Tutorial 1

Fluid properties

1. A reservoir of glycerin has a mass of 1100kg and a volume of 0.9 m^3. Calculate its weight, mass density, specific weight and specific gravity.

Solution:
Mass of glycerin (m) = 1100kg
Volume (V) = 0.9 m^3
Weight (W) = ?
Mass density (ρ) = ?
Specific weight (γ) = ?
Specific gravity (S) = ?

W = mg = 1100x9.81 = 10791N
ρ = m/V = 1100/0.9 = 1222.22 Kg/m^3
γ = ρg = 1222.22x9.81 = 11990 N/m^3
S = $\gamma_{glycerine}/\gamma_{water}$ = 11990/9810 = 1.22

2. A liquid compressed in a cylinder has a volume of 2000 cm^3 at 2MN/m^2 and a volume of 1990 cm^3 at 4MN/m^2. What is its bulk modulus of elasticity?

Solution:
Initial volume (V) = 2000 cm^3
Final volume (V1) = 1990 cm^3
Change in volume (ΔV) = 1990-2000 = -10 cm^3
Change in pressure(ΔP) = 4-2 = 2MN/m^2
Bulk modulus of elasticity (K) = ?

$$K = -\frac{\Delta P}{\Delta V/V} = -\frac{2}{-10/2000} = 400\ MN/m^2$$

3. If the bulk modulus of elasticity of water is 2.2 Gpa (GN/m^2), what pressure is required to reduce a volume by 0.8%?

Solution:
Bulk modulus of elasticity (K) = 2.2 Gpa = 2.2×10^9 Pa
Reduction in volume ($\Delta V/V$) = -0.8% = -0.008
Pressure (P) = ?

$$K = -\frac{\Delta P}{\Delta V/V}$$
$$2.2x10^9 = -\frac{P-0}{-0.008}$$

P = 17600 KPa

4. At a depth of 7.5km in the ocean, the pressure is 75Mpa. Assume a specific weight at the surface of 10 KN/m^2 and an average bulk modulus of elasticity of 2.5 Gpa for that pressure range. Find (a) the change in specific volume between the surface and 7.5km (b) the specific volume at 7.5km and (c) the specific weight at 7.5km.

Solution:
Pressure at 7.5km (P2) = 75 Mpa = 75x10^6 N/m^2
Specific weight at the surface (γ_1) = 10 KN/m^2 = 10x1000 = 10000 N/m^2
Bulk modulus of elasticity at the surface (K) = 2.5 Gpa = 2.5x10^9 N/m^2
Change in specific volume (Δv_s) = ?
Specific volume at 7.5km (vs_2) = ?
Specific weight at 7.5km (γ_2) = ?

(a) Density at the surface (ρ_1) = γ_1/g = 10000/9.81 = 1019.4 kg/m^3
Specific volume at the surface (vs_1) = $1/\rho_1$ = 1/1019.4 = 0.000981 m^3/kg
Bulk modulus in terms of specific volume is

$$K = -\frac{\Delta P}{\Delta v_s/vs_1}$$
$$2.5x10^9 = -\frac{75x10^6 - 0}{\Delta v_s/0.000981}$$

Δv_s = -0.0000294 m^3/kg

(b) vs_2 = vs_1 + Δv_s = 0.000981-0.0000294 = 0.000951 m^3/kg
(c) Density at 7.5km (ρ_2) = $1/vs_2$ = 1/0.000951 = 1051.5 kg/m^3
$\gamma_2 = \rho_2 g$ = 1051.5x9.81 = 10315 N/m^3

5. The surface tension of mercury and water at 60^0 c are 0.47N/m and 0.0662N/m respectively. What capillary height change will occur in these two fluids when they are in contact with air in a glass tube of radius 0.30mm? Use θ = 130^0 for mercury and 0^0 for water.

Solution:
Radius of tube (r) = 0.30 mm = 0.3/1000 = 0.0003m
Surface tension for mercury (σ_m) = 0.47N/m
Surface tension for water (σ) = 0.0662N/m
θ = 130^0 for mercury and 0^0 for water
Capillary height change for mercury (hm) = ?

Capillary height change for water (h) = ?

$hm = \frac{2\sigma_m Cos\theta}{\rho_m gr} = \frac{2x0.47xCos130}{13600x9.81x0.0003}$ = -0.0149m = -14.9mm

$h = \frac{2\sigma Cos\theta}{\rho gr} = \frac{2x0.0662xCos0}{1000x9.81x0.0003}$ = 0.0445m = 44.5mm

6. In a fluid the velocity measured at a distance of 75mm from the boundary is 1.125m/s. The fluid has absolute viscosity 0.048 NS/m^2 and relative density 0.9. What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution. Also calculate kinematic viscosity.

Solution:
Change in velocity (du) =1.125-0 = 1.125m/s
Change in distance (dy) = 75 – 0 = 75mm = 75/1000 m = 0.075m
Absolute viscosity (μ) = 0.048 NS/m^2
relative density (S)= 0.9
velocity gradient (du/dy) = ?
Shear stress (τ) = ?
Kinematic viscosity (υ) = ?

du/dy = 1.125/0.075 =15 s^{-1}

$\tau = \mu \frac{du}{dy} = 0.048x15$ = 0.72N/m^2

Density of fluid (ρ) = Sxρ_{water} = 0.9x1000 = 900kg/m^3
υ = μ/ρ = 0.048/900 = 5.3x10^{-5} m^2/s

7. The velocity distribution of a viscous liquid (dynamic viscosity = 9 Poise) flowing over a fixed plate is given by u = 0.85y - y^2 (u is velocity in m/s and y is the distance from the plate in m). What are the shear stresses at the plate surface and at y=0.3 m?

Solution:
u = 0.85y - y^2
Dynamic viscosity (μ) = 9 Poise =9/10 = 0.9 NS/m^2
Shear stress (τ) at plate (for y = 0) = ?
Shear stress (τ) for y = 0.3m = ?

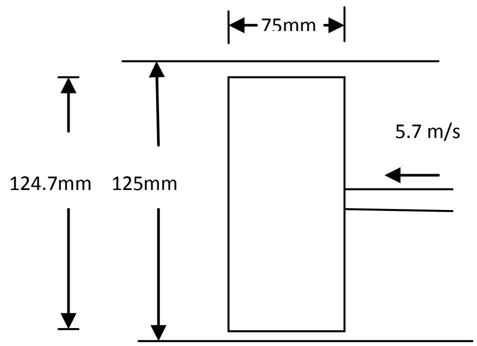
du/dy = d(0.85y - y^2)/dy = 0.85-2y

$$\tau = \mu \frac{du}{dy} = 0.9(0.85 - 2y)$$

For y = 0
Shear stress (τ) = 0.9(0.85-2x0) = 0.765 N/m^2

For y = 0.3m
Shear stress (τ) = 0.9(0.85-2x0.3) = 0.225 N/m^2

8. A piston is moving through a cylinder at a speed of 5.7m/s as shown in fig. The film of oil separating the piston from the cylinder has a viscosity of 0.95 Ns/m^2. What is the force required to maintain this motion?

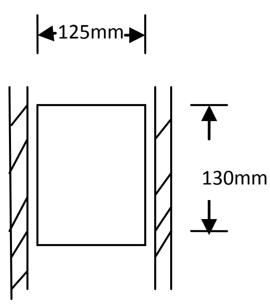


Solution:
Speed of piston = 5.7m/s
Viscosity of oil (μ) = 0.95 NS/m^2
Diameter of piston (D) =124.7mm = 0.1247m
Length of piston (L) = 75mm = 0.075m
dv = 5.7m/s
dy = dr = (125-124.7)/2 mm = 0.15mm = 0.00015m
Frictional Force (F) = ?

F = Shear stress(τ) at the piston surface x surface area of piston (A)

$F = \mu \frac{du}{dy} (\pi DL) = 0.95x \frac{5.7}{0.00015} (\pi x 0.1247 x 0.075)$ = 1060.7N

9. A piston of weight 90N slides in a lubricated pipe. The clearance between piston and pipe is 0.025mm. If the piston decelerates at 0.6m/s^2 when the speed is 0.5m/s, what is the viscosity of the oil?



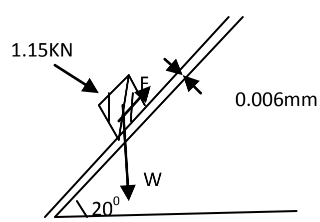
Solution:
Weight of piston (W) = 90N
Clearance (dy)= 0.025mm = 0.000025m
Deceleration (a) = 0.6m/s^2
Change in velocity (du) = 0.5m/s
Diameter of piston (D) = 125mm = 0.125m
Length of piston (L) = 130mm = 0.13m
Viscosity of oil (μ) = ?

Frictional force (F) = Shear stress(τ) at the piston surface x surface area of piston (A)
$F = \mu \frac{du}{dy} (\pi DL) = \mu x \frac{0.5}{0.000025} (\pi x 0.125 x 0.13) = 1021.02\mu$
Summing the forces

$$\sum F = ma$$
$$W - F = \frac{W}{g} a$$
$$90 - 1021.02\mu = \frac{90}{9.81}(-0.6)$$

μ = 0.094 NS/m^2

10. A square block weighing 1.15KN and 250mm on an edge slides down an incline on a film of oil 6μm thick. Assuming a linear velocity profile in the oil, calculate the terminal speed of the block. The viscosity of the oil is 0.007 NS/m^2.



Solution:
Weight of block (W) = 1.15KN =1150N
Side of block (L) = 250mm = 0.25m
Thickness (dy) = 6μm = 6x10^{-6}m
Viscosity of oil (μ) = 0.007 NS/m^2
Terminal velocity (u) = ?

Frictional force (F) = Shear stress(τ) at block surface x surface area of the block (A)
$F = \mu \frac{du}{dy} (L^2) = 0.007x \frac{u}{6x10^{-6}} (0.25^2) = 72.9u$
Component of W in the direction of F is WSin20

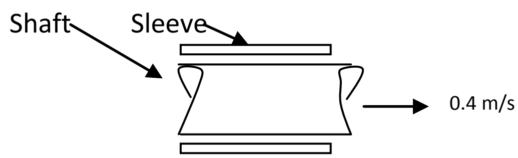
At the terminal condition, equilibrium occurs.

F = Wsin20

72.9u = 1150xSin20

u = 5.4 m/s

11. A shaft 70mm in diameter is being pushed at a speed of 0.4m/s through a bearing sleeve 70.2mm in diameter and 250mm long. The clearance, assumed uniform, is filled with oil of kinematic viscosity 0.005 m^2/s and sp gr 0.9. Find the force exerted by the oil on the shaft.



Solution:

Diameter of shaft (D) = 70mm = 0.07m

Length of shaft (L) = 250mm = 0.25m

Change in velocity (du) = 0.4m/s

Kinematic viscosity of oil (υ) = 0.005 m^2/s

Sp gr of oil (S) = 0.9

Density of oil (ρ) = 0.9x1000 = 900kg/m^3

Dynamic viscosity (μ) = υρ = 0.005x900 = 4.5 NS/m^2

Clearance (dr) = dy = (70.2-70)/2 mm = 0.1mm = 0.0001m

Force exerted by the oil on the shaft (F) = ?

F = Shear stress(τ) at the shaft x surface area of shaft (A)

$F = \mu \frac{du}{dy} (\pi DL) = 4.5x\frac{0.4}{0.0001}(\pi x 0.07x0.25)$ = 990N

12. A shaft 75mm in diameter is fixed axially and rotated inside a sleeve of diameter 75.2mm at 200rpm. The length of the shaft is 200mm. Determine the resisting torque exerted by the oil and the power required to rotate the shaft. Take viscosity of oil = 5 NS/m^2.

Solution:

Diameter of shaft (D) = 75mm = 0.075m

Radius of shaft (r) = 0.075/2 = 0.0375m

Length of shaft (L) = 200mm = 0.2m

Speed of shaft (N) = 200rpm

Dynamic viscosity of oil (μ) = 5 NS/m^2

Clearance (dr) = dy = (75.2-75)/2 mm = 0.1mm = 0.0001m

Torque exerted by the oil on the shaft (T) = ?

Power required to rotate shaft (P) = ?

Angular velocity (ω) = $\frac{2N\pi}{60} = \frac{2x200x\pi}{60}$ = 20.94rad/s
Tangential velocity (u) = $r\omega$ = 0.0375x20.94 = 0.785m/s
du = 7.85m/s

Frictional force (F) = Shear stress(τ) at the shaft x surface area of shaft = $\mu\frac{du}{dy}\ (\pi DL)$
$= 5x\frac{0.785}{0.0001}(\pi x0.075x0.2)$ = 1849.6N
$T = Fr$ = 1849.6x0.0375= 69.4Nm
$P = F\ u$ = 1849.6x7.85 = 145194W = 1451.9 KW (or use P = Tω)

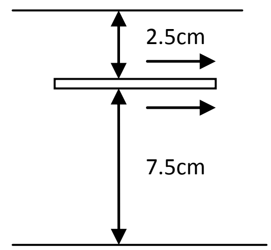
13. A cylinder of 12cm radius rotates concentrically inside a fixed cylinder of 12.6cm radius. Both cylinders are 0.3m long. Determine the viscosity of the liquid that fills the space between the cylinders if a torque of 0.9 Nm is required to maintain an angular velocity of 60rpm.

Solution:
Radius of outer cylinder (r_1) = 12.6cm = 0.126m
Radius of inner cylinder (r_2) = 12cm = 0.12m
dr =dy = 0.126-0.12 = 0.006m
Length of cylinder (L) = 0.3m
Torque (T) = 0.9Nm
N= 60rpm
Viscosity of the liquid (μ) = ?

Angular velocity (ω) = $\frac{2N\pi}{60} = \frac{2x60x\pi}{60}$ = 6.28rad/s
Tangential velocity of inner cylinder (u) = $r_2\omega$ = 0.12x6.28 = 0.75m/s
du = 0.75m/s
Frictional force (F) = Shear stress(τ) x surface area (A)
$T = F\ r_2 = \mu\frac{du}{dy}\ (2\pi r_2 L)r_2$
$0.9 = \ \mu\frac{0.75}{0.006}(2\pi x0.120x0.3)x0.120$
μ = 0.265 NS/m^2

14. A flat plate $0.3m^2$ in area moves edgewise through oil between large fixed parallels 10cm apart. If the velocity of plate is 0.6m/s and the oil has a kinematic viscosity of 0.45 stokes and specific gravity 0.8, calculate the drag force when (i) the plate is 2.5cm from one of the planes and (ii) the plate is equidistant from both the planes.

Solution:
Area of plate (A) = 0.3m^2
Velocity of plate (u) = 0.6m/s
dv = 0.6m/s
Kinematic viscosity (υ) = 0.45 stokes = 0.45x10^{-4} m^2/s
Sp. gr. of fluid (S) = 0.8



Density of fluid (ρ) = 0.8x1000 = 800kg/m^3
Dynamic viscosity (μ) = υρ = 0.45x10^{-4}x800 = 0.036 NS/m^2

(I) dy1 = 2.5cm = 0.025m
dy2 = 10-2.5 = 7.5cm = 0.075m
Total force (F) = Force on side1 (F_1) + Force on side2 (F_2) = τ_1 A + τ_2 A = (τ_1 + τ_2) A
$F = \left(\mu \frac{du}{dy1} + \mu \frac{du}{dy2}\right) A = \left(0.036 \frac{0.6}{0.025} + 0.036 \frac{0.6}{0.075}\right) 0.3$ = 0.345N

(I) If the plate is equidistant, $dy_1 = dy_2 = dy$ = 10/2 = 5cm = 0.05m
Total force (F) = Force on side1 (F_1) + Force on side2 (F_2) = τ_1 A + τ_2 A = (τ_1 + τ_2) A
$= \left(\mu \frac{du}{dy} + \mu \frac{du}{dy}\right) A = 2\mu \frac{du}{dy} A = 2x0.036x \frac{0.6}{0.05} x0.3$ = 0.26N

15. A Newtonian fluid fills the gap between a shaft and a concentric sleeve. When a force of 780N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 2m/s. If a 1400N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

Solution:
Force (F1) = 780N
Velocity (u1) = 2m/s
Force (F2) = 1400N
Velocity (u2) = ?

$\tau = \mu \frac{du}{dy}$
Also, $\tau = \frac{F}{A}$

$$\mu \frac{du}{dy} = \frac{F}{A}$$

$\frac{F}{du} = \mu \frac{A}{dx}$ = const.
Therefore, $\frac{F1}{du1} = \frac{F2}{du2}$

$\frac{780}{2} = \frac{1400}{u2}$

u2 = 3.6m/s

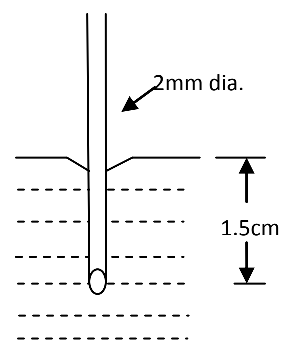
16. The surface tension of water in contact with air is 0.0725N/m. The pressure outside the droplet of water of diameter 0.02mm is atmospheric ($1.032x10^5$ N/m^2). Calculate the pressure within the droplet of water.

Solution:
Diameter of droplet = $0.02x10^{-3}$ m
Radius of droplet (r) = $0.01x10^{-3}$ m
Surface tension (σ) = 0.0725N/m
Pressure outside droplet (P_o) = $1.032x10^5$ N/m^2
Pressure within droplet (P_d) = ?

Pressure inside droplet in excess of outside pressure (P) = $\frac{2\sigma}{r} = \frac{2x0.0725}{0.01x10^{-3}}$ = 14500 N/m^2
P_d = P+P_o = 14500+$1.032x10^5$ = 117700 N/m^2

17. The tip of glass tube with an internal diameter of 2mm is immersed to a depth of 1.5cm into a liquid of sp. gr. 0.85. Air is forced into the tube to form a spherical bubble just at the lower end of the tube. Estimate surface tension of liquid if the pressure in the bubble is 200Pa.

Solution:
Radius of bubble (r) = 1mm = 0.001m
Pressure inside bubble (P_i) = 200Pa
Depth of liquid (h) = 1.5cm = 0.015m
Specific weight of liquid (γ) = $sp\ gr\ x\gamma_{water}$ = 0.85x9810 = 8338.5Pa
Surface tension (σ) = ?



Pressure outside the bubble (P_o) = γh = 8338.4x0.015 = 125.07Pa

Pressure within droplet in excess of outside pressure (P) = 200-125.07 = 74.93Pa

$P = \frac{2\sigma}{r}$

$\sigma = \frac{Pr}{2} = \frac{74.93x0.001}{2}$ = 0.0375N/m

18. Three cylindrical tubes of 0.5m length are placed co-axially and the central tube is rotated at 5 rpm applying a torque of 6 Nm. Determine the viscosity of oil which fills the space between tubes. Take r_1, r_2 and r_3 as 0.15m, 0.152m and 0.154m.

Solution:

Radius of cylinder1 (r_1) = 0.15m

Radius of cylinder2 (r_2) = 0.152m

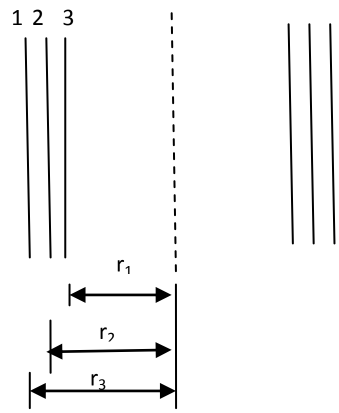
Radius of cylinder3 (r_3) = 0.154m

Length of cylinders (L) = 0.5m

Applied torque to cylinder2 (T) = 6 Nm

rpm of cylinder2 (N) = 6

Viscosity of oil (μ) =?



Torque is transmitted through oil layer from tube 2 to tube 1 and tube 3.

Angular velocity of tube 2 (ω) = $\frac{2N\pi}{60} = \frac{2x5x\pi}{60}$ = 0.5236rad/s

Tangential velocity tube2 (u_2) = $r_2\omega$ = 0.152x0.5236= 0.07958m/s

du = 0.07958m/s

F_1 = Viscous force on tube 1, F_2 = Viscous force on tube 2

For small space between cylinders, the velocity gradient may be assumed to be a straight line and average radius r can be taken.

$r_a = (r_1+r_2)/2 = 0.151$m, $r_b = (r_2+r_3)/2 = 0.153$m

$dy_1 = r_2-r_1 = 0.002$m, $dy_2 = r_3-r_2 = 0.002$m

$$T = F_1 r_a + F_2 r_b$$

$$T = \mu \frac{du}{dy_1} (2\pi r_a L)\, r_a + \mu \frac{du}{dy_2} (2\pi r_b L)\, r_b$$

$$6 = \mu \frac{0.07958}{0.002} (2\pi x 0.151 x 0.5)\, 0.151 + \mu \frac{0.07958}{0.002} (2\pi x 0.153 x 0.5)\, 0.153$$

μ = 1.038 NS/m^2

19. A fluid of absolute viscosity of 0.045 Pa-s and sp gr of 0.91 flows over a flat plate. The velocity of fluid at 70mm height over the plate is 1.13m/s. Calculate the shear stress at the solid boundary and at points 20mm above the plate considering (a) linear velocity distribution, and (b) parabolic velocity distribution with vertex at point 70mm away from the surface.

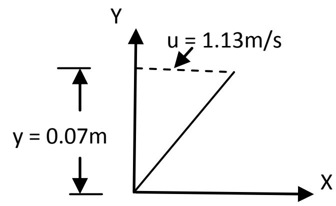
Solution:

Absolute viscosity (μ) =0.045 Pa-s

sp gr = 0.91

Velocity at 70mm from the plate = 1.13 m/s

(a) Linear velocity distribution



Equation: u = ay

a = tanθ = 1.13/0.7 = 16.1

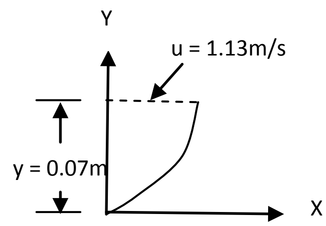
u = 16.1y

$\frac{du}{dy} = 16.1$

As the velocity gradient is constant, shear stress is also constant throughout.

Shear stress $(\tau) = \mu\frac{du}{dy}$ = 0.045x16.1 = 0.726 Pa (throughout)

(b) Parabolic velocity distribution



Equation: $u = ay^2 + by + c$

At y = 0, u = 0

c = 0

At y = 0.07m, $\frac{du}{dy} = 0$

$\frac{du}{dy} = 2ay + b$

2ax0.07+b = 0

b = -0.14a

At y = 0.07m, u = 1.13m/s

$1.13 = ax0.07^2 + bx0.07 + c$

Substituting the values of b and c

$1.13 = ax0.07^2 - 0.14ax0.07 + 0$

a =-230.61

b = -0.14x-230.61 = 32.29

Hence, $u = -230.61y^2 + 32.29y$

$\frac{du}{dy} = -461.22y + 32.29$

At y = 0, $\frac{du}{dy} = 32.29$

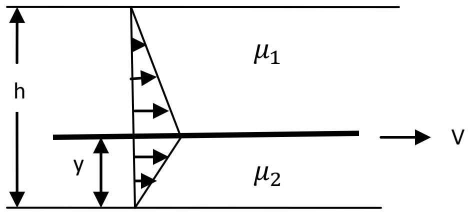
At y = 0.02m, $\frac{du}{dy} = -461.22x.02 + 32.29$ = 23.0656

Shear stress at y = 0, $(\tau_0) = \mu\frac{du}{dy}$ = 0.045x32.29 = 1.453 Pa

Shear stress at y = 0.02m, $(\tau_{0.02}) = \mu\frac{du}{dy}$ = 0.045x23.0656 = 1.038 Pa

20. Through a narrow gap of height h, a thin plate of large extent is pulled at a velocity V. On one side of the plate is oil of viscosity μ_1 and on the other side oil of viscosity μ_2. Calculate the position of the plate so that (a) the shear force in the two sides of the plate is equal, and (b) the pull required to drag the plate is minimum.

Solution:



(a) shear stress on upper face = $\tau_1 = \mu_1 \frac{du}{dy} = \mu_1 \frac{V}{h-y}$

shear stress on lower face = $\tau_2 = \mu_2 \frac{du}{dy} = \mu_2 \frac{V}{y}$

$\tau_1 = \tau_2$

$\mu_1 \frac{V}{h-y} = \mu_2 \frac{V}{y}$

$y = \frac{\mu_2 h}{\mu_1 + \mu_2}$

(b)F = pull required to drag the plate per unit area

$F = \tau_1 + \tau_2$

$F = \mu_1 \frac{V}{h-y} + \mu_2 \frac{V}{y}$

For F to be minimum,

dF/dy = 0

$\frac{dF}{dy} = \mu_1 \frac{V}{(h-y)^2} - \mu_2 \frac{V}{y^2}$

$0 = \mu_1 \frac{V}{(h-y)^2} - \mu_2 \frac{V}{y^2}$

$\frac{\mu_1}{\mu_2} = \frac{h^2 - 2hy + y^2}{y^2}$

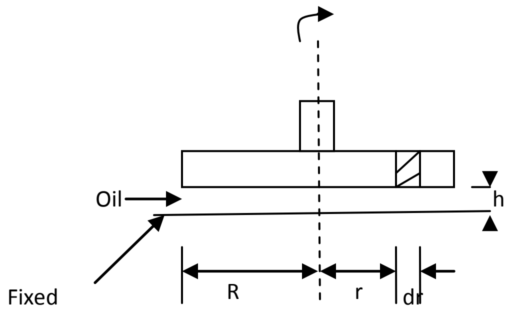
$\frac{h^2}{y^2} - \frac{2h}{y} + \left(1 - \frac{\mu_1}{\mu_2}\right) = 0$

Solving for h/y and neglecting –ve root

$\frac{h}{y} = 1 + \sqrt{\frac{\mu_1}{\mu_2}}$

$y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}}$

21. A 120mm circular disc rotates on a table separated by an oil of film of 2mm thickness. Find the viscosity of the oil if the torque required to rotate the disc at 60 rpm is 4×10^{-4} Nm. Assume the velocity gradient in the oil film to be linear.

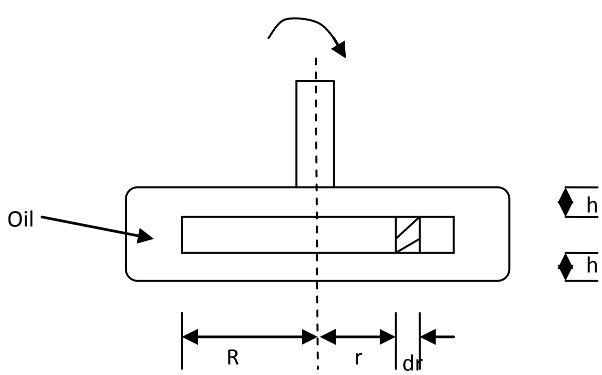


Solution:
Radius of disc (R) = 120/2 = 60mm = 0.06m
Thickness (h) = 2mm = 2x10^{-3}m
Torque (T) = 4x10^{-4} Nm
N = 60 rpm
Angular velocity (ω) = $\frac{2N\pi}{60} = \frac{2x60x\pi}{60}$ = 6.28rad/s
Viscosity of oil (μ) = ?

Consider an elementary ring or disc at radius r and having a width dr.

Torque on the element (dT) = Shear force x r
$= \tau dAr = \mu\frac{du}{dy}(2\pi rdr)r = \mu\frac{u}{h}(2\pi rdr)r = \mu\frac{r\omega}{h}(2\pi rdr)r = \frac{2\pi\mu\omega}{h}r^3dr$
Total torque (T) = $\int_o^R \frac{2\pi\mu\omega}{h}r^3dr = \frac{\pi\mu\omega R^4}{2h}$
$4x10^{-4} = \frac{\pi\mu x6.28x0.06^4}{2x2x10^{-3}}$
μ = 0.0062 Ns/m^2

22. A disk of radius R rotates at an angular velocity ω inside an oil bath of viscosity μ as shown in figure. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.



Radius of disc = R

Clearance = h on both sides
Torque = T
N = rpm
Angular velocity = (ω) = $\frac{2N\pi}{60}$
Viscosity of oil = μ

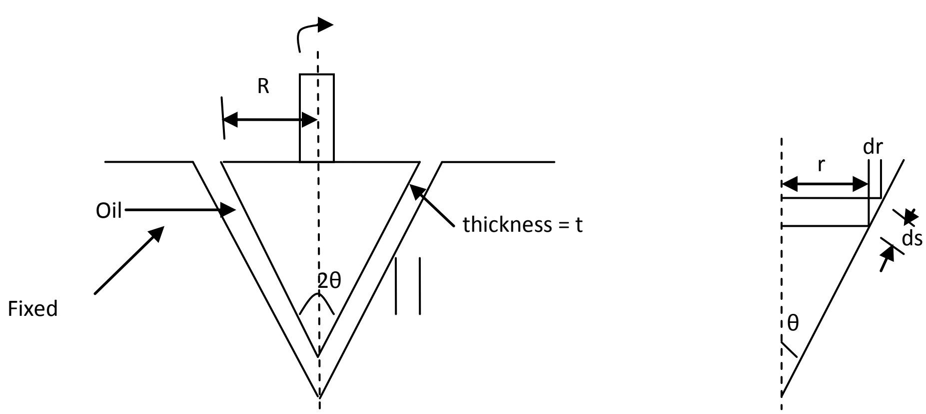
Consider an elementary ring or disc at radius r and having a width dr.

Torque on the element (dT) = Shear force x r
$= 2\tau dAr = 2\mu\frac{du}{dy}(2\pi rdr)r = 2\mu\frac{u}{h}(2\pi rdr)r = 2\mu\frac{r\omega}{h}(2\pi rdr)r = \frac{4\pi\mu\omega}{h}r^3dr$
Total torque (T) = $\int_o^R \frac{4\pi\mu\omega}{h}r^3dr = \frac{\pi\mu\omega R^4}{h}$

23. Distilled water stands in a glass tube of 10mm diameter at a height of 25mm. What is the true static height? Take surface tension of water = 0.074 N/m and angle of contact = 0^0.

Solution:
Diameter of tube (d) = 10 mm = 0.01m
Surface tension of water (σ) = 0.074N/m
θ = 0^0 for water
Capillary height change for water (h) is
$h = \frac{4\sigma Cos\theta}{\rho gd} = \frac{4x0.074xCos0}{1000x9.81x0.01}$ = 0.003m = 3mm
True static height = 25-3 = 22mm

24. A solid cone of maximum radius R and vertex angle is to rotate at an angular velocity ω . An oil of viscosity μ and thickness t fills the gap between the cone and the housing, Derive an expression for the torque required and the rate of heat dissipation in the bearing.



Solution:

Maximum radius of cone = R

thickness of oil = t

Angular velocity = (ω) = $\frac{2N\pi}{60}$

Viscosity of oil = μ

Consider an elementary area dA at radius r of the cone.

Torque on the element (dT) = Shear force x r

$$= \tau dAr = \mu\frac{du}{dy}(2\pi r dr)r = \mu\frac{u}{t}(2\pi r ds)r = \mu\frac{r\omega}{t}\left(2\pi r\frac{dr}{sin\theta}\right)r = \frac{2\pi\mu\omega}{tsin\theta}r^3dr$$

Total torque (T) = $\int_o^R \frac{2\pi\mu\omega}{tsin\theta}r^3dr = \frac{\pi\mu\omega R^4}{2tsin\theta}$

Rate of heat dissipation = power utilized in overcoming resistance = $T\omega = \frac{\pi\mu\omega^2R^4}{2tsin\theta}$