EE392 - ELECTRICAL ENGINEERING PRACTICE

MECHANICAL COMPONENT

Static Mechanics:

Shearing Force and Bending Moments in Beams

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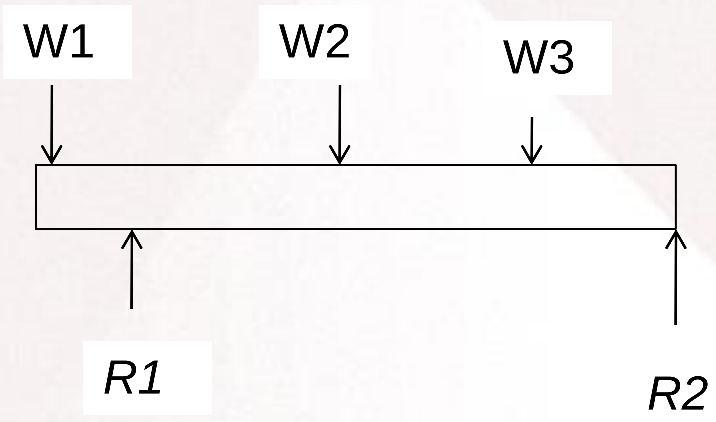
Shearing Force in beams

4.1 : Shearing Force

- The shearing force at any section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

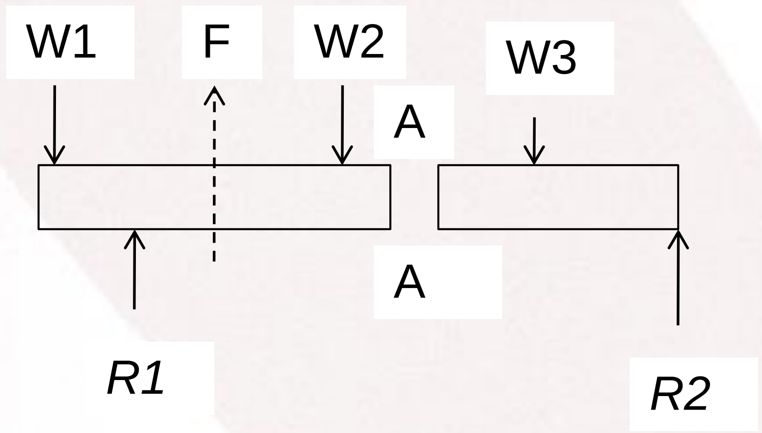


A beam carrying loads W1, W2, and W3 is simply supported at two points.

The reactions at the supports being R1 and R2.

Shearing force continued…

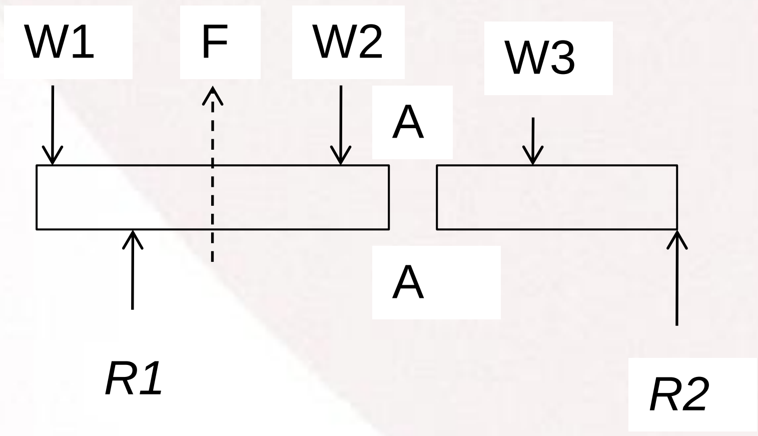
- Now imagine the beam to be divided into two portions by a section at AA.



- The resultant of the loads and reactions to the left of AA is F vertically upwards, and since the whole beam is in equilibrium, the resultant of the forces to the right of AA must also be F, acting downwards.

Shearing force continued..

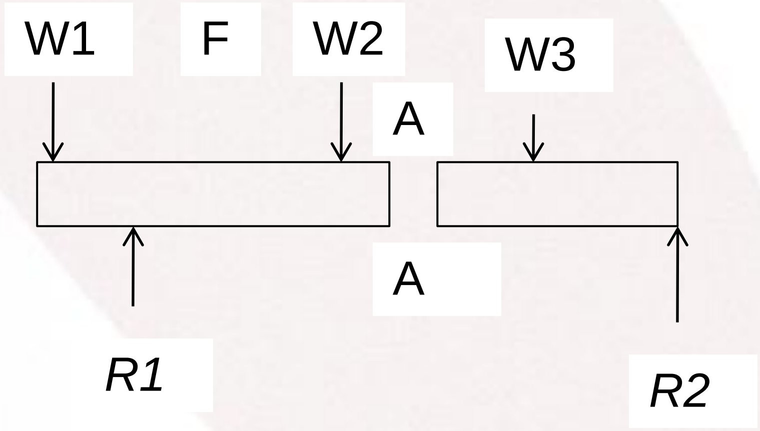
- F is called the Shearing Force (abbrev. S.F.) at the section AA and maybe defined as follows: **the shearing force at any section of a beam is the algebraic sum of the lateral components of the forces acting on either side of the section.**



- **Shearing force continued …**
- Where a force is in neither the axial nor the lateral direction it must be resolved in the usual way, the lateral component being taken into account in the shearing.
- Shearing force will be considered positive when the resultant of the forces to the left is upwards, or to the right is downwards.
- **Shearing force diagram** is one which shows the variation of shearing force along the length of the beam.

4.2 Bending Moment

- In a similar manner it can be argued that the moment about the section AA of the forces to the left is ***M*** clockwise, then the moment of the forces to the right of AA must be ***M*** anticlockwise.



- ***M*** is called the **Bending Moment** (abbr. B.M.) at AA, and is defined as: the algebraic sum of the moments about the section of all the forces acting on either side of the section.

- **Bending Moment continued …**
- Bending moment will be considered positive when the moment on the left portion is clockwise, and on the right portion anticlockwise.
- This is referred to as sagging bending moment since it tends to make the beam concave upwards at AA.
- Negative bending moment is termed hogging.
- A **bending moment diagram** is one which shows the variation of bending moment along the length of the beam.

4.3 Types of Load

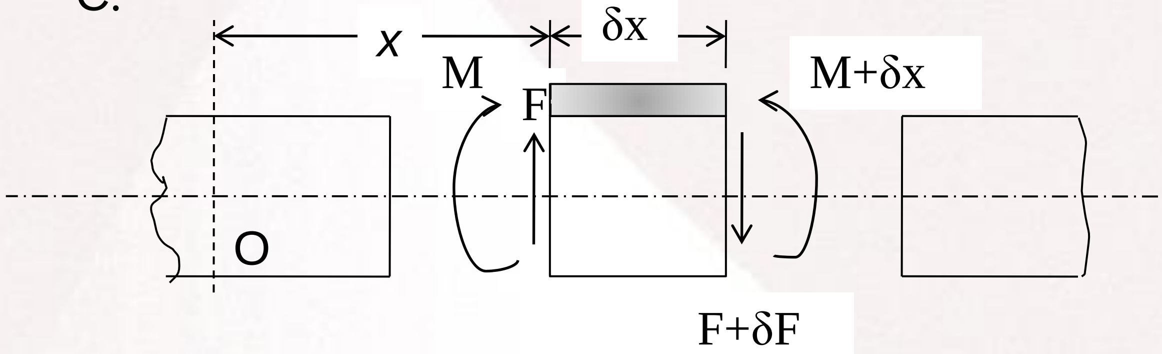
- A **beam** is normally horizontal, the loads being vertical, other cases which occur being looked upon as exceptions
- A **concentrated load** is one which is considered to act at a point, although in practice it must really be distributed over a small area.
- A **distributed load** is one which is spread in some manner over the length of the beam.
- The **rate of loading w** maybe uniform, or may vary from point to point along the beam.

4.4 Types of Support

- A **simple or free support** is one on which the beam is rested, and which exerts a reaction on the beam.
- A **built-in or encastre support** is frequently met with, **the effect being to fix the direction of the beam at the support**.
- In order to do this the support must exert a ‘fixing” moment M and a reaction R on the beam.
- A beam thus fixed at one end is called a cantilever

4.5 Relations between w, F, and M

- Consider a short length δx imagined to be a slice cut out from a loaded beam at a distance x from a fixed origin O
- Let the shearing force at the section x be F, and at $x + \delta x$ be $F + \delta F$.
- Similarly the bending moment is M at x, and $M + \delta M$ at $x + \delta x$.
- If w is the mean rate of loading on the length δx, the total load is $w\delta x$, acting approximately (exactly, if uniformly distributed) through the center C.



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Relations between w, F, and M Continued …

- The elements must be in equilibrium under the action of these forces and couples, and the following equations are obtained.
- Taking moments about C:
- $M + F.\delta x/2 + (F + \delta F)\delta x/2 = M + \delta M$
- Neglecting the product $\delta F.\delta x$, and taking the limit, gives
- $F = dM/dx$ ---------------------(1)

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- **Relations between w, F, and M Continued …**
- Resolving vertically

$$w\delta x + F + \delta F = F$$

$$w = -\,dF/dx \text{ --------------------- } (2)$$

$$= -\,d^2M/dx^2 \text{ from (1) ----------- } (3)$$

- **Relations between w, F, and M Continued …**
- From equation (1) it can be seen that, if M is varying continuously, zero shearing force corresponds to maximum or minimum bending moment.
- $F = dM/dx$ ---------------------(1)
- It will be seen later however that “peaks” in the bending moment diagram frequently occur at concentrated loads or reactions, and are not then given by $F = dM/dx = 0$, although they may represent the greatest bending moment on the beam.
- Consequently it is not always sufficient to instigate the points of zero shearing force when determine the maximum bending moment.
- At a point on the beam where the type of bending is changing from sagging to hogging, the bending moment must be zero, and this is called a point of inflection or contraflecture.

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- **Relations between w, F, and M Continued …**
- By integrating equation (1) between two values of x = a and x = b, then $M_b - M_a = \int F dx$
- Showing that the increase in bending moment between two sections is given by the area under the shearing force diagram

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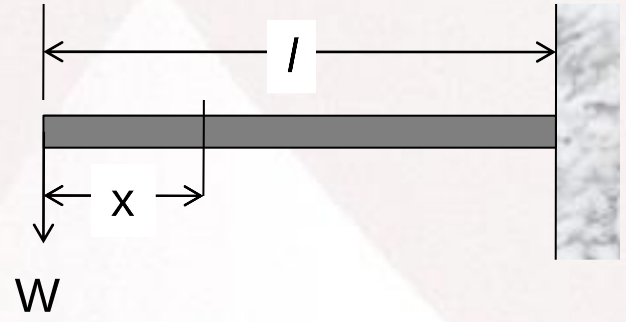
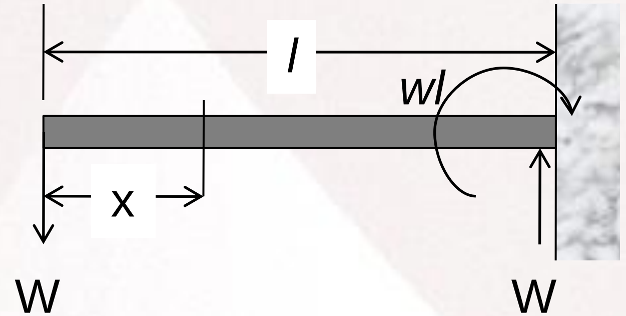
- **Relations between w, F, and M Continued …**
- Similarly, integrating equation (2)
- $F_a – F_b = \int w\,dx$
- = the area under the load distribution diagram.
- Integrating equation (3) gives
- $M_a – M_b = \iint w\,dx.dx$
- These relations prove very valuable when the rate of loading cannot be expressed in algebraic form, and provide a means of graphical solution.

Examples

- **4.6 Concentrated Loads**
- **Example 1.** A cantilever of length l carries a concentrated load W at its free end. Draw the S.F. and B.M. diagrams



- **Example 1 continued …**
- **Solution:**
- At a section a distance x from the free end, consider the forces to the left.
- Then F = -W and is constant along the whole beam (i.e for all x).



- Taking moments about the section gives M = -Wx, so that the maximum bending moment occurs at the fixed end., i.e. Mmax = Wl (hogging)
- From equilibrium considerations, the fixing moment applied at the built in end is Wl, and the reaction is W.

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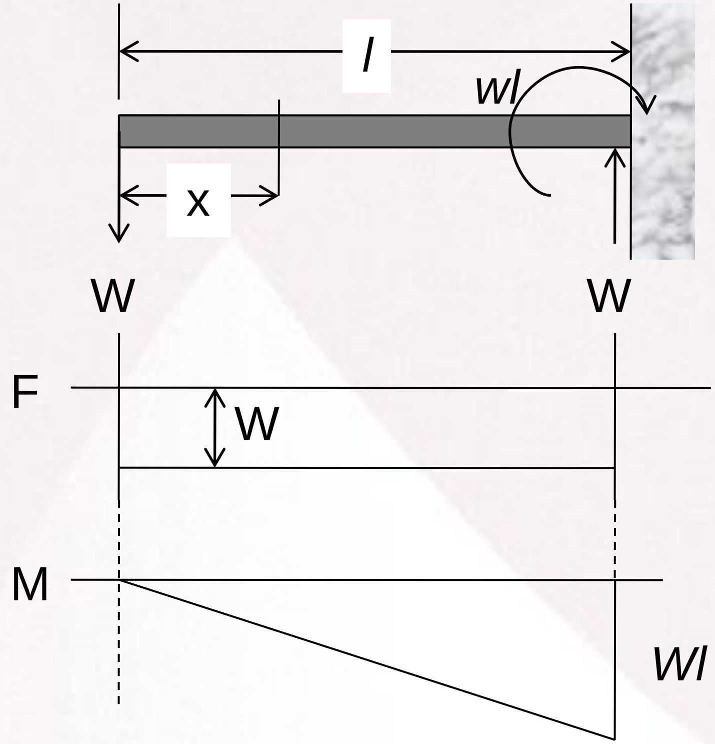
Concentrated load continued

- The S.F. and B.M. diagrams are therefore as shown:



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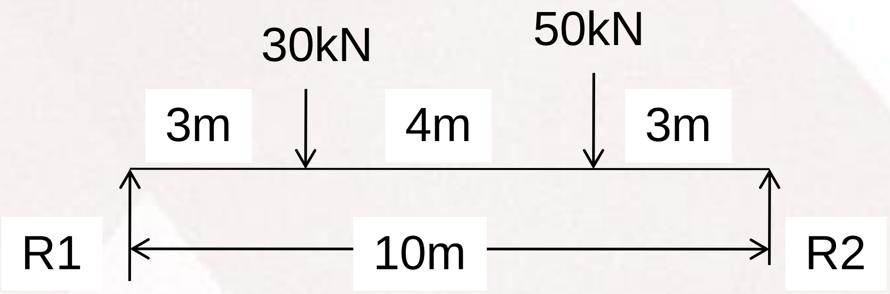
- **Example 2.** A beam 10m long is simply supported at its ends and carries concentrated loads of 30kN and 50kN at distances of 3 m from each end. Draw the S.F. and B.M. diagrams.



- **SOLUTION**
- First calculate the reactions R1 and R2 at the supports

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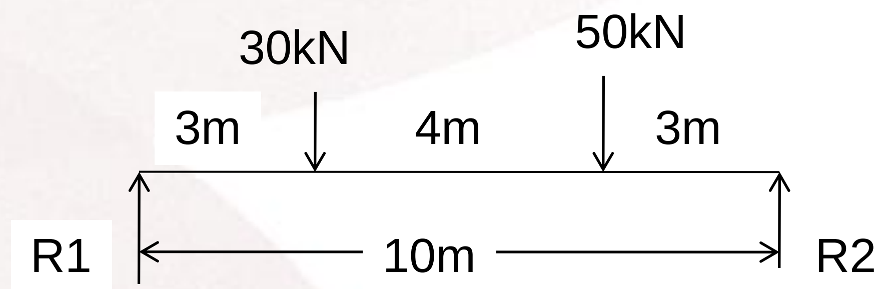

- By moments about R2:
- R1 x 10 = 30x7+50x3
- Therefore R1 = 36kN
- and R2 = 30+50-R1=44kN
- Let x be the distance of the section from the left – hand end
- **Shearing force:**

  0 < x < 3, **F** = **R1** = 36kN

  3 < x < 7, **F** = **R1**-30=6kN

  7 < x < 10, **F** = **R1** – 30 – 50 = -44kN
- Note that the last value = -R2, which provides a check on the working

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- **Bending Moment:**

  $0 < x < 3$, $M = R1x = 36x$ kNm

  $3 < x < 7$, $M = R1x – 30(x-3) = 6x + 90$ kNm

  $7 < x < 10$, $M = R1x – 30(x-3) – 50(x – 7)$

  $= -44x + 440$ kNm