Fluid Mechanics CEE 3311

LECTURE 3

Forces on submerged surfaces

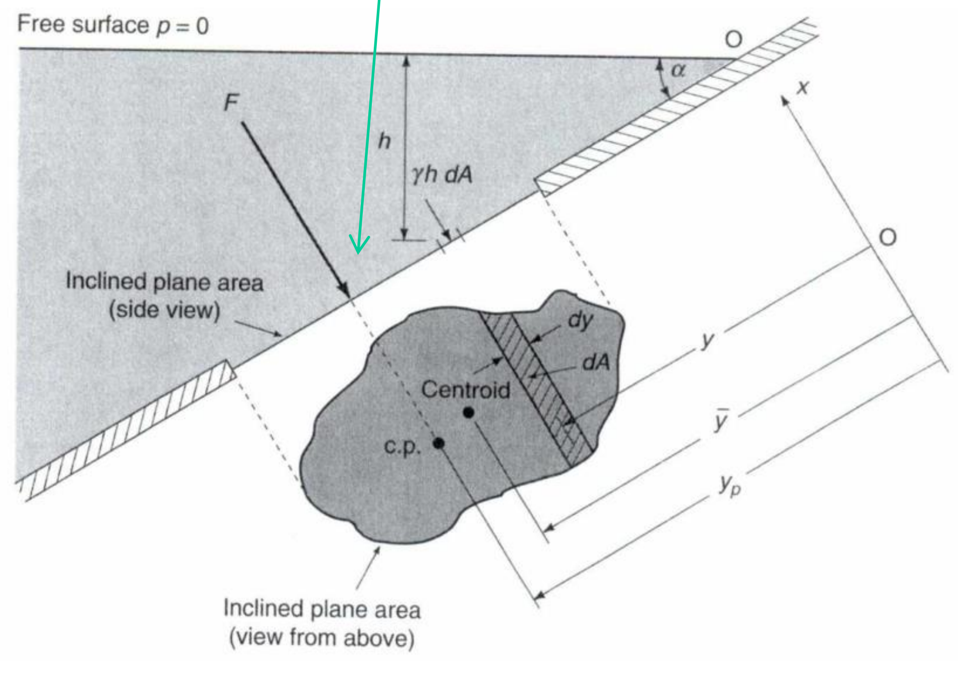
L. Handia

Forces on submerged surfaces

- Pressure was discussed in the previous lecture,. This lecture uses the knowledge on pressure from the previous lecture to compute the magnitude and location of the force due to that pressure.
- In the design of devices and objects that are submerged, it is necessary to calculate the magnitudes and locations of forces that act on both plane and curved surfaces.
- Examples:
  - dams
  - flow obstructions
  - surfaces on ships and submarines
  - holding tanks

Forces on Plane surfaces

The forces on one side of a plane surface is **always** *normal* to the surface, no matter what inclination the surfaces takes to the surface of the fluid in which it is submerged.



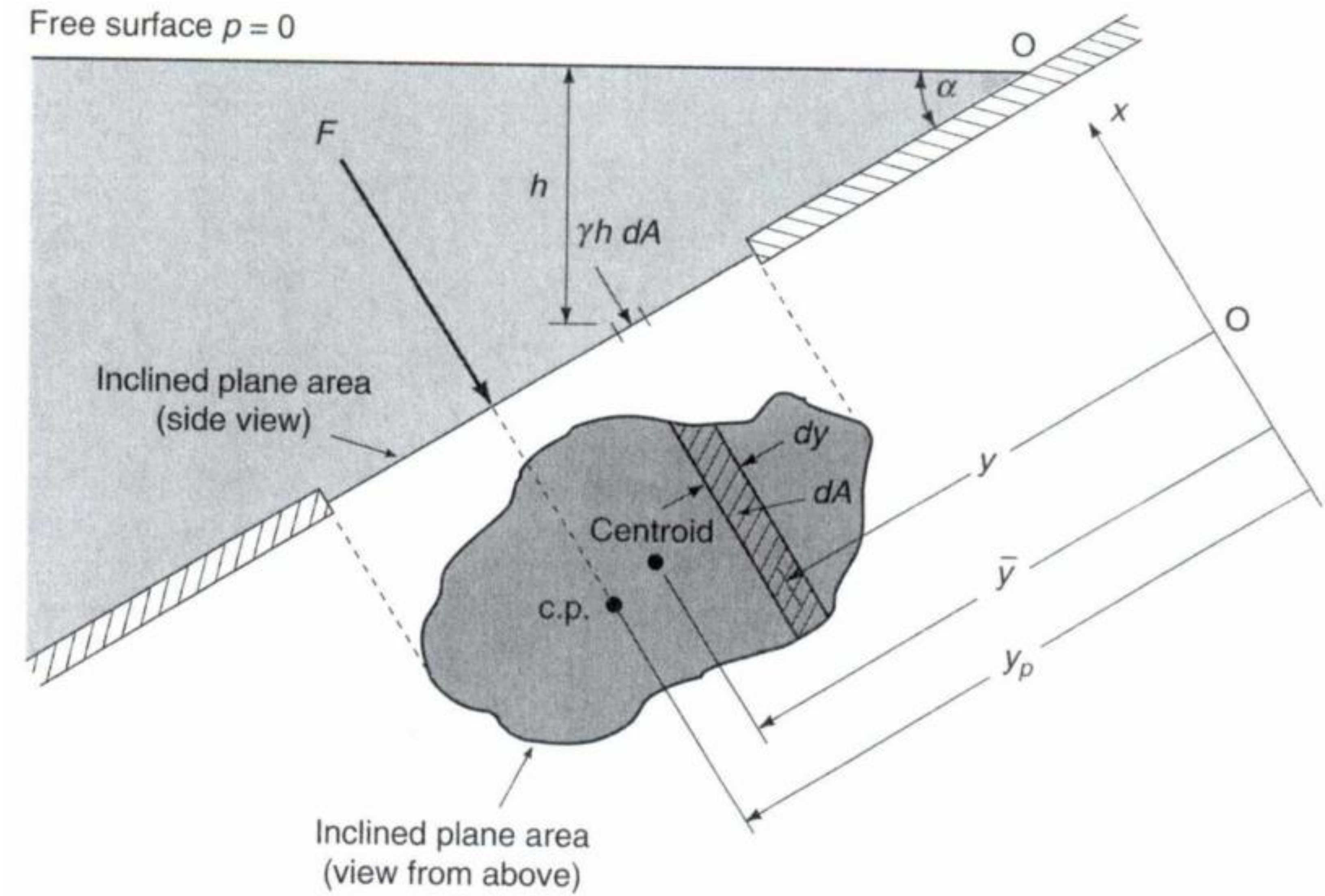
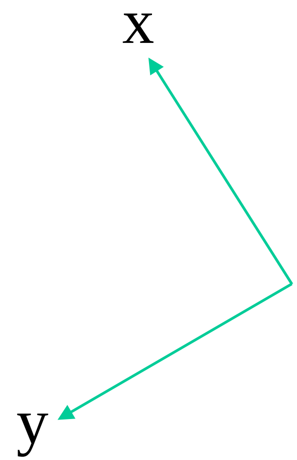
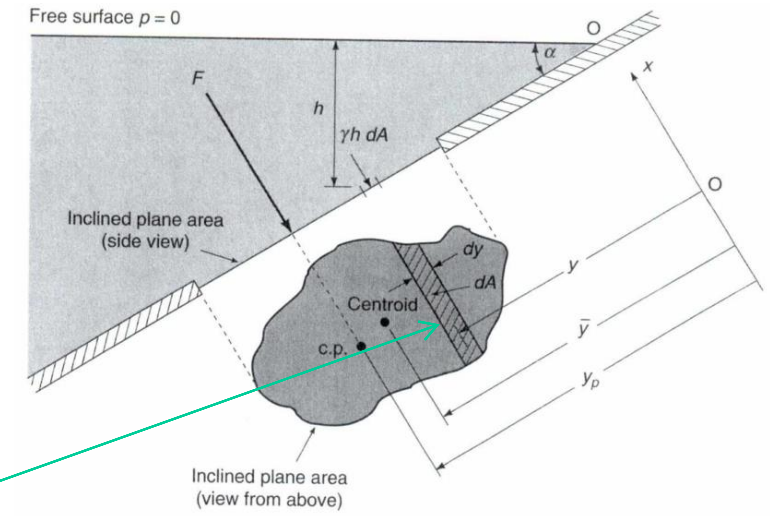


Figure 3.1 Forces on an inclined plane area

Centroid is the center of mass. If you cut a shape out of a piece of card it will balance perfectly on its centroid.





- The total force of the liquid on the plane surface is found by integrating the pressure over the area, i.e.

$$F = \int_A p\, dA \qquad 1$$

where we usually use gage pressure. Atmospheric pressure cancels out since it acts on both sides of the area.

- The x and y coordinates are in the plane of the plane surface, as shown. Assuming that p = 0 at h = 0, we know that

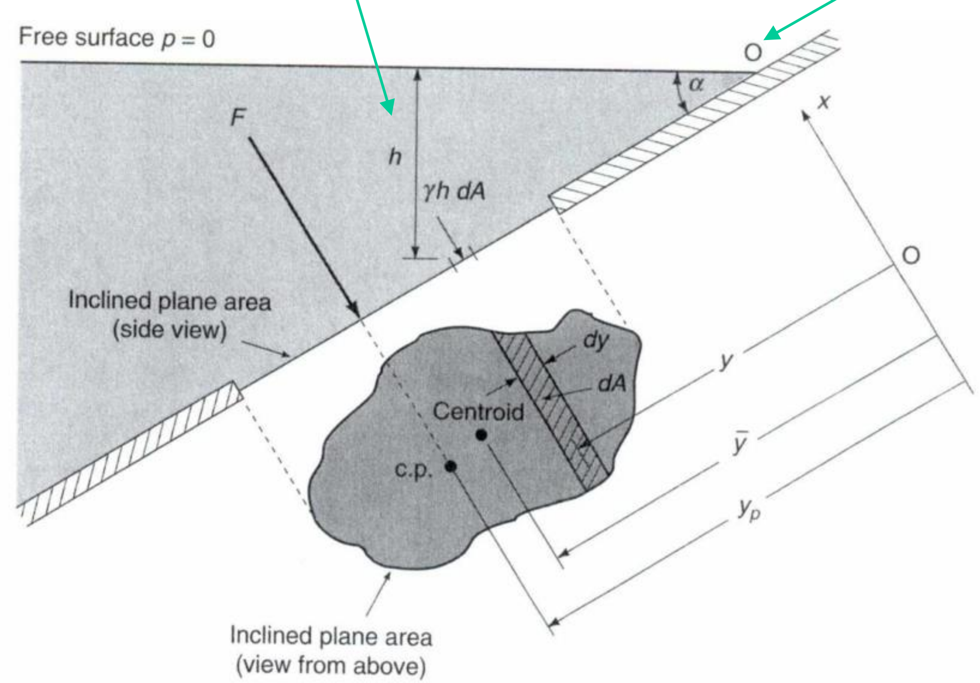
$$p = \gamma h$$

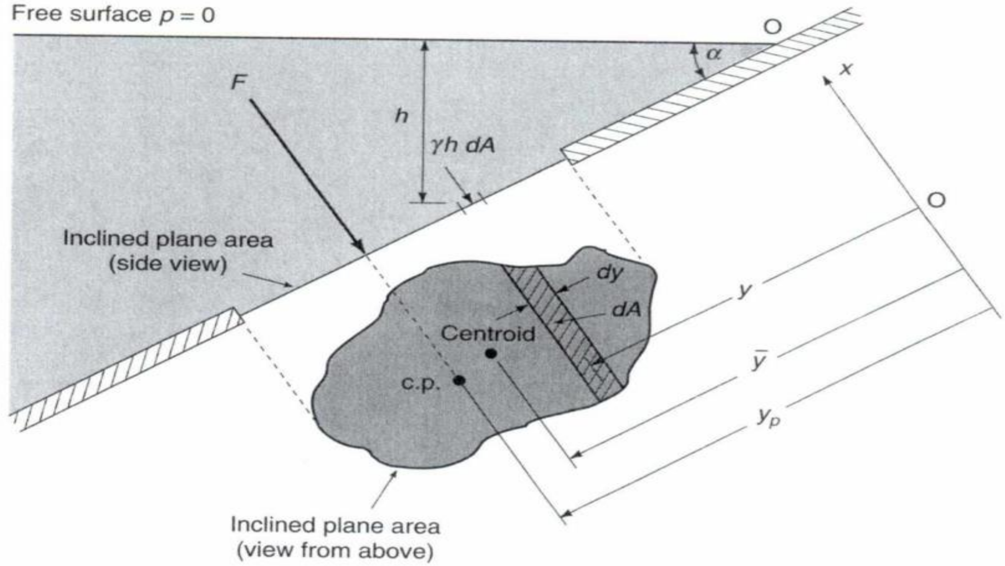
Since h = y sinα

$$= \gamma y sin\alpha \qquad 2$$

Where h is measured vertically down from the free surface to the elemental area dA and y is measured from point O on the free surface.

The force is: $$F = \int_A p\,dA = \int_A \gamma y \sin\alpha\, dA = \gamma \sin\alpha \int_A y\,dA \qquad 3$$





$$F = \gamma sin\alpha \int_A y\, dA \qquad 3$$

The distance to a centroid is defined as

$$\bar{y} = \frac{1}{A}\int_A y\, dA \qquad 4$$

Making $\int_A y\, dA$ the subject of eqn 4 and substituting in eqn 3, the expression for the force then becomes

$$F = \gamma\bar{y}A sin\alpha \qquad \text{(and since } \bar{y} sin\alpha = \bar{h}\text{)}$$

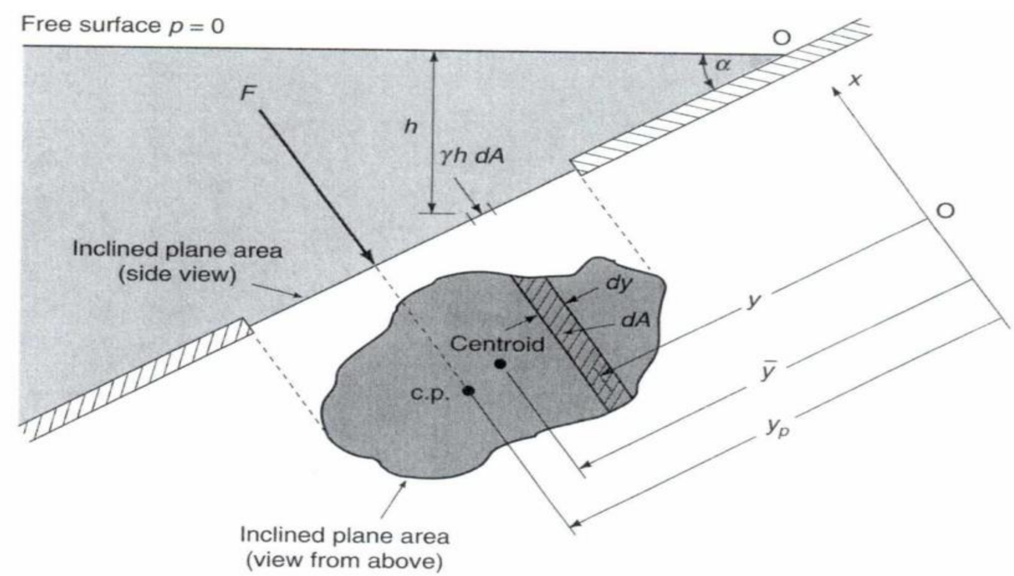
$$= \gamma\bar{h}A = p_c A \qquad 5$$

$$F = \gamma \bar{h} A = p_c A \qquad 6$$

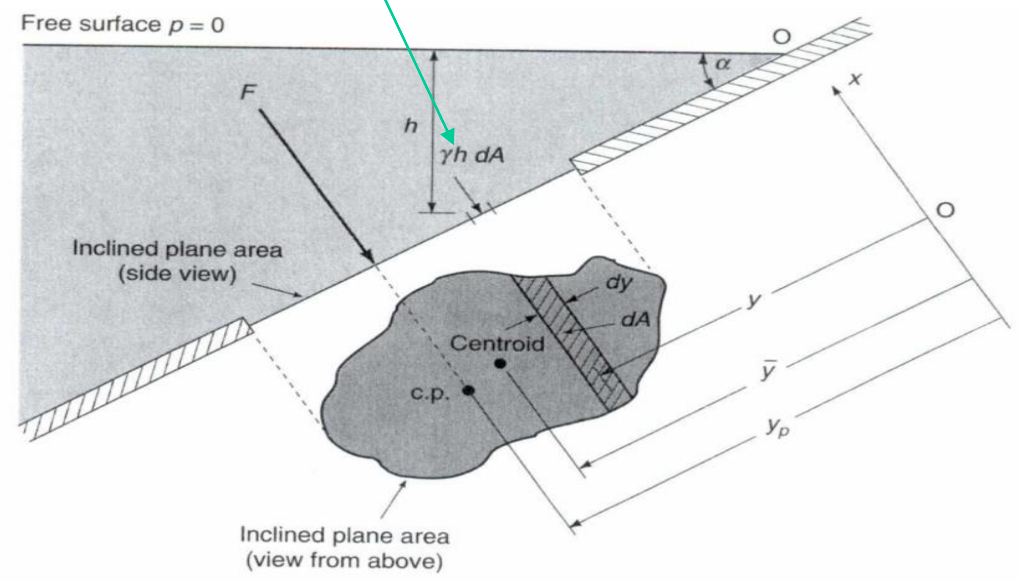
where $\bar{h}$ is the vertical distance from the free surface to the centroid of the area

p_c is the pressure at the centroid

- Thus we see that the magnitude of the force on a plane surface is the pressure *at the centroid* multiplied by the area.



- The force does *not*, in general, act at the *centroid* (but the *center of pressure*).
- To find the location of the resultant force F, we note that the sum of the ***moments of all the infinitesimal pressure forces*** (*p dA*) acting on the area A ($\int_A yp\,dA$) must ***equal*** the ***moment of the resultant force*** ($y_p F$).
- Let the force F act at the point (x_p, y_p), ***the center of pressure*** (c.p.).



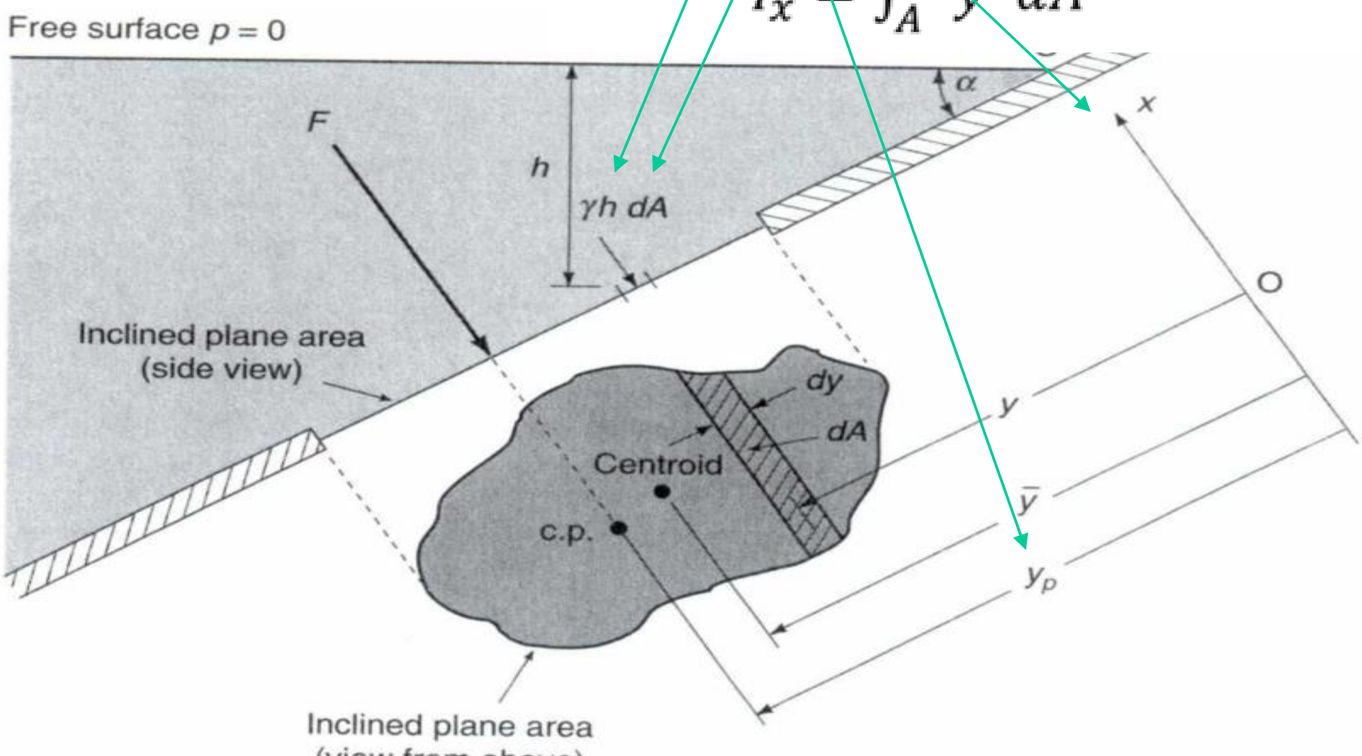
- The value of y_p can be obtained by equating moments about the x-axis: $y_p F = \int_A y(pdA)$

$$y_p F = \int_A ypdA = \int_A y(\gamma y \sin\alpha)dA = \gamma \sin\alpha \int_A y^2 dA = \gamma I_x \sin\alpha \qquad 7$$

(since $p = \gamma y sin\alpha$ eqn 2)

where the second moment of the area about the x-axis is

$$I_x = \int_A y^2 dA \qquad 8$$



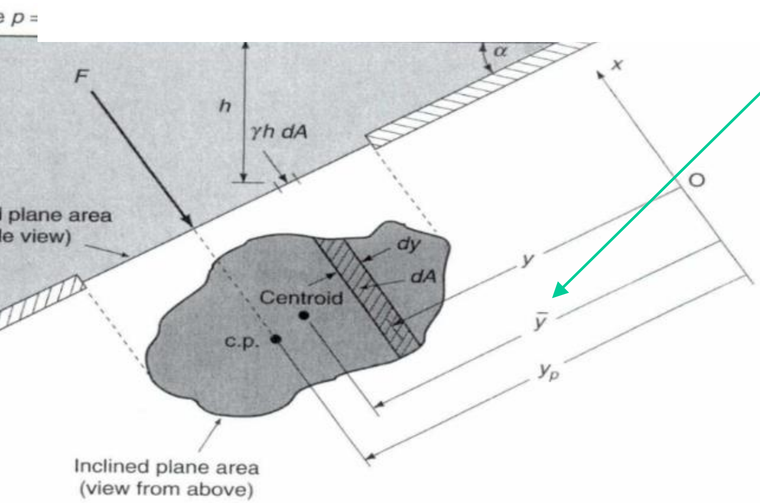
- The second moment of an area can be determined from the second moment of an area $\bar{I}$ *about the centroidal* axis by the parallel-axis-transfer theorem,

$$I_x = \bar{I} + A\bar{y}^2 \qquad 9$$

- Substitute eqns 6 and 9 into eqn 7, and obtain

$$y_p = \frac{\gamma(\bar{I} + A\bar{y}^2)sin\alpha}{\gamma\bar{y}Asin\alpha}$$

$$= \bar{y} + \frac{\bar{I}}{A\bar{y}} \qquad 10$$

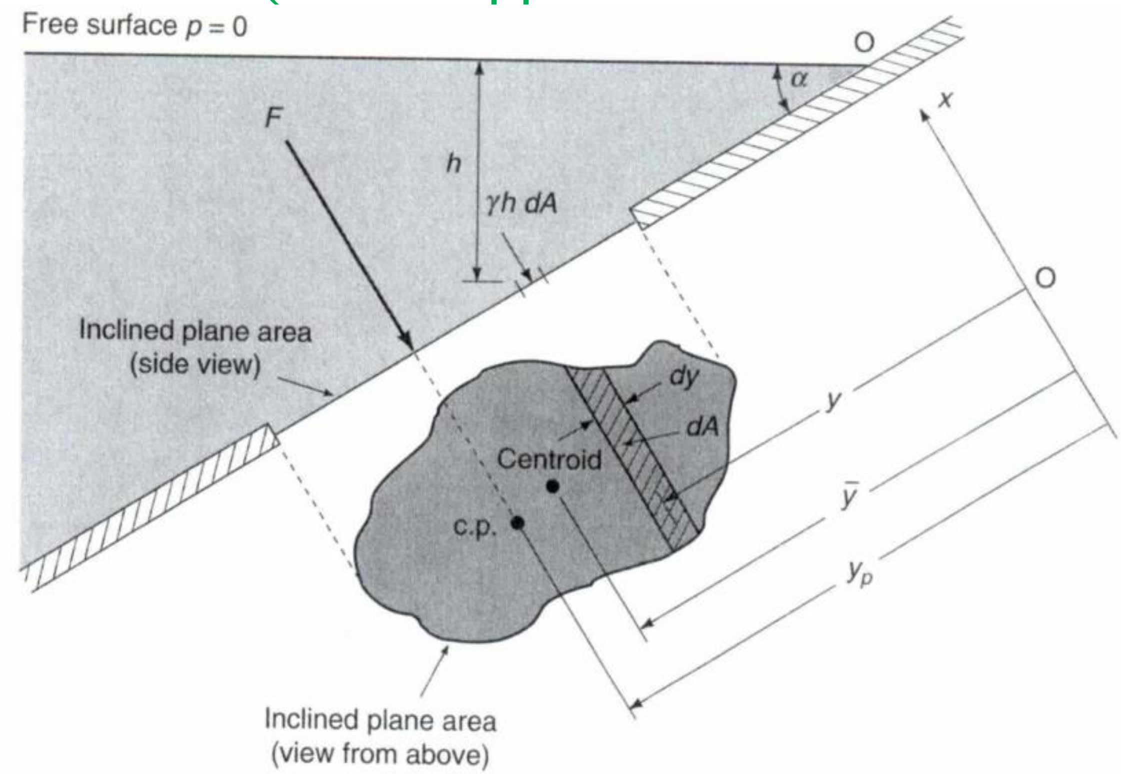


$$F = \gamma\bar{h}A = p_c A \qquad 6$$

$$y_p F = \gamma I_x \sin\alpha \qquad 7$$

$$y_p F = \gamma I_x \sin\alpha \Rightarrow y_p = \frac{\gamma I_x \sin\alpha}{F} = \frac{\gamma\left(\bar{I} + A\bar{y}^2\right)\sin\alpha}{\gamma\bar{h}A} = \frac{\gamma\left(\bar{I} + A\bar{y}^2\right)\sin\alpha}{\gamma\bar{y}A\sin\alpha}$$

where $\bar{y}$ is measured parallel to the plane area along the y-axis. (Insert Appendix C or refer to tables with formulas)



$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} \qquad 10$$

- Using eqn 10, we can show that the force on a rectangular gate, with the top edge even with the liquid surface acts two-thirds of the way down.

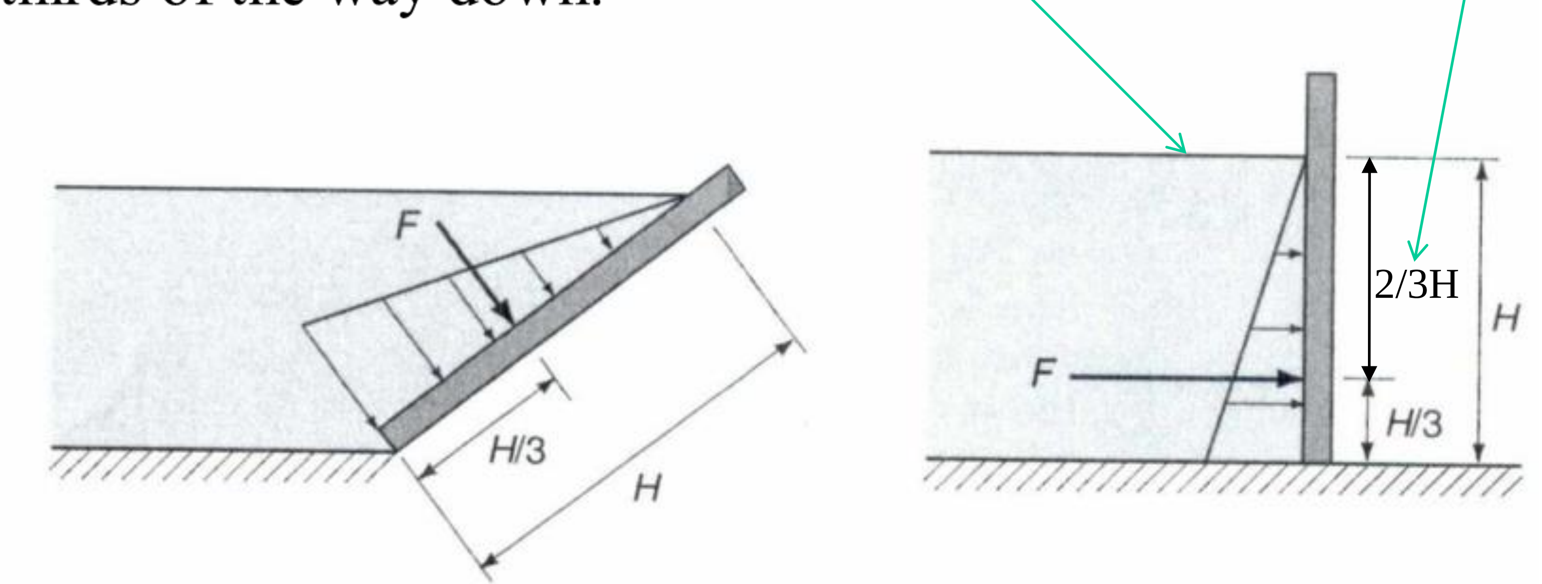


Fig 3.2 Force on a plane area with top edge in a free surface

- Eqn 10 shows us that y_p is always greater than $\bar{y}$ i.e., the resultant force of the liquid on a plane surface always acts below the centroid of the area

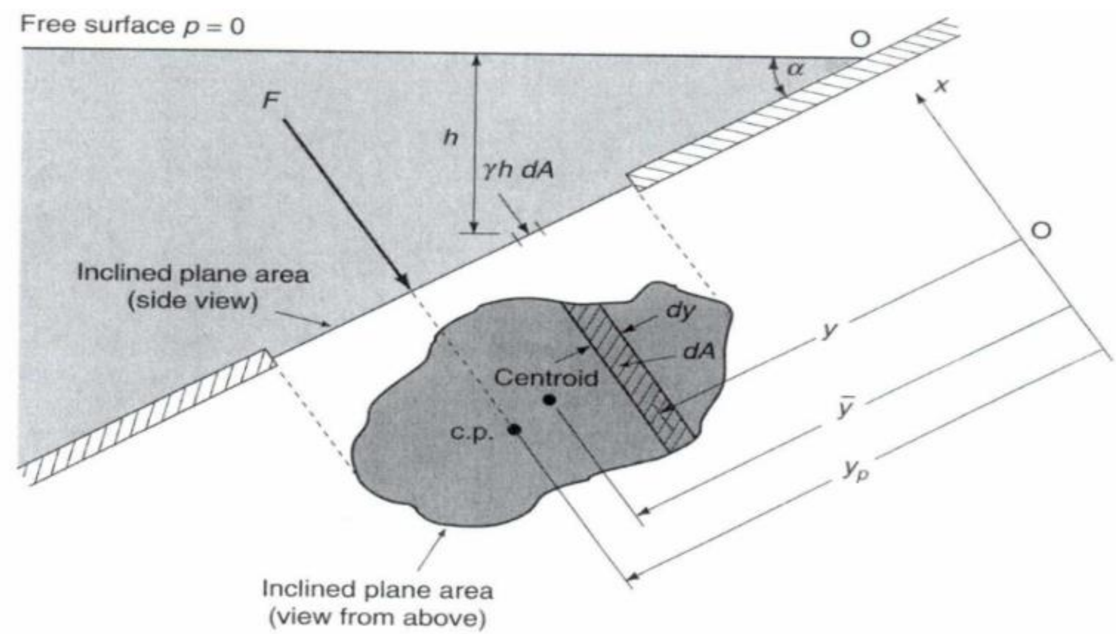
- Similarly, to locate the x-coordinate x_p of the c.p., we write

$$x_pF = \int_A \; xp\, dA \quad (\text{since } p = \gamma y sin\alpha \qquad \text{eqn 2})$$

$$= \gamma sin\alpha \int_A \; xydA = \gamma I_{xy} sin\alpha \qquad 11$$

where the product of inertia of the area A is

$$I_{xy} = \int_A \; xydA \qquad 12$$



- Using the transfer theorem for the product of inertia,

$$I_{xy} = \bar{I}_{xy} + A\bar{x}\bar{y} \quad 13$$

- Substitute eqns 6 & 13 into eqn 11 & obtain

$$x_p = \bar{x} + \frac{\bar{I}_{xy}}{Ay} \quad 14$$

We now have coordinates for locating the center of pressure

$$F = \gamma\bar{h}A = p_c A \quad 6$$

$$x_p F = \int_A xp\, dA \text{ (since } p = \gamma y sin\alpha \text{ eqn 2)}$$

$$= \gamma sin\alpha \int_A xydA = \gamma I_{xy} sin\alpha \quad 11$$

- Finally we should note that the force F in Fig 3.1 is the result of a ***pressure prism*** acting on the area.

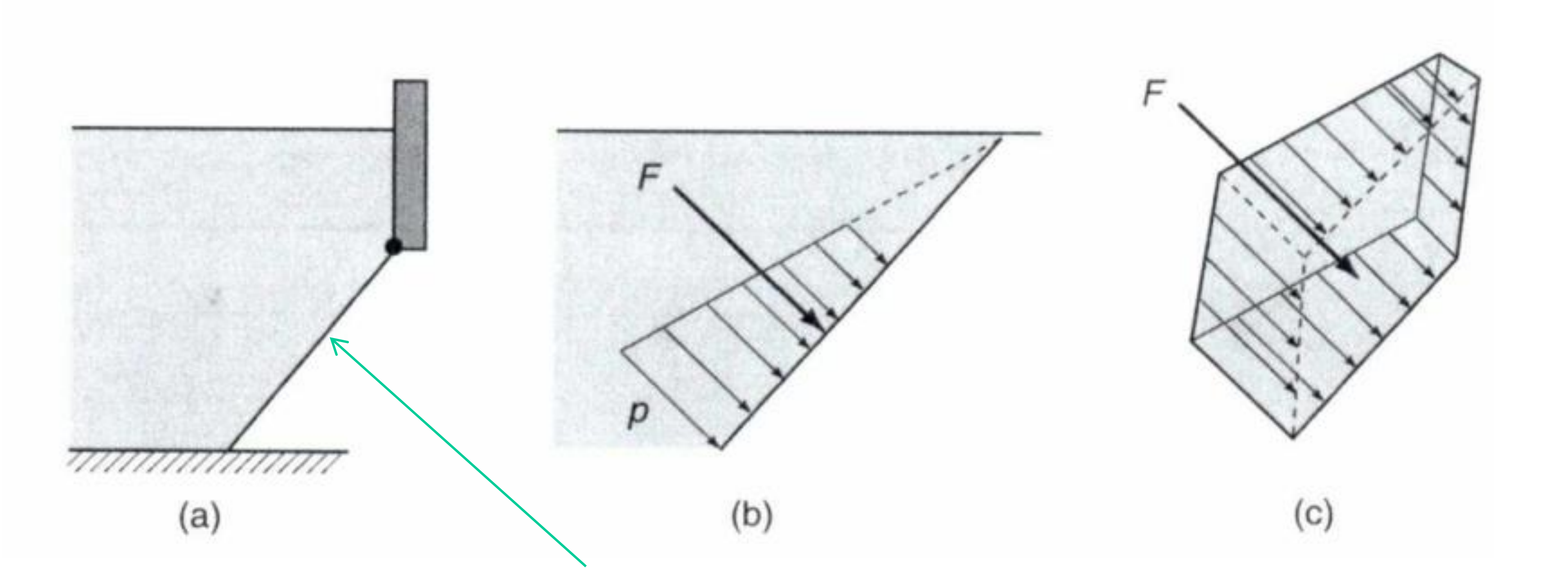
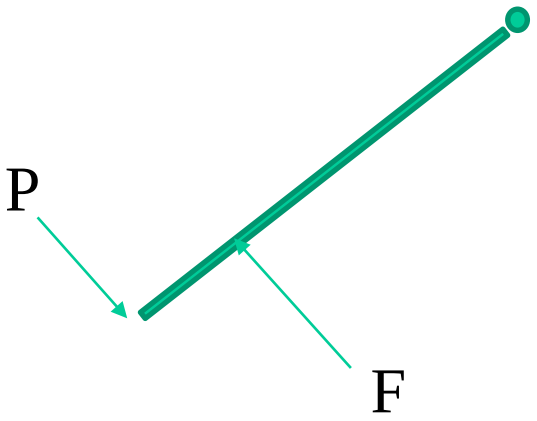


Fig 3.3 Pressure prism: (a) rectangular area (b) pressure distribution (c) pressure prism

- If we form the integral $\int \quad p\, dA$, we obtain the volume of the pressure prism, which equals the force F acting on the area in Fig 3.3 (c).
- The force acts through the centroid of the volume of the pressure prism   Note: not centroid of plane surface

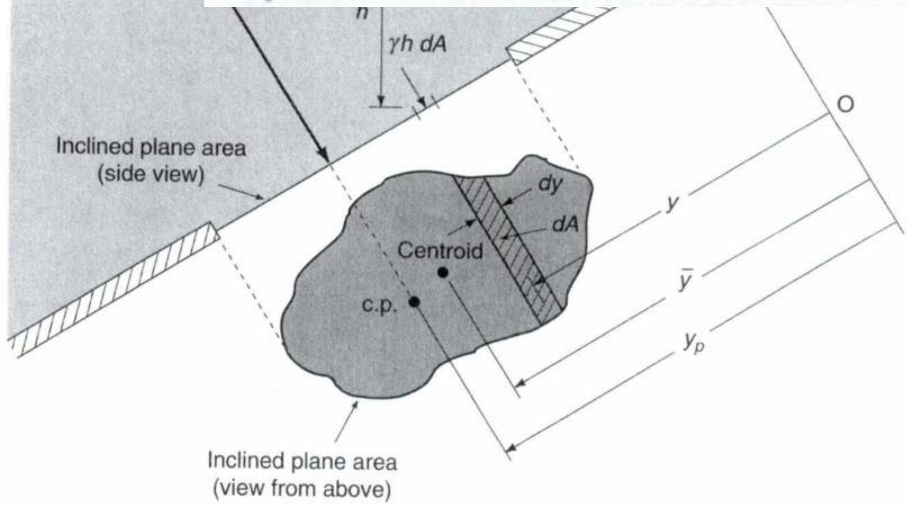
Example

A plane area of 80 cm × 80 cm acts as a window on a submersible in the Great Lakes. If it is on a 45° angle with the horizontal, what force applied at the bottom edge is needed to just open the window, if it is hinged at the top edge when the top edge is 10 m below the surface? The pressure inside the submersible is assumed to be atmospheric.



Example

A plane area of 80 cm × 80 cm acts as a window on a submersible in the Great Lakes. If it is on a 45° angle with the horizontal, what force applied at the bottom edge is needed to just open the window, if it is hinged at the top edge when the top edge is 10 m below the surface? The pressure inside the submersible is assumed to be atmospheric.



Solution: The force of the water acting on the window is

$$F = \gamma \bar{h} A$$

$$= 9810(10 + 0.4 \times \cos 45^\circ)(0.8 \times 0.8) = 64\,560 \text{ N}$$

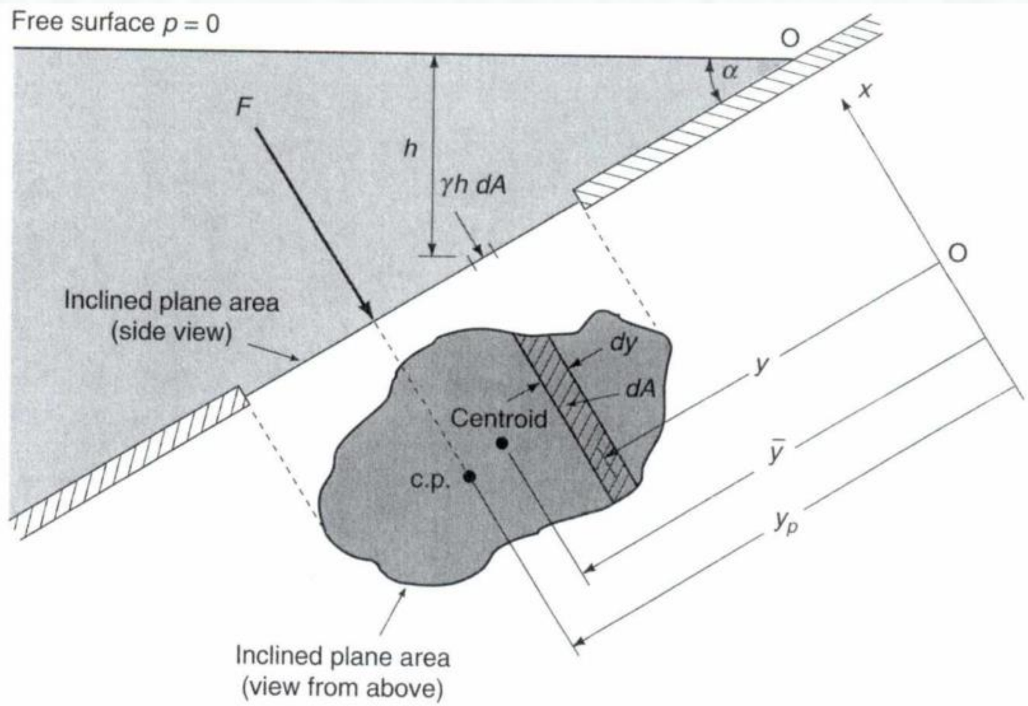
The distance $\bar{y}$ (see Fig. 2.8) is

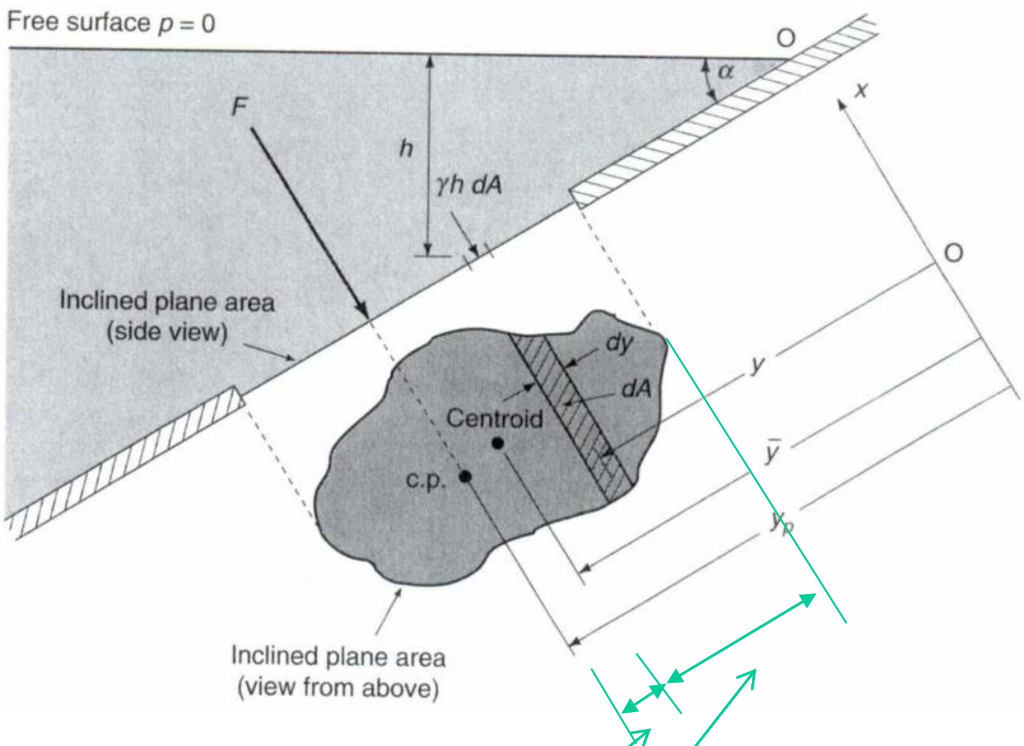
$$\bar{y} = \frac{\bar{h}}{\cos 45^\circ} = \frac{10 + 0.4 \times \cos 45^\circ}{\cos 45^\circ} = 14.542 \text{ m}$$

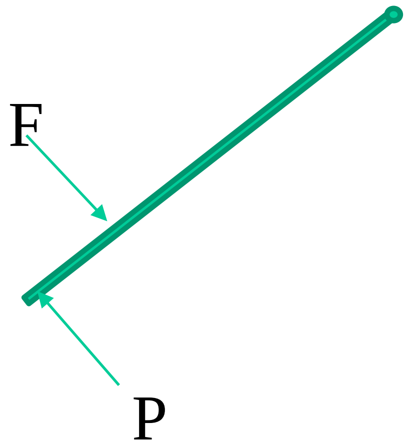
so that

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}}$$

$$= 14.542 + \frac{0.8 \times 0.8^3/12}{(0.8 \times 0.8) \times 14.542} = 14.546 \text{ m}$$







Taking moments about the hinge provides the needed force P to open the window

$$0.8P = (y_p - \bar{y} + 0.4)F$$

$$\therefore \quad P = \frac{14.546 - 14.542 + 0.4}{0.8} 64\,560 = 32\,610 \text{ N}$$

Note: P has to be smaller because of the moment arm