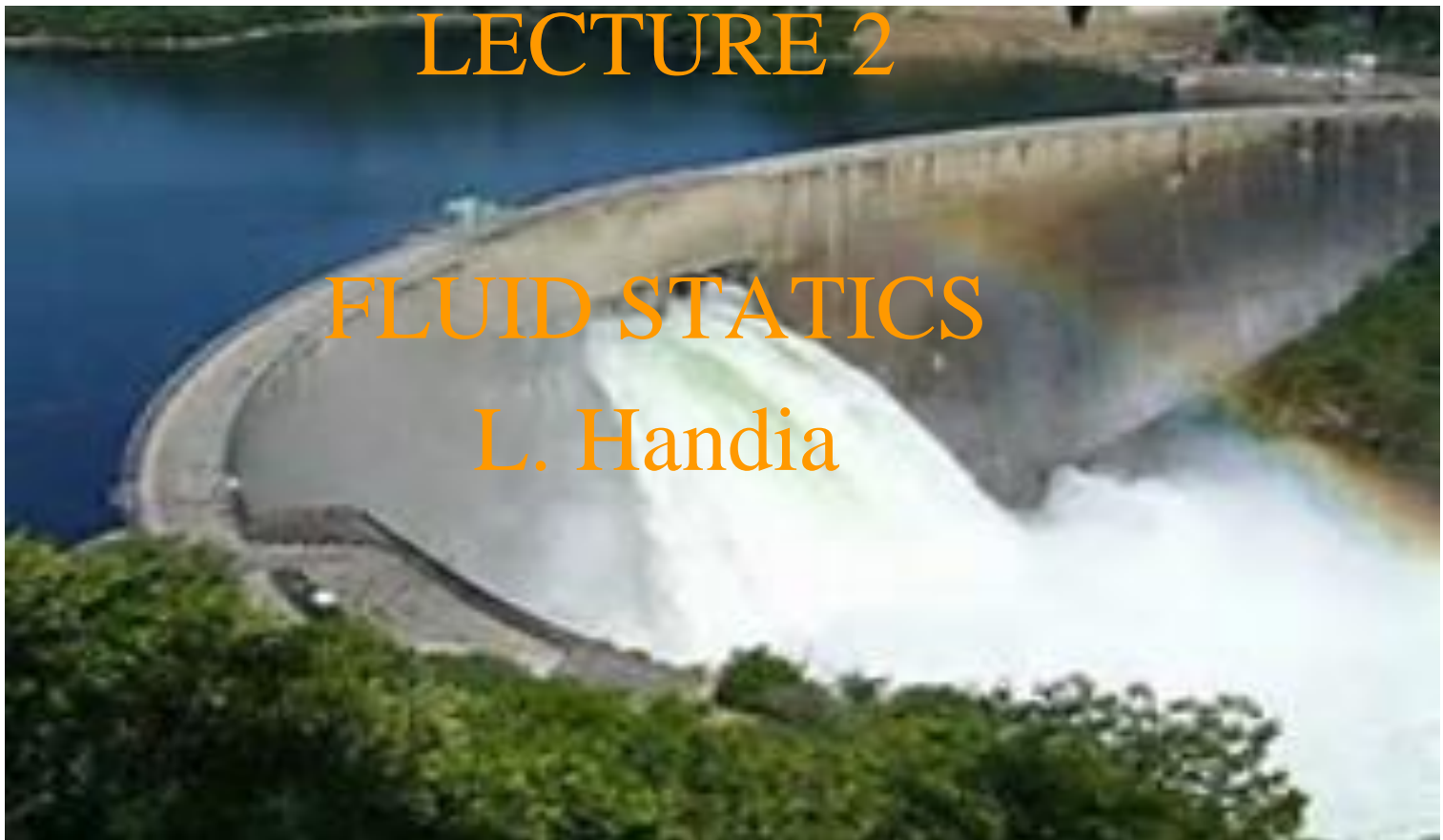


# Fluid Mechanics CEE 3311

## LECTURE 2

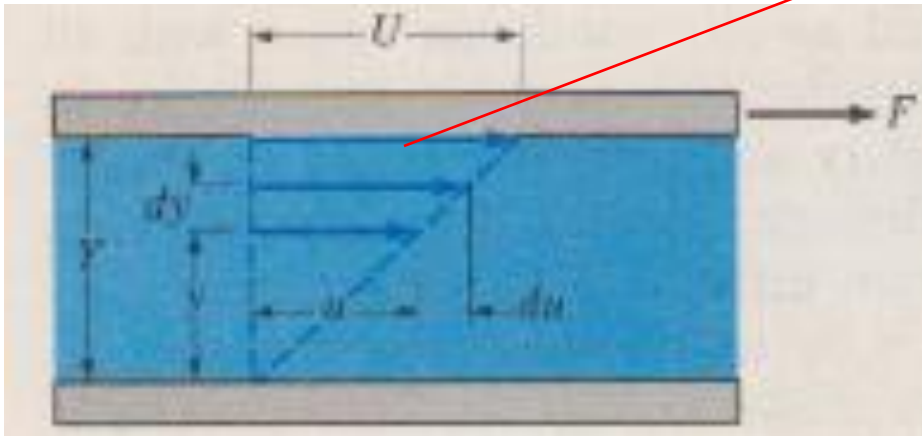
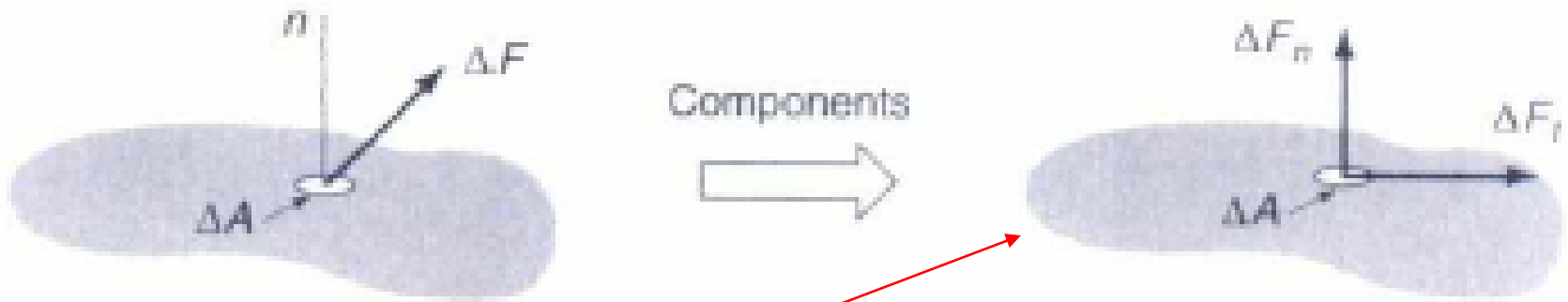
### FLUID STATICS

L. Handia



# FLUID STATICS

There are no shear stresses in fluids at rest ( $\Delta F_t = 0$ ); hence only normal pressure forces ( $\Delta F_n \neq 0$ ); **are present.**



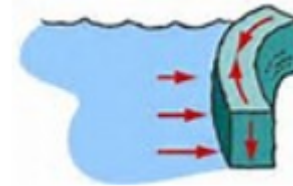
When  $F = 0$ , then velocity is zero. From  $\tau = \frac{F}{A} = \mu \frac{du}{dy}$

$$\tau = \frac{0}{A} = \mu \frac{0}{dy} = 0$$

# FLUID STATICS

- **Normal forces** produced by static fluids are often very important:

- water pushing against dam wall (they tend to overturn concrete dams)
- burst pressure vessels
- break lock gates on canals



- Obviously, to design such facilities, we need to be able to compute the **magnitudes** and **locations** of normal pressure **forces**.

# FLUID STATICS

- Note that normal pressure forces alone can occur in a moving fluid if the fluid is moving in bulk *without deformation*, i.e., as if it were solid or rigid.
- The average pressure intensity  $p$  is defined as the force exerted on a unit area.

# FLUID STATICS

- If  $F$  represents the total normal pressure force on some finite area  $A$ , while  $dF$  represents the force on an infinitesimal area  $dA$ , the pressure is

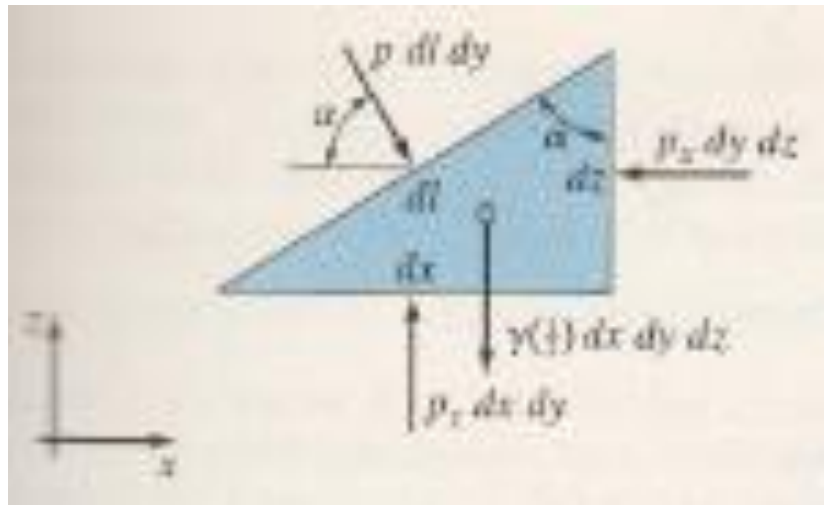
$$p = \frac{dF}{dA}$$

- If the pressure is uniform over the total area, then  $p = F / A$

# Pressure

## Pressure at a point the same in all directions

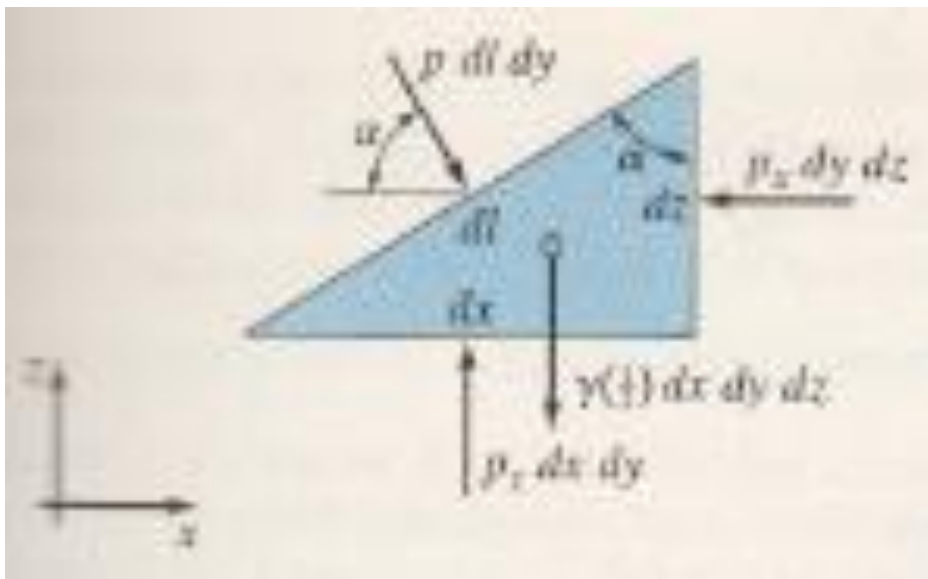
- No tangential stresses can exist in a fluid at rest, and the **only forces** between adjacent surfaces are **pressure forces normal** to the surfaces.
- Therefore the pressure at any point in a fluid at rest is the same in every direction.
- This can be proved by reference to Fig., showing a very small wedge-shaped element of fluid at rest whose thickness perpendicular to the plane of the paper is constant and equal to  $dy$ .



# Pressure

## Pressure at a point the same in all directions

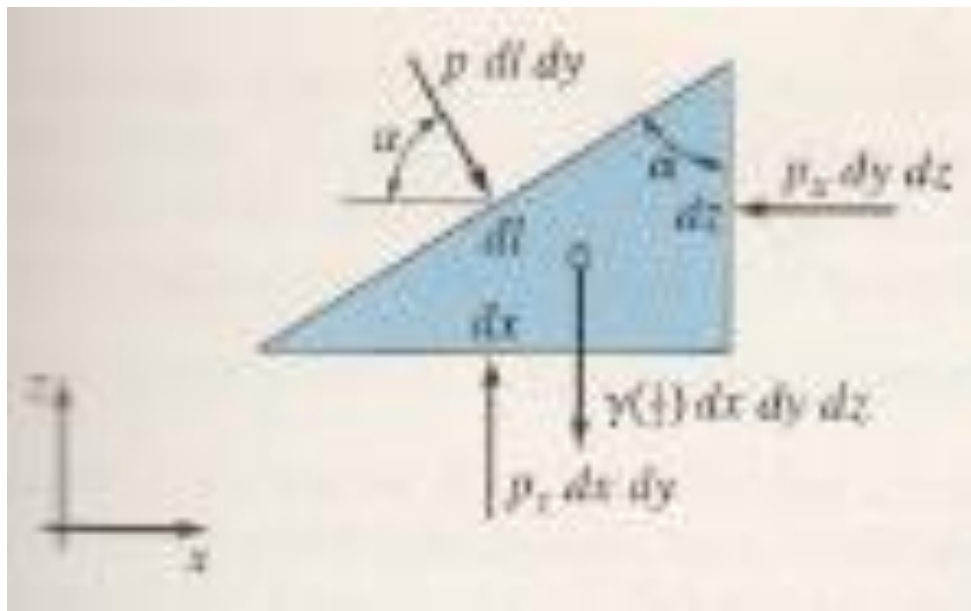
- Let  $p$  be the average pressure in any direction in the plane of the paper, let  $\alpha$  be defined as shown, and let  $p_x$  and  $p_z$  be the average pressures in the horizontal and vertical directions
- The forces acting on the element of fluid, with the exception of those in the  $y$  direction on the two faces parallel to the plane of the paper, are shown in the diagram.



# Pressure

## Pressure at a point the same in all directions

- For our purpose, forces in the  $y$  direction need not be considered because they cancel.
- Since the fluid is at rest, *no tangential forces* are involved. As this is a condition of equilibrium, ***the sum of the force components*** on the element ***in any direction*** must be equal to zero.



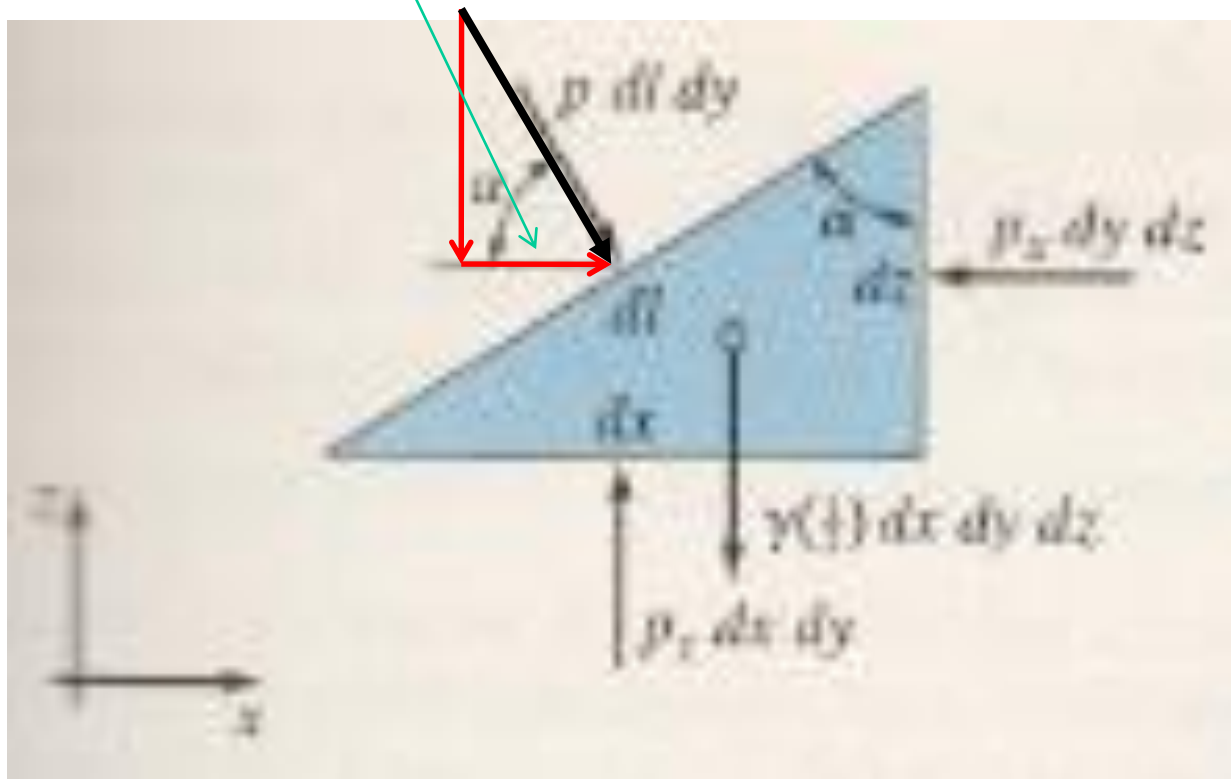
# Pressure

## Pressure at a point the same in all directions

- Writing such an equation for the components in the x direction

$$p \, dl \, dy \, \cos \alpha - p_x \, dy \, dz = 0.$$

- Since  $dz = dl \, \cos \alpha$ , it follows from above eqn. that  $p = p_x$ .



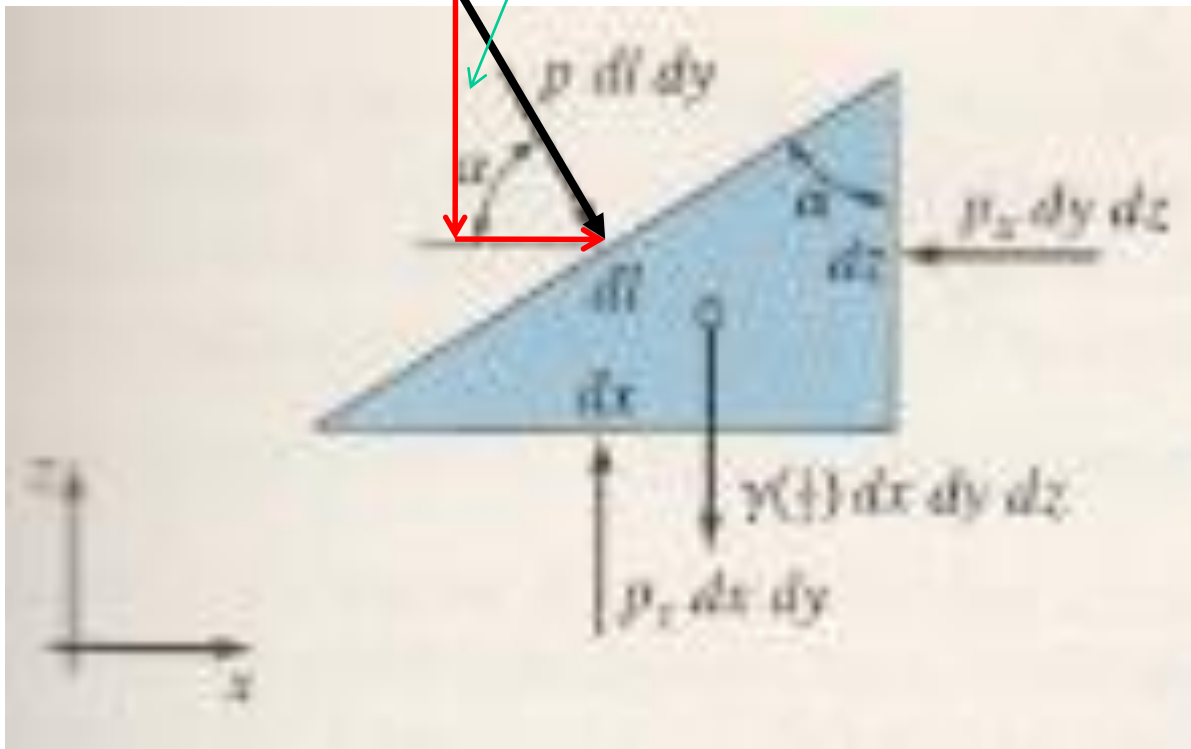
# Pressure

## Pressure at a point the same in all directions

➤ Similarly, summing forces in the z direction gives

$$p_z dx dy - p dl dy \sin \alpha - \frac{1}{2} \gamma dx dy dz = 0.$$

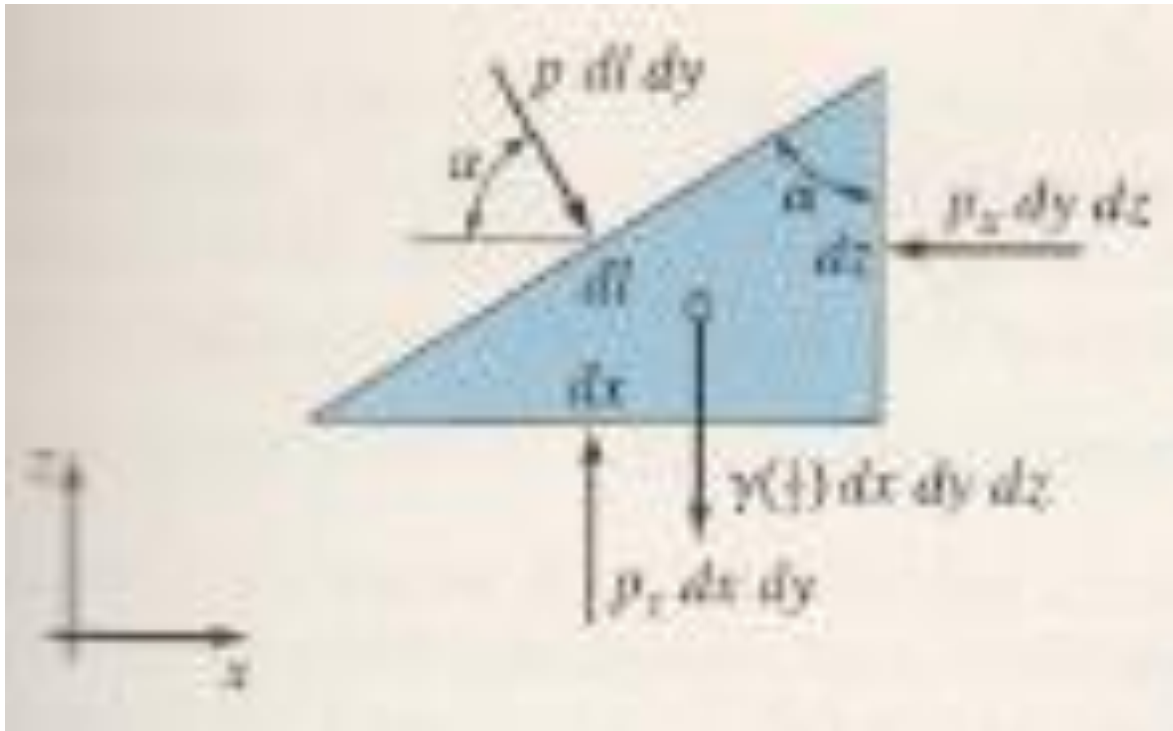
The third term is of higher order than the other two terms and so may be neglected. It follows from this that  $p = p_z$ .



# Pressure

## Pressure at a point the same in all directions

- It can also be proved that  $p = p_y$  by considering a three-dimensional case.
- The results are *independent of  $\alpha$* ; hence the pressure at any point in a fluid at rest is the same in all directions (i.e.,  $p_x = p_y = p_z$ )



# Pressure

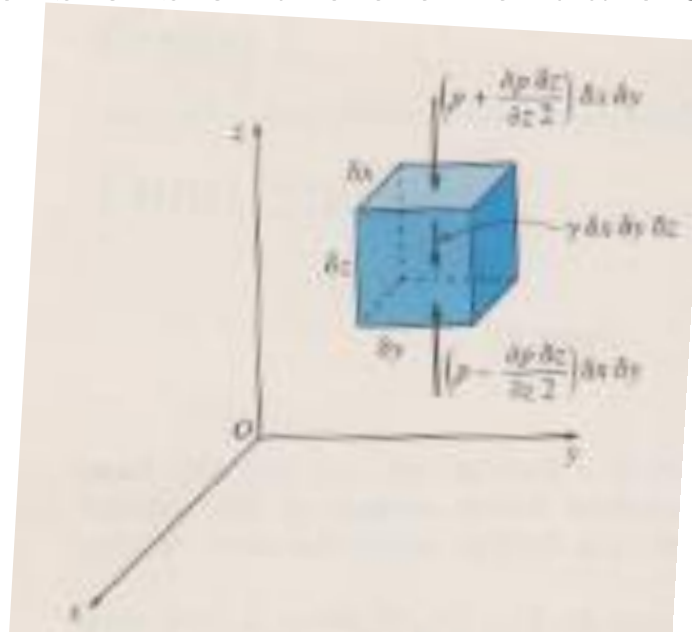
## Pressure at a point the same in all directions

➤ **Pascal's Law** states that the pressure at a point in a fluid at rest is the same in all directions *independent of the orientation* of the surface around that point.

# Pressure

## Variation of pressure in a static fluid

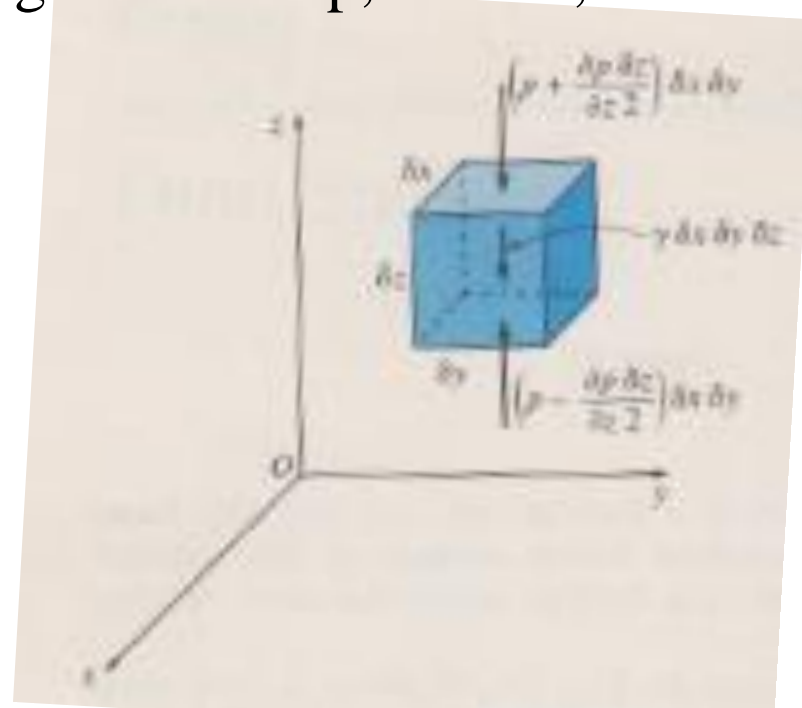
- Consider the differential element of static fluid shown in Fig below.
- Since the element is very small, we can *assume* that the density of the fluid within the element is constant.
- Assume that the pressure at the center of the element is  $p$  and that the dimensions of the element are  $\delta x$ ,  $\delta y$  and  $\delta z$ .



# Pressure

## Variation of pressure in a static fluid

- The forces acting on the fluid element in the vertical direction are
- the *body force*, the action of gravity on the mass within the element, and
  - the *surface forces*, transmitted from the surrounding fluid and acting at right angles against the top, bottom, and sides of the element.



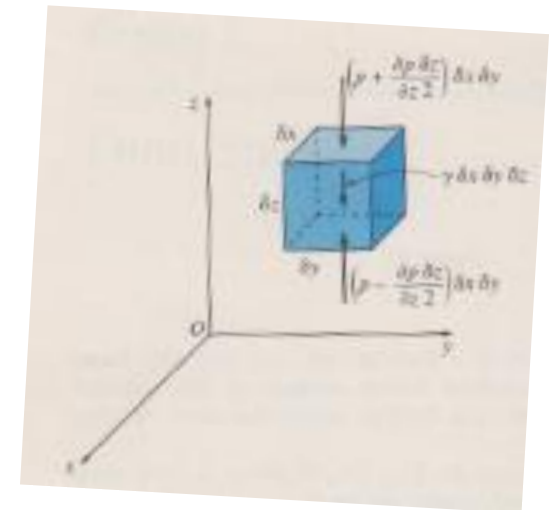
# Pressure

## Variation of pressure in a static fluid

- Because the fluid is at *rest*, the element *is in equilibrium* and the *summation of forces* acting on the element in any direction must be *zero*.
- If forces are summed in the *horizontal* direction, that is, x or y, the *only forces* acting are the pressure forces on the *vertical faces* of the element.
- To satisfy  $\sum F_x = 0$  and  $\sum F_y = 0$ , the pressures on the opposite vertical faces must be equal. Thus

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \quad \text{for the case of the fluid at rest}$$

(i.e., there is no change in pressure in the x-direction and also in the y-direction, the horizontal direction)



# Pressure

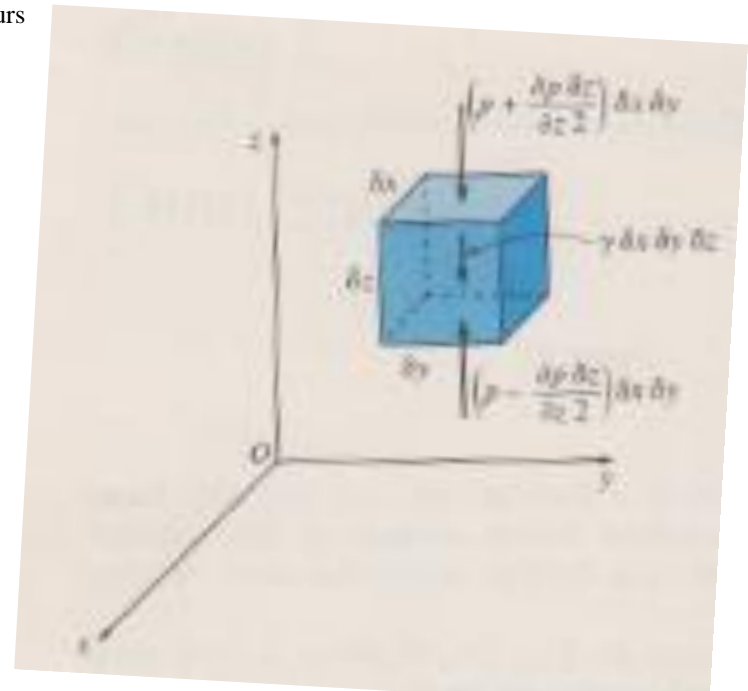
## Variation of pressure in a static fluid

Summing forces in the vertical direction and setting the sum equal to zero

$$\sum F_z = \left( p - \frac{\partial p}{\partial z} \frac{\delta z}{2} \right) \delta x \delta y - \left( p + \frac{\partial p}{\partial z} \frac{\delta z}{2} \right) \delta x \delta y - \gamma \delta x \delta y \delta z = 0$$

Rate of change of p in z direction multiplied by distance over which change occurs

This results in  $\frac{\partial p}{\partial z} = -\gamma$ , which, since p is independent of x and y, can be written as  $\frac{dp}{dz} = -\gamma$



# Pressure

## Variation of pressure in a static fluid

$$\frac{dp}{dz} = -\gamma$$

- This is the general expression that relates *variation of pressure* in a static fluid to *vertical position*.
- The minus sign indicates that ***as z gets larger (increasing elevation), the pressure gets smaller.***
- To evaluate pressure variation in a fluid at rest, one must integrate the above equation between appropriately chosen limits.
- For incompressible fluids ( $\gamma = \text{constant}$ . This is earlier assumption), above equation can be integrated directly. For compressible fluids, however,  $\gamma$  must be expressed algebraically **as a function of z or p** if one is to determine pressure accurately as a function of elevation.

# Pressure

## Variation of pressure in a static fluid

$$\frac{dp}{dz} = -\gamma; dp = -\gamma dz; \int_{p_1}^p dp = -\gamma \int_{z_1}^z dz$$

Where subscript 1 indicates conditions at a reference elevation. Integration gives

$$p - p_1 = -\gamma(z - z_1)$$

where  $p$  is the *pressure at* an elevation  $z$ . This expression is generally applicable to liquids, since they are *only slightly compressible*. Only where there are large changes in elevation, as in the ocean, need the *compressibility of the liquid be considered*, to arrive at an accurate determination of pressure variation. For small changes in elevation, equation will give accurate results

# Pressure

## Variation of pressure in a static fluid

For the case of a liquid at rest, it is **convenient** to measure distances **vertically downward** *from the free liquid surface*. If  $h$  is the distance below the free liquid surface and if the pressure of air and vapor on the surface is arbitrarily taken to be zero, equation  $p - p_1 = -\gamma(z - z_1)$  can be written as

$$p = \gamma h$$

As there must always be some pressure on the surface of any liquid, the *total pressure* at any depth  $h$  is given by the above equation *plus the pressure on the surface*.

In many situations this surface pressure may be disregarded

# Pressure

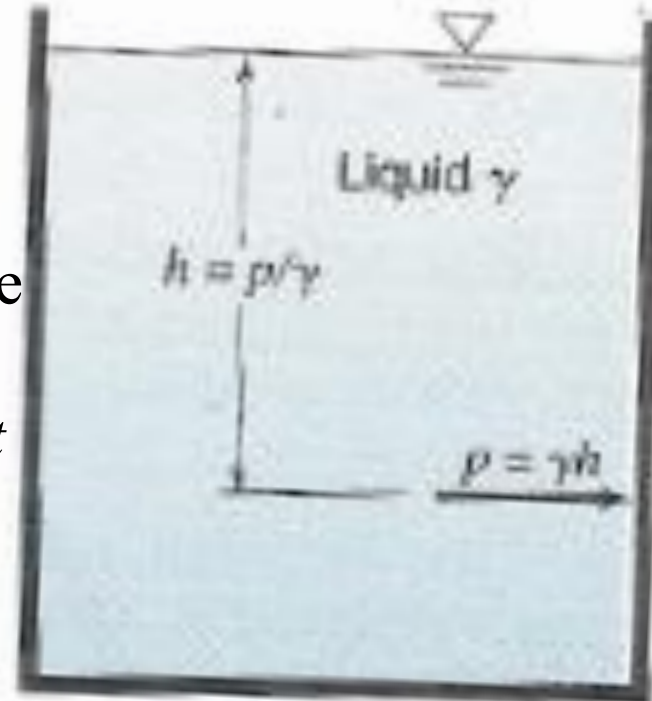
## Variation of pressure in a static fluid

- From  $p = \gamma h$ , it may be seen that all points in a connected body of constant density fluid at rest are under the *same pressure* if they are at the *same depth* below the liquid surface (Pascal's law).
- This indicates that a *surface of equal pressure* for a liquid at rest is a *horizontal plane*.
- Strictly speaking, it is a surface everywhere *normal to the direction of gravity* and is approximately a spherical surface concentric with the earth. For *practical* purposes, a *limited portion* of this surface may be *considered* a *plane area*.

# Pressure

## Pressure expressed in height of fluid

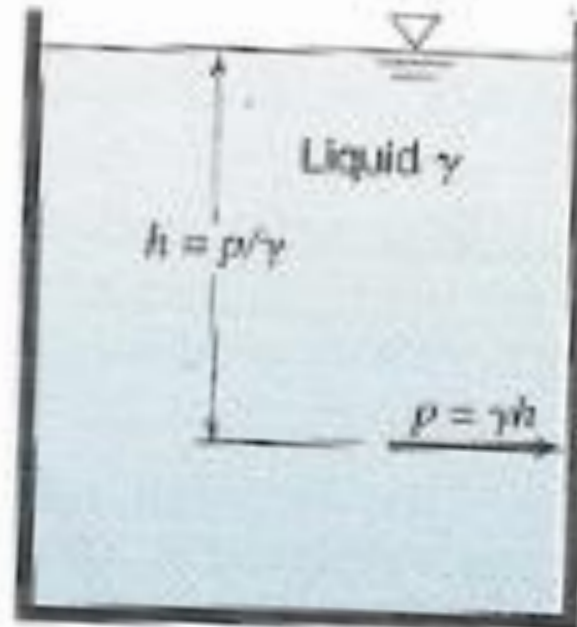
- Imagine an open tank of liquid upon whose surface there is no pressure, though in reality the minimum pressure upon any liquid surface is the pressure of its own vapor.
- Disregarding this for the moment, by the previous equation, the pressure at any depth  $h$  is  $p = \gamma h$ .
- If  $\gamma$  is assumed constant, there is a definite relation between  $p$  and  $h$ . That is, pressure (force per unit area) is equivalent to a *height  $h$  of some fluid* of constant specific weight  $\gamma$ . It is often **more convenient** to express pressure in terms of a **height of a column of fluid** rather than in pressure per unit area e.g., pressure in the pipeline is 10m of water.



# Pressure

## Pressure expressed in height of fluid

➤ Even if the surface of the liquid is *under some pressure*, it is necessary only to convert this pressure into an **equivalent height** of **the fluid in question** and *add this to the value of  $h$*  (Fig) to obtain the **total pressure**.



# Pressure

## Pressure expressed in height of fluid

➤ The preceding discussion has been applied to a liquid, but it is equally possible to use it for a *gas or vapor* by specifying some *constant specific weight*  $\gamma$  for the gas or vapor in question. Thus **pressure**  $p$  may be expressed in the **height of a column of any fluid** by the relation

$$h = \frac{p}{\gamma}$$

➤ When pressure is expressed *in meters*, it is commonly referred to as ***pressure head*** e.g., pressure head in the pipeline is 10m of water.

# Pressure

## Pressure expressed in height of fluid

➤ It is convenient to express pressures occurring in one fluid in terms of height of another fluid, e.g., barometric pressure in millimeters of mercury.

➤ Equation  $p - p_1 = -\gamma (z - z_1)$  may be expressed as follows:

$$\frac{p}{\gamma} + z = \frac{p_1}{\gamma} + z_1 = \text{constant}$$

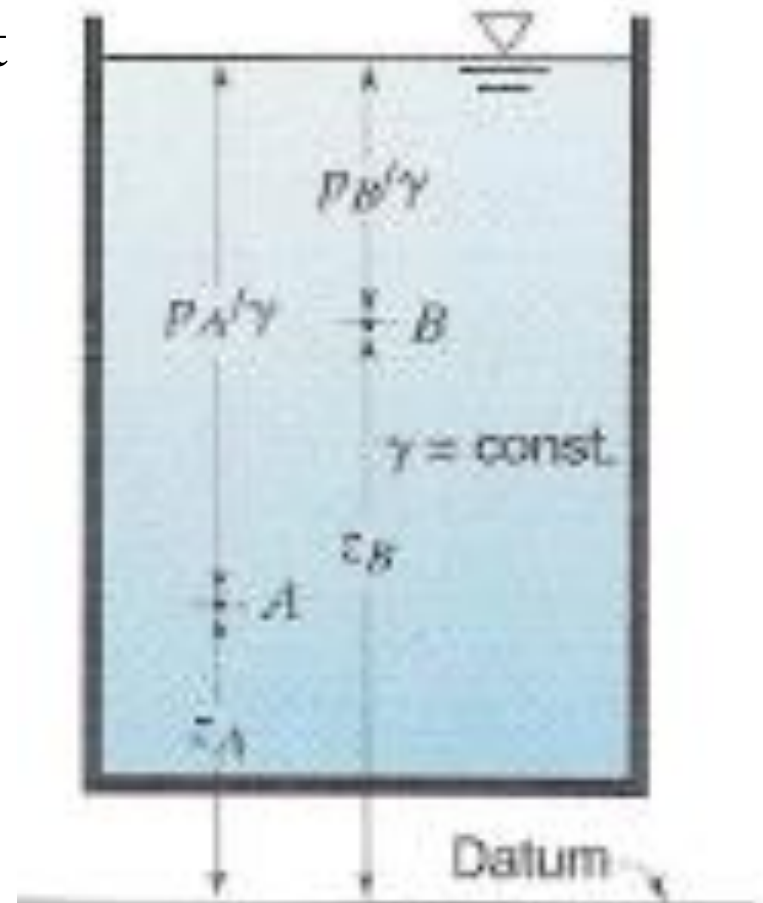
➤ This shows that **for an incompressible fluid** at rest, at any point in the fluid the sum of the elevation  $z$  and the pressure head  $p/\gamma$  is *equal* to the sum of these two quantities *at any other point*. The significance of this statement is that, in a fluid at rest, with an *increase in elevation there is a decrease in pressure head, and vice versa*. This concept is depicted in Fig below.

# Pressure

## Pressure expressed in height of fluid

$$\frac{p}{\gamma} + z = \frac{p_1}{\gamma} + z_1 = \text{constant}$$

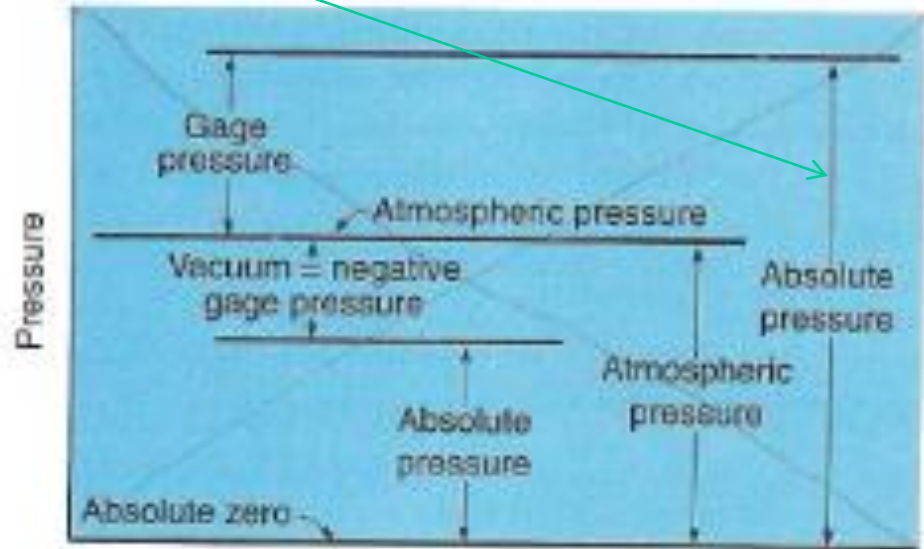
$$\frac{p_A}{\gamma} + z_A = \frac{p_B}{\gamma} + z_B = \text{constant}$$



# Pressure

## Absolute and gage pressures

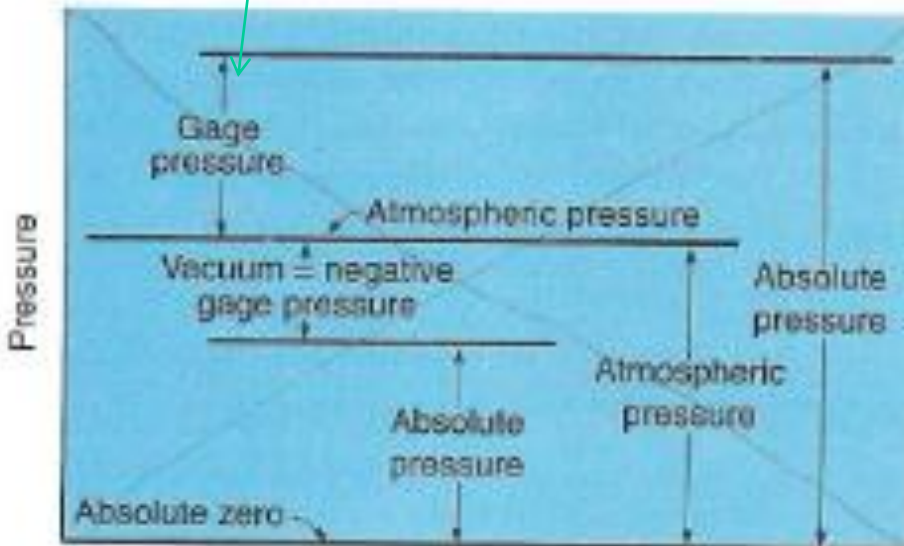
➤ If pressure is measured relative to absolute zero, it is called **absolute pressure**;



# Pressure

## Absolute and gage pressures

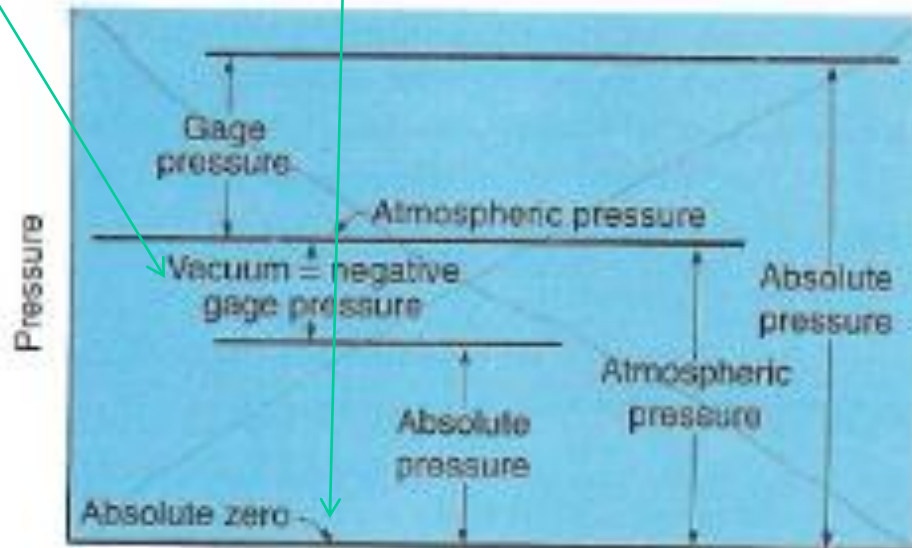
- when measured relative to atmospheric pressure as a base, it is called **gage pressure**.
- This is because practically all pressure gages register zero when open to the atmosphere, and hence measure the difference between the pressure of the fluid to which they are connected and that of the surrounding air.



# Pressure

## Absolute and gage pressures

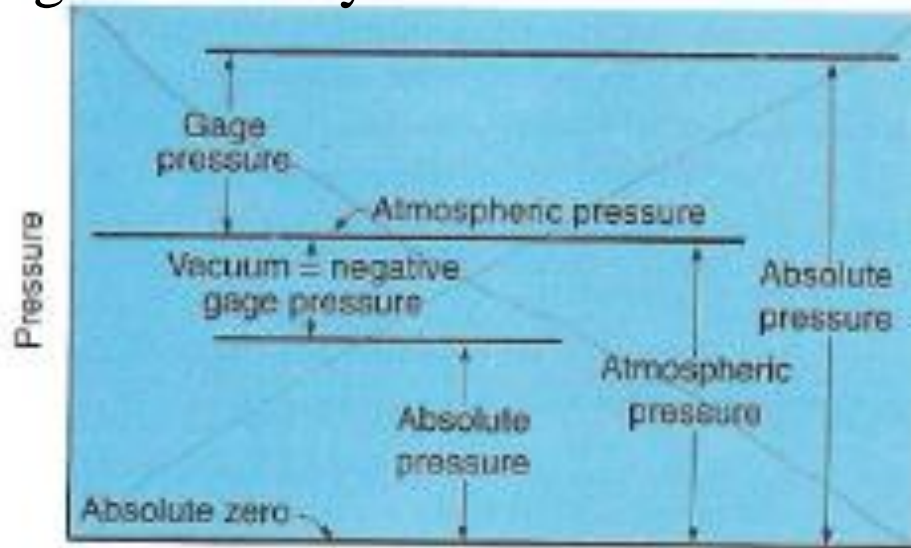
➤ If the pressure is below that of the atmosphere, it is designated as a **vacuum**, and its gage value is the amount by which it is below that of the atmosphere. What is called a “high vacuum” is really a low absolute pressure. A perfect vacuum would correspond to absolute zero pressure.



# Pressure

## Absolute and gage pressures

➤ Gage pressures are positive if they are above that of the atmosphere and negative if they are vacuum.



➤ It can be seen from the foregoing discussion that the following relation holds:

$$p_{abs} = p_{atm} + p_{gage}$$

where  $p_{gage}$  may be positive or negative (vacuum).

# Pressure

## Measurement of *absolute pressure*

### BAROMETER

- The *absolute pressure* of the atmosphere is measured with a barometer.
- Types of barometers. (a) Mercury barometer. (b) Aneroid barometer.

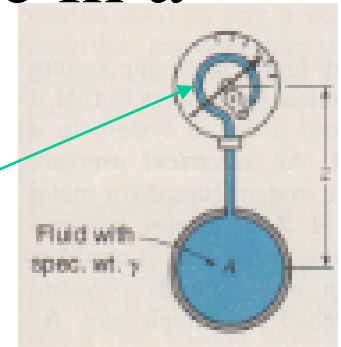
# Pressure

## Pressure measurement *in a fluid*

There are many ways by which pressure in a fluid may be measured.

### 1. Bourdon gage

- Pressures or vacuums are commonly measured by the bourdon gage
- A curved tube of elliptical cross section will change its curvature with changes in pressure within the tube



# Pressure

## Pressure measurement

### 2. Pressure transducer (a device that converts variations in a physical quantity, such as pressure, into an electrical signal, or vice versa)

#### CS456

#### Titanium SDI-12/RS-232 Pressure Transducer

Water Level, Stage, and Flow Sensors / CS456

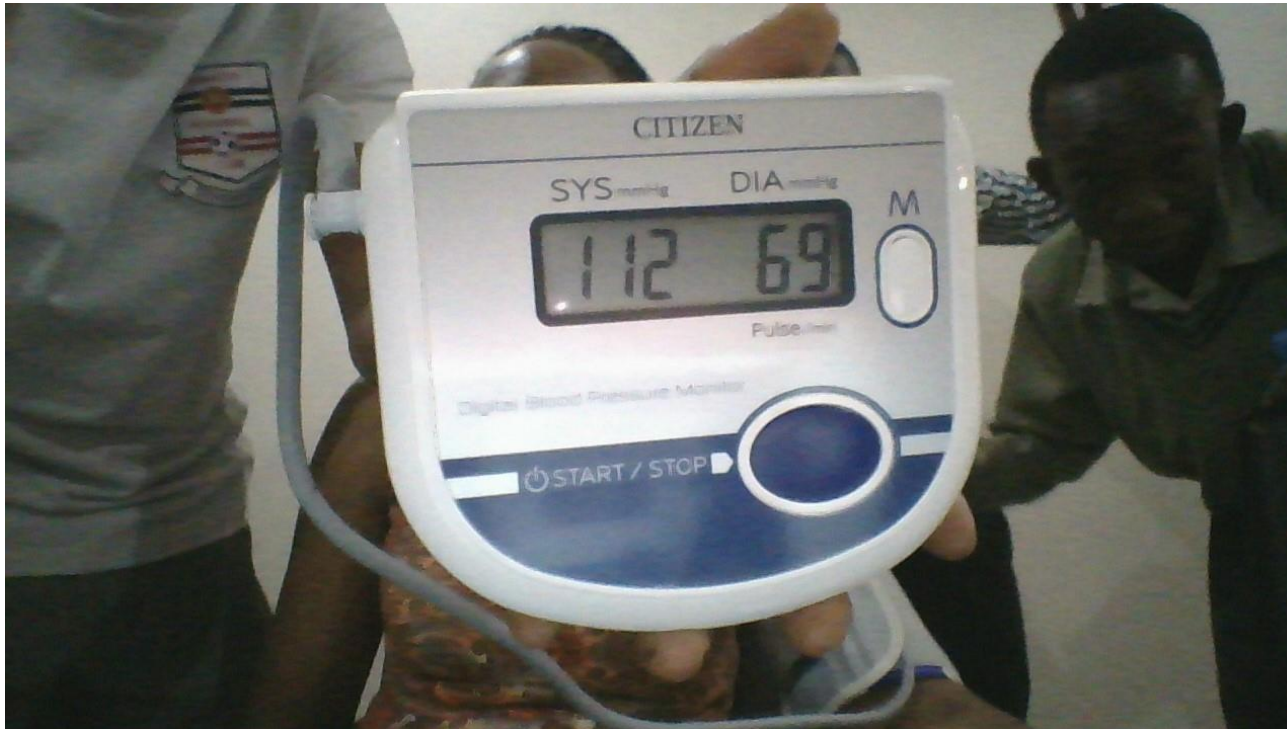


The CS456 is a pressure transducer for water-level measurements in canals, wells, ponds, harbors, lakes, streams, and tanks. It has a rugged titanium case that allows it to be used in saltwater and other harsh environments. The CS456 outputs either a digital SDI-12 or RS-232 signal to indicate observed pressure and temperature. This output can be read by many of our dataloggers. The CS456 replaces the CS455 transducer. The new transducers have a smaller gap between the water ports and the diaphragm so that less air is trapped than the user must remove during deployment. Trapped air causes the transducer's readings to drift as the air slowly dissolves into the water.

# Pressure

## Pressure measurement

### 2. Pressure transducer (a device that converts variations in a physical quantity, such as pressure, into an electrical signal, or vice versa)



# Pressure

## Pressure measurement

### 3. Manometers

- Manometers are instruments that use **columns of liquids** to measure pressures

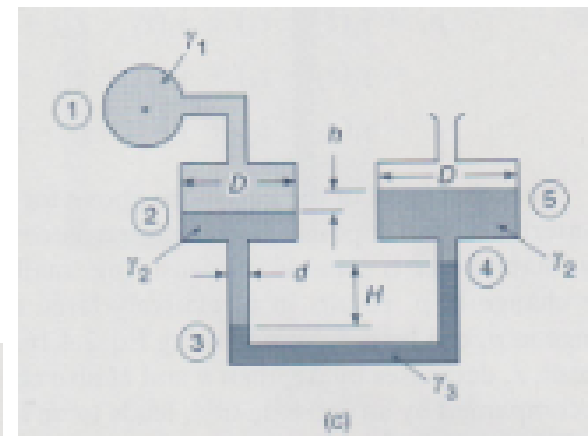
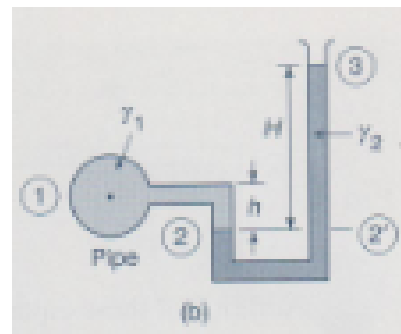
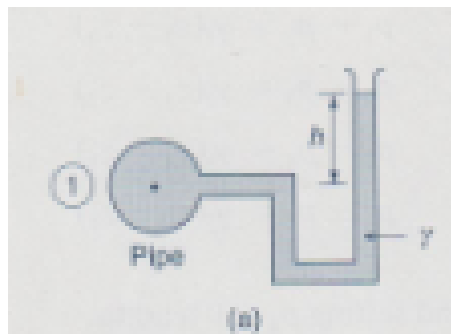


Figure 2.7 Manometers: (a) U-tube manometer (small pressures); (b) U-tube manometer (large pressures); (c) micromanometer (very small pressure changes).

# Pressure

## Pressure measurement

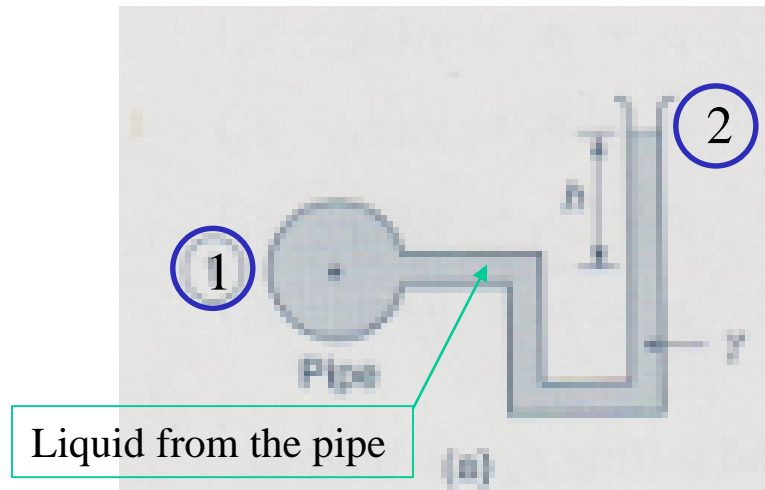
### a. U-tube Manometer (small pressures)

- The datum from which  $z_1$  and  $z_2$  are measured is located at any desired position such as through point 1.
- Since  $p_2 = 0$  (gage pressure is selected; if absolute pressure is desired, we would select  $p_2 = p_{\text{atm}}$ ) &  $z_2 - z_1 = h$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$$\frac{p_1}{\gamma} + 0 = \frac{0}{\gamma} + h$$

$$p_1 = \gamma h$$

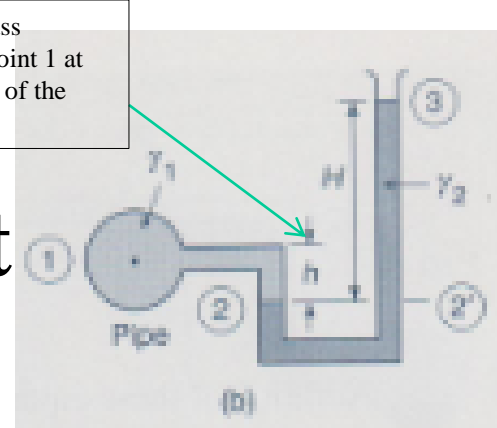


By measuring  $h$ , one computes  $p_1$ , the pressure in the pipe

# Pressure

## Pressure measurement

Should pass through point 1 at the centre of the pipe



### b. U-tube Manometer (large pressures)

- Manometer used to measure relatively large pressures since we can select  $\gamma_2$  to be quite large (e.g., mercury  $\gamma_2 = 13.6 \gamma_{\text{water}}$ )
- The pressure can be determined by introducing three points as indicated. This is necessary because  $\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$  *applies throughout one fluid*;  $\gamma$  must be constant. The value of  $\gamma$  changes abruptly at point 2; hence we write

$$\frac{p_1}{\gamma_1} + z_1 = \frac{p_2}{\gamma_1} + z_2 \quad \text{for } \gamma_1$$

$$\frac{p_2}{\gamma_2} + z_2 = \frac{p_3}{\gamma_2} + z_3 \quad \text{for } \gamma_2$$

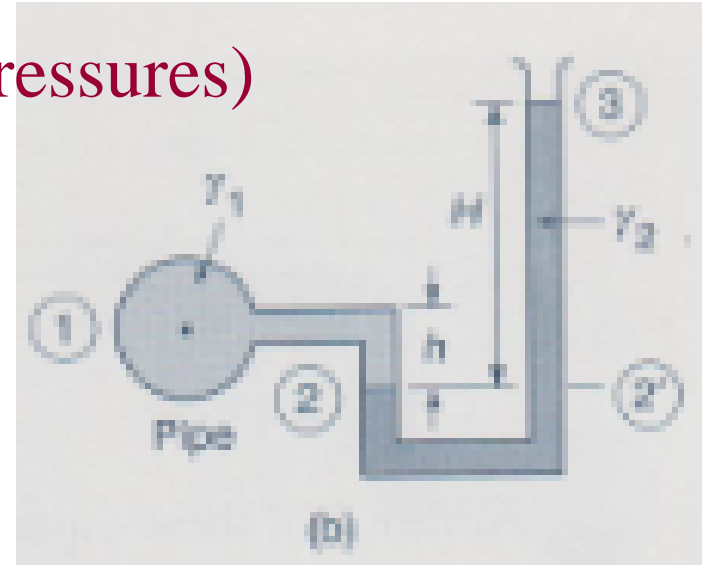
# Pressure

## Pressure measurement

### b. U-tube Manometer (large pressures)

$$\frac{p_1}{\gamma_1} + z_1 = \frac{p_2}{\gamma_1} + z_2$$

$$\frac{p_2}{\gamma_2} + z_2 = \frac{p_3}{\gamma_2} + z_3$$



where  $\gamma_1$  and  $\gamma_2$  are for liquids 1 and 2, respectively.

- Addition of these equations and setting  $p_3 = 0$  (gage pressure is used) results in

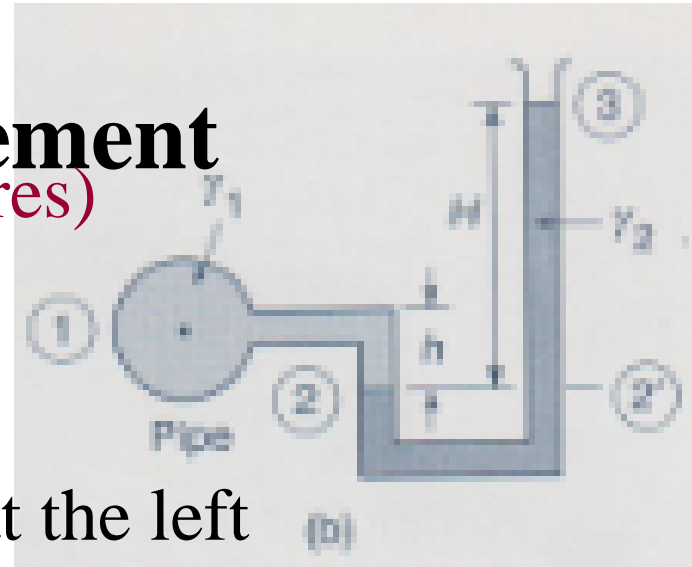
$$p_1 = \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) = -\gamma_1 h + \gamma_2 H$$

- In the analysis above We recognize that the pressure at point 2' is equal to the pressure at point 2, since points 2 and 2' are at the *same elevation* in the *same fluid* above a datum passing through the horizontal bottom of the manometer.

# Pressure

## Pressure measurement

### b. U-tube Manometer (large pressures)



$$p_1 = \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) = -\gamma_1 h + \gamma_2 H$$

- Interpret the manometer by starting at the left water pipe, **add** pressure when the elevation **decreases**, and **subtract** pressure when the elevation **increases** until the end is encountered.

$$p_1 = \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) = -\gamma_1 h + \gamma_2 H \Rightarrow p_1 + \gamma_1 h - \gamma_2 H = p_3$$

$$p_1 + \gamma_1 h - \gamma_2 H = p_3$$

$$h = - (z_2 - z_1)$$

# Pressure

## Pressure measurement

- c. Micromanometer (very small pressure changes)
- Example

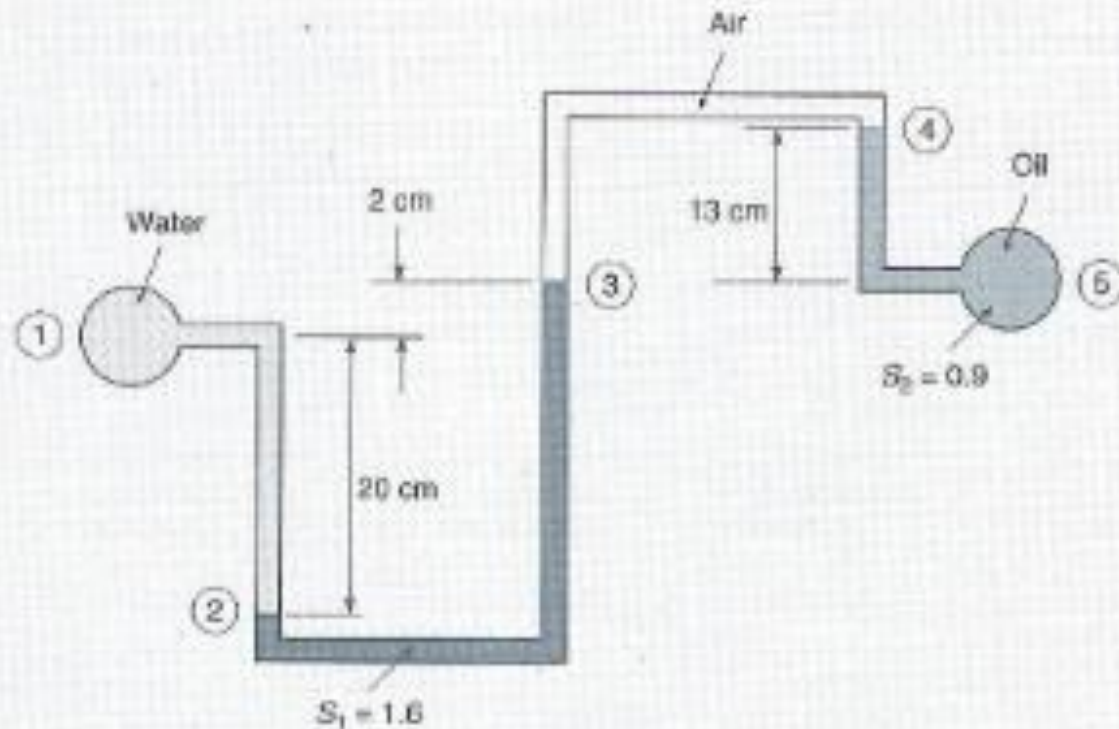


Figure E2.3

$$\begin{aligned}
 p_1 - p_5 &= (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4) + (p_4 - p_5) \\
 &= (z_2 - z_1) + \gamma S_1 (z_3 - z_2) + \gamma S_{air} (z_4 - z_3) + \gamma S_2 (z_5 - z_4)
 \end{aligned}$$

where  $\gamma = 9810 \text{ N/m}^3$ ,  $S_1 = 1.6$ ,  $S_2 = 0.9$ , and  $S_{air} \approx 0$ . Thus

$$\begin{aligned}
 p_1 - p_5 &= 9810(-0.20 + 1.6 \times 0.22 + 0 \times 0.13 + 0.9 \times -0.13) \\
 &= 343 \text{ Pa}
 \end{aligned}$$

Note that by neglecting the weight of the air, the pressure at point 3 is equal to the pressure at point 4.

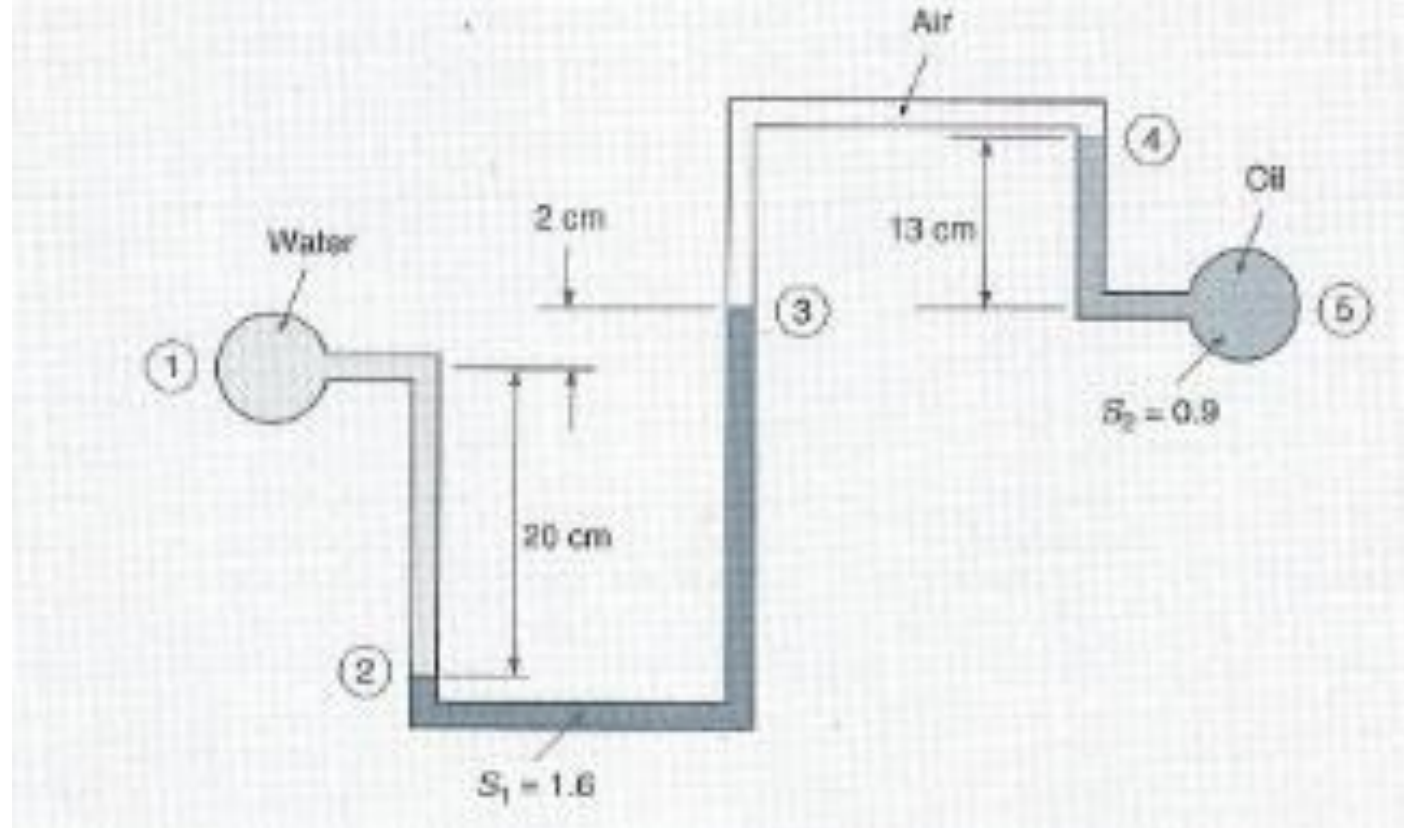
# Pressure

## Pressure measurement

- An alternative method of interpreting the manometer is to start at the left water pipe, add pressure when the elevation **decreases**, and subtract pressure when the elevation **increases** until the pipe at the right is encountered (From

$$p_1 = \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) = -\gamma_1 h + \gamma_2 H \Rightarrow p_1 + \gamma_1 h - \gamma_2 H = p_3 \quad )$$
$$p_1 + \gamma_1 h - \gamma_2 H = p_3$$

$$p_1 + \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) + \gamma_3(z_4 - z_3) + \gamma_4(z_5 - z_4) = p_5$$



$$p_1 + \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) + \gamma_3(z_4 - z_3) + \gamma_4(z_5 - z_4) = p_5$$

- Note that by neglecting the weight of the air (, the pressure at point 3 is equal to that at point 4 i.e.,  $\gamma_3(z_4 - z_3) = 0$  (if you want, you can include it)

$$p_1 + 9810 \times 0.2 - (1.6 \times 9810) \times (0.2 + 0.02) + (0.9 \times 9810) \times 0.13 = p_5$$

$$p_1 - p_5 = 343 \text{ Pa} \gg \text{ This yields the same answer}$$