

14.5 BALANCING OF SEVERAL MASSES IN DIFFERENT PLANES

Let there be a rotor revolving with a uniform angular velocity ω [Fig. 14.6(a)]. m_1 , m_2 and m_3 are the masses attached to the rotor at radii r_1 , r_2 and r_3 respectively. The masses m_1 , m_2 and m_3 rotate in planes 1, 2 and 3 respectively. Choose a reference plane at O so that the distances of the planes 1, 2 and 3 from O are l_1 , l_2 and l_3 respectively.

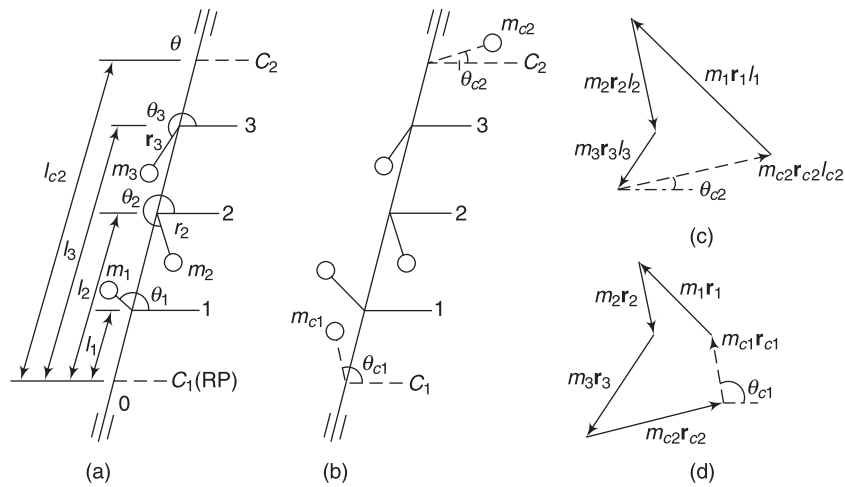


Fig. 14.6

Transference of each unbalanced force to the reference plane introduces the like number of forces and couples.

The unbalanced forces in the reference plane are $m_1 r_1 \omega^2$, $m_2 r_2 \omega^2$, and $m_3 r_3 \omega^2$ acting radially outwards.

The unbalanced couples in the reference plane are $m_1 r_1 \omega^2 l_1$, $m_2 r_2 \omega^2 l_2$ and $m_3 r_3 \omega^2 l_3$ which may be represented by vectors parallel to the respective force vectors, i.e. parallel to the respective radii of m_1 , m_2 and m_3 .

For complete balancing of the rotor, the resultant force and the resultant couple both should be zero.

$$\text{i.e.} \quad m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 = 0 \quad (14.5)$$

$$\text{and} \quad m_1 r_1 l_1 \omega^2 + m_2 r_2 l_2 \omega^2 + m_3 r_3 l_3 \omega^2 = 0 \quad (14.6)$$

If the Eqs. (14.5) and (14.6) are not satisfied, then there are unbalanced forces and couples. A mass placed in the reference plane may satisfy the force equation but the couple equation is satisfied only by two equal forces in different transverse planes. Thus, in general, two planes are needed to balance a system of rotating masses.

Therefore, in order to satisfy Eqs. (14.5) and (14.6), introduce two counter-masses m_{c1} and m_{c2} at radii r_{c1} and r_{c2} respectively. Then Eq. (14.5) may be written as,

$$m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 + m_{c1} r_{c1} \omega^2 + m_{c2} r_{c2} \omega^2 = 0 \quad (14.7)$$

$$\text{or } m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0 \quad (14.7a)$$

$$\text{In general } \Sigma m\mathbf{r} + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0 \quad (14.8)$$

Let the two countermass be placed in transverse planes at axial locations O and Q , i.e. the countermass m_{c1} be placed in the reference plane and the distance of the plane of m_{c2} be l_{c2} from the reference plane.

Equation (14.6) modifies to (taking moments about O)

$$m_1\mathbf{r}_1 l_1 \omega^2 + m_2\mathbf{r}_2 l_2 \omega^2 + m_3\mathbf{r}_3 l_3 \omega^2 + m_{c2}\mathbf{r}_{c2} l_{c2} \omega^2 = 0 \quad (14.9)$$

$$\text{or } m_1\mathbf{r}_1 l_1 + m_2\mathbf{r}_2 l_2 + m_3\mathbf{r}_3 l_3 + m_{c2}\mathbf{r}_{c2} l_{c2} = 0 \quad (14.9a)$$

$$\text{In general, } \Sigma m\mathbf{r}l + m_{c2}\mathbf{r}_{c2}l_{c2} = 0 \quad (14.10)$$

Thus, Eqs. (14.8) and (14.10) are the necessary conditions for dynamic balancing of the rotor. Again the equations can be solved mathematically or graphically.

Dividing Eq. (14.10) into component form

$$\Sigma mrl \cos \theta + m_{c2}r_{c2}l_{c2} \cos \theta_{c2} = 0$$

$$\text{and } \Sigma mrl \sin \theta + m_{c2}r_{c2}l_{c2} \sin \theta_{c2} = 0$$

$$\text{or } m_{c2}r_{c2}l_{c2} \cos \theta_{c2} = -\Sigma mrl \cos \theta \quad (i)$$

$$\text{and } m_{c2}r_{c2}l_{c2} \sin \theta_{c2} = -\Sigma mrl \sin \theta \quad (ii)$$

Squaring and adding (i) and (ii)

$$m_{c2}r_{c2}l_{c2} = \sqrt{(\Sigma mrl \cos \theta)^2 + (\Sigma mrl \sin \theta)^2} \quad (14.11)$$

Dividing (ii) by (i),

$$\tan \theta_{c2} = \frac{-\Sigma mrl \sin \theta}{-\Sigma mrl \cos \theta} \quad (14.12)$$

After obtaining the values of m_{c2} and θ_{c2} from the above equations, solve Eq. (14.8) by taking its components,

$$m_{c1}r_{c1} = \sqrt{(\Sigma mr \cos \theta + m_{c2}r_{c2} \cos \theta_{c2})^2 + (\Sigma mr \sin \theta + m_{c2}r_{c2} \sin \theta_{c2})^2} \quad (14.13)$$

and

$$\tan \theta_{c1} = \frac{-(\Sigma mr \sin \theta + m_{c2}r_{c2} \sin \theta_{c2})}{-(\Sigma mr \cos \theta + m_{c2}r_{c2} \cos \theta_{c2})} \quad (14.14)$$

To solve Eqs (14.8) and (14.10) graphically, Eq. (14.10) is solved first and a couple polygon is made by adding the known vectors and considering each vector parallel to the radial line of the mass. Then the closing vector will be $m_{c2}\mathbf{r}_{c2}l_{c2}$, the direction of which specifies the angular position of the counter mass m_{c2} [Fig. 14.6(c)] in the plane at point Q . Then solve Eq. (14.8) and make a force polygon by adding the known vectors (along with the vector $m_{c2}\mathbf{r}_{c2}$). The closing vector is $m_{c1}\mathbf{r}_{c1}$, identifying the magnitude and the direction of the counter mass m_{c1} [Fig. 14.6(d)]. Figure 14.6(b) represents the position of the balancing masses on the rotating shaft.

Example 14.3 A rotating shaft carries three unbalanced masses 4 kg, 3 kg and 2.5 kg at radial distances 75 mm, 85 mm and 50 mm and at the angular positions of 45° , 135° and 240° respectively. The second and the third masses are in the planes at 200 mm and 375 mm from the plane of the first mass. The angular positions are measured counter-clockwise from the reference line along x -axis and viewing the shaft from first mass end.

The shaft length is 800 mm between bearings and the distance between the plane of the first mass and the bearing at that end is 225 mm. Determine the amount of the counterweights in planes at 75 mm from the bearings for the complete balance of the shaft. The first counterweight is to be in a plane between the first mass and the bearing and the second counterweight in a plane between the third mass and the bearing at that end.

Solution

Figure 14.7(a) shows the planes of unbalanced masses as well as the planes of the counterweights. Plane C_1 is to be taken as the reference plane and the various distances are to be considered from this plane.

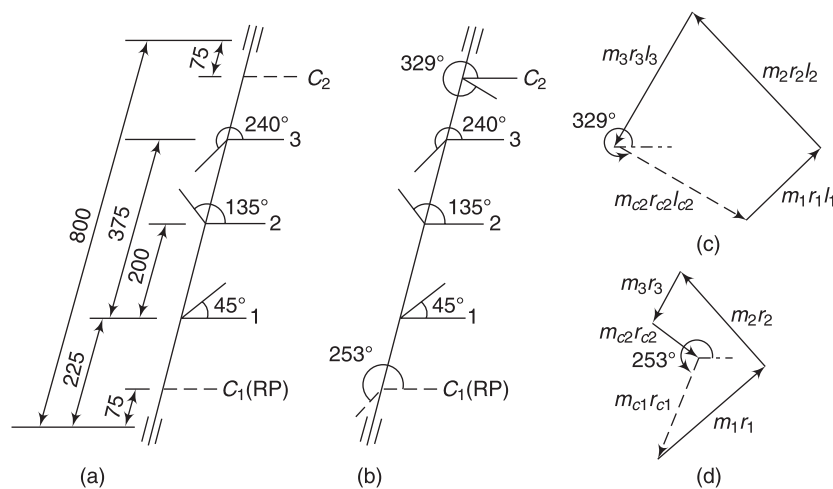


Fig. 14.7

Analytical solution

$l_{c2} = (800 - 75 \times 2) = 650$ mm, $l_1 = 225 - 75 = 150$ mm, $l_2 = 150 + 200 = 350$ mm,
 $l_3 = 150 + 375 = 525$ mm

$$\begin{aligned} m_1 r_1 l_1 &= 4 \times 75 \times 150 = 45\,000 & m_1 r_1 &= 4 \times 75 = 300 \\ m_2 r_2 l_2 &= 3 \times 85 \times 350 = 89\,250 & m_2 r_2 &= 3 \times 85 = 255 \\ m_3 r_3 l_3 &= 2.5 \times 50 \times 525 = 65\,625 & m_3 r_3 &= 2.5 \times 50 = 125 \end{aligned}$$

$$\Sigma m r l + m_{c2} r_{c2} l_{c2} = 0$$

or $45\,000 \cos 45^\circ + 89\,250 \cos 135^\circ + 65\,625 \cos 240^\circ + m_{c2} r_{c2} l_{c2} \cos \theta_{c2} = 0$

and $45\,000 \sin 45^\circ + 89\,250 \sin 135^\circ + 65\,625 \sin 240^\circ + m_{c2} r_{c2} l_{c2} \sin \theta_{c2} = 0$

On solving,

$$\begin{aligned} m_{c2} r_{c2} l_{c2} &= \left[(45\,000 \cos 45^\circ + 89\,250 \cos 135^\circ + 65\,625 \cos 240^\circ)^2 \right. \\ &\quad \left. + (45\,000 \sin 45^\circ + 89\,250 \sin 135^\circ + 65\,625 \sin 240^\circ)^2 \right]^{1/2} \\ &= [(-64\,102)^2 + (38\,096)^2]^{1/2} \end{aligned}$$

$$\text{or } m_{c2} \times 40 \times 650 = 74\,568, \quad m_{c2} = 2.868 \text{ kg}$$

$$\tan \theta_{c2} = \frac{-38\,096}{-(-64\,102)} = -0.594, \quad \theta_{c2} = 329.3^\circ \text{ or } 329^\circ 18'$$

$$\text{Now, } \Sigma m\mathbf{r} + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0$$

$$\text{or } 300 \cos 45^\circ + 255 \cos 135^\circ + 125 \cos 240^\circ + m_{c1}r_{c1} \cos \theta_1 + 2.868 \times 40 \cos 329.3^\circ = 0$$

$$\text{and } 300 \sin 45^\circ + 255 \sin 135^\circ + 125 \sin 240^\circ + m_{c1}r_{c1} \sin \theta_1 + 2.868 \times 40 \sin 329.3^\circ = 0$$

On solving,

$$m_{c1}r_{c1} = \left[\begin{array}{l} (300 \cos 45^\circ + 255 \cos 135^\circ + 125 \cos 240^\circ + 2.868 \\ \times 40 \cos 329.3^\circ)^2 + (300 \sin 45^\circ + 255 \sin 135^\circ \\ + 125 \sin 240^\circ + 2.868 \times 40 \sin 329.3^\circ)^2 \end{array} \right]^{1/2}$$

$$m_{c1} \times 75 = [(67.96)^2 + (225.62)^2]^{1/2} = 235.63, \quad m_{c1} = 3.14 \text{ kg}$$

$$\tan \theta_{c1} = \frac{-225.62}{-67.96} = 3.32; \quad \theta_{c1} = 253.2^\circ \text{ or } 253^\circ 12'$$

Graphical solution

The graphical solution has also been shown in Figs 14.7(c) and (d). From Fig. 14.7(c),

$$m_{c2}r_{c2}l_{c2} = 74\,000 \quad \therefore m_{c2} = \frac{74\,000}{40 \times 650} = 2.846 \text{ kg at } 329^\circ$$

From Fig. 14.7(d),

$$m_{c1}r_{c1} = 235, \quad \therefore m_{c1} = \frac{235}{75} = 3.13 \text{ kg at } 253^\circ$$

Figure 14.7(b) represents the position of the balancing masses on the rotating shaft.

Solution by using complex numbers

$$m_1r_1l_1 \angle \theta_1 = (4 \times 75 \times 150) \angle 45^\circ = 45\,000 \angle 45^\circ \\ = 31\,820 + j\,31\,820$$

$$m_2r_2l_2 \angle \theta_2 = (3 \times 85 \times 350) \angle 135^\circ = 89\,250 \angle 135^\circ \\ = -63\,109 + j\,63\,109$$

$$m_3r_3l_3 \angle \theta_3 = (2.5 \times 50 \times 525) \angle 240^\circ = -65\,625 \angle 240^\circ \\ = -32\,813 - j\,56\,833$$

Now,

$$m_1r_1l_1 \angle \theta_1 + m_2r_2l_2 \angle \theta_2 + m_3r_3l_3 \angle \theta_3 + m_{c2}r_{c2}l_{c2} \angle \theta_{c2} = 0 \\ (31\,820 + j\,31\,820) + (-63\,109 + j\,63\,109) \\ + (-32\,813 - j\,56\,833) + m_{c2}r_{c2}l_{c2} \angle \theta_{c2} = 0$$

$$m_{c2}r_{c2}l_{c2} \angle \theta_{c2} = 64\,102 - j\,38\,096 = 74\,568 \angle 329.3^\circ$$

$$m_{c2} \times 40 \times 650 = 74\,568, \quad m_{c2} = 2.868 \text{ kg}$$

Similarly,

$$m_1r_1 \angle \theta_1 = (4 \times 75) \angle 45^\circ = 300 \angle 45^\circ = 212.1 + j\,212.1$$

$$m_2r_2 \angle \theta_2 = (3 \times 85) \angle 135^\circ = 255 \angle 135^\circ = -180.3 + j\,180.3$$

$$m_3r_3 \angle \theta_3 = (2.5 \times 50) \angle 240^\circ = 125 \angle 240^\circ = -62.5 - j\,108.3$$

$$m_{c2}r_{c2} \angle \theta_{c2} = (2.868 \times 40) \angle 329.3^\circ = 114.72 \angle 329.3^\circ = 98.6 - j\,58.6$$

Now,

$$m_1r_1 \angle \theta_1 + m_2r_2 \angle \theta_2 + m_3r_3 \angle \theta_3 + m_{c2}r_{c2} \angle \theta_{c2} + m_{c1}r_{c1} \angle \theta_{c1} = 0 \\ (212.1 + j\,212.1) + (-180.3 + j\,180.3) + (-62.5 - j\,108.3) \\ + (98.6 - j\,58.6) + m_{c1}r_{c1} \angle \theta_{c1} = 0$$

$$m_{c1}r_{c1}\angle\theta_{c1} = -67.9 - j 225.5$$

$$= 235.5 \angle 253.2^\circ$$

or $m_{c1} \times 75 = 235.63, \quad m_{c1} = 3.14 \text{ kg}$

Example 14.4 A shaft supported in bearings 1.6 m apart projects 400 mm beyond bearings at each end. It carries three pulleys one at each end and one at the centre of its length. The masses of the end pulleys are 40 kg and 22 kg and their centres of mass are at 12 mm and 18 mm respectively from the shaft axes. The mass of the centre pulley is 38 kg and its centre of mass is 15 mm from the shaft axis. The pulleys are arranged in a manner that they give static balance. Determine

- the relative angular positions of the pulleys
- the dynamic forces developed on the bearings when the shaft rotates at 210 rpm.

Solution

Figure 14.8(a) shows the planes of the three pulleys as well as of the two bearings. Let plane of pulley 1 be the reference plane.

$$m_1r_1 = 40 \times 0.012 = 0.48$$

$$m_2r_2l_2 = 38 \times 0.015 \times 1.2 = 0.684 \quad m_2r_2 = 38 \times 0.015 = 0.57$$

$$m_3r_3l_3 = 22 \times 0.018 \times 2.4 = 0.95 \quad m_3r_3 = 22 \times 0.018 = 0.396$$

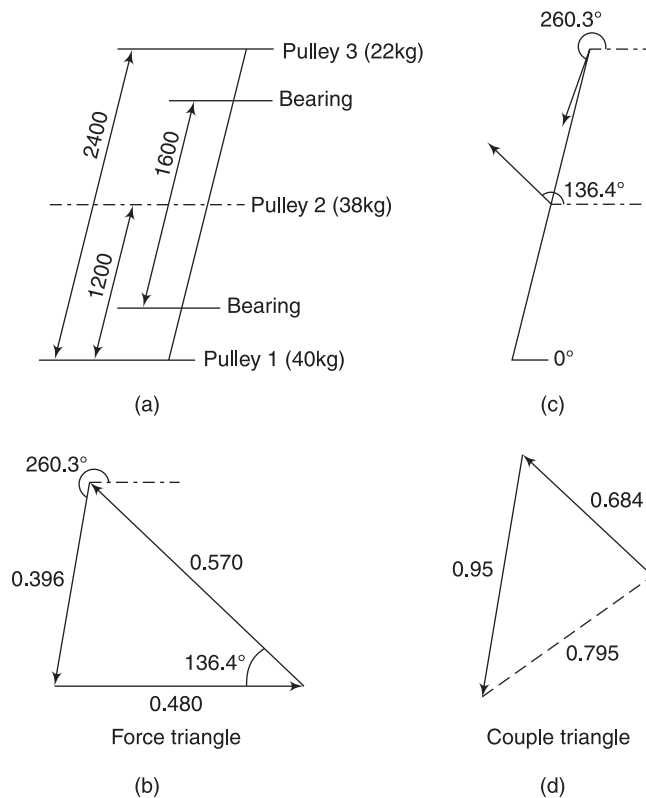


Fig. 14.8

Complete the force triangle as the three sides are known [Fig. 14.8(b)]. The mass at plane 1 is chosen at 0° angle. By completing it, the directions of the other two masses are known which have been marked in Fig. 14.8(c).

Now, as the shaft is in complete static balance, there is only unbalanced couple which is to be the same about all planes. Thus, reactions due to the unbalanced couple are to be equal and opposite on the two bearings.

To find the magnitude of the unbalanced couple, add the two couple vectors as shown in Fig. 14.8(d). The closing side shown in dotted line represents the magnitude of the unbalanced couple.

The magnitude, $mrl = 0.795$ on measurement.

$$\therefore \text{Unbalanced couple} = mr\omega^2 \cdot l = 0.795 \times \left(\frac{2\pi \times 210}{60} \right)^2 = 384.5 \text{ Nm}$$

$$\text{The reaction on each bearing} = \frac{384.5}{1.6} = 240.3 \text{ N}$$

Example 14.5 Four masses A , B , C and D are completely balanced. Masses C and D make angles of 90° and 195° respectively with that of mass B in the counter-clockwise direction. The rotating masses have following properties:

$m_b = 25 \text{ kg}$, $m_c = 40 \text{ kg}$, $m_d = 35 \text{ kg}$, $r_a = 150 \text{ mm}$, $r_b = 200 \text{ mm}$, $r_c = 100 \text{ mm}$, $r_d = 180 \text{ mm}$

Planes B and C are 250 mm apart. Determine

- (i) the mass A and its angular position with that of mass B
- (ii) the positions of all the planes relative to plane of mass A .

Solution

Refer Fig.14.9(a).

$$m_b r_b = 25 \times 100 = 5000$$

$$m_c r_c = 40 \times 100 = 4000$$

$$m_d r_d = 35 \times 180 = 6300$$

For complete balance, taking $\theta_b = 0^\circ$

$$\Sigma mr \cos \theta = 0 \quad \text{and} \quad \Sigma mr \sin \theta = 0$$

$$\text{i.e.,} \quad m_a \times 150 \times \cos \theta_a + 5000 \cos 0^\circ + 4000 \cos 90^\circ + 6300 \cos 195^\circ = 0$$

$$\text{or} \quad m_a \times 150 \times \cos \theta_a + 5000 + 0 - 6085 = 0$$

$$\text{or} \quad 150 m_a \cos \theta_a = 1085 \tag{i}$$

$$\text{and} \quad m_a \times 150 \times \sin \theta_a + 5000 \sin 0^\circ + 4000 \sin 90^\circ + 6300 \sin 195^\circ = 0$$

$$\text{or} \quad m_a \times 150 \times \sin \theta_a + 0 + 4000 - 1631 = 0$$

$$\text{or} \quad 150 m_a \sin \theta_a = -2369 \tag{ii}$$

Squaring and adding (i) and (ii),

$$22\,500 m_a^2 = (1085)^2 + (-2369)^2$$

$$\text{or} \quad m_a^2 = 30\,175 \quad \text{or} \quad m_a = 17.37 \text{ kg}$$

Dividing (ii) by (i),

$$\tan \theta_a = \frac{-236.9}{108.5} = -2.184$$

$$\text{or} \quad \theta_a = 294.6^\circ \quad \text{or} \quad 294^\circ 36'$$

For complete balance, the couple equations are,

$$\Sigma mrl \cos \theta = 0 \quad \text{and} \quad \Sigma mrl \sin \theta = 0$$

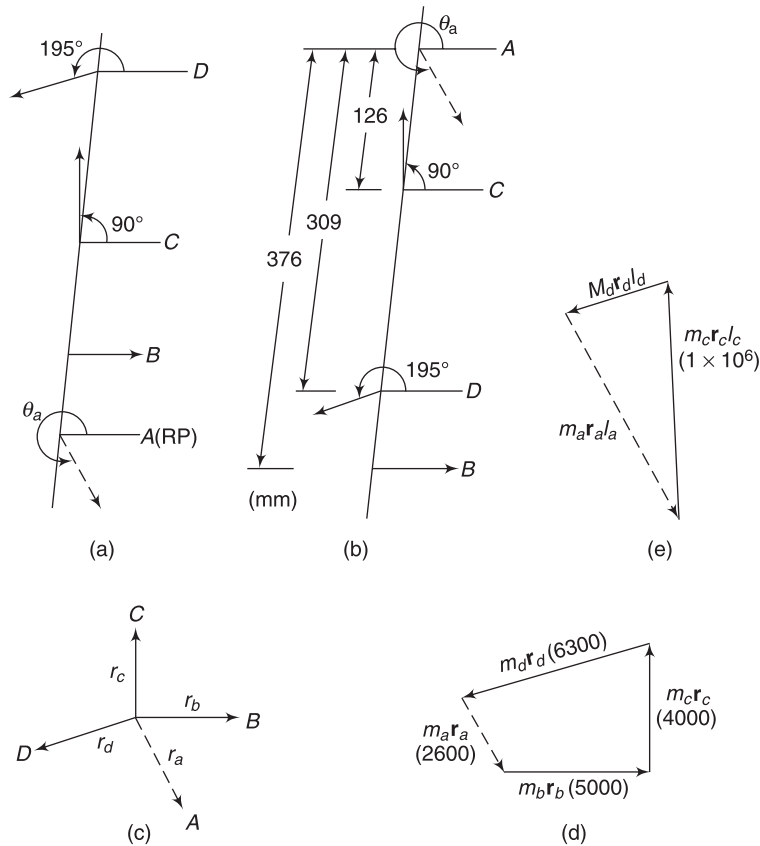


Fig. 14.9

Taking A as the reference plane,

$$5000 l_b \cos 0^\circ + 4000 l_c \cos 90^\circ + 6300 l_d \cos 195^\circ = 0$$

$$\text{or } 5000 l_b = 6085 l_d \text{ or } l_b = 1.217 l_d$$

$$\text{and } 5000 l_b \sin 0^\circ + 4000 l_c \sin 90^\circ + 6300 l_d \sin 195^\circ = 0$$

$$\text{or } 4000 l_c = 1631 l_d \text{ or } l_c = 0.4078 l_d$$

$$\text{or } l_b + 250 = 0.4078 l_d$$

$$\text{or } 1.217 l_d + 250 = 0.4078 l_d \text{ or } 0.8092 l_d = -250$$

$$\text{or } l_d = -309 \text{ mm}$$

$$l_b = 1.217 l_d = 1.217 \times (-309) = -376 \text{ mm}$$

$$l_c = l_b + 250 = -376 + 250 = -126 \text{ mm}$$

The correct positions of the planes have been shown in Figs. 14.9(b) and (c).

To solve the problem graphically, $m_a r_a$ is obtained from the vector sum of $m_b r_b$, $m_c r_c$ and $m_d r_d$ [Fig. 14.9(c)]. On measuring,

$$m_a r_a = 2600, \therefore m_a \frac{2600}{150} = 17.3 \text{ kg} \text{ and } \theta_a = 294.5^\circ$$

Now, $m_a \mathbf{r}_a l_a = 4000 \times 250 = 1 \times 10^6$, taking B as the reference plane. Take the vector $m_c \mathbf{r}_c l_c$ and from its two ends, draw lines parallel to $m_a \mathbf{r}_a$ and $m_d \mathbf{r}_d$, thus forming a triangle. Measuring the two sides

$$m_a \mathbf{r}_a l_a = 985\,000, l_a = \frac{985\,000}{17.3 \times 150} = 379 \text{ mm}$$

$$m_d \mathbf{r}_d l_d = 437\,000, l_d = \frac{437\,000}{6300} = 69 \text{ mm}$$

l_a and l_d establish the relative positions of the planes.

14.6 FORCE BALANCING OF LINKAGES

Balancing of a linkage implies that the total centre of its mass remains stationary so that the vector sum of all the frame forces always remains zero. Figure 14.10 shows a four-link mechanism. a , b , c and d represent the magnitudes of the links AB , BC , CD and DA respectively. The link masses are m_a , m_b and m_c , located at G_1 , G_2 and G_3 respectively. Let the coordinates g_i , ϕ_i describe the position of these points within each link.

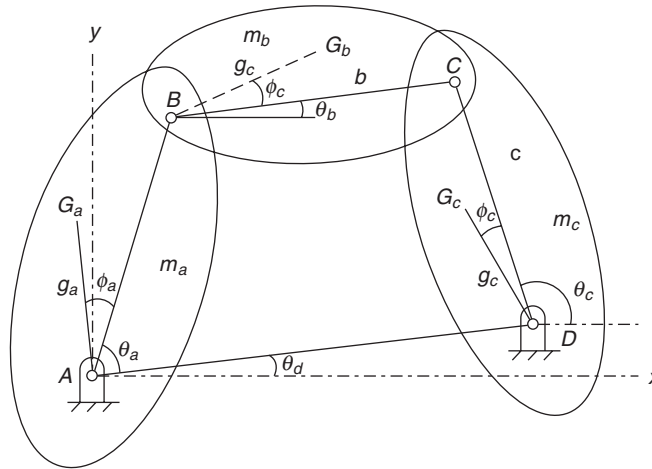


Fig. 14.10

As in any configuration of the mechanism, the figure must enclose, the links of the mechanism can be considered as vectors. Thus, vector displacement relationship can be written as

$$ae^{i\theta_a} + be^{i\theta_b} - ce^{i\theta_c} - de^{i\theta_d} = 0 \quad (14.15)$$

or
$$e^{i\theta_b} = \frac{1}{b} (de^{i\theta_d} - ae^{i\theta_a} + ce^{i\theta_c}) \quad (14.15a)$$

Let the vector \mathbf{g} represent the position of total centre of mass and m the total mass of the mechanism,

$$m = m_a + m_b + m_c \quad (14.16)$$

Then

$$m\mathbf{g} = m_a\mathbf{g}_a + m_b\mathbf{g}_b + m_c\mathbf{g}_c \quad (14.17)$$

where \mathbf{g}_a , \mathbf{g}_b and \mathbf{g}_c are the vectors representing the positions of masses m_a , m_b and m_c respectively.

$$\begin{aligned} m\mathbf{g} &= m_a g_a e^{i(\theta_a + \varphi_a)} + m_b [a e^{i\theta_a} + g_b e^{i(\theta_b + \varphi_b)}] + m_c [d e^{i\theta_d} + g_c e^{i(\theta_c + \varphi_c)}] \\ &= m_a g_a e^{i\theta_a} e^{i\varphi_a} + m_b a e^{i\theta_a} + m_b g_b e^{i\theta_b} e^{i\varphi_b} + m_c d e^{i\theta_d} + m_c g_c e^{i\theta_c} e^{i\varphi_c} \end{aligned}$$

Inserting the value of $e^{i\theta_b}$ from (14.15a)

$$\begin{aligned} m\mathbf{g} &= m_a g_a e^{i\theta_a} e^{i\varphi_a} + m_b a e^{i\theta_a} + m_b g_b \frac{1}{b} (d e^{i\theta_d} - a e^{i\theta_a} + c e^{i\theta_c}) e^{i\varphi_b} \\ &\quad + m_c d e^{i\theta_d} + m_c g_c e^{i\theta_c} e^{i\varphi_c} \\ &= (m_a g_a e^{i\varphi_a} + m_b a - m_b g_b \frac{a}{b} e^{i\varphi_b}) e^{i\theta_a} + (m_c g_c e^{i\varphi_c} + m_b g_b \frac{c}{b} e^{i\varphi_b}) e^{i\theta_d} \\ &\quad + (m_c d + m_b g_b \frac{d}{b} e^{i\varphi_b}) e^{i\theta_d} \end{aligned}$$

The centre of mass can be made stationary at the position

$$\mathbf{g} = (m_c d + m_b g_b \frac{d}{b} e^{i\varphi_b}) e^{i\theta_d}$$

if the remaining two terms in the brackets can be made zero. Let the vector \mathbf{g}'_a represent the position of the counter mass m_a to be added to the input link and vector \mathbf{g}'_c represent the position of the counter mass m_c to be added to the output link to have complete force balancing.

Thus, the equations may be written as

$$m_a g_a e^{i\varphi_a} + m_b a - m_b g_b \frac{a}{b} e^{i\varphi_b} + m'_c g'_c e^{i\varphi'_c} = 0 \quad (14.18)$$

$$\text{and} \quad m_c g_c e^{i\varphi_c} + m_b g_b \frac{c}{b} e^{i\varphi_b} + m'_a g'_a e^{i\varphi'_a} = 0 \quad (14.19)$$

from which magnitudes and the locations of the counter masses can be obtained.

Example 14.6 The following data relate to a four-link mechanism:

$a = 55 \text{ mm}$	$m_a = 0.045 \text{ kg}$	$g_a = 28 \text{ mm}$	$\varphi_a = 0^\circ$
$b = 165 \text{ mm}$	$m_b = 0.13 \text{ kg}$	$g_b = 85 \text{ mm}$	$\varphi_b = 15^\circ$,
$c = 80 \text{ mm}$	$m_c = 0.05 \text{ kg}$	$g_c = 42 \text{ mm}$	$\varphi_c = 0^\circ$
$d = 150 \text{ mm}$			

Complete force balancing by adding counter masses to the input and the output links is desired. Determine the mass-distance values and angular position of each counter mass.

Solution

$$\text{We have } m_a g_a e^{i\varphi_a} + m_b a - m_b g_b \frac{a}{b} e^{i\varphi_b} + m'_c g'_c e^{i\varphi'_c} = 0$$

$$\begin{aligned} 0.045 \times 0.028 \cos 0^\circ + 0.13 \times 0.055 - 0.13 \times 0.085 (0.055/0.165) \cos 15^\circ \\ + m'_c g'_c \cos \varphi'_c = 0 \end{aligned}$$

$$0.00126 + 0.00715 - 0.00356 + m'_a g'_a \cos \phi'_a = 0$$

$$m'_a g'_a \cos \phi'_a = -0.004853 \quad (i)$$

$$0.045 \times 0.028 \sin 0^\circ - 0.13 \times 0.085 (0.055/0.165) \sin 15^\circ + m'_a g'_a \sin \phi'_a = 0$$

$$0 - 0.000954 + m'_a g'_a \sin \phi'_a = 0$$

$$m'_a g'_a \sin \phi'_a = 0.000954 \quad (ii)$$

Squaring and adding (i) and (ii),

$$(m'_a g'_a)^2 = 0.00002446$$

or $m'_a g'_a = 0.004946 \text{ kg.m}$

Dividing (ii) by (i),

$$\tan \phi'_a = \frac{0.000954}{-0.004853} = 0.1966$$

$$\phi'_a = 168.9^\circ$$

$$m_c g_c e^{i\phi_c} + m_b g_b \frac{c}{b} e^{i\phi_b} + m'_c g'_c e^{i\phi'_c} = 0$$

$$0.05 \times 0.042 \cos 0^\circ + 0.13 \times 0.085 (0.08/0.165) \cos 15^\circ + m'_c g'_c \cos \phi'_c = 0$$

$$0.0021 + 0.005175 + m'_c g'_c \cos \phi'_c = 0$$

$$m'_c g'_c \cos \phi'_c = -0.007275 \quad (iii)$$

$$0.05 \times 0.042 \sin 0^\circ + 0.13 \times 0.085 (0.08/0.165) \sin 15^\circ + m'_c g'_c \sin \phi'_c = 0$$

$$0 + 0.001387 + m'_c g'_c \sin \phi'_c = 0$$

$$m'_c g'_c \sin \phi'_c = -0.001387 \quad (iv)$$

Squaring and adding (iii) and (iv),

$$(m'_c g'_c)^2 = 0.00006386$$

or $m'_c g'_c = 0.00799 \text{ kg.m}$

Dividing (iv) by (iii),

$$\tan \phi'_c = \frac{-0.001387}{-0.007275} = 0.19065$$

$$\phi'_c = 190.8^\circ$$

Figure 14.11 shows the complete linkage with the two counterweights added.

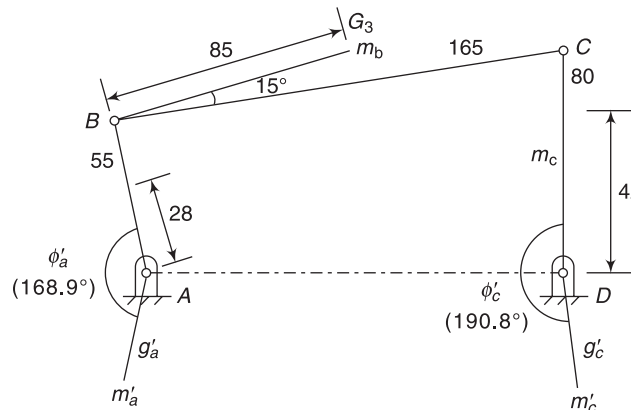


Fig. 14.11