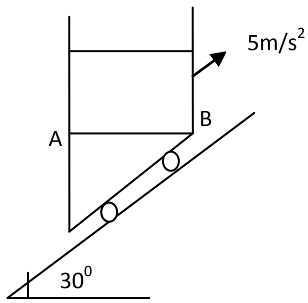
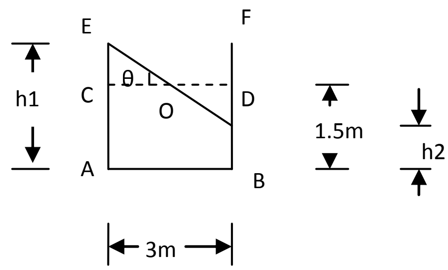
Tutorial 5
Relative equilibrium

1. An open rectangular tank 3m long and 2m wide is filled with water to a depth of 1.5m. Find the slope of the water surface when the tank moves with an acceleration of $5 m/s^2$ up a 30^0 inclined plane. Also calculate the pressure on the bottom at both ends.





Solution:

Inclination of surface (α) = 30^0

Acceleration (a) = 5 m/s^2

Slope of water surface (θ) = ?

Pressure at A (P_A) = ?, Pressure at B (P_B) = ?

Components of acceleration

$a_x = 5\cos 30 = 4.33\ m/s^2$

$a_z = 5\sin 30 = 2.5\ m/s^2$

$$tan\theta = \frac{a_x}{g+a_z} = \frac{4.33}{9.81+2.5}$$

θ = 19.38^0

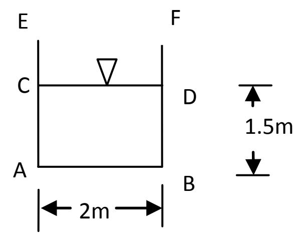
Depth of water at rear end (h1) = 1.5+(3/2)xtan19.38 = 2.03m

Depth of water at front end (h2) = 1.5-(3/2)xtan19.38 = 0.97m

$$P_A = \gamma h1\left(1+\frac{a_z}{g}\right) = 9810x2.03\left(1+\frac{2.5}{9.81}\right) = 24989\ N/m^2$$

$$P_B = \gamma h2\left(1+\frac{a_z}{g}\right) = 9810x0.97\left(1+\frac{2.5}{9.81}\right) = 11941\ N/m^2$$

2. A rectangular tank 2m long, 1.5m wide and 1.5m deep is filled with oil of specific gravity 0.8. Find the force acting on the bottom of the tank when (a) the vertical acceleration $5m/s^2$ acts upwards (b) the vertical acceleration $5m/s^2$ acts downwards.



Solution:

a) Acceleration (a_z) = 5 m/s^2 (vertically upwards)

Depth of oil (h) = 1.5m

Force acting on the bottom (F_{AB}) = ?

$P_A = \gamma_{oil} h \left(1 + \frac{a_z}{g}\right) = 0.8x9810x1.5\left(1 + \frac{5}{9.81}\right)$ = 17772 N/m^2

F_{AB} = P_A x Area at bottom = 17772x2x1.5 = 53316N

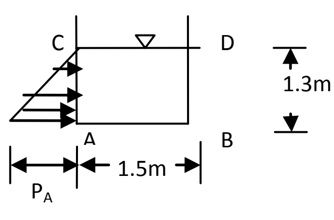
b) Acceleration (a_z) = -5 m/s^2 (vertically downwards)

Force acting on the bottom (F_{AB}) = ?

$P_A = \gamma_{oil} h \left(1 - \frac{a_z}{g}\right) = 0.8x9810x1.5\left(1 - \frac{5}{9.81}\right)$ = 5772 N/m^2

F_{AB} = P_{AB} x Area at bottom = 5772x2x1.5 = 17316N

3. An open cubical tank with each side 1.5m contains oil of specific weight 7.5KN/m^3 up to a depth of 1.3m. Find the forces acting on the side of the tank when it is being moved with an acceleration of 4m/s^2 in vertically upward and downward direction.



a) Acceleration (a_z) = 4 m/s^2 (vertically upwards)

Depth of oil (h) = 1.3m

Force acting on side (F1) = ?

$P_A = \gamma_{oil} h \left(1 + \frac{a_z}{g}\right) = 7500x1.3\left(1 + \frac{4}{9.81}\right)$ = 13725.5 N/m^2

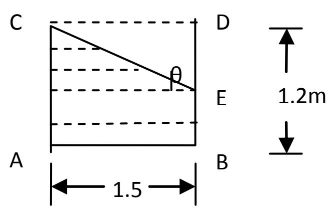
F_1= Area of pressure diagram x width = 0.5x13725.5x1.3x1.5 = 13382N

b) Acceleration (a_z) = -4 m/s^2 (vertically downwards)

Depth of oil (h) = 1.3m
Force acting on side (F2) = ?

$P_A = \gamma_{oil} h \left(1 + \frac{a_z}{g}\right) = 7500x1.3\left(1 - \frac{4}{9.81}\right)$ = 5774.46 N/m^2
F_2= Area of pressure diagram x width = 0.5x5774.46x1.3x1.5 = 5630N

4. An open rectangular tank 1.5mx1mx1.2m high is completely filled with water when at rest. Determine the volume spilled after the tank acquired a linear uniform acceleration of 0.6 m/s^2 in the horizontal direction.

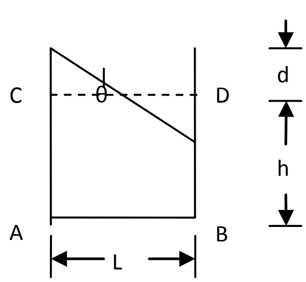


Solution:
Acceleration (a_x) = 0.6 m/s^2
Volume of water spilled=?

Slope of surface = tanθ = a_x/g = 0.6/9.81 = 0.0611
θ = 3.5^0
DE = 1.5tan3.5 =0.091m

Volume of water spilled=Area of traingeCDEx width
= 0.5x1.5x0.091x1 = 0.06825 m^3

5. What distance must the sides of a tank be carried above the surface of water contained in it if the tank is to undergo a uniform horizontal acceleration of 3m/s^2 without spilling any water? (0.1529L)



Solution:

d =?

Slope of surface = $\tan\theta = a_x/g$ = 3/9.81 = 0.306

$\theta = 17^0$

d = 0.5Ltan17 =0.153L

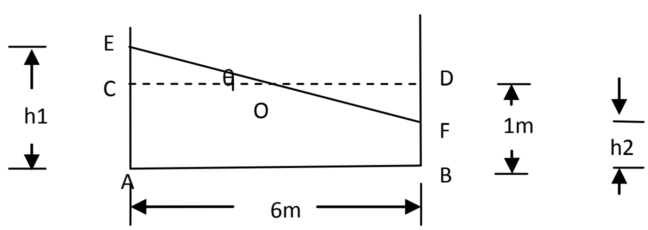
The tank must be carried 0.153L above the surface of water.

6. A rectangular tank 6m long, 2m wide and 2m deep contains water to a depth of 1m. It is accelerated horizontally at 2.5 m/s^2 in the direction of its length. Determine

a) Slope of the free surface

b) Maximum and minimum pressure intensities at bottom

c) Total force due to water acting on each end of tank. Check the difference between these forces by calculating the inertia force of the accelerated mass.



Solution:

Acceleration (a_x) = 2.5 m/s^2

Depth of water (h) = 1m

a) Slope of free surface (θ) = ?

Slope of surface = $\tan\theta = a_x/g$ = 2.5/9.81 = 0.255

$\theta = 14.3^0$

b) Max pressure (P_A) =?

Min. pressure (P_B) = ?

h1= 1+3tanθ = 1.7646m

h2= 1-3tanθ = 0.2353m

$P_A = \gamma h1$ = 9810x1.7646 = 17310.7 N/m^2

$P_B = \gamma h2$ = 9810x0.2353 = 2308.3 N/m^2

c) Force on each side (F_{AE}, F_{BD}) = ?

F_{AE} = Area of pressure diagram x width = 0.5x17310.7x1.764x2 = 30546N

F_{BD} = Area of pressure diagram x width = 0.5x2308.3x0.2353x2 = 543N

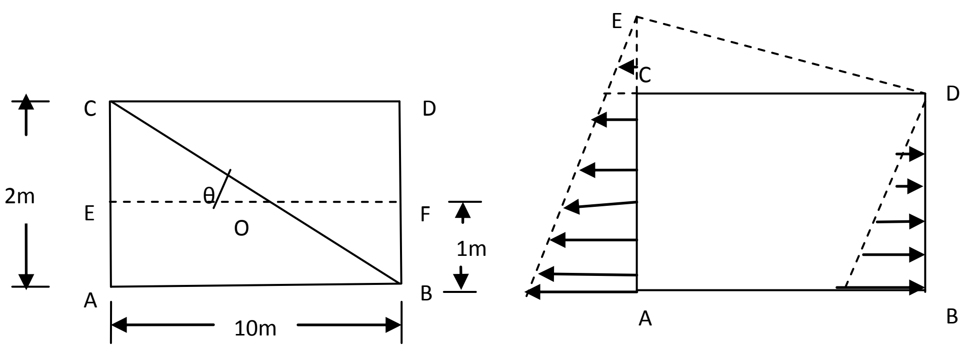
Difference in force (F_D) = 30546-543= 30003N

Force needed (F) = mass of water x linear acceleration = $\rho x\ Volume\ of\ water\ x a_x$

= 1000x(6x2x1)X2.5 = 30000N

F $\approx F_D$

7. An oil tanker 3m wide, 2m deep and 10m long contains oil of density 800 kg/m^3 to a depth of 1m. Determine the maximum horizontal acceleration that can be given to the tanker such that the oil just reaches its top end. If the tanker is closed and completely filled with the oil and accelerated horizontally at $3m/s^2$, determine the total liquid thrust (hydrostatic force) on the front and rear end.



h2

Solution:

a) Maximum horizontal acceleration (a_x) = ?

When the oil touches the top end, the water surface rises 1m at the left side and falls 1m at the right side.

Maximum permissible slope ($\tan\theta$) $= \frac{1}{5} = 0.2$

$\theta = 11.3^0$

$tan\theta = \frac{a_x}{g}$

$0.2 = \frac{a_x}{9.81}$

$a_x = 1.962 m/s^2$

b) $a_x = 3m/s^2$

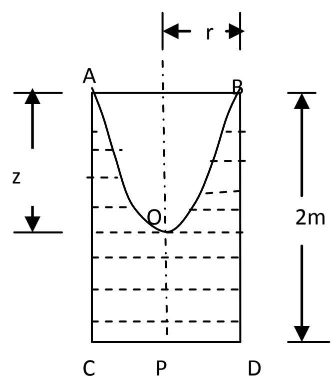
When the tanker is completely filled and closed, there will be pressure built up at the rear end equivalent to the virtual oil column CE that would assume a slope of a_x/g = 3/9.81 = 0.306.

CE = 10tanθ1 = 10x0.306 = 3.06m

Pressure at B = $\gamma x2$ = 9810x2 = 19620N/m^2
Pressure at A = $\gamma x(2 + 3.06)$ = 9810x5.06 = 49638.6N/m^2
Virtual pressure at C = $\gamma x3.06$ = 9810x3.06 = 30018.6N/m^2

Force at front end = Area of pressure diagram for BDx width
= 0.5x19620x2x3 = 58860N
Force at rear end = Area of pressure diagram for ACx width
= 0.5x(49638.6+30018.6)x2x3 = 238972N

8. An open circular cylinder of 1m diameter and 2m depth is completely filled with water and rotated about its axis about 45 rpm. Determine the depth at the axis and amount of water spilled. Also find the speed of rotation at which the central axial depth is zero.



Solution:
Radius (r) = 0.5m
rpm (N) = 45
Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x45x\pi}{60}$ = 4.712 rad/s
a) Depth at axis (PO) =?
Amount of water spilled = ?

$z = \frac{r^2\omega^2}{2g} = \frac{0.5^2 4.712^2}{2x9.81}$ =0.283m
PO = 2-0.283 = 1.717m

Amount of water spilled = Volume of paraboloid AOB
$= \frac{1}{2} x\pi r^2 z = \frac{1}{2} x\pi x 0.5^2 x 0.283$ = 0.111m^3

b) When O touches P, z becomes 2m.
speed of rotation (N) =?

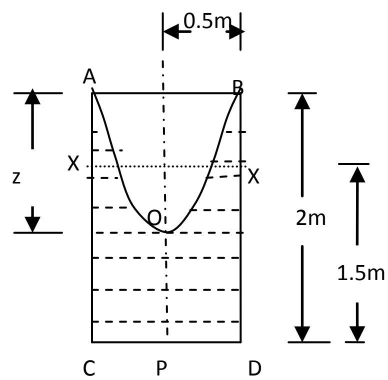
$z = \frac{r^2\omega^2}{2g}$

$2 = \frac{0.5^2\omega^2}{2x9.81}$

ω = 12.53 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x12.53}{2\pi}$ = 120rpm

9. An open circular vessel is 1m in diameter and 2m height. It contains water filled to a depth of 1.5m. If the cylinder rotates about its vertical axis, (a) what constant angular velocity can be obtained without spilling, (b) what is the pressure intensity at the center and at the corner of the bottom if ω = 6 radians/seconds.



Solution:

Radius (r) = 0.5m

a) Angular velocity (ω) =

If no water is spilled,

Volume above XX = volume of paraboloid AOB

$\pi x 0.5^2 x 0.5 = \frac{1}{2} x \pi x 0.5^2 x z$

z = 1m

When the water surface just touches the top rim,

Rise of liquid at the edge above XX = Fall of liquid at the center below XX = z/2 = 0.5m

$z = \frac{r^2\omega^2}{2g}$

$1 = \frac{0.5^2\omega^2}{2x9.81}$

ω = 8.858 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x8.858}{2\pi}$ = 85 rpm

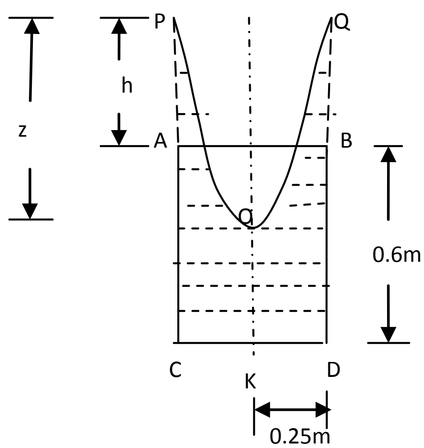
b) For ω = 6 rad/s

$z = \frac{r^2\omega^2}{2g} = \frac{0.5^2 6^2}{2g}$ = 0.46m

Origin O is 0.46/2 = 0.23m below XX.
The value of PO and CA are
PO = h1= 1.5-0.23 = 1.27m
CA = h2= 1.5+0.23 = 1.73m

Pressure at center (P_P) = $\gamma h1$ = 9810x1.27 = 12459 Pa
Pressure at corner (P_c) = $\gamma h2$ = 9810x1.73 =16971 Pa

10. A cylindrical vessel of 0.5 m diameter and 0.6 m height is completely filled with water under a pressure of 9.81 KN/m^2. It is rotated at 300 rpm about its vertical axis. Determine the pressure at point adjacent to the wall of the vessel.



Solution:
P = 9.81 KP = 9810Pa
Radius (r) = 0.25m
Rpm(N) = 300
Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x300x\pi}{60}$ = 31.41 rad/s
$z = \frac{r^2\omega^2}{2g} = \frac{0.25^2 31.41^2}{2g}$ = 3.14m

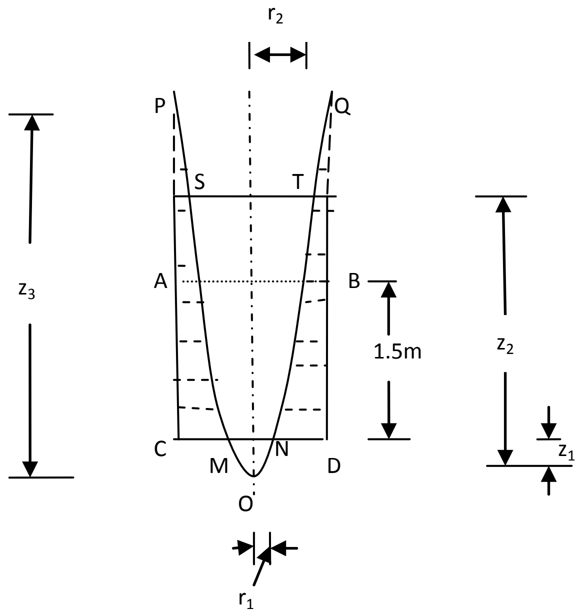
If the vessel is completely filled with water under a pressure and rotated about its vertical axis, the liquid will rise with a virtual height of h.

Volume above AB = volume of paraboloid POQ
$\pi x 0.25^2 x h = \frac{1}{2} x \pi x 0.25^2 x 3.14$
h = 1.57m

CP=h1= 0.6+1.57 = 2.17m

Pressure at C = $P + \gamma h1$ = 9810+9810X2.17 = 31098 Pa

11. A closed cylindrical vessel of 1m diameter and 2m height contains water filled to a depth of 1.5m. If the vessel is rotating at 20radians/sec, how much of the bottom of the vessel is uncovered?



Solution:
Radius (r) = 0.5m
Angular velocity (ω) = 20 rad/s

$z_3 = \frac{r^2\omega^2}{2g} = \frac{0.5^2 20^2}{2g}$ = 5.1m

$z_1 = \frac{r_1^2\omega^2}{2g} = \frac{r_1^2 20^2}{2g}$ = 20.38r_1^2 (a)

$z_2 = \frac{r_2^2\omega^2}{2g} = \frac{r_2^2 20^2}{2g}$ = 20.38r_2^2 (b)

Also, $z_2 = z_1$+1.5 = 20.38r_1^2 + 2 (c)
Equating b and c,
20.38r_1^2 + 2= 20.38r_2^2 (d)
$r_2^2 - r_1^2$ = 0.098 (e)

Volume of air above AB = Volume of paraboliod (POQ-MON)
$\pi x 0.5^2 x 0.5 = \frac{1}{2} x\pi x r_2^2 z_2 - \frac{1}{2} x\pi x r_1^2 z_1$ (f)
From a , b and f

$0.25 = r_2^2 x 20.38 r_2^2 - r_1^2 x 20.38 r_1^2$

$r_2^4 - r_1^4 = 0.0122$ (g)

solving e and g

$(r_2^2 - r_1^2)(r_2^2 + r_1^2) = 0.0122$

$0.098 (r_2^2 + r_1^2) = 0.0122$

$r_2^2 + r_1^2 = 0.1244$

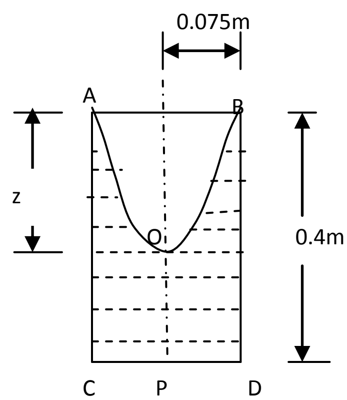
$r_2^2 - r_1^2 = 0.098$ (from e)

Solving above two equations

$r_1 = 0.115$m

Area uncovered = $\pi r_1^2 = \pi x 0.115^2$ = 0.0415m^2

12. A 400mm high open cylinder and 150mm in diameter is filled with water and rotated about its vertical axis at an angular speed of 33.5 rad/s. Determine (a) the depth of water in the cylinder when it is brought to rest, and (b) the volume of water that remains in the cylinder if the speed is doubled.



Solution:

Radius (r) = 0.075m

Angular velocity (ω) = 33.5 rad/s

a) Depth of water at rest (d) = ?

$z = \frac{r^2\omega^2}{2g} = \frac{0.075^2 33.5^2}{2x9.81}$ =0.32m

Amount of water spilled = Volume of paraboloid AOB

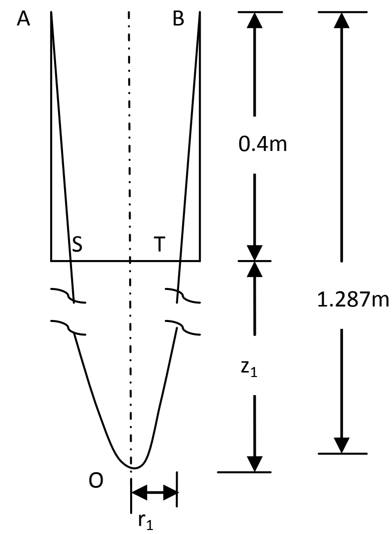
$= \frac{1}{2} x \pi r^2 z = \frac{1}{2} x \pi x 0.075^2 x 0.32$ = 0.002827m^3

Original volume of water $= \pi r^2 h = \pi x 0.075^2 x 0.4$ = 0.007069 m^3

Remaining volume of water (V_r) = 0.007069-0.002827 = 0.004242 m^3

$V_r = \pi r^2 d$

$0.004242 = \pi x 0.075^2 x d$
d = 0.24m

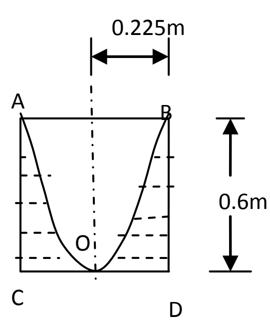


b) If the speed is doubled,
Angular velocity (ω) = 2x33.5 = 67 rad/s
$z = \frac{r^2\omega^2}{2g} = \frac{0.075^2 67^2}{2x9.81}$ = 1.287m
z_1 = 1.287-0.4 = 0.887m

$z_1 = \frac{{r_1}^2\omega^2}{2g}$
$0.887 = \frac{{r_1}^2 67^2}{2g}$
r_1 = 0.062m

Volume of water spilled = Volume of paraboloid (AOB-SOT)
$= \frac{1}{2} x \pi x 0.075^2 x 1.287 - \frac{1}{2} x \pi x 0.062^2 x 0.887$ = 0.006016 m^3
Original volume of water = 0.007069 m^3
Volume of water left = 0.007069-0.006016 = 0.00105 m^3

13. A cylindrical tank is spun at 300 rpm with its axis vertical. The tank is 0.6m high and 45cm diameter and is completely filled with water before spinning. Calculate (a) the speed at which the water surface will just touch the top rim and center bottom of the tank, and (b) the level to which the water will return when the tank stops spinning and the amount of water lost.



Solution:
Radius (r) = 0.225m
a) When the water surface touches the top rim and center bottom,

z = 0.6m

$z = \frac{r^2\omega^2}{2g}$

$0.6 = \frac{0.225^2\omega^2}{2g}$

ω = 15.25 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x15.25}{2\pi}$ = 146 rpm

b) Amount of water lost = ?

Depth of water after rest (d) = ?

Amount of water lost = Volume of paraboloid AOB

$= \frac{1}{2}x\pi x0.225^2x0.6$ = 0.0477m^3

Original volume of water $= \pi x0.225^2x0.6$ = 0.0954 m^3

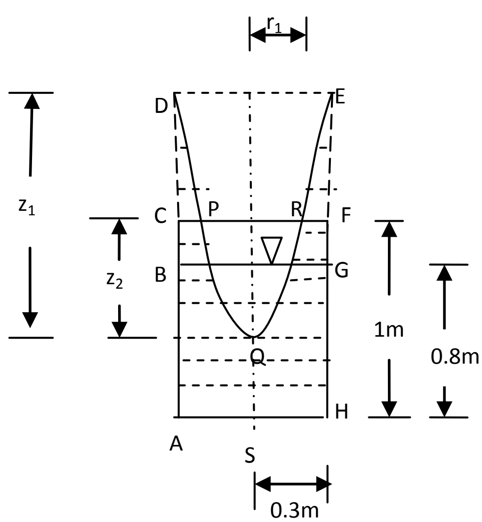
Volume of water left (V_r) = 0.0954-0.0477 = 0.0477 m^3

$V_r = \pi r^2 d$

$0.0477 = \pi x0.225^2 d$

d =0.3m

14. A cylindrical vessel closed at the top and bottom is 300mm inn diameter, 1m long and contains water up to a depth of 0.8m. The air above the water surface is at a pressure of 60 KPa. If the vessel is rotated at a speed of 250n rpm about its vertical axis, find the pressure head at the bottom of the vessel at the center point and at the edge.



Solution:

Pressure (Pa) = 60 Kpa

Head due to pressure (h) = $\frac{P_a}{\gamma} = \frac{60000}{9810}$ = 6.11m of water

Radius of cylinder (r) = 0.15m

N = 250 rpm
Pressure head at center and edge at the bottom = ?

Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x250x\pi}{60}$ = 26.16 rad/s

$z_1 = \frac{r^2\omega^2}{2g} = \frac{0.15^2 26.16^2}{2x9.81}$ =0.785m
$z_2 = \frac{r_1^2\omega^2}{2g} = \frac{r_1^2 26.16^2}{2x9.81} = 34.88r_1^2$ (a)

Volume of air above BG = Volume of parabola PQR

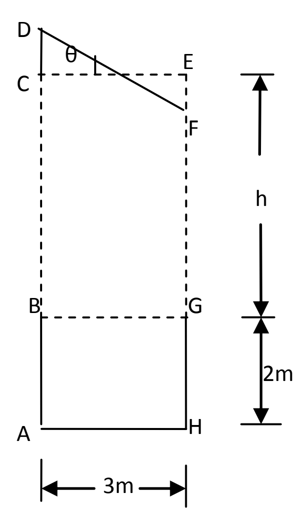
$$\pi x0.15^2x0.2 = \frac{1}{2}x\pi x r_1^2 x z_2$$

$r_1^2 z_2 = 0.009$ (b)
Solving a and b
r_1 =0.13m, Z_2 = 0.59m

QS = 1-Z2 = 1-0.59 = 0.41m
AD = AC+CD = 1+(0.785-0.59) = 1.195m

Pressure head at center = h+QS = 6.11+0.41 = 6.52m
Pressure head at edge = h+AD = 6.11+1.195 = 7.305m

15. A closed rectangular tank full of water is 3m long, 2m wide and 2m deep. The pressure at the top of water is raised to 98.1 Kpa. If now the tank is accelerated horizontally along its length at 6m/s^2, find the forces on the front and rear ends of the tank. Check your results by Newton's law too.



Solution:
Acceleration (a_x) = 6m/s^2
Pressure (P) = 98.1Kpa

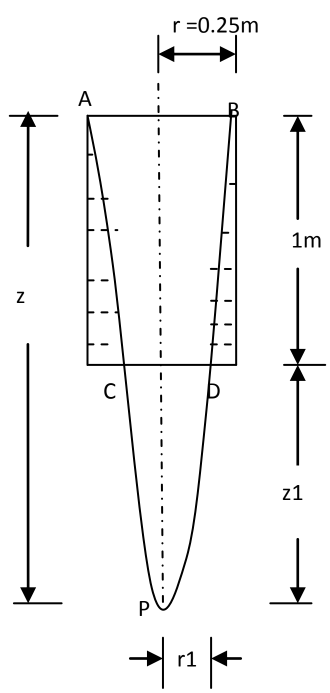
Head due to pressure (h) = $\frac{P}{\gamma} = \frac{98100}{9810}$ = 10m of water

$$tan\theta = \frac{a_x}{g} = \frac{6}{9.81}$$

CD = EF =$1.5tan\theta = 1.5x\frac{6}{9.81}$ = 0.917m
Pressure force at rear end (F_1) = $\gamma A\overline{x1}$ = 9810x(2x2)x(0.917+10+1) = 467623N
Pressure force at front end (F_2) = $\gamma A\overline{x2}$ = 9810x(2x2)x(10-0.917+1) = 395657N
Net force (F_x) = $F_1 - F_2$ = 467623 - 395657 = 71966N
According to Newton's law
F = m. a_x = ρ Volume a_x = 1000x(3x2x2)x6 = 72000 N
Hence F_x is equal to F.

16. An open cylinder tank 0.5m in diameter and 1m height is completely filled with water and rotated about its axis at 240 rpm. Determine the radius up to which the bottom will be exposed and the volume of water spilled out of the tank.

Solution:



Radius (r) = 0.25m
N = 240rpm
Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x240x\pi}{60}$ = 25.13 rad/s

$z = \frac{r^2\omega^2}{2g} = \frac{0.25^2 25.13^2}{2g}$ = 2.01m

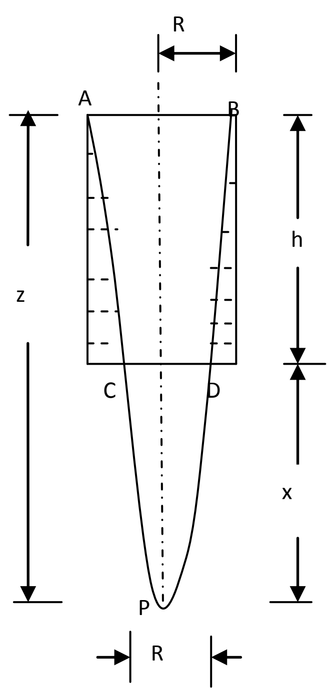
Z1 = 2.01-1= 1.01m
r1 = ?
Volume of water spilled = ?

$z_1 = \frac{{r_1}^2\omega^2}{2g}$
$1.01 = \frac{{r_1}^2 25.13^2}{2g}$
r_1 = 0.18m

Volume of water spilled = Volume of paraboloid (APB-CPD)
$= \frac{1}{2} x\pi x 0.25^2 x 2.01 - \frac{1}{2} x\pi x 0.18^2 x 1.01$ = 0.146 m^3

17. An open circular cylindrical pipe of radius R and height h is completely filled with water with its axis vertical and is rotated about its axis at an angular velocity ω. Determine the value of ω in terms of R and h such that the diameter of the exposed center portion is equal to the radius of the cylinder.

Solution:



Radius of cylinder = R
Diameter of exposed central portion = R
Radius of exposed central portion = R/2
For parabola APB

$z = \frac{R^2\omega^2}{2g}$

$x + h = \frac{R^2\omega^2}{2g}$ (a)

For parabola CPD

$x = \frac{(R/2)^2\omega^2}{2g} = \frac{R^2\omega^2}{8g}$ (b)

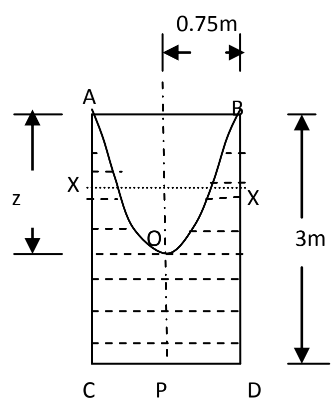
From a and b

$h = \frac{R^2\omega^2}{2g} - \frac{R^2\omega^2}{8g} = \frac{3R^2\omega^2}{8g}$

$\omega = \sqrt{\frac{8gh}{3R^2}}$

18. A cylindrical tank 1.5m in diameter and 3m in height contains water to a depth of 2.5m. Find the speed of the tank so that 20% of the original volume is spilled out.

Solution:



Radius of the tank (r) = 0.75m

After 20% of the original quantity of water spills out and if the depth is brought to rest, the depth of water will be reduced to 20%.

Depth of water when the tank comes to rest = 80% of original depth = 0.8x2.5 = 2m

Rise in water level at the end = 3-2 = 1m = fall at the center

i. e. z = 2m

$z = \frac{r^2\omega^2}{2g}$

$2 = \frac{0.75^2\omega^2}{2g}$

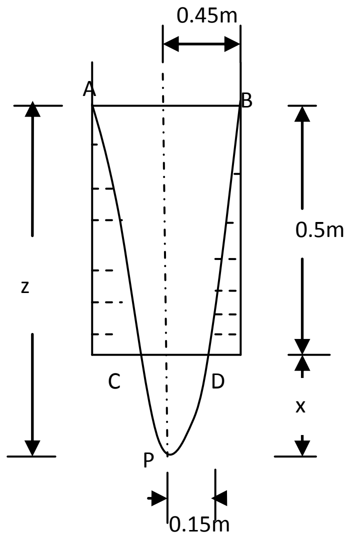
ω = 8.35 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x8.35}{2\pi}$ = 80 rpm

19. Determine the speed of rotation of a cylinder 900mm diameter when the liquid contained in it rises to 500mm height at sides and leaves a circular space 300mm diameter on the bottom uncovered. Taking

the liquid as water, calculate the total pressure on the bottom. Find also the depth when the vessel is stationary.

Solution:



Radius of cylinder (r) = 0.45m

Radius at bottom (r1) = 0.15m

For APB

$z = \frac{r^2\omega^2}{2g}$

$x + 0.5 = \frac{0.45^2\omega^2}{2g} = 0.0103\omega^2$ (a)

For CPD,

$x = \frac{r1^2\omega^2}{2g}$

$x = \frac{0.15^2\omega^2}{2g} = 0.001147\omega^2$ (b)

From a and b

ω = 7.4 rad/s, x = 0.063m

$N = \frac{60\omega}{2\pi} = \frac{60x7.4}{2\pi}$ = 70.6 rpm

Volume of water spilled = Volume of paraboloid (APB-CPD)

$= \frac{1}{2}x\pi x0.45^2x(0.5 + 0.063) - \frac{1}{2}x\pi x0.15^2x0.063$ = 0.1768 m^3

Volume of water in the tank before rotation = $\pi x0.45^2x0.5$ = 0.318 m^3

Volume of water left (V_r) =0.318-0.1768 = 0.1412 m^3

Total pressure on the bottom = Weight of water in the tank = γV_r = 9810x0.1412 = 1385N

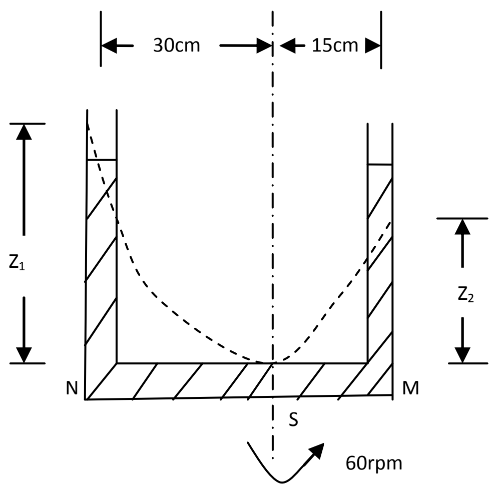
$V_r = \pi r^2 d$

$0.1412 = \pi x0.45^2 d$

d = 0.22m

Depth of water when the vessel is stationary = 0.22m

20. A U-tube shown in figure is filled with a liquid of specific gravity 1.25 to a height of 15cm in both the limbs. It is rotated about a vertical axis 15cm from one limb and 30cm from the other. If the speed of rotation is 60rpm, find the difference in the liquid levels in the two limbs. Also find the pressure at points M and N at the base of U-tube.



Solution:

Distance from the axis of rotation to left side (r_1) = 0.3m

Distance from the axis of rotation to right side (r_2) = 0.15m

Speed of rotation (N) = 60 rpm

Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x60x\pi}{60}$ = 6.28 rad/s

$z_1 = \frac{r_1^2\omega^2}{2g} = \frac{0.3^2x6.28^2}{2g}$ = 0.181m

$z_2 = \frac{r_2^2\omega^2}{2g} = \frac{0.15^2x6.28^2}{2g}$ = 0.045m

Difference in level = 0.181-0.045 = 0.136m

Sum of Z_1 and Z_2 = 0.226

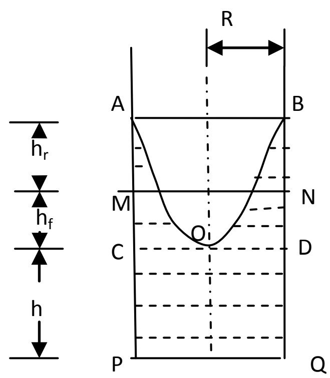
But total height of water in two limbs before rotation = 0.15+0.15 = 0.3m

The difference (0.3-0.226 = 0.074) is equally divided for two limbs.

Pressure at N = $\gamma(Z_1 + 0.074/2)$ = 1.25x9810x0.218 = 2673 Pa

Pressure at M = $\gamma(Z_2 + 0.074/2)$ = 1.25x9810x0.082 = 1005.5 Pa

21. Prove that in case of forced vortex, rise of liquid levels at the end is equal to the fall of liquid level at the axis of rotation.



Solution:

R = Radius of cylinder

MN = Water level at absolute equilibrium (original water level)

After rotation, AOB is the profile of the liquid surface.

h_r = Rise of liquid at end

h_f = Fall of liquid at the end

Volume of liquid before rotation = $\pi R^2(h + h_f)$

Volume of liquid rotation = Volume of cylinder ABQP- Volume of paraboloid AOB

$$= \pi R^2(h + h_f + h_r) - \frac{1}{2}\pi R^2(h_f + h_r)$$

Volume of liquid before rotation = Volume of liquid after rotation

$$\pi R^2(h + h_f) = \pi R^2(h + h_f + h_r) - \frac{1}{2}\pi R^2(h_f + h_r)$$

$$h_r = h_f$$

Hence, rise of liquid level at the end = fall of liquid level at the axis of rotation