Tutorial 8

Flow measurement

1. A jet of water issuing from 5mm diameter orifice working under a head of 2.0m, was found to travel horizontal and vertical distances of 2.772m and 1m respectively. If C_C = 0.61, determine discharge.

Solution:

Diameter of jet (d) = 5mm = 0.005m

C/s of jet (a) = $\frac{\pi}{4}x0.005^2$ = $1.96X10^{-5}$ m^2

Head (H) = 2m

x = 2.772m, y = 1m

C_C = 0.61

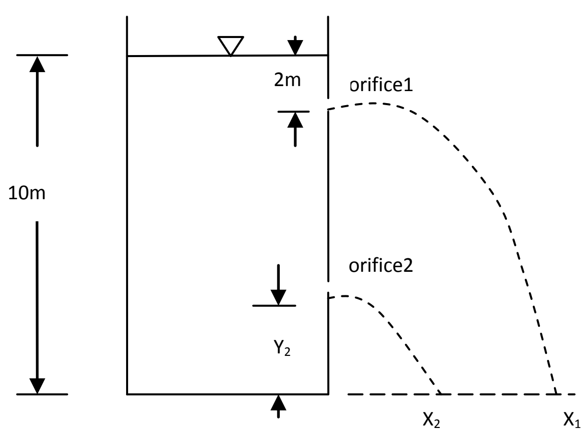
Discharge (Q) = ?

$C_v = \frac{x}{\sqrt{4yH}} = \frac{2.772}{\sqrt{4x1x2}}$ = 0.98

$C_d = C_C\ C_V$ = 0.61x0.98 = 0.5978

$Q = C_d a\sqrt{2gH} = 0.5978x1.96x10^{-5}\sqrt{2gx2}$ = $7.34X10^{-5}$ m^3/S = 0.0734 lps

2. For the two orifices shown in the figure below, determine Y_2 such that $X_2 = \frac{3X_1}{4}$.



Solution:

H_1 = 2m, Y_1 = 10-2 = 8m, H_2 = 10-Y_2

$X_2 = \frac{3X_1}{4}$

Y_2 = ?

Coefficient of velocity for orifice 1 $(C_{v1}) = \frac{X_1}{\sqrt{4Y_1H_1}}$

Coefficient of velocity for orifice 2 $(C_{v2}) = \frac{X_2}{\sqrt{4Y_2H_2}}$

Since the two orifices are identical

$C_{v1} = C_{v2}$

$\frac{X_1}{\sqrt{4Y_1H_1}} = \frac{X_2}{\sqrt{4Y_2H_2}}$

$\frac{X_1^2}{4Y_1H_1} = \frac{X_2^2}{4Y_2H_2}$

$\frac{X_1^2}{4x2x8} = \frac{\left(\frac{3X_1}{4}\right)^2}{4Y_2(10-Y_2)}$

$Y_2^2 - 10Y_2 + 9 = 0$

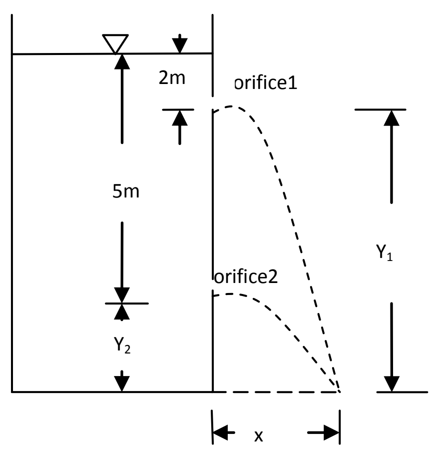
Solving for Y_2

$Y_2 = 1, 9$

As $Y_1 = 8$m, $Y_2 = 9$ ($>Y_1$) is not feasible.

Hence $Y_2 = 1$m

3. A vessel has two identical orifices provided in one of its sides as shown in the figure. Locate the point of intersection of two jets. Take C_v =0.98 for both orifices.



Solution:

$C_v = \frac{x}{\sqrt{4yH}}$

$C_{v1} = C_{v2}$

$\frac{x}{\sqrt{4Y_1H_1}} = \frac{x}{\sqrt{4Y_2H_2}}$

$Y_1H_1 = Y_2H_2$

$Y_1x2 = Y_2x5$

$Y_1 = 2.5Y_2$ (a)

Here,

$Y_1 - Y_2 = H_1 - H_2 = 3$

From a and b

$Y_1 = 5$m, $Y_2 = 2$m

$x = C_v\sqrt{4Y_1H_1} = 0.98\sqrt{4x5x2}$ = 6.2m

4. A rectangular orifice 1.0m wide and 1.5m deep is discharging water from a vessel. The top edge of the orifice is 0.8m below the water surface in the vessel. Calculate the discharge through the orifice if C_d = 0.6. Also calculate the percentage error if the orifice is treated as a small orifice.

Solution:
Width of orifice (b) = 1m
Depth of orifice (d) = 1.5m
H_1 = 0.8m, $H_2 = H_1$+d = 0.8+1.5 = 2.3m
C_d = 0.6
Discharge (Q) through large orifice = ?
error in discharge treating as small orifice = ?

$$Q = \frac{2}{3}C_d\, b\sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)$$

$$= \frac{2}{3}x0.62x\ 1\sqrt{2x9.81}\left(2.3^{3/2} - 0.8^{3/2}\right) = 5.076\ m^3/s$$

For small orifice, h = H_1 + d/2 = 0.8+1.5/2 = 1.55m
Discharge (Q1) through small orifice $(Q1) = C_d a\sqrt{2gH}$
$= 0.62x1x1.5\sqrt{2x9.81x1.55}$ = 5.13 m^3/s

% error in measuring discharge = $\frac{5.13-5.076}{5.076}x100$= 1.06%

5. A rectangular orifice of 1.5m wide and 1.2m deep is fitted in one side of a large tank. The water level on one side of the orifice is 2m above the top edge of the orifice, while on the other side of the orifice, the water level is 0.4m below its top edge. Calculate the discharge through the orifice if C_d = 0.62.

Solution:
Width of orifice (b) = 1.5m
Depth of orifice (d) = 1.2m
Height of water from top edge of orifice (H_1) = 2m
Difference of water level on both sides (H) =2+0.4 = 2.4m
Height of water from bottom edge of orifice (H_2) = H1+d = 2+1.2 = 3.2
C_d = 0.62
Discharge through partially submerged orifice (Q) = ?

$$Q = \frac{2}{3}C_d\, b\sqrt{2g}\left(H^{3/2} - H_1^{3/2}\right) + C_d b(H_2 - H)\sqrt{2gH}$$

$$= \frac{2}{3}x0.62x1.5\sqrt{2x9.81}\left(2.4^{3/2} - 2^{3/2}\right) + 0.62x1.5(3.2 - 2.4)\sqrt{2x9.81x2.4}$$

= 7.55 m^3/s

6. A horizontal venturimeter in a water main has a 20cm diameter at one end and tapers to 10cm at its throat. A piezometer installed at the inlet reads 30cm, while the one at the throat reads 18cm. Determine the discharge through the main, if C_d = 0.98.

Solution:
Diameter at inlet (d_1) = 20cm = 0.2m
C/s area of inlet (A_1) = $\frac{\pi}{4}x0.2^2$ = 0.0314m^2
Diameter at throat (d_2) = 10cm = 0.1m
C/s area of throat (A_2) = $\frac{\pi}{4}x0.1^2$ = 0.00785m^2
Pressure head at inlet ($P_1/\rho g$) =30cm =0.3m
Pressure head at throat ($P_2/\rho g$) =18cm =0.18m
Head (h) = $\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ = 0.12m
C_d = 0.98
Discharge (Q)=?

$$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.98\sqrt{2x9.81x0.12}\frac{0.0314x0.00785}{\sqrt{0.0314^2 - 0.00785^2}}$$
= 0.0122 m^3/s

7. A horizontal venturimeter is used to measure the flow of water in a 200mm diameter pipe. The throat diameter of the venturimeter is 80mm and the discharge coefficient is 0.85. If the pressure difference between the two measurement points is 10cm of mercury, calculate the average velocity in the pipe.

Solution:
Diameter at inlet (d_1) = 200mm = 0.2m
C/s area of inlet (A_1) = $\frac{\pi}{4}x0.2^2$ = 0.0314m^2
Diameter at throat (d_2) = 80mm = 0.08m
C/s area of throat (A_2) = $\frac{\pi}{4}x0.08^2$ = 0.00503m^2
Pressure difference (P_1-P_2) = 10cm of Hg = $\gamma_{Hg}h$ = 13.6x9810x0.1 Pa
$Z_1 = Z_2$
Head (h) = $\frac{P_1 - P_2}{\gamma} = \frac{13.6x9810x0.1}{9810}$ = 1.36m
C_d = 0. 85
Average velocity in the pipe (V) = ?

$$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}$$

$= 0.85\sqrt{2x9.81x1.36}\frac{0.0314x0.00503}{\sqrt{0.0314^2-0.00503^2}}$

= 0.0223 m^3/s

V = Q/A_1 = 0.0223/0.0314 = 0.71m/s

8. An orificemeter is provided in a vertical pipeline of 250mm diameter carrying oil of sp.gr. 0.9, the flow being upwards. The difference in elevation of the upstream and downstream ends of the manometer on the orificemeter is 350mm. The differential U-tube manometer shows a gauge deflection of 200mm. Calculate the discharge of oil. The diameter of the orifice is 150mm. Take C_d = 0.65.

Solution:

Diameter of pipe (d_1) = 250mm = 0.25m

C/s Area of pipe (A_1) = $\frac{\pi}{4}x0.25^2$ = 0.049 m^2

Diameter of orifice (d_2) = 150mm = 0.15m

C/s Area of pipe (A_2) = $\frac{\pi}{4}x0.15^2$ = 0.0176 m^2

Sp.gr. of oil (S_0) =0.9

Difference in elevation (Z_1-Z_2) = 350mm =0.35m

Reading of manometer (x) = 200mm = 0.2m

Sp.gr. of mercury (S) = 13.6

C_d =0.65

Discharge of oil (Q) =?

Head h is given by

$h = x\left(\frac{S}{S_0} - 1\right)$= $0.2\left(\frac{13.6}{0.9} - 1\right)$=2.82m

$$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}$$

$= 0.65\sqrt{2x9.81x2.82}\frac{0.049x0.0176}{\sqrt{0.049^2-0.0176^2}}$

= 0.091 m^3/s

9. A 20cmx10cm venturimeter is mounted in a vertical pipeline carrying oil of sp.gr. 0.8 flowing upwards. The throat section is 20cm above the entrance section of the venturimeter. The differential U-tube manometer shows a gauge deflection of 25cm. Calculate the discharge of the oil and the pressure difference between the entrance and the throat section. Take C_d =0.96

Solution:

Diameter at inlet (d_1) = 20cm = 0.2m

C/s area of inlet (A_1) =$\frac{\pi}{4}x0.2^2$ = 0.0314m^2

Diameter at throat (d_2) = 10cm = 0.1m

C/s area of throat (A_2) = $\frac{\pi}{4}x0.1^2$ = 0.00785m^2

Sp.gr. of oil (S_0) =0.8
Density of oil (ρ) =0.8x1000 = 800 kg/m^8
Difference in elevation (Z_2-Z_1) = 20cm =0.2m
Reading of manometer (x) = 25mm = 0.25m
Sp.gr. of mercury (S) = 13.6
Discharge of oil (Q) =?
Pressure difference (P_1-P_2) =?

Head h is given by

$h = x\left(\frac{S}{S_0} - 1\right)$= $0.25\left(\frac{13.6}{0.8} - 1\right)$= 4m

$Q = C_d\sqrt{2gh}\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$

$= 0.96\sqrt{2x9.81x4}\frac{0.0314x0.00785}{\sqrt{0.0314^2 - 0.00785^2}}$

= 0.0689 m^3/s

h is also expressed as

$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2)$

$\frac{P_1 - P_2}{\rho g} = 4 + 0.2$

P_1-P_2 = 4.2x800x9.81 = 32962 N/m^2

10. A venturimeter with a throat diameter of 100mm is fitted in a vertical pipeline of 200mm diameter with oil of sp.gr. 0.88 flowing upwards. The venturimeter coefficient is 0.98. The pressure gauges are fitted at tapping points, one at the throat and the other in the inlet pipe 320mm below the throat. The difference between two pressure gauge readings is 28 KN/m^2. Working from Bernoulli's equation, determine (a) the volume rate of oil through the pipe, (b) the difference in level in the two limbs of mercury if it is connected to the tapping points and connecting pipes are filled with same oil.

Solution:
Diameter at inlet (d_1) = 20cm = 0.2m
C/s area of inlet (A_1) =$\frac{\pi}{4}x0.2^2$ = 0.0314m^2
Diameter at throat (d_2) = 10cm = 0.1m
C/s area of throat (A_2) = $\frac{\pi}{4}x0.1^2$ = 0.00785m^2
Sp.gr. of oil (S_0) =0.88
Density of oil (ρ) =0.88x1000 = 880 kg/m^8
Difference in elevation (Z_2-Z_1) = 320cm =0.32m
Difference in pressure (P_1-P_2)= 28KN/m^2 = 28000N/m^2
Sp.gr. of mercury (S) =13.6

C_d =0.98
Discharge of oil (Q) =?
Manometer reading (x) =?

a. Applying Bernoulli's equation between inlet (1) and throat (2)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{P_1-P_2}{\rho g}+(Z_1-Z_2)=\frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

$$\frac{28000}{880x9.81}+(-0.32)=\frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

$$\frac{V_2^2}{2g}-\frac{V_1^2}{2g}=2.92 \qquad \text{(a)}$$

According to continuity equation
$A_1 V_1=A_2V_2$

$$V_2=\frac{A_1V_1}{A_2}=\frac{0.0314V_1}{0.00785}=4V_1 \qquad \text{(b)}$$

Solving a and b

$$\frac{(4V_1)^2}{2g}-\frac{V_1^2}{2g}=2.92$$

V_1 = 1.95m/s
Discharge (Q) =A_1V_1 = 0.0314x1.95 = 0.612 m^3/s
Actual discharge = C_d Q = 0.98x0.612 = 0.599 m^3/s

b. $h=\frac{P_1-P_2}{\rho g}+(Z_1-Z_2)=\frac{28000}{880x9.81}+(-0.32)$ =2.92

$$h=x\left(\frac{S}{S_0}-1\right)$$

$$2.92=x\left(\frac{13.6}{0.88}-1\right)$$

x = 0.2m

11. A venturimeter is used for measurement of discharge of water in a horizontal pipeline. The ratio of the upstream pipe diameter and throat is 2:1 and upstream diameter is 300mm. Mercury manometer connected at the pipe and throat shows the reading of 0.24m and the loss of head through the meter is 1/8 of the throat velocity head. Calculate the discharge in the pipe using the continuity and energy equations.

Solution:
Pipe diameter (d_1): throat diameter (d_2) = 2:1
Pipe diameter (d_1) = 300mm = 0.3m
Throat diameter (d_2) =d_1/2 = 0.15m

C/s area of inlet (A_1) = $\frac{\pi}{4}x0.3^2$ = 0.0707m^2

C/s area of throat (A_2) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Manometer reading (x) =0.24m

Sp.gr. of mercury (S) =13.6

Velocity at inlet =V_1

Velocity at throat =V_2

Head loss (h_L) =$\frac{1}{8}.\frac{V_2^2}{2g}$

Discharge in the pipe (Q) =?

Head h is given by

$h = x\left(\frac{S}{S_0} - 1\right)$= $0.24\left(\frac{13.6}{1} - 1\right)$=3.02m

Applying Bernoulli's equation between inlet (1) and throat (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + \frac{1}{8}.\frac{V_2^2}{2g} \qquad \text{(a)}$$

$$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) \qquad \text{(b)}$$

From a and b

$$h = \frac{9V_2^2}{16g} - \frac{V_1^2}{2g}$$

$$\frac{9V_2^2}{16g} - \frac{V_1^2}{2g} = 3.02 \qquad \text{(c)}$$

According to continuity equation

$A_1 V_1$=A_2V_2

$V_2 = \frac{A_1V_1}{A_2}$ =$\frac{0.0707V_1}{0.01767}$ =4V_1 (d)

Solving c and d

$$\frac{9(4V_1)^2}{16g} - \frac{V_1^2}{2g} = 3.02$$

V_1 = 1.86m/s

Discharge (Q) = A_1V_1 = 0.0707x1.86 = 0.131 m^3/s

12. A horizontal venturimeter with inlet and throat diameters of 400mm and 200mm respectively, is connected to a pipe. If the pressure in the inlet portion is 200KPa and the vacuum pressure (negative) on the throat is 400mm of mercury, find the rate of flow in the pipe taking C_d = 0.97.

Solution:

Diameter at inlet (d_1) = 400mm = 0.4m

C/s area of inlet (A_1) = $\frac{\pi}{4}x0.4^2$ = 0.1256m^2

Diameter at throat (d_2) = 200mm = 0.2m

C/s area of throat (A_2) = $\frac{\pi}{4}x0.2^2$= 0.0314m^2

Pressure at inlet (P_1) =200 Kpa = 200x10^{3}N/m^2

Pressure head at throat = -400 mm of mercury

C_d =0.97

Discharge (Q) =?

Pressure head at inlet $\left(\frac{P_1}{\rho g}\right)$ =$\frac{200000}{1000x9.81}$ =20.38m of water

Pressure head at throat $\left(\frac{P_2}{\rho g}\right)$ = -0.4 m of mercury =-0.4x13.6m of water =-5.44 m of water

Z_1=Z_2

Head h is given as

$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2)$ =20.38-(-5.44)+0 = 25.82m

$$Q = C_d\sqrt{2gh}\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.97\sqrt{2x9.81x25.82}\frac{0.1256x0.0314}{\sqrt{0.1256^2 - 0.0314^2}}$$

= 0.708 m^3/s

13. A Venturimeter is to be fitted in a horizontal pipe of 0.15m diameter to measure a flow of water which may be anything up to 240m^3/hour. The pressure head at the inlet for this flow is 18m above atmospheric and the pressure head at the throat must not be lower than 7m below atmospheric. Between the inlet and the throat there is an estimated frictional loss of 10% of the difference in pressure head between these points. Calculate the minimum allowable diameter for the throat.

Solution:

Diameter of pipe (d_1) = 0.15m

C/s area of pipe (A_1) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Discharge (Q) = 240m^3/hour = $\frac{240}{3600}$ = 0.0667m^3/s

Velocity at inlet (V_1) = Q/A_1 = 0.0667/0.01767 = 3.77m/s

Pressure head at inlet $\left(\frac{P_1}{\rho g}\right)$ =18m

Pressure head at throat $\left(\frac{P_2}{\rho g}\right)$ = -7 m

Z_1=Z_2

Head loss (h_L) = 0.1x$\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right)$ = 0.1(18+7) = 2.5m

Diameter of throat (d_2) = ?

Applying Bernoulli's equation between inlet (1) and throat (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L$$

$18 + 7 = \frac{V_2^2}{2g} - \frac{3.77^2}{2g} + 2.5$

V_2 = 21.34 m/s

C/S area of inlet (A_2) = Q/V_2 = 0.0667/21.334 = 0.003126 m^2

$A_2 = \frac{\pi}{4} x d_2^2$

$0.003126 = \frac{\pi}{4} x d_2^2$

d_2 = 0.063m = 63mm

14. A Venturimeter of throat diameter 0.07m is fitted in a 0.15m diameter vertical pipe in which liquid of relative density 0.8 flows downwards. Pressure gauges are fitted to the inlet and to the throat sections. The throat being 0.9m below the inlet. Taking the coefficient of the meter as 0.96 find the discharge a) when the pressure gauges read the same b) when the inlet gauge reads 15170 N/m^2 higher than the throat gauge.

Solution:

Diameter of pipe (d_1) = 0.15m

C/s area of pipe (A_1) = $\frac{\pi}{4} x 0.15^2$= 0.01767m^2

Diameter at throat (d_2) = 0.07m

C/s area of throat (A_2) = $\frac{\pi}{4} x 0.07^2$= 0.00385m^2

$Z_1 - Z_2$ = 0.9m

C_d = 0.96

Discharge (Q) = ?

a) $P_1 = P_2$

$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) = 0 + 0.9$ =0.9m

$Q = C_d \sqrt{2gh} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$

$= 0.96\sqrt{2x9.81x0.9} \frac{0.01767x0.00385}{\sqrt{0.01767^2 - 0.00385^2}}$

= 0.016 m^3/s

b) $P_1 = P_2$ +15170

$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) = \frac{15170}{0.8x1000x9.81} + 0.9$ = 2.83m

$Q = C_d \sqrt{2gh} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$

$= 0.96\sqrt{2x9.81x2.83} \frac{0.01766x0.00385}{\sqrt{0.01766^2 - 0.00385^2}}$

= 0.0285 m^3/s

15. Water is flowing over a sharp-crested rectangular weir of width 50cm into a tank with cross-sectional area 0.6m^2. After a period of 30s the depth of water in the tank is 1.4m. Assuming a discharge coefficient of 0.9, determine the height of the water above the weir.
If the rectangular weir is replaced by a 90^0 notch weir with the same head and a discharge coefficient of 0.8, calculate the depth increase of the water in the tank after 30s.

Solution:
a) Width of rectangular weir (L) = 50cm = 0.5m
C/S area (A) = 0.6m^2
Depth of water in tank (h) = 1.4m
Volume of water in 30 Sec = Ah = 0.6x1.4 = 0.84m^3
Discharge (Q) = Volume/time = 0.84/30 = 0.028 m^3/s
C_d = 0.9
Height of water (H) =?

$Q = \frac{2}{3} C_d \sqrt{2g}\, LH^{3/2}$
$0.028 = \frac{2}{3} x0.9\sqrt{2x9.81}\, x0.5xH^{3/2}$
H = 0.076m

b) With 90^0 V-notch, C_d = 0.8
Head (H) = 0.076m
Increase in depth after 30s (h_1) = ?
$Q = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2)\, H^{5/2}$
$Q = \frac{8}{15} x0.8\sqrt{2x9.81} \tan(90/2)\, 0.076^{5/2}$ = 0.003 m^3/s

$Q = \frac{Ah_1}{t}$
$0.003 = \frac{0.6xh_1}{30}$
h_1 = 0.15m

16. A sharp edged weir is to be constructed across a stream in which the normal flow is 200 litres/sec. If the maximum flow likely to occur in the stream is 5 times the normal flow then determine the length of weir necessary to limit the rise in water level to 40cm above that for normal flow. C_d=0.61.

Solution:
Normal discharge (Q_1) = 200 lps = 0.2 m^3/s
Maximum discharge (Q_2) = 5x0.2 = 1 m^3/s
C_d=0.61
Water level for normal flow = y
For y+0.4, Length of weir (L) = ?

$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{3/2}$

$Q_1 = \frac{2}{3} x 0.61 \sqrt{2x9.81}\, L y^{3/2}$
$0.2 = 1.801\, L y^{3/2}$
$L = 0.111\, y^{-3/2}$ (a)

$Q_2 = \frac{2}{3} x 0.61 \sqrt{2x9.81}\, L(y + 0.4)^{3/2}$
$1 = 1.801\, L(y + 0.4)^{3/2}$
$0.555 = L(y + 0.4)^{3/2}$ (b)

From a and b
$0.555 = 0.111\, y^{-3/2} (y + 0.4)^{3/2}$
$\frac{(y+0.4)^{3/2}}{y^{3/2}} = 5$
$\frac{y+0.4}{y} = 2.924$
y = 0.208m
$L = 0.111\, y^{-3/2} = 0.111\ 0.208^{-3/2}$ = 1.17m

17. Compute the flow rate if the measured head above the bottom of the V-notch is 35cm, when θ= 60^0 and C_d=0.6. If the flow is wanted within an accuracy of 2%, what are the limiting values of the head.

Solution:
Head (H) = 35cm = 0.35m
θ= 45^0
C_d=0.6
Flow rate (Q) = ?

$Q = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2)\, H^{5/2}$
$Q = \frac{8}{15} x 0.6 \sqrt{2x9.81} \tan(60/2)\, H^{5/2} = 0.818 H^{5/2}$
$= 0.818 x 0.35^{5/2}$ = 0.059 m^3/s

With $\pm 2\%$ error
Q_1 = 0.059+0.02X0.059= 0.0602 m^3/s
Q_2 = 0.059-0.02X0.059= 0.0578 m^3/s

$Q = 1.417 H^{5/2}$
$Q_1 = 1.417 H_1^{5/2}$
$0.0602 = 0.818 H_1^{5/2}$

H_1 = 0.352 m

$Q_2 = 1.417H_2^{5/2}$
$0.0578 = 0.818H_2^{5/2}$
H_2 = 0.346 m

18. Water is flowing in a rectangular channel of 1m wide and 0.8m deep. Find the discharge over a rectangular weir of crest length 60cm if the head of water over the crest of weir is 30cm and water from channel flows over the weir. Take C_d = 0.62. Take velocity of approach into consideration.

Solution:
Area of channel (A) = 1x0.8 = 0.8 m^2
Length of weir (L) = 0.6m
Head of water (H_1) = 0.3m
C_d = 0.62

Discharge over a rectangular weir without velocity of approach is

$$Q = \frac{2}{3} C_d\, L\, \sqrt{2g} H_1^{3/2}$$
$$= \frac{2}{3} x0.62x0.6\, \sqrt{2x9.81} x0.3^{3/2} = 0.18 \text{ m}^3/\text{s}$$

Velocity of approach (V_a) = Q/A = 0.18/0.8 = 0.225 m/s
Velocity head $(h_a) = \frac{V_a^2}{2g} = \frac{0.225^2}{2x9.81} =$ 0.00258m

$$Q = \frac{2}{3} C_d\, L\, \sqrt{2g} \left[(H_1 + h_a)^{3/2} - h_a^{3/2}\right]$$
$$= \frac{2}{3} x0.62x0.6\, \sqrt{2x9.81}\left[(0.3 + 0.00258)^{3/2} - 0.00258^{3/2}\right] = 0.183 \text{ m}^3/\text{s}$$

19. A broad-crested weir of length 40m, has 400mm height of water above its crest. Find the maximum discharge neglecting velocity of approach. If the velocity of approach is taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 40m^2 on the upstream side. Take C_d =0.6.

Solution:
Length of weir (L) = 40m
Head of water (H_1) = 0.4m
C_d = 0.6

Maximum discharge neglecting velocity of approach

$$Q_{max} = 1.705 C_d\, L H^{3/2}$$
$$= 1.705x0.6x40x0.4^{3/2} = 10.352 \text{ m}^3/\text{s}$$

Taking velocity of approach into consideration

Area of channel (A) = 40 m^2
Velocity of approach (V_a) = Q_{max}/A = 10.35/40 = 0.25875 m/s
Velocity head $(h_a) = \frac{V_a^2}{2g} = \frac{0.25875^2}{2x9.81}$ = 0.0034m
Maximum discharge considering velocity of approach

$$Q_{max} = 1.705 C_d\, L \left[(H + h_a)^{3/2} - h_a^{3/2}\right]$$
$$= 1.705x0.6x40\left[(0.4 + 0.0034)^{3/2} - 0.0034^{3/2}\right] = 10.476\ m^3/s$$

20. The heights of water on the upstream and downstream side of a submerged weir of length 3.5m are 300mm and 150mm respectively. If C_d for free and drowned portion is 0.6 and 0.8 respectively, find the discharge over the weir.

Solution:
Height of water on upstream side (H) = 0.3m
Height of water on downstream side (h) = 0.15m
Length of weir (L) = 3.5m
C_{d1} = 0.6, C_{d2} = 0.8
Discharge through drowned orifice (Q)= ?

$$Q = \frac{2}{3} C_{d1} L \sqrt{2g}(H - h)^{3/2} + C_{d2} L h \sqrt{2g(H - h)}$$
$$= \frac{2}{3} x0.6x3.5\sqrt{2x9.81}(0.3 - 0.15)^{3/2} + 0.8x3.5x0.15\sqrt{2x9.81(0.3 - 0.15)} = 1.08\ m^3/s$$

21. A 1.25m diameter circular tank contains water up to a height of 5m. At the bottom of the tank, an orifice of 50mm diameter is provided. Find the height of water above the orifice after 1.5 minutes. Take C_d = 0.62.

Solution:
Diameter of tank (D) =1.25m
Area of tank (A) = $\frac{\pi}{4} x1.25^2$ = 1.227 m^2
Diameter of orifice (d) =50mm = 0.05m
Area of orifice (a) = $\frac{\pi}{4} x0.05^2$ = 0.00196 m^2
Coeff. of discharge (C_d) = 0.62
Initial head (H_1) = 5m
Time (t) =1.5 minutes = 90 Sec
Final head (H_2) =?

From continuity
-Q dt = Adh
$t = -\int_{H_1}^{H_2} \frac{A}{Q} dh$

$$= -\int_{H_1}^{H_2} \frac{A}{C_d a_0 \sqrt{2gh}} dh$$

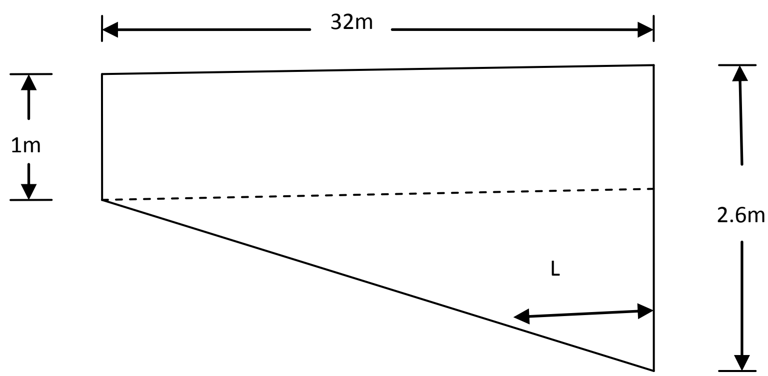
$$= \frac{2A\left(\sqrt{H_1}-\sqrt{H_2}\right)}{C_d.a\sqrt{2g}}$$

$$90 = \frac{2x1.227\left(\sqrt{5}-\sqrt{H_2}\right)}{0.62x0.00196\sqrt{2x9.81}}$$

$$0.197 = \left(2.236 - \sqrt{H_2}\right)$$

H_2 = 4.15m

22. A rectangular swimming pool is 1m deep at one end and increases uniformly in depth to 2.6m at the other end. The pool is 8m wide and 32m long and is emptied through an orifice of area 0.224m^2, at the lowest point in the side of the deep end. Taking C_d for the orifice as 0.6, find a) the time for the depth to fall by 1m b) the time to empty the pool completely.



Solution:

Area of orifice (a_0) = 0.224m^2

Cd = 0.6

a) For 1m fall in depth

H_1= 2.6m, H_2 = 1.6m

Time to fall 1m depth (t1) = ?

A = 32x8 = 256m^2

From continuity

-Q dt = Adh

$$t1 = -\int_{H_1}^{H_2} \frac{A}{Q} dh$$

$$= -\int_{H_1}^{H_2} \frac{A}{C_d a_0 \sqrt{2gh}} dh$$

$$= \frac{2A\left(\sqrt{H_1}-\sqrt{H_2}\right)}{C_d.a\sqrt{2g}}$$

$= \frac{2x256(\sqrt{2.6}-\sqrt{1.6})}{0.6x0.224\sqrt{2x9.81}}$
= 299 S

b) Time to completely empty the tank (t) = ?
Let us find out the time to empty the tank (t2) from H_1 = 1.6m to H_2 = 0m
We need A in terms of h.
A =8L
$\frac{L}{h} = \frac{32}{1.6}$
L = 20h
A = 8x20h = 160h

-Q dt = Adh
$t2 = -\int_{H_1}^{H_2} \frac{A}{Q} dh$

$= -\int_{H_1}^{H_2} \frac{160h}{C_d a_0 \sqrt{2gh}} dh = \frac{160}{C_d a_0 \sqrt{2g}} \int_{H_1}^{H_2} h^{1/2}\, dh$

$= \frac{160}{0.6x0.224\sqrt{2x9.81}} x \frac{2}{3} \left(H_1^{3/2} - H_2^{3/2}\right)$
$= 179.2\left(1.6^{3/2} - 0\right)$ = 363 S

Time to completely empty the tank (t1) = t1 + t2 = 299 + 363 = 662S

23. A cylindrical tank of internal diameter 0.6m, length 1.5m and axis vertical has a 5cm diameter sharp-edged orifice (C_d = 0.6) in the bottom, open to atmosphere. The tank is open at the top and empty. If water were admitted into the tank from above at a constant rate of 14lps, how long will it take to just fill the tank? How much water will escape through the orifice during that period?

Solution:
Diameter of tank (D) =0.6m
Area of tank (A) = $\frac{\pi}{4} x 0.6^2$ = 0.2827 m^2
Diameter of orifice (d) =5cm = 0.05m
Area of orifice (a) = $\frac{\pi}{4} x 0.05^2$ = 0.00196 m^2
Coeff. of discharge (C_d) = 0.6
$K = Cd.a\sqrt{2g} = 0.6x0.00196x\sqrt{2x9.81}$=0.0052
Inflow (Q_i) = 14 lps =0.014 m^3/s
Initial head (H_1) = 0m
Final head (H_2) = 1.5m
Time required to fill the tank (t) =?
Water escaped through orifice =?

$$t = \frac{2A}{K^2}\left[Q_i\, ln\frac{Q_i - K\sqrt{H_2}}{Q_i - K\sqrt{H_1}} + K\left(\sqrt{H_2} - \sqrt{H_1}\right)\right]$$

$$t = \frac{2x0.2827}{0.0052^2}\left[0.014\, ln\frac{0.014 - 0.0052\sqrt{1.5}}{0.014 - 0.0052\sqrt{0}} + 0.0052\left(\sqrt{1.5} - \sqrt{0}\right)\right]$$

t = 44.46 Sec

Inflow volume = Q_i X t = 0.014x44.46 = 0.6224 m^3
Volume of water contained in the tank = A x H_2 = 0.2827x1.5 = 0.424 m^3
Water escaped through orifice = 0.6224 – 0.424 = 0.1984 m^3

24. A vertical cylindrical tank 2m diameter has, at the bottom, 0.05m diameter sharp-edged orifice (Cd = 0.6).

(I) If the water enters the tank at a constant rate of 0.0095 cumecs, find the depth of water above the orifice when the level in the tank becomes stable.
(II) Find the time for the level to fall from 3m to 1m above the orifice when the inflow is turned off.
(III) If water now runs into the tank at 0.02cumecs, the orifice remaining open, find the rate of rise in water level when the level has reached a depth of 1.7m above the orifice.

Solution:
Diameter of tank (D) =2m
Area of tank (A) = $\frac{\pi}{4}x2^2$ = 3.14 m^2
Diameter of orifice (d) =5cm = 0.05m
Area of orifice (a) = $\frac{\pi}{4}x0.05^2$ = 0.00196 m^2
Coeff. of discharge (Cd) = 0.6

General equation for tank with inflow (Q_i) and outflow (Q_o)

$$(Q_i - Q_o)dt = Adh$$

Where $Q_o = Cd.a\sqrt{2gh} = k\sqrt{h}$ =Discharge through orifice
$K = Cd.a\sqrt{2g} = 0.6x0.00196x\sqrt{2x9.81}$=0.0052

I) Q_i = 0.0095 m^3/s
Depth of water above orifice (h) =?
For stable condition, dh =0
$Q_i = Q_o = K\sqrt{h}$
$0.0095 = 0.0052\sqrt{h}$
h = 3.34 m

II) Initial Head (H_1) = 3m

Final Head (H_2) = 1m
Time required to lower the head from H_1 to H_2 (t) =?
When the inflow is turned off, Q_i = 0

$$(-Q_o)dt = Adh$$

$$dt = -\frac{Adh}{Q_o} = -\frac{Adh}{K\sqrt{h}}$$

$$t = \int_{H_1}^{H_2} -\frac{Adh}{K\sqrt{h}} = \frac{2A}{K}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$

$$= \frac{2x3.14}{0.0052}\left(\sqrt{3} - \sqrt{1}\right)$$

= 885Sec

III) Q_i = 0.2 m^3/s
Head (h) =1.7m
Rate of rise in water level (dh/dt) =?

$$(Q_i - Q_o)dt = Adh$$

$$\frac{dh}{dt} = \frac{Q_i - Q_o}{A} = \frac{Qi - K\sqrt{h}}{A}$$

$$\frac{dh}{dt} = \frac{0.2 - 0.0052\sqrt{1.7}}{3.14}$$

= 0.0615 m/s

25. A cylindrical tank is placed with its axis vertical and is provided with a circular orifice of 4cm diameter at the bottom. A steady inflow and free discharge at the bottom of the orifice causes the depth of water in the tank to rise from 0.59m to 0.75m in 106 Sec. Further it is observed that the depth rises from 1.2m to 1.29m in 129 Sec. Determine the inflow rate and the diameter of the tank. Assume C_d = 0.62.

Solution:

Diameter of orifice (d) = 4cm = 0.04m
Area of orifice (a) = $\frac{\pi}{4}x0.04^2$ = 0.001257 m^2
Coeff. of discharge (C_d) = 0.62
$K = Cd.a\sqrt{2g} = 0.62x0.001257x\sqrt{2x9.81}$=0.00345
Inflow rate (Q_i) =?
Diameter of tank (D) =?

First case
dh = 0.75-0.59 = 0.16m
dt = 107S
Average head (h) = 0.67m
dh/dt = 0.001495

Second case

dh = 1.29-1.2 = 0.09m

dt = 129S

Average head (h) = 1.245m

dh/dt = 0.000698

$$(Q_i - Q_o)dt = Adh$$

$$\frac{dh}{dt} = \frac{Q_i - Q_o}{A} = \frac{Qi - K\sqrt{h}}{A}$$

Substituting values for both cases

$$0.001495 = \frac{Qi - 0.00345\sqrt{0.67}}{A}$$

Q_i=0.001495A+0.002824 (a)

$$0.000698 = \frac{Qi - 0.00345\sqrt{1.245}}{A}$$

Q_i=0.000698A+0.003849 (b)

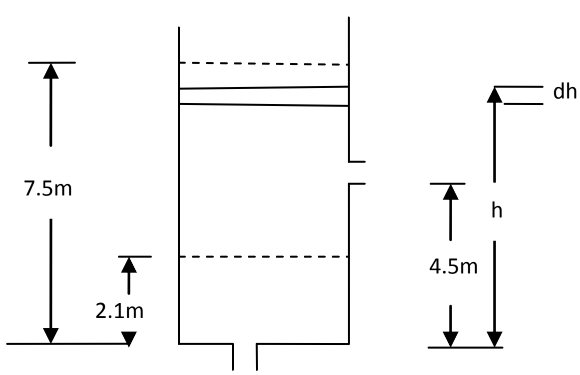
Solving a and b

A =1.285 m^2

Q_i = 0.0047 m^3/s

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4x1.285}{\pi}} = 1.28m$$

26. A tank of constant cross-sectional area of 2.8m^2 has two orifices each 9.3x10^{-4} m^2 in area as shown in the figure. Calculate the time taken to lower the water level from 7.5m to 2.1m above the bottom of the tank. Assume C_d = 0.62.



Solution:

C/s of tank (A) = 2.8m^2

C/s of orifice (a) = 9.3x10^{-4} m^2

Let the water level is at height h above the lower orifice at time t. dh is the decrease in water level in time dt.
Q = discharge
Qdt = -Adh
dt = -Adh/Q
Discharge from top orifice (Q1) = $C_d a\sqrt{2g(h-4.5)}$
Discharge from lower orifice (Q2) = $C_d a\sqrt{2gh}$
Q = Q1+Q2 = $C_d a\sqrt{2g(h-4.5)} + C_d a\sqrt{2gh}$
H1 = 7.5m, H2 = 4.5m
T1 = Time taken to lower the water from 7.5m to 4.5m when both orifices are discharging
T2 = Time taken to lower the water from 4.5m to 2.1m when only lower orifice is discharging

Finding T1

$$dt = -\frac{Adh}{Q} = -\frac{2.8dh}{C_d a\sqrt{2g(h-4.5)}+C_d a\sqrt{2gh}}$$

$$dt = -\frac{2.8dh}{C_d a\sqrt{2g}\left[\sqrt{(h-4.5)}+\sqrt{h}\right]} = -\frac{2.8dh}{0.62x9.3x10^{-4}\sqrt{2g}\left[\sqrt{(h-4.5)}+\sqrt{h}\right]}$$

$$dt = -1096.3\frac{dh}{\left[\sqrt{(h-4.5)}+\sqrt{h}\right]}$$

Integrating

$$T1 = -1096.3\int_{4.5}^{7.5}\frac{dh}{\left[\sqrt{(h-4.5)}+\sqrt{h}\right]}$$

$$T1 = -1096.3\int_{4.5}^{7.5}\frac{\left[\sqrt{(h-4.5)}-\sqrt{h}\right]dh}{-4.5}$$

= 942Sec

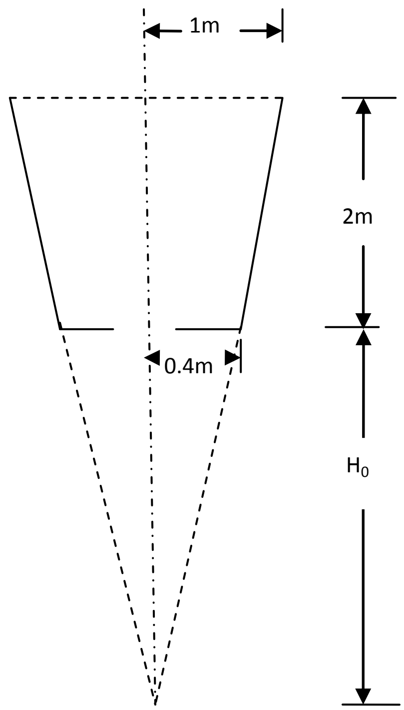
Finding T2 using direct formula
H1 = 4.5m, H2 = 2.1m

$$T2 = \frac{2A}{C_d a\sqrt{2g}}\left[\sqrt{H1}-\sqrt{H2}\right]$$

$T2 = \frac{2x2.8}{0.62x9.3x10^{-4}\sqrt{2g}}\left[\sqrt{4.5}-\sqrt{2.1}\right]$ = 1474 Sec

Total time taken to empty the tank from 7.5m to 2.1m = T1+T2 = 2416Sec = 40.26 min

27. A tank is in the form of frustum of a cone having top diameter of 2m, a bottom diameter of 0.8m and height 2m and is full of water. Find the time of emptying the tank through an orifice 100mm in diameter provided at the bottom. Take C_d = 0.625.



Solution:

C_d = 0.625

Area of orifice (a) = $\frac{\pi}{4} x 0.1^2$= 0.00785 m^2

H_1 = 2m, H_2 = 0m, R_1 = 1m, R_0 = 0.4m

From similar triangles,

$$\frac{1}{2+H_0} = \frac{0.4}{H_0}$$

H_0 = 1.33m

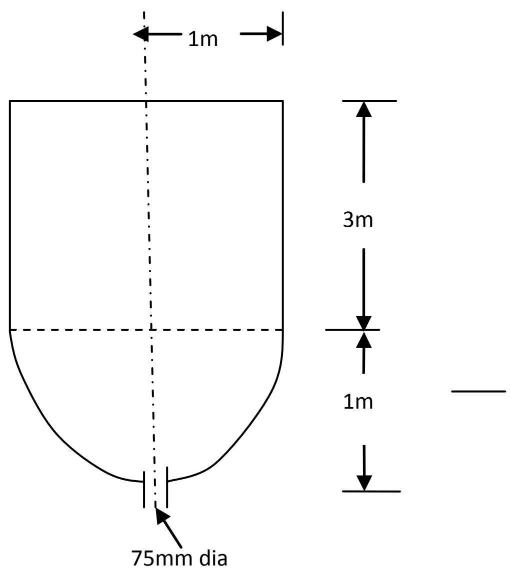
$K = \frac{1}{C_d a\sqrt{2g}} \frac{\pi R_1^2}{(H_1+H_0)^2} = \frac{1}{0.625x0.00785\sqrt{2x9.81}} \frac{\pi x 1^2}{(2+1.33)^2}$ = 13.03

Time of emptying the tank is

$$T = K\left[\frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right) + \frac{4}{3}H_0\left(H_1^{3/2} - H_2^{3/2}\right) + 2H_0^2\left(H_1^{1/2} - H_2^{1/2}\right)\right]$$

$= 13.03\left[\frac{2}{5}\left(2^{5/2} - 0^{5/2}\right) + \frac{4}{3}x1.33\left(2^{3/2} - 0^{3/2}\right) + 2x1.33^2\left(2^{1/2} - 0^{3/2}\right)\right]$ = 160 Sec

28. A tank is in the form of hemisphere of 2m diameter and having a cylindrical upper part of 2m diameter and 3m height. Find the time of emptying the tank through an orifice of 75mm diameter at its bottom if the tank is initially full of water. Take C_d = 0.62.



Solution:

C_d = 0.625

Area of cylinder (A) = $\frac{\pi}{4} x2^2$= 3.14 m^2

Area of orifice (a) = $\frac{\pi}{4} x0.075^2$= 0.004418 m^2

T1 = time taken to lower water from 4m to 1m in the cylindrical part

T2 = time taken to empty the hemispherical part from 1m to 0m

Computing T1

H1 = 4m, H2 = 1m

$$T1 = \frac{2A}{C_d a\sqrt{2g}}\left[\sqrt{H1} - \sqrt{H2}\right]$$

$$T1 = \frac{2x3.14}{0.62x0.004418\sqrt{2g}}\left[\sqrt{4} - \sqrt{1}\right]$$ = 518Sec

Computing T2

H1 = 1m, H2 = 0m

$$T2 = \frac{\pi}{C_d a\sqrt{2g}}\left[\frac{4R}{3}\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

$$T2 = \frac{\pi}{0.62x0.004418\sqrt{2g}}\left[\frac{4x1}{3}\left(1^{3/2} - 0\right) - \frac{2}{5}\left(\left(1^{5/2} - 0\right) - 0\right)\right]$$ = 242 Sec

Total time taken to empty the tank = T1+T2 = 518+242 = 760 Sec