

Quadric Surface Examples

Example 1 (12.6.30).

$$4x^2 - y + 2z^2 = 0 \Leftrightarrow y = \frac{x^2}{1/4} + \frac{z^2}{1/2}$$

This is a paraboloid with vertex at the origin, opening along the y -axis.

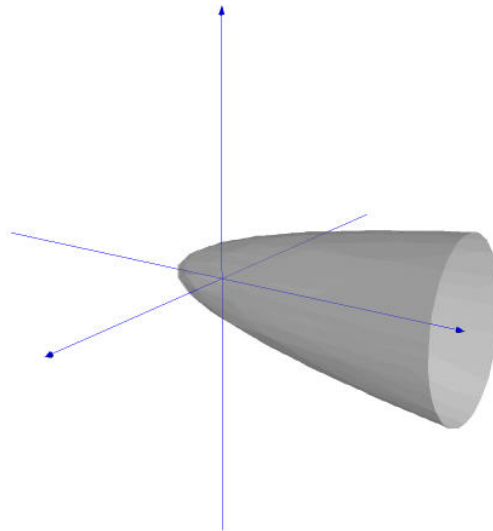
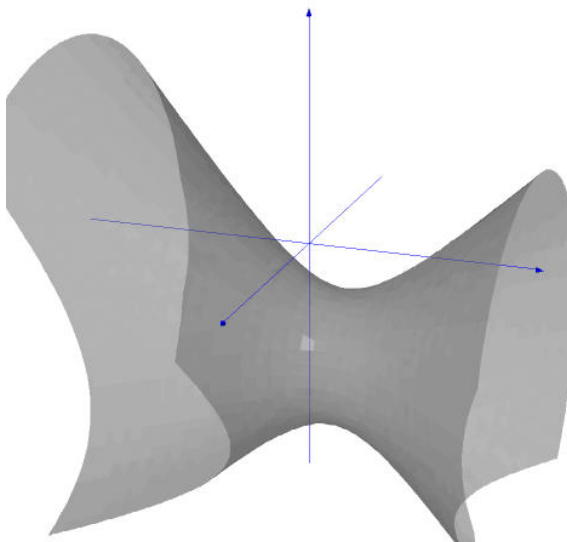


Figure 1: Paraboloid

Example 2 (12.6.36).

$$x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0 \Leftrightarrow \frac{(x-1)^2}{2} - \frac{(y-1)^2}{2} + \frac{(z+2)^2}{2} = 1$$

This is a hyperboloid of one sheet centered at $(1, 1, -2)$.



Example 3 (12.6.15).

$$-x^2 + 4y^2 - z^2 = 4 \Leftrightarrow$$

This is a hyperboloid of two sheets centered at the origin.

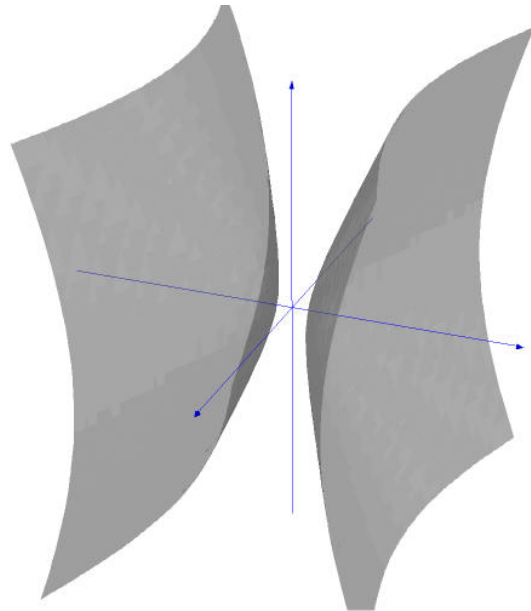


Figure 3: Hyperboloid of two sheets

Example 4 (12.rev.35).

$$4x^2 + 4y^2 - 8y + z^2 = 0 \Leftrightarrow$$

This is an ellipsoid centered at $(0, 1, 0)$.

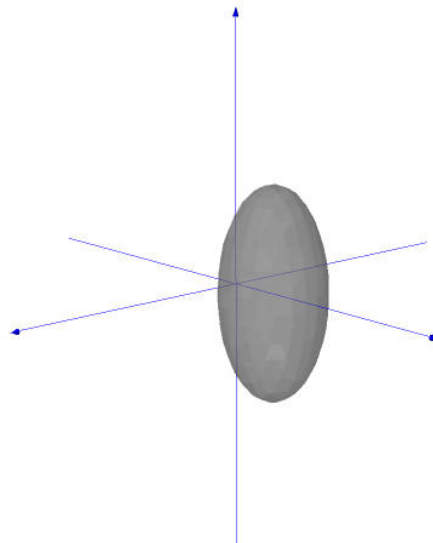


Figure 4: Ellipsoid

Example 5 (12.rev.31).

$$x^2 = y^2 + 4z^2 \Leftrightarrow$$

This is a cone with vertex at the origin and axis parallel to the x -axis.

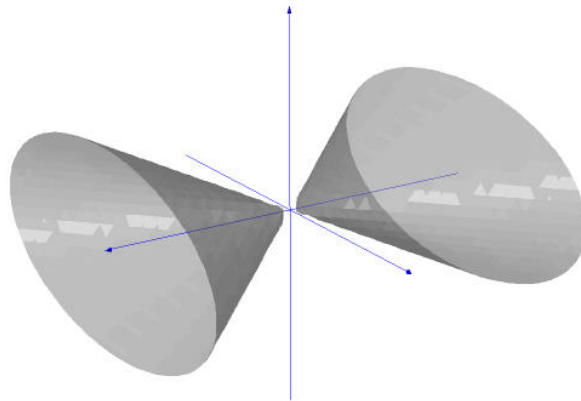
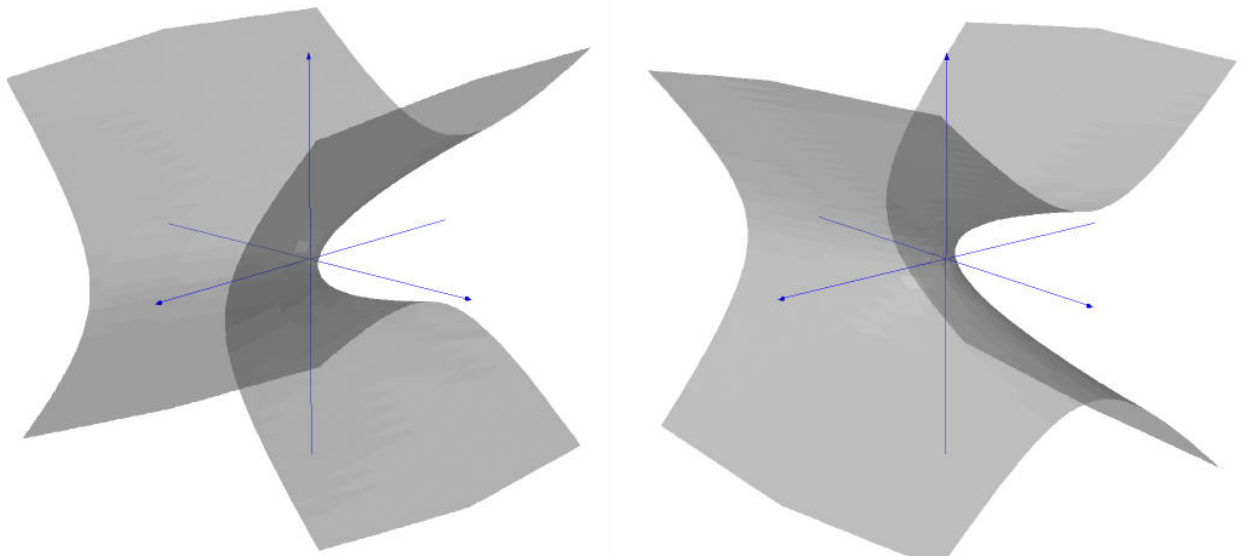


Figure 5: Cone

Example 6 (12.6.20).

$$x = y^2 - z^2 \text{ vs. } y = z^2 - x^2$$

This are both hyperbolic paraboloids, one first with the x -axis perpendicular to the 'saddle' and the other with the y -axis perpendicular to the 'saddle'.



(a) x -axis perpendicular to the 'saddle'

(b) y -axis perpendicular to the 'saddle'

Figure 6: Hyperbolic Paraboloids