



Fluid Mechanics CEE 3311
LECTURE 10

Conservation of energy

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The energy equation will be derived starting from the first law of thermodynamics which states that for steady flow, the external work done on any system plus the thermal energy transferred into or out of the system is equal to the change of energy of the system. In other words, for steady flow during time Δt

Another definition of the first law of thermodynamics: It states that the change in the internal energy ΔU of a closed system is equal to the amount of heat Q supplied *to* the system, minus the amount of work W done *by* the system on its surroundings.

In thermodynamics, the **internal energy** of a system is the total energy contained within the system. It is the energy necessary to create or prepare the system in any given state, but does not include the kinetic energy nor the potential energy of the system.

$$\Delta E = Q - W$$

10.1

where Q is thermal energy or heat transferred into a system & W is external work done

The total energy E consists of kinetic energy E_K , potential energy E_p and internal energy E_u . Thus the total energy of the system is

$$E = E_k + E_p + E_u$$

10.2

Then continuity equation becomes

E is an extensive property of the system. Then the corresponding intensive property is given by e, which is made up of e_k , e_p , and u .

In applying the control volume equation

$$\frac{dN_{\text{sys}}}{dt} = \frac{d}{dt} \int_{\text{cv}} \eta \rho \, dV + \int_{\text{cs}} \eta \rho (\vec{v} \cdot d\vec{A}) \quad 10.3$$

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt} \int_{\text{cv}} e \rho \, dV + \int_{\text{cs}} e \rho (\vec{v} \cdot d\vec{A}) \quad 10.4$$

Or

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt} \int_{cv} (e_k + e_p + u)\rho dV + \int_{cs} (e_k + e_p + u)\rho (\vec{v} d\vec{A}) \quad 10.5$$

in which $\frac{dQ}{dt}$ = rate of flow of heat into the system

$\frac{dW}{dt}$ = rate of work done by the system on its surrounding

The kinetic energy per unit mass e_k can be written as

$$e_k = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2 \quad \text{while} \quad e_p = \frac{mgz}{m} = gz \quad 10.6$$

Substituting these into eq.

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt} \int_{cv} \left(\frac{1}{2}v^2 + gz + u\right)\rho dV + \int_{cs} \left(\frac{1}{2}v^2 + gz + u\right)\rho (\vec{v} d\vec{A}) \quad 10.7$$

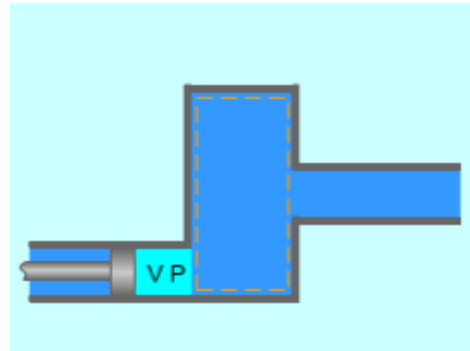
For convenience of analysis, work W is divided into flow work W_f and shaft work W_s

Flow Work

- Work is needed to push the fluid into or out of the boundaries of a control volume if mass flow is involved. This work is called the flow work (flow energy). Flow work is necessary for maintaining a continuous flow through a control volume.

Flow Work

- Consider a fluid element of volume V , pressure P , and cross-sectional area A as below.



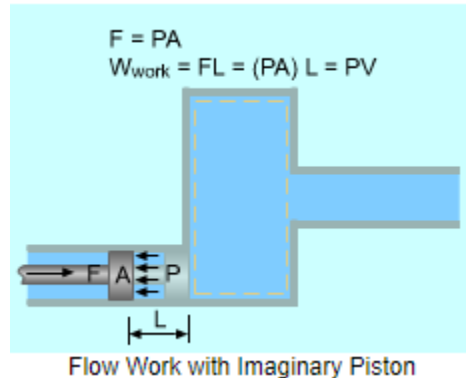
A Flow Element

- The flow immediately upstream will force this fluid element to enter the control volume, and it can be regarded as an imaginary piston.

Flow Work

- The force applied on the fluid element by the imaginary piston is:

$$F = PA$$



- The work done due to pushing the entire fluid element across the boundary into the control volume is

$$W_{\text{flow}} = FL = PAL = PV$$

- The work done due to pushing the fluid element out of the control volume is the same as the work needed to push the fluid element into the control volume.

Flow work

- Flow work, also called external work, may be done on the fluid system in various ways.
- It is done when the pressure forces acting on the boundaries move, in our case when $p_1 A_1$ and $p_2 A_2$ at the end sections move through Δs_1 and Δs_2 , respectively.
- Consider a system moving through a closed conduit

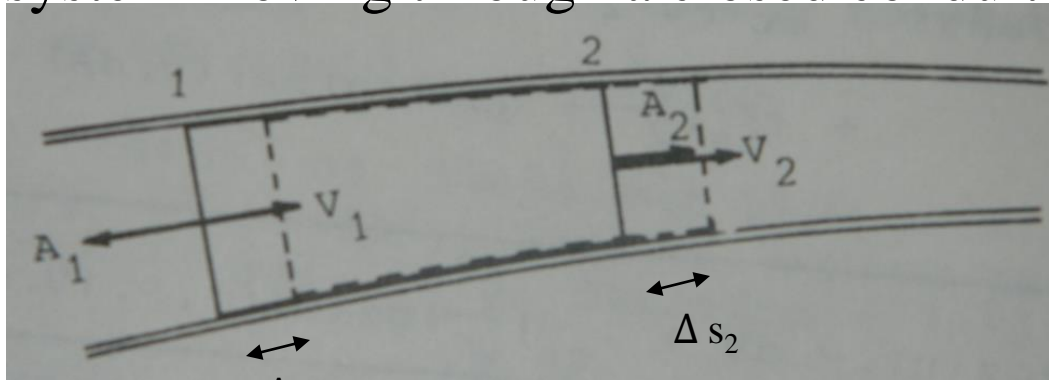


Fig 10.1 System moving through a closed conduit

- At area A_2 , the force acting upon the surrounding fluid will be $p_2 A_2$ and the distance travelled by the area in the time Δt will be $\Delta l = v_2 \Delta t$. Hence the flow work done ($W = \text{Force} \times \text{distance}$) by the system on the surrounding fluid in time Δt will be

$$\Delta W_{f,2} = p_2 A_2 v_2 \Delta t \quad 10.8$$

The rate of flow work (W/t) $\frac{dW_{f,2}}{dt} = p_2 v_2 A_2 \quad 10.9$

And similarly $\frac{dW_{f,1}}{dt} = -p_1 v_1 A_1 \quad 10.10$

The negative sign indicates that the force and displacement are in opposite directions. In terms of vector dot product, eqn can be written as

$$\frac{dW_f}{dt} = p \cdot (\vec{v}\vec{A}) \quad 10.11$$

Then the rate at which flow work is done on the system's surroundings is obtained by summing eq 10.11 for all streams passing the control surface; in general

$$\frac{dW_f}{dt} = \sum_{cs} p \cdot (\vec{v}\vec{A}) \quad 10.12$$

Shaft work

Shaft work is defined as any work other than flow work. It is usually in the form of a shaft that either

- Takes energy out of the system (work on a mechanism like a turbine blade)
- Puts energy into the system (shaft attached to a mechanism like a pump that does work on the system)

In the latter case, the fluid system is doing negative work on its surrounding.

If we substitute for $\frac{dW}{dt}$ in eq 10.7 the sum of the shaft work $\frac{dW_s}{dt}$ and the flow work rate eq 10.12, we obtain

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt} \int_{cv} \left(\frac{1}{2} v^2 + gz + u \right) \rho dV + \int_{cs} \left(\frac{1}{2} v^2 + gz + u \right) \rho (\vec{v} d\vec{A}) \quad 10.7$$

$$\frac{dW}{dt} = \frac{dW_f}{dt} + \frac{dW_s}{dt} \qquad \frac{dW_f}{dt} = \sum_{cs} p \cdot (\vec{v} \vec{A}) \quad 10.12$$

$$\frac{dQ}{dt} - \frac{dW_s}{dt} - \int_{cs} \frac{p}{\rho} \rho (\vec{v} d\vec{A}) = \frac{d}{dt} \int_{cv} \left(\frac{v^2}{2} + gz + u \right) \rho dV + \int_{cs} \left(\frac{v^2}{2} + gz + u \right) \rho (\vec{v} d\vec{A})$$

Or

$$\frac{dQ}{dt} - \frac{dW_s}{dt} = \frac{d}{dt} \int_{cv} \left(\frac{v^2}{2} + gz + u \right) \rho dV + \int_{cs} \left(\frac{v^2}{2} + gz + u + \frac{p}{\rho} \right) \rho (\vec{v} d\vec{A}) \quad 10.13$$

Equation 10.13 is the basic form of the **energy equation**. Usually some simplifications are allowed to be made.