CHAPTER

2

PRESSURE MEASUREMENT

2.1. *Pressure of a liquid.* 2.2. *Pressure head of a liquid.* 2.3. *Pascal's law.* 2.4. *Absolute and gauge pressures.* 2.5. *Measurement of pressure–Manometers–Mechanical gauges. Highlights–Objective Type Questions – Theoretical Questions–Unsolved Examples.*

2.1 Pressure of a Fluid

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The *force per unit area is called* **pressure.**

If, P = The force, and

A = Area on which the force acts; then intensity of pressure, $p = \frac{P}{A}$ ...(2.1)

The pressure of a fluid on a surface will *always act normal to the surface.*

2.2 Pressure Head of a Liquid

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

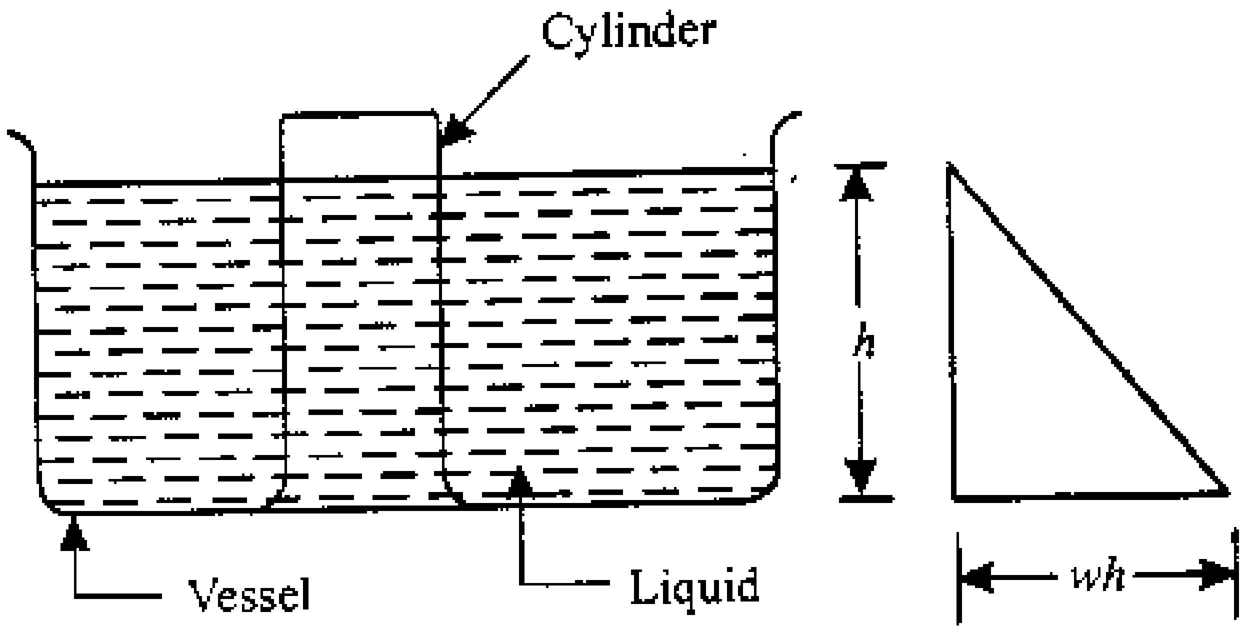


Fig. 2.1. Pressure head.

Consider a vessel containing liquid, as shown in Fig. 2.1. The liquid will exert pressure on all sides and bottom of the vessel. Now, let cylinder be made to stand in the liquid, as shown in the figure.

Let, h = Height of liquid in the cylinder,

A = Area of the cylinder base,

w = Specific weight of the liquid,

and, p = Intensity of pressure.

Now, Total pressure on the base of the cylinder = Weight of liquid in the cylinder

i.e., $p.\ A. = wAh$

$$p = \frac{wAh}{A} = wh \qquad i.e., \qquad p = wh \qquad ...(2.2)$$

As $p = wh$, *the intensity of pressure in a liquid due to its depth will vary directly with depth.*

As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a liquid pressure by the *height of the free surface* which would cause the pressure, *i.e.,*

$$h = \frac{p}{w} \qquad \text{[from eqn. (2.2)]}$$

The height of the free surface above any point is known as the *static head* at that point. In this case, static head is h.

Hence, the intensity of pressure of a liquid may be expressed in the following two ways:

1. As a force per unit area (i.e., N/mm^2, N/m^2), and
2. As an equivalent static head (i.e., metres, mm or cm of liquid).

Alternatively:

Pressure variation in fluid at rest :

In order to determine the pressure at any point in a fluid at rest "*hydrostatic law*" is used; the law states as follows:

"*The rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.*"

The proof of the law is as follows.

Refer to Fig. 2.2

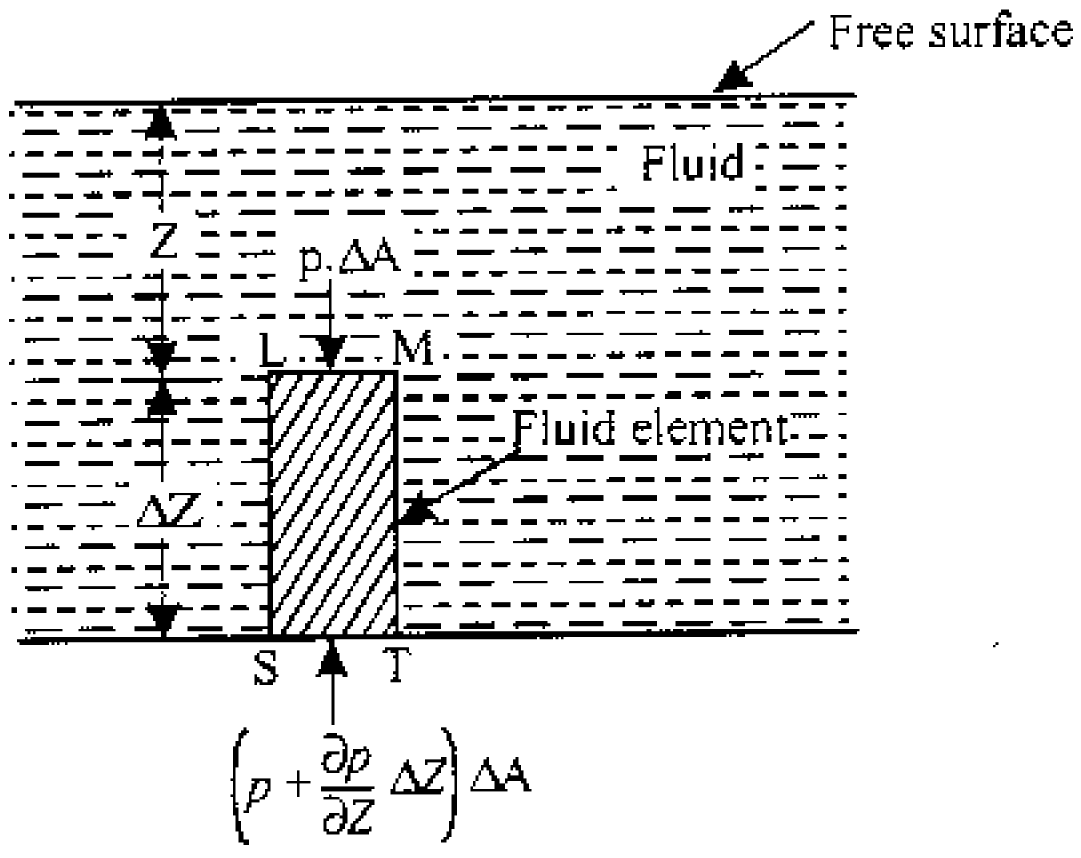


Fig. 2.2. Forces acting on a fluid element

Let, p = Intensity of pressure on face LM,

ΔA = Cross-sectional area of the element,

Z = Distance of the fluid element from free surface, and

ΔZ = Height of the fluid element.

The forces acting on the element are:

(*i*) Pressure force on the face $LM = p \times \Delta A$ ...(acting downward)

(*ii*) Pressure force on the face $ST = \left(p + \frac{\partial p}{\partial Z} \times \Delta Z\right) \times \Delta A$ ... (acting upward)

(*iii*) Weight of the fluid element = Weight density × volume

$= w \times (\Delta A \times \Delta Z)$

(*iv*) Pressure forces on surfaces MT and LS ..... are equal and opposite.

For equilibrium of the fluid element, we have

$$p \times \Delta A - \left[p + \frac{\partial p}{\partial Z} \times \Delta Z\right] \times \Delta A + w \times (\Delta A \times \Delta Z) = 0$$

or,
$$p \times \Delta A - p \times \Delta A - \frac{\partial p}{\partial Z} \times \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or,
$$\frac{\partial p}{\partial Z} \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or,
$$\frac{\partial p}{\partial Z} = w \text{ (cancelling } \Delta Z \times \Delta A \text{ from both the sides)}$$

or,
$$\frac{\partial p}{\partial Z} = \rho \times g \qquad (\because w = \rho \times g) \qquad ...(2.3)$$

Eqn. (2.3.) states *that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point.* This is "**hydrostatic law**".

On integrating the eqn. (2.3), we get

$$\int dp = \int \rho g . dZ$$

or,
$$p = \rho\, g . Z \,(= wZ) \qquad ...(2.4)$$

where, p is the pressure above atmospheric pressure.

From eqn. (2.4), we have

$$Z = \frac{p}{\rho.g}\left(=\frac{p}{w}\right) \qquad ...[2.5]$$

Here Z is known as *pressure head.*

Example 2.1. *Find the pressure at a depth of 15 m below the free surface of water in a reservoir.*

Solution. Depth of water, $h = 15$ m

Specific weight of water, $w = 9.81$ kN/m^3

Pressure p:

We know that, $p = wh = 9.81 \times 15 = 147.15$ kN/m^2

i.e., $p = 147.15$ kN/m^2 = **147.15 kPa (Ans.)**

Example 2.2. *Find the height of water column corresponding to a pressure of 54 kN/m^2.*

Solution. Intensity of pressure, $p = 54$ kN/m^2

Specific weight of water, $w = 9.81$ kN/m^3

Height of water column, h:

Using the relation: $p = wh$; $h = \frac{p}{w} = \frac{54}{9.81} =$ **5.5 m (Ans.)**

2.3. Pascal's Law

The **Pascal's law** states as follows :

"*The intensity of pressure at any point in a liquid at rest, is the same in all directions*".

Proof. Let us consider a very small wedge shaped element LMN of a liquid, as shown in Fig. 2.3.

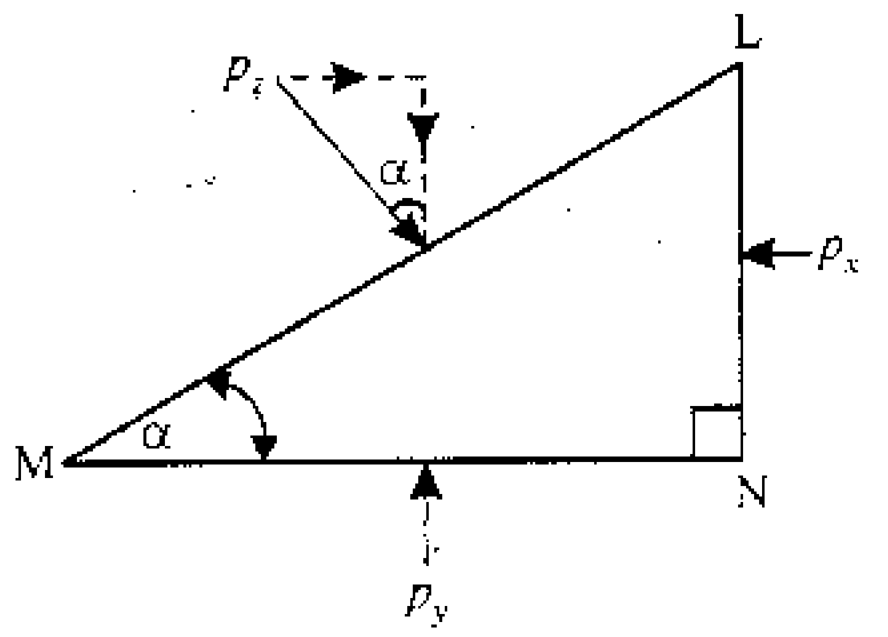


Fig. 2.3. Pressure on a fluid element at rest

Let, p_x = Intensity of horizontal pressure on the element of liquid,

p_y = Intensity of vertical pressure on the element of liquid,

p_z = Intensity of pressure on the diagonal of the right angled triangular element,

α = Angle of the element of the liquid,

P_x = Total pressure on the vertical side LN of the liquid,

P_y = Total pressure on the horizontal side MN of the liquid, and

P_z = Total pressure on the diagonal LM of the liquid.

Now, $P_x = p_x \times LN$ ...(*i*)

and, $P_y = p_y \times MN$ ...(*ii*)

and, $P_z = p_z \times LM$ ...(*iii*)

As the element of the liquid is at rest, therefore the *sum of horizontal and vertical components of the liquid pressures must be equal to zero.*

Resolving the forces *horizontally*:

$$P_z \sin\alpha = P_x$$

$$p_z . LM. \sin\alpha = p_x . LN \qquad (\because P_z = p_z . LM)$$

But, $LM . \sin\alpha = LN$ ... From Fig 2.3

$\therefore$ $p_z = p_x$ ...(*iv*)

Resolving the forces *vertically*:

$$P_z .\cos\alpha = P_y - W$$

(where, W = weight of the liquid element)

Since the element is very small, neglecting its weight, we have

$$P_z \cos\alpha = P_y \quad \text{or} \quad p_z \,.\, LM\cos\alpha = p_y \,.MN$$

But, $LM\cos\alpha = MN$ ...From Fig 2.3

$\therefore$ $p_z = p_y$ ...(*v*)

From (*iv*) and (*v*), we get $p_x = p_y = p_z$

which is independent of α.

Hence, at *any point in a fluid at rest the intensity of pressure is exerted equally in all directions, which is called* **Pascal's law**.

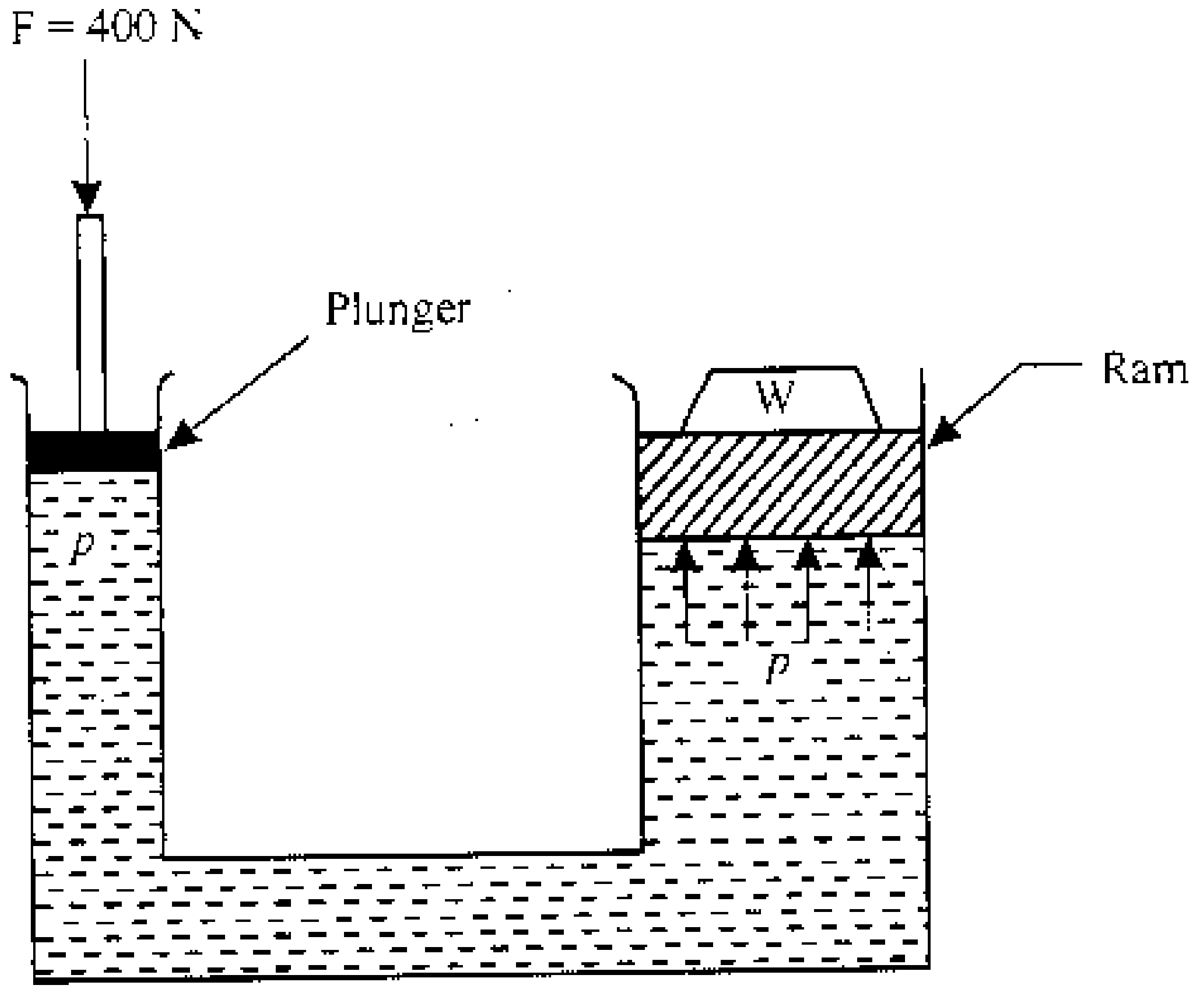
Example 2.3. *The diameters of ram and plunger of an hydraulic press are 200 mm and 30 mm respectively. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N.*

Solution. Diameter of the ram,

$$D = 200\text{ mm} = 0.2\text{ m}$$

Diameter of the plunger, $d = 30\text{ mm} = 0.03\text{ m}$

Force on the plunger, $F = 400\text{ N}$



Hydraulic press

Fig. 2.4

Load lifted, W:

Area of ram,

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\times 0.2^2 = 0.0314\text{ m}^2$$

Area of plunger,

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times 0.03^2 = 7.068\times 10^{-4}\text{ m}^2$$

Intensity of pressure due to plunger,

$$p = \frac{F}{a} = \frac{400}{7.068\times 10^{-4}} = 5.66\times 10^5\ \text{N/m}^2$$

Since the intensity of pressure will be equally transmitted (due to Pascal's law), therefore the intensity of pressure at the ram is also

$$= p = 5.66\times 10^5\ \text{N/m}^2$$

But intensity of pressure at the ram $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{\text{W}}{\text{A}} = \dfrac{W}{0.0314}\text{N/m}^2$

$\therefore$ $\dfrac{W}{0.0314} = 5.66\times 10^5$ or $W = 0.0314\times 5.66\times 10^5\text{ N} = 17.77\times 10^3\text{ N}$ or **17.77 kN (Ans.)**

Example 2.4. *For the hydraulic jack shown in Fig. 2.5 find the load lifted by the large piston when a force of 400 N is applied on the small piston. Assume the specific weight of the liquid in the jack is 9810 N/m³.*

Solution. Diameter of small piston, $d = 30 \text{ mm} = 0.03 \text{ m}$

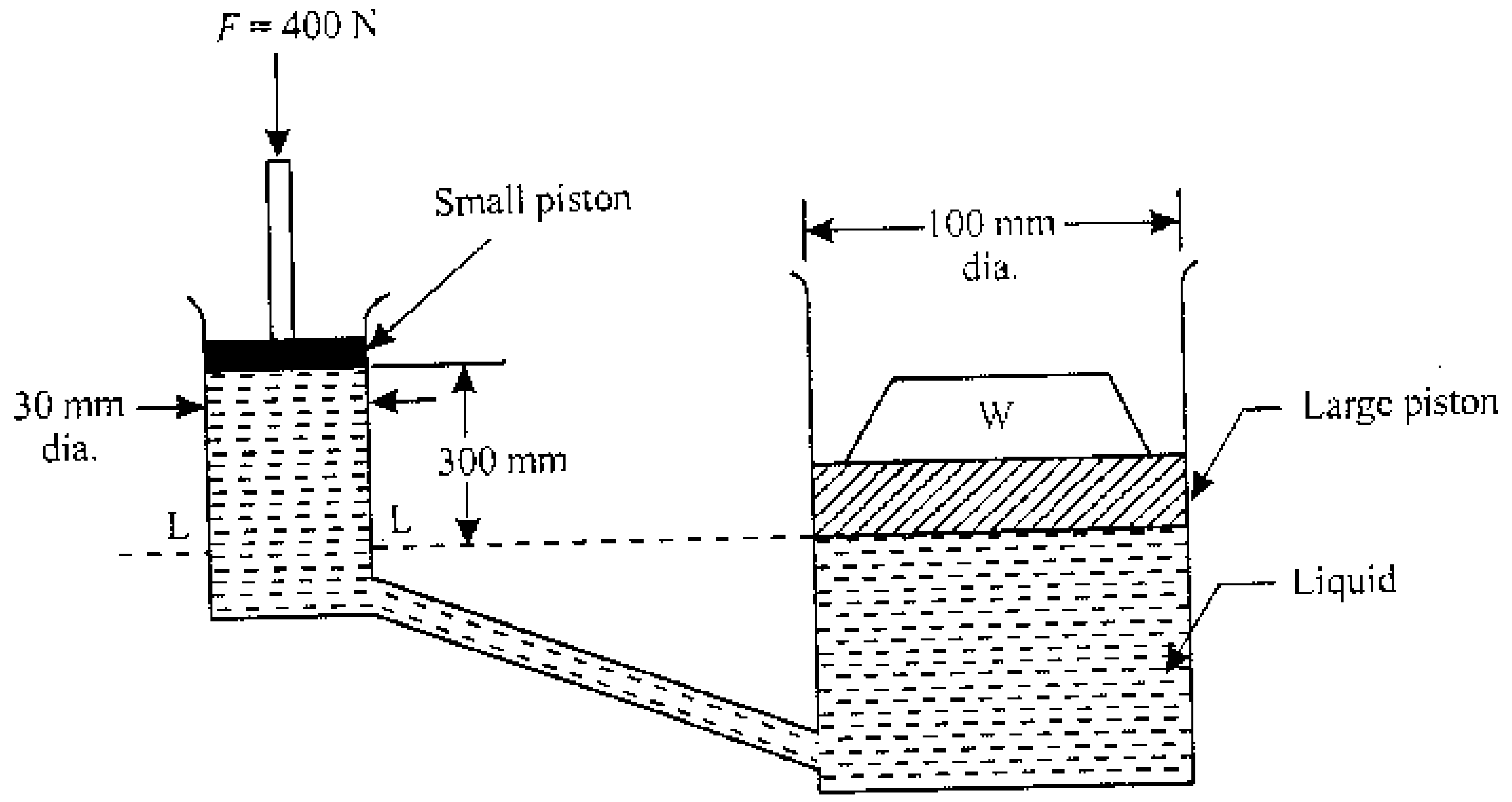


Fig. 2.5

Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.03^2 = 7.068 \times 10^{-4} \text{ m}^2$

Diameter of the large piston, $D = 100 \text{ mm} = 0.1 \text{ m}$

Area of large piston, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.1^2 = 7.854 \times 10^{-3} \text{ m}^2$

Force on small piston, $F = 400 \text{ N}$

Load lifted, W:

Pressure intensity on small piston, $p = \frac{F}{a} = \frac{400}{7.068 \times 10^{-4}} = 5.66 \times 10^5 \text{ N/m}^2$

Pressure intensity at section LL,

$$p_{LL} = \frac{F}{a} + \text{Pressure intensity due to height of 300 mm of liquid}$$

$$= \frac{F}{a} + wh = 5.66 \times 10^5 + 9810 \times \frac{300}{1000}$$

$$= 5.66 \times 10^5 + 2943 = 5.689 \times 10^5 \text{ N/m}^2$$

Pressure intensity transmitted to the large piston = $5.689 \times 10^5 \text{ N/m}^2$

Force on the large piston = Pressure intensity × area of large piston

$$= 5.689 \times 10^5 \times 7.854 \times 10^{-3} = 4468 \text{ N}$$

Hence, *load lifted by the large piston* = **4468 N (Ans.)**

2.4. Absolute and Gauge Pressures

Atmospheric pressure:

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as *atmospheric pressure*. The atmospheric pressure is also known as *'Barometric pressure'*.

The atmospheric pressure at sea level (above absolute zero) is called *'Standard atmospheric pressure'*.

Note. The local atmospheric pressure may be a little lower than these values if the place under question is higher than sea level, and higher values if the place is lower than sea level, due to the corresponding decrease or increase of the column of air standing, respectively.

Example 2.5. *Given that:*

Barometer reading = 740 mm of mercury;

Specific gravity of mercury = 13.6; Intensity of pressure = 40 kPa.

Express the intensity of pressure in S.I. units, both gauge and absolute.

Solution. Intensity of pressure, $p = 40$ kPa

Gauge pressure:

(*i*) $p = 40\text{ kPa} = 40\text{ kN/m}^2 = 0.4 \times 10^5\text{ N/m}^2 = \mathbf{0.4\ bar}$ **(Ans.)**

$(1\text{ bar} = 10^5\text{ N/m}^2)$

(*ii*) $$h = \frac{p}{w} = \frac{0.4 \times 10^5}{9.81 \times 10^3} = \mathbf{4.077\ m\ of\ water}\ \textbf{(Ans.)}$$

(*iii*) $$h = \frac{p}{w} = \frac{0.4 \times 10^5}{9.81 \times 10^3 \times 13.6} = \mathbf{0.299\ m\ of\ mercury}\ \textbf{(Ans.)}$$

$$\left[\begin{array}{l}\text{Where, } w = \text{specific weight;}\\ \text{For water} : w = 9.81\text{ kN/m}^3\\ \text{For mercury} : w = 9.81 \times 13.6\text{ kN/m}^3\end{array}\right]$$

Absolute pressure:

Barometer reading (atmospheric pressure)

$$= 740\text{ mm of mercury} = 740 \times 13.6\text{ mm of water}$$

$$= \frac{740 \times 13.6}{1000} = 10.06\text{ m of water}$$

Absolute pressure ($p_{abs.}$) = Atmospheric pressure ($p_{atm.}$) + gauge pressure (p_{gauge}).

$\therefore\ p_{abs.} = 10.06 + 4.077 = \mathbf{14.137\ m\ of\ water}$ **(Ans.)**

$= 14.137 \times (9.81 \times 10^3) = \mathbf{1.38 \times 10^5\ N/m^2}$ **(Ans.)** $(p = wh)$

$= \mathbf{1.38\ bar}$ **(Ans.)** $(1\text{ bar} = 10^5\text{ N/m}^2)$

$$= \frac{14.137}{13.6} = \mathbf{1.039\ m\ of\ mercury}.\ \textbf{(Ans.)}$$

Example 2.6. *Calculate the pressure at a point 5 m below the free water surface in a liquid that has a variable density given by relation*

$$\rho = (350 + Ay)\ kg/m^3$$

where $A = 8$ kg/m⁴ and y is the distance in metres measured from the free surface.

Solution. As per hydrostatic equation

$$dp = \rho{\cdot}g{\cdot}dy = g\,(350 + Ay)dy$$

Integrating both sides, we get

$$\int dp = \int_0^5 g(350 + Ay)\,dy = g\int_0^5 (350 + 8y)\,dy$$

$$\mathbf{p} = g\left|350y + 8 \times \frac{y^2}{2}\right|_0^5$$

$$= 9.81\,(350 \times 5 + 8 \times \frac{5^2}{2}) = 18148\text{ kN/m}^2 = \mathbf{18.15 kN/m^2}\ \textbf{(Ans.)}$$

Example 2.7. *On the suction side of a pump a gauge shows a negative pressure of 0.35 bar. Express this pressure in terms of:*

(*i*) *Intensity of pressure, kPa.*
(*ii*) *N/m^2 absolute.*
(*iii*) *Metres of water gauge.*
(*iv*) *Metres of oil (specific gravity 0.82) absolute.*
(*v*) *Centimetres of mercury gauge.*

Take atmospheric pressure as 76 cm of Hg and relative density of mercury as 13.6.

Solution. *Given*: Reading of the *vacuum* gauge = 0.35 bar

(*i*) **Intensity of pressure, kPa:**

$$\text{Gauge reading} = 0.35 \text{ bar} = 0.35 \times 10^5 \text{ N/m}^2$$
$$= 0.35 \times 10^5 \text{ Pa} = \mathbf{35 \text{ kPa (Ans.)}}$$

(*ii*) **N/m^2 absolute:**

Atmospheric pressure, $p_{atm.} = 76$ cm of Hg

$$= (13.6 \times 9810) \times \frac{76}{100} = 101396 \text{ N/m}^2$$

Absolute pressure = Atmospheric pressure − *Vacuum* pressure

$$p_{abs.} = p_{atm} - p_{vac.}$$
$$= 101396 - 35000 = \mathbf{66396 \text{ N/m}^2 \text{ absolute (Ans.)}}$$

(*iii*) **Metres of water gauge:**

$$p = \rho gh = wh$$

$$\therefore \quad h_{water}\text{(gauge)} = \frac{p}{w} = \frac{0.35 \times 10^5}{9810} = \mathbf{3.567 \text{ m}}\ (\textit{gauge})\ \mathbf{(Ans.)}$$

iv) **Metres of oil (sp. gr. = 0.82) absolute:**

$$h_{oil}\text{(absolute)} = \frac{66396}{0.82 \times 9810} = \mathbf{8.254 \text{ m of water}}\ (\textit{absolute})\ \mathbf{(Ans.)}$$

(*v*) **Centimetres of mercury gauge:**

$$h_{mercury}\text{(gauge)} = \frac{0.35 \times 10^5}{13.6 \times 9810} = 0.2623 \text{ m of mercury}$$
$$= \mathbf{26.236 \text{ cm of mercury (Ans.)}}$$

Example 2.8. *The inlet to pump is 10.5 m above the bottom of sump from which it draws water through a suction pipe. If the pressure at the pump inlet is not to fall below 28 kN/m^2 absolute, work out the minimum depth of water in the tank.*

Assume atmospheric pressure as 100 kPa.

Solution. *Given*: $p_{atm.} = 100$ kPa $= 100$ kN/m^2; $p_{abs.} = 28$ kN/m^2.

Minimum depth of water in the tank:

Let, $p_{vac.}$ = The vacuum (suction) pressure at the pump inlet.

Then, $p_{vac.} = p_{atm} - p_{abs.}$
$= (100 - 28) = 72$ kN/m^2 or 72000 N/m^2

Further, let h be the distance between the pump inlet and free water surface in the sump.

Invoking hydrostatic equation, we have

$$p = wh$$
$$72000 = 9810 \times h$$

or
$$h = \frac{72000}{9810} = 7.339 \text{ m}$$

∴ Minimum depth of water in the tank

$$= 10.5 - 7.339 = \mathbf{3.161\ m\ (Ans.)}$$

Example 2.9. *A cylindrical tank of cross-sectional area 600 mm² and 2.6 m height is filled with water upto a height of 1.5 m and remaining with oil of specific gravity 0.78. The vessel is open to atmospheric pressure. Calculate:*

(*i*) *Intensity of pressure at the interface.*

(*ii*) *Absolute and gauge pressures on the base of the tank in terms of water head, oil head and N/m².*

(*iii*) *The net force experienced by the base of the tank.*

Assume atmospheric pressure as 1.0132 bar.

Solution. *Given:* Area of cross-section of the tank, $A = 600\ \text{mm}^2 = 600 \times 10^{-6}$;

sp.gr. of oil = 0.78; $p_{\text{atm.}} = 1.0132$ bar.

(*i*) **Intensity of pressure at the interface:**

The pressure intensity at the interface between the oil and water is due to 1.1 m of oil and is given by,

$$p_{\text{interface}} = wh$$
$$= (0.78 \times 9810) \times 1.1$$
$$= \mathbf{8417\ N/m^2\ (Ans.)}$$

(*ii*) **Absolute and gauge pressure on the base of the tank:**

Pressure at the base of the tank

= Pressure at the interface (due to 1.1 m of oil) + pressure due to 1.5 m of water

i.e., $p_{\text{base (gauge)}} = 8417 + (9810 \times 1.5)$

$$= \mathbf{23132\ N/m^2\ (gauge)\ (Ans.)}$$

$$= \frac{23132}{9810} = \mathbf{2.358\ m\ of\ water\ (gauge)\ (Ans.)}$$

$$= \frac{23132}{0.78 \times 9810} = \mathbf{3.023\ m\ of\ oil\ (gauge)\ (Ans.)}$$

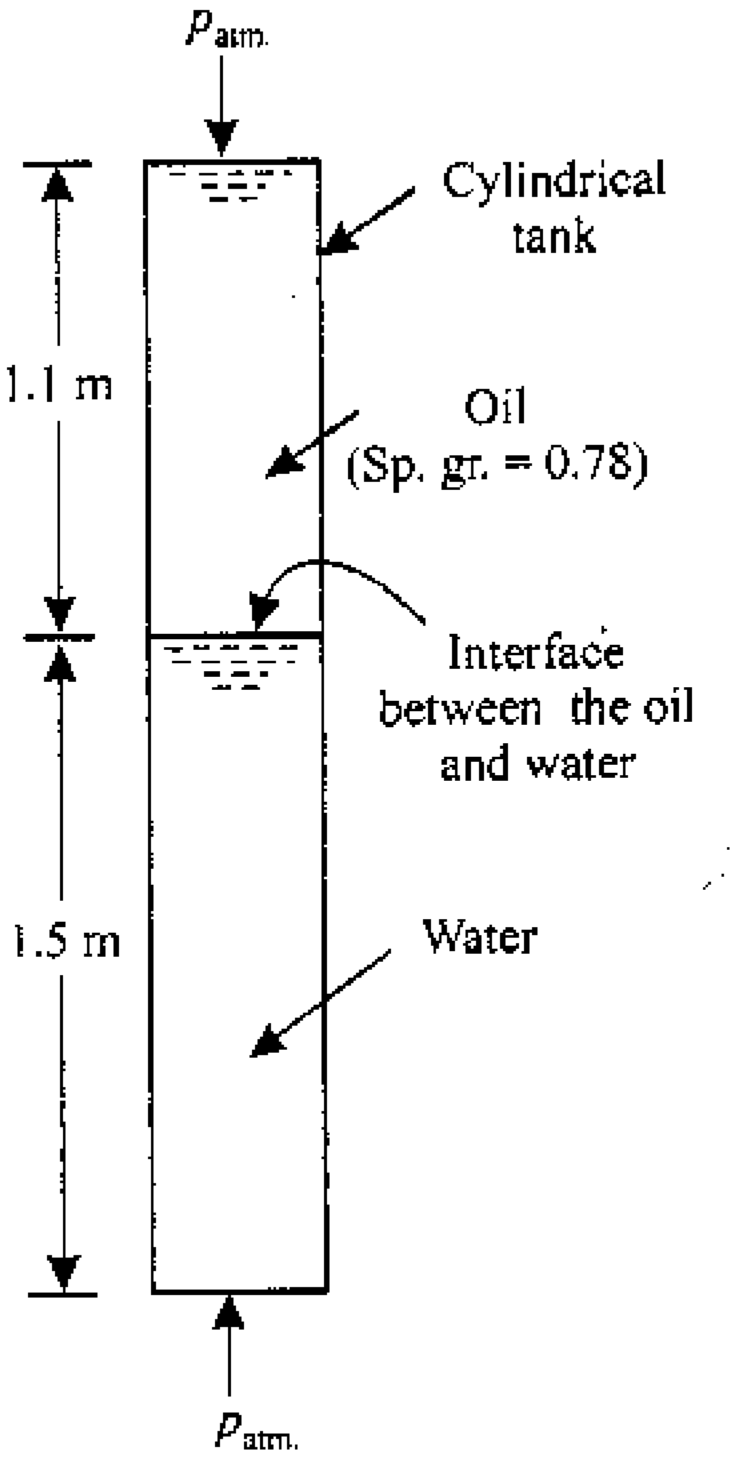


Fig. 2.7

Atmospheric pressure, $p_{\text{atm.}} = 1.0132$ bar

$= 1.0132 \times 10^5\ \text{N/m}^2$

$$= \frac{1.0132 \times 10^5}{9810} = 10.328\ \text{m of water}$$

$$= \frac{1.0132 \times 10^5}{0.78 \times 9810} = 13.241\ \text{m of oil}$$

Absolute pressure = Atmospheric pressure + gauge pressure

$$p_{\text{base}}\text{(absolute)} = 10.328 + 2.358 = \mathbf{12.686\ m\ of\ water\ (Ans.)}$$
$$= 13.241 + 3.023 = \mathbf{16.264\ m\ of\ oil\ (Ans.)}$$
$$= 101320 + 23132 = \mathbf{124452\ N/m^2\ (Ans.)}$$

(*iii*) **The net force experienced by the base of the tank:**

$$F\ (= P) = p_{\text{base}}\text{(gauge)} \times \text{cross-sectional area}$$
$$= 23132 \times 600 \times 10^{-6} = \mathbf{13.879\ N\ (Ans.)}$$

Example 2.10. (*a*) *What is hydrostatic paradox?*

(*b*) *A cylinder of 0.25 m diameter and 1.2 m height is fixed centrally on the top of a large cylinder of 0.9 m diameter and 0.8 m height. Both the cylinders are filled with water. Calculate:*

(*i*) *Total pressure at the bottom of the bigger cylinder, and*

(*ii*) *Weight of total volume of water.*

What is hydrostatic paradox between the two results and how this difference can be reconciled?

Solution. (*a*) **Hydrostatic paradox:**

Fig. 2.8 shows three vessels 1, 2 and 3 having the same area A at the bottom and each filled a liquid upto the same height h.

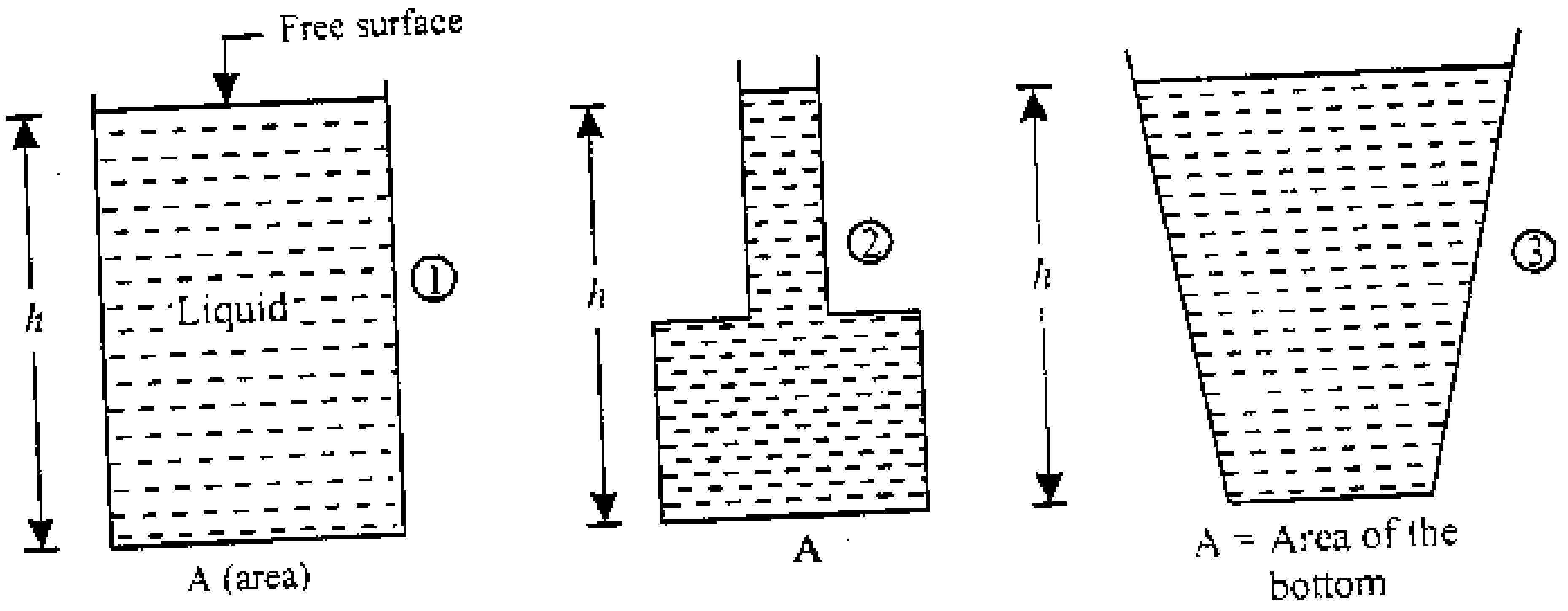


Fig. 2.8. Hydrostatic paradox

According to the **hydrostatic equation,** $p = wh$; the intensity of pressure (p) *depends only on the height of the column and not at all upon the size of the column*. Thus, in all these vessels of different shapes and sizes, the same intensity of pressure would be exerted on the bottom of each of these vessels. Since each of the vessels has the same area A at the bottom, the pressure force $P = p \times A$ on the base of each vessel would be same. This is independent of the fact that the weight of liquid in each vessel is different. *This situation is referred as* **hydrostatic paradox.**

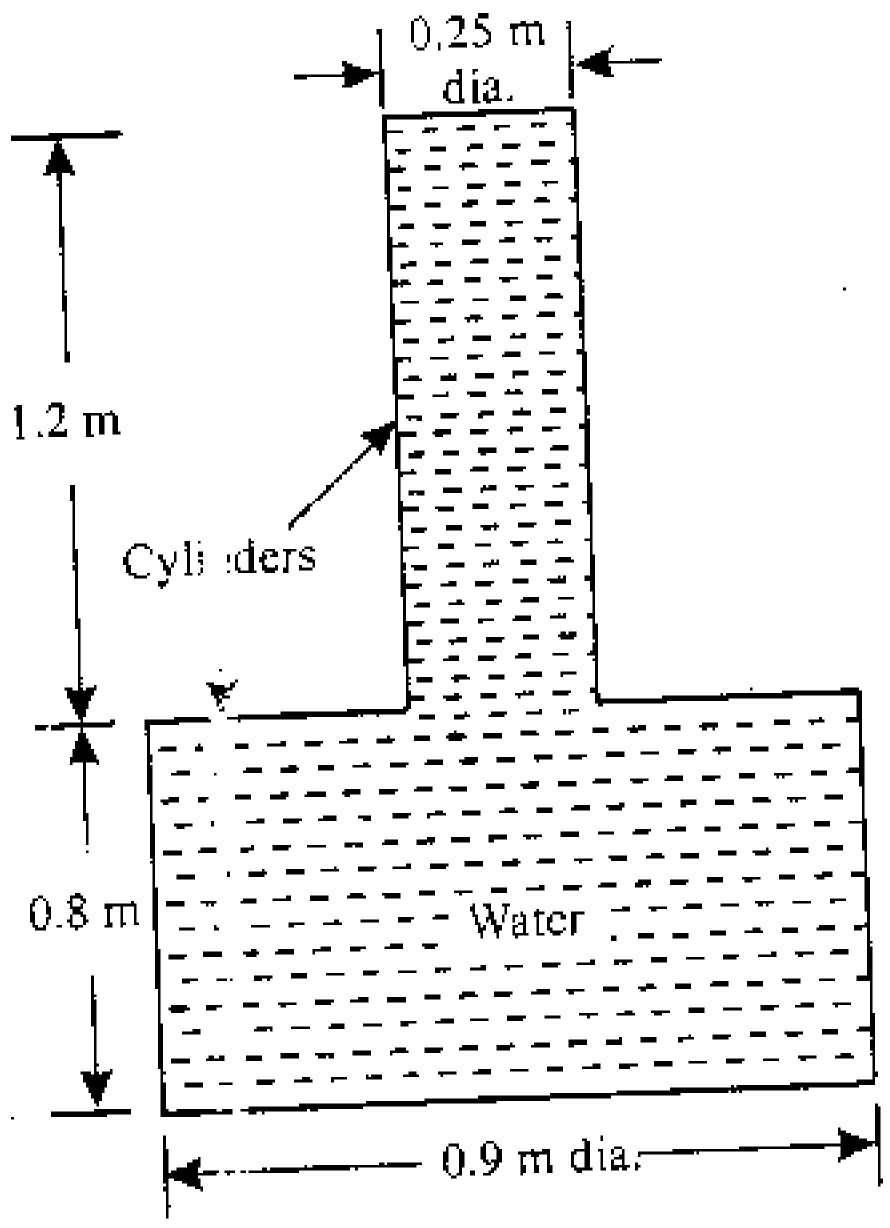


Fig. 2.9

(*b*) Area at the bottom,

$$A = \frac{\pi}{4} \times (0.9)^2 = 0.6362 \text{ m}^2$$

Intensity of pressure at the bottom

$$p = wh = 9810 \times (1.2 + 0.8)$$

$$= 19620 \text{ N/m}^2$$

Total pressure force at the bottom

$$P = p \times A = 19620 \times 0.6362 = 12482 \text{ N}$$

Weight of total volume of water contained in the cylinders,

$$W = w \times \text{volume of water}$$

$$= 9810 \left[\frac{\pi}{4} \times 0.9^2 \times 0.8 + \frac{\pi}{4} \times 0.25^2 \times 1.2 \right] = 5571 \text{ N}$$

From the above calculations it may be observed that the *total pressure force at the bottom of the cylinder is greater than the weight of total volume of water (W) contained in the cylinders.* This is **hydrostatic paradox.**

The following is the *explanation of the hydrostatic paradox*: Refer to Fig. 2.9.

Total pressure force on the bottom of bigger tank = 12482 N (downward). A reaction at the roof of the lower tank is caused by the upward force which equals,

$$wAh = 9810 \times \frac{\pi}{4}(0.9^2 - 0.25^2) \times 1.2 = 6911\,\text{N} \;\; \text{(upward)}$$

The distance h corresponding to depth of water in the cylinder fixed centrally on the top of larger cylinder.

Net downward force exerted by water = 12482 – 6911 = 5571 N and it equals the weight of water in the two cylinder.

2.5 Measurement of Pressure

The pressure of a fluid may be measured by the following devices:

1. Manometers:

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid. These are classified as follows:

(*a*) Simple manometers:

(*i*) Piezometer, (*ii*) U-tube manometer, and (*iii*) Single column manometer.

(*b*) Differential manometers.

2. Mechanical gauges:

These are the devices in which the pressure is measured by balancing the fluid column by spring (elastic element) or dead weight. Generally these gauges are used for measuring *high pressure* and where *high precision is not required*. Some commonly used mechanical gauges are:

(*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,

(*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.

2.5.1 Manometers

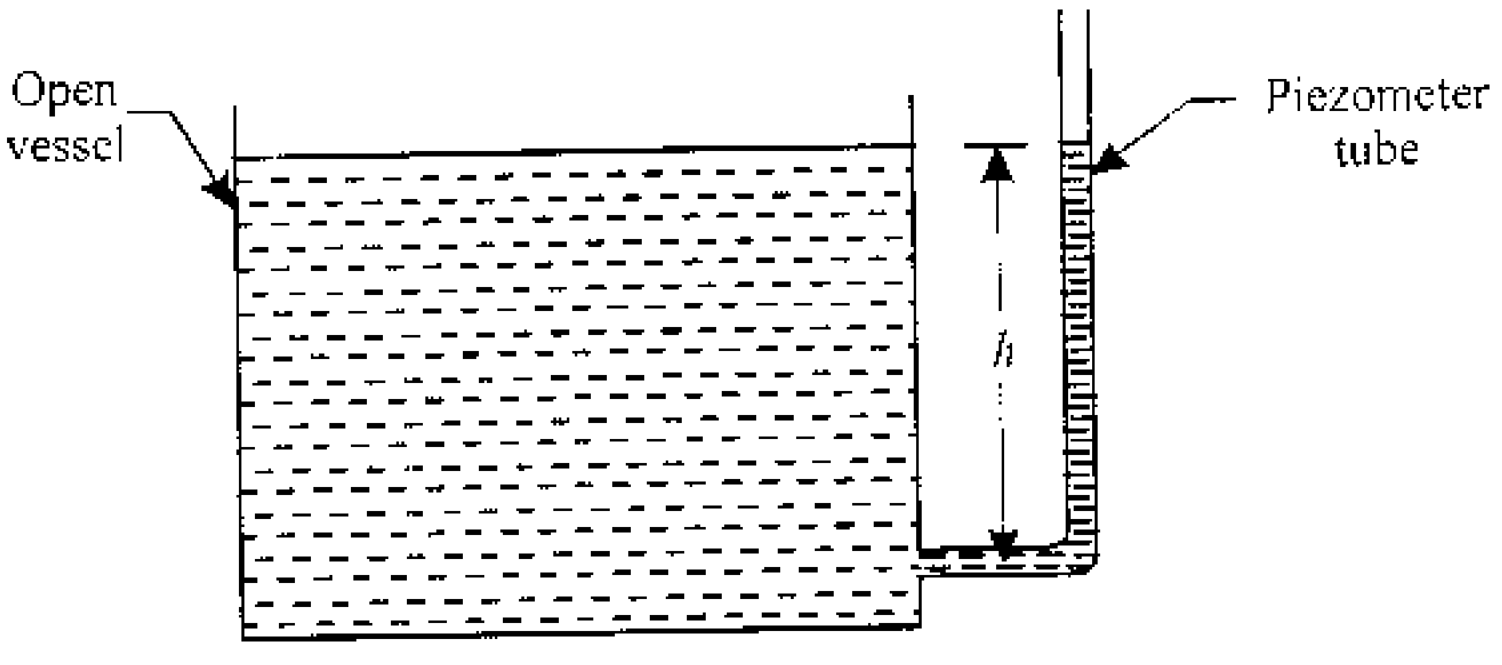
2.5.1.1. Simple manometers

A "simple manometer" is one which consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other end remains open to atmosphere.

Common types of simple manometers are discussed below:

1. Piezometer:

A piezometer is the simplest form of manometer which can be used for measuring *moderate pressures* of liquids. It consists of a glass tube (Fig 2.10) inserted in the wall of a vessel or of a pipe, containing liquid whose pressure is to be measured. The tube extends vertically upward to such a height that liquid can freely rise in it without overflowing. The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point, which can be read on the scale attached to it. Thus if w is the specific weight of the liquid, then the pressure at point $A(p)$ is given by



(a)

Fig. 2.10. (*a*) Piezometer tube fitted to open vessel.

$$p = wh$$

Piezometers measure *gauge pressure only* (at the surface of the liquid), since the surface of the liquid in the tube is subjected to atmospheric pressure. A piezometer tube is *not suitable* for measuring *negative pressure*; as in such a case the air will enter in pipe through the tube.

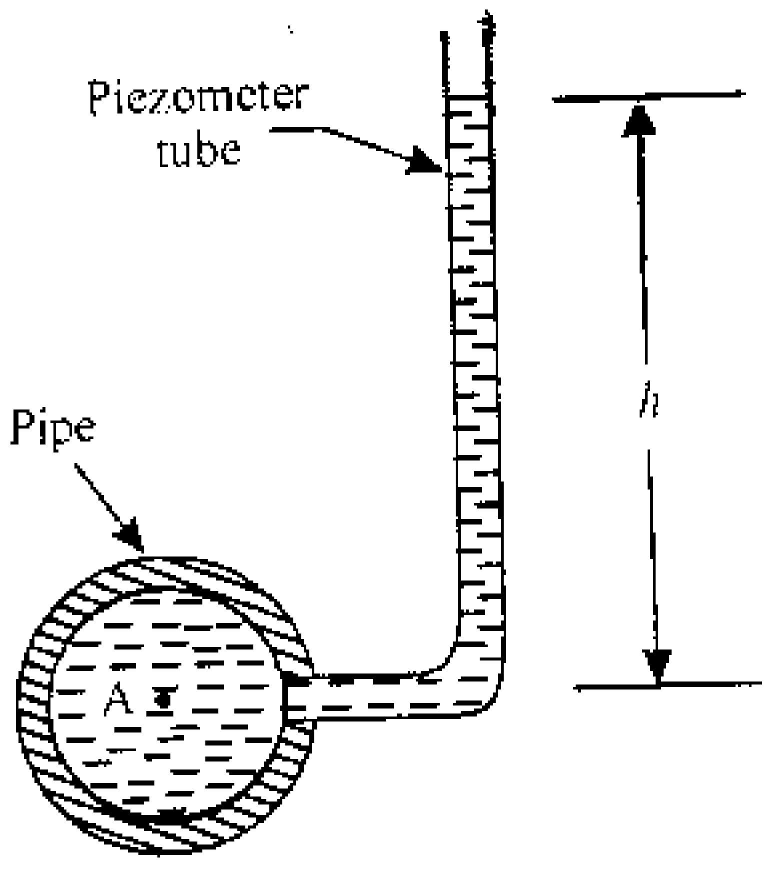


Fig. 2.10. (*b*) Piezometer tube fitted to a closed pipe.

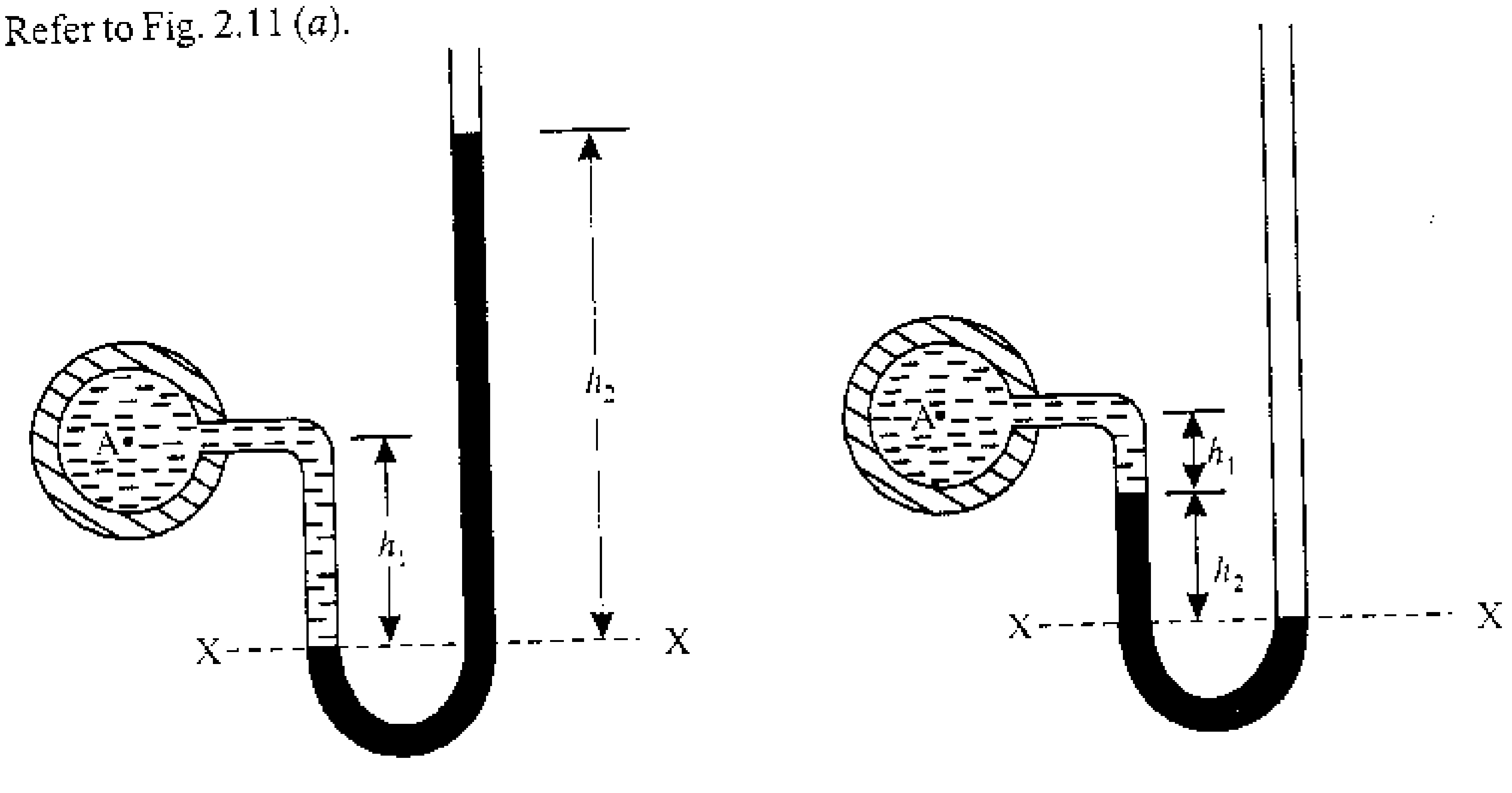
2. U-tube manometer:

Piezometers cannot be employed when large pressures in the *lighter liquids* are to be measured, since this would require *very long tubes*, which cannot be handled conveniently. Furthermore gas pressures cannot be measured by the piezometers because a *gas forms no free atmospheric surface*. These limitations can be overcome by the use of U-tube manometers.

A U-tube manometer consists of a glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.11.

(*i*) **For positive pressure:**

Refer to Fig. 2.11 (*a*).



(*a*) (*b*)

Fig. 2.11. U-tube manometer.

Let, A be the point at which pressure is to be measured. X–X is the datum line as shown in Fig. 2.11 (*a*).

Let, h_1 = Height of the light liquid in the left limb above the datum line,
h_2 = Height of the heavy liquid in the right limb above the datum line,
h = Pressure in pipe, expressed in terms of head,
S_1 = Specific gravity of the light liquid, and
S_2 = Specific gravity of the heavy liquid.

The pressures in the left limb and right limb above the datum line X–X are equal (*as the pressures at two points at the same level in a continuous homogeneous liquid are equal*).

Pressure head above X–X in the *left limb* = $h + h_1 S_1$

Pressure head above X–X in the *right limb* = $h_2 S_2$

Equating these two pressures, we get

$$h + h_1 S_1 = h_2 S_2 \quad \text{or} \quad h = h_2 S_2 - h_1 S_1 \qquad ...(2.6)$$

(*ii*) **For negative pressure:**

Refer to Fig. 2.11 (*b*)

Pressure head above *X–X* in the *left limb* = $h + h_1 S_1 + h_2 S_2$

Pressure head above *X–X* in the *right limb* = 0.

Equating these two pressures, we get

$$h + h_1 S_1 + h_2 S_2 = 0 \quad \text{or} \quad h = -(h_1 S_1 + h_2 S_2) \qquad \text{...(2.7)}$$

Example 2.11. *In a pipeline water is flowing. A manometer is used to measure the pressure drop for flow through the pipe. The difference in level was found to be 20 cm. If the manometric fluid is* CCl_4*, find the pressure drop in S.I units (density of* CCl_4 *= 1.596 g/cm³). If the manometric fluid is changed to mercury (ρ = 13.6gm/cm³) what will be the difference in level?* **(AMIE Winter, 1996)**

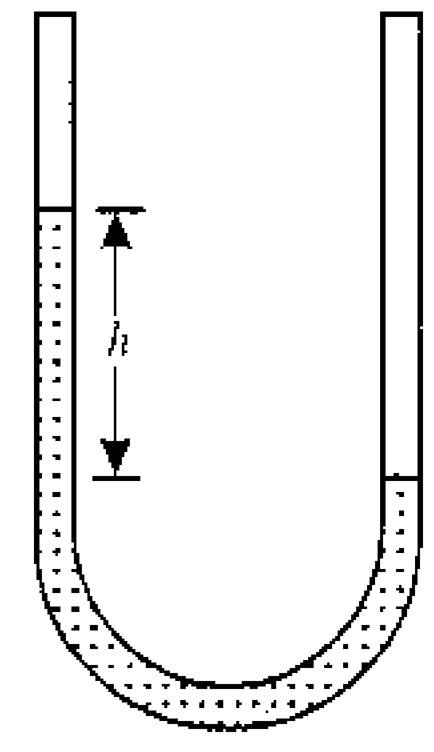


Fig. 2.12

Solution. *Given:* $h_{CCl_4} = 20\,\text{cm} = 0.2\,\text{m}$; $\rho_{CCl4} = 1.596\ \text{g/cm}^3$

$= 1.596 \times 10^3\ \text{kg/m}^3$

$\rho_{Hg} = 13.6 \times 10^3\ \text{kg/m}^3$

Pressure drop, $\Delta p = \rho_{CCl_4}\, g\, h_{CCl_4}$

$= 1.596 \times 10^3 \times 9.81 \times 0.2\ \text{N/m}^2$

$= 3131.3\ \text{N/m}^2$ or Pa = **3.131 kPa (Ans.)**

The difference in level with mercury,

$$h_{Hg} = h_{CCl_4} \times \frac{\rho_{CCl_4}}{\rho_{Hg}} = 0.20 \times \frac{1.596 \times 10^3}{13.6 \times 10^3}$$

$= 0.02347$ m or **2.347 cm (Ans.)**

Example 2.12. *A U-tube manometer is used to measure the pressure of oil of specific gravity 0.85 flowing in a pipe line. Its left end is connected to the pipe and the right-limb is open to the atmosphere. The centre of the pipe is 100 mm below the level of mercury (specific gravity = 13.6) in the right limb. If the difference of mercury level in the two limbs is 160 mm, determine the absolute pressure of the oil in the pipe.*

Solution. Specific gravity of oil, $S_1 = 0.85$

Specific gravity of mercury, $S_2 = 13.6$

Height of the oil in the left limb,

$h_1 = 160 - 100 = 60\,\text{mm} = 0.06\,\text{m}$

Difference of mercury level,

$h_2 = 160\,\text{mm} = 0.16\,\text{m}$

Absolute pressure of oil:

Let, h_1 = Gauge-pressure in the pipe in terms of head of water, and

p = Gauge pressure in terms of kN/m².

Equating the pressure heads above the datum line *X–X*, we get

$$h + h_1 S_1 = h_2 S_2$$

or, $h + 0.06 \times 0.85 = 0.16 \times 13.6 = 2.125$ m

The pressure *p* is given by,

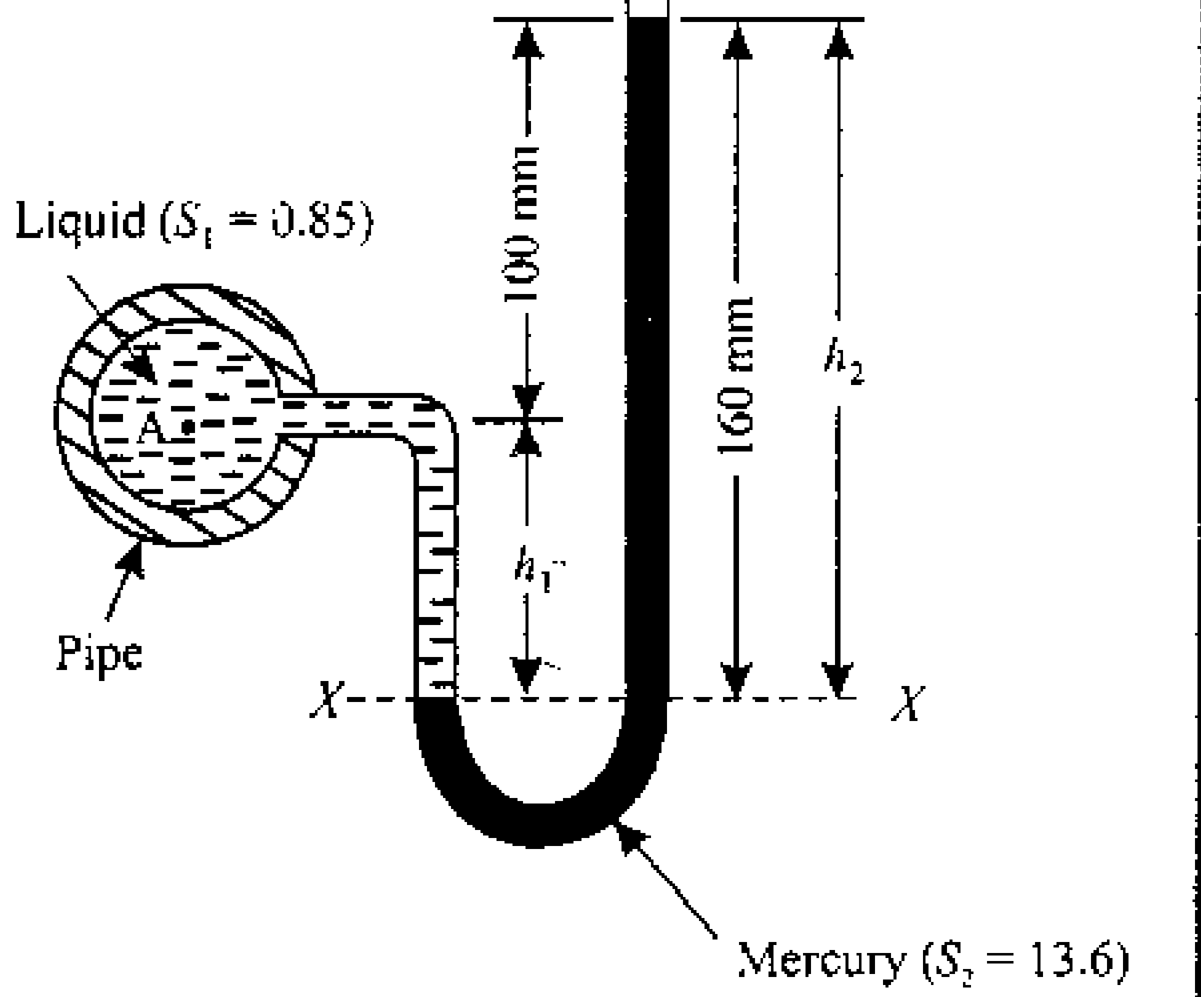


Fig. 2.13

$$p = wh$$
$$= 9.81 \times 2.125 \text{ kN/m}^2$$
$$= 20.84 \text{ kPa} \quad (\because w = 9.81 \text{ kN/m}^3 \text{ in S.I. units})$$

Absolute pressure of oil in the tube,

$$p_{abs.} = p_{atm.} + p_{gauge}$$
$$= 100 + 20.84 = \mathbf{120.84\ kPa\ (Ans.)}$$

Example 2.13. *U-tube manometer containing mercury was used to find the negative pressure in the pipe, containing water. The right limb was open to the atmosphere. Find the vacuum pressure in the pipe, if the difference of mercury level in the two limbs was 100 mm and height of water in the left limb from the centre of the pipe was found to be 40 mm below.*

Solution. Specific gravity of water, $S_1 = 1$

Specific gravity of mercury, $S_2 = 13.6$

Height of water in the left limb,

$h_1 = 40 \text{ mm} = 0.04 \text{ m}$

Height of mercury in the left limb,

$h_2 = 100 \text{ mm} = 0.1 \text{ m}$

Let, h = Pressure in the pipe in terms of head of water (*below* the atmosphere).

Equating the pressure heads above the datum line X–X, we get

$$h + h_1S_1 + h_2S_2 = 0$$

or,

$$h = -(h_1S_1 + h_2S_2)$$
$$= -(0.04 \times 1 + 0.1 \times 13.6)$$
$$= -1.4 \text{ m of water}$$

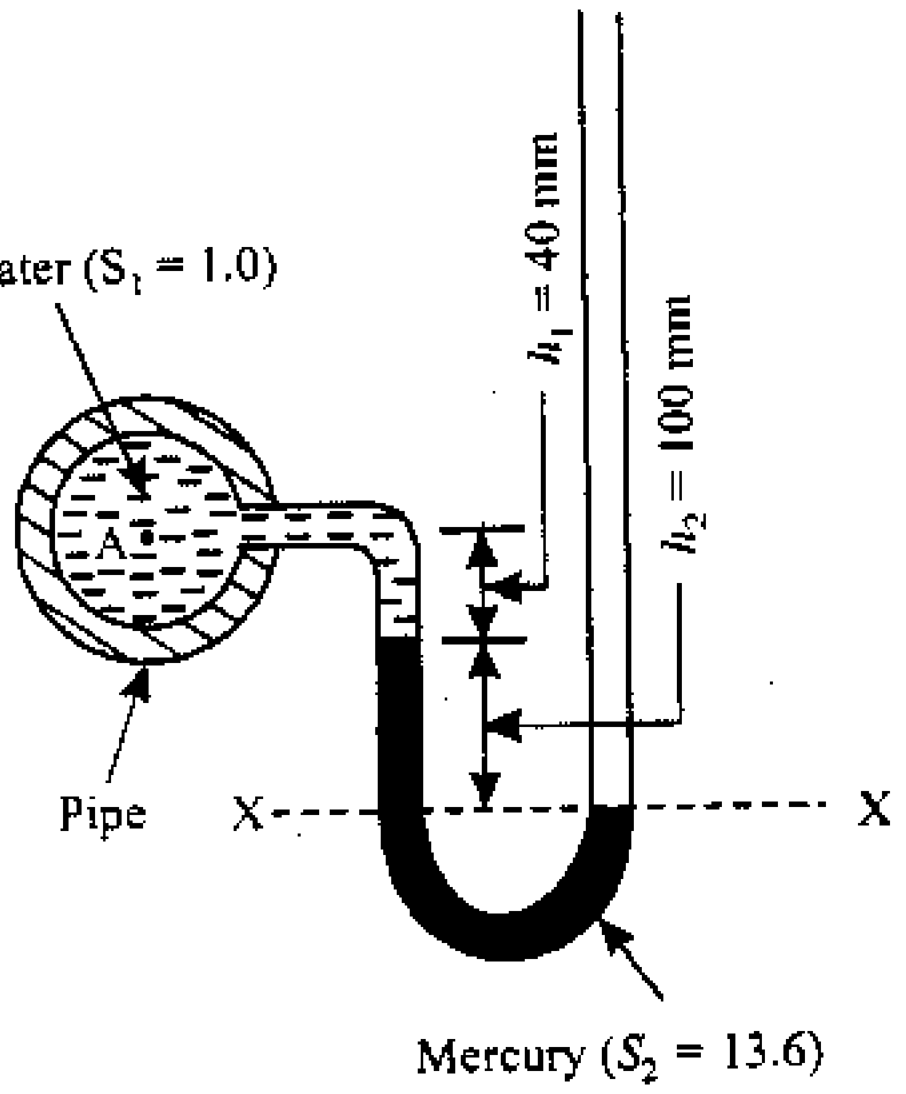


Fig. 2.14

Pressure p is given by,

$$p = wh$$
$$= 9.81 \times (-1.4) \text{ kN/m}^2$$
$$= -13.73 \text{ kPa}$$
$$= \mathbf{13.73\ kPa\ (vacuum)\ (Ans.)}$$

Example 2.14. *A simple U-tube manometer is installed across an orificemeter. The manometer is filled with mercury (sp. gravity = 13.6) and the liquid above the mercury is carbon tetrachloride (sp. gravity = 1.6). The manometer reads 200 mm. What is the pressure difference over the manometer in newtons per square metre.*

Solution. Specific gravity of heavier liquid, $S_{hl} = 13.6$

Specific gravity of lighter liquid, $S_{ll} = 1.6$

Reading of the manometer, $y = 200$ mm

Pressure difference over the manometer : p

Differential head,

$$h = y\left[\frac{S_{hl}}{S_{ll}} - 1\right]$$

$$200\left[\frac{13.6}{1.6} - 1\right] = 1500 \text{ mm of carbon tetrachloride}$$

Pressure difference over manometer,

$$p = wh = (1.6 \times 9810) \times \left(\frac{1500}{1000}\right) \text{N/m}^2$$

or $\quad \mathbf{p = 23544\ N/m^2}$ **(Ans.)**

Example 2.15. *In Fig. 2.15 is shown a conical vessel having its outlet at L to which U-tube manometer is connected. The reading of the manometer given in figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.*

Solution. When vessel is empty: (Refer to Fig. 2.15)

Let, h_1 = Height of water above X–X

Specific gravity of water, $S_1 = 1.0$

Specific gravity of mercury, $S_2 = 13.6$

Equating the pressure heads about the datum line X–X, we get

$$h_1 S_1 = h_2 S_2 \quad \text{or} \quad h_1 \times 1.0 = 150 \times 13.6 \quad \text{or} \quad h_1 = 2040 \text{ mm}$$

When vessel is full of water:

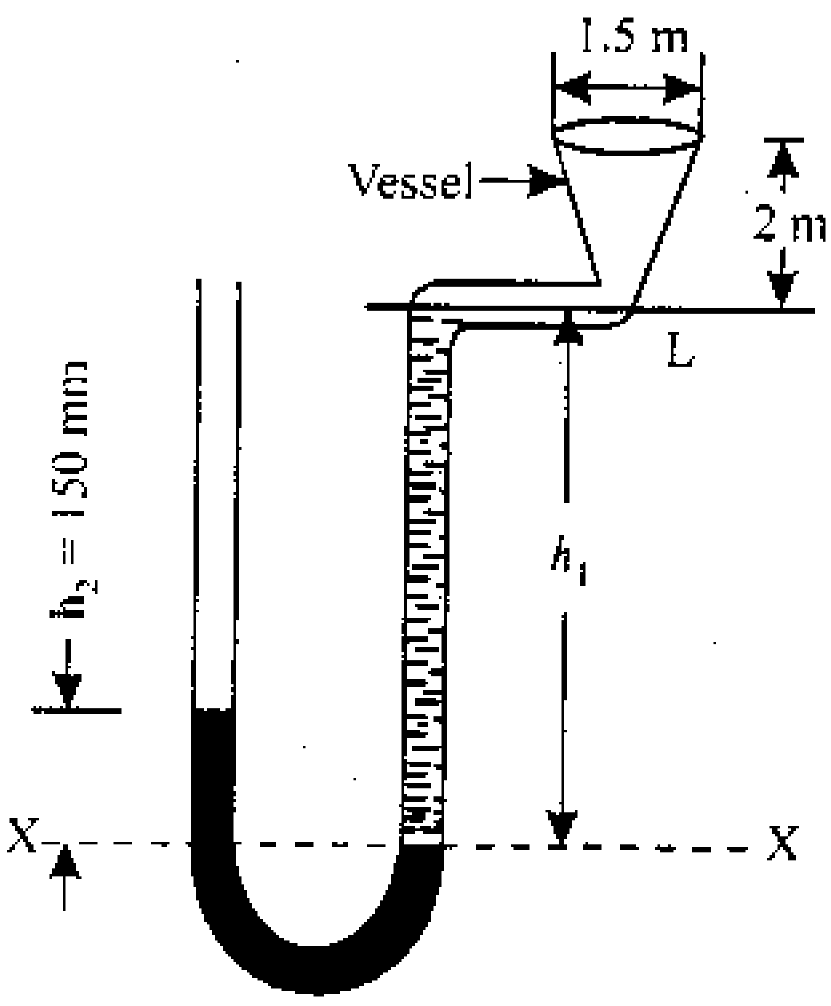


Fig. 2.15. Vessel is empty.

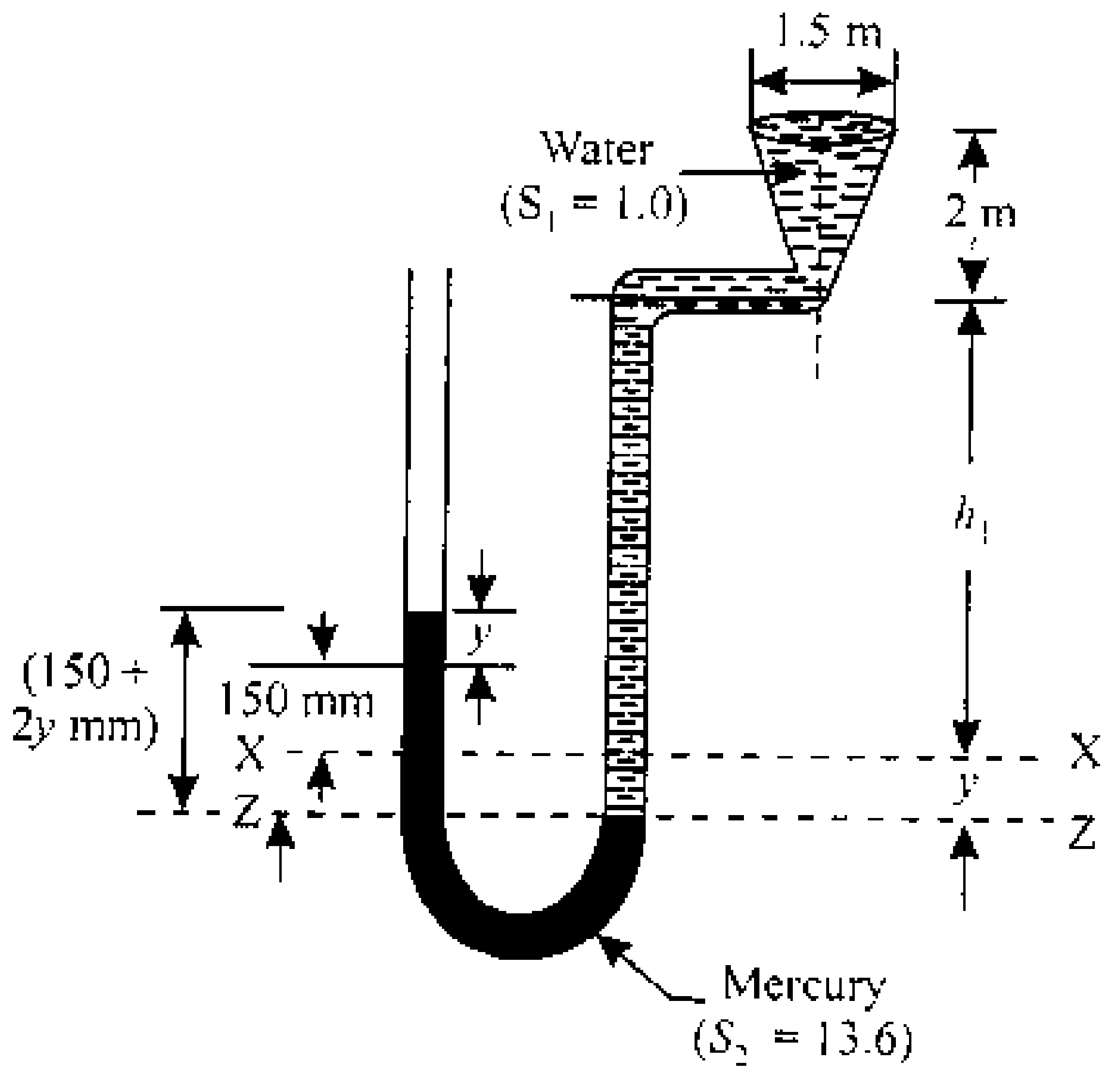


Fig. 2.16. Vessel is full of water.

Refer to Fig. 2.16. Consider the vessel to be completely filled with water. As a result of this let the mercury level go down by y mm in the right limb, and the mercury level go up by the same amount in the left limb. Now the datum line is Z–Z.

Equating the pressure heads above the datum line Z–Z, we get

$$(150 + 2y) \times 13.6 = (h_1 + y + 2000) \times 1$$

or, $\quad 150 \times 13.6 + 2y \times 13.6 = 2040 + y + 2000 \qquad [\because h_1 = 2040 \text{ mm, calculated earlier}]$

or, $\quad 2040 + 27.2y = 4040 + y = 76.3 \text{ mm}$

Thus the reading of the manometer when the vessel is completely filled with water

$$= (150 + 2y) = 150 + 2 \times 76.3 = 302.6 \text{ mm}$$

Hence, *reading of the manometer* 302.6 mm or **0.3026 m (Ans.)**

Example. 2.16. *Fig. 2.17 shows a pressure gauge with the following particulars:*

Cross-sectional area of each of the bulbs L and M = 1200 mm²;

Cross-sectional area of each vertical limb = 30 mm²;

Specific gravity of the liquid filled in bulb M = 0.9;

If the surface of separation is in the limb attached to M find the displacement of surface of separation when the pressure on the surface in M is greater than that in L by an amount equal to 20 mm of head of water.

Solution. Cross-sectional area of each vertical limb, $a = 30\ \text{m}^2$

Specific gravity of water, $S_1 = 1.0$

Specific gravity of the liquid, $S_2 = 0.9$.

Let, X–X = Initial level of separation,

h_L = Height of water above X–X, and

h_M = Height of liquid ($S_2 = 0.9$) above X–X.

Pressure head above X–X in the left limb $= h_L$

Pressure head above X–X in the right limb $= S_2 h_M = 0.9\, h_M$

Equating the pressure heads above X–X, we get

$$h_L = 0.9\, h_M \qquad \ldots(i)$$

When the pressure on the surface in bulb M is increased by 20 mm of water, let the separation level fall by an amount equal to y. Then Z–Z is the *new separation level.*

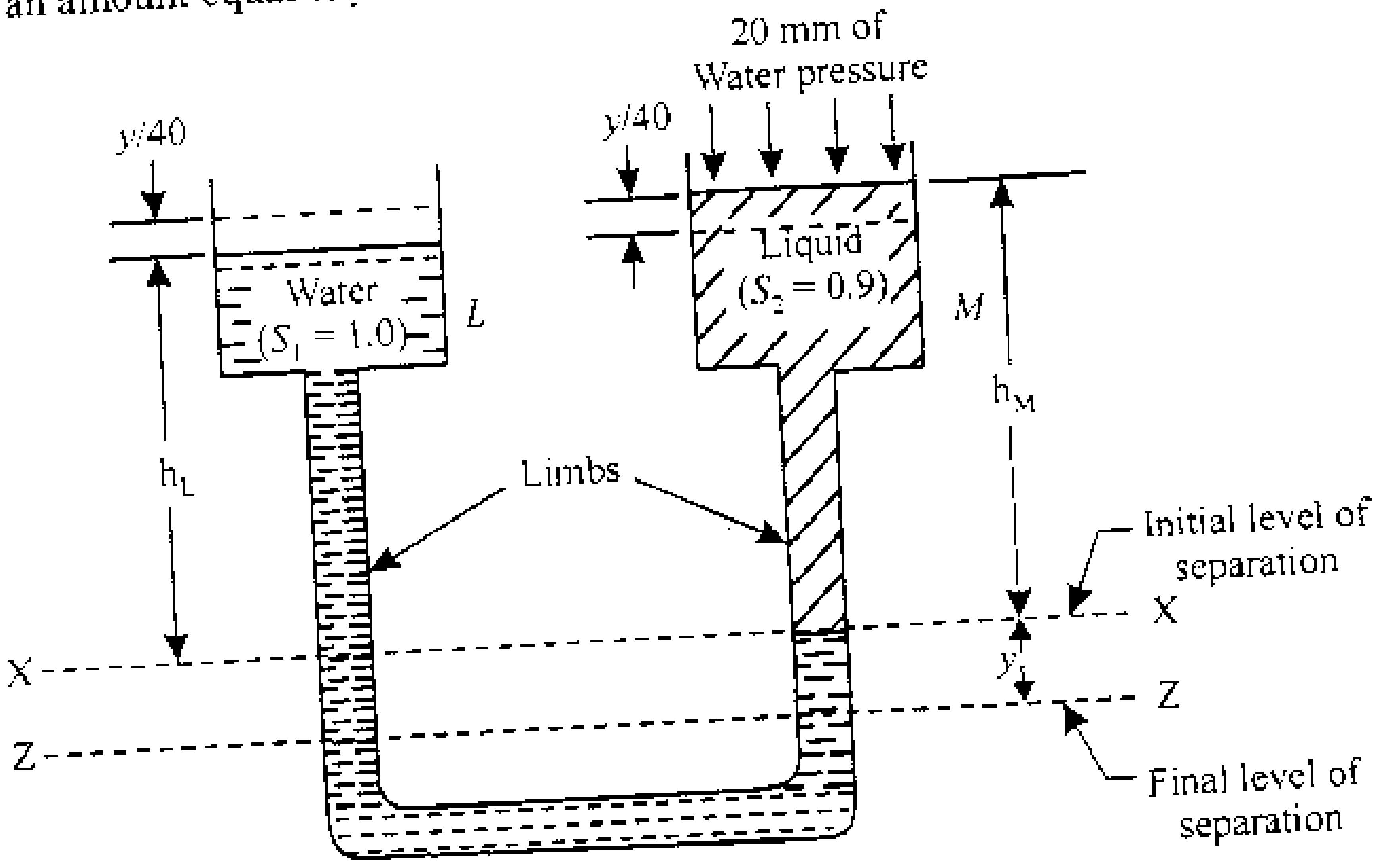


Fig. 2.17

Now, A × fall in separation level in bulb $M = a$ × fall in separation level in the limb (y).

Fall in separation level in bulb M

$$= \frac{a \times y}{A} = \frac{30 \times y}{1200} = \frac{y}{40}$$

Also, fall in separation level in bulb M = rise in surface level of $L = \frac{y}{40}$

Considering pressure heads above Z–Z, we have

Pressure head in the left limb $= \left[\frac{y}{40} + h_L + y\right]$

Pressure head in the right limb $= \left(h_M + y - \frac{y}{40}\right) \times 0.9 + 20$

Equating the pressure heads, we get

$$\left[\frac{y}{40} + h_L + y\right] = \left[h_M + y - \frac{y}{40}\right] \times 0.9 + 20$$

or, $$\frac{y}{40} - 0.9\,h_M + y = 0.9h_M + \frac{39\,y}{40} \times 0.9 + 20 \qquad (\because h_L = 0.9h_M)$$

or, $$\frac{41y}{40} = \frac{39\,y}{40} \times 0.9 + 20 \quad \text{or} \quad \frac{41y}{40} - \frac{39\,y}{40} \times 0.9 = 20$$

or, $$1.025y - 0.877y = 20 \quad \text{or} \quad y = 135.1 \text{ mm}$$

Hence, *displacement of the surface of separation* = **135.1 mm (Ans.)**

3. Single column manometer (micro-manometer):

The U-tube manometer described above usually requires reading of fluid levels at two or more points since a change in pressure causes a rise of liquid in one limb of the manometer and a drop in the other. This difficulty is however overcome by using single column manometers. A single column manometer is a modified form of a U-tube manometer in which a shallow reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one limb of the manometer, as shown in Fig. 2.18. *For any variation in pressure, the change in the liquid level in the reservoir will be so small that it may be neglected, and the pressure is indicated by the height of the liquid in the other limb.* As such only *one reading in the narrow limb of the manometer need be taken for all pressure measurements.* The narrow limb may be vertical or inclined. Thus there are two types of single column manometer as given below:

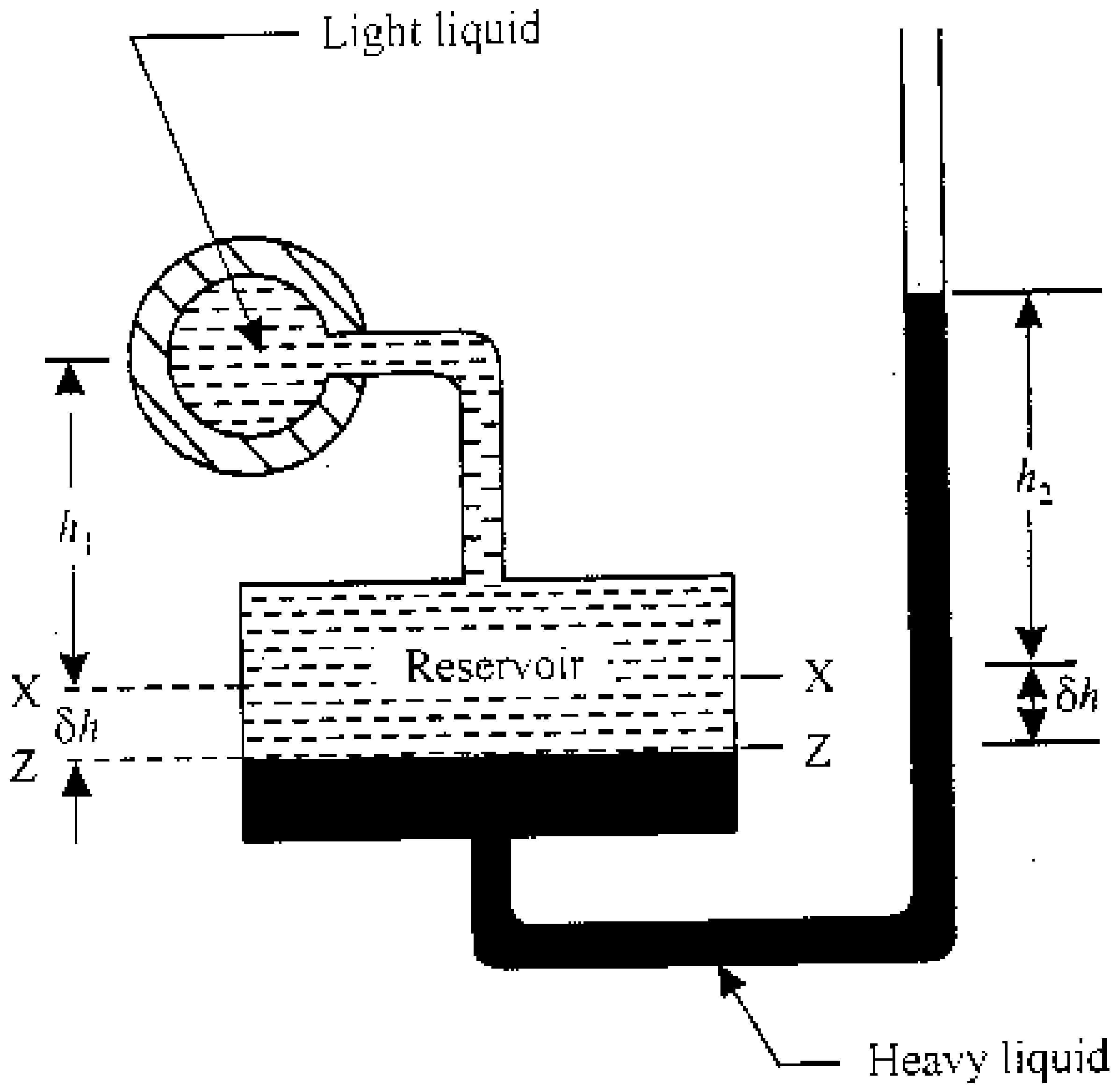


Fig. 2.18. Vertical single column manometer.

(*a*) Vertical single column manometer, and

(*b*) Inclined single column manometer.

(*a*) **Vertical single column manometer:**

Refer to Fig. 2.18

Let X–X be the datum line in the reservoir when the single column manometer is not connected to the pipe. Now consider that the manometer is connected to a pipe containing light liquid under a very high pressure. The pressure in the pipe will force the light liquid to push the heavy liquid in the reservoir downwards. As the area of the reservoir is very large, the fall of the heavy liquid level will be very small. This downward movement of the heavy liquid, in the reservoir, will cause a considerable rise of the heavy liquid in the right limb.

Let, h_1 = Height of the centre of the pipe above X–X,

h_2 = Rise of heavy liquid (after experiment) in the right limb, *Take specific gravity of mercury as 13.6.*

Solution. Specific gravity of liquid in the pipe, $S_1 = 0.8$.

Specific gravity of mercury, $S_2 = 13.6$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

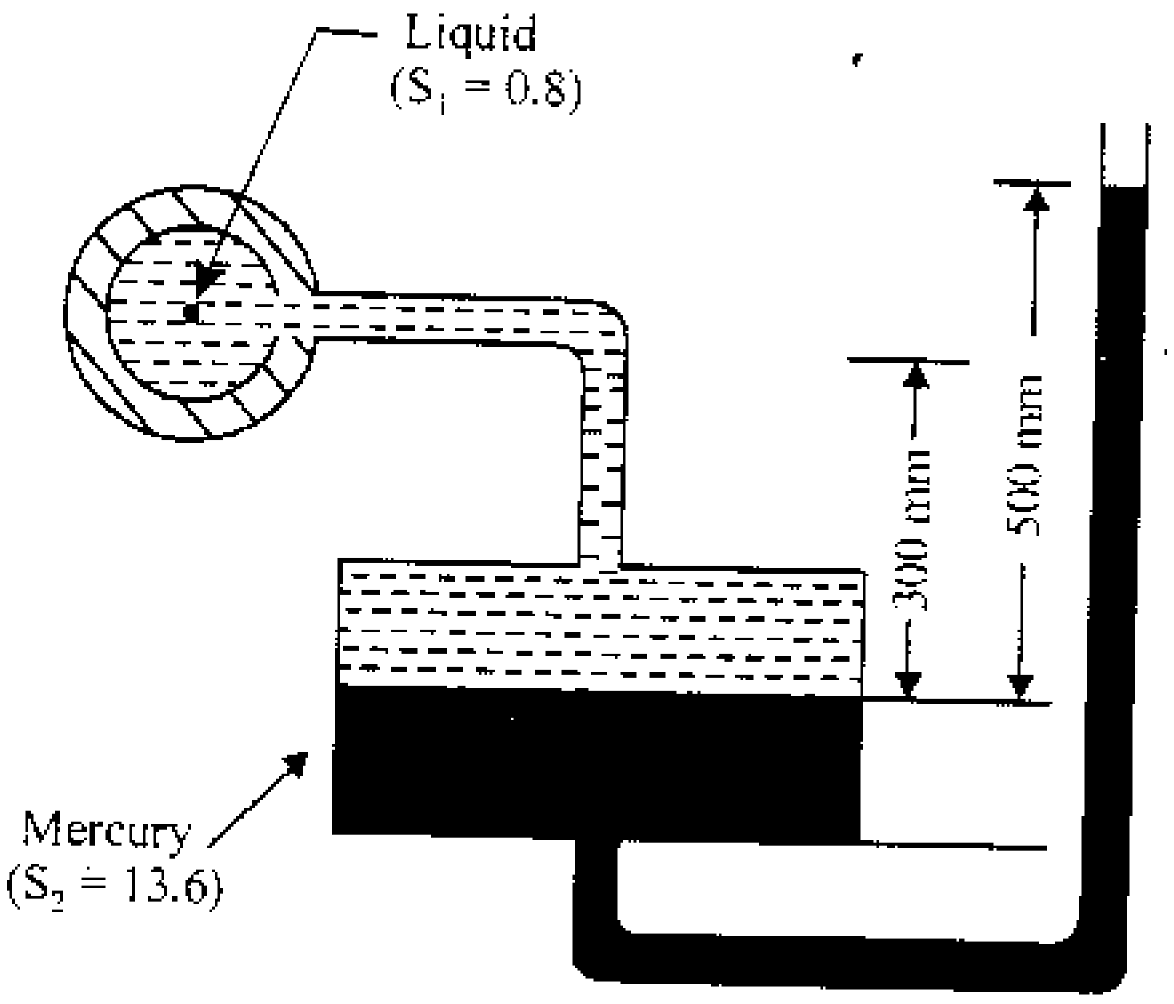


Fig. 2.20

Height of the liquid in the left limb,

$$h_1 = 300 \text{ mm}$$

Height of mercury in the right limb,

$$h_2 = 500 \text{ mm}$$

Let, h = Pressure head in the pipe.

Using the relation:

$$h = \frac{a}{A} h_2 (S_2 - S_1) + h_2 S_2 - h_1 S_1$$

or, $$h = \frac{1}{100} \times 500\,(13.6 - 0.8) + 500 \times 13.6 - 300 \times 0.8 \text{ mm of water}$$

$$= 6624 \text{ mm of water} \quad \text{or} \quad 6.624 \text{ m of water}$$

Pressure, $$p = wh = 9.81 \times 6.624$$

$$= 64.98 \text{ kN/m}^2 \text{ or } 64.98 \text{ kPa}$$

i.e., $\boldsymbol{p = 64.98}$ **kPa (Ans.)**

Example 2.18. *A manometer consists of an inclined glass tube which communicates with a metal cylinder standing upright; liquid fills the apparatus to a fixed zero mark on the tube when both the cylinder and the tube are open to atmosphere. The upper end of the cylinder is then connected to a gas supply at a pressure p and manometric liquid rises through a distance l in the tube. Establish the relation:*

$$h = Sl\left[\sin\alpha + \left(\frac{d}{D}\right)^2\right]$$

for the pressure head h of water column in terms of inclination α of the tube, specific gravity S of the liquid, and ratio of diameter d of th tube to the diameter D of the cylinder.

Also determine the value of $\left(\frac{D}{d}\right)$ *so that the error due to disregarding the change in level in the cylinder will not exceed 0.1 percent when* $\alpha = 25°$.

2.5.1.2. Differential Manometers

A **differential manometer** *is used to measure the difference in pressures between two points in a pipe, or in two different pipes.* In its simplest form a differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressures is required to be found out. Following are the most commonly used types of differential manometers.

1. U-tube differential manometer.
2. Inverted U-tube differential manometer.

1. U-tube differential manometer:

A U-tube differential manometer is shown in Fig. 2.21.

Case I. Fig. 2.21 (*a*) shows a differential manometer whose two ends are connected with two different points A and B at the same level and containing same liquid.

Let, h = Difference of the centre of A, from the mercury level in the right limb,

$S_1\ (= S_2)$ = Specific gravity of liquid at the two points A and B

S = Specific gravity of heavy liquid or mercury in the U-tube,

h_A = Pressure head at A, and

h_B = Pressure head at B,

We know that the pressures in the left limb and right limb, above the datum line, are equal. Pressure head in the *left limb*

$$= h_A + (h_1 + h)\, S_1$$

Pressure head in the *right limb*

$$= h_B + h_1 \times S_1 + h \times S$$

$$h_A + (h_1 + h)S_1 = h_B + h_1 S_1 + hS$$

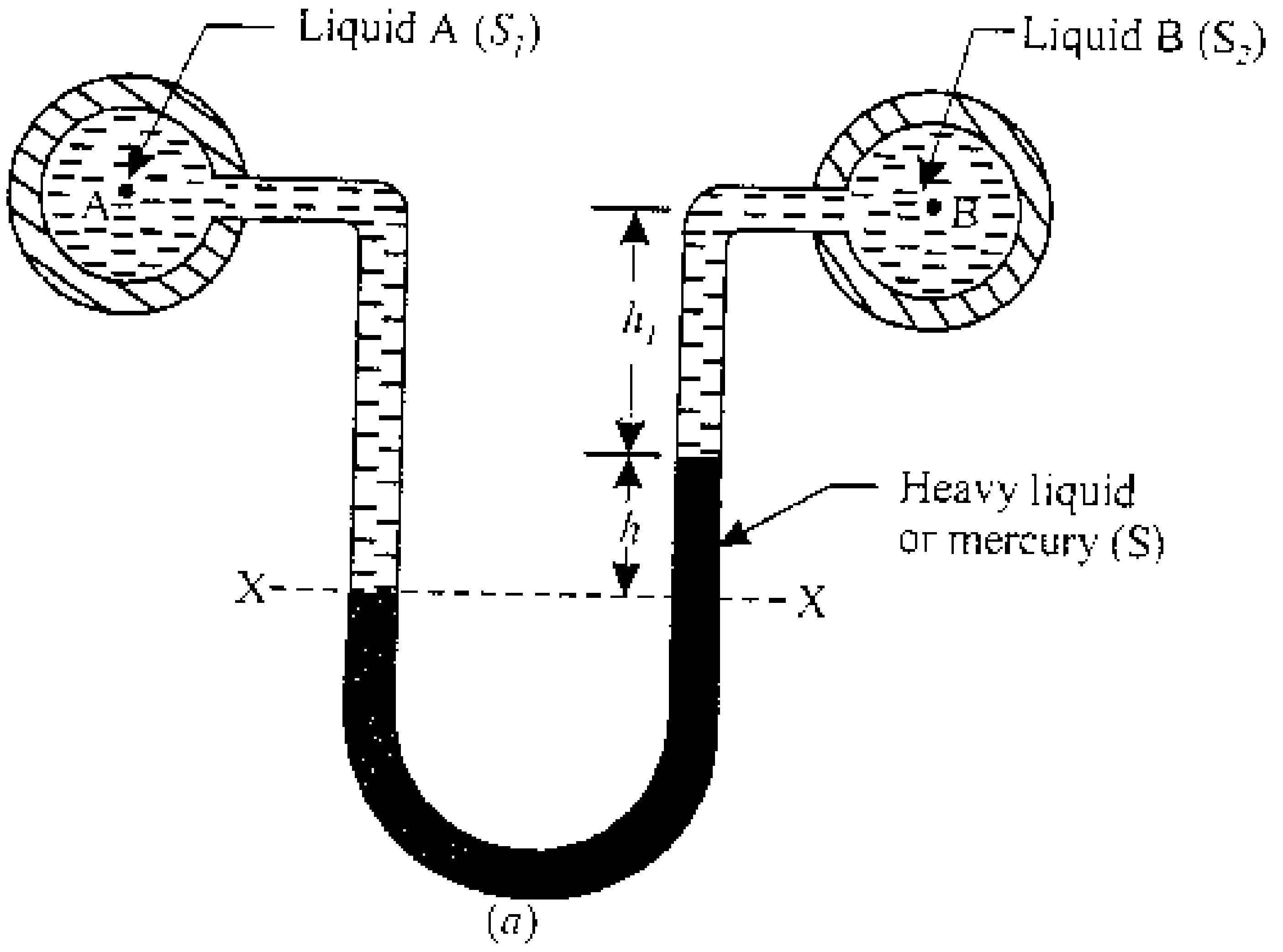
or,
$$h_A - h_B = h_1 S_1 + hS - (h_1 + h)\, S_1$$

$$= h_1 S_1 + hS - h_1 S_1 - hS_1 = h\,(S - S_1)$$

i.e., Difference of pressure head,

$$h_A - h_B = h\,(S - S_1) \qquad \text{...(2.11)}$$

Case II. Fig. 2.21 (*b*) shows a differential manometer whose two ends are connected to two different points A and B at different levels and containing different liquids.



(*a*)

(*a*) Two pipes at same level.

Fig. 2.21. U-tube differential manometers.

between two points in
consists of a U-tube,
ifference of pressures
ferential manometers.

re connected with two

level in the right limb,
d *B*
he *U*-tube,

datum line, are equal.

...(2.11)
ds are connected to two
.

B (S_2)

id
(S)

Let, h = Difference of mercury level (heavy liquid) in the U-tube,

h_1 = Distance of the centre of A, from the mercury level in the left limb,

h_2 = Distance of the centre of B, from the mercury level in the right limb,

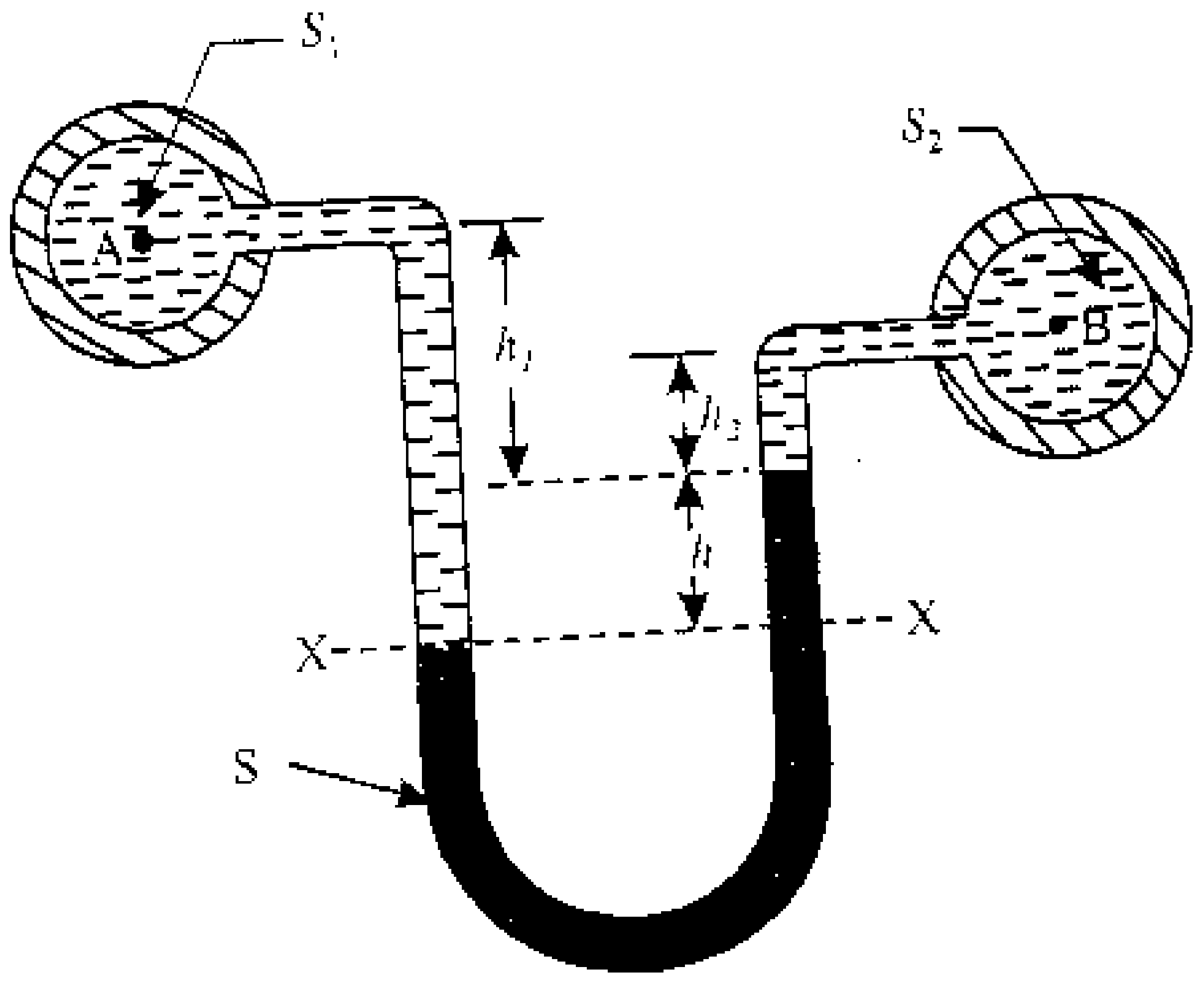
S_1 = Specific gravity of liquid in pipe A,

S_2 = Specific gravity of liquid in pipe B,

S = Specific gravity of heavy liquid or mercury,

h_A = Pressure head at A, and

h_B = Pressure head at B.



(*b*)

Fig. 2.21. (*b*) Two pipes at differential levels.

Considering the pressure heads above the datum line *X–X*, we get

Pressure head in the *left limb* = $h_A + (h_1 + h)\,S_1$

Pressure head in the *right limb* = $h_B + h_2 \times S_2 + h \times S$

Equating the above pressure heads, we get

$$h_A + (h_1 + h)\,S_1 = h_B + h_2 \times S_2 + h \times S$$

$$(h_A - h_B) = h_2 \times S_2 + h \times S - (h_1 + h)\,S_1$$

$$= h_2 \times S_2 + h \times S - h_1 S_1 - hS_1 = h\,(S - S_1) + h_2 S_2 - h_1 S_1$$

i.e., Difference of pressure head at A and B, ...(2.12)

$$h_A - h_B = h\,(S - S_1) + h_2 S_2 - h_1 S_1$$

Example 2.19. *A differential manometer connected at the two points A and B in a pipe containing an oil of specific gravity of 0.9, shows a difference in mercury levels as 150 mm. Find the difference in pressures at the two points.*

Solution. Specific gravity of oil, $S_1 = 0.9$

Specific gravity of mercury, $S = 13.6$

Difference of mercury levels. $h = 150$ mm

Let, $h_A - h_B$ = Difference of pressures between A and B, in terms of head of water, and

$p_A - p_B$ = Difference of pressures between A and B.

[Eqn. (2.11)]

Using the relation: $h_A - h_B = h\,(S - S_1)$

$= 150\,(13.6 - 0.9) = 1905$ mm = **1.905 m of water (Ans.)**

Now, using the relation,

$$p_A - p_B = wh, \text{ we have, } p_A - p_B = 9.81 \times 1.905 = 18.68 \text{ kN/m}^2 = \mathbf{18.68\ kPa\ (Ans.)}$$

Example 2.20. *Fig 2.22 shows a U-tube differential manometer connecting two pressure pipes at A and B. The pipe A contains a liquid of specific gravity 1.6 under a pressure of 110 kN/m². The pipe B contains oil of specific gravity 0.8 under a pressure of 200 kN/m². Find the difference of pressure measured by mercury as fluid filling U-tube.*

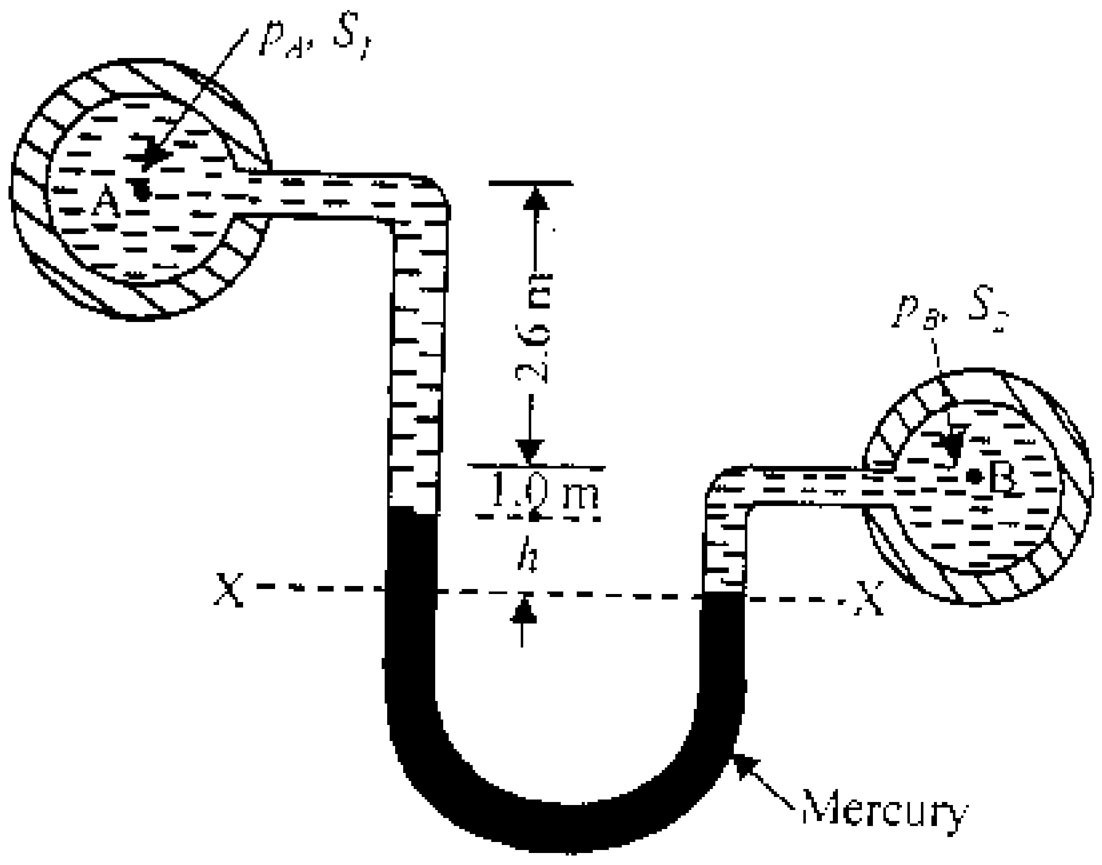


Fig. 2.22

Solution. Specific gravity of liquid at A, $S_1 = 1.6$

Specific gravity of liquid at B, $S_2 = 0.8$

Pressure at A, $p_A = 110 \text{ kN/m}^2$

Pressure head at A,

$$h_A = \frac{p_A}{w} = \frac{110}{9.81} = 11.21 \text{ m of water}$$

Pressure at B, $p_B = 200 \text{ kN/m}^2$

Pressure head at B,

$$h_B = \frac{p_B}{w} = \frac{200}{9.81} = 20.38 \text{ m of water}$$

Taking $X-X$ as the datum line:

Pressure head above X–X in the *left limb* $= h_A + (2.6 + 1.0)\, S_1 + h \times 13.6$ m of water

Pressure head above X–X in the *right limb* $= h_B + (1.0 + h) \times S_2$ m of water

Equating the above pressure heads, we get

$$h_A + (2.6 + 1.0)\, S_1 + h \times 13.6 = h_B + (1.0 + h)\, S_2$$

$$11.21 + 5.76 + 13.6\, h = 20.38 + (0.1 + h) \times 0.8$$

or, $$16.97 + 13.6\, h = 20.38 + 0.8 + 0.8\, h \text{ or } 12.8h = 4.21$$

or, $$h = 0.329 \text{ m or } \mathbf{329\ mm\ (Ans.)}$$

Example 2.21. *Fig. 2.23. shows a differential manometer connected at two points A and B. At A air pressure is 100 kN/m². Find the absolute pressure at B.*

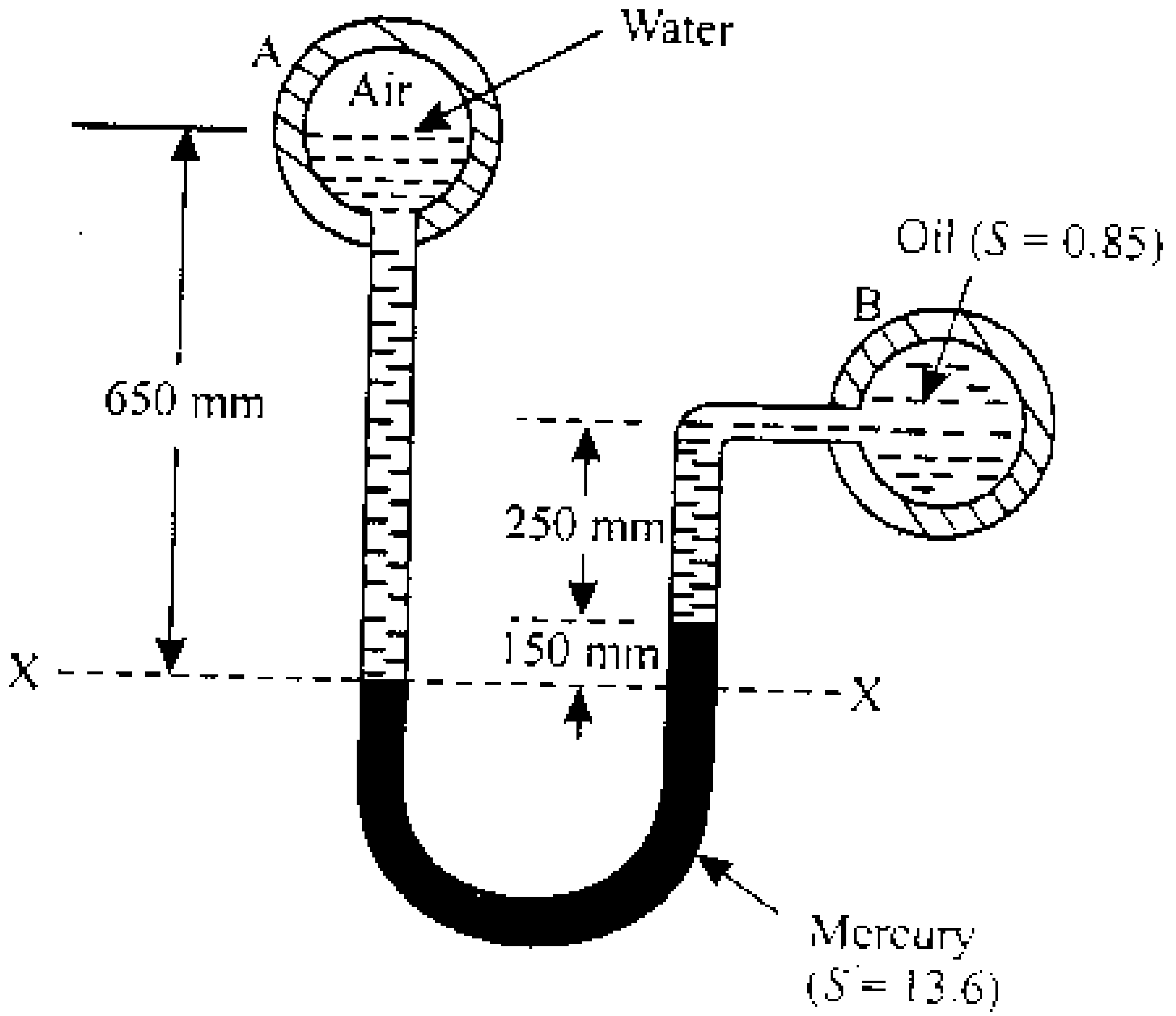


Fig. 2.23

Solution. Pressure of air at A,

$$p_A = 100 \text{ kN/m}^2$$

Pressure head at A,

$$h_A = \frac{100}{9.81} = 10.2 \text{ m}$$

Let the pressure at B is p_B.

Then, pressure head at $B = \frac{p_B}{w}$

Considering pressure heads above the datum line X–X, we have

Pressure head in the *left limb*

$$= \frac{650}{1000} + h_A = 0.65 + 10.2 = 10.85 \text{ m}$$

Pressure head in the *right limb*

$$= h_B + \frac{250}{1000} \times 0.85 + \frac{150}{1000} \times 13.6$$

$$h_A - h_1 \times S_1 = h_B - h_2 \times S_2 - h \times S$$

$$h_A - h_B = h_1 \times S_1 - h_2 \times S_2 - h \times S$$

i.e.,
$$h_A - h_B = h_1S_1 - h_2S_2 - hS \quad \text{...(2.13)}$$

Example 2.22. *Fig. 2.25 shows an inverted differential manometer having an oil of specific gravity 0.8 connected to two different pipes carrying water under pressure. Determine the pressure in the pipe B. The pressure in pipe A is 2.0 metres of water.*

Solution. Height of water in the left limb,

$$h_1 = 300 \text{ mm}$$

Height of water in the right limb,

$$h_2 = 100 \text{ mm}$$

Height of light liquid in right limb,

$$h = 150 \text{ mm}$$

Pressure in pipe A, $h_A = 2.0$ m of water

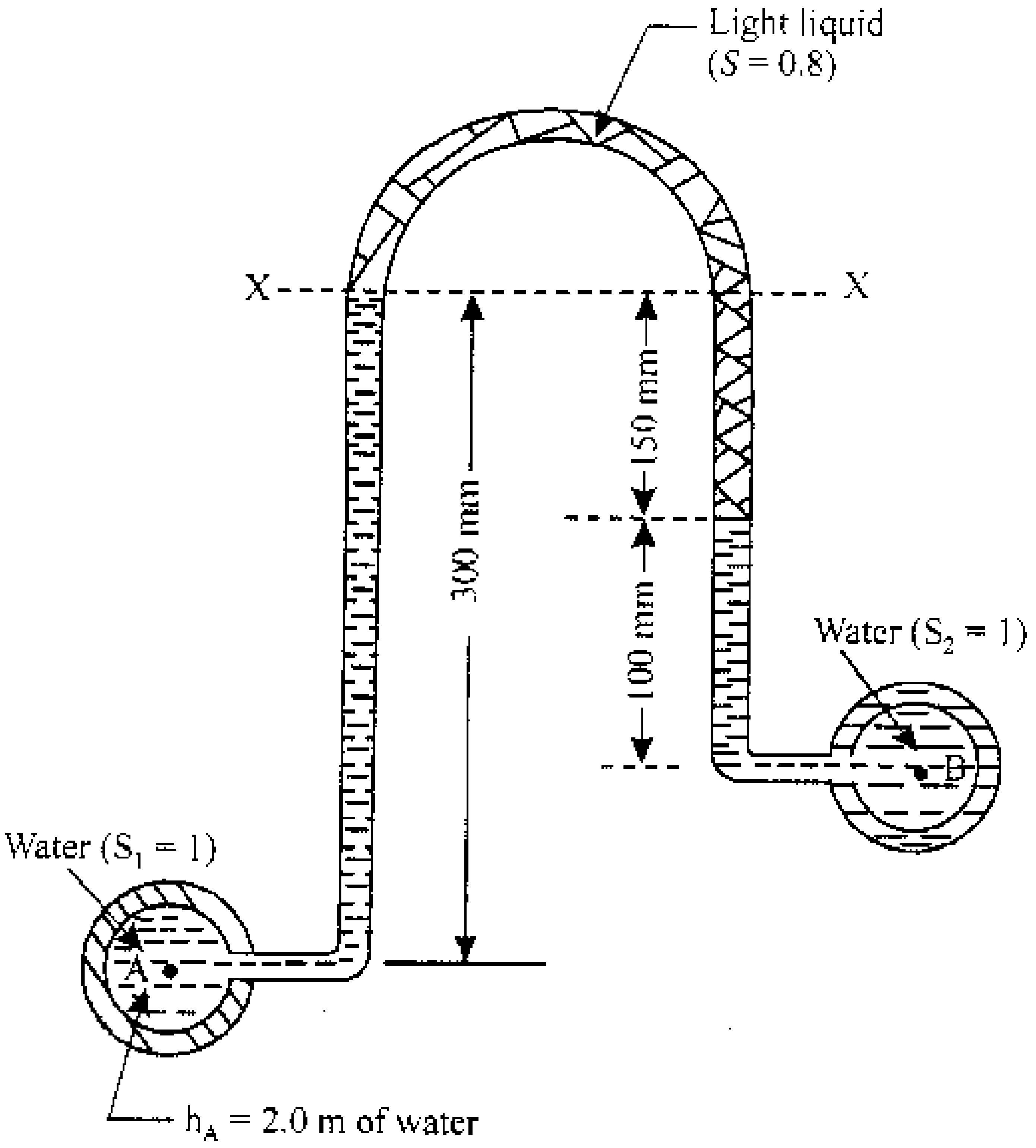


Fig. 2.25

Let, $S_1, S_2 = 1$ (sp. gr. of water)

We know that pressure heads in the left and right limbs below the datum line X–X are *equal*.

Pressure head in the *left limb* below X–X

$$= h_A - h_1S_1$$

$$= 2.0 - \frac{300}{1000} \times 1 = 1.7 \text{ m}$$

Pressure head in the right limb below X–X

$$= h_B - h_2S_2 - hS$$

$$= h_B - \frac{100}{1000} \times 1 - \frac{150}{1000} \times 0.8$$

$$= h_B - 0.1 - 0.12 = h_B - 0.22$$

Equating the two pressure heads, we get $1.7 = h_B - 0.22$

or,. $h_B = \mathbf{1.92\ m}$ **(Ans.)**

Also, $p_B = wh_B = 9.81 \times 1.92 = 18.8\ \text{kN/m}^2$

$= \mathbf{18.8\ kPa}$ **(Ans.)**

Example 2.23. *An inverted differential manometer is connected to two pipes A and B carrying water under pressure as shown in Fig. 2.26. The fluid in the manometer is oil of specific gravity 0.75. Determine the pressure difference between A and B.*

Solution. Specific gravity of oil, $S = 0.75$

Specific gravity of water, $S_1, S_2 = 1$

Difference of oil in the two limbs

$$= (450 + 200) - 450 = 200\ \text{mm}$$

We know that pressure heads on the left and right limbs below the datum line X–X are *equal.*

Pressure head in the left limb below X–X

$$= h_A - \frac{450}{1000} \times 1 = h_A - 0.45$$

Pressure head in the right limb below X–X

$$= h_B - \frac{450}{1000} \times 1 - \frac{200}{1000} \times 0.75$$

$$= h_B - 0.45 - 0.15 = h_B - 0.6$$

Equating the two pressure heads, we get

$$h_A - 0.45 = h_B - 0.6$$

$$h_B - h_A = \mathbf{0.15\ m}\ \textbf{(Ans.)}$$

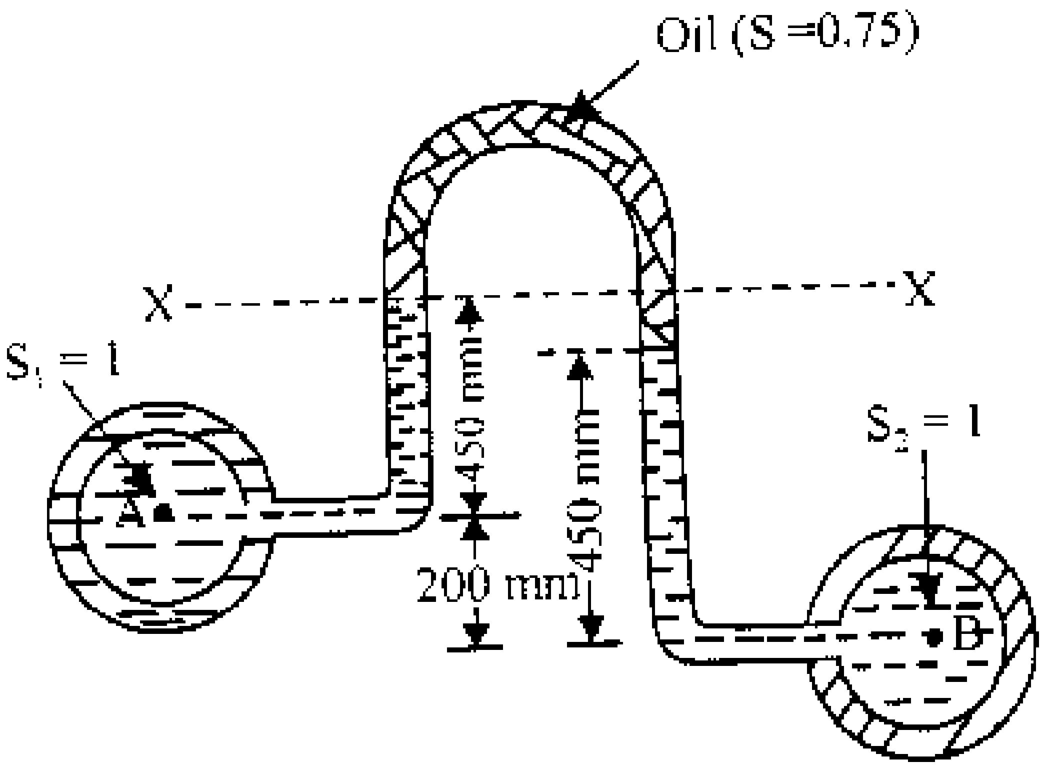


Fig. 2.26

or, $\frac{p_B}{w} - \frac{p_A}{w} = 0.15$ or $p_B - p_A = w \times 0.15 = 9.81 \times 0.15 = 1.47\ \text{kN/m}^2 = \mathbf{1.47\ kPa}$ **(Ans.)**

Example 2.24. *Describe giving a sketch, a micromanometer. Explain how it could be used for measuring small pressure difference.* **(AMIE Summer, 2001)**

Solution: *Micromanometer.* It is shown in the Fig. 2.27 and is *used for measuring small pressure differences.* It utilizes two manometer liquids which are *immiscible with each other and also with the fluid whose pressure difference is to be measured.* The heavier liquid fills the lower part of the U-tube upto 0-0 and then the lighter liquid is added on both sides filling the tanks C and D upto the level X–X. The fluid (liquid or a gas) whose pressure difference is to be measured fills the space above X–X. When the pressure p_A is slightly greater than p_B, the liquid levels will be as shown in the figure. The volume of the liquid displaced in each tank is equal to the volume of liquid displaced in the U-tube. If a is the cross-sectional area of the U-tube, and A that of the tank, then

$$A\,\Delta Z = \frac{h}{2}a \qquad \ldots(i)$$

Let be the S_1 be the specific gravity of heavier manometric liquid, and S_2 be that of the lighter manometric liquid. An expression relating p_A and p_B may be obtained by equating pressures along L–L in the U-tube. If w is specific weight of water, then

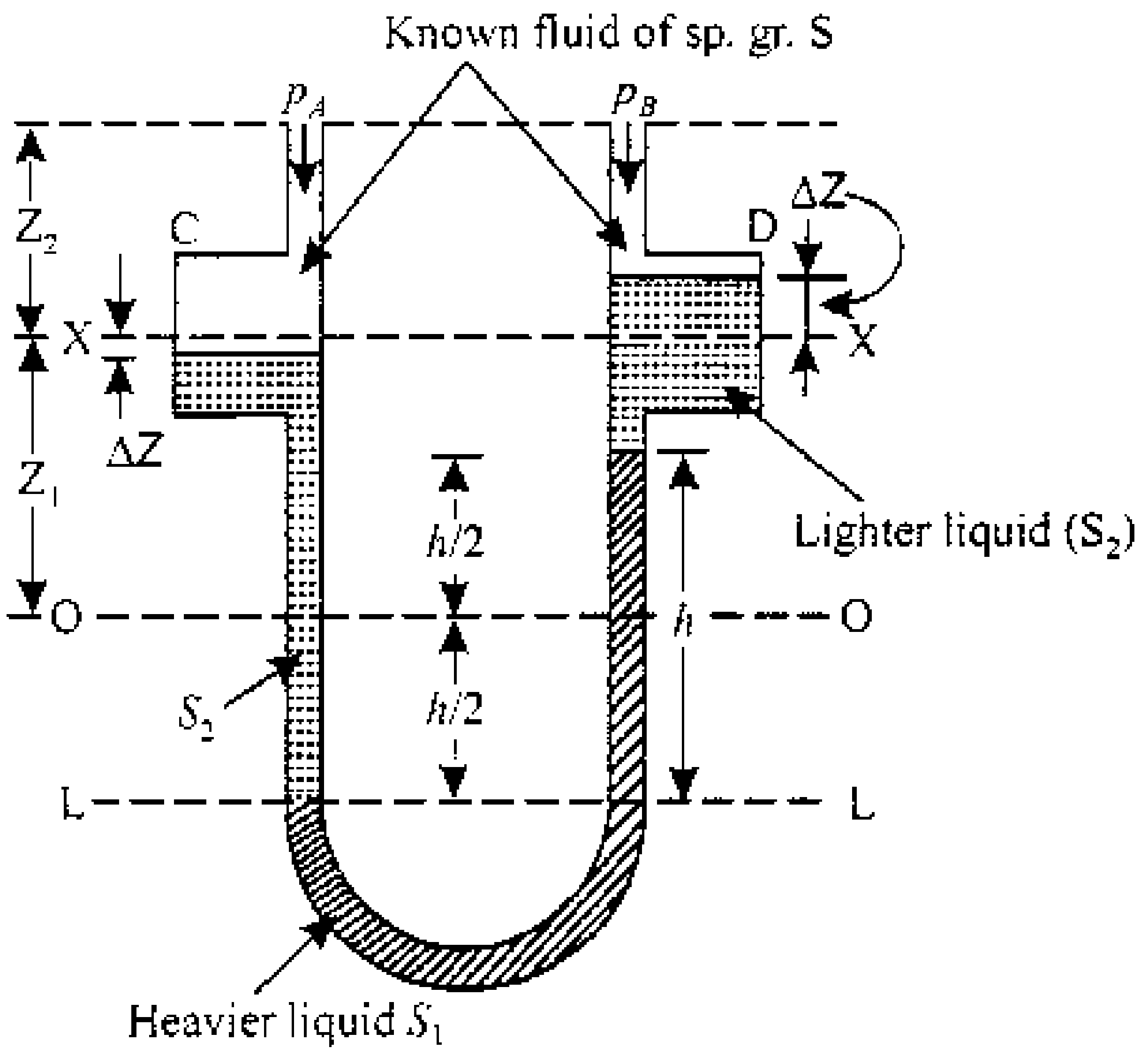


Fig. 2.27

$$\frac{p_A}{w} + \left(Z_2 + \Delta Z \right) S + \left(Z_1 - \Delta Z + \frac{h}{2} \right) S_2$$

$$= \frac{p_B}{w} + \left(Z_2 + \Delta Z \right) S + \left(Z_1 + \Delta Z - \frac{h}{2} \right) S_2 + hS_1$$

or,
$$= \frac{p_A}{w} + Z_2 S + \Delta Z \,.\, S + Z_1 S_2 - \Delta Z\, S_2 + \frac{h}{2} S_2$$

$$= \frac{p_B}{w} + Z_2 S - \Delta Z\, S + Z_1 S_2 + \Delta Z\, S_2 - \frac{h}{2} S_2 + hS_1$$

$\therefore$ Pressure difference, $\frac{p_A - p_B}{w} = \Delta Z\,(S_2 - S + S_2 - S) + h\left(S_1 - \frac{S_2}{2} - \frac{S_2}{2} \right)$

Substituting for $\Delta Z = \frac{ha}{2A}$ (from (*i*)) and simplifying, we get

or,
$$\frac{p_A - p_B}{w} = \frac{ha}{2A}\left[(2S_2 - 2S) \right] + h\left[S_1 - S_2 \right]$$

$$= h\left[\frac{a}{A}(S_2 - S) + (S_1 - S_2) \right] = hK \textbf{ (Ans.)}$$

The quantity K within the bracket is a *constant* for a given manometer and given manometric liquids of specific gravities S_1, S_2 and known fluid of specific gravity S.

Example 2.25. *Fig. 2.28. is shows a fuel gauge, for a gasoline tank in car, which reads proportional to the bottom gauge. The tank is 30 cm deep and accidently contains 1.8 cm of water in addition to the gasoline. Determine the height of air remaining at the top when the gauge erroneously reads full.*

Take: $w_{gasoline} = 6.65 \text{ kN/m}^3$, *and*
$w_{air} = 0.0118 \text{ kN/m}^3$. **(Panjab University)**

Solution. When the tank is full of gasoline,

$$p_{gauge} = wh = 6.65 \times \frac{30}{100} = 1.995 \text{ kN/m}^2$$

hS_1

$- hS_1$

$- \frac{S_2}{2} \Big)$

d given manometric

in car, which reads
ains 1.8 cm of water
top when the gauge

(Panjab University)

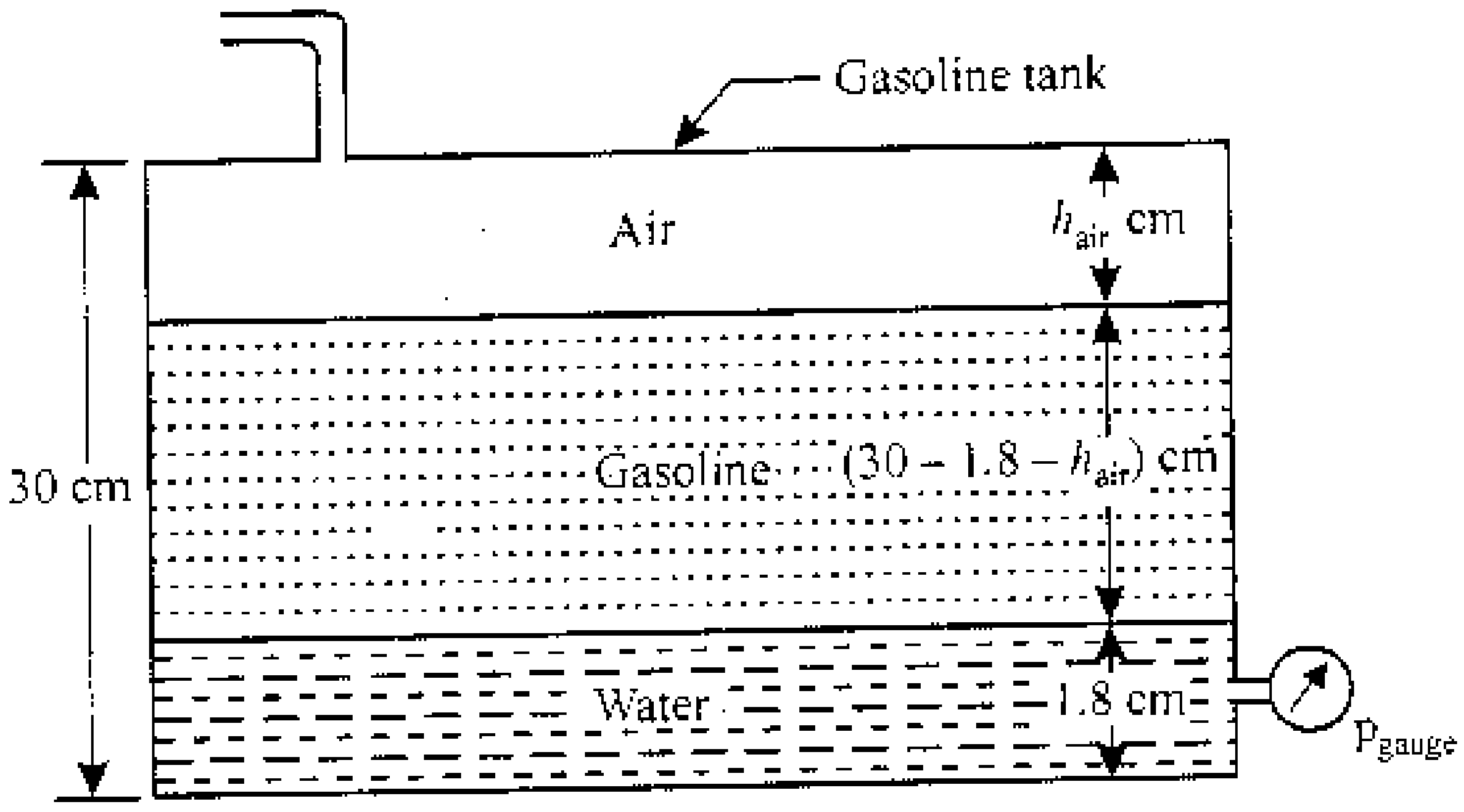


Fig. 2.28

The gauge would erroneously read 1.995 kN/m² even when h cm of air remains at the top; evidently when water is also accidently present.

∴ Pressure due to h cm height of air + pressure due to $[30 - 1.8 - h]$ cm height of gasoline + pressure due to 1.8 cm of water = 1.995

or,
$$0.0118 \times \frac{h_{air}}{100} + 6.65 \times \frac{(30 - 1.8 - h_{air})}{100} + 9.81 \times \frac{1.8}{100} = 1.995$$

or,
$$0.0118\, h_{air} + 187.53 - 6.65\, h_{air} + 17.658 = 199.5$$

or,
$$h_{air} = \frac{187.53 + 17.658 - 199.5}{6.638} = \mathbf{0.857\,cm\ (Ans.)}$$

Example 2.26. *For the Fig. 2.29 determine the pressure difference between pipes A and B. Take $Z_1 = 0.45$ m, $Z_2 = 0.225$ m, $Z_3 = 0.675$ m and $Z_4 = 0.3$ m.*

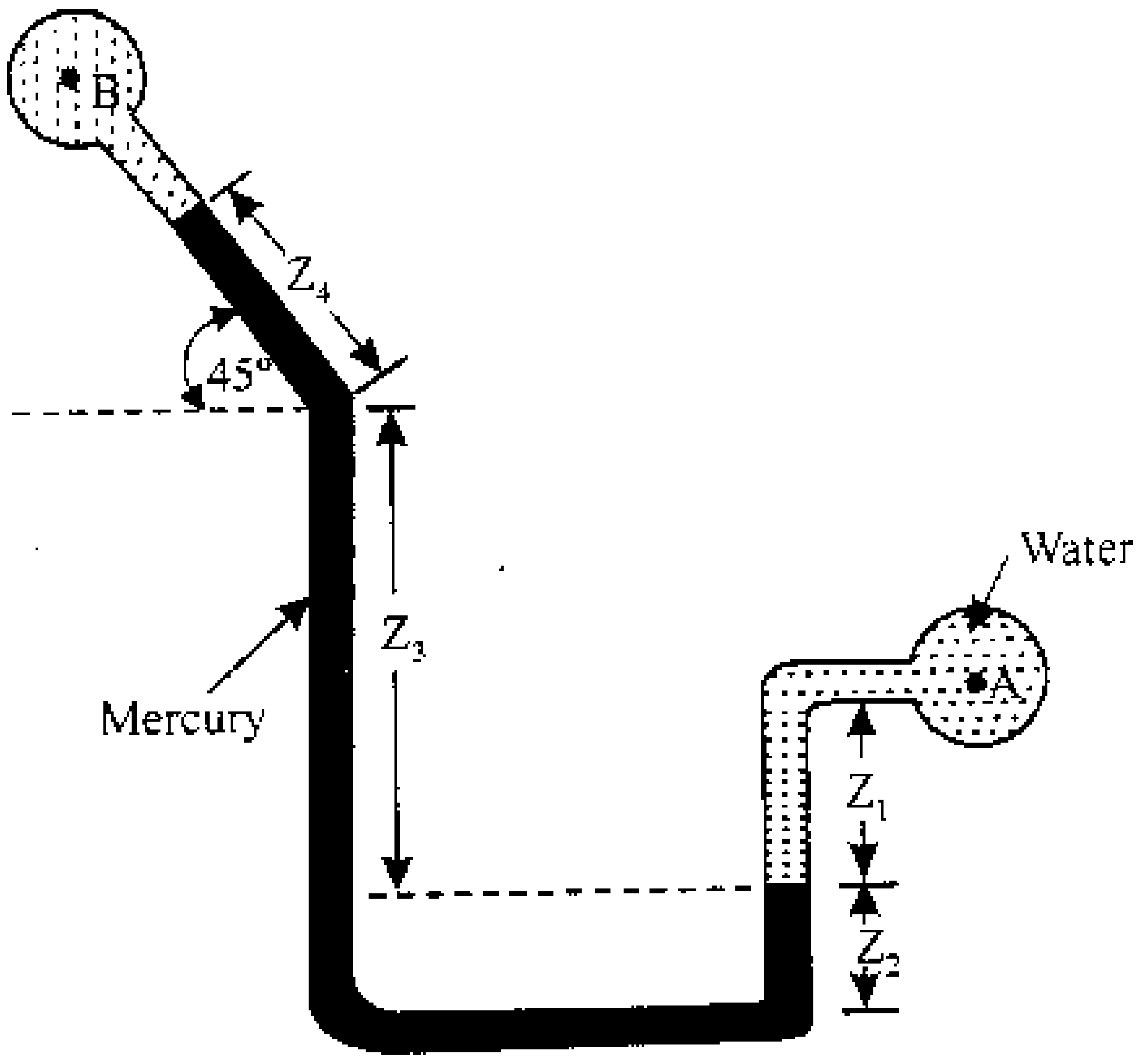


Fig. 2.29

Neglect pressure due to pressure of air column in the inclined tube.

Solution. Starting from point A, the governing manometric equation is:

$$p_A + w_w Z_1 - w_m(Z_3 + Z_4 \sin 45°) = p_B$$

∴ Pressure difference,

$$p_A - p_B = -w_m Z_1 + w_m(Z_3 + Z_4 \sin 45°)$$

$$= -9.81 \times 0.45 + 13.6 \times 9.81\,(0.675 + 0.3 \sin 45°)$$

$$= -4.414 + 118.357 = \mathbf{113.943\ kN/m^2}\ \textbf{(Ans.)}$$

Example 2.27. *From the Fig. 2.30 determine the absolute pressure in pipe A that contains oil of specific gravity = 0.88. Take $Z_1 = 0.66$ m, $Z_2 = 0.33$ m, $Z_3 = 0.165$ m and $Z_4 = 0.11$ m.*

Assume an atmospheric pressure 105 kPa. **(Madras University)**

Solution. Starting from F.W.S (free water surface) in tank (at atmospheric pressure), we get

$$p_{atm} - w_w Z_1 - w_w Z_2 - w_m Z_3 + w_0(Z_3 + Z_4) = p_A$$

$$105 + 9.81 \times 0.66 - 9.81 \times 0.33 - 13.6 \times 9.81 \times 0.165 + 0.88 \times 9.81 \times (0.165 + 0.11) = p_A$$

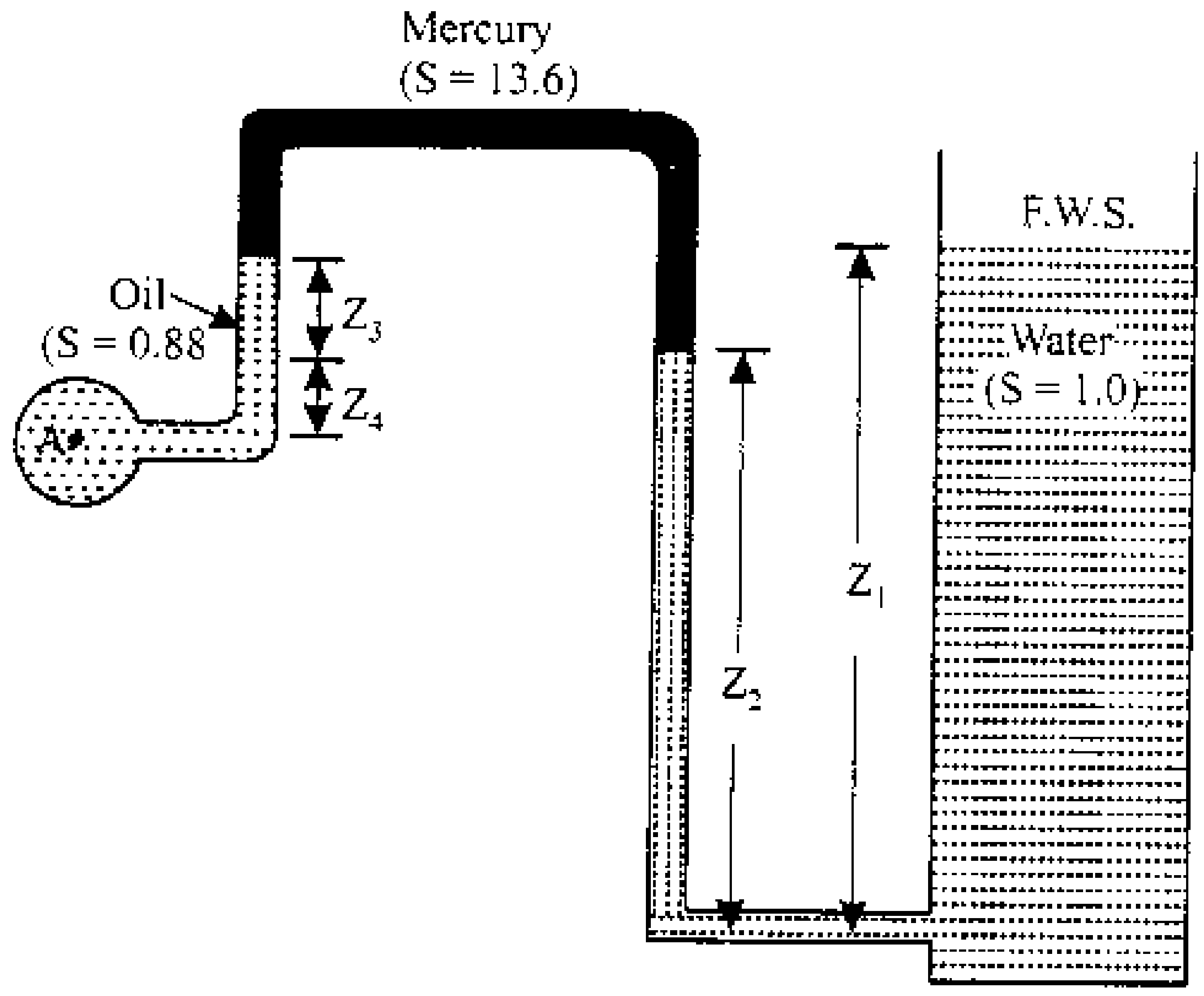


Fig. 2.30

or, $$p_A = 105 + 6.475 - 3.237 - 22.014 + 2.374$$

$$= \mathbf{88.6\ kN/m^2}\ (\textit{absolute})\ \textbf{(Ans.)}$$

Example 2.28. *Find the pressure difference between L and M in Fig. 2.31.*

Solution. $p_L - p_M$:

$$\underset{(\text{at } L)}{\frac{p_L}{w}} + \underset{(\text{at } N)}{h \times 1.5} - \underset{(\text{at } U = V)}{0.15 \times 0.8}$$

$$+ (0.15 + 0.2 - h) \times 1.5 = \frac{p_M}{w}$$

$$\frac{p_L}{w} + 1.5h - 0.12 + 0.525 - 1.5h = \frac{p_M}{w}$$

or, $$\frac{p_L - p_M}{w} = -0.405\ \text{m}$$

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, we get
.81
0.11) $= p_A$

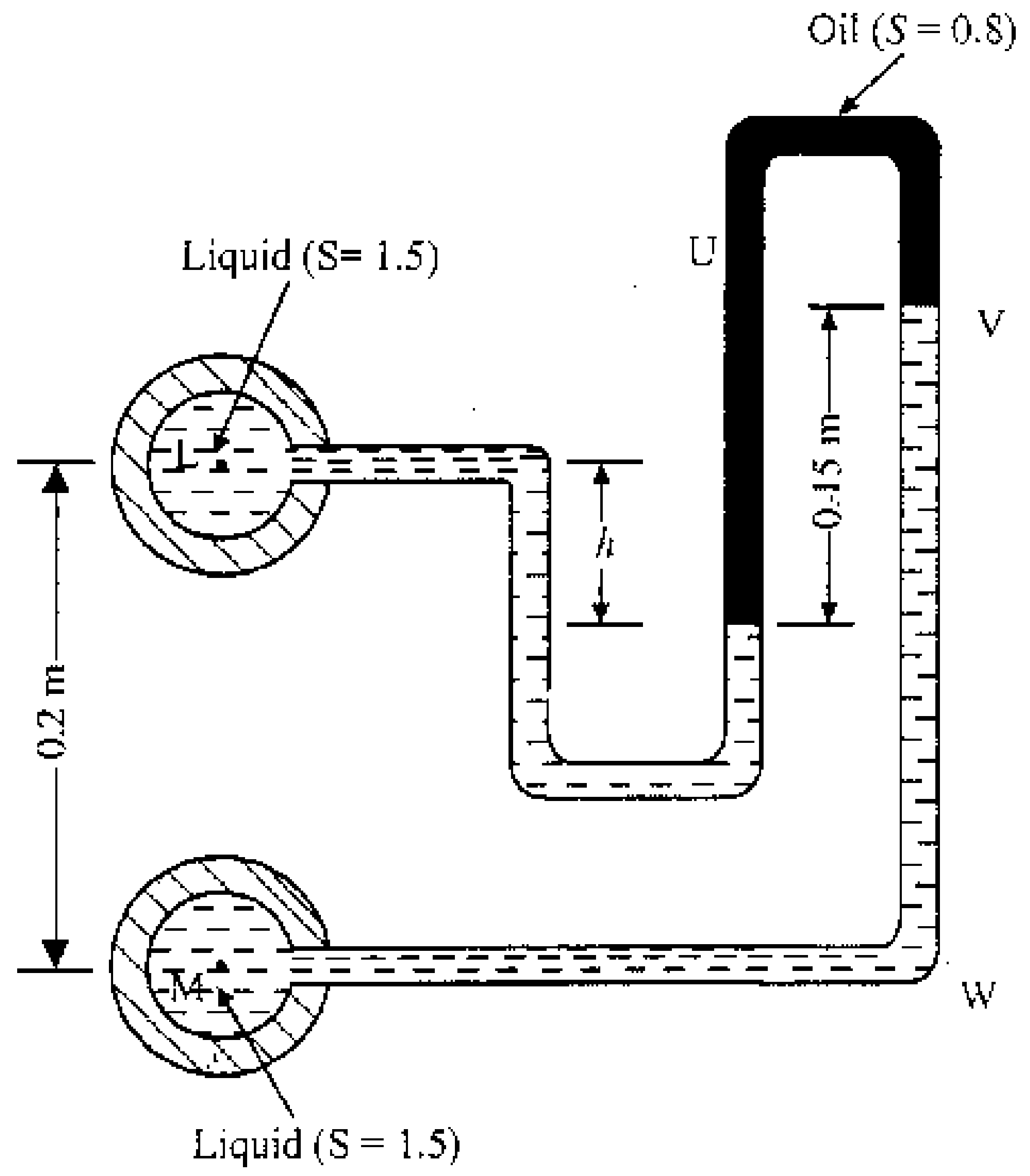

Fig. 2.31

Negative sign indicates $p_M > p_L$

i.e., $p_M - p_L = 0.405 \times 9.81$

$= \mathbf{3.97\ kN/m^2}$ **(Ans.)**

Example 2.29. *In the Fig. 2.32, if the local atmospheric pressure is 755 mm of mercury (sp. gravity = 13.6), calculate:*

(*i*) *Absolute pressure of air in the tank;*

(*ii*) *Pressure gauge reading at L.*

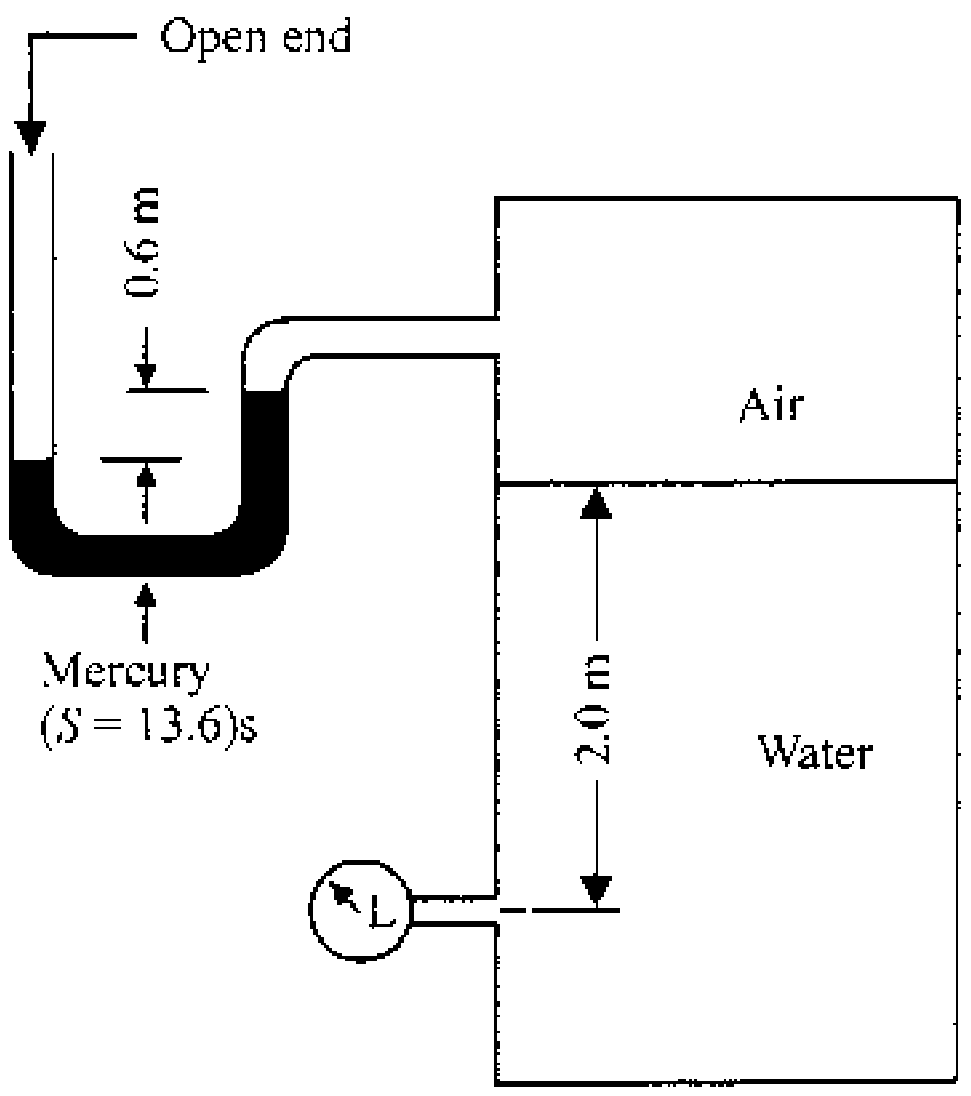

Fig. 2.32

Solution. (*i*) **Absolute pressure of air, $(p_{abs})_{air}$:**

Starting from the open end, we have

$0 - (13.6 \times w) \times 0.6 = p_{air}$ (pressure of air)

i.e., $p_{air} = -13.6 \times 9.81 \times 0.6 = -80\ \text{kN/m}^2$

$p_{atm.}$ = (atmospheric pressure)

$$= \frac{755}{1000} \times 13.6 \times 9.81 = 100.73\ \text{kN/m}^2$$

$(p_{abs.})_{air} = p_{air} + p_{atm.} = -80 + 100.73 = 20.73\ \text{kN/m}^2$

Hence $(p_{abs.})_{air} = \mathbf{20.73\ kN/m^2}$ **(Ans.)**

(*ii*) **Pressure gauge reading at L:**

Pressure at $L = p_{abs.}$ (air) + wh

$p_L = 20.73 + 9.81 \times 2 = 40.35\ \text{kN/m}^2$ *abs.*

Now, $40.35 = p_{gauge} + p_{atm.}$

$p_{gauge(L)} = 40.35 - p_{atm.} = 40.35 - 100.73$

$= -60.38 \text{ kN/m}^2$

i.e., Vacuum pressure = 60.38 kN/m²

Hence, *pressure gauge reading at* L = **60.38 kN/m² (vacuum) (Ans.)**

Example 2.30. *Find the gauge readings at L and M in Fig. 2.33 if the local atmospheric pressure is 755 mm of mercury.*

Solution. Assuming the vapour pressure of mercury (Hg) and pressure due to short column of air (w_{air} is very low) to be negligible, we have

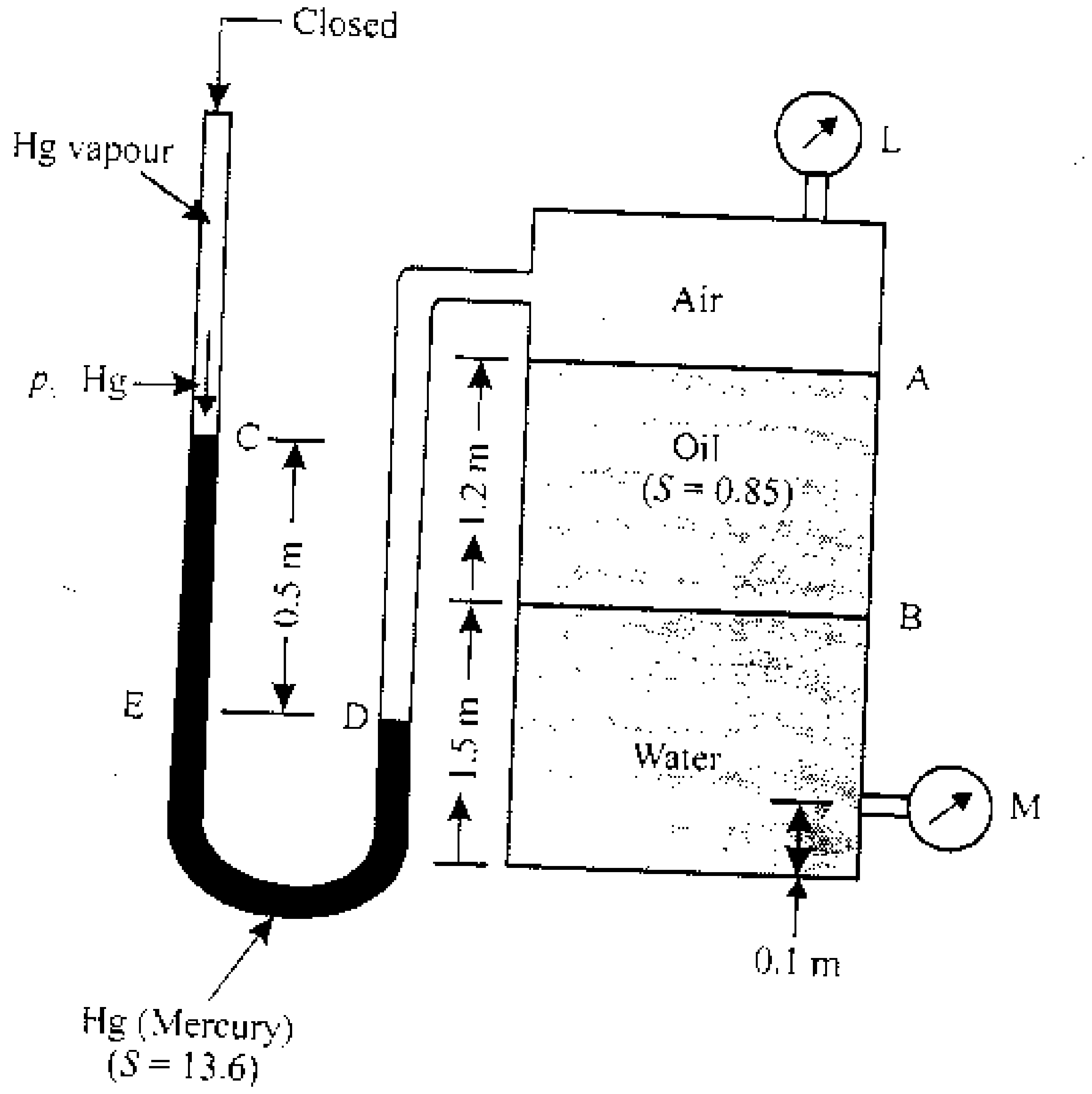

Fig. 2.33

(*i*) $(p_{gauge})_L$:

$$\underset{(\text{at } C)}{(p_v\, Hg \approx 0)} + \underset{(\text{at } D = E = A)}{0.5 \times 13.6} = \underset{(\text{at } A)}{6.8 \text{ m of water } abs.}$$

$$p_{gauge} + p_{atm.} = p_{abs.}$$

But, $$p_{atm.} = \frac{755}{1000} \times 13.6 = 10.27 \text{ m of water}$$

∴ $$p_{gauge} + 10.27 = 6.8$$

or, $$p_{gauge} = -3.47 \text{ m of water}$$
$$= -3.47 \times 9.81$$
$$= -34 \text{ kN/m}^2$$

Hence, *gauge reading at* L = **34 kN/m²** (*vacuum*) **(Ans.)**

(*ii*) $(p_{gauge})_M$:

$$\underset{(\text{at } A)}{6.8} + \underset{(\text{at } B)}{1.2 \times 0.85} + \underset{(\text{at } M)}{(1.5 - 0.1)} = 9.22 \text{ m of water } abs.$$

$$p_{gauge} + p_{atm.} = p_{abs.}$$

$$p_{gauge} + 10.27 = 9.22$$

or, $$p_{gauge} = -1.05 \text{ m of water}$$

$$= -1.05 \times 9.81 = -10.3 \text{ kN/m}^2$$

Hence, *gauge reading at M* = **10.3 kN/m²** (*vacuum*) **(Ans.)**

Example 2.31. *For the Fig 2.34 determine specific gravity of gauge liquid B if the gauge pressure at A is – 18 kN/m².*

Solution. Sp. gravity of liquid B:

Pressure at L = pressure at M

i.e., $$-18 + (1.5 \times 9.81 \times 0.6) = p_M$$

or, $$p_M = -9.17 \text{ kN/m}^2$$

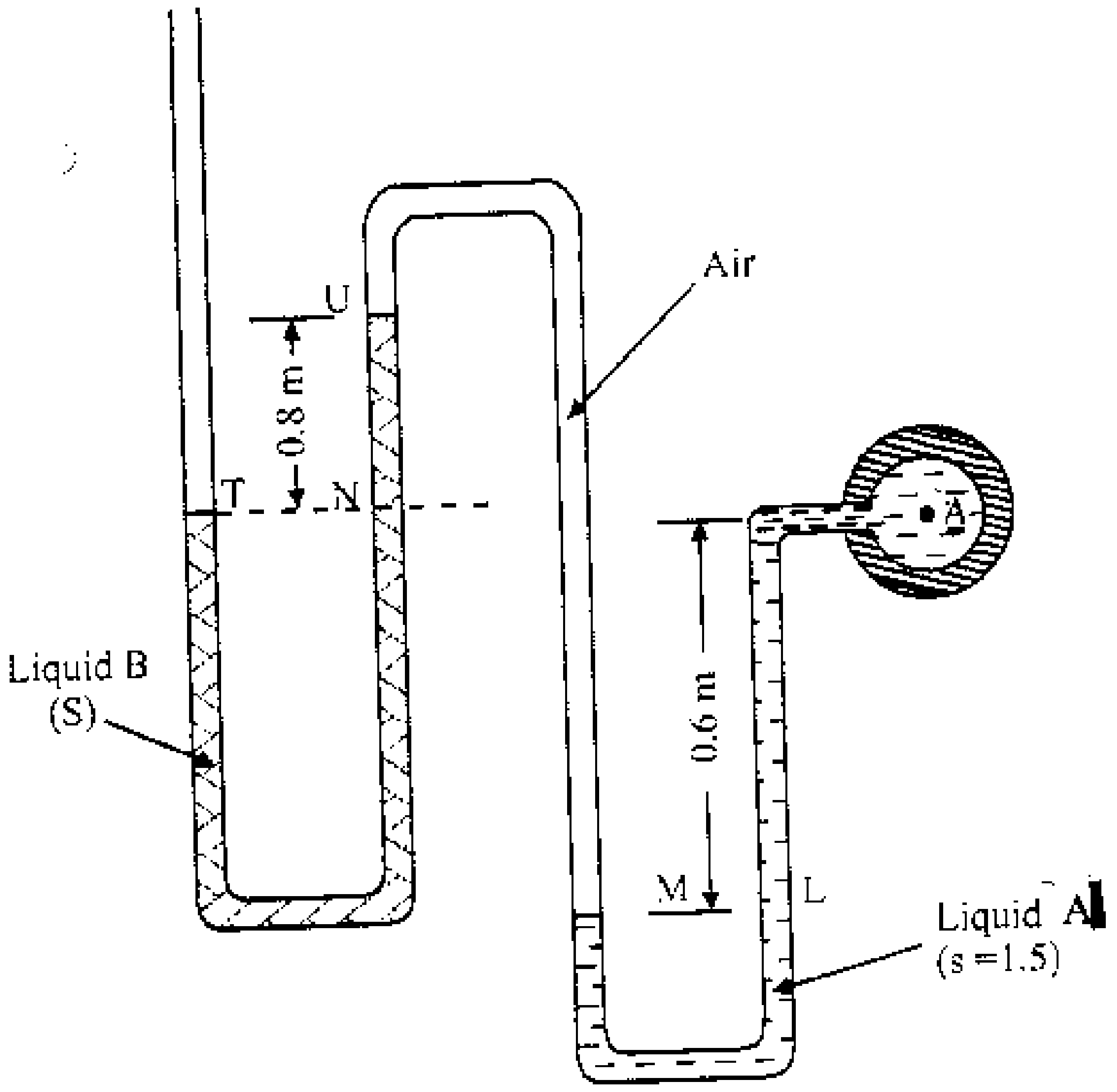

Fig. 2.34

Between points M and U, since there is an air column which can be neglected, therefore,

$$p_M = p_U (= -9.17 \text{ kN/m}^2)$$

Also, pressure at N = pressure at T.

But point T being at atmospheric pressure,

$$p_T = 0 = p_N$$

Thus $$p_N = p_U + S \times 9.81 \times 0.8 = 0$$

or $$0 = -9.17 + 7.848\,S$$

$$\mathbf{S = 1.17 \text{ (Ans.)}}$$

Example 2.32. (***Compound manometer***). *In the Fig. 2.35 is shown a compound manometer. Find the gauge pressure at A if the manometric fluid is mercury and the fluid in the pipe and in the tubing which connects the two U-tubes is water.*

Solution. Gauge pressure at A, p_A:

Pressure at B = pressure at C

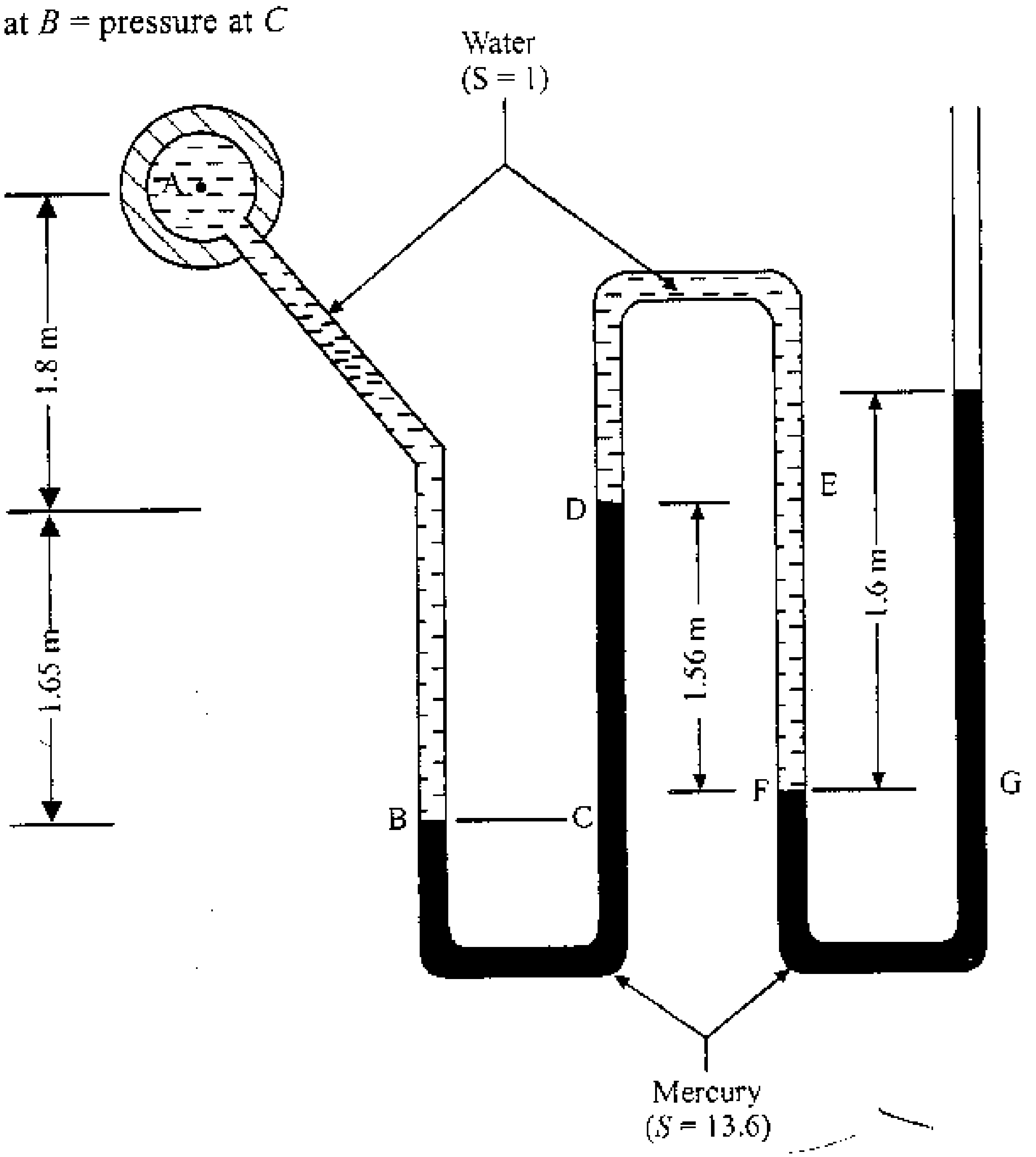

Fig. 2.35

$\therefore$
$$\frac{p_B}{w} = \frac{p_A}{w} + (1.8 + 1.65) = \frac{p_C}{w}$$

Further,

Pressure at D,

$$\frac{p_D}{w} = \frac{p_C}{w} - 1.65 \times 13.6$$

$$= \frac{p_A}{w} + (1.8 + 1.65) - 1.65 \times 13.6$$

$$= \frac{p_A}{w} + 3.45 - 22.44$$

or,
$$\frac{p_D}{w} = \frac{p_A}{w} - 18.99 \qquad \text{...(1)}$$

Also,
$$p_D = p_E \text{ and } p_F = p_G$$

But,
$$\frac{p_F}{w} = \frac{p_E}{w} + 1.56$$

and,
$$\frac{p_G}{w} = 1.6 \times 13.6 = 21.76$$

i.e., $\frac{p_E}{w} + 1.56 = 21.76$ $(\because p_F = p_G)$

or $\frac{p_E}{w} = 20.2$ or $\frac{p_D}{w} = 20.2$ $(\because p_D = p_E)$

Substituting the value of $\frac{p_D}{w}$ in eqn. (1), we get, $20.2 = \frac{p_A}{w} - 18.99$

or, $\frac{p_A}{w} = 20.2 + 18.99 = 39.19$ m of water.

i.e., $p_A = 9.81 \times 39.19 =$ **384.4 kN/m² (Ans.)**

Example 2.33. (*Compound manometer*). *In the Fig. 2.36 is shown a compound manometer. Calculate pressure difference between the points A and B. Take* $w_w = 10$ *kN/m³ for water,* $w_m = 136$ *kN/m³ for mercury and* $w_0 = 8.5$ *kN/m³ for oil.* **(Panjab University)**

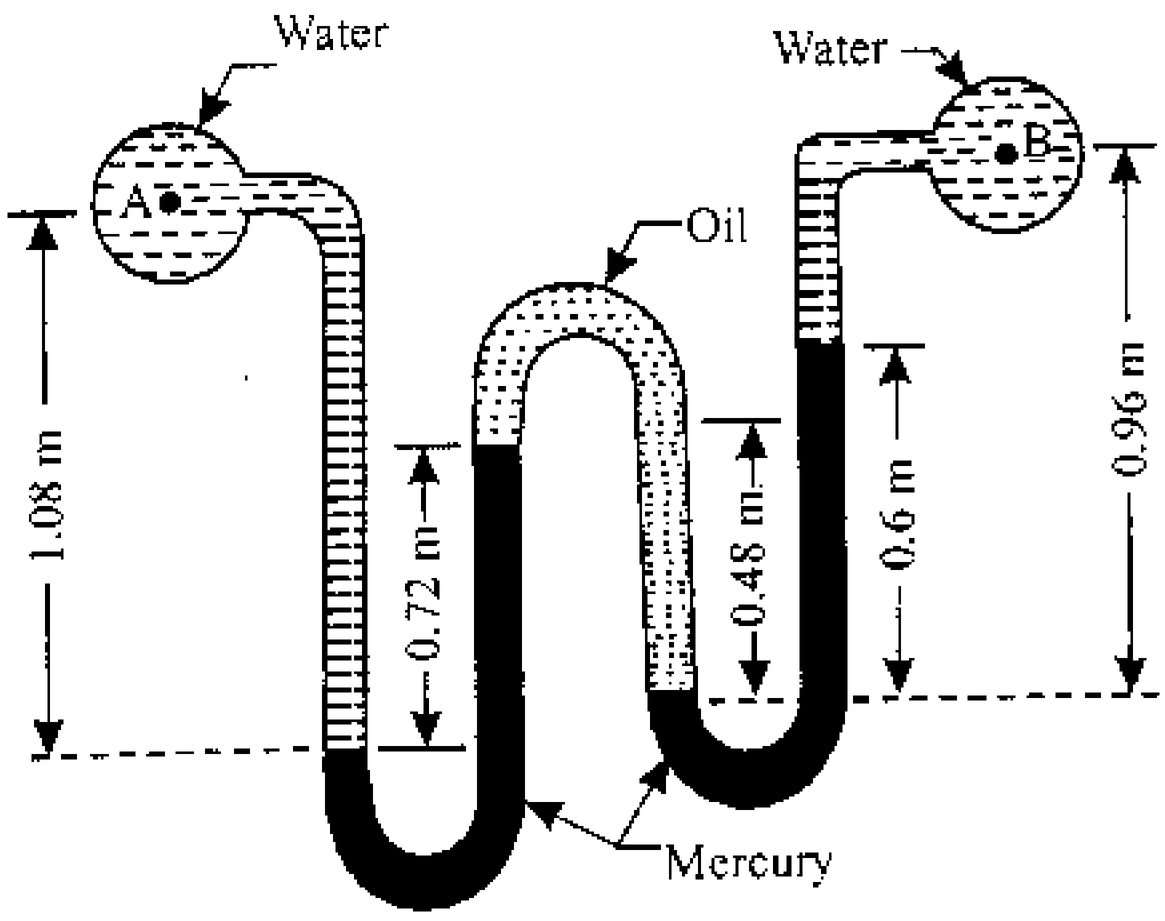

Fig. 2.36

Solution. *Given:* $w_w = 10$ kN/m³; $w_m = 136$ kN/m³; $w_0 = 8.5$ kN/m²

$p_A - p_B$:

Starting from point A, the governing manometric equation is:

$$p_A + w_w \times 1.08 - w_m \times 0.72 + w_0 \times 0.48 - w_m \times 0.6 - w_w(0.96 - 0.6) = p_B$$

or $p_A = 10 \times 1.08 - 136 \times 0.72 + 8.5 \times 0.48 - 136 \times 0.6 - 10(0.96 - 0.6) = p_B$

or $p_A + 10.8 - 97.92 + 4.08 - 81.6 - 3.6 = p_B$

or $p_A - p_B =$ **168.24 kN/m² (Ans.)**

Example 2.34. *A cylindrical bucket (empty) 450 mm in diameter and 750 mm long is forced with it open end first into water until its lower edge is 6 m below the surface. Determine the force required to maintain position, assuming the trapped air remains at constant temperature during the whole operation. Atmospheric pressure = 1.01 bar.*

The wall thickness and weight of the bucket may be considered as negligible.

Solution. Diameter of the bucket, $d = 450$ mm $= 0.45$ m

Length of the bucket, $l = 750$ mm $= 0.75$ m

Atmospheric pressure, $p_{atm} = 1.01$ bar.

Force required to maintain position, F: Refer to Fig 2.37

Let, p_{air} = Absolute pressure of compressed air trapped in the cylindrical bucket, and

y = Depth of water raised in the bucket.

Then, since the temperature of air remains constant, therefore, as per isothermal condition, we have

$$p_{atm.} \times \frac{\pi}{4} \times 0.45^2 \times 0.75 = p_{air} \times \frac{\pi}{4} \times 0.45^2 \times (0.75 - y)$$

$$(\because p_1 V_1 = p_2 V_2 \text{..... for isothermal process})$$

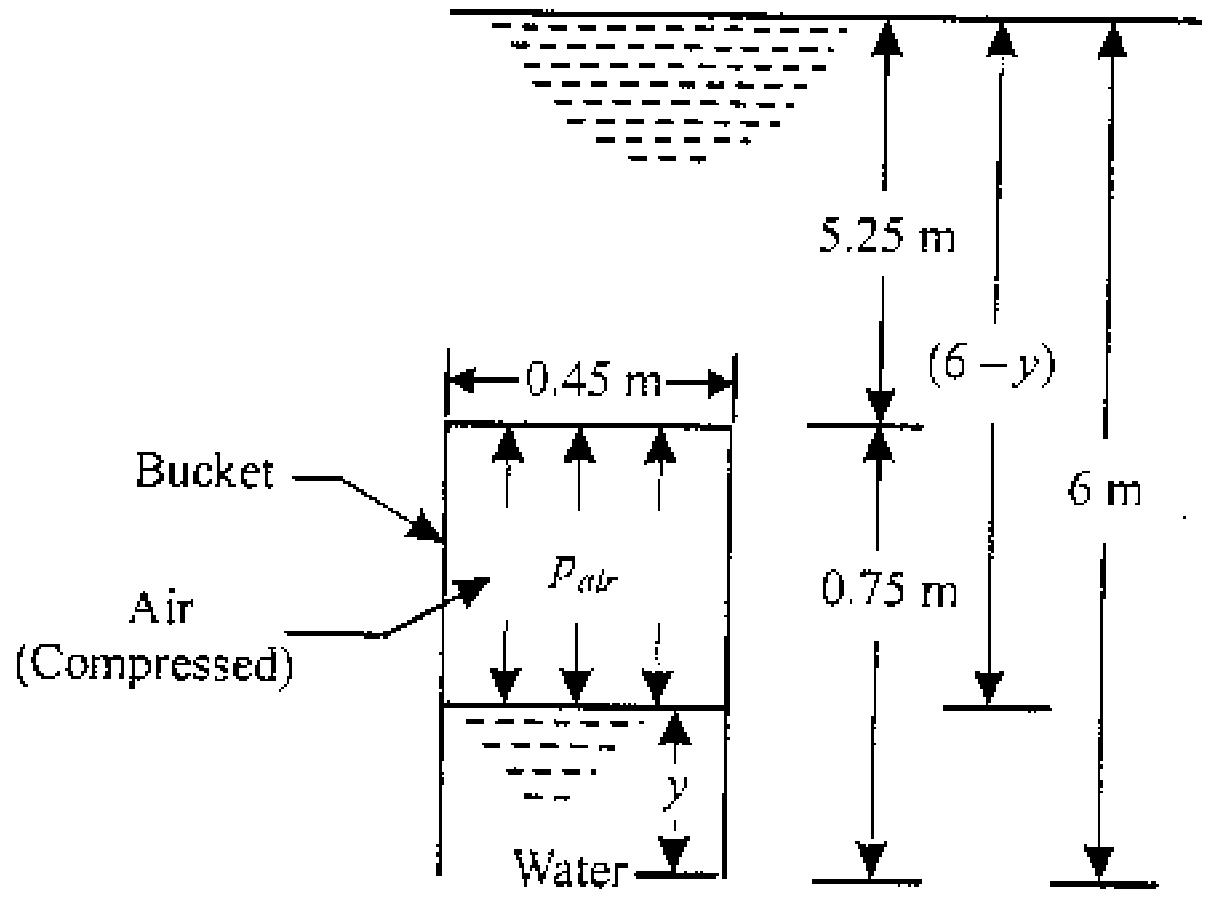

Fig. 2.37

or, $$p_{air} = \left(\frac{0.75}{0.75 - y}\right) p_{atm.} \quad \text{...}(i)$$

Also, $$p_{air} = p_{atm.} + wh = p_{atm.} + 9810 \times (6 - y) \quad \text{...}(ii)$$

$$(\because w = 9810 \text{ N/m}^3)$$

From eqn. (*i*) and (*ii*), we have

$$\left(\frac{0.75}{0.75 - y}\right) p_{atm.} = p_{atm.} + 9810(6 - y)$$

or, $$\left(\frac{0.75}{0.75 - y}\right) \times 1.01 \times 10^5 = 1.01 \times 10^5 + 9810(6 - y)$$

or, $$1.01 \times 10^5 \left(\frac{0.75}{0.75 - y} - 1\right) = 9810(6 - y)$$

or, $$\frac{1.01 \times 10^5}{9810}\left(\frac{0.75 - 0.75 + y}{0.75 - y}\right) = 6 - y$$

or, $$10.29 \times \left(\frac{y}{0.75 - y}\right) = 6 - y \quad \text{or} \quad \frac{10.29y}{0.75 - y} + y = 6$$

or, $$10.29y + y(0.75 - y) = 6(0.75 - y)$$

or, $$10.29y + 0.75y - y^2 = 4.5 - 6y \quad \text{or} \quad y^2 - 17.04y + 4.5 = 0$$

or, $$y = \frac{17.04 \pm \sqrt{17.04^2 - 4 \times 4.5}}{2} = \frac{17.04 \pm 16.5}{2} = 0.27 \text{ m}$$

(ignoring + ve sign being not possible)

Substituting the value of *y* in (*i*), we get

$$p_{air} = \left(\frac{0.75}{0.75 - 0.27}\right) \times 1.01 = 1.578 \text{ bar}$$

The force tending to move the bucket in *upward direction,*

$$P_1 = p_{air} \times \frac{\pi}{4} \times 0.45^2$$

$$= (1.578 \times 10^5) \times \frac{\pi}{4} \times 0.45^2 \times 10^{-3} \text{ kN} = 25.097 \text{ kN}$$

The force acting on the bucket in the *downward direction,*

$$P_2 = (1.01 \times 10^5 + 9810 \times 5.25) \times \frac{\pi}{4} \times 0.45^2 \times 10^{-3} \text{ kN} = 24.254 \text{ kN}$$

$\therefore$ The force required to maintain the bucket in position,

$$F = P_1 - P_2 = 25.097 - 24.254 = \mathbf{0.843 \text{ kN (Ans.)}}$$

Example 2.35. *A glass tube of uniform bore is bent into the form of a square of sides l and filled with equal amounts of three invisible liquids of densities* ρ_1, ρ_2 *and* ρ_3. *It is known that* $\rho_1 < \rho_2 < \rho_3$. *If the tube arrangement is placed in a vertical plane (i.e. two sides vertical) and if one of the vertical sides is completely filled with the liquid of density* ρ_2:

(*i*) *Show that* $\frac{1}{3}(2\rho_3 + \rho_1) > \rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1)$

(*ii*) *If the relative densities of the first and third liquids are 1.0 and 1.22 respectively, find the range of the relative densities of the second liquid which makes the above arrangement possible.*

Solution. Refer to Fig 2.38.

(*i*) **To prove,** $\frac{1}{3}(2\rho_3 + \rho_1) > \rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1)$ **:**

Let E, F and G be the interfaces, and

EA = x

Then, DE = $l - x$

Total length of the glass tube = $4l$

$\therefore$ length of each liquid = $\frac{4}{3}l$

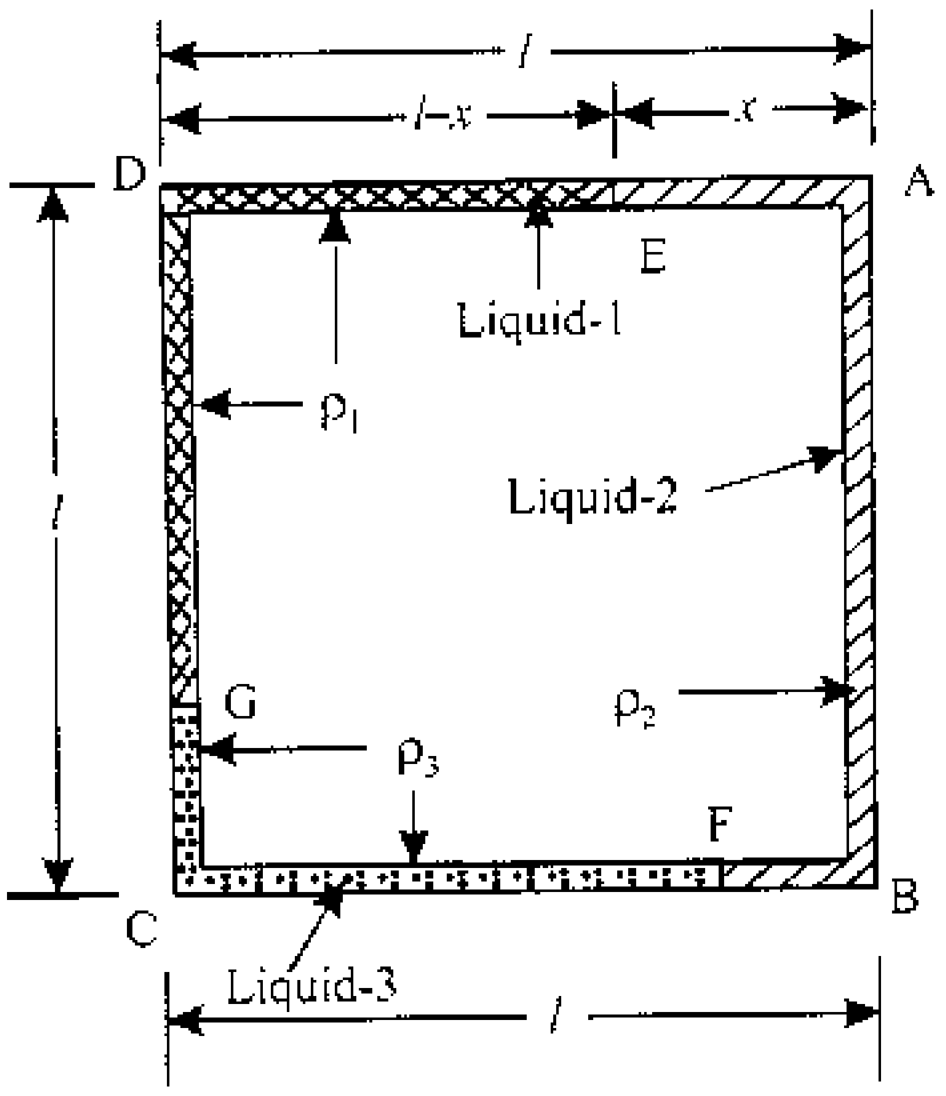

Fig. 2.38

*For liquid-*1:

$$EG = \frac{4}{3}l$$

$$DG = \frac{4}{3}l - (l - x) = \frac{1}{3}l + x$$

*For liquid-*3:

$$GC = l - \left(\frac{1}{3}l + x\right) = \frac{2}{3}l - x$$

$$FB = l - \left(\frac{2}{3}l + x\right) = \frac{1}{3}l - x$$

$$\left(\because FC = \frac{4l}{3} - GC = \frac{4l}{3} - \left(\frac{2}{3}l - x\right) = \frac{2}{3}l + x\right)$$

(Check: FB + BA + AE = $(\frac{1}{3}l - x) + l + x = \frac{4}{3}l$)

The pressure balance at the interface F is given by,
Pressure of (column DG + column GC) = pressure of column AB

$$\rho_1 g(\frac{1}{3}l + x) + \rho_3 g(\frac{2}{3}l - x) = \rho_2 g l$$

or $$\rho_1 (\frac{1}{3}l + x) + \rho_3 (\frac{2}{3}l - x) = \rho_2 l$$

$$x(\rho_1 - \rho_3) = \frac{1}{3}l\,(-\rho_1 + 3\rho_2 - 2\rho_3)$$

or $$x = \frac{\frac{1}{3}l\,(-\rho_1 + 3\rho_2 - 2\rho_3)}{(\rho_1 - \rho_3)} = \frac{\frac{1}{3}l(2\rho_3 - 3\rho_2 + \rho_1)}{(\rho_3 - \rho_1)}$$

It is known that $x > 0$ and also $x < \frac{1}{3}l$

Hence, $$0 < x < \frac{l}{3}$$

$\therefore$ $$0 < \frac{(2\rho_3 - 3\rho_2 + \rho_1)}{(\rho_3 - \rho_1)} < 1$$

Also since $\rho_1 < \rho_2 < \rho_3$ the denominator $(\rho_3 - \rho_1)$ is positive.
Hence numerator is

$$0 < (2\rho_3 - 3\rho_2 + \rho_1) < 1$$

or, $$\rho_1 + 2\rho_3 > 3\rho_2$$

or, $$\rho_2 < \frac{1}{3}(2\rho_3 + \rho_1) \qquad ...(i)$$

Also, since $$\frac{2\rho_3 - 3\rho_2 + \rho_1}{\rho_3 - \rho_1} < 1$$

or, $$2\rho_3 - 3\rho_2 + \rho_1 < (\rho_3 - \rho_1)$$

or, $$3\rho_2 > \rho_3 - 2\rho_1$$

or, $$\rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1) \qquad ...(ii)$$

Hence from inequalities (*i*) and (*ii*), we have

$$\frac{1}{3}(2\rho_3 + \rho_1) > \rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1)$$

(*ii*) **Range of relative densities of the second liquid:**

Given: $\rho_1 = 1.0;\ \ \rho_3 = 1.22$

Now, $$\rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1)$$

$$> \frac{1}{3}(1.22 + 2 \times 1.0) > 1.0733$$

Also, $$\rho_2 > \frac{1}{3}(2\rho_3 + \rho_1)$$

$$> \frac{1}{3}(2 \times 1.22 + 1.0) > 1.1467$$

Hence **$1.0733 < \rho_2 < 1.1467$ (Ans.)**

2.5.1.3. Advantages and Limitations of Manometers

Advantages of manometers:

1. Easy to fabricate and relatively inexpensive.
2. Good accuracy.
3. High sensitivity.
4. Require little maintenance.
5. Not affected by vibration.
6. Specially suitable for low pressure and low differential pressures.
7. It is easy to change the sensitivity by affecting a change in the quantity of manometic liquid in the manometer.

Limitations:

1. Usually bulky and large in size.
2. Being fragile, get broken easily.
3. Readings of the manometers are affected by changes in temperature, altitude and gravity.
4. A capillary effect is created due to surface tension of manometric fluid.
5. For better accuracy meniscus has to be measured by accurate means.

2.5.2. Mechanical Gauges

The manometers (discussed earlier) are suitable for *comparatively low pressures*. For high pressures, they become unnecessarily larger even when they are filled with heavy liquids. Therefore for measuring medium and high pressures, we make use of elastic pressure gauges. They employ different forms of elastic systems such as tubes, diaphragms or bellows etc. to measure the pressure. The elastic deformation of these elements is used to show the effect of pressure. Since these elements are deformed within the elastic limit only, therefore these gauges are sometimes called *elastic gauges*. Sometimes they are also called *secondary instruments, which implies that they must be calibrated by comparison with primary instruments such as manometers etc.*

Some of the important types of these gauges are enumerated and discussed below:

1. Bourdon tube pressure gauge,
2. Diaphragm gauge, and
3. Vacuum gauge.

1. Bourdon tube pressure gauge:

Bourdon tube pressure gauge is used for measuring high as well as low pressures. A simple form of this gauge is shown in Fig. 2.39. In this case, the pressure element consists of a metal tube of approximately elliptical cross-section. This tube is bent in the form of a segment of a circle and responds to pressure changes. When one end of the tube which is attached to the gauge case, is connected to the source of pressure, the internal pressure causes the tube to expand, whereby circumferential stress *i.e.*, hoop tension is set up. The free end of the tube moves and is in turn connected by suitable levers to a rack, which engages with a small pinion mounted on the same spindle as the pointer. Thus the pressure applied to the tube causes the rack and pinion to move. The pressure is indicated by the pointer over a dial which can be graduated in a suitable scale.

The Bourdon tube are generally made of *bronze or nickel steel*. The *former* is generally used for *ow pressures* and the *latter for high pressures*.

Depending upon the purpose for which they are required Bourdon tube gauges are made in ifferent forms, some of them are:

(*i*) *Compound Bourdon tube*–used for measuring pressures both *above and below atmospheric pressure*.

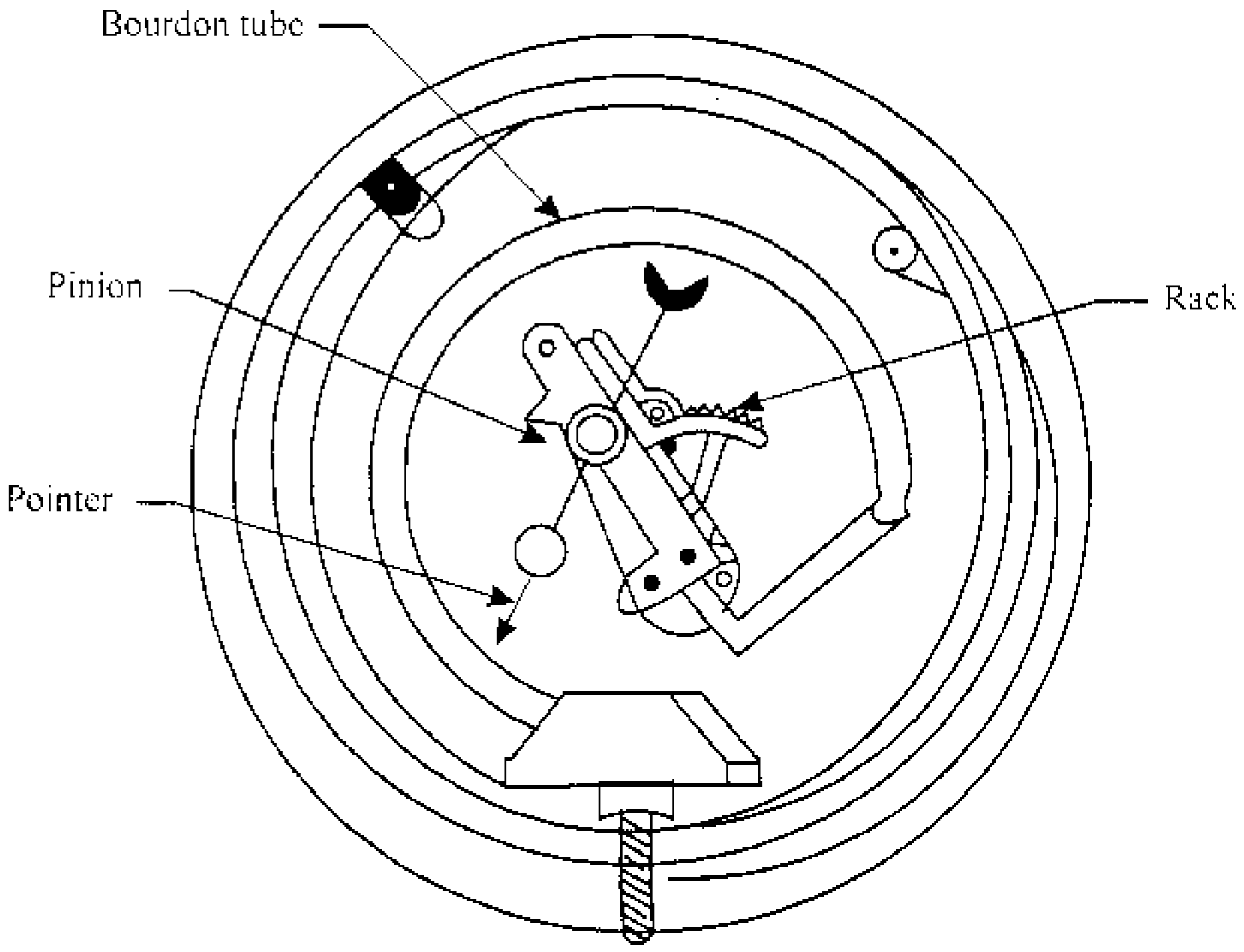

Fig. 2.39. Bourdon tube pressure gauge.

(*ii*) *Double Bourdon tube*–used where *vibrations are encountered.*

2. Diaphragm gauge:

This type of gauge employs a metallic disc or diaphragm instead of a bent tube. This disc or diaphragm is used for *actuating* the indicating device.

Refer to Fig. 2.40 When pressure is applied on the lower side of the diaphragm it is deflected upward. This movement of the diaphragm is transmitted to a rack and pinion. The latter is attached to the spindle of needle moving on a graduated dial. The dial can again be graduated in a suitable scale.

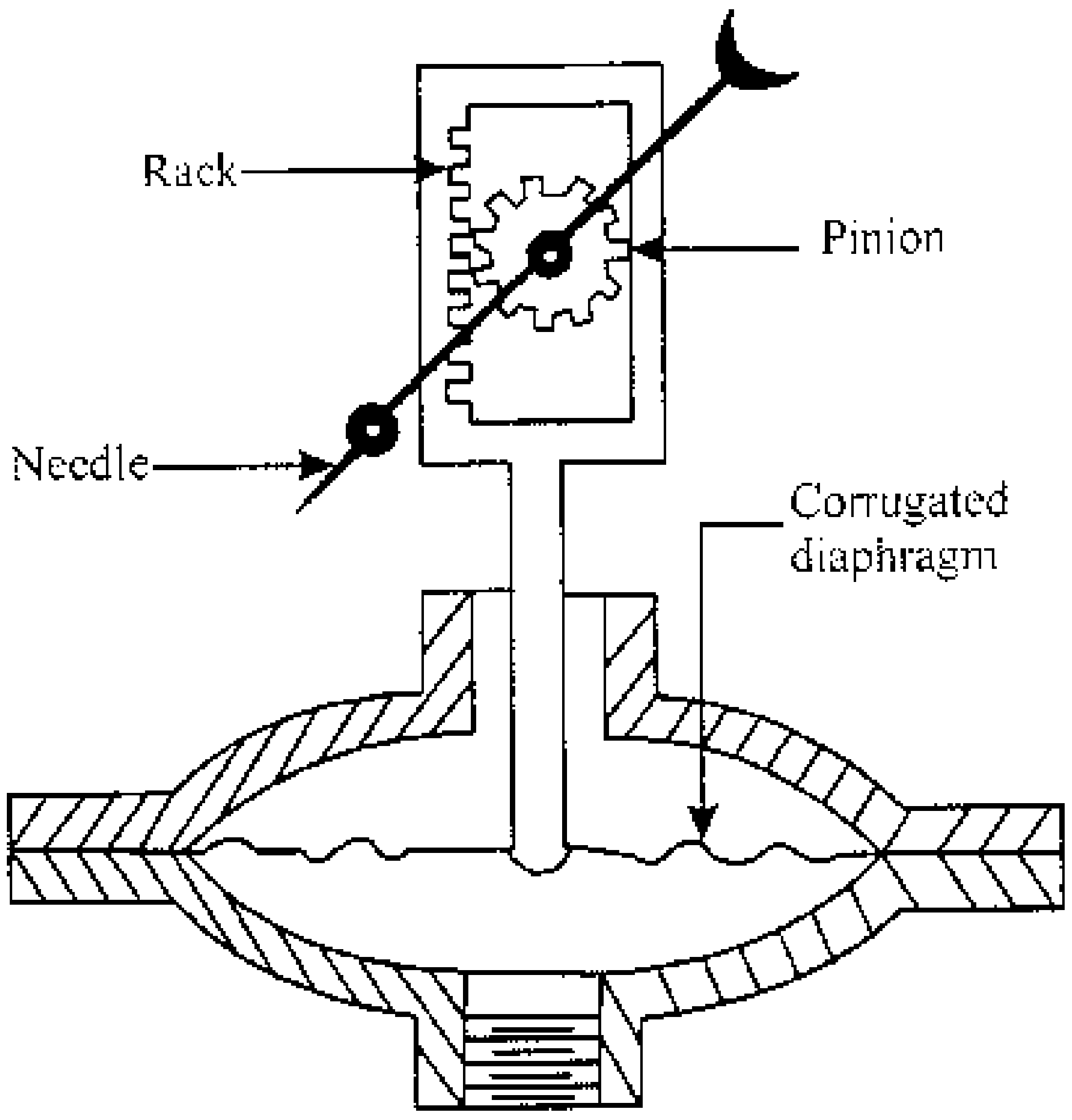

Fig. 2.40. Diaphragm gauge.

3. Vacuum gauge:

Bourdon gauges discussed earlier can be used to measure vacuum instead of pressure. Slight changes in the design are required in this purpose. Thus, in this case, the tube be *bent inward instead of outward* as in pressure gauges. *Vacuum gauges* are graduated in millimetres of mercury below atmospheric pressure. In such cases, therefore, absolute pressure in millimetres of mercury is the difference between barometer reading and vacuum gauge reading.

Vacuum gauges are used to measure the vacuum in the condensers, etc. If there is leakage, the vacuum will drop.

The pressure gauge installation requires the following considerations:

1. Flexible copper tubing and compression fittings are recommended for most installations.
2. The installation of a gauge cock and tee in the line close to the gauge is recommended because it permits the gauge to be removed for testing or replacement without having to shut down the system.
3. Pulsating pressures in the gauge line are not required.
4. The gauge and its connecting line is filled with an inert liquid and as such liquid seals are provided. Trapped air at any point of gauge lines may cause serious errors in pressure reading.

2.6 Pressure at a Point in Compressible Fluid

In case of compressible fluids, the density (ρ) changes with the change of pressure and temperature. In the fields of meteorology, oceanography and aeronautics, we come across with problems involving atmospheric air where density, pressure and temperature change with the elevation. Therefore, for fluids having variable density eqn. (2.4) cannot be intergrated unless relation between ρ and p is known.

The 'equation of state' for gases is given as:

$$p = \rho RT \qquad \text{...(2.13)}$$

or,

$$\rho = \frac{p}{RT}$$

We know that,

$$\frac{dp}{dZ} = w = \rho \times g \qquad \text{(Eqn.2.4)}$$

$$= \frac{p}{RT} \times g$$

$$\therefore \quad \frac{dp}{p} = \frac{g}{RT} \times dZ \qquad \text{...(2.14)}$$

When Z is measured *vertically upward*, the above equation reduces to:

$$\frac{dp}{p} = \frac{-g}{RT} \times dZ \qquad \text{...(2.15)}$$

Isothermal process:

In an isothermal process, temperature T remains constant, therefore, integrating eqn. (2.15) we get

$$\int_{p_0}^{p} \frac{dp}{p} = - \int_{Z_0}^{Z} - \frac{g}{RT}\, dZ = - \frac{g}{RT} \int_{Z_0}^{Z} dZ$$

or,

$$ln\left(\frac{p}{p_0}\right) = - \frac{g}{RT}(Z - Z_0)$$

where, p_0 is the pressure, and Z_0 is the height.

If the datum line is taken at Z_0, then Z_0 becomes zero and p_0 becomes the pressure at datum line.

$$ln\left(\frac{p}{p_0}\right) = - \frac{g}{RT} . Z$$

or, $$\frac{p}{p_0} = e^{(-gZ/RT)}$$

or, Pressure at a height Z is given by ...(2.16)

$$p = p_0 e^{(-gZ/RT)}$$

Adiabatic process:

When the process follows an adiabatic law, the relation between pressure and density is given by:

$$\frac{p}{\rho^{\gamma}} = \text{constant} = C \qquad ...(i)$$

where, γ is the ratio of specific heats.

$$\therefore \qquad \rho^{\gamma} = \frac{p}{C}$$

or, $$\rho = \left(\frac{p}{C}\right)^{1/\gamma} \qquad ...(ii)$$

Now eqn. (2.4) becomes

$$\frac{dp}{dZ} = -\rho g = -\left(\frac{p}{C}\right)^{1/\gamma} .g \qquad \text{.... Z measured vertically up}$$

or, $$\frac{dp}{\left(\frac{p}{C}\right)^{1/\gamma}} = -g\,.\,dZ \quad \text{or} \quad C^{1/\gamma} \times \frac{dp}{(p)^{1/\gamma}} = -g\,.dZ$$

Integrating the above equation, we get

$$C^{1/\gamma}\left[\frac{p^{\left(-\frac{1}{\gamma}+1\right)}}{-\frac{1}{\gamma}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z}$$

or, $$\left[\frac{C^{1/\gamma}p^{\left(-\frac{1}{\gamma}+1\right)}}{-\frac{1}{\gamma}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z}$$

From eqn. (i), we have, $C^{1/\gamma} = \left[\frac{p}{\rho^{\gamma}}\right]^{1/\gamma} = \frac{p^{1/\gamma}}{\rho}$ (C being constant, can be taken inside)

Substituting this value of $C^{1/\gamma}$ in the above equation, we get

$$\left[\frac{p^{1/\gamma}\,p^{\left(-\frac{1}{\gamma}+1\right)}}{\rho\left(\frac{\gamma-1}{\gamma}\right)}\right]_{p_0}^{p} = -g[Z - Z_0]$$

or, $$\frac{\gamma}{\gamma-1}\left[\frac{p}{\rho}\right]_{p_0}^{p} = -g\,(Z - Z_0)$$

$$= \frac{\gamma}{\gamma-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -g\,(Z - Z_0)$$

If datum line is taken at Z_0 (where pressure, temperature and density are p_0, T_0, ρ_0), then $Z_0 = 0$.

$$\therefore \qquad \frac{\gamma}{\gamma-1}\left[\frac{p}{\rho}-\frac{p_0}{\rho_0}\right]=-gZ$$

or,
$$\left(\frac{p}{\rho}-\frac{p_0}{\rho_0}\right)=-gZ\left(\frac{\gamma-1}{\gamma}\right)$$

or,
$$\frac{p}{\rho}=\frac{p_0}{\rho_0}-gZ\left(\frac{\gamma-1}{\gamma}\right)=\frac{p_0}{\rho_0}\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right]$$

or,
$$\frac{p}{\rho}\times\frac{\rho_0}{p_0}=\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right] \qquad \text{...(iii)}$$

But from eqn. (*i*)
$$\frac{p}{\rho^{\gamma}}=\frac{p_0}{\rho_0^{\gamma}} \quad \text{or} \quad \left(\frac{\rho_0}{\rho}\right)^{\gamma}=\frac{p_0}{p} \quad \text{or} \quad \frac{\rho_0}{\rho}=\left(\frac{p_0}{p}\right)^{1/\gamma}$$

Substituting the value of $\frac{\rho_0}{\rho}$ in eqn. (*iii*), we get

$$\frac{p}{p_0}\times\left(\frac{p_0}{p}\right)^{1/\gamma}=\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right]$$

or,
$$\frac{p}{p_0}\times\left(\frac{p}{p_0}\right)^{-1/\gamma}=\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right]$$

or,
$$\left(\frac{p}{p_0}\right)^{1-\frac{1}{\gamma}}=\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right]$$

or,
$$\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}=\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right]$$

or,
$$\frac{p}{p_0}=\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right]^{\frac{\gamma}{\gamma-1}}$$

$\therefore$ Pressure at a height Z from the ground level is given by,

$$p=p_0\left[1-\frac{\gamma-1}{\gamma}gZ\times\frac{\rho_0}{p_0}\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{...(2.17)}$$

ere, p_0 = Pressure at ground level (when $Z_0 = 0$), and

ρ_0 = Density of air at ground level.

Also, equation of state is

$$\frac{p_0}{\rho_0}=RT_0 \quad \text{or} \quad \frac{\rho_0}{p_0}=\frac{1}{RT_0}$$

Substituting the value of $\frac{\rho_0}{p_0}$ is eqn. (2.17), we get

$$p=p_0\left[1-\frac{\gamma-1}{\gamma}\times\frac{gZ}{RT_0}\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{...(2.18)}$$

- *Temperature at any point in compressible fluid in an adiabatic process is calculated as* s:

Equation of state at ground level and at a height Z from the ground level is written as

$$\frac{p_0}{\rho_0} = RT_0, \quad \text{and} \quad \frac{p}{\rho} = RT$$

Dividing these equations, we have

$$\frac{p_0}{\rho_0} \times \frac{\rho}{p} = \frac{RT_0}{RT}$$

or,
$$\frac{T}{T_0} = \frac{p}{p_0} \times \frac{\rho_0}{\rho} \qquad \ldots(iv)$$

But from eqn. (2.18), $\frac{p}{p_0}$ is given by

$$\frac{p}{p_0} = \left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right]^{\frac{\gamma}{\gamma - 1}}$$

Also for *adiabatic process* $\frac{p}{\rho^\gamma} = \frac{p_0}{\rho_0^\gamma}$ or $\left(\frac{\rho_0}{\rho}\right)^\gamma = \frac{p_0}{p}$

or,
$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{1/\gamma} = \left(\frac{p}{p_0}\right)^{-1/\gamma}$$

or,
$$\frac{\rho_0}{\rho} = \left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right]^{\left(\frac{\gamma}{\gamma - 1}\right) \times \left(-\frac{1}{\gamma}\right)}$$

or,
$$\frac{\rho_0}{\rho} = \left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right]^{-\frac{1}{\gamma - 1}}$$

Substituting the values of $\frac{p}{p_0}$ and $\frac{\rho_0}{\rho}$ in eqn. (*iv*), we get

$$\frac{T}{T_0} = \left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right]^{\frac{\gamma}{\gamma - 1}} \times \left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right]^{\frac{1}{\gamma - 1}}$$

$$= \left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right]^{\frac{\gamma}{\gamma - 1} - \frac{\gamma}{\gamma - 1}} = \left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right]$$

$$\therefore \quad T = T_0\left[1 - \frac{\gamma - 1}{\gamma} \times \frac{gZ}{RT_0}\right] \qquad \ldots(2.19)$$

Temperature lapse rate (L):

It is defined as the *rate at which the temperature changes with elevation*. It can be obtained by differentiating the eqn. w.r.t. Z as follows:

$$\frac{dT}{dZ} = \frac{d}{dZ}\left[T_0\left\{1 - \frac{\gamma - 1}{\gamma}\right\} \times \frac{gZ}{RT_0}\right]$$

where T_0, γ, g and R are constant.

$$\therefore \quad \frac{dT}{dZ} = T_0\left\{-\frac{\gamma - 1}{\gamma} \times \frac{g}{RT_0}\right\} = -\frac{g}{R}\left(\frac{\gamma - 1}{\gamma}\right)$$

Hence, temperature lapse–rate, $L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{\gamma - 1}{\gamma}\right) \qquad \ldots(2.20)$

— In this eqn., if $\gamma = 1$ the process in an isothermal one which means $\frac{dT}{dZ} = 0$; it indicates that temperature does not *change* with height.

— If $\gamma > 1$, lapse-rate is negative which means that *temperature decreases with increase of height.*

Following points are worth noting:

— The value of γ in atmosphere varies with height.

— Upto an elevation of 11km, above sea level, the temperature of air *decreases* at the rate of 0.0065° C/m. From 11 km to 32 km, the temperature remains constant but *rises* above 32 km of height.

Example 2.36. *Derive an expression for the pressure ratio in the troposphere if the absolute temperature is assumed to vary according to the law* $T = T_0 - \alpha(Z - Z_0)$, *where* T_0 *is the absolute temperature at sea level and* α *is the temperature gradient.* **(Nagpur University)**

Solution. The variation in altitude is given by:

$$dp = -\rho\, g dZ = -w dZ$$

where, ρ = Density of air, kg/m^3, and

w = Specific weight, N/m^3.

Since, $\rho = \frac{p}{RT}$, substituting in above relation, we get $dp = \left(-\frac{p}{RT}\right) g dZ$

or,
$$\frac{dp}{p} = -\frac{g}{RT} dZ \quad \text{...}(i)$$

Since,
$$T = T_0 - \alpha(Z - Z_0) \quad \text{...(Given)}$$

$\therefore$
$$dT = -\alpha\, dZ$$

Substituting for dZ in (i), we have

$$\frac{dp}{p} = \frac{g}{\alpha R} \frac{dT}{T}$$

Integrating between (p_0, p) to (Z_0, Z), where suffix 0 denotes sea level conditions, we get

$$ln\left(\frac{p}{p_0}\right) = \frac{g}{\alpha R}\, ln\left(\frac{T}{T_0}\right) = \frac{g}{\alpha R}\, ln\left[\frac{T_0 - \alpha(Z - Z_0)}{T_0}\right]$$

or,
$$\frac{p}{p_0} = \left[1 - \frac{\alpha(Z - Z_0)}{T_0}\right]$$

.....Required expression (Ans.)

Example 2.37. *The temperature of the earth's atmosphere drops about 5°C for every 1000 m of elevation above the earth's surface. If the air temperature at the ground level is 15°C and the pressure is 760 mm Hg, at what elevation is the pressure 380 mm Hg? Assume that air behaves as an ideal gas.* **(Roorkee University)**

Solution: *Given:* $T_0 = 15 + 273 = 288$ K; $p_0 = 760$ mm Hg; $p = 380$ mm Hg;

$$\frac{dT}{dZ} = -\frac{5}{1000}\,°C/\text{m}$$

Elevation Z:

Temperature lapse-rate,
$$L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{\gamma - 1}{\gamma}\right)$$

$\therefore$
$$L = -\frac{5}{1000} = -\frac{g}{R}\left(\frac{\gamma - 1}{\gamma}\right).$$

or $$\frac{5}{1000} = \frac{9.81}{287}\left(\frac{\gamma-1}{\gamma}\right)$$

∴ $$\frac{\gamma-1}{\gamma} = 0.1463$$

Using the relation: $$p = p_0\left[1-\frac{\gamma-1}{\gamma}\cdot\frac{gZ}{RT_0}\right]^{\frac{\gamma}{\gamma-1}}$$

or, $$380 = 760\left[1-0.1463\times\frac{9.81\times Z}{287\times 288}\right]^{\frac{\gamma}{0.1463}}$$

or, $$\left(\frac{380}{760}\right)^{0.1463} = 1-0.1463\times\frac{9.81\,Z}{287\times 288} = 1-1.736\times 10^{-5}\,Z$$

or, $$1.736\times 10^{-5}Z = 1-\left(\frac{380}{760}\right)^{0.1463} = 0.0964$$

or, $$Z = \frac{0.0964}{1.736\times 10^{-5}} = \mathbf{5553\,m\ (Ans)}$$

Example 2.38. *The barometric pressure at sea level is 760 mm of mercury while that on a mountain top is 735 mm. If the density of air is assumed constant at 1.2 kg/m³, what is the elevation of the mountain top?* **(Panjab University)**

Solution. *Given*: Pressure at sea level, $p_0 = 760$ mm of Hg

$$= \frac{760}{1000}\times(13.6\times 1000)\times 9.81 = 101396\ \text{N/m}^2$$

Pressure at mountain, $p = 735$ mm of Hg

$$= \frac{735}{1000}\times(13.6\times 1000)\times 9.81 = 98060\ \text{N/m}^2$$

Density of air, $\rho = 1.2$ kg/m³

Height of the mountain top from sea-level, h:

It is a known fact that as the elevation above the sea-level increases, the atmospheric pressure decreases. As the density is constant (given), hence the pressure at any height '*h*' above the sea-level is given by the equation,

$$p = p_0 - \rho g h \quad \text{or} \quad h = \frac{p_0 - p}{\rho g} = \frac{(101396 - 98060)}{1.2\times 9.81} = \mathbf{283.4\ m\ (Ans.)}$$

Example 2.39. *Determine the pressure at a height of 800 m above sea level if the atmospheric pressure is 10.139 × 10⁴ N/m² and temperature is 15°C at sea level assuming:*

(*i*) *Air is incompressible;*

(*ii*) *Pressure variation follows isothermal law;*

(*iii*) *Pressure variation follows adiabatic law.*

Take: Density of air at sea level = 1.285 kg/m³

Neglect variation of g with altitude. **(Nagpur University)**

Solution. *Given*: Height above sea-level, $Z = 8000$ m

Pressure at sea-level, $p_0 = 10.139\times 10^4$ N/m²

Temperature at sea level, $t_0 = 15°\text{C}$ ∴ $T_0 = 15 + 273 = 288$ K

Density of air, $\rho = \rho_0 = 1.285$ kg/m³.

Pressure *p*:

(*i*) *When air is incompressible*:

We know that $\frac{dp}{dz} = -\rho g$; $\quad \therefore \int_{p_0}^{p} dp = -\int_{Z_0}^{Z} \rho g \, dz$

or, $\quad p - p_0 = -pg\,(Z - Z_0)$

or, $\quad p = p_0 - \rho g Z \qquad (\because Z_0 = \text{datum line} = 0)$

$= 10.139 \times 10^4 - 1.285 \times 9.81 \times 8000 = \mathbf{543.2\ N/m^2}$ **(Ans.)**

(*ii*) *When pressure variation follows isothermal law :*

We know that $p = p_0\, e^{(-gZ/RT)}$ ...(Eqn. 2.16)

$$= p_0\, e^{(-gZ\rho_0/p_0)} \quad \left[\because \frac{p_0}{\rho_0} = RT \text{ or } \frac{\rho_0}{p_0} = \frac{1}{RT}\right]$$

$$= 10.139 \times 10^4 \times e^{(-9.81 \times 8000 \times 1.285/101390)}$$

$$= 10.139 \times 10^4 \times e^{-0.9946} = 10.139 \times 10^4 \times 0.3699$$

$= 37504\ \text{N/m}^2$ or $\mathbf{3.75\ N/cm^2}$ **(Ans.)**

(*iii*) *When pressure variation follows adiabatic law* ($\gamma = 1.4$):

Using the equation: $p = p_0\left[1 - \frac{\gamma - 1}{\gamma}\, gZ\, \frac{\rho_0}{p_0}\right]^{\gamma/\gamma - 1}$ ...[Eqn.2.17]

Substituting the values, we get

$$p = 10.139 \times 10^4 \left[1 - \frac{1.4 - 1.0}{1.4} \times 9.81 \times 8000 \times \frac{1.285}{10.139 \times 10^4}\right]^{\left(\frac{1.4}{1.4-1}\right)}$$

$$= 101390\,(1 - 0.2842)^{(1.4/0.4)} = 101390 \times (0.7158)^{3.5}$$

$= 31460\ \text{N/m}^2$ or $\mathbf{3.146\ N/cm^2}$ **(Ans)**

Example 40. *Determine the pressure and density of air at a height of 4500 m from sea-level where pressure and temperature of the air are 101400 N/m² and 15°C respectively. Density of air at sea-level is equal to 1.285 kg/m³ and the temperature lapse-rate is 0.0065°C/m.*

Solution. *Given:* Height, $Z = 4500$ m

Pressure at sea-level; $p_0 = 101400\ \text{N/m}^2$

Temperature at sea-level $T_0 = t - 273 = 15 + 273 = 288$ K

Temperature lapse–rate, $L = \frac{dT}{dZ} = -0.0065°\text{C/m}$

Density of air at sea level, $\rho_0 = 1.285\ \text{kg/m}^3$

We know that, $\quad L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{\gamma - 1}{\gamma}\right)$ ...[Eqn. (2.20)]

where, $\quad R = \frac{p_0}{\rho_0 T_0} = \frac{101400}{1.285 \times 288} = 274$

Substituting the values in the above equation, we get $-0.0065 = -\frac{9.81}{274}\left(\frac{\gamma - 1}{\gamma}\right)$

$$\therefore \quad \frac{\gamma - 1}{\gamma} = \frac{0.0065 \times 274}{9.81} = 0.1815$$

$\therefore \gamma\,(1 - 0.1815) = 1$ or $\gamma = \frac{1}{1 - 0.1815} = 1.22$

Pressure (p) and density (ρ) of air at a height of 4500 m:

We know that, $$p = p_0\left[1 - \frac{\gamma - 1}{\gamma} gZ \frac{\rho_0}{p_0}\right]^{(\gamma/\gamma-1)} \qquad \text{...[Eqn. (2.17)]}$$

Substituting the values, we get

$$p = 101400\left[1 - \frac{1.22 - 1}{1.22} \times 9.81 \times 4500 \times \frac{1.285}{101400}\right]^{\frac{1.22}{0.22}}$$

$$= 101400\,(1 - 0.1)^{5.545} = 56534 \text{ N/m}^2 \text{ or } \mathbf{5.6534\ N/cm^2\ (Ans.)}$$

Also, $\frac{p}{\rho} = RT$

where, p, ρ and T are pressure, density and temperature respectively at a height of 4500 m.

Now the value of T is calculated from temperature lapse-rate as follows :

$$t \text{ at } 4500 \text{ m} = t_0 + \frac{dT}{dZ} \times 4500 = 15 - 0.0065 \times 4500 = -14.25°\text{C}$$

$$\therefore \qquad T = 273 + (-14.25) = 258.75 \text{ K}$$

$\therefore$ Density of air at a height of 4500 m: $\rho = \frac{p}{RT} = \frac{56534}{274 \times 258.75} = \mathbf{0.797\ kg/m^3\ (Ans.)}$

Example 2.41. *Calculate the pressure round an aeroplane which is flying at an altitude of 4200 m. The temperature lapse-rate is 0.0065 K/m. The pressure, temperature and density of air at ground level are 101400 N/m², 15°C and 1.285 kg/m³ respectively.*

Variation of g with altitude may be neglected.

Solution. *Given:* Height, Z = 4200 m; Lapse-rate, $L = \frac{dT}{dZ} = -0.0065$ K/m;

$p_0 = 101400$ N/m²; $T_0 = 15 + 273 = 288$ K; $\rho_0 = 1.285$ kg/m³

Pressure round the aeroplane, p;

Let us first calculate the value of power index γ as follows:

We know that, $$L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{\gamma - 1}{\gamma}\right) \qquad \text{...[Eqn. (2.20)]}$$

where, $$R = \frac{p_0}{\rho_0 T_0} = \frac{101400}{1.285 \times 288} = 274$$

Substituting the value, we have $-0.0065 = -\frac{9.81}{274}\left(\frac{\gamma - 1}{\gamma}\right)$

or, $$\frac{\gamma - 1}{\gamma} = \frac{0.0065 \times 274}{9.81} = 0.1815 \quad \text{or} \quad \gamma = 1.22$$

Pressure round the aeroplane is given as: $$p = p_0\left[1 - \frac{\gamma - 1}{\gamma} gZ \frac{\rho_0}{p_0}\right]^{\frac{\gamma}{\gamma - 1}} \qquad \text{...[Eqn. (2.17)]}$$

$$= 101400\left[1 - \frac{1.22 - 1}{1.22} \times 9.81 \times 4200 \times \frac{1.285}{101400}\right]^{\frac{1.22}{0.22}}$$

$$= 101400\,[1 - 0.094]^{5.545} = 58656 \text{ N/m}^2 = \mathbf{5.8656\ N/cm^2\ (Ans.)}$$

HIGHLIGHTS

1. The force (P) per unit area (A) is called pressure (p); mathematically, $p = \frac{P}{A}$
2. Pressure head of a liquid, $h = \frac{p}{w}$ $\quad (\because p = wh)$

   where, w is the specific weight of the liquid.

...[Eqn. (2.17)]

$]^{\frac{1.22}{0.22}}$

$/cm^2$ (Ans.)

ght of 4500 m.

n^3 (Ans.)

at an altitude of
d density of air at

...[Eqn. (2.20)]

...[Eqn. (2.17)]

$]^{\frac{1.22}{0.22}}$

56 N/cm^2 (Ans.)

$= \frac{P}{A}$

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3. *Pascal's law* states as follows:
"The intensity of pressure at any point in a liquid at rest, is the same in all directions".
4. The atmospheric pressure at sea level (above absolute zero) is called standard atmospheric pressure.
   (*i*) Absolute pressure = atmospheric pressure + gauge pressure

$$p_{abs.} = p_{atm.} + p_{gauge}$$

   (*ii*) Vacuum pressure = Atmospheric pressure − absolute pressure (Vacuum pressure is defined as the pressure below the atmospheric pressure)
5. *Manometers* are defined as the devices used for measuring the pressure at a point in fluid by balancing the column of fluid by the same or another column of liquid.
6. *Mechanical gauges* are the devices in which the pressure is measured by balancing the fluid column by spring (elastic element) or dead weight. Some commonly used mechanical gauges are:
   (*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,
   (*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.
7. The pressure at a height Z in a static compressible fluid (gas) undergoing isothermal compression $\left(\frac{p}{\rho} = \text{const.}\right)$.

$$p = p_0 e^{(-gZ/RT)}$$

where, p_0 = Absolute pressure at sea-level or at ground level,
Z = Height from sea or ground level,
R = Gas constant, and
T = Absolute temperature.

8. The pressure and temperature at a height Z in a static compressible fluid (gas) undergoing adiabatic compression (p/ρ^{γ} = const.):

$$p = p_0\left[1 - \frac{\gamma - 1}{\gamma} gZ \frac{\rho_0}{p_0}\right]^{\frac{\gamma}{\gamma - 1}} = p_0\left[1 - \frac{\gamma - 1}{\gamma}\frac{gZ}{RT_0}\right]^{\frac{\gamma}{\gamma - 1}}$$

and, temperature, $T = T_0\left[1 - \frac{\gamma - 1}{\gamma}\frac{gZ}{RT_0}\right]$

where, p_0, T_0 are pressure and temperature at sea-level; $\gamma = 1.4$ for air.

9. The rate at which the temperature changes with elevation is known as *Temperature Lapse-Rate*. It is given by,

$$L = \frac{-g}{R}\left(\frac{\gamma - 1}{\gamma}\right)$$

if (*i*) $\gamma = 1$, temperature is zero; (*ii*) $\gamma > 1$, temperature decreases with the increase of height.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer:

1. The force per unit area is called
   (*a*) pressure (*b*) strain,
   (*c*) surface tension (*d*) none of the above.
2. The pressure of a liquid on a surface will always act ...... to the surface.
   (*a*) parallel (*b*) normal
   (*c*) 45° (*d*) 60°.
3. The pressure .... as the depth of the liquid increases.
   (*a*) increases
   (*b*) decreases
   (*c*) remain unchanged
   (*d*) none of the above.
4. The intensity of pressure in a liquid due to its depth will vary .... with depth.