Static Mechanics

Shear Stress

UNZA., Department of Mechanical Engineering

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Shear Stress

2.1 Shear Stress:

If the applied load P consists of two equal and opposite parallel forces not in the same line (as in Fig 2.1), then there is a tendency for one part of the body to slide over or shear from the other part across any section LM.

If the cross-section at LM measured parallel to the load is A, then the average shear stress $\boldsymbol{\tau} = \boldsymbol{P/A}$. **N/m2**

Notice that shear stress is tangential to the area over which it acts.

The most common occurrence of pure shear is in riveted joints.

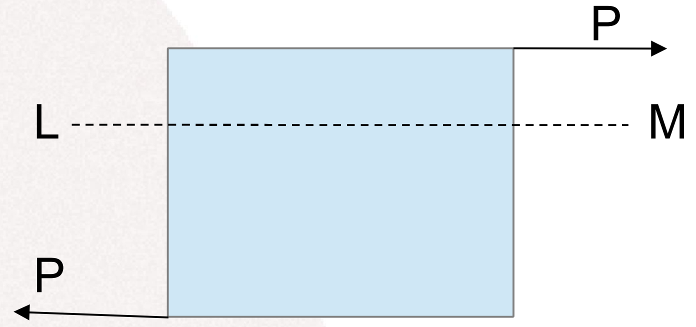

Fig. 2.1

Fig1.1

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- **2.2 Complementary Shear Stress**

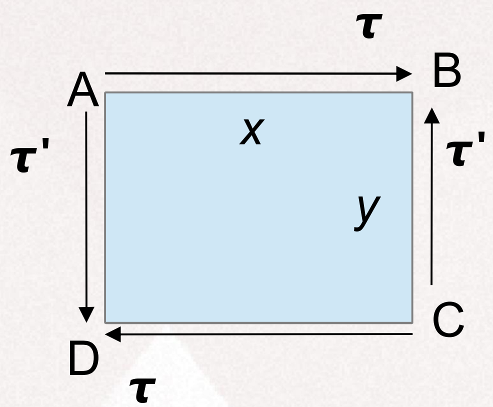


Fig 2.2

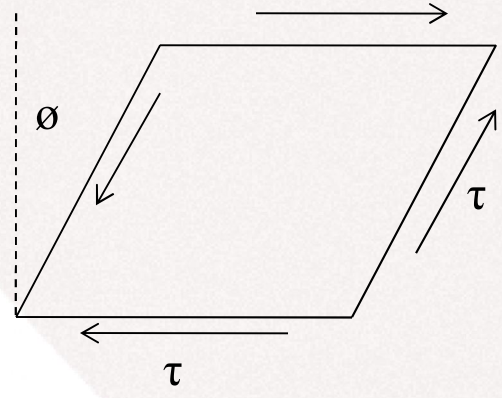
Let ABCD (Fig 2.2), be a small rectangular element of sides x,y, and z perpendicular to the figure. Let there be a shear stress τ acting on planes AB and CD.

» It is clear that these stresses will form a couple $(\tau . xz)y$ which can only be balanced by tangential forces on planes AD and BC (any normal forces which exist will balance out in pairs). These are known as complementary shear stresses.

Let τ' be the complementary shear stress induced on planes AD and BC. Then for equilibrium $(\tau . xz)y = (\tau' . yz)x$, i.e. $\tau' = \tau$ showing that every shear stress is accompanied by an equal complementary shear stress on planes at right angles.

Example 1. A flange coupling joining two sections of shaft is required to transmit 250 kW at 1000 r.p.m.. If six bolts are to be used on a pitch circle diameter of 14 cm, find the diameter of the bolts. Allowable mean shear stress 75N/mm2.

2.3 Shear Strain. Shear strain or slide *ϕ, can be defined as the change in the right angle. It is measured in radians and is dimensionless*



2.4 Modulus of Rigidity For elastic materials

- shear strain is proportional to the shear stress producing it within certain limits.
- The ratio Shear Stress / Shear Strain is called Modulus of Rigidity, i.e.

$$G = \tau / \phi \ N/mm2$$

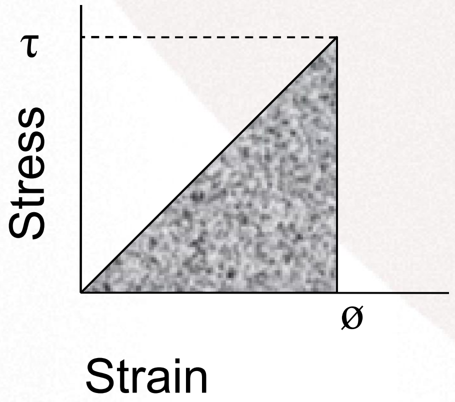
2.5 Strain Energy in shear

Within limit of proportionality stress is proportional to strain, and

Strain energy (U) = Work done in straining

= 1/2(Final couple) x(Angle turned through)

For a gradually applied stress (work done is proportional to shaded area in Fig. 2.5),



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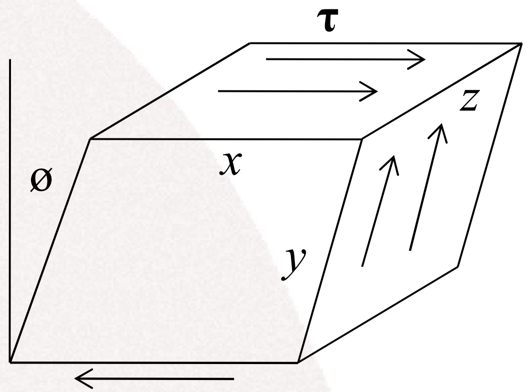
- **Strain Energy continued …**
- i.e $U = ½ (\tau yz.x)ø$

  $= ½.\tau xyz.\tau/G$

  $= (\tau^2/2G)$ x Volume
- Compare with $\sigma^2/2E$ for direct stress).
- The units again Nm (joules)



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Problem 1. pg 33 Estimate the force required to punch out circular blanks 6 cm diameter from plate 2 mm thick. Ultimate shear stress = 300 N/mm2.

2.6 Torsion – Circular Shafts

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- **Circular Shafts continued**
- For a solid shaft:

  $J = \pi D4/32$

- And the maximum stress $\tau\hat{} = 16T/\pi D^3$, at $r = D/2$
- For a hollow shaft: $J = (\pi/32)(D4 – d4)$ and $\tau\hat{} = 16D.T/\pi(D4 – d4)$, at $r = D/2$
- Torsional stiffness k is defined as torque per radian twist, i.e

  $k = T/\theta = GJ/l$

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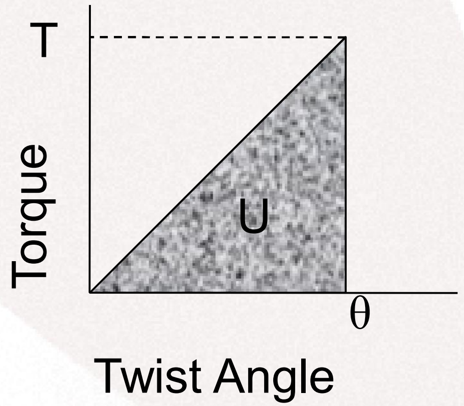
2.7 Strain Energy in Torsion

- Total strain energy of a shaft of length l under the action of a torque T is the work done in twisting, i.e
- $U = 1/2T\theta$ (most useful if T and θ have been previously found)



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- **Strain Energy in Torsion continued**
- Expressed in terms of maximum stress τ^ for a solid shaft

$$U = \tfrac{1}{2}(\pi D^3\ \tau\hat{}/16) \text{ x } (2\tau\hat{}l/GD)$$

$$= (\tau\hat{}^2/4G) x \pi D^2 l/4$$

$$= (\tau\hat{}^2/4G) \text{ x volume}$$

- Note that this gives the total strain energy over the whole shaft, for which shear stress is varying from zero at the axis to τ^ at the outside.
- The maximum strain energy per unit volume is $\tau\hat{}^2/2G$

References

- G.H Ryder, Strength of Materials,