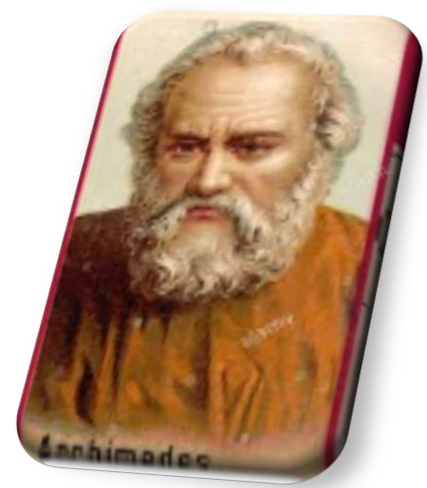
Fluid Mechanics CEE 3311

LECTURE 5

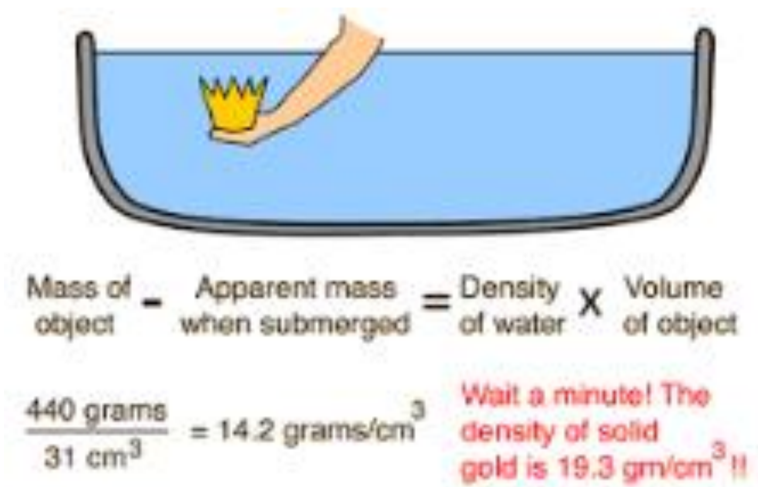
Buoyancy

L. Handia



The law of buoyancy, known as Archimedes' principle, dates back some 2200 years to the Greek philosopher Archimedes. Legend has it that Hiero, king of Syracuse, suspected that his new gold crown may have been constructed of materials other than pure gold, so he asked Archimedes to test it. Archimedes probably made a lump of pure gold that weighed the same as the crown. The lump was discovered to weigh more in water than the crown weighed in water, thereby proving to Archimedes that the crown was not pure gold. The fake material possessed a larger volume to have the same weight as gold, hence it displaced more water. **Archimedes' principle** is: There is a buoyancy force on an object equal to the weight of displaced liquid.

Principle of Archimedes: when a body is immersed in a fluid, it loses weight by an amount equal to the weight of the fluid which it displaces called its *buoyancy* (dictionary of civil engineers).



Proof of the Law of Buoyancy

- To prove the law of buoyancy, consider the submerged body (Fig 5.1 a). In part (b) a cylindrical free-body diagram is shown that includes the submerged body with weight W and liquid having a weight F_W; the cross sectional area A is the maximum cross sectional area of the body.

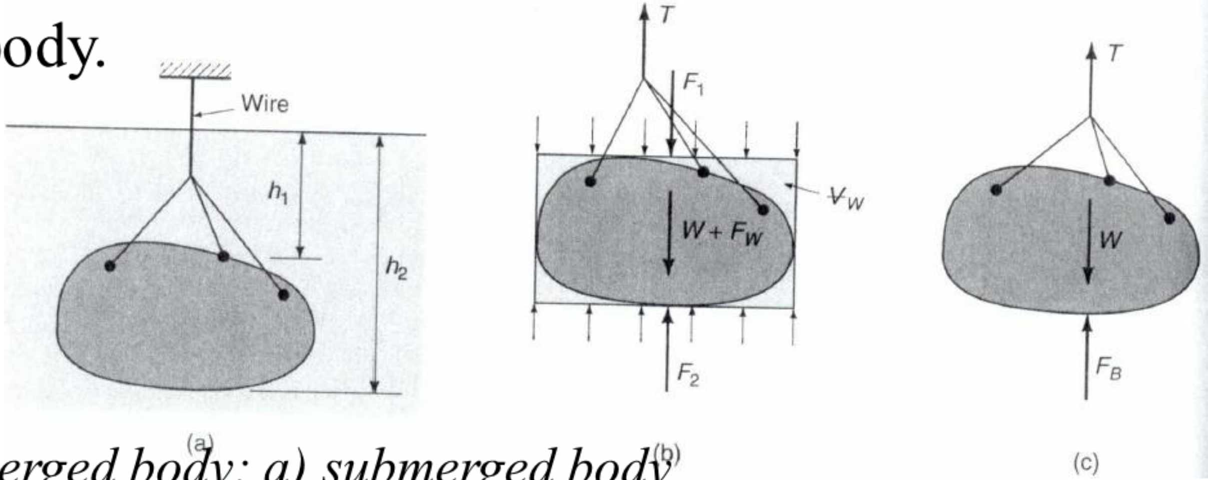


Fig 5.1 Forces on a submerged body: a) submerged body b) free-body diagram c) free-body diagram showing F_B

- From the diagram we see that the resultant vertical force acting on the free-body diagram due to the water only (do not include W as it is supported by wire, force T) is equal to
- $\sum F = F_2 - F_1 - F_W$

Proof of the Law of Buoyancy

$$\sum F = F_2 - F_1 - F_W$$

- This resultant force is by definition the buoyant force F_B. It can be expressed $F_B = p_2 A - p_1 A - \gamma V_W = \gamma h_2 A - \gamma h_1 A - \gamma V_W$

$$F_B = \gamma\,(h_2 A - h_1 A - V_W)$$ Computed using forces

- Where V_W is the liquid volume included in the free-body diagram. Recognising that the volume of the submerged body is

$$V_B = (h_2 - h_1)A - V_W)$$

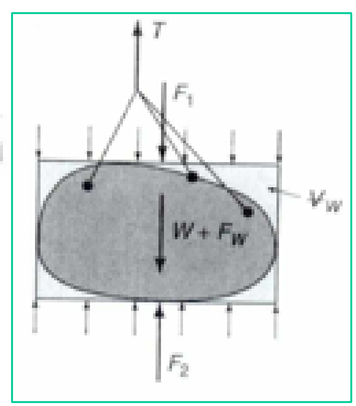
Computed using volume (geometry). Right handside equal to term in brackets of above eqn

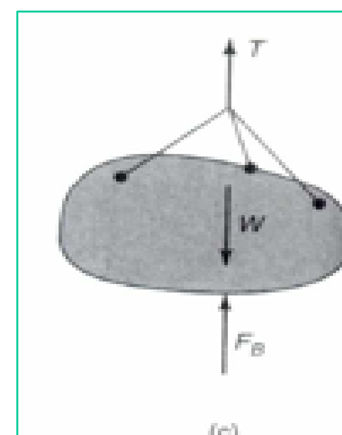
We see that $F_B = \gamma\,(h_2 A - h_1 A - V_W) = \gamma V_B$ Forces now equated to volume

$$= \gamma V_{displaced\ liquid}$$

thereby proving the law of buoyancy.

Archimedes' principle is: There is a buoyancy force on an object equal to the weight of displaced liquid.

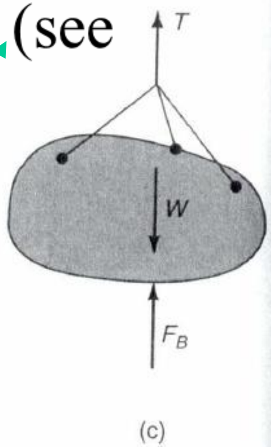




Buoyant Force

- The force necessary to hold the submerged body in place (see Fig 5.1 c) is equal to

otherwise it will sink since it is denser than water



$$T = W - F_B$$

where W is the weight of the submerged body.

- For a floating object, as in Fig 5.2, the buoyant force is

$F_B = \gamma V_{displaced\ liquid}$

- Obviously, T = 0, so that $0 = W - F_B$

since it is floating

$$F_B = W$$

where W is the weight of the floating object.

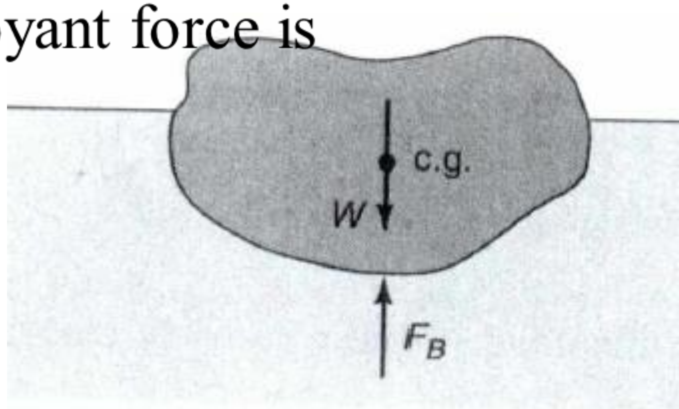


Fig 5.2 Forces on a floating object

- From the foregoing analysis it is apparent that the buoyant force F_B acts through the centroid of the displaced liquid volume. For the floating object, the weight of the object acts through the center of gravity, so the center of gravity of the object must lie on the same vertical line as the centroid of the liquid volume.

Example

The specific weight and the specific gravity of an unknown object are desired. Its weight in air is found to be 400 N and in water it weighs 300 N.

Solution: The volume is found from a force balance when submerged as follows (see Fig. 2.12c):

$$T = W - F_B$$

$$300 = 400 - 9810 \times \forall \qquad \therefore \qquad \forall = 0.0102 \text{ m}^3$$

The specific weight is then

$$\gamma = \frac{W}{\forall}$$

$$= \frac{400}{0.0102} = 39\,200 \text{ N/m}^3$$

Example

The specific gravity is found to be

$$S = \frac{\gamma}{\gamma_{water}}$$

$$= \frac{39\,200}{9810} = 4.00$$

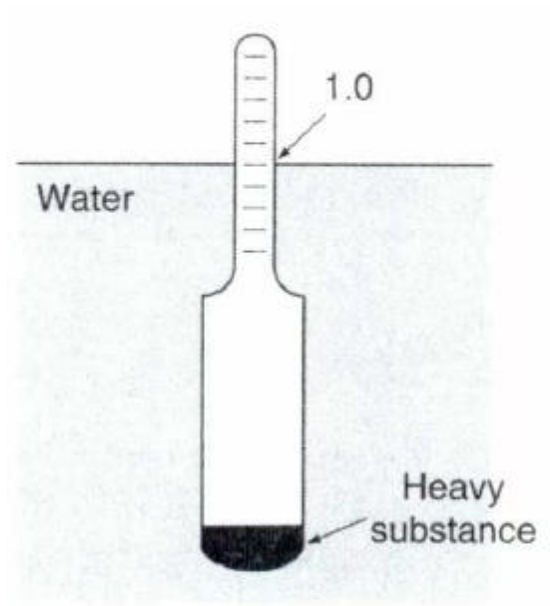
Hydrometer

A hydrometer, an instrument used to measure the specific gravity of liquids, operates on the principle of buoyancy. The upper part (Fig 5.3), the stem, has a constant diameter. When placed in pure water, the specific gravity is marked to read 1.0. the force balance is

$$W = \gamma_{water} V$$

where W is the weight of the hydrometer

V is the submerged volume below the S = 1.0 line.



(a)

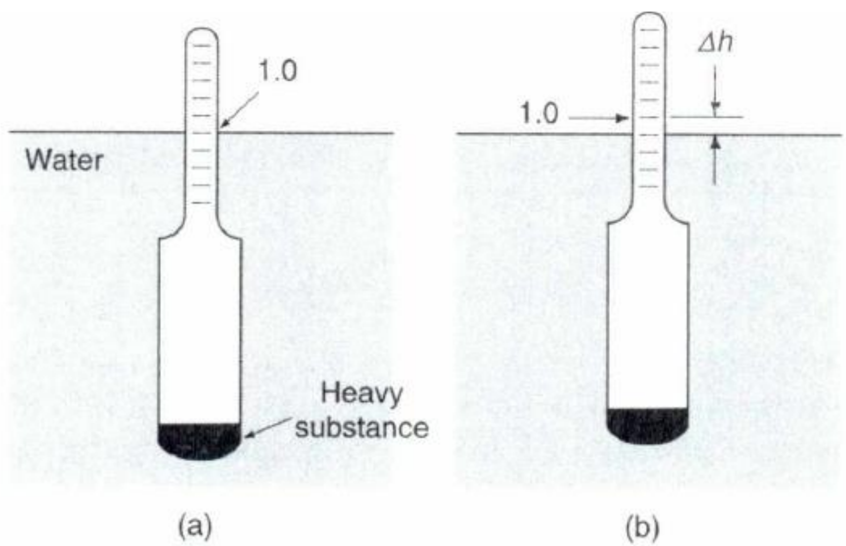
Hydrometer

In an unknown liquid of specific weight γ_x, a force balance would be

$$W = \gamma_x (V - A\Delta h)$$

where A is the cross sectional area of the stem. Equating these two expressions gives $\gamma_{water} V = \gamma_x (V - A\Delta h)$ since W is the same

$$V - A\Delta h = \frac{\gamma_{water} V}{\gamma_x}$$



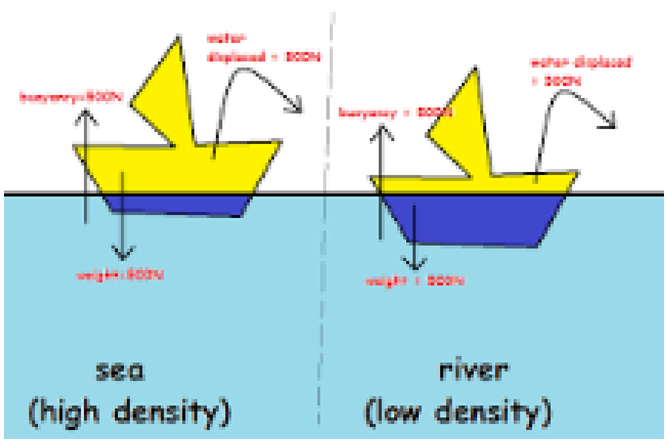


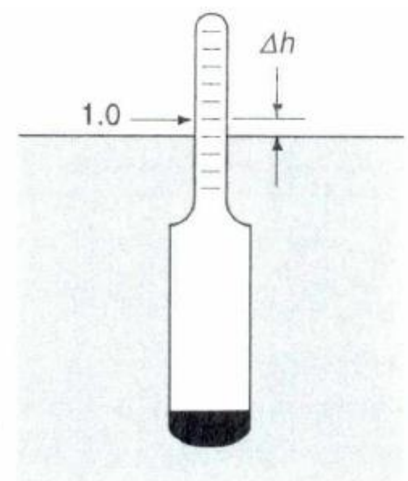
Fig 5.3 Hydrometer a) in water b) in an unknown liquid

$$V - A\Delta h = \frac{\gamma_{water} V}{\gamma_x}$$

$$\Delta h = \frac{V - \frac{\gamma_{water} V}{\gamma_x}}{A}$$

$$\Delta h = \frac{V}{A}\left(1 - \frac{\gamma_{water}}{\gamma_x}\right)$$

$$\Delta h = \frac{V}{A}\left(1 - \frac{1}{S_x}\right)$$



V is the submerged volume below the S = 1.0 line
A is the cross sectional area of the stem

where $S_x = \dfrac{\gamma_x}{\gamma_{water}}$.

For a given hydrometer, V and A are fixed so that the quantity Δh is dependent only on the specific gravity. Thus the stem can be calibrated to read S_x directly.

Application

Hydrometers are used to measure

- the amount of antifreeze in the radiator of an automobile
- the charge in a battery since the density of water changes when subjected to an electrical charge