right)^2 + Ax + B$$

Applying BC's

$$x=0; y=0; \quad \left| \quad C = -\left(\frac{R}{6}\right)0 + A(0) + B; \quad [B=0]$$

$$x=L; y=0; \quad \left| \quad 0 = -\left(\frac{R}{6}\right)L^3 + \frac{M}{2} \left(\frac{L}{3}\right)^2 + AL + 0$$

$$\Rightarrow AL = \left(\frac{M}{6L}\right)L^3 - \frac{ML^2}{18} = \frac{(3-1)ML^2}{18}$$

$$\therefore \boxed{A = \frac{ML}{9}}$$

at  $x=0$ ;  $EI \frac{dy}{dx} = -\left(\frac{R}{2}\right)x^2 + \frac{ML}{9} \Rightarrow EI \frac{dy}{dx} = \frac{ML}{9}$

i.e.  $\left[ \frac{dy}{dx} = \frac{ML}{9EI} \right]$  done.

$$L = \frac{2L}{3} \Rightarrow \left(\frac{L}{3}\right)^2$$

at  $x=L$ ;  $EI \frac{dy}{dx} = -\left(\frac{M}{6L}\right)L^3 + \frac{M}{2} \left(x - \frac{2L}{3}\right)^2 + \left(\frac{ML}{9}\right)x$

$\Rightarrow EI \frac{dy}{dx} = -\frac{ML}{6} + \frac{ML^2}{18} + \frac{ML^2}{9} = \frac{ML^2}{18}$

$$EI \frac{dy}{dx} = -\left(\frac{M}{6}\right)x^2 + M\left[x - \frac{2l}{3}\right] + \frac{ML}{9}$$

$$EI \frac{dy}{dx} = -\frac{ML}{2} + \frac{ML}{3} + \frac{ML}{9} = \left(\frac{-9+6+2}{18}\right)ML$$

$$EI \frac{dy}{dx} = -\frac{ML}{18} \quad ; \quad \left[ \frac{dy}{dx} = -\frac{ML}{18EI} \right]$$

Maximum deflection shall be at

$$x < \left(\frac{3}{2}\right)l \quad ; \quad \frac{dy}{dx} = 0 \text{ at the point.}$$

$$0 = -\frac{R}{2}x^2 + \frac{ML}{9} \quad ; \quad x^2 = \left(\frac{2}{9}\right)\frac{ML}{R} = \left(\frac{2}{9}\right)\frac{ML}{\frac{M}{l}} = \left(\frac{2}{9}\right)l^2$$

$$\Rightarrow \left[ x = \frac{\sqrt{2}}{3}l \right] \text{ done.}$$

Regina - T. Mycrango

at  $x = \frac{\sqrt{2}}{3}l$

$$EI y = -\left(\frac{R}{6}\right)x^3 + \left(\frac{ML}{9}\right)x$$

$$EI y = -\left(\frac{M}{6l}\right)\left(\frac{\sqrt{2}l}{3}\right)^3 + \left(\frac{ML}{9}\right)\left(\frac{\sqrt{2}l}{3}\right) = \left(\frac{-2\sqrt{2}l^2}{2(3)(3^2)} + \frac{l^2\sqrt{2}}{27}\right)M$$

$$EI y = +\frac{\sqrt{2}l^2 ML^2}{1} \left(\frac{-1+3}{81}\right) \quad ; \quad \left[ y = \frac{+2\sqrt{2} ML^2}{81} \right]$$

$$\therefore \left[ y = \frac{2\sqrt{2} ML^2}{81} \right] \text{ done.}$$

$$T = \frac{1}{16} = \frac{1}{16} - (iv) ; \frac{T}{C} = \frac{1}{D} \cdot (iv) \Rightarrow (v)$$

$$1 - \frac{1}{\lambda D^4} ; \frac{T}{C} = \frac{2 \left( \frac{\pi D^4}{32} \right) \left( \frac{1}{D} \right)}{\frac{\pi D^3}{16}} = \frac{1}{16} \dots (iii)$$

Maximum deflection eqn

$$\frac{T}{C} = \frac{2}{16} \left( \frac{\pi D^4 - d^4}{32} \right) ; \text{But } d = \frac{3}{4} D_1$$

$$\Rightarrow \frac{T}{C} = \left( \frac{\pi}{16} D_1 \right) \left[ D_1^4 - \left( \frac{3}{4} D_1 \right)^4 \right] = \frac{\pi D_1^3}{16} \left( \frac{175}{256} \right) \dots (iv)$$

expressing eqn (iii) and (iv)

$$\frac{\pi D^3}{16} = \frac{\pi D_1^3}{16} \left( \frac{175}{256} \right) ; D^3 = \left( \frac{256}{175} \right) D_1^3 ; D_1 = D \left( \frac{256}{175} \right)$$

$$\Rightarrow D_1 = 1.1352 D \quad \text{diameter}$$

Ratio of materials ; as  $f = \frac{8 \gamma W}{\pi D^2} \cdot l$  solid shaft

$$= \frac{8 \gamma W}{\pi \left( \frac{3}{4} D_1 \right)^2} \cdot l \quad \text{Hollow shaft}$$

$$\Rightarrow \text{Ratio} = \frac{(D_1^2 - d^2) \left( \frac{8 \gamma W}{\pi} \right)}{D^2 \left( \frac{8 \gamma W}{\pi} \right)} = \frac{(D_1^2 - \left( \frac{3}{4} D_1 \right)^2) D_1^2}{D^2} = \left( \frac{D_1}{D} \right)^2 \left( 1 - \frac{9}{16} \right)$$

$$\text{Ratio of materials} = \left( \frac{17}{16} \right) \left( \frac{D_1}{D} \right)^2 = \left( \frac{17}{16} \right) (1.1352)^2$$

$$= 0.5638$$

Ans

QUESTION 3

Material	$\gamma_{sat}$ (kN/m <sup>3</sup> )	Porosity (%)	Water Content (%)	Volume (m <sup>3</sup> )	Weight (kN)
Gravel	20	36%	18%	1860	114
Sand	20	15%	18%	1250	370
Fill	20	15%	18%	3110	480
One side				30509.1	147070.9
in Newtons				15254.55	23985.45

deflection =  $8 = 15 \text{ cm} = 0.15 \text{ m}$ . *Regina F. Ngyiro*

(a)  $\frac{8WD^3}{\pi d^4} = \frac{\sigma}{d} = \frac{6x}{\pi D^2}$  ;  $n = \frac{6x \times d^4}{\pi 8WD^3} = \frac{6(x)d^4}{8WD^3}$

$n = \frac{(83.5 \times 10^9)(0.15)(0.035)^4}{8(15254.55)(0.25)^3} = 9.856$

for  $x$  not to exceed  $15 \text{ cm}$  ;  $n = 9 \text{ coils}$  [5 marks]

$x = \frac{8WD^3 n}{6d^4} = \frac{8(15254.55)(0.25^3)(9)}{(83.5 \times 10^9)(0.035)^4}$  ;  $x = 0.137 \text{ m}$

(c) (i)  $M_u = (9.81)(1860) = 9123.3 \text{ [N]}$

$x = \frac{8W_u D^3 n}{6d^4}$  ;  $x = \frac{(8)(9123.3)(0.25^3)(9)}{(83.5 \times 10^9)(0.035)^4}$

$x = 0.0819 \text{ m}$  correct

$$K = \frac{M}{\delta}$$

$$\frac{1}{R^2}$$

$$\frac{1}{R^2} \cdot K \cdot d \cdot (\text{mm})$$

Vertical

$$K = \frac{M}{\delta}$$

$$S = R \cdot \theta$$

spring constant  
shear stress

$$K = \frac{C \cdot J \cdot \theta}{6 \pi R^3 n}$$

$$\theta = \frac{6 \pi K R^3 n}{C \cdot J}$$

$$\frac{M R^2 \cdot \pi R^4 n \cdot 3 R}{C \cdot \pi R^4}$$

$$\frac{1 \cdot L}{C \cdot J} = \theta$$

$$\frac{1}{J} = \frac{C}{L}$$

$d = 18 \text{ mm}$   
 $R = 9 \text{ mm}$   
 $n = 10$   
 $K = ?$

$$K = \frac{M}{\delta}$$



$$K = \frac{C \cdot J \cdot \theta}{6 \pi R^3 n}$$

$$\theta = \frac{6 \pi K R^3 n}{C \cdot J}$$

$$\frac{M R^2 \cdot \pi R^4 n \cdot 3 R}{C \cdot \pi R^4}$$

$$\frac{1}{J} = \frac{C}{L}$$

$$8 = 1 - 1 = \left(\frac{2}{3}\right) \frac{Wl^3}{48EI} \quad ; \quad \frac{b}{l} = \frac{8}{1} \quad ; \quad \left[\frac{b}{l} = \frac{8}{1}\right]$$

$$8 \left(\frac{b}{8}\right)^3 b = \frac{3}{8} \frac{Wl^3}{EI} \quad ; \quad \left[ l^4 = \frac{3(8)^2 Wl^3}{8EI} \right]$$

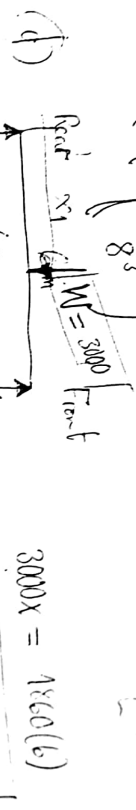
$$b^4 = \frac{3(8)^2 (23985.115)(1)^3}{(0.115)(6)(125 \times 10^9)} \quad ; \quad b^4 = 4.0935 \times 10^{-05}$$

$$b = 0.07999 \quad ; \quad \boxed{b = 0.08 \text{ m}} \quad \text{done.} \quad [5 \text{ marks}]$$

Regina - T. Alviranga

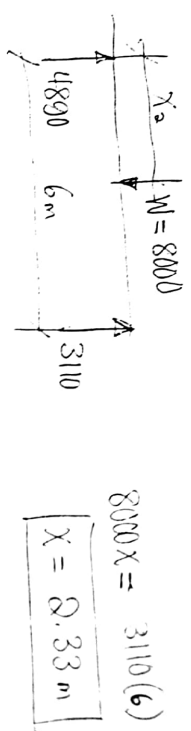
c(ii)  $MB = \frac{(1140 \times 9.81)}{8} = 5591.7 \text{ N}$

$$8 = \frac{(3)(5.5(9.81)(1)^3)}{(8)(6) \left(\frac{(0.08)^4}{8^3}\right) (125 \times 10^9)} \quad ; \quad \boxed{8 = 0.03495 \text{ m}} \quad [4 \text{ marks}]$$



$$3000x = 1860(6)$$

$$\boxed{x_1 = 3.72 \text{ m}}$$



$$8000x = 3110(6)$$

$$\boxed{x = 2.33 \text{ m}}$$

$$x_1 - x_2 = (3.72 - 2.33) = \underline{1.39 \text{ m}} \quad \text{for for sign (b)}$$

Towards the rear axle.



**Question 8**

A solid 1m shaft of radius 90 mm and density  $6 \text{ g/cm}^3$  carries 4 unbalanced masses spaced as follows: Masses A, B, C, and D are 11, 7, 10, and 13kg respectively, rotating at radii 10, 8, 9, 7cm respectively, and spaced from one end of the shaft at 20, 40, 60, and 80 cm respectively; the angular spacing from A to B, C, and D are 120, 200, and 300° respectively.

- (a) Balance the shaft using two 5 kg masses at either end (9 marks)
- (b) If before balancing, the shaft rotated at 600 rpm what will be its rotation after balancing. (5 marks)
- (c) If all masses are removed from the shaft and the power transmitted is 12 kW, determine the maximum shear stress on the shaft. (6 marks)

Note: Uniform disc or cylinder, radius  $r$ . I about the central axis is  $\frac{mr^2}{2}$  and  $k = \frac{r}{\sqrt{2}}$

**Question 9**

*Regina F. Mlyerova*

You are designing the spring damper system for a 1850 kg 4X4 Luxury vehicle. The test road is sinusoidally undulated with a maximum displacement of 36cm. If the maximum allowable displacement of the vehicle relative to the road is 42cm, determine:

- (a) the damping ratio and state whether it is over, critically or under damped (5 marks)
- (b) the stiffness of the suspension (4 marks)
- (c) the speed in km/h of the vehicle at maximum displacement (5 marks)
- (d) the maximum displacement of the vehicle if the and the vehicle is moving at 140km/h (6 marks)

$W = T \cdot 2\pi \cdot n$

$\frac{120000}{1\pi \times 93.26} = T \cdot 2\pi$

$K = \frac{111.2}{5}$

J

$618.35 \text{ kg/m}^3 \cdot \pi \cdot r^2 \cdot L$

*mass / volume*

*mass / volume*

0628558690

Question 1



Since Force must be equal and opposite, then the Moments about C (ML)  $M - PL = 0$

$$R = \frac{M}{L}$$

It follows that

$$EI \frac{d^2 y}{dx^2} = -Rx + M \left[ x - \frac{2L}{3} \right] + A$$

$$EI \frac{dy}{dx} = -\frac{R}{2} x^2 + M \left[ x - \frac{2L}{3} \right] + A$$

$$EI y = -\frac{Rx^3}{6} + \frac{M}{2} \left[ x - \frac{2L}{3} \right]^2 + Ax + B.$$

Boundary Conditions

$$x=0, y=0 \dots \dots (i)$$

$$x=L \text{ at } y=0 \dots \dots (ii)$$

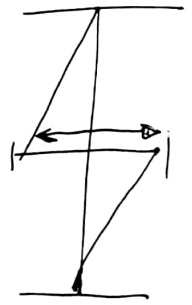
Using the first boundary condition  $B=0$ .

Using the second boundary condition

$$AL = \frac{RL^3}{6} - \left( \frac{M}{2} \right) \left( \frac{L}{3} \right)^2$$

$$= \frac{ML^3}{6} - \frac{ML^2}{18} \Rightarrow \frac{ML^2}{6} - \frac{ML^2}{18}$$

$$A = \frac{ML}{9}$$



$$\text{At } x = \frac{2L}{3}$$

$$EI \frac{dy}{dx} = \left(-\frac{R}{2}\right) \left(\frac{2L}{3}\right)^2 + \frac{mL}{9}$$

$$EI \frac{dy}{dx} = -\frac{mL}{9}$$

Slope =  $\frac{dy}{dx} = -\frac{mL}{9EI}$  and pointing downwards to the right.

$$\text{Deflection} = y = EIy = -\left(\frac{R}{2}\right) \left(\frac{2L}{3}\right)^3 + \left(\frac{mL}{9}\right) \left(\frac{2L}{3}\right)$$

$$y = \frac{2mL^3}{81EI} \text{ upwards}$$

For  $x < \frac{2L}{3}$  (a point of zero slope must exist).

$$-\frac{Rx^2}{2} + A = 0$$

$$x = \left(\frac{\sqrt{2}}{3}\right)L$$

$$\therefore \text{The maximum deflection} = \left[ -\frac{m}{6L} \left(\frac{2\sqrt{2}}{3}\right)^3 L^3 + \left(\frac{mL}{9}\right) \left(\frac{\sqrt{2}}{3}\right)L \right]$$

$EI$

$$\text{Max deflection} = \frac{2\sqrt{2} mL^3}{81EI}$$

Question 2

$$d = \frac{3}{4} D$$

J for a solid shaft =  $\frac{\pi D^4}{32}$

J for a hollow shaft =  $\frac{\pi (D^4 - d^4)}{32}$

$$\frac{T}{\tau} = \frac{J (D^4 - d^4)^2}{D^4}$$

$$= \frac{\pi}{16 D} (D^4 - d^4)$$

$$= \frac{\pi D^4}{16} \left(1 - \frac{81}{256}\right)$$

$$= \frac{\pi D^4}{16} \left(\frac{175}{256}\right)$$

Equating Diameters

$$\frac{\pi D^3}{16} = \frac{\pi D_1^3}{16} \left(\frac{175}{256}\right)$$

$$D_1 = D \cdot 1.1352$$

$$D = D_1 \sqrt[3]{\frac{175}{256}}$$

$$D = \underline{\underline{D_1 \cdot 0.881}}$$

factor of length equal length

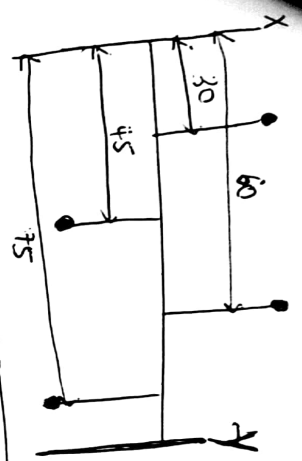
$$\frac{D_1^2 - d^2}{D_2^2}$$

$$\left(\frac{D_1}{D_2}\right) \left(1 - \frac{d^2}{D_2^2}\right)$$

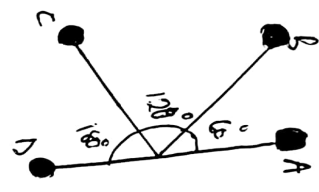
$$(1.352)^2 \left(\frac{16-d^2}{10}\right)$$

$$= 0.5637$$

quest 3



Distances in cm.



	M	r	M <sub>r</sub>	x	M <sub>r</sub> x	M <sub>r</sub> x cos θ	M <sub>r</sub> x sin θ
X	10	6.12	1.2	0	0.30	0.36	0
A	12	0.10	1.2	0.45	0.54	0.27	0.47
B	10	6.12	1.2	0.60	0.72	-0.36	0.424
C	12	0.10	1.2	0.75	0.9	-0.40	0
D	12	0.10	1.2	0.00	0.00	0.00	0.00
Σ					2.22	0.27	0.894

$$\sum M_r x \cos \theta = -0.63$$

$$\sum M_r x \sin \theta = 1.091$$

$$R = \sqrt{(0.63)^2 + (1.091)^2}$$

$$R = 1.260$$

$$\theta = \tan^{-1} \left( \frac{1.091}{-0.63} \right)$$

$$\theta = -60^\circ$$

$$\frac{222}{22} = \frac{1.260}{22}$$

$$R = 0.0573 \text{ N}$$

on the y plane, 11 kg should be placed

At a radius of 5.73 cm and one angle of 300°

	$M/r$	$Mr \cos \theta$	$Mr \sin \theta$
X	11/r <sub>1</sub>	0	0
A	1.2	1.2	0
B	1.2	0.6	1.04
C	1.2	-0.6	1.04
D	1.2	-1.2	0
Y	0.63	0.32	*0.55

$$\sum M_i \cos \theta = 0.32 \quad \sum M_i \sin \theta = 1.53$$

$$||r_1 = \sqrt{(0.32)^2 + (1.53)^2}$$

$$= \sqrt{2.4433}$$

$$||r_1 = 1.563 \quad \theta = \tan^{-1} \left( \frac{1.53}{0.32} \right)$$

$$\frac{\sum M_i r_i}{N} = \frac{1.563}{11} \quad \theta = \underline{\underline{258.19^\circ}}$$

$$r_1 = 0.142 \text{ m}$$

$$r_1 = 14.21 \text{ cm}$$

$\therefore$  The 11 kg mass must be placed at a radius of 14.21 cm from A and at 258.19° from A

$$M_1 r_1 \omega_1^2 = M_2 r_2 \omega_2^2$$

$$4.8 (2000)^2 = 6.993 (\omega_2)^2$$

$$\omega = \underline{\underline{1656.97 \text{ rpm}}}$$

$$c) I = \frac{Mr^2}{2}$$

$$K = \frac{I}{\sqrt{2}}$$

LATE

THE UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EEE 3112 - ELECTRICAL ENGINEERING PRACTISE

ASSIGNMENT ONE

NAME: PATEL ARJUN

COMPUTER NO: 13064177

LECTURER: MR. GOMA

~~$\frac{0}{40} + \frac{0}{40}$~~

~~$\frac{0}{80}$~~

~~$\frac{20}{40}$~~

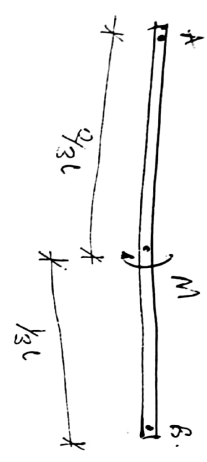
~~$\frac{1}{2}$~~

~~Arrog - boy - hand~~

~~Lasio M~~

~~$\Delta - \Delta = \Omega$~~

Section 1



72

72

55

54

48.42

30

20

6

$$\sum M_A = 0$$

$$M = R_B l$$

$$R_B = \frac{M}{l}$$

$$\sum F_y = 0$$

$$R_B = \frac{M}{l} = -R_A = \frac{M}{l}$$



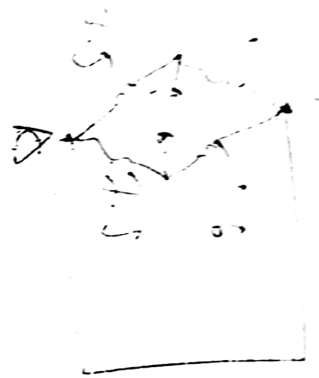
Considering a section of length \$x\$ on the right of \$xx'\$ at length \$x\$

$$\sum \frac{d^2 y}{dx^2} = M$$

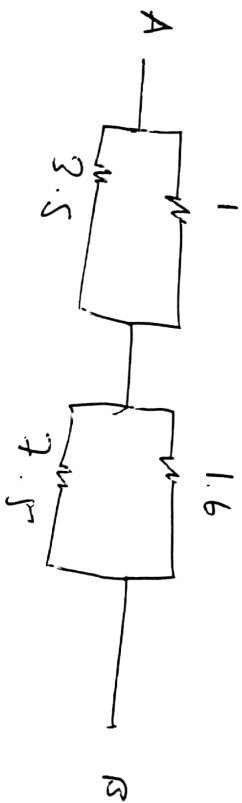
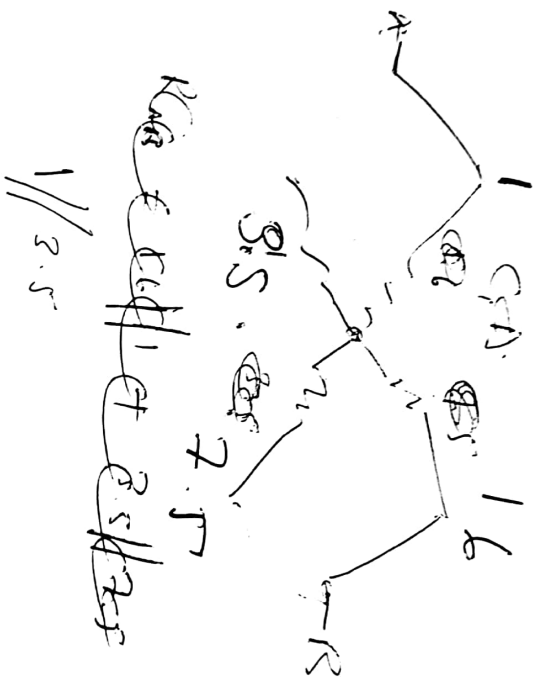
$$M = -\frac{M}{l}x + M(x - \frac{2}{3}l)^2$$

$$EI \frac{d^2 y}{dx^2} = -\frac{M}{l}x + M(x - \frac{2}{3}l)^2$$

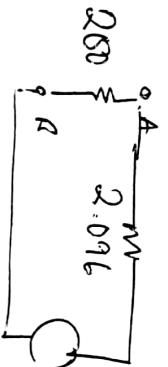
$$EI \frac{dy}{dx} = -\frac{Mx^2}{2l} + M(x - \frac{2}{3}l)^2 + C$$



1.2 mA



$2.096 = R_{TH}$



0.283

1.2 x 10<sup>-3</sup> A

$$EIy = -\frac{Mx^3}{6l} + \frac{M}{2}(x - \frac{2}{3}l)^2 + Cx + D$$

at point A where  $x=0$ ,  $y=0$ .

$$\Rightarrow D=0.$$

at point B where  $x=l$ ,  $y=0$ .

$$0 = -\frac{Ml^3}{6l} + \frac{M}{2}\left(\frac{1}{3}l\right)^2 + Cl$$

$$0 = \frac{Ml^2}{18} - \frac{Ml^2}{6} + Cl$$

$$C = \frac{Ml}{9}.$$

the equation of slope becomes

$$EI \frac{dy}{dx} = -\frac{Mx^2}{2l} + M\left(x - \frac{2}{3}l\right) + \frac{Ml}{9}$$

the equation of deflection is

$$EIy = -\frac{Mx^3}{6l} + \frac{M}{2}\left(x - \frac{2}{3}l\right)^2 + \frac{Ml}{9}x$$

the slope of the beam at the ends

at  $x=0$  and  $x=l$ .

$$E I \frac{dy}{dx} = \frac{ML}{9}$$

$$\frac{dy}{dx} \Big|_{x=0} = \frac{ML}{9EI}$$

at  $x=L$

$$E I \frac{dy}{dx} = -\frac{ML}{2L} + M\left(\frac{1}{3}L\right) + \frac{ML}{9}$$

$$E I \frac{dy}{dx} = -\frac{ML}{2} + \frac{ML}{3} + \frac{ML}{9}$$

$$E I \frac{dy}{dx} = -\frac{ML}{18}$$

$$\therefore \frac{dy}{dx} \Big|_{x=L} = \frac{-ML}{18EI}$$

for maximum deflection/ slope must be 0  
at  $x = \frac{2}{3}L$  within  $x < \frac{2}{3}L$ .

$$\frac{dy}{dx} = 0 \text{ for } x < \frac{2}{3}L$$

$$0 = -\frac{MLx^2}{2L} + \frac{ML}{9}$$

$$\frac{Mx^2}{2L} = \frac{ML}{9}$$

$$x^2 = \frac{2L^2}{9}$$

$$x = \frac{\sqrt{2}L}{3}$$

Now, for maximum deflection

$$EIy = \frac{-M \frac{2\sqrt{2}}{3} L^3}{162L} + \frac{ML^2\sqrt{2}}{27}$$

$$EIy = \frac{ML^2\sqrt{2}}{27} - ML^2\sqrt{2} \frac{2}{162}$$

$$= \frac{2\sqrt{2}}{81} ML^2$$

$$\therefore y = \frac{2\sqrt{2}}{81EI} ML^2$$

$$EIy = \frac{-M \left[ \frac{\sqrt{2}}{3} L \right]^3}{6L} + \frac{ML}{9} \frac{\sqrt{2}L}{3}$$

$$R_1 = \frac{R_m - I_{psd} R_n R_n}{V}$$

$$R_2 = \frac{I_{psd} R_m R_n}{V - I_{psd} R_n}$$

$$R_m = 100 \Omega$$

$$I_{psd} = 1 \text{ mA}$$

$$V = 3 \text{ V}$$

$$R_n = 2000 \Omega$$

$$R_1 = \frac{2000 - \frac{(1 \times 10^{-3})(100)(2000)}{3}}{1}$$

$$R_1 = 1933.3 \Omega$$

$$R_2 = \frac{(1 \times 10^{-3})(100)(2000)}{3 - (1 \times 10^{-3})(2000)}$$

$$\approx \underline{\underline{200 \Omega}}$$

3)  $5\% \text{ drop} = 5\% \times 3 = 0.15 \text{ V}$

$$\text{drop} = 3 - 0.15 = \underline{\underline{2.85}}$$

$$R_2 = \frac{I_{psd} R_m R_n}{V - I_{psd} R_n} = \frac{(1 \times 10^{-3})(100)(2000)}{2.85 - (1 \times 10^{-3})(2000)}$$

$$\approx \underline{\underline{235.51 \Omega}}$$

Question 2

Same length, same torque for a same  
Steel screw because they have same material.

Let  $\Delta_H$  = Outer diameter of hollow

$d_H$  = inner diameter of hollow

$\Delta_s$  = diameter of solid

$W_H$  = Weight of hollow

$W_s$  = Weight of solid

Comparison on ratio

$$\frac{W_H}{W_s} = \frac{\text{weight of hollow}}{\text{weight of solid}}$$

Weight  $W = mg = \rho V = \rho A l$ , for the materials  
with same length and same density

$$\frac{W_H}{W_s} = \frac{A_H}{A_s} = \frac{\frac{\pi}{4} \Delta_H^2 - \frac{\pi}{4} d_H^2}{\frac{\pi}{4} \Delta_s^2}$$

$$\frac{W_H}{W_s} = \frac{A_H}{A_s} = \frac{\Delta_H^2 - d_H^2}{\Delta_s^2}$$

for the special case

$$d_H = \frac{3}{4} \Delta_H$$

$$\frac{J_H}{R_S} = \frac{\Delta H^2 - \left[ \frac{3}{4} \Delta H \right]^2}{\Delta S^2} = \frac{\Delta H^2 \left( 1 - \left[ \frac{3}{4} \right]^2 \right)}{\Delta S^2}$$

$$= \frac{\Delta H^2 (0.41375)}{\Delta S^2}$$

from the Lorenz equation:

$$\frac{I}{R} = \frac{qE}{c} = \frac{I}{J}$$

As from Smith a given torque for same clear force.

$$\frac{J}{R} = \frac{I}{c} = \text{constant}$$

$$\frac{J_H}{R_H} = \frac{J_S}{R_S}$$

$$\frac{\frac{7}{32} \Delta H^4 - \left[ \frac{3}{4} \Delta H \right]^4}{\Delta H^{1/2}} = \frac{\frac{7}{32} \Delta S^4}{\Delta S^{1/2}}$$

$$\Delta H^3 \left( \frac{1.45}{2.56} \right) = \Delta S^3$$

$$\Delta S = 0.8209 \Delta H$$

plugging in the for  $\Delta S$

PROBLEM 2:

$$102 \text{ kN/m}^2 = 102 \times 10^3 \text{ N/m}^2$$

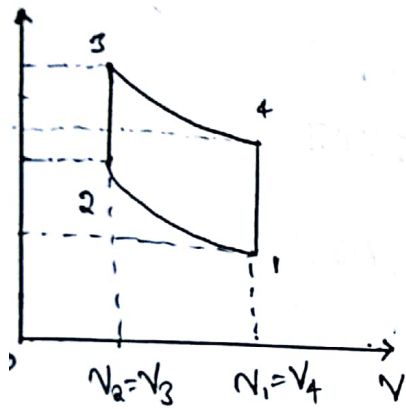
$$= 0.0025 \text{ m}^3$$

$$= 15^\circ \text{C} \equiv (15 + 273.15) \text{K} = 292.15 \text{K}$$

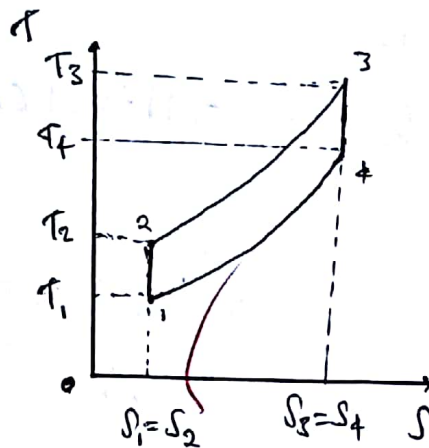
$$u = 4.6 \text{ MJ/m}^3 = 4.6 \times 10^6 \text{ J/m}^3$$

$$v = 9$$

$$p = 1.006 \text{ kJ/kgK} \quad C_v = 0.716 \text{ kJ/kgK}$$



PV Diagram



TS Diagram

eff 1 to 2:

$$= ? \quad p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$\Rightarrow p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma \quad \text{But } V_2 = \frac{V_1}{r_v}$$

$$\Rightarrow p_2 = p_1 \left( \frac{V_1}{V_1/r_v} \right)^\gamma = p_1 r_v^\gamma = 2235.1717$$

$$p_2 = 102 \text{ kN/m}^2 (9)^{1.4050}$$

$$p_2 = 2235.1717 \text{ kN/m}^2$$

$$\therefore p_2 \approx \underline{2235 \text{ kN/m}^2}$$

$$\frac{p_2}{p_1} = \left( \frac{V_1}{V_2} \right)^{\gamma-1} \Rightarrow T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = T_1 r_v^{\gamma-1} = 292.15 (9)^{0.4050}$$

$$\therefore T_2 = 711.33488 \text{K} = 438.1849^\circ \text{C} \approx \underline{438^\circ \text{C}}$$

Process 2 to 3 :

$$N_3 = \frac{N_1}{r_1} = \frac{0.0885}{9} = 2.7778 \times 10^{-4} \text{ m}^3$$

$$N_3 = N_2 = 2.78 \times 10^{-4} \text{ m}^3$$

$$P_{max} = P_3 = 4.6 \times 10^6 \text{ N/m}^2$$

$$\frac{P_3}{r_3} = \frac{P_2}{r_2}$$

$$r_3 = r_2 \left( \frac{P_3}{P_2} \right) = 711.33488 \left( \frac{4.6 \times 10^6}{2.2351717} \right)$$

$$= 1463 \text{ K} = 1190.7825^\circ\text{C}$$

$$\therefore r_3 \approx 1190^\circ\text{C}$$

(a)

Process

$$r_1 = 1$$

$$N_1 = 1$$

$$r_1 = 1$$

$$r_2 = 1$$

$$r_3 = 1$$

$$r_4 = 1$$

Process 3 to 4

$$\frac{r_3}{r_4} = \left( \frac{N_4}{N_3} \right)^{\frac{1}{\gamma-1}}$$

$$= \left( \frac{N_4}{N_3} \right)^{\frac{1}{1.4-1}}$$

$$r_4 = \frac{r_3}{r_4} = \frac{P_3}{P_4} r_1 = \frac{4.6 \times 10^6}{2.2351717} \times 2.23515 = 601.24691 \text{ (m)}$$

$$\therefore T_4 = 328.0969^\circ\text{C} \approx 328^\circ\text{C}$$

$$P_4 V_4^\gamma = P_3 V_3^\gamma$$

$$P_4 = P_3 \left( \frac{N_3}{N_4} \right)^\gamma = P_3 \left( \frac{1}{r_4} \right)^\gamma = \frac{4.6 \times 10^6 \text{ N/m}^2}{1} \left( \frac{1}{601} \right)^{1.4}$$

$$= 209916.7558 \text{ N/m}^2 \approx 210 \text{ kN/m}^2$$

from diagram  $N_1 = V_4 = 0.0025 \text{ m}^3$ .

⇒

Process 1 to 4

$$P_1 = 102 \times 10^3 \text{ N/m}^2$$

$$V_1 = 0.0025 \text{ m}^3$$

$$T_1 = 292.15 \text{ K}$$

$$P_1 = 102 \times 10^3 \text{ N/m}^2 \quad T_1 = 292.15 \text{ K} \quad 19^\circ\text{C}$$

$$R = 2835 \text{ J/m}^2 \text{K} \quad T_2 = 438^\circ\text{C}$$

$$R_2 = 4.6 \text{ MN/m}^2 \quad T_3 = 1190^\circ\text{C}$$

$$R_4 = 210 \text{ kN/m}^2 \quad T_4 = 328^\circ\text{C}$$

$$V_1 = 0.0025 \text{ m}^3$$

$$V_2 = 2.78 \times 10^{-4} \text{ m}^3$$

$$V_3 = 2.78 \times 10^{-4} \text{ m}^3$$

$$V_4 = 0.0025 \text{ m}^3$$

$$(a) \quad \eta = \left(1 - \frac{T_1}{T_2}\right) \times 100\% = \left(1 - \frac{19}{438}\right) \times 100\% = \left(1 - \frac{T_4 - T_1}{T_3 - T_2}\right) \times 100\%$$

$$= \left(1 - \frac{1}{\frac{T_3 - T_1}{T_2 - T_1}}\right) \times 100\% = \left(1 - \frac{1}{\frac{1190 - 19}{438 - 19}}\right) \times 100\%$$

$$\eta = 1 - \frac{T_1}{T_2} = \left(1 - \frac{292.15 \text{ K}}{1111.3349 \text{ K}}\right) \times 100\% = 58.929\% \approx 59\%$$

4.69 (iii) Mean effective pressure =  $\frac{W_{net}}{V} = \frac{W_{net}}{V_1 - V_2}$        $R = C_p - C_v$   
 $= 0.287 \text{ kJ/kgK}$

$$W_{net} = m c_v [(T_3 - T_2) - (T_4 - T_1)] \quad \text{and } P_1 V_1 = m R T_1$$

$$\Rightarrow m = \frac{R V_1}{R T_1} = \frac{(102 \times 10^3 \times 0.0025) \text{ Nm}}{0.287 \text{ kJ/kgK} \times 292.15 \text{ K}} = \frac{255}{847.285} = 3.0098 \times 10^{-3}$$

$$\approx 3.01 \times 10^{-3} \text{ kg}$$

$$\phi = 3.0098 \times 10^{-3} \times 716 [(1463 - 1111.3349) - (601.2469 - 292.15)]$$

$$= 953.7419 \text{ J}$$

$$\text{Mean effective pressure} = \frac{953.7419}{(0.0025 - 0.000278)} = 429 \text{ kPa} \approx 426.7 \text{ kPa}$$

$\therefore$  Mean effective pressure =  $\underline{429.2 \text{ kN/m}^2}$

(iv) Carnot efficiency =  $\left(\frac{T_3 - T_1}{T_2}\right) \times 100\%$

=  $\left(\frac{1463 \text{ K} - 292.15 \text{ K}}{1463 \text{ K}}\right) \times 100\%$

$\therefore$  Carnot efficiency =  $80.03\% \approx \underline{80.0\%}$

QUESTION 4. Same state as in question 3.

$P_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$        $P_2 = 43 \text{ bar} = 43 \times 10^5 \text{ N/m}^2$

$P_3 = 18 \text{ bar} = 18 \times 10^5 \text{ N/m}^2$        $R = 0.287 \text{ kJ/kgK}$

$T_1 = 290 \text{ K}$

$\eta = \frac{C_p}{C_v} = \frac{R + C_v}{C_v} = \frac{0.287 \text{ kJ/kgK} + 0.717 \text{ kJ/kgK}}{0.717 \text{ kJ/kgK}}$

= 1.4003

Thermal efficiency =  $1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$  and  $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

$\Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{18 \times 10^5}{1 \times 10^5}\right)^{\frac{1}{1.4003}}$

= 7.8783 = 7.88

$\eta = \left(1 - \frac{1}{7.8783}\right) \times 100\%$

= 56.23%

$\approx 56.2\%$

Mean effective pressure

$P_{me} = \frac{P_1}{\gamma} \left[ \frac{P_2}{P_1} - 1 \right]$

$\frac{P_1}{\gamma} = \frac{P_2}{\gamma} \Rightarrow P_1 = P_2$

$P_{me} = \frac{43 \times 10^5 \text{ Pa}}{1.4}$

= 3.0714 MPa

Mean effective pressure

QUESTION 7.

= 145 mm

stroke = 205 mm

Compression

clearance

$\frac{P_1}{\gamma} = \frac{P_2}{\gamma} \Rightarrow P_1 = P_2$

$$\text{Mean effective pressure} = \frac{\phi_{70}}{V_1 - V_2}$$

$$P_4 V_4^\gamma = P_3 V_3^\gamma \Rightarrow P_4 = P_3 \left( \frac{V_3}{V_4} \right)^\gamma$$

$$= 43 \times 10^5 \left( \frac{1}{7.8783} \right)^{1.4003}$$

$$= 238 \ 887.8191 \text{ N/m}^2$$

$$\frac{V_1}{V_2} = r_v \Rightarrow V_1 = r_v V_2 \quad ; \quad V_3 = V_2 \quad ; \quad V_1 = V_4$$

$$\phi_{70} = \frac{(43 \times 10^5 V_2 - 238 \ 887.8191 V_1) - (18 \times 10^5 V_2 - 1 \times 10^5 \times 7.8783 V_2)}{1.4003 - 1}$$

$$= 3.5 V_2 \text{ MJ}$$

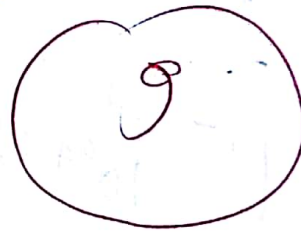
$$\text{Mean effective pressure} = \frac{3.5 V_2 \times 10^6}{r V_2 - V_2}$$

Since  $V_1 = r V_2$ .

$$= \frac{3.5 \times 10^6}{7.8783 - 1}$$

$$= 508 \ 846.6627 \text{ N/m}^2$$

$$\approx \underline{\underline{508 \text{ kN/m}^2}}$$



### \* QUESTION 7.

$$d = 145 \text{ mm}$$

$$\text{Stroke} = 205 \text{ mm}$$

clearance volume = 10% of swept volume.

$$(i) \text{ Compression ratio} = \frac{V_1}{V_2}$$

$$\text{ratio} = \frac{V_1}{0.9 V_1} = \underline{\underline{1.11}}$$

$$V_1 = V_{\text{total}} = \pi r^2 \times 205$$

$$= 205 \pi r^2 \text{ mm}^3$$

$$V_2 = \frac{100 - 10}{100} \times V_1 = 0.9 V_1$$

∴ Compressor ratio = 1.11.

(ii)

$$V_2 = \frac{6}{100} \pi r^2 h = \frac{605109.6}{100} \times \pi \times (145 \text{ mm})^2 \times (205 \text{ mm})$$

$$= 203109.8555 \text{ mm}^3$$

$$= 203.1 \times 10^6 \text{ m}^3$$

$$V_2 = N \times 6\% \times V_1$$

$$= 203.1 \times 10^6 + 0.00339$$

$$= 3.593 \times 10^{-3} \text{ m}^3$$

$$P = \frac{V_2}{V_1} \Rightarrow P = \frac{0.0035934}{0.002034} = 1.7665.$$

$$h = \left[ 1 - \frac{1}{r_2^{(n-1)}} \right] \left[ \frac{P_2 - 1}{\gamma(P_2 - 1)} \right] \times 100\%$$

$$= \left[ 1 - \frac{1}{10^{0.405}} \right] \left( \frac{1.7665^{1.405} - 1}{1.405(1.7665 - 1)} \right) \times 100\%$$

$$= 68.93\%$$

$$= \underline{\underline{68.9\%}}$$

