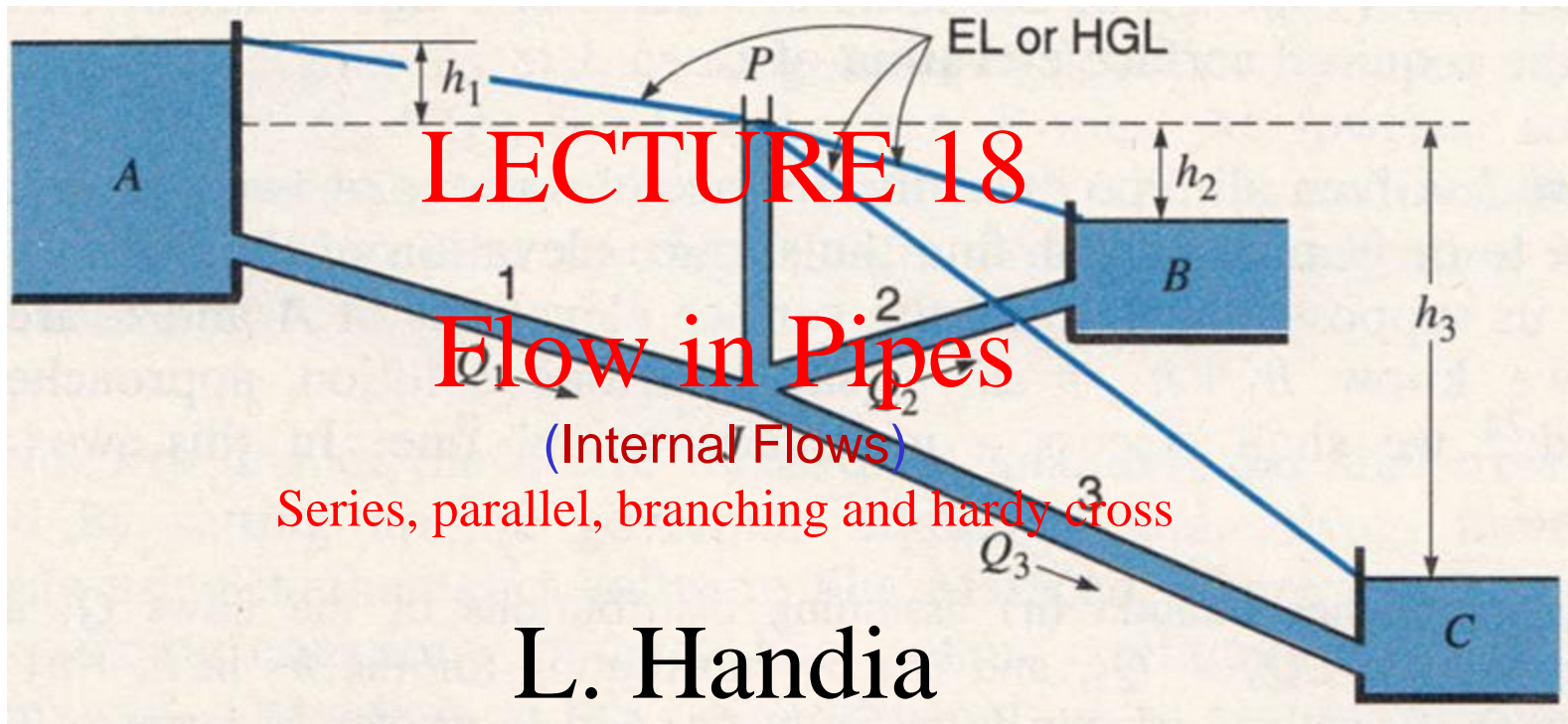
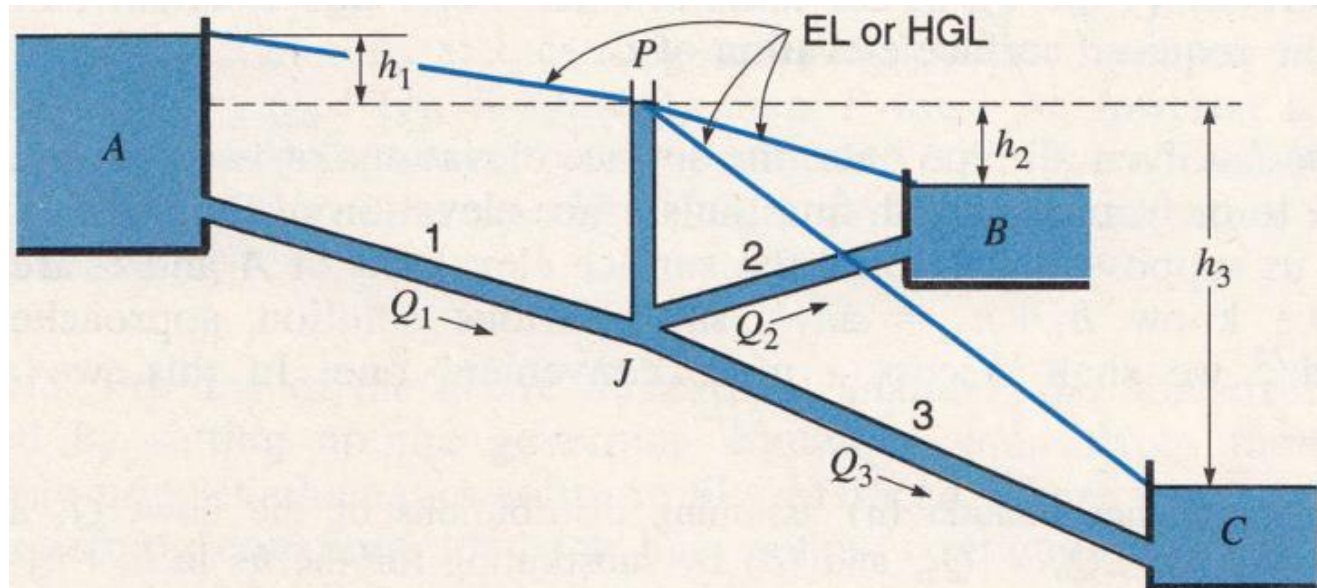


# Fluid Mechanics CEE 3311



# Branching Pipes

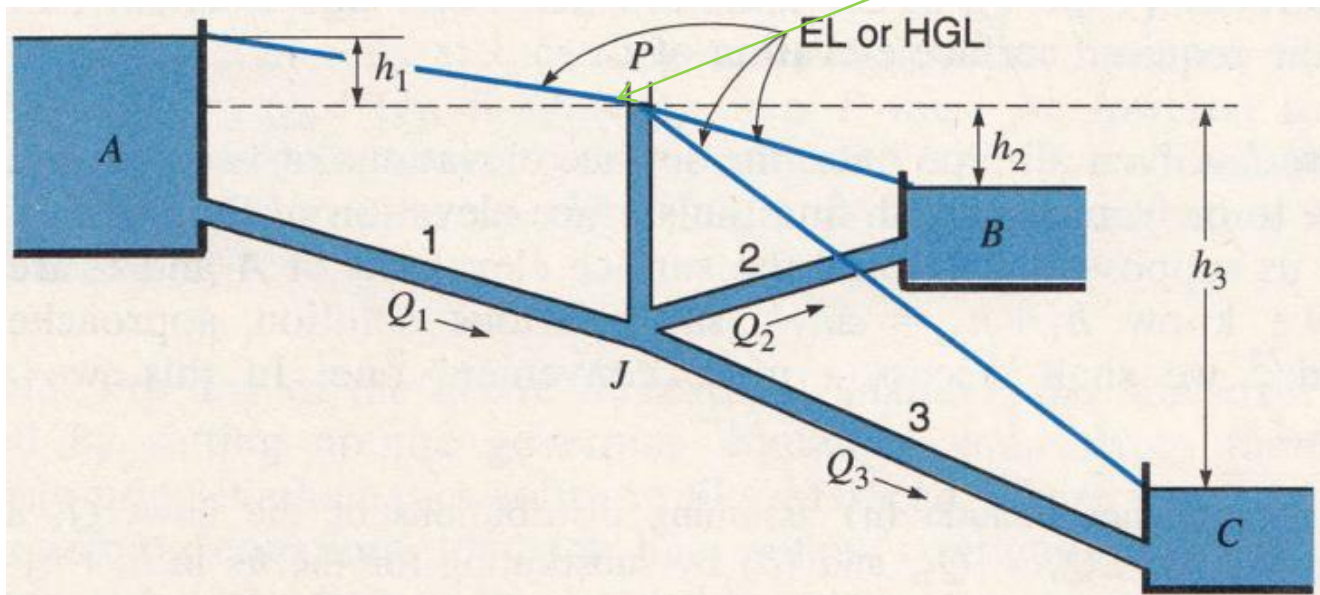
- For convenience, let us consider three pipes connected to three reservoirs as in Fig. 8.27 and connected together or branching at the common junction point J.
- Actually, any of the pipes may be considered to be connected to *some other destination* than a reservoir by simply replacing the reservoir with a *piezometer tube* in which the water level is the same as the reservoir surface.
- We shall suppose that all the pipes are *sufficiently long* (large head loss in the pipes) that *minor losses and velocity heads may be neglected*.



**Figure 8.27**  
Branching pipes.

# Branching Pipes

- We name the pipes and flows and corresponding friction losses as shown in the diagram.
- The *continuity and energy equations* require that the flow *entering* the junction (J) *equal* the flow *leaving* it and that the *pressure head at J* (which may be represented schematically by the open piezometer tube shown, with water at elevation P) *be common to all pipes*.

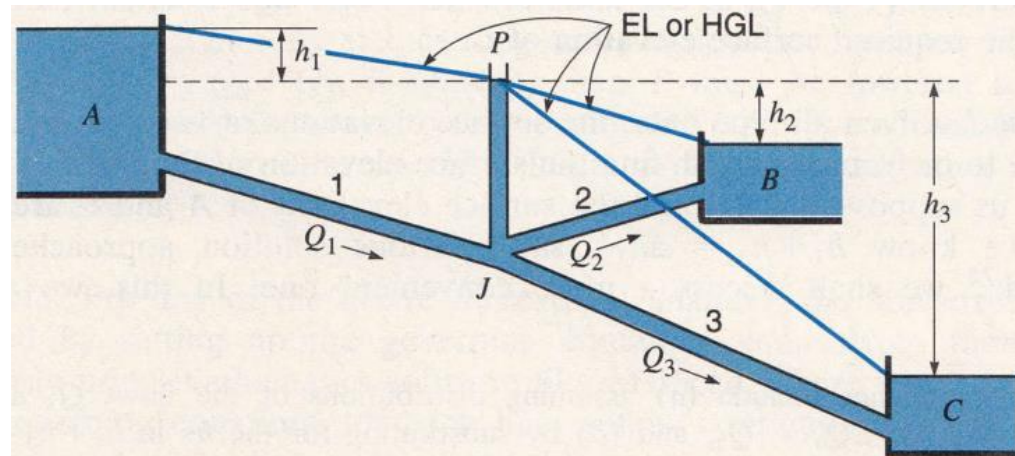


**Figure 8.27**  
Branching pipes.

# Branching Pipes

- There being no pumps, the elevation of P must lie between the surfaces of reservoirs A and C.
- If P is level with the surface of reservoir B then  $h_2$  and  $Q_2$  are both zero. If P is above the surface of reservoir B then water must flow into B and  $Q_1 = Q_2 + Q_3$ .
- If P is below the surface of reservoir B then the flow must be out of B and  $Q_1 + Q_2 = Q_3$ . So for the situation shown in Fig. 8.27 we have the following governing conditions:

1.  $Q_1 = Q_2 + Q_3$
2. Elevation of P is common to all.



**Figure 8.27**  
Branching pipes.

- The diagram suggests several different problems or cases, three of which will be discussed below using different methods of solution.

# Rigorous Solutions

- When we know the pipe wall material, we can estimate its  $e$  value (from Tables) and we know that the friction factor  $f$  varies with the  $e/D$  of the pipe and the Reynolds number of the flow.
- Because we are not considering minor losses, we can use the equations in the previous lecture.
- In particular, using only a basic scientific calculator, we can solve pipelines for  $h_L$  (Type 1 problems), for  $V$  or  $Q$  (Type 2 problems) and more rarely, we can solve for  $D$  (Type 3 problems) using a number of equations.
- These equations are preferred because they avoid trial and error, which can become quite confusing when combined with other trial-and-error techniques needed to solve for branching flow.
- These, and the variety of approaches used to “manually” solve the different types of problems that can occur, are illustrated in the following cases.

# Rigorous Solutions

**Case 1.** Given all pipe data (lengths, diameters, and materials for  $e$  values), the surface elevations of two reservoirs, and the flow to or from one of these two, **find the surface elevation of the third reservoir.**

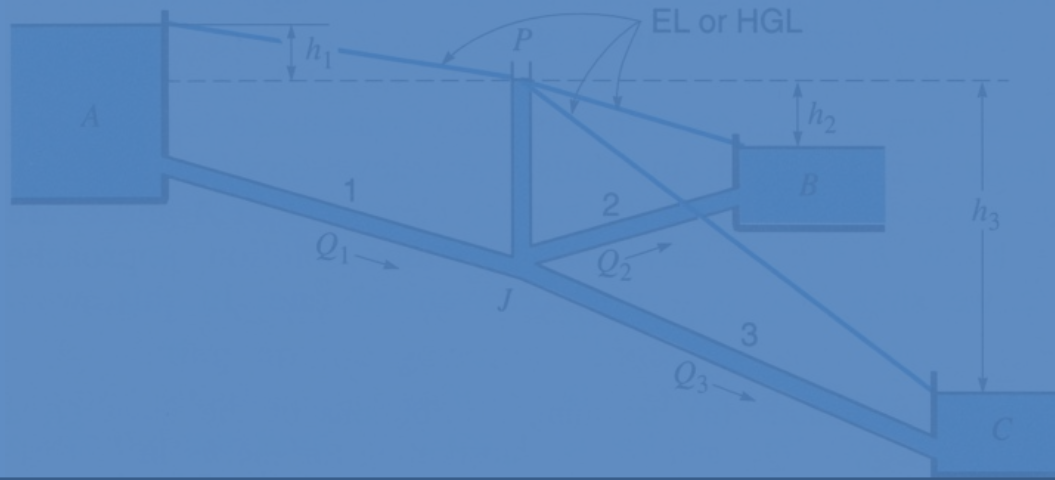
- This problem can be solved directly. Suppose that  $Q_1$  and the elevations of A and B are given.
 

Turbulent flow, all pipes:

$$\frac{1}{\sqrt{f}} = -1.8 \log \left[ \left( \frac{e/D}{3.7} \right)^{1.11} + \frac{6.9}{R} \right]$$
- Then the head loss  $h_L$  may be determined directly (Type 1), using the moody diagram or Eq. (8.41) to find the proper value of  $f$ .
 

Turbulent flow, Type 2:

$$\frac{4Q}{\pi D^2} = -2 \sqrt{\frac{2gDh_L}{L}} \log \left( \frac{e/D}{3.7} + \frac{2.51\nu}{D} \sqrt{\frac{L}{2gDh_L}} \right) \quad (8.42b)$$
- Knowing  $h_L$  fixes  $P$ , so  $h_3$  is now easily obtained. Knowing  $h_2$  enables the flow in pipe 2 to be determined directly using the Type 2 equation (8.42).
- Condition 1 (continuity at junction J) then determines  $Q_3$ , which in turn enables  $h_3$  to be found directly (Type 1), in the same manner as for line 1.
- Finally,  $P$  and  $h_3$  define the required surface elevation of C.



# Rigorous Solutions

**Case 2.** Given all pipe data, the surface elevations of two reservoirs, and the flow to or from the third, *find the surface elevation of the third reservoir.*

Let us suppose that  $Q_3$  and the surface elevations of A and C are given. Then we know  $h_1 + h_3 = \Delta h_{13}$ , say. Various solution approaches may be used; we shall discuss a more convenient one. In this, we *assume the elevation of P*, which yields values for  $h_1$  and  $h_3$ , and so  $Q_1$  and  $Q_3$  via Eq. (8.42). If these do not satisfy the discharge relation at J then P must be adjusted until they do. To help us converge on the correct elevation of P, we can plot the results of each assumption on a graph like Fig. 8.28. For  $\sum Q$  at J, inflows to J are taken as positive and outflows as negative. Two or three points, with one fairly close to the vertical axis, determine a curve that *intersects that axis at the equilibrium level of P, where  $\sum Q = 0$ , as required.* Last,  $h_2$  can be determined from  $Q_2$  and Eqs. (8.41) and (8.10), and the required surface elevation of B found.

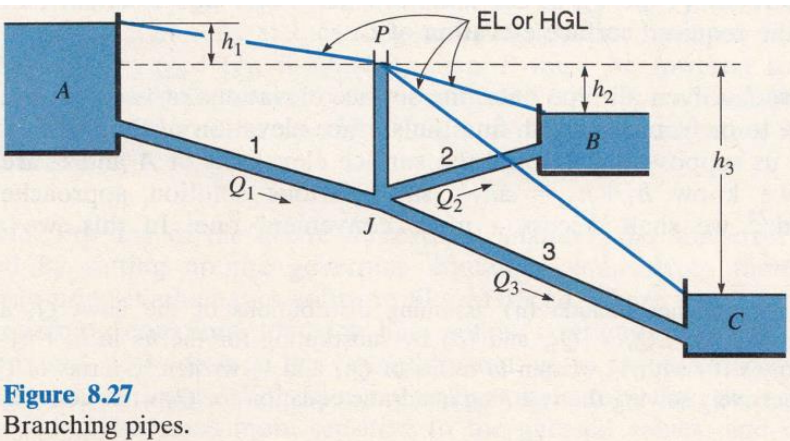


Figure 8.27  
Branching pipes.

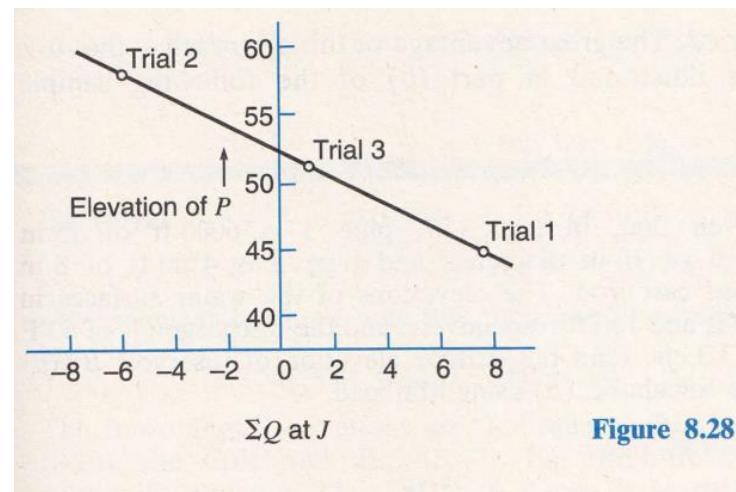


Figure 8.28

# Rigorous Solutions

**Case 3.** Given all pipe lengths and diameters and the elevations of all three reservoirs, *find the flow in each pipe.*

This is the classic three-reservoir problem, and it differs from the foregoing case that it is not immediately evident whether the flow is into or out of reservoir B. The direction is readily determined by first assuming no flow in pipe 2; that is, the piezometer level  $P$  is assumed at the elevation of the surface of B. The head losses  $h_1$  and  $h_3$  then determine the flows  $Q_1$  and  $Q_3$  via Eq. (8.42). If  $Q_1 > Q_3$  then  $P$  must be raised to satisfy continuity at  $J$ , causing water to flow into reservoir B, and we shall have  $Q_1 = Q_2 + Q_3$ ; if  $Q_1 < Q_3$  then  $P$  must be lowered to satisfy continuity at  $J$ , causing water to flow out of reservoir B, and we shall have  $Q_1 + Q_2 = Q_3$ . From here on the solution proceeds by adjusting  $P$  as for Case 2 above.

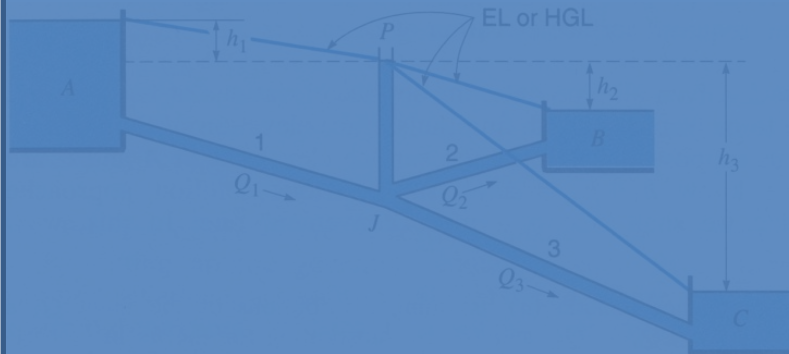
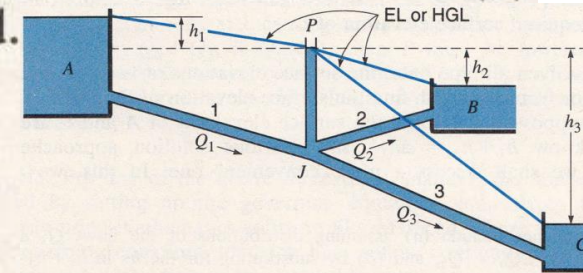


Figure 8.27  
Branching pipes.  
Branching pipes.

**SAMPLE PROBLEM 8.12** Given that, in Fig. 8.27, pipe 1 is 6000 ft of 15 in diameter, pipe 2 is 1500 ft of 10 in diameter, and pipe 3 is 4500 ft of 8 in diameter, all asphalt-dipped cast iron. The elevations of the water surfaces in reservoirs *A* and *C* are 250 ft and 160 ft respectively, and the discharge  $Q_2$  of 60°F water into reservoir *B* is 3.3 cfs. Find the surface elevation of reservoir *B*: (a) using only a basic scientific calculator; (b) using Mathcad.



**Solution**

This is a Case 2 problem.

Table A.1 for water at 60°F:  $\nu = 1.217 \times 10^{-5} \text{ ft}^2/\text{sec}$ .

Figure 8.27  
Branching pipes.

Line:	1	2	3
$L$ , ft	6000	1500	4500
$D$ , ft	1.25	10/12	8/12
$e$ , ft (Table 8.1)	0.0004	0.0004	0.0004

(a) Find the elevation of *P* by trial and error.

$L/D$	4800	1800	6750
$A$ , ft <sup>2</sup>	1.227	0.545	0.349
$e/D$	0.000 32	0.000 48	0.0006

Elevation of P lies between 160 and 250 ft. Calculate  $V$  from Eq. (8.42). Trials:

El. $P$	$h_1$	$h_3$	$V_1$	$V_3$	$Q_1$	$Q_3$	$\Sigma Q$	Move $P$ ?
200	50	40	6.444	4.481	7.907	1.564	+3.04	Up
230	20	70	4.013	5.984	4.925	2.088	-0.463	Down

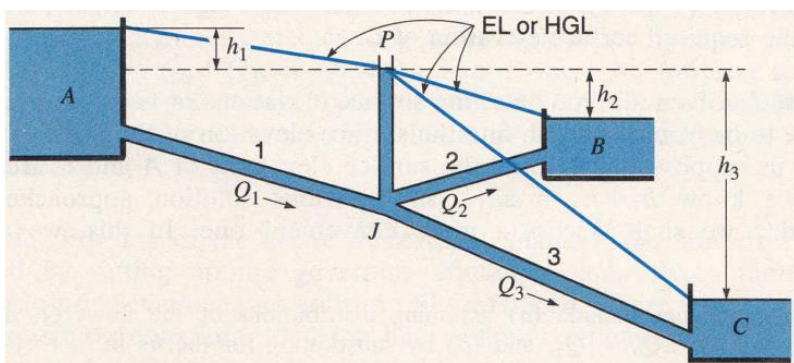
$Q_2=3.3$

$$7.907 - 3.3 - 1.564$$

**SAMPLE PROBLEM 8.12** Given that, in Fig. 8.27, pipe 1 is 6000 ft of 15 in diameter, pipe 2 is 1500 ft of 10 in diameter, and pipe 3 is 4500 ft of 8 in diameter, all asphalt-dipped cast iron. The elevations of the water surfaces in reservoirs A and C are 250 ft and 160 ft respectively, and the discharge  $Q_2$  of 60°F water into reservoir B is 3.3 cfs. Find the surface elevation of reservoir B: (a)

Turbulent flow, Type 2:

$$V = -2 \sqrt{\frac{2gDh_L}{L}} \log \left( \frac{e/D}{3.7} + \frac{2.51\nu}{D} \sqrt{\frac{L}{2gDh_L}} \right) \quad (8.42a)$$



**Figure 8.27**  
Branching pipes.

Elevation of P lies between 160 and 250 ft. Calculate  $V$  from Eq. (8.42). Trials:

$$Q_2 = 3.3$$

El. $P$	$h_1$	$h_3$	$V_1$	$V_3$	$Q_1$	$Q_3$	$\Sigma Q$	Move $P$ ?
200	50	40	6.444	4.481	7.907	1.564	+3.04	Up
230	20	70	4.013	5.984	4.925	2.088	-0.463	Down

Interpolation (Fig. 8.28):  $(230 - \text{El. } P)/(230 - 200) = 0.463/(0.463 + 3.04)$ ;  
 El.  $P = 226.03$ .

$$0.463 + 0$$

226	24	66	4.412	5.805	5.414	2.026	+0.088	Up
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Interpolation (Fig. 8.28):  $(230 - \text{El. } P)/(230 - 226) = 0.463/(0.463 + 0.088)$ ;  
 El.  $P = 226.64$ .

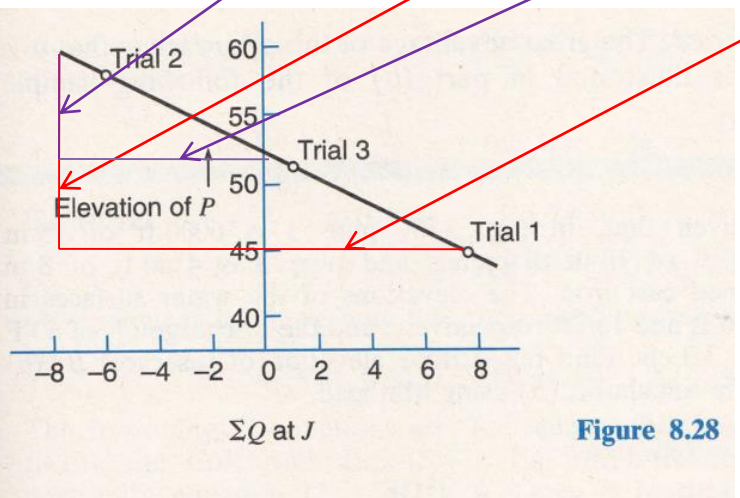


Figure 8.28

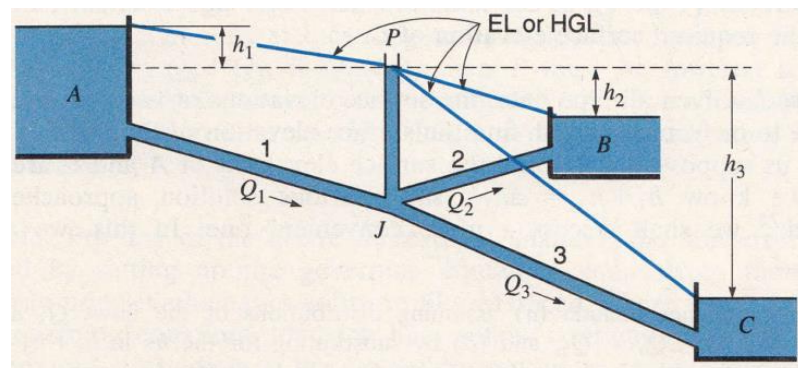


Figure 8.27  
 Branching pipes.

Close enough! *Note:* these adjustments are very suitable for making on a spreadsheet.

$$V_2 = \frac{Q_2}{A_2} = \frac{3.3}{0.545} = 6.055 \text{ fps}; \quad \mathbf{R}_2 = \frac{D_2 V_2}{\nu} = 416,500$$

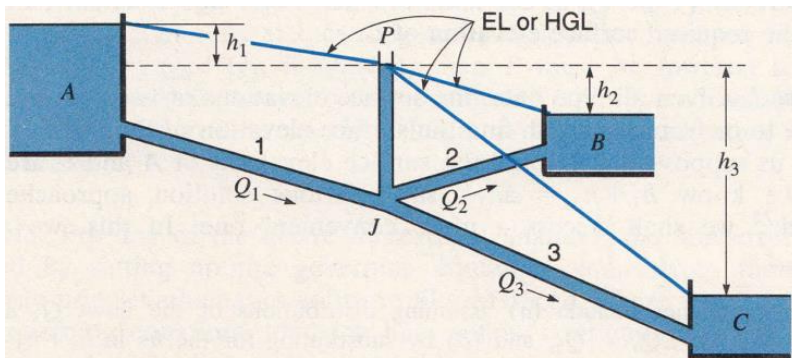
Eq. (8.41):  $f_2 = 0.01761$ ; Eq. (8.10):  $h_2 = 18.05 \text{ ft}$

El.  $B = \text{El. } P - h_2 = 226.64 - 18.05 = 208.59 \text{ ft} \quad \mathbf{ANS}$

$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

Turbulent flow,  
all pipes:

$$\frac{1}{\sqrt{f}} = -1.8 \log \left[ \left( \frac{e/D}{3.7} \right)^{1.11} + \frac{6.9}{\mathbf{R}} \right] \quad (8.41)$$



**Figure 8.27**  
Branching pipes.

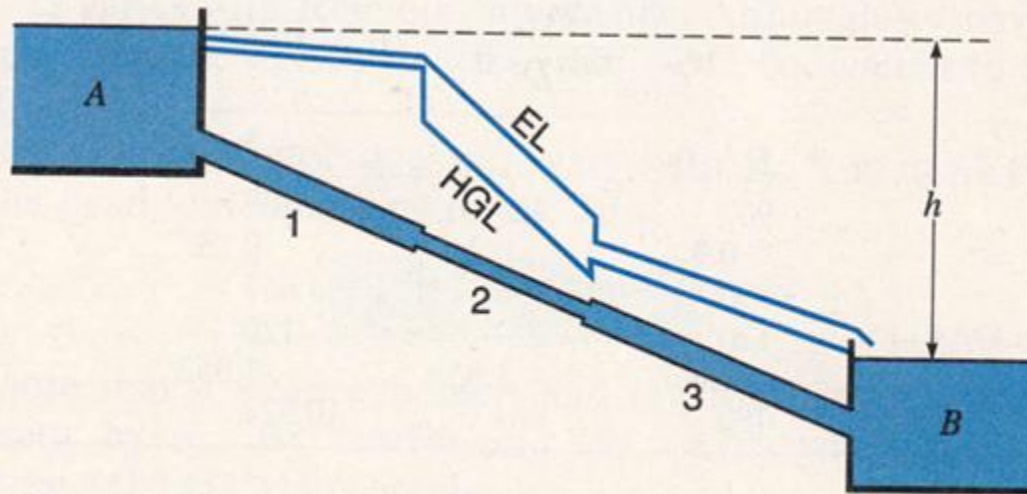
**SAMPLE PROBLEM 8.12** Given that, in Fig. 8.27, pipe 1 is 6000 ft of 15 in diameter, pipe 2 is 1500 ft of 10 in diameter, and pipe 3 is 4500 ft of 8 in diameter, all asphalt-dipped cast iron. The elevations of the water surfaces in reservoirs A and C are 250 ft and 160 ft respectively, and the discharge  $Q_2$  of 60°F water into reservoir B is 3.3 cfs. Find the surface elevation of reservoir B: (a)

# Pipes in series

If the pipe is made up of sections of different diameters, as shown in Fig. 8.29, the continuity and energy equations establish the following two simple relations that must be satisfied:

$$Q = Q_1 = Q_2 = Q_3 = \dots \quad (8.68)$$

$$h_L = h_{L1} + h_{L2} + h_{L3} + \dots \quad (8.69)$$



**Figure 8.29**  
Pipes in series.

If we wish to use the more accurate Darcy–Weisbach approach to find  $Q$ , we must note that in Eq. (8.70) each  $K$  has now become a function of a different  $f$ . The preferred manual method of solution is similar to the above, and is known as the *equivalent-velocity-head method*. Substituting from Eq. (8.10) into Eq. (8.69) and including minor losses if needed (usually if  $L/D < 1000$ ),

$$h_L = f \frac{L v^2}{D 2g} \quad h_L = h_{L1} + h_{L2} + h_{L3} \quad h_L = K \frac{v^2}{2g}$$

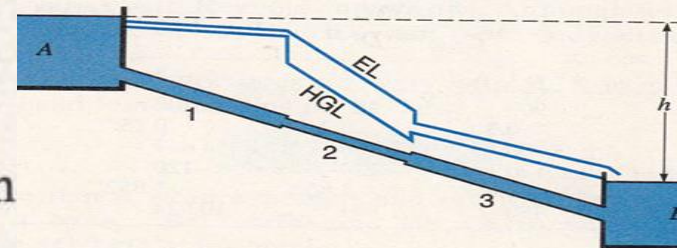
$$h_L = \left( f_1 \frac{L_1}{D_1} + \sum k_1 \right) \frac{V_1^2}{2g} + \left( f_2 \frac{L_2}{D_2} + \sum k_2 \right) \frac{V_2^2}{2g} + \dots$$

Using the equation of continuity, we know  $D_1^2 V_1 = D_2^2 V_2 = D_3^2 V_3$ , etc., from which all the velocities may be expressed in terms of one chosen velocity. So, by assuming reasonable values for each  $f$  (e.g., from Eq. (8.39) or Fig. 8.11), for any pipeline, however complex, the total head loss may be written as

$$h_L = K \frac{V^2}{2g} \quad (8.71)$$

where  $V$  is the chosen velocity. This equation may be solved for the chosen  $V$ , and so the  $V$  and  $\mathbf{R}$  and  $f$  values obtained for each pipe. For better accuracy, the assumed values of  $f$  should be replaced by the values just obtained, and an improved solution obtained. When the  $f$  values have converged  $V$  is correct, and  $Q$  may be calculated.

**SAMPLE PROBLEM 8.15** Suppose in Fig. 8.29 the pipes 1, 2, and 3 are 300 m of 30 cm diameter, 150 m of 20 cm diameter, and 250 m of 25 cm diameter, respectively, of new cast iron and are conveying 15°C water. If  $h = 10$  m, find the rate of flow from  $A$  to  $B$ .



**Solution**

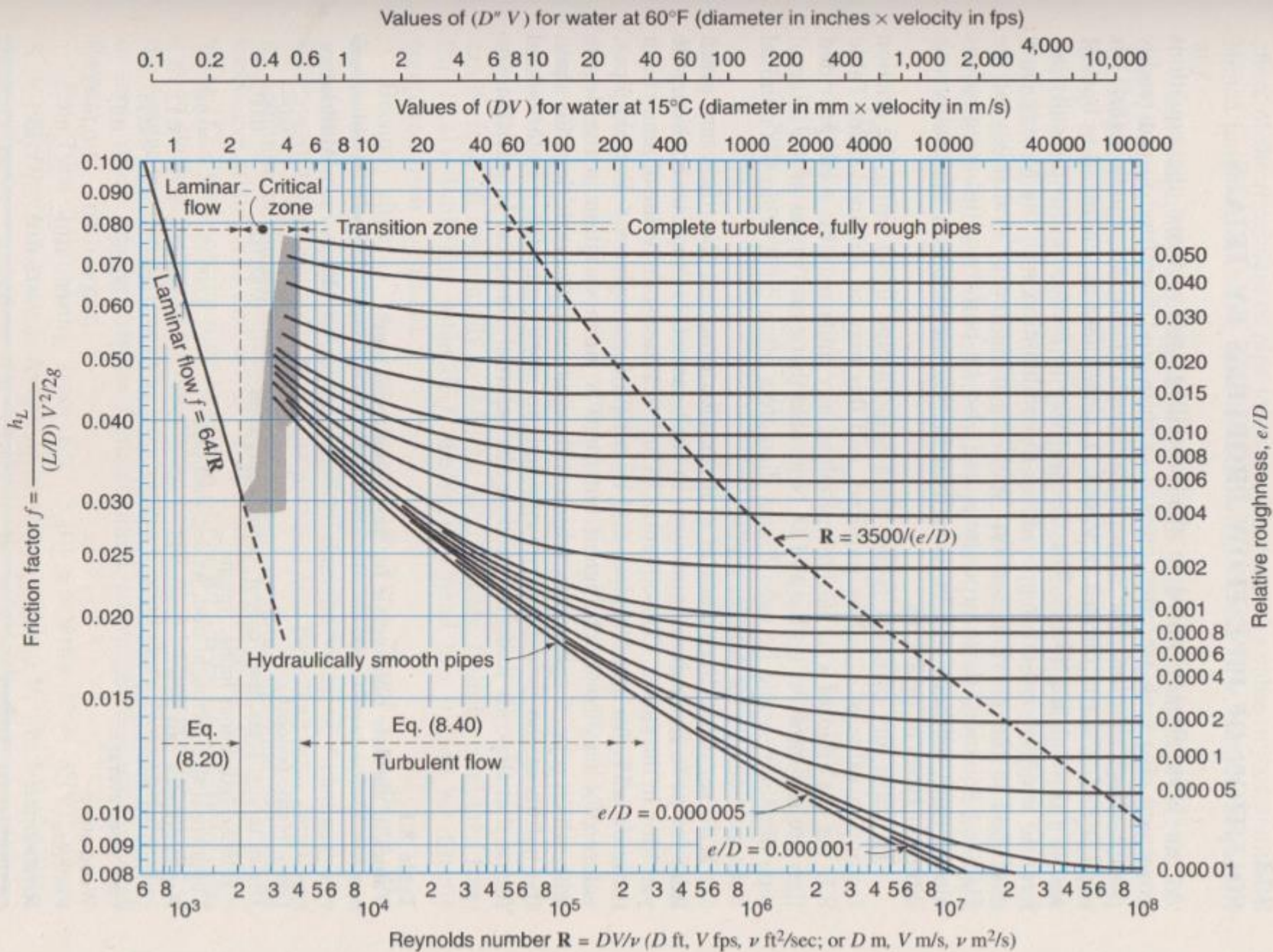
Table 8.1 for cast iron pipe:  $e = 0.25$  mm = 0.000 25 m

Pipe:	1	2	3
$L$ , m	300	150	250
$D$ , m	0.3	0.2	0.25
$e/D$	0.000 833	0.001 25	0.001 00
$f_{\min}$ (Fig. 8.11)	0.019	0.021	0.020

Assuming these friction factor values, 
$$h_L = \left( f_1 \frac{L_1}{D_1} + \sum k_1 \right) \frac{V_1^2}{2g} + \left( f_2 \frac{L_2}{D_2} + \sum k_2 \right) \frac{V_2^2}{2g} + \dots$$

$$h = 10 = 0.019 \left( \frac{300}{0.3} \right) \frac{V_1^2}{2g} + 0.021 \left( \frac{150}{0.2} \right) \frac{V_2^2}{2g} + 0.020 \left( \frac{250}{0.25} \right) \frac{V_3^2}{2g}$$

Figure 8.11  
 Moody chart for pipe friction factor (Stanton diagram).



$$Q_1 = Q_2$$

$$V_1 D_1^2 = V_2 D_2^2$$

$$V_2 = \frac{V_1 D_1^2}{D_2^2}$$

$$V_2^2 = V_1^2 \left( \frac{D_1}{D_2} \right)^4$$

From continuity, 
$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} \left( \frac{D_1}{D_2} \right)^4 = \frac{V_1^2}{2g} \left( \frac{30}{20} \right)^4 = 5.06 \frac{V_1^2}{2g}$$

Similarly,

$$\frac{V_3^2}{2g} = 2.07 \frac{V_1^2}{2g}$$

$$10 = 0.019 \left( \frac{300}{0.3} \right) \frac{V_1^2}{2g} + 0.021 \left( \frac{150}{0.2} \right) \frac{V_2^2}{2g} + 0.020 \left( \frac{250}{0.25} \right) \frac{V_3^2}{2g}$$

and thus 
$$10 = \frac{V_1^2}{2g} \left( 0.019 \frac{1000}{1} + 0.021 \frac{750}{1} 5.06 + 0.020 \frac{1000}{1} 2.07 \right)$$

from which

$$\frac{V_1^2}{2g} = 0.071 \text{ m}$$

Hence

$$V_1 = \sqrt{2(9.81 \text{ m/s}^2)(0.072 \text{ m})} = 1.18 \text{ m/s}$$

The corresponding values of  $\mathbf{R}$  are  $0.31 \times 10^6$ ,  $0.47 \times 10^6$ , and  $0.37 \times 10^6$ ; the corresponding friction factors are only slightly different from those originally

assumed, since the flow is at Reynolds numbers very close to those at which the pipes behave as rough pipes. Hence

$$Q = A_1 V_1 = \frac{1}{4} \pi (0.30)^2 1.18 = 0.083 \text{ m}^3/\text{s} \quad \mathbf{ANS}$$

Greater accuracy would have been obtained if the friction factors had been adjusted to match the pipe-friction chart more closely or were calculated by Eq. (8.41), and if minor losses had been included. In that case  $Q = 0.081 \text{ m}^3/\text{s}$ .

# Pipes in parallel

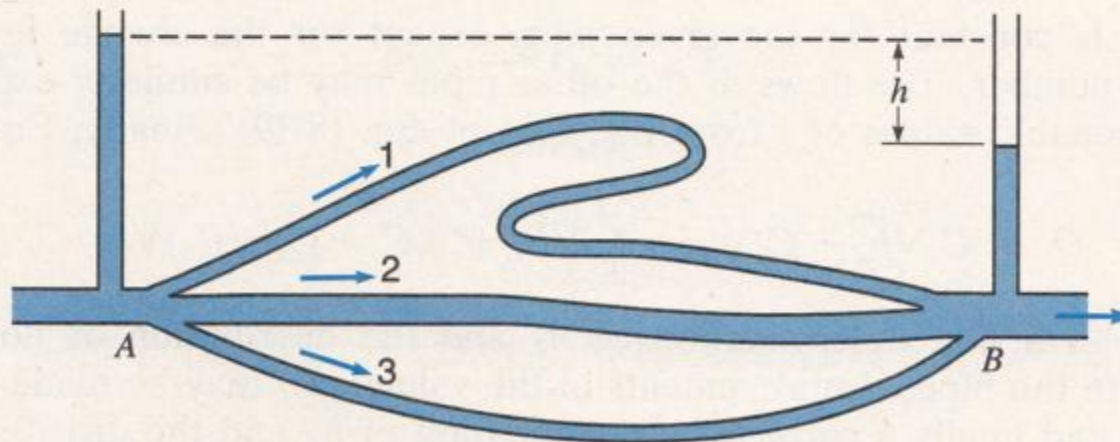
In the case of flow through two or more parallel pipes, as in Fig. 8.30, the continuity and energy equations establish the following relations that must be satisfied:

$$Q = Q_1 + Q_2 + Q_3 \quad (8.72)$$

$$h_L = h_{L1} = h_{L2} = h_{L3} \quad (8.73)$$

as the pressures at  $A$  and  $B$  are common to all pipes. Problems may be posed in various ways.

If the head loss  $h_L$  is given, the problem is straightforward. The head loss may be found directly by adding the contributions from the various pipes, as in Eq. (8.72). If empirical coefficients or constant  $f$  values are given, we can



**Figure 8.30**  
Pipes in parallel.

# Pipes in parallel

## Solutions

1. If the head loss is given, the problem is straight forward. The head loss may be found directly by adding the contributions from the various pipes, as in Eq. 8.72.
2. If empirical coefficients or constant  $f$  values are given, we can do this using Eq. 8.67 and the appropriate values of  $K$  and  $n$ .

$$h_L = KQ^n \Rightarrow Q = \left( \frac{h_L}{K} \right)^{1/n}$$

3. If, however, the pipe material or  $e$  is given, this is preferred because the Darcy-Weisbach approach is more accurate. Then we have an independent Type 2 problem for each pipe, see Lecture 17, which can be solved directly by Eq. 8.42, for example.

Turbulent flow, Type 2:

$$\frac{4Q}{\pi D^2} = -2 \sqrt{\frac{2gDh_L}{L}} \log \left( \frac{e/D}{3.7} + \frac{2.51\nu}{D} \sqrt{\frac{L}{2gDh_L}} \right) \quad (8.42b)$$

4. If the total flow  $Q$  is given and the head loss and individual flows are required, the problem is a little more involved. Using the empirical equations, we again substitute Eq. (8.67) into Eq. (8.72), to get

$$h_L = KQ^n \Rightarrow Q = \left( \frac{h_L}{K} \right)^{1/n}$$

$$Q = \left( \frac{h_{L1}}{K_1} \right)^{1/n} + \left( \frac{h_{L2}}{K_2} \right)^{1/n} + \left( \frac{h_{L3}}{K_3} \right)^{1/n} + \dots$$

$$Q = Q_1 + Q_2 + Q_3 + \dots$$

# Pipes in parallel

## Solutions

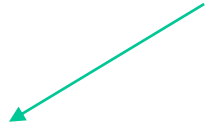
$$Q = \left(\frac{h_{L1}}{K_1}\right)^{1/n} + \left(\frac{h_{L2}}{K_2}\right)^{1/n} + \left(\frac{h_{L3}}{K_3}\right)^{1/n} + \dots$$

But since all the  $h_L$ s are equal from Eq. (8.73), this becomes

$$Q = (h_L)^{1/n} \left[ \left(\frac{1}{K_1}\right)^{1/n} + \left(\frac{1}{K_2}\right)^{1/n} + \left(\frac{1}{K_3}\right)^{1/n} + \dots \right] = (h_L)^{1/n} \sum \left(\frac{1}{K}\right)^{1/n} \quad (8.74)$$

So, knowing the pipe information and the empirical equation to be used, we can solve for  $h_L$ . We can then find the individual flows using Eq. (8.67).

$$h_L = KQ^n$$

$$Q = \left(\frac{h_L}{K}\right)^{1/n}$$


## Solutions

5. If we wish to use the more accurate Darcy-Weisbach approach to find  $h_L$

and the individual  $Q$ s, we must note that in Eq. (8.74) now each  $K$  has become a function of a different  $f$ . The preferred manual method of solution is similar to the above. Writing Eq. (8.10) for each line, including minor losses if needed,

$$h_L = \left( f \frac{L}{D} + \sum k \right) \frac{V^2}{2g}$$

$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

where  $\sum k$  is the sum of the minor-loss coefficients, which may usually be neglected if the pipe is longer than 1000 diameters. Solving for  $V$  and then  $Q$ , the following is obtained for pipe 1:

$$Q_1 = A_1 V_1 = A_1 \sqrt{\frac{2gh_L}{f_1(L_1/D_1) + \sum k_1}} = C_1 \sqrt{h_L} \quad (8.75)$$

where  $C_1$  is constant for the given pipe, except for the change in  $f$  with Reynolds number. The flows in the other pipes may be similarly expressed, using reasonable values of  $f$  from Fig. 8.11 or Eq. (8.39). Finally, Eq. (8.72) becomes

$$Q = C_1 \sqrt{h_L} + C_2 \sqrt{h_L} + C_3 \sqrt{h_L} = (C_1 + C_2 + C_3) \sqrt{h_L} \quad (8.76)$$

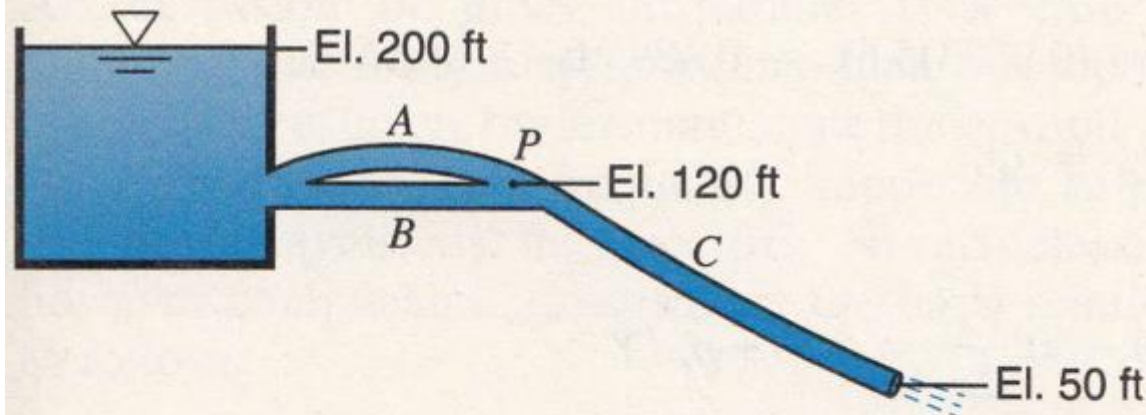
Since  $h_L$  is the same for all pipes

This enables a first determination of  $h_L$  and the distribution of flows and velocities in the pipes. Improvements in the values of  $f$  may be made next, if indicated, and finally a corrected determination of  $h_L$  and the distribution of flows.

If the turbulent flow equation (8.40) or (8.41) is used to obtain  $f$ , it is important to remember to confirm that the Reynolds number is in the turbulent range. We can pre-check the likelihood of laminar flow occurring in any of the pipes by calculating an “average” flow velocity from the total flow divided by the total area of all the pipes, and using this to obtain an indicator **R** for each pipe.

**SAMPLE PROBLEM 8.16** Three pipes  $A$ ,  $B$ , and  $C$  are interconnected as shown. The pipe characteristics are given as follows:

Pipe	$D$ (in)	$L$ (ft)	$f$
$A$	6	2000	0.020
$B$	4	1600	0.032
$C$	8	4000	0.024



**Figure S8.16**

Find the rate at which water will flow in each pipe. Find also the pressure at point  $P$ . All pipe lengths are much greater than 1000 diameters, therefore minor losses may be neglected.

$$h_L = f \frac{L v^2}{D 2g}$$

$$\alpha_1 \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \alpha_2 \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_l$$

**Solution**

Energy Eq.:  $200 - 0.020 \frac{2000 V_A^2}{6/12 \cdot 2g} - 0.024 \frac{4000 V_C^2}{8/12 \cdot 2g} = 50 + \frac{V_C^2}{2g}$

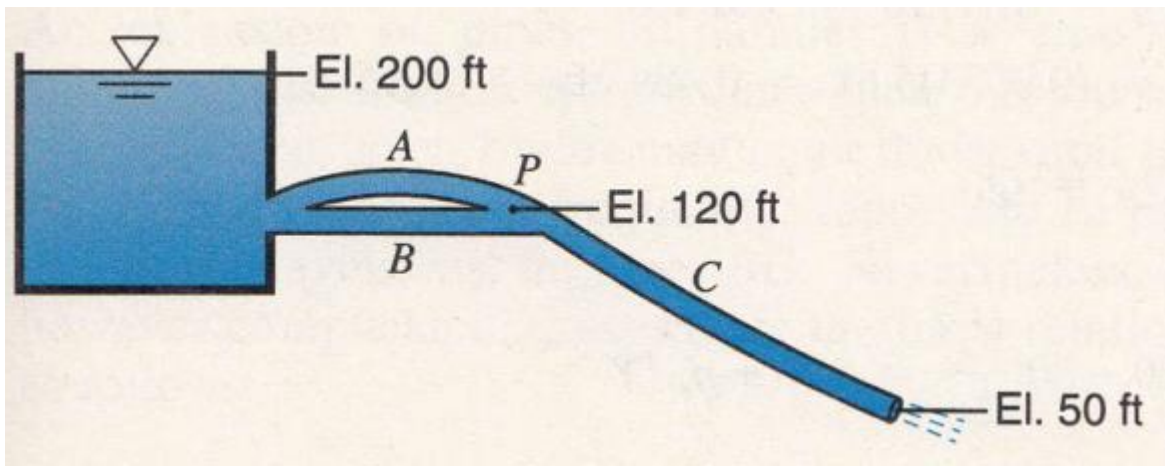
i.e.,  $150 = 80 \frac{V_A^2}{2g} + 145 \frac{V_C^2}{2g}$

Continuity:  $Q_A + Q_B = Q_C$   $6^2 V_A + 4^2 V_B = 8^2 V_C$

i.e.  $36V_A + 16V_B = 64V_C$

Also,  $h_{L_A} = h_{L_B} = 0.020 \frac{2000 V_A^2}{6/12 \cdot 2g} = 0.032 \frac{1600 V_B^2}{4/12 \cdot 2g}$

i.e.  $80V_A^2 = 153.6V_B^2, \quad V_B = 0.722V_A$



Substituting into the continuity equation,

$$36V_A + 16(0.722V_A) = 64V_C$$

$$47.5V_A = 64V_C, \quad V_A = 1.346V_C$$

Substituting into the energy equation,

$$150 = 80 \frac{(1.346V_C)^2}{2g} + 145 \frac{V_C^2}{2g} = 289.9 \frac{V_C^2}{2g}$$

$$V_C^2 = 2(32.2)150/289.9 = 33.3$$

$$V_C = 5.77 \text{ fps}, \quad Q_C = A_C V_C = (0.349)5.77 = 2.01 \text{ cfs} \quad \text{ANS}$$

$$V_A = 1.346V_C = 7.77 \text{ fps} \quad Q_A = (0.1963)7.77 = 1.526 \text{ cfs} \quad \text{ANS}$$

Continuity:

$$36(7.77) + 16V_B = 64(5.77)$$

$$279.7 + 16V_B = 369.4$$

$$V_B = 89.7/16 = 5.61 \text{ fps}$$

$$Q_B = A_B V_B = (0.0873)5.61 = 0.489 \text{ cfs} \quad \text{ANS}$$

As a check, note that  $Q_A + Q_B = Q_C$ .

To find the pressure at  $P$

$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

$$\alpha_1 \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \alpha_2 \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_l$$

$$200 - 80 \frac{V_A^2}{2g} = 120 + p_P/\gamma$$

$$p_P/\gamma = 80 - 80 \frac{(7.77)^2}{2g} = 5.01 \text{ ft}$$

Check:

$$120 + p_P/\gamma - 144 \frac{V_C^2}{2g} = 50 + \frac{V_C^2}{2g}$$

$$p_P/\gamma = 145 \frac{(5.77)^2}{2g} - 70 = 5.01 \text{ ft}$$

So  $p_P/\gamma = 5.01 \text{ ft}$  and  $p_P = (62.4/144)5.01 = 2.17 \text{ psi}$ .     **ANS**

In this example it was assumed that the values of  $f$  for each pipe were known. Actually  $f$  depends on  $\mathbf{R}$  [Fig. 8.11 or Eq. (8.41)]. Usually the absolute roughness  $e$  of each pipe is known or assumed, and an accurate solution is achieved through trial and error until the  $f$ s and  $\mathbf{R}$ s for each pipe have converged.

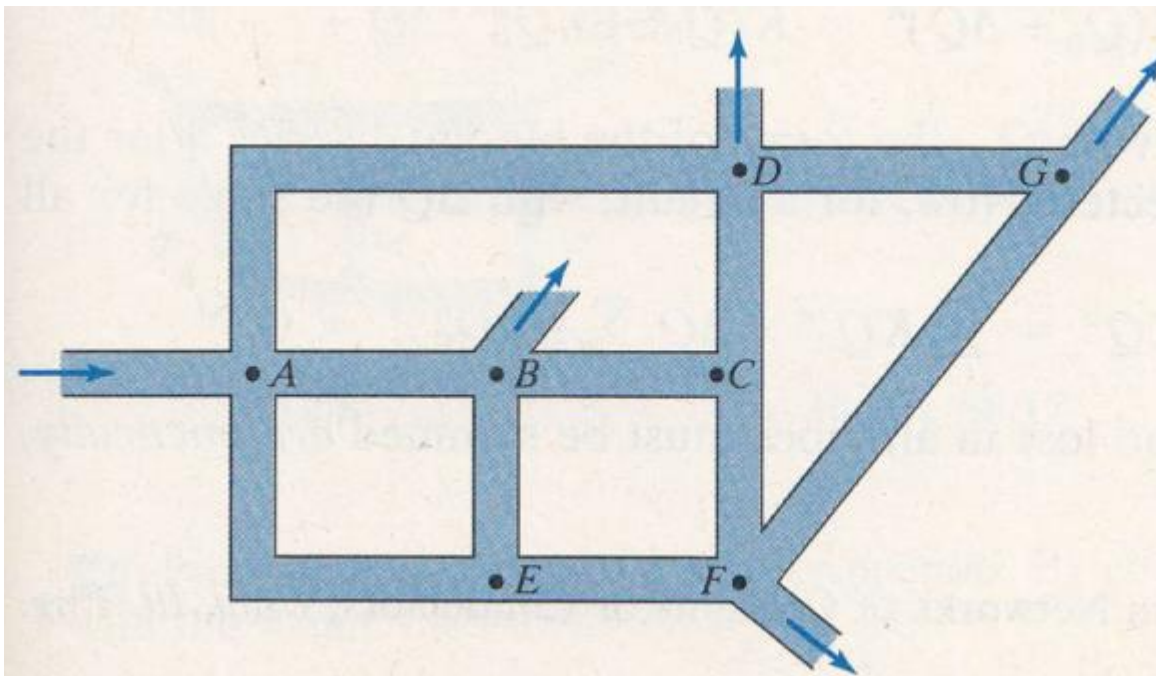


Manual iteration for  $f$  may be avoided by solving simultaneous equations using mathematics software like Mathcad. There are the usual four equations for each pipe, plus Eq. (8.72); if necessary, minor losses may easily be accounted for by using head loss equations with the form of Eq. (8.60). For the three pipes of Fig. 8.30, for example, there are therefore 13 simultaneous equations, which may be solved in the usual manner (see Sample Prob. 8.12*b*) for 13 unknowns. These unknowns are either the head loss or the total flow rate, and three values each of  $Q$ ,  $V$ ,  $\mathbf{R}$ , and  $f$ .

It is instructive to compare the solution methods for pipes in parallel with those for pipes in series. The role of the head loss in one case becomes that of the discharge rate in the other, and vice versa. The student should be already familiar with this situation from the elementary theory of dc circuits. The flow corresponds to the electrical current, the head loss to the voltage drop, and the frictional resistance to the ohmic resistance. The outstanding deficiency in this analogy occurs in the variation of potential drop with flow, which is with the first power in the electrical case ( $E = IR$ ) and with the second power in the hydraulic case ( $h_L \propto V^2 \propto Q^2$ ) for fully developed turbulent flow.

# Pipe networks: Hardy Cross method

An extension of pipes in parallel is a case frequently encountered in municipal distribution systems, in which the pipes are interconnected so that the flow to a given outlet may come by several different paths, as shown in Fig. 8.31. Indeed, it is frequently impossible to tell by inspection which way the flow travels, as in pipe *BE*. Nevertheless, the flow in any network, however complicated, must satisfy the basic relations of continuity and energy as follows:



**Figure 8.31**  
Pipe network.

1. The flow into any junction must equal the flow out of it.
2. The flow in each pipe must satisfy the pipe-friction laws for flow in a single pipe.
3. The algebraic sum of the head losses around any closed loop must be zero.

Pipe networks are generally too complicated to solve analytically, as was possible in the simpler cases of parallel pipes (Sec. 8.28). Furthermore, it is seldom possible to predict the capacity requirements of water distribution systems with high precision, and flows in them vary considerably throughout the day, so high accuracy in calculating their flows is not important. As a result, the use of empirical equations (Sec. 8.15) and constant values of  $f$  are very acceptable for this purpose. A practical procedure is the method of successive approximations, introduced by Cross.<sup>25</sup> It consists of the following elements, in order:

1. By careful inspection assume the most reasonable distribution of flows that satisfies condition 1.

1. The flow into any junction must equal the flow out of it.

2. Write condition 2 for each pipe in the form

2. The flow in each pipe must satisfy the pipe-friction laws for flow in a single pipe

$$h_L = KQ^n \quad (8.77)$$

where  $K$  and  $n$  are constants for each pipe as described in Sec. 8.26 under Approximate Solutions [Eq. (8.66), etc.]. If minor losses are important they may be included as in Eq. (8.75), which yields  $K = 1/C^2$  and  $n = 2$  for constant  $f$ . Minor losses may be included within any pipe or loop, but they are neglected at the junction points.

$$Q_1 = A_1 V_1 = A_1 \sqrt{\frac{2gh_L}{f_1(L_1/D_1) + \sum k_1}} = C_1 \sqrt{h_L}$$

$$Q = C \sqrt{h_L} \Rightarrow h_L = \frac{Q^2}{C^2} = KQ^n \therefore K = \frac{1}{C^2}, n = 2$$

3. To investigate condition 3, compute the algebraic sum of the head losses around each elementary loop,  $\sum h_L = \sum KQ^n$ . Consider losses from clockwise flows as positive, counterclockwise negative. Only by good luck will these add to zero on the first trial.

3. The algebraic sum of the head losses around any closed loop must be zero.

4. Adjust the flow in each loop by a correction  $\Delta Q$  to balance the head in that loop and give  $\sum KQ^n = 0$ . The heart of this method lies in the determination of  $\Delta Q$ . For any pipe, we may write

$$Q = Q_0 + \Delta Q$$

where  $Q$  is the correct discharge and  $Q_0$  is the assumed discharge. Then, for each pipe,

$$h_L = KQ^n = K(Q_0 + \Delta Q)^n = K(Q_0^n + nQ_0^{n-1}\Delta Q + \dots)$$

If  $\Delta Q$  is small compared with  $Q_0$ , the terms of the binomial series after the second one may be neglected. Now, for a circuit, with  $\Delta Q$  the same for all pipes,

$$\sum h_L = \sum KQ^n = \sum KQ_0^n + \Delta Q \sum KnQ_0^{n-1} = 0$$

As the corrections of head loss in all pipes must be summed *arithmetically*,

---

<sup>25</sup> H. Cross, Analysis of Flow in Networks of Conduits or Conductors, *Univ. Ill. Eng. Expt. Sta. Bull.* 286, 1936.

$$\sum h_L = \sum KQ^n = \sum KQ_0^n + \Delta Q \sum KnQ_0^{n-1} = 0$$

As the corrections of head loss in all pipes must be summed *arithmetically*, we may solve this equation for  $\Delta Q$

$$\Delta Q = \frac{-\sum KQ_0^n}{\sum |KnQ_0^{n-1}|} \quad (\text{by making } \Delta Q \text{ the subject of the formula in the above equation})$$

Since  $h_L = KQ^n \Rightarrow KQ^n = h_L$  ;and

$$h_L = KQ^n = KQ^{n+(1-1)} = KQ^{n-1}Q \Rightarrow KQ^{n-1} = \frac{h_L}{Q}$$

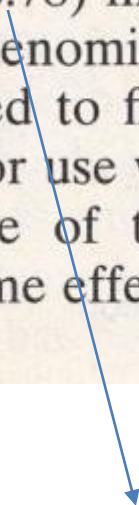
Therefore 
$$\Delta Q = \frac{-\sum KQ_0^n}{\sum |KnQ_0^{n-1}|} = \frac{-\sum h_L}{n \sum \left| \frac{h_L}{Q_0} \right|} \quad 8.78$$

It must be emphasized again that the numerator of Eq. (8.78) is to be summed algebraically, with due account of sign, while the denominator is summed arithmetically. The negative sign in Eq. (8.78) indicates that when there is an excess of head loss around a loop in the clockwise direction, the  $\Delta Q$  must be subtracted from clockwise  $Q_0$ s and added to counterclockwise ones. The reverse is true if there is a deficiency of head loss around a loop in the clockwise direction.

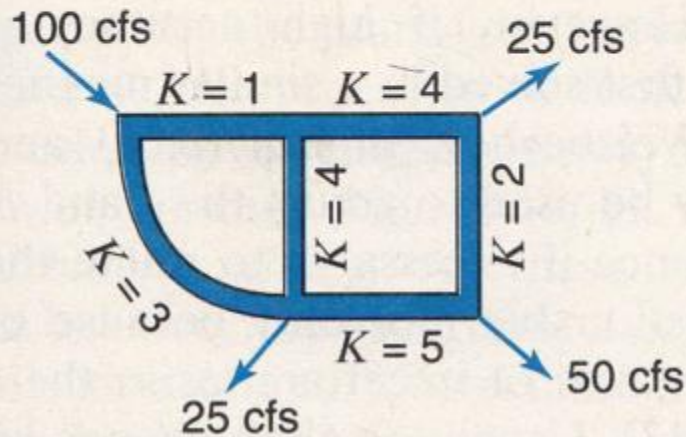
5. After each circuit is given a first correction, the losses will still not balance, because of the interaction of one circuit upon another (pipes which are common to two circuits receive two independent corrections, one for each circuit). The procedure is repeated, arriving at a second correction, and so on, until the corrections become negligible.

Either form of Eq. (8.78) may be used to find  $\Delta Q$ . As values of  $K$  appear in both numerator and denominator of the first form, values proportional to the actual  $K$  may be used to find the distribution. The second form will be found most convenient for use with pipe-friction diagrams for water pipes.

An attractive feature of the approximation method is that errors in computation have the same effect as errors in judgment and will eventually be corrected by the process.


$$\Delta Q = \frac{-\sum KQ_0^n}{\sum |KnQ_0^{n-1}|} = \frac{-\sum h_L}{n \sum \left| \frac{h_L}{Q_0} \right|}$$

**SAMPLE PROBLEM 8.17** If the flow into and out of a two-loop pipe system are as shown in Fig. S8.17, determine the flow in each pipe. The  $K$  values for each pipe were calculated from the pipe and minor loss characteristics and from an assumed value of  $f$ .



**Figure S8.17**

## Solution

As a first step, assume flow in each pipe such that continuity is satisfied at all junctions. Calculate  $\Delta Q$  for each loop, make corrections to the assumed  $Q$ s, and repeat several times until the  $\Delta Q$ s are quite small.

$$n=2$$

$$\Delta Q = \frac{-\sum KQ_0^n}{\sum |KnQ_0^{n-1}|} = \frac{-\sum h_L}{n \sum |h_L/Q_0|} \quad (8.78)$$

Left loop

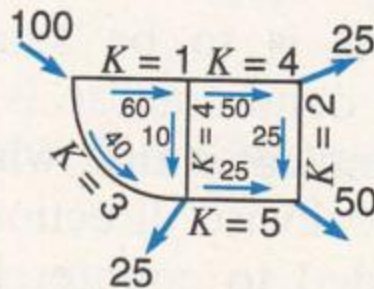
$KQ_0^n$	$ KnQ_0^{n-1} $
$1 \times 60^2 = 3,600$	$1 \times 2 \times 60 = 120$
$4 \times 10^2 = 400$	$4 \times 2 \times 10 = 80$

$3 \times 40^2 = 4,800$	$3 \times 2 \times 40 = 240$
$800$	$440$

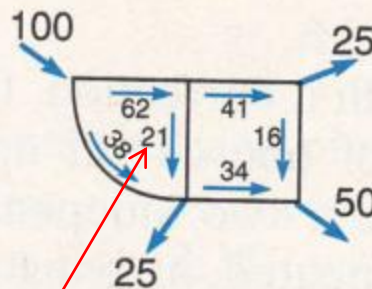
$$\Delta Q_1 = \frac{-800}{440} \approx 2$$

$1 \times 62^2 = 3,844$	$1 \times 2 \times 62 = 124$
$4 \times 21^2 = 1,764$	$4 \times 2 \times 21 = 168$

$$Q = Q_0 + \Delta Q = 10 + (2 + 9) = 21$$



First approximation



After first correction

Right loop

$KQ_0^n$	$ KnQ_0^{n-1} $
$4 \times 50^2 = 10,000$	$4 \times 2 \times 50 = 400$
$2 \times 25^2 = 1,250$	$2 \times 2 \times 25 = 100$

$4 \times 10^2 = 400$	$4 \times 2 \times 10 = 80$
$5 \times 25^2 = 3,125$	$5 \times 2 \times 25 = 250$
$7,725$	$830$

$$\Delta Q_1 = \frac{-7,725}{830} \approx 9$$

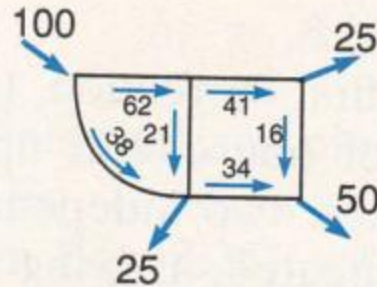
$4 \times 41^2 = 6,724$	$4 \times 2 \times 41 = 328$
$2 \times 16^2 = 512$	$2 \times 2 \times 16 = 64$

$$Q = 50 - 9 = 41$$

$$3 \times 40^2 = \frac{4,800}{800} \quad 3 \times 2 \times 40 = \frac{240}{440}$$

$$\Delta Q_1 = \frac{-800}{440} \approx 2$$

First approximation



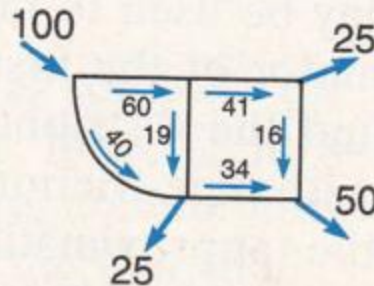
After first correction

$$1 \times 62^2 = 3,844 \quad 1 \times 2 \times 62 = 124$$

$$4 \times 21^2 = 1,764 \quad 4 \times 2 \times 21 = 168$$

$$3 \times 38^2 = \frac{4,332}{1,276} \quad 3 \times 2 \times 38 = \frac{228}{520}$$

$$\Delta Q_2 = \frac{-1,276}{520} \approx 2$$



After second correction

$$4 \times 10^2 = 400 \quad 4 \times 2 \times 10 = 80$$

$$5 \times 25^2 = \frac{3,125}{7,725} \quad 5 \times 2 \times 25 = \frac{250}{830}$$

$$\Delta Q_1 = \frac{-7,725}{830} \approx 9$$

$$4 \times 41^2 = 6,724 \quad 4 \times 2 \times 41 = 328$$

$$2 \times 16^2 = 512 \quad 2 \times 2 \times 16 = 64$$

$$4 \times 21^2 = 1,764 \quad 4 \times 2 \times 21 = 168$$

$$5 \times 34^2 = \frac{5,780}{308} \quad 5 \times 2 \times 34 = \frac{340}{900}$$

$$\Delta Q_2 = \frac{-308}{900} \approx 0$$

Further corrections can be made if greater accuracy is desired.



As noted earlier, varying demand rates usually make high accuracy unnecessary with pipe networks. However, if high accuracy is required for some reason, the problem can be first solved in a similar manner to the preceding example using the Darcy–Weisbach  $K$  in Eq. (8.66) and constant  $f$  values. Then the resulting flows may be used to adjust the  $f$  and  $K$  values, and the process repeated (more than once if necessary) to refine the answers. The value of such refinement is questionable, not only because of uncertainties in the demand flows, but also because of uncertainties in the  $e$  values (pipe roughness) to be used (see Sec. 8.13). Usually  $f$  values change by only a few percent when they are adjusted, but we can see in Fig. 8.11 that for smoother pipe it is possible for them to change by as much as a factor of five.

Simple networks can be solved without approximation and manual iteration by solving simultaneous equations using mathematics software like



Mathcad. For networks containing  $i$  pipes,  $5i$  equations are required if the Darcy–Weisbach equation with variable  $f$ , and  $2i$  equations are required if using the simplified equation (8.77) with constant friction factors. These required equations include (a) the usual (condition 2) flow equations for each pipe (four or one per pipe, depending on the equations used); (b) flow continuity equations (condition 1) at all but one of the  $j$  nodes (as these imply continuity at the last node); (c) equations for the sum of the head losses around  $i - j + 1$  loops (condition 3). The unknowns to be determined for each pipe are  $h_L$ ,  $Q$ ,  $V$ ,  $\mathbf{R}$ , and  $f$  using the Darcy–Weisbach equation, or only  $h_L$  and  $Q$  using Eq. (8.77).

The pipe-network problem lends itself well to solution by use of a digital computer. Programming takes time and care, but once set up, there is great flexibility and many hours of repetitive labor can be saved. Many software packages are now available to simulate water distribution networks; see Appendix B.