TRANSPARENCY MASTERS
Solutions to Typical Problems in

MERIAM & KRAIGE • ENGINEERING MECHANICS, VOL. 1 STATICS, 4/E

This transparency master section includes 40 problems and their solutions selected to illustrate typical applications in **Statics**. ***These problems are different from and in addition to those included in Volume 1 of the Fourth Edition of*** **Engineering Mechanics by** ***Meriam and Kraige.***

A list of the problems by chapter and article reference for Volume 1 — **Statics** ***follows below.***

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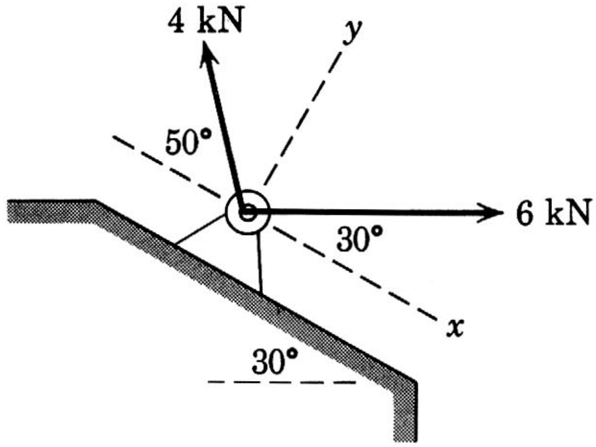
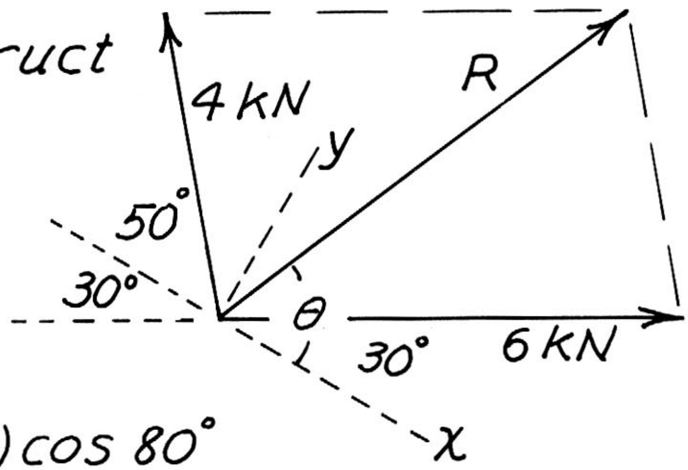
APPENDIX A AREA MOMENTS OF INERTIA
- *A/2 Definitions (2 examples)*
- *A/3 Composite Areas*
- *A/4 Products of Inertia*
- *A/4 Rotation of Axes*

Replace the two forces by a single equivalent force $\underline{R}$ and find the angle θ between $\underline{R}$ and the x-axis. Solve both geometrically and by using unit vectors $\underline{i}$ and $\underline{j}$.





Geometric

Graphical: construct parallelogram & measure R & θ.

Trigonometric:

Law of cosines

$$R^2 = 4^2 + 6^2 - 2(4)(6)\cos 80°$$

$$= 43.7, \quad \boxed{R = 6.61 \text{ kN}}$$

$$4^2 = (6.61)^2 + 6^2 - 2(6.61)(6)\cos(\theta - 30°)$$

$$\theta - 30° = \cos^{-1} 0.8029 = 36.6°, \quad \boxed{\theta = 66.6°}$$

Vector algebra

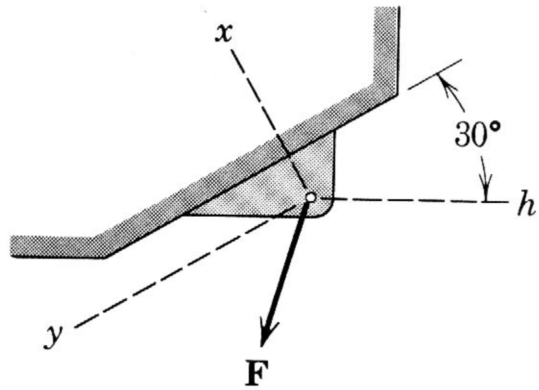
$$R_x = 6\cos 30° - 4\cos 50° = 2.63 \text{ kN}$$

$$R_y = 6\sin 30° + 4\sin 50° = 6.06 \text{ kN}$$

$$\boxed{\underline{R} = 2.63\underline{i} + 6.06\underline{j} \text{ kN}}, \quad \theta = \tan^{-1}\frac{6.06}{2.63} = \boxed{66.6°}$$

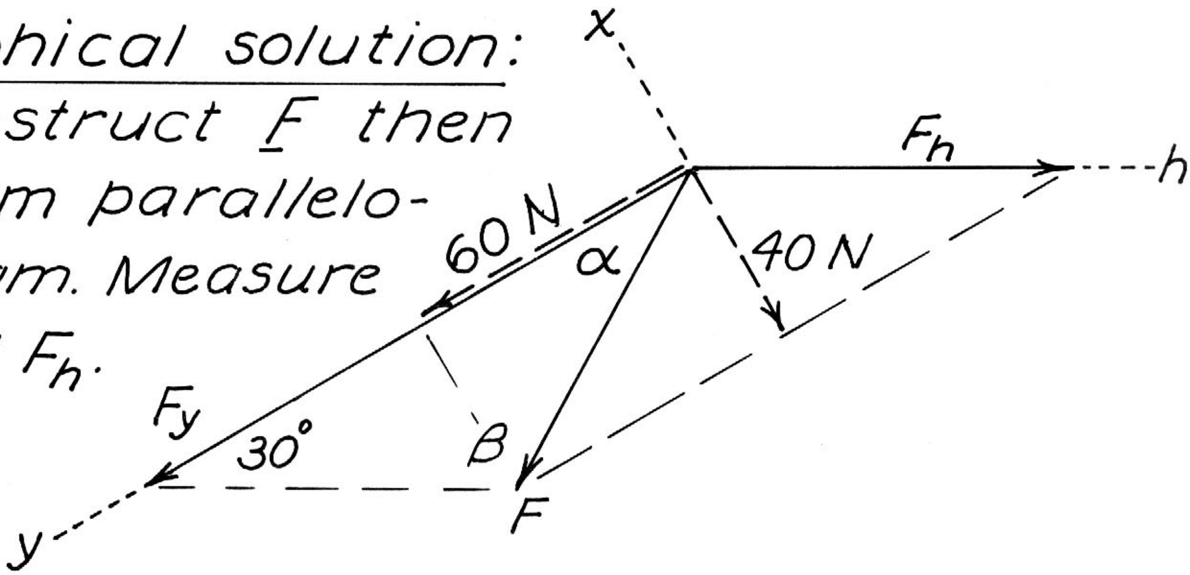
JLM

Force $\underline{F}$ in rectangular components is given by $\underline{F} = -40\underline{i} + 60\underline{j}$ N. Determine the non-rectangular components of $\underline{F}$ in the y- and h-directions.



Graphical solution:

Construct $\underline{F}$ then form parallelogram. Measure F_y & F_h.



Trigonometric solution:

$$\alpha = \tan^{-1}\frac{40}{60} = 33.7^\circ, \quad F = \sqrt{(40)^2 + (60)^2} = 72.1 \text{ N}$$

Law of sines $\dfrac{72.1}{\sin 30^\circ} = \dfrac{F_h}{\sin 33.7^\circ}$, $\boxed{F_h = 80.0 \text{ N}}$

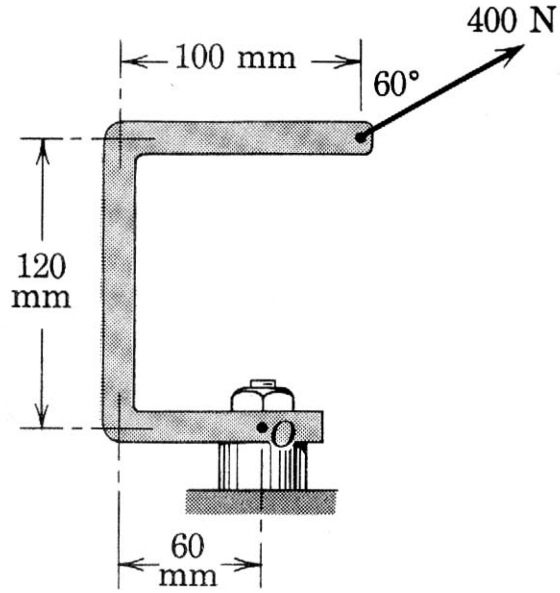
$$\beta = 180 - 30 - 33.7 = 116.3^\circ$$

$\dfrac{F_y}{\sin 116.3^\circ} = \dfrac{72.1}{\sin 30^\circ}$, $\boxed{F_y = 129.3 \text{ N}}$

JLM

ART. 2/4 MOMENT (2-D)

Calculate the moment of the 400-N force about point O in five different ways.



From the geometry

$a + 0.04 = 0.120 \tan 60°$

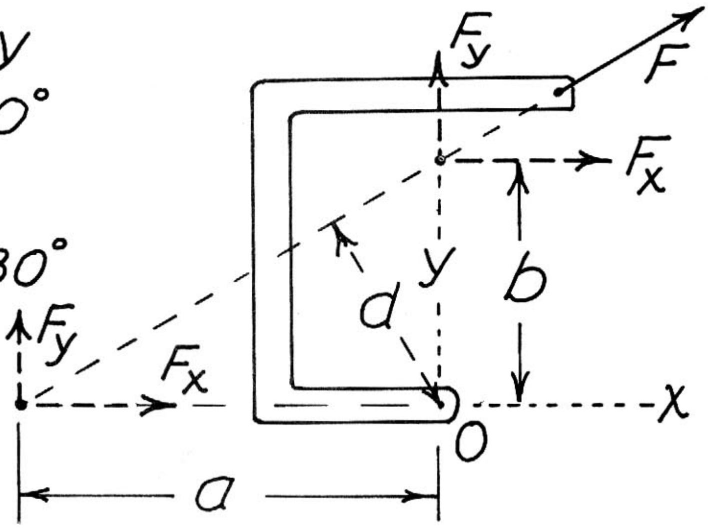
$a = 0.168$ m

$0.120 - b = 0.040 \tan 30°$

$b = 0.0969$ m

$d = b \cos 30°$

$= 0.0839$ m



(I) $M_O = Fd = 400(0.0839) = \boxed{33.6 \text{ N}\cdot\text{m}}$

(II) $M_O = 400(0.12 \sin 60° - 0.040 \cos 60°) = \boxed{33.6 \text{ N}\cdot\text{m}}$

(III) $M_O = F_x b = 400 \sin 60° (0.0969) = \boxed{33.6 \text{ N}\cdot\text{m}}$

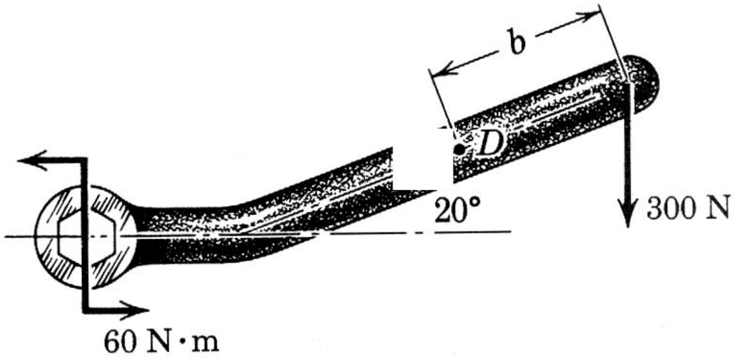
(IV) $M_O = F_y a = 400 \cos 60° (0.168) = \boxed{33.6 \text{ N}\cdot\text{m}}$

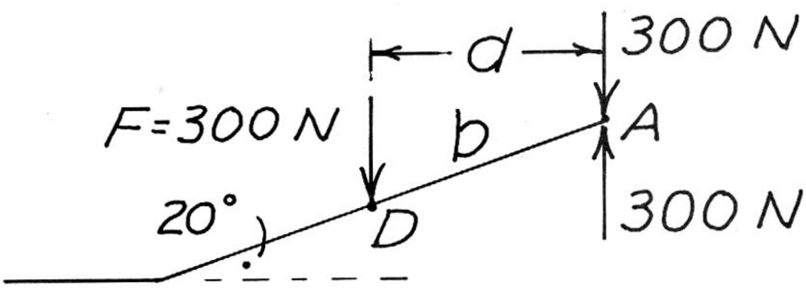
(V) $\underline{M}_O = \underline{r} \times \underline{F} = (0.04\underline{i} + 0.12\underline{j}) \times 400(\underline{i} \sin 60° + \underline{j} \cos 60°)$

$= 8\underline{k} - 41.6\underline{k} = \boxed{-33.6\underline{k} \text{ N}\cdot\text{m}}$

JLM

Replace the force and couple acting on the wrench by a single equivalent force F applied at D. Determine b.





Replace 60-N·m couple by an equivalent couple consisting of two 300-N forces a distance d apart placed to cancel the given force. Thus, resultant is $\boxed{F = 300\ N}$ located at D where

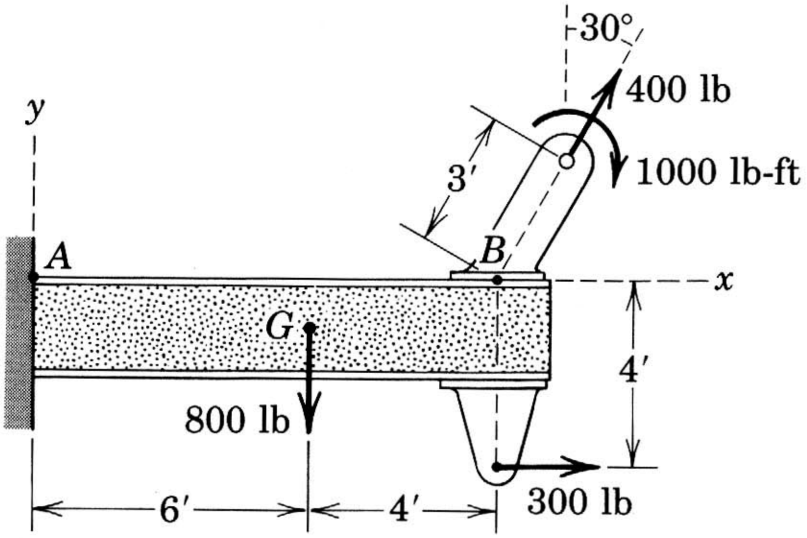
$$M_A = Fd; \quad 60 = 300d, \quad d = 0.2\ m$$

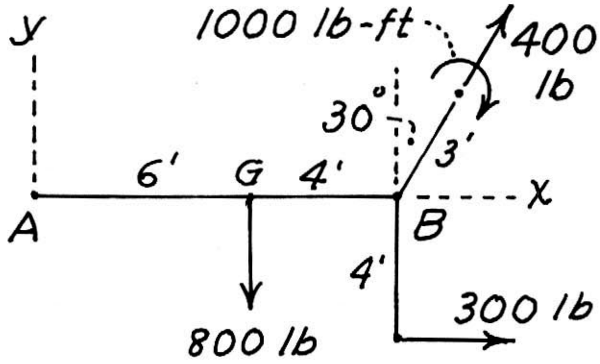
$$b = 0.2/\cos 20^\circ = 0.213\ m \text{ or } \boxed{b = 213\ mm}$$

JLM

ART. 2/6 RESULTANTS (2-D)

Represent the resultant of the three forces and one couple by an equivalent force $\underline{R}$ at A and a couple M. Find M and the magnitude and direction of $\underline{R}$.



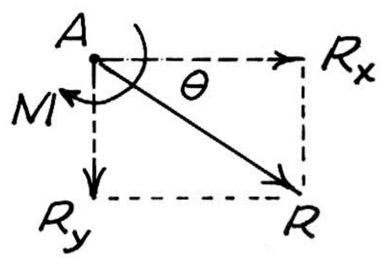


$$R_x = \Sigma F_x = 400 \sin 30^\circ + 300$$
$$= 500 \text{ lb}$$

$$R_y = \Sigma F_y = 400 \cos 30^\circ - 800$$
$$= -454 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{500^2 + 454^2}$$
$$= \boxed{675 \text{ lb}}$$

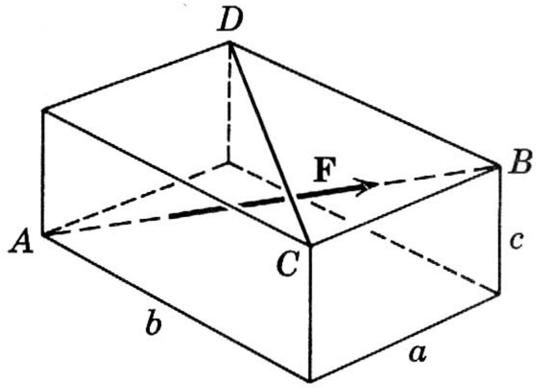
$$M = \Sigma M_A = 1000 + 800(6) - 400 \cos 30^\circ (6+4) - 300(4)$$
$$= \boxed{1136 \text{ lb-ft CW}}$$



$$\theta = \tan^{-1} \frac{454}{500}$$
$$= \boxed{42.2^\circ}$$

JLM

For $a = 3\,m$, $b = 6\,m$, $c = 2\,m$, $F = 10\,kN$, determine the magnitudes of the components of F along AC and AD and the projection of F along DC.



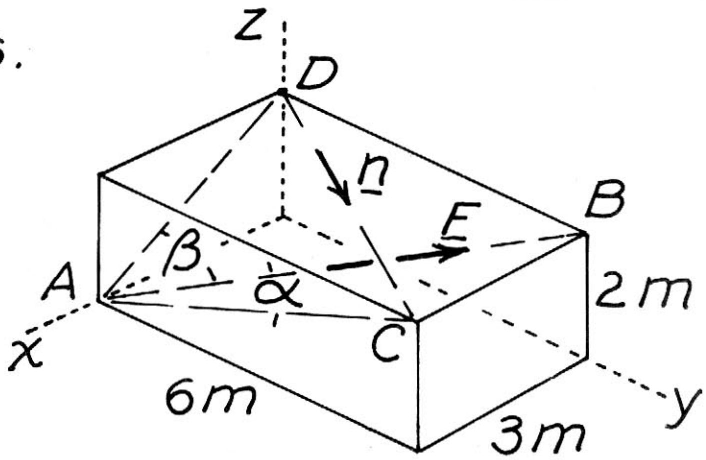
Choose x-y-z axes.

$AB = \sqrt{3^2 + 6^2 + 2^2} = 7\,m$

$AC = \sqrt{2^2 + 6^2} = 2\sqrt{10}\ m$

$AD = \sqrt{2^2 + 3^2} = \sqrt{13}\,m$

$DC = \sqrt{3^2 + 6^2} = 3\sqrt{5}\,m$



$F_{AC} = F\cos\alpha = 10\,\frac{2\sqrt{10}}{7} = \boxed{9.04\ kN}$

$F_{AD} = F\cos\beta = 10\,\frac{\sqrt{13}}{7} = \boxed{5.15\ kN}$

Let $\underline{n}$ = unit vector along DC $= \frac{3}{3\sqrt{5}}\underline{i} + \frac{6}{3\sqrt{5}}\underline{j}$

$= \frac{1}{\sqrt{5}}(\underline{i} + 2\underline{j})$

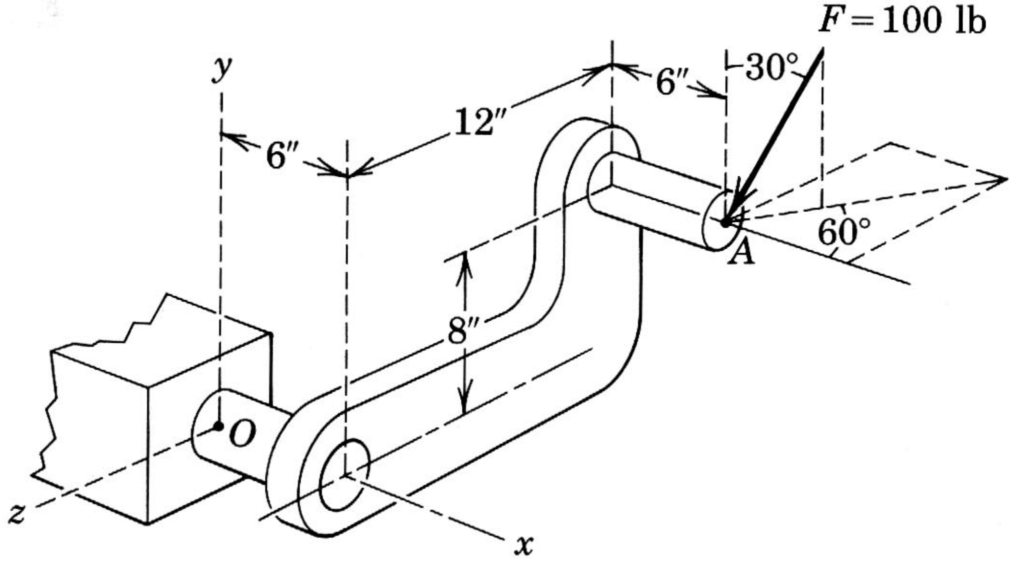
$\underline{F} = 10\left(\frac{-3}{7}\underline{i} + \frac{6}{7}\underline{j} + \frac{2}{7}\underline{k}\right)\ kN$

$F_{DC} = \underline{F}\cdot\underline{n} = \frac{10}{7}(-3\underline{i} + 6\underline{j} + 2\underline{k})\cdot\frac{1}{\sqrt{5}}(\underline{i} + 2\underline{j})$

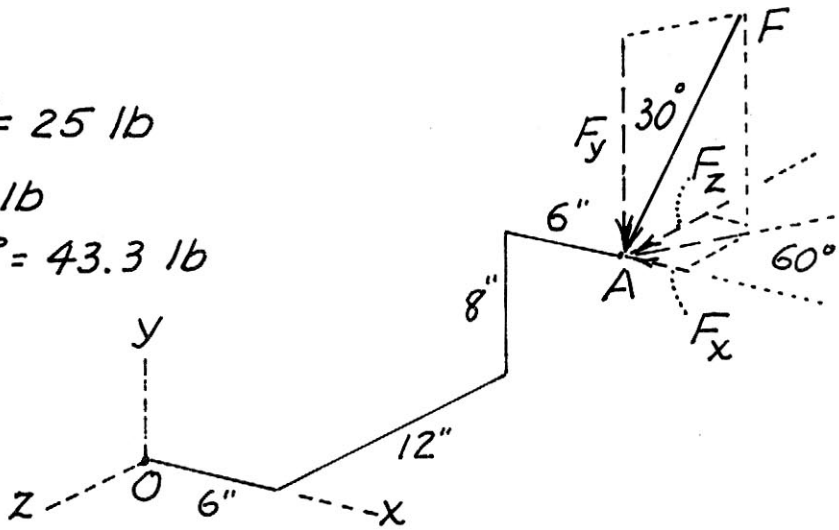
$= \frac{10}{7\sqrt{5}}(-3 + 12) = \boxed{5.75\ kN}$

JLM

Determine the moment of the 100-lb force F about the x-axis.



Scalar solution



$|F_x| = 100 \sin 30° \cos 60° = 25$ lb

$|F_y| = 100 \cos 30° = 86.6$ lb

$|F_z| = 100 \sin 30° \sin 60° = 43.3$ lb

$M_x = -86.6(12) + 43.3(8)$

$= \boxed{-693 \text{ lb-in.}}$

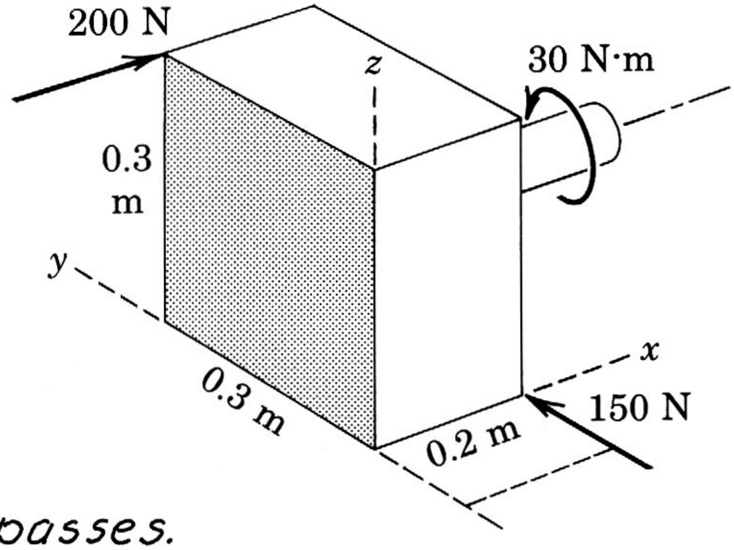
Vector solution

$\underline{M}_O = \underline{r}_A \times \underline{F}$ where $\underline{r}_A = 12\underline{i} + 8\underline{j} - 12\underline{k}$ in.

$$M_x = \underline{M}_O \cdot \underline{i} = \underline{r}_A \times \underline{F} \cdot \underline{i} = \begin{vmatrix} 12 & 8 & -12 \\ -25 & -86.6 & 43.3 \\ 1 & 0 & 0 \end{vmatrix} = \boxed{-693 \text{ lb-in.}}$$

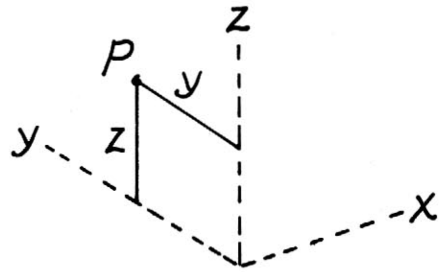
JLM

Replace the two forces and couple by a wrench. Find the moment $\underline{M}$ of the wrench and the coordinates of point P in the y-z plane through which the force of the wrench passes.



$\underline{R} = \Sigma \underline{F} = 200\underline{i} + 150\underline{j}$ N

Assume positive wrench so direction cosines of $\underline{M}$ are those of $\underline{R}$ or 0.8, 0.6, 0



$\Sigma \underline{M}_P = 200(0.3-z)\underline{j} - 200(0.3-y)\underline{k} + 150z\underline{i} + 150(0.2)\underline{k} - 30\underline{i}$

$= (-30+150z)\underline{i} + (60-200z)\underline{j} + (-30+200y)\underline{k}$ N·m

Equate direction cosines of $\Sigma\underline{M}_P$ & $\Sigma\underline{F}$ & get

$$\left.\begin{array}{l}(-30+150z)/M = 0.8\\(60-200z)/M = 0.6\\(-30+200y)/M = 0\end{array}\right\}$$ where M equals the magnitude of $\Sigma\underline{M}_P$

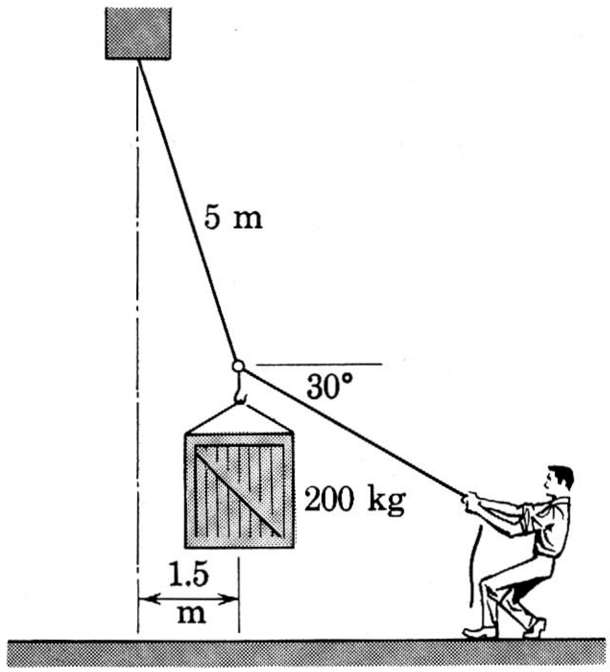
Solve & get $y = 0.15$ m or $\boxed{y = 150 \text{ mm}}$

$z = 0.264$ m or $\boxed{z = 264 \text{ mm}}$

& $M = (-30 + 150[0.264])/0.8 = 12$ N·m, $\boxed{\underline{M} = 12(0.8\underline{i} + 0.6\underline{j}) \text{ N·m}}$

JLM

Determine the pull P on the rope exerted by the man to hold the crate in the position shown. Also find the tension T in the upper rope.



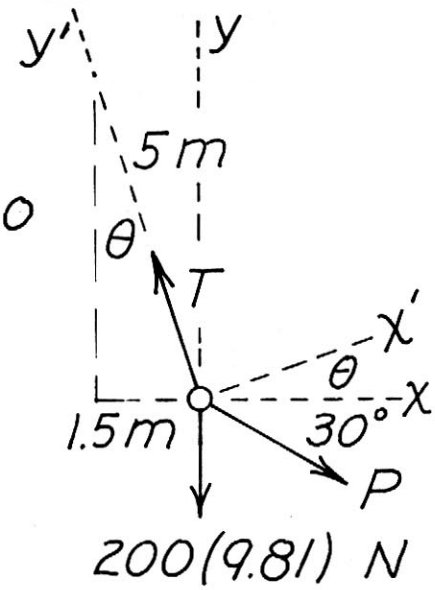
<u>Solution (I)</u> x-y axes

$\Sigma F_x = 0: \; 0.866P - \frac{1.5}{5}T = 0$

$\Sigma F_y = 0: \; -0.5P - 200(9.81) + 0.954T = 0$

Solve simultaneously & get

$\boxed{P = 871 \text{ N}, \quad T = 2513 \text{ N}}$



<u>Solution (II)</u> x'-y' axes

$\Sigma F_{x'} = 0:$

$P\cos(30° + 17.45°) - 200(9.81)\frac{1.5}{5} = 0$

$\boxed{P = 871 \text{ N}}$

$\theta = \sin^{-1}\frac{1.5}{5} = 17.45°$

$\cos 17.45° = 0.954$

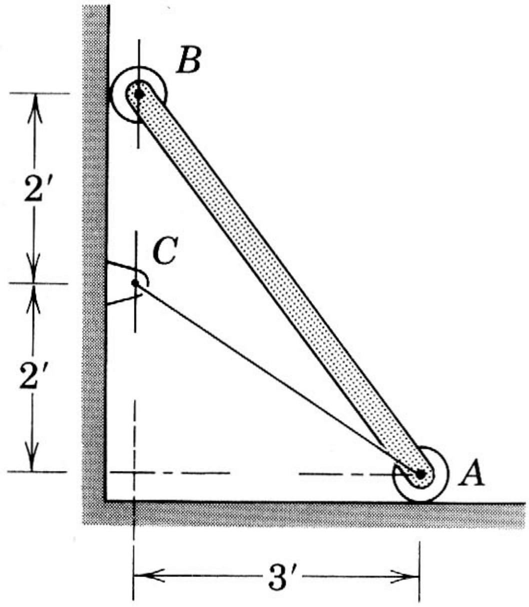
$\Sigma F_{y'} = 0:$

$T - 871\sin(30° + 17.45°) - 200(9.81)\cos 17.45° = 0$

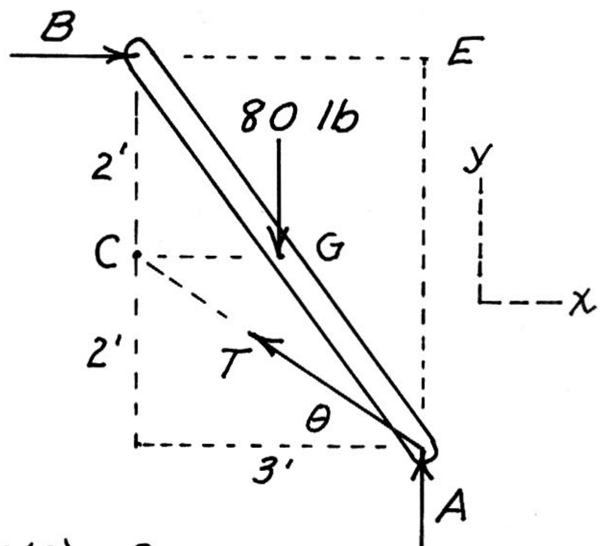
$\boxed{T = 2513 \text{ N}}$

JLM

The uniform 80-lb bar with small end rollers is supported by the horizontal and vertical surfaces and by wire AC. Calculate the tension T in the wire and the forces at A and B. Solve by using two moment equations and one force equation.



$\theta = \tan^{-1}\frac{2}{3} = 33.7°$



$\circlearrowleft + \Sigma M_A = 0: \; 4B - 80(3/2) = 0$

$\boxed{B = 30 \text{ lb}}$

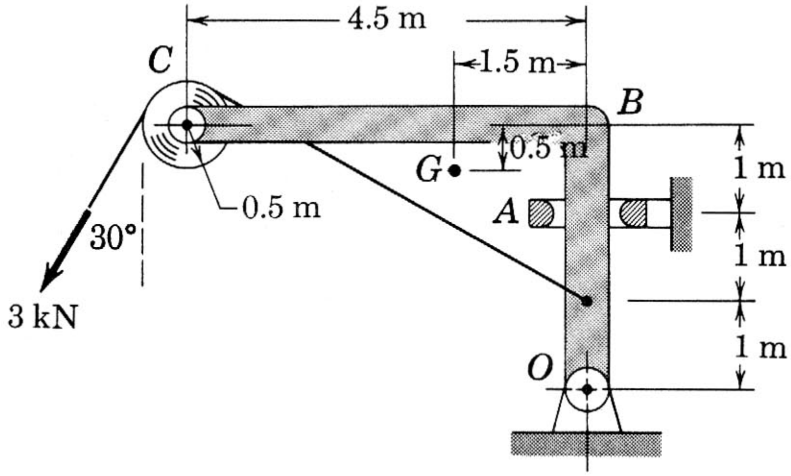
$\circlearrowleft + \Sigma M_E = 0: \; (T\cos 33.7°)4 - 80(3/2) = 0$

$\boxed{T = 36.1 \text{ lb}}$

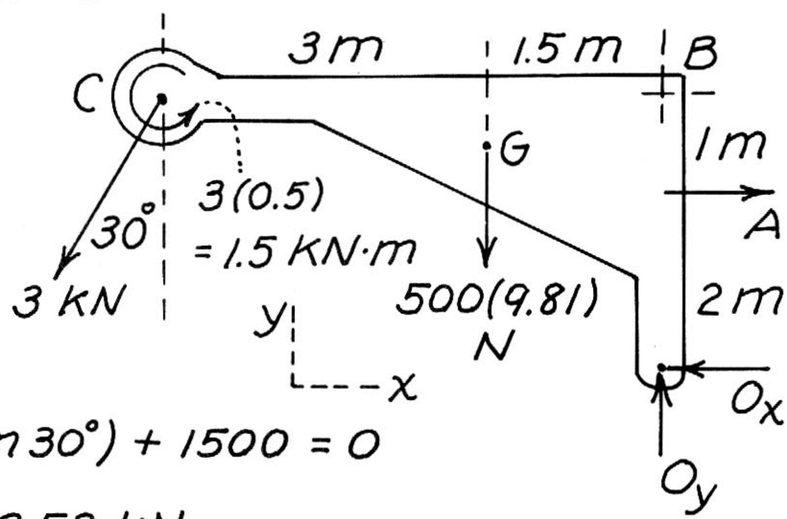
$\Sigma F_y = 0: \; A + 36.1 \sin 33.7° - 80 = 0$

$\boxed{A = 60 \text{ lb}}$

Member OBC and sheave C have a mass of 500 kg with mass center at G. Calculate the magnitude of the force supported by the pin at O. Collar A provides horizontal support only.



Replace force by force and couple at C.



$\Sigma M_O = 0$:

$500(9.81)(1.5) - 2A$

$+ 3000(4.5 \cos 30° + 3 \sin 30°) + 1500 = 0$

$A = 12\,524$ N or $A = 12.52$ kN

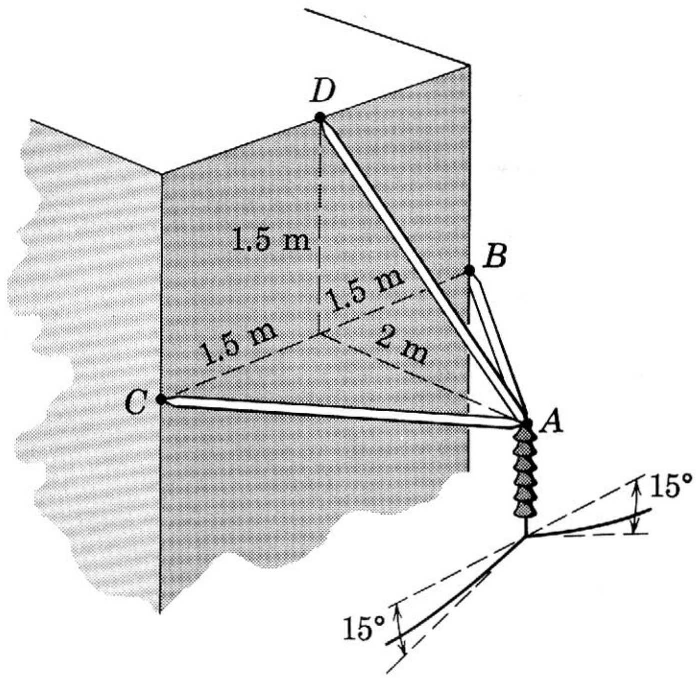
$\Sigma F_x = 0$: $12.52 - 3 \sin 30° - O_x = 0$, $O_x = 11.02$ kN

$\Sigma F_y = 0$: $O_y - 500(9.81) - 3 \cos 30° = 0$, $O_y = 7.50$ kN

$O = \sqrt{(11.02)^2 + (7.50)^2} = \boxed{13.34 \text{ kN}}$

JLM

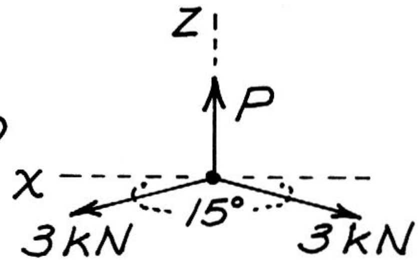
A high-voltage power line is suspended as shown. Tension in the line at the insulators is 3 kN. Calculate the tension T in link AD and the compression C in links AB and AC.

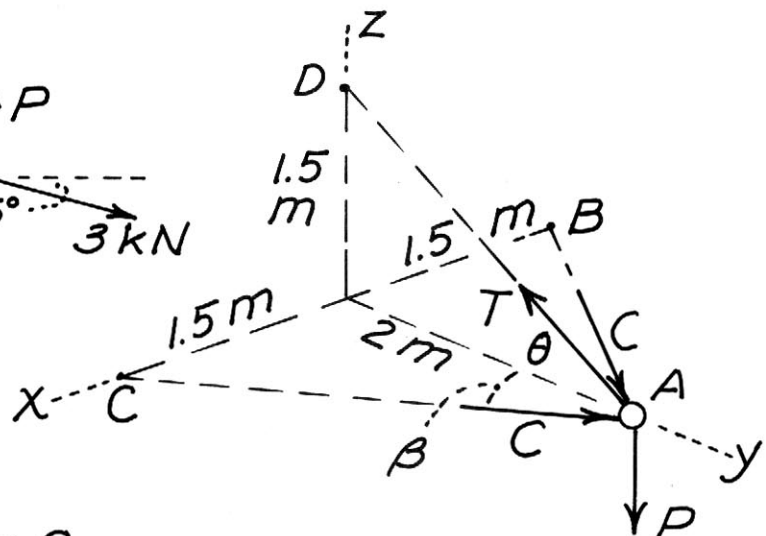


$\Sigma F_z = 0$:

$P - 2(3)\sin 15^\circ = 0$

$P = 1.553$ kN





$\overline{AC} = \overline{AD} = \sqrt{2^2 + (1.5)^2}$

$= 2.5$ m

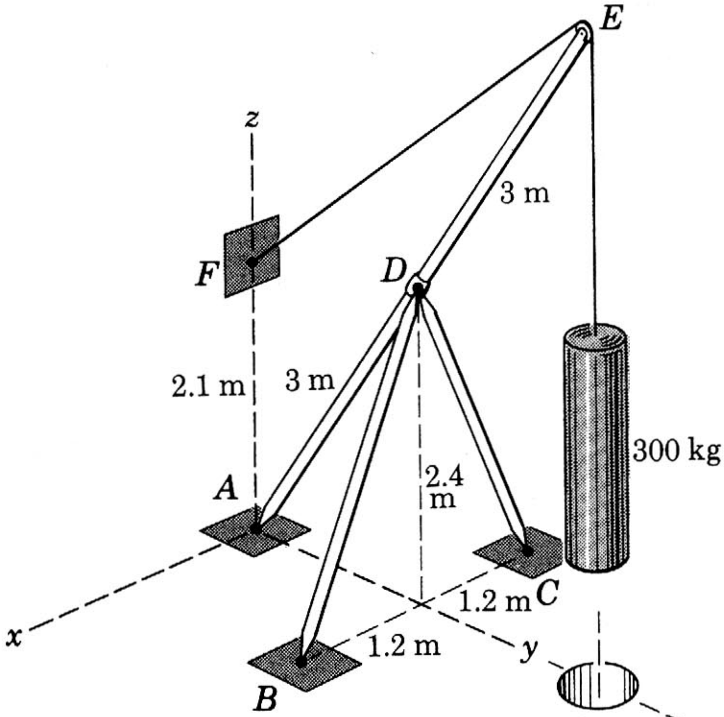
$\Sigma F_z = 0$: $T\sin\theta - 1.553 = 0$

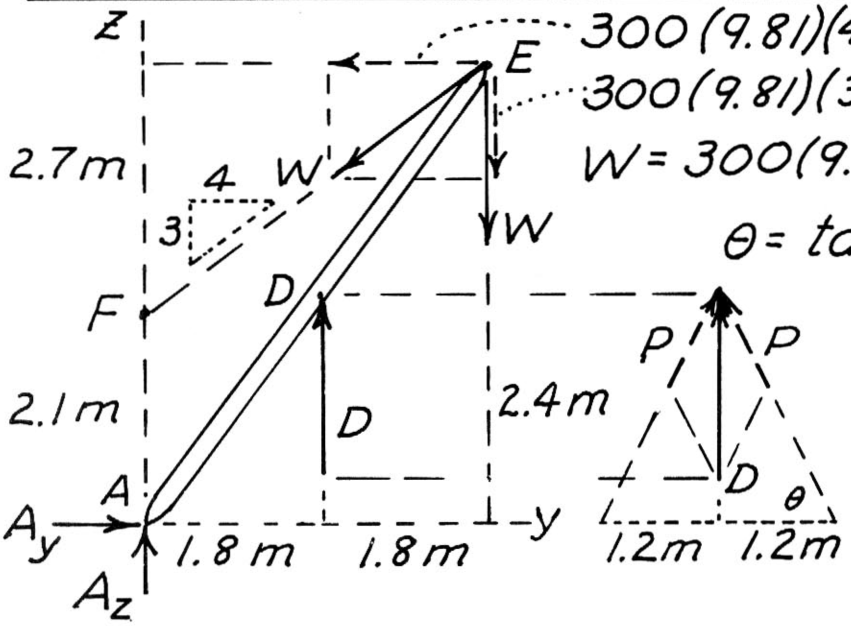
$T = \dfrac{1.553}{1.5/2.5}$, $\boxed{T = 2.59 \text{ kN}}$

$\Sigma F_y = 0$: $2C\cos\beta - 2.59\cos\theta = 0$

$C = \dfrac{2.59(2/2.5)}{2(2/2.5)}$, $\boxed{C = 1.29 \text{ kN}}$

Connections at A, B, C, D are ball & socket joints. Neglect weight of members. Find compression P in legs BD & CD and magnitude of force at A.





$300(9.81)(4/5) = 2354$ N

$300(9.81)(3/5) = 1766$ N

$W = 300(9.81) = 2943$ N

$\theta = \tan^{-1} 2.4/1.2 = 63.4°$

$2P\sin\theta = D$

$$P = \frac{D}{2\sin\theta} = \frac{\sqrt{5}}{4}D$$

$\Sigma F_y = 0$: $A_y - 2354 = 0$, $A_y = 2354$ N

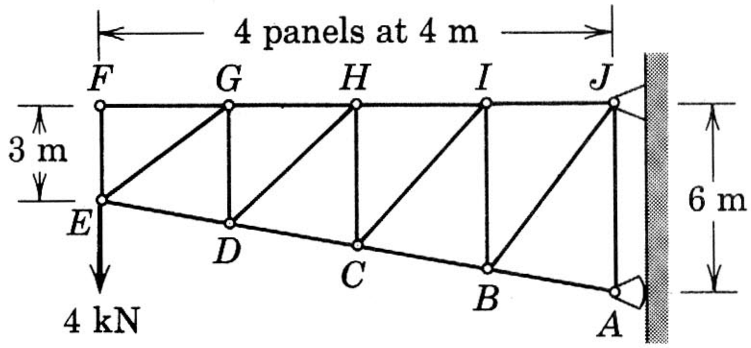
$\Sigma M_G = 0$: $\underbrace{1.8A_z - 2354(4.8) + 2943(1.8) + 1766(1.8) = 0}_{A_z = 1570\text{ N}}$,

$\Sigma F_z = 0$: $1570 + 4P/\sqrt{5} - 2943 - 1766 = 0$, $\boxed{P = 1755\text{ N}}$

$A = \sqrt{(2354)^2 + (1570)^2} = \boxed{2830\text{ N}}$

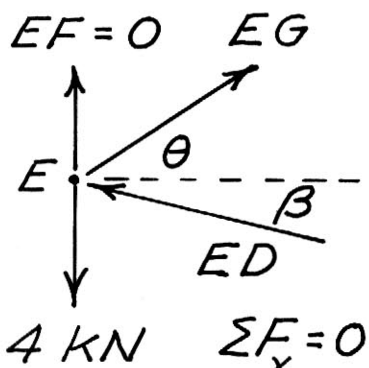
JLM

Determine the forces in members FG, EG, and GD for the simple truss.



By inspection of joint F, $FG = EF = 0$

Joint E



$\theta = \tan^{-1}\frac{3}{4}$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

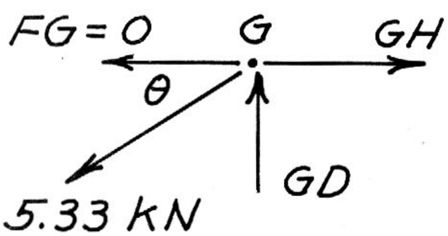
$\beta = \tan^{-1}\frac{3}{16} = 10.62°$

$\Sigma F_x = 0$: $EG\left(\frac{4}{5}\right) - ED \cos 10.62° = 0$

$\Sigma F_y = 0$: $EG\left(\frac{3}{5}\right) + ED \sin 10.62° - 4 = 0$

Solve to obtain $\boxed{EG = 5.33 \text{ kN } T}$

Joint G

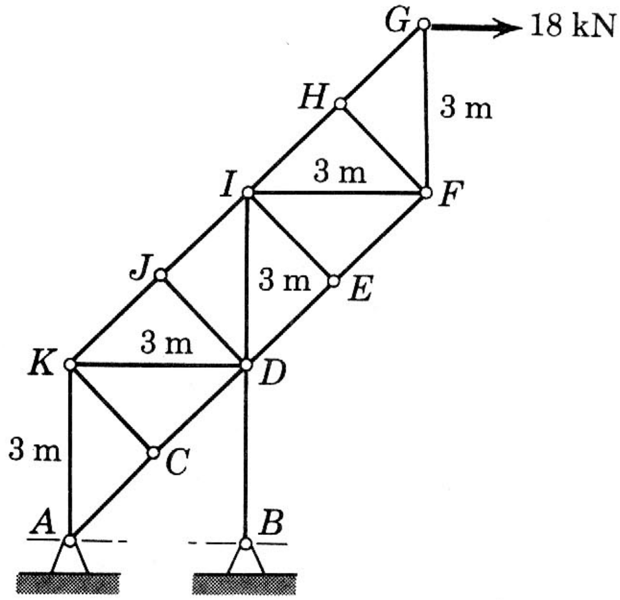


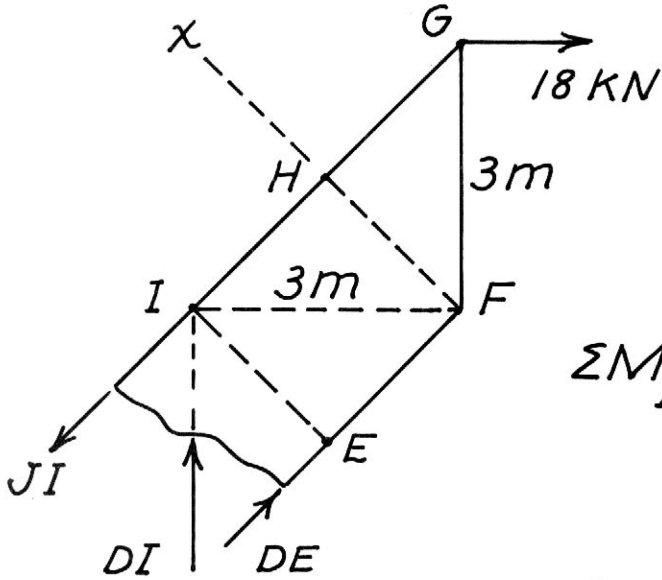
$\Sigma F_y = 0$: $GD - 5.33\left(\frac{3}{5}\right) = 0$

$\boxed{GD = 3.20 \text{ kN } C}$

JLM

Determine the forces in members DI, DE, and EI for the simple truss.





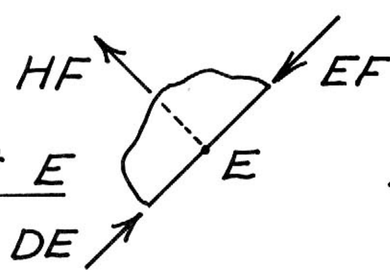
FBD of entire section

$$\Sigma M_I = 0: \; DE\,(3\cos 45^\circ) - 18\,(3) = 0$$

$$\boxed{DE = 25.5 \text{ KN C}}$$

$$\Sigma F_x = 0: \; DI\cos 45^\circ - 18\cos 45^\circ = 0$$

$$\boxed{DI = 18 \text{ KN C}}$$

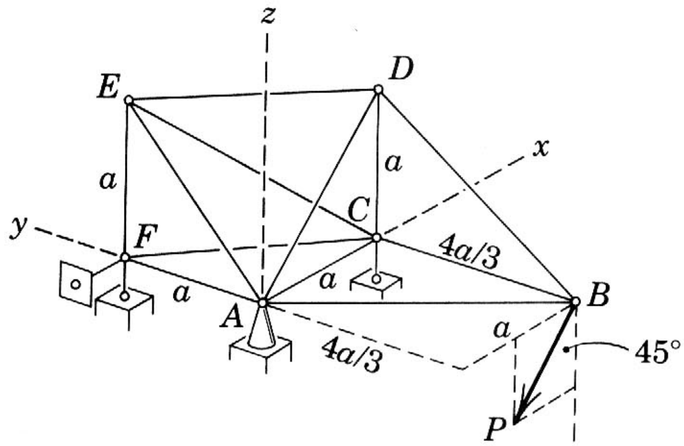


Joint E

$$\Sigma F_x = 0: \; \boxed{EI = 0}$$

JLM

Determine the forces in members AD, BD, CD, & ED of the space truss loaded and supported as shown. Verify the adequacy of internal stability.



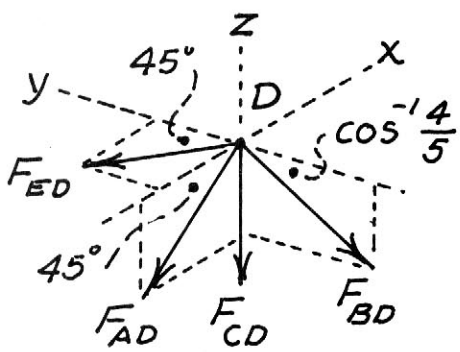
No. of members $m = 12$; No. of joints $j = 6$

$(m + 6 = 18) = (3j = 18)$ so members are adequate in number and comprise rigid tetrahedrons.

Joint B: $\Sigma F_z = 0$ gives $\frac{3}{5} F_{BD} - \frac{P}{\sqrt{2}} = 0$, $\boxed{F_{BD} = \frac{5P}{3\sqrt{2}} \quad (T)}$

All unknown forces taken (+) tension

Joint D:



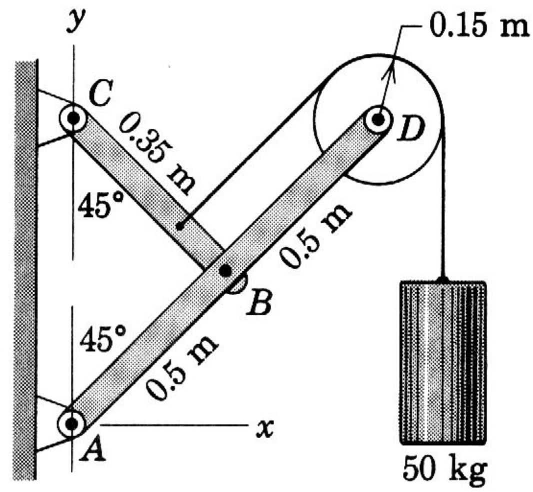
$$\left\{\begin{array}{l} \underline{F}_{BD} = F_{BD}\left(-\frac{4}{5}\underline{j} - \frac{3}{5}\underline{k}\right) = \frac{P}{3\sqrt{2}}(-4\underline{j} - 3\underline{k}) \\ \underline{F}_{CD} = F_{CD}(-\underline{k}) \\ \underline{F}_{AD} = F_{AD}\left(-\frac{\underline{i}}{\sqrt{2}} - \frac{\underline{k}}{\sqrt{2}}\right) = \frac{F_{AD}}{\sqrt{2}}(-\underline{i} - \underline{k}) \\ \underline{F}_{ED} = F_{ED}\left(\frac{\underline{j}}{\sqrt{2}} - \frac{\underline{i}}{\sqrt{2}}\right) = \frac{F_{ED}}{\sqrt{2}}(-\underline{i} + \underline{j}) \end{array}\right.$$

$\Sigma \underline{F} = \underline{0}$ yields

$$\left\{\begin{array}{ll} \underline{i}\text{-terms:} & -F_{AD}/\sqrt{2} - F_{ED}/\sqrt{2} = 0 \\ \underline{j}\text{-terms:} & -4P/(3\sqrt{2}) + F_{ED}/\sqrt{2} = 0 \\ \underline{k}\text{-terms:} & -P/\sqrt{2} - F_{CD} - F_{AD}/\sqrt{2} = 0 \end{array}\right.$$

Solve & get $\boxed{F_{AD} = -\frac{4P}{3} \quad (C)}$ $\boxed{F_{CD} = \frac{P}{3\sqrt{2}} \quad (T)}$ $\boxed{F_{ED} = \frac{4P}{3} \quad (T)}$

Determine the total force (shear) supported by the pin at B for the loaded frame.

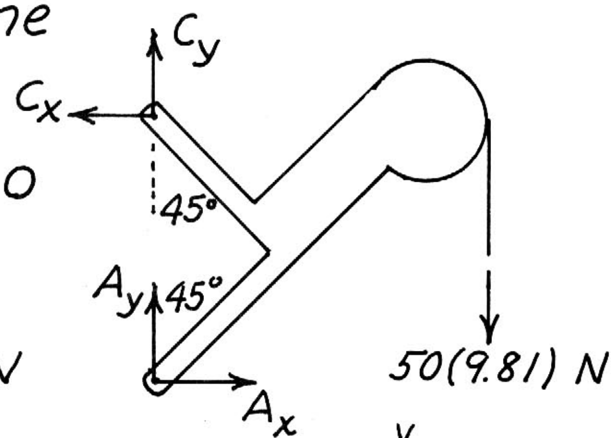


From FBD of entire frame

$\Sigma M_A = 0$:

$$C_x(0.5\sqrt{2}) - 50(9.81)\left(\frac{1}{\sqrt{2}} + 0.15\right) = 0$$

$$C_x = 595 \text{ N}$$



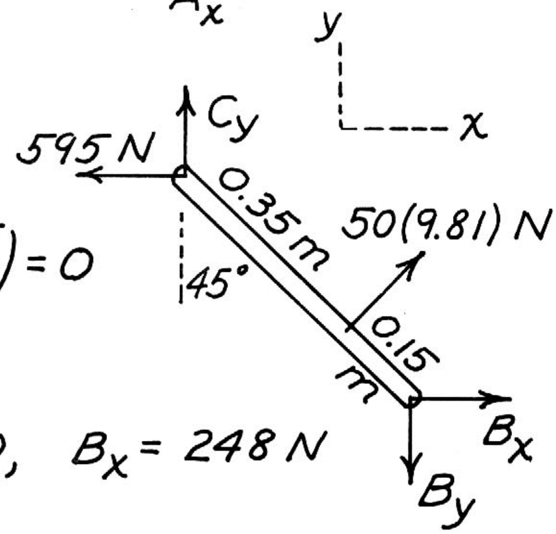
$\Sigma F_x = 0$: $A_x - 595 = 0$, $A_x = 595$ N

From FBD of member BC

$\Sigma M_B = 0$:

$$595\left(\frac{0.5}{\sqrt{2}}\right) - 50(9.81)(0.15) - C_y\left(\frac{0.5}{\sqrt{2}}\right) = 0$$

$$C_y = 386 \text{ N}$$



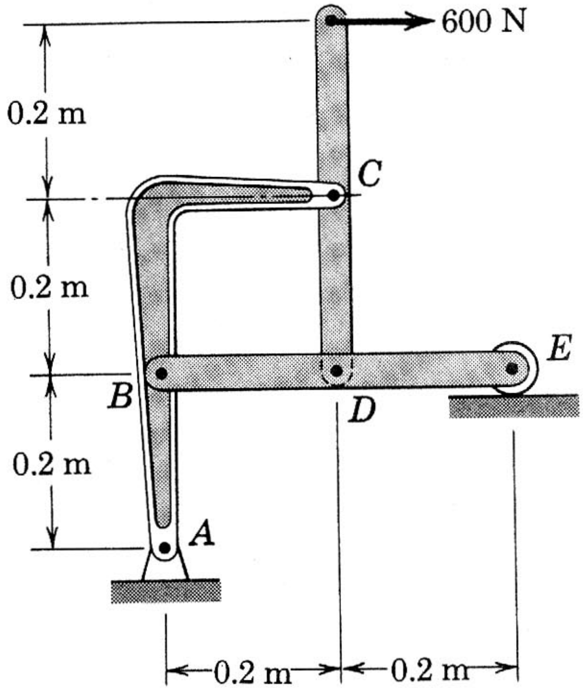
$\Sigma F_x = 0$: $B_x + 50(9.81)/\sqrt{2} - 595 = 0$, $B_x = 248$ N

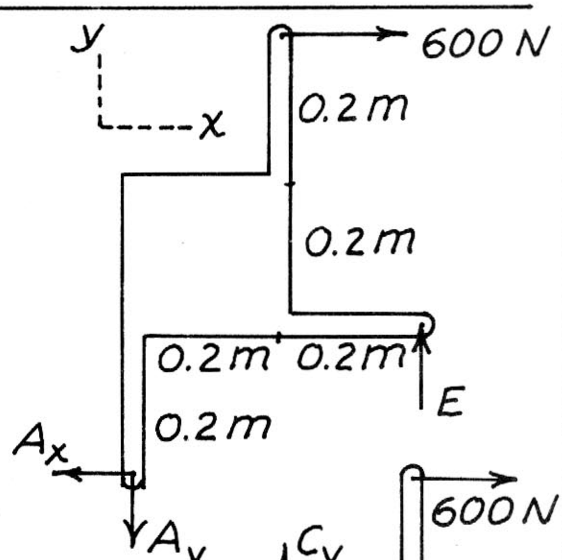
$\Sigma F_y = 0$: $386 + 50(9.81)/\sqrt{2} - B_y = 0$, $B_y = 733$ N

Total force (shear) $B = \sqrt{(248)^2 + (733)^2} = \boxed{774 \text{ N}}$

JLM

Determine the magnitude of the force supported by the pin at C.





Entire frame

$\Sigma M_A = 0: \quad 0.4E - 0.6(600) = 0$

$E = 900 \text{ N}$

$\Sigma F_x = 0: \quad A_x = 600 \text{ N}$

$\Sigma F_y = 0: \quad A_y = 900 \text{ N}$

Link CD $\quad \Sigma M_D = 0:$

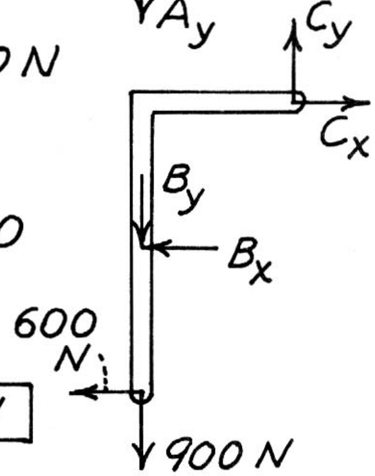
$C_x(0.2) - 600(0.4) = 0, \quad C_x = 1200 \text{ N}$

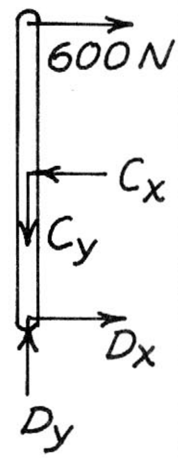
Link ABC $\quad \Sigma M_B = 0:$

$C_y(0.2) - 600(0.2) - 1200(0.2) = 0$

$C_y = 1800 \text{ N}$

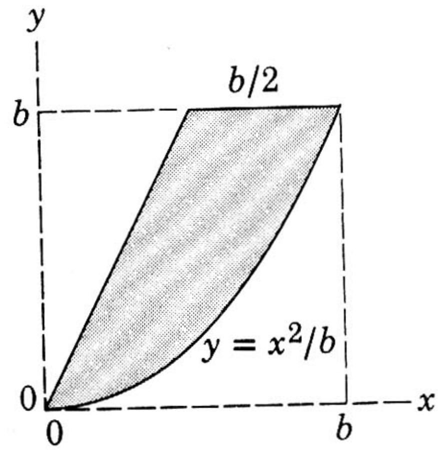
$C = \sqrt{(1200)^2 + (1800)^2} = \boxed{2160 \text{ N}}$

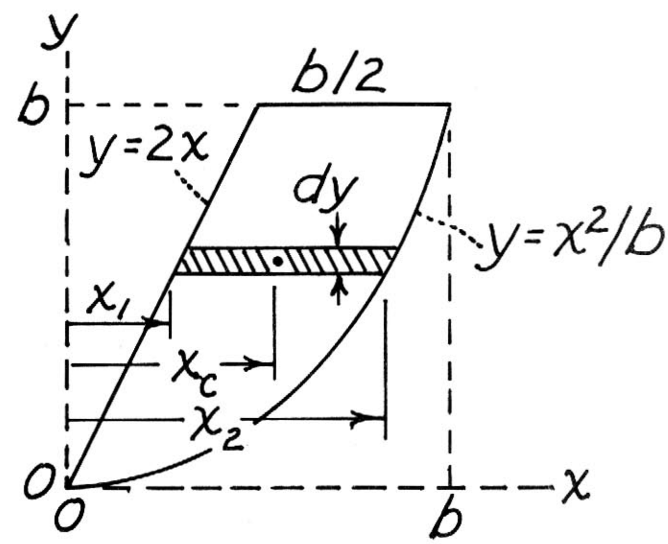




JLM

Determine the x-coordinate of the centroid of the shaded area.





$$dA = (x_2 - x_1)\,dy$$
$$= (\sqrt{by} - y/2)\,dy$$
$$A = \int_0^b (\sqrt{by} - y/2)\,dy$$
$$= \left[\frac{2}{3}\sqrt{b}\,y^{3/2} - y^2/4\right]_0^b$$
$$= \frac{5}{12}b^2$$

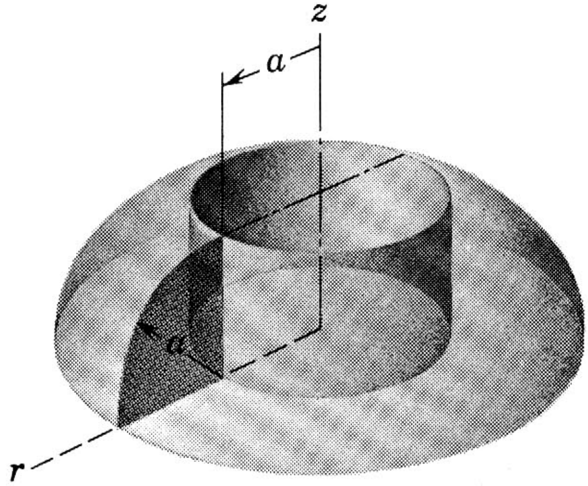
$$x_c = \frac{1}{2}(x_1 + x_2)$$

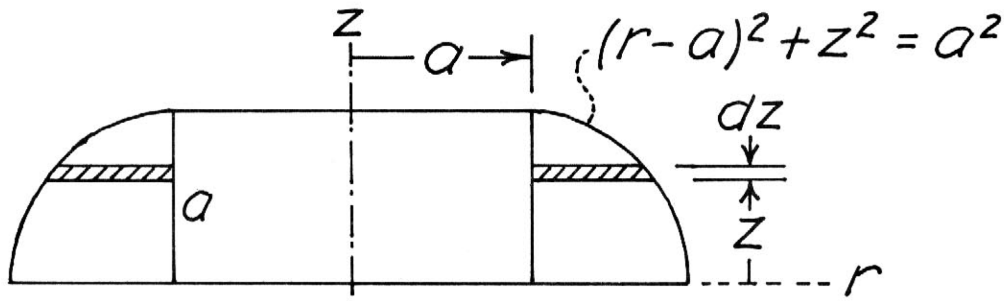
$$\int x_c\,dA = \int_0^b \frac{1}{2}(x_1 + x_2)(x_2 - x_1)\,dy = \frac{1}{2}\int_0^b (x_2^2 - x_1^2)\,dy$$
$$= \frac{1}{2}\int_0^b (by - y^2/4)\,dy = \frac{1}{2}\left(\frac{by^2}{2} - \frac{y^3}{12}\right)_0^b = \frac{5}{24}b^3$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{5b^3/24}{5b^2/12} = \boxed{\frac{b}{2}}$$

JLM

Determine the z-coordinate of the mass center of the solid obtained by revolving the quarter-circular area about the z-axis.





Differential element is a washer of radii r and a and thickness dz with volume

$$dV=\pi(r^2-a^2)\,dz=\pi\left(a^2-z^2+2a\sqrt{a^2-z^2}\right)dz$$

$$\int z\,dV=\int_0^a\pi\left(a^2z-z^3+2az\sqrt{a^2-z^2}\right)dz$$

$$=\pi\left[\frac{a^2z^2}{2}-\frac{z^4}{4}+\frac{2a}{3}\sqrt{(a^2-z^2)^3}\right]_0^a=\frac{11}{12}\pi a^4$$

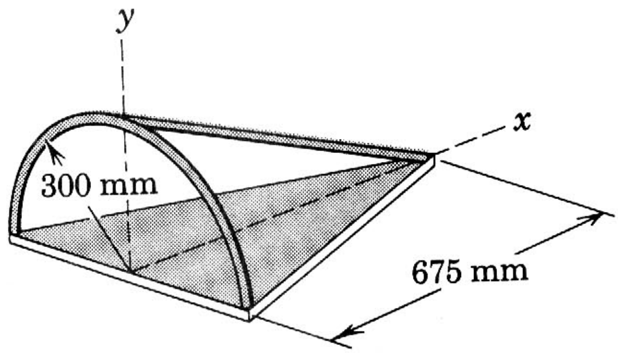
$$\int dV=\int_0^a\pi\left(a^2-z^2+2a\sqrt{a^2-z^2}\right)dz$$

$$=\pi\left[a^2z-\frac{z^3}{3}+a\left(z\sqrt{a^2-z^2}+a^2\sin^{-1}\frac{z}{a}\right)\right]_0^a=\pi a^3\left(\frac{2}{3}+\frac{\pi}{2}\right)$$

$$\bar{z}=\frac{\int z\,dV}{\int dV}=\frac{(11/12)\pi a^4}{\pi a^3(2/3+\pi/2)},\quad \boxed{\bar{z}=\frac{11a}{2(4+3\pi)}=0.410a}$$

JLM

The semicircular and straight bars are made from stock with a mass of 7.5 kg per meter of length and are welded to the triangular plate made from material with a mass of 100 kg per square meter of area. Calculate the coordinates of the mass center of the assembly.



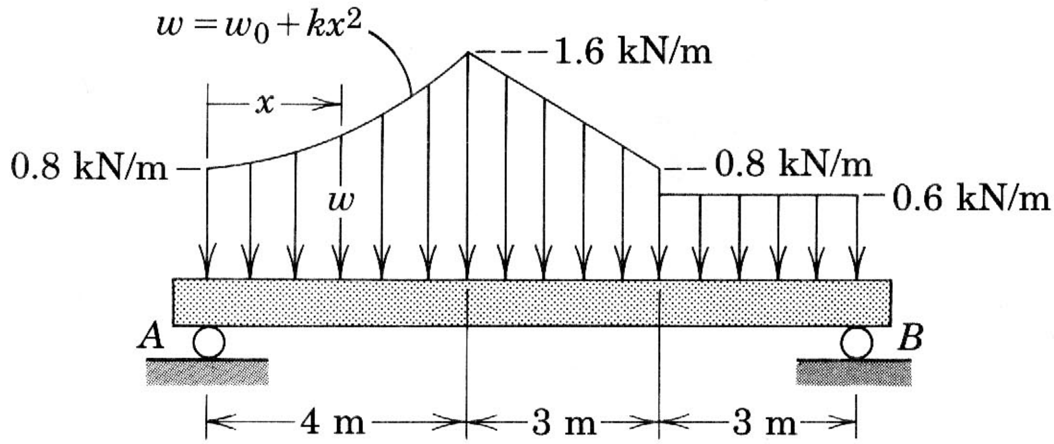
Part	m kg	$\bar{x}$ mm	$\bar{y}$ mm	$m\bar{x}$ kg·mm	$m\bar{y}$ kg·mm
Circular bar	2.25π	0	$600/\pi$	0	1350
Brace	5.54	338	150	1870	831
Base	20.25	225	0	4556	0
sums	32.86			6426	2181

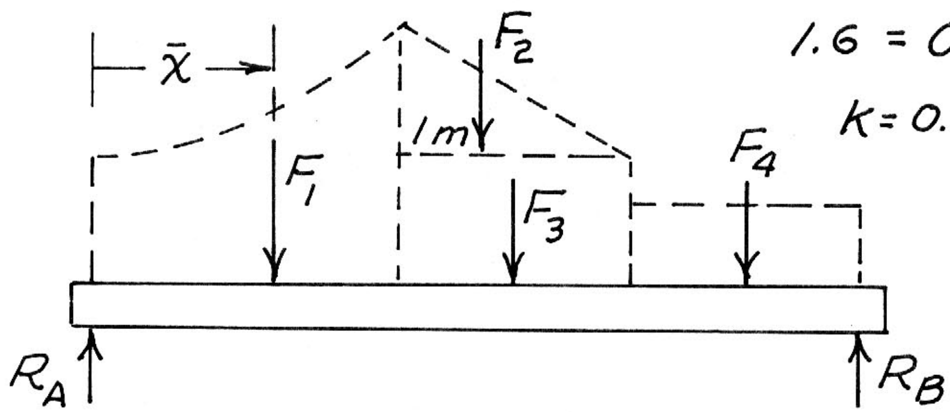
$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{6426}{32.86} = \boxed{195.6 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{2181}{32.86} = \boxed{66.4 \text{ mm}}$$

JLM

Determine the support reactions at A and B for the beam loaded as shown





$1.6 = 0.8 + k(4^2)$

$k = 0.05 \text{ kN/m}^3$

$$F_1 = \int w\,dx = \int_0^4 (0.8 + 0.05x^2)\,dx = 4.27 \text{ kN}$$

$$F_1\bar{x} = \int wx\,dx = \int_0^4 (0.8x + 0.05x^3)\,dx = 9.6 \text{ kN}\cdot\text{m}$$

$$\bar{x} = 9.6/4.27 = 2.25 \text{ m}$$

$$F_2 = \tfrac{1}{2}(1.6 - 0.8)3 = 1.2 \text{ kN}$$

$$F_3 = 0.8(3) = 2.4 \text{ kN}, \quad F_4 = 0.6(3) = 1.8 \text{ kN}$$

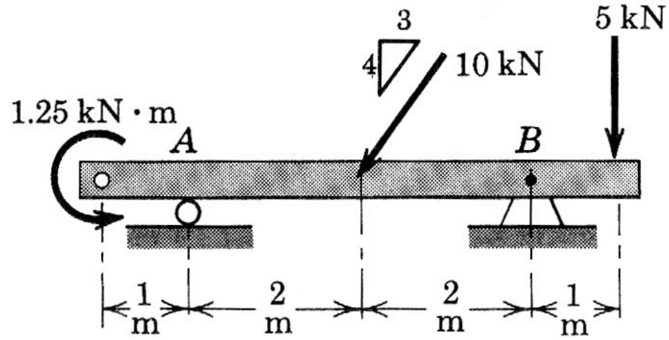
$$[\Sigma M_A = 0] \quad 4.27(2.25) + 1.2(5) + 2.4(5.5) + 1.8(8.5) - 10R_B = 0$$

$$[\Sigma F = 0] \quad R_A + R_B - (4.27 + 1.2 + 2.4 + 1.8) = 0$$

$$\boxed{R_A = 5.26 \text{ kN}} \qquad \boxed{R_B = 4.41 \text{ kN}}$$

JLM

Construct the shear and moment diagrams for the loaded beam.



$\Sigma M_A = 0$: $5(5) + 8(2) - 4B - 1.25 = 0$, $B = 9.94$ kN

$\Sigma F_V = 0$: $A + 9.94 - 13 = 0$, $A = 3.06$ kN

(I)

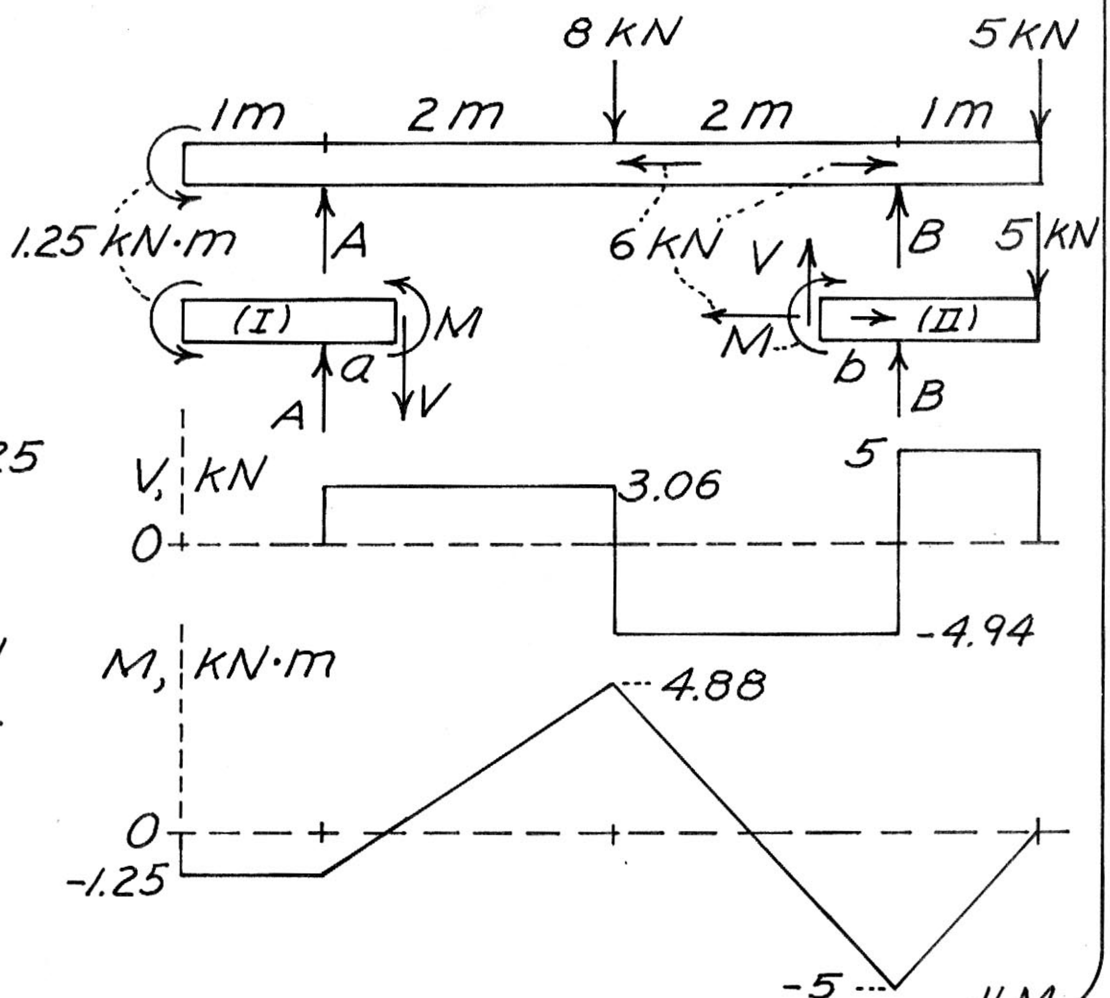
$V = 3.06$ kN

$M = 3.06a - 1.25$ kN·m

(II)

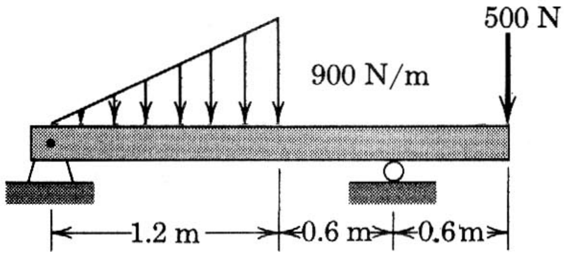
$V = -4.94$ kN

$M = 4.94b - 5$ kN·m



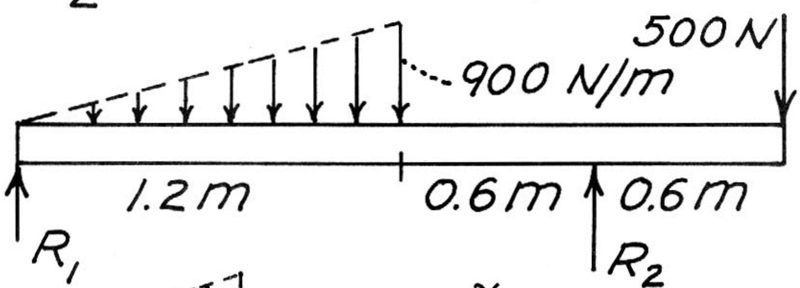
JLM

Construct the shear and moment diagrams for the loaded beam.



$\Sigma M_{R_1} = 0$: $500(2.4) - 1.8R_2 + \frac{900}{2}(1.2)(\frac{2}{3}1.2) = 0$, $R_2 = 907$ N

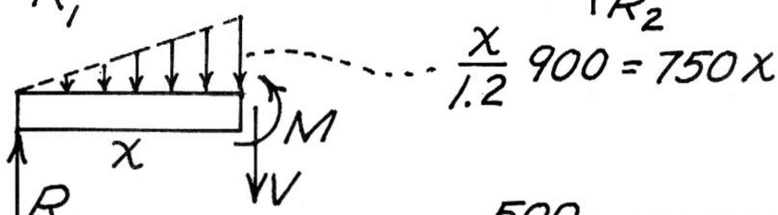
$\Sigma F_V = 0$: $R_1 + 907 - 500 - \frac{900}{2}(1.2) = 0$, $R_1 = 133.3$ N

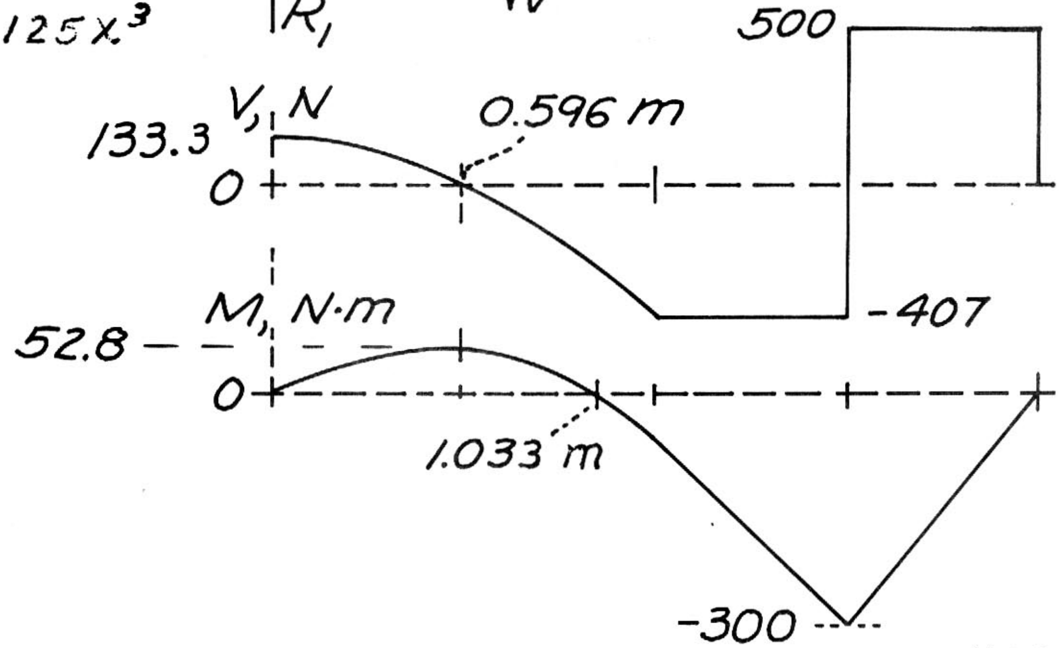


Equilibrium of section x gives

$V = 133.3 - 375x^2$

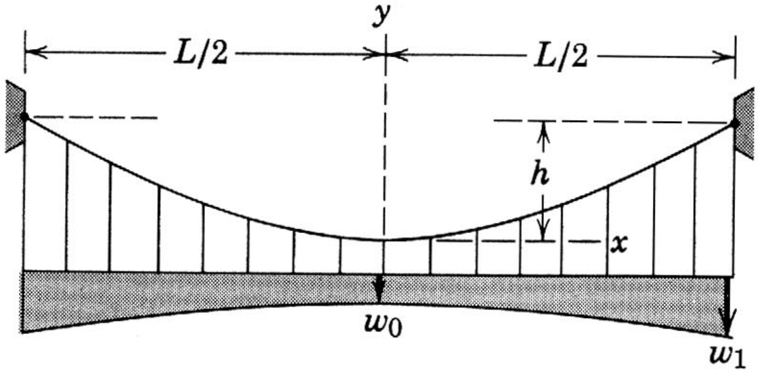
$M = 133.3x - 125x^3$





JLM

The distributed load expressed as force per unit length supported by the light cable varies from w_0 to w_1 according to $w = a + b|x^3|$. Determine the shape of the cable $y = f(x)$ and the tension T_0 at mid-span for given L and h.



When $x = 0$, $w = w_0$, so $a = w_0$

$x = L/2$, $w = w_1$, so $b = (w_1 - w_0)\frac{8}{L^3}$

Thus $w = w_0 + (w_1 - w_0)\frac{8}{L^3}|x^3|$

From D.E. of cable $d^2y/dx^2 = w/T_0$, $\frac{dy}{dx} = \frac{1}{T_0}\int_0^x w\,dx$

so $\frac{dy}{dx} = \frac{1}{T_0}\left[w_0 x + (w_1 - w_0)\frac{2x^4}{L^3}\right]$

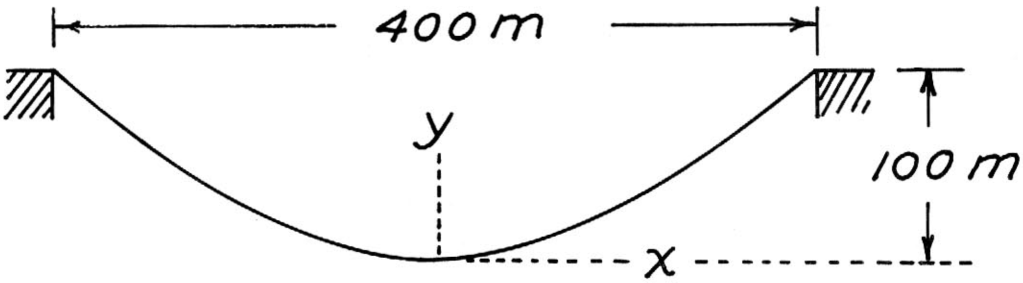
and $y = \int_0^x \frac{dy}{dx}dx$, $y = \frac{1}{T_0}\left[\frac{w_0}{2}x^2 + (w_1 - w_0)\frac{2x^5}{5L^3}\right]$

Sub. $y = h$ when $x = L/2$ & get

$$\boxed{T_0 = \frac{L^2}{80h}(9w_0 + w_1)} \qquad \boxed{y = \frac{80hx^2}{L^2(9w_0 + w_1)}\left(\frac{w_0}{2} + \frac{w_1 - w_0}{5L^3}2x^3\right)}$$

JLM

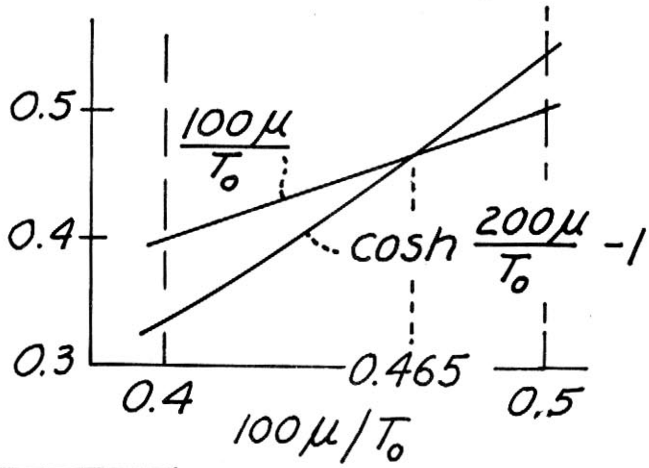
A cable hanging under its own weight is suspended between two points on the same level 400 m apart. If the sag is 100 m, find the total length S of the cable. What error results if S is calculated by using the first three terms of the series expression for the parabolic cable?



At $x = 200$ m, $y = 100$ m Eq. 5/19, $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

is $\frac{100\mu}{T_0} = \cosh\frac{200\mu}{T_0} - 1$

Solve graphically or by Newton's method & get $\frac{T_0}{\mu} = \frac{1}{0.00465} = 215$



From Eq. 5/20 $S = 2s$

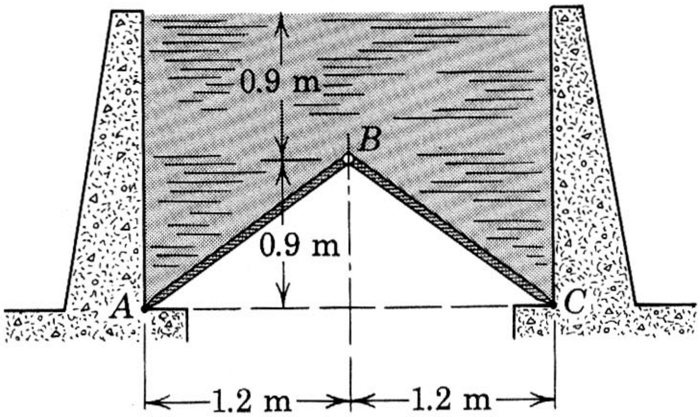
$S = 2(215)\sinh(200/215) = \boxed{460 \text{ m}}$

From Eq. 5/16, $S = 2(200)\left[1 + \frac{2}{3}\left(\frac{1}{2}\right)^2 - \frac{2}{5}\left(\frac{1}{2}\right)^4 + \cdots\right] = 457$ m

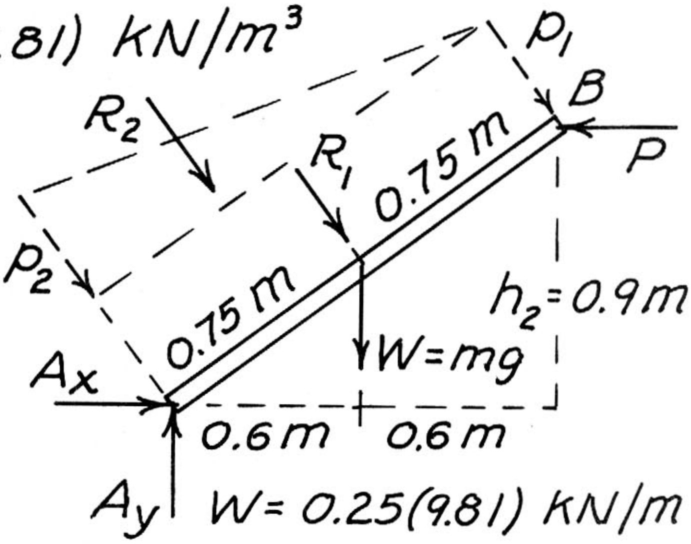
% error is $\frac{460 - 457}{460}100 = \boxed{0.65\% \text{ low}}$

JLM

Cross section of a long fresh-water channel is shown. Each of the bottom plates, hinged at B, has a mass of 250 kg per meter of channel length. Find force P per meter of channel length acting on each plate at B.



Fresh water $\rho g = 1.00(9.81)\ \text{kN/m}^3$



$R_1 = p_1 A = \rho g h_1 [\text{Area}]$

$= 1.00(9.81)(0.9)[(1.5)(1)]$

$= 13.24\ \text{kN/m}$

$R_2 = \frac{1}{2}\rho g h_2 [\text{Area}]$

$= \frac{1}{2}(1.00)(9.81)(0.9)[(1.5)(1)]$

$= 6.62\ \text{kN/m}$

$\Sigma M_A = 0:$

$$0.9P - 0.25(9.81)(0.6) - 13.24(0.75) - 6.62\,\frac{1.5}{3} = 0$$

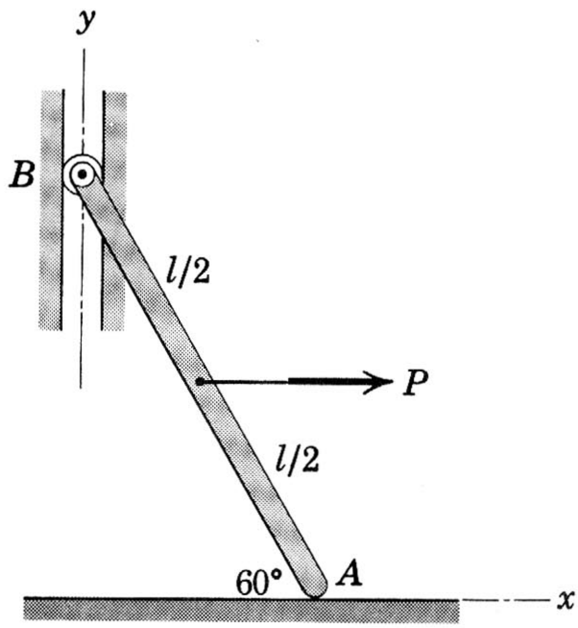
$$\boxed{P = 16.35\ \text{kN/m}}$$

JLM

Uniform 60-kg bar AB is subjected to force P. Smooth guides at B. At A, $\mu_s = 0.8$.

(a) If $P = 400$ N, find friction force at A.

(b) Find P required to cause slippage at A.



$W = mg = 60(9.81) = 589$ N

(a) $P = 400$ N. Assume equil.

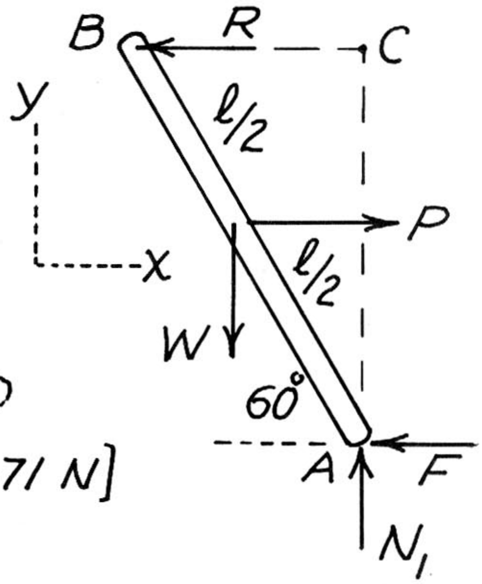
$\Sigma F_y = 0$: $N_1 - 589 = 0$, $N_1 = 589$ N

$\Sigma M_C = 0$: $400 \frac{\ell}{2} \sin 60°$

$+ 589 \frac{\ell}{2} \cos 60° - F\ell \sin 60° = 0$

$\boxed{F = 370 \text{ N}} < [\mu_s N_1 = 0.8(589) = 471 \text{ N}]$

so assumption is valid



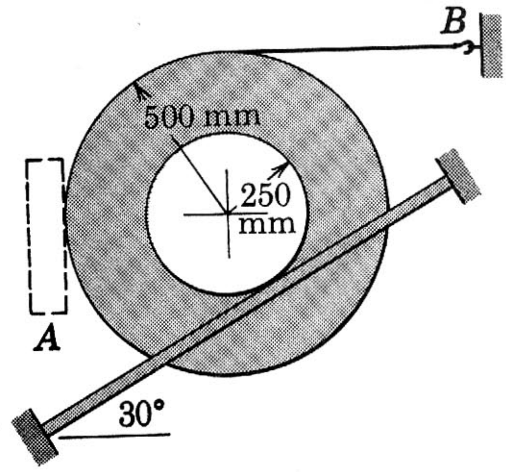
(b) $F = \mu_s N_1 = 471$ N

$\Sigma M_C = 0$; $P \frac{\ell}{2} \sin 60° + 589 \frac{\ell}{2} \cos 60° - 471(\ell \sin 60°) = 0$

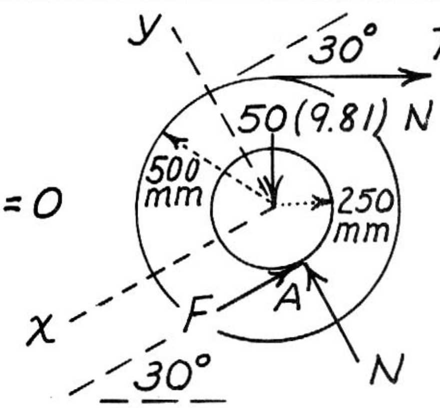
$\boxed{P = 602 \text{ N}}$

JLM

The hubs of the uniform 50-kg wheel rest on inclined rails. If support at A is removed, determine the friction force acting on the wheel if $\mu_s = 0.50$, $\mu_k = 0.40$. What would happen if $\mu_s = 0.30$ & $\mu_k = 0.25$?



First, assume equilibrium.



$\Sigma M_A = 0$:

$$T(500 + 250\cos 30°) - 50(9.81)(250 \sin 30°) = 0$$

$$T = 85.6\ N$$

$\Sigma F_y = 0$:

$$N - 50(9.81)\cos 30° - 85.6 \sin 30° = 0$$

$$N = 468\ N$$

$\Sigma F_x = 0$:

$$-F - 85.6\cos 30° + 50(9.81)\sin 30° = 0, \quad F = 171\ N$$

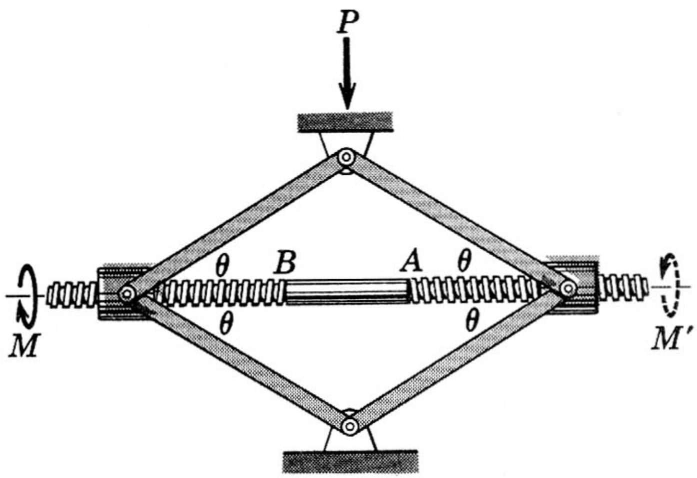
Since $(F_{needed} = 171\ N) < (F_{s\,max} = \mu_s N = 0.50(468) = 234\ N)$, equilibrium assumption is valid & $\boxed{F = 171\ N}$

If $\mu_s = 0.30$, $F_{s\,max} = 0.30(468) = 140.4\ N < 171\ N$

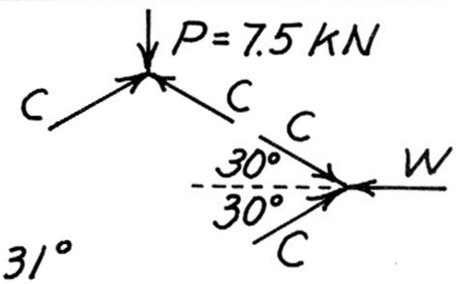
so <u>wheel will slip</u>. But $F \neq 0.25(468)\ N$ since $N \neq 468\ N$ under accelerating conditions.

JLM

Each screw of the jack has a mean diameter of 21 mm and a lead of 8 mm, one a right-hand and the other a left-hand thread. For $\theta = 30^\circ$ determine (a) the torque M required to raise the load $P = 7.5$ kN and (b) the torque M' required to lower the load. The coefficient of friction is $\mu = 0.20$.



For equilibrium



$W = 2C\cos 30^\circ, \; P = 2C\sin 30^\circ$

so $W = P \operatorname{ctn} 30^\circ = 7.5\sqrt{3} = 12.99$ kN

Friction angle $\phi = \tan^{-1} 0.20 = 11.31^\circ$

Helix angle $\alpha = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\dfrac{8}{2\pi(21/2)} = 6.91^\circ$

$M = 2Wr\tan(\phi + \alpha) = 2(12.99)(21/2)\tan(11.31^\circ + 6.91^\circ)$

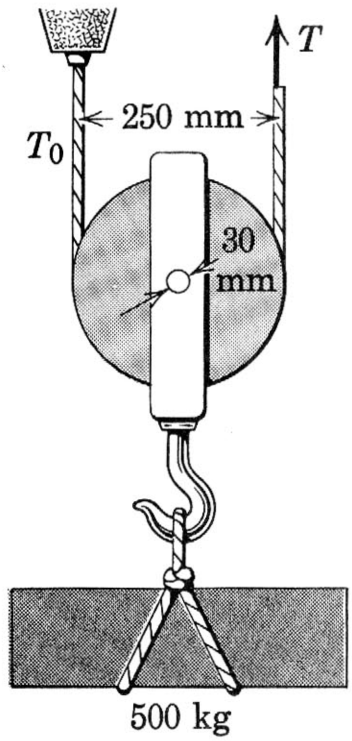
(a) $M = 89.8$ kN·mm or $\boxed{M = 89.8 \text{ N·m}}$

$M' = 2Wr\tan(\phi - \alpha) = 2(12.99)(21/2)\tan(11.31^\circ - 6.91^\circ)$

(b) $M' = 21.0$ kN·mm or $\boxed{M' = 21.0 \text{ N·m}}$

JLM

The coefficient of kinetic friction between the 30-mm-diameter pin and the pulley is 0.25. Calculate the tension T required to (a) raise the load and (b) to lower the load at a constant speed. Neglect the mass of the pulley.



$\phi = \tan^{-1} 0.25 = 14.04°$

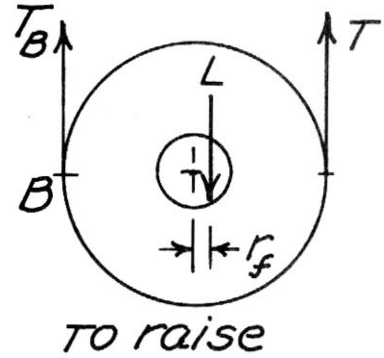
$r_f = r \sin\phi = 0.015 \sin 14.04° = 0.00364$ m

$L = 500(9.81) = 4905$ N

(a) To raise load: $\Sigma M_B = 0$

$0.25T - 4905(0.125 + 0.00364) = 0$

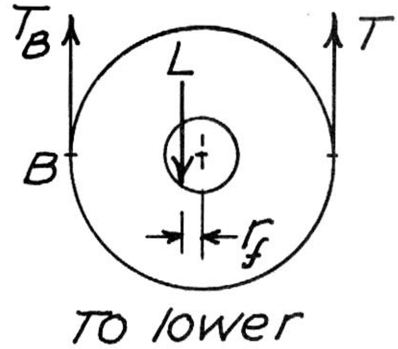
$T = 2524$ N or $\boxed{T = 2.52 \text{ kN}}$



(b) To lower load: $\Sigma M_B = 0$

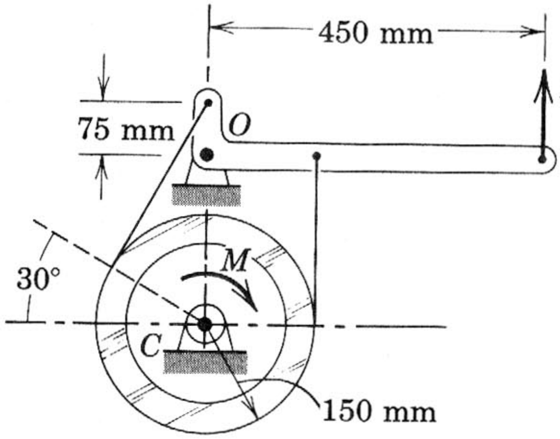
$0.25T - 4905(0.125 - 0.00364) = 0$

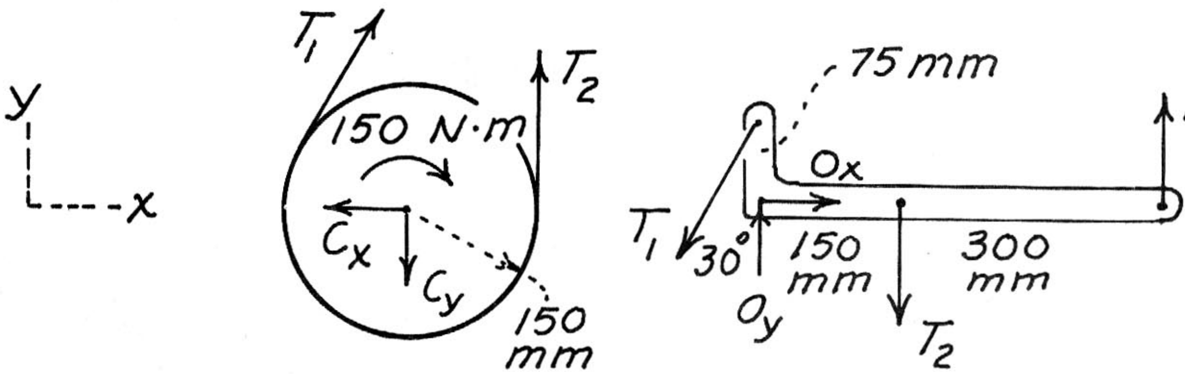
$T = 2381$ N or $\boxed{T = 2.38 \text{ kN}}$



JLM

Calculate the force P on the handle of the differential band brake that will prevent the flywheel from turning on its shaft to which the torque $M = 150$ N·m is applied. The coefficient of friction between the band and the flywheel is $\mu = 0.40$.





Band $\quad T_2 = T_1 e^{\mu\beta} \qquad T_2 = T_1 e^{0.40 \frac{7\pi}{6}} = 4.33\,T_1 \qquad \cdots (1)$

Flywheel $\Sigma M_C = 0; \quad 150 + (T_1 - T_2)(0.150) = 0$

$$T_2 - T_1 = 1000 \text{ N} \qquad \cdots (2)$$

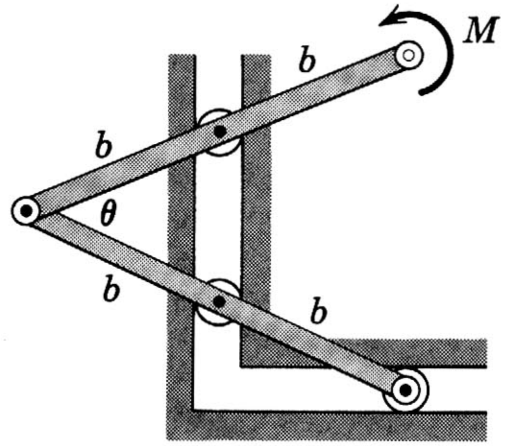
Handle $\Sigma M_O = 0; \quad 0.150\,T_2 - (T_1 \sin 30°)(0.075) - 0.450P = 0$

Solve (1) & (2) & get $T_1 = 300$ N, $T_2 = 1300$ N

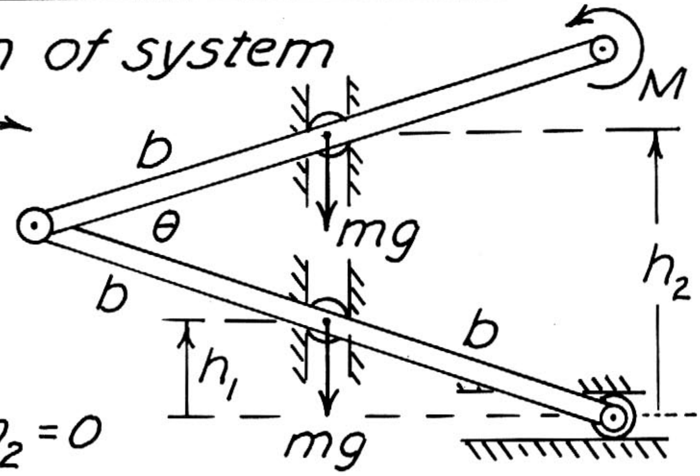
Solve for P & get $\boxed{P = 408 \text{ N}}$

JLM

The uniform bars, each of mass m, are connected as shown with their rollers of negligible mass confined to move in the vertical and horizontal guides. Determine the equilibrium angle θ resulting from the application of the couple M.



Active-force diagram of system



$h_1 = b \sin \theta/2$

$h_2 = 3b \sin \theta/2$

$\delta U = 0$:

$M\,\delta(\theta/2) - mg\,\delta h_1 - mg\,\delta h_2 = 0$

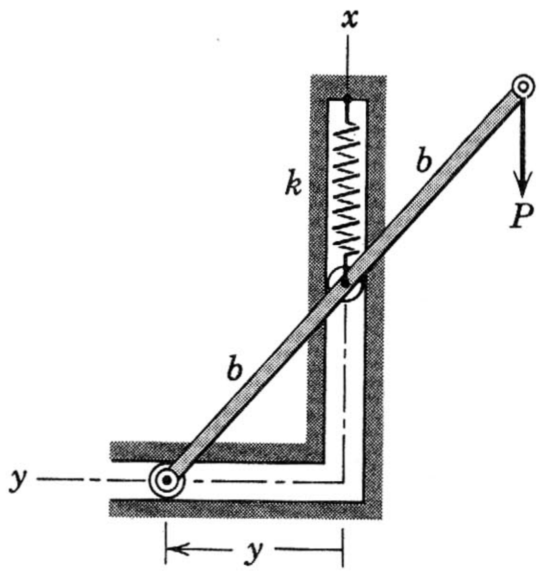
$M\,\delta(\theta/2) - mg\,\delta(b \sin \theta/2 + 3b \sin \theta/2) = 0$

$M\,\delta(\theta/2) = mg\,(4b \cos \theta/2)\,\delta(\theta/2)$

$M = 4mgb \cos \theta/2$, $\boxed{\theta = 2\cos^{-1}\dfrac{M}{4mgb}}$, $M < 4mgb$

JLM

Determine y for equilibrium of the mechanism in the vertical plane under the load P. The spring of stiffness k is unstretched when $y=0$. The mass of the uniform link is m.



With the work of the weight treated in the potential energy term, the given sketch becomes the active-force diagram. Let x = distance from y-axis to roller, so stretch of spring is $b-x$

$\delta V_e = \delta\left(\frac{1}{2}k[b-x]^2\right) = k(b-x)\delta(b-x) = -k(b-x)\delta x$

$\delta V_g = \delta(mgx) = mg\,\delta x$ where $V_g = 0$ when $x=0$

$\delta U = -P\delta(2x) = -2P\,\delta x$

$\delta U = \delta V_e + \delta V_g$ for equilibrium

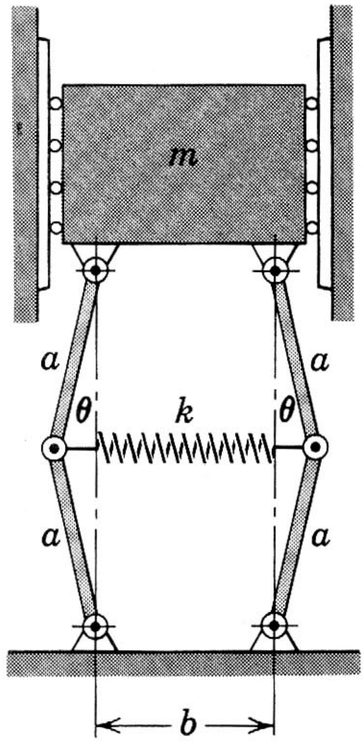
so $-2P\delta x = -k(b-x)\delta x + mg\,\delta x$

$$2P = k(b-x) + mg\ , \quad x = b - \frac{1}{k}(2P+mg)$$

But $y = +\sqrt{b^2-x^2}$. Substitute x, rearrange & get $\boxed{y = \dfrac{2P+mg}{k}\sqrt{\dfrac{2kb}{2P+mg}-1}}$

JLM

The mass m moves in smooth vertical guides and is supported by the four spring-loaded links of negligible mass. The spring of stiffness k is unstretched in the position for which $\theta = 0$. Specify the stability of the system for its equilibrium positions.



Take $V_g = 0$ through base pins

$V_g = 2mga\cos\theta$, $V_e = \frac{1}{2}kx^2 = \frac{1}{2}k(2a\sin\theta)^2$

$V = V_g + V_e = 2mga\cos\theta + 2ka^2\sin^2\theta$

$dV/d\theta = -2mga\sin\theta + 4ka^2\sin\theta\cos\theta$

$d^2V/d\theta^2 = -2mga\cos\theta + 4ka^2(2\cos^2\theta - 1)$

For equil. $dV/d\theta = 0$ gives $\theta = 0$ & $\theta = \cos^{-1}\dfrac{mg}{2ka}$

$\theta = 0$: $d^2V/d\theta^2 = \boxed{(+)\text{ stable if } k > mg/(2a)}$

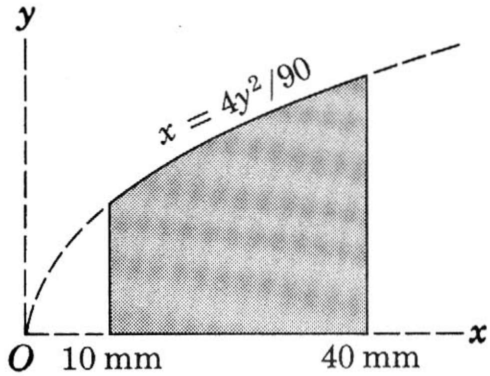
$= \boxed{(-)\text{ unstable if } k < mg/(2a)}$

$\theta = \cos^{-1}\dfrac{mg}{2ka}$: $d^2V/d\theta^2 = \dfrac{4a^2}{k}\left(\left[\dfrac{mg}{2a}\right]^2 - k^2\right)$, $k > mg/(2a)$

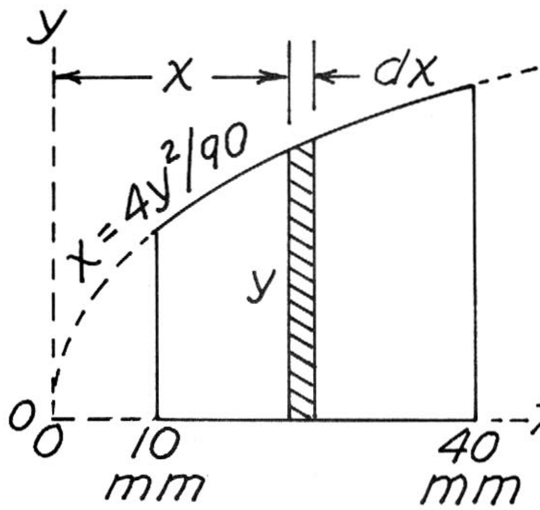
$= \boxed{(-)\text{ unstable}}$

JLM

Calculate the moment of inertia of the shaded area about the x- and y-axes. Also find the radius of gyration k_x.



For rectangular area about its base $I = \frac{1}{3}bh^3$ so



$$dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}(90x/4)^{3/2}dx$$

$$= \frac{9}{8}(10x)^{3/2}dx$$

$$I_x = \frac{9}{8}(10)^{3/2}\int_{10}^{40} x^{3/2}dx$$

$$= \frac{9}{8}(10)^{3/2}\left(\frac{2}{5}\right)x^{5/2}\Big]_{10}^{40} = \frac{9(31)}{20}(10)^4 = \boxed{13.95(10)^4 \text{ mm}^4}$$

$$\text{Area } A = \int y\,dx = \frac{3}{2}\sqrt{10}\int_{10}^{40} x^{1/2}dx = \frac{3}{2}\sqrt{10}\,\frac{2}{3}x^{3/2}\Big]_{10}^{40} = 700 \text{ mm}^2$$

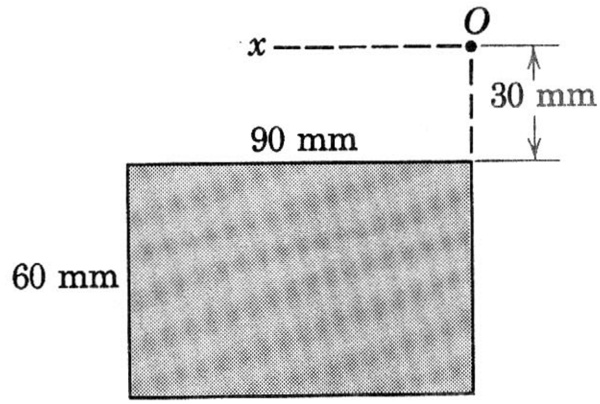
$$k_x = \sqrt{I_x/A} = \sqrt{13.95(10)^4/700} = \boxed{14.12 \text{ mm}}$$

$$I_y = \int x^2 dA = \int x^2 y\,dx = \frac{3}{2}\sqrt{10}\int_{10}^{40} x^{5/2}dx$$

$$= \frac{3}{2}\sqrt{10}\left(\frac{2}{7}\right)x^{7/2}\Big]_{10}^{40} = \boxed{54.43(10)^4 \text{ mm}^4}$$

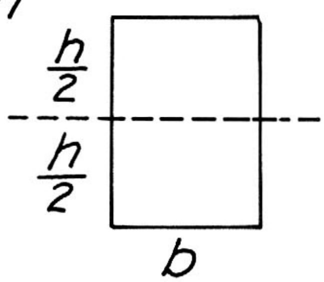
JLM

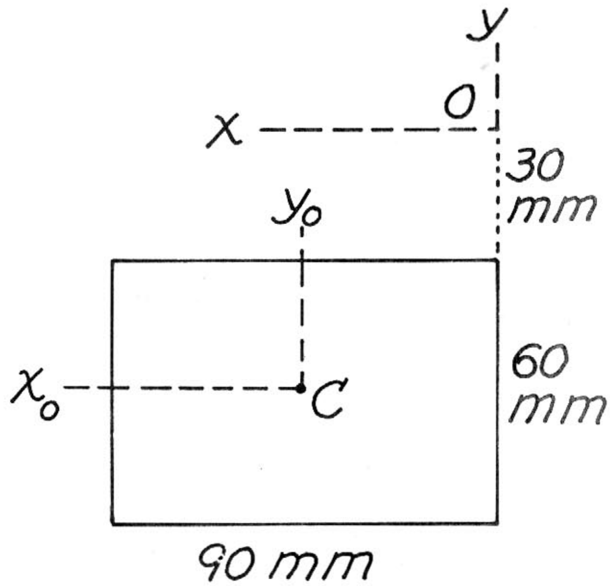
Calculate the moment of inertia of the rectangular area about the x-axis and find the polar moment of inertia about point O.



For rectangular area recall

$\bar{I} = \frac{1}{12} bh^3$



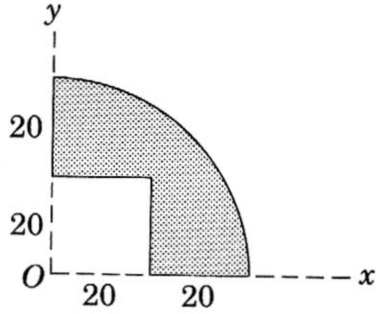


$$I_x = \bar{I}_x + A d_x^2$$
$$= \frac{1}{12}(90)(60)^3 + (90)(60)(30+30)^2 = \boxed{21.06(10)^6 \text{ mm}^4}$$

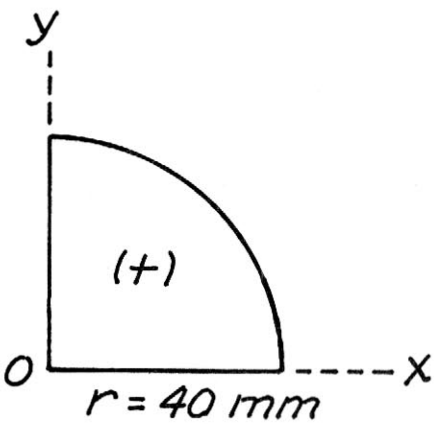
$$I_y = \bar{I}_y + A d_y^2$$
$$= \frac{1}{12}(60)(90)^3 + (90)(60)(45)^2 = 14.58(10)^6 \text{ mm}^4$$

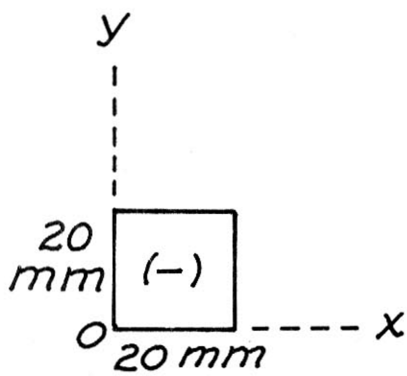
$$I_z = I_x + I_y = 21.06(10)^6 + 14.58(10)^6 = \boxed{35.64(10)^6 \text{ mm}^4}$$

Compute the moment of inertia about the x-axis and the polar radius of gyration about O for the area shown.



Dimensions in millimeters





For quarter circular area $A = \frac{\pi}{4}(40)^2 = 1257\ mm^2$

$$I_x = I_y = \frac{1}{4}\left(\frac{\pi}{4}r^4\right) = \frac{\pi}{16}(40)^4 = 503(10^3)\ mm^4$$

$$I_z = I_x + I_y = 2(503)(10^3) = 1005(10^3)\ mm^4$$

For square area $A = -20(20) = -400\ mm^2$

$$I_x = I_y = -\frac{1}{3}bh^3 = -\frac{1}{3}(20)(20)^3 = -53.3(10^3)\ mm^4$$

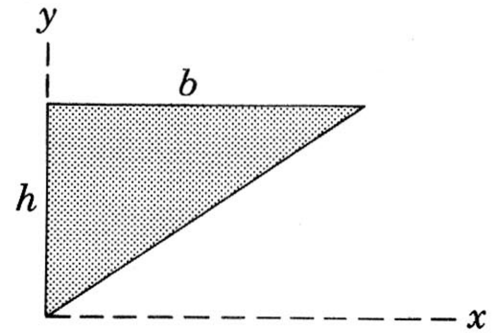
$$I_z = I_x + I_y = -2(53.3)(10^3) = -106.7(10^3)\ mm^4$$

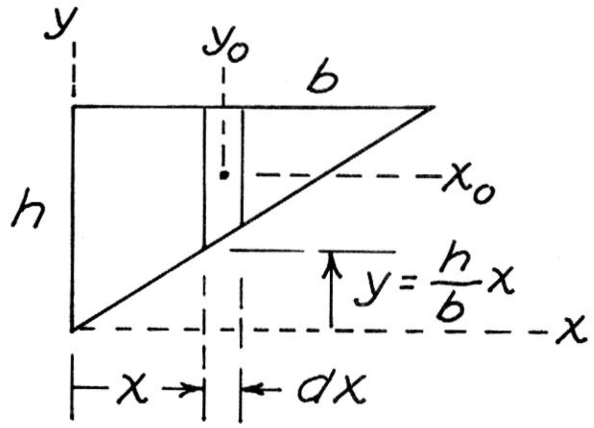
For net area $I_x = (503 - 53.3)(10^3) = \boxed{449(10^3)\ mm^4}$

$$k_z = k_O = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{(1005 - 106.7)(10^3)}{(1.257 - 0.4)(10^3)}} = \boxed{32.4\ mm}$$

JLM

Determine the product of inertia about the x-y axes for the right triangular area.





Introduce centroidal axes x_o-y_o for the differential element of area $dA = (h-y)dx$

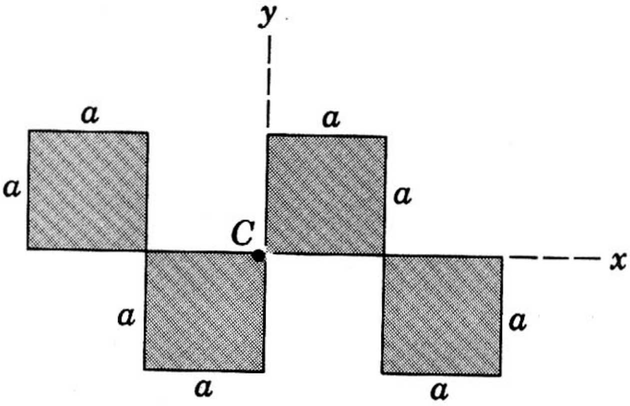
$$dI_{xy} = dI_{x_o y_o} + d_x d_y \, dA$$

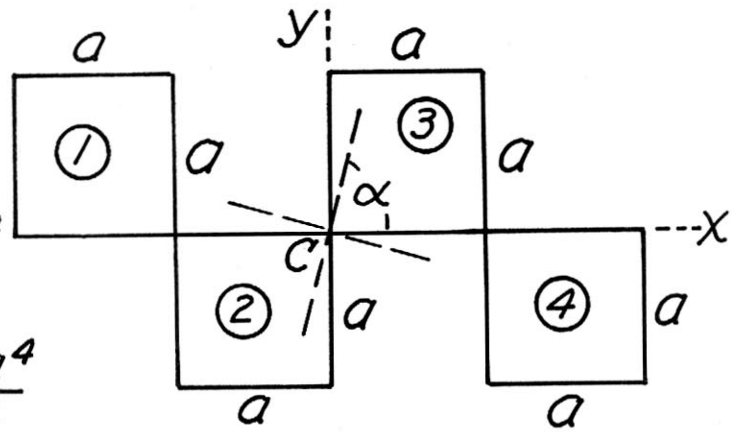
$$= 0 + x\frac{h+y}{2}(h-y)dx = \frac{h^2}{2}\left(x - \frac{x^3}{b^2}\right)dx$$

$$I_{xy} = \frac{h^2}{2}\int_0^b \left(x - \frac{x^3}{b^2}\right)dx = \boxed{\frac{1}{8}b^2h^2}$$

JLM

For the composite square areas find I_{max} & I_{min} for axes through centroid C. Find the angle α from the x-axis to the axis of I_{max}.





① ② ③ ④ $I_x = \frac{a^4}{3}$

② ③ $I_y = \frac{a^4}{3}$

① ④ $I_y = \frac{a^4}{12} + \left(\frac{3a}{2}\right)^2 a^2 = \frac{7a^4}{3}$

② ③ $I_{xy} = 0 + a^2\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{a^4}{4}$

① ④ $I_{xy} = 0 + a^2\left(\mp\frac{3a}{2}\right)\left(\pm\frac{a}{2}\right) = -\frac{3a^4}{4}$

Totals $I_x = \frac{4a^4}{3}$, $I_y = \frac{16a^4}{3}$, $I_{xy} = -a^4$

Eq. A/11 $$I_{\binom{max}{min}} = \frac{I_x + I_y}{2} \pm \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= a^4\left(\frac{10}{3} \pm \frac{1}{2}\sqrt{20}\right) = \boxed{\left(\frac{10}{3} \pm \sqrt{5}\right)a^4}$$

Eq. A/10 $$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{-2a^4}{\left(\frac{16}{3} - \frac{4}{3}\right)a^4} = -\frac{1}{2}$$

$2\alpha = 153.4°$ or $-26.6°$, $\boxed{\alpha = 76.7°}$ or $-13.3°$

JLM