The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Test 2 **17 September, 2018.**

Instructions

1. Answer all the questions.
2. Any sort of exam malpractice will be severely punished.
3. Total marks is 80.
4. Duration is **2 hours.**

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1. (a) Use a double integral to determine the area of the region that is inside $r = 4\sin\theta$ and outside $r = 2\sin\theta$. (10)

   (b) Evaluate the following integral: (10)

$$\int_{-\frac{1}{\sqrt{2}}}^{0} \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} 18y \, dz dy dx$$

2. Let

$$F(x,y,z) = (2z^4 - 2y - y^3)\,\mathbf{i} + (z - 2x - 3xy^2)\,\mathbf{j} + (6 + y + 8xz^3)\,\mathbf{k}$$

   (a) Given that the above vector field is conservative, find its potential function. (10)

   (b) Evaluate (10)

$$\int_C F \cdot dr$$

   where C is the helix given by

$$r(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \frac{2t}{5\pi}\,\mathbf{k}, \quad 0 \le t \le \frac{5\pi}{2}.$$

3. (a) Let $F(x,y) = yx^2\,\mathbf{i} - x^2\,\mathbf{j}$. Evaluate (10)

$$\int_C F \cdot dr$$

   where C is the line segment from $(0,-5)$ to $(0,5)$ followed by the path along $x^2 + y^2 = 25$ from $(0,5)$ to $(0,-5)$.

   (b) Evaluate (10)

$$\iint_D 10x^2y^3 - 6 \, dA$$

   where D is the region bounded by $x = -2y^2$ and $x = y^3$.

4. (a) Calculate the volume of the solid bounded by $z = 2 - x^2 - y^2$ and the $z = \sqrt{x^2 + y^2}$. (10)

   (b) Use a double integral to find the area of the region bounded above by $y = e^x$, below by $y = 1$, left by $x = 0$ and to the right by $x = 1$. (10)