Solutions to Quiz One

(1) $y' + 5y = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \geqslant 1 \end{cases}$ $\quad y(0) = 0$

$$y' + 5y = 1 - u(t-1)$$

$$\mathcal{L}\{y'\} + 5\mathcal{L}(y) = \mathcal{L}\{1 - u(t-1)\}.$$

$$s\mathcal{L}(y) - y(0) + 5\mathcal{L}(y) = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$\mathcal{L}\{y\}\left(s+5\right) = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$\mathcal{L}\{y\} = \frac{1}{s(s+5)} - \frac{e^{-s}}{s(s+5)}$$

$$\mathcal{L}(y) = \frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}}{s+5} - e^{-s}\left[\frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}}{s+5}\right]$$

Thus

$$y = \frac{1}{5} - \frac{1}{5}e^{-5t} - u(t-1)\left[\frac{1}{5} - \frac{1}{5}e^{-5(t-1)}\right]$$

$$y = \begin{cases} \frac{1}{5}\left(1 - e^{-5t}\right), & 0 \le t < 1 \\ \frac{1}{5}\left(1 - e^{-5t}\right) - \frac{1}{5}\left(1 - e^{-5(t-1)}\right), & t \geqslant 1 \end{cases}$$

$$y(t) = \begin{cases} \frac{1}{5}\left(1 - e^{-5t}\right), & 0 \leq t < 1 \\ \frac{1}{5} e^{-5t}\left(e - 1\right), & t \geq 1 \end{cases}$$

(2) $f(t) = \cosh^2 t$

$$f(t) = \left(\frac{e^t + e^{-t}}{2}\right)^2$$

$$f(t) = \frac{e^{2t} + 2 + e^{-2t}}{4}$$

$$f(t) = \frac{e^{2t}}{4} + \frac{1}{2} + \frac{e^{-2t}}{4}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{4}\mathcal{L}\{e^{2t}\} + \frac{1}{2}\mathcal{L}\{1\} + \frac{1}{4}\mathcal{L}\{e^{-2t}\}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{4}\,\frac{1}{s-2} + \frac{1}{2s} + \frac{1}{4}\,\frac{1}{s+2}$$

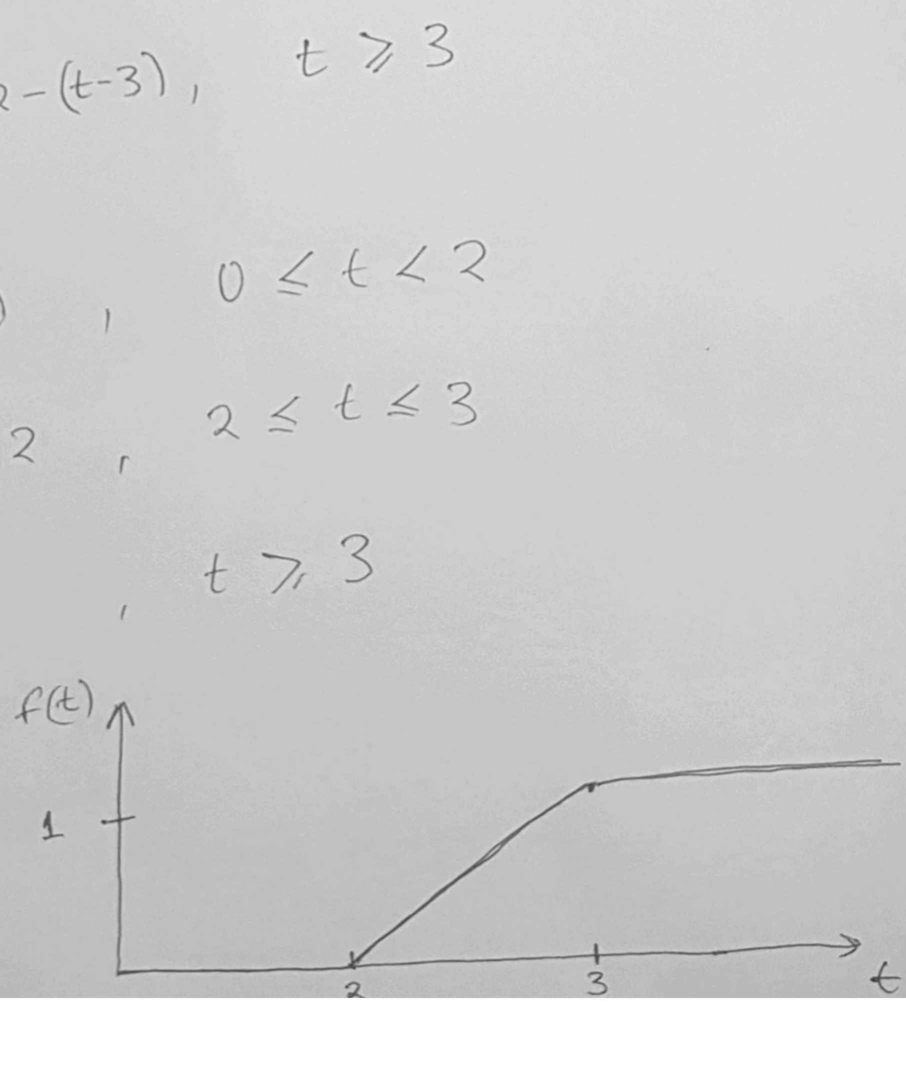
(3) $$F(s) = \frac{e^{-2s} - e^{-3s}}{s^2}$$

$$F(s) = \frac{e^{-2s}}{s^2} - \frac{e^{-3s}}{s^2}$$

$$f(t) = \mathcal{L}^{-1}\left(F(s)\right) = u(t-2)(t-2) - u(t-3)(t-3).$$

$$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ t-2, & 2 \le t \le 3 \\ t-2-(t-3), & t \geq 3 \end{cases}$$

$$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ t-2, & 2 \le t \le 3 \\ 1, & t \geq 3 \end{cases}$$



(4) $F(s) = \dfrac{1}{s^4 - 1}$

$$F(s) = \frac{1}{(s^2-1)(s^2+1)}$$

$$F(s) = \frac{1}{(s-1)(s+1)(s^2+1)}$$ decomposing into partial fractions

$$F(s) = \frac{\frac{1}{4}}{s-1} - \frac{\frac{1}{4}}{s+1} - \frac{\frac{1}{2}}{s^2+1}$$

Therefore,

$$f(t) = \mathcal{L}^{-1}\left\{F(s)\right\} = \frac{1}{4}\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) - \frac{1}{4}\mathcal{L}\left\{\frac{1}{s+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$F(t) = \frac{1}{4}e^{t} - \frac{1}{4}e^{-t} - \frac{1}{2}\sin t$$