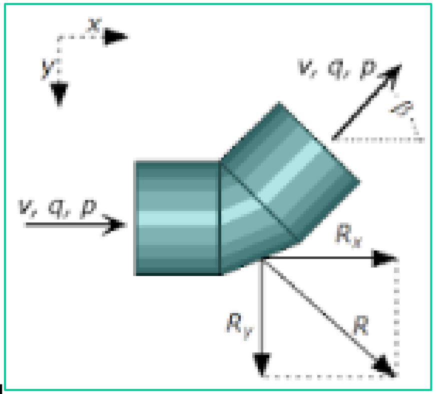
Fluid Mechanics CEE 3311

LECTURE 13



Conservation of momentum

L. Handia

mv is an extensive property of the system. Then the corresponding intensive property is given by v.

In applying the control volume equation

$$\frac{dN_{sys}}{dt} = \frac{d}{dt}\int_{cv} \eta\, \rho\, dV + \int_{cs} \eta\, \rho\, (\vec{v}\, d\vec{A}) \qquad 13.1$$

$$\frac{d(\text{momentum})}{dt} = \frac{d}{dt}\int_{cv} v\, \rho\, dV + \int_{cs} v\, \rho\, (\vec{v}\, d\vec{A}) \qquad 13.2$$

According to Newton's second law of motion, the summation of all external forces on the system is equal to the rate of change of momentum of that system $\sum \mathrm{F} = \frac{d(mv)}{dt}$ *same as d(momentum)/dt in eq. 13.2*

$$\sum \mathrm{F} = \frac{\mathrm{d(mv)}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} \mathrm{v}\,\rho\,\mathrm{dV} + \int_{cs} \mathrm{v}\,\rho\,(\vec{\mathrm{v}}\,\mathrm{d}\vec{\mathrm{A}}) \qquad 13.3$$

where ΣF represents all forces acting on the control volume. The forces include the *surface forces* F_s resulting from the surroundings acting on the control surface and *body forces* F_b that result from gravity and magnetic fields.

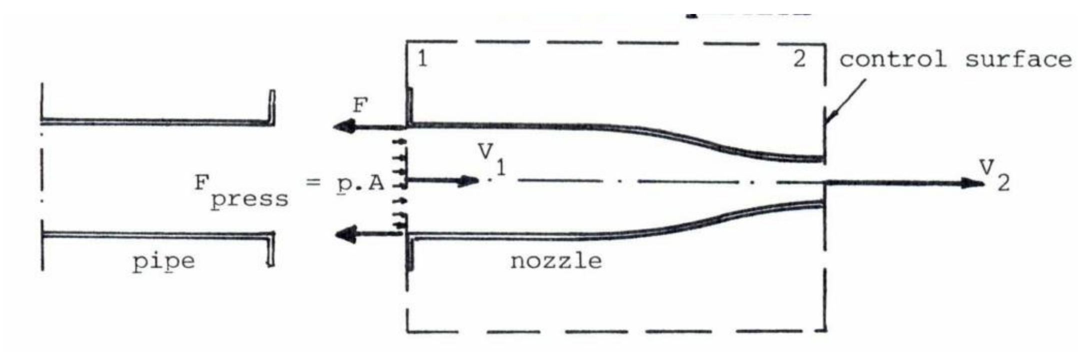


Fig 13.1 Surface forces on a nozzle control volume

$$\sum \mathrm{F_s} + \sum \mathrm{F_b} = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} \mathrm{v}\,\rho\,\mathrm{dV} + \int_{cs} \mathrm{v}\,\rho\,(\vec{\mathrm{v}}\,\mathrm{d}\vec{\mathrm{A}}) \qquad 13.4$$

This is the basic form of the momentum equation

Application

The momentum equation is often used to determine the forces induced by the flow e.g., the equation allows us to calculate the *force on the support* of the elbow in a pipeline or the *force on a submerged body* in a free-surface flow.

The vector relation may be applied for any component e.g., the x direction

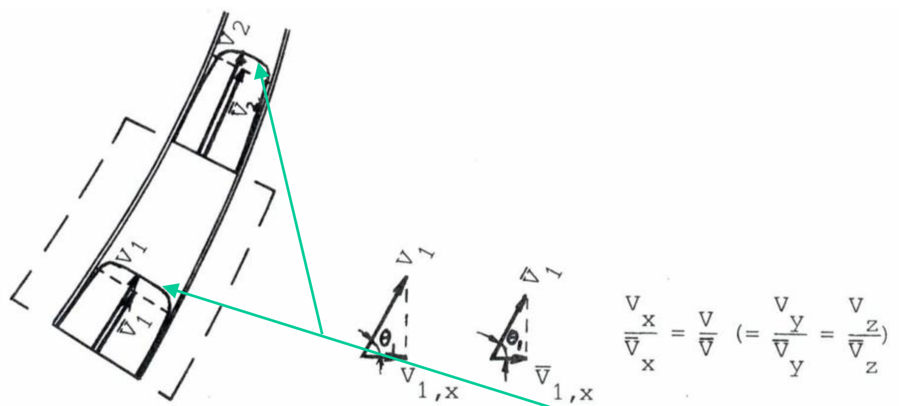


Fig 13.2 Control volume with *non-uniform* inflow and outflow normal to control surface

$$\sum F_s + \sum F_b = \frac{d}{dt}\int_{cv} v\,\rho\, dV + \int_{cs} v\,\rho\,(\vec{v}\,d\vec{A}) \quad 13.4$$

For $(\vec{v}\,d\vec{A})$ refer to Lecture 8 slides (21 to 23)

$dV = (v \cdot dt)dA\cos\theta = (\vec{v}\,d\vec{A})\,dt$

dV is the volume that has crossed dA of the cs in time dt. i.e., $v_1\,dA_1$ & $v_2\,dA_2$

$$\sum F_x = \frac{d}{dt}\int_{cv} \rho v_x\, dV + \int \rho_1 v_{1x} v_1 (-dA_1) + \int \rho_2 v_{2x} v_2\, dA_2 \quad 13.5$$

$$\sum F_x = \frac{d}{dt}\int_{cv} \rho v_x \, dV + \int \rho_1 v_{1x} v_1 (-dA_1) + \int \rho_2 v_{2x} v_2 \, dA_2 \qquad 13.5$$

Usually $\int \rho\, v_x v \, dA$ is written as $\beta \rho \bar{v}_x \bar{v} \; A$ where

$$\beta = \frac{1}{A}\int \left(\frac{v}{\bar{v}}\right)^2 dA \; = \text{mometum correction factor}$$

Hence, the momentum equation in the x, y and z directions can be written as:

$$\sum F_x = \frac{d}{dt}\int \rho v_x \, dV - \beta_1 \rho \, \bar{v}_{1x}\bar{v}_1 A_1 + \beta_2 \rho \, \bar{v}_{2x}\bar{v}_2 A_2 \qquad 13.6a$$

$$\sum F_y = \frac{d}{dt}\int \rho v_y \, dV - \beta_1 \rho \, \bar{v}_{1y}\bar{v}_1 A_1 + \beta_2 \rho \, \bar{v}_{2y}\bar{v}_2 A_2 \qquad 13.6b$$

$$\sum F_z = \frac{d}{dt}\int \rho v_z \, dV - \beta_1 \rho \, \bar{v}_{1z}\bar{v}_1 A_1 + \beta_2 \rho \, \bar{v}_{2z}\bar{v}_2 A_2 \qquad 13.6c$$

For turbulent flow in a circular pipe $1.01 < \beta < 1.07$, which is for most purposes differs negligibly from unity.

These equation are really $\sum F = \frac{d}{dt}\int \rho \, dV + \rho Q(\Delta v)$