

## PROPERTIES OF FLUIDS

1.1. Introduction. 1.2. Fluid. 1.3. Liquids and their properties. 1.4. Density-mass density-weight density-specific volume. 1.5. Specific gravity. 1.6. Viscosity-Newton's law of viscosity-types of fluids-effect of temperature on viscosity-effect of pressure on viscosity. 1.7. Thermodynamic properties. 1.8. Surface tension and capillarity. 1.9. Compressibility and bulk modulus. 1.10. Vapour pressure-Highlights-Objective Type Questions-Theoretical Questions- Unsolved Examples.

### 1.1 Introduction

#### Hydraulics:

Hydraulics (this word has been derived from a Greek word 'Hudour' which means water) may be defined as follows :

*"It is that branch of Engineering-science, which deals with water (at rest or in motion)."*

OR

*"It is that branch of Engineering-science which is based on experimental observation of water flow."*

#### Fluid Mechanics:

Fluid mechanics may be defined as *that branch of Engineering-science which deals with the behaviour of fluid under the conditions of rest and motion.*

The fluid mechanics may be divided into three parts: *Statics, kinematics and dynamics.*

**Statics.** The study of incompressible fluids under static conditions is called *hydrostatics* and that dealing with the compressible static gases is termed as *aerostatics*.

**Kinematics.** It deals with the velocities, accelerations and the patterns of flow only. Forces or energy causing velocity and acceleration are *not* dealt under this heading.

**Dynamics.** It deals with the relations between velocities, accelerations of fluid *with the forces or energy causing them.*

#### Properties of Fluids-General Aspects:

The matter can be classified on the basis of the *spacing between the molecules* of the matter as follows:

1. Solid state, and
2. Fluid state,
  - (i) Liquid state, and
  - (ii) Gaseous state.

In *solids*, the molecules are very closely spaced whereas in *liquids* the spacing between the different molecules is relatively large and in *gases* the spacing between the molecules is still large. It means that inter-molecular cohesive forces are *large* in solids, *smaller* in liquids and *extremely small* in gases, and on account of this fact, solids possess compact and rigid form, liquid molecules can move freely within the liquid mass and the molecules of gases have greater freedom of movement so that the gases fill the container completely in which they are placed.

A *solid* can resist tensile, compressive and shear stresses upto a certain limit whereas a fluid has no tensile strength or very little of it and it can resist the compressive forces only when it is kept in a container. When a fluid is subjected to a shearing force it deforms continuously as long as the force is applied. The amount of shear stress in a fluid depends on the magnitude of the rate of deformation of the fluid element.

*Liquids* and *gases* exhibit different characteristics. The liquids under ordinary conditions are quite difficult to compress (and therefore they may for most purposes be regarded as incompressible) whereas gases can be compressed much readily under the action of external pressure (and when the external pressure is removed the gases tend to expand indefinitely).

## 1.2. Fluid

A fluid may be defined as follows:

*"A fluid is a substance which is capable of flowing."*

or

*"A fluid is a substance which deforms continuously when subjected to external shearing force."*

A fluid has the following *characteristics*:

1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a liquid/fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.

A fluid may be *classified* as follows:

A. (i) *Liquid*, (ii) *Gas*, and (iii) *Vapour*. B. (i) *Ideal fluids* and (ii) *Real fluids*.

### Liquid

- A liquid is a fluid which possesses a *definite volume* (which varies only slightly with temperature and pressure).
- Liquids have bulk elastic modulus when under compression and will store up energy in the same manner as a solid. As the contraction of volume of a liquid under compression is extremely small, it is usually ignored and the *liquid is assumed to be incompressible*. A liquid will withstand a slight amount of tension due to molecular attraction between the particles which will cause an apparent shear resistance, between two adjacent layers. This phenomenon is known as **viscosity**.
- All known liquids vaporise at narrow pressures above zero, depending on the temperature.

**Gas.** It possesses *no definite volume* and is *compressible*.

**Vapour.** It is a gas whose temperature and pressure are such that it is very near the liquid state (e.g., steam).

**Ideal fluids.** An ideal fluid is one which has *no viscosity* and *surface tension* and is *incompressible*. In true sense no such fluid exists in nature. However fluids which have low viscosities such as water and air can be treated as ideal fluids under certain conditions. The assumption of ideal fluids helps in simplifying the mathematical analysis.

**Real fluids.** A *real practical fluid* is one which has *viscosity, surface tension and compressibility* in addition to the *density*. The real fluids are actually available in nature.

**Continuum.** A *continuous and homogeneous medium* is called **continuum**. From the *continuum view point*, the *overall properties and behaviour of fluids* can be studied without regard for its *atomic and molecular structure*.

## 1.3. Liquids and their Properties

- A liquid can be easily distinguished from a solid or a gas.
- A solid has a definite shape.

- A liquid takes the shape of vessel into which it is poured.
- A gas completely fills the vessel which contains it.

The properties of water are of much importance because the subject of hydraulics is mainly concerned with it. Some important properties of water which will be considered are:

- |                        |                         |                       |
|------------------------|-------------------------|-----------------------|
| (i) Density,           | (ii) Specific gravity,  | (iii) Viscosity,      |
| (iv) Vapour pressure,  | (v) Cohesion,           | (vi) Adhesion,        |
| (vii) Surface tension, | (viii) Capillarity, and | (ix) Compressibility. |

**1.4. Density**

**1.4.1 Mass density**

The *density* (also known as *mass density* or *specific mass*) of a liquid may be defined as *the mass per unit volume*  $\left(\frac{m}{V}\right)$  at a standard temperature and pressure. It is usually denoted by  $\rho$  (rho).

Its units are  $\text{kg/m}^3$  i.e. 
$$\rho = \frac{m}{V} \quad \dots(1.1)$$

**1.4.2 Weight density**

The *weight density* (also known as *specific weight*) is defined as the *weight per unit volume* at the standard temperature and pressure. It is usually denoted by  $w$ .

$$w = \rho g \quad \dots(1.2)$$

For the purposes of all calculations, relating to Hydraulics and hydraulic machines, the specific weight of water is taken as follows:

- In S.I. Units:  $w = 9.81 \text{ kN/m}^3$  (or  $9.81 \times 10^{-6} \text{ N/mm}^3$ )  
 In M.K.S. Units:  $w = 1000 \text{ kg}_f/\text{m}^3$

**1.4.3 Specific volume**

It is defined as *volume per unit mass of fluid*. It is denoted by  $v$ . Mathematically,

$$v = \frac{V}{m} = \frac{1}{\rho} \quad \dots(1.3)$$

**1.5. Specific Gravity**

*Specific gravity* is the ratio of the specific weight of the liquid to the specific weight of a standard fluid. It is dimensionless and has no units. It is represented by  $S$ .

For liquids, the standard fluid is pure water at  $4^\circ\text{C}$ .

$$\therefore \text{Specific gravity} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}} = \frac{w_{\text{liquid}}}{w_{\text{water}}}$$

**Example 1.1.** Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of  $6 \text{ m}^3$  and weight of  $44 \text{ kN}$ .

**Solution:** Volume of the liquid =  $6 \text{ m}^3$

Weight of the liquid =  $44 \text{ kN}$

**Specific weight,  $w$  :**

$$w = \frac{\text{Weight of liquid}}{\text{Volume of liquid}} = \frac{44}{6} = 7.333 \text{ kN/m}^3 \text{ (Ans.)}$$

**Specific mass or mass density,  $\rho$  :**

$$\rho = \frac{w}{g} = \frac{7.333 \times 1000}{9.81} = 747.5 \text{ kg/m}^3 \text{ (Ans.)}$$

**Specific volume,**

$$v = \frac{1}{\rho} = \frac{1}{747.5} = 0.00134 \text{ m}^3/\text{kg} \text{ (Ans.)}$$

Specific gravity,  $S$  :

$$S = \frac{W_{liquid}}{W_{water}} = \frac{7.333}{9.81} = 0.747 \text{ (Ans.)}$$

**1.6. Viscosity**

Viscosity may be defined as the *property of a fluid which determines its resistance to shearing stresses*. It is a measure of the internal fluid friction which causes resistance to flow. It is primarily due to cohesion and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid.

An ideal fluid has no viscosity. There is no fluid which can be classified as a perfectly ideal fluid. However, the fluids with very little viscosity are sometimes considered as ideal fluids.

Viscosity of fluids is due to cohesion and interaction between particles.

Refer Fig 1.1. When two layers of fluid, at a distance 'dy' apart, move one over the other at different velocities, say  $u$  and  $u + du$ , the viscosity together with relative velocity causes a shear stress acting between the fluid layers. The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to  $y$ . It is denoted by  $\tau$  (called Tau).

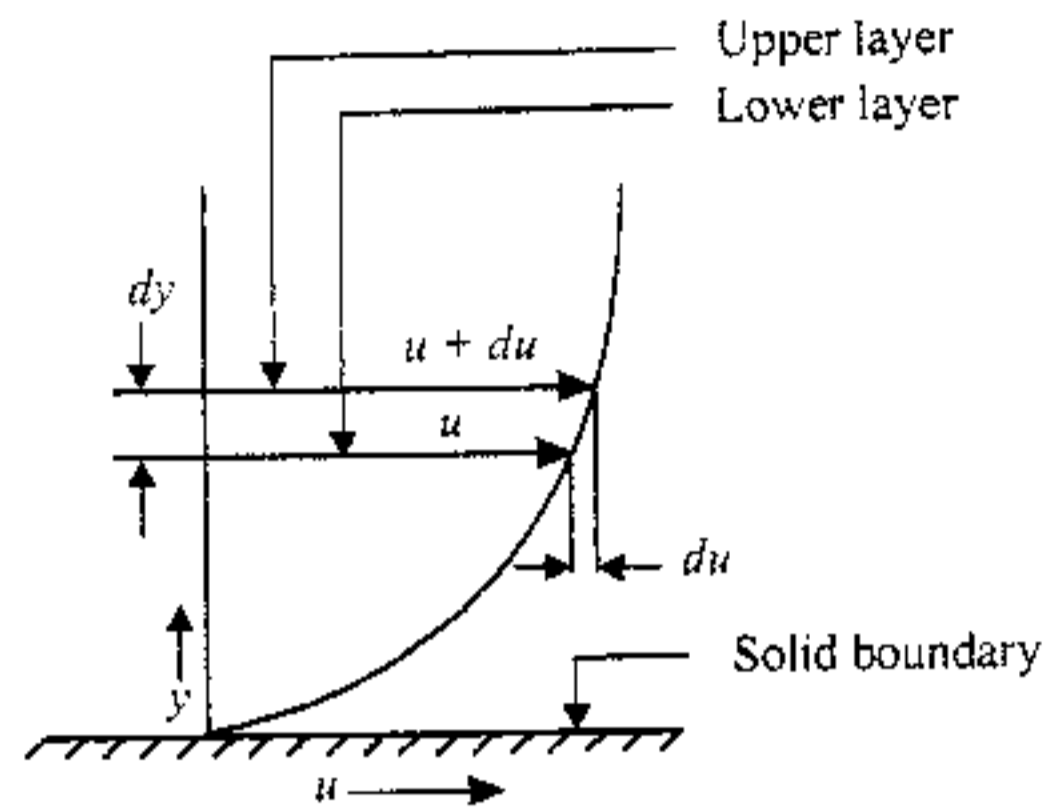


Fig. 1.1. Velocity variation near a solid boundary.

Mathematically  $\tau \propto \frac{du}{dy}$

or  $\tau = \mu \cdot \frac{du}{dy}$  ... (1.4)

where,  $\mu$  = Constant of proportionality and is known as *co-efficient of dynamic viscosity* or *only viscosity*.

$\frac{du}{dy}$  = Rate of shear stress or rate of shear deformation or velocity gradient.

From Fig. 1.1, we have  $\mu = \frac{\tau}{\left[\frac{du}{dy}\right]}$  ... (1.5)

Thus viscosity may also be defined as the *shear stress required to produce unit rate of shear strain*.

**Units of Viscosity:**

In S.I. Units:  $N.s/m^2$

In M.K.S. Units:  $kg_f \cdot sec/m^2$

$$\left[ \because \mu = \frac{\text{force/area}}{(\text{length/time}) \times \frac{1}{\text{length}}} = \frac{\text{force/length}^2}{\frac{1}{\text{time}}} = \frac{\text{force} \times \text{time}}{(\text{length})^2} \right]$$

The unit of viscosity in C.G.S. is also called *poise* =  $\frac{\text{dyne} \cdot \text{sec}}{\text{cm}^2}$ . One poise =  $\frac{1}{10} N.s/m^2$

Note. The viscosity of water at 20°C is  $\frac{1}{100}$  poise or one centipoise.

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**Kinematic Viscosity :**

*Kinematic viscosity* is defined as the *ratio between the dynamic viscosity and density of fluid*. It is denoted by  $\nu$  (called nu).

Mathematically, 
$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \quad \dots(1.6)$$

**Units of kinematic viscosity:**

In SI units:  $\text{m}^2/\text{s}$

In M.K.S. units:  $\text{m}^2/\text{sec}$ .

In C.G.S. units the kinematic viscosity is also known as stoke (=  $\text{cm}^2/\text{sec}$ .)

One stoke =  $10^{-4} \text{ m}^2/\text{s}$

**Note:** Centistoke means  $\frac{1}{100}$  stoke.

**1.6.1. Newton's Law of Viscosity**

This law states that the *shear stress ( $\tau$ ) on a fluid element layer is directly proportional to the rate of shear strain*. The constant of proportionality is called the *co-efficient of viscosity*.

Mathematically, 
$$\tau = \mu \frac{du}{dy} \quad \dots(1.7)$$

The fluids which follow this law are known as *Newtonian fluids*.

**1.6.2. Types of Fluids**

The fluids may be of the following types:

Refer to Fig.1.2.

**1. Newtonian fluids.** These fluids follow Newton's viscosity equation (i.e. eqn. 1.7). For such fluids  $\mu$  does not change with rate of deformation.

**Examples.** Water, kerosene, air etc.

**2. Non-Newtonian fluids.** Fluids which do not follow the linear relationship between shear stress and rate of deformation (given by eqn. 1.7) are termed as *Non-Newtonian fluids*. Such fluids are relatively uncommon.

**Examples.** Solutions or suspensions (slurries), mud flows, polymer solutions, blood etc. These fluids are generally complex mixtures and are studied under *rheology*, a science of deformation and flow.

**3. Plastic fluids.** In the case of a plastic substance which is non-Newtonian fluid an initial yield stress is to be exceeded to cause a continuous deformation. These substances are represented by straight line intersecting the vertical axis at the "yield stress" (Refer to Fig. 1.2).

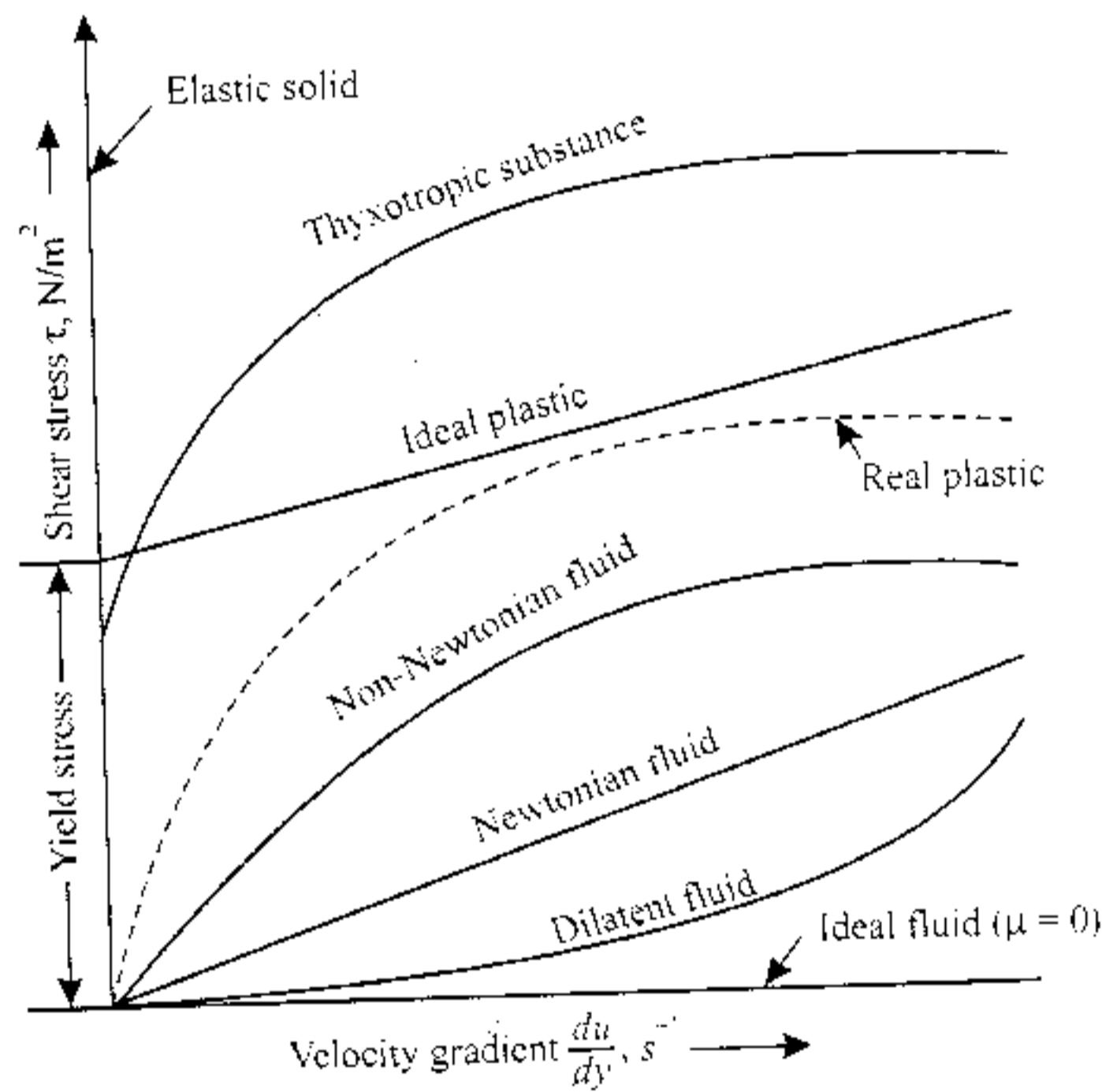


Fig. 1.2. Variation of shear stress with velocity gradient.

An **ideal plastic** (or Bingham plastic) has a definite yield stress and a constant linear relation between shear stress and the rate of angular deformation. Examples: *Sewage sludge, drilling muds* etc.

A **thixotropic substance**, which is non-Newtonian fluid, has a non-linear relationship between the shear stress and the rate of angular deformation, beyond an initial yield stress. The *printer's ink* is an example of thixotropic substance.

**4. Ideal fluid.** An ideal fluid is one which is incompressible and has zero viscosity (or in other words shear stress is always zero regardless of the motion of the fluid). Thus an ideal fluid is represented by the horizontal axis ( $\tau = 0$ ).

A *true elastic solid* may be represented by the vertical axis of the diagram.

*Summary of relations between shear stress ( $\tau$ ) and rate of angular deformation for various types of fluids:*

(i) *Ideal fluids:*  $\tau = 0$ ,

(ii) *Newtonian fluids:*  $\tau = \mu \cdot \frac{du}{dy}$ ,

(iii) *Ideal plastics:*  $\tau = \text{const.} + \mu \cdot \frac{du}{dy}$ , (iv) *Thixotropic fluids:*  $\tau = \text{const.} + \mu \left( \frac{du}{dy} \right)^n$ , and

(v) *Non-Newtonian fluids:*  $\tau = \left( \frac{du}{dy} \right)^n$ .

In case of non-Newtonian fluids, if  $n$  is less than unity then are called **pseudo-plastics** (e.g., *paper pulp, rubber suspension paints*) while fluids in which  $n$  is greater than unity are known as **dilatents**. (e.g. *Butter, printing ink*).

**Ostwald-de-Waele Equation.** It is an empirical solution to express steady-state shear stress as a function of velocity gradient, and is given as

$$\tau_{yx} = \alpha \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy}$$

If  $n = 1$ , this reduces to Newton's law of viscosity, with  $\alpha = \mu$

**Example 1.2.** (a) *What are the characteristics of an ideal fluid?*

(b) *The general relation between shear stress and velocity gradient of a fluid can be written as*

$$\tau = A \left( \frac{du}{dy} \right)^n + B$$

where  $A$ ,  $B$  and  $n$  are constants that depend upon the type of fluid and conditions imposed on the flow. Comment on the value of these constants so that the fluid may behave as:

- (i) *an ideal fluid.*
- (ii) *a Newtonian fluid and*
- (iii) *A non-Newtonian fluid.*
- (c) *Indicate whether the fluid with the following characteristics is a Newtonian or non-Newtonian.*
  - (i)  $\tau = Ay + B$  and  $u = C_1 + C_2y + C_3y^2$
  - (ii)  $\tau = Ay^{n(n-1)}$  and  $u = Cy^n$

**Solution.** (a) An ideal fluid has the following characteristics:

- No viscosity (i.e.  $\mu = 0$ )
- No surface tension.
- Incompressible (i.e.  $\rho = \text{constant}$ )

An ideal fluid can slip near a solid boundary and cannot sustain any shear force however small it may be.

$$(b) \tau = A \left( \frac{du}{dy} \right)^n + B$$

(i) *An ideal fluid:*

Since an ideal fluid has zero viscosity (*i.e.* shear stress is always zero regardless of the motion of the fluid), therefore.

$$A = B = 0$$

(ii) *A Newtonian fluid:*

Since a Newtonian fluid follows Newton's law of viscosity:

$$\tau = \mu \frac{du}{dy}, \text{ therefore:}$$

- $n = 1$  and  $B = 0$
  - The constant  $A$  takes the value of dynamic viscosity  $\mu$  for the fluid.
- Air, water, kerosene etc. behave as Newtonian fluids under normal working conditions.

(iii) *A non-Newtonian fluid:*

Depending on the value of power index  $n$ , the non-Newtonian fluids are classified as:

- If  $n > 1$  and  $B = 0$  ... **Dilatant fluids.**

**Examples:** Sugar solution, aqueous suspension and printing ink.

- If  $n < 1$  and  $B = 0$  .. **Pseudo plastic fluids.**

**Examples :** Blood, milk, liquid cement and clay.

- If  $n = 1$  and  $B = \tau_0$  ... **Bingham fluid or ideal plastic.**

An ideal plastic fluid has a definite yield stress and a constant-linear relation between shear stress developed and rate of deformation:

$$\text{i.e.} \quad \tau = \tau_0 + \mu \frac{du}{dy}$$

**Examples:** Sewage sludge, water suspension of clay and flyash etc.

$$(c) (i) \tau = Ay + B \text{ and } u = C_1 + C_2 y + C_3 y^2$$

$$\text{Now,} \quad \frac{du}{dy} = \frac{d}{dy} (C_1 + C_2 y + C_3 y^2) = C_2 + 2C_3 y$$

$$\text{For Newtonian fluid,} \quad \tau = \mu \frac{du}{dy}$$

$$\therefore \quad \tau = \mu (C_2 + 2C_3 y) = 2\mu C_3 y + \mu C_2$$

which can be rewritten as

$$\tau = Ay + B \text{ where } A = 2\mu C_3 \text{ and } B = \mu C_2$$

Since this has the same form as the given shear stress, therefore the fluid characteristics correspond to that of an *ideal fluid*.

$$(ii) \tau = Ay^{n(n-1)} \text{ and } u = Cy^n$$

$$\text{Now,} \quad \frac{du}{dy} = \frac{d}{dy} (Cy^n) = Cn(y)^{n-1}$$

$$\text{For a Newtonian fluid} \quad \tau = \mu \frac{du}{dy} = \mu Cn(y)^{n-1}$$

This expression does not conform to the value of shear stress and as such the fluid is *non-Newtonian* in character.

### 1.6.3. Effect of Temperature on Viscosity

Viscosity is effected by temperature. The viscosity of *liquids decreases* but that of *gases increases* with *increase in temperature*. This is due to the reason that in *liquids* the shear stress is due to the inter-molecular cohesion which *decreases* with increase of temperature. In *gases* the inter-molecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecules, normal to the direction of motion. The molecular activity increases with rise in temperature and so does the viscosity of gas.

$$\text{For liquids:} \quad \mu_T = Ae^{\beta/T} \quad \dots(1.8)$$

$$\text{For gases:} \quad \mu_T = \frac{bT^{1/2}}{1 + a/T} \quad \dots(1.9)$$

where,  $\mu_T$  = Dynamic viscosity at absolute temperature  $T$ ,  
 $A, \beta$  = Constants (for a given liquid), and  
 $a, b$  = Constants (for a given gas).

### 1.6.4. Effect of Pressure on Viscosity

The viscosity under ordinary conditions is not appreciably affected by the changes in pressure. However, the viscosity of some oils has been found to increase with increase in pressure.

**Example 1.3.** A plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a force of 2.2 N/m<sup>2</sup> to maintain this speed. Find the viscosity of the fluid between the plates.

**Solution:** Velocity of the moving plate,  $u = 1.2$  m/s

Distance between the plates,  $dy = 0.05$  mm =  $0.05 \times 10^{-3}$  m

Force on the moving plate,  $F = 2.2$  N/m<sup>2</sup>

Viscosity of the fluid,  $\mu$ :

$$\text{We know, } \tau = \mu \cdot \frac{du}{dy}$$

where  $\tau$  = shear stress or force per unit area = 2.2 N/m<sup>2</sup>,

$du$  = change of velocity  
 $= u - 0 = 1.2$  m/s and

$dy$  = change of distance  
 $= 0.05 \times 10^{-3}$  m.

$$\therefore 2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}}$$

$$\text{or } \mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2} = 9.16 \times 10^{-5} \text{ N.s/m}^2$$

$$= 9.16 \times 10^{-4} \text{ poise (Ans.)}$$

$$\left[ \because 1 \text{ poise} = \frac{1 \text{ N.s}}{10 \text{ m}^2} \right]$$

**Example 1.4.** A plate having an area of 0.6 m<sup>2</sup> is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.

**Solution:** Area of plate,  $A = 0.6$  m<sup>2</sup>

Weight of plate,  $W = 280$  N

Velocity of plate,  $u = 0.36$  m/s

Thickness of film,  $t = dy = 1.8$  mm =  $1.8 \times 10^{-3}$  m

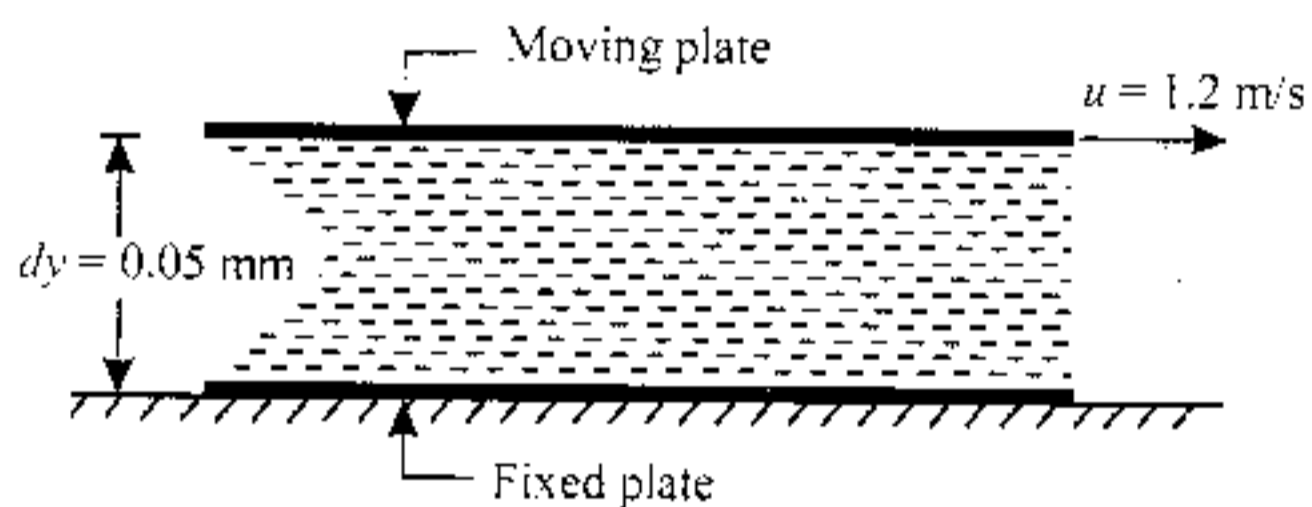


Fig. 1.3

Viscosity of the fluid,  $\mu$ :

Component of  $W$  along the plate

$$= W \sin \theta = 280 \sin 30^\circ = 140 \text{ N}$$

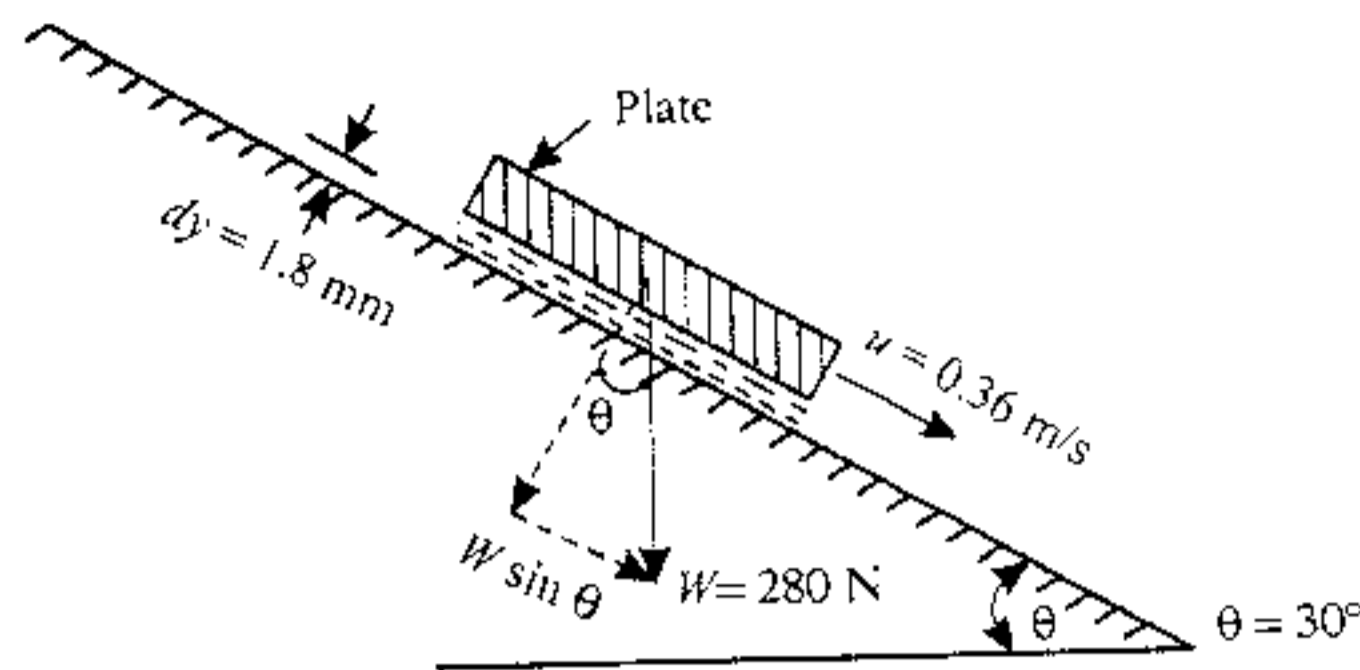


Fig. 1.4

$\therefore$  Shear force on the bottom surface of the plate,  $F = 140 \text{ N}$  and shear stress,

$$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33 \text{ N/m}^2$$

We know,

$$\tau = \mu \cdot \frac{du}{dy}$$

Where,

$$du = \text{change of velocity} = u - 0 = 0.36 \text{ m/s}$$

$$dy = t = 1.8 \times 10^{-3} \text{ m}$$

$$\therefore 233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}}$$

or

$$\mu = \frac{233.33 \times 1.8 \times 10^{-3}}{0.36} = 1.166 \text{ N.s/m}^2 = 11.66 \text{ poise (Ans.)}$$

**Example 1.5.** The space between two square flat parallel plates is filled with oil. Each side of the plate is 720 mm. The thickness of the oil film is 15 mm. The upper plate, which moves at 3 m/s requires a force of 120 N to maintain the speed. Determine:

- (i) The dynamic viscosity of the oil;
- (ii) The kinematic viscosity of oil if the specific gravity of oil is 0.95.

**Solution.** Each side of a square plate = 720 mm = 0.72 m

The thickness of the oil,  $dy = 15 \text{ mm} = 0.015 \text{ m}$

Velocity of the upper plate, = 3 m/s

$\therefore$  Change of velocity between plates,  $du = 3 - 0 = 3 \text{ m/s}$

Force required on upper plate,  $F = 120 \text{ N}$

$$\therefore \text{Shear stress, } \tau = \frac{\text{force}}{\text{area}} = \frac{120}{0.72 \times 0.72} = 231.5 \text{ N/m}^2$$

(i) **Dynamic viscosity  $\mu$ :**

We know that,

$$\tau = \mu \cdot \frac{du}{dy}, \quad 231.5 = \mu \cdot \frac{3}{0.015}$$

$$\therefore \mu = \frac{231.5 \times 0.015}{3} = 1.16 \text{ N.s/m}^2 \text{ (Ans.)}$$

(ii) **Kinematic viscosity,  $\nu$ :**

Weight density of oil,  $w = 0.95 \times 9.81 \text{ kN/m}^3 = 9.32 \text{ kN/m}^3$  or  $9320 \text{ N/m}^3$

$$\text{Mass density of oil, } \rho = \frac{w}{g} = \frac{9320}{9.81} = 950$$

$$\text{Using the relation: } \nu = \frac{\mu}{\rho} = \frac{1.16}{950} = 0.00122 \text{ m}^2/\text{s}$$

$$\text{Hence, } \nu = 0.00122 \text{ m}^2/\text{s (Ans.)}$$

**Example 1.6.** The velocity distribution for flow over a plate is given by  $u = 2y - y^2$  where  $u$  is the velocity in m/s at a distance  $y$  metres above the plate. Determine the velocity gradient and shear stress at the boundary and 1.5 m from it.

Take dynamic viscosity of fluid as  $0.9 \text{ N.s/m}^2$ .

$$\text{Solution. } u = 2y - y^2 \text{ ... (given) } \quad \therefore \frac{du}{dy} = 2 - 2y$$

(i) Velocity gradient,  $\frac{du}{dy}$ :

$$\text{At the boundary: At } y = 0, \left(\frac{du}{dy}\right)_{y=0} = 2 \text{ s}^{-1} \text{ (Ans.)}$$

$$\text{At 0.15 m from the boundary: At } y = 0.15 \text{ m, } \left(\frac{du}{dy}\right)_{y=0.15} = 2 - 2 \times 0.15 = 1.7 \text{ s}^{-1} \text{ (Ans.)}$$

(ii) Shear stress,  $\tau$ :

$$(\tau)_{y=0} = \mu \cdot \left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 2 = 1.8 \text{ N/m}^2 \text{ (Ans.)}$$

$$\text{and } (\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.9 \times 1.7 = 1.53 \text{ N/m}^2 \text{ (Ans.)}$$

[Where  $\mu = 0.9 \text{ N.s/m}^2$  ... (given)]

**Example 1.7.** A lubricating oil of viscosity  $\mu$  undergoes steady shear between a fixed lower plate and an upper plate moving at speed  $V$ . The clearance between the plates is  $t$ . Show that a linear velocity profile results if the fluid does not slip at either plate.

**Solution.** For the given geometry and motion, the shear stress  $\tau$  is constant throughout. From Newton's law of viscosity, we have

$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{constant}$$

$$\text{or } u = ly + m$$

The constants  $l$  and  $m$  are evaluated from the no slip conditions at the upper and lower plates.

$$\text{At } y = 0, u = 0 \quad \therefore m = 0$$

$$\text{At } y = t, u = V$$

$$\therefore V = lt - 0 \text{ or } l = \frac{V}{t}$$

$\therefore$  The velocity profile between plates is then given by:

$$u = \frac{Vy}{t} \text{ and is linear as indicated in Fig 1.5 (Ans.)}$$

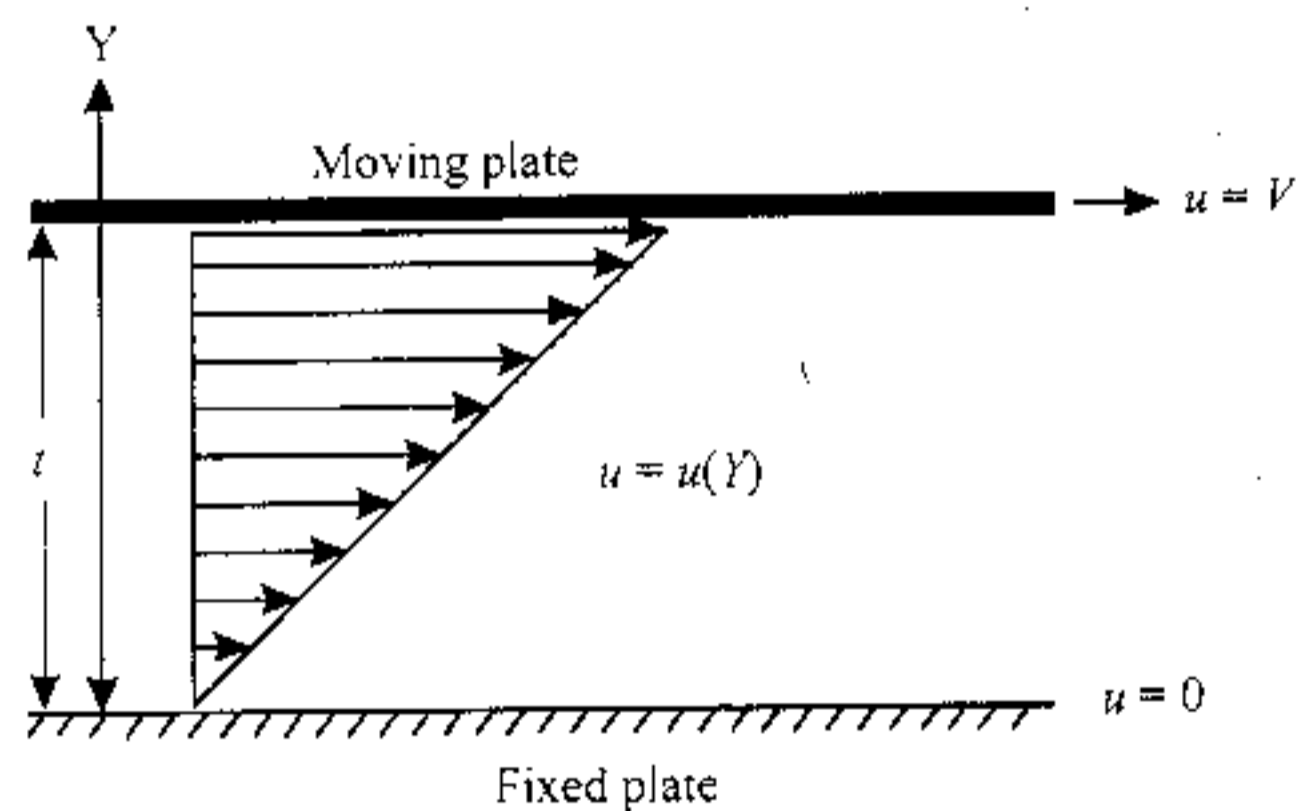


Fig. 1.5

## Properties of Fluids

**Example 1.8.** The velocity distribution of flow over a plate is parabolic with vertex 30 cm from the plate, where the velocity is 180 cm/s. If the viscosity of the fluid is  $0.9 \text{ N.s/m}^2$  find the velocity gradients and shear stresses at distances of 0, 15 cm and 30 cm from the plate.

**Solution.** Distance of the vertex from the plate = 30 cm.

Velocity at vertex,

$$u = 180 \text{ cm/s}$$

Viscosity of the fluid,

$$= 0.9 \text{ N.s/m}^2$$

The equation of velocity profile, which is parabolic, is given by

$$u = ly^2 + my + n \quad \dots(1)$$

where  $l$ ,  $m$  and  $n$  are constants. The values of these constants are found from the following boundary conditions:

(i) At  $y = 0$ ,  $u = 0$ ,

(ii) At  $y = 30 \text{ cm}$ ,

$$u = 180 \text{ cm/s and}$$

(iii) At  $y = 30 \text{ cm}$ ,  $\frac{du}{dy} = 0$ .

Substituting boundary conditions (i) in eqn. (1), we get

$$0 = 0 + 0 + n \quad \therefore n = 0$$

Substituting boundary conditions (ii) in eqn. (1), we get

$$180 = l \times (30)^2 + m \times 30 \quad \text{or} \quad 180 = 900l + 30m \quad \dots(2)$$

Substituting boundary conditions (iii) in eqn. (1), we get

$$\frac{du}{dy} = 2ly + m \quad \therefore 0 = 2l \times 30 + m \quad \text{or} \quad 0 = 60l + m \quad \dots(3)$$

Solving eqns. (2) and (3), we have  $l = -0.2$  and  $m = 12$ .

Substituting the values of  $l$ ,  $m$  and  $n$  in eqn. (1), we get  $u = -0.2y^2 + 12y$

Velocity gradients,  $\frac{du}{dy}$ :

$$\frac{du}{dy} = -0.2 \times 2y + 12 = -0.4y + 12$$

$$\text{At } y = 0, \left(\frac{du}{dy}\right)_{y=0} = 12/\text{s (Ans.)}$$

$$\text{At } y = 15 \text{ cm}, \left(\frac{du}{dy}\right)_{y=15} = -0.4 \times 15 + 12 = 6/\text{s (Ans.)}$$

$$\text{At } y = 30 \text{ cm}, \left(\frac{du}{dy}\right)_{y=30} = -0.4 \times 30 + 12 = 0 \text{ (Ans.)}$$

Shear stresses,  $\tau$ :

$$\text{We know, } \tau = \mu \frac{du}{dy}$$

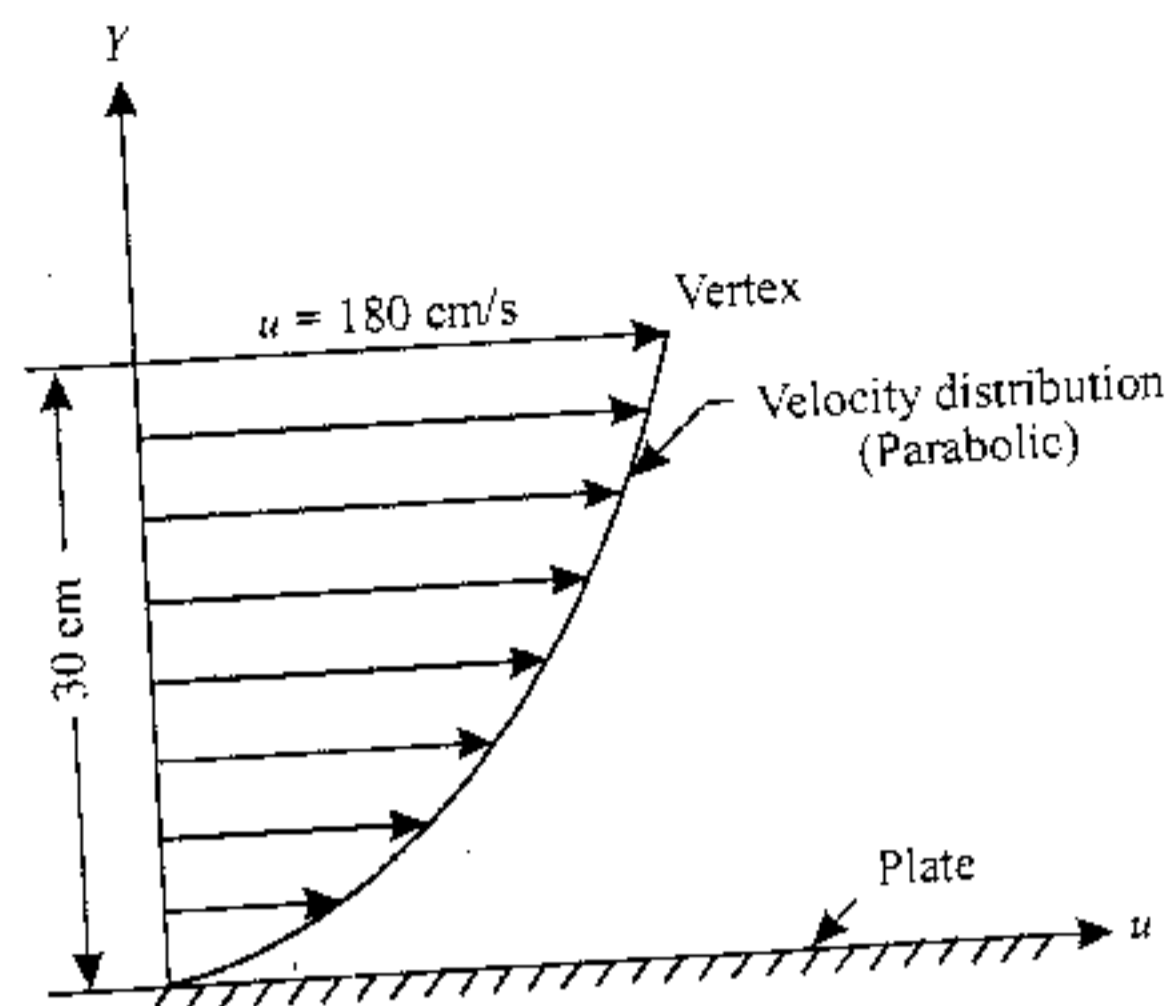


Fig. 1.6

$$\text{At } y = 0, (\tau)_{y=0} = \mu \left( \frac{du}{dy} \right)_{y=0} = 0.9 \times 12 = 10.8 \text{ N/m}^2 \text{ (Ans.)}$$

$$\text{At } y = 15, (\tau)_{y=15} = \mu \left( \frac{du}{dy} \right)_{y=15} = 0.9 \times 6 = 5.4 \text{ N/m}^2 \text{ (Ans.)}$$

$$\text{At } y = 30, (\tau)_{y=30} = \mu \left( \frac{du}{dy} \right)_{y=30} = 0.9 \times 0 = 0 \text{ (Ans.)}$$

**Example 1.9.** A fluid has an absolute viscosity of 0.048 Pa-s and a specific gravity of 0.913. For flow of such a fluid over a solid flat surface, the velocity at a point 75 mm away from the surface is 1.125 m/s. Calculate the shear stresses at the solid boundary and also at points 25 mm, 50 mm and 75 mm away from the surface in normal direction, if the velocity distribution across the surface is (i) linear, (ii) parabolic with vertex at the point 75 mm away from the surface.

(AMIE Summer, 2000)

**Solution.** (i) **Linear velocity distribution:**

If velocity distribution is linear,  $\frac{du}{dy}$  is same at every point within the boundary layer and is equal to  $\frac{du}{dy} = \frac{1.125}{0.075}$  per s.

Shear stress for all the locations,

$$\tau = \mu \frac{du}{dy} = 0.048 \times \frac{1.125}{0.075} = 0.72 \text{ N/m}^2 \text{ (Ans.)}$$

(ii) **Parabolic velocity distribution:**

For parabolic velocity distribution, let the velocity profile be  $u = ly^2 + my + n$  where the constants,  $l$ ,  $m$ , and  $n$  are found from the boundary conditions.

$$\text{At } y = 0, u = 0, \text{ giving } n = 0$$

$$\text{At } y = 0.075 \text{ m, } u = 1.125 \text{ m/s, giving}$$

$$1.125 = (0.075)^2 l + 0.075 m \quad \dots(i)$$

$$\text{or } 1.125 = 5.625 \times 10^{-3} l - 0.075 m$$

$$\text{At } y = 0.075 \text{ m, } \frac{du}{dy} = 0 = 2ly + m$$

$$\text{or } 0 = 2l \times 0.075 + m \text{ or } m = -0.15 l \quad \dots(ii)$$

Substituting (ii) in (i), we get

$$\begin{aligned} 1.125 &= 5.625 \times 10^{-3} l - 0.075 \times 0.15 l \\ &= l (5.625 \times 10^{-3} - 0.075 \times 0.15) = -0.005625 l \end{aligned}$$

$$\therefore l = -\frac{1.125}{0.005625} = -200$$

and from (ii), we have  $m = 30$ .

Hence the velocity distribution becomes  $u = -200y^2 + 30y$ , and  $\frac{du}{dy} = 30 - 400y$

Hence the shear stresses at the required locations,  $y$ , are determined in the table below:

$y$ (m)	0	0.025	0.05	0.075
$\frac{du}{dy}$ (per second)	30	20	10	0
Shear stress = $\mu \frac{du}{dy}$ N/m <sup>2</sup>	1.44	0.96	0.48	0

(Ans.)

**Example 1.10.** A 400 mm diameter shaft is rotating at 200 r.p.m. in a bearing of length 120 mm. If the thickness of oil film is 1.5 mm and the dynamic viscosity of the oil is 0.7 N.s/m<sup>2</sup>, determine:

- (i) Torque required to overcome friction in bearing;
- (ii) Power utilised in overcoming viscous resistance.

Assume a linear velocity profile.

**Solution.** Diameter of the shaft,  $d = 400 \text{ mm} = 0.4 \text{ m}$

Speed of the shaft,  $N = 200 \text{ r.p.m.}$

Thickness of the oil film,  $t = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Length of the bearing,  $l = 120 \text{ mm} = 0.12 \text{ m}$

Viscosity,  $\mu = 0.7 \text{ N.s/m}^2$

Tangential velocity of the shaft,

$$u = \frac{\pi d N}{60} = \frac{\pi \times 0.4 \times 200}{60} = 4.19 \text{ m/s}$$

- (i) Torque required to overcome friction,  $T$ :

We know,  $\tau = \mu \cdot \frac{du}{dy}$

where  $du = \text{change of velocity} = u - 0 = 4.19 \text{ m/s}$

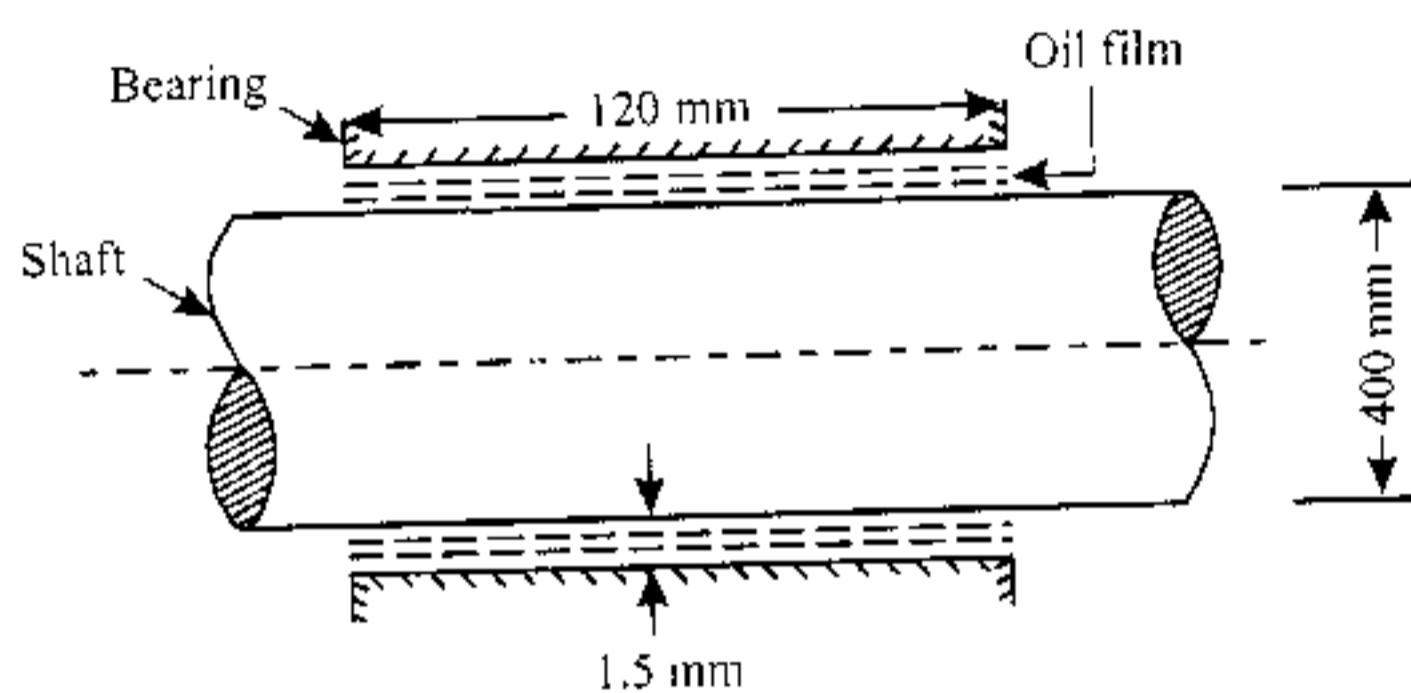


Fig. 1.7

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\tau = 0.7 \times \frac{4.19}{1.5 \times 10^{-3}} = 1955.3 \text{ N/m}^2$$

$$\begin{aligned} \therefore \text{Shear force, } F &= \text{shear stress} \times \text{area} \\ &= \tau \cdot \pi dl \\ &= 1955.3 \times \pi \times 0.4 \times 0.12 \\ &= 294.85 \text{ N} \end{aligned}$$

$$\begin{aligned} \therefore \text{Hence, viscous torque, } &= F \times d/2 = 294.85 \times \frac{0.4}{2} \\ &= 58.97 \text{ Nm (Ans.)} \end{aligned}$$

- (ii) Power utilised,  $P$ :

$$P = T \times \frac{2\pi N}{60} \text{ watts, where } T \text{ is in Nm}$$

$$P = 58.97 \times \frac{2\pi \times 200}{60} = 1235 \text{ W or } 1.235 \text{ kW (Ans.)}$$

viscosity of 0.913.  
mm the surface  
5 mm, 50 mm  
ss the surface

ummer, 2000)

y layer and is

+ n

...(i)

...(ii)

- 400 y

table below:

0.075
0
0

(Ans.)

**Example 1.11.** A 150 mm diameter shaft rotates at 1500 r.p.m. in a 200 mm long journal bearing with 150.5 mm internal diameter. The uniform annular space between the shaft and the bearing is filled with oil of dynamic viscosity 0.8 poise. Calculate the power dissipated as heat.

(AMIE Winter, 2001)

**Solution.** Given:  $d_{\text{shaft}} = 150 \text{ mm}$ ;  $d_{\text{bearing}} = 150.5 \text{ mm}$ ;  $l = 200 \text{ mm} = 0.2 \text{ m}$   
 $N = 1500 \text{ r.p.m.}$ ;  $\mu = 0.8 \text{ poise} = 0.8 \times 0.1 = 0.08 \text{ Ns/m}^2$

**Power dissipated as heat:**

$$\text{Radial thickness of the oil, } dy = \frac{(150.5 - 150)/2}{1000} \text{ m} = 0.00025 \text{ m}$$

$$\text{Tangential velocity of the shaft, } u = \frac{\pi d N}{60} = \frac{\pi \times (150 \times 10^{-3}) \times 1500}{60} = 11.78 \text{ m/s}$$

$$\therefore \text{Change of velocity, } du = u - 0 = 11.78 \text{ m/s}$$

Tangential stress in the oil layer,

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\therefore \tau = 0.08 \times \frac{11.78}{0.00025} = 3769.6 \text{ N/m}^2$$

Power dissipated as heat = Shear force  $\times$  tangential velocity of this shaft

$$= [\tau \times (\pi dl)] \times u$$

$$= 3769.6 \times \pi \times (150 \times 10^{-3}) \times 0.2 \times 11.78$$

$$= 4185 \text{ W or } 4.185 \text{ kW (Ans.)}$$

**Example 1.12.** A vertical cylinder of diameter 180 mm rotates concentrically inside another cylinder of diameter 181.2 mm. Both the cylinders are 300 mm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. Determine the viscosity of the fluid if a torque of 20 Nm is required to rotate the inner cylinder at 120 r.p.m.

**Solution.** Given: Diameter of inner cylinder,  $d = 180 \text{ mm} = 0.18 \text{ m}$

Diameter of outer cylinder,  $D = 181.2 \text{ mm} = 0.1812 \text{ m}$

Length of each cylinder,  $l = 300 \text{ mm} = 0.3 \text{ m}$

Speed of the inner cylinder,  $N = 120 \text{ r.p.m.}$

Torque,  $T = 20 \text{ Nm.}$

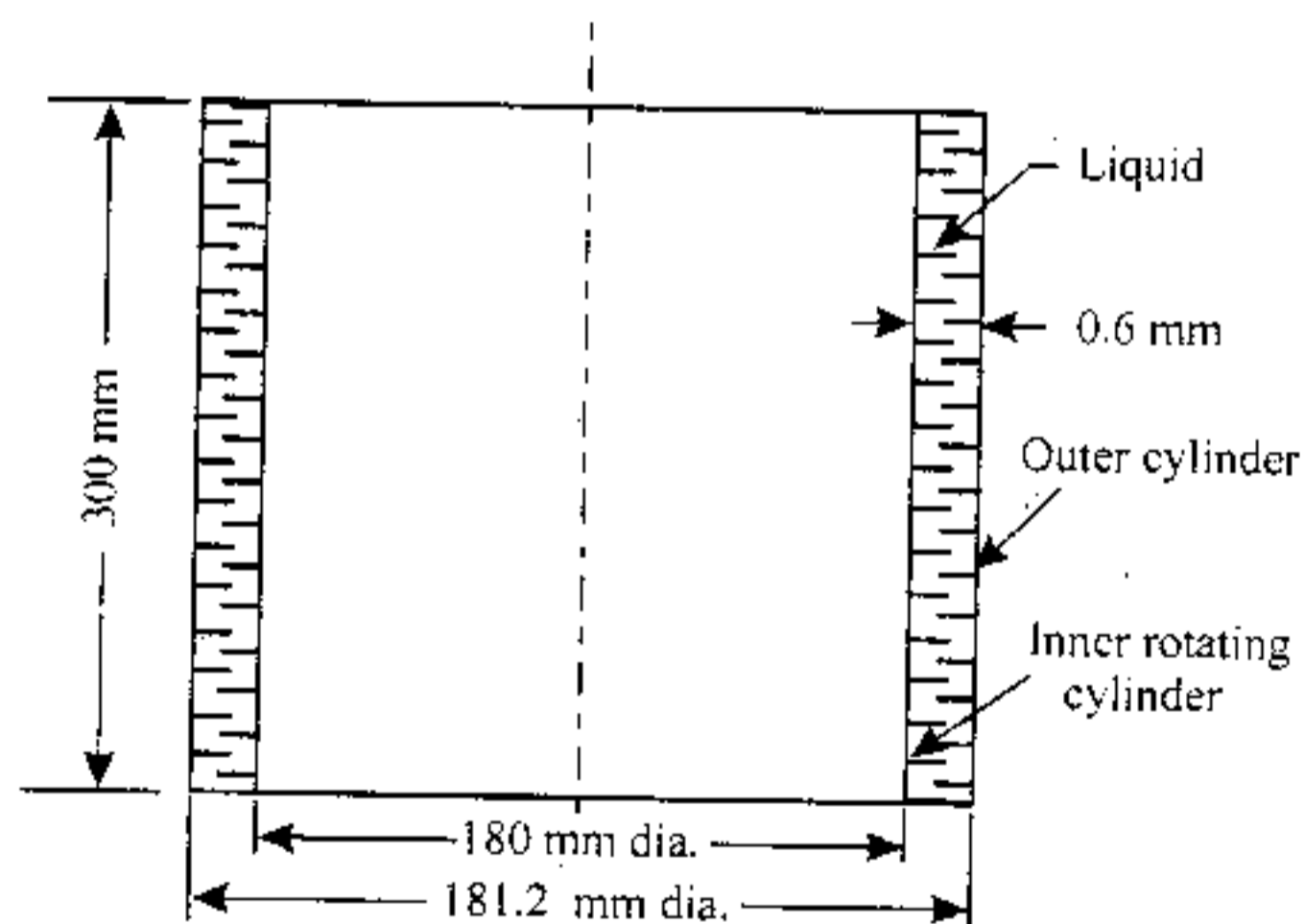


Fig. 1.8

Viscosity of the liquid,  $\mu$ :

Tangential velocity of the inner cylinder

$$u = \frac{\pi d N}{60} = \frac{\pi \times 0.18 \times 120}{60} = 1.13 \text{ m/s}$$

Surface area of the inner cylinder,

$$A = \pi d l = \pi \times 0.18 \times 0.3 \\ = 0.1696 \text{ m}^2$$

Using the relation:

$$\tau = \mu \cdot \frac{du}{dy}$$

where,

$$du = u - 0 = 1.13 - 0 \\ = 1.13 \text{ m/s, and}$$

$$dy = \frac{0.1812 - 0.180}{2} \\ = 0.0006 \text{ m}$$

$$\tau = \mu \times \frac{1.13}{0.0006}$$

$$= 1883.33\mu$$

$$\therefore \text{Shear force, } F = \tau \times A = 1883.33 \mu \times 0.1696 \text{ N}$$

$$\therefore \text{Torque, } T = F \times \frac{d}{2}$$

$$= 1883.33 \mu \times 0.1696 \times \frac{0.18}{2} \text{ or } 20 = 1883.33 \mu \times 0.1696 \times 0.09$$

$$\text{or } \mu = \frac{20}{1883.33 \times 0.1696 \times 0.09} = 0.696 \text{ N.s/m}^2$$

$$\text{i.e., } \mu = 6.96 \text{ poise (Ans.)}$$

**Example 1.13.** A circular disc of diameter  $D$  is slowly rotated in a liquid of large viscosity ( $\mu$ ) at a small distance ( $h$ ) from a fixed surface. Derive an expression of torque ( $T$ ) necessary to maintain an angular velocity ( $\omega$ ). (AMIE Winter, 2002)

**Solution.** The arrangement is shown in Fig. 1.9.

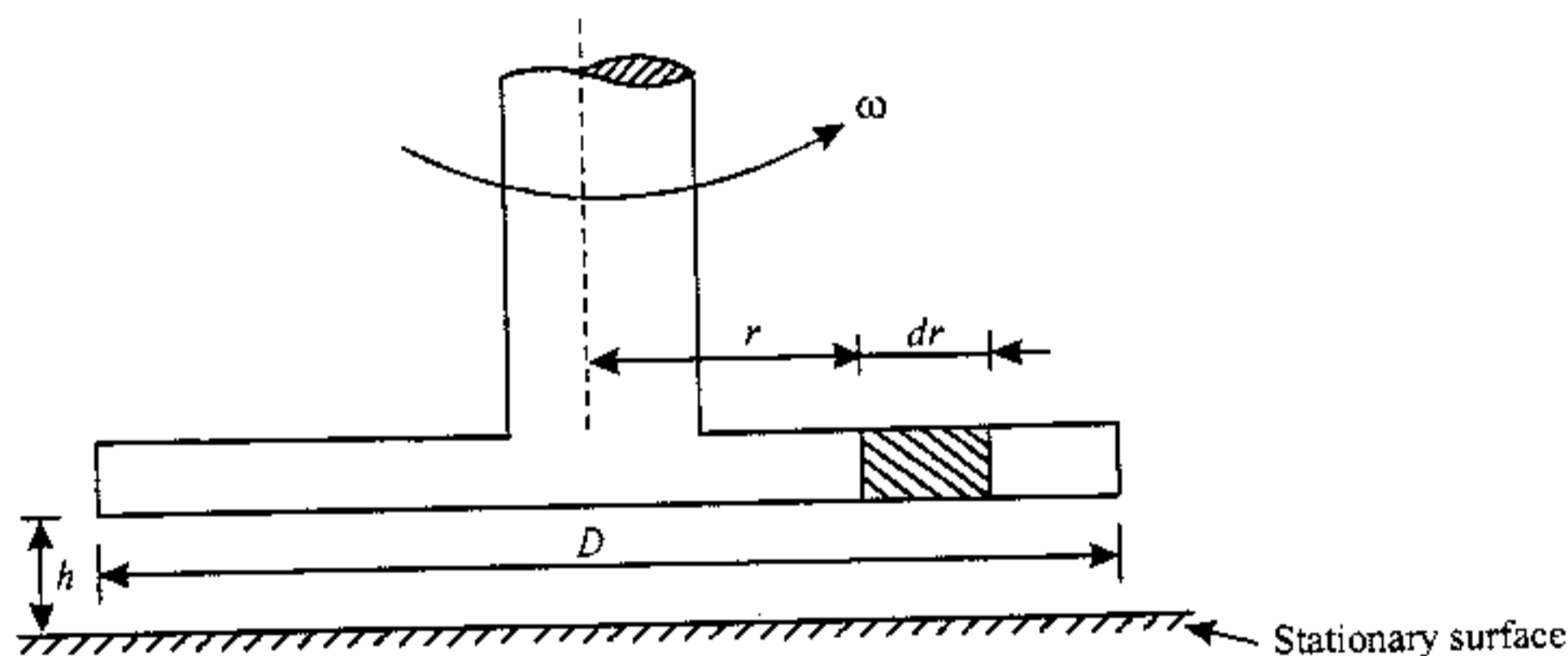


Fig.1.9

Consider an elementary ring of disc at radius  $r$  and having a width  $dr$ . Linear velocity at this ring is  $\omega r$ .

Shear stress,  $\tau = \mu \frac{du}{dy}$

Torque = shear stress  $\times$  area  $\times r$   
 $= \tau \times 2\pi r dr \times r$   
 $= \mu \frac{du}{dy} \times 2\pi r^2 \times dr$

Assuming the gap  $h$  to be small so that the velocity distribution may be assumed linear.

$$\frac{du}{dy} = \frac{\omega r}{h}$$

$\therefore$  Torque on the element

$$dT = \mu \frac{\omega r}{h} \times 2\pi r^2 \times dr = \frac{2\pi\mu\omega}{h} r^3 \times dr$$

$\therefore$  Total torque,  $T = \int_0^{D/2} \frac{2\pi\mu\omega}{h} r^3 \times dr$

or  $T = \frac{2\pi\mu\omega}{h} \left[ \frac{r^4}{4} \right]_0^{D/2} = \frac{2\pi\mu\omega}{h} \cdot \frac{1}{4} \left( \frac{D}{2} \right)^4$

or  $T = \frac{\pi\mu\omega D^4}{32h}$ , which is the required expression. (Ans.)

**Example 1.14.** A 120 mm disc rotates on a table separated by an oil film of 1.8 mm thickness. Find the viscosity of oil if the torque required to rotate the disc at 60 r.p.m is  $3.6 \times 10^{-4}$  Nm.

Assume the velocity gradient in the oil film to be linear.

**Solution.** Given: Diameter of the disc,  $D = 120$  mm = 0.12 m

Thickness of oil film,  $t = 1.8$  mm =  $1.8 \times 10^{-3}$  m

Torque,  $T = 3.6 \times 10^{-4}$  Nm

Speed of the disc,  $N = 60$  r.p.m.

$\therefore$  Angular speed of the disc,  $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 2\pi$  rad./s

Viscosity,  $\mu$ :

We know that when the velocity gradient is linear,

$$\frac{du}{dy} = \frac{u}{t}$$

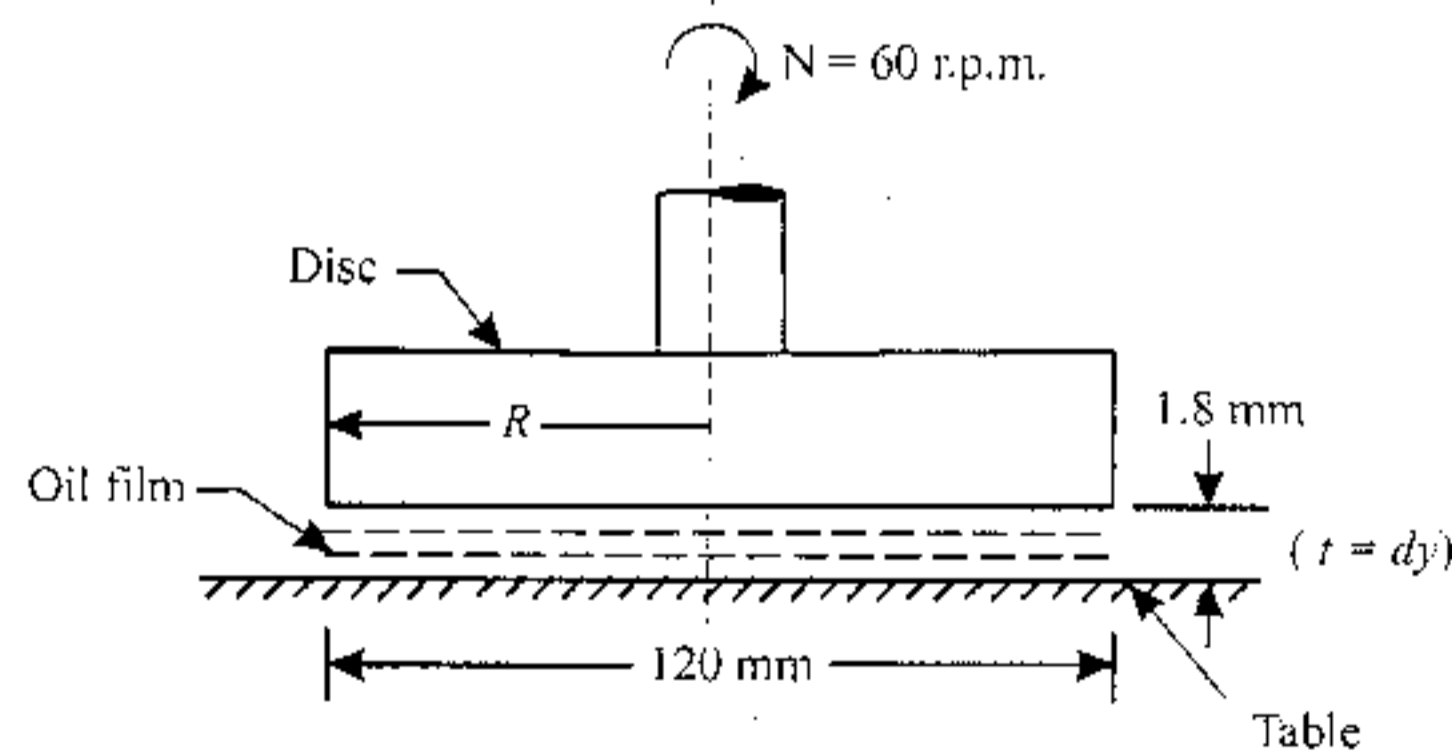


Fig. 1.10

$$\text{Shearing stress, } \tau = \mu \cdot \frac{u}{t}$$

$$\text{Shearing force} = \text{Shearing stress} \times \text{area}$$

$$= \mu \cdot \frac{u}{t} \cdot 2\pi r dr$$

(considering an element at radius  $r$  and thickness  $dr$ )

$$= \mu \cdot \frac{\omega r}{t} \cdot 2\pi r dr = \frac{2\pi\mu\omega r^2 \cdot dr}{t}$$

(where  $u = \omega r$ ,  $\omega$  being the angular velocity)

$$\therefore \text{Viscous torque} = \text{Shearing force} \times r$$

$$= \frac{2\pi\mu\omega r^2 \cdot dr}{t} \cdot r = \frac{2\pi\mu\omega r^3 \cdot dr}{t}$$

\(\therefore\) Total viscous torque,

$$T = \int_0^R \frac{2\pi\mu\omega r^3 dr}{t} = \frac{2\pi\mu\omega}{t} \int_0^R r^3 dr = \frac{\pi\mu\omega R^4}{2t} \text{ i.e., } T = \frac{\pi\mu\omega R^4}{2t}$$

Substituting the values, we get

$$3.6 \times 10^{-4} = \frac{\pi \times \mu \times 2\pi \times (0.12/2)^4}{2 \times 1.8 \times 10^{-3}}$$

$$\text{or } \mu = \frac{3.6 \times 10^{-4} \times 2 \times 1.8 \times 10^{-3}}{\pi \times 2\pi \times (0.06)^4} = 0.00506 \text{ N.s/m}^2 = 0.0506 \text{ poise.}$$

Hence,

$$\mu = 0.0506 \text{ poise (Ans.)}$$

**Example 1.15.** A solid cone of maximum radius  $R$  and vertex angle  $2\theta$  is to rotate at angular velocity  $\omega$ . An oil of viscosity  $\mu$  and thickness  $t$  fills the gap between the cone and the housing. Derive an expression for the torque required and the rate of heat dissipation in the bearing.

**Solution.** Given: Maximum radius of the cone =  $R$

Vertex angle =  $2\theta$

Viscosity of the oil =  $\mu$

Thickness of oil =  $t$

Refer Fig. to 1.11.

Consider an elementary area  $dA$  at radius  $r$  of the cone.

$$dA = 2\pi r ds = 2\pi r \times \frac{dr}{\sin\theta}$$

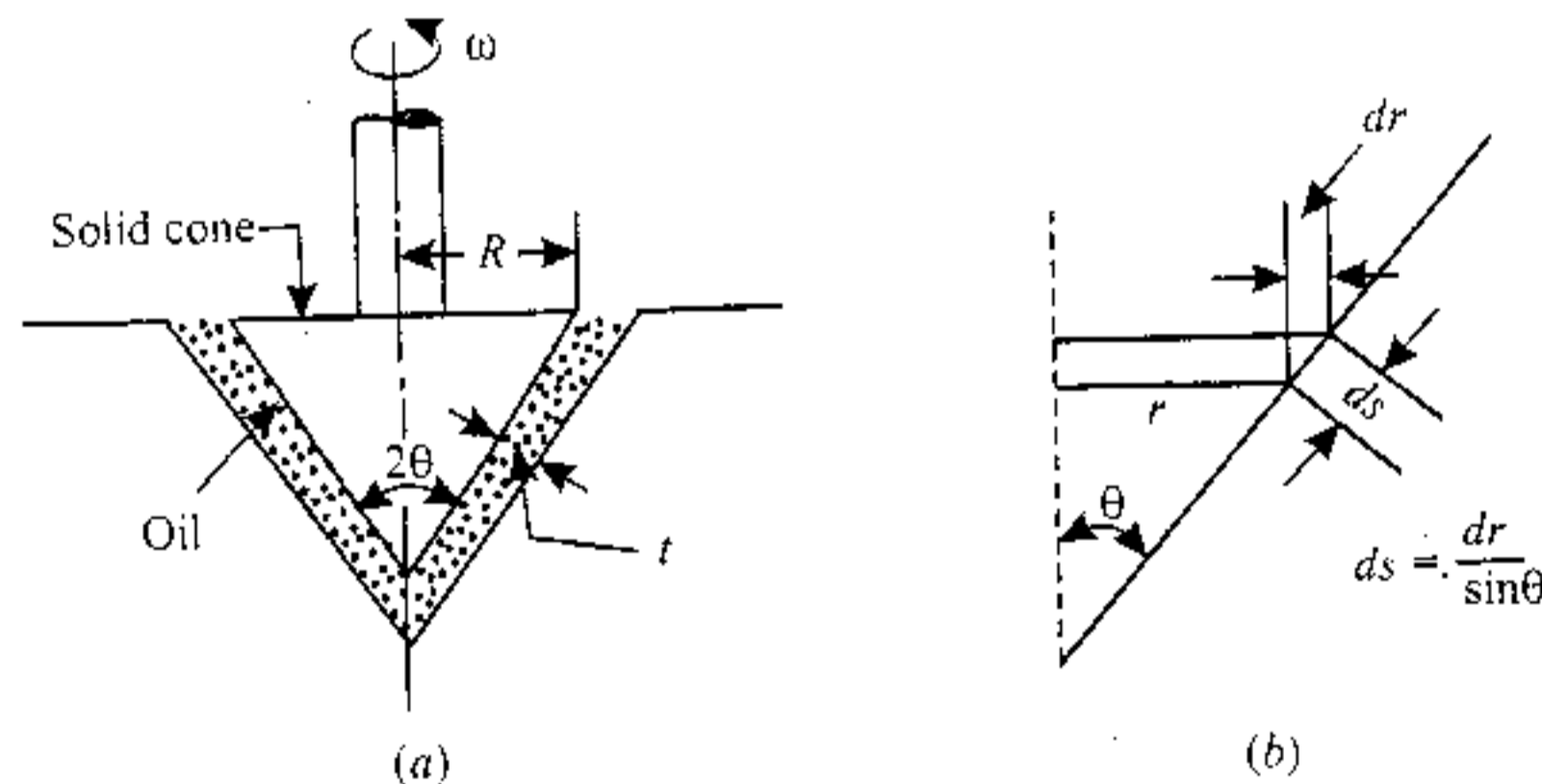


Fig. 1.11

$$\text{Shear stress} \quad \tau = \mu \frac{du}{dy} = \mu \frac{u}{t}$$

Shear force = shear stress  $\times$  area of the element

$$= \mu \frac{u}{t} \left( 2\pi r \times \frac{dr}{\sin \theta} \right)$$

$$\text{Viscous torque on the element, } dT = \mu \frac{u}{t} \left( 2\pi r \times \frac{dr}{\sin \theta} \right) \times r$$

Since the cone rotates with angular velocity  $\omega$  rad/sec., the tangential velocity,  $u = \omega r$

$$\text{or} \quad dT = \mu \frac{\omega r}{t} \left( 2\pi r \times \frac{dr}{\sin \theta} \right) \times r = \frac{2\pi\mu\omega}{t \sin \theta} r^3 dr$$

$$\therefore \text{Total torque,} \quad T = \frac{2\pi\mu\omega}{t \sin \theta} \int_0^R r^3 dr$$

$$\text{i.e.,} \quad T = \frac{2\pi\mu\omega}{t \sin \theta} \times \frac{R^4}{4} = \frac{\pi\mu\omega}{2t \sin \theta} R^4 \quad (\text{Ans.})$$

Power utilised in overcoming the resistance (or rate of heat dissipation in the bearing),

$$P = T\omega = \left( \frac{\pi\mu\omega^2}{2t \sin \theta} R^4 \right) \quad (\text{Ans.})$$

**Example 1.16.** Two large fixed parallel planes are 12 mm apart. The space between the surfaces is filled with oil of viscosity 0.972 N.s/m<sup>2</sup>. A flat thin plate 0.25 m<sup>2</sup> area moves through the oil at a velocity of 0.3 m/s. Calculate the drag force:

- When the plate is equidistant from both the planes, and
- When the thin plate is at a distance of 4 mm from one of the plane surfaces.

**Solution.** Given: Distance between the fixed parallel planes = 12 mm = 0.012 m

Area of thin plate,  $A = 0.25 \text{ m}^2$

Velocity of plate,  $u = 0.3 \text{ m/s}$

Viscosity of oil = 0.972 N.s/m<sup>2</sup>

**Drag force,  $F$ :**

- When the plate is equidistant from both the planes:

Let,  $F_1$  = Shear force on the upper side of the thin plate,

$F_2$  = Shear force on the lower side of the thin plate,

$F$  = Total force required to drag the plate ( $= F_1 + F_2$ ).

The shear  $\tau_1$ , on the upper side of the thin plate is given by:

$$\tau_1 = \mu \cdot \left( \frac{du}{dy} \right)_1$$

where,  $du = 0.3 \text{ m/s}$  (relative velocity between upper fixed plane and the plate), and  $dy = 6 \text{ mm} = 0.006 \text{ m}$  (distance between the upper fixed plane and the plate)

(Thickness of the plate neglected).

$$\therefore \tau_1 = 0.972 \times \frac{0.3}{0.006} = 48.6 \text{ N/m}^2$$

$$\therefore \text{Shear force, } F_1 = \tau_1 A = 48.6 \times 0.25 = 12.15 \text{ N}$$

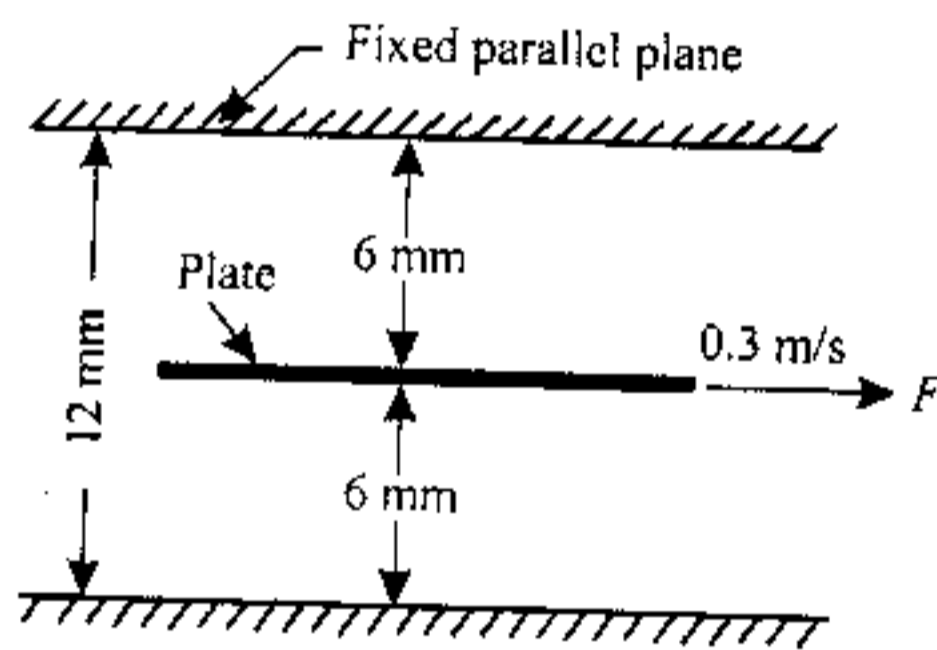


Fig. 1.12

Similarly shear stress ( $\tau_2$ ) on the lower side of the thin plate is given by

$$\tau_2 = \mu \cdot \left( \frac{du}{dy} \right)_2 = 0.972 \times \frac{0.3}{0.06} = 48.6 \text{ N/m}^2$$

and  $F_2 = \tau_2 \cdot A = 48.6 \times 0.25 = 12.15 \text{ N}$

$\therefore F = F_1 + F_2 = 12.15 + 12.15 = 24.30 \text{ N (Ans.)}$

(ii) When the thin plate is at a distance of 40 mm from one of the plane surfaces: Refer Fig. 1.13.

The shear force on the upper side of the thin plate,

$$F_1 = \tau_1 \cdot A = \mu \cdot \left( \frac{du}{dy} \right)_1 \times A$$

$$= 0.972 \times \frac{0.3}{0.008} \times 0.25 = 9.11 \text{ N}$$

The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu \cdot \left( \frac{du}{dy} \right)_2 \times A$$

$$= 0.972 \times \left( \frac{0.3}{0.004} \right) \times 0.25 = 18.22 \text{ N}$$

$\therefore$  Total force  $F = F_1 + F_2 = 9.11 + 18.22 = 27.33 \text{ N (Ans.)}$

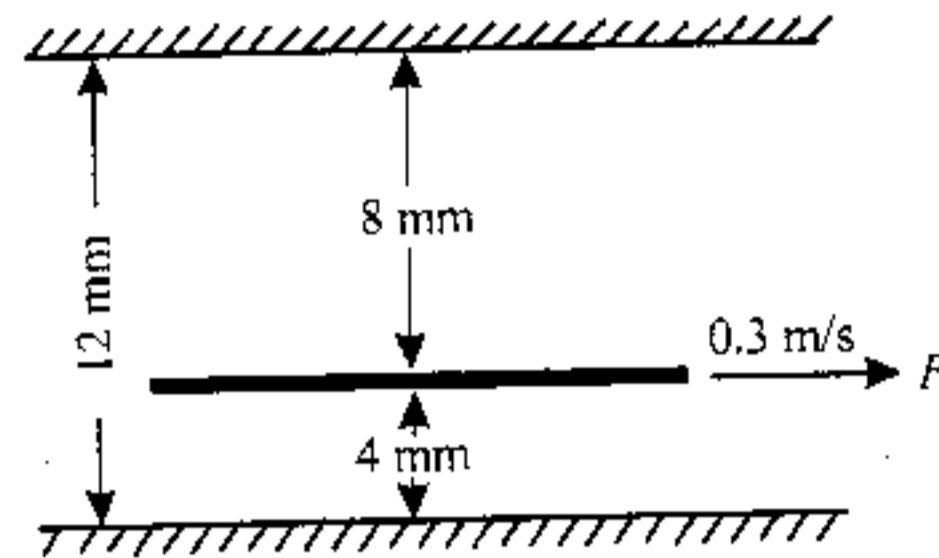


Fig. 1.13

**Example 1.17.** In the Fig. 1.14 is shown a central plate of area  $6 \text{ m}^2$  being pulled with a force of  $160 \text{ N}$ . If the dynamic viscosities of the two oils are in the ratio of  $1:3$  and the viscosity of top oil is  $0.12 \text{ N.s/m}^2$  determine the velocity at which the central plate will move.

**Solution:** Area of the plate,  $A = 6 \text{ m}^2$

Force applied to the plate,  $F = 160 \text{ N}$

Viscosity of top oil,  $\mu = 0.12 \text{ N.s/m}^2$

Velocity of the plate,  $u$ :

Let  $F_1$  = Shear force in the upper side of thin (assumed) plate,

$F_2$  = Shear force on the lower side of the thin plate, and

$F$  = Total force required to drag the plate ( $= F_1 + F_2$ )

Then,  $F = F_1 + F_2 = \tau_1 \times A + \tau_2 \times A$

$$= \mu \left( \frac{\partial u}{\partial y} \right)_1 \times A + 3\mu \left( \frac{du}{dy} \right)_2 \times A$$

(where  $\tau_1$  and  $\tau_2$  are the shear stresses on the two sides of the plate)

$$160 = 0.12 \times \frac{u}{6 \times 10^{-3}} \times 6 + 3 \times 0.12 \times \frac{u}{6 \times 10^{-3}} \times 6$$

or  $160 = 120u + 360u = 480u$  or  $u = \frac{160}{480} = 0.333 \text{ m/s (Ans.)}$

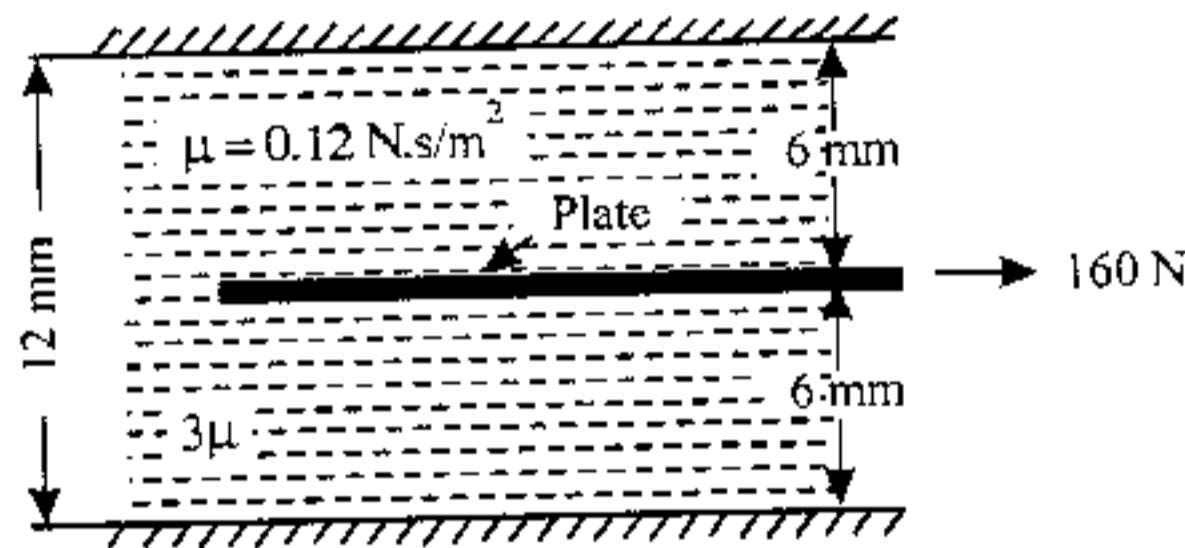


Fig. 1.14

**Example 1.18.** A metal plate  $1.25 \text{ m} \times 1.25 \text{ m} \times 6 \text{ mm}$  thick and weighing  $90 \text{ N}$  is placed midway in the  $24 \text{ mm}$  gap between the two vertical plane surfaces as shown in the Fig. 1.15. The gap is filled with an oil of specific gravity  $0.85$  and dynamic viscosity  $3.0 \text{ N.s/m}^2$ . Determine the force required to lift the plate with a constant velocity of  $0.15 \text{ m/s}$ .

**Solution.** Given: Dimensions of the plate =  $1.25 \text{ m} \times 1.25 \text{ m} \times 6 \text{ mm}$

$\therefore$  Area of the plate,

$$A = 1.25 \times 1.25 = 1.5625 \text{ m}^2$$

Thickness of the plate =  $6 \text{ mm}$

$$\therefore t_1 = t_2 = \frac{24 - 6}{2} = 9 \text{ mm}$$

(Since the plate is situated midway in the gap)

Specific gravity of oil =  $0.85$

Dynamic viscosity of oil =  $3 \text{ N.s/m}^2$

Velocity of the plate =  $0.15 \text{ m/s}$

Weight of the plate =  $90 \text{ N}$

**Force required to lift the plate:**

Drag force (or viscous resistance) against the motion of the plate,

$$F = \tau_1 \cdot A + \tau_2 \cdot A$$

(where  $\tau_1$  and  $\tau_2$  are the shear stresses on two sides of the plate)

$$= \mu \cdot \left( \frac{du}{dy} \right)_1 \times A + \mu \left( \frac{du}{dy} \right)_2 \times A$$

$$= \mu \cdot \frac{u}{t_1} \times A + \mu \cdot \frac{u}{t_2} \times A$$

$$= \mu A u \cdot \left( \frac{1}{t_1} + \frac{1}{t_2} \right)$$

$$\text{or } F = 3 \times 1.5625 \times 0.15 \left( \frac{1}{9 \times 10^{-3}} + \frac{1}{9 \times 10^{-3}} \right)$$

$$= 3 \times 1.5625 \times 0.15 \times \frac{2}{9 \times 10^{-3}} = 156.25 \text{ N}$$

Upward thrust or buoyant force on the plate = specific weight  $\times$  volume of oil displaced

$$= 0.85 \times 9810 \times (1.25 \times 1.25 \times 0.006) = 78.17 \text{ N}$$

Effective weight of the plate =  $90 - 78.17 = 11.83 \text{ N}$

$\therefore$  Total force required to lift the plate at velocity of  $0.15 \text{ m/s} = F +$  effective weight of the

$$= 156.25 + 11.83 = 168.08 \text{ N (Ans.)}$$

**Example 1.19.** A square metal plate  $1.8 \text{ m}$  side and  $1.8 \text{ mm}$  thick weighing  $60 \text{ N}$  is to be pushed through a vertical gap of  $30 \text{ mm}$  of infinite extent. The oil in the gap has a specific gravity of  $0.85$  and viscosity of  $3 \text{ N.s/m}^2$ . If the metal plate is to be lifted at a constant speed of  $0.12 \text{ m/s}$ , find the force and power required.

**Solution.** Area of metal plate,

$$A = 1.8 \times 1.8 = 3.24 \text{ m}^2$$

Thickness of the oil film,

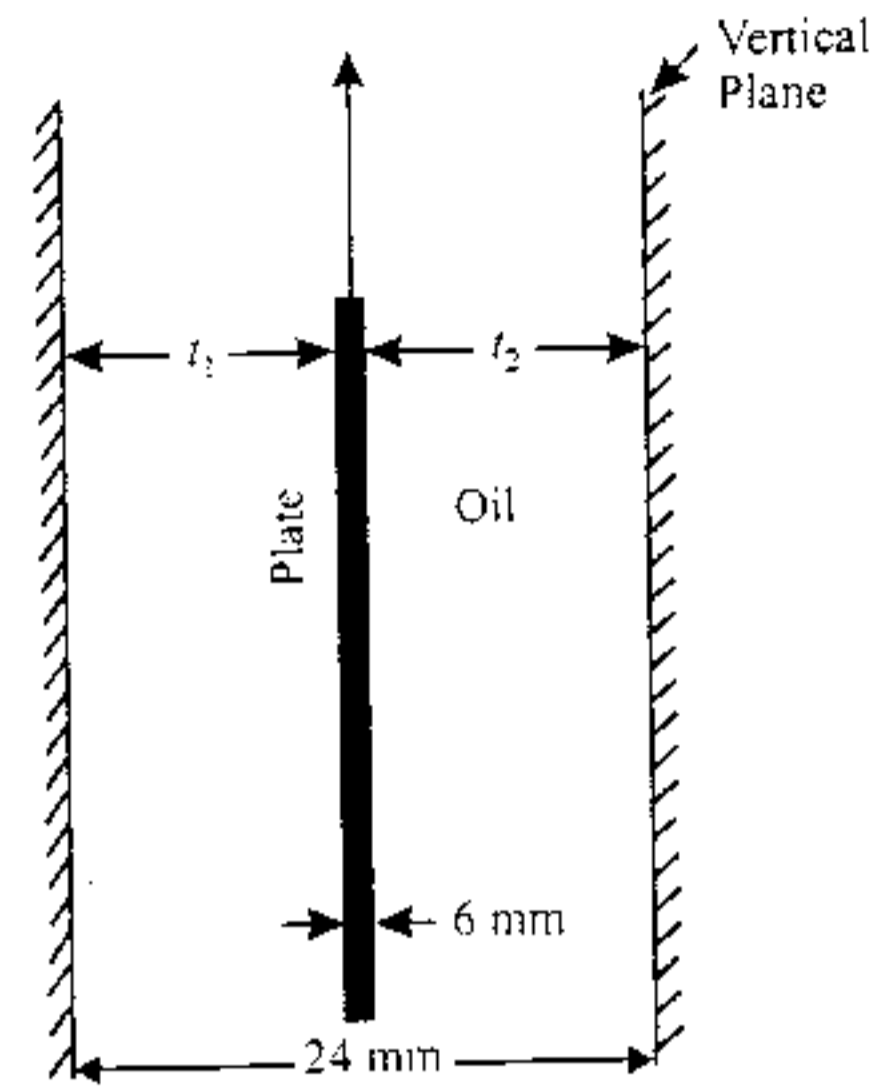


Fig. 1.15

## Properties of Fluids

$$t = dy = \frac{30 - 1.8}{2 \times 1000} = 0.0141 \text{ m}$$

Speed of the metal plate,  $u = 0.12 \text{ m/s}$ .

Change of speed,

$$du = 0.12 - 0 = 0.12 \text{ m/s}$$

Viscosity,  $\mu = 3 \text{ N.s/m}^2$

We know, shear stress,

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\therefore \tau = 3 \times \frac{0.12}{0.0141} = 25.53 \text{ N/m}^2$$

Force required,  $F$ :

$$F = W + 2(\tau \cdot A)$$

[ where  $W$  = weight of the plate

$$= 60 \text{ N (given)}$$

$$= 60 + 2 \times 25.53 \times 3.24 = 225.4 \text{ N}$$

Hence  $F = 225.4 \text{ N (Ans.)}$

Power required,  $P$ :

$$P = F \times u = 225.4 \times 0.12 = 27.05 \text{ W}$$

Hence  $P = 27.05 \text{ (Ans.)}$

**Example 1.20.** A thin plate of very large area is placed in a gap of height  $h$  with oils of viscosities  $\mu'$  and  $\mu''$  on the two sides of the plate. The plate is pulled at a constant velocity  $V$ . Calculate the position of plate so that :

- The shear force on the two sides of the plate is equal;
- The force required to drag the plate is minimum.

Assume viscous flow and neglect all end effects.

**Solution.** Given : Height of the gap =  $h$

Viscosities of oils =  $\mu'$  and  $\mu''$

Velocity of the plate =  $V$

Position of the plate,  $y$ :

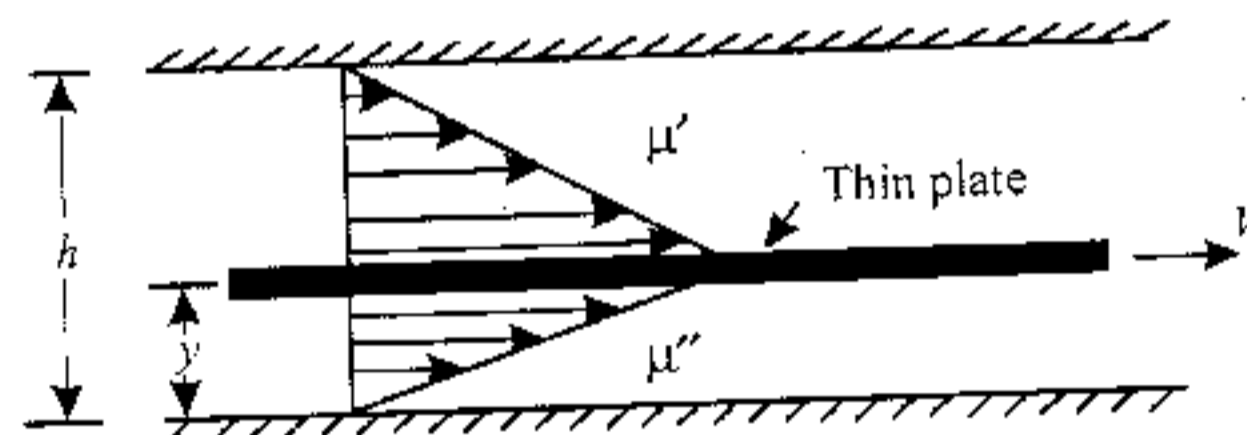


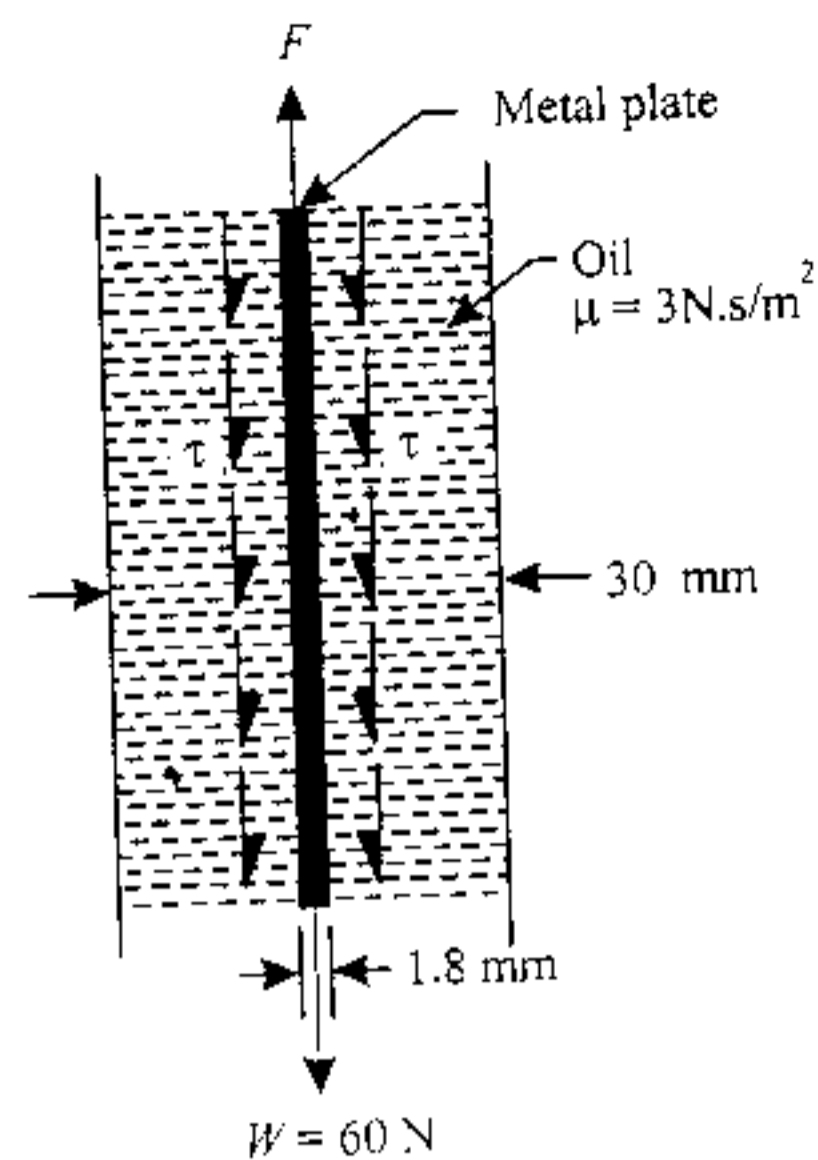
Fig. 1.17

Let  $y$  = The distance of the thin plate from one of the surfaces of the gap.

Force on the upper side of the plate,

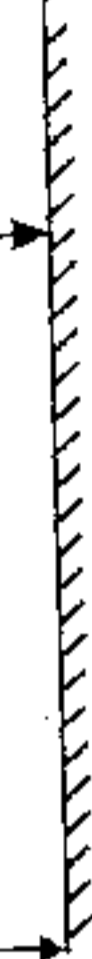
$$F_{upper} = \mu \frac{du}{dy} = \mu' \times \frac{V}{(h-y)} A$$

Force on the lower side of the plate,  $F_{lower} = \mu'' \times \frac{V}{y} A$



$N$  is placed  
g. 1.15. The  
etermine the

Vertical  
Plane



displaced

ght of the plate

$N$  is to be lifted  
gravity of 0.95  
12 m/s, find the

(i) Since the forces on the two sides of the plate are equal (given) we have,

$$\text{i.e., } F_{\text{upper}} = F_{\text{lower}}$$

$$\therefore \mu' \times \frac{V}{(h-y)} A = \mu'' \times \frac{V}{y} A$$

$$\text{or } \frac{\mu'}{h-y} = \frac{\mu''}{y} \quad \text{or } \mu' y = \mu'' h - \mu'' y$$

$$\therefore y = \frac{\mu'' h}{\mu' + \mu''} \quad (\text{Ans.})$$

(ii) Total drag force = sum of the forces on the upper and lower surfaces of the plate.

$$\text{i.e., } F = F_{\text{upper}} + F_{\text{lower}}$$

$$\text{or } F = \mu' \times \frac{V}{h-y} \times A + \mu'' \times \frac{V}{y} A$$

$$\text{For the drag force to be minimum } \frac{dF}{dy} = 0$$

$$\text{i.e., } \frac{d}{dy} \left[ \mu' \times \frac{V}{h-y} \times A + \mu'' \times \frac{V}{y} A \right] = 0$$

$$\text{or } \frac{\mu' VA}{(h-y)^2} - \frac{\mu'' VA}{y^2} = 0$$

$$\text{or } \frac{\mu'}{\mu''} = \frac{(h-y)^2}{y^2} = \frac{h^2 + y^2 - 2hy}{y^2} = \frac{h^2}{y^2} + 1 - \frac{2h}{y}$$

$$\therefore \frac{h^2}{y^2} - \frac{2h}{y} + \left( 1 + \frac{\mu'}{\mu''} \right)$$

$$\text{or } \frac{h}{y} = \frac{2 \pm \sqrt{4 - 4(1 - \mu'/\mu'')}}{2} = 1 \pm \sqrt{(\mu'/\mu'')}$$

Since  $\frac{h}{y}$  cannot be less than unity, therefore

$$\frac{h}{y} = 1 + \sqrt{\mu'/\mu''} \quad \text{or } y = \frac{h}{1 + \sqrt{\mu'/\mu''}} \quad (\text{Ans.})$$

### 1.7. Thermodynamic Properties

The thermodynamic properties need to be considered when a fluid is influenced by change of temperature. The following equation, known as the *characteristic equation of a state of a perfect gas*, is used for this purpose.

$$pV = mRT \quad \dots(1.10)$$

where,  $p$  = Absolute pressure,  $m$  = Mass of gas,  
 $V$  = Volume of  $m$  kg of gas,  $R$  = Characteristic gas constant, and  
 $T$  = Absolute temperature.

The characteristic equation in *another form*, can be derived by using *kilogram-mole as a unit*. The *kilogram-mole* is defined as a quantity of a gas equivalent to  $M$  kg of the gas, where  $M$  is the molecular weight of the gas (i.e., since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen).

As per definition of the kilogram-mole, for  $m$  kg of a gas, we have

$$m = nM \quad \dots(1.11)$$

## Properties of Fluids

where,  $n$  = No. of moles.

Note. Since the standard of mass is the kg, kilogram-mole will be written simply as mole.

Substituting for  $m$  from eqn. 1.11 in Eqn. 1.10 gives

$$pV = nMRT \quad \text{or} \quad MR = \frac{pV}{nT}$$

According to Avogadro's hypothesis the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at same temperature and pressure. Therefore,  $\frac{V}{n}$  is the same for all gases at the same value of  $p$  and  $T$ . That is the quantity  $\frac{pV}{nT}$  is a constant for all gases. This constant is called 'universal gas constant', and is given the symbol,  $R_0$ .

$$\text{i.e.,} \quad MR = R_0 = \frac{pV}{nT} \quad \text{or} \quad pV = nR_0T \quad \dots(1.12)$$

$$\text{Since} \quad MR = R_0, \text{ then } R = \frac{R_0}{M} \quad \dots(1.13)$$

It has been found experimentally that the volume of 1 mole of any perfect gas at 1 bar and  $0^\circ\text{C}$  is approximately  $22.71 \text{ m}^3$ . Therefore from eqn. 1.12,

$$R_0 = \frac{pV}{nT} = \frac{1 \times 10^5 \times 22.71}{1 \times 273.15} = 8314.3 \text{ Nm/mole K}$$

Using eqn. 1.13, the gas constant for any gas can be found when the molecular weight is known.

**Example.** For oxygen which has a molecular weight of 32, the gas constant

$$R = \frac{R_0}{M} = \frac{8314}{32} = 259.8 \text{ Nm/kg K.}$$

If the value of  $R$  is known, the specific weight of any gas can be computed at any temperature.

The density can be changed by changing temperature or pressure.

(i) When the change in the state of the fluid system is affected at *constant pressure* the process is known as **isobaric or constant pressure process**.

$$\text{Here } \frac{V}{T} = \text{constant; (Charle's law) or } \frac{v}{T} = \text{constant or } \frac{v}{T} = \frac{1}{\rho T} = \text{constant} \quad \dots(1.14)$$

(ii) When the change in the state of the fluid system is affected at *constant temperature* the process is known as **isothermal process**.

$$\text{Here } pv^\gamma = \text{constant; (Boyle's Law) or } pv = \frac{P}{\rho} = \text{constant} \quad \dots(1.15)$$

(iii) When no heat is transferred to or from the fluid during the change in the state of fluid system, the process is called **adiabatic process**.

$$\text{Here, } pv^\gamma = \text{constant or } pv^\gamma = \frac{P}{\rho^\gamma} = \text{constant} \quad \dots(1.16)$$

$$\text{where} \quad \gamma = \frac{c_p}{c_v},$$

$c_p$  = Specific heat of gas at constant pressure, and

$c_v$  = Specific heat of gas at constant volume.

$\gamma$  depends upon the molecular structure of the gas.

Note. For details regarding compression and expansion of gases please refer to chapter on "Compressible flow."

**Example 1.21.** The pressure and temperature of carbon-dioxide in a vessel are  $600 \text{ kN/m}^2 \text{ abs.}$  and  $30^\circ\text{C}$  respectively. Find its mass density, specific weight and specific volume.

change of  
a perfect

...(1.10)

as a unit.

$M$  is the  
g mole of

...(1.11)

**Solution. Given:** Pressure of  $\text{CO}_2 = 600 \text{ kN/m}^2 \text{ abs.}$

Temperature of  $\text{CO}_2 = 30 + 273 = 303 \text{ K}$

Molecular weight of  $\text{CO}_2 = 12 + 2 \times 16 = 44$

Universal gas constant,  $R_0 = 8314.3 \text{ Nm/mole K}$

$\therefore$  Characteristic gas constant,  $R = \frac{R_0}{M} = \frac{8314.3}{44} = 189 \text{ Nm/kg K}$

(i) Mass density,  $\rho$ :

$$\text{We know, } pV = mRT \quad \therefore \frac{m}{V} = \frac{p}{RT}$$

$$\text{or } \rho = \frac{p}{RT} = \frac{600 \times 10^3}{189 \times 313} = 10.14 \text{ kg/m}^3 \quad \text{i.e., } \rho = 10.14 \text{ kg/m}^3 \text{ (Ans.)}$$

(ii) Specific weight,  $w$ :

$$w = \rho g = 10.14 \times 9.81 = 99.47 \text{ N/m}^3 \text{ (Ans.)}$$

(ii) Specific volume  $v$ :

$$v = \frac{1}{\rho} = \frac{1}{10.14} = 0.0986 \text{ m}^3/\text{kg} \text{ (Ans.)}$$

## 1.8. Surface Tension and Capillarity

### 1.8.1. Surface Tension

**Cohesion.** Cohesion means intermolecular attraction between *molecules of the same liquid*. It enables a liquid to resist small amount of tensile stresses. Cohesion is a tendency of the liquid to remain as one *assemblage of particles*. "Surface tension" is due to cohesion between particles at the free surface.

**Adhesion.** Adhesion means attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a liquid to stick to another body.

Capillary action is due to both cohesion and adhesion.

Surface tension is caused by the force of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by other molecules all around and is in equilibrium. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of the molecules below it. Consequently, as shown in Fig. 1.18, there is a net inward force on the molecule. The force is normal to the liquid surface. At the free surface a thin layer of molecules is formed. This is because of this film that a thin small needle can float on the free surface (the layer acts as a membrane).

Some important examples of phenomenon of surface tension are as follows:

- (i) Rain drops (A falling rain drop becomes spherical due to cohesion and surface tension).
- (ii) Rise of sap in a tree.
- (iii) Bird can drink water from ponds.
- (iv) Capillary rise and capillary siphoning.

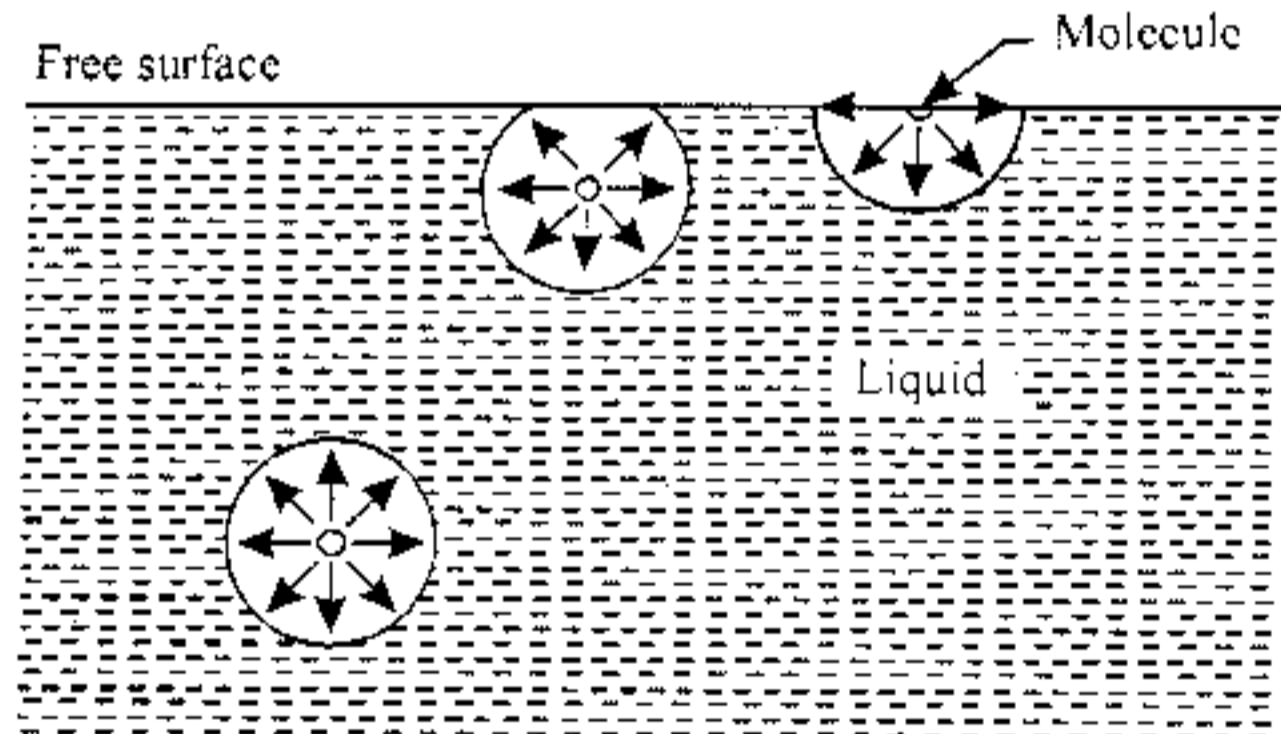


Fig. 1.18

## Properties of Fluids

- (v) Collection of dust particles on water surface.  
 (vi) Break up of liquid jets.

**Dimensional formula for surface tension:**

The dimensional formula for surface tension is given by:

$$\left[ \frac{E}{L} \right] \text{ or } \left[ \frac{M}{T^2} \right]$$

It is usually expressed in N/m. The value of surface tension depends upon the following factors:

- (i) Nature of the liquid,  
 (ii) Nature of the surrounding matter (e.g., solid, liquid or gas), and  
 (iii) Kinetic energy (and hence the temperature of the liquid molecules).

**Note.** As compared to pressure and gravitational forces surface tension forces are generally negligible; but become quite significant when there is a free surface and the boundary conditions are small as in the case of small scale models of hydraulic engineering structures.

Surface tension of water and mercury when in contact with air:

Water-air ... 0.073 N/m at 20°C;

Water-air ... 0.058 N/m at 100°C;

Mercury-air ... 0.1 N/m length.

**1.8.1.1. Pressure Inside a Water Droplet, Soap Bubble and a Liquid Jet****Case I. Water droplet:**

Let,  $p$  = Pressure inside the droplet above outside pressure (i.e.  $\Delta p = p - 0 = p$  above atmospheric pressure)

$d$  = Diameter of the droplet and

$\sigma$  = Surface tension of the liquid.

From free body diagram (Fig. 1.19 d), we have:

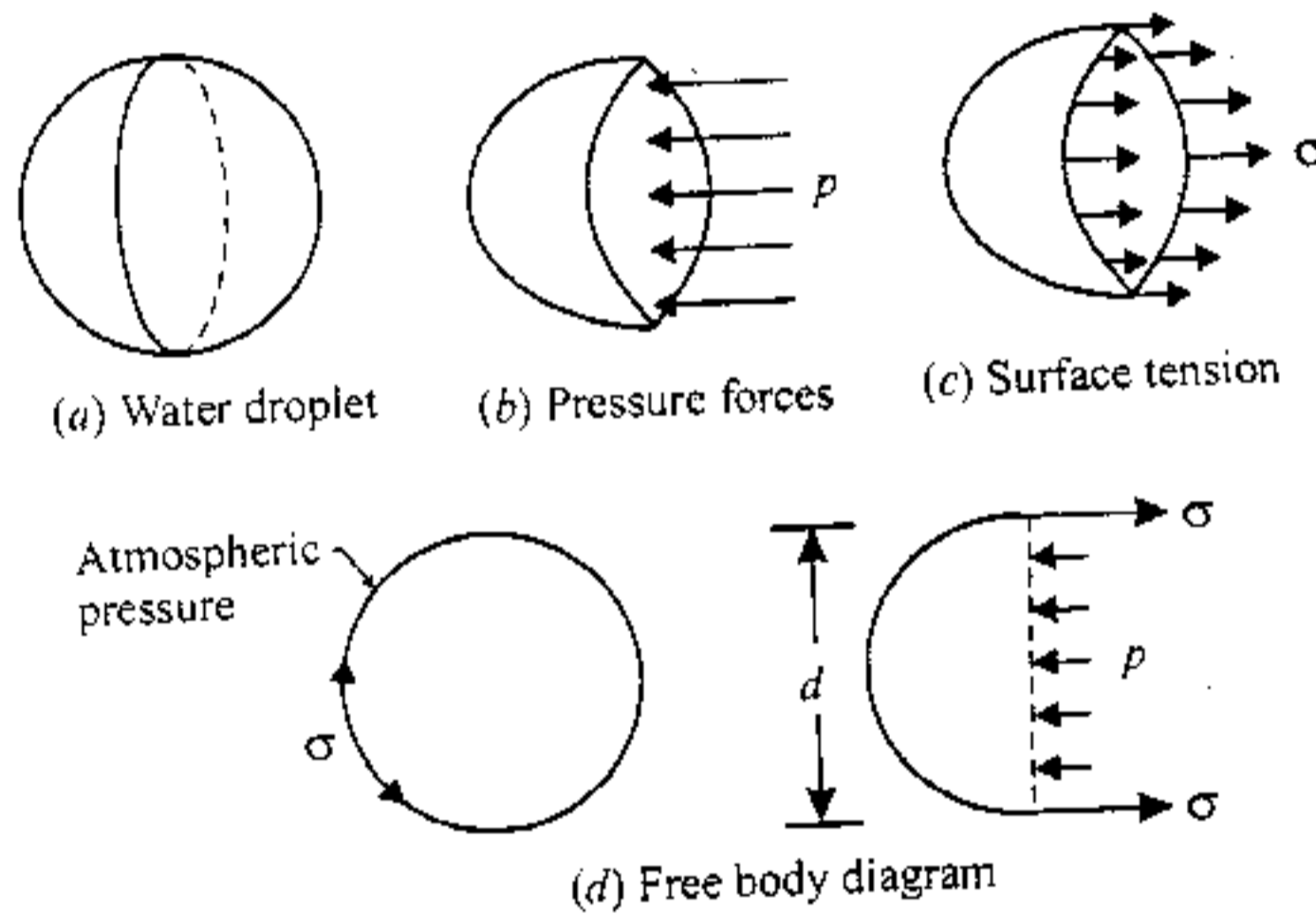


Fig. 1.19. Pressure inside a water droplet.

- (i) Pressure force =  $p \times \frac{\pi}{4} d^2$ , and  
 (ii) Surface tension force acting around the circumference =  $\sigma \times \pi d$ .  
 Under equilibrium conditions these two forces will be equal and opposite, i.e.

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\therefore p = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d} \quad \dots(1.17)$$

Eqn. 1.17 shows that with an increase in size of the droplet the pressure intensity decreases.

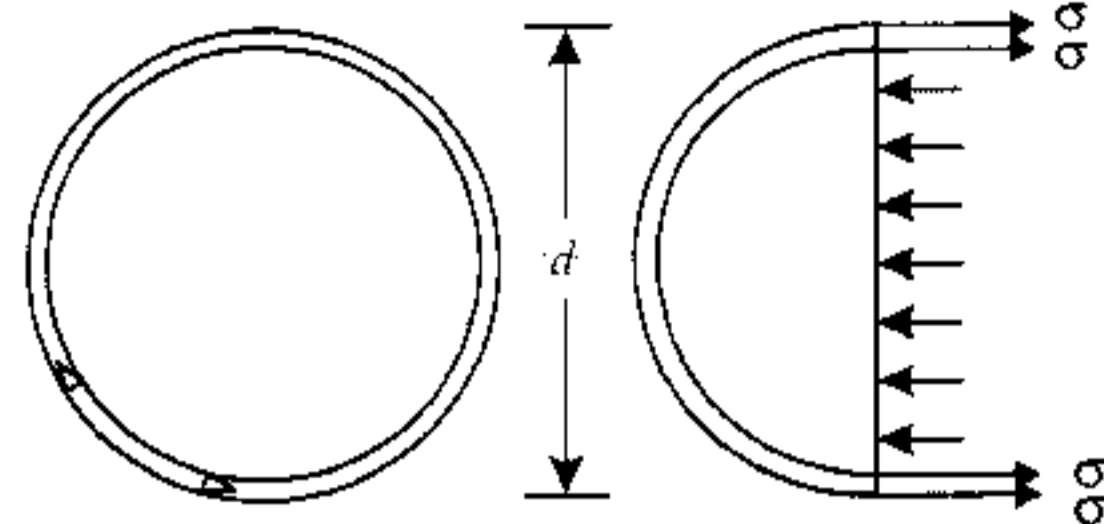
### Case II. Soap (or hollow) bubble:

Soap bubbles have two surfaces on which surface tension  $\sigma$  acts.

From the free body diagram (Fig. 1.20), we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$\therefore p = \frac{2\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \quad \dots(1.18)$$



Free body diagram

Fig. 1.20. Pressure inside a soap bubble.

Since the soap solution has a high value of surface tension  $\sigma$ , even with small pressure of blowing a soap bubble will tend to grow larger in diameter (hence formation of large soap bubbles).

### Case III. A Liquid jet:

Let us consider a cylindrical liquid jet of diameter  $d$  and length  $l$ . Fig. 1.21 shows a semi-jet.

Pressure force =  $p \times l \times d$

Surface tension force =  $\sigma \times 2l$

Equating the two forces, we have

$$p \times l \times d = \sigma \times 2l$$

$$\therefore p = \frac{\sigma \times 2l}{l \times d} = \frac{2\sigma}{d} \quad \dots(1.19)$$

**Example 1.22.** If the surface tension at air-water interface is  $0.069 \text{ N/m}$ , what is the pressure difference between inside and outside of an air bubble of diameter  $0.009 \text{ mm}$ ?

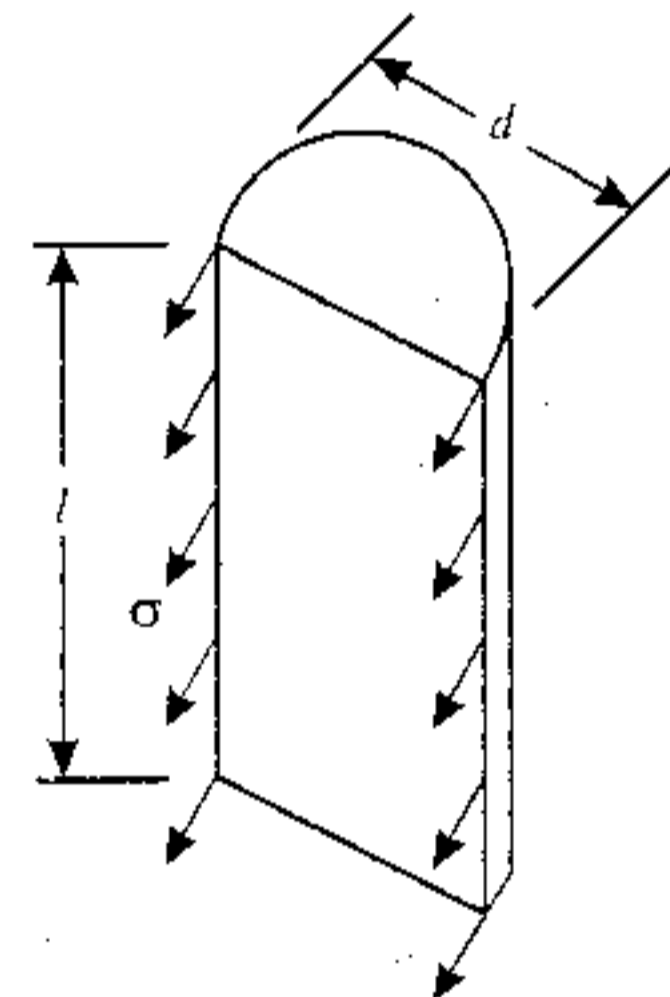
**Solution.** Given:  $\sigma = 0.069 \text{ N/m}$ ;  $d = 0.009 \text{ mm}$

An air bubble has only one surface. Hence,

$$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.069}{0.009 \times 10^{-3}} = 30667 \text{ N/m}^2$$

$$= 30.667 \text{ kN/m}^2 \text{ or kPa (Ans.)}$$



Semi-jet

Fig. 1.21. Forces on liquid jet.

**Example 1.23.** If the surface tension at the soap-air interface is  $0.09 \text{ N/m}$ , calculate the internal pressure in a soap bubble of  $28 \text{ mm}$  diameter.

**Solution.** Given:  $\sigma = 0.09 \text{ N/m}$ ;  $d = 28 \text{ mm}$ .

In a soap bubble there are two interfaces. Hence,

$$p = \frac{8\sigma}{d} = \frac{8 \times 0.09}{28 \times 10^{-3}} = 25.71 \text{ N/m}^2 \text{ (above atmospheric pressure) (Ans.)}$$

**Example 1.24.** In order to form a stream of bubbles, air is introduced through a nozzle into a tank of water at  $20^\circ\text{C}$ . If the process requires  $3.0 \text{ mm}$  diameter bubbles to be formed, by how much the air pressure at the nozzle must exceed that of the surrounding water?

What would be the absolute pressure inside the bubble if the surrounding water is at  $100.3 \text{ kN/m}^2$ ?

Take surface tension of water at  $20^\circ\text{C} = 0.0735 \text{ N/m}$ .

**Solution.** Diameter of a bubble,  $d = 3.0 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Surface tension of water at  $20^\circ\text{C}$ ,  $\sigma = 0.0735 \text{ N/m}$

The excess pressure intensity of air over that of surrounding water,  $\Delta p = p$ .

We know, 
$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0735}{3 \times 10^{-3}} = 98 \text{ N/m}^2 \text{ (Ans.)}$$

**Absolute pressure inside the bubble,  $p_{abs}$ :**

$$\begin{aligned} p_{abs} &= p + p_{atm} \\ &= 98 \times 10^{-3} + 100.3 \\ &= 0.098 + 100.3 = 100.398 \text{ kN/m}^2 \text{ (Ans.)} \end{aligned}$$

**Example 1.25.** A soap bubble  $62.5 \text{ mm}$  diameter has an internal pressure in excess of the outside pressure of  $20 \text{ N/m}^2$ . What is tension in the soap film?

**Solution.** Given: Diameter of the bubble,  $d = 62.5 \text{ mm} = 62.5 \times 10^{-3} \text{ m}$ ; Internal pressure in excess of the outside pressure,  $p = 20 \text{ N/m}^2$ .

**Surface tension,  $\sigma$ :**

Using the relation,

$$p = \frac{8\sigma}{d}, \text{ i.e., } 20 = \frac{8\sigma}{62.5 \times 10^{-3}} \therefore \sigma = 20 \times \frac{62.5 \times 10^{-3}}{8} = 0.156 \text{ N/m (Ans.)}$$

**Example 1.26.** What do you mean by surface tension? If the pressure difference between the inside and outside of the air bubble of diameter  $0.01 \text{ mm}$  is  $29.2 \text{ kPa}$ , what will be the surface tension at air-water interface? (AMIE Winter, 2000)

**Solution.** Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by the letter  $\sigma$  and is expressed as  $\text{N/m}$ .

$$p \times \frac{\pi}{4} d^2 = \sigma(\pi d)$$

or 
$$\sigma = p \times \frac{d}{4}$$

Substituting the values;  $d = 0.01 \times 10^{-3} \text{ m}$ ;  $p = 29.2 \times 10^3 \text{ Pa}$  ( or  $\text{N/m}^2$ ), we get

$$\sigma = 29.2 \times 10^3 \times \frac{0.01 \times 10^{-3}}{4} = 0.073 \text{ N/m (Ans.)}$$

### 1.8.2. Capillarity

**Capillarity** is a phenomenon by which a liquid (depending upon its specific gravity) rises into a thin glass tube above or below its general level. This phenomenon is due to the combined effect of cohesion and adhesion of liquid particles.

Fig. 1.22 shows the phenomenon of rising water in the tube of smaller diameters.

Let,  $d$  = Diameter of the capillary tube,  
 $\theta$  = Angle of contact of the water surface,  
 $\sigma$  = Surface tension force for unit length, and  
 $w$  = Weight density ( $\rho g$ ).

Now, upward surface tension force (lifting force) = weight of the water column in the tube (gravity force)

$$\pi d \sigma \cos \theta = \frac{\pi}{4} d^2 \times h \times w$$

$$\therefore h = \frac{4\sigma \cos \theta}{wd} \quad \dots(1.20)$$

For water and glass:  $\theta = 0$ .

Hence the capillary rise of water in the glass tube,

$$h = \frac{4\sigma}{wd} \quad \dots(1.21)$$

In case of mercury there is a capillary depression as shown in Fig. 1.23, and the angle of depression is  $\theta \approx 140^\circ$ . (It may be noted that here  $\cos \theta = \cos 140^\circ = \cos (180 - 40^\circ) = -\cos 40^\circ$ , therefore,  $h$  is negative indicating capillary depression).

Following points are worth noting:

- (i) Smaller the diameter of the capillary tube, greater is the capillary rise or depression.
- (ii) The measurement of liquid level in laboratory capillary (glass) tubes should not be smaller than 8 mm.
- (iii) Capillary effects are negligible for tubes longer than 12 mm.
- (iv) For *wetting liquid* (water):  $\theta < \pi/2$ . For water:  $\theta = 0$  when pure water is in contact with clean glass. But  $\theta$  becomes as high as  $25^\circ$  when water is slightly contaminated.

For *non-wetting liquid* (mercury):  $\theta > \pi/2$ . (For mercury:  $\theta$  varies between  $130^\circ$  to  $150^\circ$ )

Refer to Fig. 1.24 which illustrates the liquid gas interface with a solid surface.

- (v) The effects of surface tension are negligible in many flow problems *except those involving*:
  - capillary rise;
  - formation of drops and bubbles;
  - the break up of liquid jets, and
  - hydraulic model studies where the model or flow depth is small.

**Capillary inversion.** Due to surface tension the liquid passing out of an elliptical orifice tends

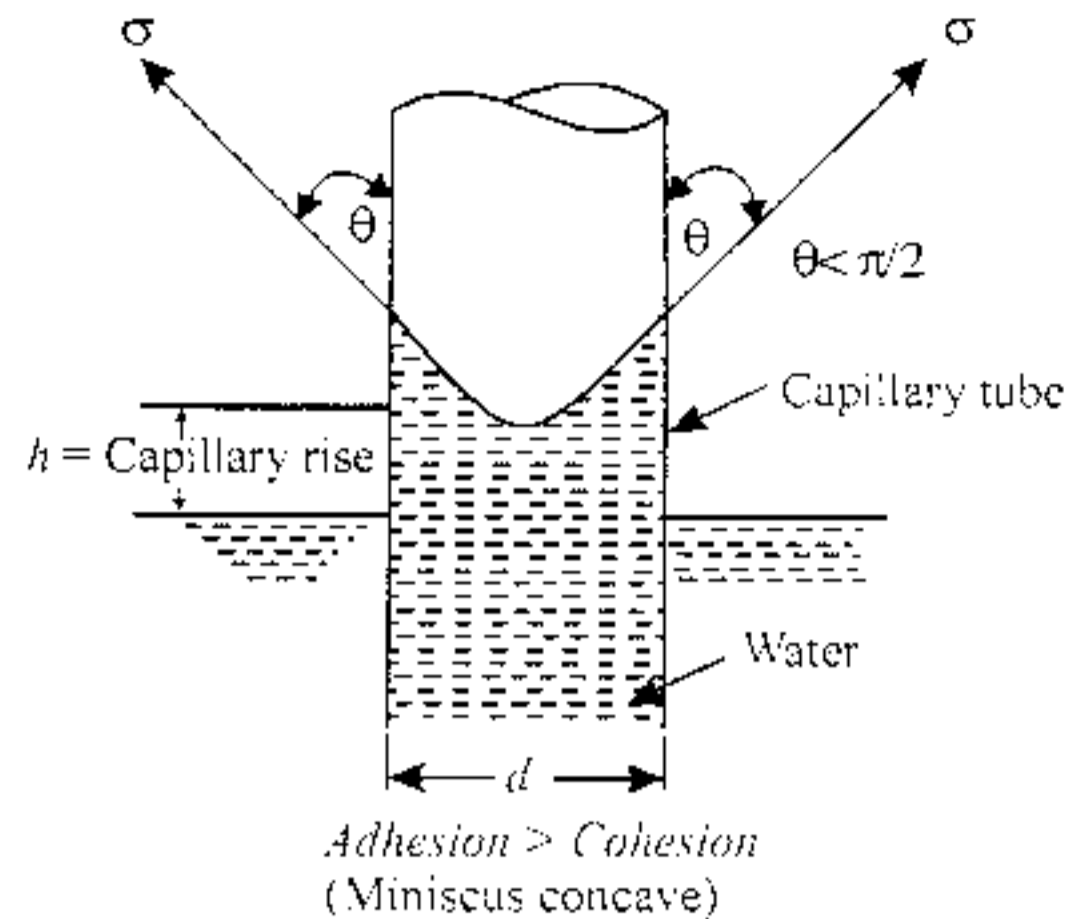


Fig. 1.22. Effect of capillarity.

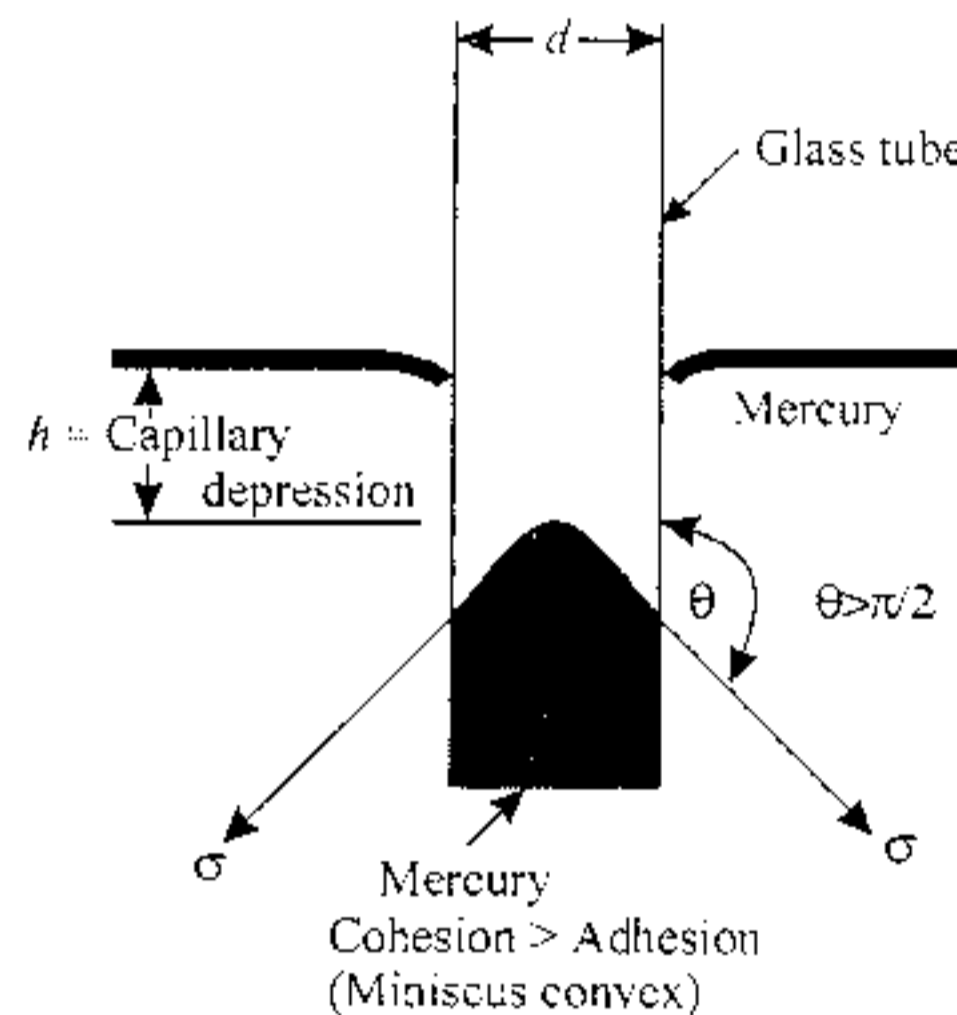


Fig. 1.23

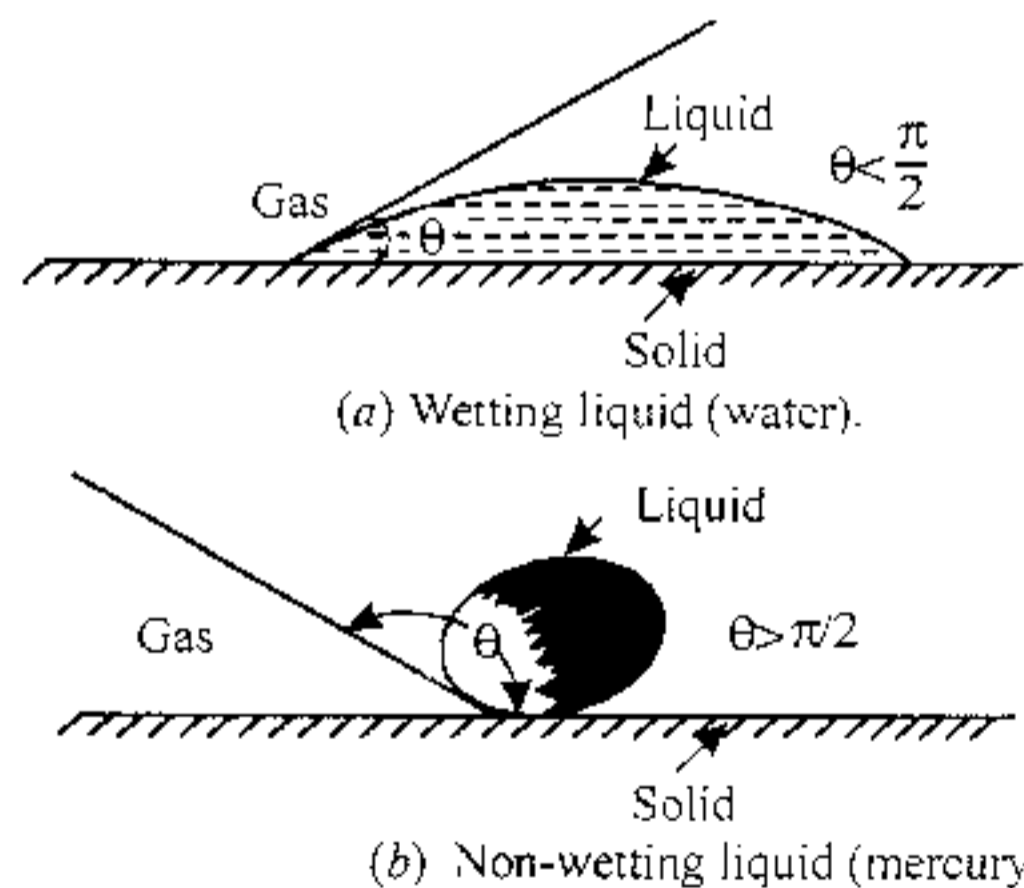


Fig. 1.24

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- molten
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**Example 1**  
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**Example 1**  
the surface tensi  
**Solution. C**  
The soap bu

to assume a circular or minimum perimeter cross-section. Here transformation of surface energy into kinetic energy takes place; the flow pattern varies as the Weber number changes and the motion continues giving rise to a series of standing waves. This phenomenon is known as *capillary inversion* of jet for orifices of *non-circular* cross-section. As shown in the Fig. 1.25 the jet issuing from a small elliptical orifice can be observed to undergo two inversion cycles in a given length.

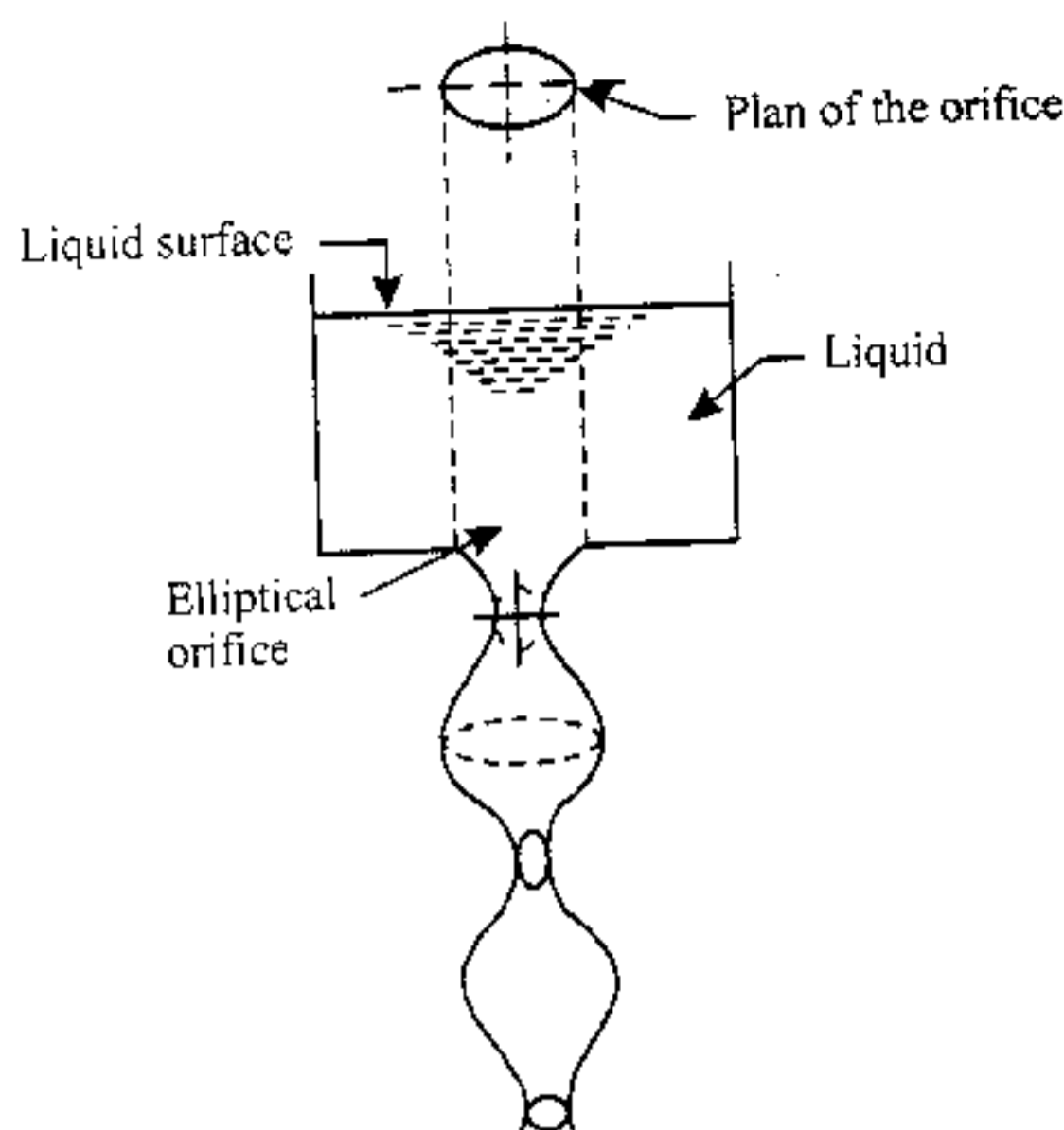


Fig. 1.25. Capillary inversion of a liquid jet.

The phenomenon of capillary inversion of jets is significant for industries involving the production and size control of liquid droplets like:

- paint,
- molten shot, and
- agricultural insecticides etc.

**Example 1.27.** A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. The angle of contact of the liquid with the glass can be assumed to be 135°. The density of the liquid = 13600 kg/m<sup>3</sup>.

What would be the level of the liquid in the tube relative to the free surface of the liquid inside the tube.

**Solution.** Given:  $d = 2.5 \text{ mm}$ ;  $\sigma = 4 \text{ N/m}$ ,  $\theta = 135^\circ$ ;  $\rho = 13600 \text{ kg/m}^3$

**Level of the liquid in the tube, h:**

The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall),

$$\begin{aligned}
 h &= \frac{4\sigma \cos \theta}{wd} && \dots(\text{Eqn. 1.20}) \\
 &= \frac{4 \times 0.4 \times \cos 135^\circ}{(9.81 \times 13600) \times 2.5 \times 10^{-3}} && (\because w = \rho g) \\
 &= -3.39 \times 10^{-3} \text{ m or } -3.39 \text{ mm}
 \end{aligned}$$

Negative sign indicates that there is a capillary depression (fall) of 3.39 mm. (Ans.)

**Example 1.28.** Assuming that the interstices in a clay are of size equal to one tenth the mean diameter of the grain, estimate the height to which water will rise in a clay soil of average grain diameter of 0.048 mm. Assume surface tension at air-water interface as 0.074 N/m.

**Solution.** Given: Diameter of the pores,  $d = \frac{1}{10} \times 0.048 = 0.0048 \text{ mm}$ ;  $\sigma = 0.074 \text{ N/m}$

Assuming  $\theta = 0^\circ$

$$h = \frac{4\sigma}{wd} = \frac{4 \times 0.074}{(9.81 \times 1000) \times 0.0048 \times 10^{-3}} = 6.286 \text{ m (Ans.)}$$

**Example 1.29.** Calculate the work done in blowing a soap bubble of diameter 100 mm. Assume the surface tension of soap solution = 0.038 N/m.

**Solution.** Given:  $d = 100 \text{ mm or } 0.1 \text{ m}$ ;  $\sigma = 0.038 \text{ N/m}$ .

The soap bubble has two interfaces.

$\therefore$  Work done = Surface tension  $\times$  total surface area

$$= 0.038 \times 4\pi \times \left(\frac{0.1}{2}\right)^2 \times 2$$

$$= 0.002388 \text{ Nm (Ans.)}$$

**Example 1.30.** Determine the minimum size of glass tubing that can be used to measure water level, if the capillary rise in the tube is not to exceed 0.3 mm. Take surface tension of water in contact with air as 0.0735 N/m.

**Solution.** Given : Capillary rise,  $h = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$

Surface tension,  $\sigma = 0.0735 \text{ N/m}$

Specific weight of water,  $w = 9810 \text{ N/m}^3$ .

Size of glass tubing,  $d$ :

$$\text{Capillary rise, } h = \frac{4\sigma \cos \theta}{wd} = \frac{4\sigma}{wd}$$

(Assuming  $\theta = 0$  for water)

$$0.3 \times 10^{-3} = \frac{4 \times 0.0735}{9810 \times d}$$

$$\therefore d = \frac{4 \times 0.0735}{0.3 \times 10^{-3} \times 9810} = 0.1 \text{ m} = 100 \text{ mm (Ans.)}$$

**Example 1.31.** A U-tube is made up of two capillaries of bores 1.2 mm and 2.4 mm respectively. The tube is held vertical and partially filled with liquid of surface tension 0.06 N/m and zero contact angle. If the estimated difference in the level of two menisci is 15 mm, determine the mass density of the liquid.

**Solution.** Given: Bores of the capillaries:

$$d_1 = 1.2 \text{ mm} = 0.0012 \text{ m}$$

$$d_2 = 2.4 \text{ mm} = 0.0024 \text{ m}$$

Difference of level,  $h_1 - h_2 = 15 \text{ mm} = 0.015 \text{ m}$ ; Angle of contact,  $\theta = 0$

Mass density of the liquid,  $\rho$ :

$$h_1 = \frac{4\sigma \cos \theta}{wd_1}, \quad \text{and} \quad h_2 = \frac{4\sigma \cos \theta}{wd_2}$$

[where  $w (= \rho g) =$  weight density of the liquid]

$$\therefore h_1 - h_2 = \frac{4\sigma}{w} \left[ \frac{1}{d_1} - \frac{1}{d_2} \right] \quad (\because \theta = 0)$$

$$0.015 = \frac{4 \times 0.06}{\rho \times 9.81} \left[ \frac{1}{0.0012} - \frac{1}{0.0024} \right] = \frac{0.02446}{\rho} \times 416.67$$

$$\therefore \rho = \frac{0.02446 \times 416.67}{0.015} = 679.45 \text{ kg/m}^3 \text{ (Ans.)}$$

**Example 1.32.** Derive an expression for the capillary rise at a liquid having surface tension  $\sigma$  and contact angle  $\theta$  between two vertical parallel plates at a distance  $W$  apart. If the plates are of glass, what will be the capillary rise of water having  $\sigma = 0.073 \text{ N/m}$ ,  $\theta = 0^\circ$ ? Take  $W = 1 \text{ mm}$ .  
(AMIE Summer, 2001)

**Solution.** Refer Fig. 1.26. Consider two vertical parallel plates immersed in a liquid whose weight density is  $w$ .

## Properties of Fluids

Given :  $\sigma$  = surface tension;

$\theta$  = contact angle.

Let  $h$  = height of liquid between plates above general liquid surface.

Under a state of equilibrium, the weight of liquid of height  $h$  is balanced by the force at the surface of liquid between the plates.

Then weight of liquid of height  $h$  is balanced by the force between the plates

$$= \text{volume of liquid of height } h \text{ between the plates} \times w$$

$$= W \times L \times h \times w \quad \dots(1)$$

where,  $L$  = length of plate, and  $w$  = weight density of the liquid.

Vertical component of surface tensile force =  $(\sigma \times \text{circumference}) \times \cos \theta$ .

$$= \sigma \times 2L \times \cos \theta \quad \dots(2)$$

For equilibrium, eqns. (1) and (2) must balance.

$$\therefore W \times L \times h \times w = \sigma \times 2L \times \cos \theta \quad \dots(3)$$

$$\text{or } h = \frac{2\sigma \cos \theta}{W \times w}$$

Eqn. (3) is the expression for capillary rise. (Ans.)

When plates are of glass,

$$\theta = 0^\circ, \sigma = 0.073 \text{ N/m}$$

$$W = 1 \text{ mm} = 0.001 \text{ m}, w = 9810 \text{ N/m}^3$$

$$\text{Capillary rise of water, } h = \frac{2\sigma \cos \theta}{W \times w}$$

$$= \frac{2 \times 0.073 \times \cos 0^\circ}{0.001 \times 9810} = 0.0149 \text{ m or } 14.9 \text{ mm}$$

Hence capillary rise = 14.9 mm (Ans.)

**Example 1.33.** A single column U-tube manometer, made of glass tubing having a nominal inside diameter of 2.4 mm, has been used to measure pressure in a pipe or vessel containing air. If the limb opened to atmosphere is 10 percent oversize, find the error in mm of mercury in the measurement of air pressure due to surface tension effects. It is stated that mercury is the manometric fluid for which surface tension  $\sigma = 0.52 \text{ N/m}$  and angle of contact  $\alpha = 140^\circ$ .

**Solution.** Given:  $d_1 = 2.4 \text{ mm}$ ;  $d_2 = 2.4 \times 1.1 = 2.64 \text{ mm}$ ;  $\sigma = 0.52 \text{ N/m}$ ;  $\alpha = 140^\circ$ .

**Error in measurement due to surface tension effects:**

The surface tension manifests the phenomenon of capillary action due to which rise or depression of manometric liquid in a tube is given by

$$h = \frac{4\sigma \cos \theta}{w d_1}$$

$$\text{Now, } h_1 = \frac{4 \times 0.52 \times \cos 140^\circ}{(13.6 \times 9810) \times (2.4 \times 10^{-3})} = -4.97 \times 10^{-3} \text{ m}$$

(Negative sign indicates capillary depression)

$$h_2 = \frac{4 \times 0.52 \times \cos 140^\circ}{(13.6 \times 9810) \times (2.64 \times 10^{-3})} = -4.52 \times 10^{-3} \text{ m}$$

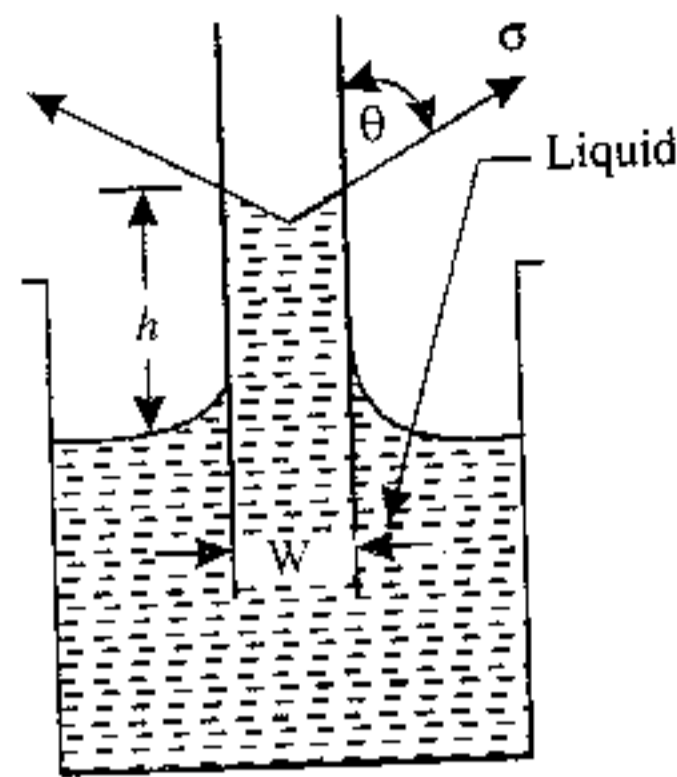


Fig. 1.26

Hence, error in measurement due to surface tension effects

$$= (4.97 - 4.52) \times 10^{-3} = 0.45 \times 10^{-3} \text{ m} = \mathbf{0.45 \text{ mm (Ans.)}}$$

**Example 1.34.** Calculate the capillary effect in millimetres in a glass tube of 4 mm diameter, when immersed in (i) water and (ii) mercury. The temperature of the liquid is 20°C and the values of surface tension of water and mercury at 20°C in contact with air are 0.0735 N/m and 0.51 N/m respectively. The contact angle for water  $\theta = 0^\circ$  and for mercury  $\theta = 130^\circ$ . Take specific weight of water at 20°C as equal to 9790 N/m<sup>3</sup>. [Engg. Services]

**Solution.** Given: Diameter of glass tube,  $d = 4 \text{ mm} = 0.004 \text{ m}$

Surface tension at 20°C,  $\sigma$ :

$$\sigma_{\text{water}} = 0.0735 \text{ N/m}, \quad \sigma_{\text{mercury}} = 0.051 \text{ N/m}$$

Specific weight of water at 20°C = 9790 N/m<sup>3</sup>

The rise or depression  $h$  of a liquid in a capillary tube is given by

$$h = \frac{4\sigma \cos \theta}{w d}$$

where,  $\sigma$  = surface tension,  $\theta$  = angle of contact, and  $w$  = specific weight.

(i) Capillary effect for water:

$$h = \frac{4 \times 0.0735 \times \cos 0^\circ}{9790 \times 0.004} \quad (\because \theta_{\text{water}} = 0^\circ \dots \text{given})$$

$$= 7.51 \times 10^{-3} \text{ m} = \mathbf{7.51 \text{ mm (rise) (Ans.)}}$$

(ii) Capillary effect for mercury:

$$h = \frac{4 \times 0.051 \times \cos 130^\circ}{(13.6 \times 9790) \times 0.004} \quad (\because \theta_{\text{mercury}} = 130^\circ \dots \text{given})$$

$$\text{or} \quad = -2.46 \times 10^{-3} \text{ m} = -2.46 \text{ mm}$$

$$\text{i.e.,} \quad h = \mathbf{2.46 \text{ mm (depression) (Ans.)}}$$

**Example 1.35.** In measuring the unit energy of a mineral oil (specific gravity = 0.85) by the bubble method, a tube having an internal diameter of 1.5 mm is immersed to depth of 12.5 mm in oil. Air is forced through the tube forming a bubble at the lower end. What magnitude of the unit surface energy will be indicated by a maximum bubble pressure intensity of 150 N/m<sup>2</sup>. [Engg. Services]

**Solution.** Sp. gravity of oil = 0.85

Internal diameter of the tube,

$$d = 1.5 \text{ mm} = 0.0015 \text{ m}$$

Depth,  $h = 12.5 \text{ mm} = 0.0125 \text{ m}$

Gauge pressure inside the bubble

$$p_i = 150 \text{ N/m}^2$$

Unit surface energy,  $\sigma$ :

Gauge pressure outside the bubble,

$$p_o = wh = (0.85 \times 9810) \times 0.0125 = 104.23 \text{ N/m}^2$$

$\therefore$  Net pressure attributable to surface tension

$$p = p_i - p_o = 150 - 104.23 = 45.77 \text{ N/m}^2$$

$$\text{Also, } p_i - p_o = \frac{4\sigma}{d}$$

Assuming diameter of bubble equal to that of the tube,

$$45.77 = \frac{4\sigma}{0.0015}$$

$$\therefore \sigma = \frac{45.77 \times 0.0015}{4} = \mathbf{0.0172 \text{ N/m (Ans.)}}$$

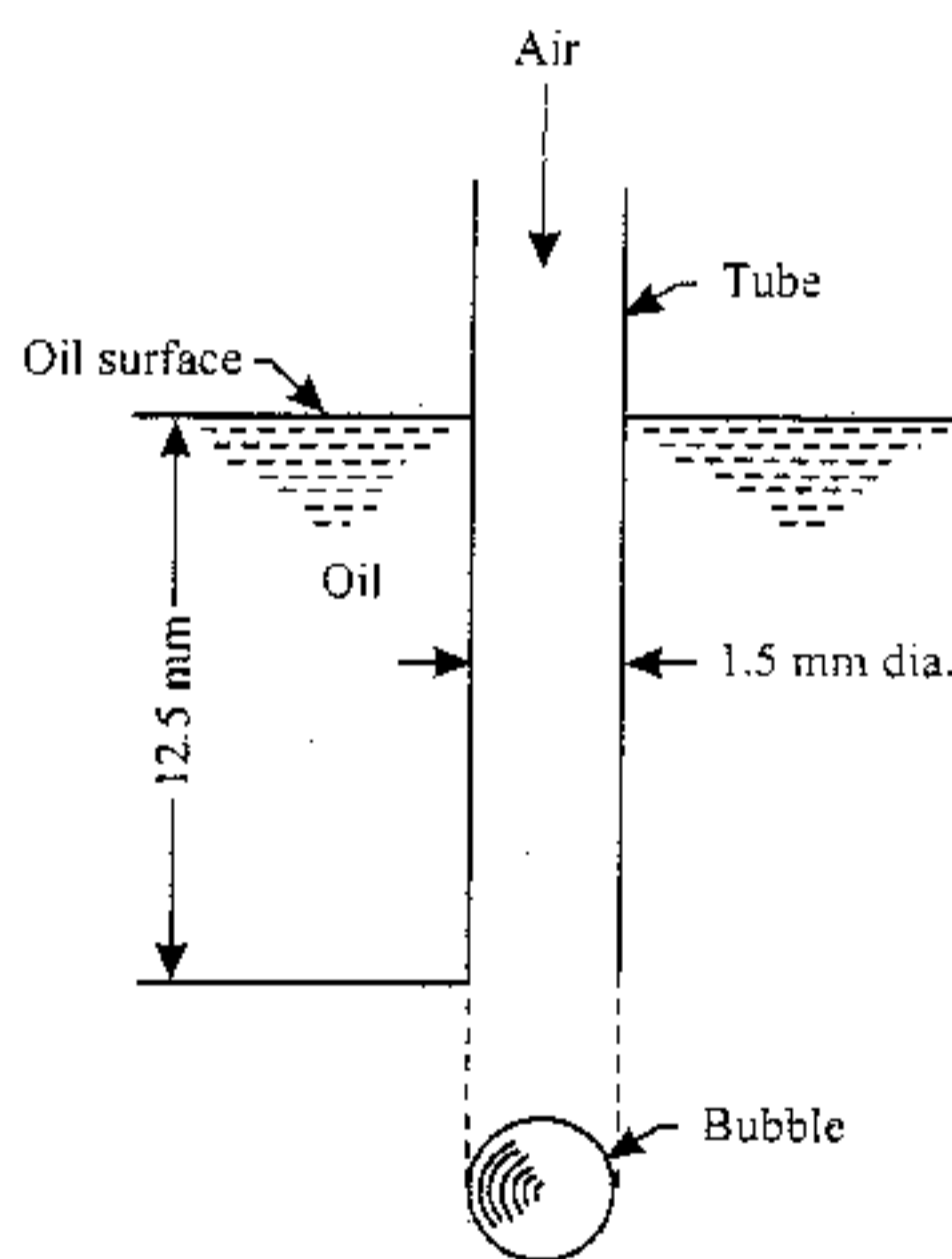


Fig. 1.27

**Example 1.36.** Two coaxial glass tubes forming an annulus with small gap are immersed in water in a trough. The inner and outer radii of the annulus are  $r_i$  and  $r_o$  respectively. What is the capillary rise if  $\sigma$  is the surface tension of water in contact with air? (AMIE Summer, 2000)

**Solution.** Refer Fig. 1.28. If the angle of contact between the liquid and the curved tube surface is  $\theta$ , the water in the annulus will continue to rise until the vertical component of the surface tension force which acts over the wetted length (outer curve of the inner tube and inner curve of the outer tube) equals the height of the water column, or

$T \cos \theta = \pi (r_o^2 - r_i^2) h \rho g$ , where  $T = \sigma \pi (r_o + r_i)$ ; substituting for T, we get

$$\sigma \pi (r_o + r_i) \cos \theta = \pi (r_o^2 - r_i^2) h \rho g$$

or  $h = \text{capillary rise} = \frac{\sigma \cos \theta}{(r_o - r_i) \rho g}$

For pure water and clean glass  $\theta \approx 0$  and  $h = \frac{\sigma}{(r_o - r_i) \rho g}$

Under actual conditions, neither water is pure, nor glass is clean. Gibson has obtained the value of  $\theta$  as  $25^\circ 32'$ .

Thus  $h = \frac{\sigma \cos 25^\circ 32'}{(r_o - r_i) \rho g} = \frac{0.902 \sigma}{(r_o - r_i) \rho g}$  (Ans.)

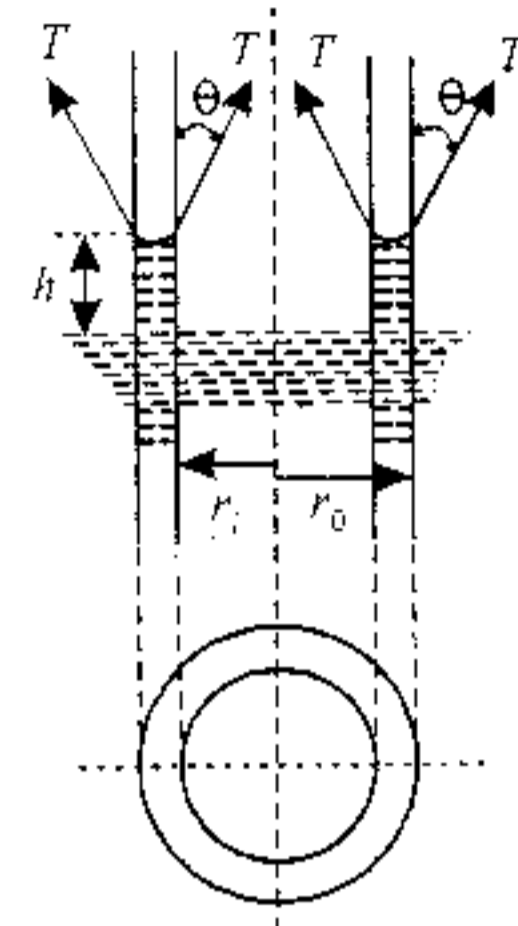


Fig. 1.28

**1.9. Compressibility and Bulk Modulus**

The property by virtue of which fluids undergo a change in volume under the action of external pressure is known as **compressibility**. It decreases with the increases in pressure of fluid as the volume modulus increases with the increase of pressure.

The variation in volume of water, with variation of pressure, is so small that for all practical purposes it is neglected. Thus, the water is considered to be an incompressible liquid. However in case of water flowing through pipes when sudden or large changes in pressure (e.g. water hammer) take place, the compressibility cannot be neglected. The compressibility in Fluid Mechanics is considered mainly when the velocity of flow is high enough reaching 20 percent of speed of sound in the medium.

Elasticity of fluids is measured in terms of **bulk modulus of elasticity (K)** which is defined as the ratio of compressive stress to volumetric strain. Compressibility is the reciprocal of bulk modulus of elasticity.

Consider a cylinder fitted with a piston as shown in Fig. 1.29

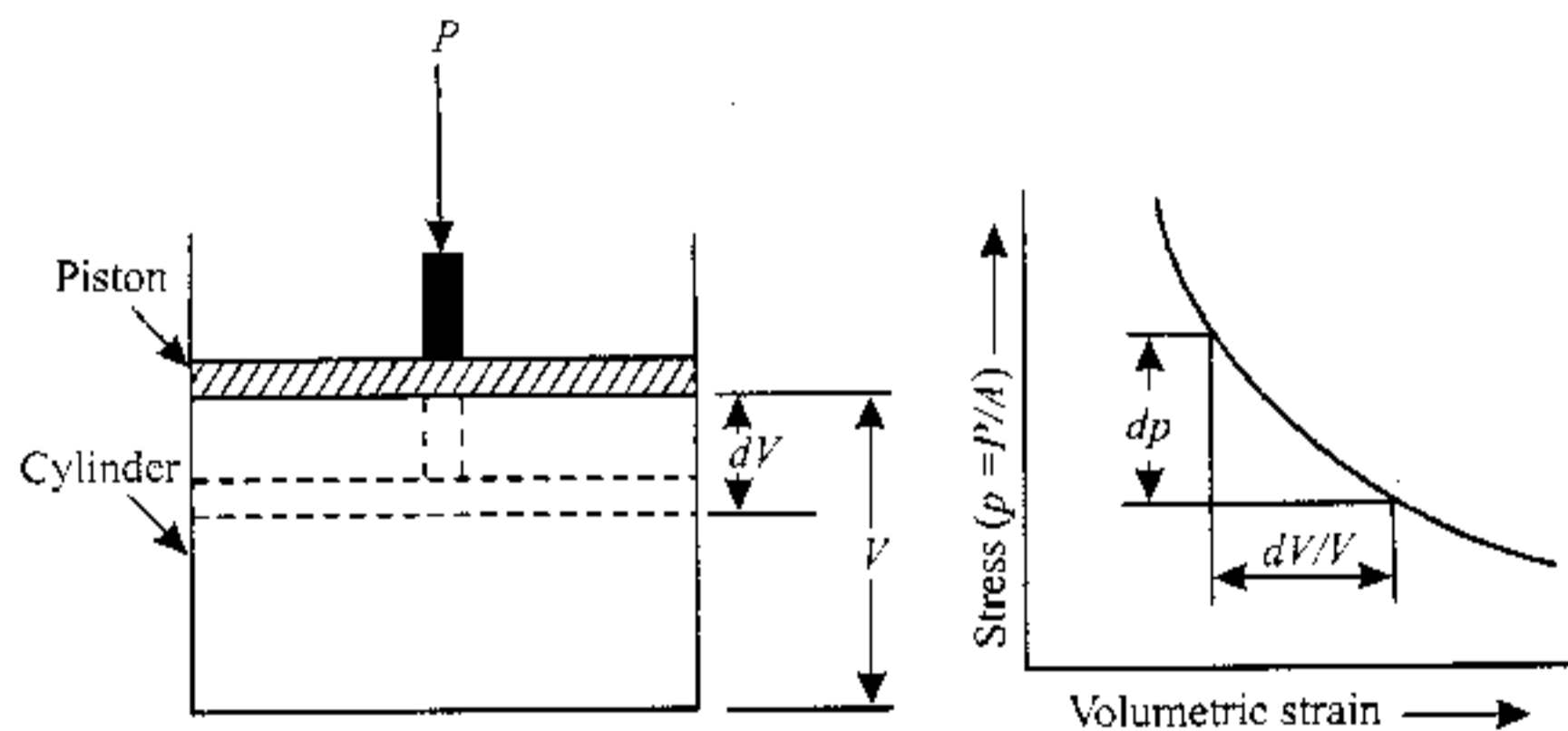


Fig. 1.29

Let,  $V$  = Volume of gas enclosed in the cylinder, and

$p$  = Pressure of gas when volume is  $V$

$$= \frac{P}{A}, \text{ where } A \text{ is the area of cross-section of the cylinder.}$$

Let the pressure is increased to  $p + dp$ , the volume of gas decreases from  $V$  to  $V - dV$ .

Then increase in pressure =  $dp$ ;      Decrease in volume =  $dV$

$$\therefore \text{Volumetric strain} = - \frac{dV}{V}$$

(Negative sign indicates *decrease in volume with increase of pressure*)

$$\therefore \text{Bulk modulus, } K = \frac{dp(\text{increase of pressure})}{-dV/V(\text{volumetric strain})}$$

$$\text{i.e., } K = \frac{dp}{-dV/V}$$

$$\left( \text{Compressibility} = \frac{1}{K} \right) \quad \dots(1.22)$$

Steepening of the curve (Fig. 1.29) with increasing pressure shows that as fluids are compressed it becomes increasingly difficult to compress them further. In other words, the *value of  $K$  increases with increase of pressure*.

The following points are worth noting:

1. The bulk modulus of elasticity ( $K$ ) of a fluid is not constant, but it increases with increase in pressure. This is so because when a fluid mass is compressed its molecules become close together and its resistance to further compression increases *i.e.*,  $K$  increases. (*e.g.* the value of  $K$  roughly doubles as the pressure is raised from 1 atmosphere to 3500 atmosphere).
2. The bulk modulus of elasticity ( $K$ ) of the fluid is affected by the temperature of the fluid. In the case of *liquids* there is a *decrease* of  $K$  with *increase of temperature*. However, for gases since pressure and temperature are inter-related and as temperature increases, pressure also increases, an *increase in temperature* results in an *increase* in the value of  $K$ .
3. At NTP (normal temperature and pressure):

$$K_{\text{water}} = 2.07 \times 10^6 \text{ kN/m}^2, \quad K_{\text{air}} = 101.3 \text{ kN/m}^2$$

**Example 1.37.** When the pressure of liquid is increased from  $3.5 \text{ MN/m}^2$  to  $6.5 \text{ MN/m}^2$  its volume is found to decrease by 0.08 percent. What is the bulk modulus of elasticity of the liquid?

**Solution.** Initial pressure =  $3.5 \text{ MN/m}^2$

Final pressure =  $6.5 \text{ MN/m}^2$

$\therefore$  Increase in pressure,  $dp = 6.5 - 3.5 = 3.0 \text{ MN/m}^2$

Decrease in volume = 0.08 percent  $\therefore - \frac{dV}{V} = \frac{0.08}{100}$

Bulk modulus ( $K$ ) is given by:

$$K = \frac{dp}{-\frac{dV}{V}} = \frac{3 \times 10^6}{\frac{0.08}{100}} = 3.75 \times 10^9 \text{ N/m}^2 \text{ or } 3.75 \text{ GN/m}^2$$

Hence  $K = 3.75 \text{ GN/m}^2$  (Ans.)

**Example 1.38.** When a pressure of  $20.7 \text{ MN/m}^2$  is applied to 100 litres of a liquid its volume decreases by 1 litre. Find the bulk modulus of the liquid and identify this liquid.

**Solution.** Net pressure applied,  $dp = 20.7 \text{ MN/m}^2$

Decrease in volume,  $dV = 1 \text{ litre}$

Initial volume,  $V = 100 \text{ litres} \therefore -\frac{dV}{V} = \frac{1}{100}$

**Bulk modulus K:**

$$K = -\frac{dp}{-dV/V} = \frac{20.7 \times 10^6}{1/100} = 20.7 \times 10^8 \text{ N/m}^2 = 2.07 \text{ GN/m}^2$$

i.e.,  $K = 2.07 \text{ GN/m}^2 \text{ (Ans.)}$

Evidently the liquid is **water (Ans.)**.

**Example 1.39.** Define compressibility of a fluid. Gas A at 125 kPa (abs.) is compressed isothermally and gas B at 100 kPa (abs.) is compressed isentropically ( $\gamma = 1.4$ ). Which gas is more compressible? **(AMIE Summer, 1999)**

**Solution.** Compressibility is the measure of relative change of volume (For density) when the fluid is subjected to a pressure change. It is the reciprocal of the bulks modulus of elasticity (K).

It is expressed mathetically as:

$$Z = \frac{1}{K} = \frac{-(dV/V)}{dp}$$

For an ideal gas, if the compression is isothermal,  $Z = \frac{1}{p}$ , and if the compression is isentropic,  $Z = \frac{1}{\gamma p}$ .

For the given gas A,

$$Z_A = \frac{1}{p} = \frac{1}{125} = 0.008 \text{ m}^2/\text{kN}$$

For the gas B,

$$Z_B = \frac{1}{\gamma p} = \frac{1}{1.4 \times 100} = 0.007143 \text{ m}^2/\text{kN}$$

Hence gas A more compressible **(Ans.)**.

**Example 1.40.** Find an expression for isothermal bulk modulus of elasticity for a gas which obeys Van der Waals' law of state according to the equation:

$$p = pRT \left[ \frac{1}{1 - b\rho} - \frac{a\rho}{RT} \right]$$

where a, b are constants and p, ρ, R and T have their usual meanings.

**(AMIE Summer, 2000)**

**Solution.** Bulk modulus of elasticity,

$K = -\frac{dp}{\left(\frac{dV}{V}\right)} = -V \frac{dp}{dV}$ , where V is volume =  $-v \frac{dp}{dv}$ , v is specific volume.

Since,  $v = \frac{1}{\rho}$  or  $\rho v = 1$ ,  $\frac{dv}{v} = -\frac{d\rho}{\rho}$

$\therefore K = \rho \frac{dp}{d\rho}$ ;  $p = \rho RT \left[ \frac{1}{1 - b\rho} - \frac{a\rho}{RT} \right]$  ... (Given)

$\therefore \frac{dp}{d\rho} = RT \left[ \frac{1}{1 - b\rho} - \frac{a\rho}{RT} \right] + \rho RT \left[ \frac{b}{(1 - b\rho)^2} - \frac{a}{RT} \right] = \frac{RT}{1 - b\rho} - a\rho + \frac{b\rho RT}{(1 - b\rho)^2} - a\rho$

$$= \frac{RT}{1 - b\rho} \left[ 1 + \frac{b\rho}{1 - b\rho} \right] - 2a\rho = \frac{RT}{(1 - b\rho)^2} - 2a\rho$$

and, 
$$K = \rho \frac{dp}{d\rho} = \frac{\rho RT}{(1 - b\rho)^2} - 2a\rho^2 \quad \dots \text{Required expression (Ans.)}$$

### 1.10. Vapour Pressure

All liquids have a tendency to evaporate or vaporize (*i.e.*, to change from the liquid to the gaseous state). Molecules are continuously projected from the free surface to the atmosphere. These ejected molecules are in a gaseous state and exert their own partial vapour pressure on the liquid surface. This pressure is known as the vapour pressure of the liquid ( $p_v$ ). If the surface above the liquid is confined, the partial vapour pressure exerted by the molecules increases till the rate at which the molecules re-enter the liquid is equal to the rate at which they leave the surface. When the equilibrium condition is reached, the vapour pressure is called saturation vapour pressure ( $p_{vs}$ ).

The following points are worth noting:

1. If the pressure on the liquid surface is *lower than or equal to the saturation vapour pressure, boiling takes place.*
2. *Vapour pressure increases with the rise in temperature.*
3. *Mercury has a very low vapour pressure and hence, it is an excellent fluid to be used in a barometer.*

Table 1.1. Summary of Fluid Characteristics

Sr. No.	Characteristics	Symbol	Definition	Dimensions	Units
1.	Mass density	$\rho$	Mass per unit volume, $\frac{m}{V}$	$ML^{-3}$	kg/m <sup>3</sup>
2.	Weight density (or specific weight)	$w$	Weight per unit volume, $\frac{W}{V}$	$FL^{-3}$	N/m <sup>3</sup>
3.	Specific volume	$v$	Volume per unit mass $\frac{V}{m} = \frac{1}{\rho}$	$L^3M^{-1}$	m <sup>3</sup> /kg
4.	Specific gravity	$S$	$\frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}}$ $= \frac{W_{\text{liquid}}}{W_{\text{water}}}$		
5.	Dynamic viscosity	$\mu$	Newton's law: $\tau = \mu \cdot \frac{du}{dy}$	$FTL^{-2}$	N.s/m <sup>2</sup> poise, centipoise
6.	Kinematic viscosity	$\nu$	$\nu = \frac{\mu}{\rho}$	$L^2T^{-1}$	m <sup>2</sup> /s stoke, centistoke
7.	Bulk modulus	$K$	$K = -\frac{\Delta p}{dV/V}$	$FL^{-2}$	N/m <sup>2</sup>
8.	Surface tension	$\sigma$	Force per unit length	$FL^{-1}$	N/m
9.	Vapour pressure	$p$	$p_v = \frac{F}{A}$	$FL^{-2}$	N/m <sup>2</sup>

Table 1.2. Properties of Some Common Fluids at 20°C and Atmospheric Pressure

Fluid	Mass density $\rho$ (kg/m <sup>3</sup> )	Specific weight $w$ (kN/m <sup>3</sup> )	Dynamic Viscosity		Kinematic Viscosity		Modulus of elasticity $E$ (N/m <sup>2</sup> )	Surface tension in contact with air, $\sigma$ (N/m)	Vapour pressure (N/m <sup>2</sup> )
			Poise	$\mu$ kg/ms	Stoke	$\nu$ m <sup>2</sup> /s			
Air	1.208	0.01185	$1.85 \times 10^{-4}$	$1.85 \times 10^{-5}$	$1.53 \times 10^{-1}$	$1.53 \times 10^{-5}$	—	—	—
Benzene	860	8.434	0.007	$7.00 \times 10^{-4}$	$8.14 \times 10^{-3}$	$8.14 \times 10^{-7}$	$1.0356 \times 10^9$	0.0255	$1.000 \times 10^4$
Castor oil	960	9.414	9.800	$9.80 \times 10^{-1}$	$1.00 \times 10^1$	$1.00 \times 10^3$	$1.441 \times 10^9$	0.0392	—
Carbon tetrachloride	1594	15.632	0.010	$1.00 \times 10^{-3}$	$6.04 \times 10^{-3}$	$6.04 \times 10^{-7}$	$1.104 \times 10^9$	0.0265	$1.275 \times 10^4$
Ethyl alcohol	789	7.737	0.012	$1.20 \times 10^{-3}$	$1.52 \times 10^{-2}$	$1.52 \times 10^{-6}$	$1.118 \times 10^9$	0.0216	$5.786 \times 10^3$
Glycerine	1260	12.356	8.350	$8.35 \times 10^{-1}$	6.63	$6.63 \times 10^{-4}$	$4.354 \times 10^9$	0.0637	$1.373 \times 10^2$
Kerosene	800	7.845	0.020	$2.00 \times 10^{-3}$	$2.50 \times 10^{-2}$	$2.50 \times 10^{-6}$	—	0.0235	—
Mercury	13550	132.880	0.016	$1.60 \times 10^{-3}$	$1.18 \times 10^{-3}$	$1.18 \times 10^{-7}$	$2.431 \times 10^{10}$	0.510	$1.726 \times 10^{-1}$

### HIGHLIGHTS

1. *Hydraulics* is that branch of Engineering science, which deals with water at rest or in motion.
2. *Fluid mechanics* may be defined as that branch of Engineering science which deals with the behaviour of fluid under the conditions of rest and motion.
3. A *fluid* is substance which is capable of flowing.
4. *Mass density* is the mass per unit volume whereas weight density (or specific weight) is the weight per unit volume at the standard temperature and pressure.
5. *Specific gravity* is the ratio of the specific weight of the liquid to the specific weight of a standard fluid. It is dimensionless and has no units.
6. *Viscosity* is the property of a fluid which determines its resistance to shearing stresses. *Newton's law of viscosity* states that the shear stress ( $\tau$ ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity.

$$\text{Mathematically, } \tau = \mu \cdot \frac{du}{dy},$$

where  $\mu$  = co-efficient of dynamic viscosity, and  $\frac{du}{dy}$  = rate of shear deformation or velocity gradient.

*Kinematic viscosity* is the ratio between the dynamic viscosity and density of fluid. It is denoted by  $\nu$  (nu).

$$\text{i.e., } \nu = \frac{\mu}{\rho}$$

7. *Cohesion and adhesion:*

*Cohesion* means intermolecular attraction between molecules of the same liquid.

*Adhesion* means attraction between molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid.

8. *Surface tension* ( $\rho$ ) is caused by the force of cohesion at the free surface. It is usually expressed in N/m.

Pressure inside:

$$(a) \text{ Water droplet : } p = \frac{4\sigma}{d}, \quad (b) \text{ Soap bubble : } p = \frac{8\sigma}{d}, \text{ and}$$

$$(c) \text{ Liquid jet : } p = \frac{2\sigma}{d} \quad (\text{where } d \text{ stands for diameter}).$$

9. *Capillarity* is a phenomenon by which a liquid (depending upon its specific gravity) rises into a thin glass tube or below its general level.

$$h = \frac{4\sigma \cos \theta}{wd}$$

where,  $h$  = Height of capillary rise,

$d$  = Diameter of the capillary tube,

$\theta$  = Angle of contact of the water surface,

$\sigma$  = Surface tension per unit length, and

$w$  = Weight density ( $\rho g$ ).

Choose

1. The

w

(a)

(c)

2. A

su

(a)

(c)

3. —

co

(a)

(c)

4. A

fol

(a)

(c)

5. The

spe

(a)

(c)

6. The

res

(a)

(c)

7. New

rela

(a)

(c)

8. Fluid

betw

term

(a)

(c)

9. The p

10. *Compressibility* is the property by virtue of which fluid undergoes a change in volume under the action of external pressure. It is the *reciprocal* of bulk modulus of elasticity ( $K$ ).

$$K = dp \text{ (increase of pressure)} / -\frac{dV}{V} \text{ (volumetric strain)}$$

$$\left( \text{Compressibility} = \frac{1}{K} \right)$$

### OBJECTIVE TYPE QUESTIONS

#### Choose the Correct Answer:

1. The branch of Engineering-science, which deals with water at rest or in motion is called  
(a) hydraulics (b) fluid mechanics  
(c) applied mechanics (d) kinematics.
2. A solid can resist which of the following stresses?  
(a) Tensile (b) Compressive  
(c) Shear (d) All of the above.
3. .... possesses no definite volume and is compressible.  
(a) Solid (b) Liquid  
(c) Gas (d) Vapour.
4. A real practical fluid possesses which of the following?  
(a) Viscosity (b) Surface tension  
(c) Compressibility (d) density.
5. The ratio of the specific weight of the liquid to the specific weight of a standard fluid is known as  
(a) specific volume (b) weight density  
(c) specific gravity (d) viscosity.
6. The property of a fluid which determines its resistance to shearing stress is called  
(a) viscosity (b) surface tension  
(c) compressibility (d) none of the above.
7. Newton's law of viscosity is given by the relation:  
(a)  $\tau = \mu^2 \frac{du}{dy}$  (b)  $\tau = \sqrt{\mu} \frac{du}{dy}$   
(c)  $\tau = \mu \cdot \frac{du}{dy}$  (d)  $\tau = (\mu)^{3/2} \frac{du}{dy}$
8. Fluids which do not follow the linear relationship between shear stress and rate of deformation are termed as .... fluids.  
(a) Newtonian (b) Non-Newtonian  
(c) dilatent (d) ideal
9. The printer's ink is an example of  
(a) Newtonian fluid (b) Non-Newtonian  
(c) Thixotropic substance  
(d) Elastic solid.
10. The viscosity of liquids .... with increase in temperature.  
(a) decreases (b) increases  
(c) first decreases and then increases  
(d) first increases and then decreases.
11. Surface tension is caused by the force of .... at the free surface.  
(a) cohesion (b) adhesion  
(c) both (a) and (b) (d) none of the above.
12. Which of the following is an example of phenomenon of surface tension?  
(a) Rain drops  
(b) Rise of sap in a tree  
(c) Break up of liquid jets  
(d) All of the above.
13. Surface tension is expressed in  
(a) N/m (b) N/m<sup>2</sup>  
(c) N<sup>2</sup>/m (d) N/m<sup>3</sup>.
14. Pressure inside a water droplet is given by the relation  
(a)  $p = \frac{4\sigma}{d}$  (b)  $p = \frac{3\sigma}{d}$   
(c)  $p = \frac{8\sigma}{d}$  (d)  $p = \frac{16\sigma}{d}$
15. .... is a phenomenon by which a liquid rises into a thin glass tube above or below its general level.  
(a) Surface tension (b) Capillarity  
(c) Cohesion (d) Adhesion.
16. The capillary rise of water in the glass tube is given by  
(a)  $h = \frac{2\sigma}{wd}$  (b)  $h = \frac{3\sigma}{wd}$   
(c)  $h = \frac{4\sigma}{wd}$  (d)  $h = \frac{6\sigma}{wd}$