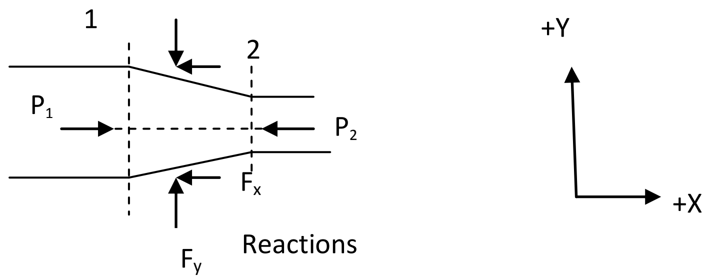
Tutorial 9

Application of Momentum Principle

1. A 60cm pipe is connected to a 30cm pipe by a standard reducer fitting. For the same flow of 0.9 m^3/s of water and a pressure of 200Kpa, what force is exerted by the water on the reducer, neglecting any lost head?



Solution:

Diameter at section 1 (d_1) = 600mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.6^2$= 0.2827m^2

Diameter at section 2(d_2) = 300mm = 0.3m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Discharge (Q) = 0.9 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = =0.9/0.2827 = 3.18 m/s

Velocity at section 2 (V_2) =Q/A_2 =0.9/0.07068 = 12.73 m/s

Pressure at section 1 (P_1) = 200Kpa = 200000N/m^2

Force exerted by water on reducer =?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{200000}{1000x9.81}+\frac{3.18^2}{2x9.81}=\frac{P_2}{1000x9.81}+\frac{12.73^2}{2x9.81}$$

P_2 = 124030 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$$(P_1\,A_1-P_2\,A_2)-F_x=\rho Q(V_{2x}-V_{1x})$$

$$(P_1\,A_1-P_2A_2)-F_x=\rho Q(V_2-V_1)$$

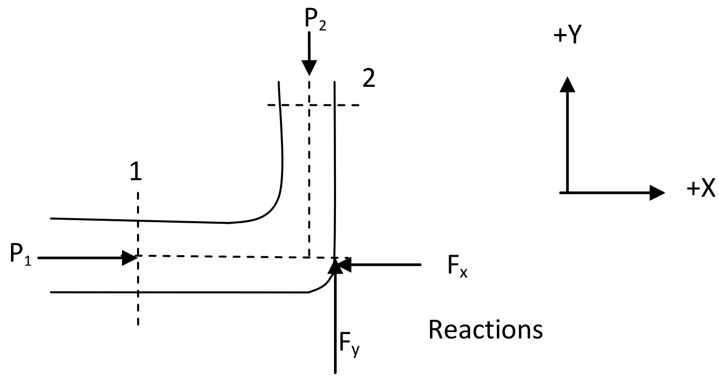
$$F_x=(P_1\,A_1-P_2A_2)+\rho Q(V_1-V_2)$$

$= (200000x\ 0.2827-124030\ x\ 0.07068)+1000x0.9(3.18-12.73)$ = 39178 N

The forces in the Y-direction will balance each other and F_y = 0.

Hence the force exerted by water on reducer is 39178 N to the right.

2. A 500mm pipe carrying 0.8 m^3/s of oil (sp gr 0.85) has a 90^0 bend in a horizontal plane. The loss of head in the bend is 1.1m of oil, and the pressure at the entrance is 290KPa. Determine the resultant force exerted by the oil on the bend.



Solution:

Diameter at section 1 ad 2 (d) = 500mm = 0.5m

Area at section 1 and 2 ($A_1 = A_2$) = $\frac{\pi}{4} x 0.5^2$= 0.19635m^2

Discharge (Q) = 0.8 m^3/s

Velocity at 1 and 2 ($V_1 = V_2$) = Q/A_1 = 0.8/0.19635 = 4.07m/s

Pressure at 1 (P_1) = 290KPa = 290000 Pa

Loss of head (h_L) = 1.1m

Resultant force exerted by the oil on the bend = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{290000}{0.85x1000x9.81} + \frac{4.07^2}{2x9.81} = \frac{P_2}{0.85x1000x9.81} + \frac{4.07^2}{2x9.81} + 1.1$$

P_2 = 280828 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$$P_1\ A_1 - F_x = \rho Q(V_{2x} - V_{1x})$$

$$P_1\ A_1 - F_x = \rho Q(0 - V_1)$$

$$F_x = P_1\ A_1 + \rho Q V_1$$

$$= 290000x0.19635 + 0.85x1000x0.8x4.07$$

= 59709 N

$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$$

$$F_y - P_2\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$$

$$F_y - P_2\ A_2 = \rho Q(V_2 - 0)$$

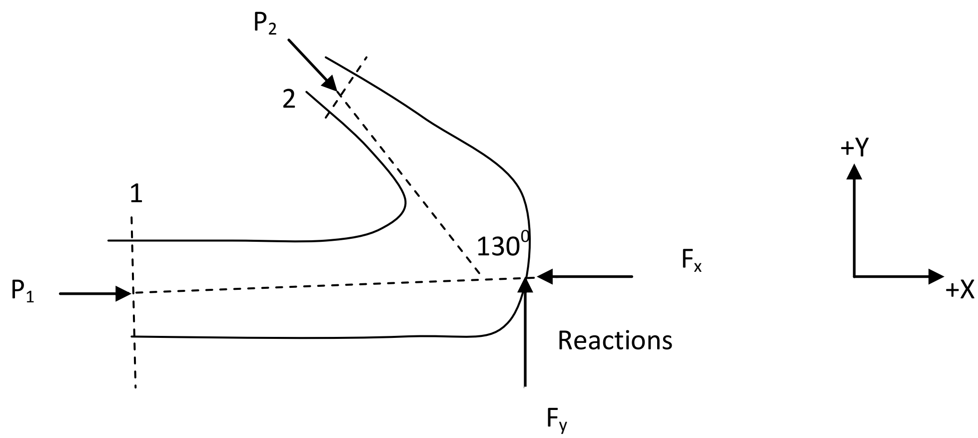
$F_y = P_2\, A_2 + \rho Q V_2$
$= 280828x0.19635 + 0.85x1000x0.8x4.07$
= 57908 N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =83177N

Resultant force exerted by the water on the bend = 83177N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{57908}{59709}$ = 44.1^0

3. The discharge of water through a 130^0 bend is 30 litres/s. The bend is lying in the horizontal plane and the diameters at the entrance and exit are 200mm and 100mm respectively. The pressure measured at the entrance is 100 kN/m^2, what is the magnitude and direction of the force exerted by the water on the bend?



Solution:
Diameter at section 1 (d_1) = 200mm = 0.2m
Area at section 1 (A_1) = $\frac{\pi}{4}x0.2^2$= 0.0314m^2
Diameter at section 2(d_2) = 100mm = 0.1m
Area at section 2 (A_2) = $\frac{\pi}{4}x0.1^2$= 0.00785m^2
Discharge (Q) = 30 lps = 0.03 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = 0.03/0.0314 = 0.95 m/s
Velocity at section 2 (V_2) =Q/A_2 = 0.03/0.00785=3.82 m/s
θ= 180-130 = 50^0
Pressure at 1 (P_1) = 100KPa = 100000 Pa
Resultant force exerted by the oil on the bend = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{100000}{1000x9.81} + \frac{0.95^2}{2x9.81} = \frac{P_2}{1000x9.81} + \frac{3.82^2}{2x9.81}$$

$P_2 = 93155\ N/m^2$

Resultant force (F_R) =?
Direction of resultant force =?

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ \ in\ X\ direction$$
$$(P_1\ A_1 + P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$
$$(P_1\ A_1 + P_2 A_2 Cos\theta) - F_x = \rho Q(-V_2 Cos\theta - V_1)$$
$$F_x = (P_1\ A_1 + P_2 A_2 Cos\theta) + \rho Q(V_1 + V_2 Cos\theta)$$
$$= (100000x0.0314 + 93155x0.00785Cos50) + 1000x0.03(0.95 + 3.82Cos50)$$
= 3712 N

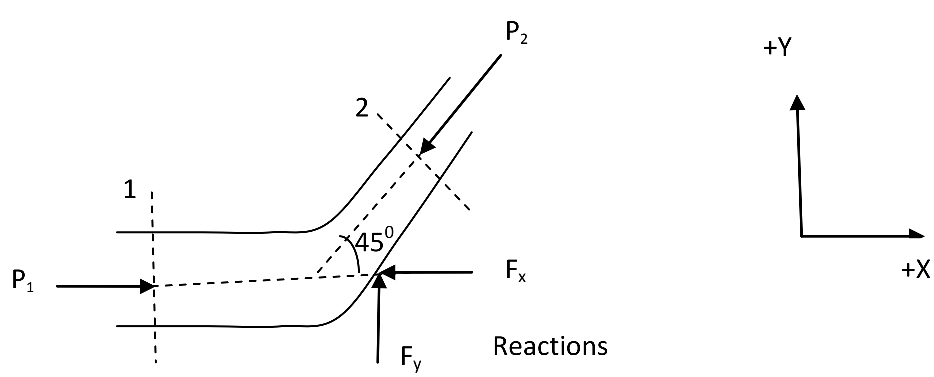
$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$$
$$F_y - P_2 Sin\theta\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$$
$$F_y - P_2\ A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$$
$$F_y = P_2\ A_2 Sin\theta + \rho Q V_2 Sin\theta$$
$$= 93155x0.00785Sin50 + 1000x0.03x3.82Sin50$$
= 648N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =3768N

Resultant force exerted by the water on the bend = 3768N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{648}{3712}$ =10^0

4. A 45^0 pipe bend tapers from 600mm diameter at inlet to 300mm diameter at outlet. The pressure at inlet is $140KN/m^2$ and the rate of flow is $0.425m^3/s$. At outlet the pressure is $123KN/m^2$ gauge. Neglecting friction, calculate the resultant force exerted by the water on the bend in magnitude and direction. The bend lies in a horizontal plane.



Solution:

Diameter at section 1 (d_1) = 600mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.6^2$= 0.2827m^2

Diameter at section 2(d_2) = 300mm = 0.3m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Discharge (Q) = 0.425 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = 1.5 m/s

Velocity at section 2 (V_2) =Q/A_2 = 6.01 m/s

Pressure at section 1 (P_1) = 140KN/m^2 = 140000N/m^2

Pressure at section 2 (P_2) =123KN/m^2 =123000N/m^2

Angle of bend (θ)= 45^0

Resultant force (F_R) =?

Direction of resultant force =?

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ \ in\ X\ direction$$

$$(P_1\ A_1 - P_2Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$

$$(P_1\ A_1 - P_2A_2Cos\theta) - F_x = \rho Q(V_2Cos\theta - V_1)$$

$$F_x = (P_1\ A_1 - P_2A_2Cos\theta) + \rho Q(V_1 - V_2Cos\theta)$$

$$= (140000x0.2827 - 123000x0.07068Cos45) + 1000x0.425(1.5 - 6.01Cos45)$$

= 32262 N

$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$$

$$F_y - P_2Sin\theta\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$$

$$F_y - P_2\ A_2Sin\theta = \rho Q(V_2Sin\theta - 0)$$

$$F_y = P_2\ A_2Sin\theta + \rho QV_2Sin\theta$$

$$= 123000x0.07068Sin45 + 1000x0.425x6.01Sin45$$

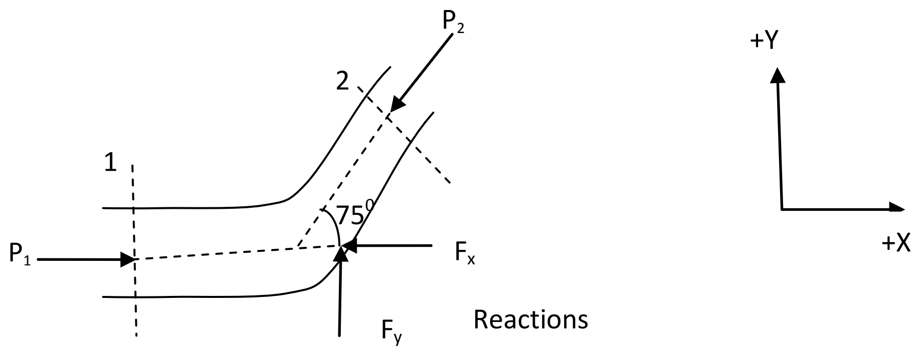
= 7953N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =33228N

Resultant force exerted by the water on the bend = 33228N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{7953}{32262}$ =13.8^0

5. A 150mm diameter pipe on the horizontal plane carries water under the head of 16m of water with the velocity of 3.5 m/s. Find the direction and magnitude of the pipe bend, if the axis of the bend was turned with angle 75^0. Assume no loss of energy at the pipe bend.



Solution:

Diameter ($d_1 = d_2$) = 150mm = 0.15m

Area at section 1 and 2 (A_1=A_2) $= \frac{\pi}{4} x 0.15^2$ = 0.01767m^2

Velocity (V_1=V_2) = 3.5 m/s

Discharge (Q) = A_1 V_1 = 0.01767x3.5 =0.0618 m3/s

Pressure head = $\frac{P}{\rho g}$ = 16 m

P =156960 N/m^2

Pressure (P=P_1=P_2) = 156960 N/m^2

Angle of bend (θ)= 75^0

Resultant force (F_R) =?

Direction of resultant force =?

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ \ in\ X\ direction$$
$$(P_1\ A_1 - P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$
$$(P_1\ A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1)$$
$$F_x = (P_1\ A_1 - P_2 A_2 Cos\theta) + \rho Q(V_1 - V_2 Cos\theta)$$
$$= (156960x0.01767 - 156960x0.01767 Cos75) + 1000x0.0618(3.5 - 3.5Cos75)$$

= 2216 N

$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$$
$$F_y - P_2 Sin\theta\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$$
$$F_y - P_2\ A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$$
$$F_y = P_2\ A_2 Sin\theta + \rho Q V_2 Sin\theta$$
$$= 156960x0.01767 Sin75 + 1000x0.0618x3.5 Sin75$$

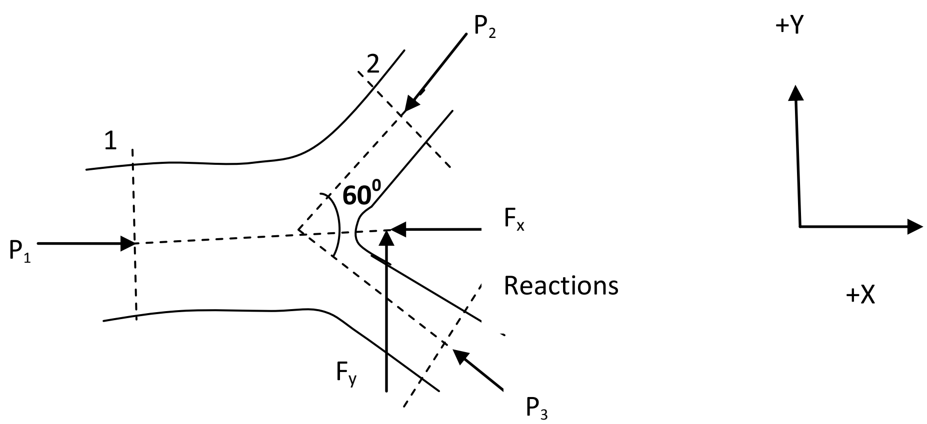
= 2888N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =3640N

Resultant force exerted by the water on the bend = 3640N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{2888}{2216}$ = 52.5^0

6. A main pipe of diameter 500mm branches in two pipes of diameter 300mm each in the horizontal plane. Angle between the branches is 60^0, which is symmetrical with respect to the main pipe. Flow discharge through the main pipe is 1.0 m^3/s, which is equally divided into the branch pipes. If the pressure intensity at the main pipe is 400KPa, find the magnitude and direction of resultant force in the bend. Assume no loss of energy due to branch junction and in pipe sections.



Diameter at section 1 (d_1) = 500mm = 0.5m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.5^2$= 0.19635m^2

Diameter at section 2 and 3 (d_2 =d_3) = 300mm = 0.3m

Area at section 2 and 3 (A2 =A3) = $\frac{\pi}{4}x0.3^2$ = 0.07068m^2

Angle of bend (θ)= 30^0 for pipe 2 and 3

Discharge through 1 (Q_1) = 1 m^3/s

Discharge through 2 and 3 (Q_2=Q_3) = $Q_1/2$ =0.5 m^3/s

Pressure at 1 (P1) = 400 KPa = 400000N/m^2

Resultant force (F_R) =?

Direction of resultant force =?

Velocity at 1 (V_1) = Q_1/A_1 = 5.09 m/s

Velocity at 2 (V_2) = Q_2/A_2 = 7.07 m/s

Velocity at 3 (V_3) = Q_3/A_3 = 7.07 m/s

Using Bernoulli's equation at 1 and 2 (Z_1=Z_2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{400000}{1000x9.81} + \frac{5.09^2}{2x9.81} = \frac{P_2}{1000x9.81} + \frac{7.07^2}{2x9.81}$$

P_2 = 387962 N/m^2

Using Bernoulli's equation at 1 and 3 (Z_1=Z_3)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3$$

$$\frac{400000}{1000x9.81} + \frac{5.09^2}{2x9.81} = \frac{P_3}{1000x9.81} + \frac{7.07^2}{2x9.81}$$

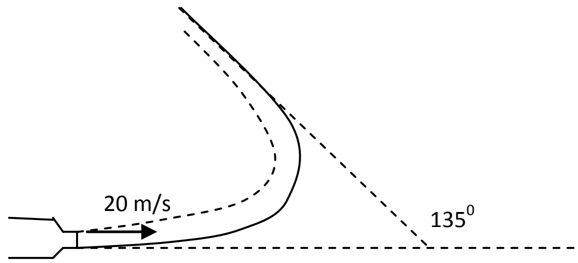
P_3 = 387962 N/m^2

$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ \ in\ X\ direction$
$(P_1\ A_1 - P_2 Cos\theta\ A_2 - P_3 Cos\theta\ A_3) - F_x = \rho\ [(Q_2\ V_{2x} + Q_3\ V_{3x}) - Q_1 V_{1x}]$
$(P_1\ A_1 - P_2 A_2 Cos\theta\ \ - P_3 A_3 Cos\theta\ ) - F_x = \rho\ [(Q_2\ V_2 Cos\theta + Q_3\ V_3 Cos\theta) - Q_1 V_1]$
$F_x = (P_1\ A_1 - P_2 A_2 Cos\theta\ \ - P_3 A_3 Cos\theta\ ) + \rho(Q_1 V_1 - Q_2\ V_2 Cos\theta - Q_3\ V_3 Cos\theta)$
$= (400000x0.19635 - 387962x0.07068Cos30 - 387962x0.07068Cos30) + 1000(1x5.09 -$
$0.5x7.07Cos30 - 0.5x7.07Cos30)$
= 30012N

$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$
$F_y - P_2 Sin\theta\ A_2 + P_3 Sin\theta\ A_3 = \rho\ \left[\left(Q_2\ V_{2y} + Q_3\ V_{3y}\right) - Q_1 V_{1y}\right]$
$F_y - P_2 A_2 Sin\theta\ \ + P_3 A_3 Sin\theta\ \ = \rho\ [(Q_2\ V_2 Sin\theta + Q_3 x -\ V_3 Sin\theta) - 0]$
$F_y = (P_2 A_2 Sin\theta - P_3 A_3 Sin\theta) + \rho\ (Q_2\ V_2 Sin\theta - Q_3\ V_3 Sin\theta)$
$= 387962x0.07068Sin30 - 387962x0.07068Sin30 + 1000x0.5x7.07Sin30 - 1000x0.5x7.07Sin30$
= 0

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =30012N
Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{0}{30012}$ = 0^0

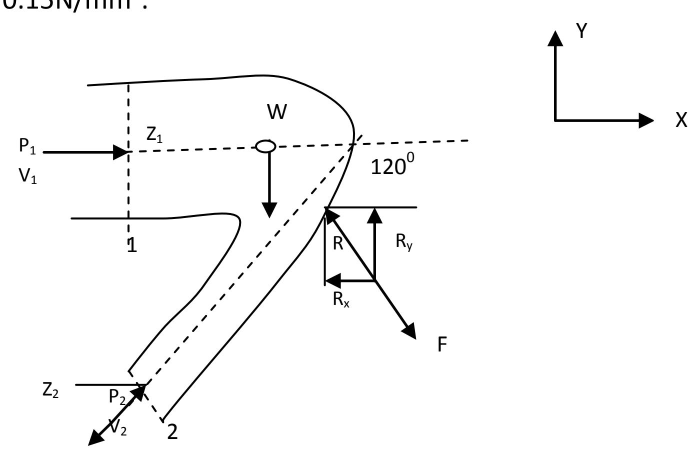
7. Determine the magnitude of resultant force and its direction on the vane shown in the figure below if a water of jet 50mm diameter and 20m/s velocity strikes the vane tangentially and deflects without friction.



Solution:
Velocity (V) = 20m/s
Diameter of jet (d) = 50mm = 0.05m
C/s of jet (A) = $\frac{\pi}{4}x0.05^2$ 0.001963 m^2
Discharge (Q) = AV = 0.001963x20 = 0.03927 m^3/s
No friction: No loss of head
Pressure is atmospheric: so no pressure force
V is constant throughout.

$F_x = \rho Q(V_{1x} - V_{2x}) = 1000x0.03927(20 - (-20cos45))$ = 1340.8 N
$F_y = \rho Q(V_{1y} - V_{2y}) = 1000x0.03927(0 - 20sin45)$ = 555.4 N
Resultant force = $\sqrt{F_x^2 + F_y^2} = \sqrt{1340.8^2 + 555.4^2}$ = 1451N
Direction = $tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{555.4}{1451}$ = 22.5^0

8. The diameter of a pipe bend is 30cm at inlet and 15cm at outlet and the flow is turned through 120^0 in a vertical plane. The axis at inlet is horizontal and the center of the outlet section is 1.5m below the center of the inlet section. Total volume of water in the bend is 0.9m^3. Neglecting friction, calculate the magnitude and direction of the force exerted on the bend by water flowing through it at 250lps and when the inlet pressure is 0.15N/mm^2.



Solution:

Diameter at section 1 (d_1) = 30cm = 0.3m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Diameter at section 2(d_2) = 15cm = 0.15m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Discharge (Q) = 250 lps = 0.25 m^3/s

Volume of water within control volume (Vol) = 0.9m^3

Weight of water within control volume (W) = $\gamma_{water}Vol$ = 9810x0.9 = 8829N

Velocity at section 1 (V_1) =Q/A_1 = 0.25/0.07068 = 3.54 m/s

Velocity at section 2 (V_2) =Q/A_2 = 0.25/0.01767=14.15 m/s

Z_2 = 0, Z_1 = 1.5m

θ= 180-120 = 60^0

Pressure at 1 (P_1) = 0.15N/mm^2 = 0.15x10^6 N/m^2

Resultant force exerted by the water on the bend = ?

Applying Bernoulli's equation between 1 and 2

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{0.15x10^6}{1000x9.81}+\frac{3.54^2}{2x9.81}+1.5=\frac{P_2}{1000x9.81}+\frac{14.15^2}{2x9.81}+0$$

P_2 = 70870 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$$(P_1\,A_1+P_2Cos\theta\,A_2)-R_x=\rho Q(V_{2x}-V_{1x})$$

$$(P_1\,A_1+P_2A_2Cos\theta)-R_x=\rho Q(-V_2Cos\theta-V_1)$$

$$R_x=(P_1\,A_1+P_2A_2Cos\theta)+\rho Q(V_1+V_2Cos\theta)$$

$$=(0.15x10^6x0.07068+70870x0.01767Cos60)+1000x0.25(3.54+14.15Cos60)$$

= 13882 N

$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$$

$$R_y-P_2Sin\theta\,A_2-W=\rho Q\left(V_{2y}-V_{1y}\right)$$

$$R_y-P_2\,A_2Sin\theta-W=\rho Q(-V_2Sin\theta-0)$$

$$R_y=P_2\,A_2Sin\theta-\rho QV_2Sin\theta+W$$

$$=70870x0.01767Sin60-1000x0.25x14.15Sin60+8829$$

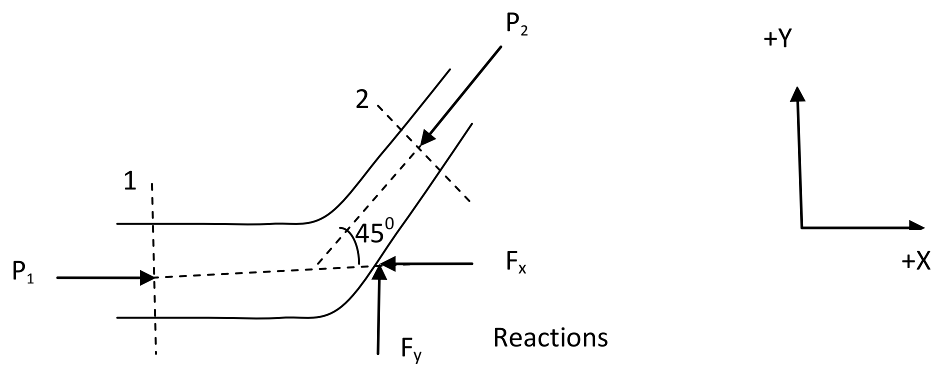
= 6850N

Resultant force $(R)=\sqrt{R_x^2+R_y^2}$ =15481N

Resultant force exerted by the water on the bend = 15481N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{R_y}{R_x}$ = $Tan^{-1}\frac{6850}{13882}$ =26.3^0

9. A 45^0 reducing bend is connected in a pipe line carrying water. The diameter at inlet and outlet of the bend is 400mm and 200mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet of the bend is $215.8 KN/m^2$. The rate of flow of water is $0.5 m^3/s$. The loss of head in the bend is 1.25m of oil of sp.gr. 0.85.



Solution:

Diameter at section 1 (d_1) = 400mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4} x 0.4^2$= $0.1256 m^2$

Diameter at section 2(d_2) = 200mm = 0.2m

Area at section 2 (A_2) = $\frac{\pi}{4} x 0.2^2$= $0.0314 m^2$

Discharge (Q) = $0.5 m^3/s$

Velocity at section 1 (V_1) =Q/A_1 = 3.98 m/s

Velocity at section 2 (V_2) =Q/A_2 = 15.92 m/s

Pressure at section 1 (P_1) = $215.8 KN/m^2$ = $215800 N/m^2$

Loss of head = 1.25m of oil of sp gr 0.85, P= 0.85x9810x1.25 N/m^2

Loss of head in terms of water (h_L) = $\frac{P}{\gamma} = \frac{0.85 x 9810 x 1.25}{9810}$ =1.0625m

Angle of bend (θ)= 45^0

Resultant force (F_R) =?

Direction of resultant force =?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{215800}{1000 x 9.81} + \frac{3.98^2}{2 x 9.81} = \frac{P_2}{1000 x 9.81} + \frac{15.92^2}{2 x 9.81} + 1.0625$$

$P_2 = 86574 N/m^2$

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$$(P_1 A_1 - P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$

$(P_1\, A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1)$
$F_x = (P_1\, A_1 - P_2 A_2 Cos\theta) + \rho Q(V_1 {-} V_2 Cos\theta)$
$= (215800x0.1256 - 86574x0.0314Cos45) + 1000x0.5(3.98 - 15.92Cos45)$
= 21544N

$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$
$F_y - P_2 Sin\theta\, A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$
$F_y - P_2\, A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$
$F_y = P_2\, A_2 Sin\theta + \rho Q V_2 Sin\theta$
$= 86574x0.0314Sin45 + 1000x0.5x15.92Sin45$
= 7551N

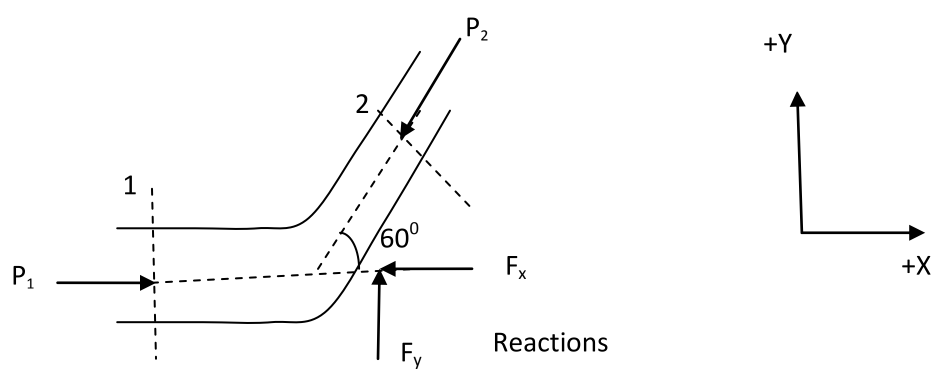
Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =22829N

Resultant force exerted by the water on the bend = 22829N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x} = Tan^{-1}\frac{7551}{21544}$ =19.3^0

10. The angle of a reducing bend is 60^0. Its initial diameter is 300mm and final diameter is 150mm and is lifted in a pipeline carrying water at a rate of 330 lps. The pressure at the commencement of the bend is 3.1 bar. The friction loss in the pipe may be assumed as 10% of kinetic energy at the exit of the bend. Determine the force exerted by the reducing bend.

Solution:



Solution:

Diameter at section 1 (d_1) = 300mm = 0.3m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Diameter at section 2(d_2) = 150mm = 0.15m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Discharge (Q) = 330lps = 0.33 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = 4.67 m/s
Velocity at section 2 (V_2) =Q/A_2 = 18.67 m/s
Pressure at section 1 (P_1) = 3.1 bar = $3.1x10^5$ N/m^2
Loss of head (h_L) = 10% of velocity head at 2 = $0.1\frac{V_2^2}{2g} = 0.1\frac{18.67^2}{2x9.81}$ = 1.77m
Angle of bend (θ)= 50^0
Resultant force (F_R) =?
Direction of resultant force =?
Finding pressure at section 2 (P_2)
Using Bernoulli's equation at 1 and 2 (Z_1=Z_2)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L$$

$$\frac{3.1x10^5}{1000x9.81}+\frac{4.67^2}{2x9.81}=\frac{P_2}{1000x9.81}+\frac{18.67^2}{2x9.81}+1.77$$

P_2 = 129256 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$
$$(P_1\,A_1 - P_2 Cos\theta\,A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$
$$(P_1\,A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1)$$
$$F_x = (P_1\,A_1 - P_2 A_2 Cos\theta) + \rho Q(V_1 - V_2 Cos\theta)$$
$$= (310000x0.07068 - 129256x0.01767Cos60) + 1000x0.33(4.67 - 18.67Cos60)$$
= 19229 N

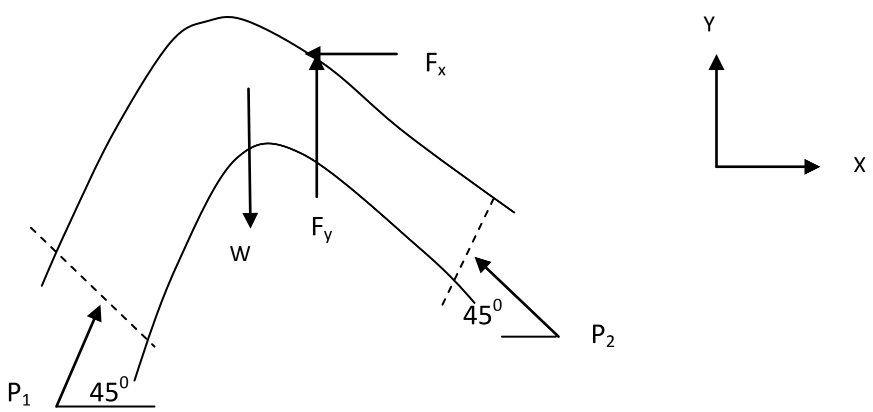
$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$$
$$F_y - P_2 Sin\theta\,A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$$
$$F_y - P_2\,A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$$
$$F_y = P_2\,A_2 Sin\theta + \rho Q V_2 Sin\theta$$
$$= 129256x0.01767Sin60 + 1000x0.33x18.67Sin60$$
= 7314N

Resultant force $(F_R) = \sqrt{F_x^2 + F_y^2}$ =20573N

Resultant force exerted by the water on the bend = 20573N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{7314}{19229}$ =20.8^0

11. A 0.4mx0.3m, 90^0 vertical bend carries $0.6m^3/s$ oil of sp gr 0.8 with a pressure of 120 Kpa at inlet to the bend. The volume of the bend is 0.1 m3. Find the magnitude and direction of the force on the bend. Neglect friction and assume both inlet and outlet sections to be at same horizontal level. Also assume that water enters the bend at 45^0 to the horizontal.



Solution:

Diameter at section 2(d_1) = 0.4m

Area at section 2 (A_1) = $\frac{\pi}{4}x0.4^2$= $0.1256m^2$

Diameter at section 1 (d_2) = 0.3m

Area at section 1 (A_2) = $\frac{\pi}{4}x0.3^2$= $0.07068m^2$

Discharge (Q) = $0.6\ m^3/s$

Weight of oil (W) = $\gamma_{oil}\ Vol$ = 0.8x9810x0.1 = 784.8N

Velocity at section 1 (V_1) =Q/A_1 = 4.8 m/s

Velocity at section 2 (V_2) =Q/A_2 = 8.5 m/s

Pressure at section 1 (P_1) = 120 Kpa

$\theta = 45^0$

Resultant force (F_R) =?

Direction of resultant force =?

Finding pressure at section 2 (P_2)

Using Bernoulli's equation at 1 and 2 (Z_1=Z_2)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{120000}{0.8x1000x9.81}+\frac{4.8^2}{2x9.81}=\frac{P_2}{0.8x1000x9.81}+\frac{8.5^2}{2x9.81}$$

P_2 = 100316 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$(P_1 Cos\theta\, A_1 - P_2 Cos\theta\, A_2) - F_x = \rho Q(V_{2x} - V_{1x})$
$(P_1\, A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1 Cos\theta)$
$F_x = (P_1\, A_1 Cos\theta - P_2 A_2 Cos\theta) + \rho Q(V_1 Cos\theta - V_2 Cos\theta)$
$= (120000x0.1256Cos45 - 100316x0.07068Cos45) + 0.8x1000x0.6(4.8Cos45 - 8.5Cos45)$
= 4388 N

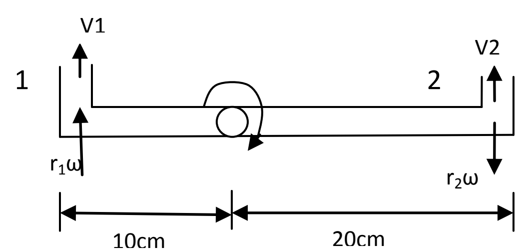
$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$
$F_y + (P_1 Sin\theta\, A_1 + P_2 Sin\theta\, A_2) - W = \rho Q\left(V_{2y} - V_{1y}\right)$
$F_y + (P_1 A_1 Sin\theta\ + P_2 A_2 Sin\theta\ ) - W = \rho Q(V_2 Sin\theta - V_1 Sin\theta)$
$F_y = -(P_1 A_1 Sin\theta\ + P_2 A_2 Sin\theta\ ) + W + \rho Q(V_2 Sin\theta - V_1 Sin\theta)$
$= -(120000x0.1256Sin45 + 100316x0.07068Sin45) + 784.8 + 0.8x1000x0.6(4.8Sin45 - 8.5Sin45)$
= -6115N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =7526N

Resultant force exerted by the water on the bend = 7526N (to the right and upward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{6115}{4388}$ =54^0

12. The lawn sprinkler shown below has nozzles of 5mm diameter and carries a total discharge of 0.20 lps. Determine the angular speed of rotation of the sprinkler and torque required to hold the sprinkler stationary. Assume no friction at the pivot.



Solution:
Diameter of nozzle (d) = 5mm =0.005m
Area of nozzle (A) = $\frac{\pi}{4}x0.005^2$ = $1.963x10^{-5}$ m^2
r_1 = 10cm =0.1m
r_2 = 20cm =0.2m
Total discharge = 0.2 lps
Discharge through each nozzle (Q) = 0.2/2 = 0.1 lps = 0.0001 m^3/s
Relative velocity at outlet of each nozzle (V_1 = V_2) = Q/A = 5.09 m/s
For torque(T) = 0, Angular speed of rotation (ω)=?
For ω =0, Torque (T) =?

a. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Jet exerts force in opposite direction at nozzle 1 and 2 (downward direction).
Torque at 1: anticlockwise, torque at 2: clockwise.
As the torque arm for 2 is greater, the sprinkler will rotate clockwise if free to rotate.

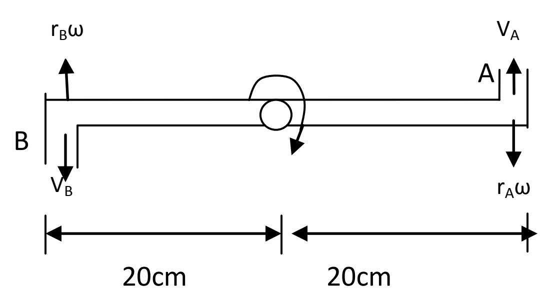
Absolute velocity at 1 (V_{1a}) = 5.09 + $r_1\,\omega$ = = 5.09 + 0.1 ω (tangential velocity and relative velocity in the same direction)
Absolute velocity at 1 (V_{2a}) = 5.09 - $r_2\,\omega$ = 5.09 - 0.2 ω (tangential velocity and relative velocity in opposite direction)

Final moment of momentum = $\rho Q V_{2a}\, r_2 - \rho Q V_{1a}\, r_1 = 0$
(Two torques in opposite direction, net torque = greater torque-smaller torque)
$V_{1a}\, r_1 = V_{2a}\, r_2$
(5.09 + 0.1 ω)0.1=(5.09 - 0.2 ω)0.2
ω = 10.18rad/s

$$\omega = \frac{2N\pi}{60}$$
N = 98 rpm

b. For ω =0, velocities are V_1 and V_2.
Torque exerted by the water on sprinkler = $\rho Q V_2 r_2 - \rho Q V_1 r_1$
= 1000x0.0001x5.09x0.2-1000x0.0001x5.09x0.1
= 0.0509 Nm
Torque required to hold the sprinkler stationary = 0.0509 Nm

13. A lawn sprinkler shown in the figure has 0.8cm diameter nozzle at the end of a rotating arm and discharges water at the rate of 12m/s. Determine the torque required to hold the rotating arm stationary. Also determine the constant speed of rotation of the arm, if free to rotate.



Solution:

Diameter of nozzle (d) = 0.8cm =0.008m

Area of nozzle (A) = $\frac{\pi}{4} x 0.008^2$= 5.026x10^{-5} m2

r_a = r_b= 20cm =0.2m

Relative velocity at A and B (V=V_A = V_B) = 12 m/s

Discharge through each nozzle (Q) = A V = 0.000603 m^3/s

For angular velocity (ω) =0, Torque required to hold the rotating arm stationary (T) =?

For torque (T) = 0, constant speed of rotation of the arm (N) =?

a. Jet exerts force in opposite direction at nozzle A and B (upward at A and downward at B).

Torque at A and B: both clockwise

(Two torques in same direction, net torque = sum of two torques)

For ω =0, velocities are V_A and V_B.

Torque exerted by the water on sprinkler = $\rho Q V_A\, r_A + \rho Q V_B\, r_B$

= 1000x0.000603x12x0.2+1000x0.000603x12x0.2

= 2.89 Nm

Torque required to hold the rotating arm stationary = 2.89 Nm

b. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Absolute velocity at A (V_{1a}) = 12- r_a ω = 12 - 0.2 ω (tangential velocity and relative velocity in opposite direction)

Absolute velocity at B (V_{2a}) = 12- r_b ω = 12 - 0.2 ω (tangential velocity and relative velocity in opposite direction)

Final moment of momentum = $\rho Q V_{1a}\, r_a + \rho Q V_{2a}\, r_b = 0$

$V_{1a}\, r_a = -V_{2a}\, r_b$

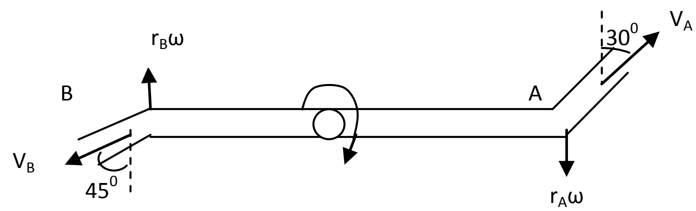
(12 - 0.2 ω)0.2 = -(12 - 0.2 ω)0.2

ω = 60 rad/s

$$\omega = \frac{2N\pi}{60}$$

N = 573 rpm

14. A lawn sprinkler with two nozzles 5mm in diameter each at 0.2m and 0.15m radii is connected across a tap capable of discharging 6 litres/min. The nozzles discharge water upwards and outwards from the plane of rotation. What torque will sprinkler exert on the hand if held stationary, and at what angular velocity will it rotate free?



Solution:

Diameter of nozzle (d) = 5mm =0.005m

Area of nozzle (A) = $\frac{\pi}{4}x0.005^2$= 1.9635x10^{-5} m2

r_A= 0.2m, r_B= 0.15m

Assuming discharge to be equally divided,

Discharge (Q_A = Q_B) = 6/2 litres/min = 3/(1000x60) m^3/s = 0.00005 m^3/s

Relative velocity at A (V_A) = Q_A/A = 0.00005/1.9635x10^{-5} = 2.54m/s

Relative velocity at B (V_B) = Q_B/A = 0.00005/1.9635x10^{-5} = 2.54m/s

Vertical component of relative velocity at A (V_{YA}) = 2.54Cos30 = 2.2m/s

Vertical component of relative velocity at B (V_{YB}) = 2.54Cos45 = 1.8m/s

For angular velocity (ω) =0, Torque required to hold the rotating arm stationary (T) =?

For torque (T) = 0, constant speed of rotation of the arm (N) =?

a. Jet exerts force in opposite direction at nozzle A and B (downward at A and upward at B).

Torque at A and B: both clockwise

(Two torques in same direction, net torque = sum of two torques)

For ω =0, velocities are V_{YA} and V_{YB}.

Torque exerted by the water on sprinkler = $\rho Q V_{YA}\, r_A + \rho Q V_{YB}\, r_B$

= 1000x0.00005x2.2x0.2+1000x0.00005x1.8x0.15

= 0.0355 Nm

Torque required to hold the rotating arm stationary = 0.0355 Nm

b. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Absolute velocity at A (V_{1a}) = V_{YA}- $r_A\omega$ = 2.2 - 0.2 ω (tangential velocity and relative velocity in opposite direction)

Absolute velocity at B (V_{2a}) = V_{YB}- $r_A\omega$ = 1.8 - 0.15 ω (tangential velocity and relative velocity in opposite direction)

Final moment of momentum = $\rho Q V_{1a}\, r_A + \rho Q V_{2a}\, r_B = 0$

$$V_{1a}\, r_a = -V_{2a}\, r_b$$

(2.2 - 0.2 ω)0.2 = -(1.8 - 0.15 ω)0.15

ω = 11.36 rad/s

$$\omega = \frac{2N\pi}{60}$$

N =109rpm

15. A flat plate is struck normally by a jet of water 50mm in diameter with a velocity of 18m/s. Calculate: a) the force on the plate when it is stationary, b) the force on the plate when it moves in the same direction as the jet with a velocity of 6m/s, and c) the work done per sec and the efficiency in case (b).

Solution:

Diameter of jet (d) = 50mm =0.05m

Area of jet (A) = $\frac{\pi}{4} x 0.05^2$ = 0.001963m^2

Velocity of jet (V) = 18 m/s

a. Force exerted by the jet on the plate $(F) = \rho A V^2$

$= 1000x0.001963x18^2$ = 636N

b. Velocity of plate (u) = 6m/s

Force exerted by the jet on the plate when the plate is moving $(Fp) = \rho A(V-u)^2$

$= 1000x0.001963x(18-6)^2$ = 283N

c. Work done per/sec (W) = Fp x distance/time = Fp x u = 283 x 6 = 1698J

Kinetic energy of jet (KE)/sec = $\frac{1}{2}(\rho A V)V^2 = \frac{1}{2}\rho A V^3$=$\frac{1}{2}x1000x0.001963x18^3$ = 5724 J

Efficiency = W/KE = 1698/5724= 0.3 = 30%

16. A jet of water 60 mm in diameter with a velocity of 15m/s strikes a flat plate inclined at an angle of 25^0 to the axis of the jet. Calculate the normal force exerted on the plate (a) when the plate is stationary, (b) when the plate is moving at 4.5 m/s in the direction of jet and (c) the work done per sec and the efficiency for case b.

Solution:

Diameter of jet (d) = 60mm =0.06m

Area of jet (A) = $\frac{\pi}{4} x 0.06^2$ = 0.00283 m^2

Velocity of jet (V) = 15 m/s
Angle of inclination of the plate with the axis of jet (θ) =25^0

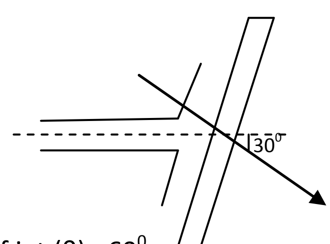
a. Normal force exerted on the plate $(F) = \rho AV^2 Sin\theta$
$= 1000x0.00283x15^2 Sin25$ = 269N

b. Velocity of plate (u) = 4.5m/s
Normal force exerted on the plate when the plate is moving $(Fp) = \rho A(V-u)^2 Sin\theta$
$= 1000x0.00283x(15-4.5)^2 Sin25$ = 132N

c. Work done per/sec (W) = Fp x distance/time = Fp x u = 132 x 4.5 = 594J
Kinetic energy of jet (KE)/sec = $\frac{1}{2}(\rho AV)V^2 = \frac{1}{2}\rho AV^3$=$\frac{1}{2}x1000x0.00283x15^3$ = 4776J
Efficiency = W/KE = 594/4776 = 0.12 = 12%

17. A 75mm diameter jet of water having a velocity of 25m/s strikes a flat plate, the normal of which is inclined at 30^0 to the jet. Find the force normal to the surface of the plate and in the direction of the jet.

Solution:
Diameter of jet (d) = 75mm =0.075m
Area of jet (A) = $\frac{\pi}{4}x0.075^2$ = 0.00442 m^2
Velocity of jet (V) = 25 m/s

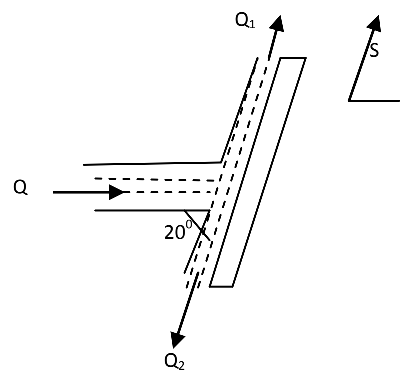


Angle made by normal with horizontal = 30^0

Angle of inclination of the plate with the axis of jet (θ) =60^0
Normal force (F_n) =?
Force in the direction of jet (F_x) = ?

$F_n = \rho AV^2 Sin\theta = 1000x0.00442x25^2 Sin60$ = 2392N
$F_x = F_n Cos30 = 2392Cos30$ = 2071N

18. A jet of 20mm in diameter moving with a velocity of 5m/s strikes a smooth plate, which is inclined at an angle of 20^0 to the horizontal. Compute the amount of flow on each side of the plate and the force exerted on the plate.



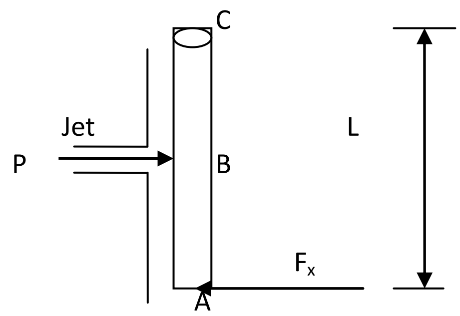
Solution:
Diameter of jet (d) = 20mm =0.02m
Area of jet (A) = $\frac{\pi}{4} x 0.02^2$ = 0.000314 m^2
Velocity of jet (V) = 5 m/s

Angle of inclination of jet with horizontal (θ) = 20^0
Q = Flow through jet before striking plate
Q_1 and Q_2 = Flow on upper and lower side of plate
A_1 and A_2 = Area of jet on upper and lower side of plate
Q = A V = 0.000314x5 = 0.00157 m^3/s
Force exerted on plate(F_X) =?

Writing momentum equation in direction of S
$\rho A_1 V^2 - \rho A_2 V^2 - \rho A V^2 Cos\theta = 0$
With Q = A V, Q_1 =A_1 V, Q2 =A_2 V
$Q_1 - Q_2 = Qcos\theta$
From continuity, $Q_1 + Q_2 = Q$
Solving above two eq.,
$Q_1 = \frac{Q}{2}(1 + Cos\theta)$ and $Q_2 = \frac{Q}{2}(1 - Cos\theta)$
$Q_1 = \frac{0.00157}{2}(1 + Cos20)$ = 0.00152 m^3/s
$Q_2 = \frac{0.00157}{2}(1 - Cos20)$ = 0.00005 m^3/s

$F_n = \rho A V^2 Sin\theta = 1000x0.000314x5^2 Sin20$ = 2.7N
$F_x = F_n Cos(90 - 20) = 2.7\ Cos70$ = 0.92N

19. A flat plate hinged about its top edge, is suspended vertically. It weighs 8KN. A jet of water, 50mm in diameter strikes the plate normally at its mid-point with a velocity of 50m/s. (a) Determine the horizontal force that should be applied to it at the bottom edge to keep it vertical. (b) Determine the angle of deflection where it stays in equilibrium, if it is allowed to rotate about the hinge.

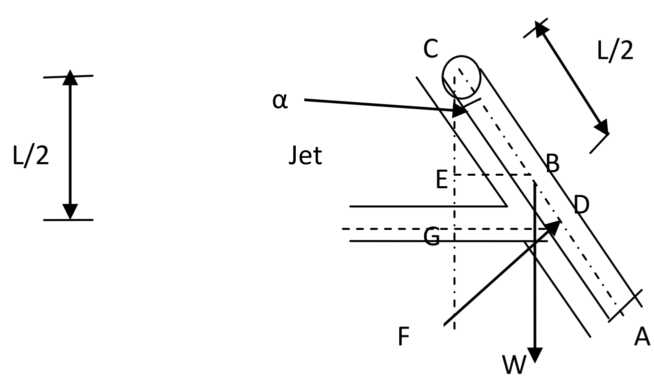


Solution:

Weight (W) =8KN = 8000N
Diameter of nozzle (d) = 50mm =0.05m
Area of nozzle (A) = $\frac{\pi}{4}x0.05^2$ = 0.001963m^2
Velocity of jet (V) = 50m/s
AB =BC =L/2

a. Horizontal force at bottom (F_x) =?
Force exerted by the jet on the plate $(P) = \rho AV^2$= $1000x0.001963x50^2$ = 4907N
Taking moment about hinge (C)
P L/2 =F_x L
F_x = P/2 = 2453.5 N

b. α = Angle of inclination of the plate with vertical



CB =L/2
As the jet strikes the plate at mid point and the plate deflects about hinge C, the vertical height from the center of the jet remains same as L/2 i.e. GC = L/2.
From ΔCGD, Perpendicular distance from C to force F = CD = CG/Cosα = L/2 $Sec\alpha$
From ΔCEB, Perpendicular distance from C to weight W = EB = CB $Sin\alpha$ = L/2 $Sin\alpha$

The axis of jet jet makes an angle (90-α) with the plate.

Force exerted by the jet normal to the plate $(F) = \rho AV^2 Sin(90-\alpha)$ = $\rho AV^2 Cos\alpha$

$= 1000x0.001963x50^2 Cos\alpha$ =$4907 Cos\alpha$

Taking moments about hinge C

$$W\frac{L}{2}Sin\alpha = F\frac{L}{2}Sec\alpha$$

$$8000\ Sin\alpha = 4907\ Cos\alpha\ Sec\alpha$$

α=37.8^0