EE3112 - ELECTRICAL ENGINEERING PRACTICE

MECHANICAL COMPONENT

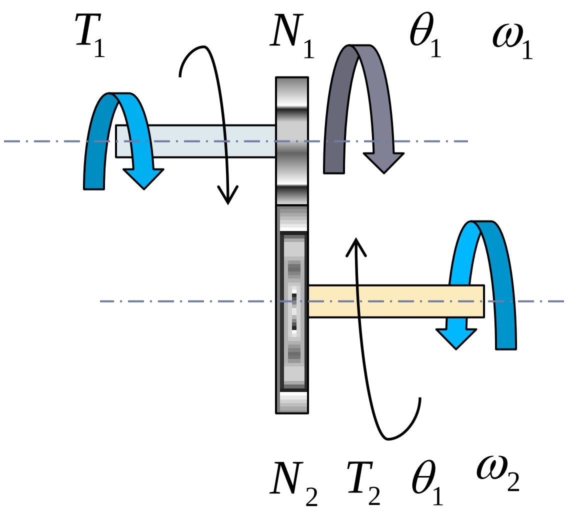
Dynamic Mechanics:

Gear Systems

Introduction

- Gear reduction systems are commonly used with motors in an attempt to best utilize the power that is available at high speeds.

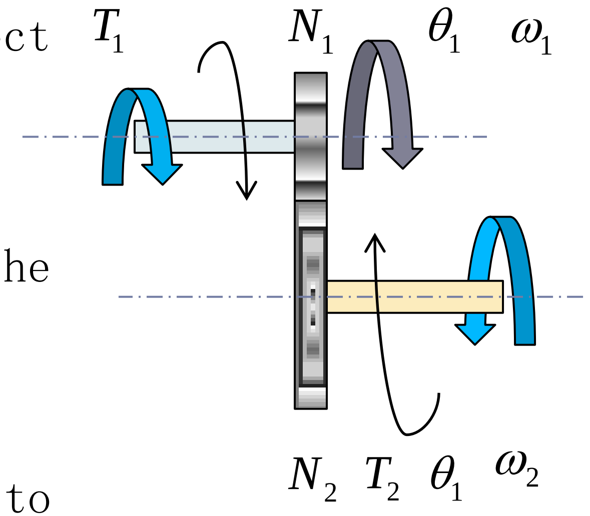
- Although there are many different types of gear configurations, the fundamental relationships are **described using a pair of spur gears as shown in the figure.**



A Pair of Spur Gears

Transmission Ratio

- Since the applied and load forces on the contacting gear teeth are equal, the magnitude of the respective torques must be in direct proportion to the radii.



A Pair of Spur Gears

- The ratio of radii is, of course, equal to the ratio of circumferences.

- Therefore, the ratio of applied torque to transmitted torque is equal to the ratio of the number of gear teeth such that:

$$\frac{T_2}{T_1} = \frac{N_2}{N_1}$$

- Where N is the number of teeth

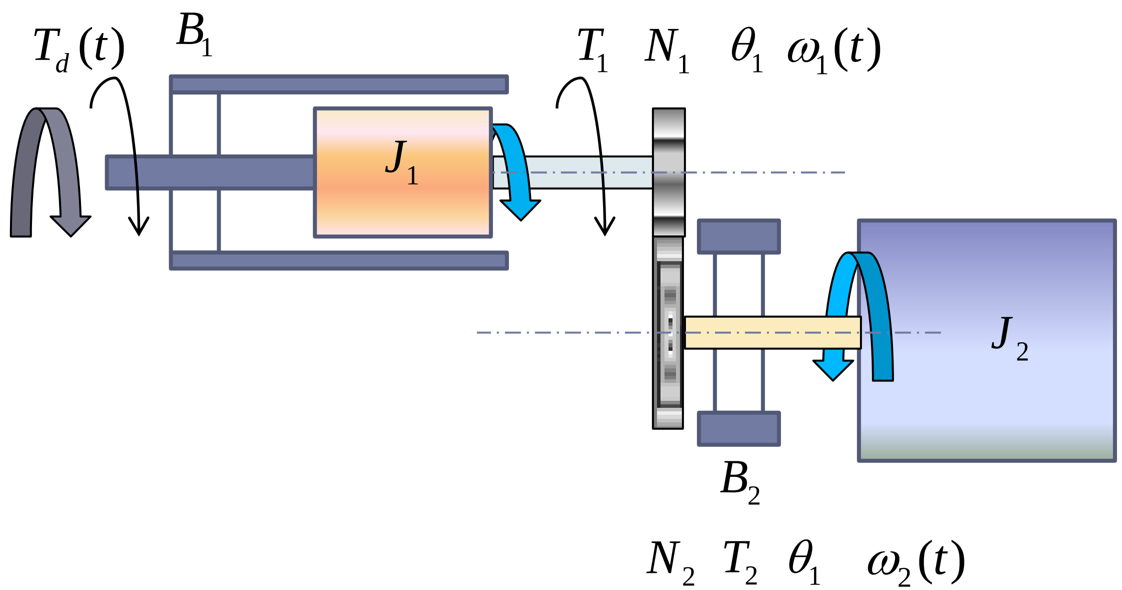
Transmission Ratio continued …

- With rotation of the gears, the distance traversed along the circumferences must be equal, and this equality can be restated in terms of the product of the change in angle and the radius such that $\theta_1 R_1 = \theta_2 R_2$
- Then a derivative with respect to time provides $\omega_1 R_1 = \omega_2 R_2$
- To avoid minus signs the assumed positive direction of motion is reversed for the second gear
- If ratios of radii are replaced with ratios of teeth, then $\dfrac{\theta_1}{\theta_2} = \dfrac{\omega_2}{\omega_1} = \dfrac{N_1}{N_2}$
- Thus, the torque changes in direct proportion to the ratio of teeth, and the angular velocity changes in reverse proportion to the ratio of teeth

Motor Rotor and Coupled Load

- If the rotor of a motor is connected to a load with gears between the motor and load, a system is obtained as shown in the figure.

- $T_d(t)$ is the developed torque of the motor, and J_1 and B_1 are the moment of inertia and viscous friction associated with the motor rotor.



Motor Rotor and Coupled Load Continued …

If T_1 is the load torque reflected back through the gears, then

$$T_d(t) = J_1 \frac{d}{dt}\omega_1(t) + B_1\omega_1(t) + T_1(t)$$

- And the load torque is $T_2(t) = J_2 \frac{d}{dt}\omega_2(t) + B_2\omega_2(t)$

- Using the torque relationship to combine the two equations then

$$T_d(t) = J_1 \frac{d}{dt}\omega_1(t) + B_1\omega_1(t) + \frac{N_1}{N_2}\left[J_2 \frac{d}{dt}\omega_2(t) + B_2\omega_2(t) \right]$$

Equivalent mass moment of inertia and equivalent friction coefficient

- The terms that are expressed in terms of the output velocity can be reflected to the source side of the gears.

- Note that angular velocity is inversely related to the ratio of teeth, and the derivatives of angular velocity must exhibit the same ratio.

- With all terms of the previous equation expressed in terms of the input velocity,

$$T_d(t) = J_{eq}\frac{d}{dt}\omega_1(t) + B_{eq}\omega_1(t) \quad \text{where } J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 \quad \text{and } B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2$$

- These equations are used when developing models for motors with loads that are coupled through gear systems.

- **Maximum acceleration**
- Gear ratio can be selected to maximize the ratio of load acceleration to applied torque.
- Assuming that the inertial loads provide the dominant components of the load torque during periods of high acceleration (or deceleration), then the friction loads can be neglected and equation 3.47 becomes

$$T_d(t) \approx J_1 \frac{d}{dt}\omega_1(t) + \frac{N_1}{N_2}\left[J_2 \frac{d}{dt}\omega_2(t) \right]$$

- To simplify the notation, replace N_2/N_1 by n and replace the load acceleration by α_2

- **Maximum acceleration continued …**
- Then if the acceleration of the motor is expressed in terms of the load acceleration and gear ratio, the ratio of the load acceleration to developed torque is

$$\frac{a_2}{T_d} = \frac{n}{n^2 J_1 + J_2}$$

- And the maximum value of this ratio occurs with $n = \sqrt{\frac{J_2}{J_1}}$

- **Maximum Velocity**
- The gear ratio can also be selected to maximize the ratio of load velocity to developed torque under steady state conditions (zero acceleration).
- The result is a gear ratio equal to the square root of the ratio of load friction to rotor friction.
- This procedure will not produce not normally generate the same ratio as obtained when maximizing acceleration.
- The final selection of gear ratio maybe a compromise that considers the relative importance of several factors.

- **Summary**
- When selecting gears, obtaining the desired gear ratio and an acceptably high torque rating in a physical package that is acceptable in view of size and weight is often difficult.
- This combination of objectives is a particular problem in robotic applications.
- Another important concern is the elimination of any significant backlash. One response to this combination is the harmonic gear.