MAT 1110 Quiz 1

SOLUTIONS - 2022

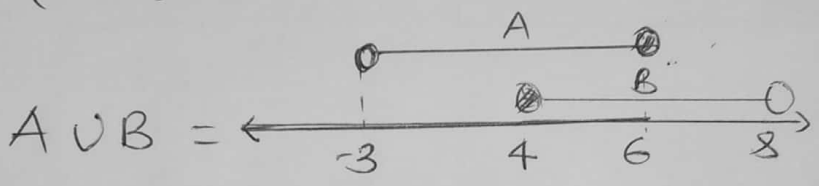
Q1 (a) $A = (-3, 6]$, $B = [4, 8)$, $C = [2, 10)$

Find $(A \cup B) - C$

Solution

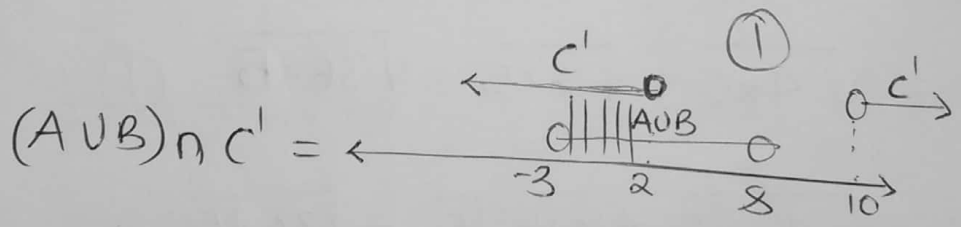
(i) $(A \cup B) - C$

$(A \cup B) \cap C'$



$A \cup B = (-3, 8)$ (1)

$C' = (-\infty, 2) \cup (10, \infty)$



(1)

$(A \cup B) \cap C' = (-3, 2)$ (1)

Hence $(A \cup B) - C = (-3, 2)$

(ii) $(A \cap B) \cup (C - (A \cup B))$

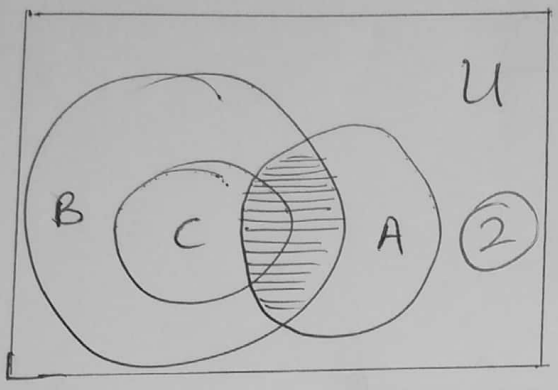
Solution

$(A \cap B) \cup (C \cap (A \cup B)')$

$(A \cap B) \cup \emptyset$ (1)

$= A \cap B$

Hence $(A \cap B) \cup (C - (A \cup B)) = A \cap B$ ①



Q2. a)i $\sqrt{20} + \sqrt{80} - \sqrt{180}$

Solution

$= \sqrt{4 \times 5} + \sqrt{16 \times 5} - \sqrt{36 \times 5}$ ①

$= \sqrt{4}\sqrt{5} + \sqrt{16}\sqrt{5} - \sqrt{36}\sqrt{5}$

$= 2\sqrt{5} + 4\sqrt{5} - 6\sqrt{5}$ ①

$= 6\sqrt{5} - 6\sqrt{5}$

$= \underline{0}$ ①

2 a ii) $\dfrac{1+\sqrt{2}}{-2+\sqrt{5}} - (\sqrt{5}+\sqrt{10}) = a + a\sqrt{a}$

Solution

$= \dfrac{(1+\sqrt{2})}{(-2+\sqrt{5})} \cdot \dfrac{(-2-\sqrt{5})}{(-2-\sqrt{5})} - (\sqrt{5}+\sqrt{10})$ ①

$= \dfrac{-2-\sqrt{5}-2\sqrt{2}-\sqrt{10}}{4+2\sqrt{5}-2\sqrt{5}-5} - (\sqrt{5}+\sqrt{10})$

$= \dfrac{-2-\sqrt{5}-2\sqrt{2}-\sqrt{10}}{4-5} - (\sqrt{5}+\sqrt{10})$ ①

$= -(-2-\sqrt{5}-2\sqrt{2}-\sqrt{10}) - (\sqrt{5}+\sqrt{10})$

$= 2+\sqrt{5}+2\sqrt{2}+\sqrt{10}-\sqrt{5}-\sqrt{10}$

$= 2+2\sqrt{2}$ ①

Hence

$2+2\sqrt{2} = a + a\sqrt{a}$

$\therefore \underline{a = 2}$ ①

Q2(b)

$$\frac{2+8i}{1-i} = x + xi + y + yi^3$$

$$= \frac{(2+8i)}{(1-i)}\frac{(1+i)}{(1+i)} = x + xi + y + yi^3 \quad (1)$$

$$\Rightarrow \frac{2+2i+8i+8i^2}{1+i-i+i^2} = x + xi + y + yi^3$$

but $i^2 = -1$, then $i^3 = i^2 \cdot i = -i$

$$\frac{2+10i-8}{1+1} = x + y + xi + y(-i) \quad (1)$$

$$\frac{-6+10i}{2} = x + y + xi - yi \quad (1)$$

$$-3 + 5i = x + y + (x-y)i \quad (1)$$

Thus $-3 = x + y$

$+\ 5 = x - y$

$$2 = 2x \quad (1)$$

$$x = 1 \Rightarrow y = -4$$

Hence $x = 1$, $y = -4$ (1)