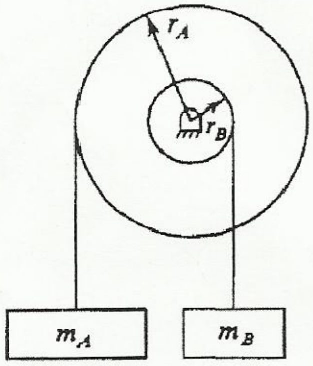
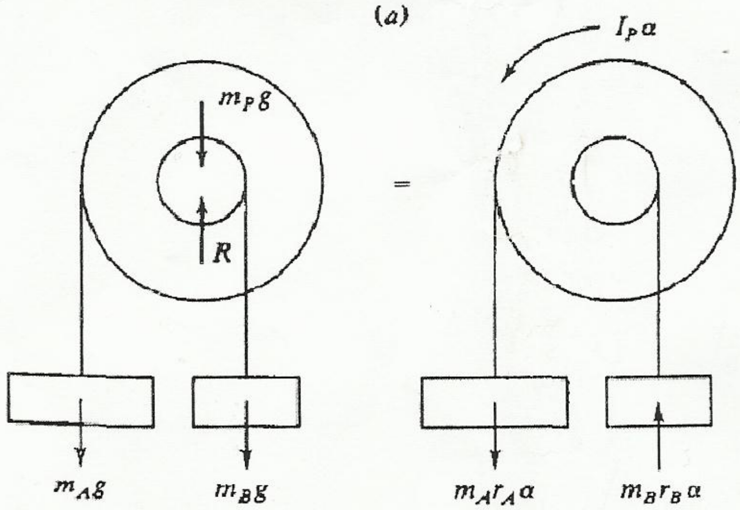
Mtonga Daniel

Q1 Determine the angular acceleration of the pulley



(a)

Solution



$$\sum Mo_{ext} = \sum Mo_{eff}$$

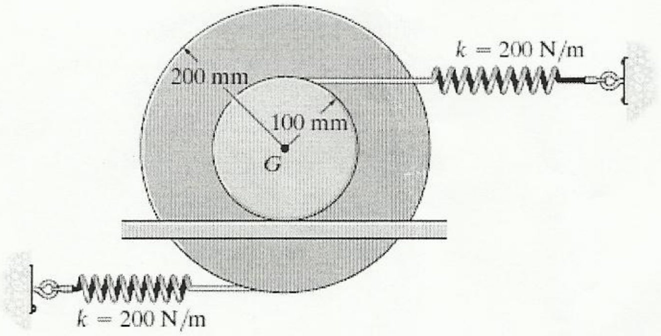
$$m_A g r_A - m_B g r_B = Ip\alpha + m_A r_A^2 \alpha + m_B r_B^2 \alpha$$

$$Where: r_A = 30cm;\ rB = 20cm;\ Ip = 0.6kg.m^2;\ m_A = 5kg;\ m_B = 3kg$$

$$\alpha = \frac{m_A g r_A - m_B g r_B}{Ip + m_A r_A^2 + m_B r_B^2}$$

$$\alpha = 7.55 rad / s^2$$

Q2 Determine the differential equation of motion of the 15-kg spool. Assume that it does not slip at the surface of contact as it oscillates. Theradius of gyration of the spool about its center of mass is K_G=125mm. The springs are originally unstretched.



solution

Energy Equation: Since the spool rolls without slipping, the stretching of both springs can be approximated as $x_1=0.1\theta$ and $x_2=0.2\theta$ when the spool is being displaced by a small angular displacement θ. Thus the elastic potential energy is

$$V_p = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2$$

$$= \frac{1}{2}(200)(0.1\theta)^2 + \frac{1}{2}(200)(0.2\theta)^2 = 5\theta^2$$

$$V = V_e = 5\theta^2$$

The mass moment of the spool about A is I_A

$$I_A = 15\left(0.125^2\right) + 15\left(0.1^2\right) = 0.384375 kg.m^2$$

The kinetic energy is

$$T = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(0.384375)\dot{\theta}^2 = 0.19211875\dot{\theta}^2$$

The total energy of the system is

$$U = T + V = 0.1921875\dot{\theta}^2 + 5\theta^2$$

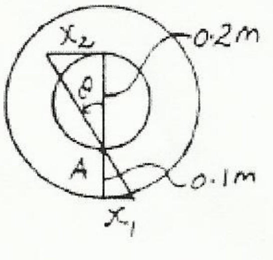
Time derivative :taking the time derivative of above eqn we have;

$$0.384375\dot{\theta}\ddot{\theta} + 10\theta\dot{\theta} = 0$$

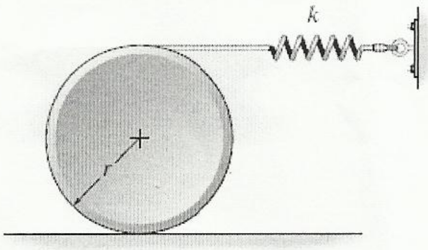
$$\dot{\theta}(0.384375\ddot{\theta} + 10\theta) = 0$$

$$\text{since } \dot{\theta} \neq 0, \textit{then the term in brackets} \rightarrow 0.384375\ddot{\theta} + 10\theta = 0$$

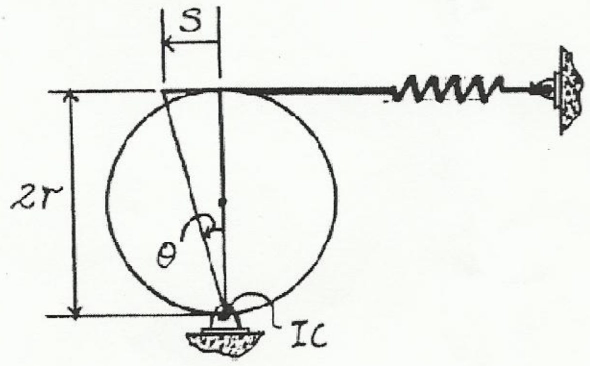
$$\ddot{\theta} + 26.0\theta = 0 \ [ANS]$$



Q3Determine the natural period of vibration of thedisk having a mass 50kg and radius 200mm.Assume the disk does notslip on the surface of contact as it oscillates. Spring constant *k*= 200N/m



SOLUTION



$T+V=\text{Constant}$

$s=(2r)\theta$

$$T+V=\frac{1}{2}\left[\frac{1}{2}mr^2+mr^2\right]\dot{\theta}^2+\frac{1}{2}k(2r\theta)^2\text{................(1)}$$

Taking time derivative of eqn 1, *we have*

$$0=\left[\frac{1}{2}mr^2+mr^2\right](2\dot{\theta}\ddot{\theta})+8kr^2\theta\dot{\theta}$$

$$mr^2\dot{\theta}\ddot{\theta}+2mr^2\dot{\theta}\ddot{\theta}+8kr^2\theta\dot{\theta}=0\rightarrow\quad 3mr^2\dot{\theta}\ddot{\theta}+8kr^2\theta\dot{\theta}=0$$

$$\ddot{\theta}+\frac{8k}{3m}\theta=0\rightarrow\text{This is a standared 2nd order diff eqn}$$

$$\text{Thus }\omega_n=\sqrt{\frac{8k}{3m}}=\sqrt{\frac{8X200}{50}}=3.26599rad/s$$

$$\text{period }\tau=\frac{2\pi}{\omega_n}=\frac{2\pi}{3.26599}$$

$\tau=1.92\sec$