Fluid Mechanics CEE 3311

LECTURE 4

Forces on curved surfaces

L. Handia

Forces on curved surfaces

- We do not use a direct method of integration to find the force due to the hydrostatic pressure on a curved surface.
- Rather, a *free body diagram* that contains the curved surface and the liquids directly above or below the curved surface is identified.
- Such a free-body diagram contains *only plane surfaces* upon which unknown fluid forces act; these unknown forces can be found as in the preceeding lecture.

Example

- Let us determine the force (P) of the curved gate on the stop.

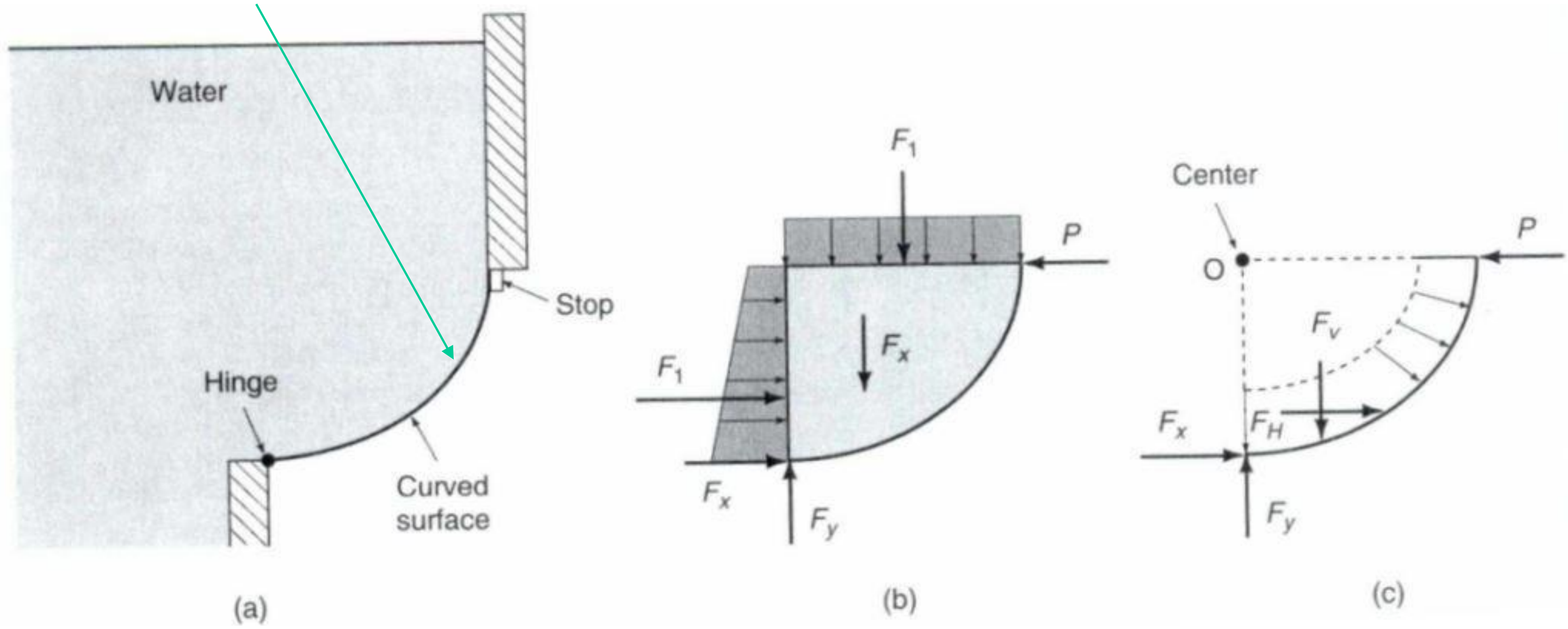


Fig 4.1 Forces on a curved surface: a) curved surface b) free-body diagram of water and gate c) free-body diagram of gate only

- The free-body diagram includes the gate and some of the water contained directly above the gate.
- The forces F_x and F_y are the horizontal and vertical components, respectively of the force acting on the hinge.
- The forces F_1 and F_2 are due to the surrounding water and are the resultant forces of the pressure distributions shown.

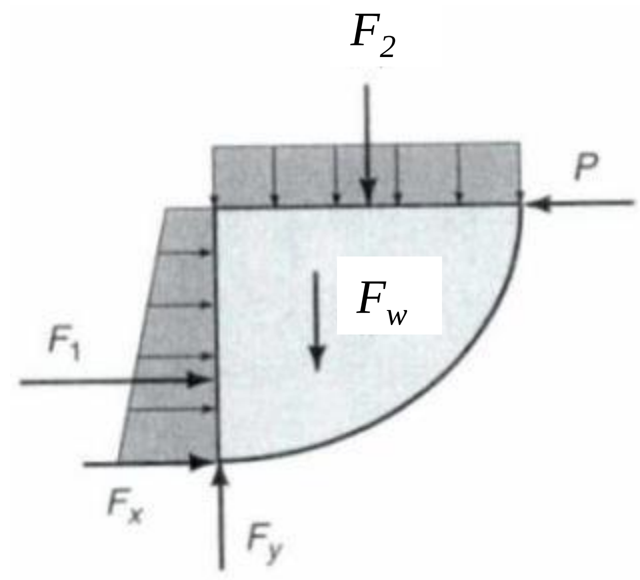


Fig 4.2 Free-body diagram of water and gate

- The body force F_w is due to the weight of the water shown.
- By summing moments about an axis through the hinge, we can determine the force P acting on the stop.

- Consider a free-body diagram of the gate only.
- The horizontal force F_H acting on the gate in Fig 4.3 is equal to F_1 of Fig 4.2.
- The component F_V in fig 4.3 is equal to the combined force $F_2 + F_W$ of Fig 4.2
- Now, $F_H + F_V$ are due to the differential pressure forces acting on the circular arc; each differential pressure force acts through the center of the circular arc, O (since forces act normal to surface).
- Hence, the resultant force $F_H + F_V$ (this the vector addition) must act through the center.
- Consequently, we can locate the components $F_H + F_V$ at the center of the quarter circle, resulting in a much simpler problem.

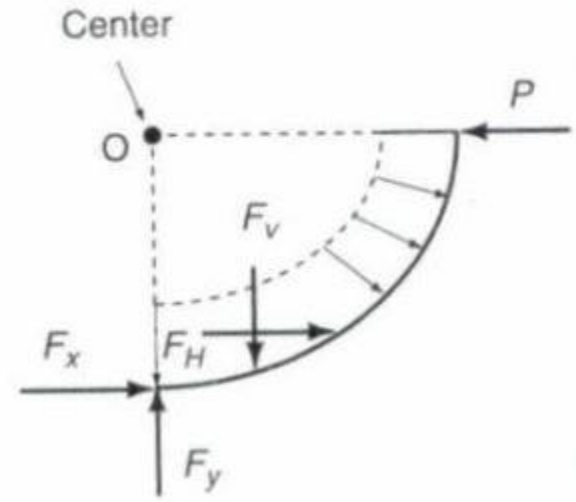


Fig 4.3 Free-body diagram of gate only

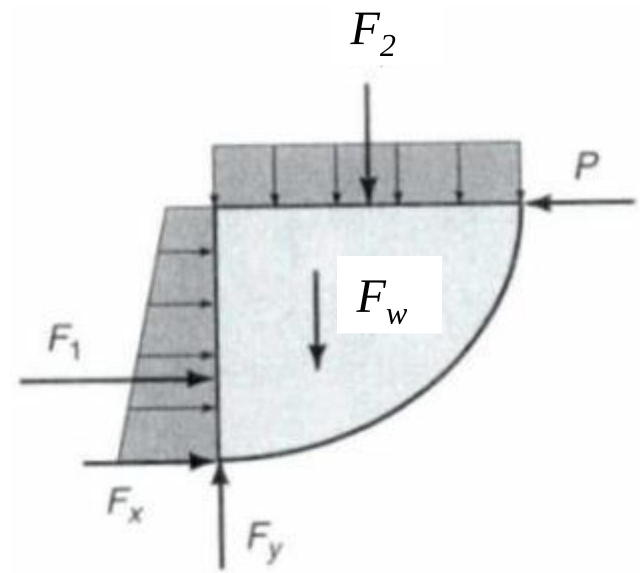


Fig 4.2 Free-body diagram of water and gate

Example 1

Calculate the force P necessary to hold the 4m-wide gate in the position shown in Fig 4.4 (a). Neglect the weight of the gate.

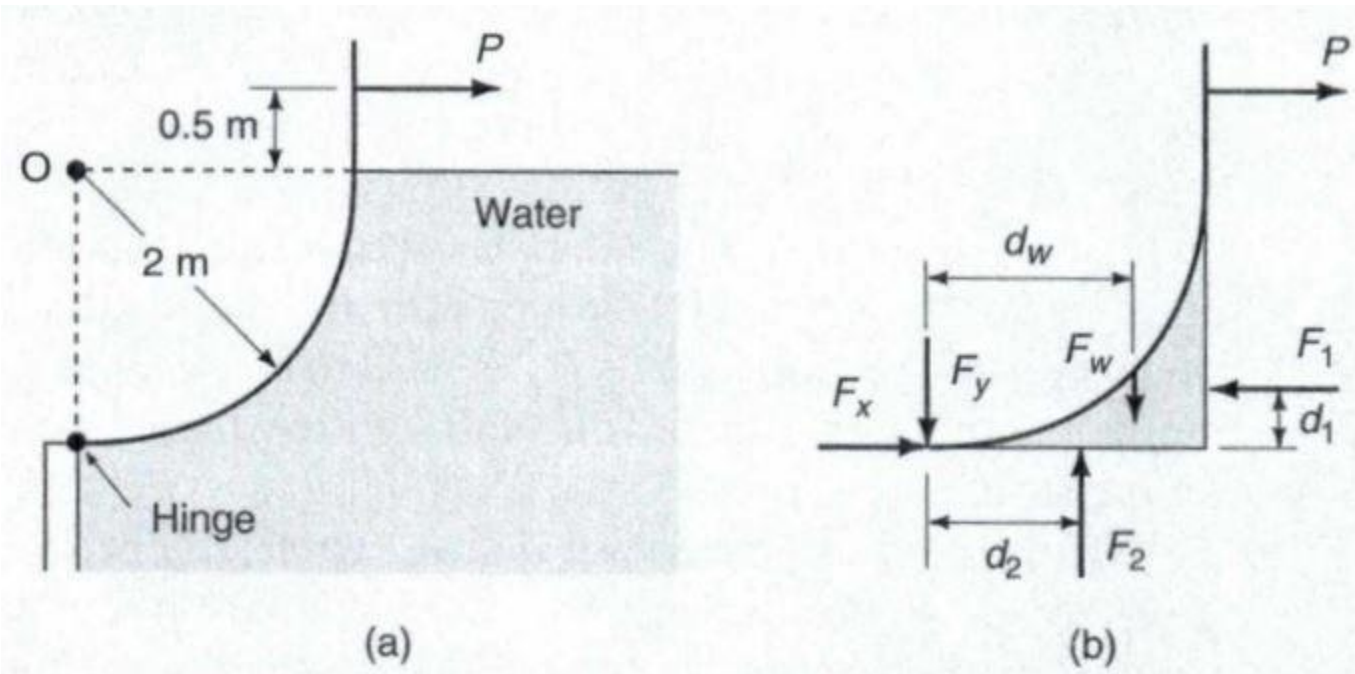
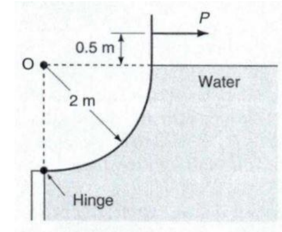


Fig 4.4 Example

Solution

The first step is to draw a free-body diagram of the gate and the water directly below the gate as shown in Fig 4.4 (b).

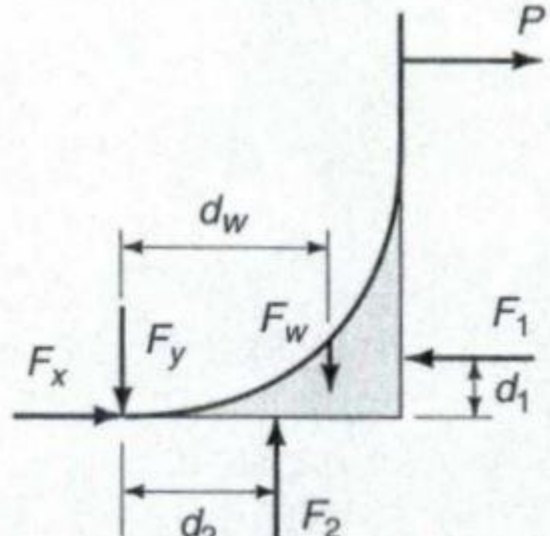
To calculate P, we must determine F_1, F_2, F_W, d_1, d_2 and d_W; then moments about the hinge will aloow us to find P. The force components are given by



(a)

$$F_1 = \gamma \bar{h} A$$
$$= 9810 \times 1 \times 8 = 78\,480 \text{ N}$$
$$F_2 = \gamma \bar{h} A$$
$$= 9810 \times 2 \times 8 = 156\,960 \text{ N}$$
$$F_W = \gamma V$$
$$= 9810 \times 4(4 - \pi) = 33\,700 \text{ N}$$

You can also use $p = \gamma h$



(b)

The distance d_w is the distance to the centroid of the volume. It can be determined by considering the area as the difference of a square and a quarter circle as shown in Fig 4.4 c-e.

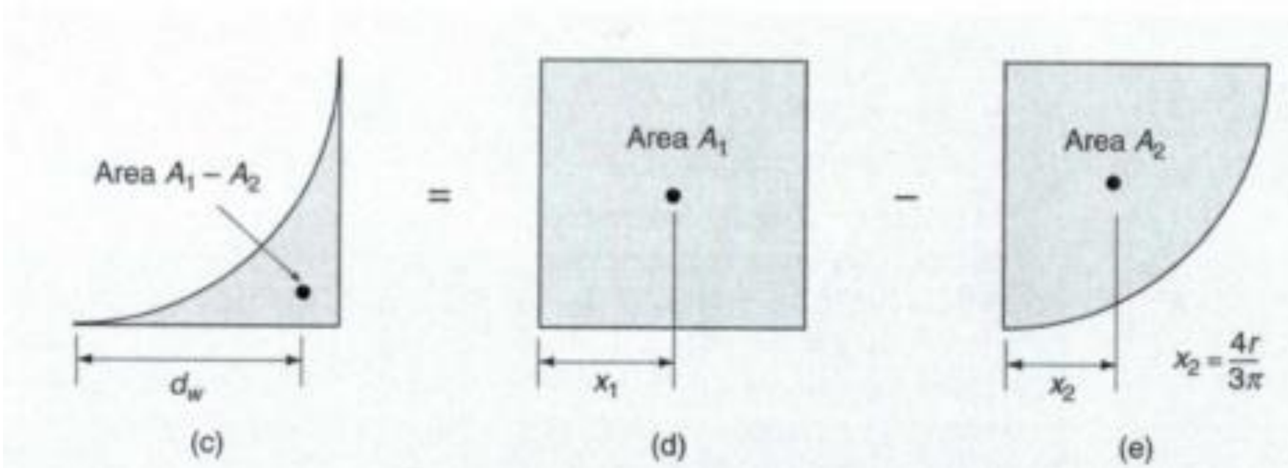


Fig 4.4 Example

Moments of area yield

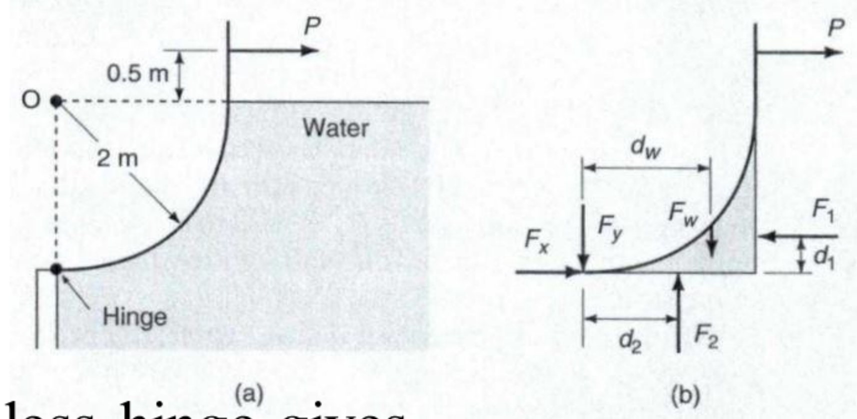
$$d_w(A_1 - A_2) = x_1A_1 - x_2A_2$$

$$d_w = \frac{x_1A_1 - x_2A_2}{A_1 - A_2}$$

$$= \frac{1 \times 4 - (4 \times 2/3\pi) \times \pi}{4 - \pi} = 1.553 \text{ m}$$

The distance d_2= 1m and because F_1 is due to a triangular pressure distribution (see Fig 4.4 (d), d_1 is given by

$$d_1 = \frac{1}{3}(2) = 0.667 \text{ m}$$

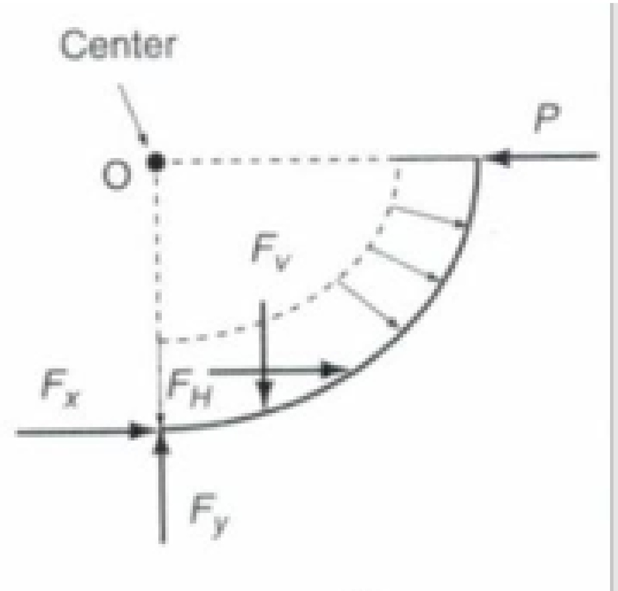


Summing moments about the frictionless hinge gives

$$2.5P = d_1F_1 + d_2F_2 - d_wF_w$$

$$P = \frac{0.667 \times 78.5 + 1 \times 157.0 - 1.553 \times 33.7}{2.5} = 62.8 \text{ kN}$$

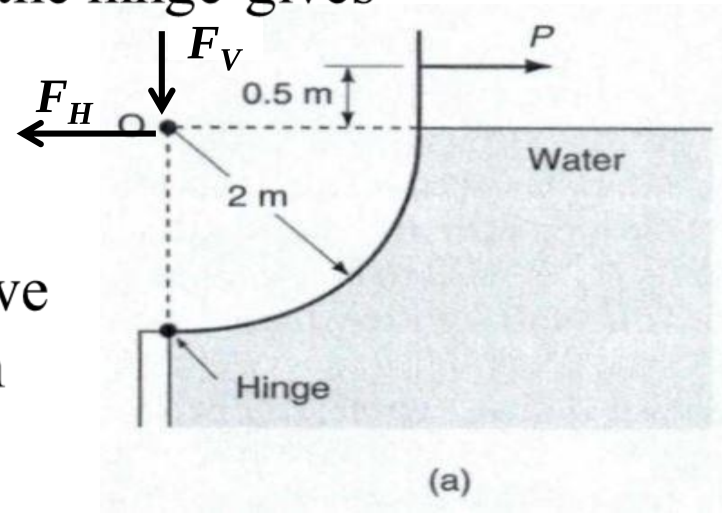
- Rather than the somewhat tedious procedure above, we could observe that all the infinitesimal forces that make up the resultant force (F_H+F_V) acting on the circular arc pass through the center O, as noted in Fig 4.3 (c).



(c)

- Since each infinitesimal force must also pass through the center, the resultant force *must also* pass through the center.
- Hence, we could have located the resultant force (F_H+F_V) at point O.
- If $F_H + F_V$ were located at O, F_V would pass through the hinge, producing no moment about the hinge. Then realising that $F_H = F_1$ and summing moments about the hinge gives

$$2.5P = 2F_H$$



- Therefore $$P = 2 \times \frac{78.48}{2.5} = 62.8 \text{ kN}$$
- This was obviously much simpler. All we needed to do was calculate F_H and then sum moments!

Example 2:Arch dam

Kariba Dam

Kariba Dam on the Zambezi River.

Credit: Ben Bird

Normal plane area where hydrostatic force is acting is

$$2r \sin\frac{\theta}{2}$$

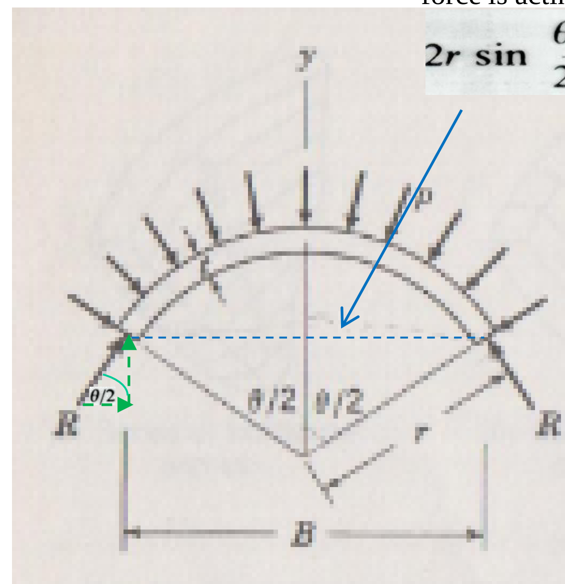


Figure 8-11 Free-body diagram of an arch rib.

$$H_h = \gamma h 2r \sin\frac{\theta}{2}$$