ENGINEERING MECHANICS
MEC107J2

SECTION C

THERMODYNAMICS and FLUID MECHANICS

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Preface

Engineering mechanics is a massive area of study encompassing analysis of forces exerted on a body to the transfer of energies between bodies and systems. The study of mechanical science is traditionally subdivided into;

- Section A Statics and strength of materials,
- Section B Dynamics and,
- Section C Thermodynamics and Fluid Mechanics.

These notes then provide background information, theories and examples for the latter section of the work, i.e. section C of the module. The examples given in the notes follow a predefined set of steps that if adhered to provide a clear and simple method for solving even the most complex of problems encountered in this module. It is however of utmost importance that you also complete the tutorials given during the semester using these methods.

Units, Prefixes and Useful Constants

Units

Description	Symbol	Units
Distance between two points	l, z	Metres, m *
Area of a surface	A	m^2
Volume of a three dimensional object	V	m^3
Time	t	Seconds s
Velocity of a body	C	m/s
Acceleration of a body	a	m/s^2
Flow rate (volumetric or mass based, the latter is used more widely in thermodynamics while volumetric flow rates are used in fluid mechanics)	$\dot{V}$, $\dot{m}$	m^3/s, kg/s
Mass (the quantity of matter in a body)	m	Kilograms kg
Density (the quantity of matter within a known volume)	r	kg/m^3
Force (Is defined from the quantity of matter, or mass, and the acceleration to which the matter is exposed)	F	kg·m/s^2 or Newtons N
Pressure (The force exerted divided by the area of application. This value is calculated in the same way as stress)	p	N/m^2
Work done (Normally defined as the energy required to move a body calculated as the force times distance moved)	W	N·m or Joules J
Power (The rate of doing work. The relationship between power and work is similar to the relationship between acceleration and velocity.)	P	J/s
Temperature (The two most common scales used in mechanical science are the Celsius and Kelvin scales. Kelvin is used almost exclusively in thermodynamics.)	T	degrees Celsius °C or degrees Kelvin °K
Heat (The transfer of energy from a hotter to a colder body)	Q	J
Heat Capacity (The energy required to change the temperature of one kilogram of matter by one degree Kelvin)	c	J/kg·K
Specific volume (All specific values are specified per kilogram of matter, therefore this is simply the volume of matter per kilogram)	v	m^3/kg
Specific work (define in much the same way)	w	J/kg

** The highlighted rows indicate base units; the remaining units are then derived from these.*

Prefixes

Multiplication Factor	Prefix	Symbol
1 000 000 000 000 = 10^{12}	terra	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ (Greek symbol mu)
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p

Rules for writing Engineering units

1. Always write a solution to four significant figures, i.e. 0.435×10^{-3}, not 0.00435 or even 0.004.
2. With the exception of kilograms, use only one prefix in the units, i.e. GN/m^2 not kN/mm^2.
3. When writing two multiplied units use a dot, for example N·m not Nm
4. Write the units in the simplest forms, for example N/m^2 not N/m/m

Useful Constants

Description	Symbol	Value
Acceleration due to gravity	g	9.80665 m/s^2
Atmospheric pressure	p_a	1.01325×10^5 N/m^2 or 1.01325 bar
Density of water	r_w	1000 kg/m^3
Specific heat capacity of water	c_w	4.186 kJ/kg·K
Specific heat capacity of air (Constant pressure)	c_p	1.005 kJ/kg·K
Specific heat capacity of air (Constant volume)	c_v	0.718 kJ/kg·K
Specific gas constant for air	R	0.287 kJ/kg·K

Compulsory Text

Applied Thermodynamics For Engineering Technologists, Eastop and McConkey, 5th Edition ISBN 0-582-09193-4

Table of Contents

Chapter 1: Introduction to Applied Thermodynamics

Introduction

To understand this section of mechanical science the meaning of Thermodynamics must first be explained, i.e. 'Thermodynamics is the science of the relations between **heat** and other (mechanical, electrical etc.) energy', OED. When this definition is committed to memory, application to every day items (car engines, Domestic central heating and solar powered calculators) becomes possible. The schematic in Figure 1, defines the interrelationship between the energy sources, **Solar**, **Geothermal**, **Chemical**, **Nuclear**, **Hydro**, **wind**, **waves**, **tidal** and the eventual conversion to **thermal**, **mechanical** and **electrical** energies. The bold lines and circles illustrate the most common links and energy source. There exists tenuous links between for instance chemical fuels and electricity (batteries), Solar and electrical (photovoltaic cells), thermal and electrical (thermocouple). Each of these sources and the conversion processes will be covered further in the next sections.

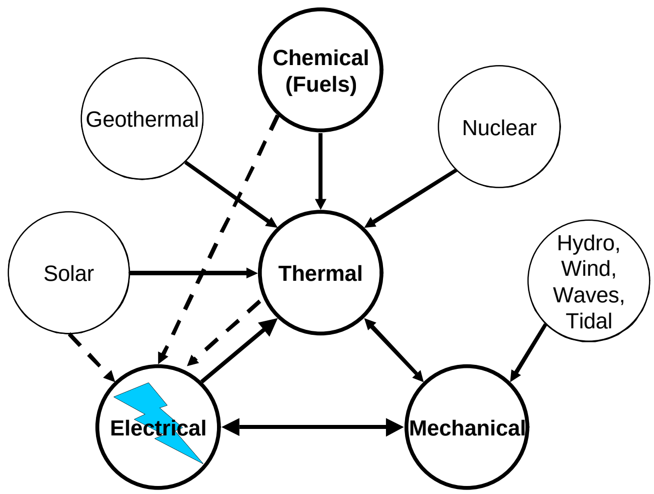


Figure 1: *Energy Conversion Diagram*[1]

The energy requirements of today's modern world are immense. The requirements range from the food required to feed over 6 billion people to the electricity required to support today's modern infrastructure. Mankind can no longer rely on the earth's finite resource of fossil fuels. It is therefore the responsibility of all of us but in particular engineers to develop renewable energy resources and more efficient systems. Renewable energy sources include solar, geothermal, wind, waves and tidal. It is also possible to create renewable chemical fuel in the form of methane, a by-product of decomposition process. Nuclear power is the most readily accessible form of renewable energy. Although nuclear energy provides an alternative to current fossil fuel power stations, it nevertheless raises questions over safety. Examples include Chernobyl, and more recently the nuclear reprocessing plant in Japan. A further disadvantage of this energy source is the heavy subsidies required from the taxpayer to support such projects.

What is Energy

The word energy comes from two Greek words meaning work within or in other words the ability to work. There are many different forms of energy e.g. the potential energy of water stored behind a dam is converted to kinetic energy as it flows through pipes towards turbines which convert the kinetic energy to mechanical energy which is then used to drive generators resulting in electrical energy. This illustrates the point that energy can neither be created nor destroyed, but rather converted from one form to another.

[1] Applied Thermodynamics For Engineering Technologists, Eastop and McConkey, 5th Edition.

There is of course a trade-off, every time energy is converted from one form to another there will be losses, e.g. heat losses, friction losses etc.. A mathematical definition of energy will be given later in this work.

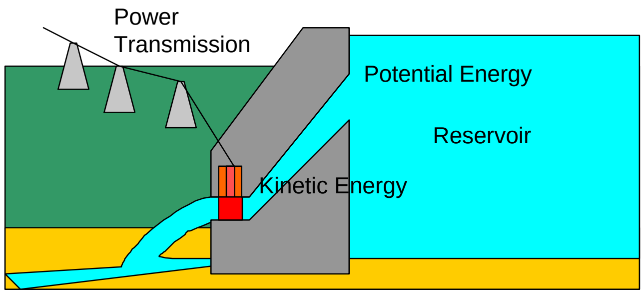


Figure 2: *Hydropower station.*

Solar Energy

The sun a is taken for granted by many of this, though poets describe beautiful images of sunrise and sunset. If, however, we view the sun from a scientific prospective we find that this giant measuring over 14 million kilometres in the under produces temperatures for of over 15 million degrees Celsius. These high temperatures are produced by the fusion of hydrogen to make helium, the opposite of the vision process used in nuclear power stations. The transfer of energy from the sun is in the form of light. Only about 70% of light from the sun reaches the surface of the earth, the remainder is reflected back into space.
Several methods exist for harnessing the energy from the sun (Figure 3);
-Heating water by using solar panels,
-Using mirrors to concentrate solar energy at one point creating temperatures in excess of 4000 degree Celsius, and
-Creating electrical energy directly, by using photovoltaic cells.

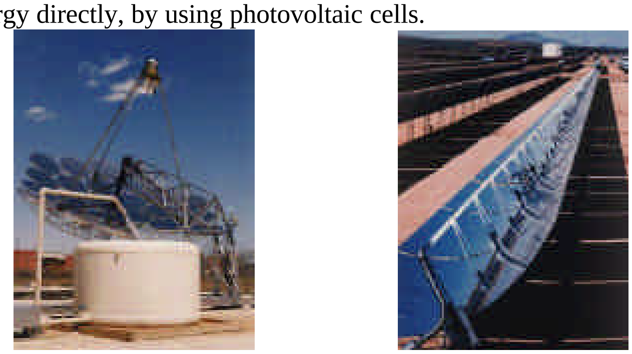


Figure 3: *Dish system for concentration sunlight, Parabolic through system.*

Geothermal Energy

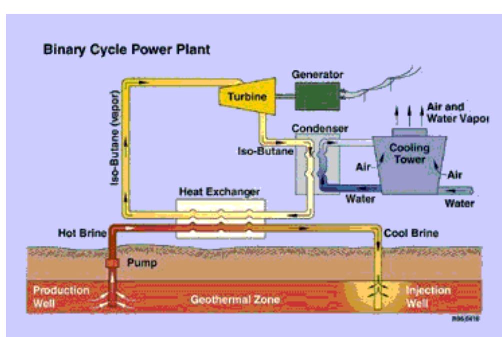


Figure 4: *Geothermal generating plant that utilises two fluids to create a vapour that drives the turbine.*

Deep inside the earth is molten core temperatures levels rise to an estimated 3000 degree Celsius. Harnessing this energy can be simple small-scale heat pump used for domestic heating during winter and cooling during the summer, or large-scale power station using the heat to create a vapour, which drive turbines to create electricity (Figure 4). There are also several schemes in Europe the use heat pumps to extract thermal energy from rivers that can then be used to heat small towns.

Chemical Fuels

Chemical fuels are produced mainly from non-renewable energy sources. These include crude oil, cool and wood. Renewable chemical fuels include methane and managed forests. Chemical fuels and in particular fossil fuels account for a major proportion of the world's energy consumption. The quantity of energy that can be extracted from these fuel sources is defined by the fuels calorific value. Most common uses of chemical fuels are the internal combustion engine both piston and turbine, and batteries. A large proportion of the world's power generation is still achieved using coal-fired power stations.

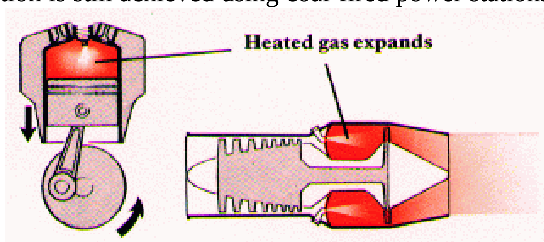


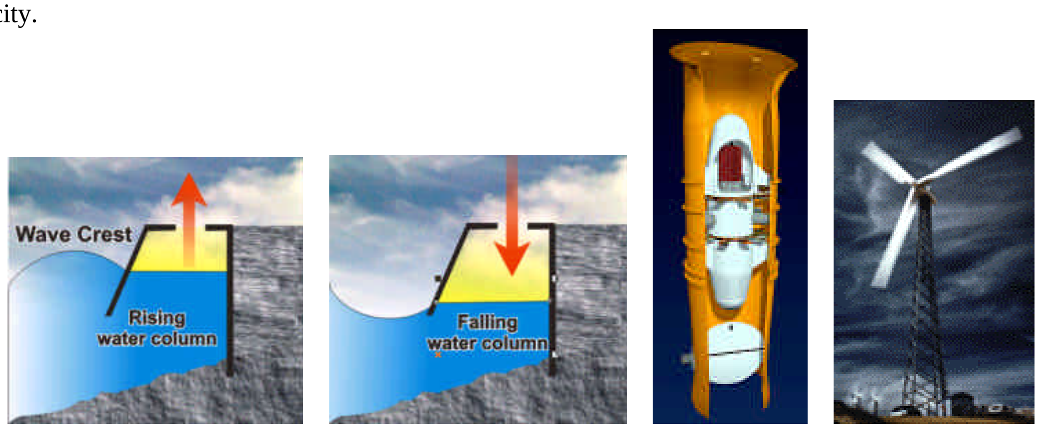
Figure 5: *Examples of the Internal Combustion Engine and Jet Engine.*[2]

Nuclear

The function of nuclear power station is not unlike that of a coal-fired power station. The difference between them is the source of heat. A coal-fired power station burns coal to provide the heat energy that converts water to steam whereas, a nuclear power station utilises nuclear fission to provide the heat source.

Other sources of energy

Over 70% of the earth's surface is covered by water. This provides a vast energy resource that can be tapped using hydroelectric power stations, wave-powered turbines and other generating devices (Figure 6). Energy from the wind has been harnessed over the centuries by using windmills. The latest designs use blades rather than old-fashioned sails. The diameter of these blades is now much larger. The computer control systems ensure that these blades are directed towards the prevailing wind and remain at a constant velocity.



(a) (b) (c) (d)

Figure 6: *(a, b, c)Wave powered turbine, (d) Wind turbine.*

[2] http://www.rolls-royce.com/education/jetengine/how.htm

Units

The discussion of energy has been kept simple so far in to illustrate the sources and uses of energies. To continue the discussion further, definitions of units and variables used to define energy are required.

Basic Units

The basic units and dimensions used in mechanical science include those listed in Table 1. The units given in these tables are base units, this is obvious for metres and seconds. The use of Kilograms for mass is not however as obvious being a multiple of 1000 from the base of grams. There are two temperature scales used in thermodynamics (Figure 7), degrees Kelvin and degrees Celsius (a further scale of Fahrenheit is used in America and meteorological applications, but will not be considered here). For each of these scales the divisions are equal, the only difference is the zero point. The Celsius scale is zero at the freezing point of water whereas the Kelvin scale is zero at 273°C below this point. This point is known as absolute zero and is defined as the theoretical point where the motions of all molecules cease. When in doubt what scale to use **ALWAYS** use the Kelvin scale.

Description	Unit	Dimension
Distance, Length	Metre (m)	[L]
Time	Seconds (s)	[T]
Mass	Kilogram (kg)	[M]
Temperature	Kelvin (K)	[θ]

Table 1: *Basic units and dimensions.*

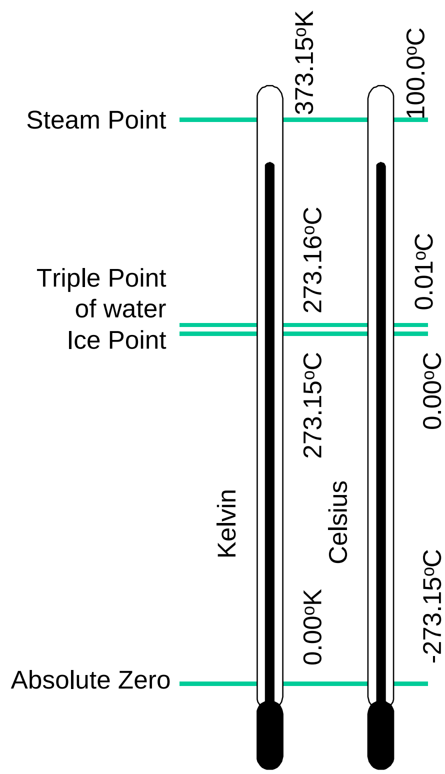


Figure 7: *Comparisons of Temperature Scales.*

Length, Area and Volume

A measurement of length *l*, using the unit metre (m) can be used to define any distance. Typical examples include height difference, *h* (m) diameter of a pipe, *d* (m) or the length of a pipe, *l* (m). Lengths can also be combined to define an area. The correct definition of an area with sides measuring *a* (m) and *b* (m) would be $a \times b = A$ (m^2). Since the units of length have been multiplied together the new unit is a multiple written as m^2. Answers to these problems should always include units. If we include a third length c (m), then we can calculate the volume by $a \times b \times c = V$ (m^3). Note that in this instance the units have now changed to m^3.

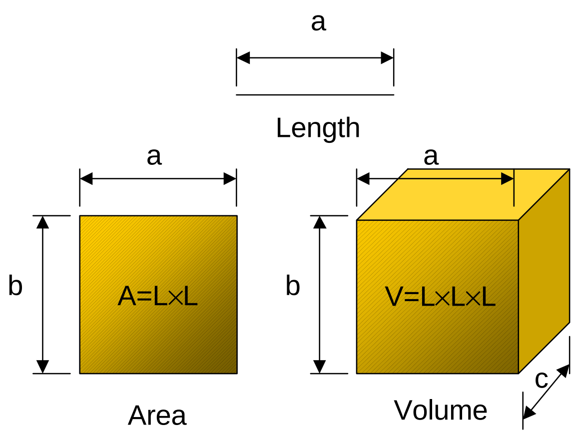


Figure 8: *Definition of length, area and volume.*

Velocity and Acceleration

If a body travels a known distance in a known time at a constant speed then the distance/time taken, or m/s defines the velocity of that body. If the velocity of the body is also changing with time then it is said to have acceleration, units m/s^2.

Density

There are three forms of density that will be encountered in Thermodynamics and fluid Mechanics: -

Mass density r, this is the mass per unit volume defined as

$$r = \frac{m}{V} \ (\text{kg/m}^3)$$

This equation can also be rearranged in terms of the mass as $m = r \times V$. if the density of aluminium can be assumed to be $2700 kg/m^3$ then for a volume of $0.5 m^3$ the mass will be 2700×0.5 which will give 1350kg.

Specific Weight w is the weight per unit volume of a substance, defined as

$$w = rg \ (\text{N/m}^3)$$

Specific density, or **relative density** s, is the ratio of the density of a substance to that of water and is defined as,

$$s = \frac{r_s}{r_w}$$

Note that there are no units for specific density as it is ratio to densities. It is commonplace to relative the density of substance in terms of relative density since the density of water is $1000 kg/m^3$. Therefore if relative density of a specific oil was defined as 0.9 then the density of the oil would be $900 kg/m^3$.

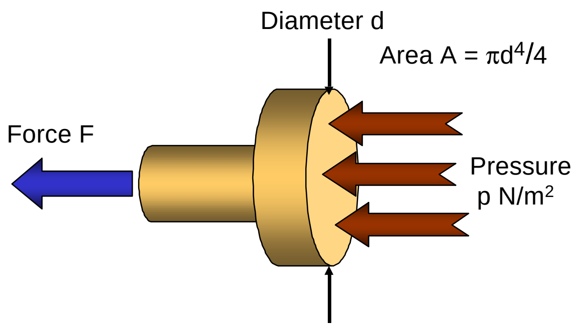
Force

From Newton’s second law force is defined as the mass of a body times the acceleration imposed on that body. The weight of a body is the force with which the earth attracts the body. Weight is therefore a measurement of force and has the units of Newton’s (N). The magnitude of this force is defined by a relationship between the mass of the body and the acceleration due to gravity g. The acceleration due to gravity is a constant of $9.81 m/s^2$. The weight of a body is then equal to the mass of the body times the acceleration due to gravity i.e. $F = mg$.

Pressure

Pressure is defined as the force per unit area with the units (N/m^2). This unit is also known as the Pascal (Pa). Both pressure and stress have the same units. The example shown below is an illustration of a cylinder with an internal pressure that in turn creates a force. A simple ketch shown below illustrates a piston being acted upon by a pressure of p N/m^2. If the area of the piston is A m^2 in the total force acting on the piston is,

$$F = p \times A \, (\text{N})$$



Note: A useful conversion for fluid mechanics is *1 bar = 100 000 N/m^2 i.e. 1 bar = 10^5 N/m^2*.
Ex. A piston 0.1 m diameter is acted upon by a gas pressure of 150 000N/m^2. What actuating force is available at the piston rod,

$$F = p \cdot A \, (\text{N})$$

Calculating the area,

$$A = \frac{p \cdot d^2}{4} \text{m}^2 = \frac{p \cdot 0.1^2}{4} = 0.00785 \, \text{m}^2$$

Thus the force is,

$$F = p \cdot A = 150000 \cdot 0.00785 = 1177.5 \, \text{N} = 1.118 \, \text{kN}$$

There are two possible pressure scales, absolute and gauge pressure. The difference can be clearly illustrated using the Bourdon gauge shown in Figure 9. Pressure is normally measured using some form of gauge, as in the case of the Bourdon gauge. The pressure measured using this gauge is known as gauge pressure. However absolute pressure measurement requires the addition of atmospheric pressure. The Bourdon gauge is exposed to atmospheric pressure that works against the pressure measurement. This reduces the pressure measurement by atmospheric pressure, typically 1 bar. Therefore to convert from gauge pressure to absolute pressure atmospheric pressure must be added to the original measurement.

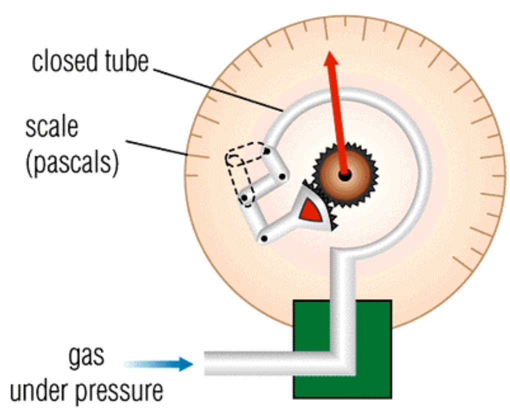


Figure 9:*Bourdon Gauge.*[3]

[3] http://ebooks.whsmithonline.co.uk/encyclopedia/13/P0005313.htm

Work

Let us assume that the force created by the above example is to be used to move a body a distance of 0.2 m. The work carried out is simply the force applied times the distance moved, in this case,

$$W = F \cdot l$$

Inserting the values,

$$W = 1177.5\ \text{N} \cdot 0.2\ \text{m} = 235.5\ \text{Nm or } 235.5\ \text{J (Joules)}$$

Note: ***Joule is the special name given to Nm.***

Energy

Energy has the same unit as work i.e. joule (J). Energy is akin to work stored. For example the energy stored in a fuel. The energy contained in a moving body in the form of kinetic energy or potential energy of a body raised a distance h. Energy can be converted from one form to another, usually with some loss of energy at every conversion; efficiency will be discussed later.

Power

Power is the rate of doing work. It is defined as the work done divided by the time taken to accomplish this work the units are in watts. Again using the above example. The piston has to move the distance given within 2 seconds then,

$$Power = \frac{\text{Work done}}{\text{Time taken}}$$

$$\therefore\ P = \frac{235.5\ \text{Nm}}{2\ \text{s}} = 114.75\ \text{J/s or W}$$

Note: ***The relationship 1W=1 J/s is used extensively when connecting quantities of energy with the rate of using or producing energy.***

Energies

Energy was previously defined as the energy stored in a body. In thermodynamics there are several unique kind of energy, including; **potential**, **kinetic**, **work**, **internal** and **heat**. Work has already been covered in detail in the previous section and will not be expanded here.

Potential and Kinetic Energy

These types of energy are the most widely known types of energy. Potential energy is created by raising a body a distance h from a known datum as shown in Figure 10. The energy that can be extracted from the body is equivalent to the mass times the acceleration due to gravity times the distance h, or *PE=mgh* (J). It is obvious that the first part of this equation is the weight of the body in N, then by multiplying it by the distance moved the work or energy is obtained.

As the body falls the energy is converted from potential energy to kinetic energy. Kinetic energy is dependent on the velocity of the body as well as it's mass, or KE= ½mV^2 (J). Both these forms are present in a fluid flowing through a system.

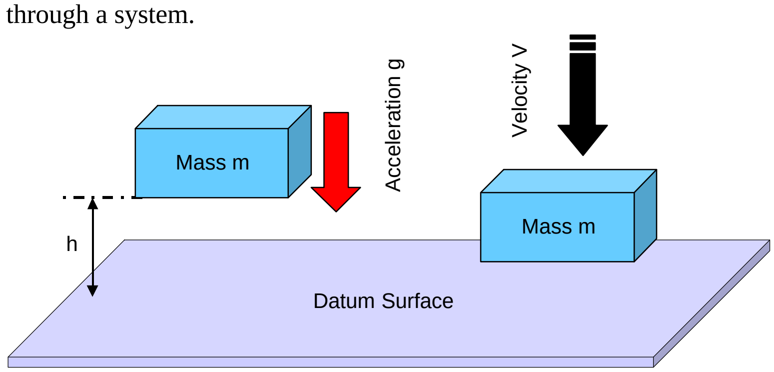


Figure 10: *conversion from Potential to Kinetic Energy.*

Internal Energy

Internal energy is the energy stored **in** a body before during and after processing. For instance when a kettle is boiled energy is transferred from the element to the water causing the molecules to vibrate. As more and more energy is added to the system the molecules vibrate more rapidly. After a while the water obtains sufficient energy to boil and change from a liquid to a vapour. Once the kettle switched off the water will still contain this internal energy which is slowly released through heat transfer.

Heat

There is a great misconception that heat is somehow the temperature of an object. Heat is **not** the temperature of a body but rather the transfer of energy from a hotter body to a cooler body. This concept if made clearer if heat is made analogous to a conversation. If knowledge is considered to be energy and heat the conversation between two people (Figure 11), then as the conversation proceeds knowledge is transferred from one person to the other. After the conversation ceases the transfer of knowledge stops, but both people have been affected by the conversation. Energy will naturally transfer from a hotter to a cooler body by the heating of the cooler body. Once both bodies reach the same temperature the heating of the cooler body stops. Temperature in this process is to heating what distance travelled would be to work.

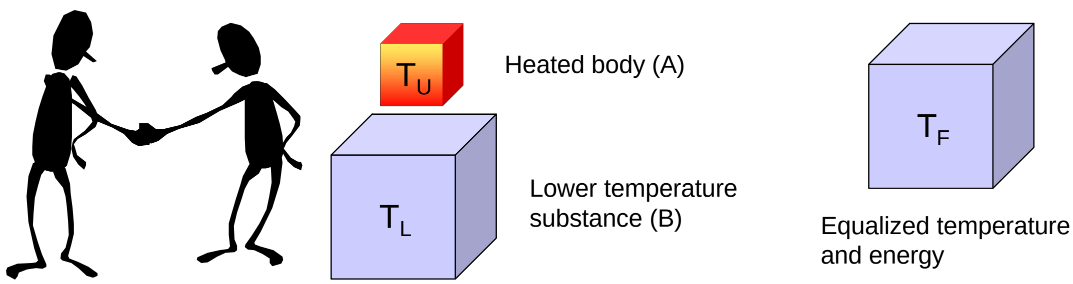


Figure 11: *Conversation analogy, and heat transfer.*

Thermodynamic Systems

A thermodynamic system can be as simple as a bicycle or as complex as a jet engine. Regardless of how complex or simple a system is, the method used to analyse the system is the same. The system is first separated from the surroundings by a boundary. This boundary is placed around the part of the system which we wish to analyse. There are two special cases of system that will be considered during this work, a closed system and open system. As the name suggested the closed system contains a known mass of the working fluid that does not cross the boundary, whereas in an open system the working fluid crosses the boundary via known input and outputs. Examples of each system are shown in Figure 12.

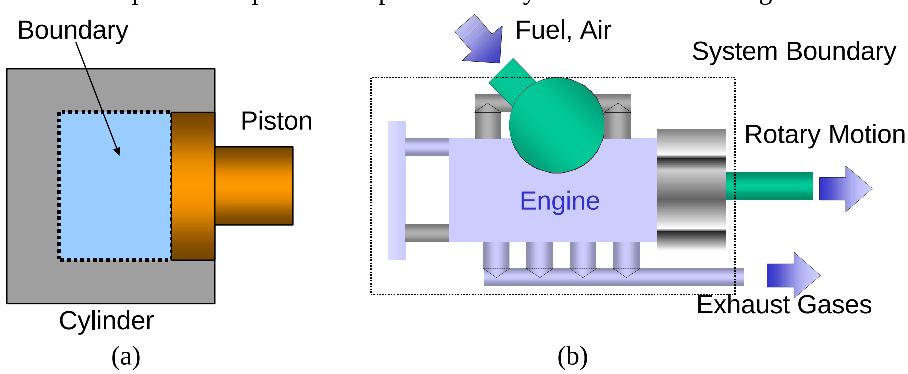


Figure 12: *(a) Closed system, (b) Open System.*

A piston and cylinder assembly is a good example of a closed system. The boundary is normally assigned to the volume of fluid contained within the assembly. Since the fluid can be compressed or allowed to expand the boundary of the system also moves. The key to this system is than the fluid does not cross the boundary.

The car engine mentioned earlier is a good example of an open system. The fuel enters the system to be burnt in the combustion chamber. Air is drawn into the engine, compressed, fuels the combustion and is then forced from the engine in the form of exhaust gasses. The boundary of the system then encompasses the entire unit. The inputs to the system are the fuel and air, the outputs are exhaust gasses and work in the form of rotary motion.

Relationship Between Work Energy and Heat

Work was defined previously as the force times the distance moved in the direction of that force. When dealing with a thermodynamic system however it is more likely to encounter pressure than force, and volume change rather than displacement. By analysing a simple closed system a relationship between work pressure and volume can be developed. A cylinder piston assembly is shown in Figure 13 (a). The working fluid exerts a pressure on the piston that results in a force that causes the piston to move. The work done is then the force times the distance moved. Assuming that the pressure remains constant,

$$Work = Force \times (L_2 - L_1)$$

Equation 1

As stated previously, however, it is better to work using pressure and volumes. Considering the working fluid contained in the system boundary (Figure 13 (b)), the volume of fluid contained in this closed system is the cross-sectional-area times the length L_1. After expansion the volume is the area times the new length L_2. Therefore for both volumes,

$$v_1 = Area \times L_1 \text{ and } v_2 = Area \times L_2$$

Equation 2

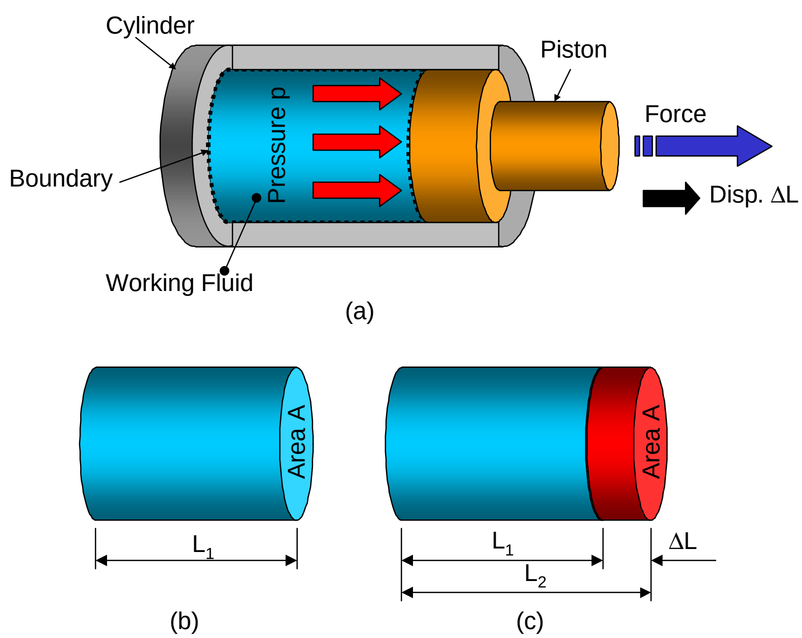


Figure 13: *Closed system (a) piston and cylinder assembly, (b) initial volume, (c) change in volume*

Both these equations can be re-arranged to make L the subject, thus changing Equation 1 to,

$$Work = Force \times \left(\frac{v_2}{Area} - \frac{v_1}{Area} \right)$$

Equation 3

Since force is equal to pressure times area then,

$$Work = pressure \times Area \times \left(\frac{v_2}{Area} - \frac{v_1}{Area} \right) \text{ or } Work = pressure \times (v_2 - v_1)$$

Equation 4

This final solution can be generalised further by plotting the process on a pressure-volume chart, or p-v diagram as shown in Figure 14. These types of plots are used widely in thermodynamics to illustrate processes.

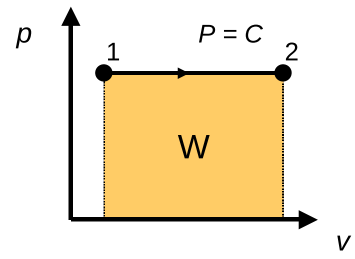


Figure 14: *Relationship between pressure volume and work.*

From Equation 4 it is apparent that work is given by the pressure times the change in volume. It is interesting to note that the area under the line described by the same constant pressure expansion shown in Figure 14 is also equal to the pressure times the change in volume or work. Then more generally we can describe the are under the curve as,

$$W = \int pdv$$

Equation 5

This derivation is a simplified version of the actual process, but nevertheless illustrates the principles involved. Now that work is described as a function or pressure and volume it is possible to combine work being done by the system (positive values) and work done on the system (negative values). If the process described in Figure 13 is now an expansion (1-2), compression (2-3), then the work done by the compression is subtracted from the work done during the expansion. This process and the resultant work is shown in Figure 15. It is important to note the direction of the curve that describes the process. Moving left to right (1-2) is work done by the system and is therefore positive, whereas moving right to left (2-3) is work done on the system.

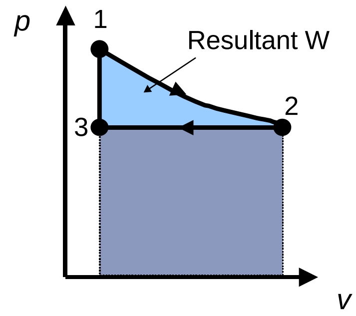


Figure 15: *Combined work values for an expansion (1-2) and compression (2-3).*

The work that we have described so far is known as reversible, i.e. if the process can be reversed to get to the original state. If however the example expansion was carried out in a different manner to produce an irreversible process, the work done by the system would be zero.

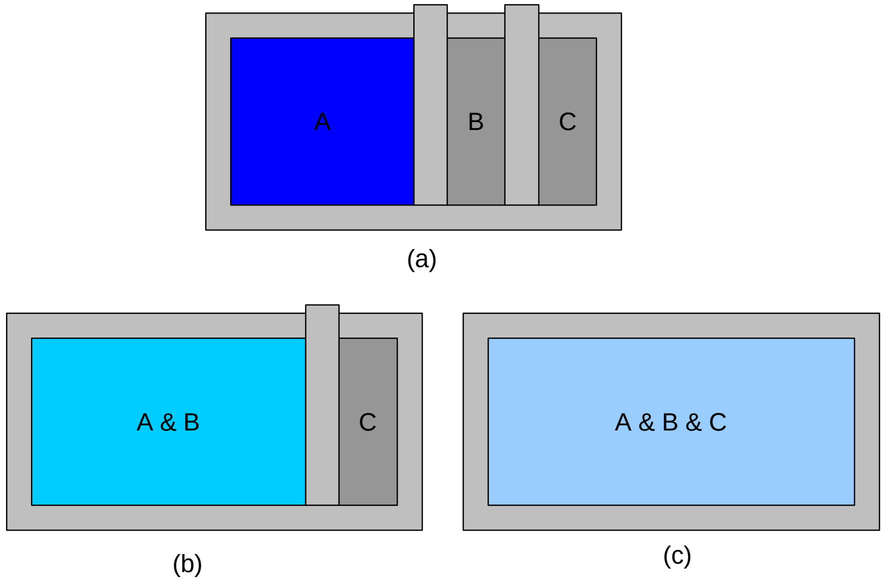


Figure 16: *Free expansion of a fluid into each compartment.*

Consider the system shown in Figure 16 (a). As each partition is removed the fluid expands to occupy all available space. Once all the partitions have been removed the fluid then occupies the volumes A, B and C. Due to the nature of this process, free expansion, the fluid cannot be returned to it's original state. It is also important to note that although the fluid have expanded no work has been carried out by the system. The representation of this process on the p-v chart is shown in .

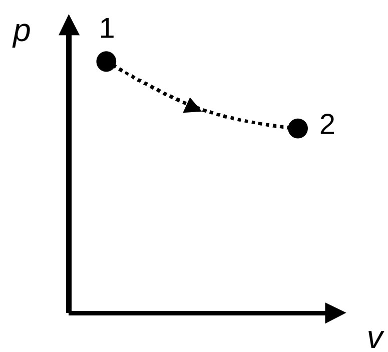


Figure 17: *p-v representation of an irreversible process.*

Work Energy and Heat

So far equations have been developed that illustrate the relationship between work, pressure and volume. For these to be of substantial use for system analysis additional relationships that also include energy and heat are required. A simple system is shown in Figure 18. There are two inputs and two outputs crossing the boundary associated with this system, work and Heat. It is possible to have systems which only perhaps contain three of these. For instance a kitchen refrigerator has a compressor (work input) that drives the refrigerant around a circuit. The refrigerant is used to cool the inside of the refrigerator by transferring energy in the form of heat. Since a refrigerator is normally cooler than the room, energy will be transferred back into the system heating the fridge. A similar example can be found in a steam plant, see Figure 19. Water is heated by the addition of energy in the boilers to create steam. The steam is then piped to a turbine where it expands and drives the turbine creating a work output. A condenser is then used to cool the steam (heat loss) so that it can be pumped (work in) back to the boilers where the cycle starts again.

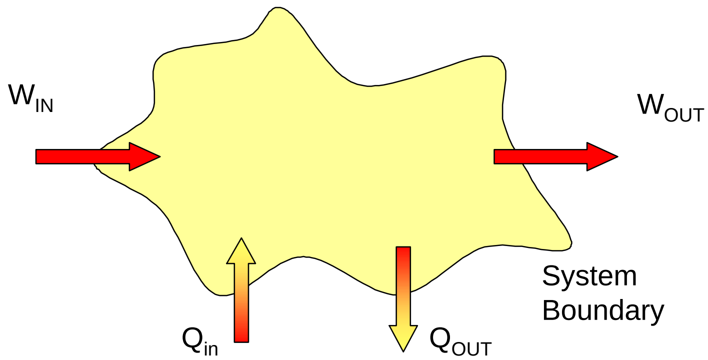


Figure 18: *System and work, and heating.*

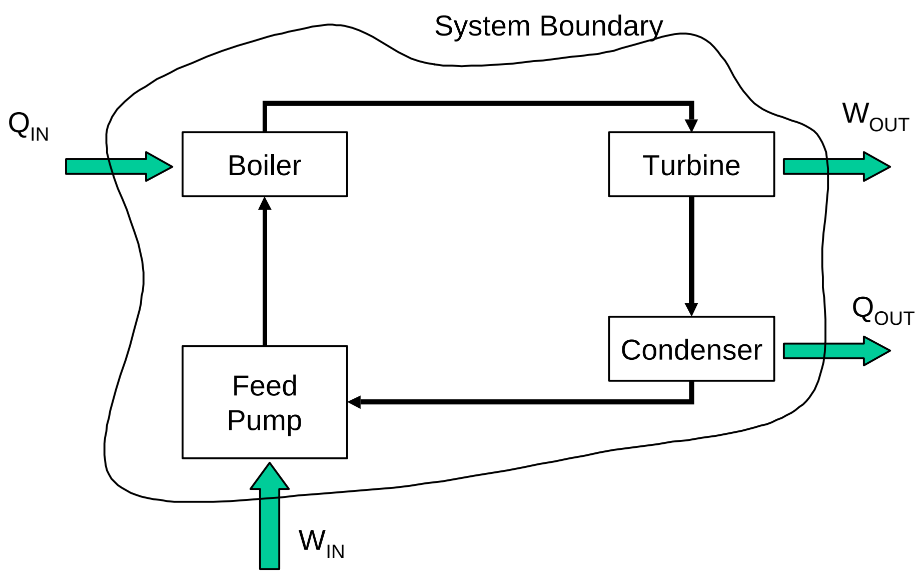


Figure 19: *A typical Steam plant operation.*

We have already discussed that energy can neither be created nor destroyed (Conservation of Energy), therefore there has to be some kind of relationship between the work and heat inputs and outputs. A thermodynamic cycle has the additional property that the intrinsic, or internal energy will be the same at the start and end of a cycle. Therefore all of the additional energy must be transferred as either work or heat. The first law of thermodynamics states that,

> *When a system undergoes a thermodynamic cycle the net heat supplied to the system from its surroundings plus the net work input to the system from its surroundings must equal zero.*

Then in mathematical form,

$$\sum Q = \sum W$$

Equation 6

The form of the equation differs slightly from that given in the text, remembering our definitions of positive and negative work.

The Non-Flow Energy Equation

There are a few special forms of Equation 6, in this case we consider what happens when the internal energy changes, i.e. not a complete cycle. The piston and cylinder assembly shown in Figure 20.

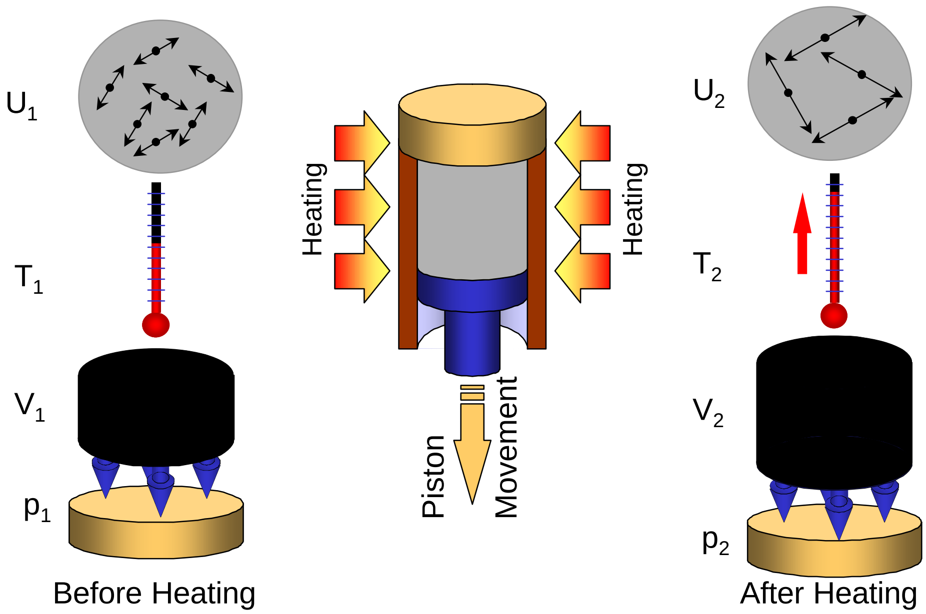


Figure 20: *Properties during a simple compression/expansion.*

This example illustrates how the properties of a fluid change during a process. If energy is added to the system by heating then the internal energy increases causing the fluid pressure to increase, moving the piston. If on the other hand the process is reversed and the fluid is pressurised, the increased internal energy will be released from an un-insulated to atmosphere by heating. A bicycle pump is a prime example. We have already related pressure volume and work (W), Temperature is related to heat transfer and internal energy (U), and heating or cooling is the heat transfer Q.
As heat is added to the above system the internal energy is increased, however some of the energy obtained during heating is used to drive the piston creating work. Therefore the heat energy supplied is equal to the change in internal energy plus the work done. Or said another way, the change in internal energy is equal to the heat supplied minus the work done. Then mathematically,

$$U_2 - U_1 = Q - W$$

Equation 7

This equation is known as the non-flow energy equation.

Steady Flow Energy Equation

The difference between a closed system and an open or steady flow system was previously defined by the boundary and the working fluid. For a closed system, as above, the working fluid does not cross the boundary, whereas the fluid crosses the boundary for an open or steady flow system. A steady flow system simply implied that the fluid is constantly flowing. Examples of steady flow processes include engines, compressors, rotary air motors etc.. A steady flow process differs from a non flow process in that the working substance crosses the boundary of the system. A typical example is shown in Figure 21.
There are now several different kinds of energy at work including, potential *mgz* (height change z in the fluid flow), kinetic $\frac{1}{2}mC^2$ (change in velocity C), Work, and heat. There is also a certain amount of energy required to 'push' the fluid into and out of the boundary. This energy is defined as pressure p times the specific volume v. In order to follow the conservation of energy principal we state that,

Initial energy + Energy Added = Final Energy

Or mathematically,

$$E_1 + (Q - W) = E_2$$

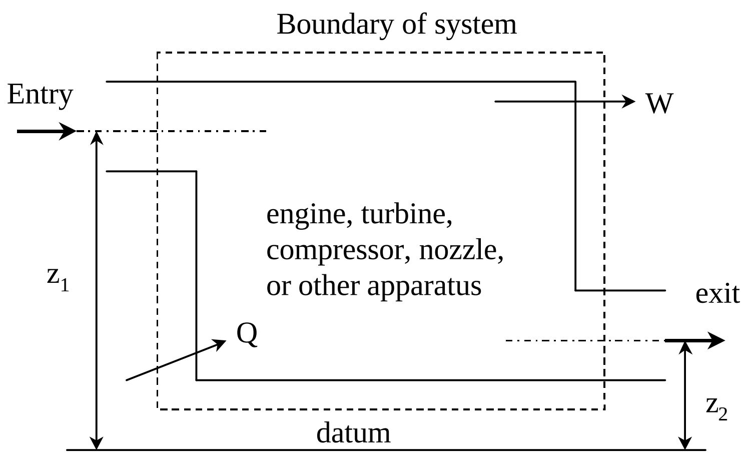


Figure 21: *A schematic of a steady flow system.*

or,

$$E_1 + Q = E_2 + W$$

The for each individual term,

$$m \cdot g \cdot z_1 + \tfrac{1}{2} \cdot m \cdot c_1^2 + U_1 + p_1 \cdot V_1 + Q = m \cdot g \cdot z_2 + \tfrac{1}{2} \cdot m \cdot c_2^2 + U_2 + p_2 \cdot V_2 + W$$

The units of this equation are J. It is more convenient to divide both sides of the equation by the mass *m*, then,

$$g \cdot z_1 + \tfrac{1}{2} \cdot c_1^2 + u_1 + p_1 \cdot v_1 + Q = g \cdot z_2 + \tfrac{1}{2} \cdot c_2^2 + u_2 + p_2 \cdot v_2 + W$$

Each of the terms in this equation has the units of J/kg. The internal energy and the energy required to push the fluid across the boundary can be combined in a variable *h* know as Specific **enthalpy.** The when the equation is re-arranged into a more convenient form,

$$Q - W = (h_2 - h_1) + \tfrac{1}{2}\left(C_2^2 - C_1^2\right) + g(z_2 - z_1)$$

This equation is the steady flow energy equation.

Class Examples

1.1 Unit mass of a fluid at a pressure of 3 bar, and with a specific volume of 0.18 m^3/kg, contained in cylinder behind a piston expands reversibly to a pressure of 0.6 bar according to the law $p = c/v^2$, where c is a constant. Calculate the work done during the process.

1.2 Units mass of a certain fluid is contained in a cylinder at an initial pressure of 20 bar. The fluid is allowed to expand reversibly behind a piston according to a law pV^2 = Constant until the volume is doubled. The fluid is then cooled reversibly until the piston regains its original position; heat is supplied reversibly with the piston firmly locked in position until the pressure rises to the original value of 20 bar. Calculate the net work done by the fluid for an initial volume of 0.05 m^3.

1.3 In a certain steam plant the turbine develops 1000 kW. The heat supplied to the steam in the boiler is 2800 kJ/kg, the heat rejected by the steam to the cooling water in the condenser is 2100 kJ/kg and the feed-pump work required to pump the condensate back into the boiler is 5 kW. Calculate the steam flow rate.

1.4 In the compression stroke of an internal combustion engine the heat rejected to the cooling water is 45 kJ/kg and the work input is 90 kJ/kg. Calculate the change in specific internal energy of the working fluid, stating whether it is gain or loss.

1.5 In the cylinder of an air motor the compressed air has a specific internal energy of 420 kJ/kg at the beginning of the expansion and a specific internal energy of 200 kJ/kg after expansion. Calculate the heat flow to or from the cylinder when the work done by the air during the expansion is 100 kJ/kg.

1.6 In the turbine of a gas turbine unit the gases flow through the turbine at 17 k/s and the power developed by the turbine is 14000 kW. The specific enthalpies of the gases at inlet and outlet are 1200 kJ/kg, and 360 kJ/kg respectively, and velocities of the gases at inlet and outlet are 60 m/s and 150 m/s respectively. Calculate the rate at which heat is rejected from the turbine. Find also the area of the inlet pipe given that the specific volume of the gases at inlet is 0.5 m^3/kg.

1.7 Air flows steadily at a rate of 0.4 kg/s through an air compressor, entering at 6 m/s with a steady pressure of 1 bar and a specific volume of 0.85 m3/kg, and leaving at 4.5 m/s with a pressure of 6.9 bar and a specific volume of 0.16 m3/kg. The specific internal energy of the air leaving is 88 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at a rate of 59 kW. Calculate the power required to drive the compressor and the inlet and outlet pipe cross-sectional areas.

Chapter 2: The Working Fluid

Introduction

In the previous section of the notes sets of equations were developed for the description of both non-flow and steady flow systems. The application of these equations was demonstrated by analysing both system types. In these examples all of the required information was given. In reality the information can be limited to only a few values. Its is therefore essential to have some method of finding the required information. This information can be obtained from a knowledge of how the working fluid behaves.

The working fluid is presented as the main topic of this section of the notes. The possible states in which the fluid can exist are illustrated both for vapours and ideal gases. Methods of determining the unknown values mentioned above are also given.

States of a Substance

There are three possible states in which a working fluid can exit; solid, liquid or vapour. To illustrate this principal an ice cube is shown in Figure 22. Initially the ice cube is held in a glass beaker (solid state). Energy is supplied to the ice cube in the form of a heat transfer. The increase in the ice cube's intrinsic energy causes the ice to melt, turning into a liquid. If the liquid is heated further it will eventually all turn into a vapour.

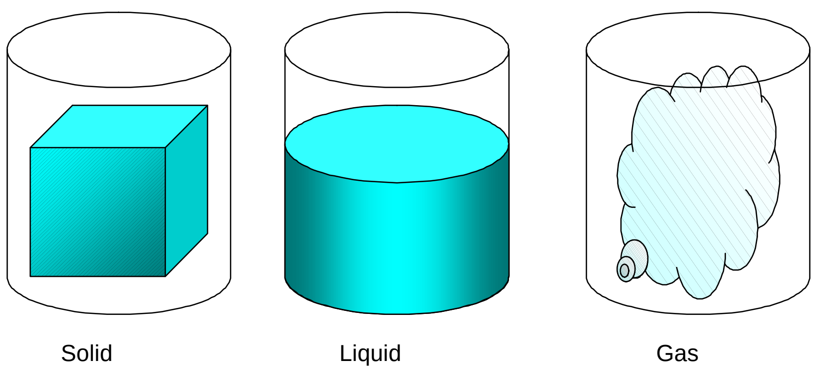


Figure 22: *States of a Substance*

State Heat and Temperature of a Substance

What shown in Figure 22 is the finished states. There are however, intermediate states between these discrete steps. As the ice (solid) is heated it's temperature will initially rise to 0°C and then remain at freezing until the all of the ice has melted. The temperature remains constant as sufficient energy is provided to melt all of the material. This energy is commonly known as latent heat, or more correctly enthalpy of fusion (Figure 23). The temperature then rises again as the liquid is heated further. Once the liquid reaches 100°C, boiling point, the temperature will again remain constant. At this point the fluid is change from a liquid to a vapour. This process is commonly known as the latent heat of evaporation, or more correctly the enthalpy of evaporation. Once all of the liquid has changed to a vapour the temperature starts to increase. Further increases in temperature will not cause additional state changes, but will increase the internal energy of the material.

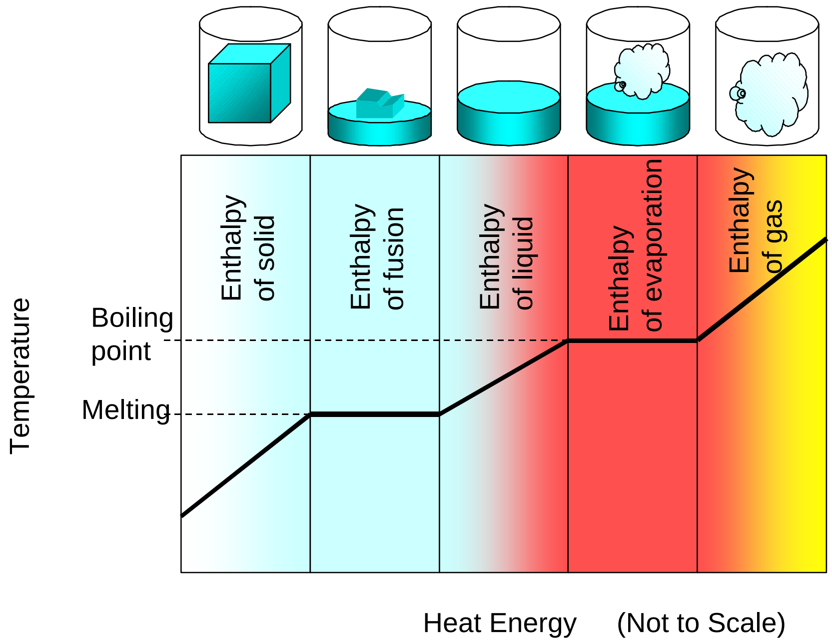


Figure 23: *Intermediate steps in the states of water.*

Boiling and Vaporisation Points

In thermodynamics we are more interested in the liquid, liquid-vapour and vapour states of the working fluid. The changes between these states i.e. liquid to liquid-vapour and liquid-vapour to vapour are also of particular interest. Consider for example boiling a kettle. Under normal atmospheric conditions for pure water , boiling starts at 100°C. A pressure cooker as the name suggests increases the pressure of the working fluid, and the temperature at which pure water boils, thus cooking the food faster. The increase in pressure actually results in a slight increase in the volume of the fluid at boiling point.

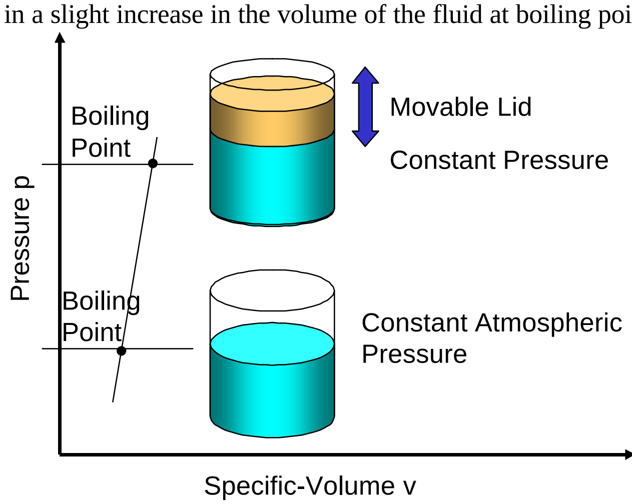


Figure 24: *Relationship between pressure, specific-volume and boiling point.*

A similar situation arises in Figure 24. For a lower pressure (atmospheric) the boiling point of pure water is 100°C. If the pressure is increased by placing a movable lid on the container then the volume and the temperature at boiling increases. Further heating will cause the water to evaporate, changing from a liquid to a vapour. The point where all of the liquid has changed to a vapour can also be identified in the same manner as shown in Figure 25.

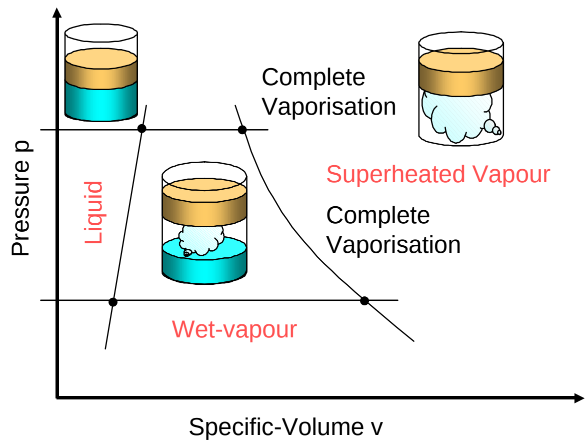


Figure 25: *Liquid, Wet-vapour and super heat regions.*

Between the regions of liquid and super heated vapour exits a state of flux where the working fluid consists of both liquid and vapour. The complete description of this region is shown in Figure 26. The lines that describe these regions are given names that relate to the transition points. The saturated liquid line describes the transition from liquid to wet-vapour. The saturated vapour line describes the transition from wet-vapour to superheated steam. The point where these two curves meet is known as the critical point. At this pressure and specific volume the fluid changes from a liquid to a vapour without the transitional stage of a wet-vapour.

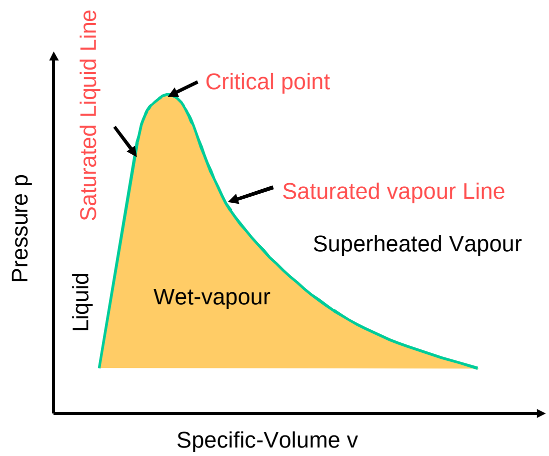


Figure 26: *The p-v chart for a vapour.*

Temperatures on the p-v Chart

The change from a liquid to a vapour occurs without a transitory phase at the critical point, however what happens above this point? Where does the liquid stop and the vapour region begin. We noted earlier that as the pressure is higher in the pressure cooker the temperature at which the water boils has also increased. This is true of every point along the saturated liquid line, i.e. as the pressure is increased so too is the temperature. We must therefore assume that some relationship exists between pressure and temperature. It is possible to visualise this relationship by constructing constant temperature (isothermal) lines on the p-v chart. The result of adding the isothermal lines is shown in Figure 27. One of the isothermal lines (T_c) in Figure 27 intersects the critical point. Above the critical point (pressures higher than this point) any fluid state that lies to the left of the Tc isothermal is a liquid. Any fluid state to the right of the Tc isothermal is a vapour. This is not true of pressures below the critical point. For pressures below the critical point, the isothermal Tc has no special significance in describing the state of the fluid.

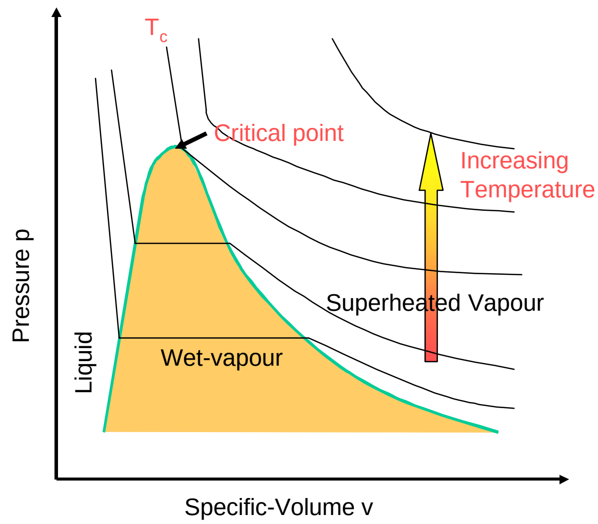


Figure 27: *Isothermal line construction on the p-v chart.*

Note the shape of the isothermals in Figure 27. Below the critical temperature isothermal, the isothermals tend to drop rapidly until reaching the saturated liquid line. At this point, the isothermals are horizontal; i.e. the pressure and temperature are constant in the wet vapour region. This fact is also obvious from Figure 23, where the temperature was shown to remain constant during the enthalpy of evaporation. Once the fluid state reaches the saturated vapour line, the temperature starts to drop again. Above the critical temperature, all of the isothermals describe the condition of a vapour.

Pressure-Temperature Relationships

It was mentioned previously that some form of relationship exists between pressure and temperature. Plotting the pressure against the temperature provides a better illustration of this relation. Three distinct solid lines are shown in Figure 28. These lines (like the saturated liquid and vapour lines) describe the zones of transition from one state to another. For instance, the transition from solid to liquid is separated by the fusion line. The line that separates the liquid and vapour zones is known as the evaporation line. Finally, the line that separates the solid state from the vapour-state is known as the sublimation line (sublimation; to convert from solid to vapour without a liquid transition).

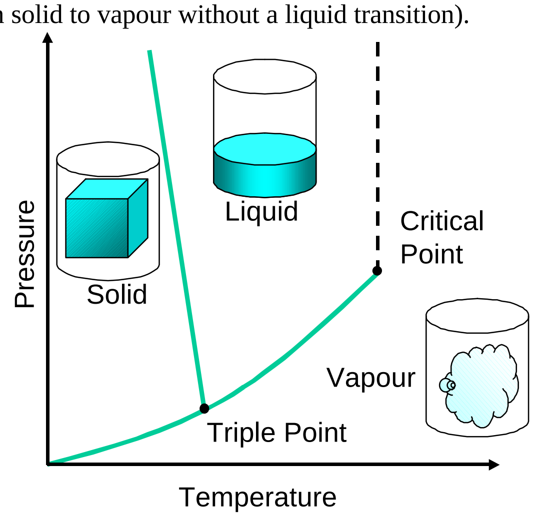


Figure 28: *Plot of equilibrium lines for pressure and temperature.*

Of special interest in this diagram is the triple point. In the first set of notes, Figure 7 illustrated that the triple point of water occurs at approximately 0°C and atmospheric pressure. At this point the working fluid exists in all three states, solid, liquid and vapour.

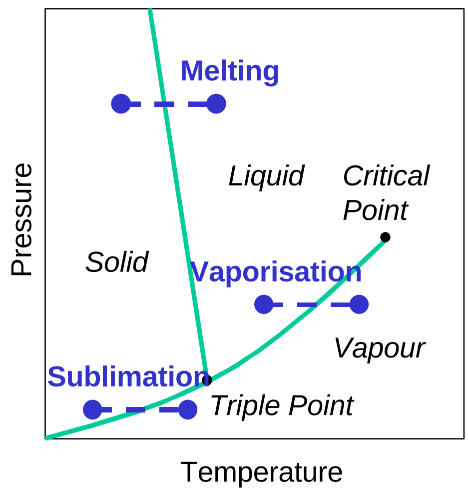


Figure 29: *Transition between the states of a substance.*

The transition between different states on a p-v chart can be seen in Figure 29. The transition from a solid or a liquid is known as melting, from a liquid to a vapour, vaporisation and from a solid to a vapour, sublimation.
As shown previously however water does not change directly from a liquid to a vapour below the critical point. The transition between these states requires a transitional liquid-vapour phase. On examination of Figure 29 it is clear that this transition is not apparent. This difficulty in describing the process in the wet-vapour region (clearly illustrated in Figure 27) highlights the need for at least two **independent** properties to fully describe the state of the fluid.

p - v - T Relationship

A clear picture of how the pressure volume and temperature are related is shown in Figure 30. A three-dimensional surface can be created using the plots shown in Figure 27 and Figure 28. The construction of the surface is shown in Figure 30 (a), the final plot is shown in Figure 30 (b).

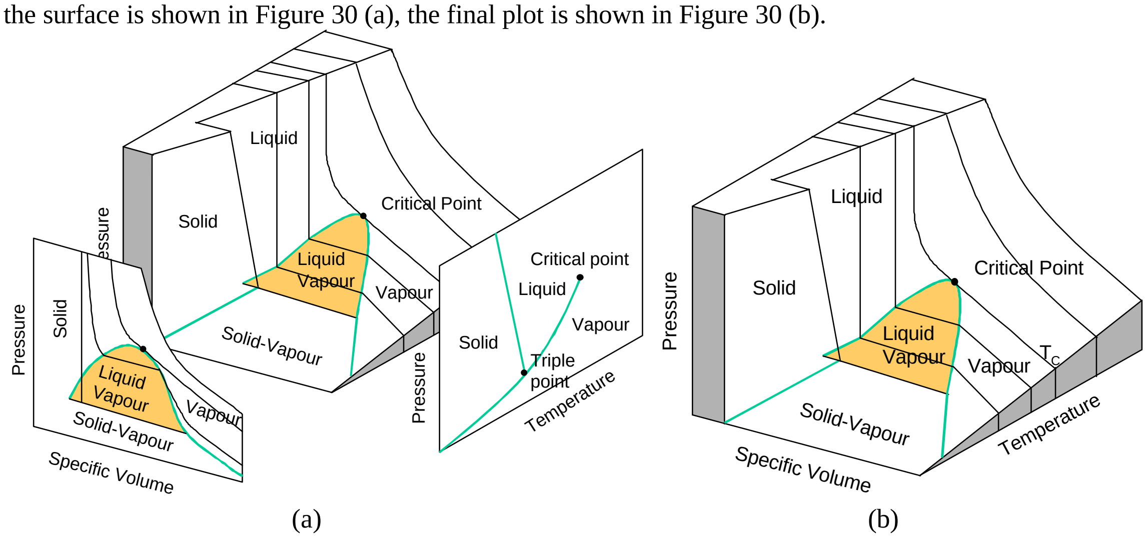


Figure 30: *p-v-T chart construction for water.*

From the p-v-T surface, it is clear that the pressure-specific volume and Pressure temperature plots are simply projections of this surface onto planes. Note also that as the fluid is cooled to a solid the volume increases, i.e. it expands. This is the reason why freezing conditions in the winter cause so much damage to copper water pipes.

Properties of a Vapour

We now have a graphical representation of how water behaves under different pressures, specific-volumes and temperatures, however actual values are required to analyse a thermodynamic system. A sample of the steam tables[4] that list these values for water is shown in Table 2. The pressure of the water is 1.0 bar (approximately Atmospheric).

p	t	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
(bar)	(°C)	(m^3/kg)	(kJ/kg)		(kJ/kg)			(kJ/kg K)		
1.00	99.6	1.694	417	2506	417	2258	2675	1.303	6.056	7.359

Table 2: *Properties of water for atmospheric pressure.*

Starting from the left of Table 2, the pressure is normally listed in bar. The temperature in °C next the pressure is the value of the isothermal shown in Figure 31. The variable v_g describes the specific-volume of the fluid where the isothermal line of 99.6°C crosses the saturated vapour line. All of the variables shown in Table 2 that have a subscript g are determined for the point where the isothermal intersects the saturated vapour line. Conversely, all of the variables that have a subscript f are determined for the point where the isothermal crosses the saturated liquid line. The variables u_f and u_g therefore describe the internal energy of the working fluid at the saturated liquid line and the saturated vapour line respectively (points f and g in Figure 31). This is also true of enthalpy (h_f and h_g). The final variable s, is the entropy of the system. This variable will not be used until second year, suffice to say that entropy describes the disorder in the fluid.

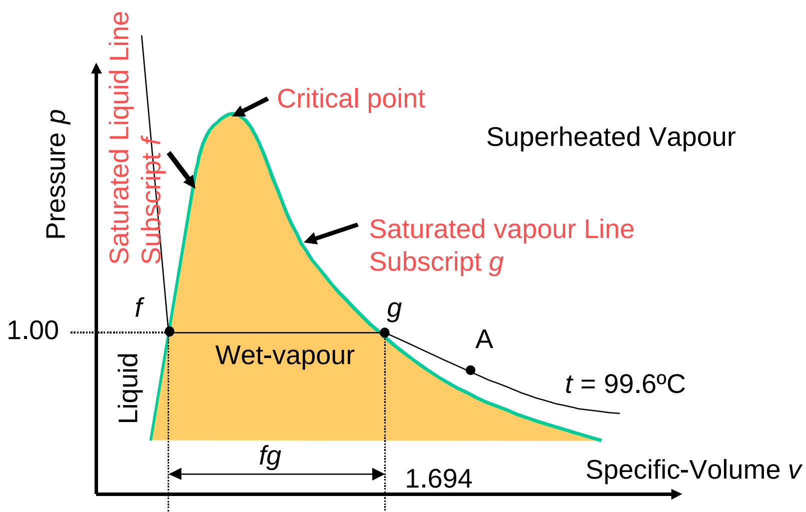


Figure 31: *Properties of water for atmospheric pressure.*

There is an additional variable used in the definition of enthalpy, i.e. h_{fg}. This variable is obtained by subtracting the enthalpy for saturated liquid h_f, from the enthalpy for saturated vapour h_g. The result is the line fg shown in Figure 31, or thinking back to Figure 23 the enthalpy of evaporation, i.e. this is the energy required to change all of the liquid to a vapour.

All of the values listed in Table 2 are for a saturated liquid or vapour. How do we determine these values of the state of the working fluid lies somewhere in the wet-vapour region? To define the condition of the working fluid in the wet-vapour region requires an additional parameter x known as the dryness fraction. For a saturated liquid the dryness fraction x is zero, i.e. there is no vapour (point f in Figure 31). For a saturated-vapour, the dryness fraction is one; i.e. there is no liquid (point g in Figure 31). Between these two extremes x can have any value from zero to one depending on the condition of the fluid.

The specific volume of the working fluid at any point in the wet-vapour region is given by,

[4] Thermodynamic and Transport Properties of Fluids SI Units Fourth Edition, Arranged by G. F. C. Rodgers and Y. R. Mayhew.

$$v = \frac{\text{volume of liquid} + \text{volume of dry vapour}}{\text{Total mass of wet vapour}}$$

Assume 1 kg of working fluid that is made up x kg of dry vapour and *(1 - x)* kg of liquid. Our equation then becomes,

$$v = v_f(1 - x) + v_g x$$

The specific volume of the liquid v_f is usually negligible in comparison with the vapour, therefore for practical problems we assume,

$$v = v_g x$$

Therefore, given that the specific volume of the working fluid is less than v_g we can calculate the dryness fraction x. Remember that *x* cannot be greater then one. For situations where the specific volume is greater than the specific volume, v_g (Point A in Figure 31) we must use superheated steam tables.
We can use the same reasoning as above when trying to find the values of internal energy u and enthalpy h in the wet vapour region, i.e. from point *f* to *g* in Figure 31. The enthalpy of wet vapour can be found from,

$$h = h_f + x h_{fg}$$

For internal energy, we need to use a slightly different form since u_{fg} is not listed. This equation takes the form,

$$u = (1 - x)u_f + x u_g$$

$$or$$

$$u = u_f + x(u_g - u_f)$$

Properties of Superheated Vapour

If we wish to know the properties for point A in Figure 31, we must use the superheated tables. This is because the values listed in Table 2 are for the saturated liquid and vapour lines. The steam tables must be used if any of the values, temperature, specific volume, internal energy or enthalpy are above the saturated vapour values. The superheated values for the same pressure are shown in Table 3. Note that no values are listed for 50 °C since this is less than the saturation temperature.

P/[bar] (t_s/[°C])		t [°C]	50	100	150	200	250	300	400	500
	v_g 1.694	*v*		1.696	1.937	2.173	2.406	2.639	3.103	3.565
1.0	u_g 2506	*u*		2506	2583	2659	2734	2881	2968	3131
99.6	h_g 2675	*h*		2676	2777	2876	2975	3075	3278	3488
	s_g 7.359	*s*		7.360	7.614	7.834	8.033	8.215	8.543	8.834

Table 3: *Superheated Steam, units are the same as Table 1.*

Table Interpolation

The tables provide many data points that enable us to determine the properties of the working fluid. The quantity of data that can be presented in this manner is however limited. Consider the data shown in Table 4. What is the internal energy for a saturated vapour at 100 °C? The only way to find this data is to interpolate between the known points. Interpolation simply assumes that a straight line describes the data between the two known data points.

p	T	v_g	u_f	u_g	H_f	h_{fg}	h_g	s_f	s_{fg}	s_g
(bar)	(°C)	(m^3/kg)	(kJ/kg)		(kJ/kg)			(kJ/kg K)		
1.00	99.6	1.694	417	2506	417	2258	2675	1.303	6.056	7.359
1.10	102.3	1.549	429	2510	429	2251	2680	1.33	5.994	7.327

Table 4: *Saturated water data for table interpolation.*

The following equation uses values for temperature and internal energy listed in Table 4,

$$u_g \text{ at } 100^{o}\text{C} = \left(u_g \text{ at } 99.6\ ^{o}\text{C}\right) + \left(\frac{100-99.6}{102.3-99.6}\right) \times \left\{\left(u_g \text{ at } 102.3^{o}\text{C}\right) - \left(u_g \text{ at } 99.6\ ^{o}\text{C}\right)\right\}$$

Inserting the internal energy values then,

$$u_g \text{ at } 100^{o}\text{C} = 2506 + \left(\frac{100-99.6}{102.3-99.6}\right) \times \left\{2510-2506\right\}$$

The internal energy is then 2506.6 kJ/kg. Your answer should always lie between the two data points.

The Perfect Gas

In the previous section, the properties of a vapour were described using a set of tabulated values. The values obtained from these tables were dependent on the state of the working fluid, i.e. liquid, wet-vapour or superheated-vapour. When dealing with a perfect gas however, it is possible to develop a set of equations that describe the properties at any state in the process. A perfect gas is assumed to remain a gas throughout the process. For instance, air is normally assumed a perfect gas although it is made up of several gases, Nitrogen N_2, Oxygen O_2, Argon Ar and Carbon dioxide CO_2. As the temperature of air is reduced these gases separate into their wet-vapour and liquid states, however at normal atmospheric conditions air is a mixed superheated-vapour.

Boyle's Law

An empirical investigation conducted by Boyle is shown in Figure 32. In the diagram, the working fluid held in the piston-cylinder assembly is compressed very slowly to keep the temperature constant. As the volume is reduced during the compression, the pressure increases. This experiment illustrated that if the temperature of a given mass of gas remained constant its volume V will vary proportionally to its pressure p, or mathematically,

$$V = constant \cdot \frac{1}{p} \text{ or } p \cdot V = constant$$

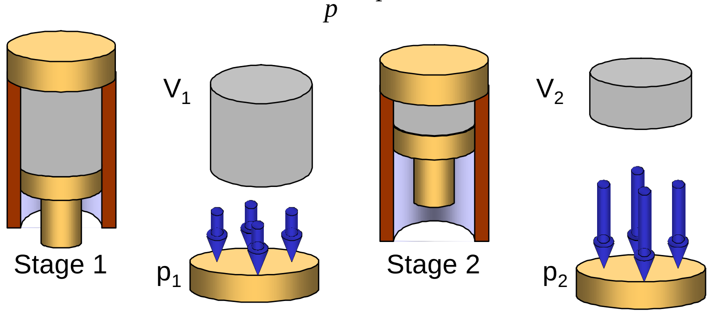


Figure 32: *Boyle's experiment to determine the relationship between pressure and volume.*

Charles' Law

Charles carried out additional empirical work that extended Boyle's investigations to include the effect of temperature on a constant volume and pressure process. Again, the fluid is contained in a piston cylinder assembly that was heated from an initial temperature as shown in Figure 33. What Charles found was if the volume of a given mass of gas remains constant, then its pressure p is proportional to the temperature T. Conversely is the pressure of a given mass of gas remains constant, then the volume V is proportional to the temperature T. Or mathematically,

$$\text{For a constant } V \quad p = constant \cdot T \text{ or } \frac{p}{T} = constant$$

$$\text{For a constant } p \quad V = constant \cdot T \text{ or } \frac{V}{T} = constant$$

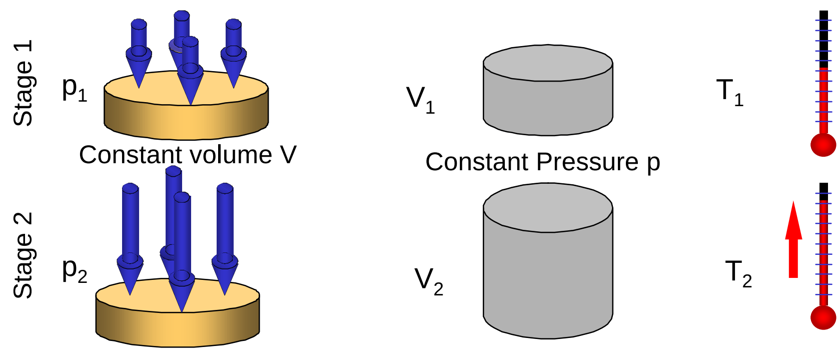


Figure 33: *Charles' experiment to determine the relationship between volume pressure and temperature.*

Ideal-Gas law

Since all of the variables, pressure, volume and temperature can change simultaneously a more general form of the previous relationships is required. By combining both Boyle's and Charles' law then,

$$\frac{p \cdot V}{T} = constant \text{ or } p \cdot V = constant \cdot T$$

The equation in its current form offers no way of dealing with different masses of gas. So introducing the mass yields,

$$\frac{p \cdot V}{T} = constant \times (\text{amount of the gas})$$

The constant is known as the ideal gas constant *R* and is given the units J/kgK, the amount of gas is defined by the mass in kg, therefore,

$$\frac{p \cdot V}{T} = R \cdot m \quad \text{or} \quad p \cdot V = m \cdot R \cdot T$$

This is the normal form of the ideal gas law, or for specific-volume v,

$$p \cdot v = R \cdot T$$

In this form of the equation, the quantity of gas is defined by a mass. There are however alternatives means of defining the quantity of matter. One such method has been defined as,

> *"The amount of a substance is that quantity which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12; the elementary entities must be specified and may be atoms, molecules, ions, electrons, or other particles, or specific groups of such particles".*

This quantity is normally defined by the variable n and assigned the units of kmol. Then relating this values to mass yields,

$$\tilde{m} = \frac{m}{n}$$

Where $\tilde{m}$ is the molar-mass, units kg/kmol. This value is also equal to the relative molecular mass, which differs only by having no units. For instance, the relative molecular mass of Oxygen is 16. It is possible then to use this definition in the ideal gas law, i.e.,

$$pV = n\tilde{m}RT \text{ or } \tilde{m}R = \frac{pV}{nT}$$

The ratio V/n is a constant for all gases at the same pressure p and temperature T. This constant is called the molar gas constant or,

$$\tilde{m}R = \tilde{R} \text{ or } R = \frac{\tilde{R}}{\tilde{m}}$$

For ideal gases $\tilde{R}$ has been shown to be 8.3145 kJ/kmol K.

Specific Heat Capacities

The standard definition of specific heat capacity is;

The quantity of energy required to raise one kg of a substance by one degree Kelvin.

This definition is acceptable when working with a solid where the pressure and volume do not change considerably during heating. For a gas however, there exists an infinite number of ways to add heat. Therefore, two standard methods have been established for determining the specific heat capacity of a gas, constant pressure and constant volume. A piston-cylinder assembly (Figure 34) is used to restrict the analysis to a reversible non-flow process. Heat is added to the system in two modes: constant volume where the piston does not move and constant pressure where the piston moves during the process.

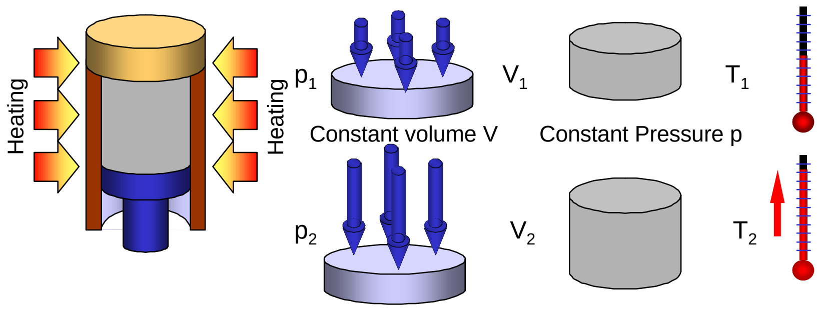


Figure 34: *Schematic of the specific heat capacity experiments.*

For a known mass of gas the relationships that relate the heating to the change in temperature are,

$$dQ = mc_p dT \text{ and } dQ = mc_v dT$$

For a perfect gas, specific heat at a constant pressure c_p and specific heat at a constant volume c_v are normally assumed to be constants. For a real gas, c_p and c_v will vary slightly with temperature. Therefore integrating,

$$Q = mc_p(T_2 - T_1) \text{ and } Q = mc_v(T_2 - T_1)$$

Joule's Law

Sets of experiments were carried out by Joule to determine the effect of volume and temperature on the internal energy of a fluid. His hypothesis was,

$$u = f(v, T)$$

i.e. the internal energy was some function of the volume and temperature. To determine the effect of volume change he devised the experiment shown in Figure 35. Two chambers A and B are filled with the same gas at different pressures surrounded by a water bath. The water bath was first allowed to reach an equilibrium state before the valve was opened between the chambers.

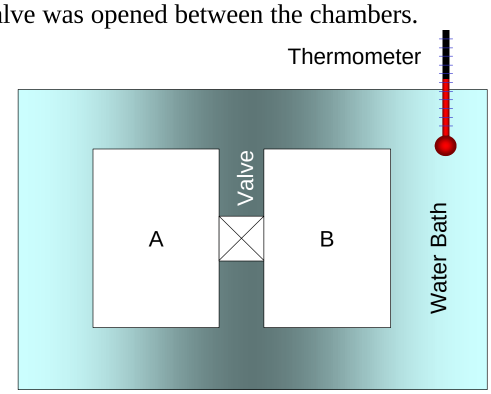


Figure 35: *Joule's experiment to determine the effect of volume change on internal energy.*

The temperature change was found to be zero. Also since the change in volume is simply a free expansion the work done was zero. It is therefore logical to conclude that the change in internal energy for a change in volume is zero, i.e.,

$$\cancel{Q} - \cancel{W} = u_2 - u_1 = 0$$

Joule then concluded that the internal energy was governed only by the change in temperature, or,

$$u = f(T)$$

If we then consider a unit mass of a perfect gas heated at a constant volume, then from our non-flow energy equation,

$$Q - \cancel{W} = u_2 - u_1$$

Since the work done in a constant volume process is zero. Therefore

$$Q = u_2 - u_1$$

Also for a perfect gas heated at a constant volume

$$Q = c_v(T_2 - T_1)$$

Thus combining the relationships,

$$u_2 - u_1 = c_v(T_2 - T_1)$$

And for a known mass of gas,

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

This equation applies to any reversible or irreversible process when the working fluid is an ideal gas.

Relationship between Specific Heat Capacities

Consider an ideal gas heated from T_1 to T_2 at a constant pressure, then,

$$Q - W = u_2 - u_1$$

Also for a perfect gas,

$$u_2 - u_1 = c_v(T_2 - T_1)$$

In a constant pressure process, the work done is described by the area under the graph shown in Figure 36.

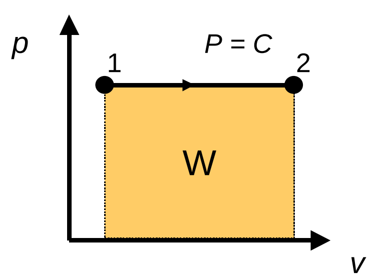


Figure 36: *Pressure-specific volume plot for a constant pressure process.*

The work done is then simply,

$$W = p(v_2 - v_1)$$

From the ideal gas law

$$pv = RT$$

$$\therefore W = R(T_2 - T_1)$$

Thus combining these relationships

$$Q - R(T_2 - T_1) = c_v(T_2 - T_1)$$

Also for a constant pressure process

$$Q = c_p(T_2 - T_1)$$

Therefore,

$$c_p(T_2 - T_1) - R(T_2 - T_1) = c_v(T_2 - T_1)$$

Dividing both sides of the equation by $(T_2 - T_1)$ yields,

$$c_p - R = c_v \text{ or } R = c_p - c_v$$

For these equations to hold true, the values of c_p must always be greater than c_v.

Specific Enthalpy of a Perfect Gas

It is possible to conduct a similar analysis for the enthalpy of a perfect gas. From our definition of enthalpy,

$$h = u + pv$$

From the ideal gas law

$$pv = RT$$

And for internal energy

$$u_2 - u_1 = c_v(T_2 - T_1)$$

If the change in enthalpy is defined as,

$$h_2 - h_1 = (u_2 - u_1) + (p_2 v_2 - p_1 v_2)$$

Then using the ideal gas law and change in internal energy,

$$h_2 - h_1 = c_v(T_2 - T_1) + R(T_2 - T_1)$$

In addition, the ideal gas constant R was defined as,

$$R = c_p - c_v$$

Therefore, substituting for R,

$$h_2 - h_1 = c_v(T_2 - T_1) + c_p(T_2 - T_1) - c_v(T_2 - T_1)$$

Simplifying yields,

$$h_2 - h_1 = c_p(T_2 - T_1)$$

For a mass m,

$$H_2 - H_1 = mc_p(T_2 - T_1)$$

If we assume that $u = 0$ when $T = 0$, then $h = 0$ at $T = 0$, thus,

$$h = c_p T \text{ or } H = mc_p T$$

Note that absolute temperature must be used in these equations.

Ratio of Specific Heat Capacities

The ratio of specific heat capacity at a constant pressure to the specific heat capacity at a constant volume is given by,

$$g = \frac{c_p}{c_v}$$

It has already been noted that $c_p > c_v$, therefore g will always be greater that unity. g is 1.4 for gases such as Carbon monoxide CO, Hydrogen H_2, Nitrogen N_2 and Oxygen O_2. Some useful relationships between c_p, c_v, R and g can now be derived. Since

$$R = c_p - c_v$$

Then dividing both sides by c_v yields,

$$\frac{R}{c_v} = \frac{c_p}{c_v} - 1$$

Therefore,

$$g - 1 = \frac{R}{c_v} \text{ or } c_v = \frac{R}{g - 1}$$

Since $c_p = gc_v$ then,

$$c_p = \frac{gR}{g - 1}$$

It is however preferable to remember the derivations from first principals rather than the results.

Class Examples

2.1 Calculate the specific volume, specific enthalpy, and specific internal energy of wet steam at 18 bar, dryness fraction 0.9.

2.2 Calculate the dryness fraction, specific volume and specific internal energy of wet steam at 7 bar, specific enthalpy 2600 kJ/kg

2.3 Steam at 110 bar has a specific volume of 0.0196 m^3/kg, calculate the temperature, the specific enthalpy and the specific internal energy.

2.4 Steam at 150 bar has a specific enthalpy of 3309 kJ/kg. Calculate the temperature, the specific volume and the specific internal energy.

2.5 Sketch a pressure-volume diagram for steam and mark on it the following points, labelling clearly the pressure, specific volume and temperature of each point.

p = 20 bar, t = 250 °C

t = 212.4 °C, v = 0.09957 m^3/kg

p = 10 bar, h = 2650 kJ/kg

p = 6 bar, h = 3166 kJ/kg

2.6 Calculate the internal energy for each of the states given in example 2.5

2.7 A vessel of volume 0.2 m^3 contains nitrogen at 1.013 bar and 15 °C. If 0.2 kg of nitrogen is now pumped into the vessel, calculate the new pressure when the vessel has returned to its initial temperature. The molar mass of nitrogen is 28 kg/kmol, and it may be assumed to be a perfect gas.

2.8 A certain gas of mass 0.01 kg occupies a volume of 0.003 m^3 at a pressure of 7 bar and a temperature of 131 °C. The gas is allowed to expand until the pressure is 1 bar and the final volume is 0.02 m^3. Calculate:

a) The molar mass of the gas

b) The final temperature.

2.9 A certain perfect gas has specific heat capacities as follows: c_p = 0.846 kJ/kg K and c_v = 0.657 kJ/kg K Calculate the gas constant and the molar mass of the gas.

2.10 A perfect gas has a molar mass of 26 kg/kmol and a value of g =1.26. Calculate the heat rejected:

a) When unit mass of the gas is contained in a rigid vessel at 3 bar and 315 °C, and is then cooled until the pressure falls to 1.5 bar;

b) When the unit mass flow rate of the gas enters a pipeline at 280 °C, and flows steadily to the end of the pipe where the temperature is 20 °C. Neglect changes in velocity of the gas in the pipeline.

Chapter 3: Reversible and Irreversible Processes

Introduction

In section 1 we developed the non-flow energy equation and steady flow energy equation for the analysis of closed and open thermodynamic system. In section 2 the steam tables and ideal gas law were used to determine the properties of a thermodynamic system. For this current section of work these previous concepts are extended and applied to specific thermodynamic processes.

Reversible Non-flow Processes

The non-flow energy equation is stated simply as,

$$Q - W = u_2 - u_1$$

Where Q defines the heat energy added (+Q) to or taken from the system (-Q), W is the work done on (-W) or by the system (+W), and u_2 - u_1 is the change in internal energy. This equation simply states that the difference between the heat added to the system and the work done by the system is equal to the change in internal energy. This equation will now be applied to a set processes that often occur in a thermodynamic system.

Constant Volume Process

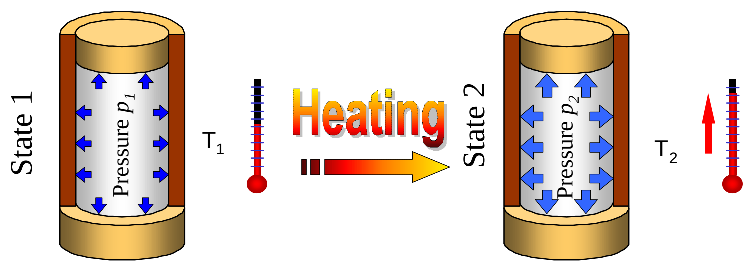


Figure 37: *Heating of a working fluid in a rigid container*

Consider the process shown in Figure 37. If the process occurs with no change in volume, then the process is classified as a constant volume process. It is clear that during the process the volume remains constant while the pressure increases as the temperature increases. For an ideal gas, the process would be represented by Charles' law. The consequence of this definition is that since no volumetric change has occurred, then work done by the system is zero. The non-flow energy equation then becomes,

$$Q = u_2 - u_1$$

Thus, the change in internal energy depends only on the heat added or subtracted from the system. This is illustrated below for both a vapour and a perfect gas.

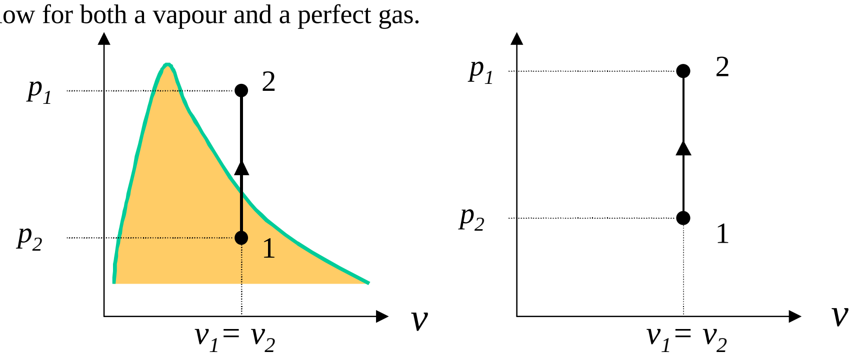


Figure 38: *p-v plots for a constant volume process, vapour and ideal gas.*

It is also worth noting that for a perfect gas,

$$Q = mc_v(T_2 - T_1)$$

Where *m* is the mass, c_v is the specific heat capacity for a constant volume and *T* is the temperature. Note that the result obtained here is similar to that developed by Joule in section 2.

Constant Pressure Process

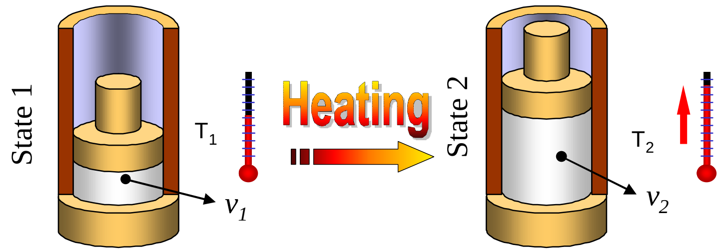


Figure 39: *Piston and cylinder assembly for a constant pressure process.*

It is possible to obtain a constant pressure process for a closed system using the simple construction shown in Figure 39. To ensure that a process is kept at a constant pressure the boundary must move as heat is added to the system. This is similar to the experiment carried out by Charles, where the system is normally visualised as a piston and cylinder assembly, where a mass is placed on the piston which is free to move. The mass then creates a constant force that results in a constant pressure. The non-flow energy equation for the constant pressure process is then,

$$Q - W = u_2 - u_1$$

The constant pressure process can also be plotted on the *p-v* chart as shown in Figure 40.

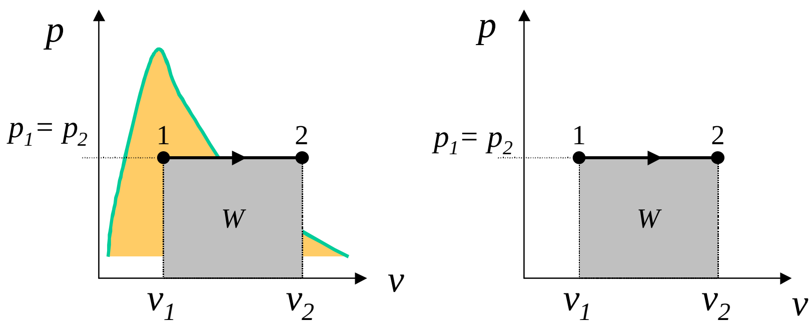


Figure 40: *p-v plots for a constant pressure process, vapour and ideal gas.*

The area beneath the lines defines the work by the system. In mathematical terms the work is,

$$W = \int_{v_1}^{v_2} p dv \text{ or } W = p(v_2 - v_1)$$

Inserting this definition of work into the non-flow equation yields

$$Q - p(v_2 - v_1) = u_2 - u_1 \text{ or } Q = (u_2 + pv_2) - (u_1 + pv_1)$$

$$\therefore Q = h_2 - h_1$$

The latter definition arises from $h = u + pv$. Therefore the heat added to or subtracted from the system is a function of the change in enthalpy. This method is normally employed for a process where water vapour is the working fluid. It is also worth noting that for a perfect gas,

$$Q = mc_p(T_2 - T_1)$$

Where *m* is the mass, c_p is the specific heat capacity for a constant pressure and *T* is the temperature

Constant Temperature/ Isothermal Process

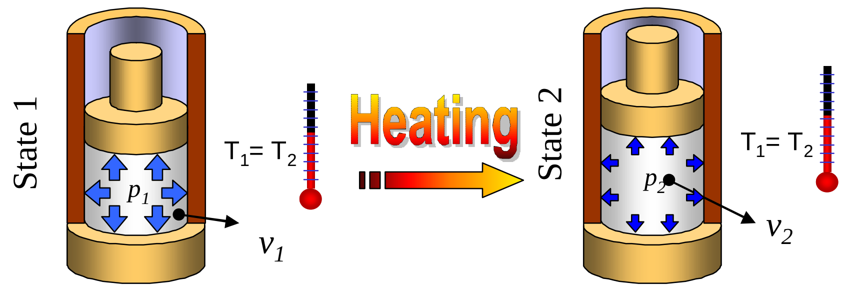


Figure 41: *Piston cylinder assembly schematic for a constant temperature process.*

For the experiments conducted by Boyle, any expansion or compression was conducted at a slow rate to ensure that any heat change in the working fluid was negligible. In normal operating conditions, however that level of control is not possible, nor desirable. It is therefore important to consider the following for a closed system;

- As the pressure changes due to expansion, there is a tendency for the temperature to fall. To create an isothermal process heat must be added during the expansion as illustrated in Figure 41.
- For compression, heat must be removed.

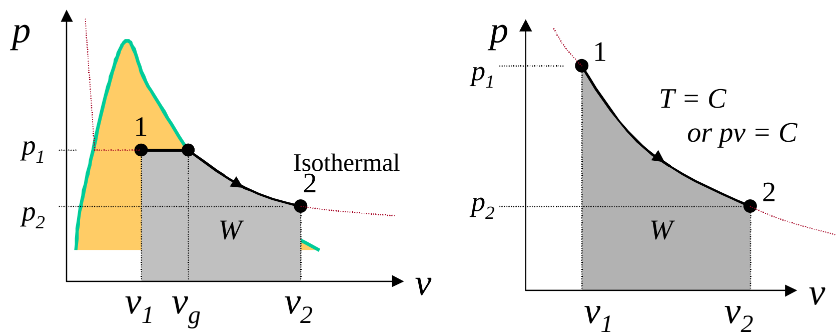


Figure 42: *p-v plots for a constant temperature process, vapour and ideal gas.*

Note that for a vapour illustrated in Figure 42, both the temperature and pressure remain constant in the wet vapour range. In the liquid and super heated range the temperatures changes relative to the pressure. Each of the isothermals on the *p-v* chart defines a constant temperature process. This process is easily defined in the wet vapour range 1-*g*, but is more difficult in the superheated range *g*-2. The heat supplied in the wet vapour range is simply $h_g - h_1$, the internal energy is $u_g - u_1$, therefore the work is defined as,

$$W = Q - (u_g - u_1) \text{ or } W = (h_g - h_1) - (u_g - u_1)$$

Note this equation **only** applies to the wet-vapour region where the pressure and temperature remain constant, i.e. a constant pressure process. For the superheat region, the internal energy is still defined by $u_2 - u_1$, whereas the quantity of heat supplied must be either provided, or found using an alternative method (this method will be covered in second year). Alternatively, the work done can be found graphically using the *p-v* chart.

For an ideal gas the relationship between pressure *p*, volume *v* and temperature *T* is given by,

$$pv = RT \text{ or } pV = mRT$$

When T is a constant, a hyperbolic relationship is obtained (identical to Boyle's work),

$$pv = \text{constant} = c$$

Since a definite constant exists it is possible to derive a relationship for the area under the $p - v$ curve,

$$W = \int_{v_1}^{v_2} p\,dv \text{ or } W = \int_{v_1}^{v_2} \frac{c}{v}\,dv$$
$$W = c[\ln v]_{v_1}^{v_2}$$
$$W = c(\ln v_2 - \ln v_1) \text{ or,}$$
$$W = c\ln\left(\frac{v_2}{v_1}\right)$$

Since p_1v_1 = constant = p_2v_2 then we can also write,

$$W = p_1v_1\ln\left(\frac{v_2}{v_1}\right) \text{or } W = p_2v_2\ln\left(\frac{v_2}{v_1}\right)$$
$$\text{Likewise } W = p_1v_1\ln\left(\frac{p_1}{p_2}\right)$$

Several formulations exist, each of which can be derived from first principals. It is also worth noting that from Joule's law for a constant temperature process,

$$U_2 - U_1 = mc_v(T_2 - T_1)$$
$$U_2 - U_1 = 0$$

Therefore, for an ideal gas **ONLY**,

$$Q - W = 0$$

Reversible Adiabatic non-flow Processes

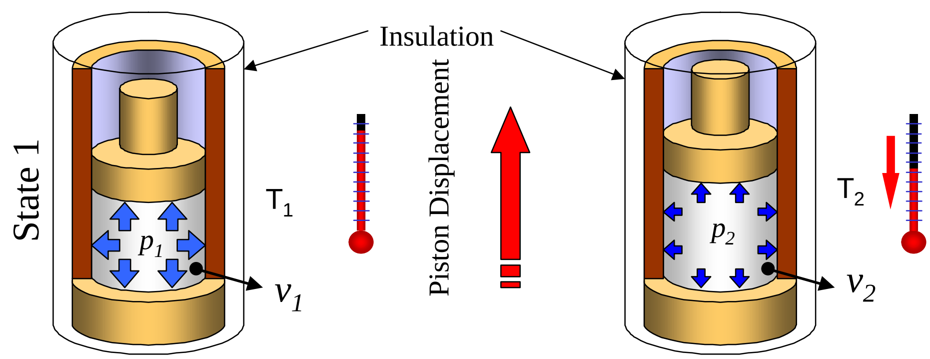


Figure 43: *Piston and cylinder assembly used to describe a non-flow adiabatic process.*

An adiabatic process is one in which no heat transfer occurs, i.e. the systems is perfectly insulated, therefore Q = 0. For the non-flow expansion shown in Figure 43, the temperature drops as the piston moves upwards. Therefore, for this system the work done is related directly to the change in internal energy. The non-flow energy equation then becomes,

$$-W = u_2 - u_1$$

It is possible to derive a relationship[5] between p and v for a perfect gas,

$$pv^g = \text{constant} = \text{c}$$

[5] See Eastop and McConkey Applied Thermodynamics for Engineering Technologists 5th Edition Section 3.2 for a complete derivation. (**remember the sign usage for work is reversed in this book**)

The work can therefore be defined as,

$$W = \int_{v_1}^{v_2} p dv \text{ or } W = \int_{v_1}^{v_2} \frac{c}{v^g} dv$$

$$W = c\left[\frac{v^{-g+1}}{-g+1}\right]_{v_1}^{v_2}$$

$$W = c\left(\frac{v_2^{-g+1} - v_1^{-g+1}}{1-g}\right)$$

$$W = \frac{p_2 v_2^g v_2^{1-g} - p_1 v_2^g v_2^{1-g}}{1-g}$$

$$W = \frac{p_2 v_2 - p_1 v_1}{1-g}$$

Each ideal gas will have a unique value of γ ($g = c_p/c_v$). The work done can also be represented graphically as illustrated in Figure 44. Note that for both a vapour and ideal gas the temperature changes during the process, for the expansion shown in Figure 43 the temperature drops.

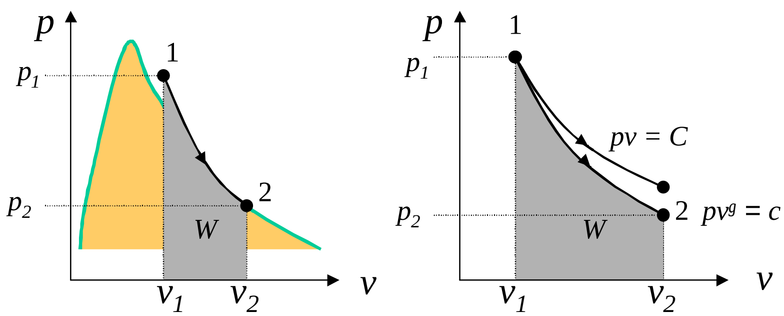


Figure 44: *p-v relationship for a vapour and ideal gas during an adiabatic process.*

Polytropic Processes

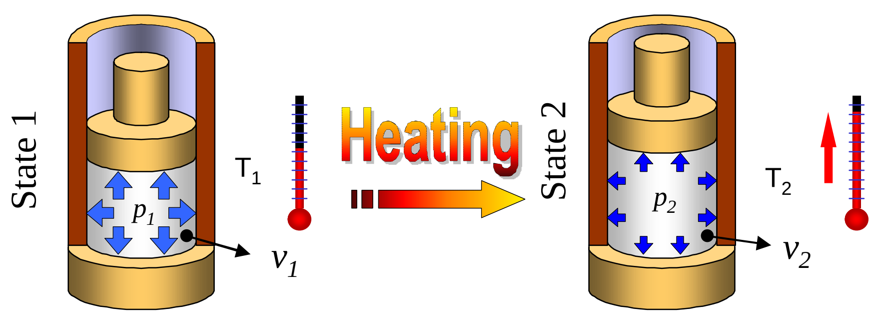


Figure 45: *A polytropic process where all the parameters can change.*

All of the processes listed so far are ideal, and therefore require assumptions about an actual process before they can be applied. In most instances expansion and compression processes occur with a change in volume, pressure, temperature and internal energy. A bicycle is a perfect example of such a process. It is found that in practice many processes using both vapours and ideal gases follow the law,

$$pv^n = \text{constant} = \text{c}$$

Where *n* is a constant determined using experimental results.

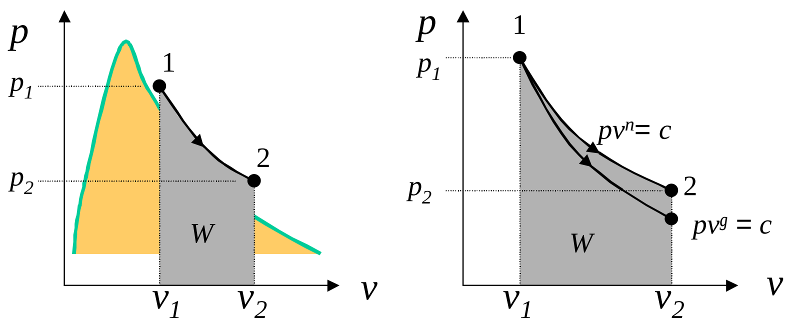


Figure 46: *p-v relationship for a vapour and ideal gas during a polytropic process.*

Since heat is transferred during a polytropic process, the adiabatic relationship no longer applies. The changes for work done are illustrated in Figure 46. Following the same integration method as used above for the adiabatic process (*n* replaces *g*) the work can be defined as,

$$W = \frac{p_2 v_2 - p_1 v_1}{1 - n}$$

Using the same methods illustrated for the isothermal process it is possible to derive further equations using $pv = RT$, detailed derivations can be found elsewhere[6]. One useful relationship that can be applied to ideal gases is,

$$Q = \left(\frac{n - g}{1 - g}\right) W$$

Then by assuming different values of *n*,

- $n = 0$, pv^0 = constant, p = constant, Constant Pressure Process.
- $n = 1$, pv^1 = constant, pv = constant, Isothermal Process.
- $n = \gamma$, pv^g = constant, Adiabatic Process.
- $n = \infty$, pv^{∞} = constant, v = constant, Constant volume Process.

All of these functions can only be applied to an ideal gas.

Reversible Flow Process

Consider a fluid flowing in a pipe section as illustrated in Figure 47. This process is normally described as a steady flow process and classed as an open system as the fluid crosses the boundary. By changing the definition of the boundary, it is possible to approximate the **processing** of the fluid to that of a non-flow process. This does not mean that the non-flow energy equation applies, this remains an open system, however it is possible to use the adiabatic equations described previously to determine the changes in the fluid during processing. Hence, we can determine state 2 by assuming

$$pv^g = \text{constant} = \text{c}$$

For a reversible adiabatic flow of an ideal gas for a unit-mass flow rate the steady-flow energy equation becomes,

$$\left(h_1 + \frac{C_1^2}{2}\right) + Q - W = \left(h_2 + \frac{C_2^2}{2}\right)$$

---

[6] See Eastop and McConkey Applied Thermodynamics for Engineering Technologists 5th Edition Section 3.3 for a complete derivation. (**remember the sign usage for work is reversed in this book**)

Therefore since Q = 0,

$$W = \left(h_1 + \frac{C_1^2}{2}\right) - \left(h_2 + \frac{C_2^2}{2}\right)$$

$$W = (h_1 - h_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right)$$

This is not the same as the previous definition of work,

$$-W = u_2 - u_1$$

I.e. although the two end states can be determined using non-flow equations, the work cannot.

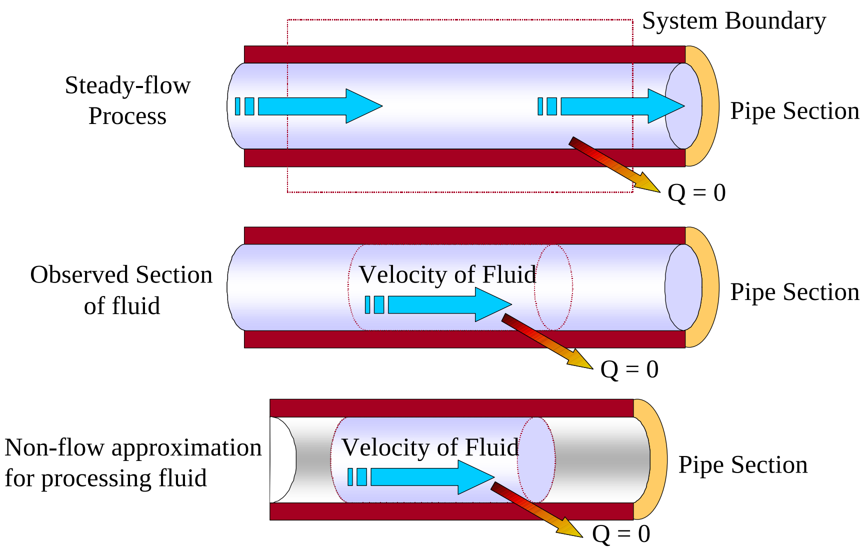


Figure 47: *Non-flow approximation for the processing of a fluid in a pipe during steady-flow.*

Irreversible Processes

For a process to be reversible, the following criteria must be satisfied;

1. The friction piston and cylinder and internal friction of the fluid must be negligible,
2. The difference between the fluid pressure and its surroundings must be infinitely small, and
3. The difference between the fluid temperature and its surroundings must be infinitely small.

Most processes are only approximately reversible, in practical circumstances a reversible process is only internally reversible, i.e. the systems undergoes a process that can be reversed but the surroundings are irreversibly changed. Certain processes cannot be thought of as internally reversible, two important ones are listed below.

Unresisted Free Expansion

Consider the two perfectly insulated interconnecting vessels shown below in Figure 48. When the valve is opened the higher-pressure fluid in A will flow into B. Since the vessel is perfectly insulated there will be no transfer of heat $Q = 0$ and since there is no volume change there will be no work done, $W = 0$. The non-flow equation then becomes,

$$u_2 - u_1 = 0 \text{ or } u_2 = u_1$$

For an ideal gas it can also be shown that $T_1 = T_2$.

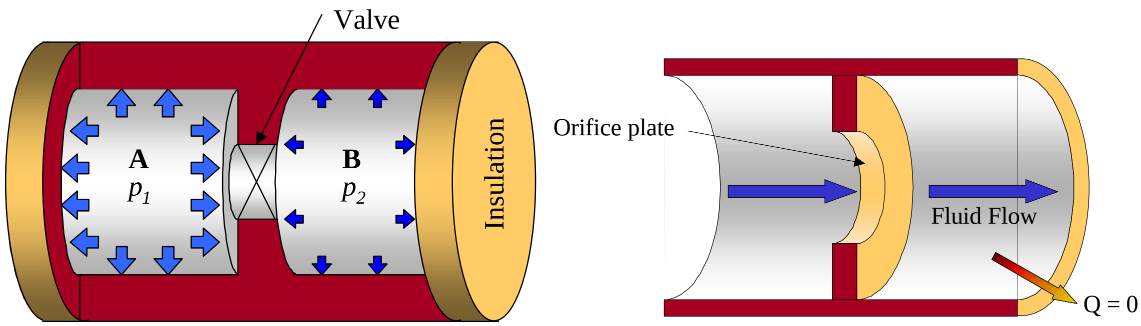


Figure 48: *Unresisted expansion and throttling*

Throttling

Throttling is a process whereby a restriction to flow (orifice plate, valve etc.) is placed in a pipeline as shown in Figure 48. Assuming that the change in velocities and heat loss is negligible then using,

$$\left(h_1 + \frac{C_1^2}{2}\right) + Q - W = \left(h_2 + \frac{C_2^2}{2}\right)$$

We can show that since $W = 0$,

$$h_1 = h_2$$

For an ideal gas it can also be shown that $T_1 = T_2$.

Class Examples

3.1 A mass of 0.05 kg of a fluid is heated at a constant pressure of 2 bar until the volume occupied is 0.0658 m^3. Calculate the heat supplied and the work done:

a) When the fluid is steam, initially dry saturated;

b) When the fluid is air, initially at 130 °C

3.2 Steam at 7 bar and dryness fraction 0.9 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. Calculate the change of internal energy and the change of enthalpy per kg of steam.

3.3 1 kg of nitrogen (molar mass 28 kg/kmol) is compressed reversibly and isothermally from 1.01 bar, 20 °C to 4.2 bar. Calculate the work done and the heat flow during the process. Assume nitrogen to be a perfect gas.

3.4 1 kg of steam at 100 bar and 375 °C expands reversibly in a perfectly thermally insulated cylinder behind a piston until the pressure is 38 bar and the steam is then dry saturated. Calculate the work done.

3.5 Air at 1.02 bar 22°C, initially occupying a cylinder volume of 0.015 m^3, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate the final temperature, the final volume and the work done on the mass of air in the cylinder.

3.7 1 kg of a perfect gas is compressed from 1.1 bar, 27 °C according to a law $pv^{1.3} = c$, until the pressure is 6.6 bar. Calculate the heat flow to or from the cylinder walls.

3.8 A gas turbine receives gases from the combustion chamber at 7 bar and 650 °C, with a velocity of 9 m/s. The gases leave the turbine at 1 bar with a velocity of 45 m/s. Assuming that the expansion is adiabatic and reversible in the ideal case, calculate the power output per unit mass flow rate. For the gases take $g = 1.333$ and $c_p = 1.11$ kJ/kgK

3.9 Air at 20 bar is initially contained in a vessel A, the volume of which can be assumed to be 1 m^3. The valve it opened and the air expands to fill vessels A and B. Assuming that the vessels are of equal volume calculate the final pressure of the air.

Chapter 4: Pressure at a Depth

Introduction

In previous chapters, the pressure exerted by the working fluid was introduced. The pressure in this instance was caused by a difference in atmospheric pressure and the pressure inside the vessel. The internal pressure within the vessel resulted from the impact of molecules on the vessel walls. The magnitude of the pressure depended simply on the internal energy of the working fluid. For most ideal gases, this pressure is assumed the same throughout the fluid. However when dealing with liquid forms of the working fluid the pressure will vary depending on the depth. Relationships governing pressure and a depth and possible engineering applications are developed within this chapter.

Pressure Measurement Scales

Pressure can be measured with two scales. Each of the scales have a different zero value. For a gauge pressure reading this value is 1.016 bar, i.e. atmospheric pressure. For absolute pressure, zero represents a perfect vacuum. Thus, the value p shown in Figure 49 can have two readings,

1) gauge = p

2) Absolute = p + Atmospheric or Absolute = p + 1.016 bar

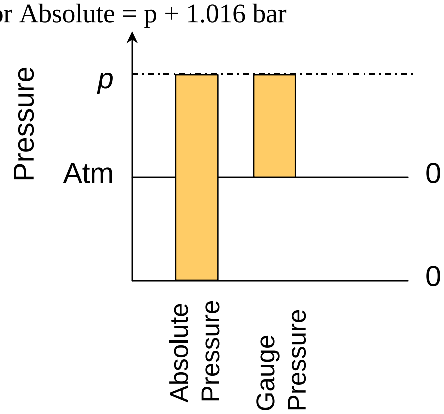


Figure 49: *Gauge and Atmospheric Pressure Measurement Scales.*

Absolute-pressure measurement values were used mainly in the thermodynamic sections of the notes. For fluid mechanics, it is more common to use the gauge pressure measurement.

Pressure at a Depth

Consider the volume of water shown in Figure 50. The mass at the top of the water column exerts a force equivalent to,

$$F = m \cdot g$$

The pressure at the top of the water p_T is therefore given by

$$p_T = \frac{F}{A}$$

At the base of the water column, the total force is that of the mass at the top and the weight of the water. Remember weight is a force, mass is simple the quantity of matter. The mass of water is given by,

$$m = \rho \cdot V$$

The force exerted by the water is,

$$F_W = m \cdot g = \rho \cdot V \cdot g$$

The pressure exerted by the water is then,

$$p_W = \frac{F_w}{A} = \frac{r \cdot V \cdot g}{A}$$

The volume of water divided by the cross-sectional area is simple the length or the depth of the water h. The equation for the pressure exerted by the water then becomes,

$$p = r \cdot g \cdot h$$

The total pressure at the base of the column of fluid is

$$p_{Total} = p_M + p_W$$

The pressure at the top of a column of fluid is normally atmospheric pressure. The gauge pressure is therefore given by the equation highlighted above, whereas the absolute pressure is,

$$p_{ABS} = p_{ATM} + r \cdot g \cdot h$$

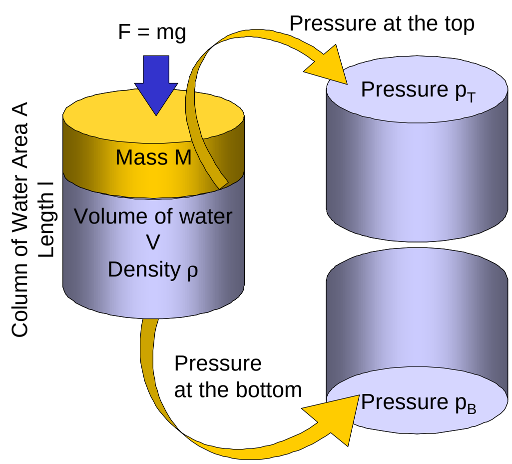


Figure 50: *Pressure in a column of liquid*

The equation for pressure at a depth implies that the pressure increases linearly relative to the depth of fluid. The derivation given above also illustrates that the area does not affect the pressure. The linear increase in pressure with depth is shown in Figure 51.

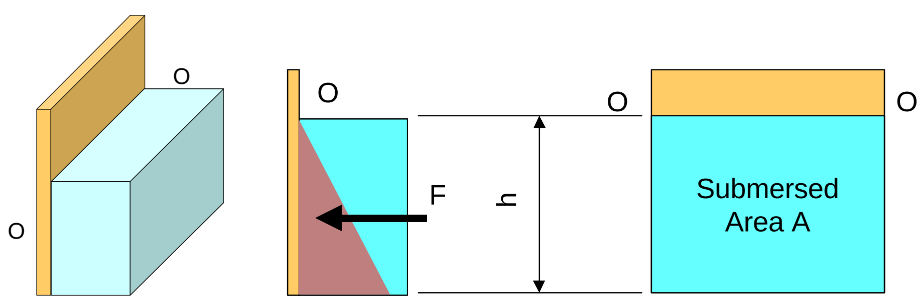


Figure 51: *Pressure on a submersed surface.*

The pressure at the top of the fluid is simply atmospheric, therefore the gauge pressure is zero. The pressure at the base of the section is shown in Figure 51 is,

$$p = r \cdot g \cdot h$$

The pressure acting on the entire surface can be assumed an average value or half the maximum pressure at the base of the area, i.e.,

$$p = \tfrac{1}{2} \cdot r \cdot g \cdot h$$

The force exerted can be found by multiplying the pressure by the area A, therefore,

$$F = \tfrac{1}{2} \cdot A \cdot r \cdot g \cdot h$$

Transmission of Fluid Pressure

Incompressible flow is assumed for all the fluid mechanics work presented in this module. This has implications for transmission of fluid pressure. Consider the 'hydraulic jack' shown in Figure 52. A force F_1 is applied to the piston of a small cylinder and forces oil or water out into the large cylinder thus raising the piston supporting the load F_2. The force F_1 acting on area A_1 produces a pressure p_1 which, is transmitted equally in all directions through the liquid. If we first consider h equal to zero i.e. both pistons at the same level, then for equilibrium p_1 must equal p_2 then mathematically,

$$p_1 = \frac{F_1}{A_1} \text{ and } p_2 = \frac{F_2}{A_2}$$

Equating the pressures,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ or } F_1 = F_2 \frac{A_1}{A_2}$$

From this equation it is clear that as the area A_1 decreases the force F_1 required to overcome F_2 reduces. In order to accommodate the height differences simply add or subtract the pressure head $p = r \cdot g \cdot h$.

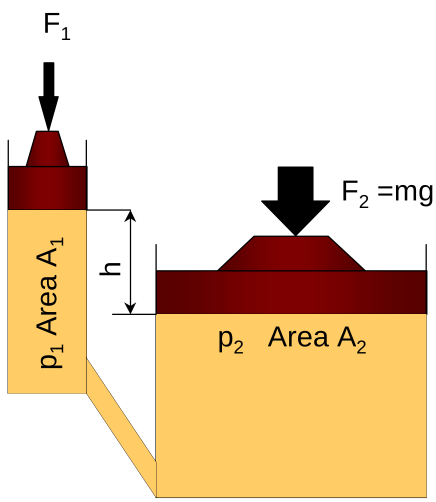


Figure 52: *Simple Hydraulic Jack assembly.*

Pressure Measurement

The pressure at a depth principal can be applied to measuring pressures. Two main applications are the manometers and barometers, used for measuring the pressure in pipelines and atmospheric pressure respectively.

Manometers

The U tube manometer shown in Figure 53 (a), is open to atmospheric pressure at each end. Since the pressure is the same at both ends of the tube the fluid in the tube remains level. The fluid used for this applications can be water, oil or in some cases mercury. The liquid used in a Manometer depends on the pressure range; a more dense fluid can be used to measure higher pressures.

If one end of the manometer is connected to a pipeline of gauge pressure p then the height **difference** h can be used to determine the gauge pressure as,

$$p = r \cdot g \cdot h$$

The absolute pressure can be obtained by adding atmospheric pressure to the gauge pressure. The measurement of atmospheric pressure is achieved using a Barometer.

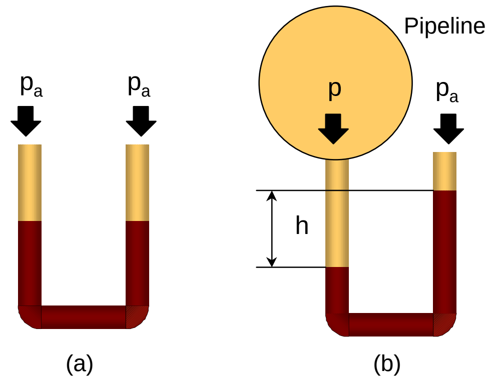


Figure 53: *Pressure measurement, using Manometers*

Barometer

The schematic in Figure 54 shows a container partly filled with liquid. A fine bore tube, open at the bottom and closed at the top, stands vertically as shown. The pressure acting at the top of the column of mercury is zero absolute. Atmospheric pressure is acting on the surface of the liquid container. The pressure of the atmosphere supports the liquid column h meters high, therefore,

$$p = r \cdot g \cdot h$$

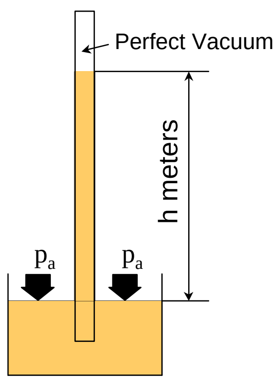


Figure 54: *Schematic of a Barometer*

The liquid normally used for a Barometer is Mercury (density 13.6×10^3 kg/m^3). Using mercury, the height of the column is only 1m. If water were used then a 10m column would be required to measure the same pressure.

Class Examples

4.1 What is the pressure at a depth of 10 m in free water, density 1000kg/m3.

4.2 The pressure in a pipe is 6 bar, if there is a small bore tube, open at both ends fixed into the pipe, to what height will the water rise in the pipe?

4.3 Two cylinders with pistons are connected by a pipe containing water. Their diameters are 75mm and 600 mm and the face of the smaller piston is 6 m above the larger. What force on the smaller piston is required to maintain a load of 3500 kg on the larger piston.

4.4 A U-tube manometer is using an oil of relative density 0.8 as the measuring fluid. If when the manometer is used to measure the pressure in a pipe the reading is 300mm what is the gas pressure? What is the absolute pressure if a mercury barometer reading of atmospheric pressure is 750mm of Hg (relative density 13.6)?

Chapter 5: Continuity of Flow

Introduction

As stated in chapter 4 all fluid flow in this module is assumed to be incompressible. This section of the notes illustrates how this principle can be used to establish a relationship for the continuity of flow and hence determine the changes in velocity and flow for complex pipe networks

Relationship between Pipe Diameter and Fluid Velocity

For incompressible flow, the amount of liquid flowing into a single pipe without branches must equal the amount of liquid flowing out of that section. Consider the pipe section shown in Figure 55. The implications of this statement are simply that,

$$Q_A = Q_B$$

Where Q_A and Q_B are the flow rates at sections A and B respectively. The units used for Q are normally m^3/s, however for mass flow rates these units change to kg/s.

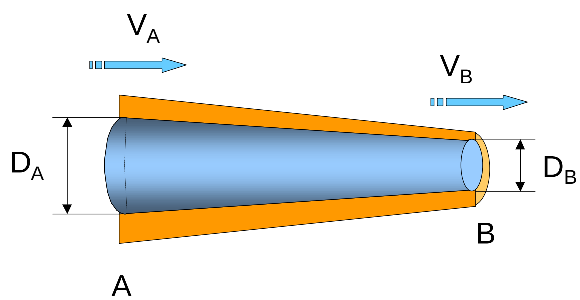


Figure 55: *Incompressible flow in a single pipe section.*

The flow rate from A to B in Figure 55 therefore remains constant. It is however obvious from Figure 55 that the section diameter of the pipes from A to B does change. The flow rate can be described as,

$$Q = A \cdot V$$

Where A is the cross sectional area of the pipe at any section and V is the velocity at that section. Therefore for this pipe section,

$$A_A \cdot V_A = A_B \cdot V_B$$

Solving for B

$$V_B = V_A \cdot \frac{A_A}{A_B}$$

It is possible to simplify this relationship further by realising that,

$$A = \frac{p \cdot D^2}{4}$$

Inserting this equation and simplifying we obtain

$$V_B = V_A \cdot \left(\frac{D_A}{D_B} \right)^2$$

Hence as the diameter of the pipe decreases then the velocity also decreases.

Continuity of Flow in Pipe Networks

In the previous section a single pipe was considered, however in reality this pipe may branch into new sections or join other pipes. In this instance, we can still apply the principles but we must develop them in a different manner. Consider the branching pipe network shown in Figure 56.

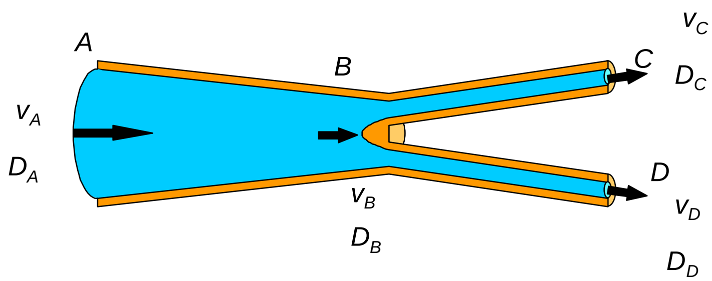


Figure 56: *Incompressible flow in a branching pipe network.*

Since the fluid is incompressible the amount of fluid entering the pipe at A, must equal that branching at B, must equal the total and C and D. Or expressed mathematically

$$Q_A = Q_B = Q_C + Q_D$$

Then expanding the equation,

$$A_A \cdot V_A = A_B \cdot V_B = A_C \cdot V_C + A_D \cdot V_D$$

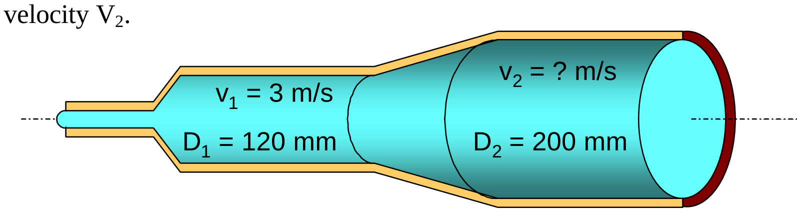
The using the area,

$$D_A^2 \cdot V_A = D_B^2 \cdot V_B = D_C^2 \cdot V_C + D_D^2 \cdot V_D$$

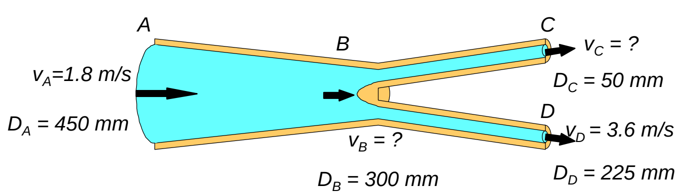
It is more difficult to see a direct relationship between the pipe diameters and the velocities for the branching network.

Class Examples

5.1 The diagram below shows a horizontal pipe through which water is flowing. For the values shown, what is the velocity V_2.



5.2 The diagram shows a pipe that divides the flow from A to B into two separate flows. The Relevant dimensions are shown. Determine the unknown values and the flow rate at each point.



Example Problem Solutions

Example 5.1 Solution

For continuity of flow,

$$Q_1 = Q_2 = A_1 v_1 = A_2 v_2$$

$$\therefore v_2 = \frac{v_1 A_1}{A_2}$$

The areas are,

$$A_1 = 0.120^2 \frac{p}{4} = 11.31\times10^{-3}\, m^2$$

$$A_2 = 0.200^2 \frac{p}{4} = 31.42\times10^{-3}\, m^2$$

The velocity at point 2 is then,

$$v_2 = \frac{3\times11.31\times10^{-2}}{31.42\times10^{-2}} = 1.08 m/s$$

Example 5.2 Solution:
Since the velocity and diameter are known at point A then the flow rate at A is,

$$Q_A = A_A v_A = 0.45^2 \frac{p}{4}\times1.8$$
$$= 0.286 m^3/s$$

The flow rate at A will equal that at B. To find the velocity at C we must first establish the flow rate at D,

$$Q_D = A_D v_D = 0.225^2 \frac{p}{4}\times3.6$$
$$= 0.143 m^3/s$$

For continuity of flow,

$$Q_A = Q_B = Q_D + Q_C$$

Therefore,

$$Q_B = 0.286 m^3/s$$

And

$$Q_C = Q_A - Q_D$$
$$= 0.286 - 0.143$$
$$= 0.143 m^3/s$$

The velocity at B and C are then,

$$v_B = \frac{Q_B}{A_B} = \frac{0.286}{\left(0.3^2 \frac{p}{4}\right)} = 4.05 m/s$$

$$v_C = \frac{Q_C}{A_C} = \frac{0.143}{\left(0.15^2 \frac{p}{4}\right)} = 8.10 m/s$$

Chapter 6: Analysis of Pipe Networks

Introduction

In Chapter 5 a series of equations were developed to describe the changes in fluid velocity with changes in pipeline diameters. In this Chapter, changes in pipe size, height and internal pressure are brought together in the form of Bernoulli's equation. This equation is based on the energies found in a typical pipe network.

Energies in a Pipe Network

A liquid may possess three forms of energy,

- *Potential energy* due to a height difference (Figure 57)
- *Kinetic energy* due to the velocity of the fluid (Figure 57)
- *Pressure energy* due to the fluid flowing in a continuous stream under pressure.

$P.E. = m \cdot g \cdot h$ (J) $\qquad K.E. = \frac{1}{2} \cdot m \cdot v^2$ (J)

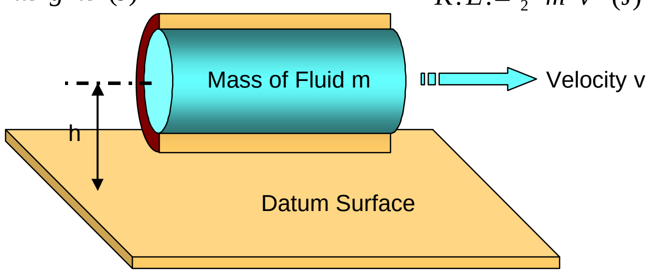


Figure 57: *Potential and Kinetic Energy during incompressible flow*

Pressure energy, when a fluid flows in a continuous stream cross-sectional area A (m^3) under a pressure p (N/m^2), the force will equal,

$$F = p \cdot A$$

Our standard definition of work is,

$$\text{Work} = \text{Force} \cdot \text{Distance moved}$$

The volume of the liquid is given as,

$$\text{Volume} = \frac{m}{r}$$

The distance moved by the fluid is,

$$\text{Distance moved} = \frac{\text{Volume}}{\text{Cross-sectional area}} = \frac{V}{A} = \frac{m}{r \cdot A}$$

Since work is equal to the force exerted times the distance moved,

$$\text{Work done or Pressure energy} = p \cdot A \times \frac{m}{r \cdot A} = \frac{p \cdot m}{r} \text{ (J)}$$

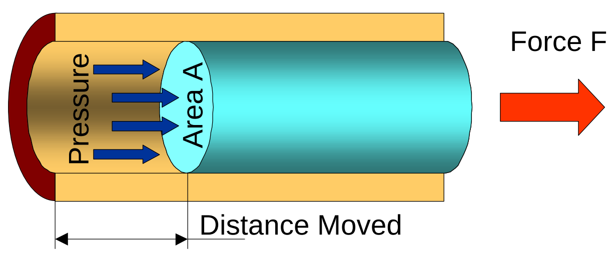


Figure 58: *Work required to move the fluid.*

Bernoulli's Equation

We have previously developed an equation for a steady flow system (Page 14 of introduction to thermodynamics), i.e.

$$E_1 + Q = E_2 + W$$

Where E_1 and E_2 are the energies at the start and end of the process, Q is the energy transfer by heating and W is the work done. If we apply the same reasoning to the pipe system shown in Figure 59 and consider the system to be open, heat transfer, work and change in internal energy all equal zero, therefore

$$E_A = E_B$$

Or the total energy at A must equal the total energy at B.

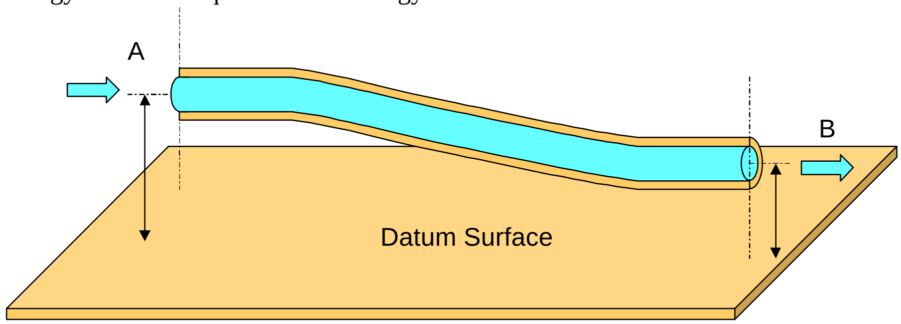


Figure 59: *Simple pipe system.*

The energies contained at A include potential energy (PE), kinetic energy (KE), and the energy required to force the fluid through the pipe. Our statement of energies then becomes,

$$\therefore\ m \cdot g \cdot h_A + \tfrac{1}{2} \cdot m \cdot v_A^2 + \frac{p_A \cdot m}{r} = m \cdot g \cdot h_B + \tfrac{1}{2} \cdot m \cdot v_B^2 + \frac{p_B \cdot m}{r}$$

Where m is the mass of the fluid, g is the acceleration due to gravity, h_A and h_B are the heights of points A and B, v_A and v_B are the velocities of the fluid at point A and B and p_A and p_B are the pressures at points A and B respectively. As noted for the manometers it is easier to measure height differences. Therefore dividing both sides of the equation by mg yields,

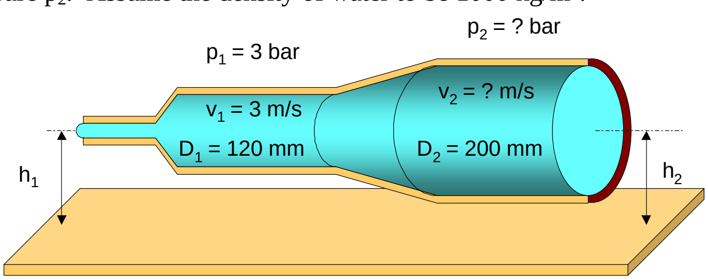
$$h_A + \frac{v_A^2}{2 \cdot g} + \frac{p_A}{r \cdot g} = h_B + \frac{v_B^2}{2 \cdot g} + \frac{p_B}{r \cdot g}$$

Equation 8

Equation 8 is known as Bernoulli's equation and is used to analyse flow in pipe systems.

Class Examples

6.1 The diagram below shows a horizontal pipe through which water is flowing. For the values shown, what is the pressure p_2. Assume the density of water to be 1000 kg/m^3.



6.2 A pipe 300 m long tapers from 1.2 m diameter to 0.6 m diameter at its lower end and slopes downward at 1 in 100, i.e. for every 100 m of travel the pipe drops 1 m. The pressure at the upper end is 69 kN/m^2. Neglecting friction losses, find the pressure at the lower end of the pipe, when the flow rate is 0.092 m^3/min.

Chapter 7: Empirical Measurement of Flow rate

Introduction

Flow rate can be measured by timing how long it takes to collect a known volume or mass of a liquid. Unfortunately, this method of flow measurement interrupts the flow. More cost effective and less intrusive methods of flow measurement are introduced in this chapter.

Venturi and Orifice Plate Meters

The Venturi and Orifice plate meters operate on the principal of pressure difference. In chapters 5 and 6 a series of equations were developed for continuity of flow and pressure difference using Bernoulli's equation. If a liquid flowing through a single pipe section is force through a reduced section then by continuity of flow the velocity is increased. The increase in velocity reduces the pressure. This pressure difference is normally measured using Manometers as shown below in Figure 60. This concept is developed below using Bernoulli's equation.

Development of Theory for Flow analysis

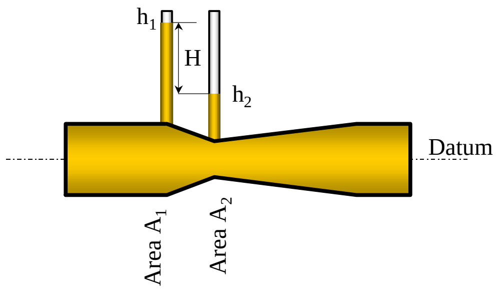


Figure 60: *Schematic of the Venturi meter.*

Consider the schematic of the Venturi meter in Figure 60. As the fluid flows into the meter the reduced cross sectional area increases the velocity of the fluid. The increased velocity reduces the pressure at this section to ensure continuity of flow (the difference in height of the manometers). Applying Bernoulli's equation we obtain,

$$\frac{v_1^2}{2\cdot g}+\frac{p_1}{r\cdot g}=\frac{v_2^2}{2\cdot g}+\frac{p_2}{r\cdot g}$$

Grouping variables

$$\frac{v_2^2}{2\cdot g}-\frac{v_1^2}{2\cdot g}=\frac{p_1}{r\cdot g}-\frac{p_2}{r\cdot g}$$

Factorising

$$\frac{1}{2\cdot g}\cdot\left(v_2^2-v_1^2\right)=\frac{1}{r\cdot g}\cdot\left(p_1-p_2\right)$$

Recall that for a manometer $p=r\cdot g\cdot h$,

$$\therefore\ \frac{1}{2\cdot g}\cdot\left(v_2^2-v_1^2\right)=\frac{1}{r\cdot g}\cdot\left(r\cdot g\cdot h_1-r\cdot g\cdot h_2\right)$$

Simplifying

$$\therefore\ \frac{1}{2\cdot g}\cdot\left(v_2^2-v_1^2\right)=\left(h_1-\cdot h_2\right)=H$$

$$\therefore\ \left(v_2^2-v_1^2\right)=2\cdot g\cdot H$$

We have defined the volumetric flow rate as $Q = Av$

$$\therefore\ v_1 = \frac{Q}{A_1} \text{ and } v_2 = \frac{Q}{A_2}$$

$$\therefore \left(\frac{\mathrm{Q}}{\mathrm{A}_2}\right)^2 - \left(\frac{\mathrm{Q}}{\mathrm{A}_1}\right)^2 = 2 \cdot g \cdot H$$

$$\therefore \frac{\mathrm{Q}^2}{\mathrm{A}_2^2} - \frac{\mathrm{Q}^2}{\mathrm{A}_1^2} = 2 \cdot g \cdot H$$

$$\therefore Q^2 \cdot \left(\frac{1}{\mathrm{A}_2^2} - \frac{1}{\mathrm{A}_1^2}\right) = 2 \cdot g \cdot H$$

$$\therefore Q^2 \cdot \left(\frac{\mathrm{A}_1^2 - \mathrm{A}_2^2}{\mathrm{A}_2^2 \cdot \mathrm{A}_1^2}\right) = 2 \cdot g \cdot H$$

$$\frac{Q^2 \cdot \mathrm{A}_2^2}{\mathrm{A}_2^2 \cdot \mathrm{A}_1^2} \cdot \left(\frac{\mathrm{A}_1^2}{\mathrm{A}_2^2} - 1\right) = 2 \cdot g \cdot H$$

$$Q^2 = \frac{\mathrm{A}_1^2 \cdot 2 \cdot g \cdot H}{\left(\frac{\mathrm{A}_1^2}{\mathrm{A}_2^2} - 1\right)}$$

And finally

$$Q^2 = \frac{\mathrm{A}_1^2 \cdot 2 \cdot g \cdot H}{\left(\frac{\mathrm{A}_1^2}{\mathrm{A}_2^2} - 1\right)} \quad \therefore Q = \sqrt{\frac{\mathrm{A}_1^2 \cdot 2 \cdot g \cdot H}{\left(\frac{\mathrm{A}_1^2}{\mathrm{A}_2^2} - 1\right)}} \quad \therefore Q = \mathrm{A}_1 \cdot \sqrt{\frac{2 \cdot g \cdot H}{\left(\frac{\mathrm{A}_1^2}{\mathrm{A}_2^2} - 1\right)}}$$

Separating all of the constants on the right hand side

$$Q = \mathrm{A}_1 \cdot \sqrt{\frac{2 \cdot g}{\left(\frac{\mathrm{A}_1^2}{\mathrm{A}_2^2} - 1\right)}} \cdot \sqrt{\mathrm{H}}$$

$$\therefore \mathrm{Q} = \mathrm{k} \cdot \sqrt{\mathrm{H}}$$

$$\text{where } \mathrm{k} = \mathrm{A}_1 \cdot \sqrt{\frac{2 \cdot g}{\left(\frac{\mathrm{A}_1^2}{\mathrm{A}_2^2} - 1\right)}}$$

Where k is a constant for a particular meter. These equations apply equally well to the orifice plate meter.

Coefficient of Discharge

The equations developed above assume that the meters have no losses, whereas in reality both friction and turbulence contribute to losses in the system. The smooth transition of the Venturi meter reduces the effect of the losses and is therefore the most effective of the two meters. The Orifice plate however introduces large losses by creating turbulent flow around the plate (Figure 61). We compensate for these losses by calibrating the meters and introducing a constant known as the coefficient of discharge Cd. Then mathematically

$$Q = C_d \cdot k \cdot \sqrt{H} \quad m^3/s$$

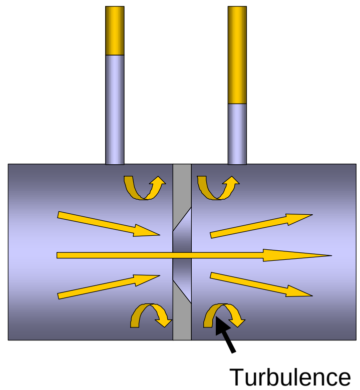


Figure 61: *Illustration of turbulence in the Orifice plate meter.*

Class Examples

7.1 A horizontal Venturi meter has inlet and throat diameters of 350 mm and 150 mm respectively. If the coefficient of discharge is 0.96 and the difference in head between inlet and outlet is 1.27 m of water, determine the volume flow rate in m^3/s.

Important Formulae

Thermodynamics

Energy Equations

Non-Flow Energy Equation $Q - W = u_2 - u_1$

Steady Flow Energy Equation $Q - W = (h_2 - h_1) + \frac{1}{2}\left(C_2^2 - C_1^2\right) + g(z_2 - z_1)$

Enthalpy $h = u + pv$

From the steam tables: Wet Vapour

Specific Volume $v = v_g x$

Enthalpy $h = h_f + x h_{fg}$

Internal Energy $u = u_f + x\left(u_g - u_f\right)$

From the steam tables: Table Interpolation

p	T	v_g	u_f	u_g	H_f	h_{fg}	h_g	s_f	s_{fg}	s_g
(bar)	(°C)	(m^3/kg)	(kJ/kg)		(kJ/kg)			(kJ/kg K)		
1.00	99.6	1.694	417	2506	417	2258	2675	1.303	6.056	7.359
1.10	102.3	1.549	429	2510	429	2251	2680	1.33	5.994	7.327

$$u_g \text{ at } 100^{\circ}\text{C} = \left(u_g \text{ at } 99.6\ ^{\circ}\text{C}\right) + \left(\frac{100 - 99.6}{102.3 - 99.6}\right) \times \left\{\left(u_g \text{ at } 102.3^{\circ}\text{C}\right) - \left(u_g \text{ at } 99.6\ ^{\circ}\text{C}\right)\right\}$$

Perfect Gases

Ideal Gas Law $p \cdot V = m \cdot R \cdot T$ or $p \cdot v = R \cdot T$

Molar mass $R = \dfrac{\tilde{R}}{\tilde{m}}$

Specific Heat Capacities and Heat Transfer $Q = mc_p(T_2 - T_1)$ and $Q = mc_v(T_2 - T_1)$

Joules Law (IDEAL GAS ONLY) $u_2 - u_1 = c_v\left(T_2 - T_1\right)$

Specific Enthalpy of a Perfect Gas $h = c_p T$ or $H = mc_p T$

Note that absolute temperature must be used in these equations.

Ratio of Specific Heat Capacities $g = \dfrac{c_p}{c_v}$

Reversible Processes

<u>For BOTH Ideal gases and Steam</u>

Work done $W = \int_{v_1}^{v_2} p\,dv$

<u>For An ideal gas only</u>

Constant Pressure Process $p = constant$,

Constant Volume Process $v = constant$, W = 0

Constant Temperature Process $pv = constant$

Zero Heat Transfer (Adiabatic) Process $pv^g = constant$, Q = 0

Polytropic (real) Process $pv^n = constant$,

Also for a Polytropic Process $Q = \left(\frac{n-g}{1-g}\right)W$

Reversible Adiabatic flow of a vapour/gas $W = \left(h_1 - h_2\right) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right)$

Fluid Mechanics

Pressure at a depth $p = r \cdot g \cdot h$

The volume flow rate = $Q = Av$ (m^3/s)

Continuity of Flow $Q_A = Q_B$

Bernoulli's Equation $h_A + \frac{v_A^2}{2 \cdot g} + \frac{p_A}{r \cdot g} = h_B + \frac{v_B^2}{2 \cdot g} + \frac{p_B}{r \cdot g}$

Venturi/Orifice Plate Meters

$$Q = A_1 \cdot \sqrt{\frac{2 \cdot g}{\left(\frac{A_1^2}{A_2^2} - 1\right)}} \cdot \sqrt{H}$$

$$\therefore Q = k \cdot \sqrt{H}$$

$$\text{where } k = A_1 \cdot \sqrt{\frac{2 \cdot g}{\left(\frac{A_1^2}{A_2^2} - 1\right)}}$$

Losses In a Flow Meter $Q = C_d \cdot k \cdot \sqrt{H} \quad m^3/s$