

# Tutorial 7

## Fluid Dynamics

1. Oil with sp gr 0.75 is flowing through a 15cm diameter pipe under a pressure of 105KN/m<sup>2</sup>. If the total energy relative to a datum plane 2.5m below the center of the pipe is 18m, determine the flow rate of oil.

Solution:

Pressure (P) = 105 Kpa

Diameter of pipe (d) = 15cm = 0.15m

C/S Area of pipe (A) =  $\frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$

Datum head (Z) = 2.5m

Total energy (E) = 18m

Flow rate (Q) = ?

According to Bernoulli's equation,

$$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z$$

$$18 = \frac{105000}{0.75 \times 9810} + \frac{V^2}{2g} + 2.5$$

$$V = 4.91 \text{ m/s}$$

$$Q = AV = 0.01767 \times 4.91 = 0.0867 \text{ m}^3/\text{s}$$

2. A fluid is flowing in a 20cm diameter pipe at a pressure of 28 KN/m<sup>2</sup> with a velocity of 2.4m/s. The elevation of center of pipe above a given datum is 4m. Find the total energy head above the given datum if the fluid is (a) water, (b) oil of sp gr 0.82, and (c) gas with a specific weight of 6.4 N/m<sup>3</sup>.

Solution:

Pressure (P) = 28 Kpa

Velocity (V) = 2.4m/s

Datum head (Z) = 4m

Energy head (E) = ?

a) For water,  $\gamma = 9810 \text{ N/m}^3$

$$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{28000}{9810} + \frac{2.4^2}{2g} + 4 = 7.15 \text{ m}$$

b) For oil,  $\gamma = 0.82 \times 9810 = 8044.2 \text{ N/m}^3$

$$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{28000}{8044.2} + \frac{2.4^2}{2g} + 4 = 7.77 \text{ m}$$

b) For gas,  $\gamma = 6.4 \text{ N/m}^3$

$$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{28000}{6.4} + \frac{2.4^2}{2g} + 4 = 4379.3\text{m}$$

3. A pipe, through which water is flowing is having diameters 40cm and 20cm at sections 1 and 2 respectively. The velocity of water at section 1 is 5m/s. Find the velocity head at sections 1 and 2 and also compute discharge.

Solution:

Diameter of pipe at section1 ( $d_1$ ) = 40cm = 0.4m

C/S Area of pipe at section1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.4^2 = 0.1256 \text{ m}^2$

Velocity of pipe at section1 ( $V_1$ ) = 5m/s

Diameter of pipe at section2 ( $d_2$ ) = 20cm = 0.2m

C/S Area of pipe at section2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Velocity heads = ?

Discharge (Q) = ?

$Q = A_1 V_1 = 0.1256 \times 5 = 0.628 \text{ m}^3/\text{s}$

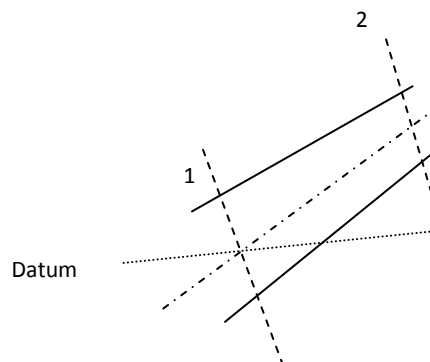
$Q = A_1 V_1 = A_2 V_2$

$V_2 = Q/A_2 = 0.628/0.0314 = 20\text{m/s}$

Velocity head at section 1 =  $\frac{V_1^2}{2g} = \frac{5^2}{2g} = 1.274\text{m}$

Velocity head at section 2 =  $\frac{V_2^2}{2g} = \frac{20^2}{2g} = 20.38\text{m}$

4. The water is flowing through a taper pipe of length 50m having diameters 40cm at the upper end and 20cm at the lower end, at the rate of 60 lps. The pipe has a slope of 1 in 40. Find the pressure at the lower end if the pressure at the higher level is 24.525 N/cm<sup>2</sup> (a) assuming no loss of energy (b) a loss of 0.2m.



Solution:

Diameter of pipe at section1 ( $d_1$ ) = 20cm = 0.2m

C/S Area of pipe at section1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Diameter of pipe at section 2 ( $d_2$ ) = 40cm = 0.4m

C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.4^2 = 0.1256 \text{ m}^2$

Discharge ( $Q$ ) = 60lps =  $60 \times 10^{-3} \text{ m}^3/\text{s} = 0.06 \text{ m}^3/\text{s}$

Velocity at section 1 ( $V_1$ ) =  $Q/A_1 = 0.06/0.0314 = 1.91\text{m/s}$

Velocity at section 2 ( $V_2$ ) =  $Q/A_2 = 0.06/0.1256 = 0.47\text{m/s}$

Pressure at section 2 ( $P_2$ ) =  $24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Pressure at section 1 ( $P_1$ ) = ?

Slope =  $\tan\theta = 1/40$

$\theta = 1.43^\circ$

Taking datum head at section 1 ( $Z_1$ ) = 0

$Z_2 = 50\text{Sin}1.43 = 1.25\text{m}$

a. Applying Bernoulli's equation at section 1 and 2 (considering no loss)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{9810} + \frac{1.91^2}{2g} + 0 = \frac{24.525 \times 10^4}{9810} + \frac{0.47^2}{2g} + 1.25$$

$$P_1 = 255799 \text{ N/m}^2$$

b. Applying Bernoulli's equation at section 1 and 2

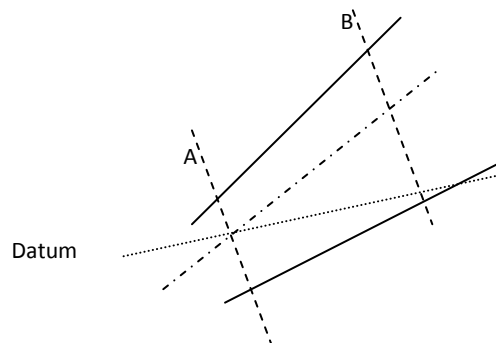
Loss of head ( $h_L$ ) = 0.1m

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{9810} + \frac{1.91^2}{2g} + 0 = \frac{24.525 \times 10^4}{9810} + \frac{0.47^2}{2g} + 1.25 + 0.1$$

$$P_1 = 256780 \text{ N/m}^2$$

5. A pipe line carrying oil of sp.gr. 0.8, changes in diameter from 300mm at a position A to 500mm at position B which is 5m at a higher level. If the pressures at A and B are  $19.62 \text{ N/cm}^2$  and  $14.91 \text{ N/cm}^2$  respectively, and the discharge is 150 lps, determine the loss of head and the direction of flow.



Solution:

Diameter of pipe at section A ( $d_1$ ) = 300mm = 0.3m

$$\text{C/S Area of pipe at section A } (A_1) = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

$$\text{Diameter of pipe at section B } (d_2) = 500\text{mm} = 0.5\text{m}$$

$$\text{C/S Area of pipe at section B } (A_2) = \frac{\pi}{4} \times 0.5^2 = 0.1963 \text{ m}^2$$

$$\text{Discharge } (Q) = 150\text{ps} = 150 \times 10^{-3} \text{ m}^3/\text{s} = 0.15 \text{ m}^3/\text{s}$$

$$\text{Velocity at section A } (V_1) = Q/A_1 = 0.15/0.0707 = 2.12\text{m/s}$$

$$\text{Velocity at section B } (V_2) = Q/A_2 = 0.15/0.1963 = 0.764\text{m/s}$$

$$\text{Datum at section A } (Z_1) = 0$$

$$\text{Datum at section B } (Z_2) = 5\text{m}$$

$$\text{Pressure at section A } (P_1) = 19.62 \text{ N/cm}^2 = 19.62 \times 10^{-4} \text{ N/m}^2$$

$$\text{Pressure at section B } (P_2) = 14.91 \text{ N/cm}^2 = 14.91 \times 10^{-4} \text{ N/m}^2$$

$$\text{Loss of head } (h_L) = ?$$

$$\text{Direction of flow} = ?$$

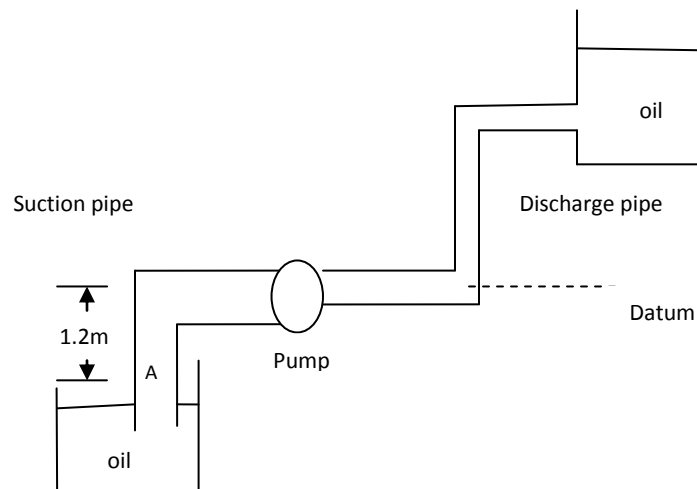
$$\text{Energy head at A } (E_A) = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{196200}{0.8 \times 9810} + \frac{2.12^2}{2g} + 0 = 25.229\text{m}$$

$$\text{Energy head at B } (E_B) = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{149100}{0.8 \times 9810} + \frac{0.764^2}{2g} + 5 = 24.028$$

$$h_L = E_A - E_B = 25.229 - 24.028 = 1.201\text{m}$$

As  $E_A$  is greater than  $E_B$ , the flow takes from A to B.

6. A 100mm diameter suction pipe leading to a pump carries a discharge of  $0.03 \text{ m}^3/\text{s}$  of oil (sp gr = 0.85). If the pressure at point A in the suction pipe is a vacuum of 180mmHg, find the total energy head at point A w.r.t a datum at the pump.



Solution:

$$\text{Diameter of pipe at section A } (d) = 100\text{mm} = 0.1\text{m}$$

$$\text{C/S Area of pipe at section A } (A) = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

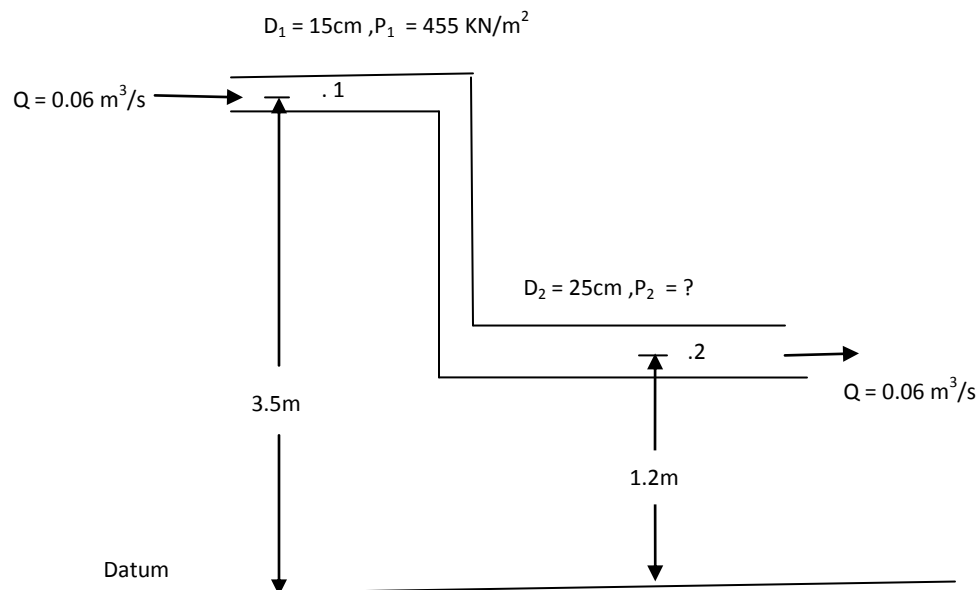
$$\text{Discharge } (Q) = 0.03 \text{ m}^3/\text{s}$$

Velocity at point A ( $V$ ) =  $Q/A = 0.03/0.00785 = 3.82\text{m/s}$   
 Pressure head at A ( $h$ ) =  $-180\text{mm Hg} = -0.18\text{ m of Hg}$   
 Pressure at A ( $P$ ) =  $\gamma_{Hg}h = 13.6 \times 9810 \times (-0.18) = -24015\text{ Pa}$   
 Datum head ( $Z$ ) =  $-1.2$   
 Total energy head at A ( $E$ ) = ?

According to Bernoulli's equation,

$$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{-24015}{0.85 \times 9810} + \frac{3.82^2}{2g} - 1.2 = -3.336\text{m}$$

7. Oil (sp gr = 0.84) is flowing in a pipe under the conditions shown in the fig. If the total head loss from point 1 to point 2 is 0.9m, find the pressure at point 2.



Solution:

Diameter of pipe at section 1 ( $d_1$ ) =  $15\text{cm} = 0.15\text{m}$   
 C/S Area of pipe at section 1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.15^2 = 0.0176\text{ m}^2$   
 Diameter of pipe at section 2 ( $d_2$ ) =  $25\text{cm} = 0.25\text{m}$   
 C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.25^2 = 0.049\text{ m}^2$   
 Discharge ( $Q$ ) =  $0.06\text{ m}^3/\text{s}$   
 Velocity at point 1 ( $V_1$ ) =  $Q/A_1 = 0.06/0.0176 = 3.4\text{m/s}$   
 Velocity at point 2 ( $V_2$ ) =  $Q/A_2 = 0.06/0.049 = 1.22\text{m/s}$   
 Datum head at 1 ( $Z_1$ ) =  $3.5\text{m}$   
 Datum head at 2 ( $Z_2$ ) =  $1.2\text{m}$   
 Head loss ( $h_L$ ) =  $0.9\text{m}$   
 Pressure at 1 ( $P_1$ ) =  $455\text{ Kpa} = 455000\text{ Pa}$

Pressure at 2 ( $P_2$ ) = ?

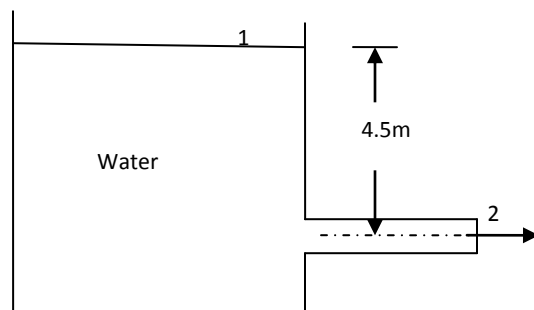
Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{455000}{0.84 \times 9810} + \frac{3.4^2}{2g} + 3.5 = \frac{P_2}{0.84 \times 9810} + \frac{1.22^2}{2g} + 1.2 + 0.9$$

$$P_2 = 470767 \text{ Pa} = 470.76 \text{ KPa}$$

8. A 20cm diameter horizontal pipe is attached to a reservoir as shown in fig. If the total head loss between the water surface in the reservoir and the water jet at the end of the pipe is 1.8m, what are the velocity and flow rate of water being discharged from the pipe?



Solution:

Diameter of pipe at section 2 ( $d_2$ ) = 20cm = 0.2m

C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Velocity at point 1 ( $V_1$ ) = 0 m/s

Datum head at 1 ( $Z_1$ ) = 4.5m

Datum head at 2 ( $Z_2$ ) = 0

Head loss ( $h_L$ ) = 1.8m

Pressure at 1 ( $P_1$ ) = 0 (atmospheric)

Pressure at 2 ( $P_2$ ) = 0 (atmospheric)

Velocity at point 2 ( $V_2$ ) = ?

Discharge from point 1 ( $Q_2$ ) = ?

Applying Bernoulli's equation at 1 and 2

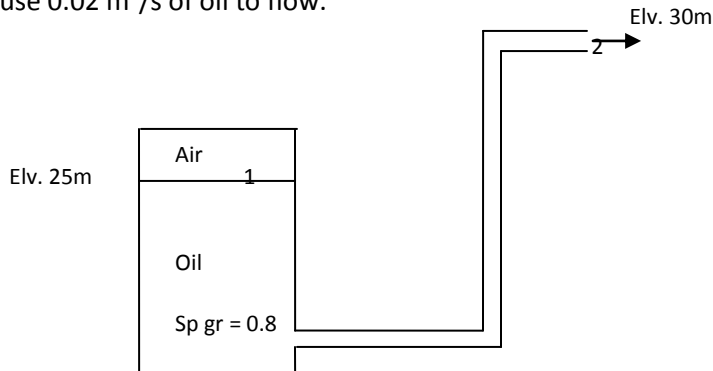
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$0 + 0 + 4.5 = 0 + \frac{V_2^2}{2g} + 0 + 1.8$$

$$V_2 = 7.28 \text{ m/s}$$

$$Q_2 = A_2 V_2 = 7.28 \times 0.0314 = 0.228 \text{ m}^3/\text{s}$$

9. Oil flows from a tank through 140m of 15cm diameter pipe and then discharge into air as shown in the fig. If the head loss from point 1 to point 2 is 0.55m of oil, determine the pressure needed at point 1 to cause 0.02 m<sup>3</sup>/s of oil to flow.



Solution:

Diameter of pipe at section 2 ( $d_2$ ) = 15cm = 0.15m

C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.15^2 = 0.0176 \text{ m}^2$

Velocity at point 1 ( $V_1$ ) = 0 m/s

Datum head at 1 ( $Z_1$ ) = 25m

Datum head at 2 ( $Z_2$ ) = 30m

Head loss ( $h_L$ ) = 0.55m

Pressure at 2 ( $P_2$ ) = 0 (atmospheric)

Discharge ( $Q$ ) = 0.02 m<sup>3</sup>/s

Velocity at point 2 ( $V_2$ ) =  $Q/A_2 = 0.02/0.0176 = 1.14\text{m/s}$

Pressure at 1 ( $P_1$ ) = ?

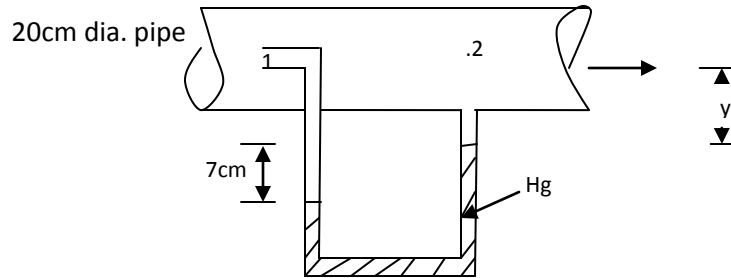
Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{0.8 \times 9810} + 0 + 25 = 0 + \frac{1.14^2}{2g} + 30 + 0.55$$

$$P_1 = 43037 \text{ Pa} = 43.037 \text{ Kpa}$$

10. In the fig., one end of a U-tube is oriented directly into the flow so that the velocity of the stream is zero at this point (stagnation point). Neglecting friction, determine the flow of water in the pipe.



Solution:

Diameter of pipe ( $d$ ) = 20cm = 0.2m

C/S Area of pipe ( $A$ ) =  $\frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Velocity at point 1 ( $V_1$ ) = 0 m/s

Taking center of pipe as datum

Datum head at 1 ( $Z_1$ ) = 0

Datum head at 2 ( $Z_2$ ) = 0

Pressure at 1 =  $P_1$

Pressure at 2 =  $P_2$

Velocity at point 2 =  $V_2$

Flow rate ( $Q$ ) = ?

Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{\gamma} + 0 + 0 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0$$

$$V_2 = \sqrt{\frac{2g}{\gamma} (P_1 - P_2)}$$

Writing manometric equation

$$P_1 + \gamma_{\text{water}}(y + 0.07) = P_2 + \gamma_{\text{water}}y + \gamma_{\text{Hg}} \times 0.07$$

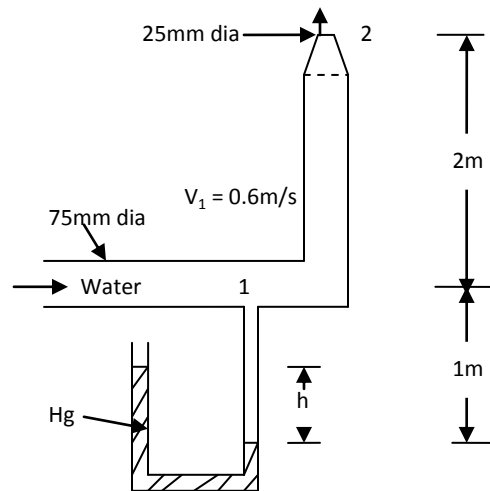
$$P_1 + 9810 \times 0.07 = P_2 + 13.6 \times 9810 \times 0.07$$

$$P_1 - P_2 = 8652.4 \text{ Pa}$$

$$V_2 = \sqrt{\frac{2g}{\gamma} (P_1 - P_2)} = \sqrt{\frac{2g}{9810} \times 8652.4} = 4.15 \text{ m/s}$$

$$Q = A V_2 = 0.0314 \times 4.15 = 0.13 \text{ m}^3/\text{s}$$

11. Find the manometer reading (h) in the lossless system of the fig.



Solution:

Diameter of pipe at section 1 ( $d_1$ ) = 75mm = 0.075m

C/S Area of pipe at section 1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.075^2 = 0.00442 \text{ m}^2$

Diameter of pipe at section 2 ( $d_2$ ) = 25mm = 0.025m

C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.025^2 = 0.00049 \text{ m}^2$

Velocity at point 1 ( $V_1$ ) = 0.6 m/s

Discharge ( $Q$ ) =  $A_1 V_1 = 0.00442 \times 0.6 = 0.00265 \text{ m}^3/\text{s}$

Velocity at point 2 ( $V_2$ ) =  $Q/A_2 = 0.00265/0.00049 = 5.4 \text{ m/s}$

Pressure at 2 ( $P_2$ ) = 0 (atmospheric)

Pressure at 1 =  $P_1$

Taking datum through point 1

Datum head at 1 ( $Z_1$ ) = 0

Datum head at 2 ( $Z_2$ ) = 2m

$h = ?$

Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{9810} + \frac{0.6^2}{2g} + 0 = 0 + \frac{5.4^2}{2g} + 2$$

$$P_1 = 34020 \text{ Pa}$$

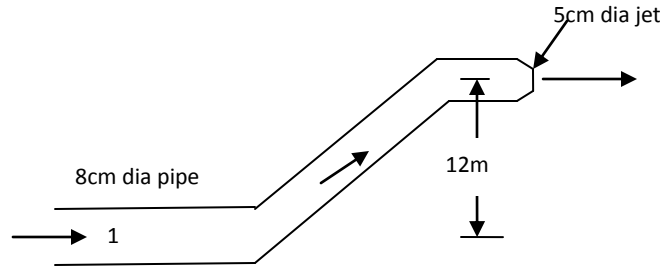
Writing manometric equation

$$P_1 + \gamma_{\text{water}} x 1 = 0 + \gamma_{\text{Hg}} x h$$

$$34020 + 9810 x 1 = 13.6 x 9810 x h$$

$$h = 0.328 \text{ m}$$

12. In the fig., the fluid is water and the pressure at point 1 is 180Kpa gage. If the mass flux is 15kg/s, what is the head loss between 1 and 2? (flux = flow rate)



Solution:

Diameter of pipe at section 1 ( $d_1$ ) = 8cm = 0.08m

C/S Area of pipe at section 1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.08^2 = 0.00503 \text{ m}^2$

Diameter of pipe at section 2 ( $d_2$ ) = 5cm = 0.05m

C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.05^2 = 0.00196 \text{ m}^2$

Mass flux ( $M$ ) = 15kg/s

Velocity at point 1 ( $V_1$ ) =  $\frac{M}{\rho A_1} = \frac{15}{1000 \times 0.00503} = 2.98 \text{ m/s}$

Velocity at point 2 ( $V_2$ ) =  $\frac{M}{\rho A_2} = \frac{15}{1000 \times 0.00196} = 7.65 \text{ m/s}$

Pressure at 1 ( $P_1$ ) = 180 Kpa

Pressure at 2 ( $P_2$ ) = 0 (atmospheric)

Taking datum through point 1

Datum head at 1 ( $Z_1$ ) = 0

Datum head at 2 ( $Z_2$ ) = 12m

Head loss ( $h_L$ ) = ?

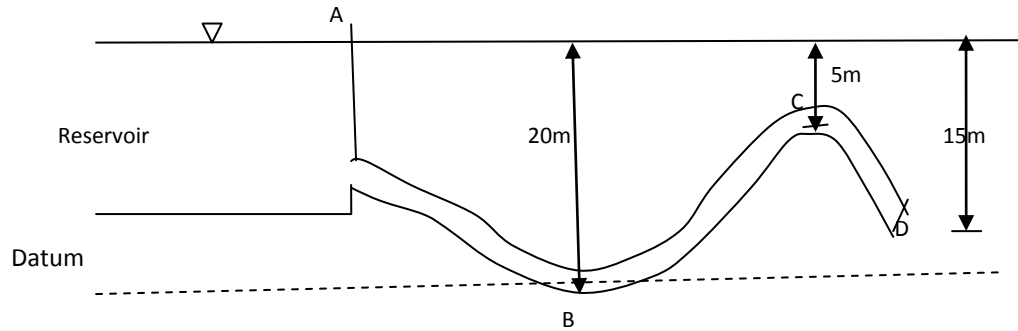
Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{180000}{9810} + \frac{2.98^2}{2g} + 0 = 0 + \frac{7.65^2}{2g} + 12 + h_L$$

$$h_L = 3.82 \text{ m}$$

13. A pipeline connected to a reservoir discharges water to the atmosphere. The loss of head is 1 times velocity head from A to B, 1.5 times velocity head from B to C and 0.5 times velocity head from C to D. If the pipe is 150mm in diameter, calculate the pressure heads at B and C. Also compute discharge.



Solution:

Diameter of pipe ( $d$ ) = 15cm = 0.15m

C/S Area of pipe at section (A) =  $\frac{\pi}{4} \times 0.15^2 = 0.0176 \text{ m}^2$

Velocity at A ( $V_A$ ) = 0

$V$  = Velocity of water through pipe =  $V_B = V_C = V_D$

$hL_{AB} = \frac{V^2}{2g}$ ,  $hL_{BC} = \frac{1.5V^2}{2g}$ ,  $hL_{CD} = \frac{0.5V^2}{2g}$

Total head loss ( $hL$ ) =  $\frac{3V^2}{2g}$

Datum head at A ( $Z_A$ ) = 20m

Datum head at B ( $Z_B$ ) = 0

Datum head at C ( $Z_C$ ) = 15m

Datum head at D ( $Z_D$ ) = 5m

Pressure at A ( $P_A$ ) = 0 (atmospheric)

Pressure at D ( $P_D$ ) = 0 (atmospheric)

Pressure head at B ( $P_B/\gamma$ ) = ?

Pressure head at C ( $P_C/\gamma$ ) = ?

Discharge ( $Q$ ) = ?

Applying Bernoulli's equation at A and D

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D + hL$$

$$0 + 0 + 20 = 0 + \frac{V^2}{2g} + 5 + \frac{3V^2}{2g}$$

$$V = 8.57 \text{ m/s}$$

$$V_B = V_C = V_D = 8.57 \text{ m/s}$$

Applying Bernoulli's equation at A and B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + hL_{AB}$$

$$0 + 0 + 20 = \frac{P_B}{\gamma} + \frac{V^2}{2g} + 0 + \frac{V^2}{2g}$$

$$20 = \frac{P_B}{\gamma} + \frac{8.57^2}{2g} + \frac{8.57^2}{2g}$$

$$\frac{P_B}{\gamma} = 12.5m$$

Applying Bernoulli's equation at B and C

$$\frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C + hL_{BC}$$

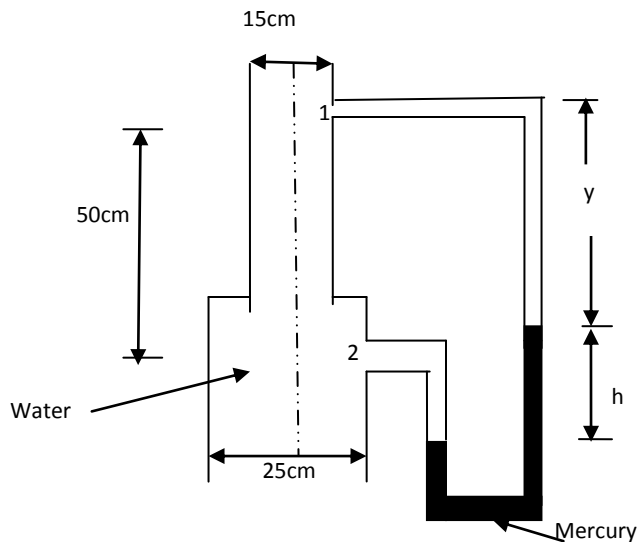
$$12.5 + \frac{V^2}{2g} + 0 = \frac{P_C}{\gamma} + \frac{V^2}{2g} + 15 + \frac{1.5V^2}{2g}$$

$$-2.5 = \frac{P_C}{\gamma} + \frac{1.5 \cdot 8.57^2}{2g}$$

$$\frac{P_C}{\gamma} = -8.11m$$

$$Q = AV = 0.0176 \times 8.57 = 0.1508 \text{ m}^3/\text{s}$$

14. A 15cm diameter pipe is expanded to 25cm diameter suddenly at a section. The head loss at a sudden expansion from section 1 to 2 is given by  $h_L = (V_1 - V_2)^2 / 2g$ . For a discharge of 45 lps, calculate the manometer reading h.



Solution:

Diameter of pipe at section 1 ( $d_1$ ) = 15cm = 0.15m

C/S Area of pipe at section 1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.15^2 = 0.0176 \text{ m}^2$

Diameter of pipe at section 2 ( $d_2$ ) = 25cm = 0.25m

Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.25^2 = 0.049 \text{ m}^2$

Discharge (Q) = 45 lps =  $0.045 \text{ m}^3/\text{s}$

Velocity at point 1 ( $V_1$ ) =  $Q/A_1 = 0.045/0.0176 = 2.55 \text{ m/s}$

Velocity at point 2 ( $V_2$ ) =  $Q/A_2 = 0.045/0.049 = 0.92 \text{ m/s}$

Pressure at 1 =  $P_1$

Pressure at 2 =  $P_2$

Taking datum through point 2

Datum head at 1 ( $Z_1$ ) = 0.5 m

Datum head at 2 ( $Z_2$ ) = 0

Head loss ( $h_L$ ) =  $\frac{(V_1 - V_2)^2}{2g} = \frac{(2.55 - 0.92)^2}{2g} = 0.14 \text{ m}$

$h = ?$

Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{9810} + \frac{2.55^2}{2g} + 0.5 = \frac{P_2}{9810} + \frac{0.92^2}{2g} + 0 + 0.14$$

$$P_1 - P_2 = -6359.6 \text{ Pa}$$

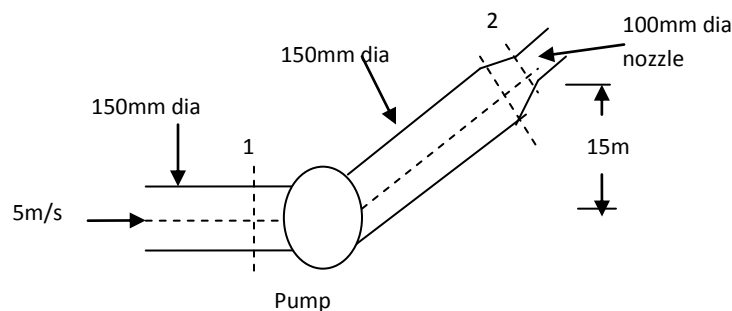
Writing manometric equation

$$P_1 + \gamma_{\text{water}} \times y + \gamma_{\text{Hg}} \times h = P_2 + \gamma_{\text{water}} \times h$$

$$-6359.6 + 9810(0.5 - h) + 13.6 \times 9810h = 0$$

$$h = 0.0117 \text{ m}$$

15. Water is pumped from a reservoir through 150mm diameter pipe and is delivered at a height of 15m from the centerline of pump through a 100mm nozzle connected to 150mm discharge line as shown in the figure. If the pressure at the pump inlet is  $210 \text{ KN/m}^2$  absolute, inlet velocity of  $5 \text{ m/s}$  and the jet is discharged into atmosphere, determine the energy supplied by the pump. Take atmospheric pressure =  $101.3 \text{ KN/m}^2$  and assume no friction.



Solution:

Diameter of pipe at section 1 ( $d_1$ ) = 150mm = 0.15m

$$\text{C/S Area of pipe at section 1 } (A_1) = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

$$\text{Diameter of pipe at section 2 } (d_2) = 100 \text{ mm} = 0.1 \text{ m}$$

$$\text{C/S Area of pipe at section 2 } (A_2) = \frac{\pi}{4} \times 0.1^2 = 0.007854 \text{ m}^2$$

$$\text{Velocity at point 1 } (V_1) = 5 \text{ m/s}$$

$$\text{Discharge } (Q) = A_1 V_1 = 0.01767 \times 5 = 0.08835 \text{ m}^3/\text{s}$$

$$\text{Velocity at point 2 } (V_2) = Q/A_2 = 0.08835/0.007854 = 11.25 \text{ m/s}$$

$$\text{Pressure at 1 } (P_1) = 210 \text{ kN/m}^2$$

$$\text{Pressure at 2 } (P_2) = 101.3 \text{ kN/m}^2 \text{ (atmospheric)}$$

Taking datum through centerline of pump

$$\text{Datum head at 1 } (Z_1) = 0$$

$$\text{Datum head at 2 } (Z_2) = 15 \text{ m}$$

Energy supplied by pump = ?

$h_p$  = Head supplied by the pump

Applying Bernoulli's equation at 1 and 2

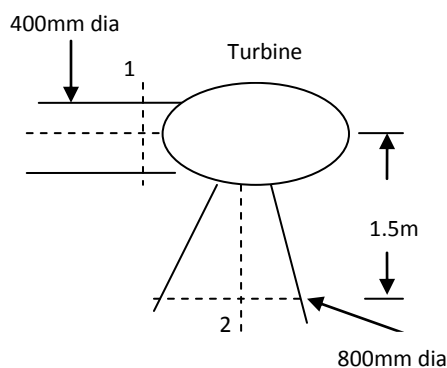
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{210000}{9810} + \frac{5^2}{2g} + 0 + h_p = \frac{101300}{9810} + \frac{11.25^2}{2g} + 15$$

$$h_p = 9.09 \text{ m}$$

$$\text{Energy supplied by the pump} = \gamma Q h_p = 9810 \times 0.08835 \times 9.09 = 7878 \text{ W}$$

16. For a turbine shown in the figure,  $P_1 = 200 \text{ Kpa}$ ,  $P_2 = -35 \text{ Kpa}$ ,  $Q = 0.3 \text{ m}^3/\text{s}$ . Determine the energy output of the machine if its efficiency is 80%.



Solution:

$$\text{Diameter of pipe at section 1 } (d_1) = 400 \text{ mm} = 0.4 \text{ m}$$

$$\text{C/S Area of pipe at section 1 } (A_1) = \frac{\pi}{4} \times 0.4^2 = 0.1256 \text{ m}^2$$

Diameter of pipe at section 2 ( $d_2$ ) = 800mm = 0.8m

C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.8^2 = 0.5026 \text{ m}^2$

Discharge ( $Q$ ) =  $0.3 \text{ m}^3/\text{s}$

Velocity at point 1 ( $V_1$ ) =  $Q/A_2 = 0.3/0.1256 = 2.38 \text{ m/s}$

Velocity at point 2 ( $V_2$ ) =  $Q/A_2 = 0.3/0.5026 = 0.6 \text{ m/s}$

Pressure at 1 ( $P_1$ ) = 200 Kpa

Pressure at 2 ( $P_2$ ) = -35Kpa

Taking datum at 2,

Datum head at 1 ( $Z_2$ ) = 0

Datum head at 2 ( $Z_1$ ) = 1.5m

Efficiency ( $\eta$ ) = 0.8

Energy output of machine = ?

$h_t$  = Head extracted by turbine

Applying Bernoulli's equation at 1 and 2

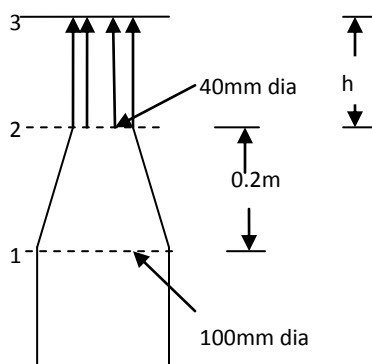
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 - h_t = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{200000}{9810} + \frac{2.38^2}{2g} + 1.5 - h_t = \frac{-35000}{9810} + \frac{0.6^2}{2g} + 0$$

$$h_t = 25.72\text{m}$$

$$\text{Energy output} = \eta(\gamma Q h_t) = 0.8 \times (9810 \times 0.3 \times 25.72) = 60555\text{W}$$

17. A jet of water issues vertically upwards from 0.2m high nozzle whose inlet and outlet diameters are 100mm and 40mm respectively. If the pressure at the inlet is 20 Kpa above the atmospheric pressure, determine the discharge and the height to which the jet will rise. Assume no friction



Solution:

Inlet diameter ( $d_1$ ) = 100mm = 0.1m

C/S Area at inlet ( $A_1$ ) =  $\frac{\pi}{4} \times 0.1^2 = 0.007854 \text{ m}^2$

Outlet diameter ( $d_2$ ) = 40mm = 0.04m

$$C/S \text{ Area at outlet } (A_2) = \frac{\pi}{4} \times 0.04^2 = 0.001257 \text{m}^2$$

$$\text{Pressure at 1 } (P_1) = 20 \text{ KN/m}^2$$

$$\text{Pressure at 2 and 3 } (P_2, P_3) = 0$$

$$\text{Discharge } (Q) = ?$$

$$\text{Height to which jet will rise } (h) = ?$$

Applying Bernoulli's equation at 1 and 2 (Taking datum at 1)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{20000}{9810} + \frac{V_1^2}{2g} + 0 = 0 + \frac{V_2^2}{2g} + 0.2$$

$$V_1^2 - V_2^2 = -36.076 \quad (a)$$

From continuity

$$A_1 V_1 = A_2 V_2$$

$$0.007854 V_1 = 0.001257 V_2$$

$$V_1 = 0.16 V_2 \quad (b)$$

From a and b

$$(0.16 V_2)^2 - V_2^2 = -36.076$$

$$V_2 = 6.08 \text{m/s}$$

$$V_1 = 0.97 \text{m/s}$$

$$Q = A_1 V_1 = 0.007854 \times 0.97 = 0.007618 \text{ m}^3/\text{s}$$

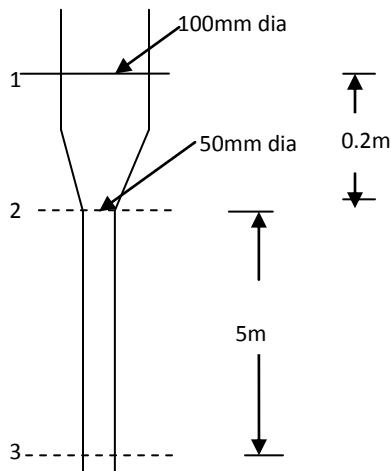
Applying Bernoulli's equation at 2 and 3 (Taking datum at 2)

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$0 + \frac{6.08^2}{2g} + 0 = 0 + 0 + h$$

$$h = 1.88 \text{m}$$

18. A jet of water coming out from 50mm diameter rounded nozzle attached to 100mm diameter pipe is directed vertically downwards. If the pressure in the 100mm diameter pipe 0.2m above the nozzle is 200 Kpa gauge, determine the diameter of jet 5m below the nozzle level.



Solution:

Diameter of pipe at section 1 ( $d_1$ ) = 100mm = 0.1m

C/S Area of pipe at section 1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.1^2 = 0.007854 \text{ m}^2$

Diameter of pipe at section 2 ( $d_2$ ) = 50mm = 0.05m

C/S Area of pipe at section 2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.05^2 = 0.001963 \text{ m}^2$

Pressure at 1 ( $P_1$ ) = 200 Kpa

Pressure at 2 and 3 ( $P_2, P_3$ ) = 0

Applying Bernoulli's equation at 1 and 2 (Taking datum at 2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{20000}{9810} + \frac{V_1^2}{2g} + 0.2 = 0 + \frac{V_2^2}{2g} + 0$$

$$V_1^2 - V_2^2 = -404 \quad (\text{a})$$

From continuity

$$A_1 V_1 = A_2 V_2$$

$$0.007854 V_1 = 0.001963 V_2$$

$$V_1 = 0.25 V_2 \quad (\text{b})$$

From a and b

$$(0.25 V_2)^2 - V_2^2 = -404$$

$$V_2 = 20.75 \text{ m/s}$$

$$V_1 = 5.18 \text{ m/s}$$

$$Q = A_1 V_1 = 0.007854 \times 5.18 = 0.0407 \text{ m}^3/\text{s}$$

Applying Bernoulli's equation at 2 and 3 (Taking datum at 3)

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$0 + \frac{20.75^2}{2g} + 5 = 0 + \frac{V_3^2}{2g} + 0$$

$$V_3 = 23 \text{ m/s}$$

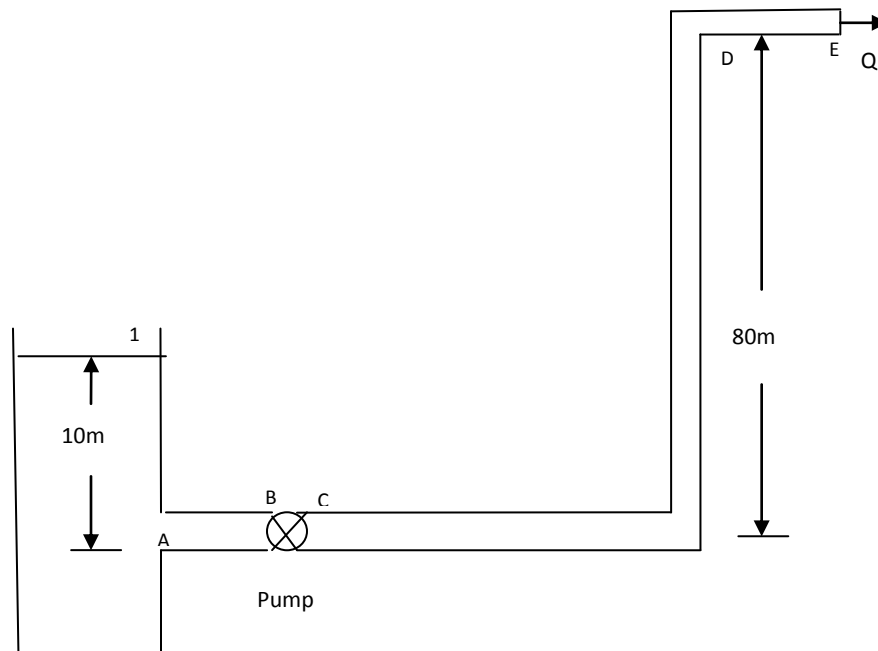
$$A_3 = Q/V_3 = 0.0407/23 = 0.00177 \text{ m}^2$$

$$A_3 = \frac{\pi}{4} d_3^2$$

$$0.00177 = \frac{\pi}{4} d_3^2$$

$$d_3 = 0.047 \text{ m} = 47 \text{ mm}$$

19. The pipe flow in the figure is driven by the pump. What gauge pressure is needed to be supplied by the pump to provide water flow rate of  $Q = 60\text{m}^3/\text{h}$ ? Neglect head loss from A to B. Head loss from C to D  $= 30 \frac{V_{CD}^2}{2g}$ ; Head loss from D to E  $= 20 \frac{V_{CD}^2}{2g}$ ;  $d_{AB}$  (diameter of pipe AB)  $= d_{CD} = 5\text{cm}$ ;  $d_{DE} = 2\text{cm}$ . where  $V_{CD}$  = velocity in pipe CD and  $V_{DE}$  = velocity in pipe DE.



Solution:

$$\text{Discharge (Q)} = 60\text{m}^3/\text{h} = 60/3600 \text{ m}^3/\text{s} = 0.0167 \text{ m}^3/\text{s}$$

$$\text{Diameter of pipe AB and CD} = 5\text{cm} = 0.05\text{m}$$

$$\text{C/s Area of pipe AB and CD (A}_{AB}, A_{CD}) = \frac{\pi}{4} \times 0.05^2 = 0.001963\text{m}^2$$

$$\text{Diameter of pipe DE} = 2\text{cm} = 0.02\text{m}$$

$$\text{C/s Area of pipe DE (A}_{DE}) = \frac{\pi}{4} \times 0.02^2 = 0.000314\text{m}^2$$

$$\text{Velocity of flow through AB and CD (V}_{AB} = V_{CD}) = Q/A_{AB} = 0.0167/0.001963 = 8.5\text{m/s}$$

$$\text{Velocity of flow through DE (V}_{DE}) = Q/A_{DE} = 0.0167/0.000314 = 53.2\text{m/s}$$

$$\text{Head loss between A and B (h}_{L_{AB}}) = 0$$

$$\text{Head loss from C to D (h}_{L_{CD}}) = 30 \frac{V_{CD}^2}{2g} = 30 \frac{8.5^2}{2g} = 110.5\text{m}$$

$$\text{Head loss from D to E (h}_{L_{DE}}) = 20 \frac{V_{CD}^2}{2g} = 20 \frac{8.5^2}{2g} = 73.65\text{m}$$

$$\text{Total head loss (h}_L) = 0 + 110.5 + 73.65 = 184.15\text{m}$$

$h_p$  = Head supplied by the pump

At point 1,  $V_1 = 0$ ,  $P_1 = 0$  (atm)

At point E,  $P_E = 0$  (atm)

Applying Bernoulli's equation between 1 and E (Taking datum through A)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + h_p = \frac{P_E}{\gamma} + \frac{V_E^2}{2g} + Z_E + h_L$$

$$0 + 0 + 10 + h_p = 0 + \frac{53.2^2}{2g} + 80 + 184.15$$

$$h_m = 398.4\text{m}$$

Applying Bernoulli's equation between B and C

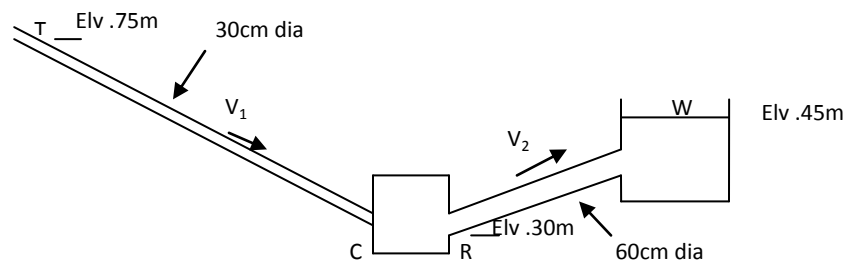
$$\frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_p = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C$$

$$V_B = V_C, Z_B = Z_C$$

$$P_C - P_B = \gamma h_p = 9810 \times 398.4 = 3908304 \text{ Pa} = 3908.304 \text{ Kpa}$$

Gauge pressure to be supplied by the pump = 3908.304 Kpa

20. The head extracted by turbine CR in the fig. is 60m and the pressure at T is 505 KN/m<sup>2</sup>. For losses of  $2V_2^2/2g$  between W and R and  $3V_1^2/2g$  between C and T, determine (a) how much water is flowing and (b) the pressure head at R. Draw the energy gradient line and hydraulic gradient line.



Solution:

$$\text{Diameter of pipe TC } (d_1) = 30\text{cm} = 0.3\text{m}$$

$$\text{C/S Area of pipe TC } (A_1) = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

$$\text{Diameter of pipe RW } (d_2) = 60\text{cm} = 0.6\text{m}$$

$$\text{C/S Area of pipe RW } (A_2) = \frac{\pi}{4} \times 0.6^2 = 0.2827 \text{ m}^2$$

$$\text{Pressure at T } (P_T) = 505 \text{ KPa}$$

$$\text{Velocity at T } (V_T) = V_1$$

$$h_{L_{TC}} = 3V_1^2/2g$$

$$h_{L_{RW}} = 2V_2^2/2g$$

$$h_t = 60\text{m}$$

$$\text{Datum head at T } (Z_T) = 75\text{m}$$

$$\text{Datum head at R } (Z_R) = 30\text{m}$$

$$\text{Datum head at W } (Z_W) = 45\text{m}$$

$$\text{Pressure at W } (P_W) = 0$$

$$\text{Velocity at W } (V_W) = 0$$

$$\text{Flow rate } (Q) = ?$$

$$\text{Pressure head at R } (P_R/\gamma) = ?$$

a) Applying Bernoulli's equation at T and W

$$\frac{P_T}{\gamma} + \frac{V_T^2}{2g} + Z_T - h_t = \frac{P_W}{\gamma} + \frac{V_W^2}{2g} + Z_W + \text{Total head loss}$$

$$\frac{505000}{9810} + \frac{V_1^2}{2g} + 75 - 60 = 0 + 0 + 45 + \frac{3V_1^2}{2g} + \frac{2V_2^2}{2g}$$

$$V_1^2 + V_2^2 = 210.7 \quad (\text{a})$$

Continuity equation

$$A_1 V_1 = A_2 V_2$$

$$0.0707 V_1 = 0.2827 V_2$$

$$V_1 = 4 V_2 \quad (\text{b})$$

From a and b

$$V_2 = 3.52 \text{ m/s}$$

$$V_1 = 14.08 \text{ m/s}$$

$$Q = A_1 V_1 = 0.0707 \times 14.08 = 0.995 \text{ m}^3/\text{s}$$

b) Applying Bernoulli's equation at R and W

$$\frac{P_R}{\gamma} + \frac{V_R^2}{2g} + Z_R = \frac{P_W}{\gamma} + \frac{V_W^2}{2g} + Z_W + hL_{RW}$$

$$\frac{P_R}{\gamma} + \frac{V_2^2}{2g} + 30 = 0 + 0 + 45 + \frac{2V_2^2}{2g}$$

$$\frac{P_R}{\gamma} = 15 + \frac{V_2^2}{2g} = 15 + \frac{3.52^2}{2g} = 15.63 \text{ m}$$

Finding the elevation of EGL and HGL

$$\text{EGL at T} = \frac{P_T}{\gamma} + \frac{V_T^2}{2g} + Z_T = \frac{505000}{9810} + \frac{14.08^2}{2g} + 75 = 136.6 \text{ m}$$

$$\text{EGL at C} = \text{EGL at T} - h_{L_{TC}} = 136.6 - \frac{3 \times 14.08^2}{2g} = 106.3 \text{ m}$$

$$\text{EGL at R} = \text{EGL at C} - h_{CR} = 106.3 - 60 = 46.3 \text{ m}$$

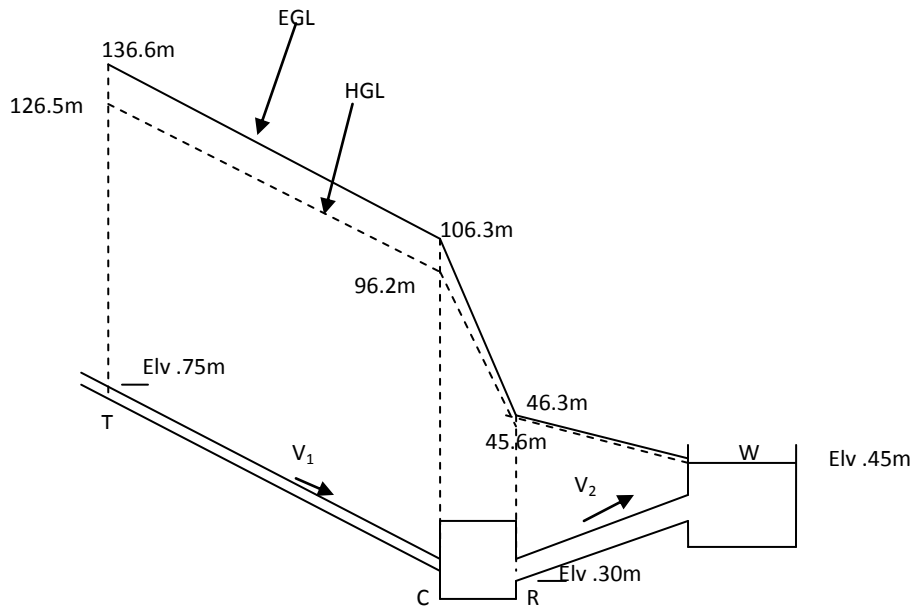
$$\text{EGL at W} = \text{EGL at R} - h_{L_{RW}} = 46.3 - \frac{2 \times 3.52^2}{2g} = 45 \text{ m}$$

$$\text{HGL at T} = \text{EGL at T} - \frac{V_1^2}{2g} = 136.6 - \frac{14.08^2}{2g} = 126.5 \text{ m}$$

$$\text{HGL at C} = \text{EGL at C} - \frac{V_1^2}{2g} = 106.3 - \frac{14.08^2}{2g} = 96.2 \text{ m}$$

$$\text{HGL at R} = \text{EGL at R} - \frac{V_2^2}{2g} = 46.3 - \frac{3.52^2}{2g} = 45.6 \text{ m}$$

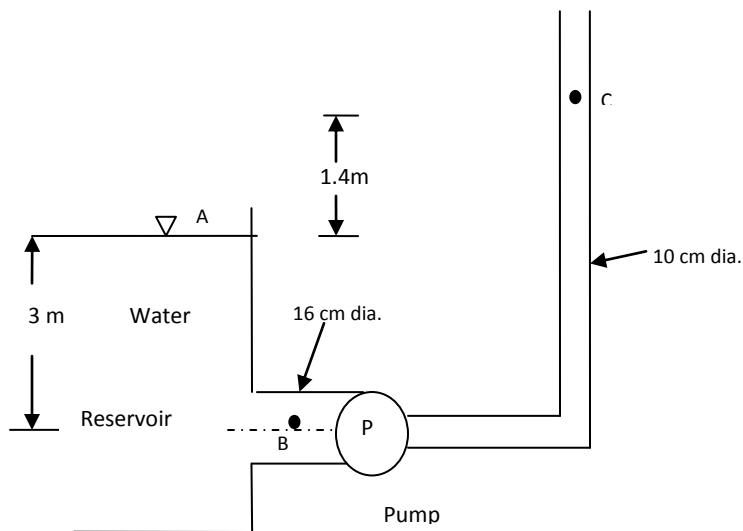
$$\text{HGL at W} = \text{EGL at W} - \frac{V_W^2}{2g} = 45 - 0 = 45 \text{ m}$$



21. The figure below shows a pump P pumping 90 lps of water from a tank.

(a) What will be the pressure at points B and C when the pump delivers 14.5KW of power to the flow? Assume the losses in the system to be negligible.

(b) What will be the pressure at C when the loss in the inlet to the pump is negligible and between the pump and the point C, a loss equal to 2 times the velocity head at B takes place.



Solution:

$$\text{Discharge (Q)} = 90 \text{ lps} = 0.09 \text{ m}^3/\text{s}$$

$$\text{Diameter of large pipe (d}_B\text{)} = 16\text{cm} = 0.16\text{m}$$

$$\text{Diameter of smaller pipe (d}_C\text{)} = 10\text{cm} = 0.1\text{m}$$

$$\text{(a) Power (P)} = 14.5 \text{ KW} = 14500\text{W}$$

$$\text{Pressure at point B (P}_B\text{)} = ?$$

$$\text{Pressure at point C (P}_C\text{)} = ?$$

$$\text{Head supplied by the pump} = h_p$$

$$P = \gamma Q h_p$$

$$14500 = 9810 \times 0.09 \times h_p$$

$$h_p = 16.42\text{m}$$

$$V_A = 0, P_A = 0 \text{ (atm. pr.)}$$

Using continuity equation at B and C

$$V_B = \frac{Q}{A_B} = \frac{0.09}{\frac{\pi}{4} 0.16^2} = 4.47\text{m/s}$$

$$V_C = \frac{Q}{A_C} = \frac{0.09}{\frac{\pi}{4} 0.1^2} = 11.46\text{m/s}$$

Applying Bernoulli's equation between A and B (taking B as datum)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B$$

$$0 + 0 + 3 = \frac{P_B}{9810} + \frac{4.47^2}{2 \times 9.81} + 0$$

$$P_B = 19440 \text{ Pa}$$

Applying Bernoulli's equation between A and C (taking B as datum)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A + h_p = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C$$

$$0 + 0 + 3 + 16.42 = \frac{P_C}{9810} + \frac{11.46^2}{2 \times 9.81} + 4.4$$

$$P_C = 81680 \text{ Pa}$$

b) Loss between A and B = 0

$$\text{Loss between B and C} = 2 \frac{V_B^2}{2g} = 2 \frac{4.47^2}{2 \times 9.81} = 2.04\text{m}$$

$$\text{Total loss of head (h}_L\text{)} = 2.04\text{m}$$

$$\text{Pressure at point C (P}_C\text{)} = ?$$

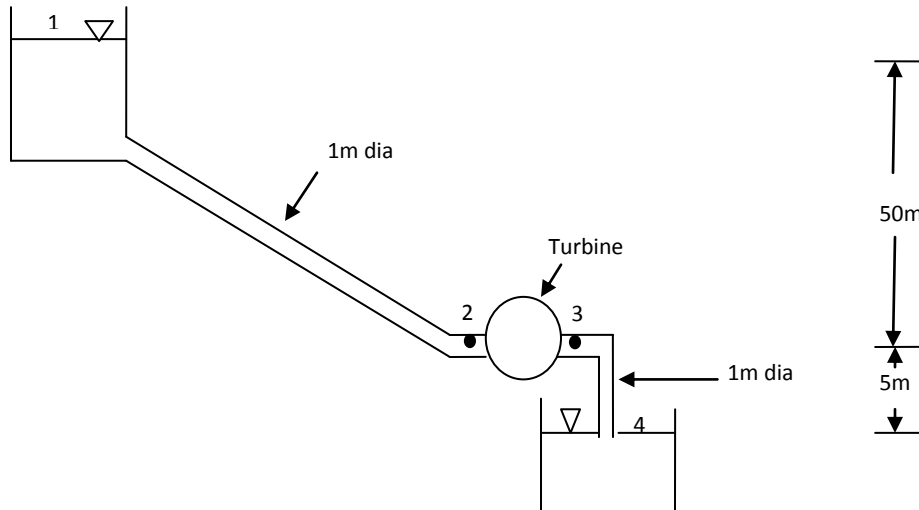
Applying Bernoulli's equation between A and C (taking B as datum)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A + h_p = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C + h_L$$

$$0 + 0 + 3 + 16.42 = \frac{P_C}{9810} + \frac{11.46^2}{2 \times 9.81} + 4.4 + 2.04$$

$$P_C = 61668 \text{ Pa}$$

22. The figure below shows a pipe connecting a reservoir to a turbine which discharges water to the tailrace through another pipe. The head loss between the reservoir and the turbine is 8 times kinetic head in the pipe and that from the turbine to tailrace is 0.4 times the kinetic head in the pipe. The rate of flow is  $1.36\text{m}^3/\text{s}$  and the pipe diameter in both cases is 1m. Determine (a) the pressure at inlet and exit of turbine, and (b) the power generated by the turbine.



Solution:

Diameter of pipe 1-2 and 3-4 ( $d$ ) = 1m

Discharge ( $Q$ ) =  $1.36\text{m}^3/\text{s}$

As  $d$  is same, velocity ( $V$ ) is also same for pipe 1-2 and 3-4.

$$V = \frac{Q}{A} = \frac{1.36}{\frac{\pi \cdot 1^2}{4}} = 1.73 \text{ m/s}$$

$$\text{Head loss between reservoir and turbine } (h_{La}) = 8 \frac{V^2}{2g} = 8 \frac{1.73^2}{2 \times 9.81} = 1.22\text{m}$$

$$\text{Head loss between turbine and tailrace } (h_{Lb}) = 0.4 \frac{V^2}{2g} = 0.4 \frac{1.73^2}{2 \times 9.81} = 0.06\text{m}$$

$$\text{Total loss of head } (h_L) = 1.22 + 0.06 = 1.28\text{m}$$

$$P_1 = 0, P_4 = 0, V_1 = 0, V_4 = 0$$

(a) Pressure at 2 ( $P_2$ ) = ?

Pressure at 3 ( $P_3$ ) = ?

Applying Bernoulli's equation between 1 and 2 (taking datum through 2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{La}$$

$$0 + 0 + 50 = \frac{P_2}{9810} + \frac{1.73^2}{2 \times 9.81} + 0 + 1.22$$

$$P_2 = 477035 \text{ Pa}$$

Applying Bernoulli's equation between 3 and 4 (taking datum through 4)

$$\frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 = \frac{P_4}{\gamma} + \frac{V_4^2}{2g} + Z_4 + h_{Lb}$$

$$\frac{P_3}{9810} + \frac{1.73^2}{2 \times 9.81} + 5 = 0 + 0 + 0 + 0.06$$

$$P_3 = -49958 \text{ Pa}$$

b) Power generated by the turbine (P) = ?

Head extracted by turbine =  $h_T$

Applying Bernoulli's equation between 2 and 3 (taking datum through 2)

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 - h_T = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

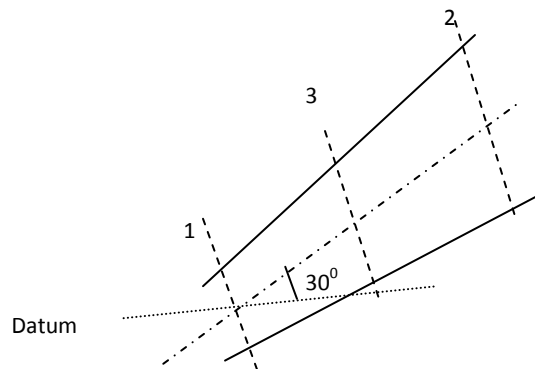
$$\frac{477035}{9810\gamma} + \frac{1.73^2}{2 \times 9.81} + 0 - h_T = \frac{-49958}{9810} + \frac{1.73^2}{2 \times 9.81} + 0$$

$$h_T = 53.7 \text{ m}$$

$$P = \gamma Q h_T = 9810 \times 1.36 \times 53.7 = 711176 \text{ W} = 711.176 \text{ KW}$$

A 2.5m long pipeline tapers uniformly from 10cm diameter to 20cm diameter at its upper end. The pipe centerline slopes upwards at an angle of  $30^\circ$  to the horizontal and the flow direction is from smaller to bigger cross-section. If the pressure at lower and upper ends of the pipe are 2bar and 2.4bar respectively, determine the flow rate and the pressure at the mid-length of the pipeline. Assume no energy losses.

Solution:



Solution:

Diameter of pipe at section1 ( $d_1$ ) = 10cm = 0.1m

C/S Area of pipe at section1 ( $A_1$ ) =  $\frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Diameter of pipe at section2 ( $d_2$ ) = 20cm = 0.2m

C/S Area of pipe at section2 ( $A_2$ ) =  $\frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Pressure at section 1 ( $P_1$ ) = 2 bar =  $2 \times 10^5 \text{ N/m}^2$

Pressure at section 2 ( $P_2$ ) = 2.4bar =  $2.4 \times 10^5 \text{ N/m}^2$

Discharge (Q) = ?

Pressure at mid length ( $P_3$ ) = ?

Taking datum head at section 1 ( $Z_1$ ) = 0

$$Z_2 = 2.5 \sin 30 = 1.25 \text{ m}$$

$$Z_3 = 1.25 \sin 30 = 0.625 \text{ m}$$

a. From continuity equation

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times 0.1^2 V_1 = \frac{\pi}{4} \times 0.2^2 V_2$$

$$V_1 = 4 V_2$$

Applying Bernoulli's equation at section 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{2 \times 10^5}{9810} + \frac{(4V_2)^2}{2g} + 0 = \frac{2.4 \times 10^5}{9810} + \frac{V_2^2}{2g} + 1.25$$

$$V_2 = 2.64 \text{ m/s}$$

$$V_1 = 4 \times 2.64 = 10.56 \text{ m/s}$$

$$Q = A_1 V_1 = 0.00785 \times 10.56 = 0.083 \text{ m}^3/\text{s}$$

b. At mid length  $d_3 = 15 \text{ cm} = 0.15 \text{ m}$

$$\text{C/S Area of pipe at section 3 } (A_3) = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

$$V_3 = Q/A_3 = 0.083/0.01767 = 4.7 \text{ m/s}$$

Applying Bernoulli's equation at section 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$\frac{2 \times 10^5}{9810} + \frac{10.56^2}{2g} + 0 = \frac{P_3}{9810} + \frac{4.7^2}{2g} + 0.625$$

$$P_3 = 238581 \text{ N/m}^2 = 238.581 \text{ Kpa}$$