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THE EFFECT OF RANGE OF STRESS ON THE TORSIONAL FATIGUE STRENGTH OF STEEL

BY
JAMES O. SMITH

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THE EFFECT OF RANGE OF STRESS ON THE TORSIONAL FATIGUE STRENGTH OF STEEL

I. INTRODUCTION

1. *Purpose of Investigation.*—Various investigators have shown that the maximum tensile or compressive stress to which a steel member may be repeatedly subjected without causing fracture (the endurance limit) depends upon the range of stress applied, that is, upon the values of the maximum and minimum stresses of a repeated cycle, but very few experimental data are available involving varying ranges of torsional shearing stress. The research committee of the American Society for Testing Materials reported in 1937 that "There is a paucity of data available covering fatigue tests showing the effect of range of stress on shearing endurance limit." The limited data presented in the report of this research committee indicate that for unnotched specimens of several steels the minimum range of shearing stress required to cause a repeated stress (fatigue) fracture is nearly constant. Therefore, as the minimum shearing stress is increased, the maximum value in the range of stress that causes rupture after the application of a large number of cycles of stress (the endurance limit for the given range of stress) is also increased by a nearly constant amount, provided that the shearing elastic strength of the material is not exceeded. Little information, however, is available indicating the effect of range of stress on the shearing endurance limit for specimens of steel containing a notch, hole, fillet, keyway, or other type of "stress raiser."

Most of the data available on the torsional shearing fatigue strength of steel concern the endurance limit for a range of stress in which the stress alternates in a cycle from a maximum value to a minimum value of equal magnitude but of opposite sign; this range or cycle of stress will be referred to as a completely reversed range or cycle of stress.

There are many torque-resisting steel members in service, such as helical springs, crankshafts, cam-shafts, propeller shafts, etc., which are subjected to repeated torsional shearing stresses such that the stress in each cycle varies from a minimum stress to a maximum stress of a different magnitude. Moreover, in many of these torque-resisting members there are stress concentrations caused by fillets, oil holes, keyways, or other discontinuities.

TABLE 1
CHEMICAL ANALYSIS OF STEEL TESTED

Material	Chemical Analysis per cent						
	C	Mn	P	S	Si	Ni	Cr
S.A.E. 3140 steel.	0.37	0.75	0.017	0.030	0.178	1.33	0.65

The purpose of the investigation herein reported was to determine the effect of the range of stress upon the torsional shearing endurance limit of S.A.E. 3140 steel for specimens free from stress concentration, and also for specimens containing stress concentrations caused by a transverse hole drilled through the specimens. The steel was tested in the hot-rolled condition and also in a heat-treated condition. Thus the tests show the effect of range of torsional shearing stress for four possible conditions of use of the material.

2. *Acknowledgments.*—The investigation was carried on as a part of the work of the Engineering Experiment Station of which DEAN M. L. ENGER is director and of the Department of Theoretical and Applied Mechanics of which PROFESSOR F. B. SEELY is the head.

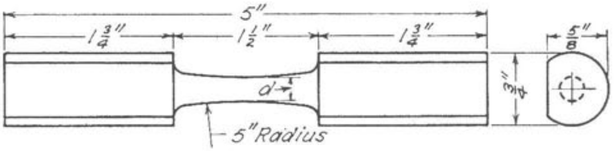
The author is indebted to PROFESSOR SEELY and PROFESSOR T. J. DOLAN for many helpful suggestions, and for their interest and encouragement in carrying out these tests.

Appreciation is expressed for the generous cooperation of PROFESSOR H. F. MOORE and MR. N. J. ALLEMAN in making available the equipment of the Fatigue of Metals Laboratory used in this investigation.

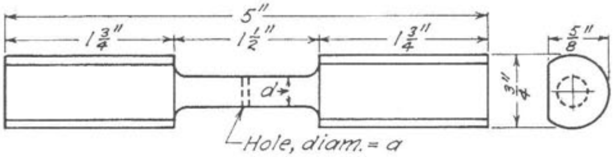
A considerable number of the tests reported herein was made under the direction of PROFESSOR T. J. DOLAN by MR. D. D. STREID and MR. R. A. MACGREGOR in satisfying the requirement for thesis work for the degree of Bachelor of Science in Mechanical Engineering.

II. MATERIALS AND SPECIMENS USED, METHOD OF TESTING, AND TEST DATA

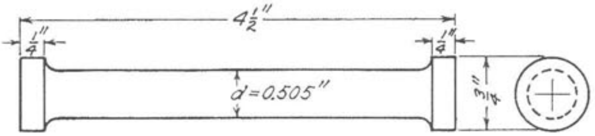
3. *Materials and Specimens Used.*—S.A.E. 3140 steel was chosen for these tests because of its frequent use in service for parts subjected to ranges of repeated stress covered by the investigation. This steel is generally used in a heat-treated condition, but tests were made on it in both the hot-rolled and a heat-treated condition.



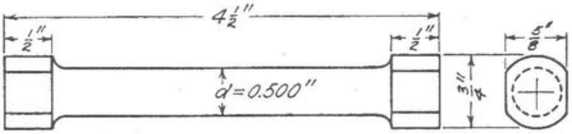
(a)-Unnotched Specimen for Torsional Fatigue Tests.



(b)-Notched Specimen for Torsional Fatigue Tests.



(c)-Tension Specimen for Static Tests



(d)-Torsion Specimen for Static Tests
Dimensions "a" and "d" varied slightly (See Table 3)

FIG. 1. DETAILS OF TEST SPECIMENS

The chemical composition of the steel as taken from the heat analysis is given in Table 1.

The specimens used were cut from seven 7/8-inch round bars received from the manufacturer in the hot-rolled condition, having a Brinell hardness averaging about 245. Some of the specimens were given a heat-treatment consisting of heating to 1510 deg. F. for twenty minutes, quenching in light oil at 85 deg. F., and tempering in a salt ($NaN0_3$-$KN0_3$) bath for about half an hour at 920 deg. F.

The general details of the specimens tested are given in Fig. 1. The type of specimen shown in Fig. 1(a), which had a solid cross-section and was free from abrupt changes in cross-section, is referred to as the unnotched specimen. The diameter *d* of the unnotched specimen ranged from 0.25 inch to 0.32 inch depending on the strength of the steel. A variation in diameter was necessary in order to keep the load within the capacity of the testing machine. The type of specimen shown in Fig. 1(b) with the transverse hole is referred to as the notched specimen.* The diameter of the hole, a, was made such that as nearly as possible the ratio of a to d was 0.1 ($a/d = 0.10$). The nominal diameter d of the notched specimens varied from 0.32 inch to 0.40 inch according to the strength of the steel (see Table 3 for further details of the specimens).

The specimens which were heat-treated were rough turned to about 0.050 inch (diameter) oversize, heat-treated, and then finished.

The surface in the neighborhood of the critical section of each specimen was finished circumferentially with No. 00 polishing paper, but since in torsion a fatigue crack may start on a longitudinal, transverse, or diagonal plane it is not possible, by polishing in a particular direction, to minimize the effect of surface scratches to the same extent as is possible by polishing bending specimens in the longitudinal direction. For the specimens with a hole the finishing operation was nearly completed before the hole was drilled and thus the sharp edge of the hole was rounded only slightly in completing the polishing.

The photomicrographs of Fig. 2 show the structure of the steel in the conditions as tested. In the hot-rolled condition, as shown in Fig. 2(a), the steel is composed of medium to large size grains of pearlite and ferrite, while for the same steel in the heat-treated condition Fig. 2(b) shows a very fine-grained troostitic-sorbitic structure, with traces of ferrite at high magnification (x2000).

4. *Method of Testing.*—Three repeated-torsion testing machines were used in making the tests for the data reported herein; each machine had a capacity of 300 in. lb. twisting moment. Figure 3 shows one of these machines† which is arranged to produce a controlled angular rotation of one end of the specimen through the chuck A' by means of cam C and lever arm B. The torque thus produced is transferred through the specimen S to chuck A'' and thence to the calibrated bar D, the twist of which is proportional to the torque, and is measured by the dial gage M. By adjusting the set screws F, which act on a lever fastened to the left end of bar D, the desired range of twisting moment, and therefore the range of stress, may be obtained. These machines operate at 1400 r.p.m., and cut off automatically when a specimen fractures.

The nominal shearing unit-stress, s_s, in all specimens was calculated by means of the torsion formula, $s_s = Tc/J$, in which T is the

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*In fatigue testing any discontinuity such as a v notch, fillet, keyway, transverse hole, etc., causing an abrupt change in the cross-section of the specimen is frequently referred to as a notch.

†See University of Illinois Engineering Experiment Station Circular No. 23 for further description of repeated torsion machine.

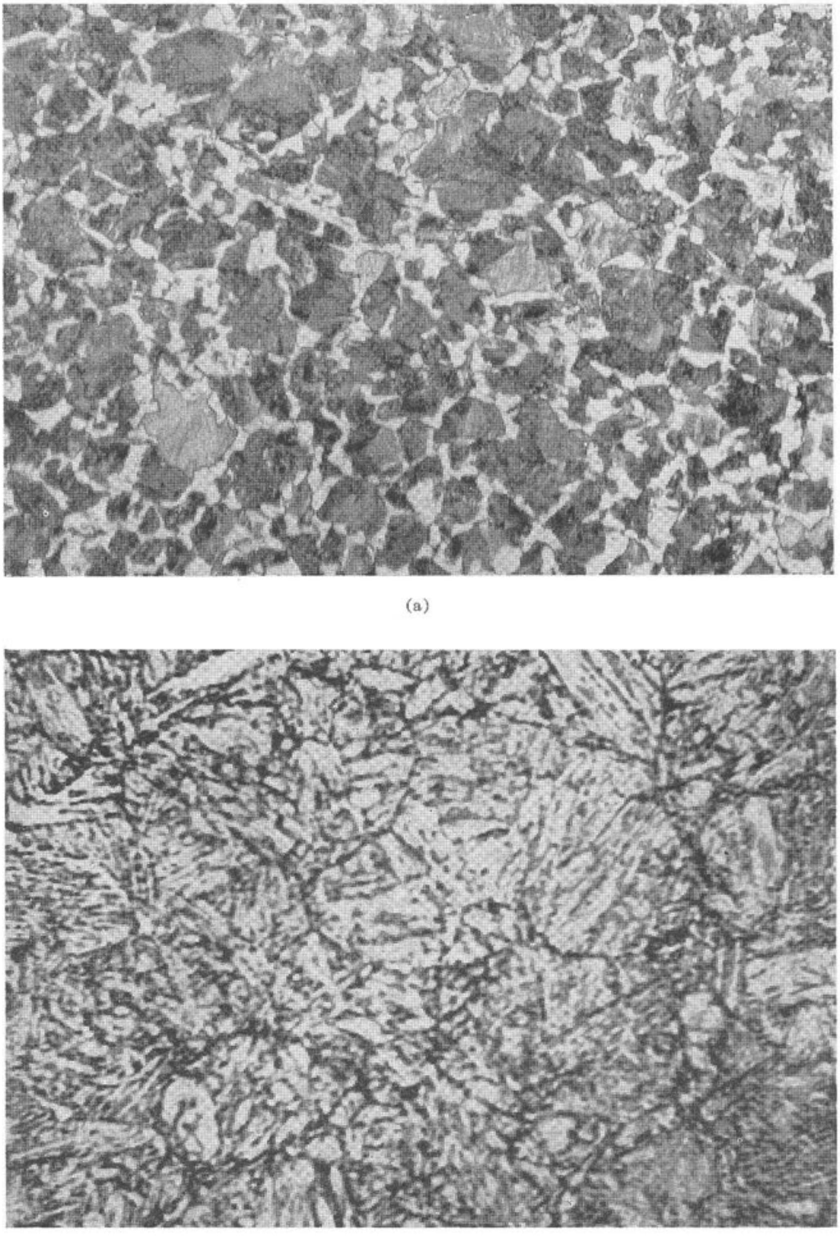

FIG. 2. PHOTOMICROGRAPHS OF STEEL TESTED, TRANSVERSE SECTION

(a) S.A.E. 3140 steel, hot-rolled (x 400, etched 2 per cent nital)
(b) S.A.E. 3140 steel, quenched and tempered (x 2000, etched 2 per cent nital)

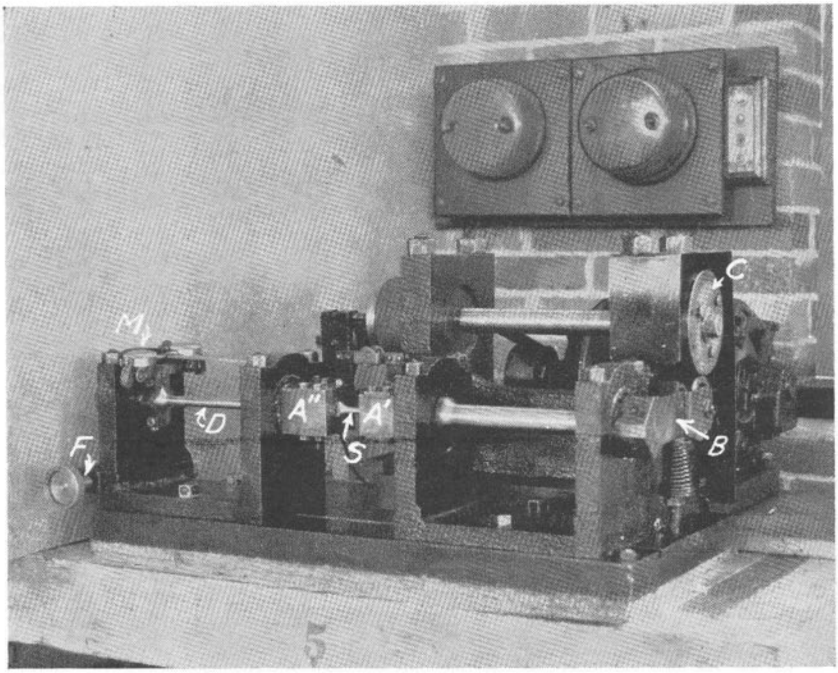

FIG. 3. REPEATED TORSION TESTING MACHINE

twisting moment, J is the polar moment of inertia of the circular section where the critical stress occurs in the specimen, c is the radius of the critical cross-section and s_s is the nominal shearing unit-stress. In making computations for shearing unit-stress in the specimens with a transverse hole no allowance was made for the hole.

An endurance limit for each of four ranges of stress was established for the steel in each of four conditions, namely, (1) the hot-rolled unnotched, (2) the hot-rolled notched, (3) the heat-treated unnotched and (4) the heat-treated notched conditions. All endurance limits except those for completely reversed ranges of stress were established by keeping the minimum stress of the range constant and varying the maximum stress. All endurance limits are based upon 10 million cycles of stress, except where conditions made a higher number of cycles necessary.

5. *Test Data.*—Figure 4 shows the lower portions of the static torsion stress–strain curves for the steel as obtained from solid unnotched specimens. It will be observed that the steel did not have a

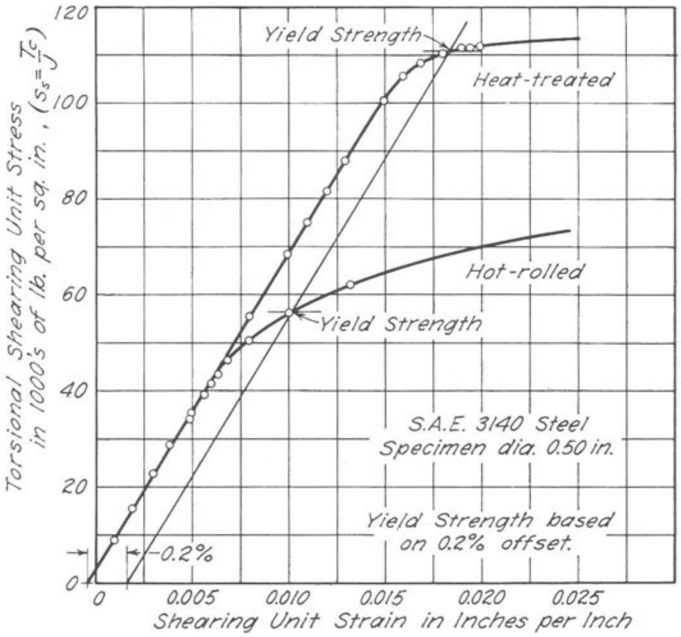

FIG. 4. STRESS-STRAIN CURVE FOR TORSIONAL STATIC TEST

pronounced yield point in either the hot-rolled or the heat-treated condition. The results of static tension and torsion tests are shown in Table 2. Drawings of specimens used in these tests are shown in Figs. 1(c) and 1(d).

The static torsional yield strengths reported in Table 2 were obtained from cylindrical specimens of solid cross-section rather than of hollow cross-section because the solid form of specimen corresponds more closely to members as used in service. Furthermore, it will be noted that the values of yield strength reported in Table 2 are based on an arbitrary offset of 0.2 per cent. However, the yield strength of the material used in these tests was obtained using hollow specimens as well as solid specimens, and the ratio of the yield strengths of hollow and solid specimens was found to be from 0.80 to 0.85. This ratio corresponds closely to that obtained by Seely and Putnam* for six steels.

Two types of failure were observed in the repeated stress tests, namely, a progressive fracture or fatigue failure, and failure by general yielding before a fatigue crack started. The type of failure depended

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*F. B. Seely and W. J. Putnam, Bulletin 115, Engineering Experiment Station, University of Illinois, p. 41.

TABLE 2
PHYSICAL PROPERTIES OF STEEL TESTED

Material and Condition	Static Tension Tests				Static Torsion Tests Solid Cylindrical Specimens	
	Yield Point lb. per sq. in.	Ultimate Tensile Strength lb. per sq. in.	Elongation in 2 in. per cent	Reduction of Area per cent	Yield Point lb. per sq. in.	Modulus of Rupture lb. per sq. in.
S.A.E. 3140 steel, hot-rolled....................	74 000*	115 000	25.0	66.6	56 000*	103 000
S.A.E. 3140 steel quenched from 1510 deg. F., tempered for 30 min. in salt bath at 920 deg. F......	153 000*	162 000	16.5	60.0	110 500*	137 000

*Yield strength for 2/10 per cent offset; no definite yield point. A.S.T.M. Standards E6-36 largely on the range of stress to which the specimens were subjected. Photographs of specimens indicating these two types of failure are shown in Fig. 5.

Figures 5(a) and 5(b) show typical fatigue fractures which occurred in these repeated stress tests. In a fatigue failure of an unnotched specimen (Fig. 5b) the fracture usually began as a longitudinal shear failure, and then followed a helicoidal path approximately along the plane of maximum tensile stress in the specimen. For specimens with a transverse hole the fatigue fracture always was first detected along the plane of maximum tensile stress and followed a helicoidal path. These facts agree with results of tests by Southwell and Gough,* in which they found that when no stress concentration was present a shear failure occurred in low carbon steel specimens subjected to repeated torsional stress, but when stress concentrations were developed (due to flaws in the material, or to abrupt change in the shape of the specimen) a helicoidal fracture resulted, with fracture progressing approximately along planes of maximum tensile stress. This helicoidal type of fracture is characteristic of a tensile failure in members subjected to static torsional loads.

Figures 5(c) and 5(d) are from photographs of unnotched hot-rolled and heat-treated specimens, respectively, which have withstood 10 million cycles of torsional shearing stress at or slightly below the endurance limit. It is evident from what follows that these specimens, although they have not developed a fatigue crack, may be said to have failed because they are so badly distorted. This type of failure will be referred to as general yielding. The hot-rolled specimen, Fig. 5(c), was tested with a range of torsional shearing stress of from +30 000 to a calculated stress of +95 000 lb. per sq. in. (calculated by the formula $s_s = Tc/J$), although the static yield strength of this steel was only 56 000 lb. per sq. in. It will be noticed that the specimen was permanently twisted through an angle of about 20 degrees, as can be seen from the relative positions of the flat surfaces on the ends of the specimen which were originally in the same plane. At the upper end of the specimen the flat surface is almost perpendicular to the plane of the paper. Figure 5(d) shows an unnotched specimen of the heat-treated steel which was subjected to 10 million cycles of torsional shearing stress from +40 000 to a calculated stress of 122 000 lb. per sq. in., the yield strength of the heat-treated steel being 110 000 lb. per sq. in. The permanent angle of twist for this specimen was 11.5 degrees.

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*R. V. Southwell and H. J. Gough, "On the Concentration of Stress in the Neighborhood of a Small Spherical Flaw; and on the Propagation of Fatigue Fractures in Statistically Isotropic Materials," Philosophical Magazine, Vol. 1, Jan. 1926.

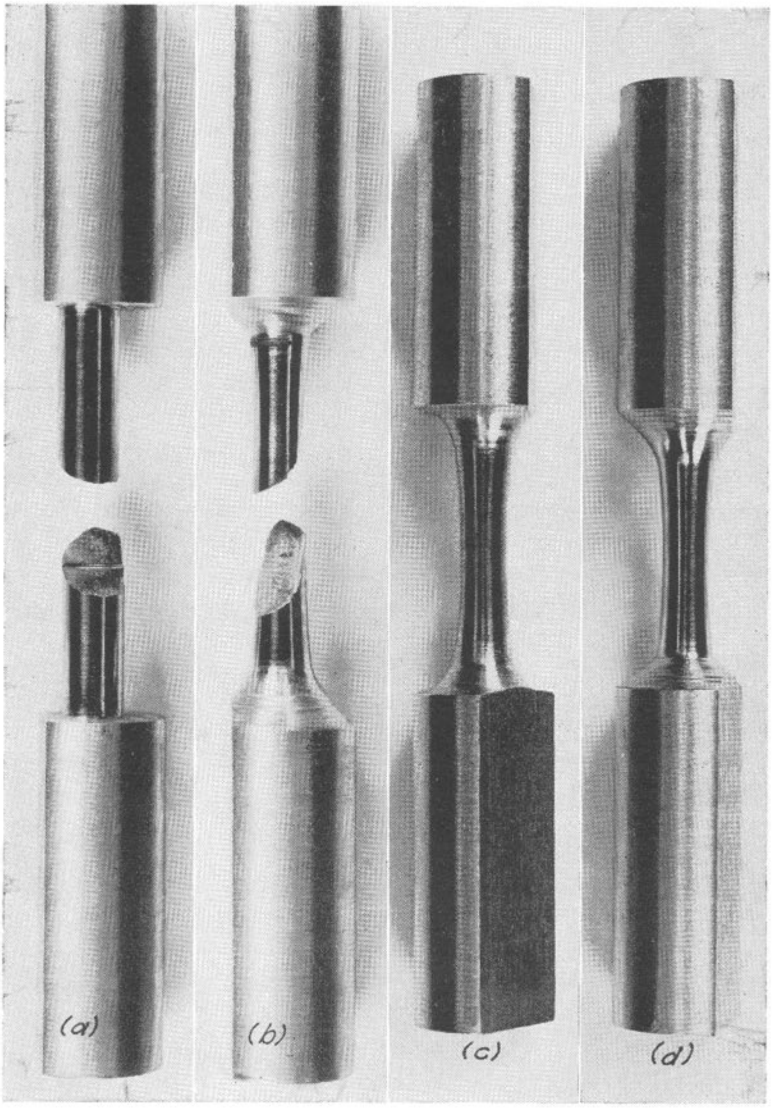

FIG. 5. PHOTOGRAPHS OF SPECIMENS AFTER TESTING UNDER REPEATED TORSIONAL STRESS

(a) Fatigue failure of notched specimen
(b) Fatigue failure of unnotched specimen
(c) Failure by general yielding, hot-rolled unnotched specimen
(d) Failure by general yielding, heat-treated unnotched specimen

It is evident, therefore, that the endurance limit has little significance as a criterion of failure for ranges of stress that permit the member to fail by general yielding before the endurance limit is reached. This fact will be emphasized further in the section on interpretation of test data.

It is recognized that values for s_s calculated by the formula $s_s = Tc/J$ for values of torque above the static elastic strength of the material are not the stresses which actually exist, but for means of comparison this method of calculation seems permissible.

The *S-N* diagrams shown in Figs. 6 and 7 give the individual results for each specimen tested under repeated torsional shearing stress. The ordinates of the points of these diagrams are the nominal maximum torsional shearing unit-stress as calculated by the formula $s_s = Tc/J$, and the abscissas represent the number of cycles of stress required to fracture the specimen, except that plotted points with arrows attached indicate that the specimen did not fracture. Table 3 gives a summary of the data taken from these curves. Columns 1 and 2 of Table 3 give information concerning the details of the specimens used to establish each endurance limit. Columns 3 and 4 give the results of the repeated torsional shearing stress tests.

III. Interpretation of Test Data and Discussion of Results of Tests

6. *Method of Plotting and Meaning of Terms or Symbols.*—For convenience in plotting the data to show the effect of range of stress upon the torsional shearing endurance limit, three methods of specifying a range of stress have been chosen. These three methods of specifying a range of stress and the method of plotting the data corresponding to each method are described in the following paragraphs.

First, a range of stress is the magnitude of the change of stress in passing from the minimum stress to the maximum stress of a cycle. A range of stress is not specified completely, however, unless either the maximum or minimum stress is given in addition to the magnitude of the range. Thus in Fig. 8 a range of stress may be specified by stating the maximum stress, $s_{\max}$, and the minimum stress, $s_{\min}$, or by stating the magnitude of the range, Δs, with either $s_{\max}$ or $s_{\min}$. This method of specifying a range of stress will be used in plotting the data on a modified Goodman diagram which will be described in detail in the next section (see Fig. 10 or 11).

Second, a range of stress may be thought of as being made up of a steady stress s_m with an alternating stress s_a superimposed upon it,

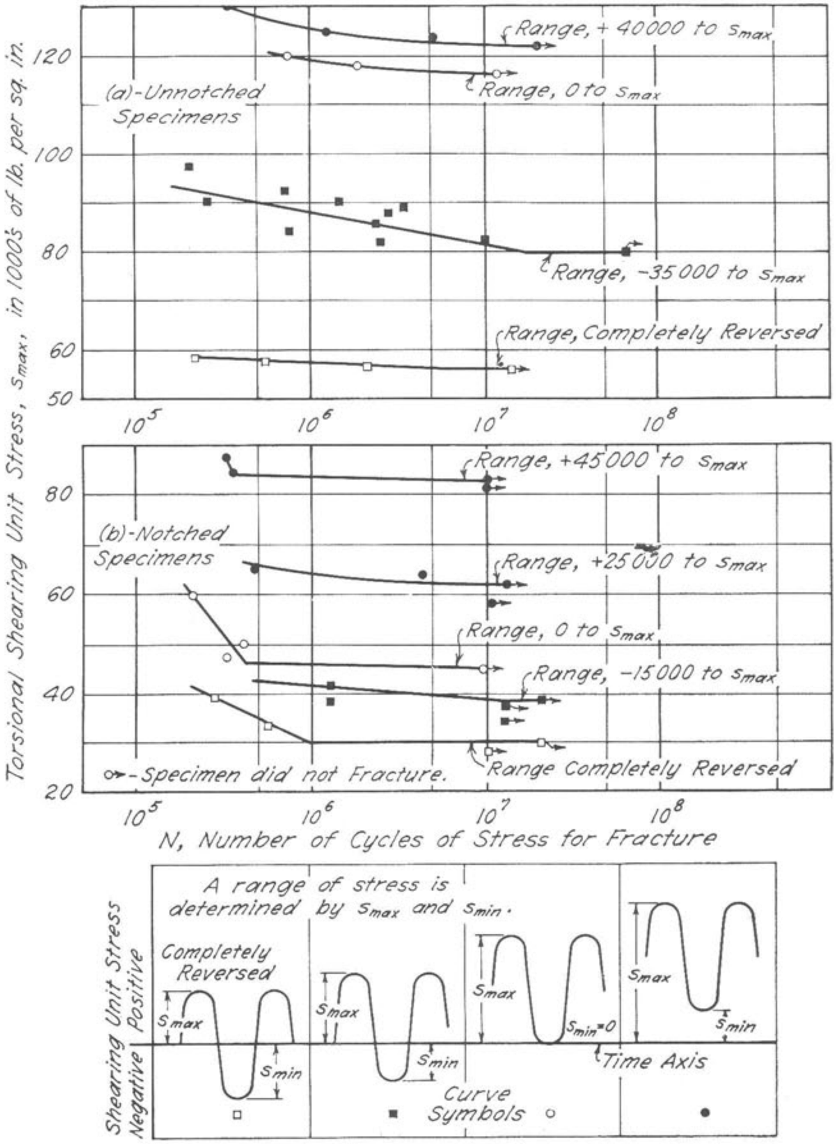

FIG. 6. S-N DIAGRAMS FOR HEAT-TREATED S.A.E. 3140 STEEL FOR VARIOUS RANGES OF TORSIONAL SHEARING STRESS

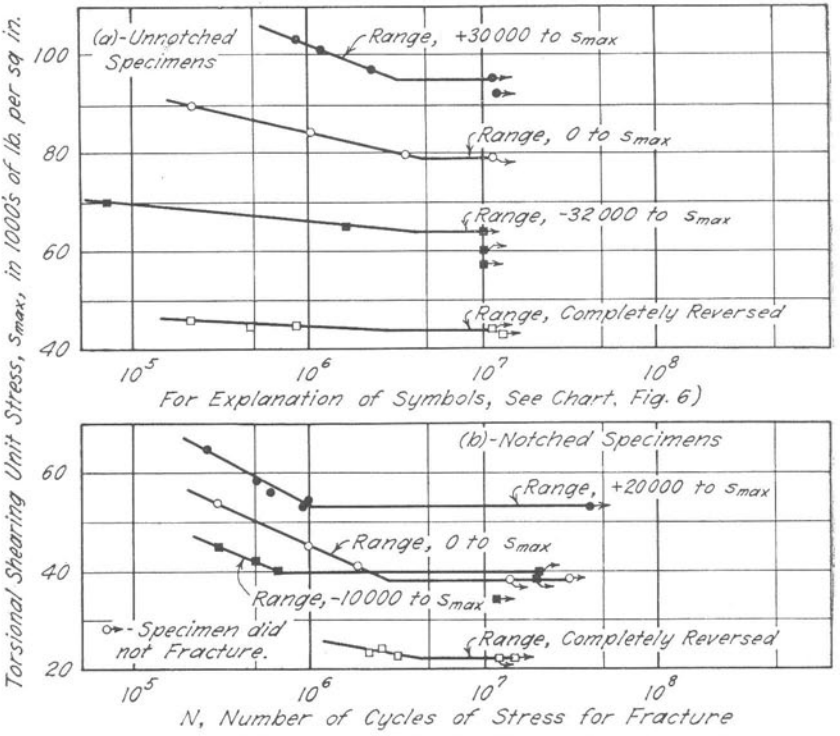

FIG. 7. S-N DIAGRAMS FOR HOT-ROLLED S.A.E. 3140 STEEL FOR VARIOUS RANGES OF TORSIONAL SHEARING STRESS

so that s_a is completely reversed around s_m. For example, in Fig. 8 a range of stress is specified by choosing s_m and s_a. This method of specifying a range of stress will be used to show the effect of range of stress upon the endurance limit by plotting s_m as abscissas and s_a as ordinates. This diagram will be referred to as the "mean stress–alternating stress" diagram.

Third, a range of stress may be specified by stating the ratio of the minimum stress, $s_{\min}$, to the maximum stress, $s_{\max}$. This ratio of the minimum to the maximum stress is called the "range ratio," and is denoted by r, which may be positive or negative; in addition, either the maximum stress, $s_{\max}$, or the minimum stress, $s_{\min}$, must be given. For example, in Fig. 8 the range of stress is specified by stating the value of $s_{\max}$ and r. This third method of specifying a range of stress is used to show the effect of range of stress upon the endurance limit by plotting as abscissas the values of r and as ordinates the values of the endurance limit, $s_{\max}$, in terms of the endurance limit,

TABLE 3
RESULTS OF REPEATED TORSIONAL SHEARING STRESS TESTS

Specimen Details		Range of Stress	
Minimum Diameter d in. (1)	Diameter of Hole a in. (2)	Minimum Stress s_{min} lb. per sq. in. (3)	Endurance Limit or Maximum Stress s_{max} lb. per sq. in. (4)
I. S.A.E. 3140 steel, hot-rolled, unnotched			
0.320	No hole	−44 000	+44 000
0.250	" "	−32 000	+64 000
0.320	" "	0	+79 000
0.250	" "	+30 000	+95 000
II. S.A.E. 3140 steel, hot-rolled, notched			
0.400	0.040	−22 000	+22 000
0.320	0.032	−10 000	+38 000
0.400	0.040	0	+39 000
0.320	0.032	+20 000	+53 000
III. S.A.E. 3140 steel, heat-treated, unnotched			
0.280	No hole	−56 000	+ 56 000
0.250	" "	−35 000	+ 80 000
0.280	" "	0	+116 000
0.250	" "	+40 000	+122 000
IV. S.A.E. 3140 steel, heat-treated, notched			
0.380	0.036	−30 000	+30 000
0.320	0.032	−15 000	+38 000
0.400	0.040	0	+45 000
0.320	0.032	+25 000	+62 000
0.320	0.032	+45 000	+83 000

s_{-1}, for the same type of specimens for completely reversed cycles of stress. This diagram will be referred to as the "range ratio–endurance limit" diagram.

The symbols which are used in this report are illustrated by Fig. 8 in part and defined as follows:

s_{max} = The maximum shearing stress for a range of torsional shearing stress. If this range of stress is the maximum range that can be repeated an indefinitely large number of times without causing fracture then s_{max} becomes the endurance limit of the material.

s_{min} = The minimum shearing stress for any range of torsional shearing stress. s_{min} may be positive or negative.

s_{-1} = The endurance limit for completely reversed torsional shearing stress. This symbol applies to the results obtained from either unnotched or notched specimens.

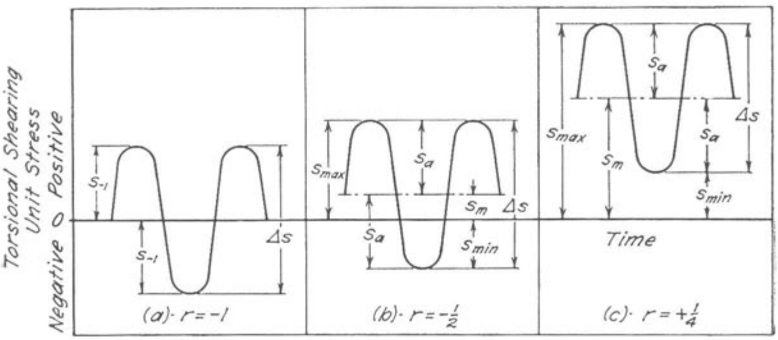

FIG. 8. STRESS SYMBOLS FOR VARYING RANGE OF STRESS

s_m, s_a = The mean or steady torsional shearing stress s_m upon which an alternating stress s_a is superimposed to create a range of stress.

Note: It will be observed that for completely reversed cycles of stress, $s_m = 0$ and $s_{\max} = -s_{\min} = s_a$.

r = The ratio of the minimum stress, $s_{\min}$, to the maximum stress, $s_{\max}$, for a given range of stress; r may be positive or negative and is called the range ratio.

s_y = The torsional static yield strength obtained from tests of solid cylindrical specimens based on 0.2 per cent offset.

s_u = The torsional modulus of rupture as obtained from tests of solid cylindrical specimens.

Δs = The magnitude of a range of stress; the algebraic difference between $s_{\max}$ and $s_{\min}$. When the magnitude Δs is such that $s_{\max}$ is the endurance limit, Δs is called the magnitude of the endurance range of stress.

7. *Modified Goodman Diagram.*—A diagram such as Fig. 9 for representing the effect of range of stress upon the endurance limit was devised by Goodman* based chiefly on test results by Wöhler. In this diagram the minimum stress, $s_{\min}$, of an endurance range of stress is plotted as an ordinate to the line of zero stress with an arbitrary abscissa such that one extremity of the ordinate lies on a line DB making an angle of 45 degrees with the line of zero stress. He found that, if the maximum stress, $s_{\max}$, of the endurance range is plotted as an ordinate at the same abscissa, the upper ends of the ordinates (values of $s_{\max}$) lie approximately on the line CB where

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*"Fatigue of Metals," H. J. Gough, D. Van Nostrand, New York, 1926, p. 67, 68.

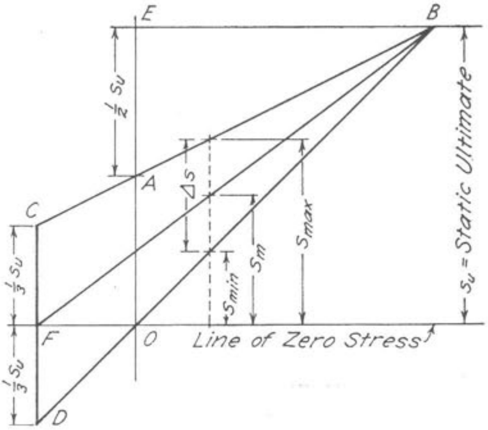

FIG. 9. GOODMAN DIAGRAM SHOWING EFFECT OF RANGE OF STRESS UPON ENDURANCE LIMIT

FC is $\frac{1}{3}s_u$ and $OA = AE = \frac{1}{2}s_u$. From similar triangles in Fig. 9 the following equation may be found

$$\frac{\Delta s}{2} = \frac{1}{3}\,(s_u - s_m) \qquad (1)$$

This equation has been used extensively to represent range of stress data. Equation (1) may also be written in the form

$$\frac{\Delta s}{2} = \frac{1}{3}s_u\left(1 - \frac{s_m}{s_u}\right) \qquad (2)$$

Further, it has been found by other investigators that a better agreement with experimental results for steels may be obtained if Equation (2) is modified by replacing the factor $\frac{1}{3}s_u$ by the endurance limit, s_{-1},* for the same type of specimen for completely reversed cycles of stress. Equation (2) thus becomes

$$\frac{\Delta s}{2} = s_{-1}\left(1 - \frac{s_m}{s_u}\right) \qquad (3)$$

Equation (3) gives the magnitude of the endurance range, Δs, when the mean stress s_m of the endurance range is chosen, but it is commonly more convenient to have an equation from which the en-

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*Moore and Kommers, "Fatigue of Metals," McGraw Hill Co., 1927, pp. 185-186.

durance limit, $s_{\max}$, may be calculated more directly. From Fig. 9 it is noted that

$$\frac{\Delta s}{2} = s_{\max} - s_m$$

Hence Equation (3) may be written

$$s_{\max} = s_m + s_{-1}\left(1 - \frac{s_m}{s_u}\right) \qquad (4)$$

Equation (4) is represented by a straight line on the Goodman diagram similar to line *CB* in Fig. 9 in which, however, s_{-1} is not equal to $\frac{1}{3}s_u$, but is the actual endurance limit obtained from fatigue tests, and hence the name "Modified Goodman Equation"* is given to it, and diagrams plotted from Equation (4) are called "Modified Goodman Diagrams." Figures 10(a) and 10(b) are modified Goodman diagrams plotted from test data for the notched steel specimens, that is, specimens with a transverse hole. The line *CB* is fixed by two points *C* and *B* determined from Equation (4). For point *C*, $s_m = 0$, $s_{\max} = s_{-1}$. For point *B*, $s_m = s_u$, $s_{\max} = s_u$. The test data are plotted in the same manner as on a Goodman diagram. It may be noted that the straight line *CB* represented by Equation (4) agrees satisfactorily with the test data. A comparison of values of endurance limit calculated by the modified Goodman equation with the test data is made in Table 4, columns (4) and (5). Column (4) shows the computed values, and column (5) shows the ratio of the computed values to the test results.

Figures 11(a) and 11(b) are modified Goodman diagrams plotted with the data for the unnotched steel specimens. It is clear from Fig. 11 that the modified Goodman diagram does not agree well with the test data for the unnotched specimens. The test data show that line *CA* drawn parallel to the line *DB* such that the magnitude of the endurance range of stress, Δs, is constant agrees satisfactorily with the test data, provided that the torsional static yield strength represented by line *AE* is not exceeded. Based on the hypothesis of a constant endurance range of stress within the yield strength of the material, the usable endurance limit may be written as

$$s_{\max} = s_m + s_{-1} \leqq s_y \qquad (5)$$

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*Note: The "Modified Goodman Equation" used here is not the same as the "Modified Goodman-Johnson Formula" used by Moore and Kommers.

TABLE 4

COMPARISON OF TORSIONAL FATIGUE STRENGTH AS CALCULATED BY VARIOUS METHODS WITH ACTUAL TEST VALUES

Material S.A.E. 3140 Steel	Actual Test Values			Endurance Limit as Calculated by Various Methods, lb. per sq. in.							
	Range Ratio r (1)	Mean Stress s_m lb. per sq. in. (2)	Endurance Limit s_{max} lb. per sq. in. (3)	Modified Goodman Equation (4) $s_{max} = s_m + s_{-1}\left(1 - \frac{s_m}{s_u}\right)$ (4)	Ratio Col. 4 to Col. 3 (5)	Mean Stress–Alternating Stress $s_{max} = s_m + s_{-1}\left(1 - \frac{s_m}{s_y}\right)$ (6)	Ratio Col. 6 to Col. 3 (7)	Equation (11) $s_{max} = \frac{2.7s_{-1}}{1.7 - r}$ (8)	Ratio Col. 8 to Col. 3 (9)	Moore-Kommers Equation $s_{max} = \frac{3s_{-1}}{2 - r}$ (10)	Ratio Col. 10 to Col. 3 (11)
Hot-rolled notched specimens	−1.00	0	22 000	22 000	1.00	22 000	1.00	22 000	1.00	22 000	1.00
	−0.29	13 500	38 000	32 600	0.86	30 200	0.80	30 000	0.78	29 000	0.76
	0.00	19 500	39 000	37 300	0.96	33 700	0.86	35 000	0.90	33 000	0.85
	+0.38	36 500	53 000	50 600	0.96	44 900	0.85	45 000	0.85	41 000	0.77
Heat-treated* notched specimens	−1.00	0	30 000	30 000	1.00	30 000	1.00	30 000	1.00	30 000	1.00
	−0.40	11 500	38 000	39 000	1.03	40 000	1.02	38 000	1.01	37 500	0.99
	0.00	22 500	45 000	47 900	1.06	46 400	1.03	47 500	1.06	45 000	1.00
	+0.40	43 500	62 000	64 000	1.03	61 600	0.99	62 500	1.01	56 000	0.90
	+0.56	63 000	81 000	79 200	0.98	75 800	0.94	71 000	0.88	62 500	0.77

*Heat-treatment was quench, (1510 deg. F.), temper ($NaNO_3$—KNO_3, 920 deg. F.)

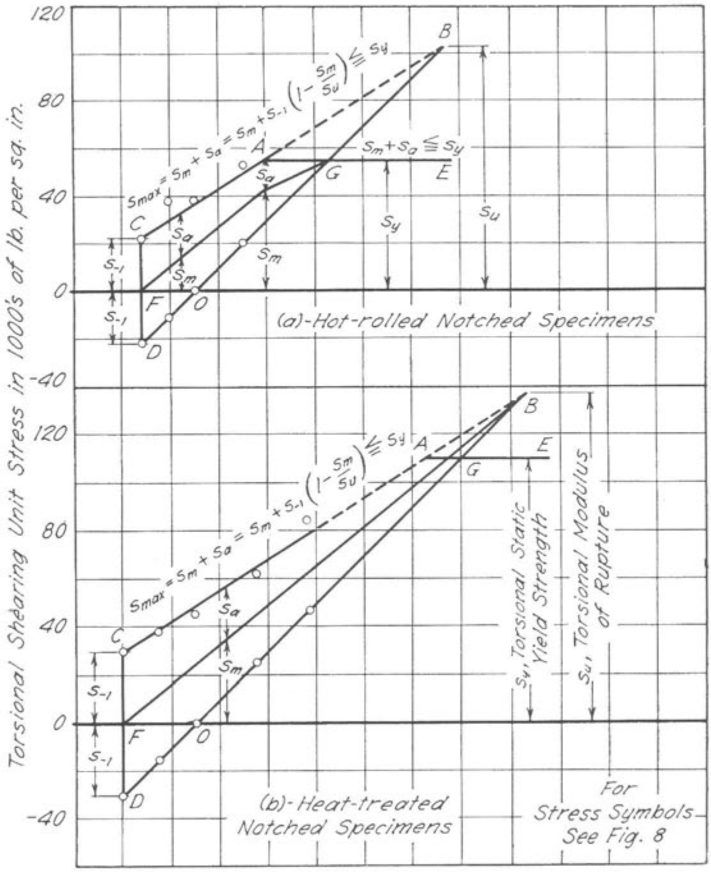

FIG. 10. MODIFIED GOODMAN DIAGRAMS SHOWING EFFECT OF RANGE OF TORSIONAL SHEARING STRESS UPON ENDURANCE LIMIT OF S.A.E. 3140 STEEL

A range of stress in which Δs is a constant provided that the maximum stress of the range is within the elastic strength, is in agreement with the results already reported by various investigators for steel free from notches and stress concentrations (see Fig. 14).

8. *Mean Stress–Alternating Stress Diagram.*—Soderberg* proposed to represent range of stress data by a diagram in which the mean stress s_m of a cycle is expressed as a fraction of the torsional static yield strength and is plotted as an abscissa, and the superimposed alternating stress of the cycle, s_a, is expressed as a fraction of

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*Trans. A.S.M.E., Applied Mechanics Division, July-Sept. 1933, Vol. No. 3, A.P.M. 55-16.

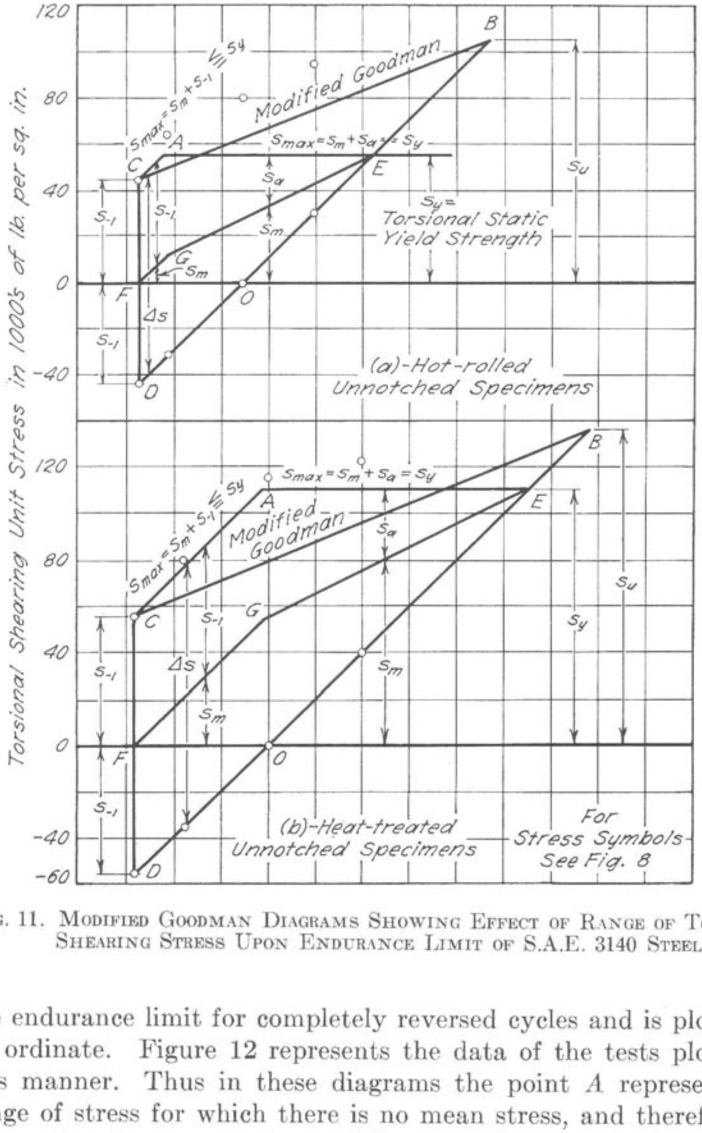

FIG. 11. MODIFIED GOODMAN DIAGRAMS SHOWING EFFECT OF RANGE OF TORSIONAL SHEARING STRESS UPON ENDURANCE LIMIT OF S.A.E. 3140 STEEL

the endurance limit for completely reversed cycles and is plotted as an ordinate. Figure 12 represents the data of the tests plotted in this manner. Thus in these diagrams the point *A* represents the range of stress for which there is no mean stress, and therefore the alternating stress s_a equals s_{-1}. The point *B* represents the range of stress for which the mean stress s_m is equal to s_y, the torsional static yield strength as obtained from tests of solid cylindrical specimens, and consequently no alternating stress is allowable. Now let a straight line be drawn joining *A* to *B*, and let the point *C* on *AB* represent an intermediate range of stress. Then the mean stress s_m is given by the abscissa *OE* and the corresponding alternating

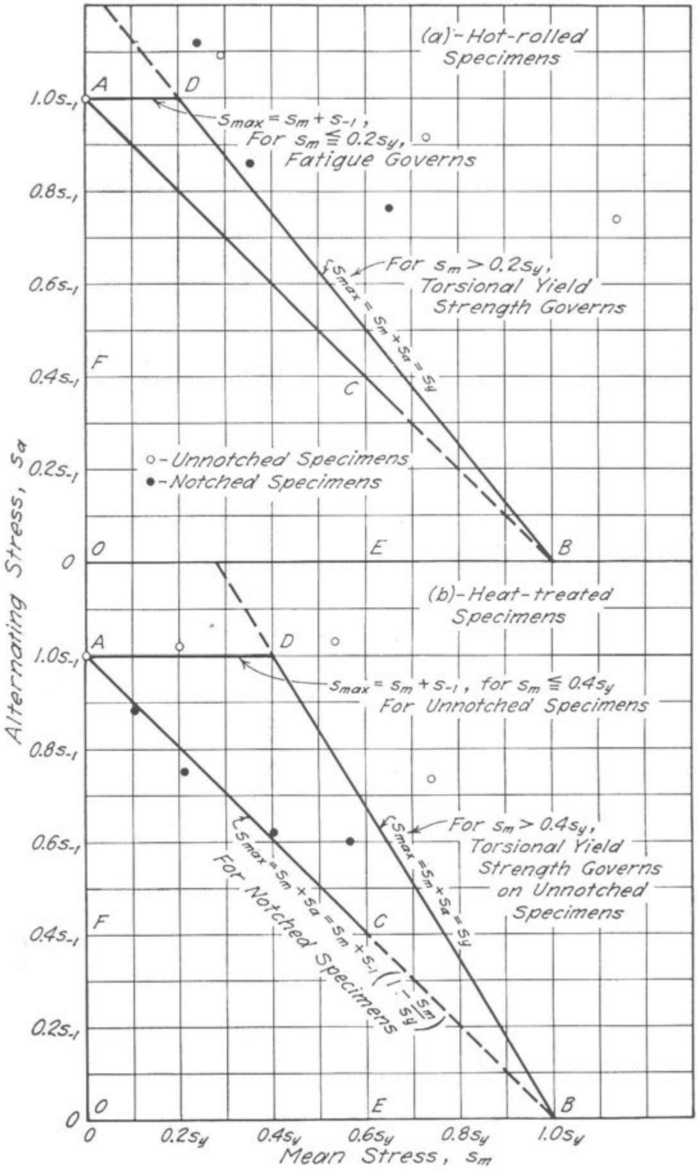

FIG. 12. MEAN STRESS–ALTERNATING STRESS DIAGRAMS SHOWING EFFECT OF RANGE OF TORSIONAL SHEARING STRESS UPON ENDURANCE LIMIT OF S.A.E. 3140 STEEL

stress s_a is given by the ordinate *OF* according to this straight line representation.

The equation of line *AB* in Figs. 12(a) and 12(b) may be written in the intercept form as

$$\frac{s_m}{s_y} + \frac{s_a}{s_{-1}} = 1 \tag{6}$$

If Equation (6) is rewritten as

$$s_a = s_{-1}\left(1 - \frac{s_m}{s_y}\right) \tag{7}$$

an expression for the alternating stress is obtained. As already pointed out, it is frequently more desirable to have an equation giving the endurance limit. If the mean stress s_m is added to both members of Equation (7), the following equation for the endurance limit $s_{\max}$ is obtained:

$$s_{\max} = s_a + s_m = s_m + s_{-1}\left(1 - \frac{s_m}{s_y}\right) \tag{8}$$

It will be noted that, if in Equation (8) s_y is replaced by s_u, the result is identical with the modified Goodman equation (4). In fact, if the straight line *CG* is drawn in Figs. 10(a) and 10(b) it will be represented by Equation (8). Equation (8) will be referred to as the "mean stress–alternating stress" equation. Column 6 of Table 4 shows a summary of values of the endurance limit for notched specimens of steel as computed by Equation (8) for the mean stresses (see column 2) s_m of the endurance ranges established by the fatigue tests. Column 7 of Table 4 shows the ratio of the endurance limits computed by Equation (8) to the endurance limits established by the fatigue tests. It will be noted that the average deviation of the computed values of endurance limit is -6.0 per cent and the greatest single deviation is -20 per cent, both values being on the side of safety.

In the mean stress–alternating stress diagram, Fig. 12, it will be noted that the straight line *AB* represented by Equation (8) agrees more closely with the data for the notched steel specimens than with those of the unnotched steel specimens. It was decided that the data for the unnotched specimens is more closely represented by a line *AD* parallel to the mean stress axis drawn from a point *A* where $s_{\max} = s_a = s_{-1}$ and $s_m = 0$ to a point *D* where

$s_{\max} = s_m + s_{-1} \leqq s_y$. The line *AD* is represented by Equation (5), which has already been seen to give values of endurance limit corresponding to a range of stress with a constant magnitude Δs. Since endurance limits in excess of the torsional static yield strength s_y permit the material to fail by general yielding, the line *DB* is drawn in the diagram such that points on this line represent cycles of stress in which the sum of the mean stress and the alternating stress is equal to s_y.

The mean stress–alternating stress diagram for representing endurance range of stress data from fatigue tests of metals under direct tensile and compressive stresses or under combined direct stresses and flexural stresses was used by various investigators, including Soderberg, Haigh* and Wahl.†

9. *Range Ratio–Endurance Limit Diagram.*—In a study of range of stress data for specimens with no abrupt change of section of several steels subjected to fatigue tests under stresses consisting of a combination of direct tensile stress and flexural stress, Howell‡ used a diagram in which he plotted values of the ratio r of $s_{\min}$ to $s_{\max}$ as abscissas, and as ordinates values of the endurance limit, $s_{\max}$, expressed in terms of s_{-1}, the endurance limit for the range of completely reversed stress. Moore and Kommers¶ used the same diagram for a study of range of stress data from torsional shearing fatigue tests of several steels in which they used specimens with no abrupt change of section. The data from the tests reported herein are plotted in this way in Figs. 13(a) and 13(b). Moore and Jasper§ and McAdam** found that for fatigue tests in torsional shear of polished steel specimens with no abrupt change of section, the magnitude of the range of stress was nearly constant. They derived an equation on this hypothesis as follows:

$$s_{\max} = \Delta s + s_{\min} = 2s_{-1} + s_{\min}$$

whence,

$$s_{\max} = \frac{2s_{-1}}{1 - r} \tag{9}$$

Equation (9) will be referred to as the constant range equation.

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*Haigh, B. P., Journal, Society Chemical Industry (British), Jan. 11, 1929, p. 33.

†Wahl, A. M., "Analysis of Effect of Wire Curvature on Allowable Stresses in Helical Springs," Preprint of paper presented to The American Society of Mechanical Engineers, Dec. 1938.

‡Bulletin No. 136, University of Illinois Engineering Experiment Station, p. 67-89.

¶Moore and Kommers, "Fatigue of Metals," McGraw-Hill, 1927, p. 185.

§Bulletin No. 142, University of Illinois Engineering Experiment Station, p. 72.

**Proceedings of American Society for Testing Materials, Vol. 24, pt. II, p. 574, 1924.

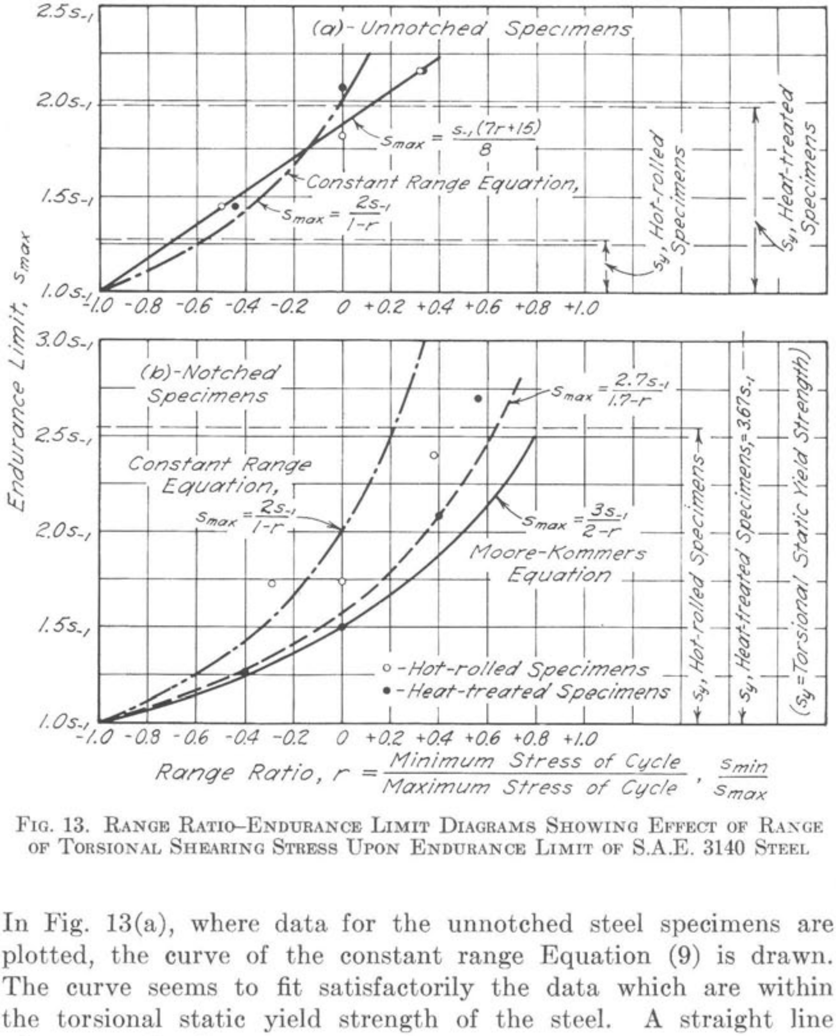

FIG. 13. RANGE RATIO–ENDURANCE LIMIT DIAGRAMS SHOWING EFFECT OF RANGE OF TORSIONAL SHEARING STRESS UPON ENDURANCE LIMIT OF S.A.E. 3140 STEEL

In Fig. 13(a), where data for the unnotched steel specimens are plotted, the curve of the constant range Equation (9) is drawn. The curve seems to fit satisfactorily the data which are within the torsional static yield strength of the steel. A straight line whose equation is

$$s_{\max} = \frac{s_{-1}(7r + 15)}{8} \tag{10}$$

seems to fit all the data for the unnotched specimens. Since endurance limits greater than the static yield strength of the material are of little importance, it would seem that the constant range equation, which agrees with data for unnotched polished steel specimens within the torsional static yield strength, can be depended upon without serious error where there is no stress concentration. The straight line representation of the data seems unimportant unless endurance limits above the yield strength of the steel are desired.

The data for the notched specimens, Fig. 13(b), are now considered. First, the constant range curve of Equation (9) is drawn and is seen to give values of endurance limit which are too large and hence unsafe. It was thought that a curve whose equation is of the same general form as the constant range Equation (9) could be used to represent these data. Accordingly, an empirical equation was written as follows:

$$s_{\max} = \frac{2.7s_{-1}}{1.7 - r} \tag{11}$$

Equation (11) is plotted in Fig. 13(b) and seems to represent the data satisfactorily for these fatigue tests of notched specimens of steel. Further, since data for notched specimens of only two steels are available, it was thought that a third and still more conservative curve representing the data might be chosen. Accordingly, a curve whose equation is of the same form as Equations (10) and (11) was chosen, as follows:

$$s_{\max} = \frac{3s_{-1}}{2 - r} \tag{12}$$

Equation (12) was also used by Moore and Kommers for representing range of stress data for unnotched specimens of several steels subjected to fatigue tests under a combination of direct tensile stress and of flexural stresses. In Table 4, columns 8 to 11, inclusive, give some comparisons of values of endurance limit computed by Equations (11) and (12) with the actual experimental values.

10. *Comparison of Results Obtained by Various Methods of Interpretation.*—Three methods of interpreting the range of stress data of this report have been used. By each method of interpretation an equation for calculating the endurance limit was found. A study of Table 4 shows that with these formulas a computed value of endurance limit agreeing satisfactorily with the experimental value may be expected. There are, however, certain other comparisons of these interpretations which should be made.

First of all, by each method of interpretation there are certain experimental constants for the steel necessary for the computation of an endurance limit. In the modified Goodman equation, Equation (4), the experimental constants required are s_{-1} and s_u, which are the endurance limit for specimens with the same type of stress concentration for completely reversed stress and the static torsional modulus of rupture, respectively. In the mean stress–alternating stress equation, Equation (8), the experimental constants necessary are s_{-1} and s_y, the torsional static yield strength as obtained from tests of solid cylindrical specimens. In Equations (9), (10), (11) and (12) where the range ratio r is used, only one experimental constant is necessary, namely, s_{-1}. It is conceivable that the available experimental constants might be a dominant factor in the choice of a method of computation of the endurance limit. For any one of the methods chosen it is necessary to have at least the experimental value s_{-1} for the type of stress concentration to be used.

It is frequently desirable to compare range of stress data for two or more materials. The adaptability of a method of interpreting range of stress for the comparison of test data for various materials may therefore be important. The mean stress–alternating stress and the range ratio–endurance limit diagrams are both used for comparing range of stress data for two or more materials. Figure 14 is a range ratio–endurance limit diagram showing range of stress data of the tests herein reported, and also data previously reported by various investigators for several steels and cast irons. Although the properties of the various materials* varied widely, this diagram shows that for small unnotched polished specimens of steel the constant range of stress curve fits the data satisfactorily.

IV. CONCLUSIONS

11. *Summary.*—The formulas for computing the endurance limit for any range of stress in torsional shear are divided into two classes depending upon whether the steel has a stress concentration or not. The symbols used in these formulas have been given in Section 6.

The first class of formulas consists of those which may be used to compute the endurance limit for any range of stress in torsional shear for steel members not having a stress concentration. These formulas are

$$s_{\max} = s_m + s_{-1} \tag{I}$$

$$s_{\max} = \frac{2s_{-1}}{1 - r} \tag{II}$$

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*For the tests of cast iron see, Moore and Picco, "Fatigue Tests of High Strength Cast Iron," Trans. American Foundryman's Association, Vol. 42, 1934, p. 525.

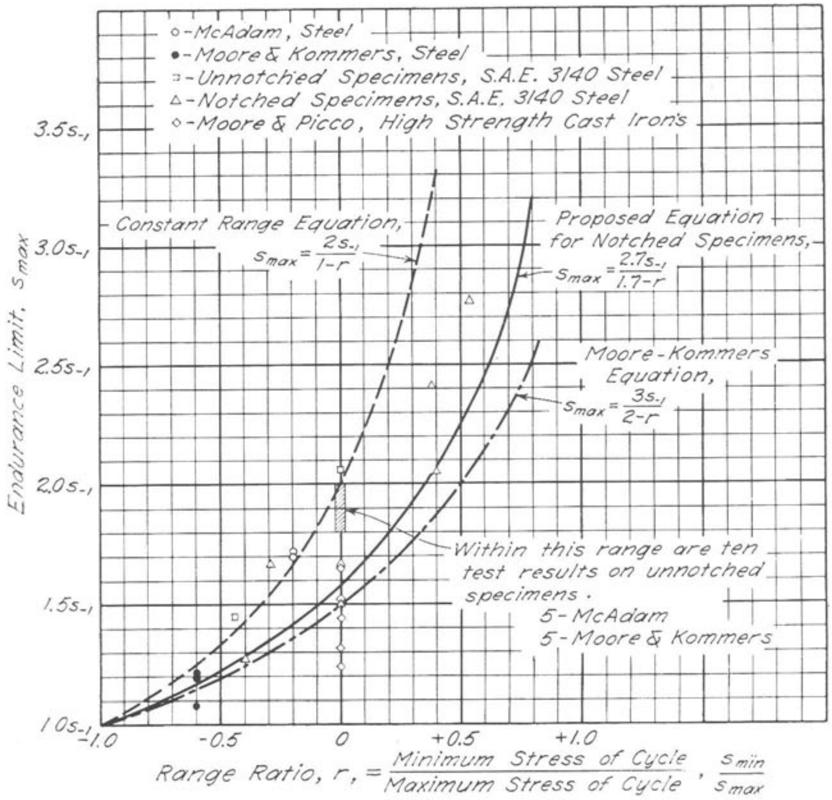

FIG. 14. RANGE RATIO–ENDURANCE LIMIT DIAGRAM SHOWING EFFECT OF RANGE OF STRESS UPON TORSIONAL ENDURANCE LIMIT AS REPORTED BY VARIOUS INVESTIGATORS

Formulas I and II are based on the hypothesis of a constant magnitude of range of stress, a hypothesis which has been supported by data from several investigators for small polished steel specimens with no abrupt change of section.

The second class of formulas consists of those which may be used to compute the endurance limit for any range of stress in torsional shear for steel members having a stress concentration caused by a transverse hole.

These formulas are:

$$s_{\max} = s_m + s_{-1}\left(1 - \frac{s_m}{s_u}\right) \qquad \text{(III) (modified Goodman formula)}$$

$$s_{\max} = s_m + s_{-1}\left(1 - \frac{s_m}{s_y}\right) \qquad \text{(IV) (mean stress–alternating stress formula)}$$

$$s_{\max} = \frac{2.7s_{-1}}{1.7 - r} \qquad \text{(V)}$$

$$s_{\max} = \frac{3s_{-1}}{2 - r} \qquad \text{(VI)}$$

(range ratio–endurance limit formulas)

Formulas (III), (IV), (V), (VI) are all based on the results of fatigue tests in torsional shear of small specimens of S.A.E. 3140 steel in the hot-rolled and a heat-treated condition with a stress concentration caused by a small transverse hole. Formula (VI) is the same formula as that developed by Moore and Kommers from results of fatigue tests of polished unnotched steel specimens subjected to combined flexural and tensile stresses.

In all formulas given for computing the endurance limit the experimental constant s_{-1} is required. If a formula for computing the endurance limit for a range of stress in a steel with a stress concentration caused by a transverse hole is to be used, then the value of s_{-1} is required, and must be experimentally determined for specimens with this stress concentration. However, should information be available for the endurance limit for completely reversed cycles of stress for specimens without a stress concentration, then a stress concentration factor may be used to estimate s_{-1} for the specimens with the stress concentration. If these formulas are thought to be satisfactory for use where the stress concentration may be of some form other than that due to a transverse hole the same procedure as before may be used to find s_{-1}. For the purpose of finding s_{-1} for various stress concentrations, some factors are given in Tables 5 and 6.

The values of static torsional yield strength, s_y, to be used in Formula (IV) should be obtained from tests of solid cylindrical specimens based on an arbitrary value of offset of 0.2 per cent. If the value of s_y is obtained from a hollow specimen, or if a smaller value than 0.2 per cent offset is used, the calculated value of the endurance limit, $s_{\max}$, will be somewhat less than the endurance limit that the material may be expected to have.

The specimens used in these tests varied in diameter from 0.25 inch to 0.40 inch, but no study of the effect of size was made since that was not the purpose of the investigation. However, data obtained by Mailander and Bauersfeld* for the effect of size show that for specimens of chrome-nickel-tungsten steel with no stress

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*R. Mailander and W. Bauersfeld, "Einfluss der Probengrösse Und Probenform auf die Dreh-Schwingungsfestigkeit Von Stahl," Techniche Mitteilungen Krupp, Vol. 2, Dec. 1934, Pages 143-152.

TABLE 5

STRESS CONCENTRATION FACTORS FOR TRANSVERSE HOLES IN SPECIMENS SUBJECTED TO COMPLETELY REVERSED CYCLES OF TORSIONAL SHEARING STRESS

Material	Specimen Diameter d in.	Diameter of Hole a in.	Ratio a/d	Stress Concentration Factor K	Endurance Limit of Material in Reversed Torsion lb. per sq. in.
Results obtained by Dolan*					
S.A.E. 1020 steel, hot-rolled	0.400 0.400 0.400	0.04 0.07 0.10	0.10 0.175 0.250	1.31 1.43 1.62	20 100
Rail steel, hot-rolled	0.400 0.400	0.04 0.10	0.10 0.25	1.64 2.26	37 000
S.A.E. 3140 steel, hot-rolled	0.400 0.400	0.04 0.10	0.10 0.25	2.00 2.25	44 000
S.A.E. 3140 steel, quenched and tempered	0.380	0.036	0.095	1.87	56 000
Results obtained by Armbruster†					
N-steel	0.394	0.059	0.15	1.39	22 800
V-steel	0.394	0.059	0.15	1.31	24 900
E-steel	0.394	0.059	0.15	1.66	42 000
Results obtained by Mailander and Bauersfeld‡					
Cr-Ni-W steel	0.55 1.18 1.77	0.078 0.118 0.197	0.14 0.10 0.11	1.60 1.88 1.74	39 800 33 400 28 400
Results obtained from "Plaster Model" method*					
Pottery plaster	2.0 2.0 2.0	0.125 0.25 0.50	0.063 0.125 0.250	1.86 1.89 2.12	Tested in static torsion

*Dolan, T. J., University of Illinois Engineering Experiment Station, Bulletin 293, Table 5, p. 27, 1937.

†Armbruster, E., "Einfluss der Oberflächenbeschaffenheit auf den Spannungsverlauf und die Schwingungsfestigkeit," Ver. Deutsch. Ing. Verlag 1931.

‡Mailander, R. and Bauersfeld, W., "Einfluss der Probengrösse und Probenform auf die Dreh-Schwingungsfestigkeit von Stahl," Technische Mitteilungen Krupp, Vol. 2, Dec. 1934, pages 143-152. Allowance was made in these tests for the area removed by the hole; all other values in this table were computed on the basis of the gross cross-section of the specimen.

concentration the endurance limit for completely reversed cycles of torsional shearing stress for specimens of diameters 0.55 inch, 1.18 inch and 1.77 inch decreased 16 per cent in passing from the 0.55 inch diameter to the 1.18 inch diameter, and decreased 29 per cent in passing from the 0.55 inch diameter to the 1.77 inch diameter. They also found that for specimens of the same steel with stress concentration caused by a transverse hole the endurance limit for completely reversed cycles of torsional shearing stress decreased 19 per cent in passing from a diameter of 0.55 inch to one of 1.18 inch, and decreased 28 per cent in passing from a diameter of 0.55 inch to

TABLE 6

STRESS CONCENTRATION FACTORS FOR FILLETS IN SPECIMENS SUBJECTED TO COMPLETELY REVERSED CYCLES OF TORSIONAL SHEARING STRESS

Material	Minimum Diameter d in.	Ratio D/d	Ratio r/D	Stress Concentration Factor K	Theoretical Value of K[*]
Results obtained by Dolan[†]					
S.A.E. 1020 steel, hot-rolled	0.375	2.0	0.0053	1.15	> 3.3
S.A.E. 3140 steel, hot-rolled	0.300	2.5	0.027	1.54	1.95
	0.300	2.5	0.067	1.57	1.52
S.A.E. 3140 steel, quenched and tempered	0.300	2.5	0.0087	1.51	3.00
Results obtained by Armbruster[‡]					
N-steel	0.394	1.4	0.17	0.96	1.45
N-steel	0.394	1.4	0.03	0.99	2.36
E-steel	0.394	1.4	0.17	1.11	1.45
E-steel	0.394	1.4	0.03	1.66	2.36
Results obtained by Mailander and Bauersfeld[¶]					
Cr-Ni-W steel	0.55	1.8	0.14	1.10	1.56
Cr-Ni-W steel	1.18	1.8	0.10	1.17	1.70
Cr-Ni-W steel	1.77	1.8	0.11	1.03	1.65

[*] As obtained by Jacobsen Trans A.S.M.E. Vol. 47, No. 1974, p. 619, using the "Electric Analogy" method.

[†] Dolan, T. J., University of Illinois Engineering Experiment Station, Bulletin 293, 1937.

[‡] Armbruster, E., "Einfluss der Oberflächenbeschaffenheit auf den Spannungsverlauf und die Schwingungsfestigkeit," Ver. Deutsch. Ing. Verlag 1931.

[¶] Mailander, R. and Bauersfeld, W., "Einfluss der Probengrösse und Probenform auf die Dreh-Schwingungsfestigkeit von Stahl," Technische Mitteilungen Krupp, Vol. 2, Dec. 1934, pages 143-152.

one of 1.77 inch. The effect of size should be kept in mind when use is made of the results of these tests.

12. *Conclusions.*—The following conclusions may be drawn:

(1) For ranges of torsional shearing stress other than completely reversed cycles of stress, steel is likely to cease to perform satisfactorily as a structural or machine member by either one of two types of failure. First, for some ranges of stress it may fail by developing large permanent deformation at the torsional static yield strength before the endurance limit stress is reached. Second, for some ranges of stress it may fail by progressive fracture (fatigue) at a nominal or calculated stress below the torsional static yield strength. These two types of failure are illustrated in Fig. 5. Therefore, endurance limits for various ranges of stress have little significance if their values exceed the static elastic strength of the steel.

(2) For small polished specimens free from stress concentrations, the torsional shearing endurance limit of S.A.E. 3140 steel in both

the hot-rolled (as received condition) and the quenched and tempered condition follows the constant range relation; that is, the magnitude of any endurance range of stress is constant and equal to the magnitude of the endurance range of stress for completely reversed cycles of stress, provided that the maximum stress in the range does not exceed the torsional static yield strength of the steel. In order to possess a high endurance limit for various ranges of torsional shearing stress a steel which is to be used without stress concentration must also possess a high static elastic strength in addition to a high endurance limit for completely reversed cycles of stress. Therefore, heat-treated steels are particularly desirable for use for ranges of stress other than completely reversed cycles of stress, since heat-treatment usually raises the static strength properties more than it does the fatigue strength properties. The facts on which this conclusion is based are shown by the diagrams of Figs. 11, 12 and 13(b) and by Formulas (I) and (II) of the summary.

(3) For small specimens with a stress concentration caused by a small transverse hole the torsional shearing endurance limit of S.A.E. 3140 steel in both the hot-rolled (as received) condition and the heat-treated condition is not independent of the range of stress. The magnitude of the endurance range of stress is decreased as the maximum stress in the range increases, that is, the difference between the maximum and minimum stresses of the endurance range decreases as the maximum stress is increased. The effect of the range of stress may be shown satisfactorily by any one of three methods of interpretation of the test data. The facts on which this conclusion is based are shown by the diagrams of Figs. 10, 12 and 13(a) and by Formulas (III-VI), inclusive, in the summary. These formulas and diagrams indicate that steels with high usable torsional shearing endurance limit for various ranges of stress must possess high elastic strengths as well as relatively low notch sensitivity; this last requirement insures a relatively high endurance limit for completely reversed cycles of torsional shearing stress.

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