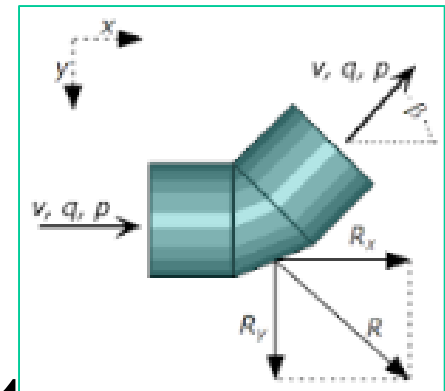


Fluid Mechanics CEE 3311

LECTURE 13



Conservation of momentum

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mv is an extensive property of the system.
Then the corresponding intensive property is given by v .

In applying the control volume equation

$$\frac{dN_{\text{sys}}}{dt} = \frac{d}{dt} \int_{\text{cv}} \eta \rho dV + \int_{\text{cs}} \eta \rho (\vec{v} \cdot d\vec{A}) \quad 13.1$$

$$\frac{d(\text{momentum})}{dt} = \frac{d}{dt} \int_{\text{cv}} v \rho dV + \int_{\text{cs}} v \rho (\vec{v} \cdot d\vec{A}) \quad 13.2$$

According to Newton's second law of motion, the summation of all external forces on the system is equal to the rate of change of momentum of that system

$$\sum F = \frac{d(mv)}{dt} \quad \begin{array}{l} \text{same as} \\ d(\text{momentum})/dt \\ \text{in eq. 13.2} \end{array}$$

$$\sum F = \frac{d(mv)}{dt} = \frac{d}{dt} \int_{cv} v \rho dV + \int_{cs} v \rho (\vec{v} d\vec{A}) \quad 13.3$$

where $\sum F$ represents all forces acting on the control volume. The forces include the *surface forces* F_s resulting from the surroundings acting on the control surface and *body forces* F_b that result from gravity and magnetic fields.

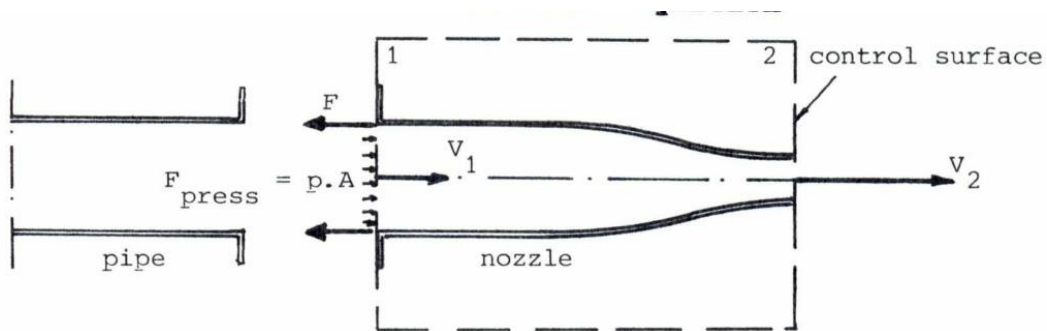


Fig 13.1 Surface forces on a nozzle control volume

$$\sum F_s + \sum F_b = \frac{d}{dt} \int_{cv} v \rho dV + \int_{cs} v \rho (\vec{v} d\vec{A}) \quad 13.4$$

This is the basic form of the momentum equation

Application

The momentum equation is often used to **determine the forces induced by the flow** e.g., the equation allows us to calculate the *force on the support* of the elbow in a pipeline or the *force on a submerged body* in a free-surface flow.

The vector relation may be applied for any component e.g., the x direction

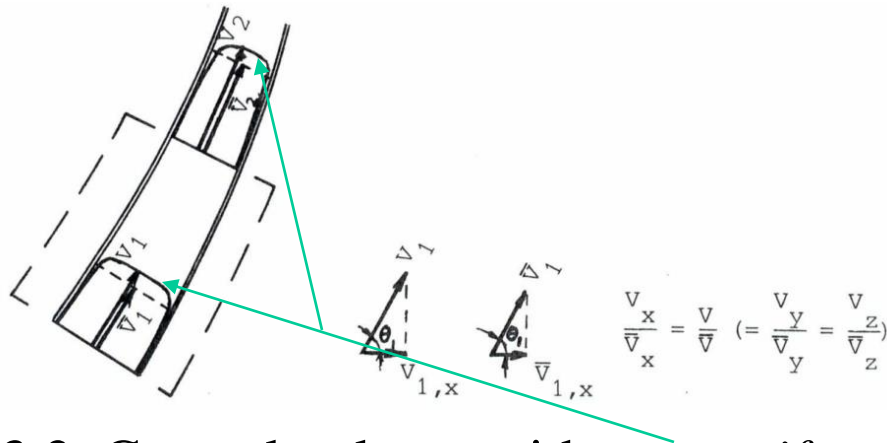


Fig 13.2 Control volume with *non-uniform* inflow and outflow normal to control surface

$$\sum F_s + \sum F_b = \frac{d}{dt} \int_{cv} v \rho dV + \int_{cs} v \rho (\vec{v} d\vec{A}) \quad 13.4$$

For $(\vec{v} d\vec{A})$ refer to Lecture 8 slides (21 to 23)

$dV = (v \cdot dt) dA \cos\theta = (\vec{v} d\vec{A}) dt$
 dV is the volume that has crossed dA of the cs in time dt . i.e., $v_1 dA_1$ & $v_2 dA_2$

$$\sum F_x = \frac{d}{dt} \int_{cv} \rho v_x dV + \int \rho_1 v_{1x} v_1 (-dA_1) + \int \rho_2 v_{2x} v_2 dA_2 \quad 13.5$$

$$\sum F_x = \frac{d}{dt} \int_{cv} \rho v_x dV + \int \rho_1 v_{1x} v_1 (-dA_1) + \int \rho_2 v_{2x} v_2 dA_2 \quad 13.5$$

Usually $\int \rho v_x v dA$ is written as $\beta \rho \bar{v}_x \bar{v} A$ where

$$\beta = \frac{1}{A} \int \left(\frac{v}{\bar{v}} \right)^2 dA = \text{momentum correction factor}$$

Hence, the momentum equation in the x, y and z directions can be written as:

$$\sum F_x = \frac{d}{dt} \int \rho v_x dV - \beta_1 \rho \bar{v}_{1x} \bar{v}_1 A_1 + \beta_2 \rho \bar{v}_{2x} \bar{v}_2 A_2 \quad 13.6a$$

$$\sum F_y = \frac{d}{dt} \int \rho v_y dV - \beta_1 \rho \bar{v}_{1y} \bar{v}_1 A_1 + \beta_2 \rho \bar{v}_{2y} \bar{v}_2 A_2 \quad 13.6b$$

$$\sum F_z = \frac{d}{dt} \int \rho v_z dV - \beta_1 \rho \bar{v}_{1z} \bar{v}_1 A_1 + \beta_2 \rho \bar{v}_{2z} \bar{v}_2 A_2 \quad 13.6c$$

For turbulent flow in a circular pipe $\beta \approx 1.07$, which is for most purposes differs negligibly from unity.