

The University of Zambia
Department of Mathematics and Statistics
Academic Year 2019/20
Mat 3110 Test on Double and Triple Integrals

Duration : 2 hours

Instructions: Answer all the questions.

1. (a) Determine the volume that lies under $f(x, y) = 10 + xy \sin(x^2 - y^2)$ and above the rectangle given by $[-3, 0] \times [1, 3]$ in xy -plane.
- (b) Use a double integral to determine the volume of the region bounded by $z = 3 - 2y$, the surface $y = 1 - x^2$ and the planes $y = 0$ and $z = 0$.
- (c) Evaluate the following double integral.

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2 + y^2} \, dy dx$$

2. (a) Evaluate

$$\iiint_E 6z^2 \, dV$$

where E is the region behind $x + 6y + 2z = 8$ that is in front of the region in the xy -plane bounded by $z = 2y$ and $z = \sqrt{4y}$.

- (b) Evaluate the following triple integral

$$\int_{-4}^4 \int_0^{\sqrt{16-y^2}} \int_0^{6+x} 6yx^2 \, dz dx dy$$

- (c) Evaluate

$$\iiint_E z^2 \, dV$$

where E is the region between spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 25$ and inside $z = -\sqrt{\frac{x^2}{3} + \frac{y^2}{3}}$.

The University of Zambia
 Department of Mathematics and Statistics
 Academic Year 2018/19
 Mat 3110 Test 2

Duration : 2 hours

September 2, 2019.

Instructions: There are eight questions in this test. Each question carries 10 marks. Answer all the questions and show all the necessary working to earn full marks.

1. Find the area of the region bounded above by $y = e^x$, below by $y = 1$, left by $x = 0$ and right by $x = 1$. (10)

2. Evaluate the double integral (10)

$$\iint_D e^{-x-2y} dA$$

where D is the region in the 1st quadrant in which $y \leq x$.

3. Evaluate the following triple integral. (10)

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^6 \frac{dz dy dx}{\sqrt{x^2+y^2}}$$

4. Evaluate the triple integral (10)

$$\int_0^2 \int_0^4 \int_z^2 yze^{x^3} dx dy dz$$

5. Find the volume of the solid that is between the $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 25$, and inside $z = -\sqrt{\frac{1}{3}x^2 + \frac{1}{3}y^2}$. (10)

6. Evaluate (10)

$$\int_C z^3(x+7) - 2y ds$$

where C is a circle in space of radius 1 centered at $(-3, 0, 0)$ and lying on the plane $x = -3$.

7. Let $F(x, y, z) = \frac{x}{\sqrt{x^2+z^2}} \mathbf{i} + (2yz - 6y) \mathbf{j} + \left(y^2 + \frac{z}{\sqrt{x^2+z^2}}\right) \mathbf{k}$. (10)

(a) Prove that F is conservative.

(b) Evaluate

$$\int_C F \cdot dr$$

where C the line segment from $(1, 0, -1)$ to $(2, -4, 3)$.

$\cos \theta \cos 3 + \sin \theta \sin 3$

$\frac{28}{17}$

$\frac{28-11}{4}$

$\frac{28-11}{4}$

The University of Zambia

Department of Mathematics and Statistics

Academic Year 2018/19

Mat 3110 Test 1

$\frac{\sqrt{4^2 - 4^2}}{\sqrt{2}}$

Duration : 2 hours

12th April, 2019.

Instructions:

- This test contains 2 pages and 6 questions Total marks is 60.
- Answer all the questions.

$\frac{1}{17} - \frac{17}{16}$

1. Solve the initial value problem below using Laplace transforms.

$y'' + 4y' = \cos(t - 3) + 4t, \quad y(3) = 0, \quad y'(3) = 7.$

2. Determine the Laplace transforms of the functions below.

(a) $f(t) = t^2 \cosh t$

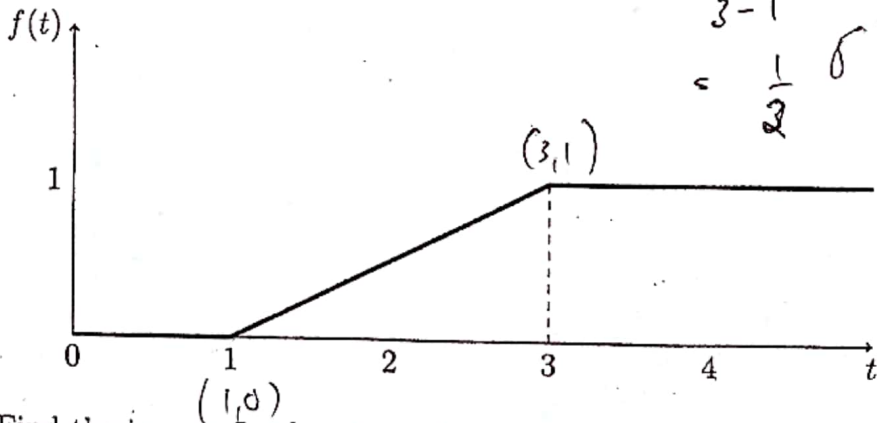
(b) The function with graph shown below.

$m = \frac{1-0}{3-1} = \frac{1}{2}$

$s^3(s+4)$

$\frac{4s^2 + 12s + 7s^2}{s^2(s^2+4s)}$

$\frac{4s^2 + 12s + 7s^2}{(s^2+4s)s^2}$



3. (a) Find the inverse Laplace transform of function

$F(s) = \frac{8}{3s^2 + 12} + \frac{3}{s^2 - 49}$

$s(s+4)$

(b) Find and sketch the inverse Laplace transform of

$f(\theta) = \frac{1}{2}\theta - \frac{1}{2}$

$X = \frac{3}{2} - \frac{1}{2}\theta \quad Y = \frac{1}{2}\theta + C$

$F(s) = \frac{1}{e^{2s}s^2}$

$\sigma = \frac{1}{2}$

$L(\theta-1) + X = 1$

$\frac{11}{2}\theta - \frac{3}{2} - \frac{28}{17}$

f

The University of Zambia
 Department of Mathematics and Statistics
 Academic Year 2018/19
 Mat 3110 Test 2

Duration : 2 hours

September 2, 2019.

Instructions: There are eight questions in this test. Each question carries 10 marks. Answer all the questions and show all the necessary working to earn full marks.

1. Find the area of the region bounded above by $y = e^x$, below by $y = 1$, left by $x = 0$ and right by $x = 1$. (10)

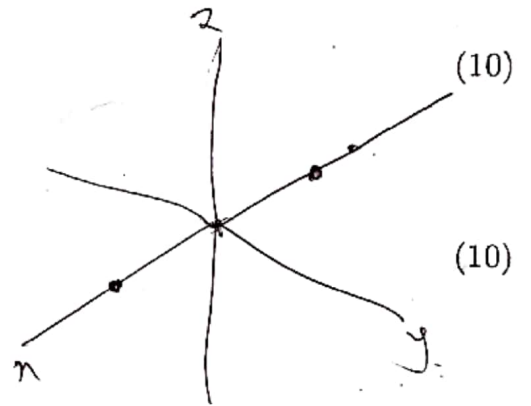
2. Evaluate the double integral

$$\iint_D e^{-x-2y} dA$$

where D is the region in the 1st quadrant in which $y \leq x$.

3. Evaluate the following triple integral.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^6 \frac{dz dy dx}{\sqrt{x^2+y^2}}$$



4. Evaluate the triple integral

$$\int_0^2 \int_0^4 \int_z^2 yze^{x^3} dx dy dz$$

$$x^2 + y^2 = r^2$$

5. Find the volume of the solid that is between the $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 25$, and inside $z = -\sqrt{\frac{1}{3}x^2 + \frac{1}{3}y^2}$. (10)

6. Evaluate

$$\int_C z^3(x+7) - 2y ds$$

$$y^2 + z^2 = 1$$

where C is a circle in space of radius 1 centered at $(-3, 0, 0)$ and lying on the plane $x = -3$.

7. Let $F(x, y, z) = \frac{x}{\sqrt{x^2+z^2}} \mathbf{i} + (2yz - 6y) \mathbf{j} + \left(y^2 + \frac{z}{\sqrt{x^2+z^2}}\right) \mathbf{k}$.

$$\cos t + \sin t$$

(a) Prove that F is conservative.

(b) Evaluate

$$\int_C F \cdot dr$$

$$x = -3i + \cos t j + \sin t k$$

where C the line segment from $(1, 0, -1)$ to $(2, -4, 3)$.

The University of Zambia
School of Natural Sciences
Department of Mathematics and Statistics

2019/20 Academic Year Examinations

MAT 3110 Engineering Mathematics II

November 30, 2020.

Duration: Three hours

Instructions:

- Do not open this booklet until you are told to do so.
 - This paper consists of six questions, each carry 20 marks. Attempt any FIVE questions.
 - Show all the essential working to earn full marks.
 - Non programmable calculators are allowed.
-

1. (a) Find a power series solution in powers of x of the differential equation (9)

$$(1 - x^2)y'' - 2xy' + 2y = 0.$$

- (b) Solve the following initial value problem (4)

$$x^2y'' - 5xy' + 8y = 0, \quad y(1) = 5, \quad y'(1) = 18.$$

- (c) Find the general solution to the following system of first order differential value problem. (7)

$$\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -5 \end{pmatrix} \mathbf{x}.$$

2. (a) Solve the initial value problem below using Laplace transforms. (12)

$$y'' + 9y = \begin{cases} 8 \sin t, & 0 \leq t < \pi \\ 0, & t \geq \pi. \end{cases} \quad y(0) = 0, \quad y'(0) = 4.$$

- (b) Determine the Laplace transform of (4)

$$f(t) = \sinh t \cos t.$$

- (c) Determine the inverse Laplace transform of (4)

$$F(s) = \frac{6}{(s+1)^3}.$$

3. (a) Evaluate the following double integrals. (5)

i.

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$$

ii.

$$\int_{-3}^0 \int_1^3 10 + xy \sin(x^2 - y^2) \, dy \, dx$$

- (b) Use a double integral to determine the volume of the region bounded by $z = 3 - 2y$, the surface $y = 1 - x^2$ and the planes $y = 0$ and $z = 0$. (5)

x, y

$z = 3 - 2y(1 - x^2)$

Page 2

- (c) Determine the volume that lies under $f(x, y) = 9x^2 + 4xy + 4$ and above the rectangle $R = \{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq 2\}$ in the xy -plane.

Surf = 2 sin cos φ

4. (a) Evaluate the following triple integrals.

i.

$$\int_{-4}^4 \int_0^{\sqrt{16-y^2}} \int_0^{6+x} 6yx^2 \, dz \, dx \, dy$$

ii.

$$\int_0^2 \int_0^4 \int_z^2 yze^{x^3} \, dx \, dy \, dz$$

- (b) Find the volume of the solid that is between $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 25$, and inside $z = -\sqrt{\frac{1}{3}x^2 + \frac{1}{3}y^2}$.

- (c) Convert the triple integral

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz \, dz \, dy \, dx$$

into a triple integral in cylindrical coordinates and evaluate.

5. (a) Evaluate

$$\int_C (3x^2 - 2y) \, ds$$

where C is the line segment from $(3, 6)$ to $(1, -1)$.

- (b) Let $\vec{F}(x, y) = (2xy - 4 - \frac{1}{2} \sin(\frac{x}{2}) \sin(\frac{y}{2})) \mathbf{i} + (x^2 + \frac{1}{2} \cos(\frac{x}{2}) \cos(\frac{y}{2})) \mathbf{j}$.

i. Prove that \vec{F} is conservative.

ii. Evaluate

$$\int_C \vec{F} \cdot d\mathbf{r}$$

where C the portion of the circle centered at the origin, having radius 2 and is in the 1st quadrant and oriented counter clockwise.

- (c) Use Green's theorem to evaluate the line integral

$$\int_C \vec{F} \cdot d\mathbf{r}$$

2 P sin φ

2 P sin φ

where $\vec{F}(x, y) = xy^2\mathbf{i} + (1 - xy^3)\mathbf{j}$ and C is the parallelogram with vertices $(-1, 2)$, $(-1, -1)$, $(1, 1)$ and $(1, 4)$. The orientation of C is counterclockwise.

6. (a) Evaluate

$$\int_S 6xy \, dS$$

(6)

where S is the portion of the plane $x + y + z = 1$ that lies in the 1st octant and is in front of the yz -plane.

(b) Use Stoke's theorem to evaluate the integral

$$\int_S \text{curl } \vec{F} \cdot d\mathbf{S}$$

(7)

where $\vec{F}(x, y, z) = 2y\mathbf{i} + 3x\mathbf{j} + (z - x)\mathbf{k}$ and S is the portion of $y = 11 - 3x^2 - 3z^2$ in front of $y = 5$ with orientation in the positive y -axis direction.

(c) Use the divergence theorem to evaluate

$$\int_S \vec{F} \cdot d\mathbf{S}$$

(7)

where $\vec{F}(x, y, z) = (4x - z^2)\mathbf{i} + (x + 3z)\mathbf{j} + (6 - z)\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 36$ and the plane $z = -2$ and $z = 3$. Note that all the three surfaces of the solid are included in S .

————— *End of Examination* —————

The University of Zambia
 Department of Mathematics and Statistics
 Academic Year 2018/19
 Mat 3110 Test 3

Duration : 1 hour 30 minutes October 28, 2019.

Instructions: There are three questions in this test. Answer all the questions and show all the necessary working to earn full marks.

1. During an epidemic of a certain disease a doctor is consulted by 110 people suffering from symptoms commonly associated with the disease. Of the 110 people, 45 are female of whom 20 actually have the disease and 25 do not. Fifteen males have the disease and the rest do not.
- (a) A person is selected at random, The event that this person is female is denoted by A and the event that this person is suffering from the disease is denoted by B . Evaluate
- (i) $P(A)$ (ii) $P(A \cup B)$, (iii) $P(A \cap B)$, (iv) $P(A | B)$.
- (b) If three different people are selected at random without replacement, what is the probability of
- (i) all three having the disease, (ii) exactly one of the three having the disease, (iii) one of the three being a female with the disease, one a male with the disease and one a female without the disease?
- (c) Of people with the disease 96% react positively to a test for diagnosing the disease as do 8% of people without the disease. What is the probability of a person selected at random
- (i) reacting positively, (ii) having the disease given that he or she reacted positively?

2. Using Stoke's theorem to evaluate the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $F(x, y, z) = (zx^3 - 2z) \mathbf{i} + xz \mathbf{j} + xy \mathbf{k}$ and C is the triangle with vertices $(0, 0, 4)$, $(0, 2, 0)$ and $(2, 0, 0)$. Direction is $(2, 0, 0)$ to $(0, 0, 4)$, then $(0, 0, 4)$ to $(0, 2, 0)$ and then $(0, 2, 0)$ to $(2, 0, 0)$.

3. Evaluate the surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

$$\frac{\mathbf{r}_x \times \mathbf{r}_y}{\|\mathbf{r}_x \times \mathbf{r}_y\|} \times dA$$

$$\int \mathbf{F} \cdot \mathbf{N} \, dS$$

The University of Zambia
Department of Mathematics and Statistics
Academic Year 2018/19
Mat 3110 Test 3

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where $F(x, y, z) = (zx^3 - 2z) \mathbf{i} + xz \mathbf{j} + xy \mathbf{k}$ and C is the triangle with vertices $(0, 0, 4)$, $(0, 2, 0)$ and $(2, 0, 0)$. Direction is $(2, 0, 0)$ to $(0, 0, 4)$, then $(0, 0, 4)$ to $(0, 2, 0)$ and then $(0, 2, 0)$ to $(2, 0, 0)$.

3. Evaluate the surface integral

$$\int_S \mathbf{F} \cdot d\mathbf{S}$$

where $F(x, y, z) = -xy \mathbf{i} + (z - 1) \mathbf{j} + z^3 \mathbf{k}$ and S the surface of the solid bounded by $y = 4x^2 + 4z^2 - 1$ and the plane $y = 7$. Note that both of the surfaces of this solid are included in S .

End of Test!

The University of Zambia
Department of Mathematics and Statistics
Academic Year 2018/19
Mat 3110 Test 3

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(a) A person is selected at random, The event that this person is female is denoted by A and the event that this person is suffering from the disease is denoted by B . Evaluate

(i) $P(A)$ (ii) $P(A \cup B)$, (iii) $P(A \cap B)$, (iv) $P(A | B)$.

(b) If three different people are selected at random without replacement, what is the probability of

(i) all three having the disease, (ii) exactly one of the three having the disease,

(iii) one of the three being a female with the disease, one a male with the disease and one a female without the disease?

(c) Of people with the disease 96% react positively to a test for diagnosing the disease as do 8% of people without the disease. What is the probability of a person selected at random

(i) reacting positively, (ii) having the disease given that he or she reacted positively?

2. Using Stoke's theorem to evaluate the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $F(x, y, z) = (zx^3 - 2z) \mathbf{i} + xz \mathbf{j} + xy \mathbf{k}$ and C is the triangle with vertices $(0, 0, 4)$, $(0, 2, 0)$ and $(2, 0, 0)$. Direction is $(2, 0, 0)$ to $(0, 0, 4)$, then $(0, 0, 4)$ to $(0, 2, 0)$ and then $(0, 2, 0)$ to $(2, 0, 0)$.

3. Evaluate the surface integral

$$\int_S \mathbf{F} \cdot d\mathbf{S}$$

The University of Zambia
 Department of Mathematics and Statistics
 Academic Year 2018/19
 Mat 3110 Quiz 3

Duration : 1 hour

August 13, 2019.

1. Evaluate the double integral

$$\iint_D e^{-x-2y} dA$$

where D is the region in the 1st quadrant in which $x \leq y$.

2. Evaluate the following triple integral.

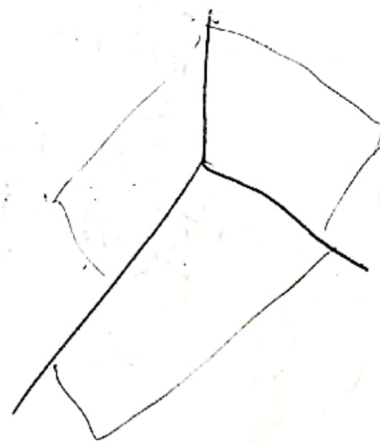
$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{1+x^2+y^2+z^2}$$

3. Evaluate the triple integral

$$\iiint_E (x^2 + y^2)^{\frac{3}{2}} dV$$

where E is the solid bounded above by $z = \frac{1}{2}(x^2 + y^2)$, below by the xy -plane, and laterally (in the sides) by $x^2 + y^2 = 25$.

4. Find the volume of the solid enclosed by the surfaces $z = 0$, $x = 0$, $y = 0$, $z = \sqrt{xy + z^2}$ and $z = \sqrt{1 - x^2 - y^2}$.



$\sqrt{1-x^2-y^2}$
 $z = \sqrt{xy + z^2}$
 z

3

2c
(no)

The University of Zambia
Department of Mathematics and Statistics
Academic Year 2018/19
Mat 3110 Quiz 2

Duration : 1 hour

June 7, 2019.

1. Find the area of the region inside $r^2 = -2 + 4r \sin \theta$ and outside $r = \sqrt{2}$. (10)
(Note that these are curves in plane written using polar coordinates).

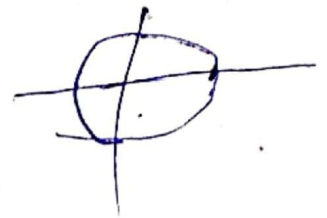
2. Evaluate the following double integral. (10)

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{1+y^3} dy dx$$

3. Use a double integral to find the volume of the solid that is bounded by $z = 3 - 2y$, $y = 1 - x^2$, $y = 0$ and the plane $z = 0$. (10)

4. Use a double integral to find the volume of the solid that is bounded by $z = 12 - 3x^2 - 3y^2$ and $z = x^2 + y^2 - 8$. (10)

The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II



Test 2

17 September, 2018.

Instructions

1. Answer all the questions.
2. Any sort of exam malpractice will be severely punished.
3. Total marks is 80.
4. Duration is 2 hours.

sin 3π = -1

1. (a) Use a double integral to determine the area of the region that is inside $r = 4 \sin \theta$ and outside $r = 2 \sin \theta$. (10)
- (b) Evaluate the following integral: (10)

✓

$$\int_{-\frac{1}{\sqrt{2}}}^0 \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} 18y \, dz \, dy \, dx$$

2. Let

$$F(x, y, z) = (2z^4 - 2y - y^3) \mathbf{i} + (z - 2x - 3xy^2) \mathbf{j} + (6 + y + 8xz^3) \mathbf{k}$$

- (a) Given that the above vector field is conservative, find its potential function. (10)
- (b) Evaluate (10)

$$\int_C F \cdot dr$$

where C is the helix given by

$$r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + \frac{2t}{5\pi} \mathbf{k}, \quad 0 \leq t \leq \frac{5\pi}{2}$$

3. (a) Let $F(x, y) = yx^2 \mathbf{i} - x^2 \mathbf{j}$. Evaluate

$$\int_C F \cdot dr$$

where C is the line segment from $(0, -5)$ to $(5, 0)$ followed by the path along $x^2 + y^2 = 25$ from $(0, 5)$ to $(0, -5)$.

- (b) Evaluate

$$\iint_D 10x^2y^3 - 6 \, dA$$

where D is the region bounded by $x = -2y^2$ and $x = y^3$.

1/2

2nd of 3/2

0

-2

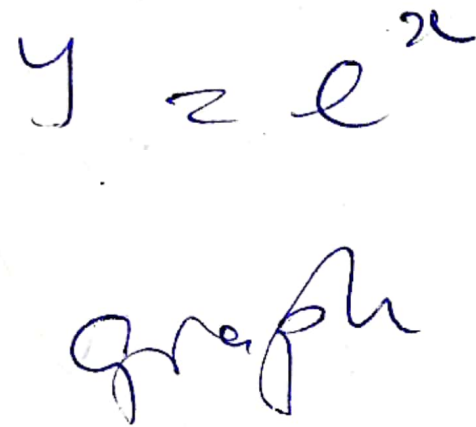
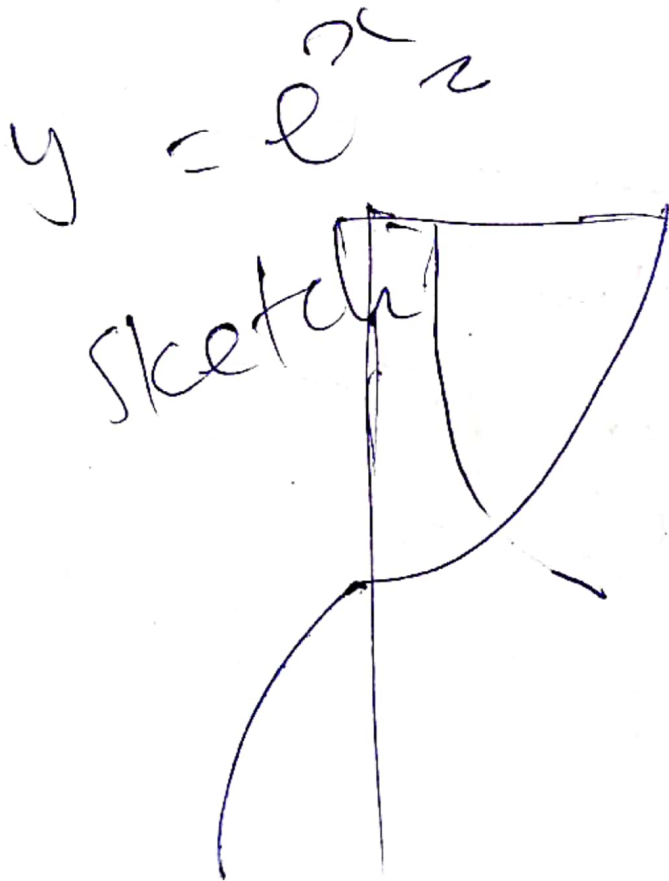
2y^2

y^3

622.1538462

1.25

4. (a) Calculate the volume of the solid bounded by $z = 2 - x^2 - y^2$ and the $z = \sqrt{x^2 + y^2}$.
- (b) Use a double integral to find the area of the region bounded above by $y = e^x$, below by $y = 1$, left by $x = 0$ and to the right by $x = 1$.



The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Test 2

17 September, 2018.

Instructions

1. Answer all the questions.
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3. Total marks is 80.
4. Duration is 2 hours.

1. (a) Use a double integral to determine the area of the region that is inside $r = 4 \sin \theta$ and outside $r = 2 \sin \theta$. (10)

- (b) Evaluate the following integral: (10)

$$\int_{-\frac{1}{\sqrt{2}}}^0 \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} 18y \, dz \, dy \, dx$$

2. Let

$$F(x, y, z) = (2z^4 - 2y - y^3) \mathbf{i} + (z - 2x - 3xy^2) \mathbf{j} + (6 + y + 8xz^3) \mathbf{k}$$

- (a) Given that the above vector field is conservative, find its potential function. (10)

- (b) Evaluate (10)

$$\int_C F \cdot dr$$

where C is the helix given by

$$r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + \frac{2t}{5\pi} \mathbf{k}, \quad 0 \leq t \leq \frac{5\pi}{2}$$

3. (a) Let $F(x, y) = yx^2 \mathbf{i} - x^2 \mathbf{j}$. Evaluate (10)

$$\int_C F \cdot dr$$

where C is the line segment from $(0, -5)$ to $(0, 5)$ followed by the path along $x^2 + y^2 = 25$ from $(0, 5)$ to $(0, -5)$.

- (b) Evaluate (10)

$$\iint_D (10x^2y^3 - 6) \, dA$$

where D is the region bounded by $x = -2y^2$ and $x = y^3$.

21 920

$\frac{5\pi}{2} \times \frac{1+2t}{5\pi}$
 $2(1+2t)$

The University of Zambia
 Department of Mathematics and Statistics
 Mat 3110 Test 3

Wallah Noh

Duration : 2 hours

November 11, 2018.

Instructions:

- This test contains 2 pages and 8 questions Total marks is 80.
- Answer all the questions.
- Any sort of malpractice will be severely punishment.

1. How many 11 letter words can you make from the letters of the word PROBABILITY that are such that the two I's are separated?
2. If X is a random variable with a distribution of $N(\mu, \sigma^2)$, find:
 $P(\mu - 3\sigma \leq X \leq \mu + 3\sigma)$
3. In an experiment two bags A and B, containing red and green marbles are used. Bag A contains four red marbles and one green marble and bag B contains two red marbles and seven green marbles. An unbiased coin is tossed. If a head turns up, a marble is drawn at random from bag A while if a tail turns up, a marble is drawn at random from bag B. Calculate the probability that a red marble is drawn in a single trial. Given that a red marble is selected, calculate the probability that when the coin was tossed a head was obtained.
4. Before starting to play the game 'Snakes and Ladders', a player throws an ordinary unbiased die until a six is obtained. The number of throws before a player starts is the random variable Y , where Y takes the values 1, 2, 3,
 Find $P(Y = 2)$.

5. The probabilities of events A and B are $P(A)$ and $P(B)$ respectively.

$P(A) = \frac{5}{12}, P(A \cap B) = \frac{1}{6}, P(A \cup B) = q.$

Find, in terms of q ,

(a) $P(B)$,

$$\frac{q - \frac{3}{12}}{1} = \frac{12q - 3}{12}$$

How many
 $\log E = 10^4$
 $E = 10^4$

$$\frac{49 - 1}{4}$$

$4 \times 3 (49 - 7)$

(b) $P(A|B)$.

Given that A and B are independent events,

(c) find the value of q .

6. The number of customers entering a certain branch of a bank on a Monday lunchtime may be modelled by a Poisson distribution with mean 2.4 per minute.

Find the probability that, during a particular minute, four or more customers enter the branch.

7. In practising the high jump a certain athlete has five attempts at a particular height. The probability that she succeeds at any one attempt is p . Find an expression, in terms of p , for the probability that she succeeds

- (a) exactly four times,
- (b) exactly two times.

The probability that she succeeds exactly four times is twice the probability that she succeeds exactly two times. Find the value of p .

8. An IQ test shows that the scores follow a distribution of $N(65, 16)$. It is desirable to arrange the participants into three groups (low intelligence, average intelligence and high intelligence) so that 20% of the participants are filled under the first group, 65% in the second and 15% in the third. What are the scores that differentiate each intelligence group from one another?

$P(A|B)$.

Given that A and B are independent events

Find the value of q .

6. The number of q entering a certain branch of a bank during lunchtime may be modelled by a poisson distribution. Find the probability. of the value.

$N(65, 16)$.

University of Zambia
School of Natural Sciences
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Test 1

March 29, 2017.

- Instructions :** 1. Answer all the questions.
2. Duration is 2 hours.

1. Let $f(x) = x$ for $-2 < x < 2$ and $f(x+4) = f(x)$. (10)

(a) Find the Fourier series of $f(x)$.

(b) Deduce from the Fourier series of $f(x)$ that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

2. (a) Find the general solution the following Euler equation. (10)

$$x^2 y'' + xy' - 4y = 0.$$

(b) Show that

$$\mathcal{L}\{\sinh t \cos t\} = \frac{s^2 - 2}{s^4 + 4}$$

3. Find and sketch the inverse Laplace transform of (10)

$$\frac{e^{-2s} - e^{-4s}}{s^2}$$

4. (a) Find the power series solution of the ordinary differential equation (10)

$$y'' - y = x^2$$

(b) Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}.$$

5. Let (10)

$$f(t) = \begin{cases} t, & 0 \leq t < 1 \\ t-1, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases}$$

(a) Write $f(t)$ in terms of unit step functions.

(b) What is the Laplace transform of $f(t)$?

(c) Solve the initial value problem;

$$y' + 2y = f(t), \quad y(0) = 0.$$

The End!

The University of Zambia
Department of Mathematics and Statistics
Academic Year 2018/19
Mat 3110 Test 1

Duration : 2 hours

12th April, 2019.

Instructions:

- This test contains 2 pages and 6 questions Total marks is 60.
- Answer all the questions.

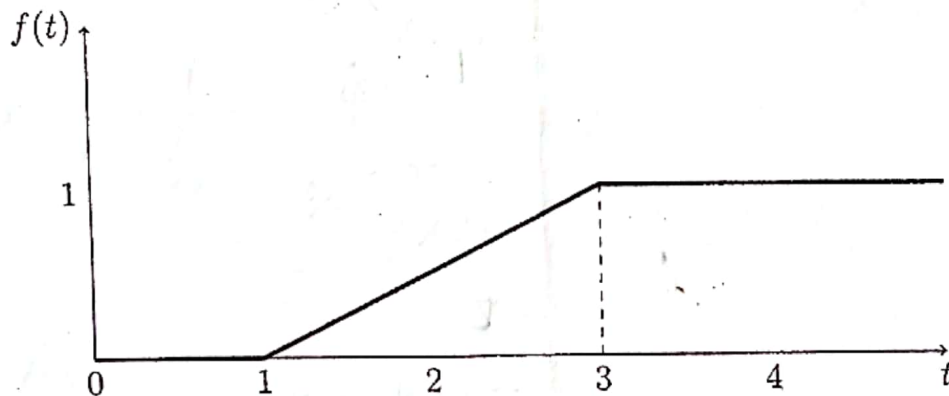
1. Solve the initial value problem below using Laplace transforms.

$$y'' + 4y' = \cos(t - 3) + 4t, \quad y(3) = 0, \quad y'(3) = 7.$$

2. Determine the Laplace transforms of the functions below.

(a) $f(t) = t^2 \cosh t$

(b) The function with graph shown below.



3. (a) Find the inverse Laplace transform of function

$$F(s) = \frac{8}{3s^2 + 12} + \frac{3}{s^2 - 49}$$

(b) Find and sketch the inverse Laplace transform of

$$F(s) = \frac{1}{e^{2s}s^2}$$

4. (a) Find the eigenvalue and corresponding eigenvectors of the reflection matrix
Solve the following initial value problem

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- (b) Find the general solution to the following differential equation.

$$x^2 y'' - 7xy' + 16y = 0.$$

5. Determine the constants α , β , y_0 and y'_0 so that $Y(s) = \frac{s}{(s+1)^2}$ is the Laplace transform of the solution to the initial value problem

$$y'' + \alpha y' + \beta y = 0, \quad y(0) = y_0, \quad y'(0) = y'_0.$$

6. Find the solution of the following initial value problem.

$$(x+2)y'' + xy' - y = 0, \quad y(0) = 1, \quad y'(0) = -1.$$

END OF TEST!

$$x = 1$$

$$1 + 3 - 1$$

$$x = 2$$

$$-2$$

$$-8 + 6 - 1$$

$$=$$

$$-27 + 27 - 1$$

(a) $\frac{e^{-3s}}{(s-1)^3}$
 (d) $\frac{e^{-3s}}{s^4}$

(b) $\frac{6(1-e^{-\pi s})}{(s^2+9)}$
 (e) $\frac{2(e^{-s}-e^{-3s})}{(s^2-4)}$

(c) $\frac{4(e^{-2s}-2e^{-5s})}{s}$
 (f) $\frac{(1+e^{-2\pi(s+1)})(s+1)}{((s+1)^2+1)}$

7. Solve the following initial value problems with a discontinuous force.

(a) $y'' + 9y = \begin{cases} 8 \sin t, & : 0 < t < \pi \\ 0, & : t \geq \pi \end{cases} \quad y(0) = 0, y'(0) = 4$

(b) $y'' + 3y' + 2y = \begin{cases} 4t, & : 0 < t < 1 \\ 8, & : t \geq 1 \end{cases} \quad y(0) = 0, y'(0) = 0$

(c) $y'' + y' - 2y = \begin{cases} 3 \sin t - \cos t, & : 0 < t < 2\pi \\ 3 \sin 2t - \cos 2t, & : t \geq 2\pi \end{cases} \quad y(0) = 1, y'(0) = 0$

(d) $y'' + 3y' + 2y = \begin{cases} 1, & : 0 < t < 1 \\ 0, & : t \geq \pi \end{cases} \quad y(0) = 0, y'(0) = 0$

(e) $y'' + 2y' + 5y = \begin{cases} 10 \sin t, & : 0 < t < 2\pi \\ 0, & : t \geq 2\pi \end{cases} \quad y(\pi) = 1, y'(\pi) = 2e^{-\pi} - 2$

(f) $y'' + 4y = \begin{cases} 8t^2, & : 0 < t < 5 \\ 0, & : t \geq 5 \end{cases} \quad y(1) = 1 + \cos 2, y'(1) = 4 - 2 \sin 2$

8. Find

(a) $1 * \sin \omega t$

(b) $e^t * e^{-t}$

(c) $(\cos \omega t) * (\cos \omega t)$

(d) $(\sin \omega t) * (\cos \omega t)$

(e) $t * e^t$

ω is a constant.

9. Solve the following integral equations using Laplace transforms.

(a) $y(t) + 4 \int_0^t y(\tau) (t-\tau) d\tau = 2t$

(b) $y(t) - \int_0^t y(\tau) d\tau = 1$

(c) $y(t) - \int_0^t y(\tau) \sin 2(t-\tau) d\tau = \sin 2t$

(d) $y(t) + \int_0^t y(\tau) \cosh(t-\tau) d\tau = t + e^t$

(e) $y(t) + 2e^t \int_0^t y(\tau) e^{-\tau} d\tau = te^t$

(f) $y(t) - \int_0^t y(\tau) (t-\tau) d\tau = 2 - \frac{1}{2}t^2$

10. Find the inverse Laplace transform of the following functions. ω is a constant.

(a) $\frac{2\pi s}{(s^2+\pi^2)^2}$

(b) $\frac{\omega}{s^2(s^2+\omega^2)}$

(c) $\frac{18s}{(s^2+36)^2}$

$(t=0) + (t-\pi) + 0$
 $(t=0)^2$

$\frac{1}{s^2} * \frac{1}{s^2+\omega^2}$

$0 + 9 + 0 - (-\frac{250}{3} - 3) = \frac{286}{3}$
 $9 + 2 \frac{9}{2} + 3 = 18$
 $\frac{286}{3}$
 $P=2$
 $s=3$
 $f=21$

$\int uv = \frac{1}{k} \frac{d}{dt} [(k-s) + \int (e^{-s}) + 3]$
 $(k-s)^2 + B(e^{-s}) + 2s$

74870
 6119

$sL(y) - y(0) - y'(0)$
 $sL(y) - s$
 $L[u(t-a)f(t-a)] = e^{-cs} Lf(s)$

The University of Zambia
 Department of Mathematics and Statistics
 Mat 3110-Engineering Mathematics II

Tutorial Sheet 1 - Laplace transforms March, 2019.

Submit question 7 for assessment: Submit on 15th March, 2019 during class from 11-13hrs.

1. Find the Laplace transforms of the following functions. Assume that a, b, ω and θ are constants.

- (a) $(a - bt)^2$
- (b) $3t + 12$
- (c) $\cos^2 \omega t$
- (d) $e^{2t} \sinh t$
- (e) $e^{-t} \sin 4t$
- (f) $\sin(\omega t + \theta)$
- (g) $1.5 \sin(3t - \frac{\pi}{2})$
- (h) $t^2 e^{-3t}$
- (i) $ke^{-at} \cos \omega t$
- (j) $\sinh t \cos t$

2. Find the inverse Laplace transforms of the following functions. Assume that a, b, n and L are constants.

- (a) $\frac{5s+1}{s^2-25}$
- (b) $\frac{s^2}{L^2 s^2 + n^2 \pi^2}$
- (c) $\frac{12}{s^4} - \frac{228}{s^6}$
- (d) $\frac{4s+32}{s^2-16}$
- (e) $\frac{s+10}{s^2-s-2}$
- (f) $\frac{1}{(s+a)(s+b)}$
- (g) $\frac{\pi}{(s+\pi)^2}$
- (h) $\frac{6}{(s+1)^3}$
- (i) $\frac{4}{s^2-2s-3}$
- (j) $\frac{\pi}{s^2+10\pi s+24\pi^2}$
- (k) $\frac{2s-1}{s^2-6s+18}$

3. Solve the following initial value problems by using Laplace transforms.

- (a) $y'' + 2y = 0, y(0) = \frac{3}{2}$
- (b) $y'' - y' - 6y = 0, y(0) = 11, y'(0) = 28$
- (c) $y'' + 9y = 10e^{-t}, y(0) = 0, y'(0) = 0$
- (d) $y'' - 4y' + 3y = 6t - 8, y(0) = 0, y'(0) = 0$

4. Solve the following shifted data initial value problems by using Laplace transforms.

- (a) $y' - 6y = 0, y(-1) = 4$
- (b) $y'' - 2y' - 3y = 0, y(4) = -3, y'(4) = -17$
- (c) $y'' + 2y' + 5y = 10t - 100, y(2) = -4, y'(2) = 14$

5. Sketch or graph the given function, which is assumed to be zero outside the given interval. Represent it, using unit step functions. Find its Laplace transform.

- (a) $t (0 < t < 2)$
- (b) $t - 2 (t < 2)$
- (c) $\cos 4t (0 < t < \pi)$
- (d) $e^t (0 < t < \frac{\pi}{2})$
- (e) $\sin \pi t (2 < t < 4)$
- (f) $e^{-\pi t} (2 < t < 4)$
- (g) $t^2 (1 < t < 2)$
- (h) $t^2 (0 < t < \frac{3}{2})$
- (i) $\sin t (\frac{\pi}{2} < t < \pi)$

6. Find and sketch the inverse Laplace transforms of the following functions.

$\frac{s^2 - 6s + (-3)^2 - (-3)^2 + 13}{(s-3)^2 + 9}$

The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

S.A. ...
A = ∫∫ dA

Tutorial Sheet 4- Triple Integrals

July, 2019.

As part of continuous assessment, submit Question 6 on 2 August, 2019 during class time.

1. Evaluate the following triple integrals.

(a) $\int_1^2 \int_0^2 \int_{-1}^1 2 + z^2 - xy \, dz \, dx \, dy$

(b) $\int_0^2 \int_{x^2}^2 \int_0^{xz} y^2 - 6z \, dy \, dz \, dx$

(c) $\int_{-1}^2 \int_0^1 \int_0^{2z} 3x - \sqrt{1+z^2} \, dx \, dz \, dy$

In each of the above, can you tell over which solid we are integrating?

2. Evaluate the following triple integrals over the given solids.

(In as much as you are not required to sketch the solids over which you are integrating, you are encouraged to do so as it will enhance your understanding.)

(a) Evaluate $\int \int \int_E 12y \, dV$ where E is the region below $6x + 4y + 3z = 12$ in the first octant.

(b) Evaluate $\int \int \int_E 5x^2 \, dV$ where E is the region below $x + 2y + 4z = 8$ in the first octant.

(c) Evaluate $\int \int \int_E 10z^2 - x \, dV$ where E is the solid below $z = 8 - y$ and above the region in the xy -plane bounded by $y = 2x$, $x = 3$ and $y = 0$.

(d) Evaluate $\int \int \int_E 4y^2 \, dV$ where E is the solid below $z = -3x^2 - 3y^2$ and above $z = -12$.

(e) Evaluate $\int \int \int_E 2y - 9z \, dV$ where E is the solid behind the plane $6x + 3y + 3z = 15$ and in front of the triangle in the xz -plane with vertices, in (x, z) form: $(0, 0)$, $(0, 4)$ and $(2, 4)$.

(f) Evaluate $\int \int \int_E 18x \, dV$ where E is the solid behind the surface $y = 4 - x^2$ that is in front of the solid in the xz -plane bounded by $z = -3x$, $z = 2x$ and $z = 2$.

(g) Evaluate $\int \int \int_E 20x^3 \, dV$ where E is the solid bounded by $x = 2 - y^2 - z^2$ and $x = 5y^2 + 5z^2 - 6$.

(h) Evaluate $\int \int \int_E 6z^2 \, dV$ where E is the solid behind $x + 6y + 2z = 8$ that is in front of the region in the yz -plane bounded by $z = 2y$ and $z = \sqrt{4y}$.

(i) Evaluate $\int \int \int_E 8y \, dV$ where E is the solid between $x + y + z = 6$ and $x + y + z = 10$ above the triangle in the xy -plane with vertices, in (x, y) form: $(0, 0)$, $(1, 2)$ and $(1, 4)$.

3. Use triple integrals to find the volumes of the following solids.

- (a) The volume of the solid is below the plane $z = 8 - y$ and above the region in the xy -plane bounded by $y = 2x$, $x = 3$ and $y = 0$.
- (b) The volume of the solid in the 1st octant that is below $4x + 8y + z = 16$.
- (c) The volume of the solid behind $6x + 3y + 3z = 15$ and in front of the triangle in the xz -plane with vertices, in (x, z) form: $(0, 0)$, $(0, 4)$ and $(2, 4)$.
- (d) The solid enclosed by $y = x^2 + z^2$ and $y = \sqrt{x^2 + z^2}$.
- (e) The solid behind $x + 6y + 2z = 8$ that is in front of the region in the yz -plane bounded by $z = 2y$ and $z = \sqrt{4y}$.

4. Evaluate the following integrals

(a) $\int_0^2 \int_{-\sqrt{4-x^2}}^0 \int_{\sqrt{5x^2+5y^2}}^{\sqrt{24-x^2-y^2}} 7x \, dz \, dy \, dx$

(b) $\int_{-4}^4 \int_0^{\sqrt{16-y^2}} \int_0^{6+x} 6yx^2 \, dz \, dx \, dy$

(c) $\int_{-\sqrt{5}}^{\sqrt{5}} \int_0^{\sqrt{5-y^2}} \int_{-\sqrt{x^2+y^2}}^{-\sqrt{10-x^2-y^2}} 3xz^2 \, dz \, dx \, dy$

In each of the above, can you tell over which solid we are integrating?

$x^2 + y^2 + z^2 = 10$

5. Evaluate the following triple integrals over the given solids.

(a) Evaluate $\int \int \int_E 8z \, dV$ where E is the solid bounded by $z = 2x^2 + 2y^2 - 4$ and $z = 5 - x^2 - y^2$

(b) Evaluate $\int \int \int_E 4y^2 \, dV$ where E is the solid enclosed by the sphere $x^2 + y^2 + z^2 = 9$.

(c) Evaluate $\int \int \int_E 9yz^3 \, dV$ where E is the solid between $x = -\sqrt{9y^2 + 9z^2}$ and $x = \sqrt{y^2 + z^2}$ inside the cylinder $y^2 + z^2 = 1$.

(d) Evaluate $\int \int \int_E 2yz \, dV$ where E is the solid below $x^2 + y^2 + z^2 = 16$ and inside $z = \sqrt{3x^2 + 3y^2}$ that is in the 1st octant.

(e) Evaluate $\int \int \int_E x + 2 \, dV$ where E is the solid between the two planes $2x + y + z = 6$ and $6x + 3y + 3z = 12$ inside the cylinder $x^2 + z^2 = 16$.

(f) Evaluate $\int \int \int_E 5y^2 \, dV$ where E is the portion of the solid sphere $x^2 + y^2 + z^2 = 4$ with $x \leq 0$.

6. Use triple integrals to find the volumes of the following solids.

- (a) The solid that is below the plane $x = z + 3$, above $x = -z - 6$ and inside the cylinder $y^2 + z^2 = 4$.
- (b) The solid bounded by $y = \sqrt{x^2 + z^2}$, and $x^2 + y^2 + z^2 = 2$ in the 1st octant.

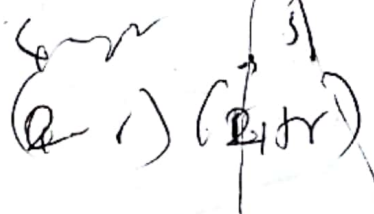
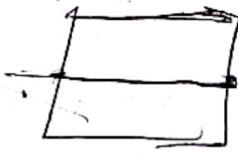
7. Use triple integrals in cylindrical coordinates to derive the volume of a cylinder of height h and radius a .

8. Use a triple integral in spherical coordinates to derive the volume of a sphere with radius a .

$V = \iiint_E dV$

$\int \cos^2 \alpha \cdot \sin^3 \alpha \, d\alpha$

$\rho = \sqrt{x^2 + y^2 + z^2}$
 $\frac{z - \sqrt{x^2 + y^2}}{\rho} = \cos \alpha$



$x^2 + y^2 + z^2 = a^2$
 $(a - \rho)^2$

$$e^{-x-x} = e^{-3x}$$

$$\frac{2}{3} \sqrt[3]{3} = 3$$

$$\sqrt[3]{2} + \frac{1}{3}$$

$$\sqrt[3]{2} = \frac{1}{2} \sqrt[3]{\frac{6}{2}} = \frac{1}{2} \sqrt[3]{\frac{6}{2}}$$

The University of Zambia
 Department of Mathematics and Statistics
 Academic Year 2018/19
 Mat 3110 Quiz 3

For 5 = 6

Duration : 1 hour

August 13, 2019.

1. Evaluate the double integral

$$\iint_D e^{-x-2y} dA$$

where D is the region in the 1st quadrant in which $x \leq y$.

2. Evaluate the following triple integral.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{1+x^2+y^2+z^2}$$

3. Evaluate the triple integral

$$\iiint_E (x^2 + y^2)^{\frac{3}{2}} dV$$

where E is the solid bounded above by $z = \frac{1}{2}(x^2 + y^2)$, below by the xy -plane, and laterally (in the sides) by $x^2 + y^2 = 25$.

4. Find the volume of the solid enclosed by the surfaces $z = 0$, $x = 0$, $y = 0$, $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{1 - x^2 - y^2}$.

Handwritten solutions for the problems:

Problem 1: $\int_0^{\pi/2} \int_0^y e^{-x-2y} dx dy$

Problem 2: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{1+x^2+y^2+z^2}$

Problem 3: $\int_0^{2\pi} \int_0^{2\pi} \int_0^{\frac{1}{2}(x^2+y^2)} (x^2+y^2)^{\frac{3}{2}} dz r dr d\theta$

Problem 4: $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\min(r, \sqrt{1-r^2})} r dz dr d\theta$

Final answers for problem 4: $\frac{\pi}{12}$ and $\frac{\pi d}{6}$

Due : Monday

Assignment No. 1

Discuss the merits and demerits of having a

- (i) Fee Simple land holding and;
- (ii) Lease hold Land Tenure systems.

Cimamba Banda Dominic

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$$\nabla f = \frac{df}{dx}i + \frac{df}{dy}j + \frac{df}{dz}k = \boxed{F}$$

Tutorial Sheet 5- Line Integrals

August, 2019.

1. Evaluate the following line integrals.

$$\int_C f(x, y, z) \cdot |r'(t)| dt$$

(a) $\int_C x + 12xz \, ds$ where C is given by

$$r(t) = ti + \frac{t^2}{2}j + \frac{t^4}{4}k, \quad 0 \leq t \leq 1.$$

(b) $\int_C y^2 - 10xy \, ds$ where C is the left half of the circle centered at the origin of radius 6 with counter clockwise rotation.

$$y = \frac{\sqrt{36-x^2}}{2}$$

$$\text{let } x = \frac{t}{2}$$

$$y = \frac{\sqrt{36-t^2}}{2}$$

(c) $\int_C z^3 - 4x + 2y \, ds$ where C is the line segment from $(2, 4, 1)$ to $(1, 1, 0)$.

2. Evaluate the following line integrals.

(a) $\int_C F \cdot dr$ where C is the portion of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ that is in the 1st, 4th and 3rd quadrants in the clockwise direction.

$$F(x, y) = 2x^2 i + (y^2 - 1) j.$$

(b) $\int_C F \cdot dr$ where C is the portion of $y = x^3 + 2$ from $x = -1$ to $x = 2$.

$$F(x, y) = (x^3 - y) i + (x^2 + 7x) j.$$

(c) $\int_C F \cdot dr$ where C is the line segment from $(1, 4, -2)$ to $(3, 4, 6)$.

$$F(x, y, z) = (3x - 3y) i + (y^3 - 10) j + yz k.$$

let $v = \sin t$
 $du = \cos t dt$
 $dt = \frac{du}{\cos t}$
 $3 \cos t \cdot u^2 du$
 u^3

3. Evaluate the following integrals

(a) $\int_C \nabla f \cdot dr$ where $f(x, y) = 4x + 3xy^2 - \ln(x^2 + y^2)$ and C is the upper half of $x^2 + y^2 + 1$ with clockwise rotation followed by the right half of $(x-1)^2 + \frac{(y-2)^2}{4} = 1$ with counter clockwise rotation.

(b) $\int_C \nabla f \cdot dr$ where $f(x, y, z) = zx^2 + x(y-2)^2$ and C is the line segment from $(1, 2, 0)$ to $(-3, 10, 9)$ followed by line segment from $(-3, 10, 9)$ to $(6, 0, 2)$.

(c) $\int_C \nabla f \cdot dr$ where $f(x, y) = 20y \cos(x+3) - yx^3$ and C is the right of $(x+3)^2 + \frac{(y-1)^2}{4} = 1$ with clockwise rotation.

(d) $\int_C \nabla f \cdot dr$ where $f(x, y, z) = \frac{3x-8y}{z-6}$ and C is given by

$$r(t) = 6t i + 4j + (9-t^3)k$$

with $-1 \leq t \leq 3$.

$x = 6t =$
 $y = 4$
 $z = 9 - t^3$

$\int_C F \cdot dr = \int_C f(x, y, z) \cdot |r'(t)| dt$

$\int_C \nabla f \cdot dr = f(b) - f(a)$
 determine if smooth & clockwise if

Banda Dominic
 $x = -6, y = 4, z = 10$
 $t = -3$

(e) $\int_C \nabla f \cdot dr$ where $f(x,y) = 5x - y^2 + 10xy + 9$ and C is given by

$$r(t) = \frac{2t}{t^2+1} i + (1-8t)j$$

with $-2 \leq t \leq 0$.

Handwritten notes:

$$z = \frac{-9}{4+1} = -\frac{9}{5}$$

$$y = 1 - 16 = -15$$

4. For the following vector fields, determine if the vector fields are conservative. If a vector field is conservative, find its potential function.

- (a) $F(x,y) = (6x - 5y^2 + 2xy^3 - 10) i + (3x^2y^2 - 10xy) j$
- (b) $F(x,y) = (8 - 14xy^2 + 2ye^{2x}) i + (e^{2x} - 14x^2y) j$
- (c) $F(x,y) = -(3 - (1+2y)e^{x-1}) i + (3y^2 + 2e^{x-1}) j$
- (d) $F(x,y) = (y^2 - 4y + 5) i + (2xy - 4x - 9) j$

Handwritten notes for problem 4:

$$\frac{df}{dy} = -10y + 6xy^2$$

$$\frac{df}{dx} = 6xy^2 - 10y$$

Since $\frac{df}{dy} = \frac{df}{dx}$

5. Evaluate the following line integrals.

- (a) $\int_C F \cdot dr$ where $F(x,y) = (6x - 5y^2 + 2xy^3 - 10) i + (3x^2y^2 - 10xy) j$ and C is the upper half of $x^2 + y^2 = 1$ in the clockwise direction, followed by $y = \sin(\pi x)$ from $(1,0)$ to $(2,0)$, followed by the lower half of $(x-3)^2 + y^2 = 1$ in the counter clockwise direction.
- (b) $\int_C F \cdot dr$ where $F(x,y) = (8 - 14xy^2 + 2ye^{2x}) i + (e^{2x} - 14x^2y) j$ and C is a sequence of line segments: $(-1,2)$ to $(0,0)$ then $(0,0)$ to $(2,1)$ and then $(2,1)$ to $(4,-2)$.
- (c) $\int_C F \cdot dr$ where $F(x,y) = -(3 - (1+2y)e^{x-1}) i + (3y^2 + 2e^{x-1}) j$ and C is the portion of $y = x^3 + 1$ from $x = -2$ to $x = 1$.
- (d) $\int_C F \cdot dr$ where $F(x,y) = (y^2 - 4y + 5) i + (2xy - 4x - 9) j$ and C is the upper half of $\frac{x^2}{36} + \frac{y^2}{16} = 1$ with clockwise direction.

Plan use $\int_C F \cdot dr = \int \dots - \int \dots$

6. The following three-dimensional vector fields are conservative. Find their potential functions.

- (a) $F(x,y,z) = (2z^4 - 2y - y^3) i + (z - 2x - 3xy^2) j + (6 + y + 8xz^3) k$
- (b) $F(x,y,z) = (\frac{2xy}{z^3}) i + (2y - z^2 + \frac{x^2}{z^3}) j - (4z^3 + 2yz + \frac{3x^2y}{z^4}) k$
- (c) $F(x,y,z) = 2xy^3z^4 i + 3x^2y^2z^4 j + 4x^2y^3z^3 k$
- (d) $F(x,y,z) = (2x \cos y - 2z^3) i + (3 + 2ye^z - x^2 \sin y) j + (y^2e^z - 6xz^2) k$

7. Evaluate the following line integrals.

- (a) $\int_C F \cdot dr$ where C is a the following sequence of straight lines: $(0,0)$ to $(1,3)$, then $(1,3)$ to $(1,5)$ and lastly $(1,5)$ to $(0,0)$.

$$F(x,y) = (6y - 3y^2 + x) i + yx^3 j$$

Handwritten notes for problem 7(a):

$$6y - 3y^2 + 3yx^3$$

- (b) $\int_C F \cdot dr$ where C is a the path consisting of the straight line from $(0,0)$ to $(0,-4)$, then along $x^2 + y^2 = 16$ from $(0,-4)$ to $(4,0)$ and lastly the straight line from $(4,0)$ to $(0,0)$.

$$F(x,y) = (y^3 - xy^2) i + (2 - x^3) j$$

Handwritten notes for problem 7(b):

$$3xy^2 - 6 - 6y$$

- (c) $\int_C F \cdot dr$ where C is the parallelogram with vertices $(1,1)$, $(-1,2)$, $(1,4)$ and $(-1,-1)$.

Test Coverage
Double integrals
Polar and Change

+ Line Integrals
→

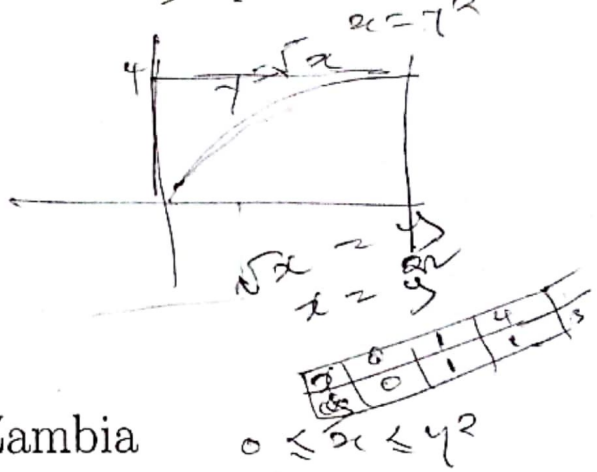
$$F(x,y) = xy^2 i + (1 - xy^3) j$$

Handwritten notes for problem 7(c):

$$\sin 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 1 - \sin^2 \theta - \sin^2 \theta$$

$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \left(\frac{dx}{dt} \mathbf{i} + \frac{dy}{dt} \mathbf{j} + \frac{dz}{dt} \mathbf{k} \right) \cdot \mathbf{F}(x, y, z) dt$
 $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$ (1+8)



The University of Zambia
 Department of Mathematics and Statistics
 Academic Year 2018/19
 Mat 3110 Quiz 2

$x^2 - 13 - 2^2 = 2$
 $y^2 - 4y + (-2)^2 - (-2)^2$
 $(y-2)^2 - 4 = 2$

Duration : 1 hour

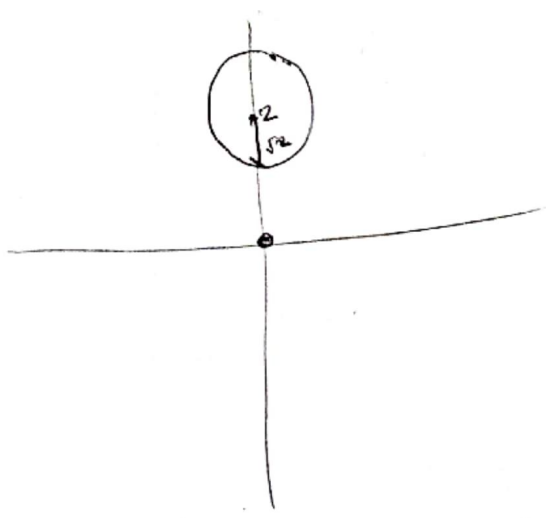
June 7, 2019.

$x^2 + y^2 = -2 + 4y$ $x^2 + y^2 - 4y = -2$

1. Find the area of the region inside $r^2 = -2 + 4r \sin \theta$ and outside $r = \sqrt{2}$. (10)
 (Note that these are curves in plane written using polar coordinates).
2. Evaluate the following double integral. (10)

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{1+y^3} dy dx$$

3. Use a double integral to find the volume of the solid that is bounded by $z = 3 - 2y$, $y = 1 - x^2$, $y = 0$ and the plane $z = 0$. (10)
4. Use a double integral to find the volume of the solid that is bounded by $z = 12 - 3x^2 - 3y^2$ and $z = x^2 + y^2 - 8$. (10)



$x^2 + y^2 = -2 + 4y$
 $x^2 + y^2 - 4y = -2$
 $x^2 + (y-2)^2 - 4 = -2$
 $x^2 + (y-2)^2 = 2$ (Circle)

$r = \sqrt{2}$
 $z = 12 - 3x^2 - 3y^2$
 $z = x^2 + y^2 - 8$
 $dz = 2y dy$
 $dz = 2x dx$
 $dz = 2y dy$
 $dz = 2x dx$
 $dz = 2y dy$
 $dz = 2x dx$

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Tutorial Sheet 3 - Double Integrals

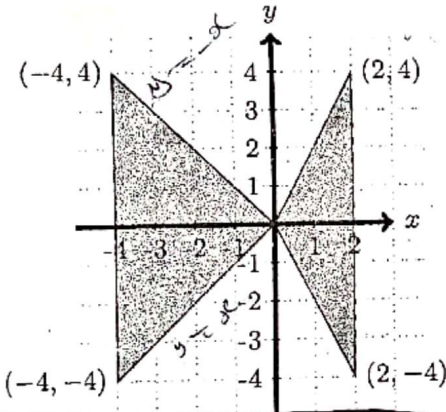
June, 2019.

1. Compute the given double integrals over the indicated regions.

(a) $\iint_D 6y\sqrt{x} - 2y^3 \, dA$, where D is the unit circle centered at the origin.

(b) $\iint_D 8yx^3 \, dA$, where $D = \{(x, y) \mid -1 \leq y \leq 2, -1 \leq x + y^2\}$.

(c) $\iint_D xy - y^2 \, dA$ where D is shown below.



2. Evaluate the following double integrals.

(a) $\int_{-1}^0 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} x^2 \, dx \, dy$

(b) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$

(c) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$

3. Use double integrals to determine the area of the following regions in the xy -plane.

(a) The region bounded by $y = x^2 + 1$ and $y = \frac{1}{2}x^2 + 3$.

(b) The region bounded by $x = -y^2$ and $x = y - 6$.

(c) The region bounded by $y = x^2 + 1$ and $y = \frac{1}{2}x^2 + 3$.

4. Use a double integral to determine the volume of the following solids.

(a) The solid bounded by the planes $z = 4 - 2x - 2y$, $y = 2x$, $x = 0$ and $z = 0$.

(b) The solid that is inside both the cylinder $x^2 + y^2 = 9$ and $x^2 + y^2 + z^2 = 16$.

(c) The solid that is bounded by $z = 12 - 3x^2 - 3y^2$ and $z = x^2 + y^2 - 8$.

(d) The solid that is below $z = 9 - 4x^2 - 4y^2$ and above the xy -plane.

(e) The solid in the first octant that is below the plane given by $2x + 6y + 4z = 8$.

(f) The solid that is bounded by the plane $z = 3 - 2y$, the surface $y = 1 - x^2$, the plane $x = 0$ and the plane $z = 0$.

$y = 0$
 $\cos 2A = \cos^2 A - (\sin A)^2$

Amamba Banda *zminic*

$$y = e^{-t} \left[a \cos 2t + b \sin 2t \right] + c e^{2t} \left[d \cos 2t + e \sin 2t \right]$$

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Mat 3110-Engineering Mathematics II

Tutorial Sheet 2 - Further Ordinary Differential Equations

May, 2019.

1. Apply the power series method to solve the following differential equations.

- (a) $y' = 2y$
- (b) $y'' + y = 0$
- (c) $y' = ky$ k is a constant
- (d) $(1-x)y' = y$
- (e) $(x+1)y' = 3y$
- (f) $(1+x)y' + y = 0$
- (g) $y' + 2xy = 0$
- (h) $y' = 3x^2y$
- (i) $y'' - y = 0$
- (j) $y'' + 2xy = 0$
- (k) $y'' - y' = 0$
- (l) $y'' - 9y = 0$

2. Show that

$$\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} = \sum_{j=1}^{\infty} (j+1)j a_{j+1} x^{j-1} = \sum_{m=2}^{\infty} (s+1)(s+1)a_{s+2} x^s$$

$$\frac{dy}{dx} = 2y$$

3. For each of the series below, shift the index so that the power under the summation sign is x^m .

- (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n} x^{n+2}$
- (b) $\sum_{s=1}^{\infty} \frac{s(s+1)}{s^2+1} x^{s-1}$
- (c) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{6^k} x^{k-3}$

4. Find the general solution of the following differential equations.

- (a) $xy' = 3y + 3$
- (b) $(x-3)y' - xy = 0$
- (c) $y' = 2xy$
- (d) $(1-x^4)y' = 4x^3y$
- (e) $(x+1)y' - (2x+3)y = 0$
- (f) $(1+x)y'' - y = x$
- (g) $y'' - 3y' + 2y = 0$
- (h) $y'' - 4xy' + (4x^2 - 2)y = 0$
- (i) $(1-x^2)y'' - 2xy' + 2y = 0$
- (j) $y'' - xy + y = 0$

5. Find the general solution of the following differential equations.

- (a) $x^2y'' - 6y = 0$ rs/E
- (b) $x^2y'' + 4y' = 0$ rs
- (c) $x^2y'' - 2xy' + 2y = 0$ E
- (d) $x^2y'' + 9xy' + 16y = 0$ E
- (e) $x^2y'' + xy' - y = 0$ E
- (f) $x^2y'' + 3xy' + y = 0$ E
- (g) $x^2y'' + 3xy' + 5y = 0$ E
- (h) $x^2y'' + xy' + y = 0$ E

6. Solve the following initial value problems.

- (a) $x^2y'' - 4xy' + 4y = 0, y(1) = 4, y'(1) = 13$
- (b) $4x^2y'' - 4xy' - y = 0, y(4) = 2, y'(4) = \frac{1}{4}$
- (c) $x^2y'' - 5xy' + 8y = 0, y(1) = 5, y'(1) = 18$

$$\frac{dx}{dy} = 3y + 3$$

$$x dx = (3y + 3) dy$$

$$\int (x dx - (3y + 3) dy) = 0$$

$$\frac{dx}{x} - \frac{dy}{y} = 0$$

$$rs^2 + c_1rs + c_2 = m \alpha x^{m-1}$$

$$(1 - 4c_1)(2)$$

$$s = 1 \quad s =$$

7. Convert each of the following linear ordinary differential equation into a system of first linear ordinary differential equations

(a) $y'' - 4y' + 5y = 0$

(b) $y''' - 5y'' + 9y = t \cos 2t$

(c) $y'''' = 3y'''' - \pi y'' + 2\pi y' - 6y = 11$

8. Rewrite each of the systems you found in the question above into a matrix-vector form

9. Find the eigenvalues and corresponding eigenvectors of the following matrices.

(a) $\begin{pmatrix} 5 & -2 \\ 9 & -6 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

(e) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

10. Find the general solution of each system below.

(a) $\mathbf{x}' = \begin{pmatrix} 2 & 7 \\ -5 & -10 \end{pmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{pmatrix} -3 & 6 \\ -3 & 3 \end{pmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{pmatrix} 8 & -4 \\ 1 & 4 \end{pmatrix} \mathbf{x}$

(d) $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -5 \end{pmatrix} \mathbf{x}$

(e) $\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}$

(f) $\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -4 \end{pmatrix} \mathbf{x}$

11. Give the equations of the following lines.

(a) The line throught the points $(2, -4, 1)$ and $(0, 4, -10)$.

(b) The line throught the point $(-7, 2, 4)$ and parallel to the line given by $x = 5 - 8t, y = 6 + t, z = -12t$

$v = [-8i + j - 12k]$

12. For the following, determine the intersection point of the two lines or show that they do not intersect.

(a) The line given by $x = 8 + t, y = 5 + 6t, z = 4 - 2t$ and the line given by

$$\frac{x+7}{12} = \frac{y-3}{-1} = \frac{z-14}{8}$$

(b) The line passing through the points $(1, -2, 13)$ and $(2, 0, -5)$ and the line given by $x = 2 + 4t, y = -1 - t, z = 3$

13. Write down the equation of the following planes.

(a) The plane containing the points $(6, -3, 1), (5, -4, 1)$ and $(3, -4, 0)$.

(b) The plane containing the point $(1, -5, 8)$ and orthogonal to the line given by $x = -3 + 15t, y = 14 - t, z = 9 - 3t$

(c) The plane containing the point $(-8, 3, 7)$ and parallel to the plane given by $4x + 8y - 2z = 45$.

(d) The plane containing the point $(2, 0, 8)$ and containing the line given by $x = 8t, y = -1 - 5t, z = 4 - t$.

$(x-x_0) + b(y-y_0) + c(z-z_0) = 0$
 $(x_1, y_1, z_1) = (x_2, y_2, z_2) + t(a + b + c)$
 $x - x_1 = y - y_1 = z - z_1$



- (e) The plane containing the two lines given by $x = 7 + 5t, y = 2 + t, z = 6t$ and $x = 7 - 6t, y = 2 - 2t, z = 10t$
14. Convert the following Cartesian coordinates for the point into cylindrical coordinates.
 (a) $(-3, 5, -8)$ (b) $(4, 1, 7)$
15. Convert the equation written in Cylindrical coordinates into an equation in Cartesian coordinates.
 (a) $zr^3 \cos(\theta) = 4r + 8$ (b) $r^2 - 3 \sin(\theta) = z^3 \sqrt{r^2 + 1}$ (c) $\tan(\theta) + 2z = 1 - r^2$
16. Convert the following Cartesian coordinates for the point into spherical coordinates.
 (a) $(6, 2, -8)$ (b) $(-1, 5, 2)$ (c) $(-3, -2, 1)$
17. Convert the equation written in spherical coordinates into an equation in Cartesian coordinates.
 (a) $(\rho \cos \phi) \sin \phi \sin \theta = 3$ (b) $\rho - \cos \phi = 2 + \cos^2 \phi$ (c) $\tan \phi (\cos \theta - \sin \theta) = 4$

Submit the following questions for assessment: Submit on 24th May, 2019 during class from 11-13hrs.

18. Does the line given by $x = 9 + 21t, y = -7, z = 12 - 11t$ intersect the xy -plane? If so, give the point.
19. Does the line given by $x = 9 + 21t, y = -7, z = 12 - 11t$ intersect the xz -plane? If so, give the point.
20. Find the line of intersection of the plane given by $4x + y + 10z = -2$ and the plane given by $-8x + 2y + 3z = -8$.

✓ 21. Identify the surface generated by the given equation.

(a) $z = -4r, z < 0$ (b) $2r + 6 \cos \theta + 18 \sin \theta = 5$ (c) $\theta = \frac{\pi}{3}$

✓ 22. Identify the surface generated by the given equation.

(a) $\cos^2 \phi - \sin^2 \phi = 0$ (b) $\sin \phi \cos \theta + \sin \phi \sin \theta + \cos \phi = \frac{1}{\rho}$

(21)
 a) $z = -4 \sqrt{x^2 + y^2}$
 → Cone below xy plane

b) $2r + 6 \cos \theta + 18 \sin \theta = 5$
 $2r + 6 \cos \theta + 3 \cdot 6 \sin \theta = 5$
 $2r + 6 (\cos \theta + 3 \sin \theta) = 5$

22 a) $x^2 - y^2 = 0$

⊙ $\tan^{-1}(\frac{y}{x}) = \frac{\pi}{3}$
 $\frac{y}{x} = \tan \frac{\pi}{3}$
 $y = \sqrt{3} \tan \frac{\pi}{3} x$
 Plane in \mathbb{R}^3

~~$x + y + z = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$~~
 $3 \sin \theta \cos \theta + 3 \sin \theta \sin \theta + 3 \cos \theta = 1$
 $x + y + z = 1$
 (plane in \mathbb{R}^3)
 ~~$x + y + z = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$~~