Mechanical Springs

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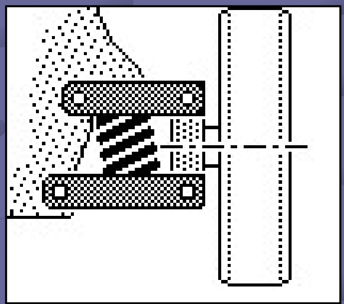
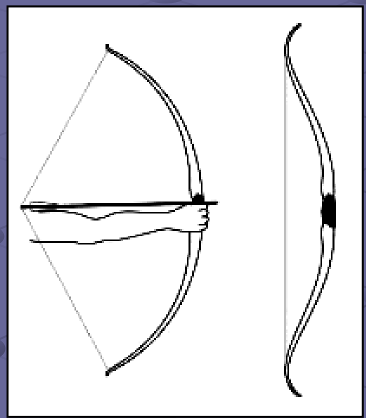
Content

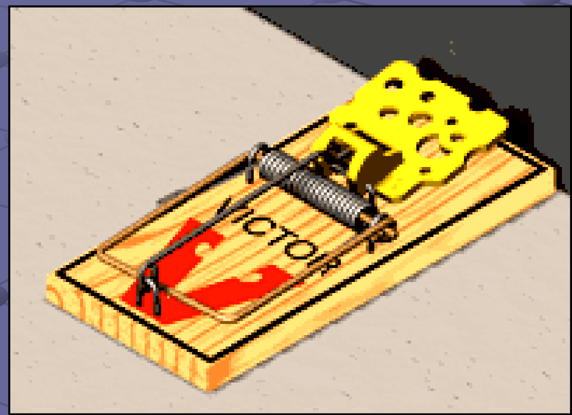
- Definition and Working Principle
- Types of Springs
- Manufacturing
- Spring Materials
- Stresses in Helical Springs
- Deflection of Helical Springs
- Specialities of Compression and Tension Springs
- Buckling
- Critical Frequencies
- Fatigue Loading
- Case Study

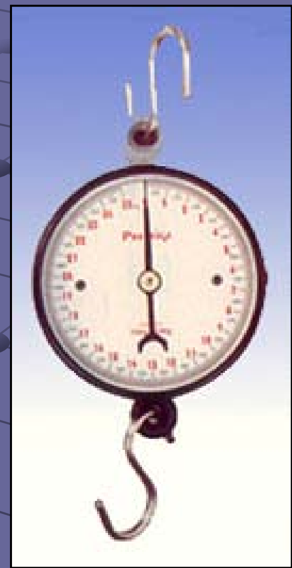
Definition and Working Principle

Springs are flexible machine elements used for controlled application of force (or torque) or for storing and release of mechanical energy.





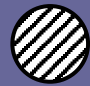


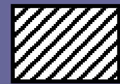


Flexibility (elastic deformation) is enabled due to cleverly designed geometry or by using of flexible material.

Types of Springs

- **Helical springs**

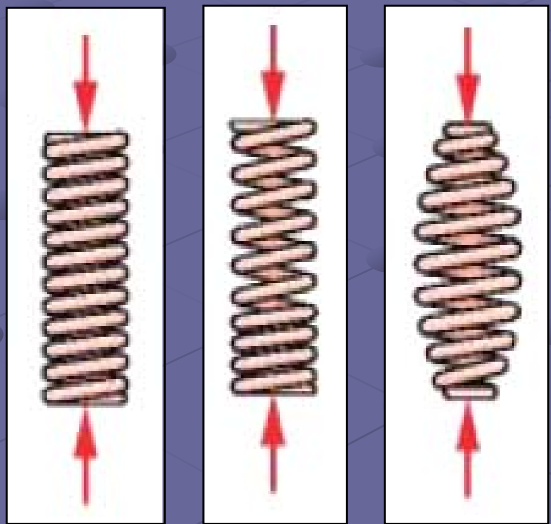
Round wire 

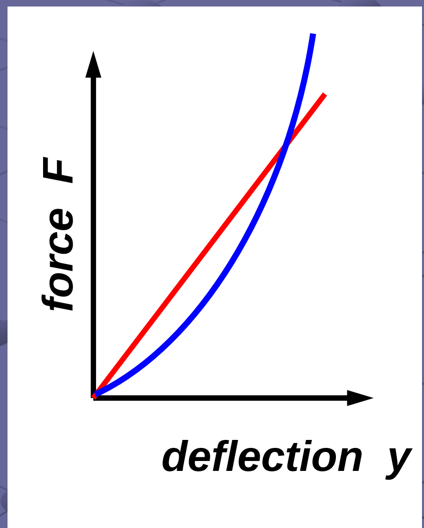
Rectangular wire 

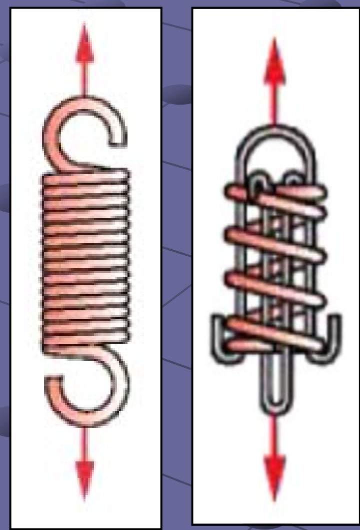
Compression

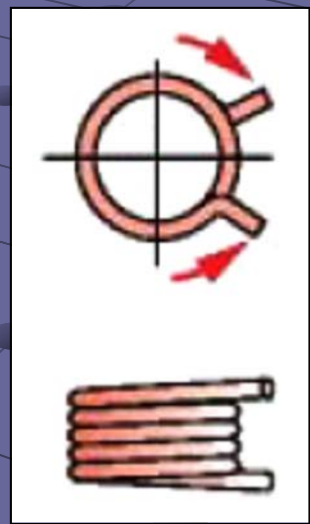
Tension

Torsion



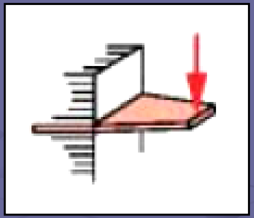


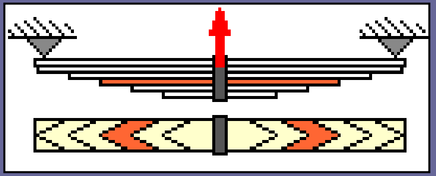




Types of Springs

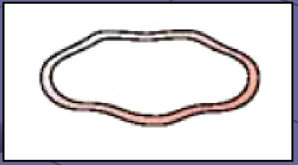
- Leaf springs





- Spiral springs

- Spring washers



Bellewille spring

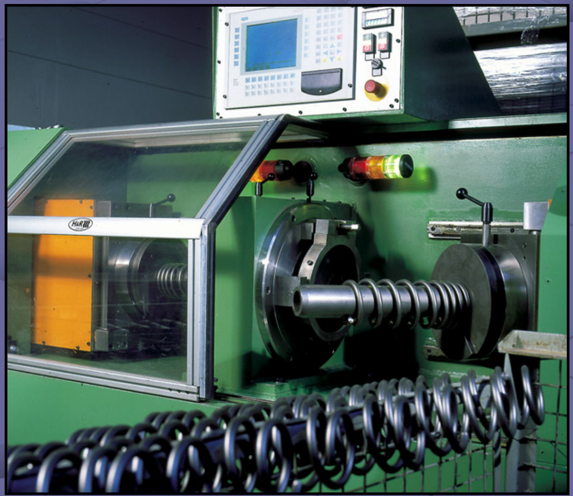
- Special springs

Manufacturing of Springs

Preparation

Winding

(Thermal treatment) Grinding



Calibration

Coating

Testing

Spring Materials

Springs are relatively highly stressed machine parts, which requires materials of high tensile strength and high yield strength.

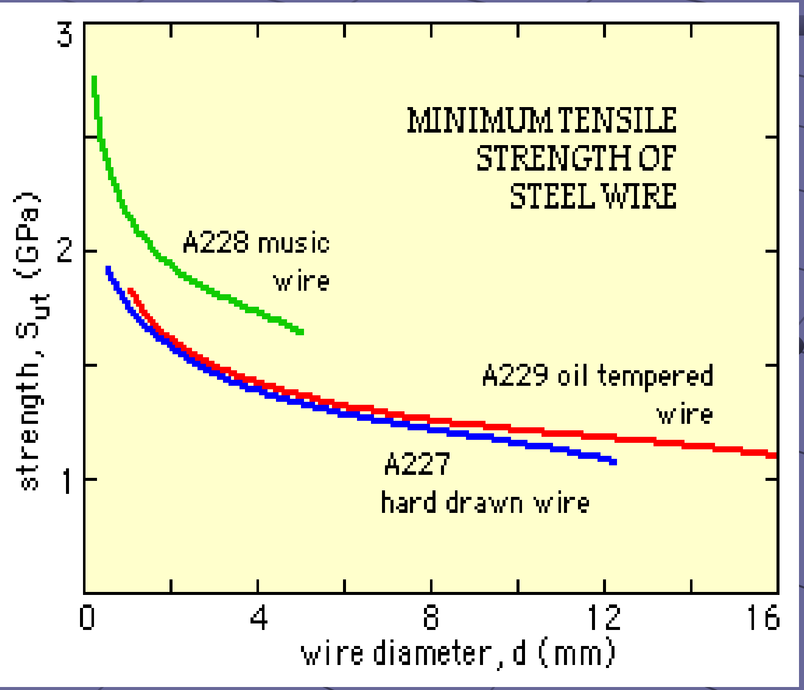
Most used materials are carbon steels, alloy steels, corrosion-resisting steels, phosphor bronze, spring brass and others.

Tensile strength of spring material depends on wire diameter, material and processing.

$$S_{ut} = \frac{A}{d^m}$$

$$0{,}35 S_{ut} \le S_{sy} \le 0{,}52 S_{ut}$$

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi·in^m	Diameter, mm	A, MPa·mm^m
Music wire	A228	0.145	0.004–0.256	201	0.10–6.5	2211
OQ&T wire	A229	0.187	0.020–0.500	147	0.5–12.7	1855
Hard-drawn wire	A227	0.190	0.028–0.500	140	0.7–12.7	1783
Chrome-vanadium wire	A232	0.168	0.032–0.437	169	0.8–11.1	2005
Chrome-silicon wire	A401	0.108	0.063–0.375	202	1.6–9.5	1974
302 Stainless wire	A313	0.146	0.013–0.10	169	0.3–2.5	1867
		0.263	0.10–0.20	128	2.5–5	2065
		0.478	0.20–0.40	90	5–10	2911
Phosphor-bronze wire	B159	0	0.004–0.022	145	0.1–0.6	1000
		0.028	0.022–0.075	121	0.6–2	913
		0.064	0.075–0.30	110	2–7.5	932

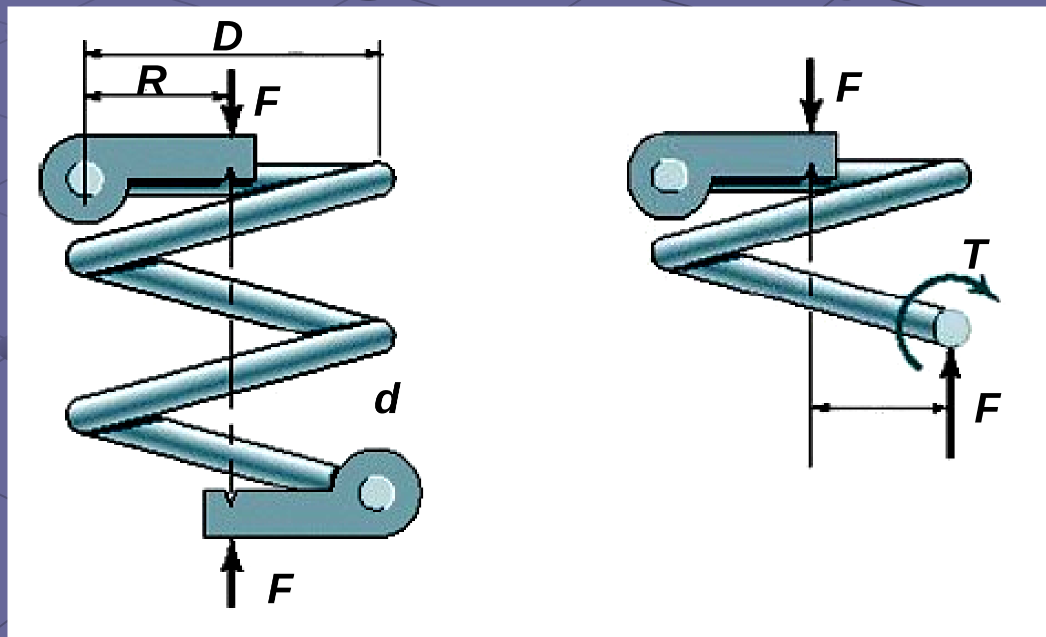


Stresses in Helical Springs

Generally, coils of loaded spring are under combined stress (compression/tension, bending, torsion and shear).

In most cases springs have relatively small *helix angle* and small *Spring index (C=D/d),* therefore compression stress and bending stress can be omitted.

Maximum stress can be computed by superposition of *torsion stress* and *direct shear stress*.



Stresses in Helical Springs

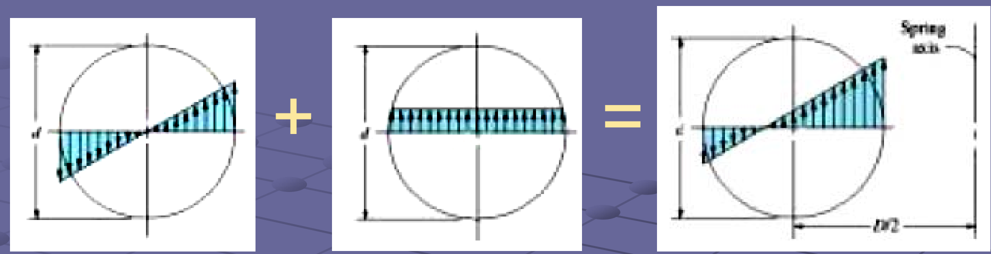
$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$T = \dfrac{FD}{2}$ **torque**

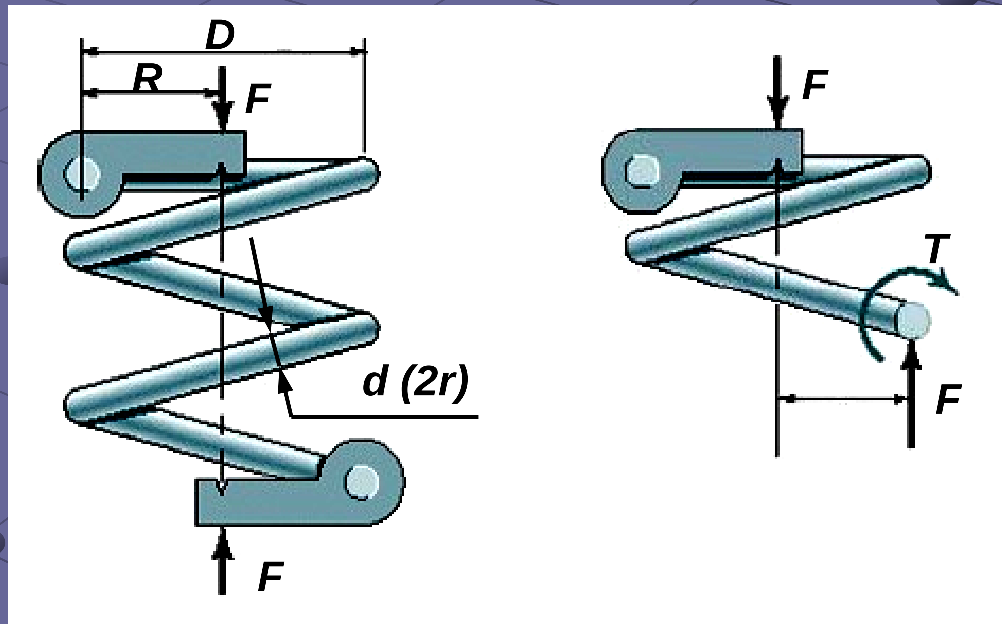
$J = \dfrac{\pi d^4}{32}$ **polar moment**

$A = \dfrac{\pi d^2}{4}$ **cross-section area**

d **– wire diameter**
D **– helix spring diameter**
F **– applied force**



Torsion stress | *Shear stress (direct)* | *Maximum stress*



Stresses in Helical Springs

$$C = \frac{D}{d}$$ spring index

$$\tau_{max} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$ maximum stress

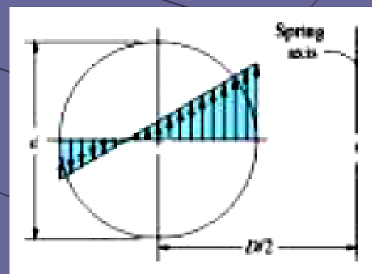
(C = 3 ÷12)

$$\tau_{max} = \frac{8FD}{\pi d^3}\left(1 + \frac{0,5}{C}\right)$$

$$\left(1 + \frac{0,5}{C}\right) = K_S$$ shear-stress augmentation factor

$$\boxed{\tau_{max} = K_S \frac{8FD}{\pi d^3}}$$

Maximum shear stress
(for a case of straight wire)



Stresses in Helical Springs

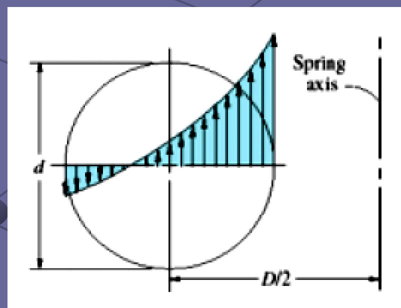
Curvature effect **- curvature of the wire (helix shape) increases the stress on the inside of the spring and decreases it on outside.**

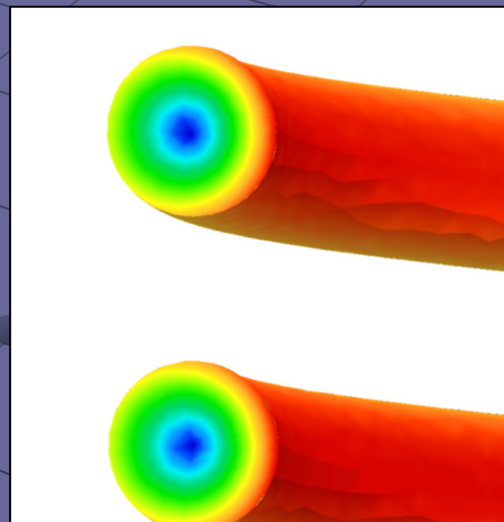
Curvature effect **together with** ***shear-stress augmentation factor*** **can be defined by** ***Wahl factor*** **or** ***Bergsträsser factor.*** **Because results of these factors differ by less than 1%, Bergsträsser factor is preferred.**

$$K_B = \frac{4C+2}{4C-3}$$

$$\left(K_W = \frac{4C-1}{4C-4} + \frac{0{,}651}{C} \right)$$

$$\tau_{\max} = K_B \frac{8FD}{\pi d^3}$$





Deflection of Helical Springs

Deflection–force relation is easily obtained from Castiliano's theorem.

$$U = U_T + U_S = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

$$J = \frac{\pi d^4}{32} \qquad T = \frac{FD}{2} \qquad A = \frac{\pi d^2}{4} \qquad l = \pi DN$$

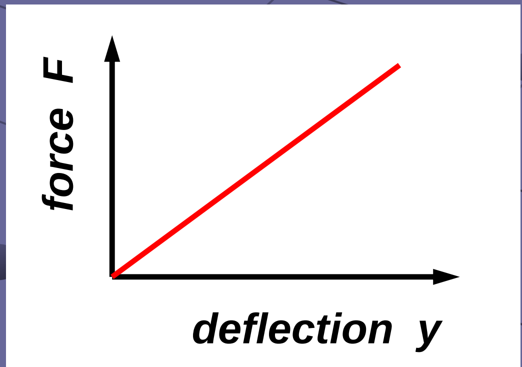
$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

$$\boxed{y = \frac{\partial U}{\partial F}} = \frac{8FD^3 N}{d^4 G} + \cancel{\frac{4FDN}{d^2 G}} \Longrightarrow \boxed{y \approx \frac{8FD^3 N}{d^4 G}}$$

$$\boxed{k = \frac{F}{y} = \frac{d^4 G}{8D^3 N}}$$

Spring rate

Spring characteristic

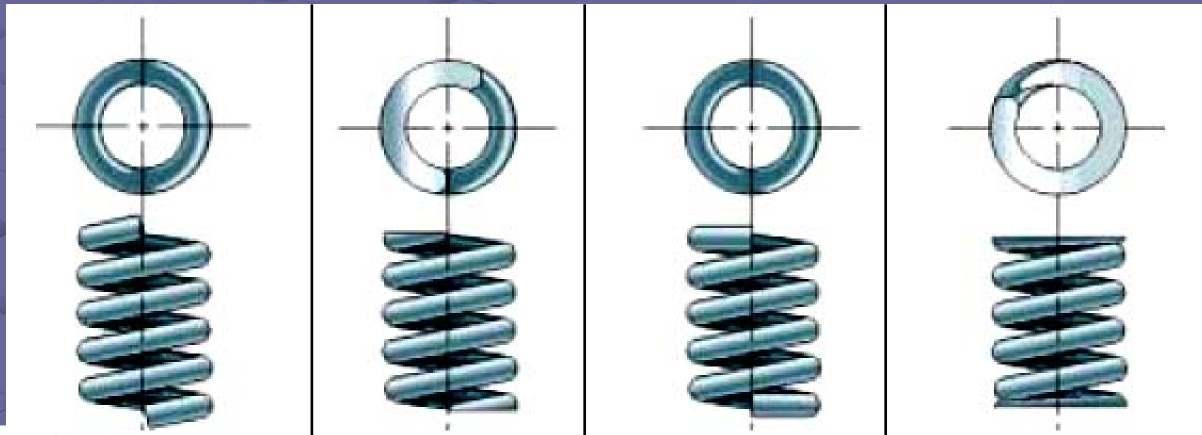


Compression Springs

Quality of load transfer from the support to the spring depends on the type of spring ends.

$$N_a = N_t - N_e$$

N_a – number of active coils

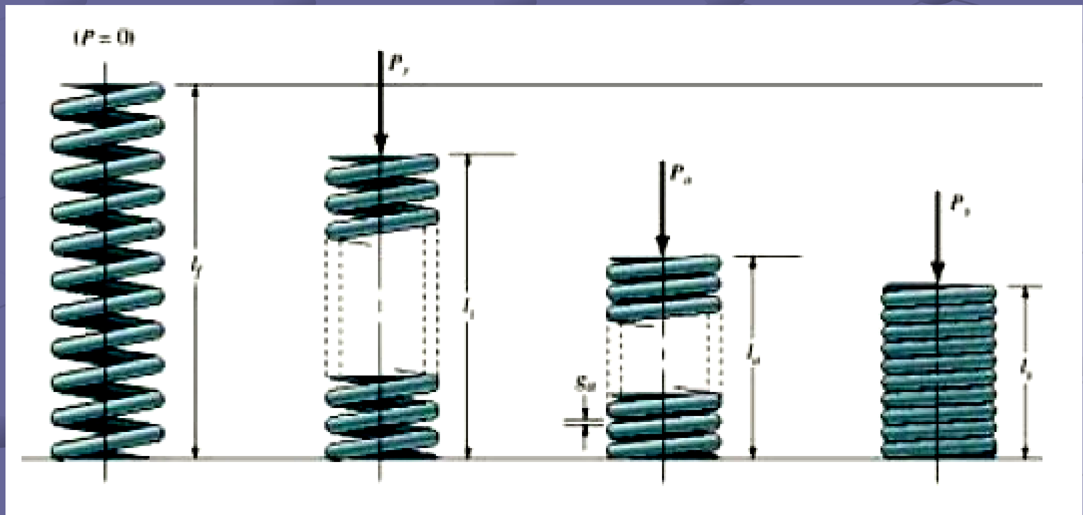


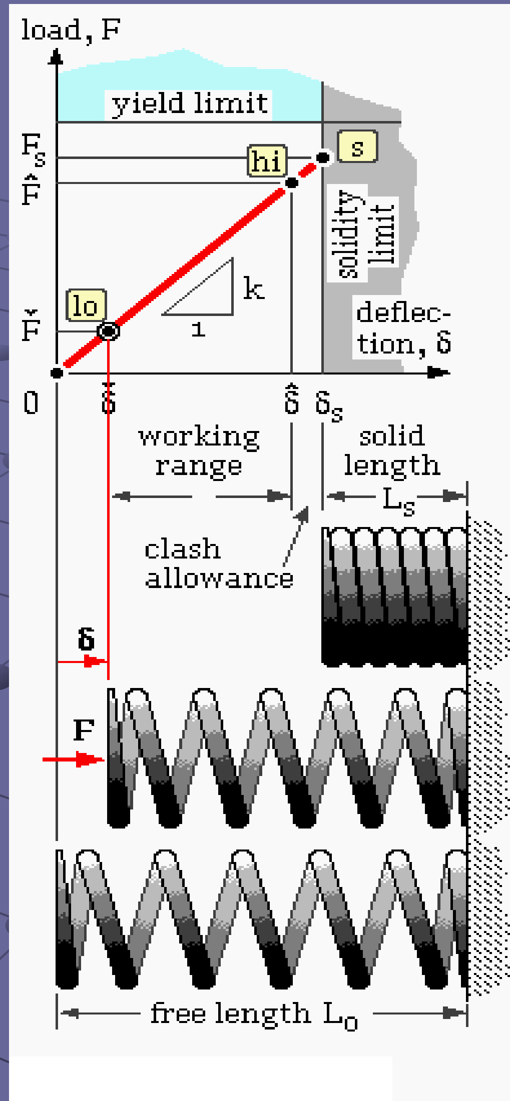
Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
		Type of Spring Ends		
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Compression Springs

Basic states of compression springs.

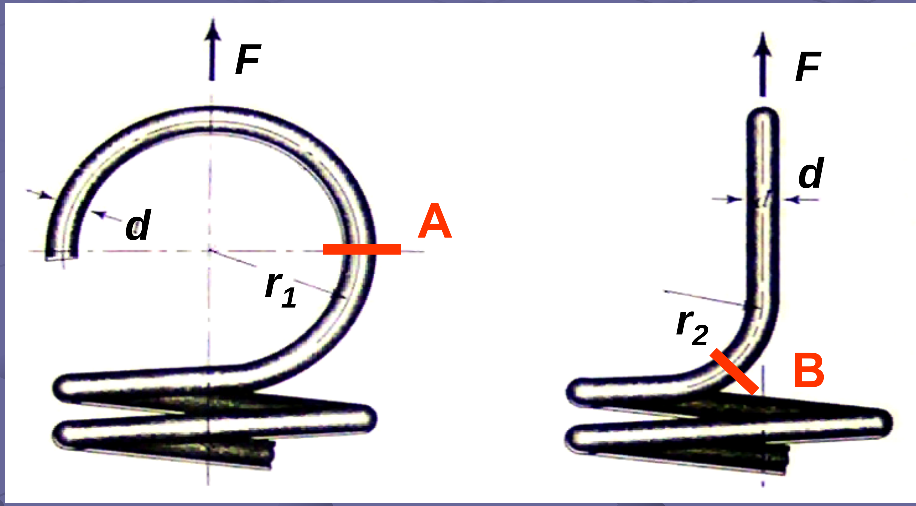
free initial working solid





Tension Springs

Stresses in the hook ends of tension springs are usually higher than stresses in the coils of the spring. Stress-augmentation factors are used to determine these stresses.

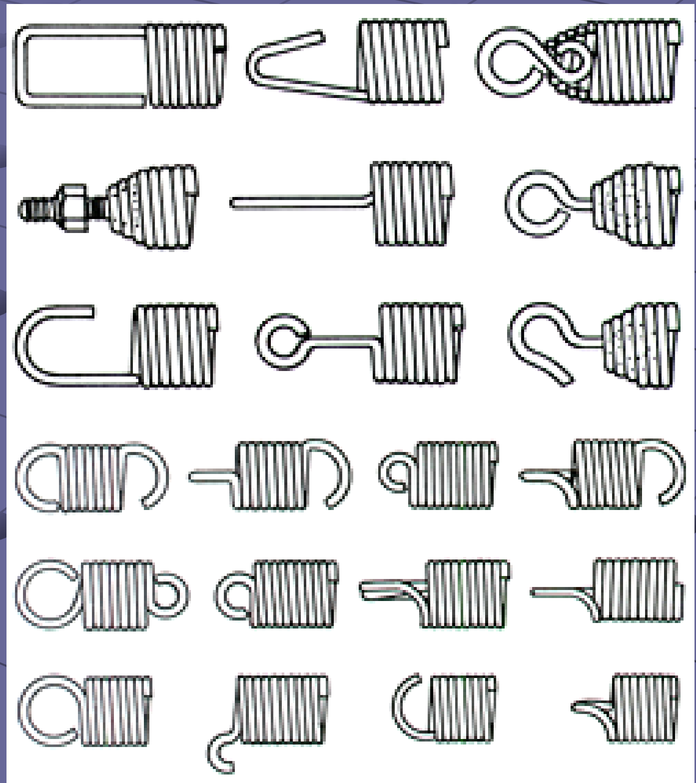


$$K_A = \frac{4C_1^{\,2} - C_1 - 1}{4C_1(C_1 - 1)}$$

$$C_1 = \frac{2r_1}{d}$$

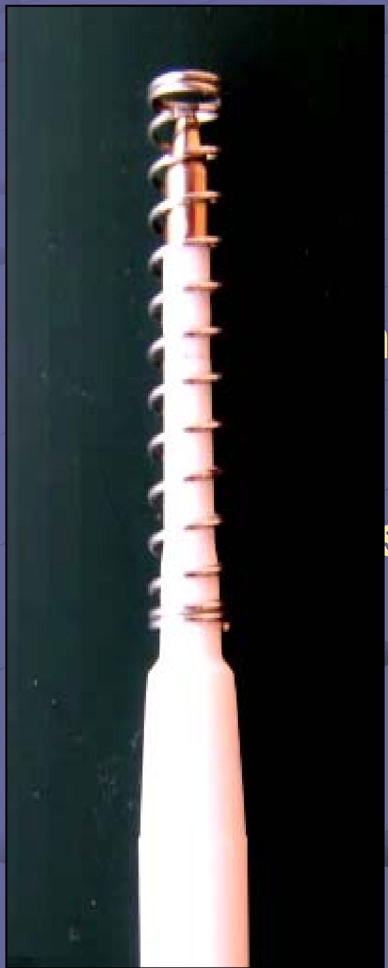
$$K_B = \frac{4C_2 - 1}{4C_2 - 4}$$

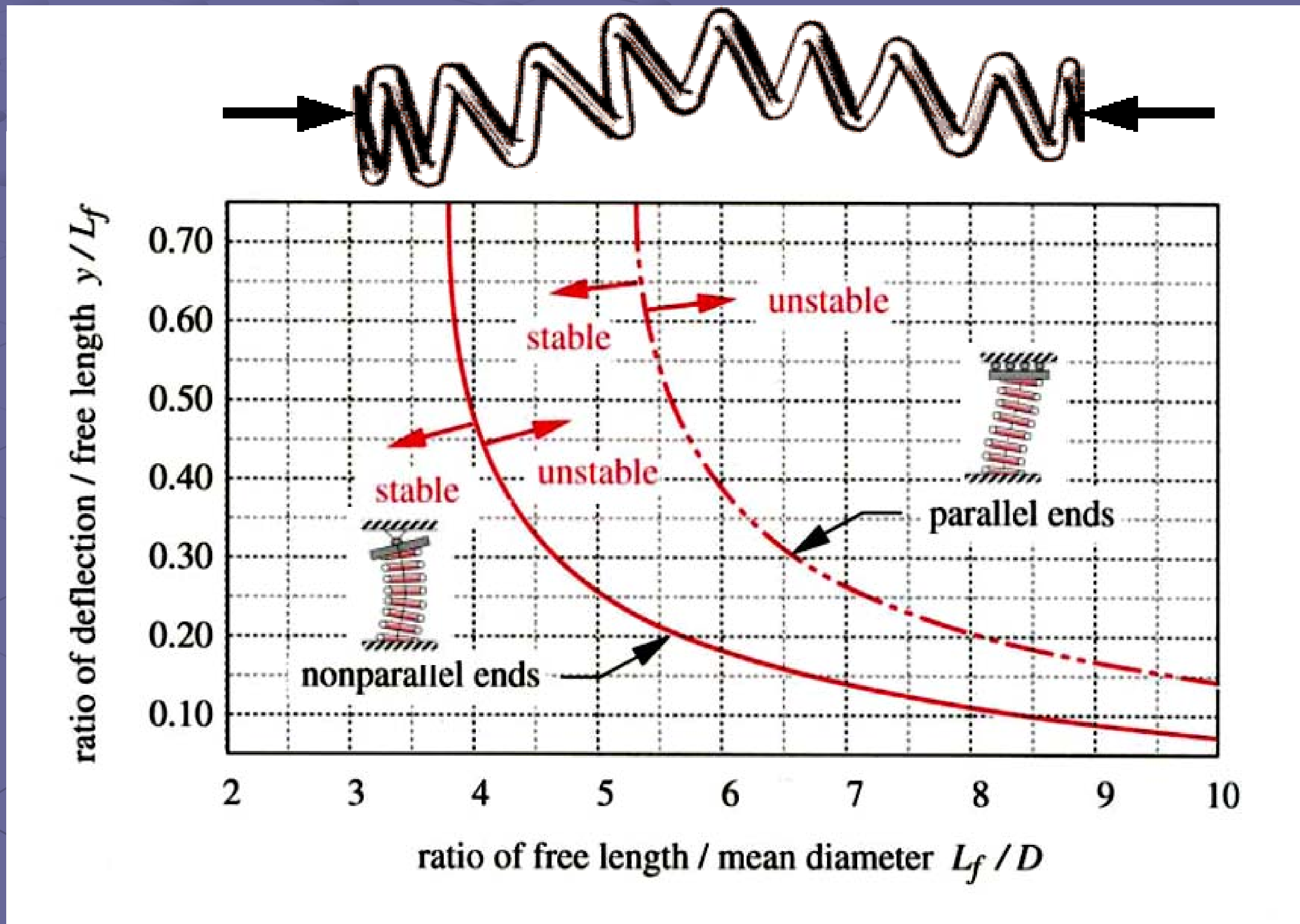
$$C_2 = \frac{2r_2}{d}$$



Buckling

If deflection of slender compression spring exceeds some critical value than the spring will buckle.





Critical Frequencies

Spring surge occurs when the end of compression spring is disturbed. If spring is used in applications requiring rapid reciprocating motion, then natural frequencies of the spring have to be investigated if they are not close to the frequency of disturbing, by this we avoid the danger of resonance.

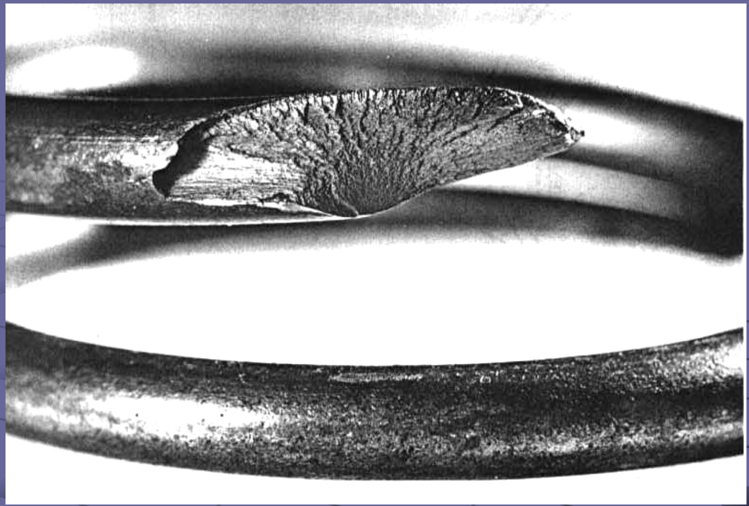
Natural frequencies for the spring between two parallel plates:

$$f = m\frac{1}{2}\sqrt{\frac{ka}{W}}$$

m = 1,2,3…
k – spring rate
a – acceleration (of gravity, general acc. of machine)
W – weight of spring

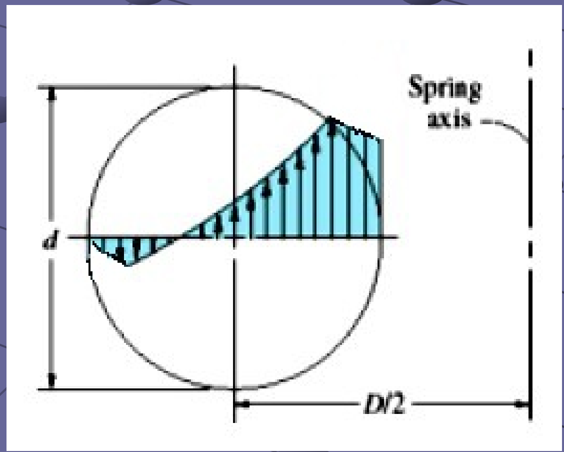
Fatigue Loading

In not a few cases, springs are subject to fatigue loading.



Fatigue cracks are usually initiated on the highly stressed inner surface of the wire or in the stress raisers.

Fatigue strength of spring wire may be increased by *peening* – bombarding the wire with pellets. This sets up surface localized residual stresses which counteract and thus decrease the stress on the surface when the spring is loaded.



Fatigue Loading

Zimmerli has found out that for carbon and alloy steel springs with the wire diameter smaller than *10mm* there are no differences in the flat part of SN curve.

Strength component	Unpeened	Peened
Ssa [MPa]	241	398
Ssm [MPa]	379	534

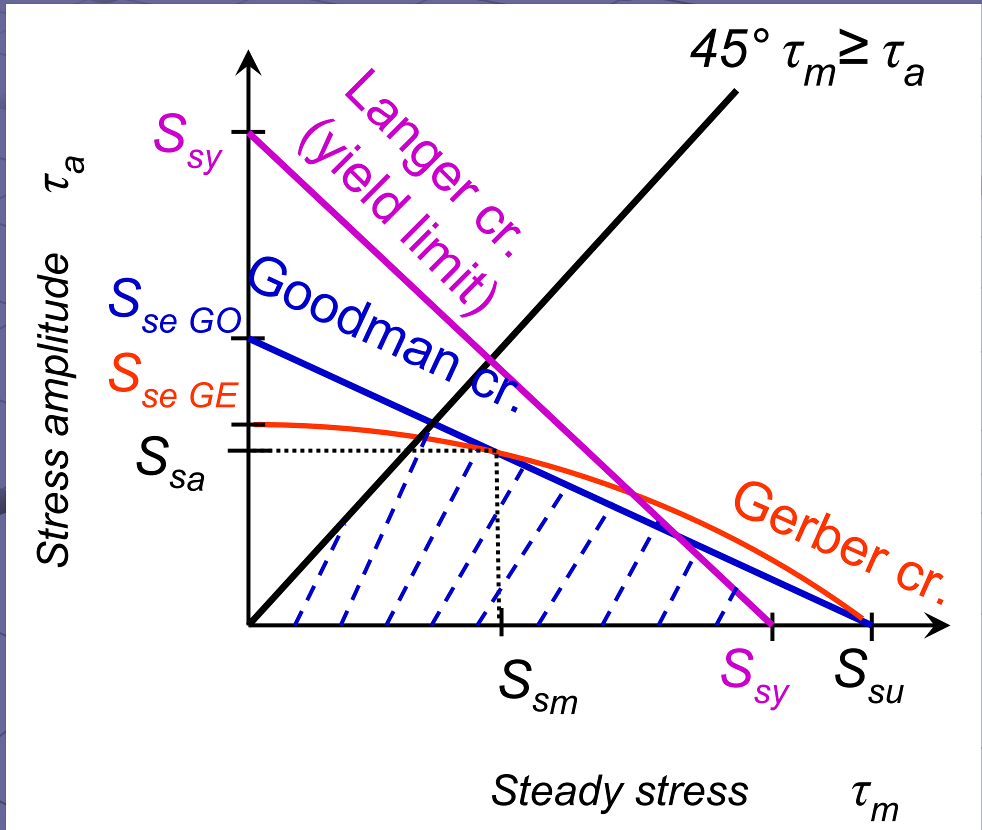
S_{se} (fatigue strength in reversed shear) can be calculated for selected criterion or can be found in the material tables.

$$S_{se} = \frac{S_{sa}}{1-\left(\frac{S_{sm}}{S_{su}}\right)^2}$$

Gerber cr.

$$S_{se} = \frac{S_{sa}}{1-\left(\frac{S_{sm}}{S_{su}}\right)}$$

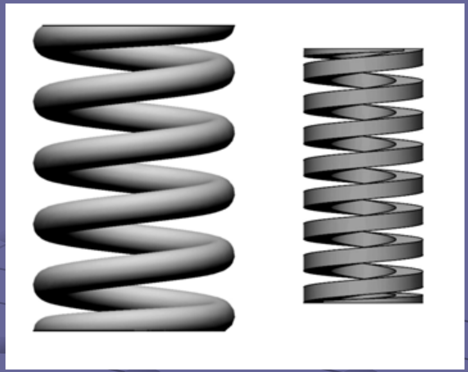
Goodman cr.



Case Study - Task

Two springs – one of round wire

– one of rectangular wire



1. *Experiment* – measure dimensions, measure spring characteristics on a spring tester.

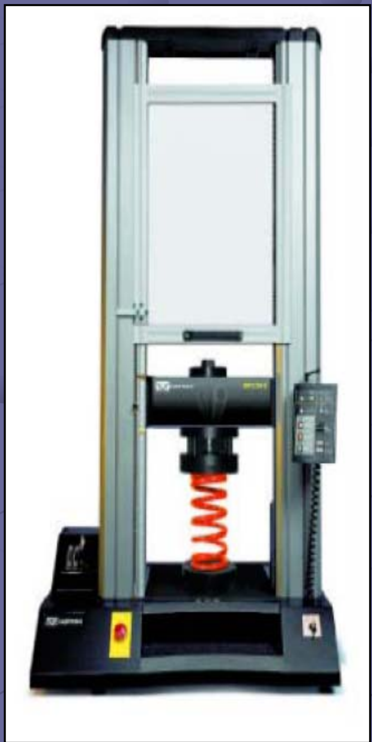
2. *FEA* – create 3D models of springs, investigate spring behaviour (deflection) under load.

3. *Analytical calculation* – use equations from theory and calculate spring characteristics analytically.

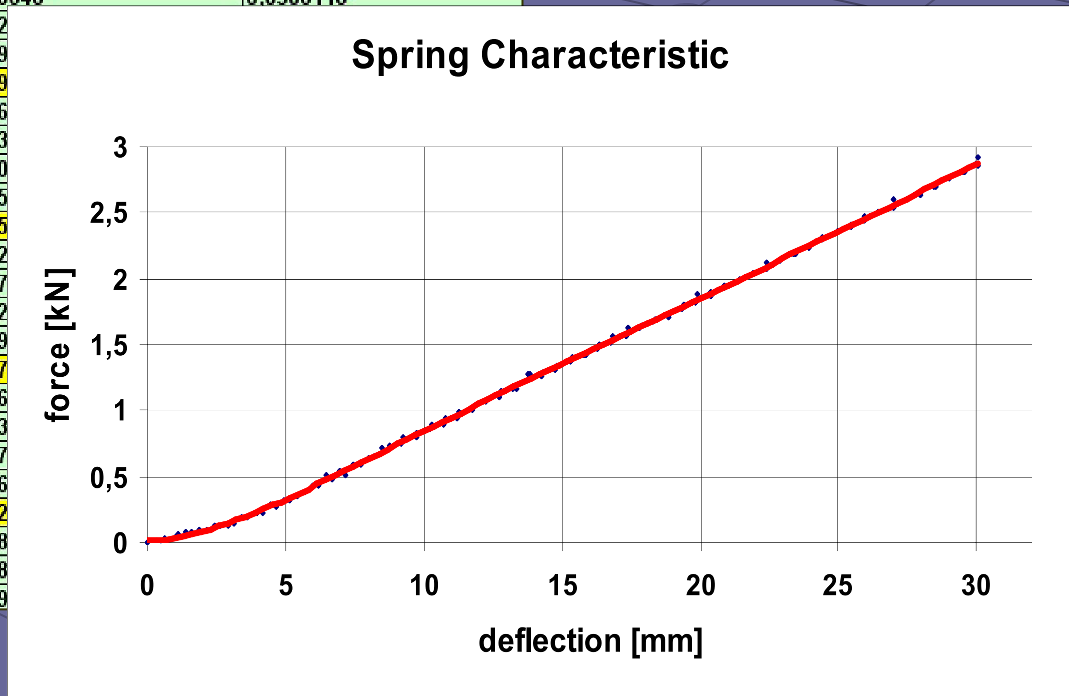
4. *Comment and compare results* – explain differences between results of individual approaches.

Experiment

$d\,,\; D\,,\; l_0\,,\; p\,,\; N_a,\; N_e\,,\; N_t$

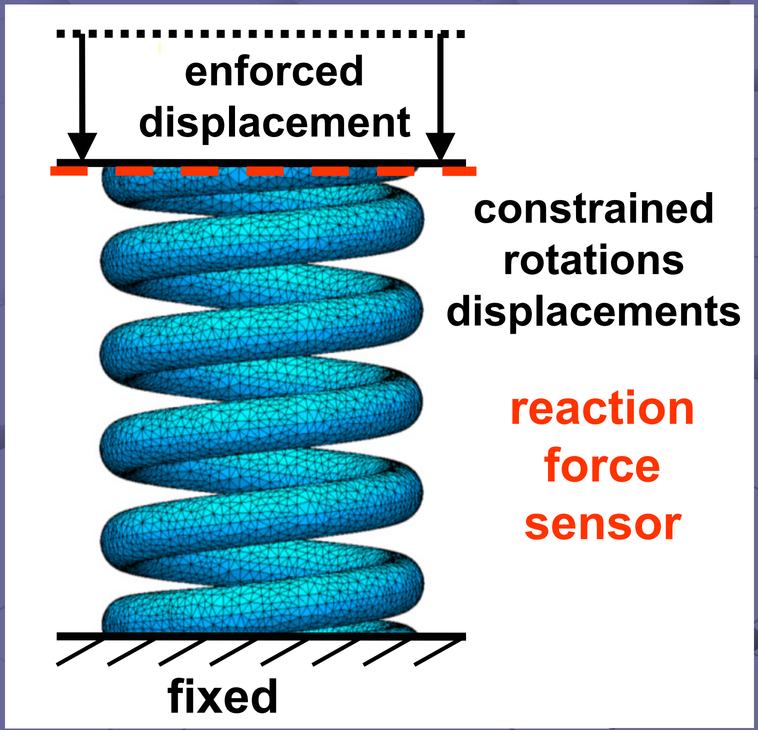


displacement (mm) - exp.	force (kN) - exp.
0	0
0,519678	0,0130287
1,020648	0,0508118
1,392	
1,899	
2,409	
2,916	
3,423	
3,930	
4,435	
4,945	
5,452	
5,957	
6,462	
6,969	
7,477	
7,986	
8,493	
8,757	
9,266	
9,772	
10,28	
10,78	
11,29	



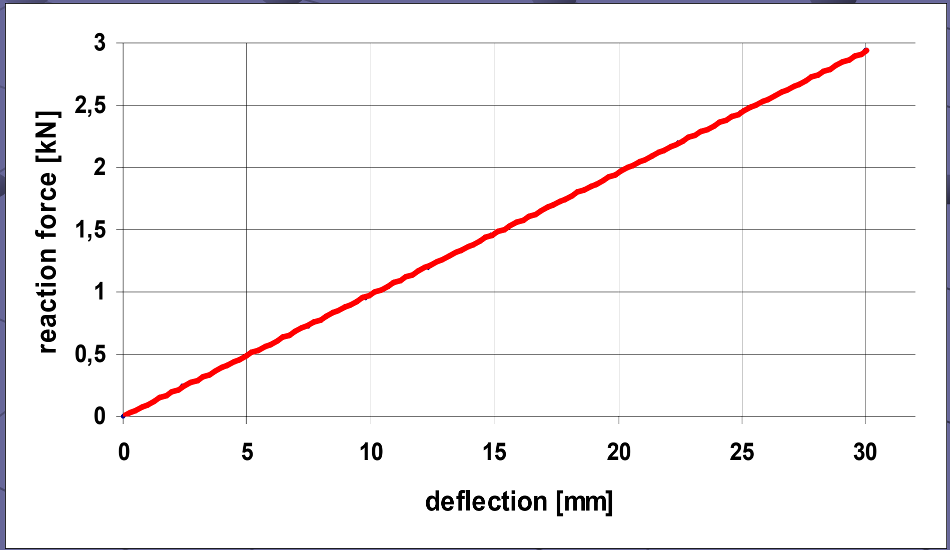
FEA

Creating 3D model with dimensions measured in experimental part. Modification and simplification of solid model in order to meet conditions of experiment. Setting of boundary conditions and loads. Setting of material properties, element type and size. Analysis.

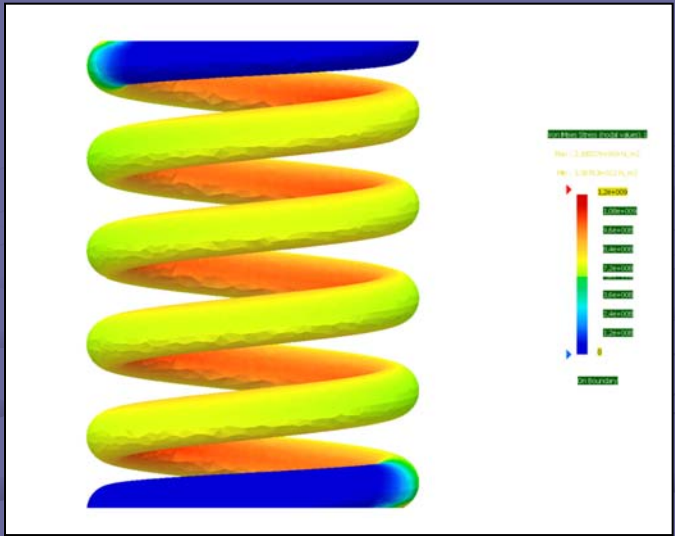


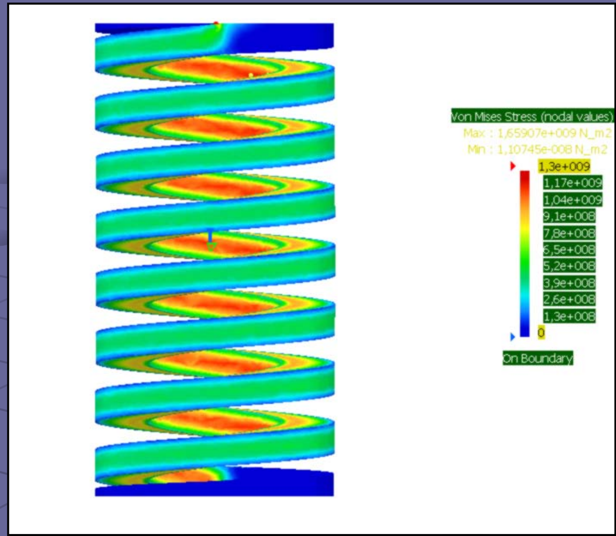
Analysis input : enforced displacement (deflection)

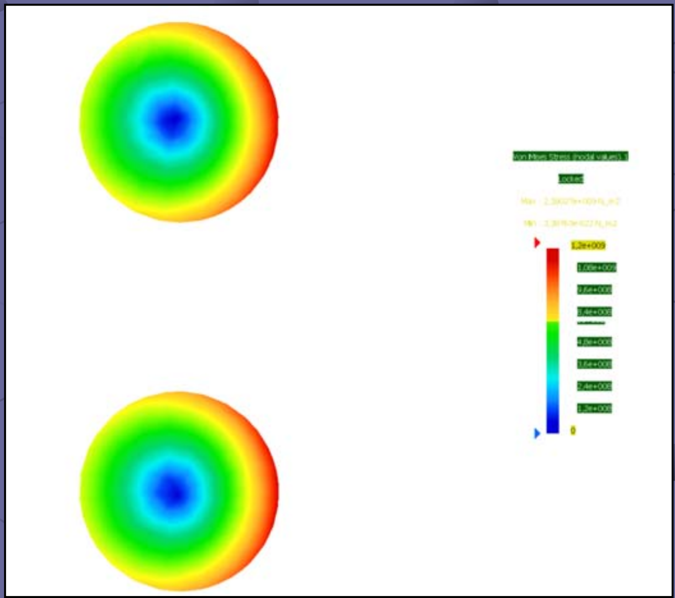
Analysis output : reaction force

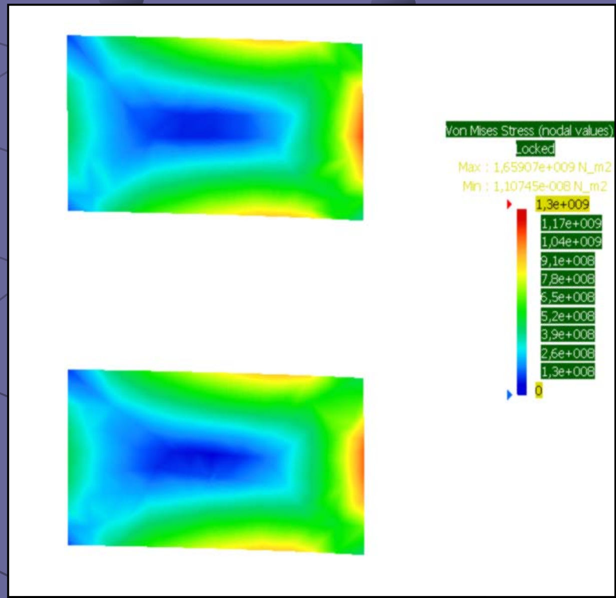


FEA – Stress Distribution









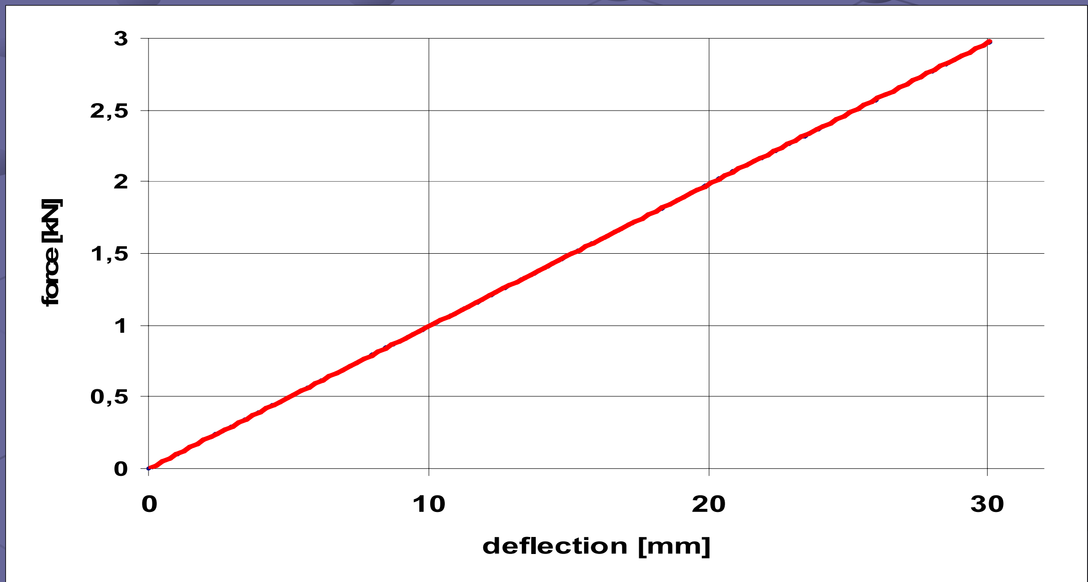
Analytical Calculation

Round wire:

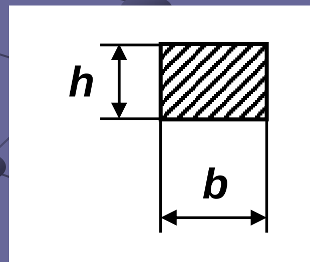
$$y = \frac{8FD^3N}{d^4G} \Longrightarrow F = \frac{yd^4G}{8D^3N}$$

Rectangular wire:

$$y = \frac{\varepsilon D^3NF}{b^2h^2G} \Longrightarrow F = \frac{yb^2h^2G}{\varepsilon D^3N}$$



ε – depends on b/h ratio, (from tables)

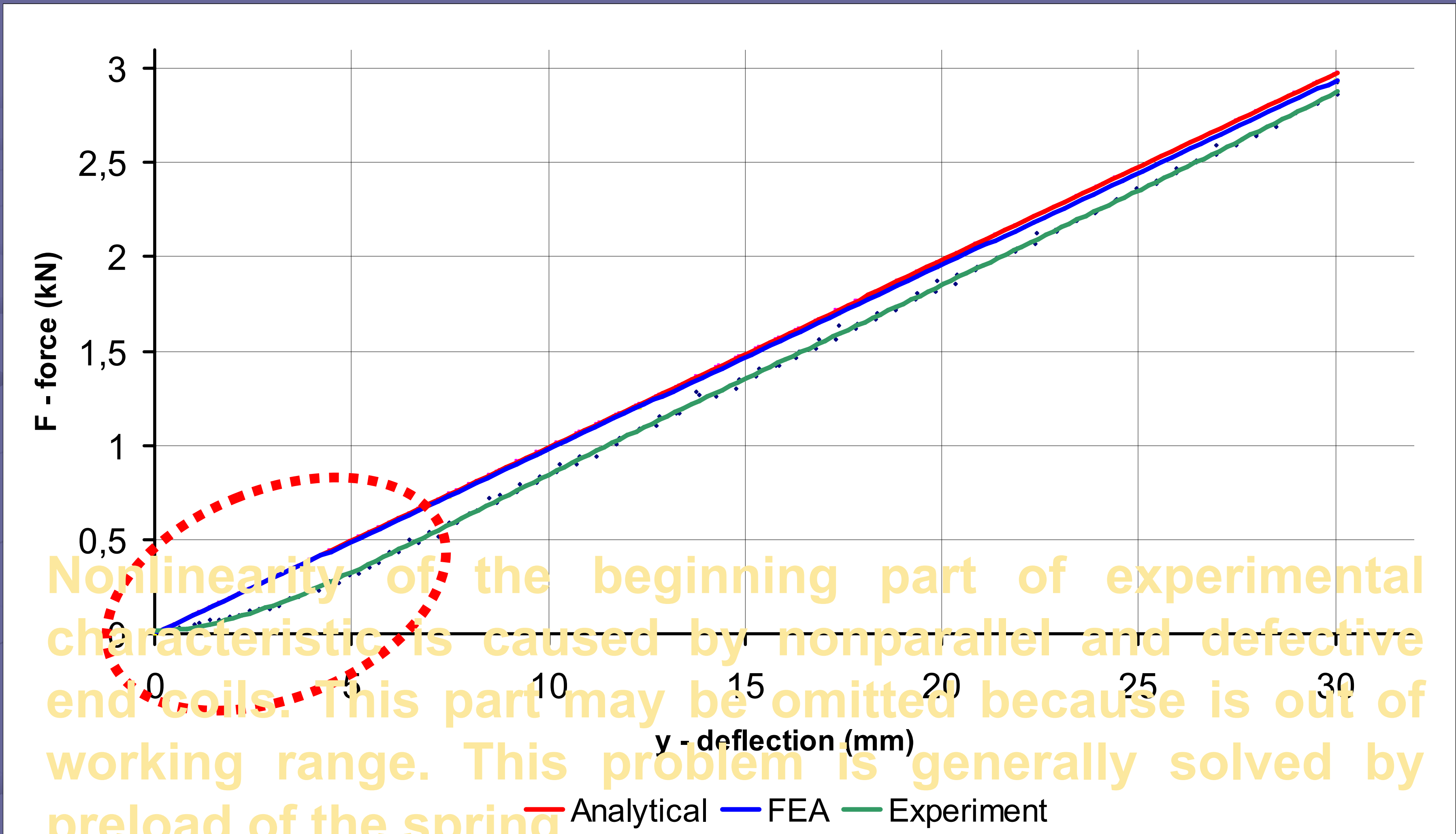


Calculation procedure

Position mm	Load kN	y (mm) - exp.	F (kN) - exp.	F (kN) - analyt.	F (kN) - FEA	τmax (MPa) - analyt.	σHMH (MPa) - analyt.	σHMH (MPa) - FEA
26,031711	9,8218734	0	0	0	0	0	0	0
25,512033	9,8088447	0,519678	0,0130287	0,05146147				
25,011063	9,7710616	1,020648	0,0508118	0,101070367				
24,638974	9,7489131	1,392737	0,0729603	0,137916736				
24,131768	9,7332788	1,899943	0,0885946	0,188143157				
23,622483	9,7007072	2,409228	0,1211662	0,238575453	0,236	67,08723576	116,1985009	124,863
23,115278	9,6928899	2,916433	0,1289835	0,288801776				
22,608072	9,6303525	3,423639	0,1915209	0,339028198				
22,100866	9,5951753	3,930845	0,2266981	0,389254619				
21,595739	9,5417579	4,435972	0,2801155	0,439275166				
21,086455	9,5091865	4,945256	0,3126869	0,489707363	0,483	137,7053376	238,5126413	256,298
20,579249	9,4674949	5,452462	0,3543785	0,539933785				
20,074122	9,3893232	5,957589	0,4325502	0,589954332				
19,568995	9,3189685	6,462716	0,5029049	0,63997488				
19,061789	9,2850942	6,969922	0,5367792	0,690201301				
18,554584	9,2342824	7,477127	0,587591	0,740427624	0,732	208,2076839	360,6262871	387,517
18,045299	9,1821679	7,986412	0,6397055	0,79085992				
17,538093	9,1039962	8,493618	0,7178772	0,841086342				
17,274097	9,0883619	8,757614	0,7335115	0,867228726				
16,764813	9,0245218	9,266898	0,7973516	0,917660923				
16,259685	8,9854358	9,772026	0,8364376	0,96768157	0,956	272,1113204	471,3106322	506,455
15,748322	8,9242014	10,283389	0,897672	1,018319641				
15,243195	8,8786012	10,788516	0,9432722	1,068340188				
14,733911	8,8395153	11,2978	0,9823581	1,118772385				
14,226705	8,7795837	11,805006	1,0422897	1,168998807				
13,719499	8,7339834	12,312212	1,08789	1,219225229	1,204	342,8452057	593,8253154	638,106

Comparing results

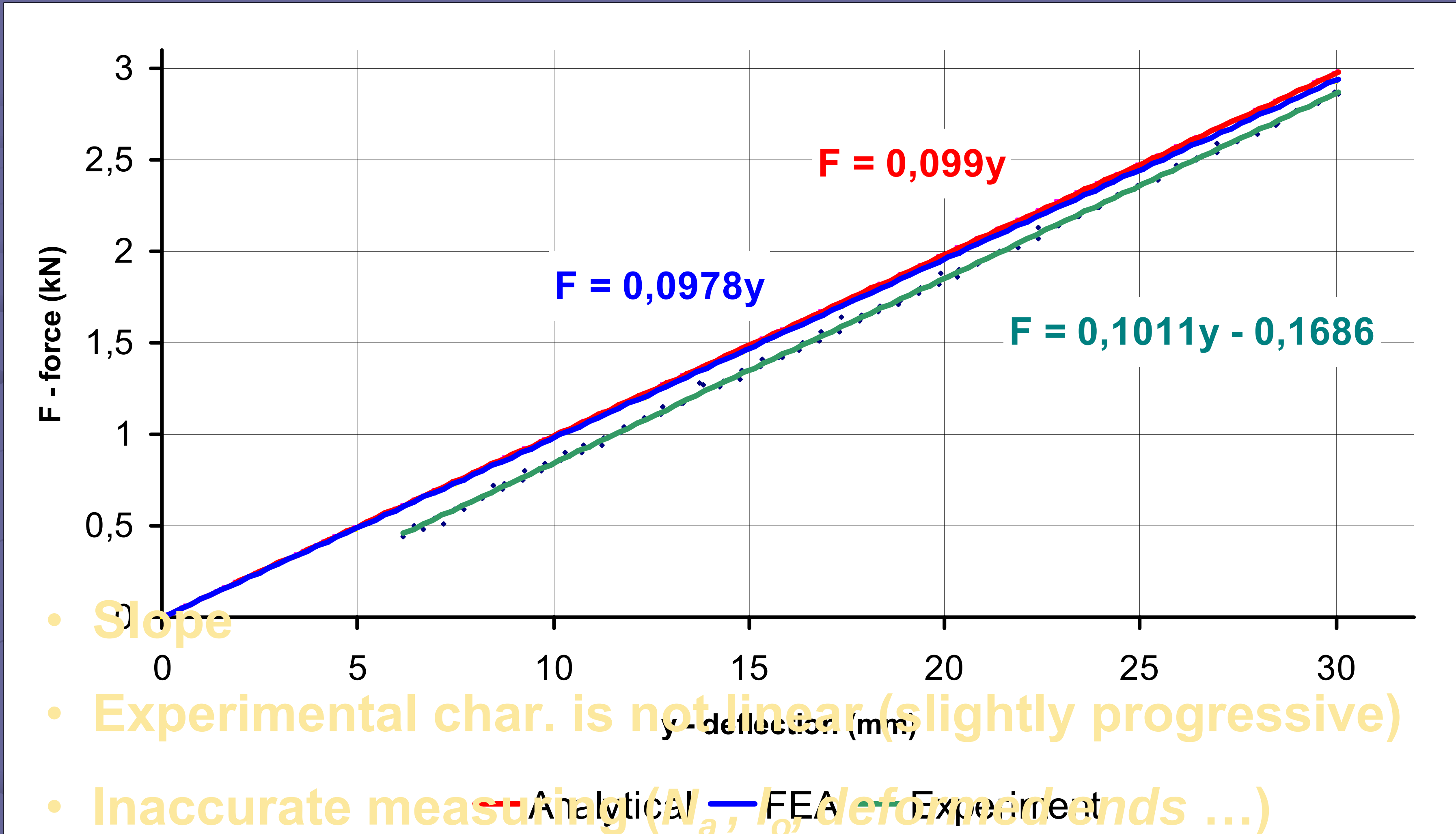
Round wire spring – Force / Deflection relation



Nonlinearity of the beginning part of experimental characteristic is caused by nonparallel and defective end coils. This part may be omitted because is out of working range. This problem is generally solved by preload of the spring.

Comparing results

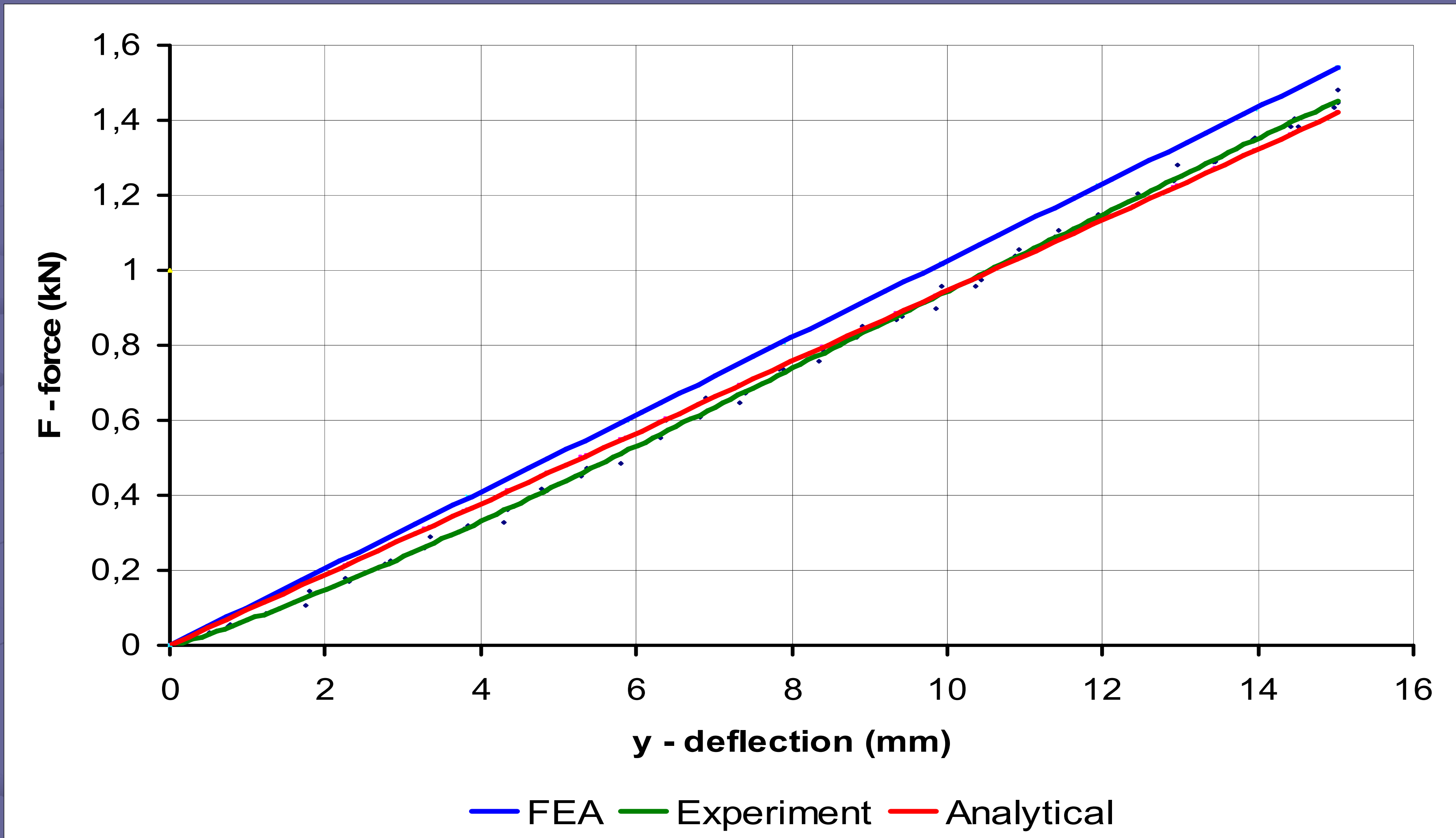
Round wire spring – Force / Deflection relation



- Slope
- Experimental char. is not linear (slightly progressive)
- Inaccurate measuring (N_a, l_o, *deformed ends* …)

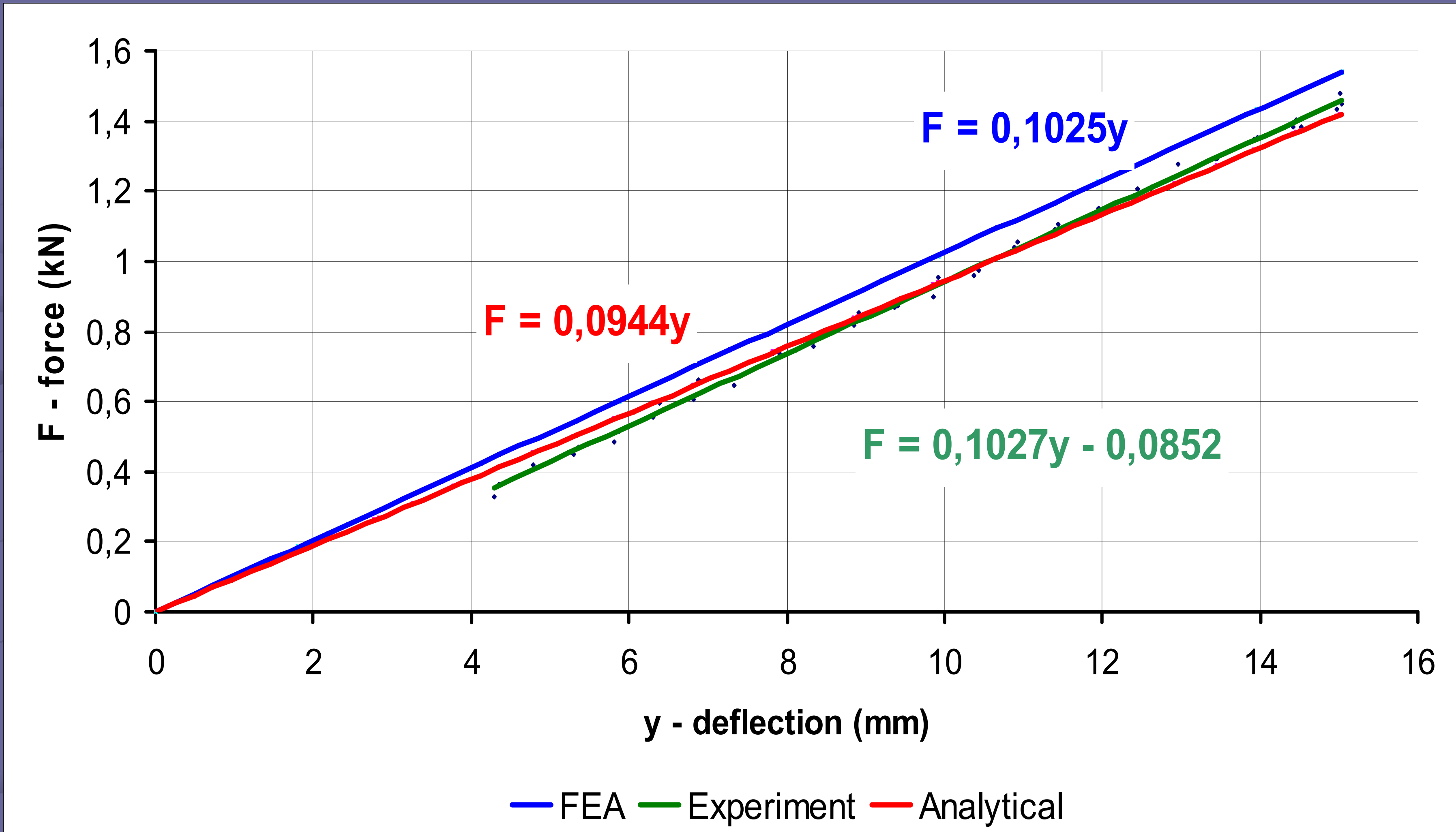
Comparing results

Rectangle wire spring – Force / Deflection relation



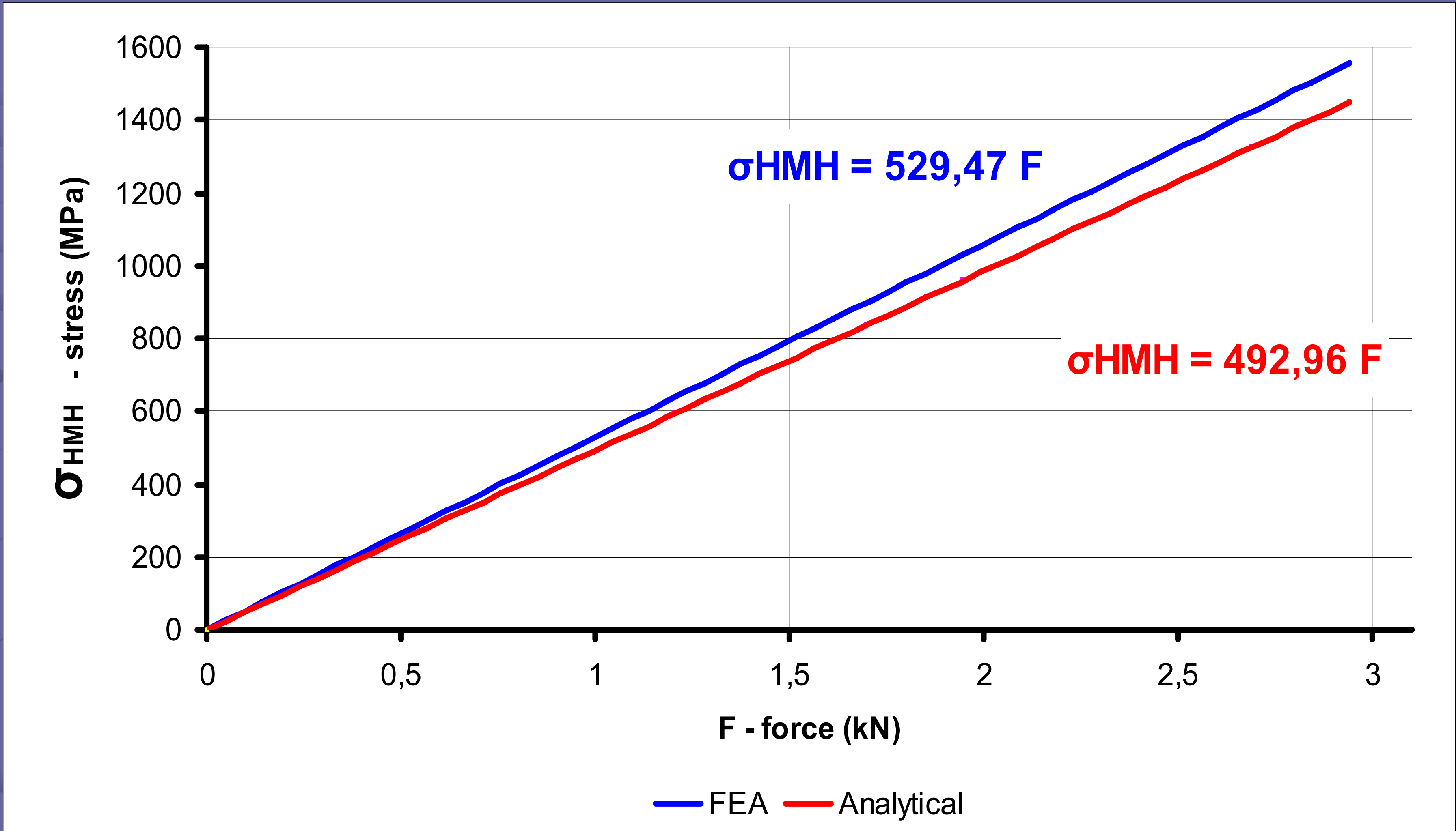
Comparing results

Rectangle wire spring – Force / Deflection relation



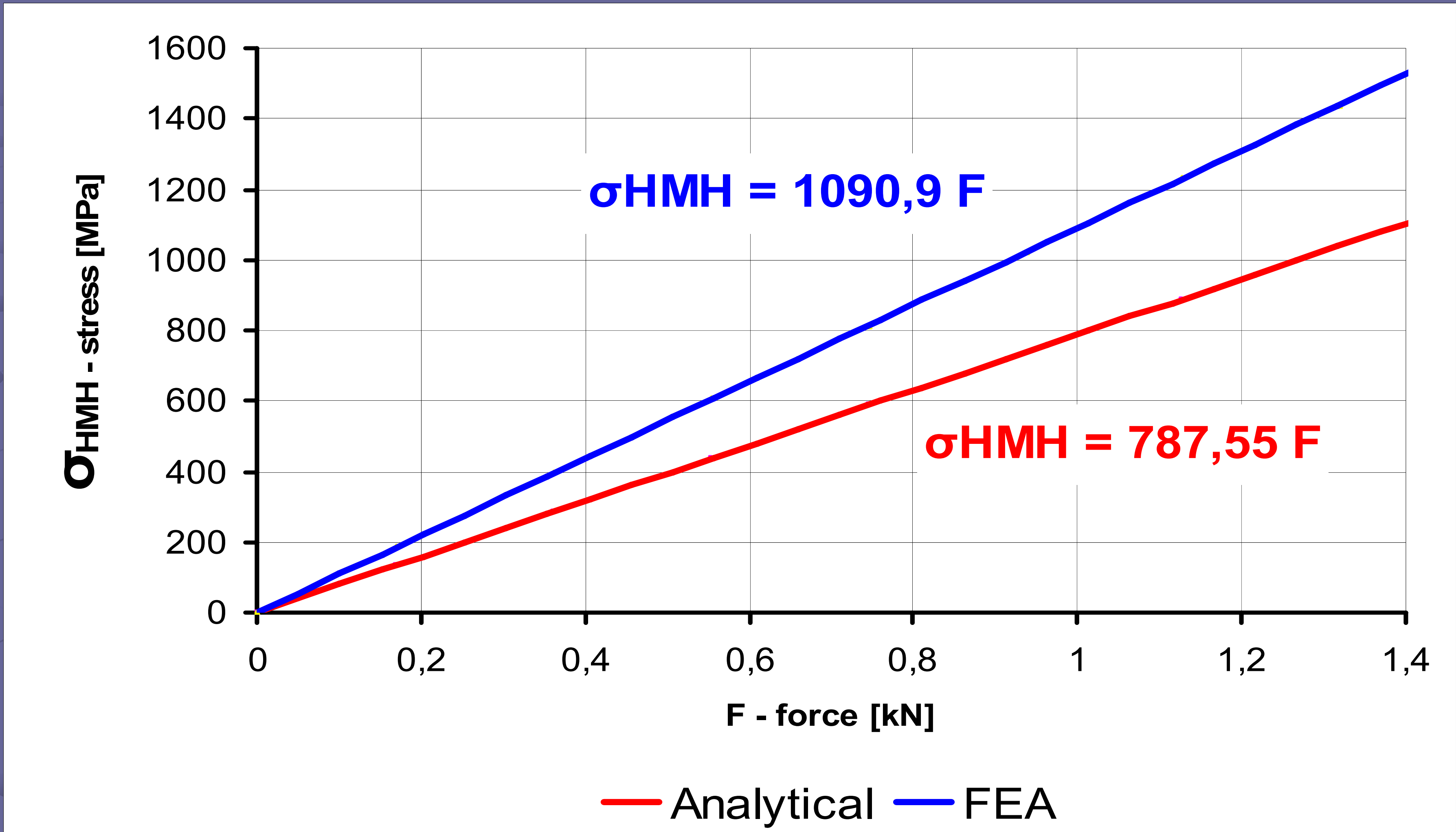
Comparing results

Round wire spring – Stress / Force relation



Comparing results

Rectangle wire spring – Stress / Force relation



Comparing results

- Analytical approach is not so developed for rectangular wire like for round wire. Parameters from tables etc.
- Results from FEA are higher because the other types of stresses are involved – compression and bending. These are omitted in analytical approach.
- Analytical approach is not so developed for rectangular wire like for round wire.

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