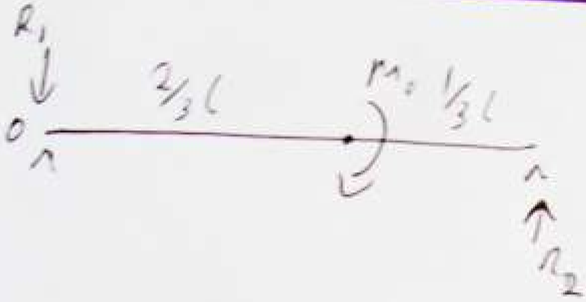
Q1)



static balance

moments about O

$$R_2 l = M_0$$

forces

$$R_1 = R_2$$

$$R_1 = \frac{M_0}{l}$$

$$EI\frac{d^2y}{dx^2} = M = -R_1 x + M_0\left[x - \tfrac{2}{3}l\right]^0$$

$$= -\frac{M_0}{l}x + M_0\left[x - \tfrac{2}{3}l\right]^0$$

$$EI\frac{dy}{dx} = -\frac{M_0 x^2}{2l} + M_0\left[x - \tfrac{2}{3}l\right] + A$$

$$EIy = -\frac{M_0 x^3}{6l} + \frac{M_0}{2}\left[x - \tfrac{2}{3}l\right]^2 + Ax + B$$

BC's

$x=0,\ y=0 \quad \therefore B=0$

$x=l,\ y=0$

$$0 = -\frac{M_0 l^2}{6} + \frac{M_0}{2} \cdot \frac{1}{9} l^2 + Al$$

$$Al = \frac{M_0 l^2}{6} - \frac{M_0 l^2}{18}$$

$$A = M_0 \cdot \frac{l}{9}$$

Slope at $x=0$

$$EI\frac{dy}{dx} = 0 + 0 + \frac{M_0 l}{9}$$

$$\frac{dy}{dx} = \frac{M_0 l}{9EI} \quad //$$

Slope at $x=l$

$$EI\frac{dy}{dx} = -\frac{M_0 l}{2} + M_0 \frac{l}{3} + \frac{M_0 l}{9}$$

$$EI\frac{dy}{dx} = M_0 l\left(-\frac{9}{18} + \frac{6}{18} + \frac{2}{18}\right)$$

$$\frac{dy}{dx} = -\frac{M_0 l}{18EI} \quad //$$

max deflection:

$$EIy = -\frac{M_0 x^3}{6l} + \frac{M_0}{2}\left[x - \tfrac{2}{3}l\right]^2 + \frac{M_0 l}{9}x$$

$$EI\frac{dy}{dx} = 0$$

$$0 = -\frac{M_0 x^2}{2l} + M_0\left[x - \tfrac{2}{3}l\right] + \frac{M_0 l}{9}$$

max deflection occurs $x < \tfrac{2}{3}l$ hence:

$$0 = -\frac{M_0 x^2}{2l} + \frac{M_0 l}{9}$$

$$x^2 = \frac{2l^2}{9}$$

$$x = \frac{\sqrt{2}\,l}{3} \quad //$$

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$$EI\,y = -\frac{M_0}{6l}\left(\frac{\sqrt{2}}{3}l\right)^3 + \frac{M_0 l}{9}\cdot\frac{\sqrt{2}}{3}l$$

$$y = \frac{M_0}{EI}\left(\frac{\sqrt{2}\,l^2}{27} - \frac{2^{3/2}\,l^2}{162}\right)$$

$$y = \frac{M_0}{EI}\cdot\left(\frac{6\sqrt{2} - 2\sqrt{2}}{162}\right)l^2$$

$$y = \frac{2\sqrt{2}}{81}\cdot\frac{M_0 l^2}{EI}$$

Q2) Solid shaft D_1

Hollow " D_2 out

$\frac{3}{4}D_2$ in

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

Solid shaft

$J = \frac{\pi D_1^4}{32}$ max shear stress $\hat{\tau} = \frac{16T}{\pi D_1^3}$

Hollow shaft

$J = \frac{\pi}{32}\left(D_2^4 - \left(\frac{3}{4}D_2\right)^4\right)$, $\hat{\tau} = \frac{16 D_2 \cdot T}{\pi\left(D_2^4 - \left(\frac{3}{4}D_2\right)^4\right)}$

$$\frac{16T}{\pi D_1^3} = \frac{16 D_2 T}{\pi\left(D_2^4 - \left(\frac{3}{4}D_2\right)^4\right)}$$

$$D_1^3 = \frac{\left(D_2^4 - \left(\frac{3}{4}D_2\right)^4\right)}{D_2}$$

$$D_1^3 = \frac{D_2^4 - \frac{81 D_2^4}{256}}{D_2} = \frac{175}{256}D_2^3$$

$$D_1^3 = \frac{175}{256} D_2^3$$

$$D_1 = 0.8809\, D_2$$

weight ratio:

$$V_1 = \rho \pi D_1^2 l, \quad V_2 = \pi\left(D_2^2 - \left(\tfrac{3}{4}D_2\right)^2\right) l$$

Ratio: $$\frac{V_1}{V_2}$$

$$= \frac{\pi D_1^2 l}{\pi\left(D_2^2 - \frac{9}{16}D_2^2\right) l}$$

$$= \frac{D_1^2}{\left(D_2^2 - \frac{9}{16}D_2^2\right)}$$

$$= \frac{(0.8809)^2 D_2^2}{0.4375\, D_2^2}$$

$$= \underline{\underline{1.774}}$$

(or inverse is fine)