EEE3112 - ELECTRICAL ENGINEERING PRACTICE
MECHANICAL COMPONENT

LECTURER: Mr. K. H. MOONGA

MOBILE: 0972026390

EMAIL: khmoonga@unza.zm

OFFICE: ROOM SR5 ANNEX BUILDING

Contents

- **PART I: STATIC MECHANICS**
- Torsional stresses and twisting in circular shafts, helical and leaf springs
- Deflection of beams with concentrated and distributed loading
- Static equilibrium of coplanar and 3-D force and torque systems

- **PART II: DYNAMIC MECHANICS**
- Rotating bodies moments of inertia of plane figures and 3-Dimensional symmetrical objects
- Radius of gyration, Energy, momentum and impulses
- Systems with couplings and gearing
- Introduction to Mechanical vibration, free vibration, resonance, damping and vibration isolation
- **Qualitative description of failure mechanism**

- RYDER G.H, Strength of Materials
- MERIUM J.L. Engineering Mechanics Vol. II: Dynamics, 1986, Macmillan
- MERIUM J. L. Engineering Mechanics Vol. I. Statics, 1986, Macmillan
- Hibbeler R.C., Engineering Mechanics: Dynamics, Prentice Hall; 12 edition 2009
- Hibbeler R.C., Mechanics of Materials, Printice Hall: 8 edition 2011

Grades and distribution of marks

Course management

- E-learning website to be confirmed
- Assignment shall be due the following week on the same day the assignment was given unless prior arrangement is made for any changes
- Quizzes will be handed in at the end of the session
- Labs will be due 7 days from date of experiment

Conclusion

This has been the introduction to the mechanical component of the course.

Questions?

EEE3112 - ELECTRICAL ENGINEERING PRACTICE

MECHANICAL COMPONENT

TORSIONAL STRESSES AND TWISTING OF SHAFTS

Introduction:

FIRST WE LOOK AT : DIRECT STRESS

Contents – DIRECT STRESS

1.1 Load:

Any engineering design built of a number of members is in equilibrium under the action of external forces and reactions at the points of support.

These forces constitute **load** on the member. Since the member is in equilibrium all forces must sum up to zero.

Simplest type of load is direct pull or push technically as tension or compression illustrated in fig. 1.1

If a member is in motion, the load maybe caused partly by dynamic or inertia forces.

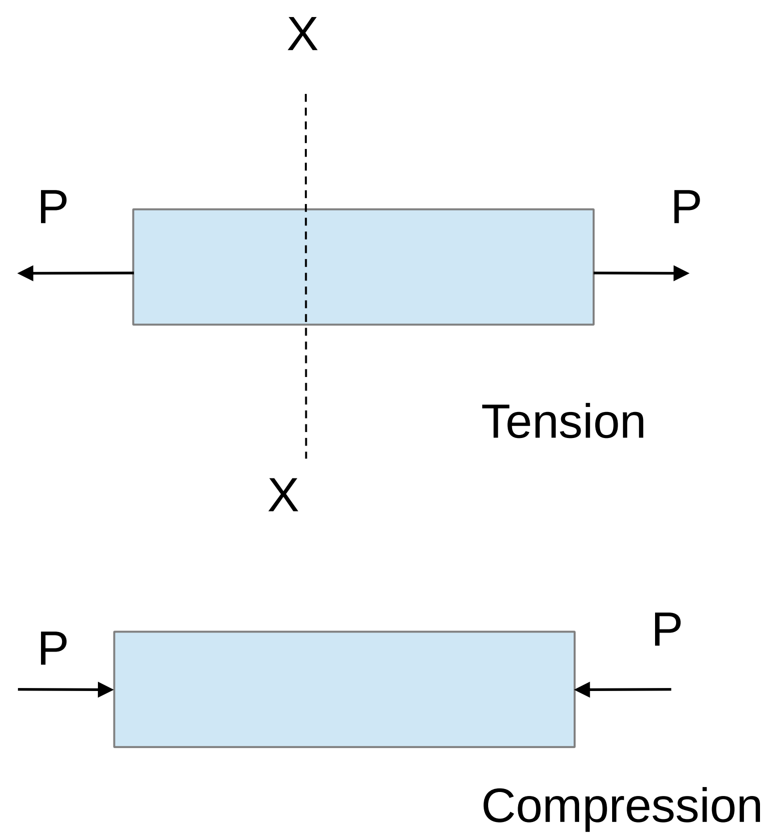


Fig1.1

- **1.2 Stress**

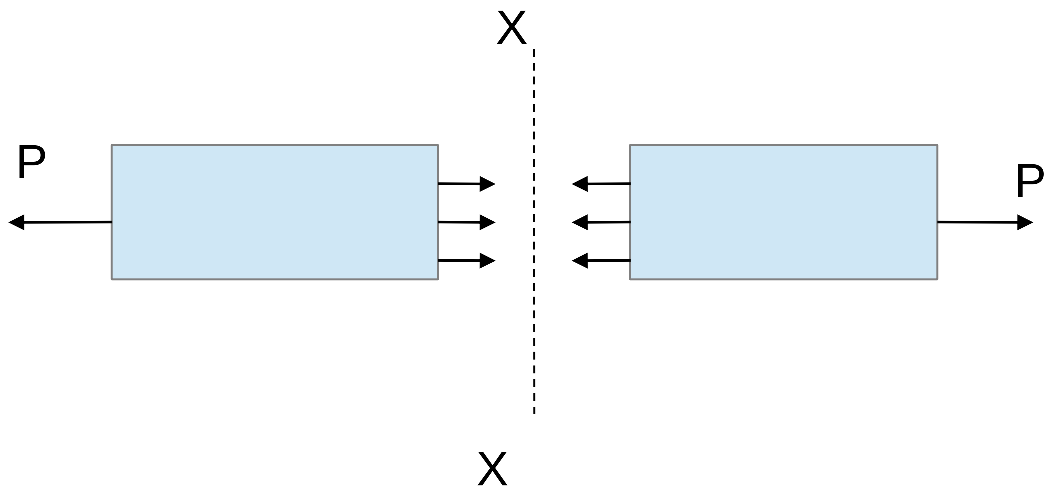


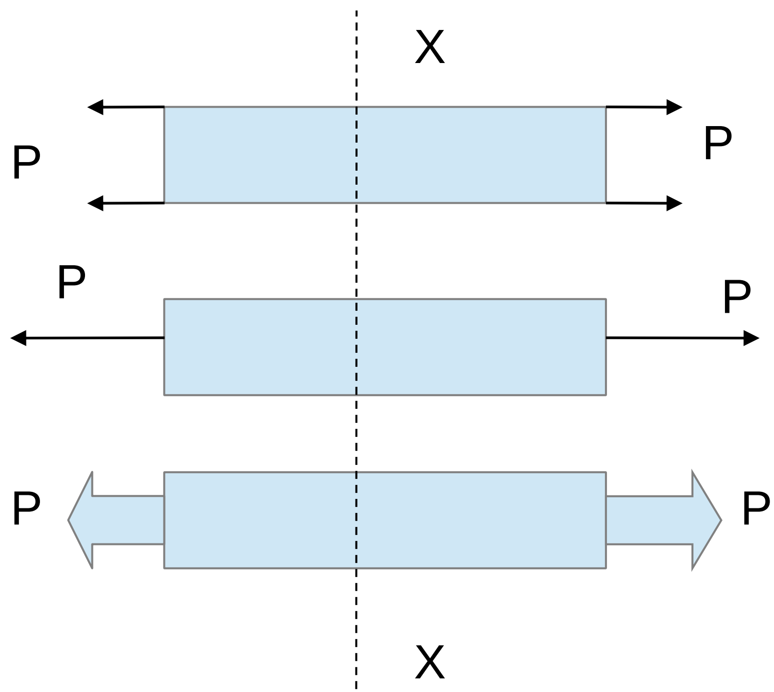
Fig 1.2

Consider any section X-X of the member, the total force carried by such a section must equal the load *P*

Stress $\sigma = \frac{P}{A}$, where A is the area.

Stress is force per unit area, in SI units **N/m2**

1.3 Principle of St• Venant. States that the actual distribution of the load over the surface of its application will not affect the distribution of stress or strain on sections of the body which are at an appreciable distance (relative to the dimensions) away from the load.



Statically equivalent, where section X-X is at an appreciable distance away from the load

1.4 Strain. Strain is a measure of the deformation produced in the member by the load. Direct stress produce a change in length in the direction of stress. If a rod of length l is in tension and the stretch or elongation produced is x, then the direct strain ε is defined as the ratio

Elongation / original length or $\varepsilon = x/l$

Normally tensile strains will be considered positive and compressive strains (I.e decrease in length) negative.

Note that strain is a ratio hence dimensionless

1.5 Hooke's law. Principle of Superposition This states that strain is proportional to the stress producing it. It is obeyed within certain limits of stress by most ferrous alloys and can usually be assumed to apply with sufficient accuracy to other engineering materials such as timber, concrete and non-ferrous alloys.

In general a material is said to be elastic if all the deformations are proportional to the load.

Where a number of loads are acting together on an elastic material, the principle of superposition states that the resultant strain will be the sum of individual strains caused by each load acting separately

1.6 Modulus of Elasticity (Young's Modulus)

- Within the limits for which Hooke's law is obeyed, the ratio of the direct stress to the strain produced is called Young's Modulus or Modulus of Elasticity (E), i.e. $E = \sigma / \varepsilon$ [N/m2]

- For a bar of uniform cross section A and length l this can be written as

  $$E = Pl / Ax,$$

  E is therefore a constant for a given material, and is usually assumed to be the same in tension or compression.

- Young's Modulus represents the stress required to cause unit strain, I.e provided Hooke's Laws continued to be obeyed.

1.7 Tensile Test

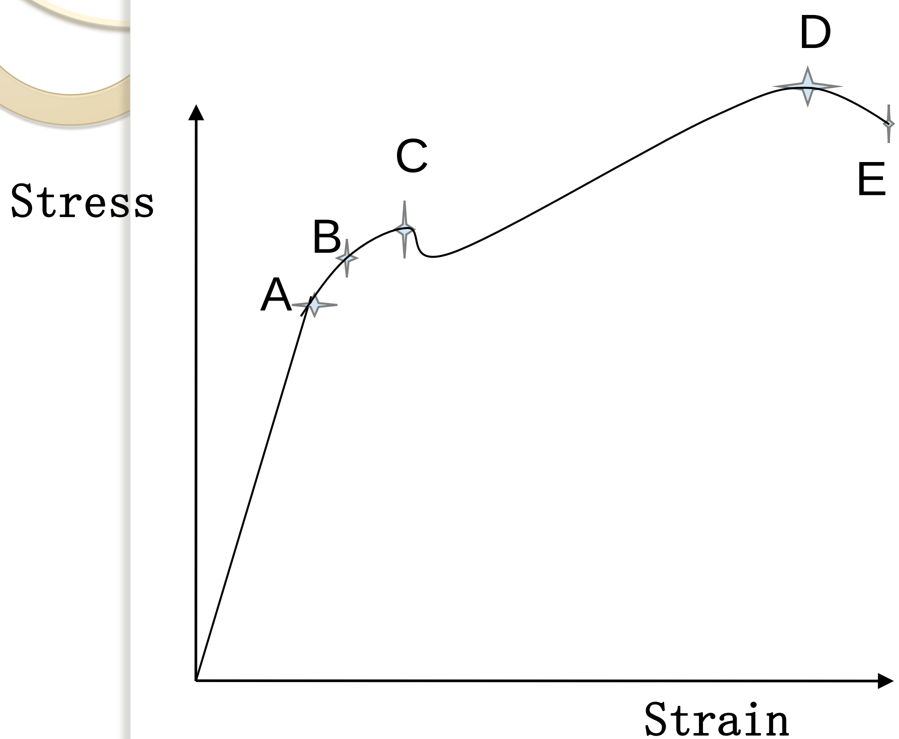


Fig 1.3

Mainly apply to the behavior of mild steel

A – Limit of proportionality (Hooke's Law obeyed)

B – Elastic Limit (Hooke's Law not obeyed but still in elastic region)

C – Yield point, metal shows appreciable strain even without further increase in load
(some material will show upper and lower yield point)

D – maximum or Ultimate tensile stress calculated by dividing the load at D by original cross section area.

E– Rapture

•

- **Tensile Test continued …**
- The capacity for being drawn out plastically before breaking is called the **ductility** of the material measured by the following two quantities:
- (1) percentage elongation = total increase in gauge length / original length expressed as a percentage
- (2) percentage reduction in area or contraction – reduction in cross sectional area at the neck, expressed as a percentage of original area.

1.8 Factor of Safety

Factor of Safety = Ultimate stress / Working Stress

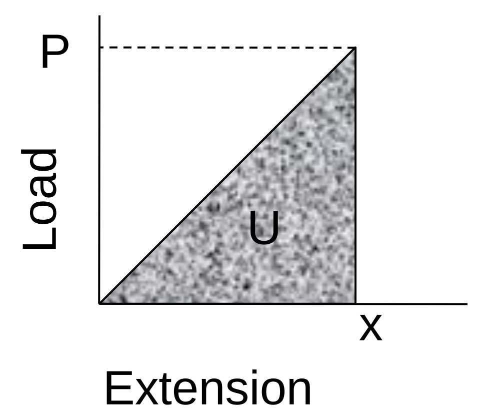
1.9 Strain Energy, Resilience

When a tensile or compressive load *P* is applied to a bar there is a change in length *x* which, for an elastic material is proportional to the load

The strain energy of a bar is defined as the work done by the load in straining it.

Under static or gradually applied load work done is represented by the shaded area, giving

$U = 1/2Px$ --------------- (1)



Strain Energy, Resilience continued …

- In terms of stress and dimensions, for a bar of uniform section A and length l substitute P = σA and $x = \sigma l/E$, giving

$$U = \tfrac{1}{2}.\sigma A.\sigma l/E$$

$$= (\sigma 2/2E)Al \text{ ------------------ } (2)$$

- But Al is the volume of the bar, and hence equation (2) can be stated: "the strain energy per unit volume (usually called the resilience) in simple tension or compression is $\sigma 2/2E$"

- Strain energy is always positive quantity, being units, will be expressed in Nm (ie Joules).

- Solve questions from G.H Ryder, Strength of Materials,

Next WE LOOK AT: Shear Stress

SHEAR STRESS

2.1 Shear Stress:

If the applied load P consists of two equal and opposite parallel forces not in the same line (as in Fig 2.1), then there is a tendency for one part of the body to slide over or shear from the other part across any section LM.

If the cross-section at LM measured parallel to the load is A, then the average shear stress $\boldsymbol{\tau} = P/A$. N/m2

Notice that shear stress is tangential to the area over which it acts.

The most common occurrence of pure shear is in riveted joints.

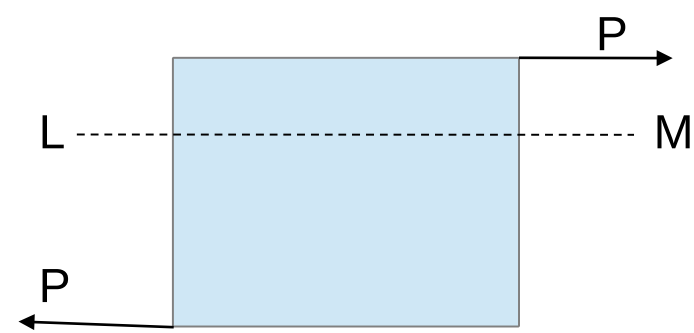


Fig. 2.1

Fig1.1

- **2.2 Complementary Shear Stress**

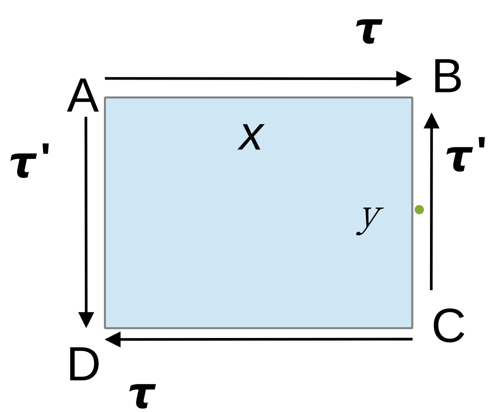


Fig 2.2

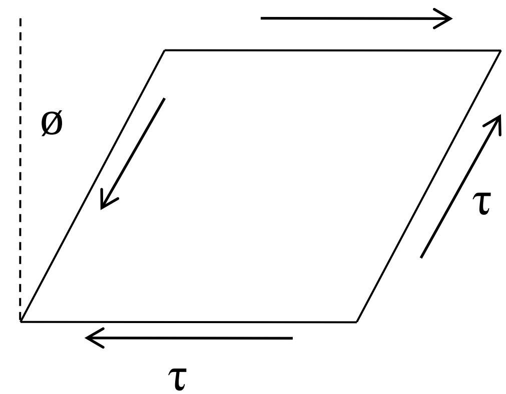
Let ABCD (Fig 2.2), be a small rectangular element of sides x, y, and z perpendicular to the figure. Let there be a shear stress τ acting on planes AB and CD.

It is clear that these stresses will form a couple $(\tau.xz)y$ which can only be balanced by tangential forces on planes AD and BC (any normal forces which exist will balance out in pairs). These are known as complementary shear stresses.

Let τ' be the complementary shear stress induced on planes AD and BC. Then for equilibrium $(\tau.xz)y=(\tau'.yz)x$, i.e. $\tau'=\tau$ showing that every shear stress is accompanied by an equal complementary shear stress on planes at right angles.

Example 1. A flange coupling joining two sections of shaft is required to transmit 250 kW at 1000 r.p.m.. If six bolts are to be used on a pitch circle diameter of 14 cm, find the diameter of the bolts. Allowable mean shear stress 75N/mm2.

2.3 Shear Strain. Shear strain or slide ϕ, *can be defined as the change in the right angle. It is measured in radians and is dimensionless*



2.4 Modulus of Rigidity For elastic materials

- shear strain is proportional to the shear stress producing it within certain limits.

- The ratio Shear Stress / Shear Strain is called Modulus of Rigidity, i.e.

$$G = \tau/\phi \ N/mm2$$

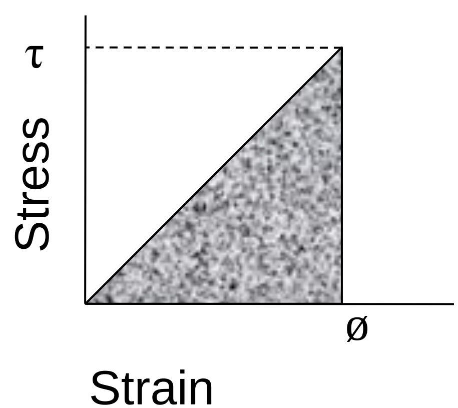
2.5 Strain Energy in shear

Within limit of proportionality stress is proportional to strain, and

Strain energy (U) = Work done in straining

= 1/2(Final couple) x(Angle turned through)

For a gradually applied stress (work done is proportional to shaded area in Fig. 2.5),



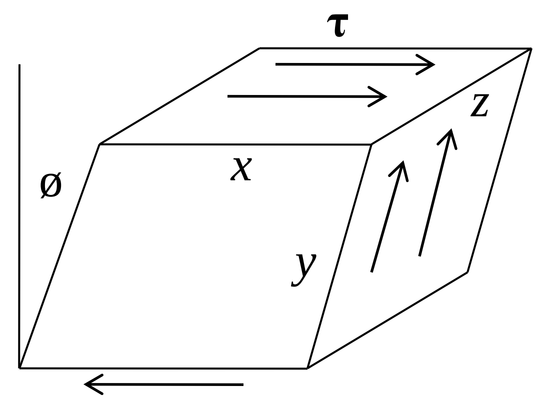
- **Strain Energy continued …**
- i.e U = ½ (τyz.x)ø

  = ½.τxyz.τ/G

  = $(\tau^2/2G)$ x Volume

- Compare with $\sigma^2/2E$ for direct stress).

- The units again Nm (joules)



Problem 1. pg 33 Estimate the force required to punch out circular blanks 6 cm diameter from plate 2 mm thick. Ultimate shear stress = 300 N/mm2.

2.6 Torsion – Circular Shafts

Having covered direct stress and shear stress we are now ready to tackle torsion in shafts

Torsion

8.1. Circular Shafts. If a shaft is acted upon by a pure torque T about its polar axis, shear stresses will be set up in directions perpendicular to the radius on all transverse sections (Fig. 8.1).

The complementary shear stress on longitudinal planes will cause a distortion of filaments which were originally in the longitudinal direction. It will be assumed that points lying on a radius before twisting will remain on a radius, the angle of twist being θ over a length l of shaft. This assumption is justified by the symmetry of the cross-section.

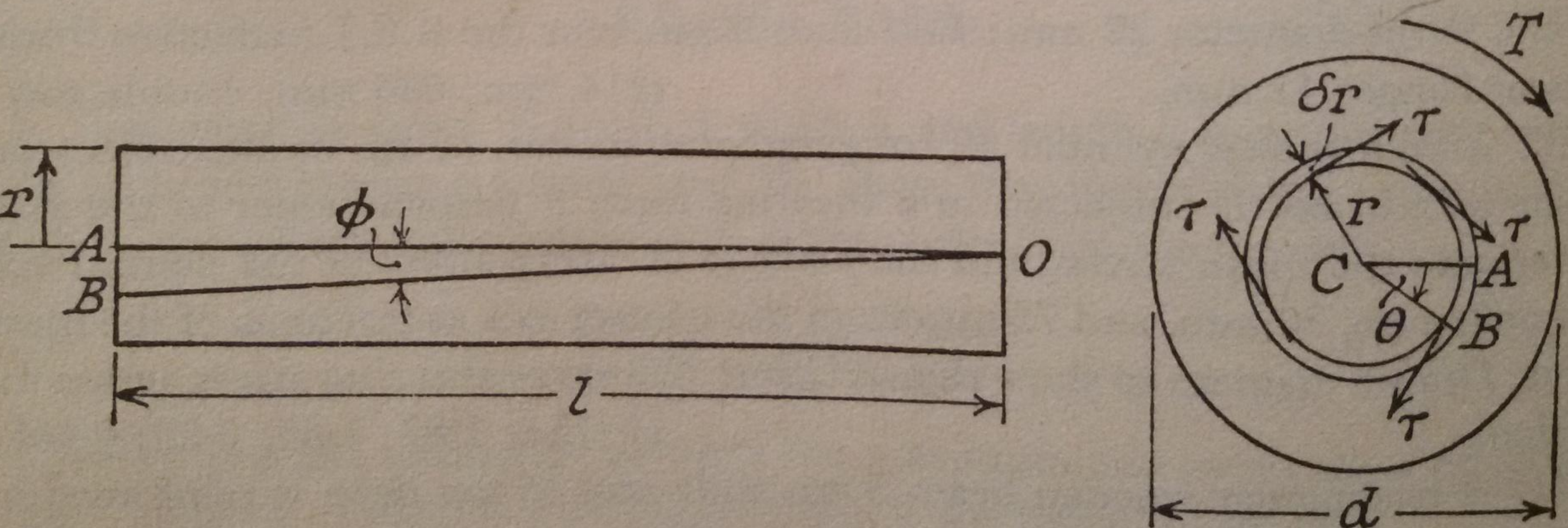


Fig. 8.1

The left-hand figure shows the shear strain ϕ of elements at a distance r from the axis (ϕ is constant for constant T), so that a line originally OA twists to OB, and $\angle ACB = \theta$, the relative angle of twist of cross-sections a distance l apart.

$$\text{Arc AB} = r\theta = l\phi \text{ approx.}$$

But $\phi = \tau/G$, where G is the modulus of rigidity (Para. 2.4).

By substitution and rearranging

$$\tau/r = G\theta/l \qquad (1)$$

The torque can be equated to the sum of the moments of the tangential stresses on the elements $2\pi r\delta r$, i.e.

$$\begin{aligned} T &= \int \tau(2\pi r\,dr)r \\ &= (G\theta/l)\int(2\pi r\,dr)r^2 \quad \text{from (1)} \\ &= (G\theta/l)J \qquad (2) \end{aligned}$$

where J is called the polar moment of inertia.

Combining (1) and (2)

$$T/J = \tau/r = G\theta/l \qquad (3)$$

- **Circular Shafts continued**
- For a solid shaft:

  $J = \pi D4/32$

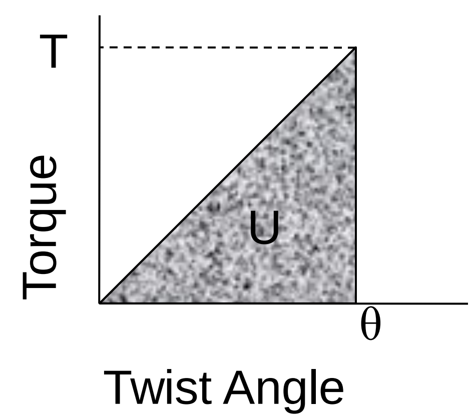
- And the maximum stress $\tau\hat{} = 16T/\pi D^3$, at $r = D/2$
- For a hollow shaft: $J = (\pi/32)(D4 – d4)$ and $\tau\hat{} = 16D.T/\pi(D4 – d4)$, at $r = D/2$
- Torsional stiffness k is defined as torque per radian twist, i.e

  $k = T/\boldsymbol{\theta} = GJ/l$

2.7 Strain Energy in Torsion

- Total strain energy of a shaft of length l under the action of a torque T is the work done in twisting, i.e

- U = 1/2Tθ (most useful if T and θ have been previously found)



- **Strain Energy in Torsion continued**

- Expressed in terms of maximum stress $\hat{\tau}$ for a solid shaft

$$U = \tfrac{1}{2}(\pi D^3 \hat{\tau}/16) \times (2\hat{\tau} l/GD)$$

$$= (\hat{\tau}^2/4G) \times \pi D^2 l/4$$

$$= (\hat{\tau}^2/4G) \times \text{volume}$$

- Note that this gives the total strain energy over the whole shaft, for which shear stress is varying from zero at the axis to $\hat{\tau}$ at the outside.

- The maximum strain energy per unit volume is $\hat{\tau}^2/2G$

SUMMARY

Torsion of Circular Shafts $T/J = \tau/r = G\theta/l$.
Maximum shear stress $\hat{\tau} = 16T/\pi D^3$, for solid shafts.
Strain Energy $U = \frac{1}{2}T\theta$

$$= T^2 l/2GJ$$

$$= (\hat{\tau}^2/4G) \times \text{volume}, \quad \text{for solid shafts.}$$

Stiffness $k = T/\theta = GJ/l$.

References

- G.H Ryder, Strength of Materials,