



Fluid Mechanics CEE 3311

LECTURE 19

Flow in Open Channels

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Open channels

- An open channel is one in which the stream is *not completely enclosed by solid boundaries* and therefore has a *free surface* subjected only to atmospheric pressure.
- The flow in such a channel is caused not by some external head, but rather **only by the gravity** component along the slope of the channel. Thus open-channel flow is often referred to as **free-surface flow** or **gravity flow**.
- For convenience in dealing with large channel systems, they are often divided into reaches. A reach is a continuous stretch of a waterway, often chosen to have reasonably uniform properties like cross section, slope, and discharge.

Reynolds number in open channels

Open-channel flow is *usually fully rough*; that is, it occurs at high Reynolds numbers. For open channels the Reynolds number is defined by $Re = R_h v / \nu$, where R_h is the hydraulic radius. Since $R_h = D/4$, the critical value of Reynolds number at which the change over occurs from laminar flow to turbulent flow in open channels is **500**, whereas in pressure conduits the critical value is 2,000.

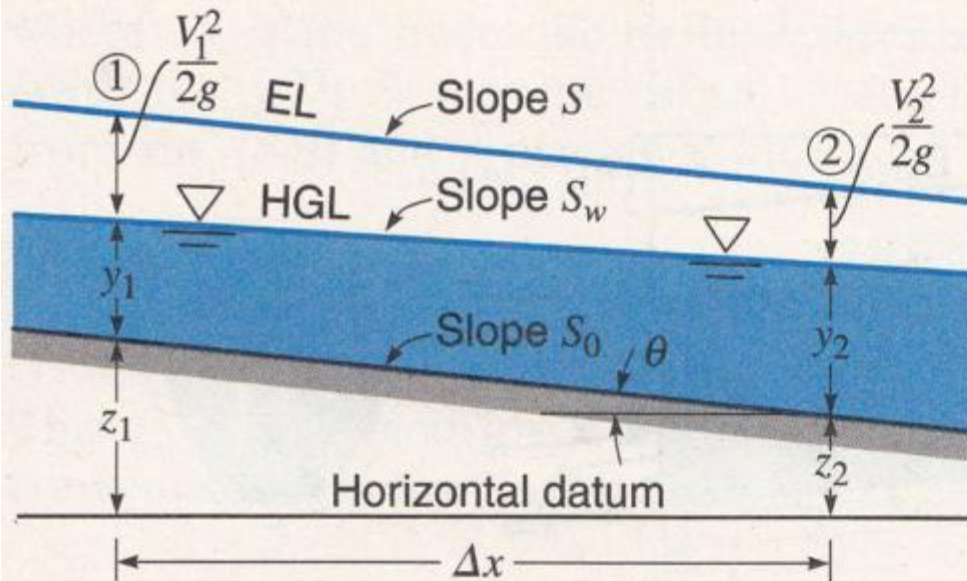


Figure 10.2

Open channel flow—definition sketch ($L =$ distance along the channel bed between sections 1 and 2).

Uniform flow

- Uniform flow means that the water *cross section and depth remain constant over a certain reach* of the channel as well as over time. This requires that the drop in potential energy due to the fall in elevation along the channel be exactly that consumed by the energy dissipation through boundary friction and turbulence.
- The depth in uniform flow is commonly referred to as the *normal flow*, y_0
- Uniform flow is an equilibrium condition that flow tends to if the channel is sufficiently long with constant slope, cross section, and roughness.

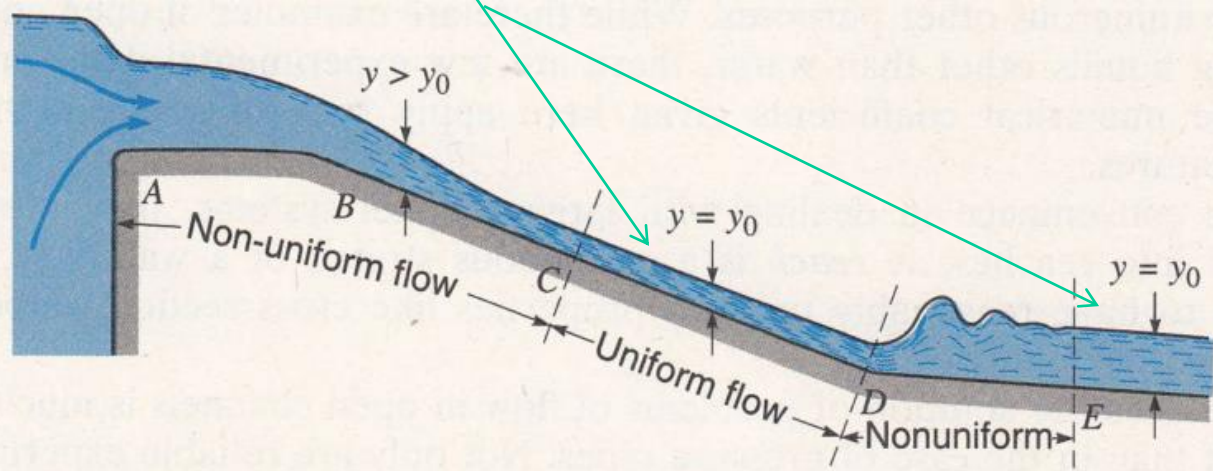


Figure 10.1

Steady flow down a chute or spillway

Uniform flow

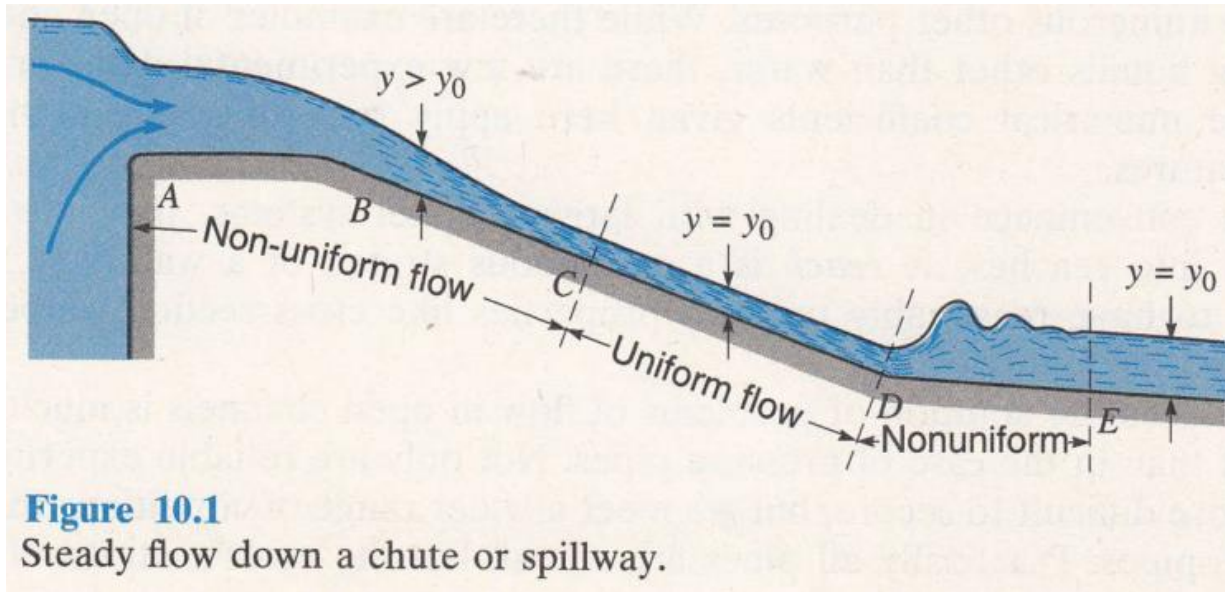


Figure 10.1
Steady flow down a chute or spillway.

TABLE 10.1 Combinations of One-Dimensional Free-Surface Flows

<i>Type of flow</i>	<i>Average velocity</i>	<i>Depth</i>
Steady, uniform	$V = \text{const.}$	$y = \text{const.}$
Steady, nonuniform	$V = V(x)$	$y = y(x)$
Unsteady, uniform	$V = V(t)$	$y = y(t)$
Unsteady, nonuniform	$V = V(x, t)$	$y = y(x, t)$

Consider the short reach of length L along the channel between stations 1 and 2 in uniform flow with water cross section of area A (Fig. 10.3). As the flow is neither accelerating nor decelerating, we may consider the body of water contained in the reach in static equilibrium.

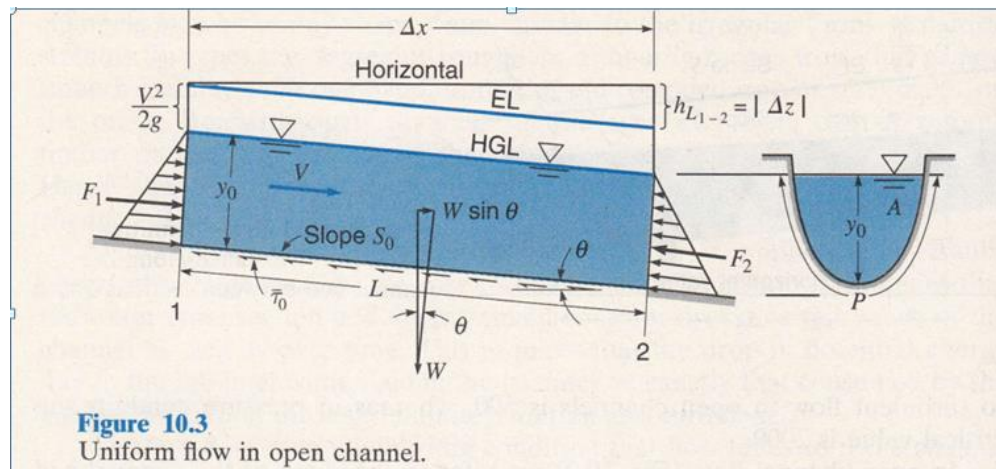
Summing forces along the channel, the hydrostatic-pressure forces F_1 and F_2 balance each other, since there is no change in the depth y between the stations. The only force in the direction of motion is the gravity component, and this must be resisted by the average boundary shear stress $\bar{\tau}_0$, acting over the area PL , where P is the wetted perimeter of the section. Thus

$$\gamma AL \sin \theta = \bar{\tau}_0 PL$$

But $\sin \theta = h_L/L = S$. Solving for $\bar{\tau}_0$, we have

$$\bar{\tau}_0 = \gamma \frac{A}{P} S = \gamma R_h S$$

(10.4:



where R_h is the hydraulic radius and for most slopes (with $\theta < 5.7^\circ$) S_0 may be taken as equal to S . Substituting the value of $\bar{\tau}_0$ from Eq. (8.8) and replacing S with S_0 ,

$$\bar{\tau}_0 = C_f \rho \frac{V^2}{2} = \gamma R_h S_0$$

$$\bar{\tau}_0 = \gamma \frac{A}{P} S = \gamma R_h S$$

This may be solved for v in terms of either the friction coefficient C_f or the conventional friction factor f [Eq. (8.11)] to give

$$V = \sqrt{\frac{2g}{C_f} R_h S_0} = \sqrt{\frac{8g}{f} R_h S_0} \quad (10.5)$$

Chezy equation

Antoine de Chézy (1718–1798), a French bridge engineer and hydraulics expert, proposed in 1775 that the velocity in an open channel varied as $\sqrt{R_h S_0}$. This led to the formula

$$V = C\sqrt{R_h S_0}$$

$$V = C \sqrt{R_h S_0} \quad (10.6)$$

which is known by his name. It has been widely used both for open channels and for pipes under pressure. Comparing Eqs. (10.6) and (10.5), it is seen that $C = \sqrt{8g/f}$. Despite the simplicity of Eq. (10.6), it has the distinct drawback that C is not a pure number but has the dimensions $L^{1/2}T^{-1}$, requiring that values of C in SI units be converted before being used with BG units in the rest of the formula.

Mannings formula

One of the best as well as one of the most widely used formulas for uniform flow in open channels is that published by the Irish engineer Robert Manning (1816-1897). Manning had found from many tests that the value of C in the Chézy formula varied approximately as $R_h^{1/6}$, and others observed that the proportionality factor was very close to the reciprocal of n, the coefficient of roughness in the previously used, but complicated and inaccurate, Kutter formula. This led to the formula that has since spread to all parts of the world. The Manning formula is

$$V(m/s) = \frac{1}{n} R_h^{2/3} S_0^{1/2} \quad (10.7a)$$

Specific energy and alternate depths

For *any* cross-section shape, the *specific energy* E at a particular section is defined as the energy head referred to the channel bed as datum. Thus

$$E = y + \alpha \frac{V^2}{2g} \quad (10.16)$$

where α is the kinetic energy correction factor (Sec. 5.1), which accounts for velocity variations across the section. Friction at the channel walls reduces velocities near the wetted perimeter, as indicated in Fig. 10.7 for which $\alpha = 1.105$. As noted in Sec. 10.4, the value of α is usually assumed to be unity; for typical velocities this results in only a small error in E .

Specific energy and alternate depths

For *rectangular* channels, provided they are not unusually narrow so that α is large, a representative average value of the flow q per unit width can be expressed as $q = Q/b$. The average velocity $V = Q/A = qb/by = q/y$ and so Eq. (10.16) with $\alpha = 1$ can be expressed as

$$E = y + \frac{1}{2g} \frac{q^2}{y^2} \quad (10.17)$$

Let us consider how E will vary with y if q remains constant.

$$E = y + \frac{1}{2g} \frac{q^2}{y^2} \quad (E - y)y^2 = \frac{q^2}{2g} = \text{constant} \quad (10.19)$$



A plot of E vs. y is hyperbola-like with asymptotes $(E - y) = 0$ (that is, $E = y$) and $y = 0$. Such a curve, shown in Fig. 10.12, is known as the **specific energy diagram**. Actually each different value of q will give a different curve, as shown in Fig. 10.12. For a particular q , we see there are two possible values of y for a given value of E . These are known as **alternate depths**. Equation (10.19) is a cubic equation with three roots, the third root being negative has no physical meaning. The two alternate depths represent two

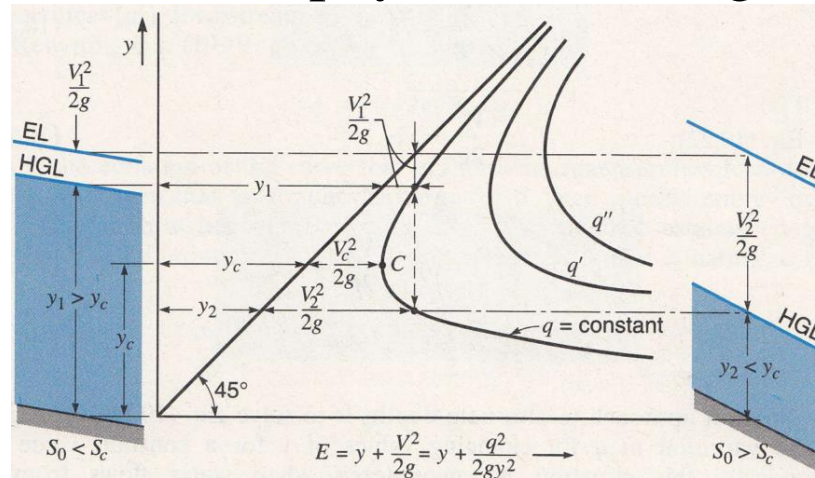


Figure 10.12

Specific-energy diagram for three constant rates of discharge in a rectangular channel. (Bed slopes are greatly exaggerated.)

The two alternate depths represent two totally different flow regimes—slow and deep on the upper limb of the curve and fast and shallow on the lower limb of the curve. Point *C* represents the dividing point between the two regimes of flow. At *C*, for a given *q*, the value of *E* is a minimum and the flow at this point is referred to as **critical flow**. The depth of flow at that point is the **critical depth** y_c and the velocity is the **critical velocity** V_c . A relation for critical depth in a wide rectangular channel can be found by differentiating *E* of Eq. (10.17) with respect to *y* to find the value of *y* for which *E* is a minimum. Thus

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} \qquad E = y + \frac{1}{2g} \frac{q^2}{y^2} \qquad (10.20)$$

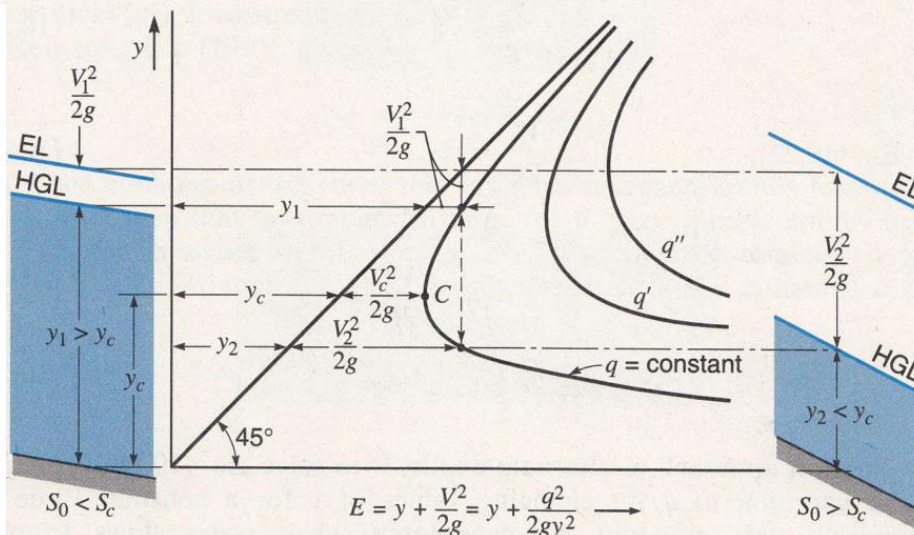


Figure 10.12

Specific-energy diagram for three constant rates of discharge in a rectangular channel. (Bed slopes are greatly exaggerated.)

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} \quad (10.20)$$

and when E is a minimum, $y = y_c$ and $dE/dy = 0$, so that

$$0 = 1 - \frac{q^2}{gy_c^3}, \quad \text{or} \quad q^2 = gy_c^3 \quad (10.21)$$

Substituting $q = Vy = V_c y_c$ gives

$$V = Q/A = qb/by = q/y \Rightarrow V^2 = \frac{q^2}{y^2} = \frac{gy^3}{y^2} = gy$$

$$V_c^2 = gy_c \quad \text{and} \quad V_c = \sqrt{gy_c} = \frac{q}{y_c} \quad (10.22)$$

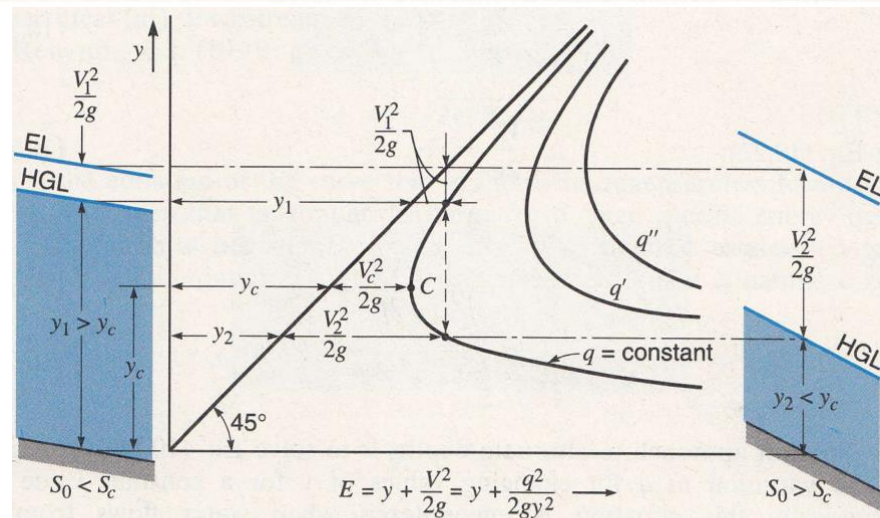


Figure 10.12

where the subscript c indicates critical flow conditions (minimum specific energy for a given q).

Equation (10.22) may also be expressed as

$$v_c = \sqrt{gy_c} = \frac{q}{y_c}$$

$$\sqrt{gy_c} = \frac{q}{y_c} \Rightarrow gy_c = \frac{q^2}{y_c^2} \Rightarrow q^2 = gy_c^3 \Rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3} \quad 10.23$$

From Eq 10.22 $v_c = \sqrt{gy_c} \Rightarrow v_c^2 = gy_c$

$$\frac{v_c^2}{2g} = \frac{gy_c}{2g} = \frac{1}{2}y_c \quad 10.24$$

Hence $E_c = E_{\min} = y_c + \frac{v_c}{2g} = y_c + \frac{1}{2}y_c = \frac{3}{2}y_c \quad 10.25$

And $y_c = \frac{2}{3}E_c = \frac{2}{3}E_{\min} \quad 10.26$

The discharge is a maximum when $y = y_c$, as indicated on the discharge curve of Fig. 10.13*b*. An expression for q_{\max} may be obtained by substituting $E = 1.5y_c$ from Eq. (10.28) into Eq. (10.27), to obtain

$$q_{\max} = \sqrt{gy_c^3} \quad (10.29)$$

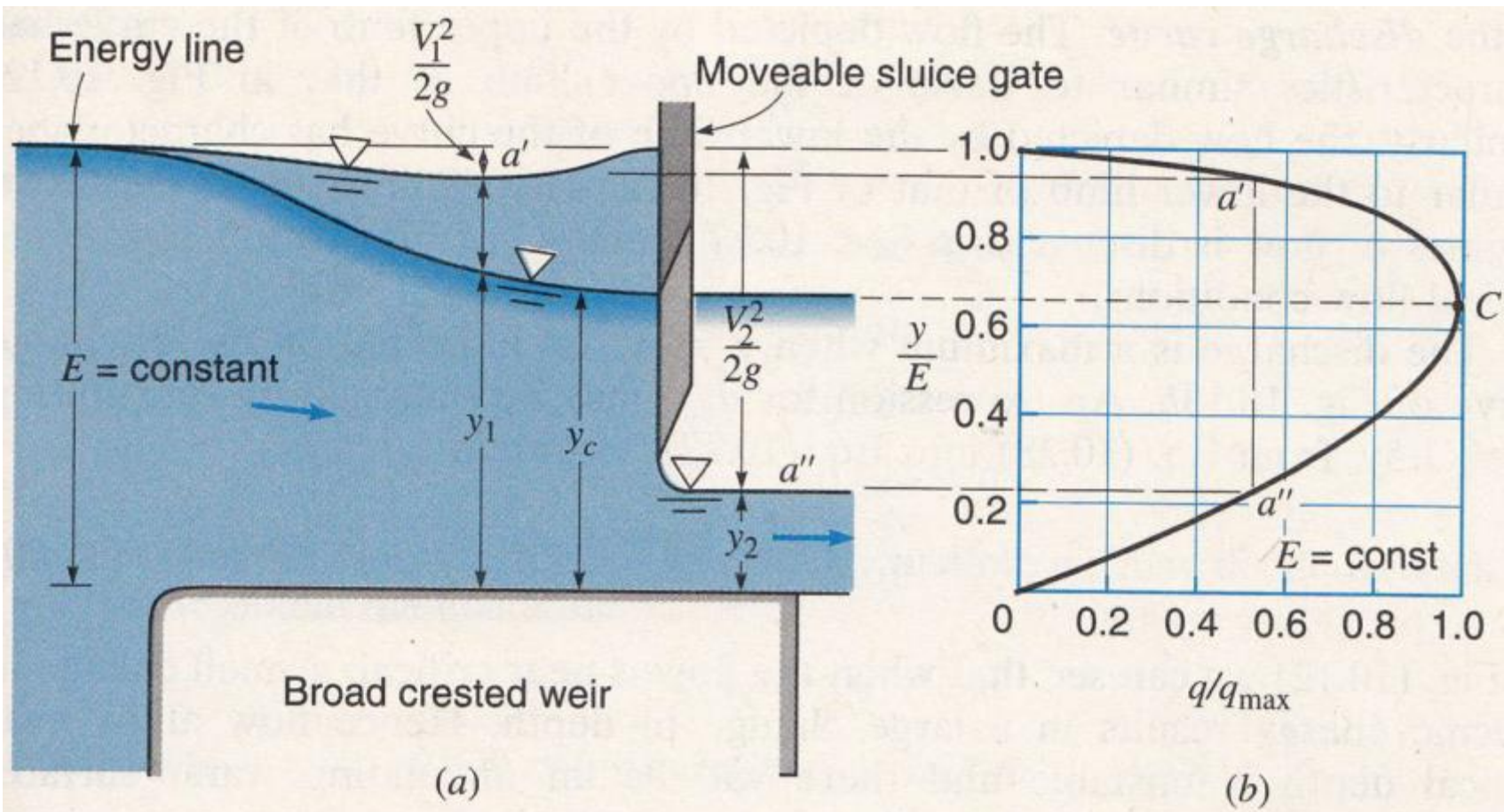


Figure 10.13

Subcritical and supercritical flows

In Sec. 10.9 we referred to the upper and lower portions of the specific-energy diagram (Fig. 10.12) and the discharge curve (Fig. 10.13). The upper limb of those curves, where velocities are less than critical, represent **subcritical** (also known as tranquil or upper-stage) flow, while the lower limb of the curves where velocities are greater than critical represent **supercritical** (also known as rapid or lower-stage) flow. We discussed how to identify the critical point separating these two portions in Sec. 10.9.

The slope required to give uniform flow at critical depth ($y_0 = y_c$) for a given discharge is known as the **critical slope** S_c . Note that S_c varies with discharge. We obtain an expression for the critical slope of a *wide and shallow* rectangular channel ($R_h = y$) when we combine Eq. (10.23) for critical flow with Eq. (10.18) for uniform flow, eliminating q as follows:

$$\begin{array}{l} \text{Wide and shallow,} \\ \text{BG units:} \end{array} \quad S_c = \left(\frac{n}{1.486} \right)^2 \frac{g}{y_c^{1/3}} \quad (10.30)$$

Substituting for y_c from Eq. (10.23), this can also be written as

$$\begin{array}{l} \text{Wide and shallow,} \\ \text{BG units:} \end{array} \quad S_c = \left(\frac{n}{1.486} \right)^2 \frac{g^{10/9}}{q^{2/9}} \quad (10.31)$$

When using SI units, we simply replace the 1.486 by 1 in these equations.

If the bed slope $S_0 > S_c$, the slope is known as a **steep slope** for the given discharge. Normal depth for uniform flow on such a slope will be less than critical depth and hence normal flow will be supercritical. In contrast, if $S_0 < S_c$, the normal depth will be greater than critical and normal flow is subcritical. Such a slope is referred to as a **mild slope**. By referring to Eq. (10.30), we see that the hydraulic steepness of a channel slope is determined by more than its elevation gradient. A steep slope for a channel with a smooth lining could be a mild slope for the same flow with a rough lining. Even for a given channel with a given boundary roughness, the slope may be mild for a low rate of discharge and steep for a higher one.

It may be recalled that the Froude number (Sec. 7.4) is defined as V/\sqrt{gL} . If for rectangular channels the depth of flow is used to represent the significant length parameter in the Froude number (that is, $\mathbf{F} = V/\sqrt{gy}$), we find by comparing this with Eq. (10.22) that the flow is critical if $\mathbf{F} = 1.0$, the flow is subcritical if $\mathbf{F} < 1.0$, and the flow is supercritical if $\mathbf{F} > 1.0$.

$$V_c = \sqrt{gy_c} = \frac{q}{y_c}$$

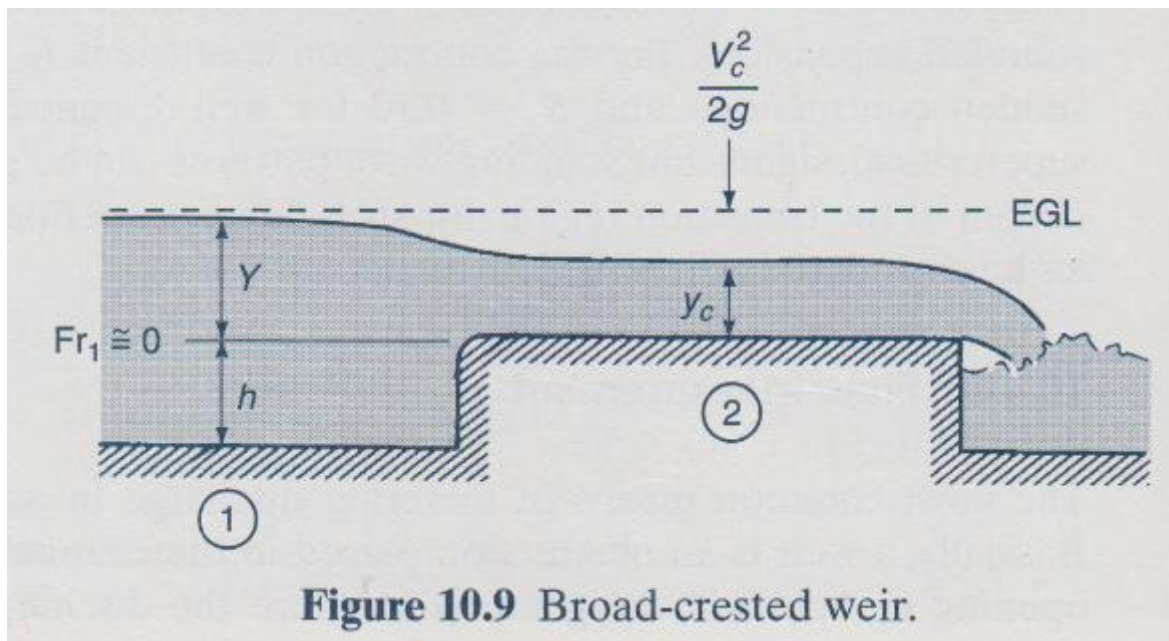
$$F = \frac{V}{\sqrt{gy}} = \frac{V_c}{\sqrt{gy_c}} = \frac{V_c}{V_c} = 1$$

Table 10.2**Characteristics of subcritical, critical, and supercritical flow in rectangular channels**

Characteristic	Subcritical	Critical	Supercritical
Depth of flow, y	$y > y_c$	$y = y_c = \left(\frac{q^2}{g}\right)^{1/3}$	$y < y_c$
Velocity of flow, V	$V < V_c$	$V = V_c = \sqrt{gy}$	$V > V_c$
Slope for uniform flow, S_0	Mild slope $S_0 < S_c$	Critical slope $S_0 = S_c$ [Eq. (10.30) if wide and shallow]	Steep slope $S_0 > S_c$
Froude number, $\mathbf{F} = \frac{V}{\sqrt{gy}} = \frac{q}{\sqrt{gy^3}}$	$\mathbf{F} < 1.0$	$\mathbf{F} = 1.0$	$\mathbf{F} > 1.0$
Disturbance waves (Sec. 10.20)	Will propagate in all directions	Will hold fast, not propagate upstream	Will form standing wave with $\sin \beta = c/V$ downstream only
Velocity head compared with half-depth	$\frac{V^2}{2g} < \frac{y}{2}$	$\frac{V^2}{2g} = \frac{y}{2}$	$\frac{V^2}{2g} > \frac{y}{2}$
Can be followed by a hydraulic jump? (Sec. 10.16)	No	No	Yes

Occurrence of critical depth

- Weir
- For thin plate weir $Q = C_d b \sqrt{2g} h^{3/2}$



Occurrence of critical depth

- Flumes

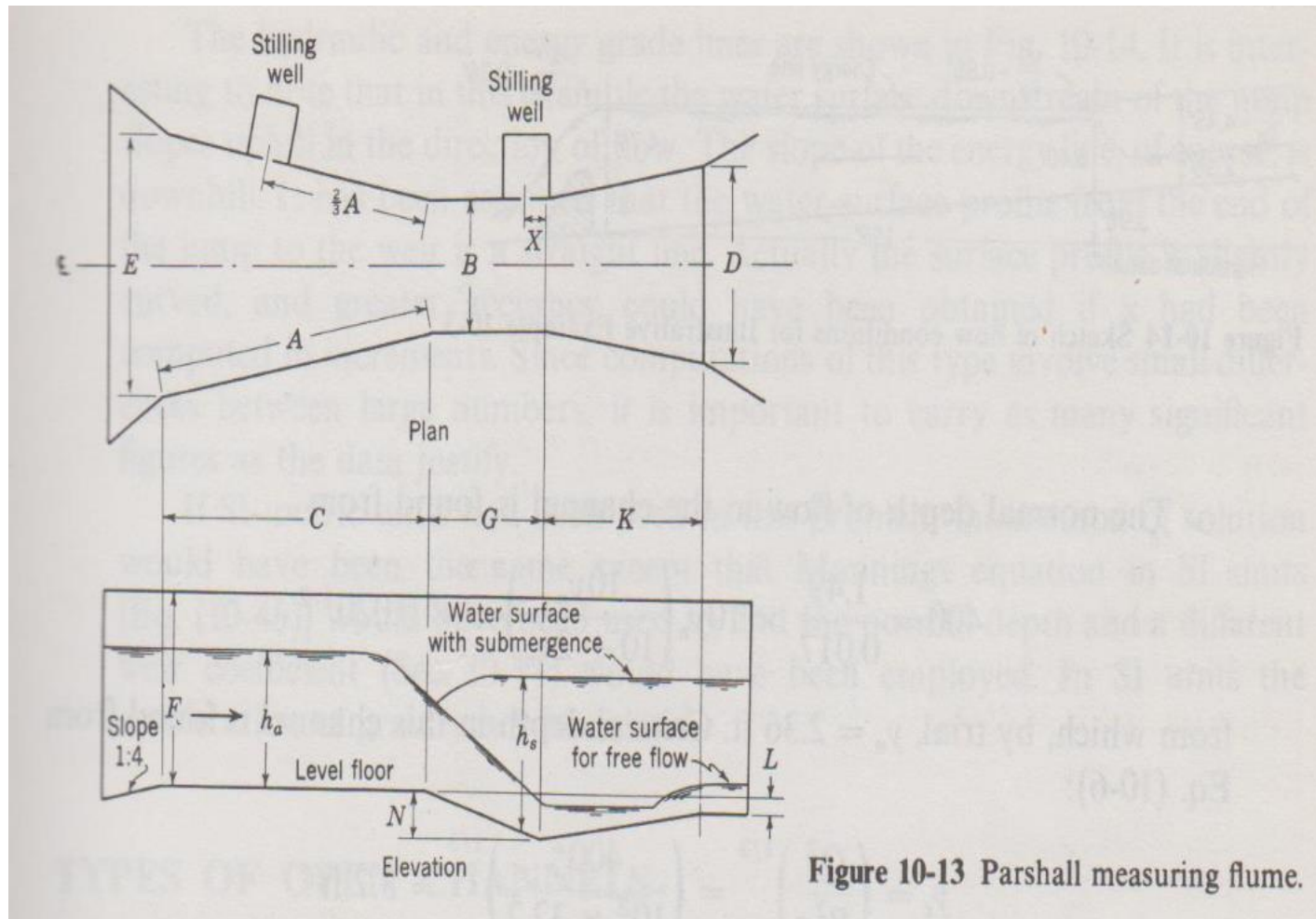
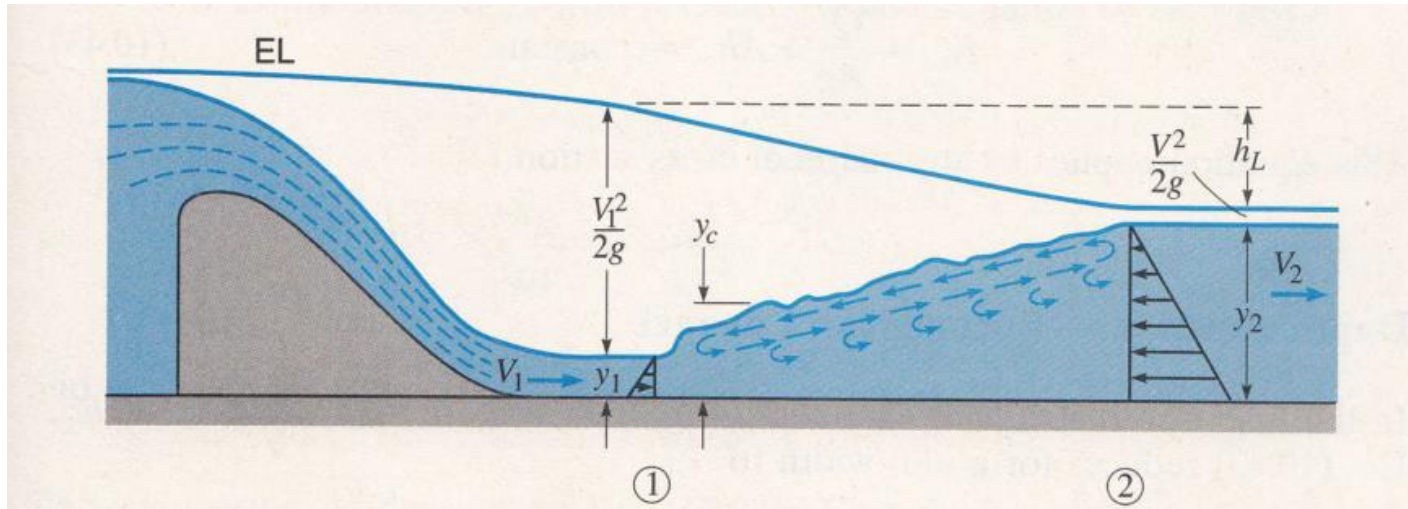


Figure 10-13 Parshall measuring flume.

Hydraulic jump

By far the most important of the local nonuniform-flow phenomena is that which occurs when supercritical flow has its velocity reduced to subcritical. We have seen in the surface profiles of Fig. 10.20 that there is no ordinary means of changing from supercritical to subcritical flow with a smooth transition, because theory calls for a vertical slope of the water surface. The result, then, is a marked discontinuity in the surface, characterized by a steep upward slope of the profile, broken throughout with violent turbulence, and known universally as the *hydraulic jump*.



Hydraulic jump

The specific reason for the occurrence of the hydraulic jump can perhaps best be explained by reference to the M_3 curve of Fig. 10.20. Downstream of the sluice gate the flow decelerates because the slope is not great enough to maintain supercritical flow. The **specific energy decreases** as the depth increases (proceeding to the left along the lower limb of the specific-energy diagram, Fig. 10.12). Were this condition to progress until the flow reached critical depth, an **increase in specific energy** would be required **as** the depth increased **from the critical to the uniform flow depth** downstream. But this is a physical impossibility. Therefore the jump forms before the necessary energy is lost.

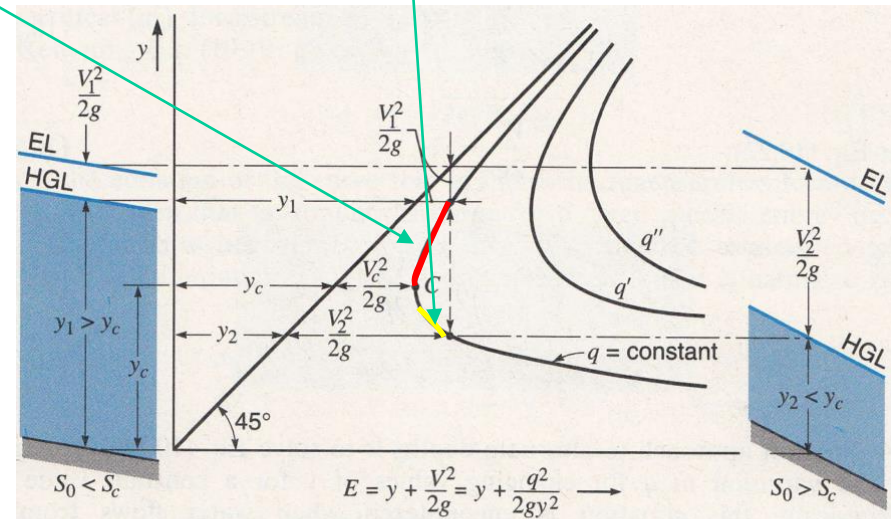
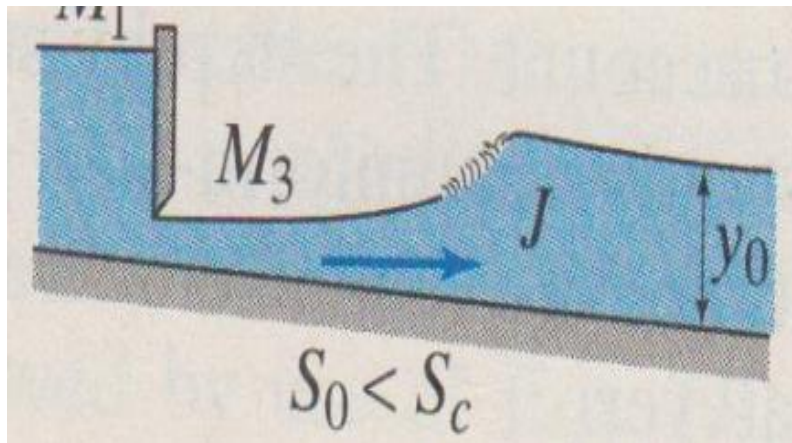
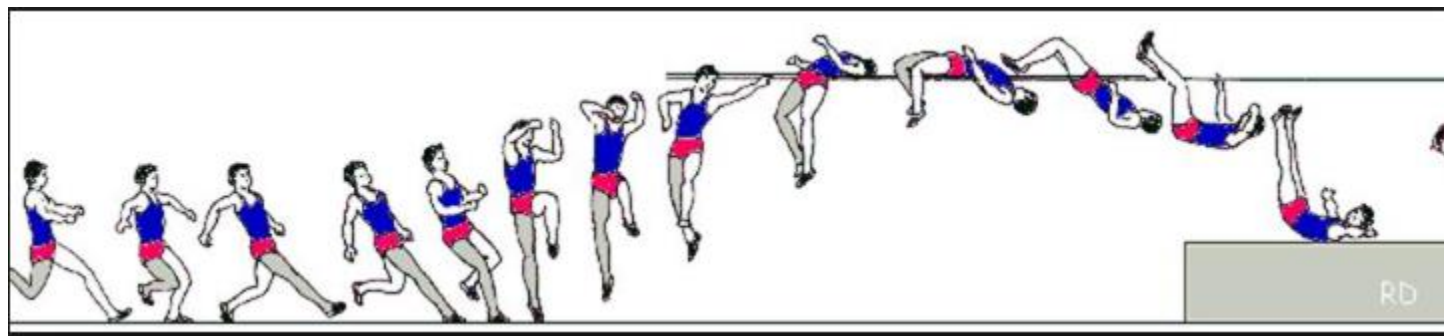
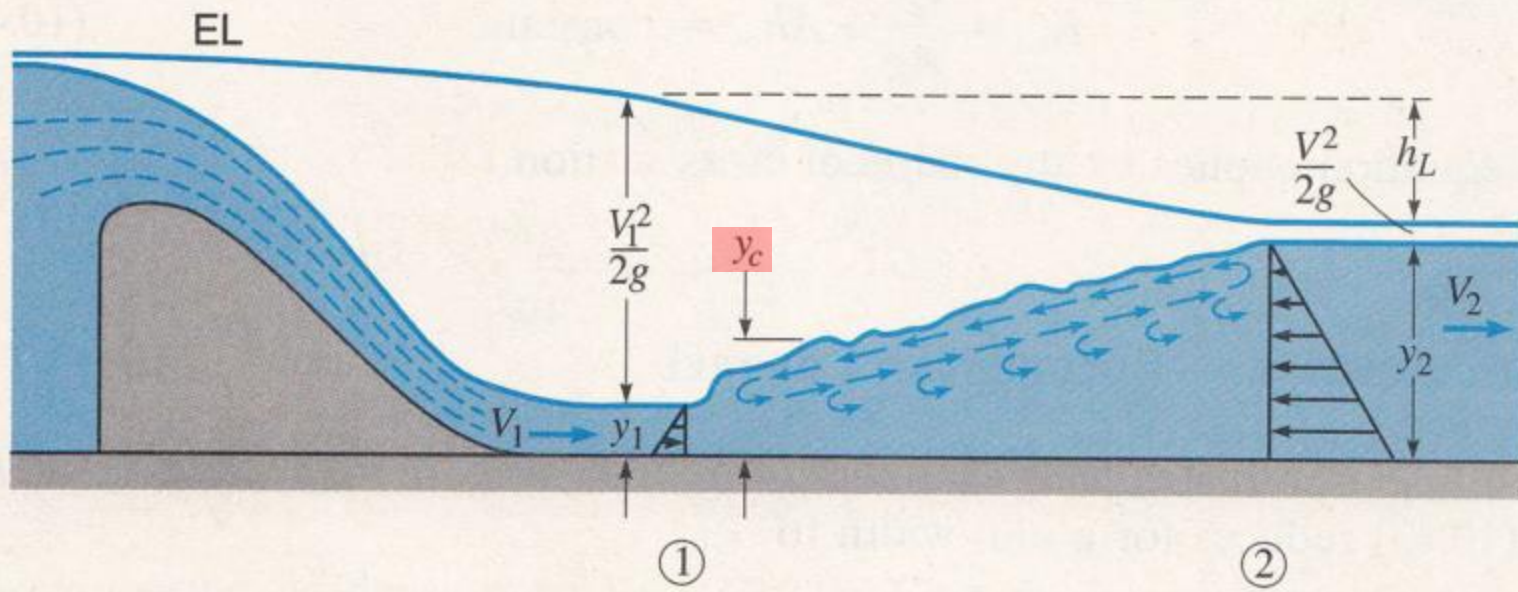


Figure 10.12 Specific-energy diagram for three constant rates of discharge in a rectangular channel. (Bed slopes are greatly exaggerated.)



A **hydraulic jump** is a phenomenon in which fluid flowing at a supercritical state will undergo a transition to a subcritical state. Boundary conditions upstream and downstream of the jump will dictate its strength as well as its location. An idealized hydraulic jump is shown in Fig. 10.15. The strength of the jump varies widely, as shown in Fig. 10.2, with relatively mild disturbances occurring at one extreme, and significant separation and eddy formation taking place at the other. As a result, the energy loss associated with the jump is considered to be unknown, so the energy equation is not used in the initial analysis. Assuming no friction along the bottom and no submerged obstacle; that is, setting F equal to zero, Eq. 10.5.2 shows that $M_1 = M_2$. For a rectangular section, Eq. 10.5.4 can be substituted into the relation, allowing one to obtain

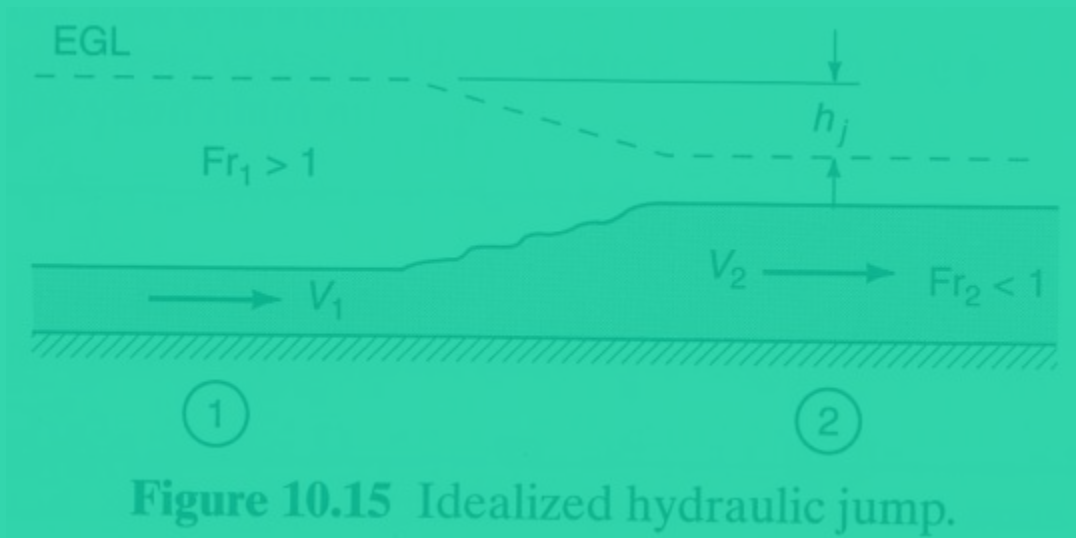


Figure 10.15 Idealized hydraulic jump.

Derivation of formula for conjugate depths-Method A



$$\frac{q^2}{g} \left(\frac{1}{y_1} - \frac{1}{y_2} \right) = \frac{1}{2} (y_2^2 - y_1^2)$$

Rearranging, factoring, and noting that $Fr_1^2 = q^2/(gy_1^3)$ there results

$$Fr_1^2 = \frac{1}{2} \frac{y_2}{y_1} \left(\frac{y_2}{y_1} + 1 \right) \quad (10.5.8)$$

This equation is dimensionless and it relates the Froude number upstream of the jump with the ratio of the downstream to upstream depths. It can be seen that Eq. 10.5.8 is quadratic with respect to y_2/y_1 provided that Fr_1 is known. Solving for y_2/y_1 , one obtains

$$\frac{y_2}{y_1} = \frac{1}{2} (\sqrt{1 + 8Fr_1^2} - 1) \quad (10.5.9)$$

The positive sign in front of the radical has been chosen to yield a physically meaningful solution. It is worth noting that Eq. 10.5.9 is also valid if the subscripts on the depths and Froude number are reversed:

$$\frac{y_1}{y_2} = \frac{1}{2} (\sqrt{1 + 8Fr_2^2} - 1) \quad (10.5.10)$$

Derivation of formula for conjugate depths-Method B

Applying the momentum equation in the x- direction

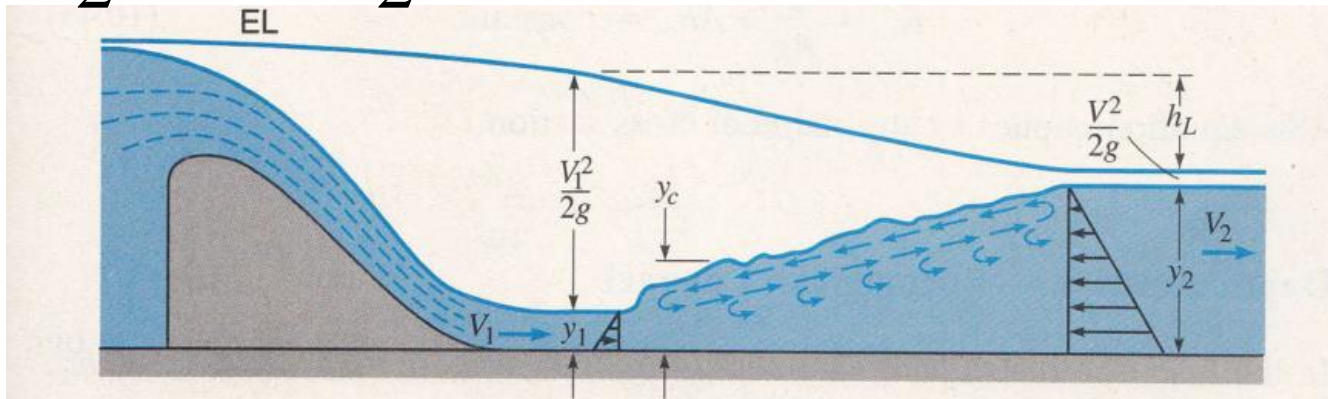
$$\sum F_x = \rho Q(\Delta v)$$

$$\sum F_x = P_1 A_1 - P_2 A_2 = \frac{\rho g y_1}{2} y_1 - \frac{\rho g y_2}{2} y_2 \quad \text{per unit width}$$

$$\rho Q(\Delta v) = \rho v_1(-A_1 v_1) + \rho v_2(-A_2 v_2)$$

$$= \rho v_1(-y_1 v_1) + \rho v_2(-y_2 v_2) \quad \text{per unit width}$$

$$\frac{\rho g y_1}{2} y_1 - \frac{\rho g y_2}{2} y_2 = \rho v_1(-y_1 v_1) + \rho v_2(-y_2 v_2)$$



Derivation of formula for conjugate depths-Method B

$$\frac{\rho g y_1}{2} y_1 - \frac{\rho g y_2}{2} y_2 = \rho v_1 (-y_1 v_1) + \rho v_2 (-y_2 v_2)$$

$$\frac{\rho g}{2} (y_1^2 - y_2^2) = \rho (v_2^2 y_2 - v_1^2 y_1)$$

$$\frac{1}{2} (y_1^2 - y_2^2) = \frac{\rho}{\rho g} (v_2^2 y_2 - v_1^2 y_1)$$

$$\frac{1}{2} (y_1^2 - y_2^2) = \frac{1}{g} (v_2^2 y_2 - v_1^2 y_1)$$

5

From the continuity equation $v_2 = v_1 y_1 / y_2$. substitute this into 5 and simplify

$$\frac{1}{2} (y_1^2 - y_2^2) = \frac{1}{g} \left(\left(\frac{v_1 y_1}{y_2} \right)^2 y_2 - v_1^2 y_1 \right)$$

$$2 \frac{v_1^2}{g y_1} = \left(\frac{y_2}{y_1} \right)^2 + \frac{y_2}{y_1}$$

Derivation of formula for conjugate depths-Method B

$$2 \frac{v_1^2}{gy_1} = \left(\frac{y_2}{y_1} \right)^2 + \frac{y_2}{y_1}$$

The term on the left hand side of eqn 6 will be recognised as being twice Fr_1^2 . Hence eqn 6 is written as

$$\left(\frac{y_2}{y_1} \right)^2 + \left(\frac{y_2}{y_1} \right) - 2Fr_1^2 = 0 \quad 7$$

By use of the quadratic formula and solving for y_2/y_1 in terms of upstream Froude number, we obtain

$$y_2 = \frac{1}{2} y_1 \left((1 + 8Fr_1^2)^{1/2} - 1 \right) \quad 8$$

Derivation of formula for conjugate depths-Method C



When the velocity of a flow in an open rectangular channel of width w is relatively large, it is possible for the flow to “jump” from a depth y_1 to a depth y_2 over a relatively short distance as shown in Fig. E4.12; this is referred to as a **hydraulic jump**. Express y_2 in terms of y_1 and V_1 ; assume a horizontal flow.

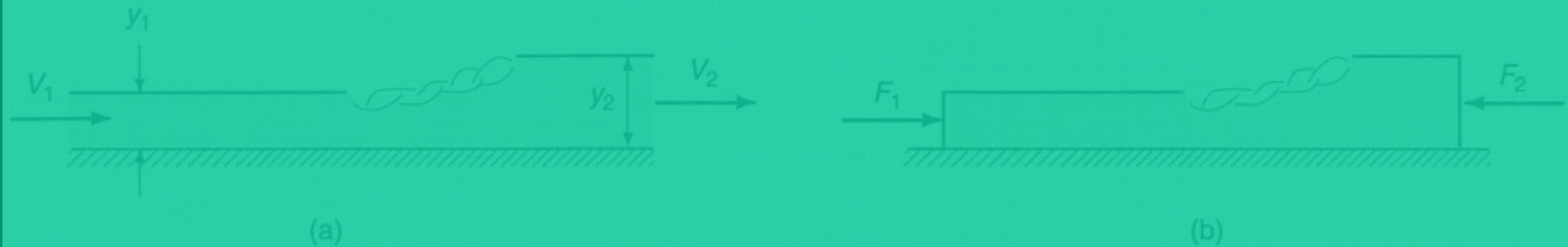


Figure E4.12

A control volume is selected with inlet and exit areas upstream and downstream of the “jump” sufficiently far that the streamlines are parallel to the wall with hydrostatic pressure distributions. Neglecting the drag that is present on the walls (if the distance between sections is relatively small, the drag force should be negligible), the momentum equation can be manipulated as follows:



$$\Sigma F_x = \dot{m} (V_{2x} - V_{1x})$$

$$F_1 - F_2 = \rho A_1 V_1 (V_2 - V_1)$$

$$\gamma \frac{y_1}{2} (y_1 w) - \gamma \frac{y_2}{2} (y_2 w) = \rho y_1 w V_1 \left(V_1 \frac{y_1}{y_2} - V_1 \right)$$

where we have expressed F_1 and F_2 using Eq. 2.4.24, and continuity in the form of Eq. 4.3.6, so that

$$V_2 = \frac{y_1}{y_2} V_1$$

The momentum equation can be simplified to

$$\frac{\gamma}{2} (y_1^2 - y_2^2) = \rho y_1 V_1^2 \frac{y_1 - y_2}{y_2}$$

or

$$\frac{g}{2} (y_1 - y_2)(y_1 + y_2) = \frac{y_1}{y_2} V_1^2 (y_1 - y_2)$$

The factor $(y_1 - y_2)$ is divided out and y_2 is found as follows:

$$\frac{g}{2} (y_1 + y_2) = \frac{y_1}{y_2} V_1^2$$



$$y_2^2 + y_1 y_2 - \frac{2}{g} y_1 V_1^2 = 0$$

$$\therefore y_2 = \frac{1}{2} \left(-y_1 + \sqrt{y_1^2 + \frac{8}{g} y_1 V_1^2} \right)$$

where the quadratic formula has been used. The energy equation could now be used to provide an expression for the losses in the hydraulic jump.



$$y_2 = \frac{y_1}{2} \left(-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}} \right) \quad (10.46a)$$

or

$$y_1 = \frac{y_2}{2} \left(-1 + \sqrt{1 + \frac{8q^2}{gy_2^3}} \right) \quad (10.46b)$$

These equations relate the depths before and after hydraulic jump (i.e., the conjugate depths) in a rectangular channel. They give good results if the channel slope is less than about 0.05. For steeper channel slopes the effect of the gravity component of the weight of liquid between sections 1 and 2 of Fig. 10.23 must be considered.

Energy loss in a hydraulic jump

The head loss h_{L_j} caused by the jump is the drop in energy from 1 to 2. Or

$$h_{L_j} = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right) \quad (10.47)$$

On Fig. 10.24 points e and c on the specific energy diagram represent the conjugate depths, and the horizontal distance between them ($= cg = fh$) is the head loss. Replacing V by q/y and using Eq. (10.45), it can be shown that also

Rectangular channel:

$$h_{L_j} = \frac{(y_2 - y_1)^3}{4y_1 y_2} \quad (10.48)$$

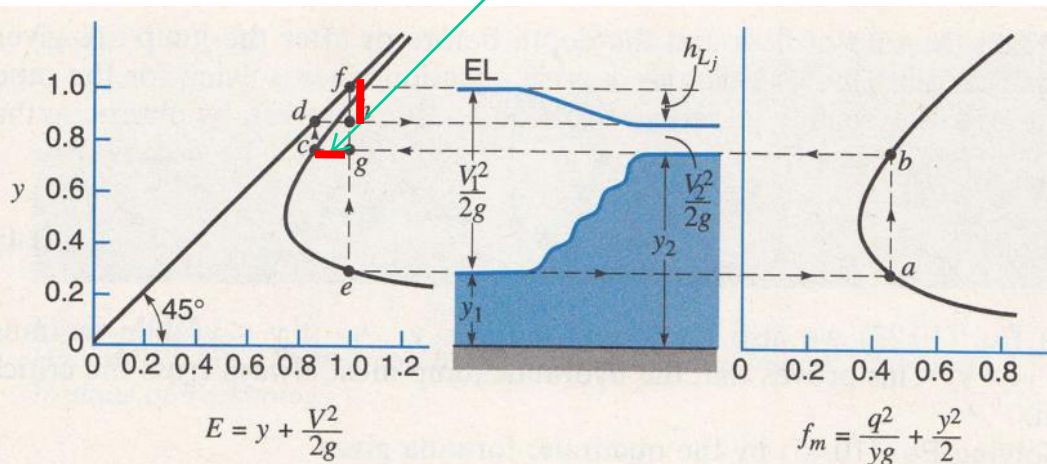
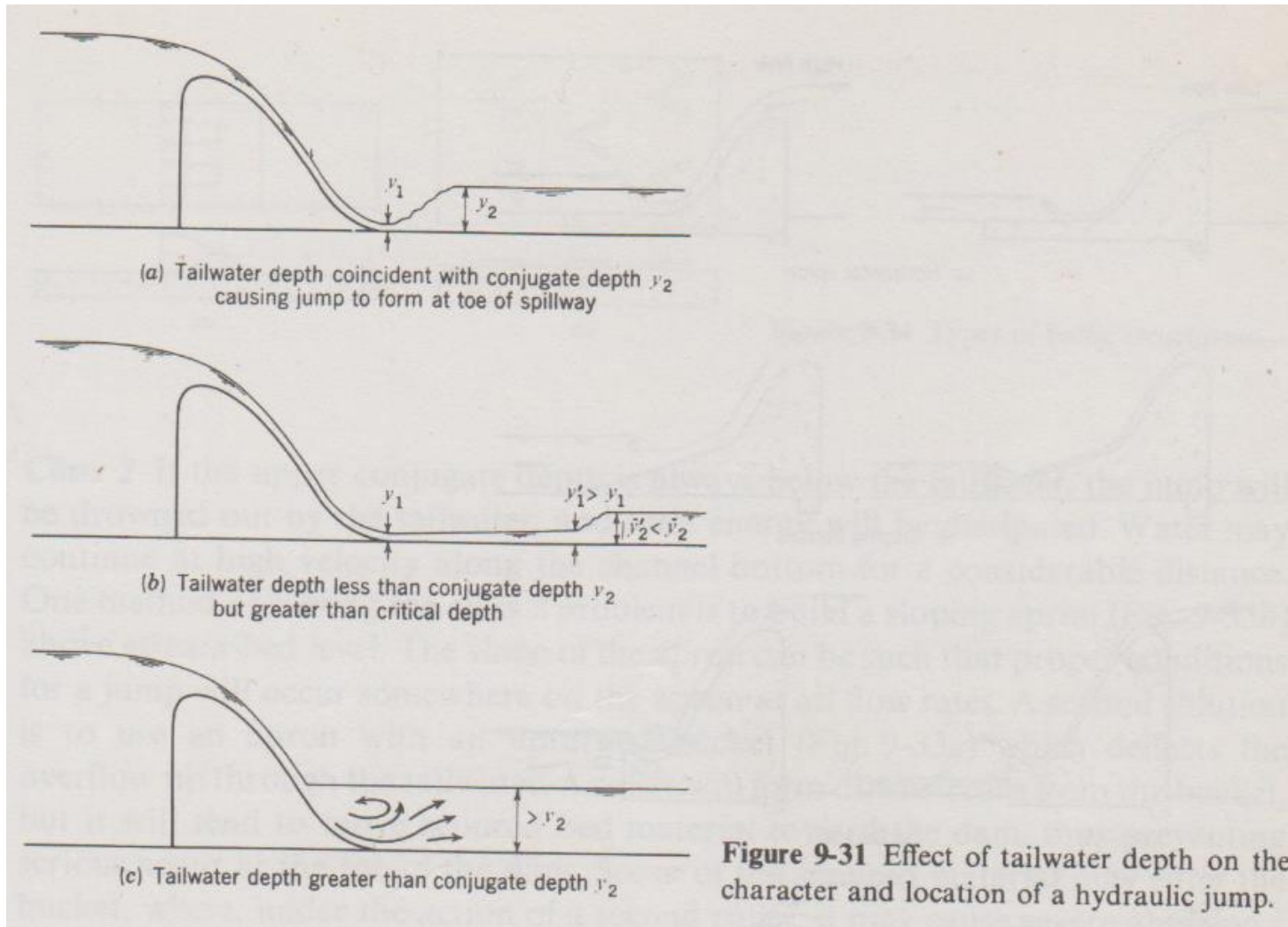


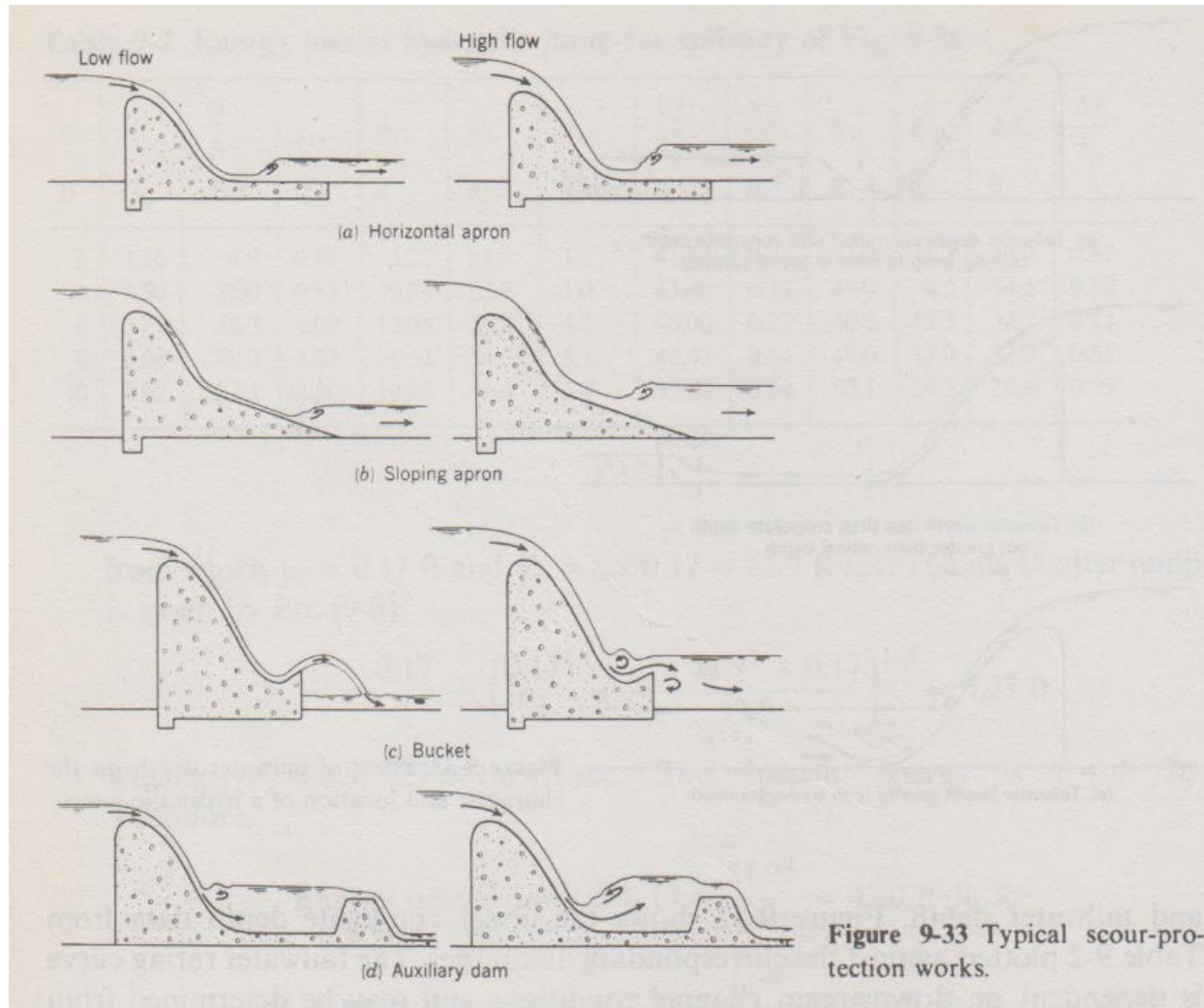
Figure 10.24
Energy and momentum relations in hydraulic jump.

Applications of hydraulic jump

1. Scour protection



Applications of hydraulic jump



Applications of hydraulic jump

2. Where velocity of fluid has to be reduced

Best wishes for the exam,
without any **turbulence**, **pressure**
drag but with all the
buoyancy