CHAPTER

12

FLOW THROUGH PIPES

12.1. Introduction

A **pipe** *is a closed conduit* (generally of circular section) *which is used for carrying fluids under pressure.* The flow in a pipe is termed *pipe flow* only when the fluid *completely fills the cross-section and there is no free surface of fluid.* The pipe running partially full (in such a case *atmospheric pressure exists* inside the pipe) behaves like an *open channel.*

12.2. Loss of Energy (or Head) in Pipes

When water flows in a pipe, it experiences some resistance to its motion, due to which its velocity and ultimately the head of water available is reduced. This loss of energy (or head) is classified as follows :

A. Major Energy Losses

This loss is due to *friction.*

B. Minor Energy Losses

These losses are due to :

1. Sudden enlargement of pipe,
2. Sudden contraction of pipe,
3. Bend of pipe,
4. An obstruction in pipe,
5. Pipe fittings, etc.

12.3. Major Energy Losses

These losses which are *due to friction* are calculated by :

1. Darcy-Weisbach formula
2. Chezy's formula.

12·3·1. Darcy-Weisbach Formula

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach formula (derived in chapter 11 Art. 11·2) which is given by,

$$h_f = \frac{4fLV^2}{D \times 2g} \qquad \text{...(12·1)}$$

where, h_f = Loss of head due to friction,

f = Co-efficient of friction, (a function of Reynolds number, Re)

$$f = \frac{0.0791}{(Re)^{1/4}} \text{ for } Re \text{ varying from 4000 to } 10^6$$

$$= \frac{16}{Re} \text{ for } Re < 2000 \text{ (laminar/viscous flow)}$$

L = Length of the pipe,

V = Mean velocity of flow, and

D = Diameter of the pipe.

12·3·2. Chezy's Formula for Loss of Head due to Friction

Refer to Fig. 11·2. An equilibrium between the propelling force due to pressure difference and the frictional resistance gives :

$$(p_1 - p_2) A = f' PLV^2$$

or

$$\frac{(p_1 - p_2)}{w} \cdot A = \frac{f'}{w} PLV^2 \qquad \text{[Refer to Art. 11.2]}$$

or

$$h_f = \frac{f'}{w} \frac{P}{A} LV^2$$

$\therefore$ Mean velocity,

$$V = \sqrt{\frac{w}{f'}} \times \sqrt{\frac{A}{P} \times \frac{h_f}{L}}$$

where, the factor $\sqrt{\frac{w}{f}}$, is called the Chezy's constant, C;

the ratio $\frac{A}{P}\left(= \frac{\text{area of flow}}{\text{wetted perimeter}}\right)$ is called the *"hydraulic mean depth"* or *"hydraulic radius"* and denoted by m (or R);

the ratio $\frac{h_f}{L}$ prescribes the loss of head per unit length of pipe and is denoted by i or S (slope).

$\therefore$ Mean velocity, $V = C\sqrt{m\,i}$ ...(12·2)

Eqn. (12·2) is known as **Chezy's formula.** This formula helps to find the head loss due to friction if the mean flow velocity through the pipe and also the value of Chezy's constant C is known.

Note : (*i*) Darcy-Weisbach formula (for loss of head) is generally used for the flow through *pipes*.

(*ii*) Chezy's formula (for loss of head) is generally used for the flow through *open channels*.

(*iii*) The values of hydraulic mean depth for a *circular pipe*,

$$m = \frac{D}{4} \left[\because m = \frac{\text{Area}}{\text{Perimeter}} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4}\right]$$

Example 12·1. *In a pipe of diameter 350 mm and length 75 m water is flowing at a velocity of 2·8 m/s. Find the head lost due to friction using :*

(*i*) *Darcy-Weisbach formula;* (*ii*) *Chezy's formula for which C = 55.*

Assume kinematic viscosity of water as 0·012 stoke.

Solution. Diameter of the pipe, $D = 350 \text{ mm} = 0{\cdot}35 \text{ m}$

Length of the pipe, $L = 75$ m

Velocity of flow, $V = 2{\cdot}8$ m/s

Chezy's constant $C = 55$

Kinematic viscocity of water, $\nu = 0{\cdot}012$ stoke $= 0{\cdot}012 \times 10^{-4}$ m²/s.

Head lost due to friction, h_f :

(*i*) **Darcy-Weisbach fomula :**

Darcy-Weisbach formula is given by,

$$h_f = \frac{4fLV^2}{D \times 2g}$$

where, f = coefficient of friction (a function of Reynolds number, Re)

$$Re = \frac{V \times D}{\nu} = \frac{2{\cdot}8 \times 0{\cdot}35}{0{\cdot}012 \times 10^{-4}} = 8{\cdot}167 \times 10^5$$

$$\therefore \quad f = \frac{0{\cdot}0791}{(Re)^{1/4}} = \frac{0{\cdot}0791}{(8{\cdot}167 \times 10^5)^{1/4}} = 0{\cdot}00263$$

$\therefore$ Head lost due to friction,

$$h_f = \frac{4 \times 0{\cdot}00263 \times 75 \times (2{\cdot}8)^2}{0{\cdot}35 \times 2 \times 9{\cdot}81} = \mathbf{0{\cdot}9 \text{ m (Ans.)}}$$

(*ii*) **Chezy's formula :**

$$V = C\sqrt{m\,i}$$

where, $$C = 55, \; m = \frac{A}{P} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4} = \frac{0{\cdot}35}{4} = 0{\cdot}0875 \text{ m}$$

$$\therefore \quad 2{\cdot}8 = 55\sqrt{0{\cdot}0875 \times i}$$

or, $$0{\cdot}0875 \times i = \left(\frac{2{\cdot}8}{55}\right)^2 = 0{\cdot}00259$$

or, $$i = 0{\cdot}0296$$

But, $$i = \frac{h_f}{L} = 0{\cdot}0296$$

$$\therefore \quad \frac{h_f}{75} = 0{\cdot}0296$$

or, $$h_f = 75 \times 0{\cdot}0296 = \mathbf{2{\cdot}22 \text{ m (Ans.)}}$$

Example 12·2. *Water flows through a pipe of diameter 300 mm with a velocity of 5 m/s. If the co-efficient of friction is given by* $f = 0{\cdot}015 + \frac{0{\cdot}08}{Re^{0{\cdot}3}}$ *where Re is the Reynolds number, find the head lost due to friction for a length of 10 m. Take kinematic viscosity of water as 0·01 stoke.*

Solution. Diameter of the pipe, $D = 300 \text{ mm} = 0{\cdot}3$ m

Velocity of water $V = 5$ m/s

Length of the pipe, $L = 10$ m

Viscosity of water, $\nu = 0{\cdot}01$ stoke $= 0{\cdot}01 \times 10^{-4}$ m²/s.

($\because$ 1 stoke = 1 cm²/s = 1×10^{-4} m²/s)

Head lost due to friction h_f:

Co-efficient of friction, $f = 0{\cdot}015 + \dfrac{0{\cdot}08}{(Re)^{0{\cdot}3}}$ ...(given)

But, Reynolds number, $Re = \dfrac{\rho VD}{\mu} = \dfrac{VD}{\nu} = \dfrac{5 \times 0{\cdot}3}{0{\cdot}01 \times 10^{-4}} = 1{\cdot}5 \times 10^6$

$$\therefore \quad f = 0{\cdot}015 + \frac{0{\cdot}08}{(1{\cdot}5 \times 10^6)^{0{\cdot}3}} = 0{\cdot}0161$$

$\therefore$ Head lost due to friction,

$$h_f = \frac{4fLV^2}{D \times 2g} = \frac{4 \times 0{\cdot}0161 \times 10 \times 5^2}{0{\cdot}3 \times 2 \times 9{\cdot}81}$$

$$= \mathbf{2{\cdot}735\ m\ (Ans.)}$$

Example 12·3. *In a pipe of 300 mm diameter and 800 m length an oil of specific gravity 0·8 is flowing at the rate of 0·45 m³/s. Find :*

(*i*) *Head lost due to friction, and*

(*ii*) *Power required to maintain the flow.*

Take kinematic viscosity of oil as 0·3 stoke.

Solution. Diameter of the pipe, $D = 300$ mm $= 0{\cdot}3$ m

Length of the pipe, $L = 800$ m

Specific gravity of oil $= 0{\cdot}8$

Kinematic viscosity of oil, $\nu = 0{\cdot}3$ stoke $= 0{\cdot}3 \times 10^{-4}$ m²/s

Discharge, $Q = 0{\cdot}45$ m³/s.

(*i*) **Head lost due to friction, h_f:**

Velocity, $$V = \frac{Q}{\text{Area}} = \frac{0{\cdot}45}{\frac{\pi}{4} \times 0{\cdot}3^2} = 6{\cdot}366 \text{ m/s}$$

$\therefore$ Reynolds number, $$Re = \frac{V \times D}{\nu} = \frac{6{\cdot}366 \times 0{\cdot}3}{0{\cdot}3 \times 10^{-4}} = 6{\cdot}366 \times 10^4$$

$\therefore$ Co-efficient of friction, $$f = \frac{0{\cdot}0791}{(Re)^{1/4}} = \frac{0{\cdot}0791}{(6{\cdot}366 \times 10^4)^{1/4}} = 0{\cdot}00498$$

$$\therefore \quad h_f = \frac{4fLV^2}{D \times 2g} = \frac{4 \times 0{\cdot}00498 \times 800 \times (6{\cdot}366)^2}{0{\cdot}3 \times 2 \times 9{\cdot}81} = \mathbf{109{\cdot}72\ m\ (Ans.)}$$

(*ii*) **Power required, P:**

Power required to maintain the flow, $P = wQh_f$

where, $w = 0{\cdot}8 \times 9{\cdot}81 = 7{\cdot}848$ kN/m³

$h_f = 109{\cdot}72$ m, $Q = 0{\cdot}45$ m³/s

$\therefore \quad P = 7{\cdot}848 \times 0{\cdot}45 \times 109{\cdot}72 =$ **387·48 kW (Ans.)**

Example 12·4. *Water is to be supplied to the inhabitants of a college campus through a supply main. The following data is given :*

Distance of the reservoir from the campus = 3000 m

Number of inhabitants = 4000

Consumption of water per day of each inhabitant = 180 litres

Loss of head due to friction = 18 m

Co-efficient of friction for the pipe, f = 0·007

If the half of the daily supply is pumped in 8 hours, determine the size of the supply main.

Solution. Distance of the reservoir from the college campus = 3000 m

Number of inhabitants = 4000

Consumption per day per inhabitant = 180 litres = $0{\cdot}18\ m^3$

$\therefore$ Total supply per day = $4000 \times 0{\cdot}18 = 720\ m^3$

Since half of this supply is to be pumped in 8 hours, therefore maximum flow for which the pipe is to be designed,

$$Q = \frac{720}{2 \times 8 \times 3600} = 0{\cdot}0125\ m^3/s$$

Loss of head due to friction, $h_f = 18$ m

Co-efficient of friction, $f = 0{\cdot}007$

Diameter of the supply line, *D* :

Using the relation :

$$h_f = \frac{4fLV^2}{D \times 2g}$$

where $$V = \frac{Q}{A} = \frac{0{\cdot}0125}{\frac{\pi}{4} \times D^2} = \frac{0{\cdot}0159}{D^2}$$

$$\therefore \quad 18 = \frac{4 \times 0{\cdot}007 \times 3000 \times (0{\cdot}159 / D^2)^2}{D \times 2 \times 9{\cdot}81}$$

or $$D^5 = \frac{4 \times 0{\cdot}007 \times 3000 \times 0{\cdot}0159^2}{18 \times 2 \times 9{\cdot}81} = 6{\cdot}013 \times 10^{-5}$$

$\therefore$ $D = 0{\cdot}143$ m or **143 mm (Ans.)**

Example 12·5. *Water flows through a pipeline whose diameter varies from 25 cm to 15 cm in a length of 10 m. If the Darcy-Weisbach friction factor is assumed constant at 0·018 for the whole pipe, determine the head loss in friction when the pipe is flowing full with a discharge of 0·06 m³/s.*

Solution. *Given :* $D_1 = 25$ cm = 0·25 m; $D_2 = 15$ cm = 0·15 m, $L = 10$ m; $f = 0{\cdot}018$; $Q = 0{\cdot}06\ m^3/s$

Consider a stretch of length dx at a distance x from the 25 cm diameter end (Fig. 12.1).

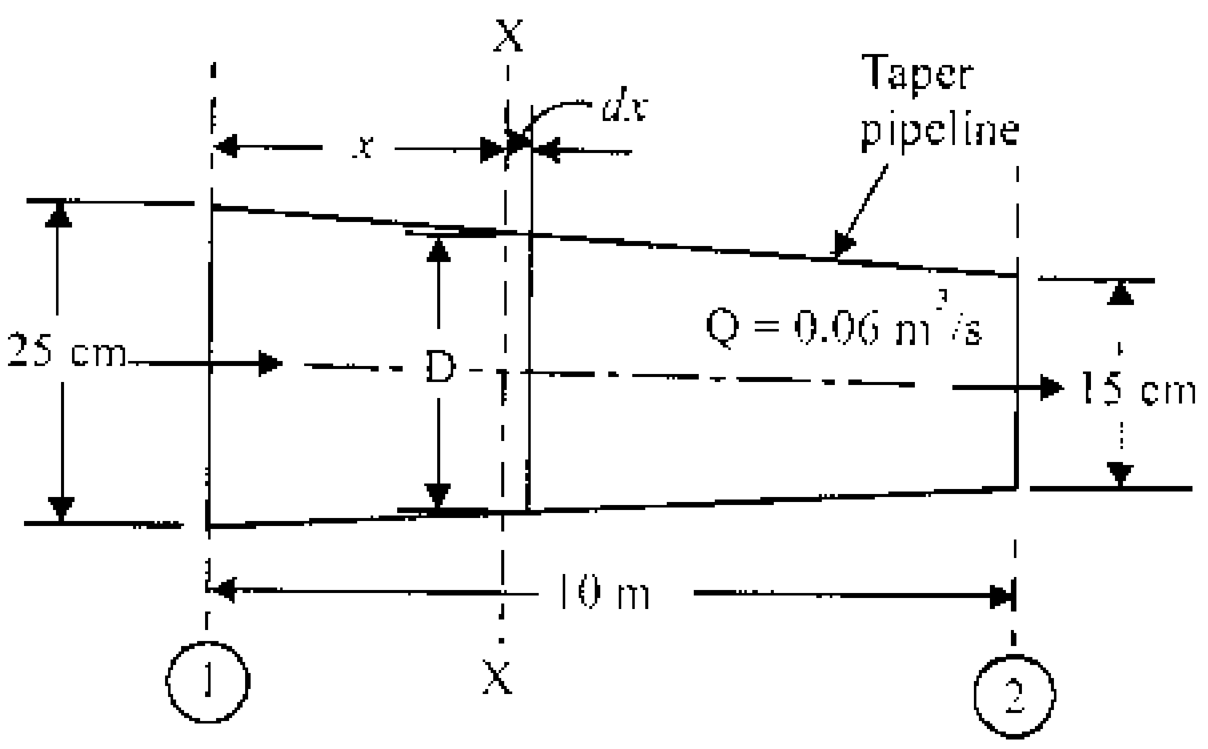

Fig. 12.1

$$dh_{fx} = \frac{f dx V^2}{D \times 2g}$$

$$= \frac{f dx \left[Q \div \left(\frac{\pi}{4} \times D^2 \right)^2 \right]}{D \times 2g} = \frac{f Q^2 dx}{2g \left(\frac{\pi}{4} \right)^2 D^5}$$

$$= 0{\cdot}08263 \times \frac{f Q^2 dx}{D^5}$$

where, D = Diameter at the section XX

$$D = \left[0{\cdot}25 - \left(\frac{0{\cdot}25 - 0{\cdot}15}{10} \right) x \right] = \frac{1}{100} (25 - x) \text{ m}$$

Hence,

$$dh_{fx} = 0{\cdot}08263 \times 0{\cdot}018\,(0{\cdot}06)^2 \times (100)^5 \times \frac{dx}{(25 - x)^5}$$

$$= 53544 \times \frac{dx}{(25 - x)^5}$$

Total head loss,

$$h_f = \int_0^{10} dh_{fx}$$

$$= 53544 \int_0^{10} (25 - x)^{-5}\, dx = 53544 \left[\frac{1}{4\,(25 - x)^4} \right]_0^{10}$$

$$= \frac{53544}{4} \left[\frac{1}{(15)^4} - \frac{1}{(25)^4} \right] = \mathbf{0{\cdot}23\ m} \quad \textbf{(Ans.)}$$

Example. 12·6. *A pipeline 50 cm diameter takes off from a reservoir whose water surface elevation is 145 m above datum. The pipe is 4500 m long and is laid completely at the datum level. In the last 1000 m of the pipe, water is withdrawn by a series of pipes at a uniform rate of 0·075 m³/s per 250 m. Find the pressure at the end of the pipeline.*

Assume f = 0·018 and the pipe to have a dead end.

Solution. *Given* : Diameter of pipeline, D = 50 cm = 0·5 m; L = 4500 m; L_0 = 1000 m; f = 0·018.

First an expression for loss of head in a pipe having a uniform withdrawal of q^* m³/s per metre length is derived.

Refer to Fig. 12.2. Consider a section at a distance x from the start of the uniform withdrawal at q^* per metre length.

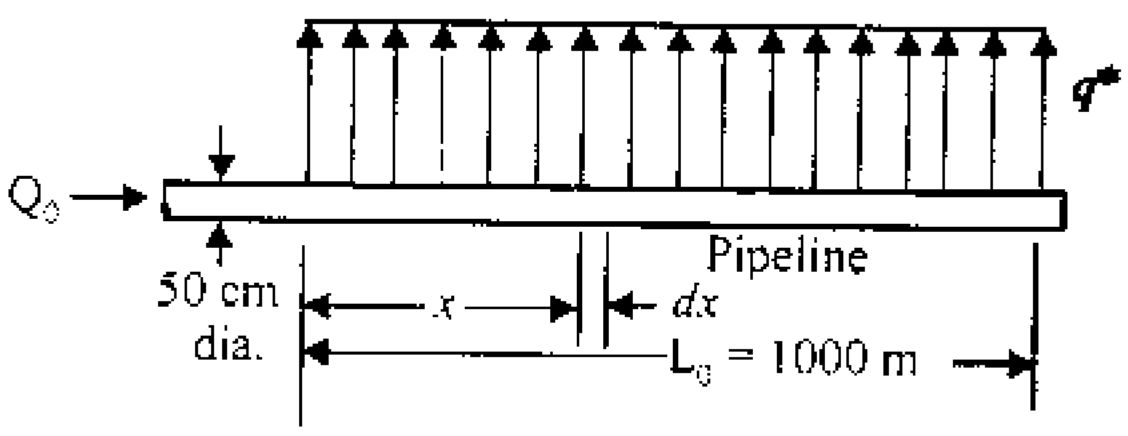

Fig. 12.2

Discharge, $Q_x = Q_0 - q^* x$

In a small distance dx

$$dh_f = \frac{f L V^2}{D \times 2g} = \frac{f}{2g} \times \left(\frac{Q_0 - q^* x}{\frac{\pi}{4} D^2} \right)^2 \times \frac{1}{D} \times dx = \frac{8f}{\pi^2 g D^5} (Q_0 - q^* x)^2 dx$$

$$\therefore \quad h_f = \int_0^{L_0} dh_f = \frac{-8f}{3\pi^2 g D^5} \times \frac{1}{q^*} \times \left[(Q_0 - q^* x)^3 \right]_0^{L_0}$$

or, $$h_f = \frac{8f}{3\pi^2 g D^5} \times \frac{1}{q^*} \times \left[Q_0^3 - (Q_0 - q^* L_0)^3\right]$$

Here, $$Q_0 = \frac{1000}{250} \times 0{\cdot}075 = 0{\cdot}3 \text{ m}^3/\text{s}$$

$$q^* = \frac{0{\cdot}075}{250} = 0{\cdot}0003 \text{ m}^3/\text{s}$$

$$L_0 = 1000 \text{ m}$$

H_L = Total head lost = [Head lost in first (4500 – 1000) m with a discharge $Q_d = 0{\cdot}3$ m³/s] + [Head lost in 1000 m with a uniform withdrawal of q^*]

$= h_{f1} + h_{f2}$

$$h_{f1} = \frac{0{\cdot}018 \times 3500 \times \left[0{\cdot}3 \div \left(\frac{\pi}{4} \times 0{\cdot}5\right)^2\right]^2}{0{\cdot}5 \times 2 \times 9{\cdot}81} = 24{\cdot}3 \text{ m}$$

$$h_{f2} = \frac{8f}{3\pi^2 g D^5} \times \frac{1}{q^*} \times \left[Q_0^3 - (Q_0 - q^* L_0)^3\right]$$

$$= \frac{8 \times 0{\cdot}018}{3\pi^2 \times 9{\cdot}81 \times (0{\cdot}5)^5} \times \frac{1}{0{\cdot}0003}\left[(0{\cdot}3)^3 - (0{\cdot}3 - 0{\cdot}0003 \times 1000)^3\right]$$

$= 1{\cdot}43$ m

Total head loss = 24·3 + 1·43 = 25·73 m

Residual head at the dead end = 145 – 25·73 = **119·27 m (Ans.)**

Example 12·7. *A pump delivers water from a tank A (water surface elevation = 110 m) to tank B (water surface elevation = 170 m). The suction pipe is 45 m long (f = 0·024) and 35 cm in diameter. The delivery pipe is 950 m long (f = 0·022) and 25 cm in diameter. The head discharge relationship for the pump is given by $H_p = (90 - 8000\,Q^2)$, where H_p is in metres and Q in m³/s. Calculate :*

(*i*) *The discharge in the pipeline.*

(*ii*) *The power delivered by the pump.*

Solution. Refer to Fig. 12·3.

Given: $D_1 = 35$ cm $= 0{\cdot}35$ m; $L_1 = 45$ m; $D_2 = 25$ cm $= 0{\cdot}25$ m; $L_2 = 950$ m; $f_1 = 0{\cdot}024$; $f_2 = 0{\cdot}022$; $H_p = 90 - 8000\,Q^2$

Suction pipe :

Head loss, $$h_{L1} = \frac{f_1 L_1 V_1^2}{D_1 \times 2g}$$

$$= \frac{0{\cdot}024 \times 45}{0{\cdot}35} \times \frac{V_1^2}{2g} = 3{\cdot}086\,\frac{V_1^2}{2g}$$

Delivery pipe :

Head loss, $$h_{L2} = \frac{f_2 L_2 V_2^2}{D_2 \times 2g}$$

$$= \frac{0{\cdot}022 \times 950}{0{\cdot}25} \times \frac{V_2^2}{2g} = 83{\cdot}6\,\frac{V_2^2}{2g}$$

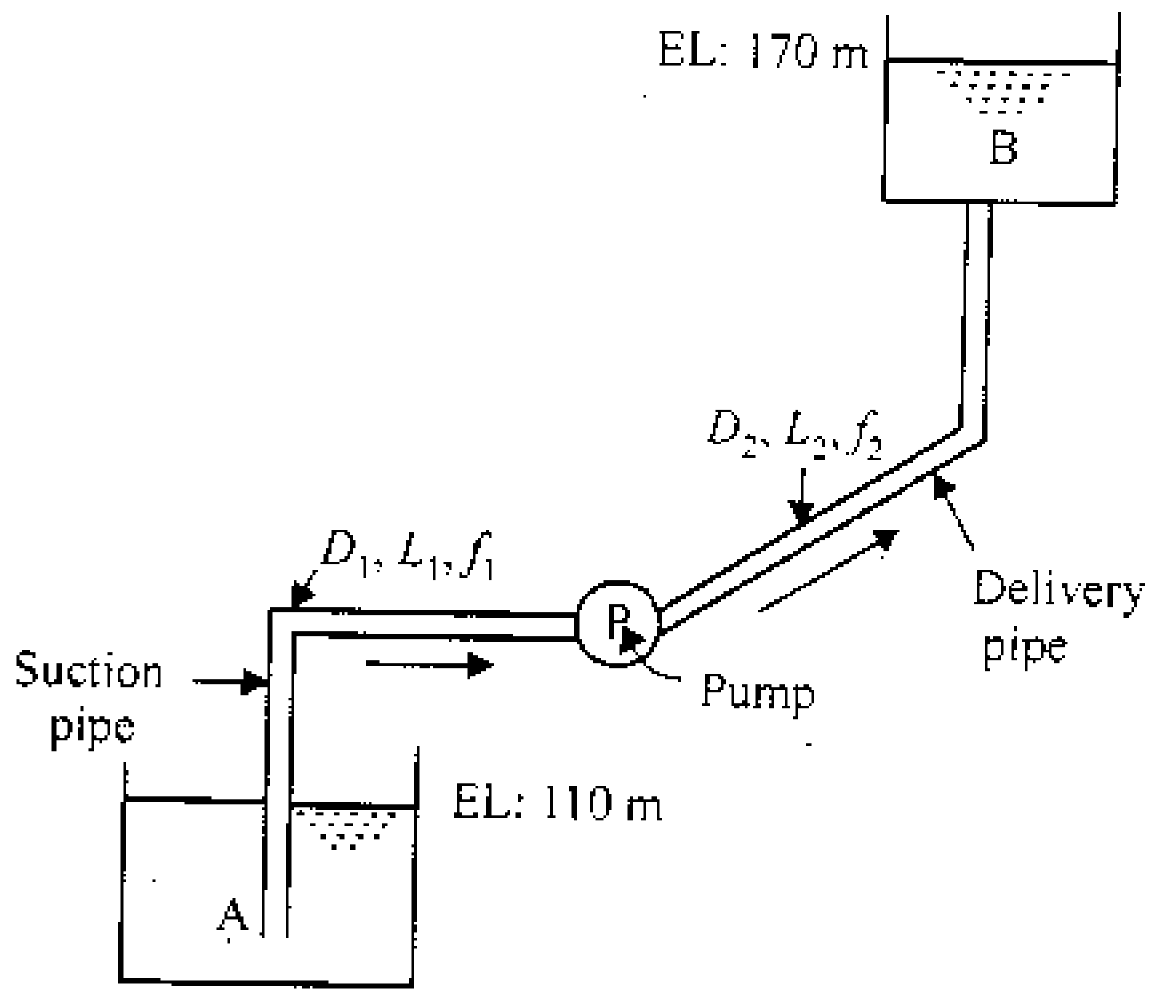

Fig. 12.3

Total head loss, $H_L = 3{\cdot}086 \dfrac{V_1^2}{2g} + 83{\cdot}6 \dfrac{V_2^2}{2g}$

By continuity equation,

$$A_1V_1 = A_2V_2$$

$$\frac{\pi}{4} \times (0{\cdot}35)^2 \times V_1 = \frac{\pi}{4} \times (0{\cdot}25)^2 \times V_2$$

or,
$$V_1 = 0{\cdot}51\ V_2$$

$$\frac{V_1^2}{2g} = 0{\cdot}26 \frac{V_2^2}{2g}$$

$\therefore$
$$H_L = 3{\cdot}086 \times 0{\cdot}26 \frac{V_2^2}{2g} + 83{\cdot}6 \frac{V_2^2}{2g} = 84{\cdot}4 \frac{V_2^2}{2g}$$

Static head = 170 – 110 = 60 m

H_p = Head delivered by the pump = Static head + friction head

$$= 60 + 84{\cdot}4 \frac{V_2^2}{2g}$$

$$= 60 + 84{\cdot}4 \times \left(\frac{Q}{\frac{\pi}{4} \times (0{\cdot}25)^2}\right)^2 \times \frac{1}{2 \times 9{\cdot}81}$$

$$= 60 + 1785{\cdot}3\ Q^2$$

Also, $H_p = 90 - 8000\,Q^2$ ...(*Given*)

$\therefore$ $90 - 8000\,Q^2 = 60 + 1785{\cdot}3\,Q^2$

or, $Q = 0{\cdot}05537\ \text{m}^3/\text{s}$

$\therefore$ $H_p = 60 + 1785{\cdot}3 \times (0{\cdot}05537)^2 = 65{\cdot}47\ \text{m}$

Hence, power delivered by the pump,

$$\mathbf{P} = wQH_p = 9{\cdot}81 \times 0{\cdot}05537 \times 65{\cdot}47 = \mathbf{35{\cdot}56\ kW}\ \textbf{(Ans.)}$$

12·4. Minor Energy Losses

Whereas the major loss of energy or head is due to friction, the minor loss of energy (or head) includes the following cases :

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head due to an obstruction in the pipe,
4. Loss of head at the entrance to a pipe,
5. Loss of head at the exit of a pipe,
6. Loss of head due to bend in the pipe, and
7. Loss of head in various pipe fittings.

12·4·1. Loss of Head due to Sudden Enlargement

Fig. 12·4. shows a liquid flowing through a pipe which has sudden enlargement. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy (or head).

Consider two sections 1 – 1(before enlargement) and 2 – 2 (after enlargement).

Now, net force = change of momentum

$\therefore \quad (p_1 - p_2) A_2 = \rho A_2 (V_2^2 - V_1V_2)$

or, $$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1V_2$$

Dividing both sides by g, we get

$$\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g}$$

or, $$\frac{p_1}{w} - \frac{p_2}{w} = \frac{V_2^2 - V_1V_2}{g} \qquad (\because \rho g = w)$$

Substituting the value of $\left(\frac{p_1}{w} - \frac{p_2}{w}\right)$ in eqn. (*i*), we get

$$h_e = \frac{V_2^2 - V_1V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g} = \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g} = \frac{(V_1 - V_2)^2}{2g}$$

$$\therefore \quad h_e = \frac{(V_1 - V_2)^2}{2g} \qquad ...(12\cdot2)$$

Example 12·8. *At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Calculate the rate of flow.* **[AMIE, Panjab University]**

Solution. Diameter of the smaller pipe, D_1 = 240 = mm = 0·24 m

Diameter of larger pipe, D_2 = 480 mm = 0·48 m

Rise of hydraulic gradient, *i.e.*

$$\left(\frac{p_2}{w} + z_2\right) - \left(\frac{p_1}{w} + z_1\right) = 10 \text{ mm} = 0\cdot01 \text{ m}$$

$$\left[\text{The term } \left(\frac{p}{w} + z\right) \text{ prescribes the hydraulic gradient}\right]$$

Rate of flow, Q:

Applying Bernoulli's equation to small and large pipe sections (1–1 and 2–2), we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_e \text{ (i.e., head lost due to sudden enlargement)} \qquad ...(i)$$

But, $$h_e = \frac{(V_1 - V_2)^2}{2g} \qquad ...(ii)$$

From continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4} \times D_2^2}{\frac{\pi}{4} \times D_1^2} \times V_2 = \left(\frac{D_2}{D_1}\right)^2 \times V_2$$

or, $$V_1 = \left(\frac{0\cdot48}{0\cdot24}\right)^2 \times V_2 = 4V_2$$

Substituting this value of V_1 in eqn. (*ii*), we get

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now, substituting the values of he and V_1 in eqn. (*i*), we have

$$\frac{p_1}{w} + \frac{(4V_2)^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

or, $$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{p_2}{w} + z_2\right) - \left(\frac{p_1}{w} + z_1\right)$$

or, $$\frac{6V_2^2}{2g} = 0.01$$

or, $$V_2 = \left(\frac{0.01 \times 2 \times 9.81}{6}\right)^{1/2} = 0.181 \text{ m/s}$$

∴ Rate of flow, $Q = A_2V_2 = \frac{\pi}{4} \times 0.48^2 \times 0.181 =$ **0.03275 m³/s (Ans.)**

Example 12·9. *A horizontal pipe 150 mm in diameter, is joined by sudden enlargement to a 225 mm diameter pipe. Water is flowing through it at the rate of 0·05 m³/s. Find :*

(*i*) *Loss of head due to abrupt expansion,*

(*ii*) *Pressure difference in the two pipes, and*

(*iii*) *Change in pressure if the change of section is gradual without any loss.*

Solution. Diameter of the smaller pipe, $D_1 = 150 \text{ mm} = 0.15 \text{ m}$

∴ Area, $A_1 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$

Diameter of the larger pipe, $D_2 = 225 \text{ mm} = 0.225 \text{ m}$

∴ Area, $A_2 = \frac{\pi}{4} \times 0.225^2 = 0.03976 \text{ m}^2$

Discharge, $Q = 0.05 \text{ m}^3/\text{s}$

(*i*) **Loss of head due to abrupt expansion, h_e**

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

where, V_1 and V_2 are the velocities of flow in the smaller and larger diameter pipes respectively.

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{0.01767} = 2.83 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{0.03976} = 1.26 \text{ m/s}$$

Hence, $$h_e = \frac{(2.83 - 1.26)^2}{2 \times 9.81} = \mathbf{0.1256 \text{ m}} \textbf{ (Ans.)}$$

(*ii*) **Pressure difference in the two pipes :**

Applying Bernoulli's equation at the smaller and the larger pipe sections, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_e$$

or $$\left(\frac{p_2 - p_1}{w}\right) = \frac{V_1^2 - V_2^2}{2g} - h_e \qquad [\because z_1 = z_2, \text{ the pipe being horizontal}]$$

or, $$\left(\frac{p_2 - p_1}{w}\right) = \frac{2{\cdot}83^2 - 1{\cdot}26^2}{2 \times 9{\cdot}81} - 0{\cdot}1256 = \mathbf{0{\cdot}202\ m\ of\ water\ (Ans.)}$$

The positive sign indicates that there is *gain* in pressure. Thus, although there is an energy loss, the pressure increases across a sudden flow of expansion.

(*iii*) **Change in pressure with gradual change of section :**

If the change of section is gradual *without loss,* then, gain in pressure,

$$\frac{p_2 - p_1}{w} = \frac{V_1^2 - V_2^2}{2g} = \frac{2{\cdot}83^2 - 1{\cdot}26^2}{2 \times 9{\cdot}81} = \mathbf{0{\cdot}327\ m\ of\ water\ (Ans.)}$$

Example 12·10. *The diameter of a horizontal pipe which is 300 mm is suddenly enlarged to 600 mm. The rate of flow of water through this pipe is 0·4 m³/s. If the intensity of pressure in the smaller pipe is 125 kN/m², determine.*

(*i*) *Loss of head, due to sudden enlargement,*

(*ii*) *Intensity of pressure in the larger pipe, and*

(*iii*) *Power lost due to enlargement.*

Solution. Diameter of the smaller pipe, D_1 = 300 mm = 0·03 m

Area, $A_1 = \frac{\pi}{4} \times 0{\cdot}3^2 = 0{\cdot}0707\ \text{m}^2$

Diameter of the longer pipe, D_2 = 600 mm = ·06 m

Area, $A_2 = \frac{\pi}{4} \times 0{\cdot}06^2 = 0{\cdot}2828\ \text{m}^2$

Rate of flow of water, $Q = 0{\cdot}4\ \text{m}^3/\text{s}$

Intensity of pressure in the smaller pipe, $p_1 = 125\ \text{kN/m}^2$

Now velocity, $$V_1 = \frac{Q}{A_1} = \frac{0{\cdot}4}{0{\cdot}0707} = 5{\cdot}66\ \text{m/s}$$

Velocity, $$V_2 = \frac{Q}{A_2} = \frac{0{\cdot}4}{0{\cdot}2828} = 1{\cdot}414\ \text{m/s}$$

(*i*) **Loss of head due to sudden enlargement, h_e :**

Loss of head due to sudden enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(5{\cdot}66 - 1{\cdot}414)^2}{2 \times 9{\cdot}81} = \mathbf{0{\cdot}918\ m\ \ (Ans.)}$$

(*ii*) **Intensity of pressure in the large pipe, p_2 :**

Applying Bernoulli's equation before and after sudden enlargement, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_e$$

But, $z_1 = z_2$ ...because pipe is horizontal

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} + h_e$$

or, $$\frac{p_2}{w} = \frac{p_1}{w} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{125}{9.81} + \frac{5.66^2}{2 \times 9.81} - \frac{1.414^2}{2 \times 9.81} - 0.918$$

$$= 12.74 + 1.63 - 0.1 - 0.918 = 13.35 \text{ m}$$

$\therefore$ $p_2 = w \times 13.35 = 9.81 \times 13.35$

$= \mathbf{130.9\ kN/m^2}$ **(Ans.)**

(*iii*) **Power lost due to sudden enlargement, P_{lost} :**

$$P_{lost} = \frac{wQh_e}{1000}\ kW$$

where, $w = 9.81 \times 1000\ N/m^2$,

$Q = 0.4\ m^3/s$, and

$h_e = 0.918$ m

$$\therefore \quad P_{lost} = \frac{(9.81 \times 1000) \times 0.4 \times 0.918}{1000}$$

$= \mathbf{3.6\ kW}$ **(Ans.)**

Example 12·11. *In a 80 mm diameter pipeline an oil of specific gravity 0·8 is flowing at the rate of 6·0125 m³/s. A sudden expansion takes place into a second pipeline of such diameter that maximum pressure rise is obtained. Find :*

(*i*) *Loss of energy in sudden expansion,*

(*ii*) *Differential gauge length indicated by an oil-mercury manometer connected between the two pipes.* **[UPSC Exams.]**

Solution. Diameter of the smaller pipe, D_1 = 80 mm = 0·08 m

Specific gravity of oil, $S = 0.8$

Discharge, $Q = 0.0125\ m^3/s$.

(*i*) **Loss of energy in sudden expansion :**

Velocity of flow, $$V_1 = \frac{Q}{\text{Area}} = \frac{0.0125}{\frac{\pi}{4} \times 0.08^2} = 2.49 \text{ m/s}$$

The pressure rise will be maximum when

$$\frac{D_1}{D_2} = \frac{1}{\sqrt{2}}$$ (where, D_2 = diameter of the larger pipe)

[For derivation of the formula, refer to Example 12·12]

or, $$D_2 = \sqrt{2}\, D_1 = \sqrt{2} \times 0.08 = 0.1131 \text{ m}$$

$$\therefore \quad V_2 = \frac{0.0125}{\frac{\pi}{4} \times (0.1131)^2} = 1.244 \text{ m/s}$$

Loss of energy (or head) in sudden expansion,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(2.49 - 1.244)^2}{2 \times 9.81} = \mathbf{0.079\ m\ of\ oil}\ \ \textbf{(Ans.)}$$

(*iii*) **Reading of the manometer :**

The energy equation is given as

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_e$$ ($z_1 = z_2$, the pipe being horizontal)

$$\left(\frac{p_2 - p_1}{w}\right) = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{2{\cdot}49^2}{2 \times 9.81} - \frac{1.244^2}{2 \times 9.81} - 0.079 = 0.158 \text{ of oil}$$

Let, h = Reading of the U-tube oil-mercury manometer where limbs are connected across the expanded transition,

Then, $$\frac{p_2 - p_1}{w} = h\left(\frac{S_m}{S_0} - 1\right)$$

[where, S_m = specific gravity of mercury (= 13·6)]

or, $$0{\cdot}158 = h\left(\frac{13{\cdot}6}{0{\cdot}8} - 1\right) = 16\,h$$

or, $$h = \frac{0{\cdot}158}{16} = 0{\cdot}009875 \text{ m} \quad \text{or} \quad \mathbf{9{\cdot}875 \text{ mm (Ans.)}}$$

Example 12·12. *For sudden expansion in a pipe flow, work out the optimum ratio between the diameter of the pipe before expansion and the diameter of pipe after expansion so that pressure rise is maximum ? Also find the maximum pressure rise.* **(UPSC Exams.)**

Solution. Applying Bernoulli's equation at sections 1 – 1 and 2 – 2, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_e$$

where, h_e (energy loss due to sudden expansion) $= \dfrac{(V_1 - V_2)^2}{2g}$

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} + \frac{(V_1 - V_2)^2}{2g}$$ ($\because z_1 = z_2$, the pipe being horizontal)

Pressure rise, $$\Delta p = (p_2 - p_1) = w\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{(V_1 - V_2)^2}{2g}\right]$$

From continuity considerations,

$$A_1V_1 = A_2V_2$$

$$V_2 = \frac{A_1V_1}{A_2} = \left(\frac{D_1}{D_2}\right)^2 V_1$$

$$\therefore \quad \Delta p = w \times \frac{V_1^2}{2g}\left[1 - \left(\frac{D_1}{D_2}\right)^4 - \left\{1 - \left(\frac{D_1}{D_2}\right)^2\right\}^2\right]$$

or $$\Delta p = w \times \frac{V_1^2}{2g}\left[2\left(\frac{D_1}{D_2}\right)^2 - 2\left(\frac{D_1}{D_2}\right)^4\right]$$

There is only one value or ratio $\left(\dfrac{D_1}{D_2}\right)$ which will provide the maximum pressure rise.

$\therefore$ For maximum pressure rise, $\dfrac{d\,(\Delta p)}{d\,(D_1/D_2)} = 0$

or, $$\frac{d\,(\Delta p)}{d\,(D_1/D_2)} = \left[4\left(\frac{D_1}{D_2}\right) - 8\left(\frac{D_1}{D_2}\right)^3\right] = 0$$

From which $\left(\dfrac{D_1}{D_2}\right)^2 = \dfrac{1}{2}$ or $\dfrac{D_1}{D_2} = \dfrac{1}{\sqrt{2}}$

$\therefore$ Diameter ratio for the maximum pressure rise is

$$\frac{D_1}{D_2} = \frac{1}{\sqrt{2}} \quad \textbf{(Ans.)}$$

Maximum pressurerise is,

$$(\Delta p)_{max} = w \times \frac{V_1^2}{2g}\left[2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)^4\right]$$

$$= \frac{wV_1^2}{2g}(1 - 0{\cdot}5) = \frac{0{\cdot}5wV_1^2}{2g} \quad \textbf{(Ans.)}$$

12·4·2. Loss of Head due to Sudden Contraction

Due to sudden contraction, the streamlines converge to a minimum cross-section called the *vena contracta* and then expand to fill the downstream pipe (Fig. 12·5.)

Let, A_c = Area of flow at section C–C,

V_c = Velocity of flow at section C–C,

A_2 = Area of flow at section 2–2,

V_2 = Velocity of flow at section 2–2, and

h_c = Loss of head due to sudden contraction.

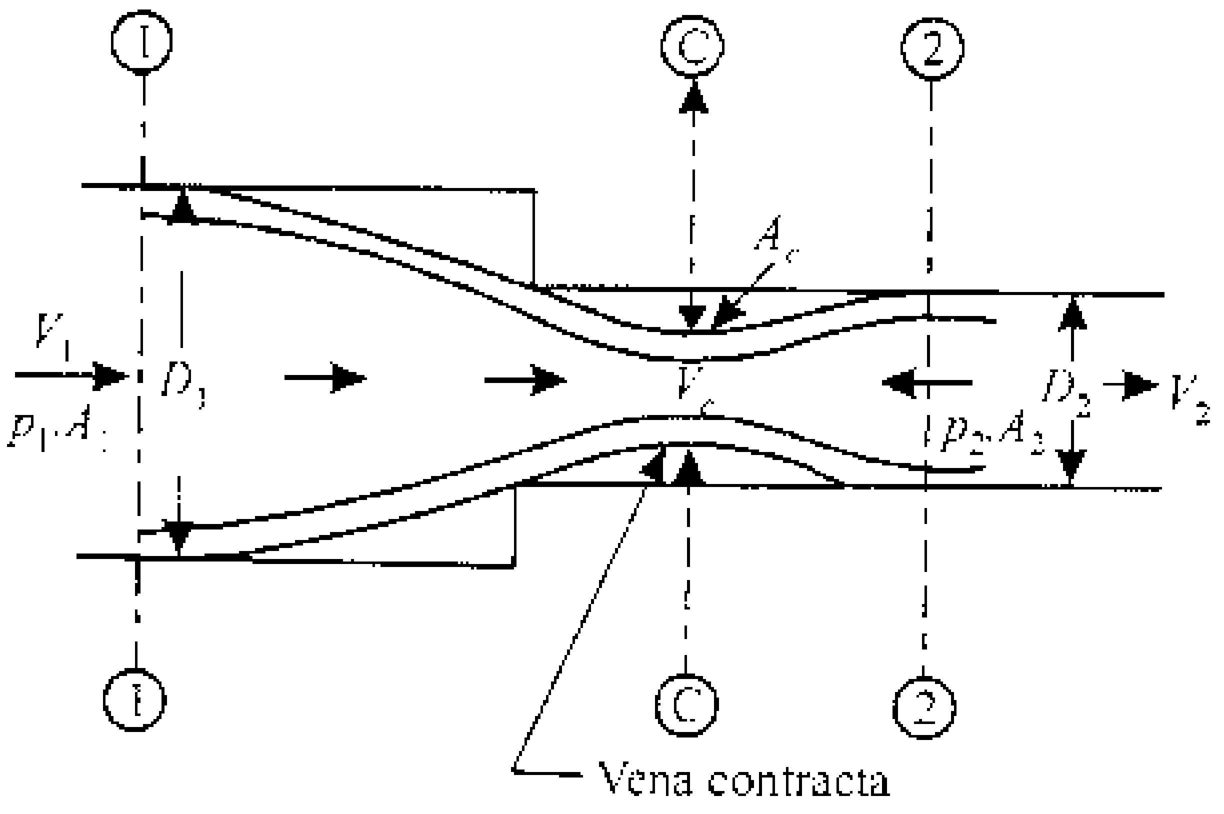

Fig. 12.5

Loss of head due to sudden contraction = Loss up to vena contracta + loss due to sudden enlargement beyond vena contracta

or, $$h_c = \text{negligibly small} + \frac{(V_c - V_2)^2}{2g} \quad \ldots (i)$$

From continuity equation, we have

$$A_cV_c = A_2V_2$$

or, $$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c / A_2)} = \frac{1}{C_c} \quad \left(\because C_c = \frac{A_c}{A_2}\right)$$

or, $$V_c = \frac{V_2}{C_c}$$

Substituting the value of V_c in eqn. (*i*), we get

$$h_c = \frac{\left(\dfrac{V_2}{C_c} - V_2\right)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

i.e., $$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 \quad \ldots (12{\cdot}3)$$

In general, $h_c = k \frac{V_2^2}{2g}$

where, $k = \left(\frac{1}{C_c} - 1\right)^2$

From experiments : $C_c = 0.62 + 0.38 \left(\frac{A_2}{A_1}\right)^3$

and thus the loss co-efficient k is a function of ratio

$$\frac{A_1}{A_2} \text{ or } \frac{D_2}{D_1}$$

and, $k = 0.375$ for $C_c = 0.62$.

For gradual contraction (conical reducers) k is a function of cone angle and $\simeq 0.1$.

Note : If the value of C_c is not given then loss of head due to contraction may be taken as $0.5 \frac{V_2^2}{2g}$

i.e., $$h_c = 0.5 \frac{V_2^2}{2g} \quad ...(12.4)$$

Example 12·13. *A horizontal pipe carries water at the rate of 0·04 m³/s. Its diameter, which is 300 mm reduces abruptly to 150 mm. Calculate the pressure loss across the contraction. Take the co-efficient of contraction = 0·62.*

Solution. Diameter of the large pipe, $D_1 = 300$ mm $= 0.3$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$

Diameter of the small pipe, $D_2 = 150$ mm $= 0.15$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times 0.15^2 = 0.1767 \text{ m}^2$

Discharge, $Q = 0.04$ m³/s.

Co-efficient of contraction, $C_c = 0.62$

Pressure loss across the contraction, $(p_1 - p_2)$:

From continuity equation, we have

$$A_1 V_1 = A_2 V_2 = Q$$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.04}{0.0707} = 0.566 \text{ m/s}$$

and, $$V_2 = \frac{Q}{A_2} = \frac{0.04}{0.01767} = 2.26 \text{ m/s}$$

Applying Bernoulli's equation before and after contraction, we get,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_c \quad ...(i)$$

But, $z_1 = z_2$ ...because the pipe is horizontal and head loss due to contraction (h_c) is given as :

$$h_c = \left[\frac{1}{C_c} - 1\right]^2 \frac{V_2^2}{2g} = \left[\frac{1}{0.62} - 1\right]^2 \times \frac{2.26^2}{2 \times 9.81} = 0.0978$$

Substituting these values in eqn. (*i*), we get,

$$\frac{p_1}{w} + \frac{0.566^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{2.26^2}{2 \times 9.81} + 0.0978$$

$$\therefore \quad \frac{p_1}{w} - \frac{p_2}{w} = \frac{2.26^2}{2 \times 9.81} + 0.0978 - \frac{0.566^2}{2 \times 9.81}$$

$$= 0.26 + 0.0978 - 0.016 = 0.3418$$

Hence, $p_1 - p_2 = w \times 0.3418 = 9.81 \times 0.3418$

$= \mathbf{3.35\ kN/m^2}$ **(Ans.)**

Example 12·14. *A pipe of diameter 225 mm is attached to a 150 mm diameter pipe by means of a flange in such a manner that the axes of the two pipes are in a straight line. Water flows through the arrangement at the rate of 0·05 m³/s. The pressure loss at the transition as indicated by differential gauge length on a water-mercury manometer connected between two pipes equals 35 mm. Calculate :*

(*i*) *The loss of head due to contraction, and*

(*ii*) *The co-efficient of contraction.*

Solution. Diameter of the large pipe, $D_1 = 225$ mm $= 0.225$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} \times 0.225^2 = 0.03976\ m^2$

Diameter of the small pipe, $D_2 = 150$ mm $= 0.15$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767\ m^2$

Discharge, $Q = 0.05\ m^3/s$

Reading of the differential gauge, $h = 35$ mm $= 0.035$ m

(*i*) **Loss of head due to contraction h_c :**

When the water-mercury manometer is connected across the contracted transition, then

$$\frac{p_1 - p_2}{w} = h\left(\frac{S_m}{S_w} - 1\right)$$

where, S_m = Sp. gr. of mercury (= 13·6), and

S_w = Sp. gr. of water (= 1).

Substituting the values in the above eqn., we get

$$\frac{p_1 - p_2}{w} = 0.035\left(\frac{13.6}{1.0} - 1\right) = 0.441 \text{ m}$$

Let V_1 and V_2 be the velocities of flow in the large diameter and small diameter pipes respectively, then

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{0.03976} = 1.26 \text{ m/s, and}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{0.01767} = 2.83 \text{ m/s}$$

Invoking Bernoulli's equation, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_c$$

[where, h_c = head loss due to contraction, and $z_1 = z_2$ ....the pipe being horizontal]

or, $$h_c = \left(\frac{p_1 - p_2}{w}\right) + \frac{V_1^2 - V_2^2}{2g}$$

$$= 0{\cdot}441 + \frac{(1{\cdot}26)^2 - (2{\cdot}83)^2}{2 \times 9{\cdot}81} = 0{\cdot}114 \text{ m of water} \quad \textbf{(Ans.)}$$

(*ii*) **The co-efficient of contraction, C_c :**

The loss of head due to contraction is given by,

$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \times \frac{V_2^2}{2g}$$

or
$$0{\cdot}114 = \left(\frac{1}{C_c} - 1\right)^2 \times \frac{2{\cdot}83^2}{2 \times 9{\cdot}81}$$

from which C_c = **0·65 (Ans.)**

Example 12·15. *When a sudden contraction is introduced in a horizontal pipeline from 500 mm diameter to 250 mm diameter, the pressure changes from 105 kN/m² to 69 kN/m². If the co-efficient of contraction is assumed to be 0·65, calculate the water flow rate.*

Following this if there is sudden enlargement from 250 mm to 500 mm and if the pressure at the 250 mm section is 69 kN/m², what is the pressure at the 500 mm enlarged portion ?

[Roorkee University]

Solution. Diameter of the large pipe, D_1 = 500 m = 0·5 m

$\therefore$ Area, $A_1 = \frac{\pi}{4} \times 0{\cdot}5^2 = 0{\cdot}1963 \text{ m}^3$

Diameter fo the small pipe, D_2 = 250 mm = 0·025 m

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times 0{\cdot}25^2 = 0{\cdot}04908 \text{ m}^2$

Pressure inside the large pipe, p_1 = 105 kN/m²

Pressure inside the small pipe, p_2 = 69 kN/m²

Co-efficient of contraction, C_c = 0·65

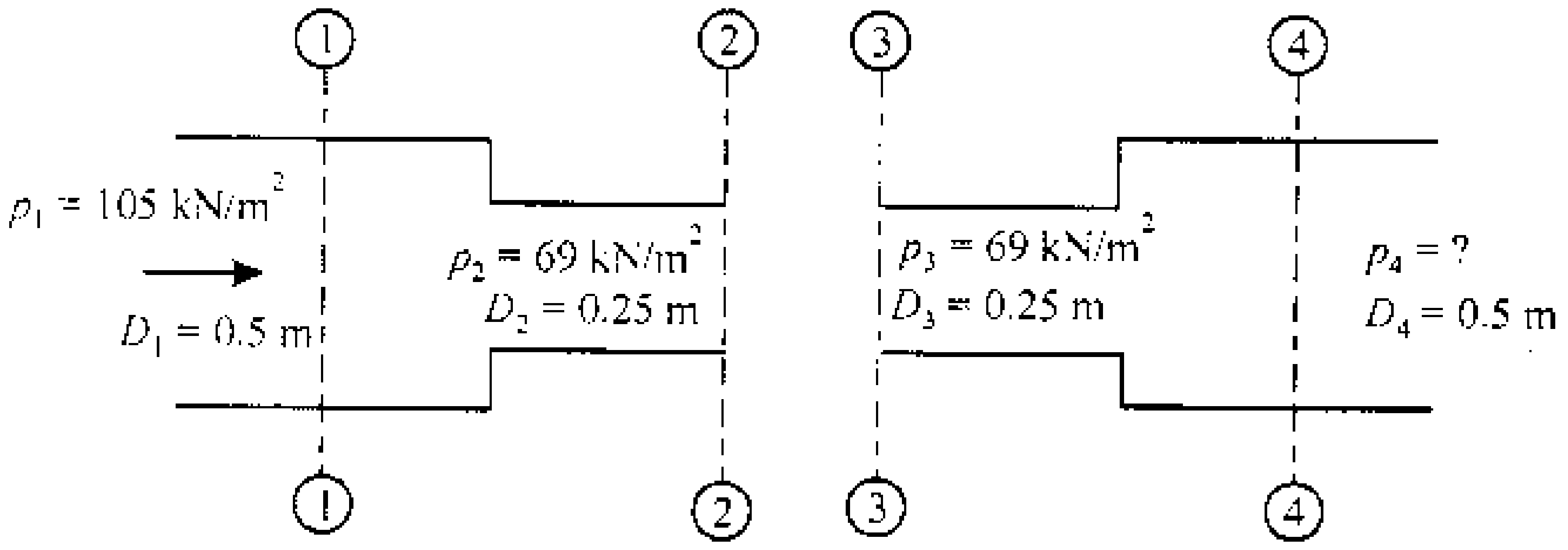

Fig. 12.6

(*i*) **Flow rate, Q :**

Head lost due to contraction is given by,

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{0{\cdot}65} - 1{\cdot}0\right)^2 = \frac{V_2^2}{2g}\left[\frac{1}{0{\cdot}65} - 1{\cdot}0\right]^2 \qquad \text{[Eqn. (12·3)]}$$

$$= 0{\cdot}2899\, \frac{V_2^2}{2g} \qquad \ldots(i)$$

From continuity considerations, we have

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2}{A_1} \times V_2 = \frac{\pi/4 \times D_2^2}{\pi/4 \times D_1^2} \times V_2$$

or $$V_1 = \left(\frac{0.25}{0.50}\right)^2 \times V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation at 1-1 and 2-2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_c$$

But $z_1 = z_2$ ...the pipe being horizontal.

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} + h_c$$

Substituting the values, we get

$$\frac{105}{9.81} + \frac{(V_2/4)^2}{2 \times 9.81} = \frac{69}{9.81} + \frac{V_2^2}{2 \times 9.81} + 0.2899\,\frac{V_2^2}{2 \times 9.81}$$

or, $$210 + \frac{V_2^2}{16} = 138 + V_2^2 + 0.2899\,V_2^2$$

or, $$72 = 1.2899\,V_2^2 - \frac{V_2^2}{16} = 1.2274\,V_2^2$$

$$V_2 = 7.66 \text{ m/s}$$

Hence, rate of flow, $Q = A_2V_2 = 0.04908 \times 7.66 = \mathbf{0.376\ m^3/s}$ **(Ans.)**

(*ii*) **Pressure at the enlarged section, p_4 :**

Applying Bernoulli's equation at the sections 3-3 and 4-4, we get

$$\frac{p_3}{w} + \frac{V_3^2}{2g} + z_3 = \frac{p_4}{w} + \frac{V_4^2}{2g} + z_4 + h_e \quad \text{(loss of head due to sudden enlargement)}$$

But, $p_3 = 69$ kN/m²

$$V_3 = V_2 = 7.66 \text{ m/s}$$

$$V_4 = V_1 = \frac{V_2}{4} = \frac{7.66}{4} = 1.915 \text{ m/s}$$

$$z_3 = z_4$$

And, $$h_e = \frac{(V_3 - V_4)^2}{2g} = \frac{(7.66 - 1.915)^2}{2 \times 9.81} \text{ m} = 1.68 \text{ m}$$

Substituting the values in the above equation, we get

$$\frac{69}{9.81} + \frac{7.66^2}{2 \times 9.81} = \frac{p_4}{9.81} + \frac{(1.915)^2}{2 \times 9.81} + 1.68$$

$$7.033 + 2.99 = \frac{p_4}{9.81} + 0.187 + 1.68$$

or $$\mathbf{p_4 = 80\ kN/m^2} \quad \textbf{(Ans.)}$$

12·4·3. Loss of Head due to Obstruction in Pipe

Refer to Fig. 12·7. The loss of energy due to an obstruction in pipe takes place on account of the reduction in the cross-sectional area of the pipe by the presence of obstruction which is followed by an abrupt enlargement of the stream beyond the obstruction.

Head loss due to obstruction ($h_{obs.}$) is given by the relation :

$$h_{obs.} = \left[\frac{A}{C_c\,(A-a)}\right]^{-2} \frac{V^2}{2g} \quad ...(12.5)$$

where, A = Area of the pipe,

a = Maximum area of obstruction, and

V = Velocity of liquid in pipe.

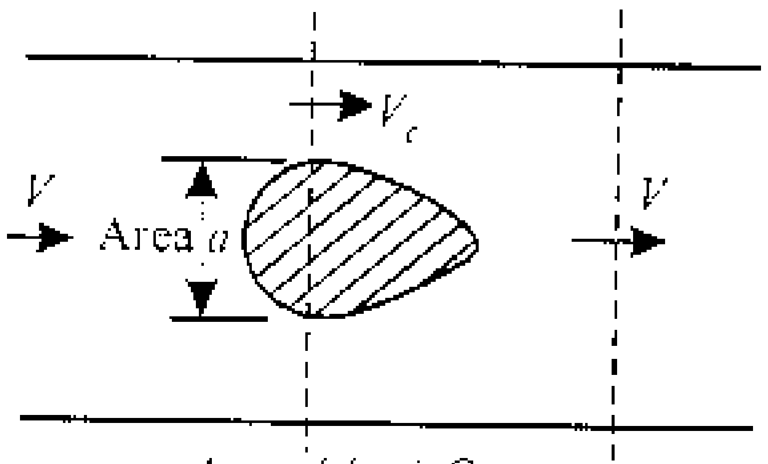

Fig. 12.7

12·4·4. Loss of Head at the Entrance to Pipe

Loss of head at the entrance to pipe (h_i) is given by the relation :

$$h_i = 0{\cdot}5\,\frac{V^2}{2g} \quad ...(12.6)$$

where, V = Velocity of liquid in pipe.

12·4·5. Loss of Dead at the Exit of a Pipe

Loss of head at the exit of a pipe is denoted by h_0 and is given by the relation.

$$h_0 = \frac{V^2}{2g} \quad ...(12.7)$$

where, V = Velocity at outlet of pipe.

12·4·6. Loss of Head due to Bend in the Pipe

In general the loss of head in bends (h_b) provided in pipes may be expressed as :

$$h_b = k\,\frac{V^2}{2g} \quad ...(12.8)$$

where, V = Mean velocity of flow of fluid, and

k = Co-efficient of bend; it depends upon *angle of bend, radius of curvature of bend* and *diameter of pipe.*

12·4·7. Loss of Head in Various Pipe Fittings

The loss of head in the various pipe fittings (such as valves, couplings, etc.) may also be represented as :

$$h_{\text{fittings}} = k\,\frac{V^2}{2g}$$

where, V = Mean velocity flow in the pipe, and k = value of the co-efficient; it depends on the type of the pipe fitting.

12·5. Hydraulic Gradient and Total Energy Lines

The concept of hydraulic gradient line and total energy line is quite useful in the study of flow of fluid in pipes. These lines may be obtained as indicated below.

Total Energy Line (T.E.L. or E.G.L.)

It is known that the *total head* (which is also total energy per unit weight) with respect to any arbitrary datum, is the sum of the elevation (potential) head, pressure head and velocity head, *i.e.,*

$$\text{Total head} = \frac{p}{w} + z + \frac{V^2}{2g}$$

When the fluid flows along the pipe, there is loss of head (energy) and the total energy decreases in the direction of flow. If the total energy at various points along the axis of the pipe is plotted and joined by a line, the line so obtained is called the ***'Energy gradient line' (E.G.L.).***

In literature, energy gradient line (E.G.L.) is also known as *'Total energy line' (T.E.L.).*

Hydraulic Gradient Line (H.G.L.)

The sum of potential (or elevation) head and the pressure head $\left(\frac{p}{w}+z\right)$ at any point is called the *piezometric head. If a line is drawn joining the piezometric levels at various points,* the line so obtained is called the ***'Hydraulic gradient line.'***

The following points are worth noting :

1. Energy gradient line (E.G.L.) *always drops in the direction of flow because of loss of head.*
2. Hydraulic gradient line (H.G.L.) may *rise* or *fall* depending on the pressure changes.
3. Hydraulic gradient line (H.G.L.) is *always below the energy gradient line (E.G.L.)* and the vertical intercept between the two is equal to the velocity head $\left(\frac{V^2}{2g}\right)$.
4. For a pipe of *uniform cross-section the slope of the hydraulic gradient line is equal to the slope of energy gradient line.*
5. *There is no relation whatsoever between the slope of energy gradient line and the slope of the axis of the pipe.*

Example 12·16. *A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur,*

(*i*) *Determine the rate of flow.*

(*ii*) *Draw the hydraulic gradient and energy gradient lines. Take f = 0·01 for both sections of the pipe.* **[M.U.]**

Solution. Total length of the horizontal pipeline, $L = 40$ m.

Length of first pipe $L_1 = 25$ m

Diameter of first pipe $D_1 = 150$ mm $= 0{\cdot}15$ m

Length of second pipe, $L_2 = 40 - 25 = 15$ m

Diameter of second pipe, $D_2 = 300$ mm $= 0{\cdot}3$ m

Height of water, $H = 8$ m

Co-efficient of friction, $f = 0{\cdot}01$

(*i*) **Rate of flow, Q :**

Applying Bernoulli's equation to the free water surface (F.W.S.) in the tank and outlet of the pipe as shown in Fig. 12·8., we get

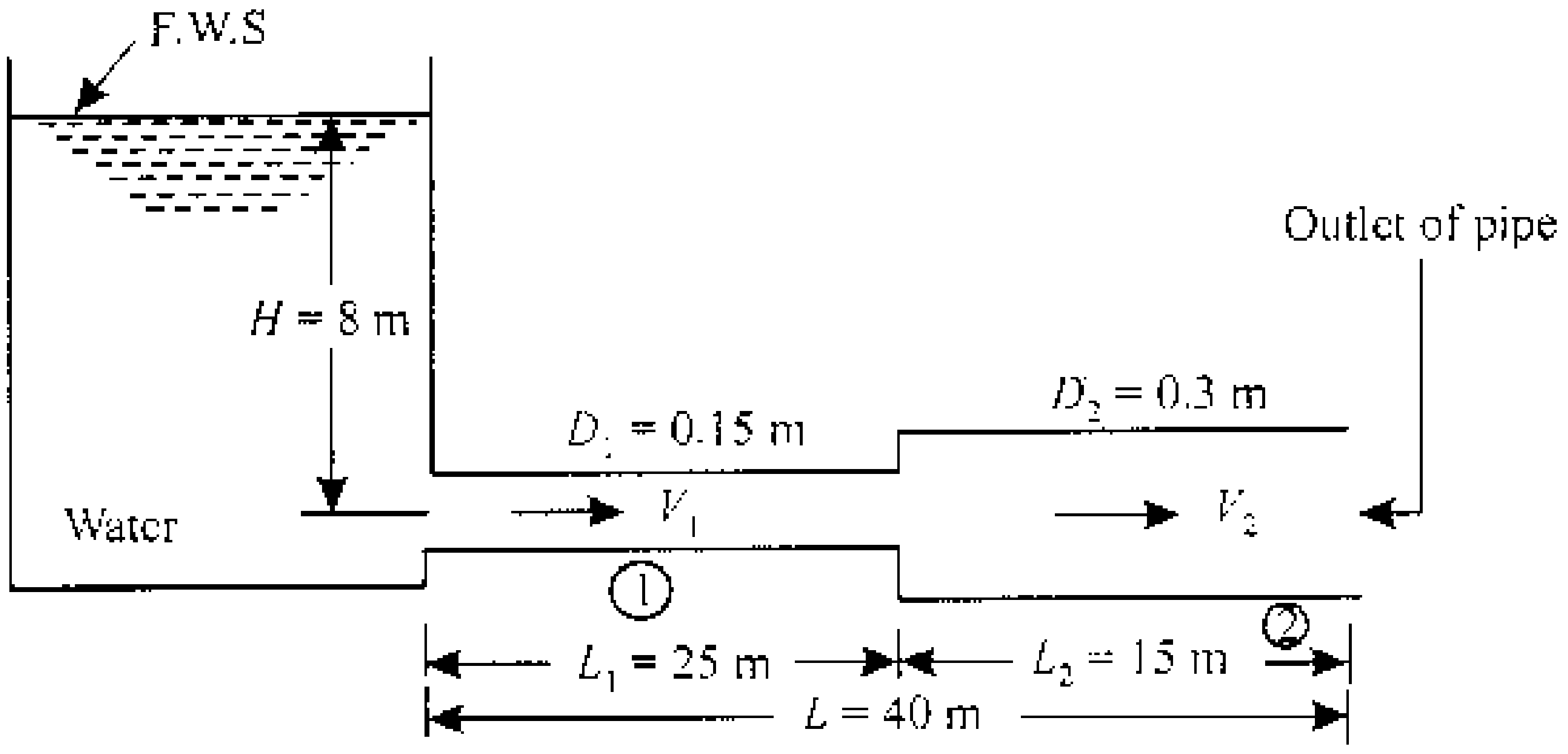

Fig. 12.8

$$\frac{p_1}{w}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{w}+\frac{V_2^2}{2g}+z_2 + \text{all losses}$$

$$0 + 0 + 8{\cdot}0 = 0 + \frac{V_2^2}{2g} + 0 + h_i + h_{f_1} + h_e + h_{f_2} \qquad \ldots(i)$$

where, V_2 = Velocity of water at the outlet of pipe,

h_i = Loss of head at entrance = $0{\cdot}5 \frac{V_1^2}{2g}$,

h_{f_1} = Head lost due to friction in pipe 1 = $\frac{4fL_1V_1^2}{D_1 \times 2g}$,

h_e = Loss of head due to sudden enlargement = $\frac{(V_1 - V_2)^2}{2g}$, and

h_{f_2} = Head lost due to friction in pipe 2 = $\frac{4fL_2V_2^2}{D_2 \times 2g}$

From continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{(\pi/4) \times D_2^2 \times V_2}{(\pi/4) \times D_1^2}$$

$$= \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0{\cdot}3}{0{\cdot}15}\right)^2 \times V_2 = 4V_2$$

Substituting the value of V_1 in different head losses, we have

$$h_i = \frac{0{\cdot}5\, V_1^2}{2g} = \frac{0{\cdot}5\,(4 \times V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f_1} = \frac{4fL_1V_1^2}{D_1 \times 2g} = \frac{4 \times 0{\cdot}01 \times 25 \times (4 \times V_2)^2}{0{\cdot}15 \times 2g} = 106{\cdot}6\, \frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f_2} = \frac{4fL_2V_2^2}{D_2 \times 2g} = \frac{4 \times 0{\cdot}01 \times 15 \times V_2^2}{0{\cdot}3 \times 2g} = \frac{2V_2^2}{2g}$$

Substituting the values of these losses in eqn. (i), we get

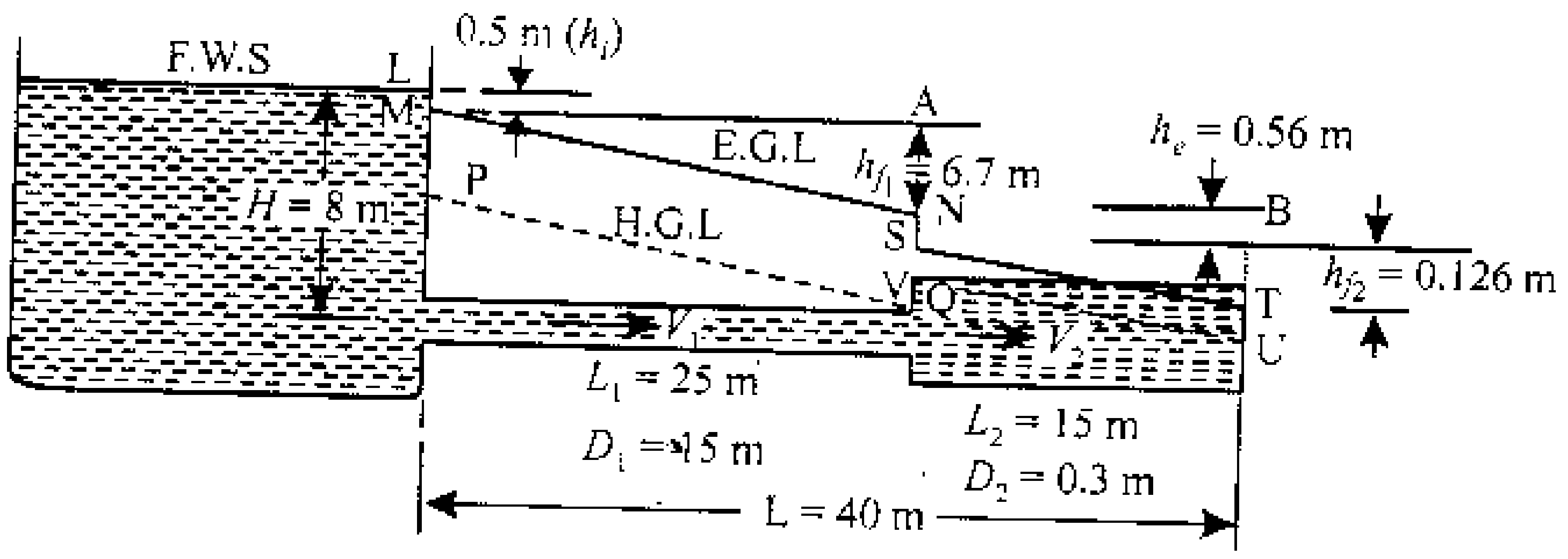

Fig. 12.9

$$8 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + \frac{106{\cdot}6\, V_2^2}{2g} + \frac{9V_2^2}{2g} + \frac{2V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}(1 + 8 + 106{\cdot}6 + 9 + 2) = 126{\cdot}6\, \frac{V_2^2}{2g}$$

$$\therefore \quad V_2 = \sqrt{\frac{8 \times 2g}{126{\cdot}6}} = \sqrt{\frac{8 \times 2 \times 9{\cdot}81}{126{\cdot}6}} = 1{\cdot}11 \text{ m/s}$$

Hence, rate of flow, $Q = A_2V_2 = \frac{\pi}{4} \times 0{\cdot}3^2 \times 1{\cdot}11 =$ **0·078 m³/s (Ans.)**

(*ii*) **E.G.L. and H.G.L. :**

The various head losses are : (Refer to Fig. 12·9)

$$h_i = \frac{8V_2^2}{2g} = \frac{8 \times (1{\cdot}11)^2}{2 \times 9{\cdot}81} = 0{\cdot}5 \text{ m}$$

$$h_{f1} = 106{\cdot}6\,\frac{V_2^2}{2g} = \frac{106{\cdot}6 \times (1{\cdot}11)^2}{2 \times 9{\cdot}81} = 6{\cdot}7 \text{ m}$$

$$h_e = \frac{9V_2^2}{2g} = \frac{9 \times (1{\cdot}11)^2}{2 \times 9{\cdot}81} = 0{\cdot}56 \text{ m}$$

$$h_{f2} = \frac{2V_2^2}{2g} = \frac{2 \times (1{\cdot}11)^2}{2 \times 9{\cdot}81} = 0{\cdot}126 \text{ m}$$

To draw E.G.L. and H.G.L. the following *procedure* is followed.

E.G.L. (Energy gradient line) :

The point L lies on F.W.S. (free water surface).

— Take $LM = h_i = 0{\cdot}5$ m

— From M draw a horizontal line. Taking MA equal to the length of the pipe (*i.e.*, L_1) draw a vertical line downward from the point A. Cut $AN = h_{f_1} = 6{\cdot}7$ m

— Join MN

— From N, draw a line NS vertically downward equal to h_e (= 0·56 m)

— From S, draw SB horizontal and from point U (which is lying on the centre of the pipe) draw a vertical line in the upward direction, meeting at B. From B take $BT = h_{f_2} = 0{\cdot}126$ m.

— Join ST

— The line $LMNST$ represents the energy gradient line (E.G.L.)

H.G.L. (Hydraulic gradient line) :

— From M, take $MP = \frac{V_1^2}{2g} = \frac{(4 \times 1{\cdot}11)^2}{2 \times 9.81} = 1{\cdot}0$ m

— Draw the line PQ parallel to the line MN

— From the point U, draw a line UV parallel to the

— Join QV

— The line $PQVU$ represents the hy

Example 12·17. *Two reservoirs a cm diameter and length 6 m and the oth water levels in the two reservoirs is 6 m, Take f = 0·04.*

Solution. Refer to Fig. 12·10. *Given : D*

$L_2 = 16$ m; total losses = 6 m; $f = 0{\cdot}04$.

From the continuing equation, we have A_1V

$$\frac{\pi}{4} \times 0{\cdot}15^2 \times V_1 = \frac{\pi}{4} \times 0{\cdot}225^2 \times V_2$$

Discharge,

Energy gradient

Example 12·18. *A pipe is horizontal as show BC are 25 m and 15 m resp line of the pipe and at C 1 m*

(i) The rate of flow, and

$\frac{V^2}{2g}$ $(\because V_B = V = 3{\cdot}073 \text{ m/s})$

$$\therefore \quad V_2 = \sqrt{\frac{8 \times 2g}{126 \cdot 6}} = \sqrt{\frac{8 \times 2 \times 9 \cdot 81}{126 \cdot 6}} = 1 \cdot 11 \text{ m/s}$$

Hence, rate of flow, $Q = A_2 V_2 = \frac{\pi}{4} \times 0 \cdot 3^2 \times 1 \cdot 11 =$ **0·078 m³/s (Ans.)**

(*ii*) **E.G.L. and H.G.L. :**

The various head losses are : (Refer to Fig. 12·9)

$$h_i = \frac{8V_2^2}{2g} = \frac{8 \times (1 \cdot 11)^2}{2 \times 9 \cdot 81} = 0 \cdot 5 \text{ m}$$

$$h_{f1} = 106 \cdot 6 \frac{V_2^2}{2g} = \frac{106 \cdot 6 \times (1 \cdot 11)^2}{2 \times 9 \cdot 81} = 6 \cdot 7 \text{ m}$$

$$h_e = \frac{9V_2^2}{2g} = \frac{9 \times (1 \cdot 11)^2}{2 \times 9 \cdot 81} = 0 \cdot 56 \text{ m}$$

$$h_{f2} = \frac{2V_2^2}{2g} = \frac{2 \times (1 \cdot 11)^2}{2 \times 9 \cdot 81} = 0 \cdot 126 \text{ m}$$

To draw E.G.L. and H.G.L. the following *procedure* is followed.

E.G.L. (Energy gradient line) :

The point L lies on F.W.S. (free water surface).

— Take $LM = h_i = 0 \cdot 5$ m

— From M draw a horizontal line. Taking MA equal to the length of the pipe (*i.e.*, L_1) draw a vertical line downward from the point A. Cut $AN = h_{f_1} = 6 \cdot 7$ m

— Join MN

— From N, draw a line NS vertically downward equal to h_e (= 0·56 m)

— From S, draw SB horizontal and from point U (which is lying on the centre of the pipe) draw a vertical line in the upward direction, meeting at B. From B take $BT = h_{f_2} = 0 \cdot 126$ m.

— Join ST

— The line $LMNST$ represents the energy gradient line (E.G.L.)

H.G.L. (Hydraulic gradient line) :

— From M, take $MP = \frac{V_1^2}{2g} = \frac{(4 \times 1 \cdot 11)^2}{2 \times 9.81} = 1 \cdot 0$ m $\qquad (\because V_1 = 4V_2)$

— Draw the line PQ parallel to the line MN

— From the point U, draw a line UV parallel to the line TS

— Join QV

— The line $PQVU$ represents the hydraulic gradient line (H.G.L.).

Example 12·17. *Two reservoirs are connected by a pipeline consisting of two pipes, one of 15 cm diameter and length 6 m and the other of diameter 22·5 cm and 16 m length. If the difference of water levels in the two reservoirs is 6 m, calculate the discharge and draw the energy gradient line. Take f = 0·04.* **(AMIE)**

Solution. Refer to Fig. 12·10. *Given :* $D_1 = 15$ cm = 0·15 m; $L_1 = 6$ m; $D_2 = 22 \cdot 5$ cm = 0·225 m; $L_2 = 16$ m; total losses = 6 m; $f = 0 \cdot 04$.

From the continuing equation, we have $A_1 V_1 = A_2 V_2$

$$\frac{\pi}{4} \times 0 \cdot 15^2 \times V_1 = \frac{\pi}{4} \times 0 \cdot 225^2 \times V_2 \qquad \therefore V_1 = 2 \cdot 25\, V_2$$

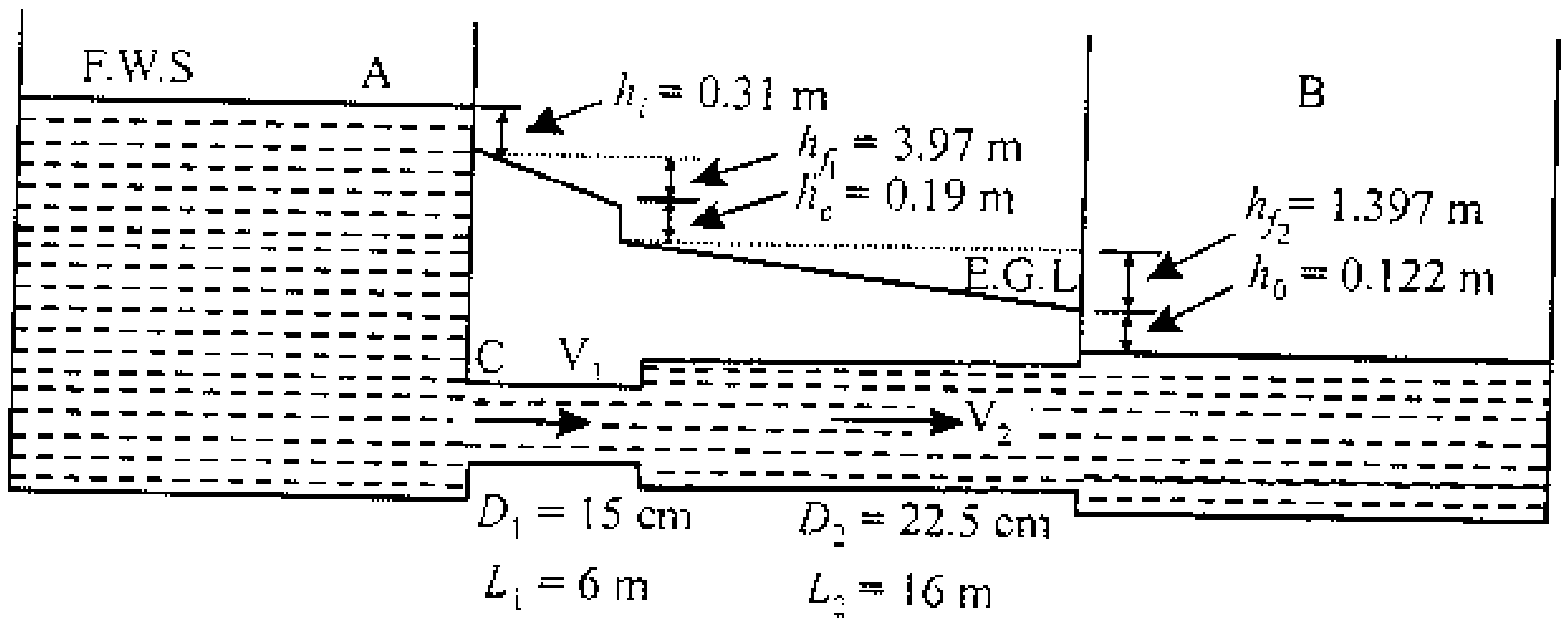

Fig. 12.10

Loss of head at entrance to a pipe, $h_i = \frac{0.5\,V_1^2}{2g}$

Loss of head due to friction in pipe AB,

$$h_{f_1} = \frac{4fL_1V_1^2}{D_1 \times 2g} = \frac{4 \times 0.04 \times 6 \times V_1^2}{0.15 \times 2g} = 6.4\,\frac{V_1^2}{2g}$$

Loss of head due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{\left(V_1 - \frac{V_1}{2.25}\right)^2}{2g} = 0.308\,\frac{V_1^2}{2g}$$

Loss of head due to friction in the pipe, BC,

$$h_{f_2} = \frac{4fL_2V^2}{D_2 \times 2g} = \frac{4 \times 0.04 \times 16 \times \left(\frac{V_1}{2.25}\right)^2}{0.225 \times 2g} = 2.25\,\frac{V_1^2}{2g}$$

Loss of head due to friction in the pipe BC,

$$h_0 = \frac{V_2^2}{2g} = \left(\frac{V_1}{2.25}\right)^2 \times \frac{1}{2g} = 0.197\,\frac{V_1^2}{2g}$$

Applying Bernoulli's equation to free water surface (F.W.S.) in the two tanks , we have

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_B^2}{w} + \frac{V_B^2}{2g} + z_B + \text{losses}$$

i.e., $p_A = p_B = 0,\ V_A = V_B = 0,\ z_A - z_B = 6$ m

Hence total losses = 6 m

i.e., $h_i + h_{f_1} + h_e + h_{f_2} + h_0 = 6$

$$\frac{0.5V_1^2}{2g} + 6.4\,\frac{V_1^2}{2g} + 0.308\,\frac{V_1^2}{2g} + 2.25\,\frac{V_1^2}{2g} + 0.197\,\frac{V_1^2}{2g} = 6$$

$$V_1 = 3.49 \text{ m}$$

Discharge, $Q = A_1V_1 = \frac{\pi}{4} \times 0.15^2 \times 3.49 =$ **0.0617 m³/s (Ans.)**

Energy gradient line is shown in the Fig. 12·10.

Example 12·18. *A pipe ABC connecting two reservoirs is 80 mm in diameter. From A to B the pipe is horizontal as shown in Fig. 12·11. and from B to C it falls by 3·5 metres. The lengths AB and BC are 25 m and 15 m respectively. If the water surface in the reservoir at A is 4 m above the centre-line of the pipe and at C 1 m above the line of the pipe, calculate :*

(*i*) *The rate of flow, and*

Neglect the loss at the bend but consider all other losses. Also draw the energy and hydraulic gradient lines. Take Darcy friction factor = 0·024 and $K_{entrance}$ = 0·5. **[IIT Delhi]**

Solution. Diameter of the pipe, $D = 80$ mm $= 0·08$ m

Area, $A = \frac{\pi}{4} \times 0·08^2 = 0·005026\ \text{m}^2$

Friction factor $(= 4f) = 0·024$

$K_{entrance} = 0·5$

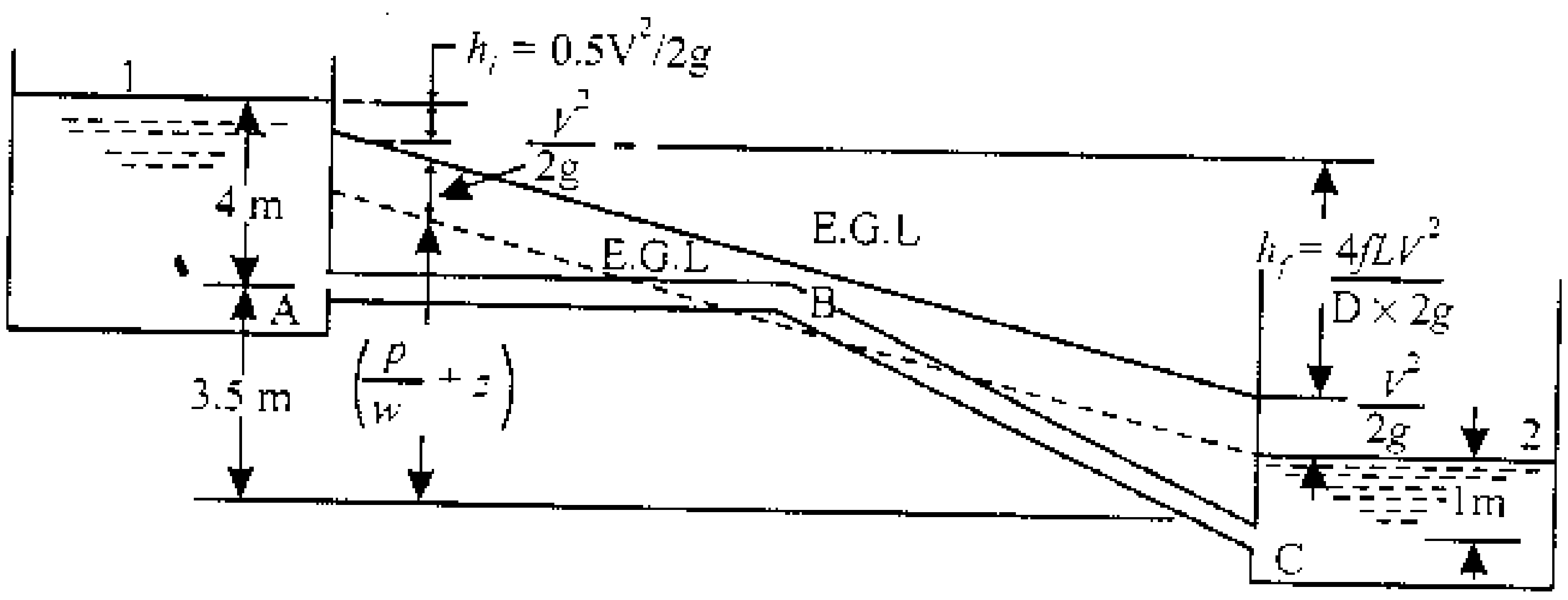

Fig. 12.11

(*i*) **The rate of flow, Q :**

Applying Bernoulli's equation between the water surfaces 1 and 2 in the two reservoirs (considering horizontal plane through C as datum), we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{loss at entrance} + h_f \text{ (loss due to friction)} + \frac{V^2}{2g}$$

$$0 + 0 + (4 + 3·5 - 1) = 0 + 0 + 0 + \frac{0·5\,V^2}{2g} + \frac{4fLV^2}{D \times 2g} + \frac{V^2}{2g}$$

(where, V = velocity of flow in the pipe)

or $$6·5 = \frac{0·5\,V^2}{2g} + \frac{0·024 \times (25 + 15) \times V^2}{0·08 \times 2g} + \frac{V^2}{2g}$$

or $$= \frac{V^2}{2g}(0·5 + 12 + 1) + 13·5\,\frac{V^2}{2g}$$

or $$V^2 = \frac{6·5 \times 2 \times 9·81}{13·5} = 9·446$$

or $V = 3·073$ **m/s (Ans.)**

$\therefore$ Flow rate $= A \times V = 0·005026 \times 3·073 =$ **0·01544 m³/s (Ans.)**

(*ii*) **Pressure head in the pipe at B, $\frac{p_B}{w}$:**

Applying Bernoulli's equation at A and B, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_B}{w} + \frac{V_B^2}{2g} + z_B + \frac{0·5\,V_B^2}{2g} + h_f$$

$$0 + 0 + 4 = \frac{p_B}{w} + \frac{V^2}{2g} + 0 + \frac{0·5\,V^2}{2g} + \frac{4fL_{AB}V^2}{D \times 2g}$$

$$4 = \frac{p_B}{w} + \frac{V^2}{2g} + \frac{0·5V^2}{2g} + \frac{0·024 \times 25 \times V^2}{0·08 \times 2g} \qquad (\because V_B = V = 3·073 \text{ m/s})$$

$$4 = \frac{p_B}{w} + \frac{V^2}{2g} + \frac{0{\cdot}5V^2}{2g} + \frac{7{\cdot}5V^2}{2g}$$

$$= \frac{p_B}{w} + \frac{9V^2}{2g}$$

or $$\frac{p_B}{w} = 4 - \frac{9V^2}{2g} = 4 - \frac{9 \times (3{\cdot}073)^2}{2 \times 9{\cdot}81}$$

$= -$ **0·33 m of water** (below atmosphere) **(Ans.)**

Energy gradient and hydraulic gradient lines (E.G.L. and H.G.L.):

— For plotting E.G.L. and H.G.L., we require the velocity head, (same throughout)

$$\frac{V^2}{2g} = \frac{(3{\cdot}073)^2}{2 \times 9{\cdot}81} = 0{\cdot}481 \text{ m}$$

— Total energy at B w.r.t. horizontal datum through C

$$= 3{\cdot}5 + \frac{p_B}{w} + \frac{V^2}{2g} = 3{\cdot}5 - 0{\cdot}33 + \frac{(3{\cdot}073)^2}{2 \times 9{\cdot}81}$$

$$= 3{\cdot}65 \text{ m}$$

Energy gradient and hydraulic gradient lines are shown firm and dotted respectively in the Fig. 12·11 H.G.L. *below the pipeline* near B indicates that pressure is *negative.*

Example 12·19. *Two reservoirs A and C having a difference of level of 15·5 m are connected by a pipeline ABC the elevation of point B being 4·0 m below the level of water in reservoir A. The length AB of the pipeline is 250 m, the pipe being made of mild steel having a friction co-efficient f_1, while the length BC is 450 m, the pipe having made of cast-iron having a friction co-efficient f_2. Both the lengths AB and BC have a diameter of 200 mm. A partially closed valve is located in the length BC at a distance of 150 m from reservoir C.*

If the flow through the pipeline is 3 m³/min, the pressure head at B is 0·5 m and the head loss at the valve is 5·0 m.

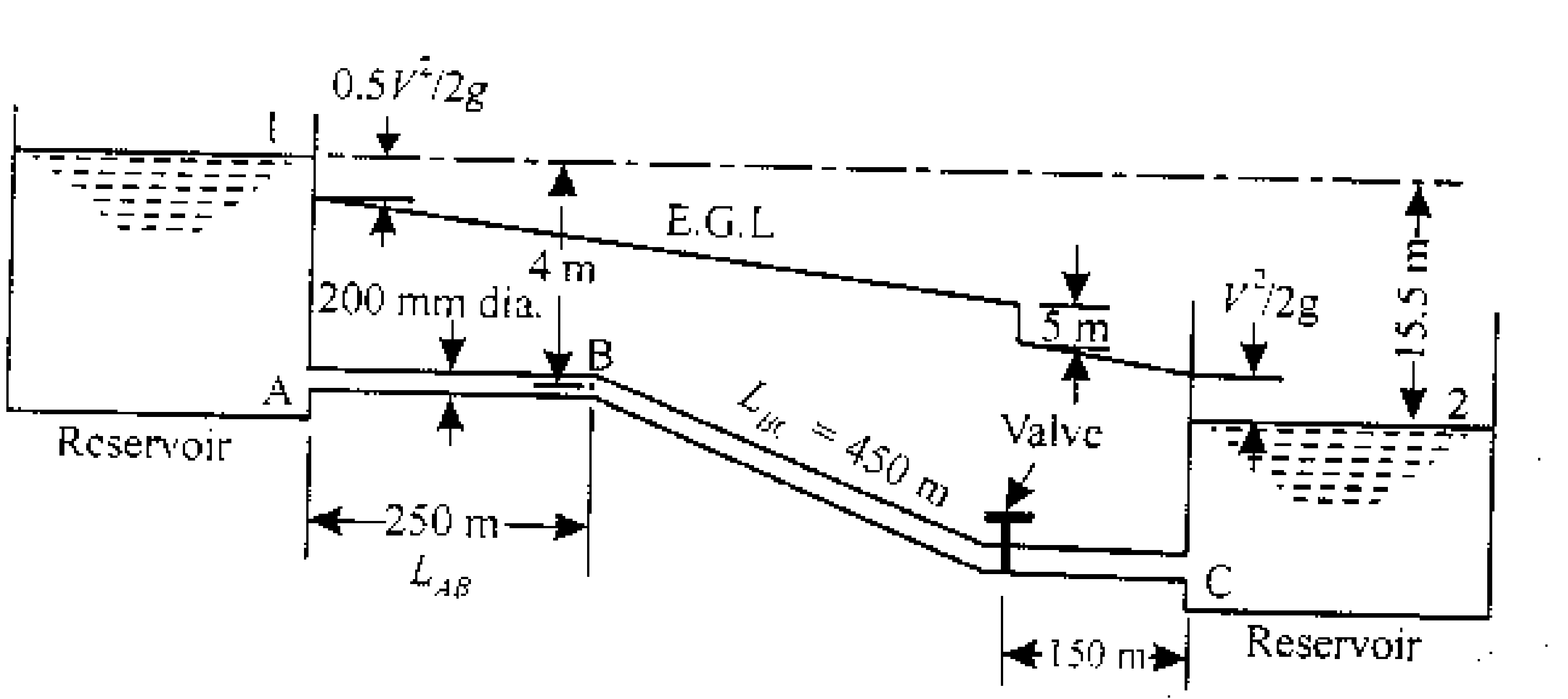

Fig. 12.12

(*i*) *Find the friction coefficients f_1 and f_2.*

(*ii*) *Draw the hydraulic grade line of the pipeline and indicate on the diagram head loss values at significant points. Take into account head loss at entrance and exit points of the pipeline.*

[UPSC Exams.]

Solution. Difference of water level between two reservoirs = 15·5 m

Diameter of the pipe ABC, D = 200 mm = 0·2 m

Length AB, L_{AB} = 250 m

Length BC, L_{BC} = 450 m

Discharge through the pipe, $Q = 3\ m^3/min = 0{\cdot}05\ m^3/s$

Pressure head at B, $h_B = \left(= \frac{p_B}{w}\right) = 0{\cdot}5$ m

Head loss at the valve = 5·0 m

(*i*) **Friction co-efficients f_1 and f_2 :**

Velocity in the pipe ABC, $V = \frac{Q}{\text{Area}} = \frac{0{\cdot}05}{\frac{\pi}{4} \times 0{\cdot}2^2} = 1{\cdot}59$ m/s

Applying Bernoulli's equation at '1' and at 'B', we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_B}{w} + \frac{V_B^2}{2g} + z_2 + \frac{0{\cdot}5V_B^2}{2g} + (h_f)_{AB}$$

$$0 + 0 + 4 = 0{\cdot}5 + \frac{V_B^2}{2g} + 0 + \frac{0{\cdot}5V_B^2}{2g} + \frac{4f_1 L_{AB} V_B^2}{D \times 2g}$$

$$4 = 0{\cdot}5 + \frac{V^2}{2g} + \frac{0{\cdot}5V^2}{2g} + \frac{4f_1 \times 250 \times V^2}{0{\cdot}2 \times 2g} \qquad (\because V_B = V)$$

or,
$$4 = 0{\cdot}5 + \frac{(1{\cdot}59)^2}{2 \times 9{\cdot}81} + \frac{0{\cdot}5 \times (1{\cdot}59)^2}{2 \times 9{\cdot}81} + \frac{4f_1 \times 250 \times (1{\cdot}59)^2}{0{\cdot}2 \times 2 \times 9{\cdot}81}$$
$$= 0{\cdot}5 + 0{\cdot}129 + 0{\cdot}0644 + 644{\cdot}3 f_1$$

or, $f_1 = $ **0·0051 (Ans.)**

Applying Bernoulli's equation between '1' and '2' and considering all losses in the pipeline ABC including the exit loss, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \frac{0{\cdot}5V^2}{2g} + \frac{4f_1 L_{AB} V^2}{D \times 2g} + \frac{4f_2 L_{BC} V^2}{D \times 2g} + 5{\cdot}0 + \frac{V^2}{2g}$$

$$0 + 0 + 15{\cdot}5 = 0 + 0 + 0 + \frac{0{\cdot}5 \times (1{\cdot}59)^2}{2 \times 9{\cdot}81} + \frac{4 \times 0{\cdot}0051 \times 250 \times (1{\cdot}59)^2}{0{\cdot}2 \times 2 \times 9{\cdot}81}$$
$$+ \frac{4f_2 \times 450 \times (1{\cdot}59)^2}{0{\cdot}2 \times 2 \times 9{\cdot}81} + 5{\cdot}0 + \frac{(1{\cdot}59)^2}{2 \times 9{\cdot}81}$$

or, $15{\cdot}5 = 0{\cdot}0644 + 3{\cdot}28 + 1159{\cdot}6 f_2 + 5{\cdot}0 + 0{\cdot}1288$

or, $f_2 = $ **0·00606 (Ans.)**

(*ii*) **H.G.L. (hydraulic gradient line) :**

Fig. 12.9 shows the E.G.L. (energy gradient line), H.G.L. will be $\frac{V^2}{2g}$ below the E.G.L.

Example 12.20. *Water is being pumped at the rate of 0·02 m³/s to an overhead tank through a 150 mm diameter 300 m long delivery pipe. In the tank, the pipe discharges freely at height of 15 m above the pump. If the Darcy-Weisbach friction factor = 0·03 for the pipe, determine :*

(*i*) *The pressure developed by the pump on its delivery side, and*

(*ii*) *The power delivered to water by the pump.*

Draw also the hydraulic gradient from the pump to the tank. Assume that the first 285 m of the delivery pipe is horizontal and the rest is vertical. **[UPSC Civil Services Exams.]**

Solution. Rate of flow, $Q = 0{\cdot}02\ m^3/s$

Diameter of the pipe, D = 150 mm = 0·15 m

Length of the pipe, L = 300 m

Darcy-Weisbach friction factor $(4f) = 0{\cdot}03$

(*i*) **The pressure developed by the pump on its delivery side :**

Velocity of flow, $V = \frac{Q}{\text{Area}} = \frac{0{\cdot}02}{\frac{\pi}{4} \times 0{\cdot}15^2} = 1{\cdot}132$ m/s

Let, p = pressure (gauge) just on the delivery side of the pump.

Applying Bernoulli's equation to the section just on the delivery side of the pump and to the discharge end of the pipeline where the gauge pressure is zero *i.e.*, sections 1 and 2, we have

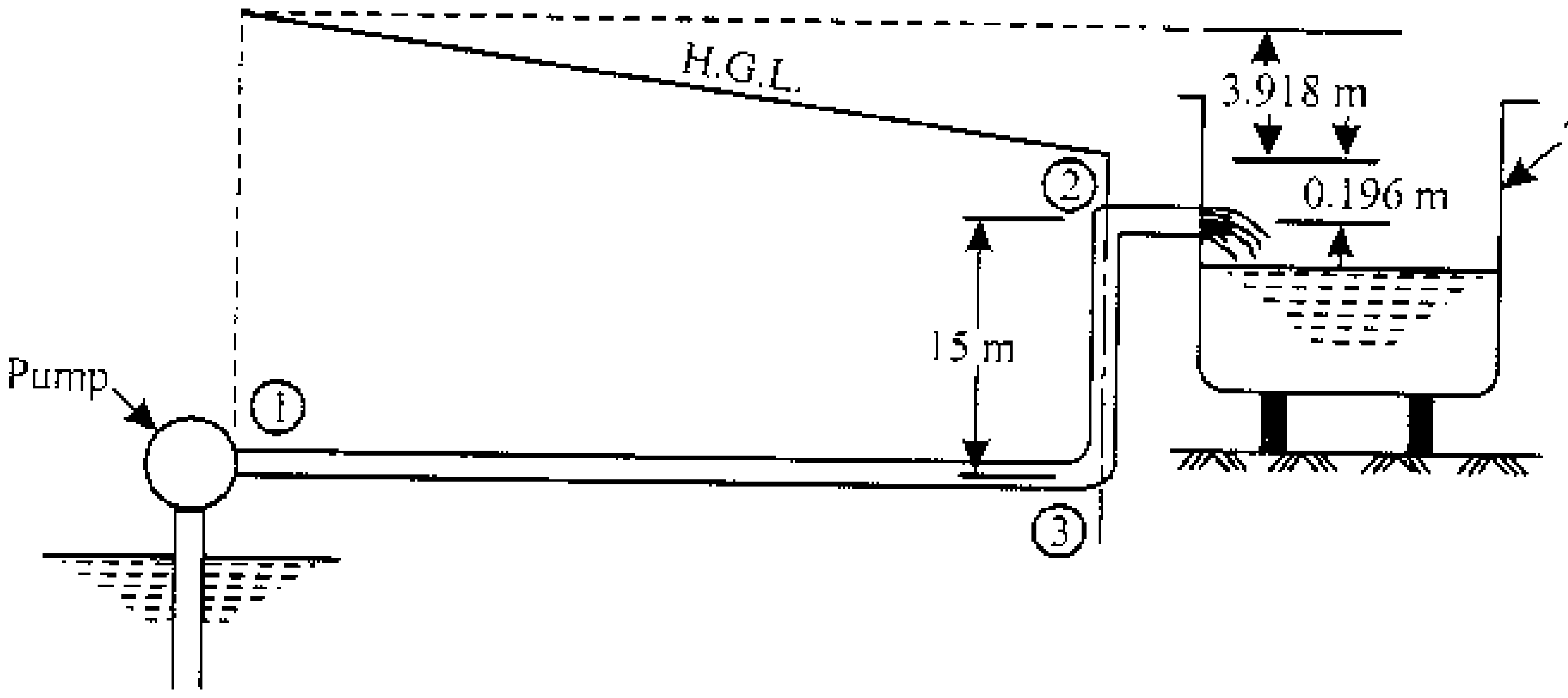

Fig. 12.13

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f \qquad (h_f = \text{head loss due to friction})$$

$$\frac{p_1}{w} + \frac{(1{\cdot}132)^2}{2 \times 9{\cdot}81} + 0 = 0 + \frac{(1{\cdot}132)^2}{2 \times 9.81} + 15 + \frac{0{\cdot}03 \times 300 \times (1{\cdot}132)^2}{0{\cdot}15 \times 2 \times 9{\cdot}81}$$

$$\left(\because\ h_f = \frac{fLV^2}{D \times 2g} \text{ where, } L = 285 + 15 = 300 \text{ m}\right)$$

$$\frac{p_1}{w} + 0{\cdot}0653 = 0{\cdot}0653 + 15 + 3{\cdot}918$$

or, $$\frac{p_1}{w} = 18{\cdot}918 \text{ m}$$

Hence the pressure developed by the pump on the delivery side

= 18·918 m of water (Ans.)

(*ii*) **The power delivered to water by the pump, P :**

$$P = wQh_f = 9{\cdot}81 \times 0{\cdot}02 \times 3{\cdot}918 \text{ kW}$$

= 0·768 kW (Ans.)

[**Note :** The power required to drive the pump $= \frac{wQh_f}{\eta}$, where η is the efficiency of the pump.]

H.G.L. (hydraulic gradient line) :

Head loss in 15 m vertical length of pipeline

$$= \frac{h_f}{300} \times 15 = \frac{3{\cdot}918}{300} \times 15 = 0{\cdot}196 \text{ m}$$

Now piezometric head $\left(\frac{p}{w} + z\right)$ at:

Section 1 = 19·114 m (*i.e.*, 18·918 + 0·196 = 19·114 m)

Section 2 = 15·196 m (*i.e.*, 19·114 – 3·198 = 15·196 m)

Section 3 = 15 m (*i.e.*, 15·196 − 0·196 = 15 m)

The HGL from the pump to the overhead tank is plotted by marking the ordinates of piezometric heads at 1, 2 and 3 (as above) and joining these by straight lines as shown in Fig. 12·13.

Example 12.21. *A pipeline ABC 200 m long, is laid on an upward slope of 1 in 40. The length of the portion AB is 100 m and its diameter is 100 mm. At B the pipe section suddenly enlarges to 200 mm diameter and remains so for the remainder of its length BC, 100 m. A flow of 0·02 m³/s. is pumped into the pipe at its lower end A and is discharged at the upper end C into a closed tank. The pressure at the supply end A is 200 kN/m².*

(*i*) *What is the pressure at C?*

(*ii*) *Draw the energy gradient and hydraulic gradient lines.*

Assume co-efficient of friction f = 0·008.

Solution. Length of pipe, $ABC = 200$ m

Slope of the pipe, = 1 in 40

Length of pipe AB, $L_{AB} = 100$ m

Diameter of the pipe AB, $D_{AB} = 100$ mm = 0·1 m

Length of pipe BC, $L_{BC} = 100$ m

Diameter of the pipe BC, $D_{BC} = 200$ mm = 0·2 m

Co-efficient of friction, $f = 0{\cdot}008$

Discharge, $Q = 0{\cdot}02\ m^3/s$

The pressure at the supply end, $p_A = 200\ kN/m^2$

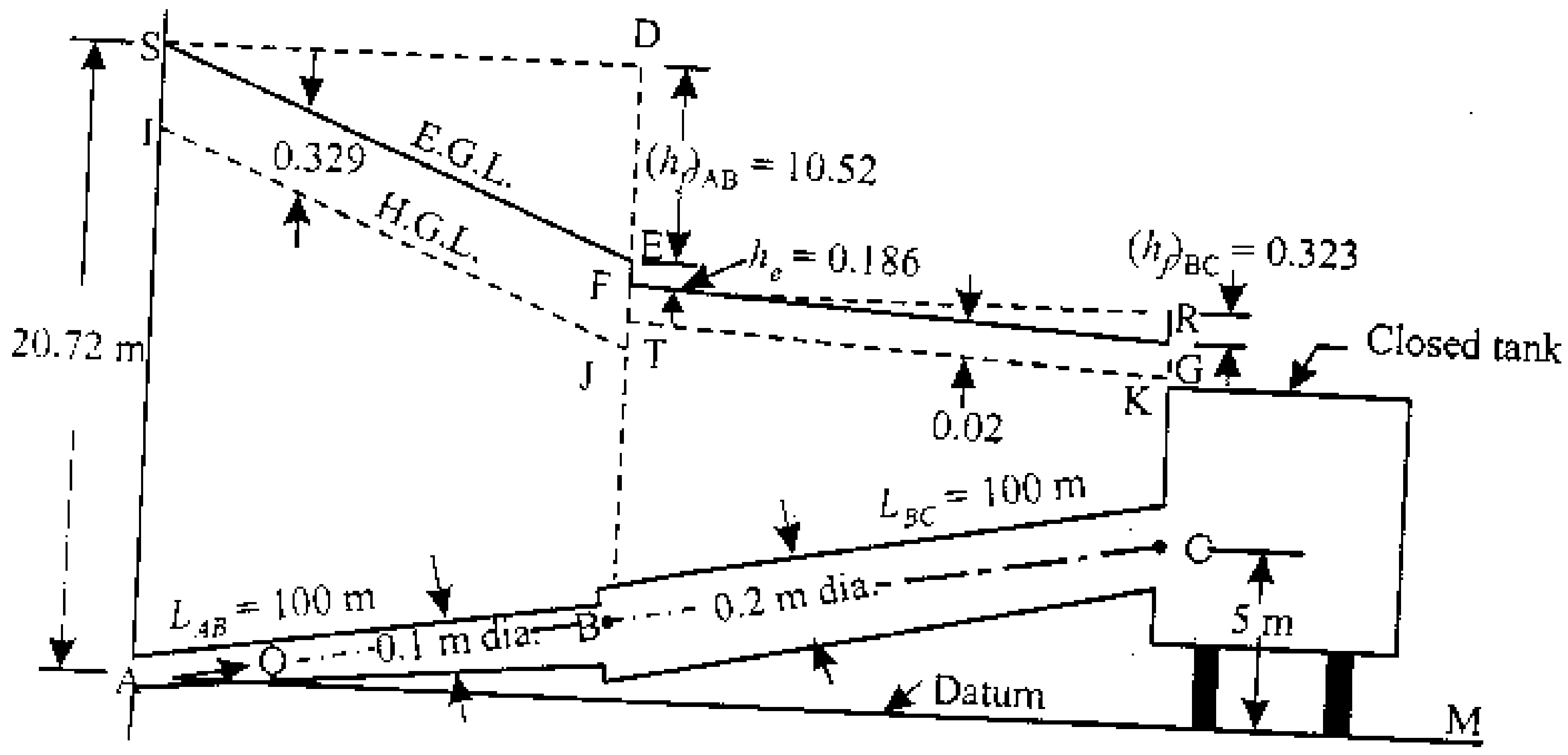

Fig. 12.14

Pressure at C, p_c :

Velocity of flow in pipe AB, $V_{AB} = \dfrac{0{\cdot}02}{(\pi/4)\times 0{\cdot}1^2} = 2{\cdot}54$ m/s

Velocity of flow in pipe BC, $V_{BC} = \dfrac{0{\cdot}02}{(\pi/4)\times 0{\cdot}2^2} = 0{\cdot}63$ m/s

Invoking Bernoulli's equation at points A and C, we have

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{w} + \frac{V_C^2}{2g} + z_C + (h_f)_{AB} + h_e + (h_f)_{BC} \quad ...(i)$$

where, $$(h_f)_{AB} = \frac{4fL_{AB}V_{AB}^2}{D_{AB} \times 2g} = \frac{4 \times 0.008 \times 100 \times 2.54^2}{0.1 \times 2 \times 9.81} = 10.52 \text{ m}$$

Loss of head due to sudden enlargement,

$$h_e = \frac{(V_{AB} - V_{BC})^2}{2g} = \frac{(2.54 - 0.63)^2}{2 \times 9.81} = 0.186 \text{ m}$$

$$(h_f)_{BC} = \frac{4fL_{BC}V_{BC}^2}{D_{BC} \times 2g} = \frac{4 \times 0.008 \times 100 \times 0.63^2}{0.2 \times 2 \times 9.81} = 0.323 \text{ m}$$

Substituting the values in eqn. (i), we get

$$\frac{200}{9.81} + \frac{(2.54)^2}{2 \times 9.81} + 0 = \frac{p_C}{w} + \frac{0.63^2}{2 \times 9.81} + 5.0 + 10.52 + 0.186 + 0.323$$

$$20.38 + 0.329 = \frac{p_C}{w} + 0.02 + 16.03$$

or $$\frac{p_C}{w} = 4.659 \text{ m}$$

or $$p_C = 9.81 \times 4.659 = \mathbf{45.7 \text{ kN/m}^2} \quad \textbf{(Ans.)}$$

(ii) **Energy gradient and hydraulic gradient lines :**

Pipe AB : Assuming the datum line passing through A, then total energy at A

$$= \frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{200}{9.81} + \frac{(2.54)^2}{2 \times 9.81} + 0 = 20.72 \text{ m}$$

Total energy at B = Total energy at $A - (h_f)_{AB}$

$$= 20.72 - 10.52 = 10.2 \text{ m}$$

Also, $$\frac{V_C^2}{2g} = \frac{(0.63)^2}{2 \times 9.81} = 0.02 \text{ m}$$

Energy gradient line (E.G.L.)

— Draw a horizontal line AM as shown in Fig. 12·14.

— Draw the centreline of the pipe in such a way that slope of the pipe is 1 in 40. Thus, point C will be at a height of $\frac{1}{40} \times 200 = 5$ m from the line AM.

— Draw a vertical line AS equal to total energy at A *i.e.*, $AS = 20.72$ m

— From point S, draw a horizontal line and from point B, a vertical line, meeting at D.

— From D, take vertical distance $DE = (h_f)_{AB} = 10.52$ m. Join SE.

— From E take $EF = h_e = 0.186$ m

— From F draw a hroizontal line and from C, a vertical line meeting at R. From R take $RG = (h_f)BC = 0.323$ m. Joint F to G. Then SEFG represents the ***energy gradient or total energy line.***

Hydraulic gradient line (H.G.L.)

— Draw the line IJ parallel to the line SE at a distance of $\frac{V_{AB}^2}{2g} = \frac{(2.54)^2}{2 \times 9.81} = 0.329$ m in the downward direction.

— Draw the line KT parallel to the line GF at a distance of $\frac{V_C^2}{2g} = \frac{(0.63)^2}{2 \times 9.81} = 0.02$ m . Join J to T.

The line IJTK represents the ***hydraulic gradient line.***

12·6. Pipes in Series or Compound Pipes

Fig. 12·15 shows a system of pipes in series.

Let, D_1, D_2, D_3 = Diameters of pipes 1, 2 and 3 respectively,

L_1, L_2, L_3 = Length of pipes 1, 2 and 3 respectively,

V_1, V_2, V_3 = Velocities of flow through pipes 1, 2 and 3 respectively

f_1, f_2, f_3 = Co-efficients of friction for pipes 1, 2 and 3 respectively, and

H = Difference of water level in the two tanks.

As the rate of flow (Q) of water through each pipe is same, therefore,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

Also, the difference in liquid surface levels = sum of the various head losses in the pipes

i.e.,
$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^3}{2g} \quad ...(i)$$

where,
h_i = Head loss at entrance $= \frac{0{\cdot}5V_1^2}{2g}$

h_{f_1} = Head loss due to friction in pipe 1 $= \frac{4f_1L_1V_1^2}{D_1 \times 2g}$

h_c = Head loss at contraction $= \frac{0{\cdot}5V_2^2}{2g}$

h_{f_2} = Head loss due to friction in pipe 2 $= \frac{4f_2L_2V_2^2}{D_2 \times 2g}$

h_e = Head loss due to enlargement $= \frac{(V_2 - V_3)^2}{2g}$

h_{f_3} = Head loss due to friction in pipe 3 $= \frac{4f_3L_3V_3^2}{D_3 \times 2g}$

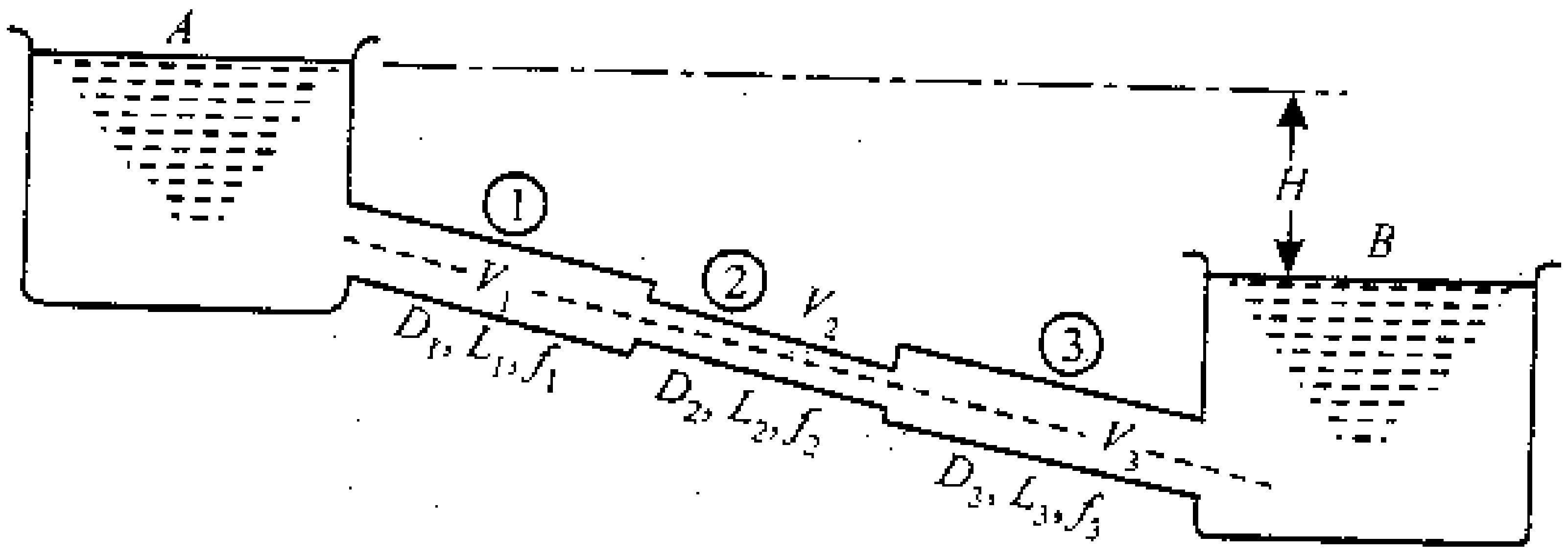

Fig. 12.15. Pipes in series.

Substituting the values in (*i*), we have

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g}$$

$$= \frac{0{\cdot}5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0{\cdot}5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g} \quad ...(12{\cdot}9)$$

If *minor losses are neglected*, then above equation becomes,

$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad ...(12{\cdot}10)$$

$f_1 = f_2 = f_3 = f$, then

$$H = \frac{4fL_1V_1^2}{D_1 \times 2g} + \frac{4fL_2V_2^2}{D_2 \times 2g} + \frac{4fL_3V_3^2}{D_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{D_1} + \frac{L_2V_2^2}{D_2} + \frac{L_3V_3^2}{D_3}\right] \quad ...(12\cdot11)$$

Example 12·22. *Three pipes of diameters 300 mm, 200 mm and 400 mm and lengths 450 m, 255 m and 315 m respectively are connected in series. The difference in water surface levels in two tanks is 18 m. Determine the rate of flow of water if co-efficients of friction are 0·0075, 0·0078 and 0·0072 respectively considering :*

(*i*) *Minor losses also, and*

(*ii*) *Neglecting minor losses.*

Solution. Pipe 1 : $L_1 = 450$ m, $= D_1 = 300$ mm $= 0{\cdot}3$ m, $f_1 = 0{\cdot}0075$

Pipe 2 : $L_2 = 255$ m, $D_2 = 200$ mm $= 0{\cdot}2$ m, $f_2 = 0{\cdot}0078$

Pipe 3 : $L_3 = 315$ m, $D_3 = 400$ mm $= 0{\cdot}4$ m, $f_3 = 0{\cdot}0072$

Difference of water level, $H = 18$ m.

(*i*) **Considering minor losses :**

Let V_1, V_2 and V_3 be the velocities in Ist, 2nd, and 3rd pipe respectively.

From continuity considerations, we have

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{(\pi/4) \times D_1^2}{(\pi/4) \times D_2^2} \times V_1 = \frac{D_1^2}{D_2^2} \times V_1 = \left(\frac{0{\cdot}3}{0{\cdot}2}\right)^2 V_1 = 2{\cdot}25\, V_1$$

and,
$$V_3 = \frac{A_1V_1}{A_3} = \frac{(\pi/4) \times D_1^2}{(\pi/4) \times D_3^2} \times V_1 = \frac{D_1^2}{D_3^2} \times V_1 = \left(\frac{0{\cdot}3}{0{\cdot}4}\right)^2 V_1 = 0{\cdot}5625\, V_1$$

We know that
$$H = \frac{0{\cdot}5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0{\cdot}5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g} \quad ...[\text{Eqn. } (12\cdot9)]$$

$$18 = \frac{0{\cdot}5V_1^2}{2g} + \frac{4 \times 0{\cdot}0075 \times 450 \times V_1^2}{0{\cdot}3 \times 2g} + \frac{0{\cdot}5 \times (2{\cdot}25\, V_1)^2}{2g} + \frac{4 \times 0{\cdot}0078 \times 255 \times (2{\cdot}25V_1)^2}{0{\cdot}2 \times 2g}$$
$$+ \frac{(2{\cdot}25\, V_1 - 0{\cdot}5625\, V_1)^2}{2g} + \frac{4 \times 0{\cdot}0072 \times 315 \times (0{\cdot}5625\, V_1)^2}{0{\cdot}4 \times 2g} + \frac{(0{\cdot}5625\, V_1)^2}{2g}$$

$$18 = \frac{V_1^2}{2g}(0{\cdot}5 + 45 + 2{\cdot}53 + 201{\cdot}4 + 2{\cdot}847 + 7{\cdot}176 + 0{\cdot}316)$$

$$= 259{\cdot}77\, \frac{V_1^2}{2g}$$

or,
$$V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{259{\cdot}77}} = 1{\cdot}166 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_1 \times V_1 = (\pi/4) \times 0{\cdot}3^2 \times 1{\cdot}166 =$ **0·0824 m³/s (Ans.)**

(*ii*) **Neglecting minor losses :**

We know that,
$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad ...[\text{Eqn. } (12\cdot10)]$$

$$18\cdot0 = \frac{V_1^2}{2g}\left(\frac{4\times 0\cdot0075\times 450}{0\cdot3}+\frac{4\times 0\cdot0078\times 255\times 2\cdot25^2}{0\cdot2}+\frac{4\times 0\cdot0072\times 315\times (0\cdot5625)^2}{0\cdot4}\right)$$

$$= \frac{V_1^2}{2g}(45 + 201\cdot4 + 7\cdot176) = 253\cdot57 \times \frac{V_1^2}{2g}$$

or, $$V_1 = \sqrt{\frac{18\times 2\times 9\cdot81}{253\cdot57}} = 1\cdot18 \text{ m}$$

$\therefore$ Discharge, $Q = A_1 V_1 = (\pi/4)\times 0\cdot3^2 \times 1\cdot18 =$ **0·0834 m³/s (Ans.)**

Example 12·23. *Two reservoirs with a difference in elevation of 15 m are connected by the three pipes in series. The pipes are 300 m long of diameter 30 cm, 150 m long of 20 cm diameter, and 200 m long of 25 cm diameter respectively. The friction factors (f) in the relation*

$$h_f = \frac{fLV^2}{D\times 2g}$$

for the three pipes are, respectively, 0·018, 0·020 and 0·019, and which account for friction and all losses. Further the contractions and expansions are sudden. Determine the flow rate in l/s. The loss co-efficient for sudden contraction from dia. 30 to 20 cm = 0·24. **(AMIE Summer, 2001)**

Solution. Refer to Fig. 12.16. *Given :* $D_1 = 30$ cm $= 0\cdot3$ m; $L_1 = 300$ m; $D_2 = 20$ cm $= 0\cdot2$ m; $L_2 = 150$ m; $D_3 = 25$ cm $= 0\cdot25$ m; $L_3 = 200$ m; $f_1 = 0\cdot018$; $f_2 = 0\cdot020$, $f_3 = 0\cdot019$.

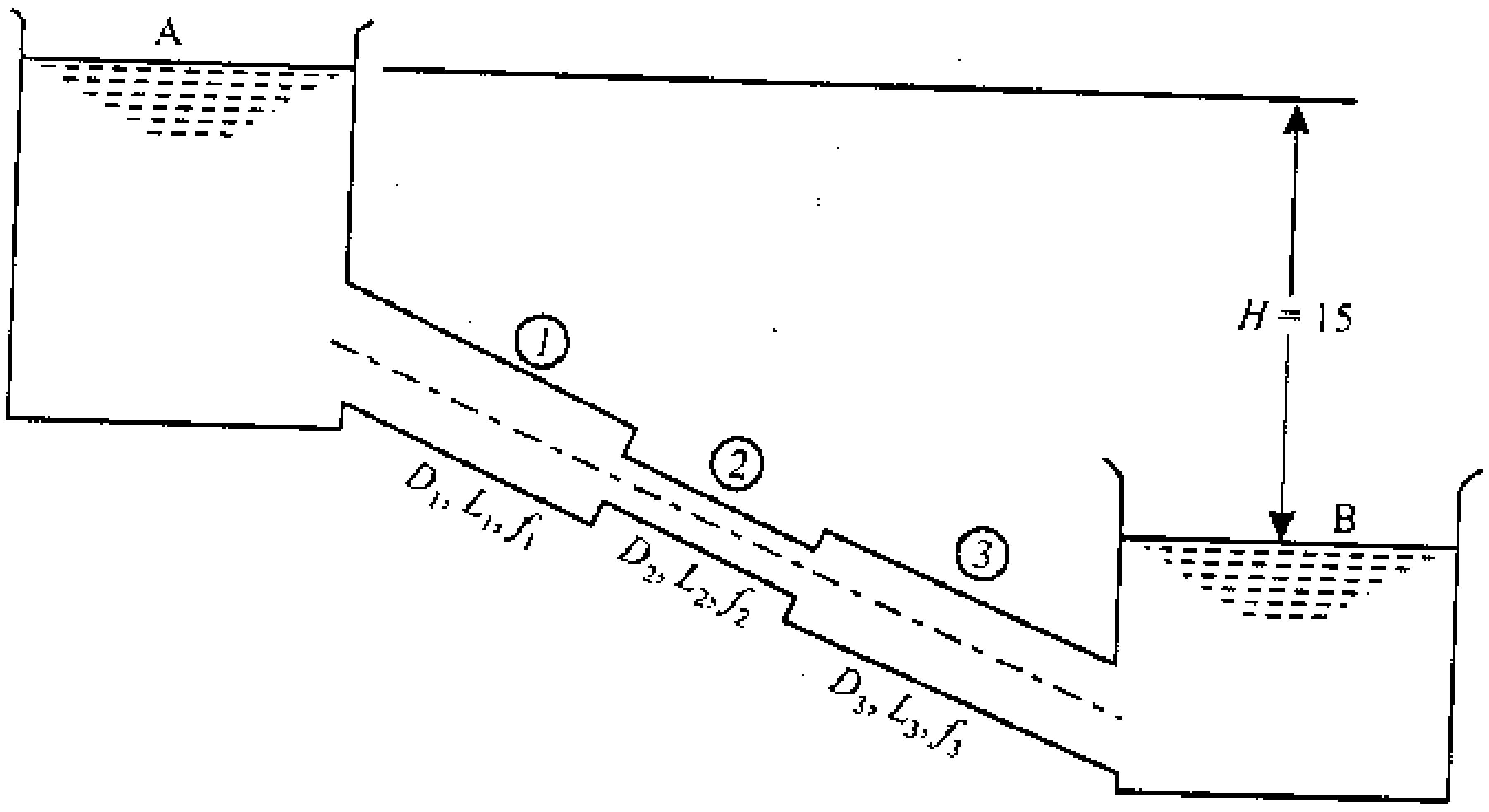

Fig. 12.16

Loss co-efficient for sudden contraction = 0·24

Flow rate in l/s, Q :

Various types of losses which occur in the pipelines 1, 2 and 3 are :

(*i*) Head loss at entrance, $h_i = 0\cdot5\times\frac{V_1^2}{2g} = \frac{0\cdot5}{2\times 9\cdot81}\times\left[\frac{4Q}{\pi\times 0\cdot30^2}\right] = 5\cdot1\,Q^2$

(*ii*) Head loss due to friction in pipe 1, $h_{f_1} = \frac{f_1 L_1 V_1^2}{D_1\times 2g} = \frac{f_1\times L_1}{D_1\times 2g}\left[\frac{4Q}{\pi D_1^2}\right]^2$

$$= \frac{0\cdot018\times 300}{0\cdot3\times 2\times 9\cdot81}\left[\frac{4Q}{\pi\times 0\cdot3^2}\right]^2 = 183\cdot6\,Q^2$$

Head loss at contraction, $h_c = 0\cdot24\,\frac{V_2^2}{2g} = \frac{0\cdot24}{2g}\left[\frac{4Q}{\pi D_2^2}\right]^2$

$$= \frac{0{\cdot}24}{2 \times 9{\cdot}81}\left[\frac{4Q}{\pi \times 0{\cdot}2^2}\right]^2 = 12{\cdot}394\ Q^2$$

(*iv*) Head loss due to friction in pipe 2, $h_{f_2} = \dfrac{f_2 L_2 V_2^2}{D_2 \times 2g} = \dfrac{f_2 \times L_2}{D_2 \times 2g}\left[\dfrac{4Q}{\pi D_2^2}\right]^2$

$$= \frac{0{\cdot}02 \times 150}{0{\cdot}2 \times 2 \times 9{\cdot}81}\left[\frac{4Q}{\pi \times 0{\cdot}2^2}\right]^2 = 774{\cdot}267\ Q^2$$

(*v*) Head loss due to sudden enlargement, $h_e = \dfrac{(V_2 - V_3)^2}{2g} = \dfrac{1}{2g}\left[\dfrac{4Q}{\pi D_2^2} - \dfrac{4Q}{\pi D_3^2}\right]^2$

$$= \frac{16Q^2}{2 \times 9{\cdot}81 \times \pi^2}\left[\frac{1}{0{\cdot}2^2} - \frac{1}{0{\cdot}25^2}\right]^2 = 6{\cdot}69\ Q^2$$

(*vi*) Head loss due to friction in pipe 3, $h_{f_3} = \dfrac{f_3 L_3 V_3^2}{D_3 \times 2g} = \dfrac{f_3 L_3}{D_3 \times 2g}\left[\dfrac{4Q}{\pi D_3^2}\right]^2$

$$= \frac{0{\cdot}019 \times 200}{0{\cdot}25 \times 2 \times 9{\cdot}81}\left[\frac{4Q}{\pi \times 0{\cdot}25^2}\right]^2 = 321{\cdot}518\ Q^2$$

(*vii*) Head loss at the exit, $h_0 = \dfrac{V_3^2}{2g} = \dfrac{1}{2g}\left[\dfrac{4Q}{\pi D_3^2}\right]^2 = \dfrac{1}{2 \times 9{\cdot}81}\left[\dfrac{4Q}{\pi \times 0{\cdot}25^2}\right]^2 = 21{\cdot}152\ Q^2$

Applying the Bernoulli's equation between the water surfaces of the two reservoirs, we get

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{w} + \frac{V_B^2}{2g} + z_B + \text{all losses.}$$

$$0 + 0 + 15 = 0 + 0 + 0 + (5{\cdot}1 + 183{\cdot}6 + 12{\cdot}394 + 774{\cdot}267 + 6{\cdot}69 + 321{\cdot}518 + 21{\cdot}152)\,Q^2$$

or, $Q = 0{\cdot}1064\ \text{m}^3/\text{s}$ or **106·4 *l/s*** **(Ans.)**

12·7. Equivalent Pipe

An **equivalent pipe** *is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.* The uniform diameter of the equivalent pipe is known as the *equivalent diameter* of the series or compound pipe.

Let, L_1, L_2, L_3 etc. = Lengths of pipes 1, 2, 3, etc.

D_1, D_2, D_3 etc. = Diameters of pipes 1, 2, 3, etc.,

H = Total head loss,

L = Length of the equivalent pipe, and

D = Diameter of the equivalent pipe.

Then, neglecting minor losses, total head loss,

$$h_f = h_{f_1} + h_{f_2} + h_{f_3} + \ldots$$

or,
$$H = \frac{4 f_1 L_1 V_1^2}{D_1 \times 2g} + \frac{4 f_2 L_2 V_2^2}{D_2 \times 2g} + \frac{4 f_3 L_3 V_3^2}{D_3 \times 2g} + \ldots \qquad \ldots(12\cdot[illegible])$$

(where, f_1, f_2 and f_3, etc. are co-efficients of friction)

Also, from continuity considerations,

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$= \frac{\pi}{4} \times D_1^2 V_1 = \frac{\pi}{4} \times D_2^2 V_2 = \frac{\pi}{4} \times D_3^2 V_3$$

$$\therefore \quad V_1 = \frac{4Q}{\pi D_1^2},\ V_2 = \frac{4Q}{\pi D_2^2},\ V_3 = \frac{4Q}{\pi D_3^2}$$

Substituting these values in eqn. (12·12), assuming $f_1 = f_2 = f_3$, etc. $= f$, we get

$$H = \frac{4fL_1 \times \left(\frac{4Q}{\pi D_1^2}\right)^2}{D_1 \times 2g} + \frac{4fL_2 \times \left(\frac{4Q}{\pi D_2^2}\right)^2}{D_2 \times 2g} + \frac{4fL_3 \times \left(\frac{4Q}{\pi D_3^2}\right)^2}{D_3 \times 2g} + \ldots$$

$$= \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\right) \qquad \ldots(12\cdot13)$$

Head loss in the equivalent pipe,

$$H = \frac{4fLV^2}{D \times 2g}$$ (assuming the same value of f as in compound pipe)

where, $$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} \times D^2} = \frac{4Q}{\pi D^2}$$

$$\therefore \quad H = \frac{4fL\left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g} = \frac{4 \times 16 fQ^2 f}{\pi^2 \times 2g}\left[\frac{L}{D^5}\right] \qquad \ldots(12\cdot14)$$

From eqns. (12·13) and (12·14), we have

$$\frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\right) = \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left(\frac{L}{D^5}\right)$$

or, $$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots \qquad \ldots(12.15)$$

Eqn. 12·15 is known as **Dupit's equation.** If the length of the equivalent pipe is equal to the length of the compound pipe *i.e.*, $L = (L_1 + L_2 + L_3 + \ldots)$, the diameter D of the equivalent pipe may be determined by using this equation. Sometimes a pipe of a given diameter D which is available may be required to be used as equivalent pipe to replace a compound pipe; in this case the length of the equivalent pipe may be required to be determined and the same may also be determined by using eqn. (12·15).

Example 12.24. *A piping system consists of three pipes arranged in series; the lengths of the pipes are 1200 m, 750 m and 600 m and diameters 750 mm, 600 mm and 450 mm respectively.*

(*i*) *Transform the system to an equivalent 450 mm diameter pipe, and*

(*ii*) *Determine an equivalent diameter for the pipe, 2550 m long.*

Solution. *Pipe 1*: $L_1 = 1200$ m; $D_1 = 750$ mm $= 0·75$ m

Pipe 2: $L_2 = 750$ m; $D_2 = 600$ mm $= 0·6$ m

Pipe 3: $L_3 = 600$ m; $D_3 = 450$ mm $= 0·45$ m

(*i*) **Equivalent length, L :**

Diameter of the equivalent pipe, $D = 450$ mm $= 0·45$ m (*Given*)

Using the relation :

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

$$= \frac{1200}{(0{\cdot}75)^5} + \frac{750}{(0{\cdot}6)^5} + \frac{600}{(0{\cdot}45)^5}, \text{ we have}$$

$$\frac{L}{(0{\cdot}45)^5} = 5056{\cdot}8 + 9645 + 32515{\cdot}4 = 47217{\cdot}2$$

or, $L = 47217{\cdot}2 \times (0{\cdot}45)^5 =$ **871·3 m (Ans.)**

(*ii*) **Equivalent diameter, D :**

Length of the equivalent pipe, $L = 2550$ m (*Given*)

Now, $$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

or, $$\frac{2550}{D^5} = \frac{1200}{(0{\cdot}75)^5} + \frac{750}{(0{\cdot}6)^5} + \frac{600}{(0{\cdot}45)^5}$$

$$= 5056{\cdot}8 + 9645 + 32515{\cdot}4 = 47217{\cdot}2$$

or, $$D = \left(\frac{2550}{47217{\cdot}2}\right)^{1/5} = 0{\cdot}5578 \text{ m or } \mathbf{557{\cdot}8\ mm}\ \textbf{(Ans.)}$$

Example. 12·25. *A compound piping system consists of 1800 m of 50 cm diameter, 1200 m of 40 cm diameter and 600 m of 30 cm diameter pipes of the same material connected in series. What is the equivalent length of a 40 cm diameter pipe of the same material ? State clearly the assumption (s) made.* **(N.U.)**

Solution. *Given :* $L_1 = 1800$ m; $D_1 = 50$ cm $= 0{\cdot}5$ m; $L_2 = 1200$ m; $D_2 = 40$ cm $= 0{\cdot}4$ m; $L_3 = 600$ m; $D_3 = 30$ cm $= 0{\cdot}3$ m.

Equivalent length of a 0·4 m diameter pipe of the same material, L :

Refer to Fig. 12·17.

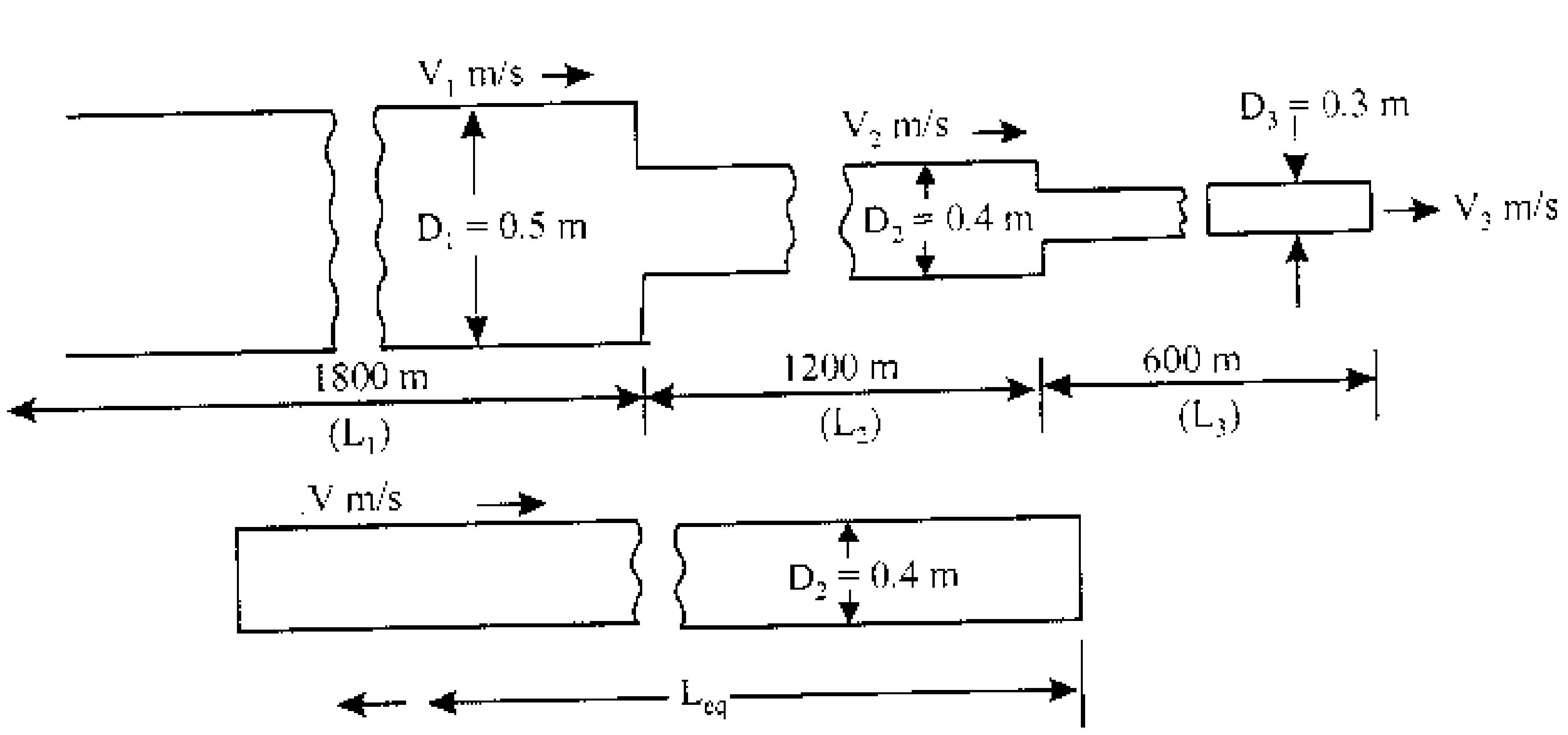

Fig. 12.17

Assumptions : 1. f is constant, and is the same for all the pipes.

2. The head loss due to contraction is ignored.

Using the relation : $$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$ ..[Eqn. (12·15)]

or, $$\frac{L}{(0{\cdot}4)^5} = \frac{1800}{(0{\cdot}5)^5} + \frac{1200}{(0{\cdot}4)^5} + \frac{600}{(0{\cdot}3)^5}$$

$$L = 1800 \times \left(\frac{0.4}{0.5}\right)^5 + 1200 + 600\left(\frac{0.4}{0.3}\right)^5$$

$$= 589.82 + 1200 + 2528.39 = \mathbf{4318.2\ m\ (Ans.)}$$

Example 12·26. *A pipe 150 mm in diameter and 15 m long is connected to the bottom of a tank, 15 metres long by 12 metres wide. The original head over the open end of the pipe is 5 metres. Find the time of emptying the tank, assuming the entrance to the pipe is sharp-edged.*

Assume $f = 0.01$ *in* $h_f = \frac{fLV^2}{D \times 2g}$. **[UPSC Exams.]**

Solution. Diameter of the pipe, $D = 150\ \text{mm} = 0.15\ \text{m}$

Length of the pipe, $L = 15$ m

Area of the tank, $= 12 \times 15 = 180\ \text{m}^2$

Friction factor, $f = 0.01$

Writing the Bernoulli's equation between the liquid surface at height 'h' and the lower end of the pipe, considering the entrance and the friction losses, we get

$$h + 15 = 0.5\frac{V^2}{2g} + \frac{V^2}{2g} + h_f$$

$$= 0.5\frac{V^2}{2g} + \frac{V^2}{2g} + \frac{0.01 \times 15 \times V^2}{0.15 \times 2g}$$

(where, V = velocity of flow in pipe)

$$= \frac{2.5V^2}{2g}$$

or, $$V = \sqrt{\frac{2g\,(h+15)}{2.5}}$$

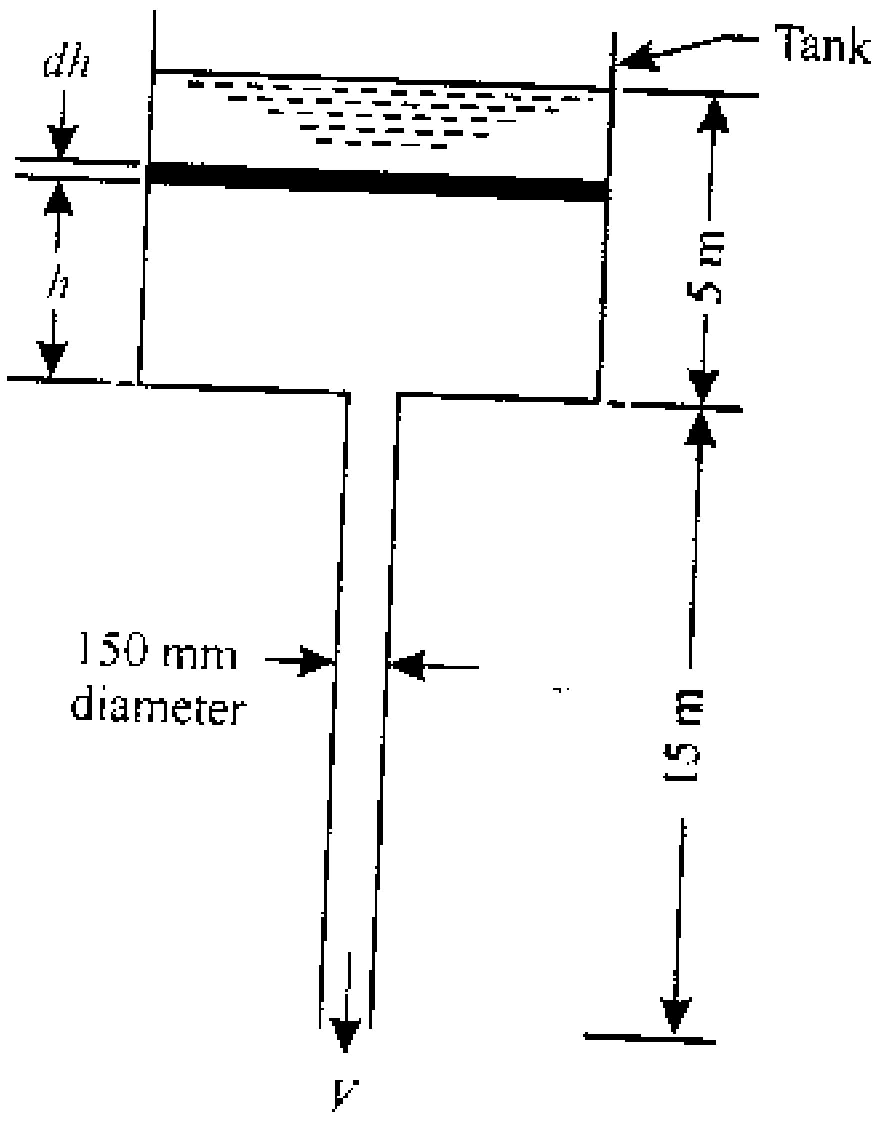

Fig. 12.18

Let us assume that the liquid surface falls a distance dh in time dt, then

$$12 \times 15 \times (-dh) = Q \cdot dt$$

or, $$180 \times (-dh) = (\pi/4) \times 0.15^2 \times \sqrt{\frac{2g\,(h+15)}{2.5}} \cdot dt$$

$$= 0.0495\sqrt{(h+15)}\ dt$$

or, $$\int_0^T dt = -\frac{180}{0.0495}\int_5^0 \frac{dh}{\sqrt{h+15}}$$

Let, $h + 15 = H$, then

$$T = -3636\int_{20}^{15}\frac{dh}{\sqrt{H}} = -3636 \times 2\left[\sqrt{H}\right]_{20}^{15} = -3636 \times 2\,(\sqrt{15} - \sqrt{20})$$

$$= 4357\ \text{s or } \mathbf{1.21\ hours\ (Ans.)}$$

2·8. Pipes in Parallel

The pipes are said to be in parallel (Fig. 12·19) when a main line divides into two or more parallel ·es which again join together downstream and continues as a main line.

It may be seen from Fig. 12·19 that the rate of discharge in the main line is equal to the pipes.

Thus, $$Q = Q_1 + Q_2 \quad ...(12.16)$$

When the pipes are arranged in parallel, the *loss of head in each pipe (branch) is same.*

$\therefore$ Loss of head in pipe 1 = Loss of head in pipe 2.

or, $$h_f = \frac{4f_1L_1V_1^2}{D_1 \times 2g} = \frac{4f_2L_2V_2^2}{D_2 \times 2g} \quad ...(12\cdot17)$$

When, $f_1 = f_2$, then

$$\frac{L_1V_1^2}{D_1 \times 2g} = \frac{L_2V_2^2}{D_2 \times 2g} \quad ...(12\cdot18)$$

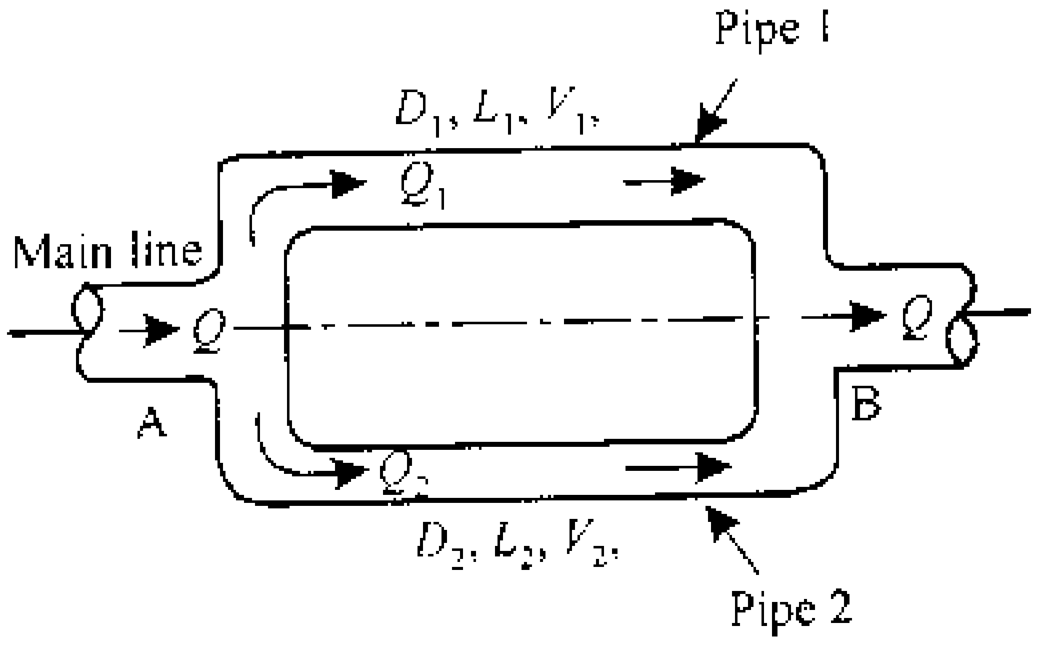

Fig. 12.19

Example 12·27. *The main pipe divides into two parallel pipes which again forms one pipe as shown in Fig. 12·19. The data is as follows :*

First parallel pipe : Length = 1000 m, diameter = 0·8 m

Second parallel pipe : Length = 1000 m, diameter = 0·6 m

Co-efficient of friction for each parallel pipe = 0·005

If the total rate of flow in the main is 2m³/s find the rate of flow in each parallel pipe.

Solution. Length of pipe 1, $L_1 = 1000$ m

Diameter of pipe 1, $D_1 = 0{\cdot}8$ m

Length of pipe 2, $L_2 = 1000$ m

Diameter of pipe 2, $D_2 = 0{\cdot}6$ m

Total rate of flow, $Q = 2$ m³/s

Co-efficients of friction, $f_1 = f_2 = 0{\cdot}005$

Rate of flow in each pipe :

Let, Q_1 = Rate of flow in pipe 1,

Q_2 = Rate of flow in pipe 2, and

Q = Total rate of flow (in main line).

Then, $Q = Q_1 + Q_2$ (Eqn. 12·16)

Also, $$h_f = \frac{4f_1L_1V_1^2}{D_1 \times 2g} = \frac{4f_2L_2V_2^2}{D_2 \times 2g} \quad ...[\text{Eqn. } 12\cdot17]$$

$f_1 = f_2\,(= 0{\cdot}005)$ and $L_1 = L_2\,(= 1000 \text{ m})$

The above equation reduces to :

$$\frac{V_1^2}{D_1} = \frac{V_2^2}{D_2} \quad \text{or} \quad \frac{V_1^2}{0{\cdot}8} = \frac{V_2^2}{0{\cdot}6}$$

or $$V_1 = \sqrt{\frac{0{\cdot}8}{0{\cdot}6} \times V_2^2} = 1{\cdot}15\, V_2 \quad ...(i)$$

Now, $$Q_1 = \frac{\pi}{4} \times D_1^2 \times V_1$$

$$= \frac{\pi}{4} \times 0{\cdot}8^2 \times 1{\cdot}15\, V_2 = 0{\cdot}578\, V_2$$

and, $$Q_2 = \frac{\pi}{4} \times D_2^2 \times V_2 = \frac{\pi}{4} \times 0{\cdot}6^2 \times V_2 = 0{\cdot}283\, V_2$$

Substituting the values of Q_1 and Q_2 in eqn. (*i*), we get

$$2 = 0{\cdot}578\, V_2 + 0{\cdot}283\, V_2$$

or, $$V_2 = \frac{2}{(0{\cdot}578 + 0{\cdot}283)} = 2{\cdot}32 \text{ m/s}$$

Substituting the values of V_2 in eqn. (*ii*), we get,

or, $$V_2 = 1{\cdot}15 \times 2{\cdot}32 = 2{\cdot}67 \text{ m/s}$$

Hence, $$Q_1 = A_1 V_1 = \frac{\pi}{4} \times 0{\cdot}8^2 \times 2{\cdot}67 = \mathbf{1{\cdot}342 \ m^3/s} \ \ \textbf{(Ans.)}$$

$\therefore$ $$Q_2 = Q - Q_1 = 2 - 1{\cdot}342$$
$$= \mathbf{0{\cdot}658 \ m^3/s} \ \ \textbf{(Ans.)}$$

Example 12·28. *A pipeline of 600 mm diameter is 1·5 km long. To increase the discharge another line of the same diameter is introduced parallel to the first in the second-half of the length. If f = 0·01 and head at inlet is 300 mm calculate the increase in discharge.*

Neglect minor losses. **[M.U.]**

Solution. Diameter of the pipeline, $D = 0{\cdot}6$ m

Length of the pipeline, $L = 1{\cdot}5 \text{ km} = 1{\cdot}5 \times 1000 = 1500$ m.

Co-efficient of friction, $f = 0{\cdot}01$

Head at inlet, $h = 0{\cdot}3$ m

Head at outlet (= atmospheric head) = 0

$\therefore$ Head lost, $h_f = 0{\cdot}3$

Length of another parallel pipe, $L_2 \ (= L_1) = \frac{1500}{2} = 750$ m

Diameter of another parallel pipe, $D_2 \ (= D_1) = 0{\cdot}6$ m

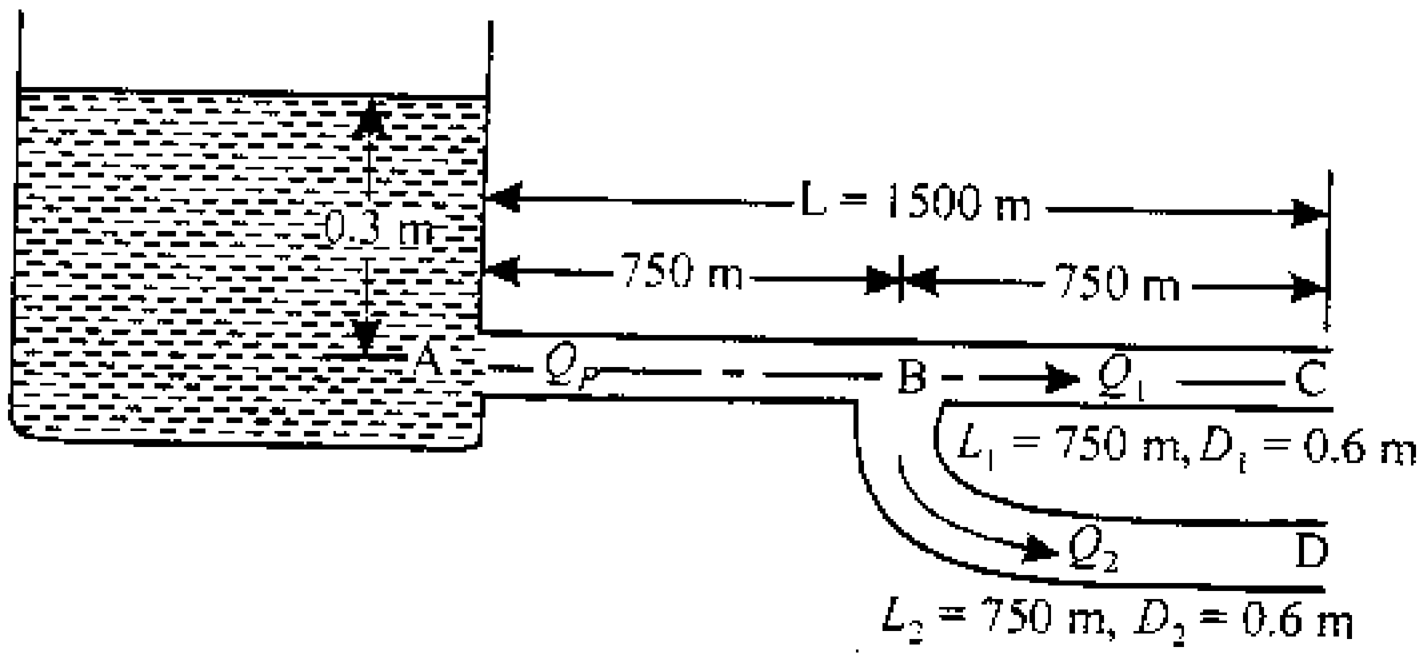

Fig. 12.20

The arrangement of the pipe system is shown in Fig. 12·20.

Increase in discharge :

Case. I. *Discharge (Q) for a single pipe of length 1500 m and diameter 0·6 m:*

The head lost due to friction in single pipe is given as :

$$h_f = \frac{4fLV^2}{D \times 2g}$$

(where, V = velocity of flow for a single pipe)

$\therefore$ $$0{\cdot}3 = \frac{4 \times 0{\cdot}01 \times 1500 \times V^2}{0{\cdot}6 \times 2 \times 9{\cdot}81}$$

or, $$V = \left[\frac{0{\cdot}3 \times 0{\cdot}6 \times 2 \times 9.81}{4 \times 0{\cdot}01 \times 1500}\right]^{\frac{1}{2}} = 0{\cdot}243 \text{ m/s}$$

$\therefore$ Discharge, $$Q = A \times V = \frac{\pi}{4} \times 0{\cdot}6^2 \times 0{\cdot}243 = 0{\cdot}0687 \text{ m}^3/\text{s} \quad \ldots(i)$$

Case. II. *When an additional pipe of length 750 m and diameter 0·6 m is connected in parallel with the last half length of the pipe:*

Let, Q_1 = Discharge in first parallel pipe,

Q_2 = Discharge in second parallel pipe,

Q_p = Discharge in the main pipe (when pipes are connected in parallel)

Then, $$Q_p = Q_1 + Q_2 \quad \ldots(\text{Fig. } 12{\cdot}20)$$

As the pipes in parallel have the same diameter and length,

$\therefore$ $$Q_1 = Q_2 = \frac{Q_p}{2}$$

Consider the flow through ABC or ABD;

Head lost (due to friction) in ABC

$$= \text{Head lost in } AB + \text{head lost in } BC \quad \ldots(ii)$$

Head lost in ABC = 0·3 m (*given*)

Now, head lost in $$AB = \frac{4 \times 0{\cdot}01 \times 750 \times V_{AB}^2}{0{\cdot}6 \times 2 \times 9{\cdot}81}$$

But, $$V_{AB} = \frac{Q_p}{\text{Area}} = \frac{Q_p}{(\pi/4) \times 0{\cdot}6^2} = 3{\cdot}54\, Q_p$$

$\therefore$ *Head Lost (due to friction) in AB*

$$= \frac{4 \times 0{\cdot}01 \times 750 \times (3{\cdot}54\, Q_p)^2}{0{\cdot}6 \times 2 \times 9{\cdot}81} = 3{\cdot}19\, Q_p^2$$

Head lost due to friction through BC

$$= \frac{4 f L_1 V_{BC}^2}{D_1 \times 2g}$$

$$= \frac{4 \times 0{\cdot}01 \times 750 \times (1{\cdot}77\, Q_p)^2}{0{\cdot}61 \times 2 \times 9{\cdot}81}$$

$$= 7{\cdot}98\, Q_p^2$$

$$\left[\because V_{BC} = \frac{(Q_p/2)}{\text{area}} = \frac{(Q_p/2)}{(\pi/4) \times 0{\cdot}6^2} = 1{\cdot}77\, Q_p\right]$$

Substituting these values in eqn. (*ii*), we get

$$0{\cdot}3 = 31{\cdot}9\, Q_p^2 + 7{\cdot}98\, Q_p^2$$

or, $$Q_p = \left[\frac{0{\cdot}3}{31{\cdot}9 + 7{\cdot}98}\right]^{1/2} = 0{\cdot}087 \text{ m}^3/\text{s}$$

$\therefore$ Increase in discharge $= Q_p - Q$

$= 0{\cdot}087 - 0{\cdot}0687 =$ **0·0183 m³/s (Ans.)**

Example 12.29. *Two sharp ended pipes of diameters 50 mm and 100 mm respectively, each of length 100 m respectively, are connected in parallel between two reservoirs which have a difference of level of 10 m. If the friction factor for each pipe is 0·32, calculate :*

(*i*) *Rate of flow for each pipe, and*

(*ii*) *The diameter of a single pipe 100 m long which would give the same discharge, if it were substitued for the original two pipes.* **[Allahabad University]**

Solution. Diameter of pipe 1, $D_1 = 50 \text{ mm} = 0{\cdot}05 \text{ m}$

Diameter of pipe 2, $D_2 = 100 \text{ mm} = 0{\cdot}1 \text{ m}$

Length of pipe 1, $L_1 = 100 \text{ m}$

Length of pipe 2, $L_2 = 100 \text{ m}$

Difference in level, $h = 10 \text{ m}$

Friction factor, $(4f) = 0{\cdot}32$

(*i*) **Rate of flow for each pipe :**

Let, V_1 = Velocity of flow in pipe 1, and

V_2 = Velocity of flow in pipe 2.

Since the pipes are connected in *parallel*, therefore the *loss of head will be same* in both the pipes.

For the *pipe 1*, the loss of head,

$$10 = h_f = \frac{4fL_1V_1^2}{D_1 \times 2g} = \frac{0{\cdot}32 \times 100 \times V_1^2}{0{\cdot}05 \times 2 \times 9.81} = 32{\cdot}62\ V_1^2 \qquad (\because 4f = 0{\cdot}32)$$

or, $$10 = 32{\cdot}62\ V_1^2$$

or, $$V_1 = \left(\frac{10}{32{\cdot}62}\right)^{1/2} = 0{\cdot}55 \text{ m/s}$$

∴ Rate of flow in pipe 1,

$$Q_1 = A_1V_1 = \frac{\pi}{4} \times (0{\cdot}05)^2 \times 0{\cdot}55$$

$$= \mathbf{0{\cdot}00108\ m^3/s} \quad \textbf{(Ans.)}$$

For the *pipe* 2 the loss of head is given by,

$$10 = \frac{4fL_2V_2^2}{D_2 \times 2g} = \frac{0{\cdot}32 \times 100 \times V_2^2}{0{\cdot}1 \times 2 \times 9{\cdot}81} = 16{\cdot}31\ V_2^2$$

or, $$V_2 = \left(\frac{10}{16{\cdot}31}\right)^{1/2} = 0{\cdot}78 \text{ m/s}$$

∴ Rate of flow in pipe 2,

$$Q_2 = A_2V_2 = \frac{\pi}{4} \times 0{\cdot}1^2 \times 0{\cdot}78 = \mathbf{0{\cdot}00613\ m^3/s\ (Ans.)}$$

(*ii*) **Diameter of the single pipe, D :**

Let, D = Diameter of the single pipe,

L = Length of the single pipe = 100 m,

V = Velocity of liquid in the single pipe, and

Q = Discharge through the single pipe.

Now, $$Q = Q_1 + Q_2$$
$$= 0{\cdot}00108 + 0{\cdot}00613 = 0{\cdot}00721 \text{ m}^3/\text{s}$$

∴ $$V = \frac{Q}{A} = \frac{0{\cdot}00721}{(\pi/4) \times D^2} = \frac{0{\cdot}00918}{D^2} \text{ m/s}$$

Loss of head through the single pipe,

$$10 = h_f = \frac{4fLV^2}{D \times 2g} = \frac{0{\cdot}32 \times 100 \times \left(\frac{0{\cdot}00918}{D^2}\right)^2}{D \times 2 \times 9{\cdot}81}$$

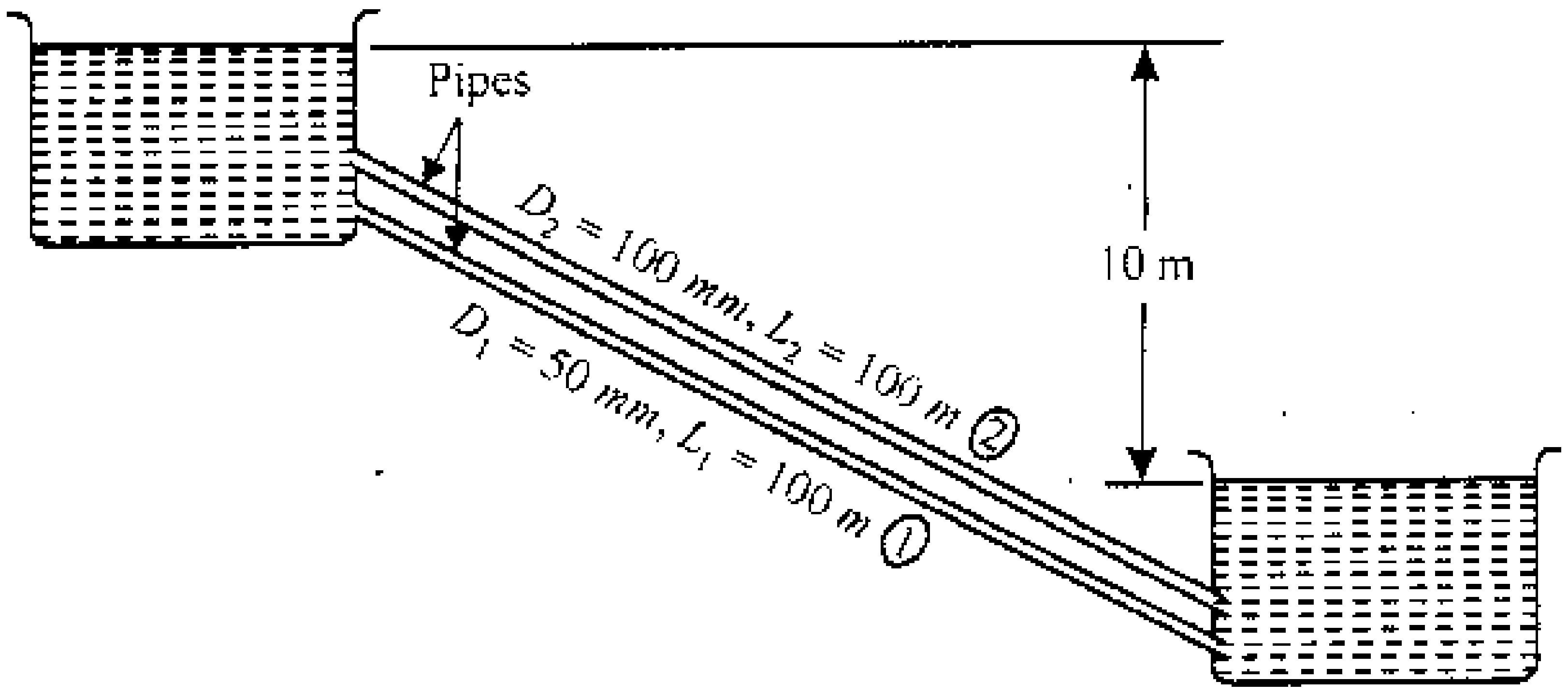

Fig. 12.21

or, $$10 = \frac{0{\cdot}32 \times 100 \times (0{\cdot}00918)^2}{2 \times 9.81 \times D^5} = \frac{0{\cdot}0001375}{D^5}$$

or $$D = \left[\frac{0{\cdot}0001375}{10}\right]^{1/5} = 0{\cdot}1066 \text{ m} = 106{\cdot}6 \text{ mm}$$

i.e., $D =$ **106·6 mm (Ans.)**

Example 12.30. *A 250 mm diameter, 3 km long straight pipe runs between two reservoirs of surface elevations 135 m and 60 m. A 1·5 km long, 300 mm diameter pipe is laid parallel to the 250 mm diameter pipe from its mid-point to the lower reservoir. Neglecting all minor losses and assuming a friction factor of 0·02 for both pipes, find the increase in discharge caused by addition of 300 diameter pipe.* **(AMIE Winter, 2000)**

Solution. Neglecting minor losses, the application of Bernoulli's equation between the water surfaces of the two reservoirs yields $(135 - 60) = \dfrac{fLV^2}{D \times 2g} = \dfrac{0{\cdot}02 \times 3000 \times V^2}{0{\cdot}25 \times 2 \times 9.81}$

or, $$V = \left[\frac{(135 - 60) \times 0{\cdot}25 \times 2 \times 9.81}{0{\cdot}02 \times 3000}\right]^{1/2} = 2{\cdot}476 \text{ m/s}$$

The discharge through the pipeline,

$$Q = \frac{\pi}{4} \times (0{\cdot}25)^2 \times 2{\cdot}476 = 0{\cdot}1215 \text{ m}^3/\text{s}$$

In case of altered pipeline (see fig. 12·22) the discharge through pipe section AB is the sum of the discharges through sections BC and BD, or

$$Q_1 = Q_2 + Q_3$$

or, $$\frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} D_2^2 V_2 + \frac{\pi}{4} D_3^2 V_3$$

$$D_1^2 V_1 = D_2^2 V_2 + D_3^2 V_3$$

$$= D_1^2 V_2 + D_3^2 V_3 \qquad \text{...}(i)$$

Also, as the end points of sections BC and CD are same (They are in parallel.),

$$h_{f_2} = h_{f_3}$$

or, $$\frac{f\,(L/2)\,V_2^2}{D_2 \times 2g} = \frac{f\,(L/2)\,V_3^2}{D_3 \times 2g}$$

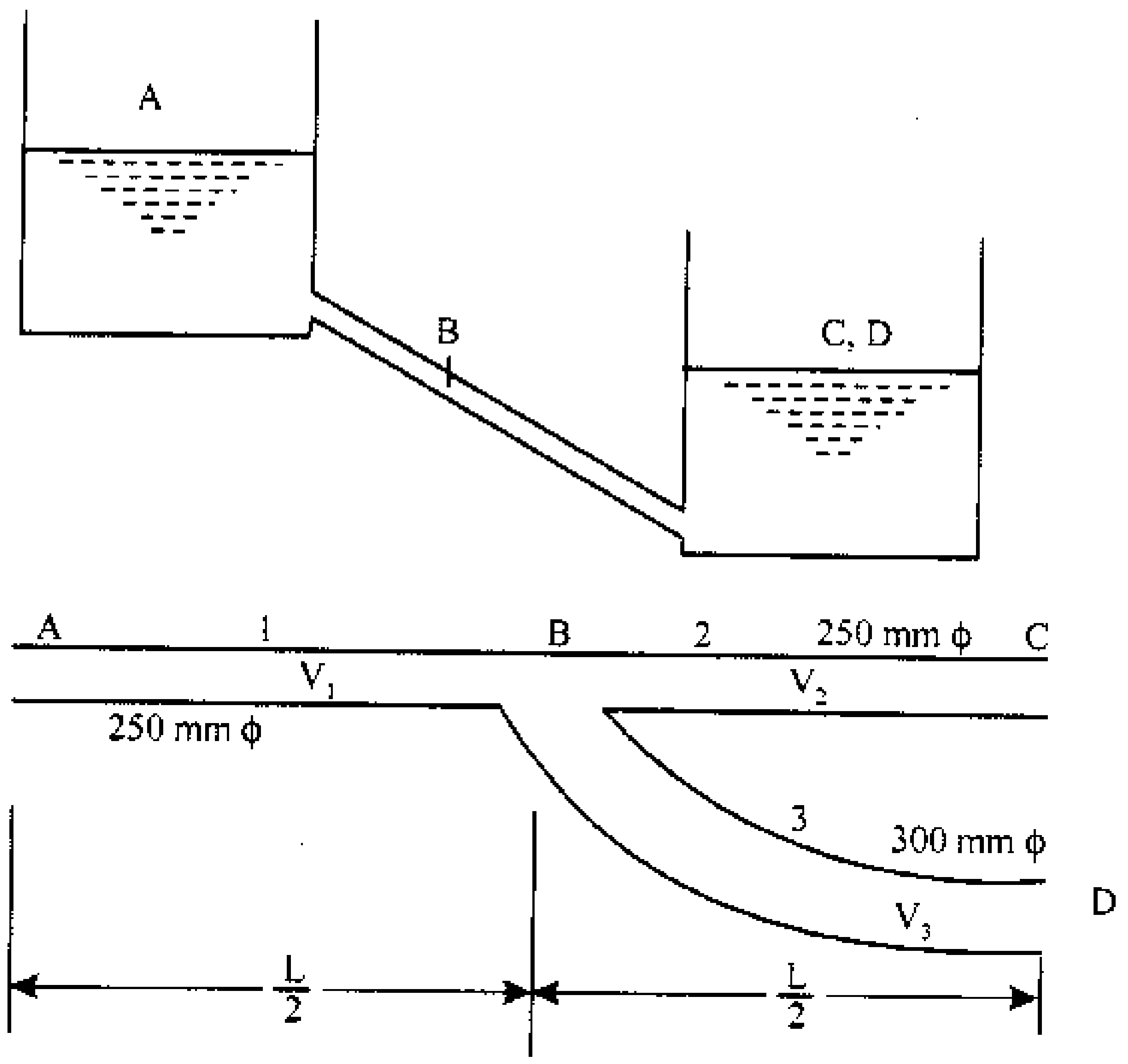

Fig. 12.22

or, $$\frac{V_2^2}{D_2} = \frac{V_3^2}{D_3}$$

or, $$V_3 = \left[V_2^2 \times \frac{D_3}{D_2}\right]^{1/2}$$

$$= \left[V_2^2 \times \frac{300}{250}\right]^{1/2}$$

$$= \sqrt{\frac{300}{250}}\, V_2 \qquad ...(ii)$$

Substituting for V_3 in (*i*), we get

or, $$(250)^2\, V_1 = (250)^2\, V_2 + (300)^2 \times \sqrt{\frac{300}{250}}\, V_2$$

or, $$(250)^2\, (V_1 - V_2) = (300)^2 \times \sqrt{\frac{300}{250}}\, V_2$$

or, $$V_1 - V_2 = 1{\cdot}5774\, V_2$$

or, $$V_1 = 2{\cdot}5774\, V_2 \text{ or } V_2 = 0{\cdot}388\, V_1$$

Again, applying Bernoulli's equation between the water surfaces of the two reservoirs through ABC, we get

$$(135 - 60) = \frac{f(L/2)V_1^2}{D_1 \times 2g} + \frac{f(L/2)V_2^2}{D_2 \times 2g}$$

$$75 = \frac{f(L/2)}{D_1 \times 2g}(V_1^2 + V_2^2) \qquad (\because D_1 = D_2)$$

$$= \frac{0{\cdot}02 \times 1500}{0{\cdot}25 \times 2 \times 9.81}\left\{V_1^2 + (0{\cdot}388\, V_1)^2\right\} = 6{\cdot}116 \times 1{\cdot}1505\, V_1^2$$

or, $$V_1 = \left(\frac{75}{6{\cdot}116 \times 1{\cdot}1505}\right)^{1/2} = 3{\cdot}26 \text{ m/s}$$

∴ Discharge $$= \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} \times 0{\cdot}25^2 \times 3{\cdot}26 = 0{\cdot}16 \text{ m}^3/\text{s}$$

∴ Increase in discharge $= 0{\cdot}16 - 0{\cdot}1215 =$ **$0{\cdot}0385$ m³/s** or **31·7 % (Ans.)**

Example 12·31. *A farmer wishes to connect two pipes of different lengths and diameters to a common header supplied with 8×10^{-3} m³/s of water from a pump. One pipe is 100 m long and 5 cm in diameter. The other pipe is 800 m long. Determine the diameter of the second pipe such that both pipes have the same flow rate. Assume the pipes to be laid on level ground and friction co-efficient for both pipes as 0·02. Also determine the head loss in metres of water in the pipes.* **(GATE)**

Solution. Refer Fig. to 12·23. *Given* : $Q = 8 \times 10^{-3}$ m³/s; $D_1 = 5$ cm $= 0{\cdot}05$ m; $L_1 = 100$ m; $L_2 = 800$ m; Friction co-efficient, $f = 0{\cdot}02$.

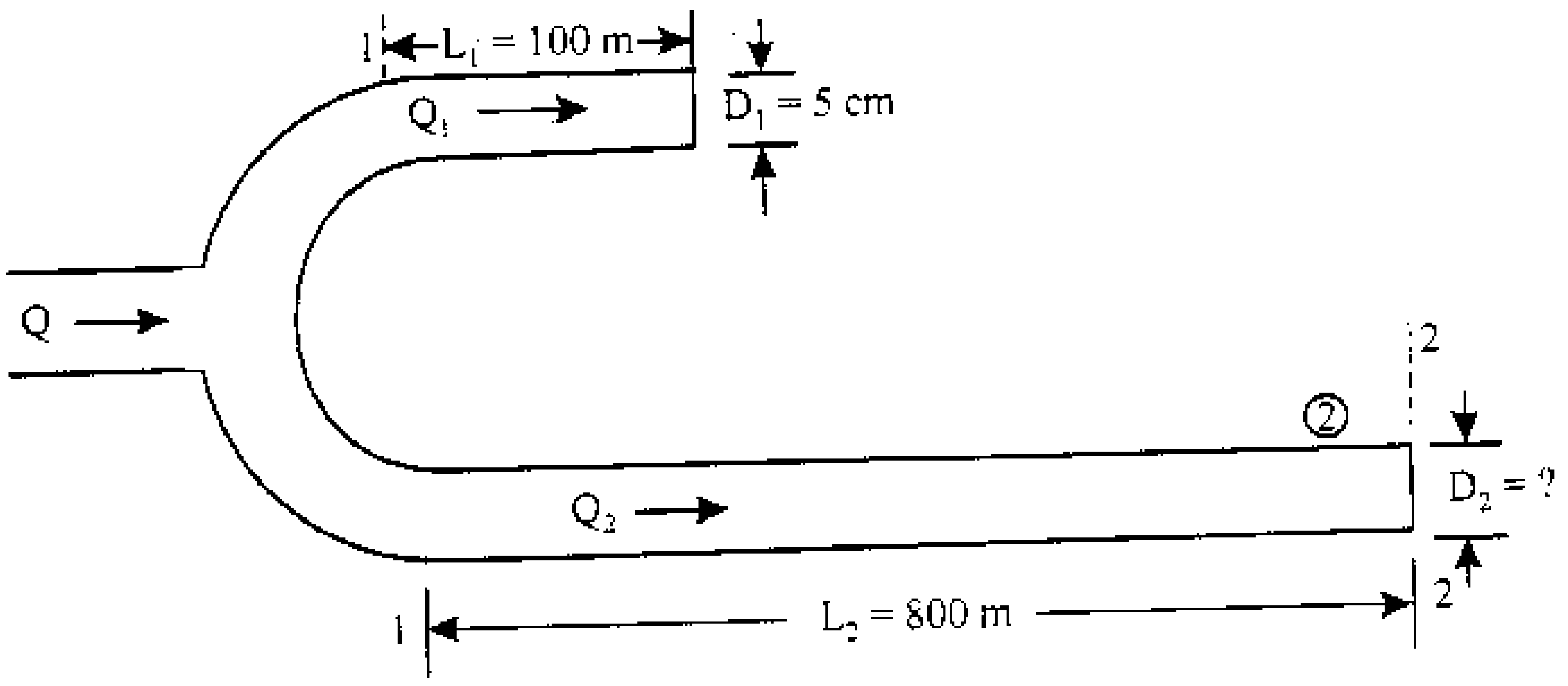

Fig. 12.23

Diameter, D_2 ;

$$Q = Q_1 + Q_2 \qquad \left[\text{where, } Q_1 = Q_2 = \frac{8 \times 10^{-3}}{2} = 4 \times 10^{-3} \text{ m}^3/\text{s } (\textit{Given})\right]$$

For pipe 1, $$h_{f_1} = \frac{4fL_1V_1^2}{D_1 \times 2g}$$

where, $$V_1 = \frac{Q_1}{(\pi/4) \times D_1^2} = \frac{4Q_1}{\pi D_1^2}$$

∴ $$h_{f_1} = \frac{4fL_1 \times \left[\frac{4Q_1}{\pi D_1^2}\right]^2}{D_1 \times 2g} = \frac{32\, fL_1Q_1^2}{\pi^2 \times D_1^5 \times g}$$

Similarly, *for pipe 2,* $$h_{f_2} = \frac{32\, fL_2Q_2^2}{\pi^2 \times D_2^5 \times g} \quad \text{...(i}$$

Equating (*i*) and (*ii*) [since $h_{f_1} = h_{f_2}$], we get

$$\frac{32\, fL_1Q_1^2}{\pi^2 \times D_1^5 \times g} = \frac{32\, fL_2Q_2^2}{\pi^2 \times D_2^5 \times g}$$

or, $$\frac{Q_1^2}{Q_2^2} = \frac{L_2 D_1^5}{L_1 D_2^5}$$

But, $$Q_1 = Q_2$$ ...(Given)

∴ $$L_2 D_1^5 = L_1 D_2^5$$

or, $$D_2 = \left(\frac{L_2 D_1^5}{L_1}\right)^{1/5} = \left[\frac{800 \times (0{\cdot}05)^5}{100}\right]^{1/5} = 0{\cdot}07578 \text{ m} = \mathbf{7{\cdot}578 \text{ cm}} \quad \text{(Ans.)}$$

Head loss :

$$h_{f1} (= h_{f2}) = \frac{32\, f L_1 Q_1^2}{\pi^2 \times D_1^5 \times g} = \frac{32 \times 0{\cdot}02 \times 100 \times (4 \times 10^{-3})^2}{\pi^2 \times (0{\cdot}05)^5 \times 9{\cdot}81} = \mathbf{33{\cdot}84 \text{ m}} \quad \text{(Ans.)}$$

Example 12.32. *A pipeline with diameter 0·8 m and length 3000 m connects two open reservoirs of water which have their water surfaces of elevations of 100 m and 70 m above a datum. In order to increase the rate of flow between the reservoirs by 20 % it is decided to lay an additional 0·8 m diameter pipeline from the upper reservoir. The second pipeline is to be parallel to the original pipeline and is to be connected to the latter at some suitable point. Determine the point of connection, assuming that the friction factor is 0·04 for each pipeline. Neglect minor losses.*

(U.P.S.C., 1998)

Solution. Refer to Fig. 12·24. *Given :* $L = 3000$ m; $D = 0{\cdot}8$ m; $f = 0{\cdot}04$

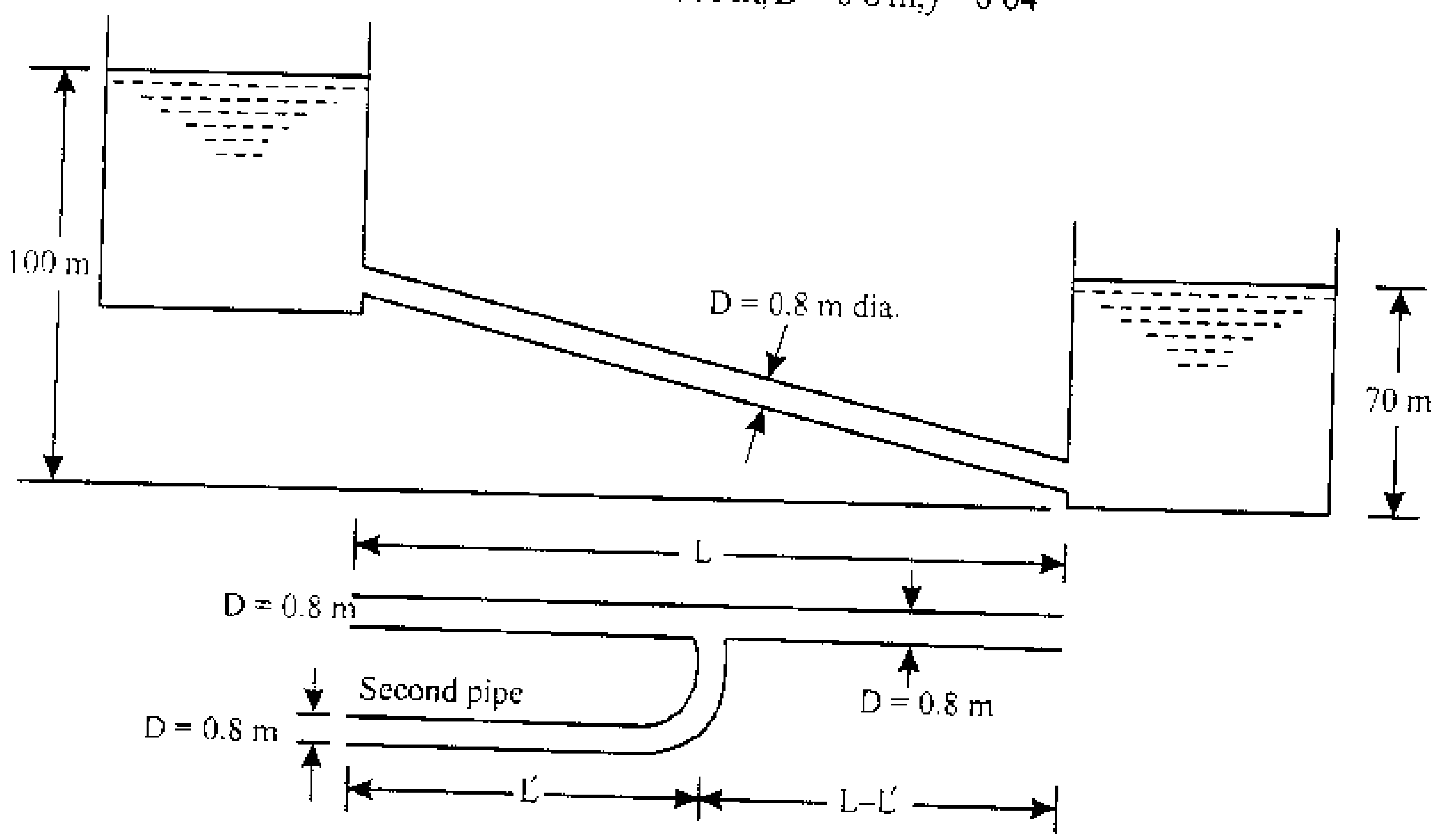

Fig. 12.24

Point of connection of second pipe, L':

Head at inlet of pipe = 100 – 70 = 30 m

Case I. *Discharge (Q_1) for a single pipe length 3000 m and diameter 0·8 m:*

The head lost due to friction in single pipe is given as :

$$h_f = \frac{f L V_1^2}{D \times 2g}$$

where, $$V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\frac{\pi}{4} \times D^2} = \frac{4Q_1}{\pi D^2}$$ [where, f = friction factor (= 4 × co-efficient of friction]

$$\therefore \qquad h_f = \frac{fL \times \left(\frac{4Q_1}{\pi D^2}\right)^2}{D \times 2g} = \frac{8fLQ_1^2}{\pi^2 D^5 \times g} = \frac{fLQ_1^2}{12D^5}$$

Substituting the values, we get

$$30 = \frac{0{\cdot}04 \times 3000 \times Q_1^2}{12 \times (0{\cdot}8)^5}$$

or,

$$Q_1 = \left[\frac{30 \times 12 \times (0{\cdot}8)^5}{0{\cdot}04 \times 3000}\right]^{1/2} = 0{\cdot}99 \text{ m}^3/\text{s}$$

Case II. *When another pipeline of length L′ is added :*

Total discharge, $Q_2 = 1{\cdot}2\, Q_1 = 1{\cdot}2 \times 0{\cdot}99 = 1{\cdot}188 \text{ m}^3/\text{s}$

Discharge through each pipeline = $\dfrac{Q_2}{2}$

In this case, total head loss = sum of head losses in two pipes

i.e.,

$$h_f = \frac{fL'V_1^2}{D \times 2g} + \frac{f(L - L')V_2^2}{D \times 2g}$$

or,

$$30 = \frac{fL' \times \left(\frac{Q_2}{2}\right)^2}{12\, D^5} + \frac{f(L - L')Q_2^2}{12\, D^5}$$

or,

$$30 = \frac{fL'Q_2^2}{4 \times 12\, D^5} + \frac{f(L - L')Q_2^2}{12\, D^5}$$

Substituting the values, we get

or,

$$\frac{30 \times 12 \times (0{\cdot}8)^5}{0{\cdot}04 \times (1{\cdot}188)^2} = \frac{L'}{4} - L' + 3000$$

$$2089{\cdot}6 = \frac{L'}{4} - L' + 3000$$

or,

$$L' - \frac{L'}{4} = 3000 - 2089{\cdot}6 = 910{\cdot}4$$

$\therefore$ $\quad L' = \mathbf{1213{\cdot}87\ m}$ **(Ans.)**

Example 12.33. *Two reservoirs have a constant difference of levels of 70 m and are connected by a 250 mm diameter pipe which is 4 km long. The pipe is tapped mid-way between the reservoirs and water is drawn at the rate of 0·04 m³/s. Assuming friction factor = 0·04, determine the rate at which water enters the lower reservoir.*

Solution. Diameter of the pipe, $D = 250 \text{ mm} = 0{\cdot}25 \text{ m}$

Difference of level, $h = 70 \text{ m}$

Friction factor, $4f = 0{\cdot}04$

Rate at which water enters the lower reservoir :

Let, Q = Discharge entering the lower reservoir.

Then, discharge at the inlet = $(Q + 0{\cdot}04)\ \text{m}^3/\text{s}$.

Now, from the application of Bernoulli's equation, we have

$$h = h_{f_1} + h_{f_2}$$

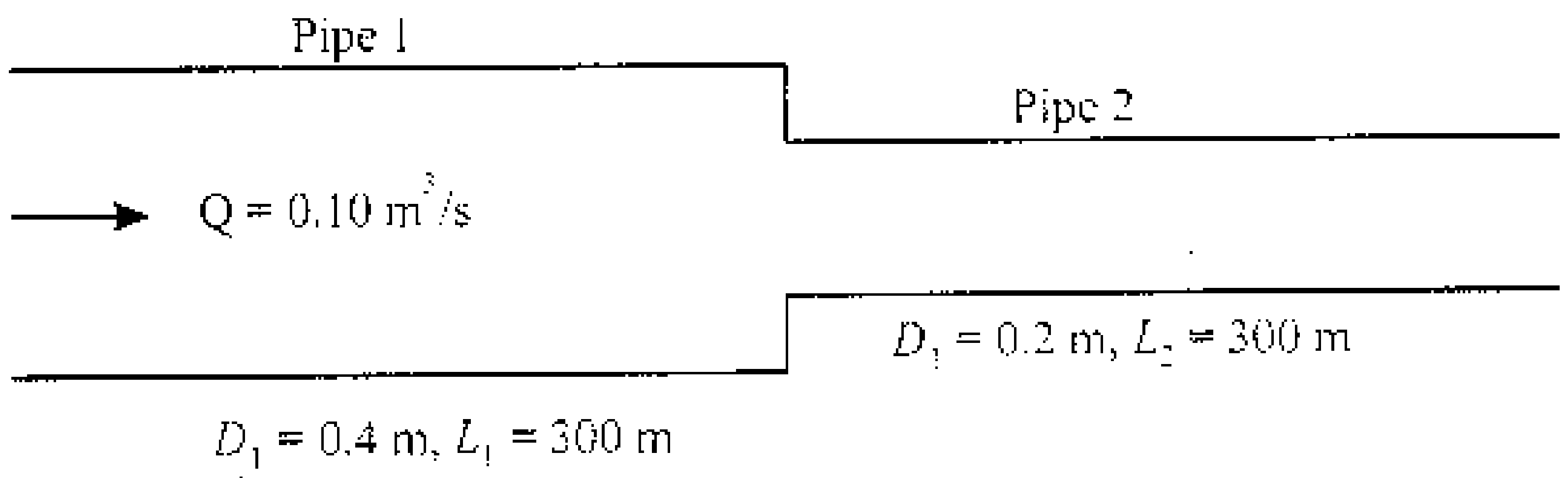

Fig. 12.26

Solution. Diameter of the pipe 1, $D_1 = 400$ mm $= 0{\cdot}4$ m

Length of the pipe 1, $L_1 = 300$ m

Diameter of the pipe 2, $D_2 = 200$ mm $= 0{\cdot}2$ m

Length of the pipe 2, $L_2 = 300$ m

Friction factor for each pipe $(4f) = 0{\cdot}0075$

Discharge, $Q = 0{\cdot}1$ m³/s

(*i*) **Pipes connected in series – Loss of head :**

Velocity of flow in pipe 1, $V_1 = \dfrac{0{\cdot}1}{(\pi/4)\times 0{\cdot}4^2} = 0{\cdot}796$ m/s

Velocity of flow in pipe 2, $V_2 = \dfrac{0{\cdot}1}{(\pi/4)\times 0{\cdot}2^2} = 3{\cdot}183$ m/s

Head lost due to friction in pipe 1 $= \dfrac{4f_1L_1V_1^2}{D_1\times 2g} = \dfrac{0{\cdot}0075\times 300\times 0{\cdot}796^2}{0{\cdot}4\times 2\times 9{\cdot}81} = 0{\cdot}1816$ m

Assuming head lost due to contraction,

$$h_c = k\frac{V_2^2}{2g}$$

or,

$$h_c = 0{\cdot}33\frac{V_2^2}{2g}\left[\text{for } \frac{D_2}{D_1} = 0{\cdot}5,\ k = 0{\cdot}33 \quad \text{...(from tables)}\right]$$

$$= \frac{0{\cdot}33\times 3{\cdot}183^2}{2\times 9{\cdot}81} = 0{\cdot}17 \text{ m}$$

Head lost due to friction in pipe 2 $= \dfrac{4f_2L_2V_2^2}{D_2\times 2g} = \dfrac{0{\cdot}0075\times 300\times 3{\cdot}183^2}{0{\cdot}2\times 2\times 9.81}$

$= 5{\cdot}809$ m

$\therefore$ Head lost in the pipeline $= 0{\cdot}1816 + 0{\cdot}17 + 5{\cdot}809 =$ **6·16 m** **(Ans.)**

(*ii*) **Pipes in Parallel–Loss of head :**

From continuity consideration, we have

$$Q = Q_1 + Q_2$$

$$0{\cdot}1 = Q_1 + Q_2 \qquad \text{...(Given)}$$

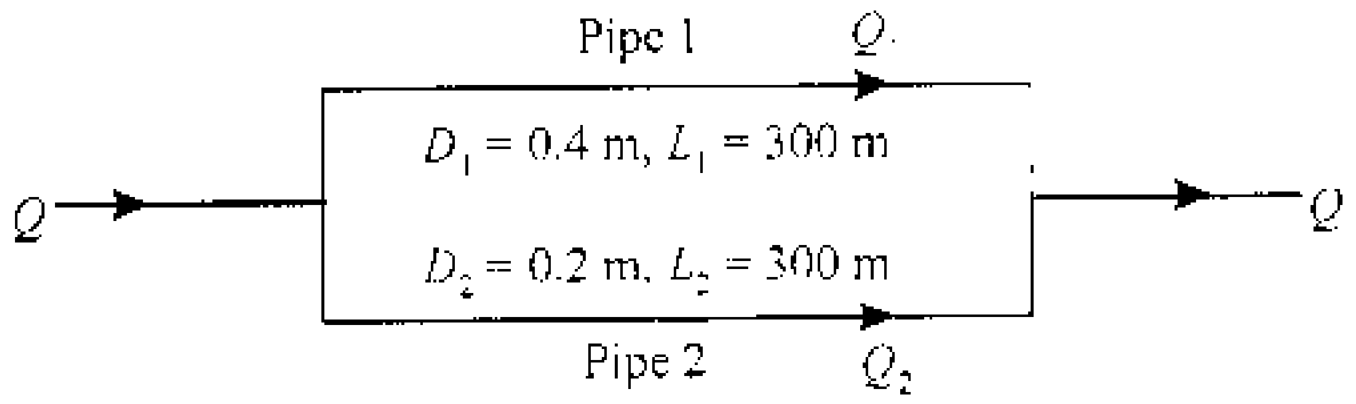

Fig. 12.27

or, $0.1 = \frac{\pi}{4} \times 0.4^2 \times V_1 + \frac{\pi}{4} \times 0.2^2 \times V_2$

$= 0.1257\, V_1 + 0.0314\, V_2$ ...(1)

Also, head lost will be same, since the pipes are connected in parallel.

$\therefore \quad h_f = \frac{4 f_1 L_1 V_1^2}{D_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{D_2 \times 2g}$

But, $f_1 = f_2$ and $L_1 = L_2$

$\therefore \quad \frac{V_1^2}{D_1} = \frac{V_2^2}{D_2}$ or $\frac{V_1^2}{V_2^2} = \frac{D_1}{D_2} = \frac{0.4}{0.2} = 2$

or, $V_1^2 = 2V_2^2$ ...(2)

Substituting the value of V_2 from (1) in (2), we get

$$V_1^2 = 2\left[\frac{0.1 - 0.1257\, V_1}{0.0314}\right]^2 = 2\,(3.185 - 4V_1)^2 = 20.29 + 32V_1^2 - 50.96V_1$$

or, $31V_1^2 - 50.96V_1 + 20.29 = 0$

or, $V_1 = \frac{50.96 \pm \sqrt{50.96^2 - 4 \times 31 \times 20.29}}{2 \times 31} = 0.97$ m/s, 0.677 m/s

Using $V_1 = 0.97$ m/s, we have

$Q_1 = (\pi/4) \times 0.4^2 \times 0.97 = 0.1219$ m³/s

Since $Q_1 > Q$, $V_1 = 0.97$ m/s is not *realistic.*

Using $V_1 = 0.677$ m/s, we have

$Q_1 = (\pi/4) \times 0.4^2 \times 0.677 = 0.085$ m³/s

$Q_2 = 0.1 - 0.085 = 0.015$ m³/s

Head lost $= \frac{4 f_1 L_1 V_1^2}{D_1 \times 2g} = \frac{0.0075 \times 300 \times 0.677^2}{0.4 \times 2 \times 9.81} =$ **0.131 m (Ans.)**

Example 12·35. *The pipes of diameter D and d of equal length L are considered. If the pipes are arranged in parallel, the loss of head for either pipe for a flow of Q is h. If the pipes are arranged in series and the same quantity Q flows through them, the loss of head is H. If d = 0·5 D, find the percentage of total flow through each pipe when placed in parallel and the ratio of H to h neglecting minor losses and assuming friction co-efficient to be constant.* **[UPSC Exams.]**

Solution. Diameter of pipe 1, $D_1 = D$

Length of pipe 1, $L_1 = L$

Diameter of pipe 2, $D_2 = d$

Length of pipe 2, $L_2 = L$

Total discharge $= Q$

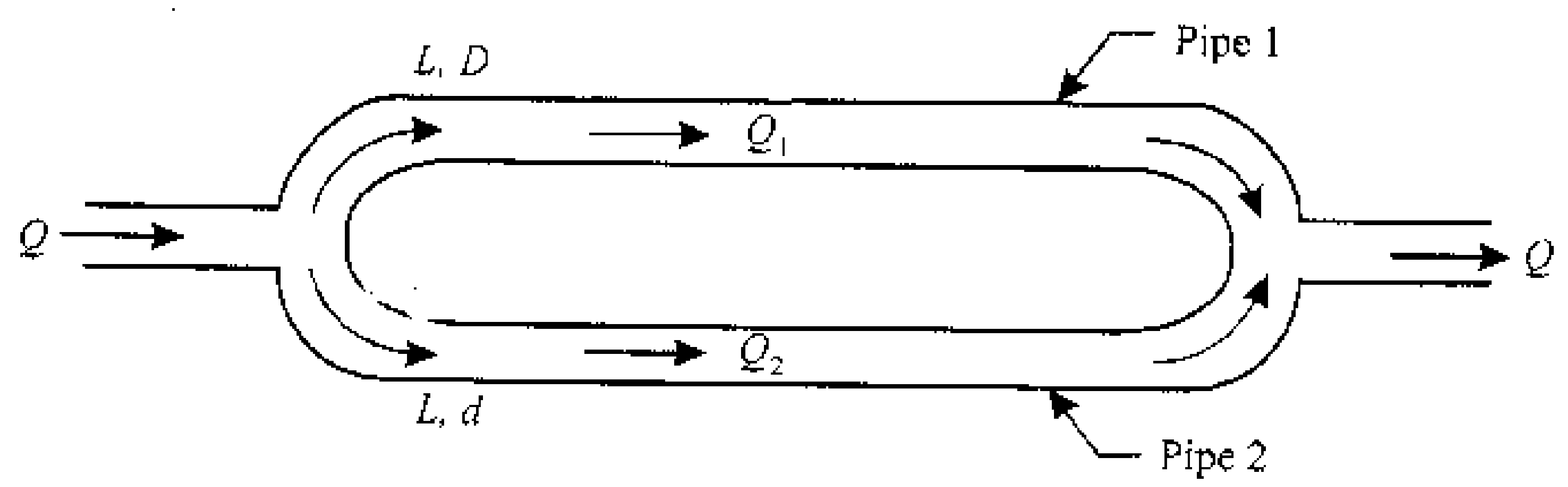

Fig. 12.28. Pipes connected in parallel.

Head lost when pipes are arranged in parallel = h

Head lost when pipe are arranged in series $= H$

$d = 0{\cdot}5\ D$ and f is constant.

Case I. Pipes connected in "parallel" :

When pipes are connected in *parallel*,

$$Q = Q_1 + Q_2 \qquad \text{...(i)}$$

Loss of head in each pipe = h

For pipe 1 : $$h = \frac{4fL_1V_1^2}{D \times 2g}$$

where, $$V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{(\pi/4) \times D^2} = \frac{4Q_1}{\pi D^2}$$

$\therefore$ $$h = \frac{4fL \times \left(\frac{4Q_1}{\pi D^2}\right)^2}{D \times 2g} = \frac{32fLQ_1^2}{\pi^2 D^5 \times g} \qquad \text{...(ii)} \qquad (\because L_1 = L)$$

For pipe 2 : $$h = \frac{32fLQ_2^2}{\pi^2 d^5 \times g} \qquad \text{...(iii)}$$

From eqns. (*ii*) and (*iii*), we have

$$\frac{32fLQ_1^2}{\pi^2 D^5 \times g} = \frac{32fLQ_2^2}{\pi^2 d^5 \times g}$$

or, $$\frac{Q_1^2}{D^5} = \frac{Q_2^2}{d^5}$$

or, $$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{D}{d}\right)^5 = \left(\frac{D}{0{\cdot}5D}\right)^5 = 32 \qquad (\because d = 0{\cdot}5D\text{...Given})$$

or, $$\frac{Q_1}{Q_2} = 5{\cdot}567 \quad \text{or} \quad Q_1 = 5{\cdot}567\, Q_2$$

Substituting the value of Q_1 in eqn. (*i*), we get

$$Q = 5{\cdot}567Q_2 + Q_2 = 6{\cdot}657Q_2$$

$\therefore$ $$Q_2 = \frac{Q}{6{\cdot}657} = 0{\cdot}15Q \qquad \text{...(iv)}$$

and, $$Q_1 = Q - 0{\cdot}15\,Q = 0{\cdot}85\,Q \qquad \text{...[From (i)]} \qquad \text{...(v)}$$

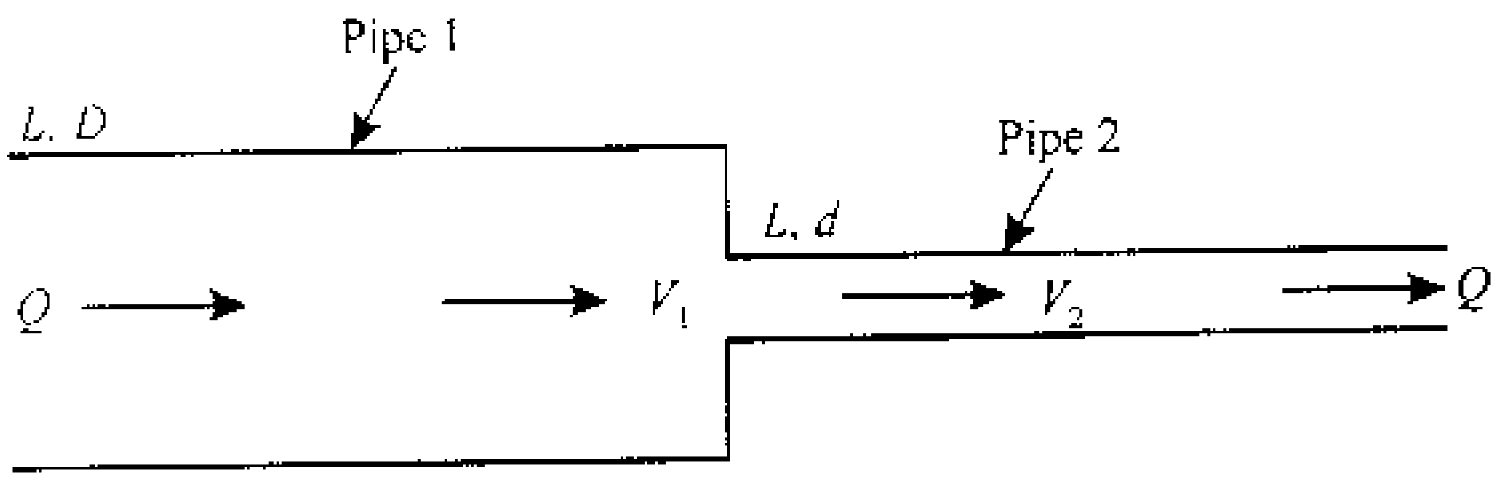

Fig. 12.29. Pipes connected in series.

Case II. Pipes connected in "series" :

In this case, total loss = sum of head losses in the two pipes

$\therefore$ $$H = \frac{4fLV_1^2}{D \times 2g} + \frac{4fLV_2^2}{d \times 2g}$$

where, $$V_1 = \frac{Q}{(\pi/4) \times D^2} = \frac{4Q}{\pi D^2}$$

$$V_2 = \frac{Q}{(\pi/4) \times d^2} = \frac{4Q}{\pi d^2}$$

$\therefore$ $$H = \frac{4fL \times \left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g} + \frac{4fL \times \left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} \quad ...(vi)$$

or, $$H = \frac{32\, fLQ^2}{\pi^2 D^5 \times g} + \frac{32\, fLQ^2}{\pi^2 d^5 \times g}$$

From eqn. (*ii*) $$\frac{32\, fL}{\pi^2 D^5 \times g} = \frac{h}{Q_1^2}$$

and, from eqn. (*iii*) $$\frac{32\, fL}{\pi^2 d^5 \times g} = \frac{h}{Q_2^2}$$

Substituting these values in eqn. (*vi*), we get

$$H = Q^2 \times \frac{h}{Q_1^2} + Q^2 \times \frac{h}{Q_2^2} = h\left(\frac{Q^2}{Q_1^2} + \frac{Q^2}{Q_2^2}\right)$$

$\therefore$ $$\frac{H}{h} = \frac{Q^2}{Q_1^2} + \frac{Q^2}{Q_2^2}$$

But from eqn. (*iv*) and (*v*)

$$Q_1 = 0{\cdot}85\, Q \text{ and } Q_2 = 0{\cdot}15\, Q$$

$\therefore$ $$\frac{H}{h} = \frac{Q^2}{(0{\cdot}85Q)^2} + \frac{Q^2}{(0{\cdot}15Q)^2} = \mathbf{45{\cdot}828} \text{ (Ans.)}$$

Example 12·36. *A pumping plant forces water through a 600 mm diameter main, the friction head being 27 m. In order to reduce the power consumption, it is proposed to lay another main of appropriate diameter along the side of the existing one, so that the two pipes may work in parallel for the entire length and reduce the friction head to 9·6 m only. Find the diameter of the new main if with the exception of diameter, it is similar to the existing one in every respect.*

[Panjab University]

Solution. Diameter of single main pipe, $D = 600 \text{ mm} = 0{\cdot}6 \text{ m}$

Friction head, $h_f = 27$ m

Friction head for two parallel pipes = 9·6 m

Diameter of the new main :

Case I. *Single main :*

$$h_f = \frac{4fLV^2}{D \times 2g}$$

$$27 = \frac{4fLV^2}{0{\cdot}6 \times 2 \times 9{\cdot}81}$$

$$fLV^2 = \frac{27 \times 0{\cdot}6 \times 2 \times 9{\cdot}81}{4} = 79{\cdot}461$$

But, $$V = \frac{Q}{A}$$

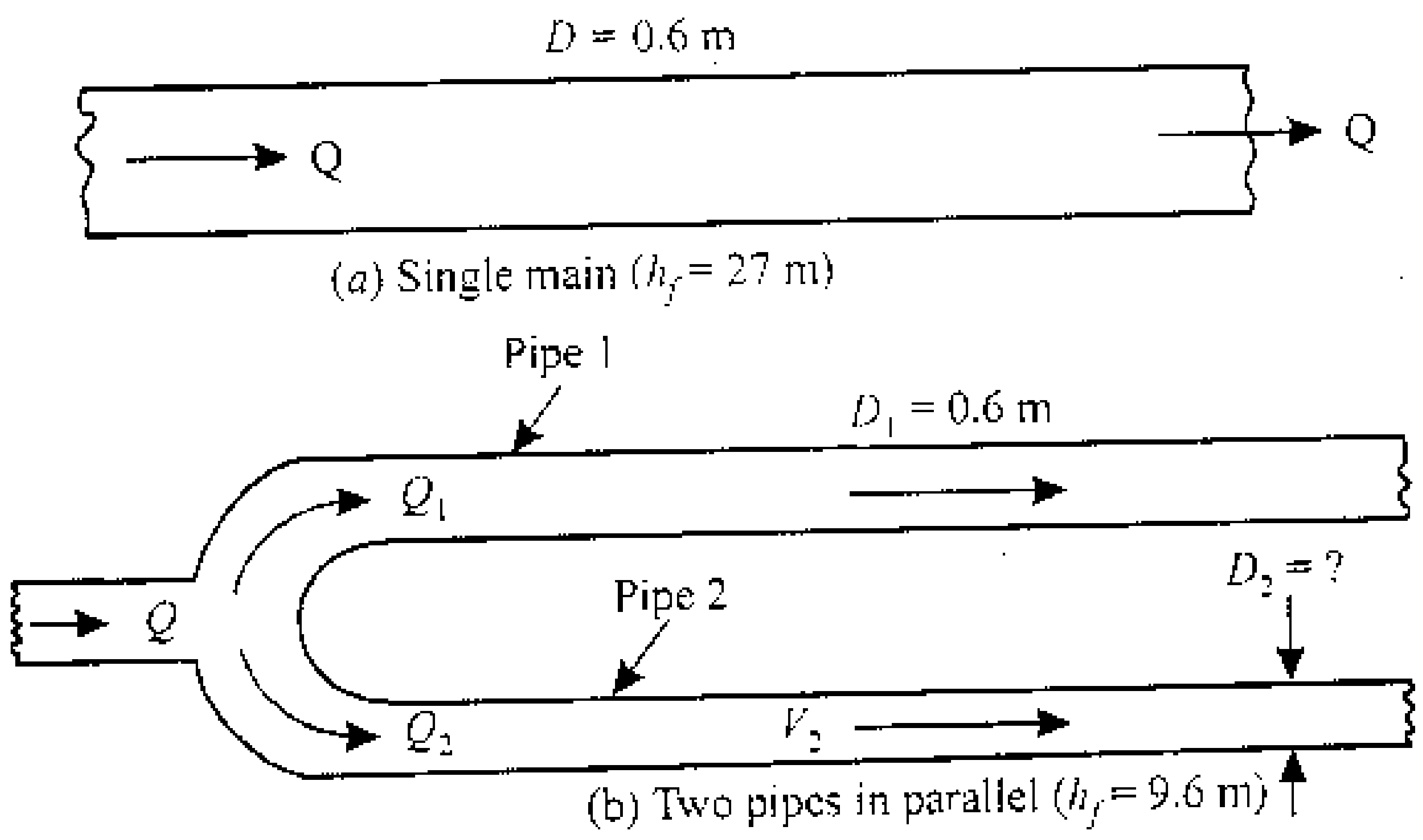

Fig. 12.30

$$\therefore \quad fL\frac{Q^2}{A^2} = 79{\cdot}461 \qquad \ldots(i)$$

Case II. *Two pipes in parallel :*

Loss of head, $h_f = 9{\cdot}6$ m

For pipe 1 :

$$h_{f_1} = \frac{4fL_1V_1^2}{D_1 \times 2g} = 9{\cdot}6$$

But,

$$L_1 = L,\ V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{A} \qquad (\because A_1 = A)$$

$$D_1 = D = 0{\cdot}6 \text{ m}$$

$$\therefore \quad \frac{4fL}{0{\cdot}6 \times 2 \times 9{\cdot}81} \times \frac{Q_1^2}{A^2} = 9{\cdot}6$$

or,

$$fL\frac{Q_1^2}{A^2} = \frac{9{\cdot}6 \times 0{\cdot}6 \times 2 \times 9{\cdot}81}{4} = 28{\cdot}25 \qquad \ldots(ii)$$

For pipe 2 :

$$h_{f_2} = \frac{4fL_2V_2^2}{D_2 \times 2g} = 9{\cdot}6$$

where,

$$L_2 = L,\ V_2 = \frac{Q_2}{A_2}$$

$$\therefore \quad \frac{4fLQ_2^2}{D_2 \times 2g \times A_2^2} = 9{\cdot}6$$

or,

$$\frac{fLQ_2^2}{D_2A_2^2} = \frac{9{\cdot}6 \times 2 \times 9{\cdot}81}{4} = 47{\cdot}09 \qquad \ldots(iii)$$

Dividing (*i*) by (*iii*), we get

$$\frac{Q^2}{Q_1^2} = \frac{79{\cdot}461}{28{\cdot}25} = 2{\cdot}81$$

or,

$$\frac{Q}{Q_1} = 1{\cdot}67$$

or, $$Q_1 = \frac{Q}{1{\cdot}67} = 0{\cdot}59\,Q$$

But, $$Q_1 + Q_2 = Q$$

$\therefore$ $$Q_2 = Q - Q_1 = Q - 0{\cdot}59Q = 0{\cdot}41Q$$

Dividing (*ii*) by (*iii*), we get

$$\frac{Q_1^2 \times D_2 \times A_2^2}{Q_2^2 \times A^2} = \frac{28{\cdot}25}{47{\cdot}09} = 0{\cdot}6$$

or, $$\frac{Q_1 \times D_2 \times (\pi/4 \times D_2^2)^2}{Q_2^2 \times \left[\pi/4 \times (0{\cdot}6)^2\right]^2} = 0{\cdot}6$$

$$\left(\frac{0{\cdot}59Q}{0{\cdot}41Q}\right)^2 \times \frac{D_2^5}{(0{\cdot}36)^2} = 0{\cdot}6$$

or, $$D_2^5 = 0{\cdot}6 \times (0{\cdot}36)^2 \times \left(\frac{0{\cdot}41}{0{\cdot}59}\right)^2 = 0{\cdot}03755$$

or, $$D_2 = 0{\cdot}518\text{ m} = \mathbf{518\text{ mm (Ans.)}}$$

Example 12·37. *Two pipes A and B are connected in parallel between two points. Pipe A is 180 m long and has a diameter of 12 cm. Pipe B is 120 m long and has a diameter of 10 cm. Both the pipes have the same friction factor of 0·017. A partially closed valve in pipe A causes the discharge in the two pipes to be the same (Fig. 12·31.). Neglecting all other minor losses, calculate the value of the valve co-efficient.*

Solution. *Given* : $L_A = 180$ m; $D_A = 12$ cm $= 0{\cdot}12$ m; $L_B = 120$ m; $D_B = 10$ cm $= 0{\cdot}1$ m; Friction factor; $f = 0{\cdot}017$.

Value of the valve co-efficient, K_v :

Since the discharges are same in both the pipes,

$$A_A V_A = A_B V_B$$

or, $$\frac{\pi}{4} \times (0{\cdot}12)^2 \times V_A = \frac{\pi}{4} \times (0{\cdot}1)^2 \times V_B$$

$\therefore$ $$V_B = 1{\cdot}44\, V_A$$

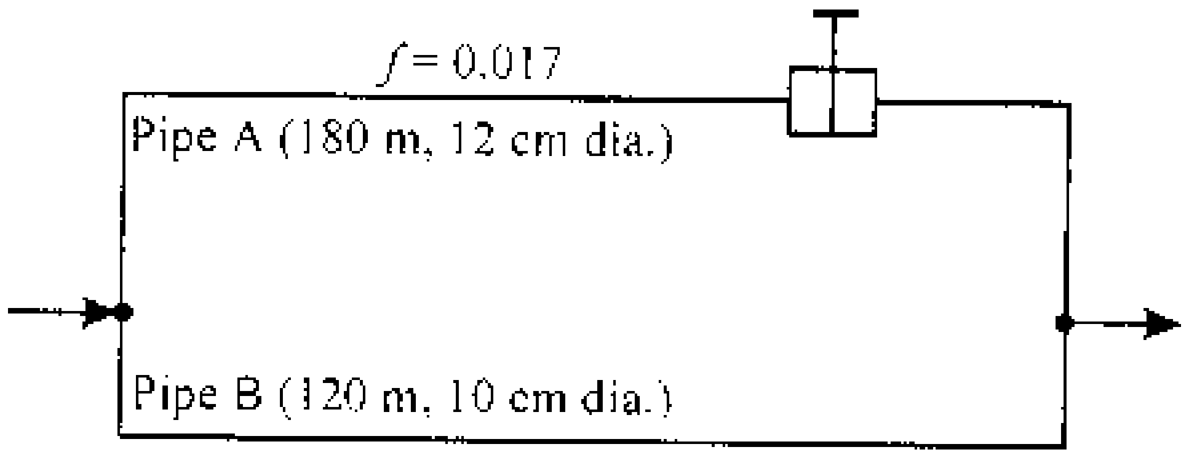

Fig. 12.31

Let the losses in the valve be $K_v \dfrac{V_A^2}{2g}$.

Head losses in both the pipes are same.

Hence, $$\frac{f_A L_A V_A^2}{D_A \times 2g} + \frac{K_v V_A^2}{2g} = \frac{f_B L_B V_B^2}{D_B \times 2g}$$

$$\frac{0{\cdot}017 \times 180}{0{\cdot}12} \times \frac{V_A^2}{2g} + K_v \frac{V_A^2}{2g} = \frac{0{\cdot}017 \times 120 \times (1{\cdot}44)^2}{0{\cdot}10} \times \frac{V_A^2}{2g}$$

$$25{\cdot}5 + K_v = 42{\cdot}30$$

$\therefore$ $$\mathbf{K_v = 16{\cdot}8 \text{ (Ans.)}}$$

Example 12·38. *Two pipes 1 and 2, each of 12 cm diameter branch off from a point A in a pipeline and rejoin at B. Pipe 1 is 480 m long and pipe 2 is 720 m long. So total head at A is 36 m. A short pipe 10 cm diameter is fitted at B and the flow is discharged into atmosphere through it as shown in Fig. 12·32. Assuming f = 0·018 for both the pipes, Calculate :*

(*i*) *Total discharge, and*

(*ii*) *Distribution of discharge in pipes 1 and 2.*

Solution. *Given* : $D_1 = 12$ cm $= 0{\cdot}12$ m; $L_1 = 480$ m; $D_2 = 12$ cm $= 0{\cdot}12$ m, $L_2 = 720$ m; $D_3 = 10$ cm $= 0{\cdot}1$ m; $f = 0{\cdot}018$.

(*i*) **Total discharge, Q :**

As 10 cm diameter pipe is short, the friction loss in it can be neglected.

$$H_B \,(= \text{Head at } B) = \frac{V_3^2}{g}$$

$$H_A - H_B = 36 - \frac{V_3^2}{2g}$$

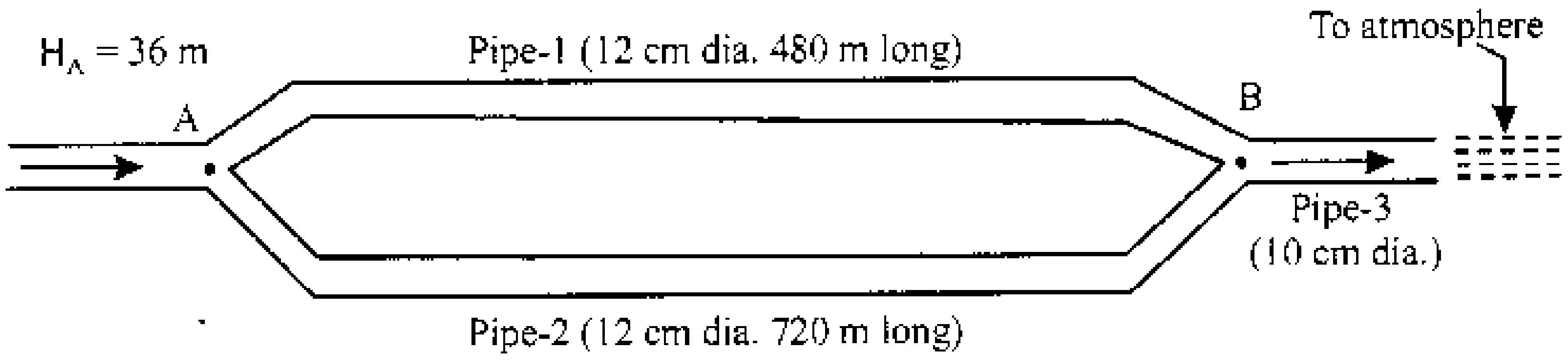

Fig. 12.32

Consider an equivalent pipe $D_{eq.} = 0{\cdot}1$ m and $f_{eq.} = 0{\cdot}018$ to replace the parallel pipes 1 and 2. Then,

$$\left(\frac{D_{eq.}^5}{f_{eq.}\, L_{eq.}}\right)^{\frac{1}{2}} = \left(\frac{D_1^5}{f_1\, L_1}\right)^{\frac{1}{2}} + \left(\frac{D_2^5}{f_2\, L_2}\right)^{\frac{1}{2}}$$

Since $\quad f_{eq.} = f_1 = f_2$ and $D_1 = D_2 = 0{\cdot}12$ m

$$\therefore \quad \frac{(0{\cdot}10)^{5/2}}{(L_{eq.})^{1/2}} = (0{\cdot}12)^{5/2}\left[\frac{1}{\sqrt{480}} + \frac{1}{\sqrt{720}}\right]$$

$$= 0{\cdot}004988\,(0{\cdot}04564 + 0{\cdot}03727) = 0{\cdot}0004136$$

$$\therefore \quad L_{eq.} = \left[\frac{(0{\cdot}10)^{5/2}}{0{\cdot}0004136}\right]^2 = 58{\cdot}46 \text{ m}$$

As $\quad D_{eq.} = 0{\cdot}1$ m, velocity in this pipe $= V_3 = V_{eq.}$

$$\therefore \text{ Head loss} \quad = H_A - H_B = 36 - \frac{V_3^2}{2g} = \frac{f_{eq.}\, L_{eq.}\, V_{eq.}^2}{D_{eq.} \times 2g}$$

$$= \frac{0{\cdot}018 \times 58{\cdot}46}{0{\cdot}1} \times \frac{V_3^2}{2g} = 10{\cdot}52\,\frac{V_3^2}{2g}$$

$$\therefore \quad \frac{V_3^2}{2g} = (10{\cdot}52 + 1) = 36$$

$$V_3 = \left(\frac{36}{11{\cdot}52} \times 2 \times 9{\cdot}81\right)^{1/2} = 7{\cdot}83 \text{ m/s}$$

Total discharge, $\quad Q = \frac{\pi}{4} \times (0{\cdot}10)^2 \times 7{\cdot}83 = \mathbf{0{\cdot}06149\ m^3/s}$ **(Ans.)**

(*ii*) **Division of discharge in pipes 1 and 2; Q_1, Q_2**

$$H_A - H_B = 36 - \frac{(7{\cdot}83)^2}{2 \times 9{\cdot}81} = 32{\cdot}875 \text{ m} = h_{f_1} = h_{f_2}$$

$$\therefore \quad \frac{f_1 L_1 V_1^2}{D_1 \times 2g} = \frac{0{\cdot}018 \times 480}{0{\cdot}12} \times \frac{V_1^2}{2g} = 32{\cdot}875$$

$$\therefore \quad V_1 = \left(\frac{32{\cdot}875 \times 2 \times 9{\cdot}81 \times 0{\cdot}12}{0{\cdot}018 \times 480}\right)^{1/2} = 2{\cdot}993 \text{ m/s}$$

$$\therefore \quad \mathbf{Q_1} = \frac{\pi}{4} \times (0{\cdot}12)^2 \times 2{\cdot}993 = \mathbf{0{\cdot}03385\ m^3/s\ \ (Ans.)}$$

Again,
$$\frac{f_2 L_2}{D_2} \times \frac{V_2^2}{2g} = \frac{0{\cdot}018 \times 720}{0{\cdot}12} \times \frac{V_2^2}{2g} = 32{\cdot}875$$

or,
$$V_2 = \left(\frac{32{\cdot}875 \times 2 \times 9{\cdot}81 \times 0{\cdot}12}{0{\cdot}018 \times 720}\right)^{1/2} = 2{\cdot}444 \text{ m/s}$$

$$\therefore \quad \mathbf{Q_2} = \frac{\pi}{4} \times (0{\cdot}12)^2 \times 2{\cdot}444 = \mathbf{0{\cdot}02764\ m^3/s\ \ (Ans.)}$$

[*Check* : $Q_1 + Q_2 = 0{\cdot}03385 + 0{\cdot}02764 = 0{\cdot}06149$ m³/s]

Example 12·39. *Two reservoirs A and B are connected through a piping system consisting of 50 cm diameter, pipe, 450 m long branching two pipes of 35 cm diameter and 25 cm diameter, each 650 m long. A pump situated at reservoir A pumps 0·35 m³/s of water through this pipe system to reservoir B whose water surface elevation is 50 m above that of A. Assuming pump efficiency as 60 percent and f = 0·018, determine the input power for the pump.*

Solution. Refer to Fig. 12·33. *Given* : $D_1 = 50$ cm $= 0{\cdot}5$ m, $L_1 = 450$ m; $D_2 = 35$ cm $= 0{\cdot}35$ m, $L_2 = 650$ m; $D_3 = 25$ cm $= 0{\cdot}25$ m, $L_3 = 650$ m, $Q = 0{\cdot}35$ m³/s; $\eta_{pump} = 60$ %; $f = 0{\cdot}018$.

Consider equivalent pipe of diameter 0·5 m to replace the two parallel pipes. The equivalent pipe ($D_{eq.}$, $L_{eq.}$, $f_{eq.}$) to replace a set of parallel pipes (D_2, L_2, f_2) and (D_3, L_3, f_3) is given by,

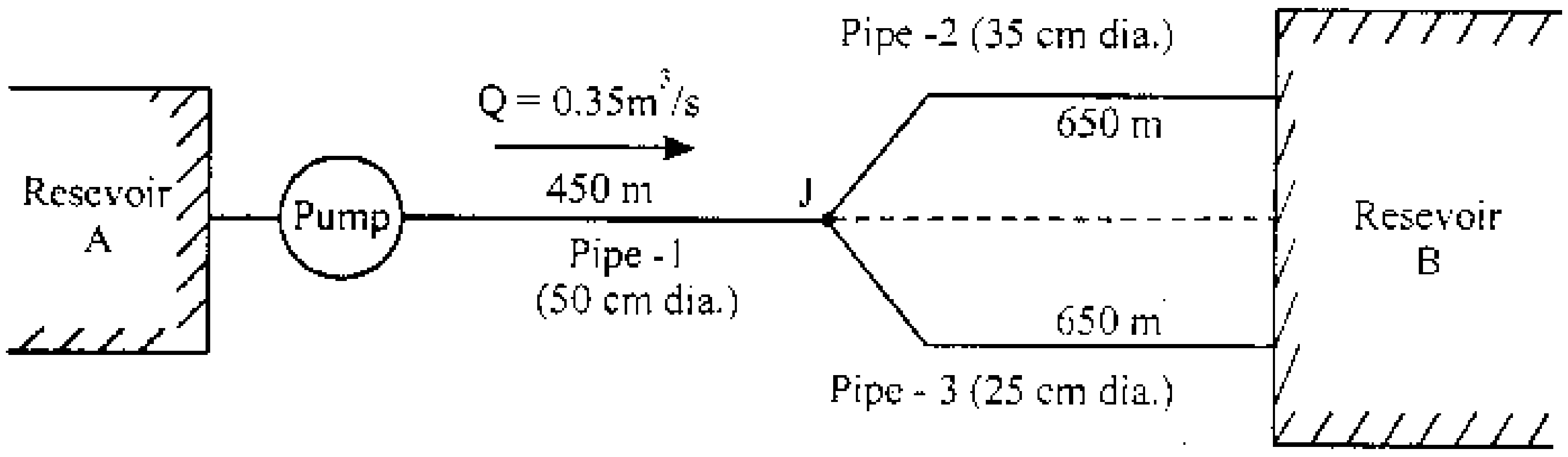

Fig. 12.33

$$\left(\frac{D_{eq.}^5}{f_{eq.}\,L_{eq.}}\right)^{1/2} = \left(\frac{D_2^5}{f_2\,L_2}\right)^{1/2} + \left(\frac{D_3^5}{f_3\,L_3}\right)^{1/2}$$

Here, $\quad f_{eq.} = f_1 = f_2$

Substituting the various values in the above eqn., we get

$$\left[\frac{(0{\cdot}5)^5}{L_{eq.}}\right]^{1/2} = \left[\frac{(0{\cdot}35)^5}{650}\right]^{1/2} + \left[\frac{(0{\cdot}25)^5}{650}\right]^{1/2}$$

or,
$$\frac{0{\cdot}1768}{(L_{eq.})^{1/2}} = 0{\cdot}002843 + 0{\cdot}001226$$

or, $L_{eq.} = 1888$ m = Equivalent length of 0·5 m diameter pipe to replace the parallel pipe.

The total equivalent length of 0·5 m pipe is now

$$= 450 + 1888 = 2338 \text{ m}$$

$$V = \frac{Q}{A} = \frac{0{\cdot}35}{\frac{\pi}{4} \times (0{\cdot}5)^2} = 1{\cdot}78 \text{ m/s}$$

$$h_f = \frac{f_{eq.}\, L_{eq.}\, V^2}{D \times 2g} = \frac{0{\cdot}018 \times 2338 \times (1{\cdot}78)^2}{0{\cdot}5 \times 2 \times 9{\cdot}81} = 13{\cdot}59 \text{ m}$$

$$h_t = \text{Total pumping head} = 50 - 13{\cdot}59 = 63{\cdot}59 \text{ m}$$

Power input for the pump,

$$P = \frac{wQh_f}{\eta_{pump}} = \frac{9{\cdot}81 \times 0{\cdot}35 \times 63{\cdot}59}{0{\cdot}6} = \mathbf{363{\cdot}9 \text{ kW}} \quad \textbf{(Ans.)}$$

Note : This question could also be solved without considering the equivalent pipe. First the discharge through the each pipe is determined and then the total frictional loss is calculated. However, the calculations are definitely less with the equivalent pipe method.

Example 12·40. (Flow through branched pipes). *The water levels in the two reservoirs A and B are 104·5 m and 100 m respectively above the daum. A pipe joins each to a common point D, where pressure is 98·1 kN/m² gauge and height is 83·5 m above datum. Another pipe connects D to another tank C. What will be the height of water level in C assuming the same value of 'f' for all pipes. Take friction co-efficient = 0·0075. The diameters of the pipes AD, BD and CD are 300 mm, 450 mm, 600 mm respectively and their lengths are 240 m, 270 m, 300 m respectively.* **[IIT Delhi]**

Solution. For pipe AD : $D_{AD} = 300 \text{ mm} = 0{\cdot}3 \text{ m}$

$L_{AD} = 240 \text{ m}$

For pipe BD : $D_{BD} = 450 \text{ mm} = 0{\cdot}45 \text{ m}$

$L_{BD} = 270 \text{ m}$

For pipe CD : $D_{CD} = 600 \text{ mm} = 0{\cdot}6 \text{ m}$

$L_{CD} = 300 \text{ m}$

Friction co-efficient for each pipe, $f = 0{\cdot}0075$

Pressure at D, $p_0 = 98{\cdot}1 \text{ kN/m}^2$

Height of water level in tank C :

The pressure head at $D = \frac{p_D}{w} = \frac{98{\cdot}1}{9{\cdot}81} = 10$ m of water

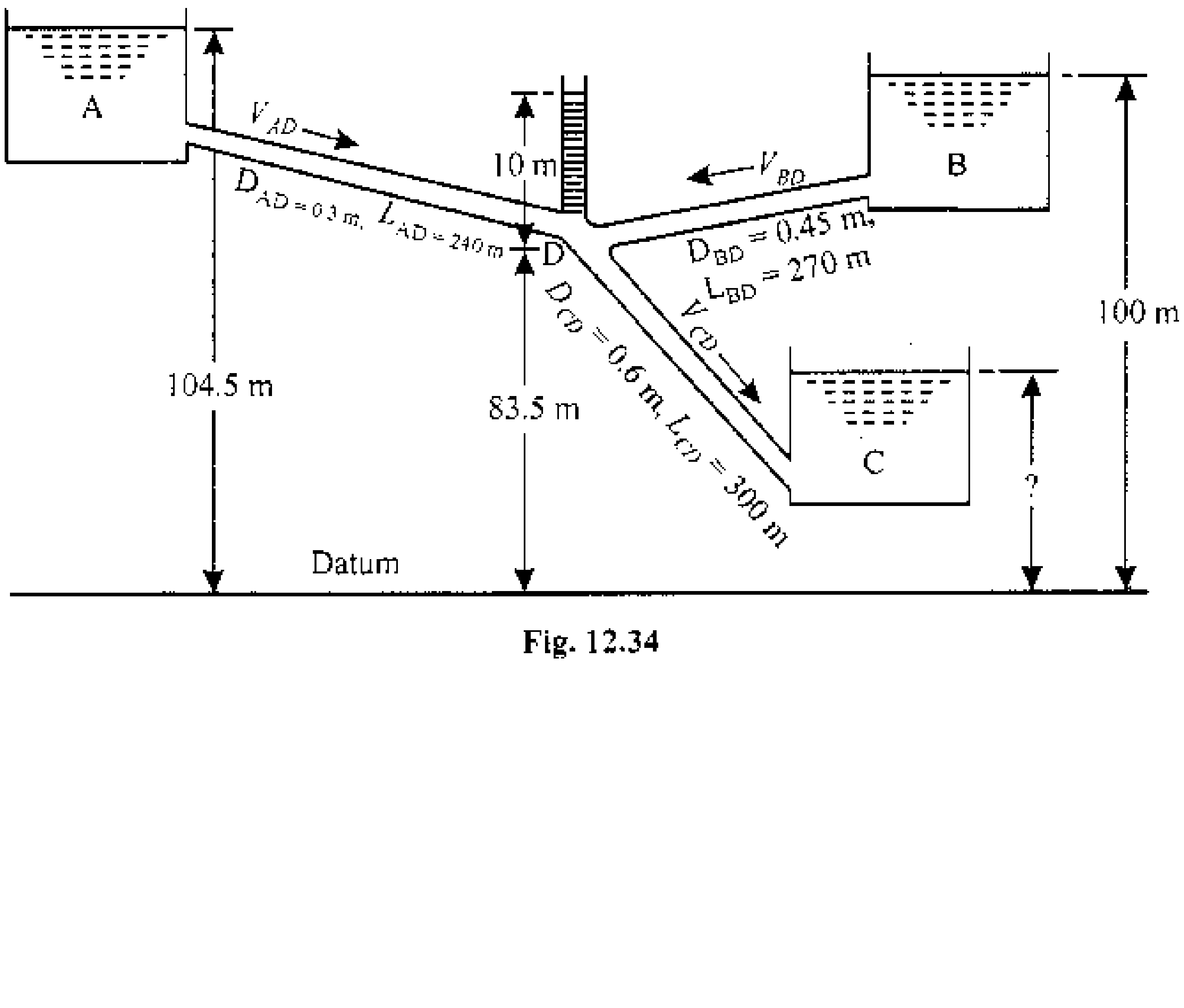

Fig. 12.34

$\therefore$ The piezometric head at $D = 83{\cdot}5 - 10 = 93{\cdot}5$ m

Head loss between A and $D = 104{\cdot}5 - 93{\cdot}5 = 11{\cdot}0$ m

Head loss between B and $D = 100 - 93{\cdot}5 = 6{\cdot}5$ m

Using Darcy-Weisbach equation, we get

For pipe AD :
$$11 = \frac{4fL_{AD}V_{AD}^2}{D_{AD} \times 2g} = \frac{4 \times 0{\cdot}0075 \times 240 \times V_{AD}^2}{0{\cdot}3 \times 2 \times 9{\cdot}81}$$

or,
$$V_{AD}^2 = \frac{11 \times 0{\cdot}3 \times 2 \times 9{\cdot}81}{4 \times 0{\cdot}0075 \times 240} = 8{\cdot}99$$

or,
$$V_{AD} = 3 \text{ m/s}$$

For pipe BD :
$$6{\cdot}5 = \frac{4fL_{BD}V_{BD}^2}{D_{BD} \times 2g} = \frac{4 \times 0{\cdot}0075 \times 270 \times V_{BD}^2}{0{\cdot}45 \times 2 \times 9{\cdot}81}$$

or,
$$V_{BD}^2 = \frac{6{\cdot}5 \times 0{\cdot}45 \times 2 \times 9{\cdot}81}{4 \times 0{\cdot}0075 \times 270} = 7{\cdot}085$$

or,
$$V_{BD} = 2{\cdot}66 \text{ m/s}$$

From continuity considerations, we have

$$Q_{AD} + Q_{BD} = Q_{CD}$$

or,
$$Q_{CD} = (\pi/4) \times D_{AD}^2 \times V_{AD} + (\pi/4) \times D_{BD}^2 \times V_{BD}$$
$$= (\pi/4) \times (0{\cdot}3)^2 \times 3 + (\pi/4) \times (0{\cdot}45)^2 \times 2{\cdot}66 = 0{\cdot}635 \text{ m}^3\text{/s}$$

$\therefore$ Velocity of flow in pipe CD,

$$V_{CD} = \frac{Q_{CD}}{(\pi/4) \times D_{CD}^2} = \frac{0{\cdot}635}{(\pi/4) \times 0{\cdot}6^2} = 2{\cdot}24 \text{ m/s}$$

Head loss in pipe $CD = \dfrac{4fL_{CD}V_{CD}^2}{D_{CD} \times 2g} = \dfrac{4 \times 0{\cdot}0075 \times 300 \times 2{\cdot}24^2}{0{\cdot}6 \times 2 \times 9{\cdot}81} = 3{\cdot}84$ m

$\therefore$ Water level in tank $C = 93{\cdot}5 - 3{\cdot}84 =$ **89·66 m (Ans.)**

Example 12·41. (Flow through branched pipes). *Fig. 12·35 shows three reservoirs connected by pipes. Each pipe is 300 mm in diameter and 1500 m long. Assuming co-efficient of friction for each pipe, f = 0·01 find the discharge in each pipe.*

Solution. Diameter of each pipe, $D_1 = D_2 = D_3 = 300 \text{ mm} = 0{\cdot}3$ m

Length of each pipe, $L_1 = L_2 = L_3 = 1500$ m

Co-efficient of friction for each pipe, $f = 0{\cdot}1$

Discharge in each pipe :

To find out the direction of flow in pipe 2, let us *assume* that no flow occurs in pipe 2. That is, the piezometric level is 30 m,

$\therefore$ Head loss in pipe 1, $h_{f_1} = 70 - 30 = 40$ m

Also,
$$h_{f_1} = \frac{4fL_1V_1^2}{D_1 \times 2g}$$

$\therefore$
$$40 = \frac{4 \times 0{\cdot}01 \times 1500V_1^2}{0{\cdot}3 \times 2 \times 9.81}$$

or,
$$V_1^2 = \frac{40 \times 0{\cdot}3 \times 2 \times 9{\cdot}81}{4 \times 0{\cdot}01 \times 1500} = 3{\cdot}924$$

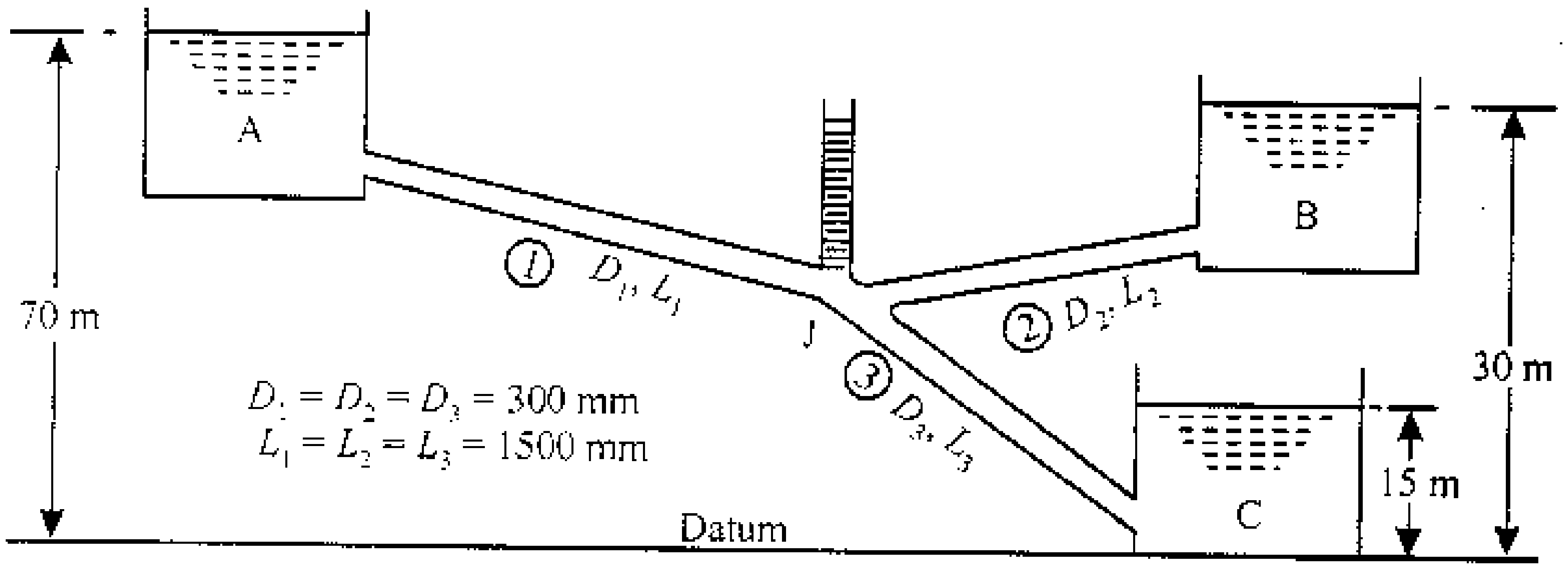

Fig. 12.35

or, $V_1 = 1{\cdot}981$ m/s

∴ Discharge through the pipe 1,

$$Q_1 = A_1 V_1 = \frac{\pi}{4} \times 0{\cdot}3^2 \times 1{\cdot}981 = 0{\cdot}14 \text{ m}^3\text{/s}$$

Again, head loss in pipe 3, $h_{f_3} = 30 - 15 = 15$ m

But,
$$h_{f_3} = \frac{4fL_3V_3^2}{D_3 \times 2g}$$

∴
$$15 = \frac{4 \times 0{\cdot}01 \times 1500 \times V_3^2}{0{\cdot}3 \times 2 \times 9{\cdot}81}$$

or,
$$V_3^2 = \frac{15 \times 0{\cdot}3 \times 2 \times 9{\cdot}81}{4 \times 0{\cdot}01 \times 1500} = 1{\cdot}471$$

or, $V_3 = 1{\cdot}213$ m/s

∴ Discharge through the pipe 3,

$$Q_3 = A_3 V_3 = \frac{\pi}{4} \times 0{\cdot}3^2 \times 1{\cdot}213 = 0{\cdot}0857 \text{ m}^3\text{/s}$$

Since $Q_1 > Q_3$, the *direction of flow is from J to B.*

Considering the flow from reservoir A and B, we have

$(70 - 30)$ = Head loss in pipe 1 + head loss in pipe 2

or,
$$40 = h_{f_1} + h_{f_2} = \frac{4fL_1V_1^2}{D_1 \times 2g} + \frac{4fL_2V_2^2}{D_2 \times 2g}$$

$$40 = \frac{4 \times 0{\cdot}01 \times 1500 \times V_1^2}{0{\cdot}3 \times 2 \times 9{\cdot}81} + \frac{4 \times 0{\cdot}01 \times 1500 \times V_2^2}{0{\cdot}3 \times 2 \times 9{\cdot}81}$$

or,
$$40 = 10{\cdot}2\,(V_1^2 + V_2^2)$$

or,
$$V_1^2 + V_2^2 = \frac{40}{10{\cdot}2} = 3{\cdot}92$$

or,
$$V_2 = \sqrt{3{\cdot}92 - V_1^2} \qquad \text{...(i)}$$

Similarly, considering the flow from reservoir A to C, we have

$$70 - 15 = h_{f_1} + h_{f_3}$$

$$= \frac{4fL_1V_1^2}{D_1 \times 2g} + \frac{4fL_3V_3^2}{D_3 \times 2g} = \frac{4 \times 0{\cdot}01 \times 1500 \times V_1^2}{0{\cdot}3 \times 2 \times 9{\cdot}81} + \frac{4 \times 0{\cdot}01 \times 1500 \times V_3^2}{0{\cdot}3 \times 2 \times 9{\cdot}81}$$

or,
$$55 = 10{\cdot}2\,(V_1^2 + V_3^2)$$

or, $$V_1^2 + V_3^2 = \frac{55}{10 \cdot 2} = 5 \cdot 39$$

or, $$V_3 = \sqrt{5 \cdot 39 - V_1^2} \qquad \ldots(ii)$$

From continuity considerations, we have

$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

But, $$A_1 = A_2 = A_3 \qquad (\because D_1 = D_2 = D_3)$$

$\therefore$ $$V_1 = V_2 + V_3 \qquad \ldots(iii)$$

From eqns. (*i*), (*ii*) and (*iii*), we have

$$V_1 = \sqrt{3 \cdot 92 - V_1^2} + \sqrt{5 \cdot 39 - V_1^2} \qquad \ldots(iv)$$

By trial and error, we get, $V_1 \simeq 1 \cdot 9$ m/s

From eqn. (*i*) : $$V_2 = \sqrt{3 \cdot 92 - 1 \cdot 9^2} = 0 \cdot 56 \text{ m/s}$$

From (*ii*) : $$V_3 = \sqrt{5 \cdot 39 - 1 \cdot 9^2} = 1 \cdot 34 \text{ m/s}$$

Thus, $Q_1 = (\pi/4) \times 0 \cdot 3^2 \times 1 \cdot 9 =$ **$0 \cdot 134$ m³/s (Ans.)**

$Q_2 = (\pi/4) \times 0 \cdot 3^2 \times 0 \cdot 56 =$ **$0 \cdot 0396$ m³/s (Ans.)**

$Q_3 = (\pi/4) \times 0 \cdot 3^2 \times 1 \cdot 34 =$ **$0 \cdot 0947$ m³/s (Ans.)**

Example 12·42. *Fig. 12·36 shows a pump supplying water from a sump at elevation 20 m to a reservoir at elevation 30 m through a pipeline of 0·5 m diameter and length 1000 m, f = 0·005. At mid-length a branch pipe 0·3 diameter, 500 m long, f = 0·005, discharges free at elevation 25 m at the rate of 0·25 m³/s. Determine :*

(*i*) *The discharge into the reservoir,*

(*ii*) *The pressure to be maintained by the pump, and*

(*iii*) *The power of the pump assuming an overall efficiency of 70 per cent.*

Solution. Refer to Fig. 12·36.

Given : $D_1 = 0 \cdot 5$ m, $L_1 = 500$ m

$D_2 = 0 \cdot 5$ m, $L_2 = 500$ m

$D_3 = 0 \cdot 3$ m, $L_3 = 500$ m

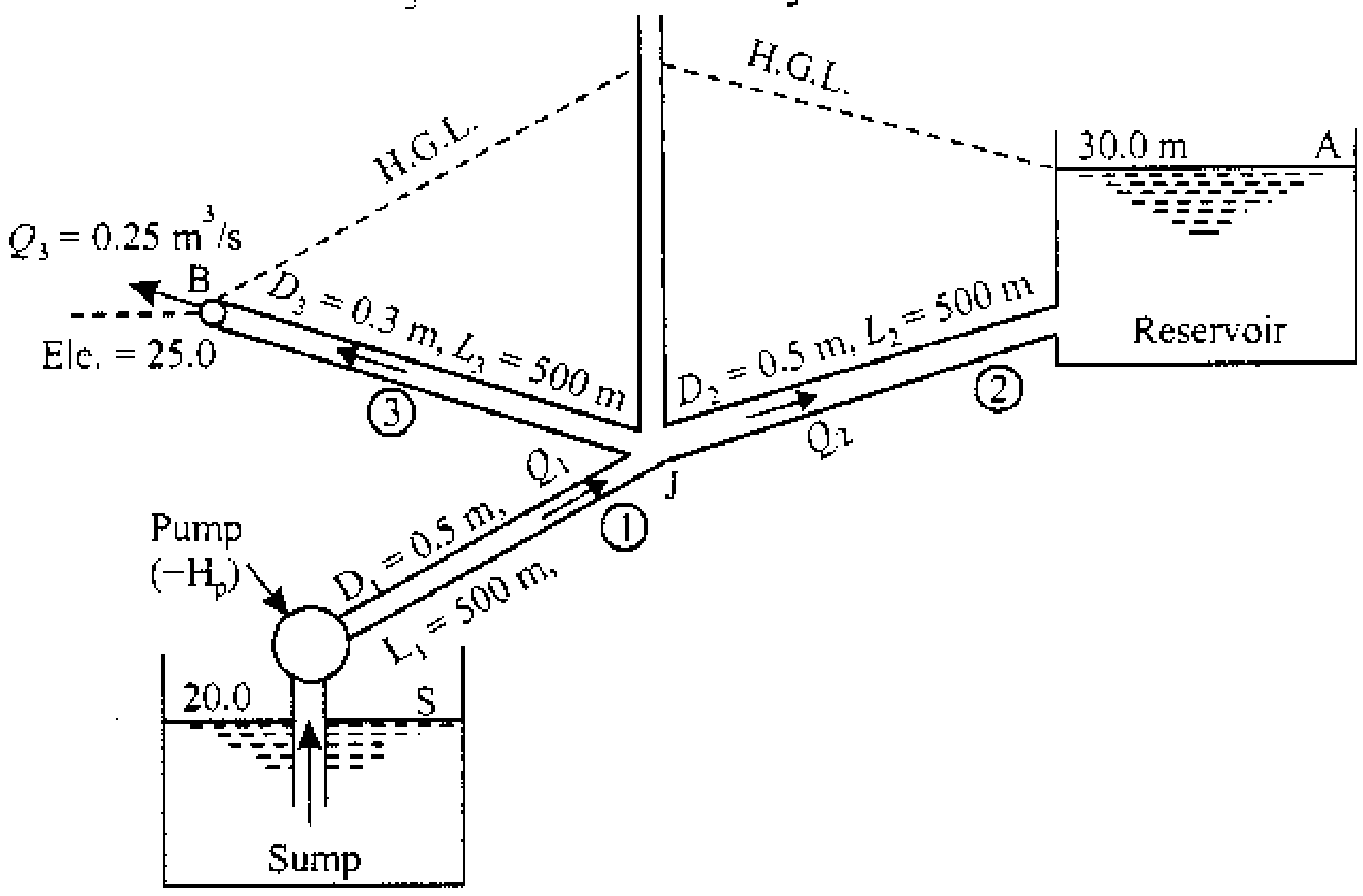

Fig. 12.36

Co-efficient of friction, $f_1 = f_2 = f_3 = f = 0{\cdot}005$

Discharge through pipe 3, $Q_3 = 0{\cdot}25$ m³/s

Overall efficiency, $\eta_0 = 70\%$.

(*i*) **The discharge into the reservoir, Q_2 :**

Energy at the joint J,

$$E_J = E_B \text{ (energy at } B) + (h_f)_{JB}$$

$$= \left(\frac{p_B}{w} + \frac{V_3^2}{2g} + z_B\right) + (h_f)_{JB}$$

$$= \frac{p_B}{w} + \frac{V_3^2}{2g} + z_B + \frac{4f_3L_3V_3^2}{D_3 \times 2g}$$

$$= 0 + \frac{V_3^2}{2g}\left(1 + \frac{4f_3L_3}{D_3}\right) + 25$$

$$= \left(\frac{0{\cdot}25}{\pi/4 \times 0{\cdot}3^2}\right)^2 \times \frac{1}{2 \times 9{\cdot}81}\left(1 + \frac{4 \times 0{\cdot}005 \times 500}{0{\cdot}3}\right) + 25$$

$$= 0{\cdot}637\,(1 + 33{\cdot}33) + 25 = 46{\cdot}87 \text{ m}$$

Head loss, $h_{f_2} = 46{\cdot}87 - 30 = 16{\cdot}87$ m

i.e. $$16{\cdot}87 = \frac{4f_2L_2V_2^2}{D_2 \times 2g} = \frac{4 \times 0{\cdot}005 \times 500 \times V_2^2}{0{\cdot}5 \times 2 \times 9{\cdot}81}$$

or, $$V_2 = \left(\frac{16{\cdot}87 \times 0{\cdot}5 \times 2 \times 9{\cdot}81}{4 \times 0{\cdot}005 \times 500}\right)^{1/2} = 4{\cdot}07 \text{ m/s}$$

$\therefore$ $Q_2 = (\pi/4) \times 0{\cdot}5^2 \times 4{\cdot}07 = \mathbf{0{\cdot}8}$ **m³/s (Ans.)**

$Q_1 = Q_2 + Q_3 = 0{\cdot}8 + 0{\cdot}25 = 1{\cdot}05$ m³/s

Head loss, $$h_{f_1} = \frac{4f_1L_1V_1^2}{D_1 \times 2g} = \frac{4 \times 0{\cdot}005 \times 500 \times (5{\cdot}35)^2}{0{\cdot}5 \times 2 \times 9{\cdot}81} = 29{\cdot}17 \text{ m}$$

$$\left(\because V_1 = \frac{Q_1}{A_1} = \frac{1{\cdot}05}{(\pi/4) \times 0{\cdot}5^2} = 5{\cdot}35 \text{ m/s}\right)$$

Applying energy equation between the sump (S) and the junction (J), we have

$$E_S + H_p = E_J + h_{f_1}$$

$$0 + 0 + 20 + H_p = 46{\cdot}87 + 29{\cdot}17 = 76{\cdot}04 \text{ m}$$

$$H_p = 56{\cdot}04 \text{ m}$$

(*i*) **The pressure to be maintained by the pump, p :**

$$p = wH_p = 9{\cdot}81 \times 56{\cdot}04 = \mathbf{549{\cdot}7 \text{ kN/m}^2} \quad \textbf{(Ans.)}$$

(*ii*) **The power of pump, P :**

$$P = \frac{wQ_1H_p}{\eta_0} = \frac{9{\cdot}81 \times 1{\cdot}05 \times 56{\cdot}04}{0{\cdot}7} = \mathbf{824{\cdot}6 \text{ kW}} \quad \textbf{(Ans.)}$$

Example 12·43. (Pipe networks). *Fig. 12·37 shows a network in which Q and h_f refer to discharges and pressure drops respectively. Subscripts 1, 2, 3, 4 and 5 designate respective values in pipe lengths AC, BC, CD, DA and AC. Subscripts A, B, C and D designate discharges entering or leaving the junction points A, B, C and D respectively.*

By sticking to the values given in the figure find the following discharges Q_B, Q_2, Q_4 and Q_5; and pressure drops h_{f_4} and h_{f_5} and give these computed values at their respective places on a neat sketch of the network along with flow directions. **[GATE]**

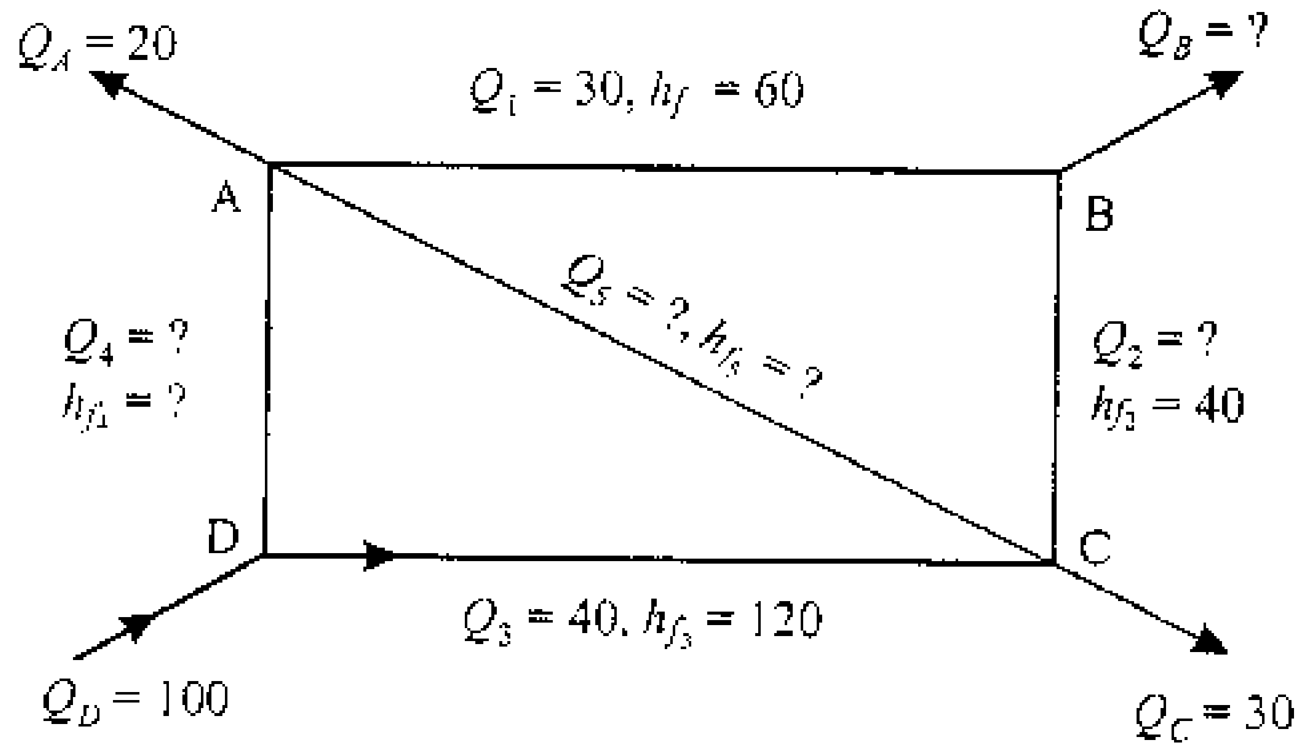

Fig. 12.37

Solution. At junctions, $\Sigma Q = 0$

i.e., Discharge entering the junction = Discharge leaving the junction

At junction D : $Q_D = Q_3 + Q_4$

or $100 = 40 + Q_4$

or $\boldsymbol{Q_4 = 100 - 40 = 60}$ ...leaving the junction

At junction A : $Q_4 = Q_A + Q_1 + Q_5$

$60 = 20 + 30 + Q_5$

$\therefore$ $\boldsymbol{Q_5 = 60 - 20 - 30 = 10}$ ...leaving the junction

At junction C : $Q_3 + Q_5 + Q_2 = Q_c$

$40 + 10 + Q_2 = 30$

$\therefore$ $\boldsymbol{Q_2 = 30 - 40 - 10 = -20}$ ...leaving the junction *C*

At junction B : $Q_1 + Q_2 = Q_B$

$30 + 20 = Q_B$

i.e., $\boldsymbol{Q_B = 50}$ ...leaving the junction *B*

For each elementary circuit, $\boldsymbol{\Sigma h_f = 0}$

Circuit ABC :

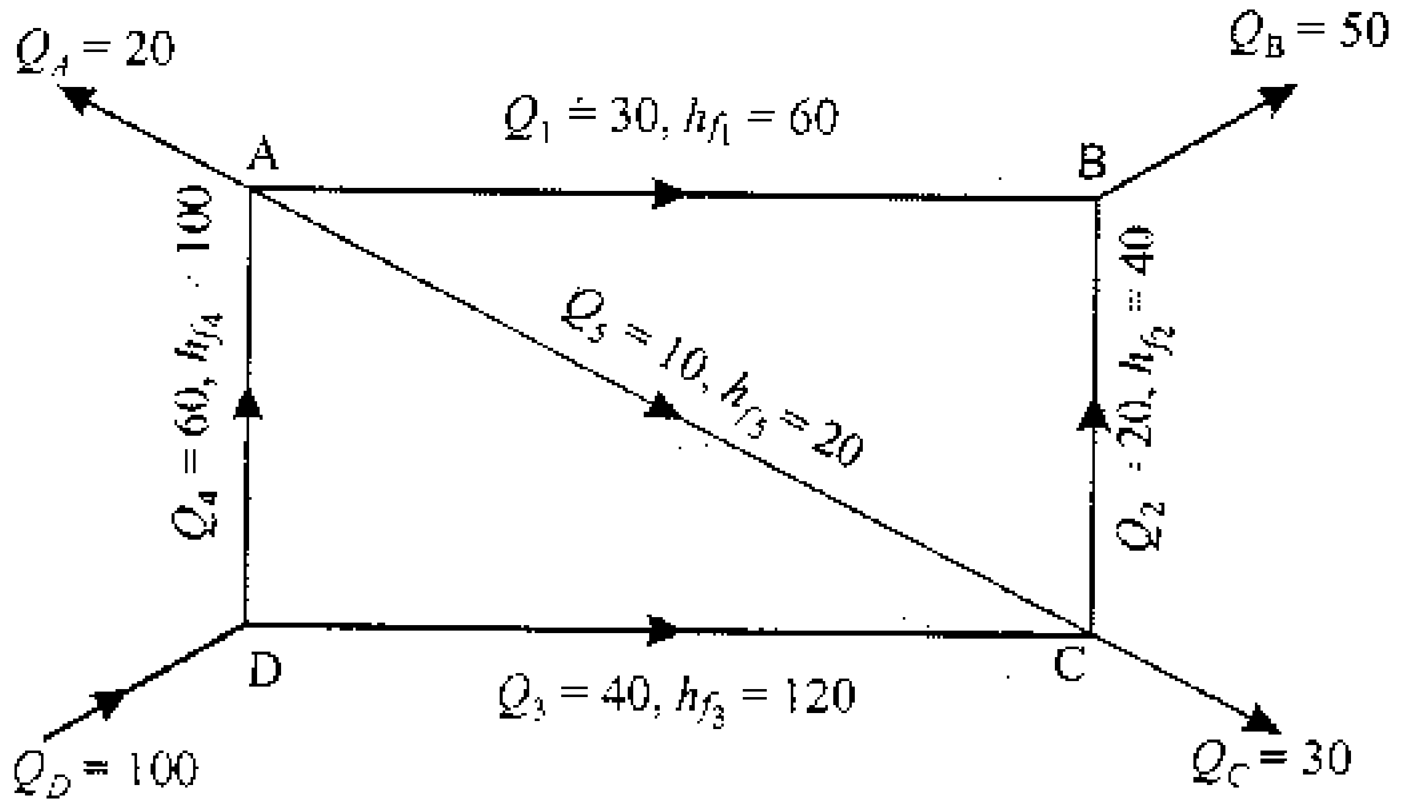

Fig. 12.38

$$h_{f_1} - h_{f_2} - h_{f_5} = 0$$

$$60 - 40 - h_{f_5} = 0$$

$$\therefore \quad h_{f_5} = 20$$

Circuit ACD :

$$h_{f_4} + h_{f_5} - h_{f_3} = 0$$

$$h_{f_4} + 20 - 120 = 0$$

$$\therefore \quad h_{f_4} = 100$$

The calculated vlaues and the flow directions are shown in Fig. 12·38.

12.9. Syphon

A **syphon** *is a long bent pipe employed for carrying water from a reservoir at a higher elevation to another reservoir at a lower elevation when the two reservoirs are separated by a hill or high level ground in between* as shown in Fig. 12·39.

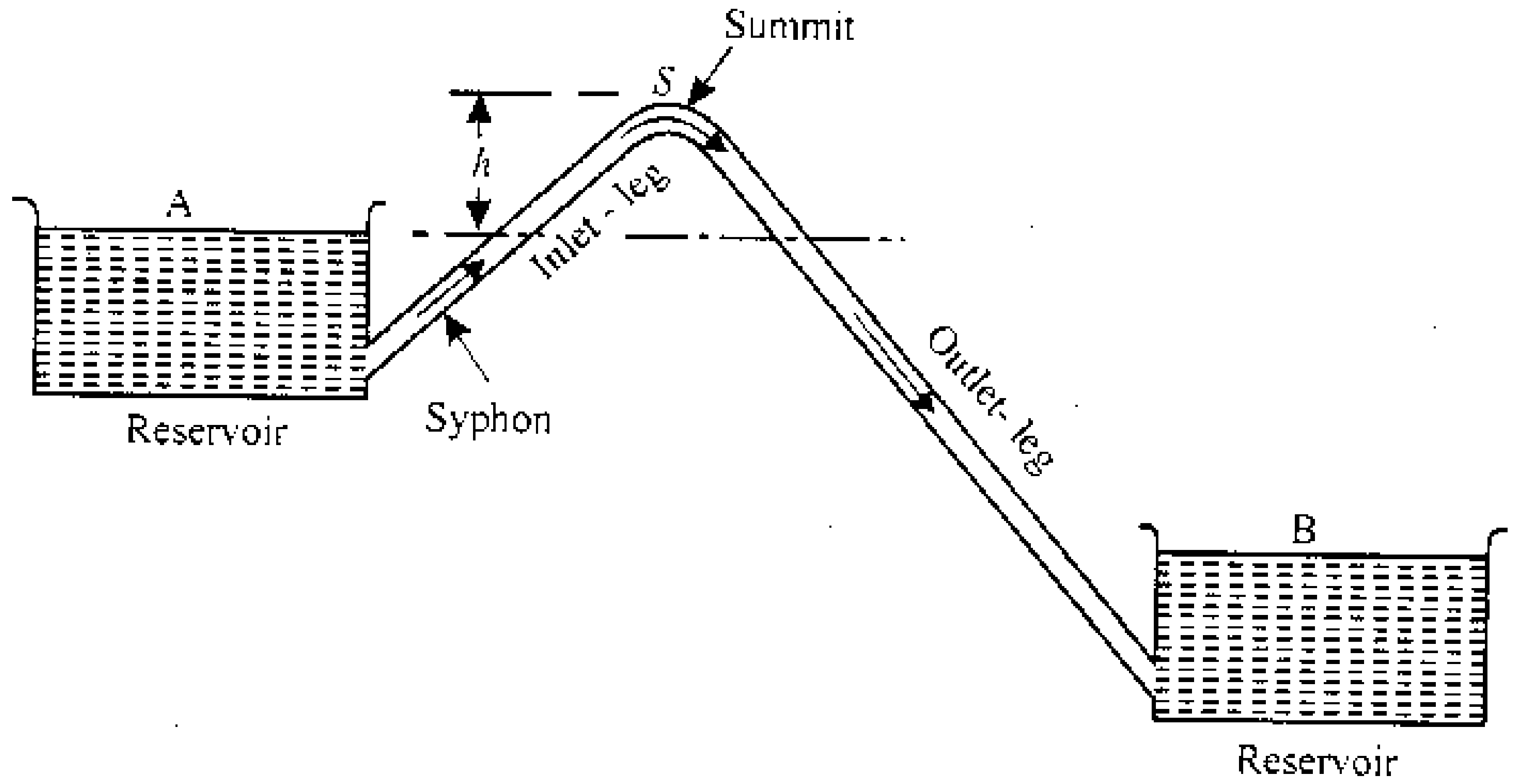

Fig. 12.39. Syphon.

The highest point (S) of the syphon is called the **summit.** The pressure at the point S is **less** *than atmospheric pressure* (since S lies above the free water surface in the tank A). The pressure at S can be reduced theoretically to − 10·3 m of water but in actual practice this pressure is only − 7·6 m of water (or 10·3 − 7·6 = 2·7 m of water *absolute*). When the pressure at S becomes less than 2·7 m of water absolute, the dissolved air and other gases would come out from water and collect at the summit. *Therefore syphon should be so laid that no section of the pipe will be more than 7·6 m above the hydraulic gradient at that section.* Moreover, in order to limit the reduction of the pressure at the summit the length of the *inlet-leg* (rising portion of the syphon) of the syphon is also required to be limited (this is so because, if the *inlet leg* is very long a considerable loss of head due to friction is caused, resulting in further reduction of the pressure at the summit).

Example 12.44. *Two reservoirs, having a difference in elevation of 15 m, are connected by a 200 mm diameter syphon. The length of the syphon is 400 m and the summit is 3 m above the water level in the upper reservoir. The length of the pipe from upper reservoir to the summit is 120 m. If the co-efficient of friction is 0·005, determine :*

(*i*) *Discharge through the syphon, and*

(*ii*) *Pressure at the summit.*

Neglect minor losses.

Solution. Diameter of the syphon, $D = 200$ mm $= 0{\cdot}2$ m

Length of the syphon, $L = 400$ m

Difference in level of the two reservoirs, $H = 15$ m

Height of the summit from upper reservoir, $h = 3$ m

Co-efficient of friction, $f = 0{\cdot}005$

(*i*) **Discharge through the syphon, Q :**

Applying Bernoulli's equation to points A and B, we get,

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{w} + \frac{V_B^2}{2g} + z_B + \text{loss of head due to friction from } A \text{ to } B$$

or, $$0 + 0 + z_A = 0 + 0 + z_B + h_f$$

$[\because p_A = p_B = \text{atmospheric pressure, and } V_A = V_B = 0]$

or, $$z_A - z_B = h_f = 15$$

But, $$h_f = \frac{4fLV^2}{D \times 2g}$$ (where, V = velocity of water in the pipe)

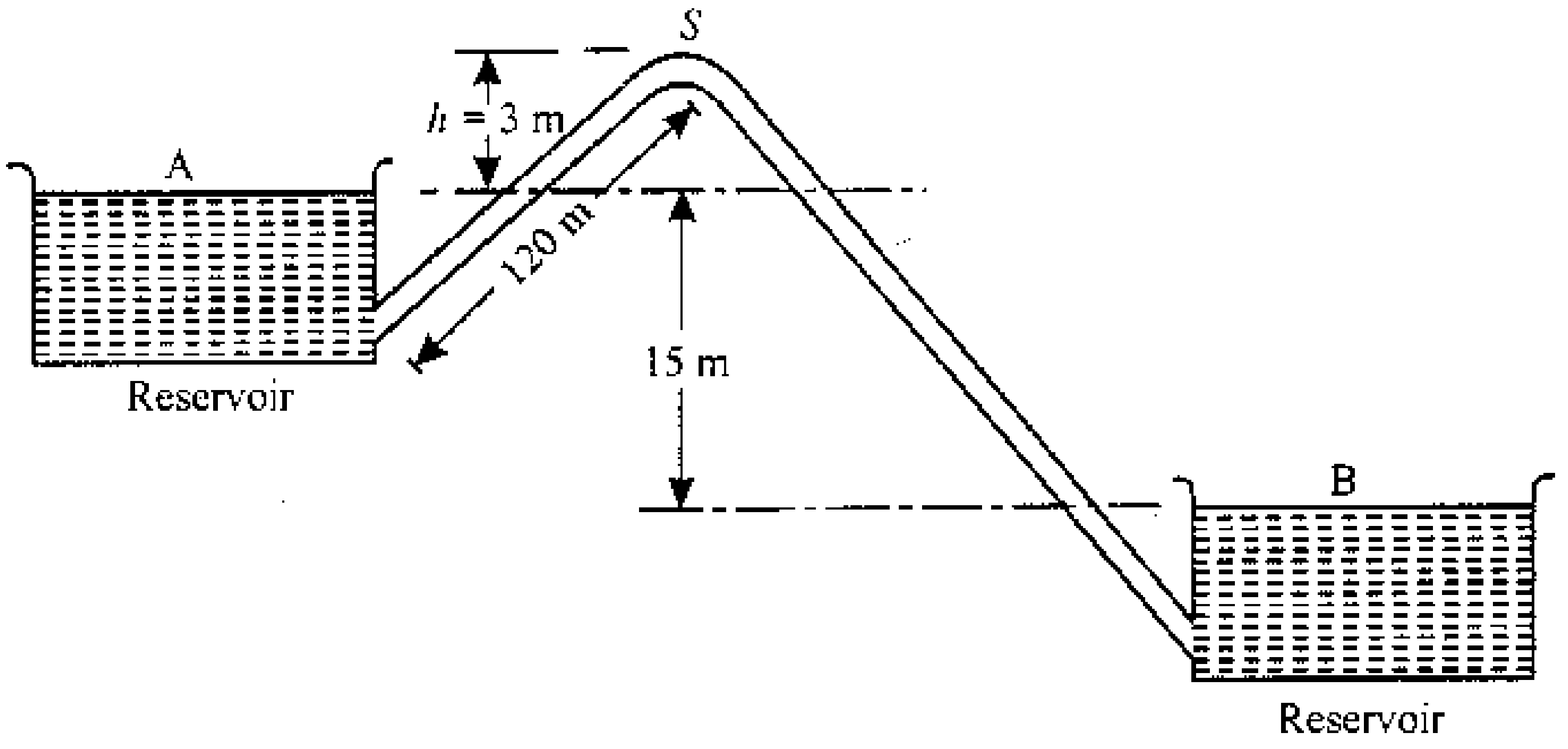

Fig. 12.40

$\therefore$ $$15 = \frac{4 \times 0{\cdot}005 \times 400 \times V^2}{0{\cdot}2 \times 2 \times 9{\cdot}81}$$

or, $$V^2 = \frac{15 \times 0{\cdot}2 \times 2 \times 9.81}{4 \times 0{\cdot}005 \times 400}$$

or, $$V = 2{\cdot}7 \text{ m/s}$$

$\therefore$ Discharge, $$Q = \text{Area} \times \text{velocity}$$

$$= \frac{\pi}{4} \times 0{\cdot}2^2 \times 2{\cdot}7 = \mathbf{0{\cdot}0848\ m^3/s}\ \textbf{(Ans.)}$$

(*ii*) **Pressure at summit :**

Applying Bernoulli's equation to points A and S, we get

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_S}{w} + \frac{V_S^2}{2g} + z_S$$

$$+ \text{ loss of head due to friction between } A \text{ and } S$$

(... assuming datum line passing through A)

$$0+0+0=\frac{p_S}{w}+\frac{2\cdot7^2}{2\times9.81}+3+\frac{4\times0\cdot005\times120\times(2\cdot7)^2}{0\cdot2\times2\times9\cdot81}$$

$$0=\frac{p_S}{w}+0\cdot37+3+4\cdot46=\frac{p_S}{w}+7\cdot83$$

$$\frac{p_S}{w}=-\ 7\cdot83 \text{ m of water (Ans.)}$$

Example 12.45. *A 200 mm diameter pipe, 4000 m long connects two reservoirs whose surface levels differ by 40 m. At a distance of 400 m from the upper reservoir, the pipe crosses a ridge the summit of which is 9 m above the level of water in the upper reservoir. Determine :*

(*i*) *The minimum depth of the pipe below the summit of the ridge, if the absolute pressure head at the summit of syphon is not to fall below 3·0 m of the water (absolute).*

(*ii*) *The discharge through the pipe.*

Take co-efficient of friction f = 0·006 and atmospheric head = 10·3 m of water. Neglect minor losses.

Solution. Diameter of the pipe, $D = 200$ mm $= 0\cdot2$ m

Total length of the pipe, $L = 4000$ m

Length of syphon from upper reservoir to the summit, $L_1 = 400$ m

Difference in levels of two reservoirs = 40 m

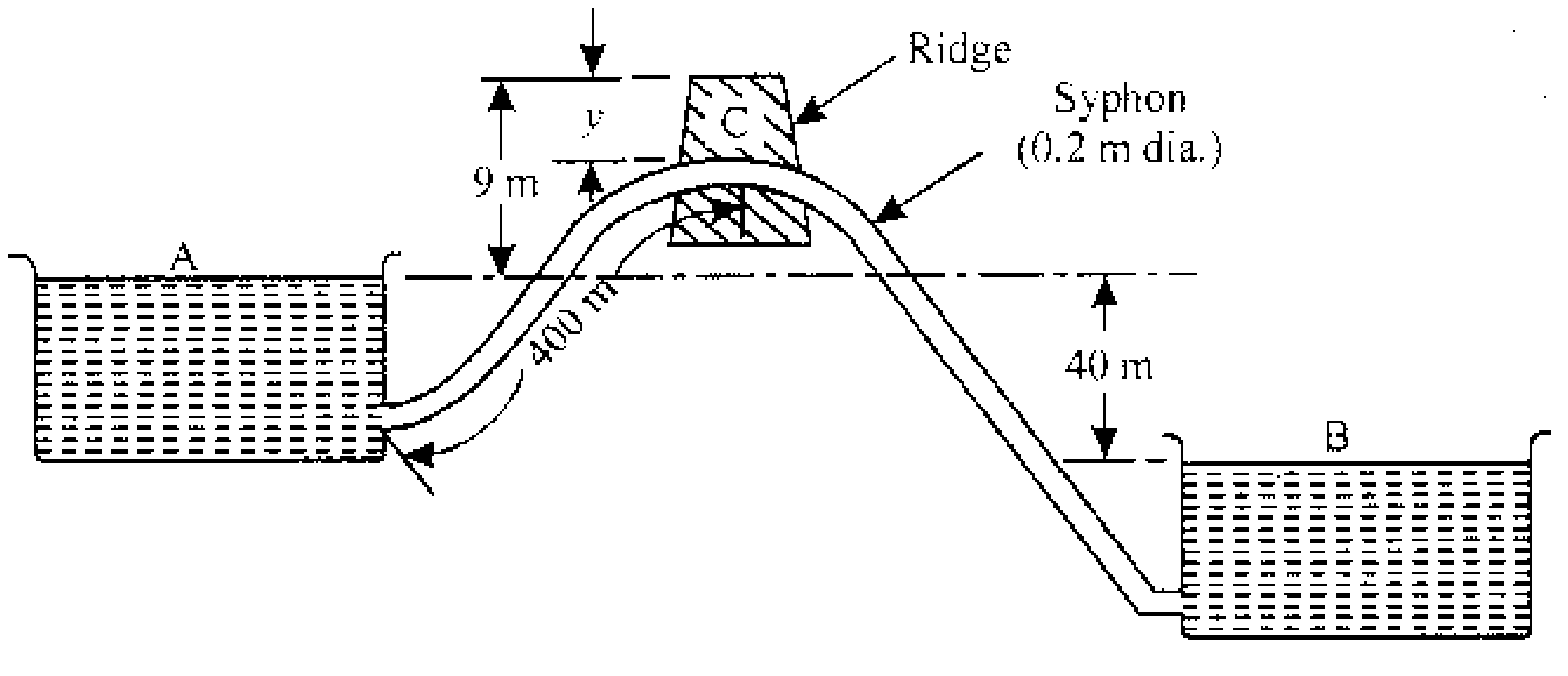

Fig. 12.41

Friction co-efficient, $f = 0\cdot006$

Atmospheric pressure head = 10·3 m of water

Pressure head at C, $\frac{p_S}{w} = 3\cdot0$ m of water (absolute)

(*i*) **Minimum depth of pipe below the summit, *y* :**

Let, y = Depth of the pipe below the summit of the ridge.

Then, height of syphon from the water surface in the upper reservoir = (9 – y)

Applying Bernoulli's equation at A and B (taking datum line passing through B), we have

$$\frac{p_A}{w}+\frac{V_A^2}{2g}+z_A=\frac{p_B}{w}+\frac{V_B^2}{2g}+z_B+(h_f)_{A-B}$$

$$0+0+40=0+0+0+\frac{4fLV^2}{D\times2g} \qquad (\because V_A=V_B=0)$$

or,
$$40=\frac{4\times0\cdot006\times4000\times V^2}{0\cdot2\times2\times9\cdot81}$$

$$\therefore \qquad V = \left(\frac{40 \times 0.2 \times 2 \times 9.81}{40 \times 0.006 \times 4000}\right)^{1/2} = 1.278 \text{ m/s}$$

Now applying Bernoulli's equation at A and C (assuming datum line passing through A), we have

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{w} + \frac{V_C^2}{2g} + z_C + (h_f)_{A-C}$$

$$10.3 + 0 + 0 = 3.0 + \frac{V^2}{2g} + (9 - y) + \frac{4fL_1V^2}{D \times 2g}$$

or, $$10.3 = 3 + \frac{(1.278)^2}{2 \times 9.81} + (9 - y) + \frac{4 \times 0.006 \times 400 \times (1.278)^2}{0.2 \times 2 \times 9.81}$$

$$= 3 + 0.0832 + (9 - y) + 3.99$$

or, $y = \mathbf{5.77\ m}$ **(Ans.)**

(*ii*) **The discharge through the pipe, Q:**

$$Q = A \times V = \frac{\pi}{4} \times 0.2^2 \times 1.278$$

$$= \mathbf{0.04\ m^3/s}\ \textbf{(Ans.)}$$

Example 12·46. *Water from a main canal is syphoned to a branch canal over an embankment by means of wrought iron pipes of 90 mm diameter. The length of pipeline up to the summit is 25 m and the total length is 65 m. Entry loss may be assumed as one-half of the velocity head in the pipe. Assume friction factor, f = 0·03. Water surface elevation in the branch canal is 10 m below that of the main canal.*

(*i*) *If the total quantity of water required to be coneyed is 0·06m^3/s, how many pipelines are needed ?*

(*ii*) *What is the maximum permissible height of the summit above the water level in the main canal so that the water pressure at summit may not fall below 20 kN/m^2 absolute, the barometric reading being 10 m of water ?* **(UPSC Exams.)**

Solution. Diameter of the pipe, D = 90 mm = 0·9 m

Total length of the pipeline, L = 65 m

The length of the pipeline up to the summit, L_1 = 25 m

Entry loss $= 0.5 \dfrac{V^2}{2g}$

Friction Factor, f = 0·03

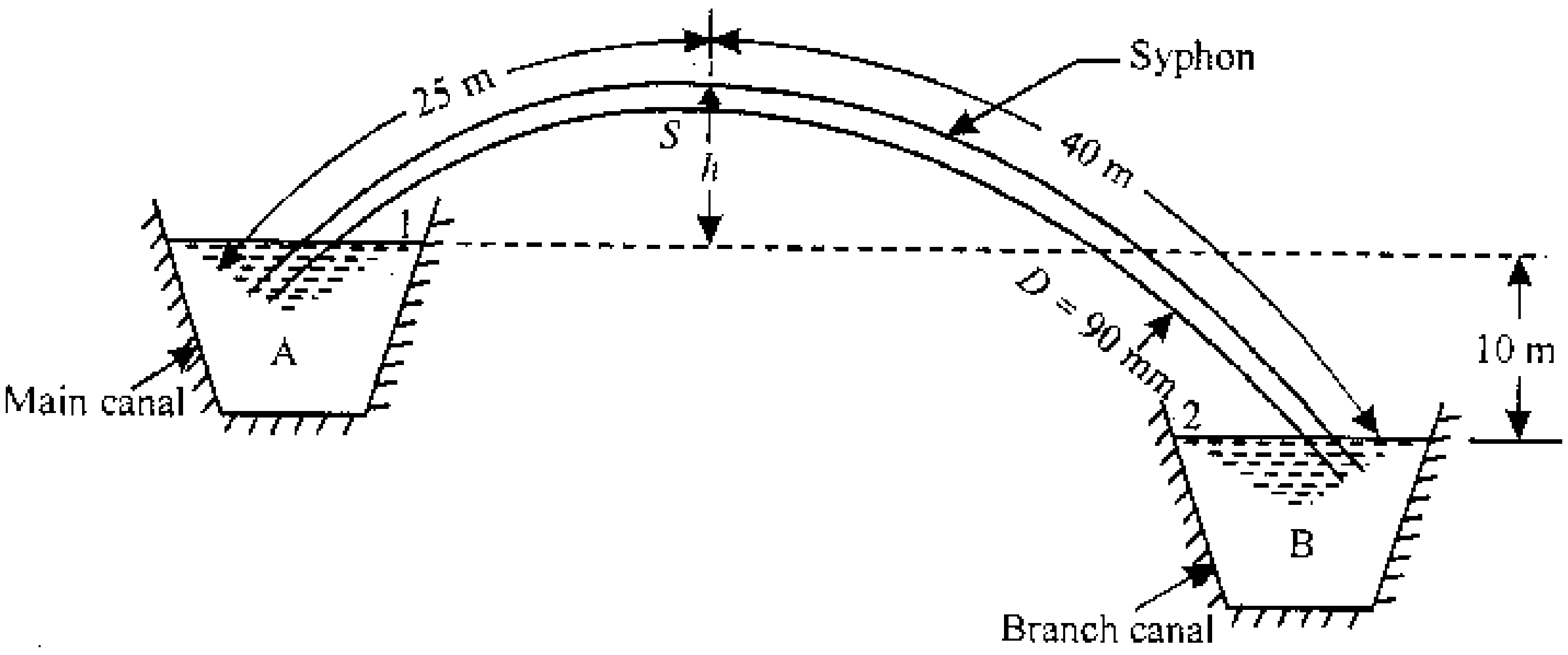

Fig. 12.42

Total discharge $Q = 0{\cdot}06$ m^3/s

Pressure at the summit, $p_S = 20$ kN/m^2 absolute

Atmospheric pressure head = 10 m of water.

(*i*) **Number of pipelines needed :**

Applying Bernoulli's equation between water surfaces (1 and 2) of two canals, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + 0{\cdot}5\frac{V^2}{2g} + \frac{fLV^2}{D \times 2g} + \frac{V^2}{2g}$$

$$0 + 0 + 10 = 0 + 0 + 0 + 1{\cdot}5\frac{V^2}{2g} + \frac{0{\cdot}03 \times 65 \times V^2}{0{\cdot}09 \times 2g}$$

or, $$10 = \frac{23{\cdot}17\, V^2}{2g} \qquad (\because V_1 = V_2 = 0)$$

or, $$V = \left(\frac{10 \times 2 \times 9{\cdot}81}{23{\cdot}17}\right)^{1/2} = 2{\cdot}91 \text{ m/s}$$

(where, V = velocity of flow through the pipe)

Discharge through a 90 mm diameter pipe,

$$= \frac{\pi}{4} \times 0{\cdot}09^2 \times 2{\cdot}91 = 0{\cdot}0185 \text{ m}^3/\text{s}$$

Number of 90 mm diameter pipes required to convey 0·06 m^3/s

$$= \frac{0{\cdot}06}{0{\cdot}185} = 3{\cdot}24, \text{ say } \mathbf{4} \quad \textbf{(Ans.)}$$

(*ii*) **Height of the summit, h :**

Invoking Bernoulli's equation between water surface1 and the summit point S

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_S}{w} + \frac{V_S^2}{2g} + z_S + \frac{0{\cdot}5V^2}{2g} + \frac{fL_1V^2}{D \times 2g}$$

$$10 + 0 + 0 = \frac{20}{9{\cdot}81} + \frac{V^2}{2g} + h + \frac{0{\cdot}5V^2}{2g} + \frac{0{\cdot}03 \times 25 \times V^2}{0{\cdot}09 \times 2g}$$

or, $$10 = 2{\cdot}038 + \frac{1{\cdot}5 \times 2{\cdot}91^2}{2 \times 9{\cdot}81} + h + \frac{0{\cdot}03 \times 25 \times (2{\cdot}91)^2}{0{\cdot}09 \times 2 \times 9.81}$$

or, $$10 = 2{\cdot}038 + 0{\cdot}647 + h + 3{\cdot}596$$

or, $$h = \mathbf{3{\cdot}72 \text{ m}} \quad \textbf{(Ans.)}$$

12·10. Power Transmission through Pipes

The transmission of power through pipes carrying water or other liquids is commonly used for working of several hydraulic machines. The hydruaulic power transmitted by a pipe however depends on (*i*) the *discharge* passing through the pipe and (*ii*) the *total head of water* (or liquid).

Consider a pipe AB *connected to a high level storage tank as shown in Fig. 12·43.*

Let, H = Head of water available at the inlet of pipe, m,

L = Length of the pipe, m,

D = Diameter of the pipe, m,

V = Velocity of water in the pipe m/s,

f = Co-efficient of friction, and

h_f = Loss of head in the pipe AB, due to friction, m.

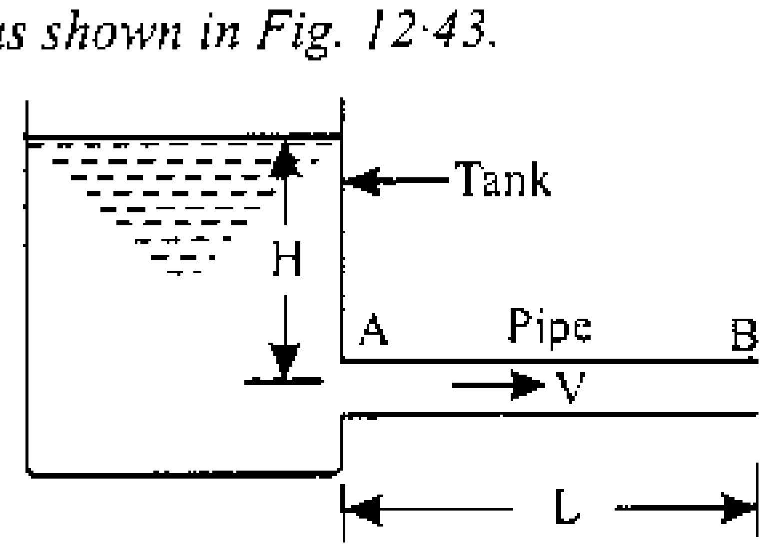

Fig. 12.43

Weight of water flowing through the pipe per second

$= wQ = wAV$ ...(*i*)

(where, Q = discharge of water through the pipe, m^3/s)

and, net head of water available at B (neglecting minor losses)

$$= H - h_f = H - \frac{4fLV^2}{D \times 2g}$$

Also, the efficiency of transmission,

$$\eta = \frac{H - h_f}{H}$$

and power,

$$P = \frac{\left\{\begin{matrix}\text{Weight of water flowing/sec.} \\ \times \text{ head of water}\end{matrix}\right\}}{1000} \text{ kW}$$

$$= wQ(H - h_f) \text{ kW}$$

(where, $w = 9{\cdot}81$ kN/m^3 for water)

$$= wAV\left(H - \frac{4fLV^2}{D \times 2g}\right) \text{ kW}$$

$$= wA\left(HV - \frac{4fLV^3}{D \times 2g}\right) \text{ kW} \quad \text{...(iii)}$$

It is evident from eqn. (*iii*) that power transmitted depends upon the velocity of water (V), as the other things are constant.

∴ Power transmitted will be *maximum*, when

$$\frac{d(P)}{dV} = 0$$

or,
$$\frac{d}{dV}\left[wA\left(HV - \frac{4fLV^3}{D \times 2g}\right)\right] = 0$$

or,
$$wA\left(H - \frac{4 \times 3fLV^2}{D \times 2g}\right) = 0$$

or,
$$H - 3 \times \frac{4fLV^2}{D \times 2g} = 0$$

or,
$$H - 3h_f = 0 \qquad \left[\because h_f = \frac{4fLV^2}{D \times 2g}\right]$$

or,
$$H = 3h_f$$

or,
$$h_f = \frac{H}{3}$$

It means that *power transmitted through the pipe is maximum, when head lost due to friction in the pipe is equal to $\frac{1}{3}$ of the total supply head.*

The *maximum efficiency* would correspond to the maximum power transmitted and hence maximum efficiency,

$$\eta = \frac{H - \frac{H}{3}}{H} = \frac{\frac{2}{3}H}{H} = \frac{2}{3} \quad \text{or} \quad 66{\cdot}7\%$$

Example 12·47. *A 2500 m long pipeline is used for transmission of power. 120 kW power is to be transmitted through the pipe in which water having a pressure of 4000 kN/m² at inlet is flowing. If the pressure drop over the length of pipe is 800 kN/m² and f = 0·006, find :*

(*i*) *Diameter of the pipe, and*

(*ii*) *Efficiency of transmission.*

Solution. Length of the pipeline, $L = 2500$ m

Power transmitted, $P = 120$ kW

Pressure at inlet, $p = 4000$ kN/m²

$$H = \frac{p}{w} = \frac{4000}{9{\cdot}81} \doteq 407{\cdot}7 \text{ m}$$

Pressure drop $= 800$ kN/m²

∴ Loss of head, $h_f = \frac{800}{9.81} = 81{\cdot}5$ m

∴ Co-efficient of friction, $f = 0{\cdot}006$.

(*i*) **Diameter of the pipe, D :**

Head available at the end of the pipe, $H - h_f = 407{\cdot}7 - 81{\cdot}5 = 326{\cdot}2$ m

Now, power transmitted is given by : $P = wQ\,(H - h_f)$ kW

$$120 = 9{\cdot}81 \times Q \times 326{\cdot}2$$

where, Q = Discharge through the pipe in m³/s, and

and, w = Specific weight of water = 9·81 kN/m³

$$\therefore \quad Q = \frac{120}{9{\cdot}81 \times 326{\cdot}2} = 0{\cdot}0375 \text{ m}^3/\text{s}$$

But,
$$Q = \frac{\pi}{4} D^2 \times V$$

$$\therefore \quad 0{\cdot}0375 = \frac{\pi}{4} D^2 \times V$$

or,
$$V = \frac{0{\cdot}0375 \times 4}{\pi D^2} = \frac{0{\cdot}0477}{D^2} \qquad \ldots(i)$$

The head lost due to friction, $h_f = \frac{4fLV^2}{D \times 2g}$

But, $h_f = 81{\cdot}5$ m (calculated above)

$$\therefore \quad 81{\cdot}5 = \frac{4 \times 0{\cdot}006 \times 2500 \times (0{\cdot}0477/D^2)^2}{D \times 2 \times 9{\cdot}81}$$

or,
$$81{\cdot}5 = \frac{4 \times 0{\cdot}006 \times 2500 \times (0{\cdot}0477)^2}{D^5 \times 2 \times 9{\cdot}81}$$

or,
$$D^5 = \frac{4 \times 0{\cdot}006 \times 2500 \times (0{\cdot}0477)^2}{81{\cdot}5 \times 2 \times 9{\cdot}81}$$

or, $D = 0{\cdot}1535$ m or **153·5 mm (Ans.)**

(*ii*) **Efficiency of transmission, η :**

$$\eta = \frac{H - h_f}{H} = \frac{407{\cdot}7 - 81{\cdot}5}{407{\cdot}7} = 0{\cdot}8 \text{ or } \mathbf{80\%} \quad \textbf{(Ans.)}$$

12·11. Flow through Nozzle at the End of a Pipe

Refer to Fig. 12·44. A *nozzle is a tapering mouthpiece, which is fitted to the outlet end of a pipe*. The total energy at the end of the pipe consists of pressure energy and kinetic energy. By fitting the nozzle at the end of a pipe, the total energy is converted into *kinetic energy*. A high velocity is required in the fields of power development, fire fighting, mining, etc.

Fig. 12·44 shows a nozzle fitted at the end of a pipe connected to a reservoir.

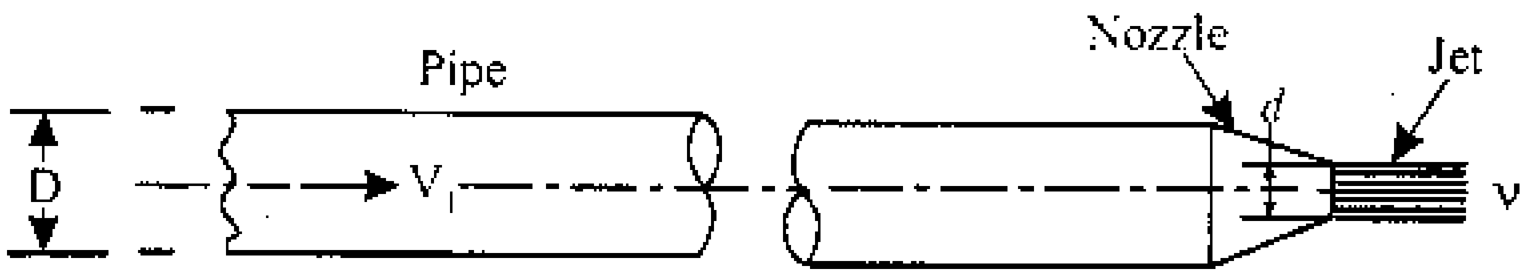

Fig. 12.44

Let, D = Diameter of the pipe,

L = Length of the pipe,

d = Diameter of the nozzle,

V = Velocity of flow in pipe,

v = Velocity of flow at the outlet of the nozzle,

f = Co-efficient of friction for the pipe, and

H = Height of water level in the reservoir above the centreline of the nozzle.

Head lost due to friction in pipe,

$$h_f = \frac{4fLV^2}{D \times 2g}$$

∴ Head available at the base of the nozzle

$$= H - h_f = H - \frac{4fLV^2}{D \times 2g}$$

Assuming the minor losses and losses in the nozzle to be negligible, we have :

Total head at the nozzle outlet $= \dfrac{v^2}{2g}$

$$\therefore \quad H = h_f + \frac{v^2}{2g} = \frac{4fLV^2}{D \times 2g} - \frac{v^2}{2g} \qquad ...(i)$$

From continuity consideration, we have

$$AV = av$$

(where A and a are the areas of the pipe and area of the nozzle at outlet respectively)

or,
$$V = \frac{av}{A}$$

Substituting the value of V in eqn. (*i*), we get,

$$H = \frac{4fLa^2v^2}{D \times 2g \times A^2} + \frac{v^2}{2g}$$

$$= \frac{v^2}{2g}\left(\frac{1 + 4fLa^2}{D \times A^2}\right)$$

$$\therefore \quad v = \sqrt{\frac{2gH}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}} \qquad ...(12.20)$$

∴ Discharge through the nozzle = $a \times v$

12·11·1. Power Transmitted through the Nozzle

Mass of liquid flowing per second at the outlet ofthe nozzle, $m = \rho a v$

The K.E. of the jet at outlet of the nozzle

$$= \frac{1}{2} m v^2 = \frac{1}{2} \times \rho a v \times v^2 = \frac{1}{2} \rho a v^3$$

∴ Power available at the outlet of nozzle $= \frac{1}{2} \rho a v^3$ watts

Also, power available at the inlet of pipe = wQH

∴ Efficiency of power transmission through the nozzle,

$$\eta = \frac{\text{Power available at the outlet of nozzle}}{\text{Power available at the inlet of pipe}} = \frac{\frac{1}{2}\rho a v^3}{wQH}$$

But, $w = \rho g$ and $Q = av$

∴
$$\eta = \frac{\frac{1}{2}\rho a v^3}{\rho g \times a v \times H} = \frac{v^2}{2gH} = \left[\frac{1}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}\right] \quad \text{...(12·21)}$$

$$\left[\because v = \sqrt{\frac{2gH}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}} \quad \text{...eqn. (12·20)}\right]$$

12·11·2. Condition for Transmission of Maximum Power Through Nozzle

We know that,
$$H = h_f + \frac{v^2}{2g} = \frac{4fLV^2}{D \times 2g} + \frac{v^2}{2g}$$

or,
$$\frac{v^2}{2g} = H - \frac{4fLV^2}{D \times 2g}$$

But power transmitted through the nozzle,

$$P = \frac{1}{2}\rho a v^3 = \frac{1}{2}\rho a v \times v^2$$

$$= \frac{1}{2}\rho a v\left[2g\left(H - \frac{4fLV^2}{D \times 2g}\right)\right]$$

$$= wav\left(H - \frac{4fLV^2}{D \times 2g}\right) \quad \text{...(12·22)}$$

From continuity consideration, we have

$$AV = av \text{ or } V = \frac{av}{A}$$

Substituting the value of V in eqn. (12·22), we get

Power transmitted through nozzle,
$$P = wav\left(H - \frac{4fL \times a^2 v^2}{D \times 2g \times A^2}\right) \quad \text{...[12·22(a)]}$$

Power transmitted will be maximum, when $\frac{dP}{dv} = 0$

$$\frac{d}{dv}\left[wav\left(H - \frac{4fL}{D \times 2g} \times \frac{a^2v^2}{A^2}\right)\right] = 0$$

or,
$$\frac{d}{dv}\left[wa\left(Hv - \frac{4fL}{D \times 2g} \times \frac{a^2v^3}{A^2}\right)\right] = 0$$

or,
$$H - 3 \times \frac{4fL}{D \times 2g} \times V^2 = 0 \qquad \left(\because \frac{a^2v^2}{A^2} = V^2\right)$$

or,
$$H - 3h_f = 0 \qquad \left(\because h_f = \frac{4fLV^2}{D \times 2g}\right)$$

or,
$$h_f = \frac{H}{3} \qquad ...(12\cdot23)$$

The eqn. (12·23) indicates that the *power transmitted by a nozzle is maximum when the head lost due to friction in pipe is equal to one-third the total head supplied at the inlet of pipe.*

12·11·3. Diameter of the Nozzle for Transmitting Maximum Power

We know that,
$$H = h_f + \frac{v^2}{2g}$$

But,
$$H = 3h_f \qquad \text{[From eqn. (12·22)]}$$

$\therefore$
$$3h_f = h_f + \frac{v^2}{2g} \quad \text{or} \quad 2h_f = \frac{v^2}{2g}$$

$$\frac{2 \times 4fLV^2}{D \times 2g} = \frac{v^2}{2g}$$

For *continuity* considerations, we have

$$AV = av \quad \text{or} \quad V = \frac{av}{A}$$

$\therefore$
$$\frac{2 \times 4fL \times a^2v^2}{D \times 2g \times A^2} = \frac{v^2}{2g}$$

or,
$$\frac{A^2}{a^2} = \frac{8fL}{D} \quad \text{or} \quad \frac{A}{a} = \sqrt{\frac{8fL}{D}} \qquad ...(12\cdot24)$$

Eqn. (12·24) gives the *ratio* between the areas of the supply pipe and the nozzle for maximum power transmission.

Substituting the values of A and a in eqn. (12·24) and squaring both sides, we have

$$\left(\frac{\frac{\pi}{4} \times D^2}{\frac{\pi}{4} \times d^2}\right)^2 = \frac{8fL}{D}$$

or,
$$\frac{D^4}{d^4} = \frac{8fL}{D} \quad \text{or} \quad D^5 = 8fLd^4$$

$\therefore$
$$d = \left(\frac{D^5}{8fL}\right)^{1/4} \qquad ...(12\cdot25)$$

Example 12·48. *A nozzle is fitted to a pipe 120 mm in diameter and 250 m long, with co-efficient of friction as 0·01. If the available head at the nozzle is 100 m find the diameter of the nozzle and the maximum power transmitted by a jet of water discharging freely out of a nozzle.*

Solution. Diameter of the pipe, $D = 120$ mm $= 0{\cdot}12$ m

Length of the pipe, $L = 250$ m

Co-efficient of friction, $f = 0{\cdot}01$

Head of water, $H = 100$ m.

(*i*) **Diameter of the nozzle for maximum power, d :**

Using the relation :

$$d = \left[\frac{D^5}{8fL}\right]^{1/4} \qquad \text{...[Eqn. (12.25)]}$$

$$= \left[\frac{0{\cdot}12^5}{8 \times 0{\cdot}01 \times 250}\right]^{1/4} = 0{\cdot}0334 \text{ m or } \mathbf{33{\cdot}4 \text{ mm}}$$

i.e., $d = \mathbf{33{\cdot}4}$ **mm (Ans.)**

(*ii*) **Maximum power transmitted by the jet, P :**

We know that for the maximum transmission of power, the head lost due to friction $= \dfrac{H}{3}$

$\therefore$ Available head, $h = 100 - \dfrac{100}{3} = \dfrac{200}{3} = 66{\cdot}67$ m

$\therefore$ Velocity of water through the nozzle,

$$v = \sqrt{2gh} = \sqrt{2 \times 9{\cdot}81 \times 66{\cdot}67} = 36{\cdot}2 \text{ m/s}$$

Now using the relation, $P = wQH$, we have $\qquad (\because Q = a.v)$

$$P = wavH$$

$$= 9{\cdot}81 \times \frac{\pi}{4} \times 0{\cdot}0334^2 \times 36{\cdot}2 \times 66{\cdot}67 = 20{\cdot}74 \text{ kW}$$

i.e., $P = \mathbf{20{\cdot}74}$ **kW (Ans.)**

Example 12.49. *Find the maximum power transmitted by a jet of water discharging freely out of nozzle fitted to a pipe 300 m long and 100 mm diameter with co-efficient of friction as 00·01. The available head at the nozzzle is 90 m.* **[Panjab University]**

Solution. Length of the pipe, $L = 300$ m

Diameter of the pipe, $D = 100$ mm $= 0{\cdot}1$ m

Area of the pipe, $A = \dfrac{\pi}{4} \times 0{\cdot}1^2 = 0{\cdot}007854$ m^2

Co-efficient of friction, $f = 0{\cdot}01$

Head available at the nozzle, $h = 90$ m

Maximum power transmitted, *P* :

Let, a = Area of the nozzle.

Also, $$\frac{A}{a} = \sqrt{\frac{8fL}{D}}$$

$$\therefore \quad \frac{0{\cdot}007854}{a} = \sqrt{\frac{8 \times 0{\cdot}01 \times 300}{0{\cdot}1}} = 15{\cdot}492$$

or, $$a = \frac{0.007854}{15.492} = 0.0005069 \text{ m}^2$$

Also, $$h = \frac{v^2}{2g}$$

$\therefore$ $$90 = \frac{v^2}{2g} \text{ or } v = \sqrt{90 \times 2 \times 9.81} = 42.02 \text{ m/s}$$

Discharge through the nozzle, $Q = av = 0.0005069 \times 42.02 = 0.0213 \text{ m}^3/\text{s}$

$\therefore$ Maximum power transmitted,

$$P = wQh = 9.81 \times 0.0213 \times 90 \text{ kW} = \mathbf{18.8\ kW} \quad \textbf{(Ans.)}$$
$$(\because w = 9.81 \text{ kN/m}^3)$$

Example 12·50. *A fire engine supplies water to a hosepipe, 75 m long and 0·075 m in diameter, at a pressure of 294 kN/m² (gauge). The discharge end of the hosepipe has a nozzle of diameter d fixed to it. Taking friction factor as 0·032, determine the diameter d of the nozzle, so that the momentum of the issuing jet may be a maximum.* **(UPSC Exams.)**

Solution. Diameter of the hosepipe, $D = 0.75$ m

Length of the hosepipe, $L = 75$ m

Pressure of water, $p = 294 \text{ kN/m}^2$ (gauge)

Friction factor $(= 4f) = 0.032$

Diameter of the nozzle, d :

Head lost in the hosepipe, $$h_f = \frac{4fLV^2}{D \times 2g} = \frac{0.032 \times 75 \times V^2}{0.075 \times 2g} = \frac{32V^2}{2g}$$

Applying the continuity equation to the pipe and the jet, we get

$$Q = AV = av$$

or, $$v = \frac{AV}{a} = \frac{\frac{\pi}{4} \times D^2 \times V}{\frac{\pi}{4} \times d^2} = \left(\frac{0.0075}{d}\right)^2 V$$

where, d = Diameter of the nozzle,

a = Area of the jet, and

A = Area of the hosepipe.

Applying Bernoulli's equation to the hosepipe at the fire engine and to the nozzle jet, considering the hosepipe and the nozzle to be in the horizontal plane, we have

$$\frac{294}{9.81} + \frac{V^2}{2g} + 0 = 0 + \frac{v^2}{2g} + 0 + h_f \text{ (head lost in the hosepipe)}$$

(Neglecting the energy loss in the nozzle, being very small)

or, $$30 + \frac{V^2}{2g} = \left(\frac{0.075}{d}\right)^4 \frac{V^2}{2g} + \frac{32V^2}{2g}$$

or, $$30 = \frac{V^2}{2g}\left[32 + \left(\frac{0.075}{d}\right)^4 - 1\right]$$

or, $$V = \sqrt{\frac{30 \times 2 \times 9.81}{31 + \left(\frac{0.075}{d}\right)^4}}$$

Momentum of issuing jet,

$$M = \rho Q v = 1000 \times \{\pi/4 \times (0{\cdot}075)^2 \times V\} \left(\frac{0{\cdot}075}{d}\right)^2 V = 0{\cdot}0248 \frac{V^2}{d^2}$$

Substituting for V, we get

$$M = \frac{0{\cdot}0248}{d^2}\left[\frac{30 \times 2 \times 9.81}{31 + (0{\cdot}075/d)^4}\right] = \frac{14{\cdot}6}{d^2\,[31 + (0{\cdot}075/d)^4]}$$

For momentum to be maximum, $\dfrac{dM}{dd} = 0$

$$\frac{d}{dd}\left[d^2\left\{31 + \left(\frac{0{\cdot}075}{d}\right)^4\right\}\right] = 0$$

or,
$$\frac{d}{dd}\left[31\,d^2 + \frac{0{\cdot}075^4}{d^2}\right] = 0$$

or,
$$62d - \frac{2 \times 0{\cdot}075^4}{d^3} = 0$$

or,
$$62d^4 - 6{\cdot}328 \times 10^{-5} = 0$$

or,
$$d = \left(\frac{6{\cdot}328 \times 10^{-5}}{62}\right)^{1/4} = 0{\cdot}03178 \text{ m} \quad \text{or} \quad \mathbf{31{\cdot}78\ mm} \quad \textbf{(Ans.)}$$

12.12. Water Hammer in Pipes

In a long pipe, when the flowing water is suddenly brought to rest by closing the valve or by any similar cause, there will be a sudden rise in pressure due to the momentum of water being destroyed. A pressure wave is transmitted along the pipe. A sudden rise in pressure has the effect of hammering action on the walls of the pipe. This phenomenon of sudden rise in presssure is known as **water hammer** or **hammer blow**. The magnitude of pressure rise depends on :

(*i*) *The speed at which valve is closed,*

(*ii*) *The velocity of flow,*

(*iii*) *The length of pipe, and*

(*iv*) *The elastic properties of the pipe material as well as that of the flowing fluid.*

The rise in pressure in some cases may be so large that the pipe may even burst and therefore it is essential to take into account this pressure rise in the design of the pipes.

12·12·1. Gradual Closure of Valve

Consider a long pipe carrying liquid (Fig. (12·45)) and provided with a valve which is closed gradually.

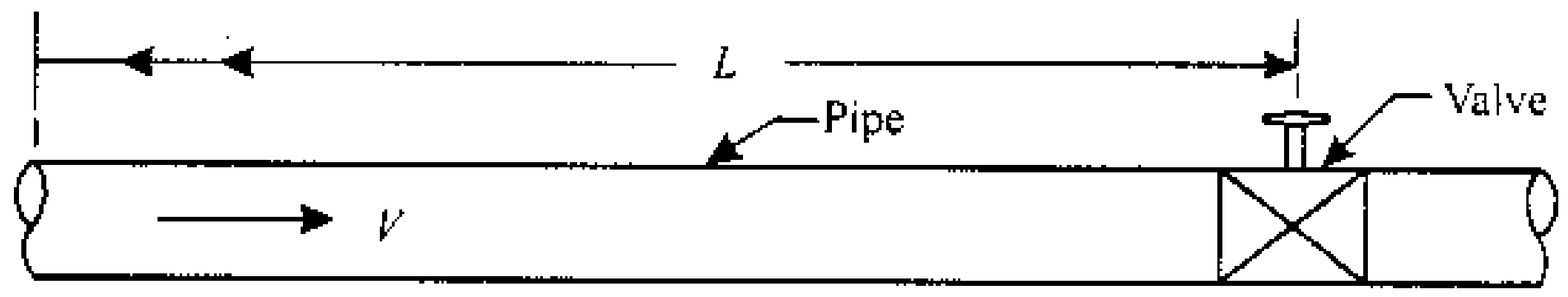

Fig. 12.45. Water hammer.

Let, A = Area of cross-section of the pipe,

L = Length of the pipe,

V = Velocity of flow of water in the pipe,

t = Time required to close the valve (in seconds), and

p = Intensity of pressure wave produced.

The mass of liquid contained in the pipe is $= \rho AL$

Assuming that the rate of closure of the valve is so adjusted that the liquid column in the pipe is brought to rest with a uniform retardation; from an intial velocity V to zero in time t seconds, we have,

Retardation of water $= \dfrac{V-0}{t} = \dfrac{V}{t}$

$\therefore$ The axial force available for producing retardation

$$= \text{Mass} \times \text{retardation}$$

$$= \rho AL \times \frac{V}{t} \qquad ...(i)$$

Also force due to pressure wave is $= p.A$ ...(*ii*)

Equating the two forces given by eqns. (*i*) and (*ii*), we have

$$\rho AL \times \frac{V}{t} = p \times A$$

or,
$$p = \frac{\rho LV}{t} \qquad ...(12{\cdot}26)$$

$\therefore$ Head of pressure,
$$H = \frac{p}{w} = \frac{\rho LV}{w \times t} = \frac{\rho LV}{\rho \cdot g \cdot t} = \frac{LV}{gt}$$

i.e.,
$$H = \frac{LV}{gt} \qquad ...(12{\cdot}27)$$

(*i*) The closure of valve is said to be *gradual* when $t < \dfrac{2L}{C}$ ...(12·28)

(*ii*) The closure of valve is said to be *instantaneous* when $t > \dfrac{2L}{C}$ ...(12·29)

where, C = velocity of the pressure wave.

12·12·2. Instantaneous Closure of Valve in Rigid Pipes

Eqn. (11·26) indicates that when the valve is closed instantaneously (*i.e.*, $t = 0$), the inertia head should rise to infinity. However, in practive, it is not possible to close the valve instantaneously, as it always takes some time. Thus, even for a very rapid closure of the valve, as observed during experimentation, the pressure rise is quite finite and measurable. Moreover, eqn. (12·26) has been derived on the *assumption that the liquid is incompressible.* This assumption is *incorrect,* because at very high pressures even liquids get compressed to *some extent* and *behave like cmpressible fluids.*

Consider a pipe of length L and area of cross-section A (Fig. 12·45) carrying water which is flowing through it at a velocity V. When the valve is closed instantaneously the K.E. of the flowing water is converted into strain energy of water (neglecting effect of friction and assuming the pipe wall to be perfectly rigid).

Loss of $K.E. = \dfrac{1}{2} mV^2 = \dfrac{1}{2}\rho AL \times V^2$ $\qquad (\because m = \rho \times A \times L)$

Gain of strain energy
$$= \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$$

[where, k = Bulk modulus of elasticity of water, and
p = Intensity of perssure wave produced.]

Equating the loss of $K.E.$ to the gain of strain energy, we get

$$\frac{1}{2}\rho AL \times V^2 = \frac{1}{2}\frac{p^2}{K} \times AL$$

or,
$$p^2 = \frac{1}{2}\rho ALV^2 \times \frac{2K}{AL} = \rho KV^2$$

$\therefore$ $$p = \sqrt{\rho K V^2} = V\sqrt{\rho K} = V\sqrt{\frac{K\rho^2}{\rho}}$$

or, $$p = V\rho C \qquad \text{...(12·30)}$$

$$\left(\text{where, } C = \sqrt{\frac{K}{\rho}}, C \text{ being the } \textit{velocity of pressure wave.}\right)$$

12·12·3. Instantaneous Closure of Valve in Elastic Pipes

As shown in Fig. 12·45, consider a pipe of length L, diameter D, thickness t (small compared to diameter).

Let, p = Increase of pressure due to water hammer,

E = Modulus of elasticity of pipe material, and

$\frac{1}{m}$ = Poisson's ratio for pipe material.

When the valve is closed intantaneously, rise of pressure takes place due to which circumferential and longitudinal stresses are produced in the pipe wall; these stresses are given as (from knowledge of strength of materials)

$$\sigma_c = \frac{pD}{2t} \quad \text{and} \quad \sigma_l = \frac{pD}{4t}$$

where, σ_c = Circumferential stress, and

σ_l = Longitudinal stress.

Also, strain energy stored in the pipe material per unit volume is

$$= \frac{1}{2E}\left(\sigma_c^2 + \sigma_l^2 - \frac{2\sigma_c\,\sigma_l}{m}\right)$$

$$= \frac{1}{2E}\left[\left(\frac{pD}{2t}\right)^2 + \left(\frac{pD}{4t}\right)^2 - \frac{2 \times \frac{pD}{2t} \times \frac{pD}{4t}}{m}\right]$$

$$= \frac{1}{2E}\left[\frac{p^2D^2}{4t^2} + \frac{p^2D^2}{16t^2} - \frac{p^2D^2}{4mt^2}\right]$$

Assuming $\frac{1}{m} = 1/4$, we have

Strain energy per unit volume $$= \frac{1}{2E}\left[\frac{p^2D^2}{4t^2} + \frac{p^2D^2}{16t^2} - \frac{p^2D^2}{16t^2}\right] = \frac{p^2D^2}{8Et^2}$$

Total strain energy stored in pipe material

$$= \frac{p^2D^2}{8Et^2} \times \text{total volume of pipe material}$$

$$= \frac{p^2D^2}{8Et^2} \times \pi\, Dt \times L = \frac{p^2 \times D^3L}{8Et}$$

$$= \frac{p^2 \times \pi D^2 \times DL}{8Et} = \frac{p^2ADL}{2Et} \quad \left[\because A \text{ (area of the pipe)} = \frac{\pi}{4} \times D^2\right]$$

Loss of K.E. of water $= \frac{1}{2}mV^2 = \frac{1}{2}\rho AL \times V^2$

Gain of strain energy in water $= \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$

Also, the loss of K.E. of water = gain of strain energy in water + strain energy stored in material.

$$\therefore \qquad \frac{1}{2}\rho AL \times V^2 = \frac{1}{2}\frac{p^2}{K} \times AL + \frac{p^2 ADL}{2Et}$$

Dividing both sides by $\frac{AL}{2}$, we get

$$\rho V^2 = \frac{p^2}{K} + \frac{p^2 D}{Et} = p^2\left(\frac{1}{K} + \frac{D}{Et}\right)$$

$$\therefore \qquad p^2 = \frac{\rho V^2}{\left(\frac{1}{K} + \frac{D}{Et}\right)}$$

or,

$$p = \sqrt{\frac{\rho V^2}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} = V \times \sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} \qquad \text{...(12·31)}$$

12·12·4. Time required by Pressure Wave to travel from the Valve to the Tank and from Tank to Valve.

Time taken,

$$t = \frac{\text{Distance travelled from valve to tank and back}}{\text{Velocity of pressure wave}}$$

$$= \frac{L + L}{C} = \frac{2L}{C} \quad i.e., \quad t = \frac{2L}{C} \qquad \text{...(12·32)}$$

where, L = Length of the pipe, and

C = Velocity of pressure wave.

Example 12.51. *In a pipe 600 mm diameter and 3000 m length, provided with a valve at its end, water is flowing with a velocity of 2 m/s. Assuming velocity of pressure wave C = 1500 m/s, find :*

(*i*) *The rise in pressure if the valve is closed in 20 seconds, and*

(*ii*) *The rise in pressure if the valve is closed in 2·5 seconds. Assume the pipe to be rigid one and take bulk modulus of water as 2 GN/m².*

Solution. Diameter of the pipe, D = 600 mm = 0·6 m

Length of the pipe, L = 300 m

Velocity of water, V = 2 m/s

Velocity of pressure wave, C = 1500 m/s.

(*i*) **Rise in pressure, p :**

Time taken to close the valve, t = 20 s

Now, the ratio, $\frac{2L}{C} = \frac{2 \times 3000}{1500} = 4$

The close of valve is said to be *gradual* if,

$$t > \frac{2L}{C} \qquad \text{...[Eqn. (12·28)]}$$

Hence, the valve is closed *gradually*.

The rise in pressure (p), for gradual closure of valve, is given by

$$p = \frac{\rho L V}{t} \qquad \text{...[Eqn. (12·26)]}$$

$$= \frac{1000 \times 3000 \times 2}{20} = 300 \times 10^3 \text{ N/m}^2 \text{ or } \mathbf{300\ kN/m^2\ (Ans.)}$$

(*ii*) **Rise in pressure, p :**

Time taken to close the valve, $t = 2{\cdot}5$ s

Bulk modulus of water, $K = 2$ GN/m^2

Velocity of pressure wave is given by,

$$C = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2 \times 10^9}{1000}} = 1414{\cdot}2 \text{ m/s}$$

The ratio, $$\frac{2L}{C} = \frac{2 \times 3000}{1414{\cdot}2} = 4{\cdot}24 \text{ s}$$

$$\therefore \qquad t < \frac{2L}{C}$$

Thus, the valve is closed *intantaneously* [From eqn. (12·29)]

When pipe is rigid, the rise in pressure due to instantaneous closure of the valve is given by (eqn. 12·30),

$$p = V\rho C = 2 \times 1000 \times 1414{\cdot}2 \text{ N/m}^2 \text{ or } \mathbf{2828{\cdot}4 \text{ kN/m}^2 \text{ (Ans.)}}$$

Example 12·52. *Water is flowing in a pipe of 150 mm diameter with a velocity of 2·5 m/s when it is suddenly brought to rest by closing the valve. Find the pressure rise assuming pipe is elastic, E = 206 GN/m², Poisson's ratio = 0·25 and K for water = 2·06 GN/m². Pipe wall is 5 mm thick.*

Solution. Diameter of the pipe, $D = 150$ mm $= 0{\cdot}15$ m

Thickness of the pipe, $t = 5$ mm $= 0{\cdot}005$ m

Velocity of water, $V = 2{\cdot}5$ m/s

Modulus of elasticity, $E = 206$ GN/m^2

Bulk modulus of water, $K = 2{\cdot}06$ GN/m^2

Poisson's ratio, $\frac{1}{m} = 1/4$

Pressure rise, p :

Using the relation :

$$p = V\sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} \qquad \text{...[Eqn. (12·31)]}$$

$$= 2{\cdot}5\sqrt{\frac{1000}{\left(\frac{1}{2{\cdot}06 \times 10^9} + \frac{0{\cdot}15}{2{\cdot}06 \times 10^9 \times 0{\cdot}005}\right)}} = 2{\cdot}5\sqrt{\frac{10^{12}}{0{\cdot}485 + 0{\cdot}1456}}$$

$$= \mathbf{3148 \text{ kN/m}^2 \text{ (Ans.)}}$$

Example 12·53. *In a pressure penstock 4500 m long water is flowing at 4 m/s. If the velocity of the pressure wave travelling in the pipe due to sudden complete closure of a valve at the downstream end is given as 1500 m/s, find :*

(*i*) *The maximum pressure rise, and*

(*ii*) *The period of oscillation.*

Show how the pressure changes with time at the middle point of the penstock length. All friction losses may be neglected. **(UPSC Exams)**

Solution. Length of the penstock, $L = 4500$ m

Velocity of water, $V = 4$ m/s

Velocity of the pressure wave, $C = 1500$ m/s (*i*) **The maximum pressure rise :**

Maximum pressure is given by

$$p = V\rho C \quad \text{...[Eqn. (12·39)]}$$

$$= 4 \times 1000 \times 1500 \text{ N/m}^2 \text{ or } \mathbf{6\ MN/m^2\ (Ans.)}$$

(*ii*) **The period of oscillation :**

$$\text{The period of oscillation } = \frac{2L}{C} = \frac{2 \times 4500}{1500} = \mathbf{6\ seconds\quad (Ans.)}$$

Pressure changes with time at the middle point of the penstock length :

The pressure wave reaches at the middle point of the penstock length in 1·5 s $\left(\frac{4500}{2 \times 1500} = 1{\cdot}5 \text{ s}\right)$; at this instant the pressure at the middle point rises by 6 MN/m² and remains unchanged until the pressure wave returns as a wave of rarefaction of negative pressure. This happens at 4·5 s after the closure of the valve when the pressure in the penstock at the middle point is reduced by an equal amount. Fig. 12·46 shows the pressure changes at the middle point with respect to time.

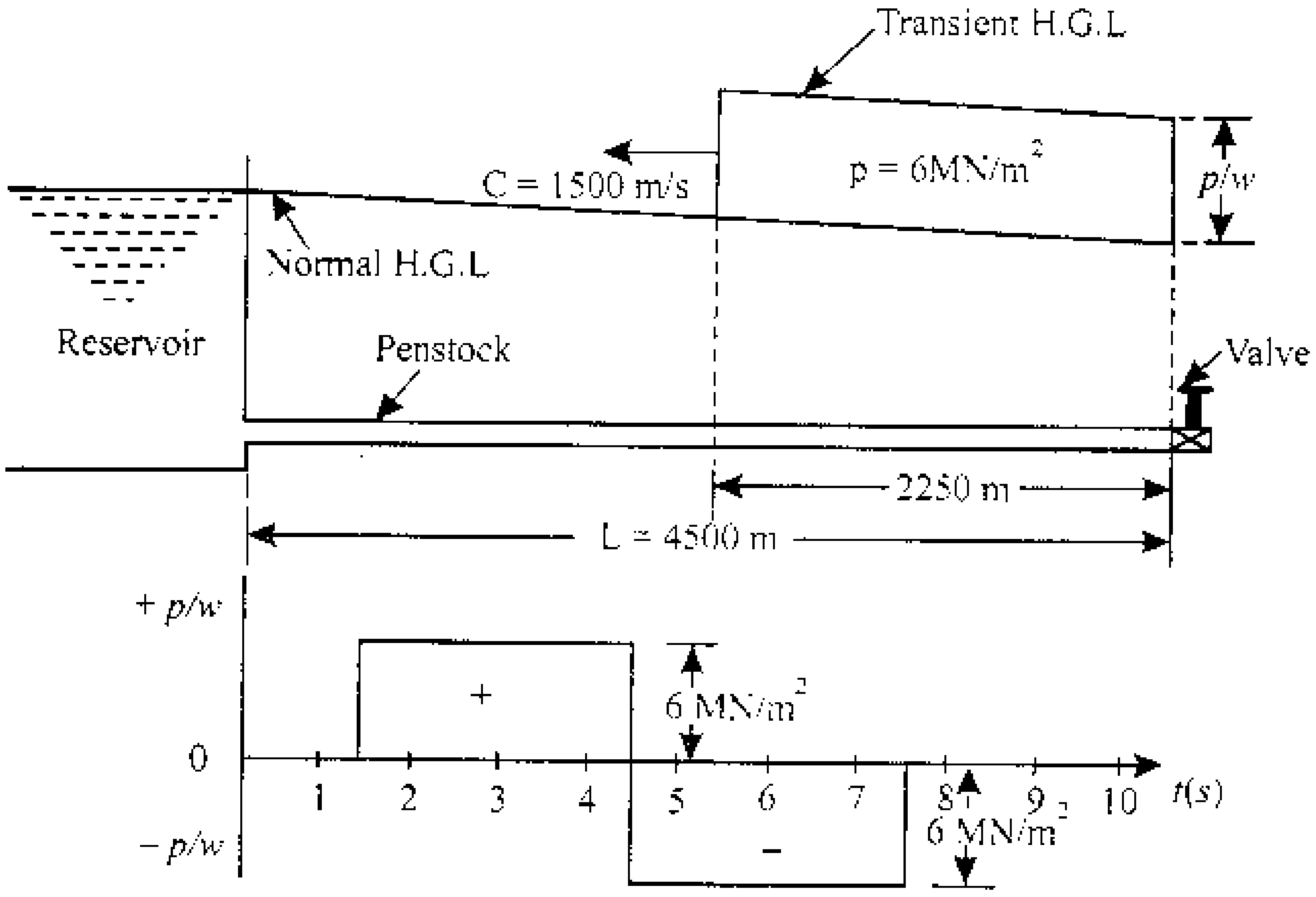

Fig. 12.46. Pressure changes at the middle point of the penstock length.

HIGHLIGHTS

1. A pipe is a closed conduit (generally of circular section) which is used for carrying fluids under pressure.
2. On the basis of experiments Reynolds discovered that :
   (*i*) In case of *laminar flow :* The loss of pressure head $\propto$ velocity (V)
   (*ii*) In case of *turbulent flow :* the loss of pressure head $\propto V^2$ (approximately)
3. *Energy (or head) losses :*
   A. *Major energy losses.......due to frictrion.*
   B. *Minor energy loses.*
   These losses are due to

(*i*) Sudden enlargement of pipe
(*ii*) Sudden contraction of pipe,
(*iii*) Bend in pipe,
(*iv*) An obstruction in pipe, and
(*v*) Pipe fittings, etc.

4. Major energy losses (due to friction), Important formulae :

(*i*) *Darcy-Weisbach formula* (for loss of head due to friction)

$$h_f = \frac{4fLV^2}{D \times 2g} = \frac{f_1 L V^2}{D \times 2g}$$

where, f = co-efficient of friction, f_1 = friction factor (= $4f$)

(*ii*) Chezy's formula (for loss of head due to friction)

$$V = C\sqrt{mi} \left(\text{where, } i = \frac{h_f}{L} \right)$$

[$\therefore h_f = i \times L$, where i is obtained from Chezy's formula.]

5. Minor energy losses; Important formulae :

(*i*) Loss of head due to *sudden enlargement*

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

(*ii*) Loss of head due to *sudden contraction*.

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

(*iii*) Loss of head due to *obstruction in pipe*,

$$h_{obs.} = \left[\frac{A}{C_c\,(A - a)} - 1\right]^2 \frac{V^2}{2g}$$

where, a = Maximum area of obstruction, and
A = Area of the pipe.

(*iv*) Loss of head at the entrance to pipe,

$$h_i = 0{\cdot}5\,\frac{V^2}{2g}$$ where, V = velocity of liquid in pipe.

(*v*) Loss of head at the *exit of pipe*,

$$h_o = \frac{V^2}{2g}$$ where, V = velocity at outlet of pipe.

(*vi*) Loss of head due to *bend in the pipe*,

$$h_b = k\,\frac{V^2}{2g}$$ where, k = co-efficient of bend.

(*vii*) Loss of head in *various pipe fittings*.

$$h_{fittings} = k\,\frac{V^2}{2g}$$

where, k = value of co-efficient; it depends on the type of pipe fittings.

6. *Energy gradient line* (E.G.L.). If the total energy at various points along the axis of the pipe is plotted and joined by a line, the line so obtained is called the '*Eenergy gradient line*'.

7. *Hydraulic gradient line* (H.G.L.). If a line is drawn joining the piezometric levels at various points, the line so obtained is called the '*Hydraulic gradient line*'.
8. *Equivalent pipe.* It is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe constisting of several pipes of different lengths and diameters. To determine the size of the equivalent pipe Dupit's equation, given below, is used :

$$\frac{L}{D^5} = \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\right]$$

9. In case of *parallel pipes* :
   (*i*) Rate of discharge in the main line = Sum of the discharges in each of the parallel pipes.
   i.e., $Q = Q_1 + Q_2 + \ldots$
   (*ii*) The loss of head in each pipe is same.
10. A *syphon* is a long bent pipe employed for carrying water from a reservoir at a higher elevation to another reservoir at lower elevation when the two reservoirs are separated by a hill or high level ground in between.
11. *Power transmission through pipes :*

Efficiency, $\eta = \frac{H - h_f}{H}$

Power, $P = wQ(H - h_f)$ kW (where, $w = 9{\cdot}81$ kN/m³ for water)

Power transmitted will be maximum when, $h_f = \frac{H}{3}$

Then, $\eta_{max} = \frac{H - H/3}{H} = 66{\cdot}7\%$

12. Flow through nozzles; Important formulae :

(*i*) Velocity, $v = \sqrt{\dfrac{2gh}{1 + \dfrac{4fL}{D} \cdot \dfrac{a^2}{A^2}}}$

(*ii*) Power, $P = wav\left[H - \frac{4fL}{D \times 2g}\left(\frac{a^2v^2}{A^2}\right)\right]$ kW (where, $w = 9{\cdot}81$ kN/m³ for water)

(*iii*) Condition for maximum power transmission : $h_f = \frac{H}{3}$

(*iv*) Diameter of nozzle for maximum power transmission,

$$d = \left(\frac{D^5}{8fL}\right)^{1/4}$$

13. *Water hammer in pipes.* The phenomenon of sudden rise in pressure in a pipe when water flowing in it is suddenly brought to rest by closing the valve is known as *water hammer* or *hammer blow.*

14. Valve closure is *gradual* when $t > \frac{2L}{C}$

Valve closure is *sudden* when $t < \frac{2L}{C}$

where, $C = \sqrt{\frac{K}{\rho}}$, C being the velocity of of pressure wave produced due to water hammer.

15. The intensity of pressure rise due to water hammer is given by $p = \frac{\rho L V}{t}$ ... when valve is closed gradually (where, t = time required to close the valve),

$p = V\sqrt{\rho K}$ ...when the valve is closed suddenly and pipe is assumed *rigid*, and

$p = V \times \sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}}$ ...when valve is closed suddenly and the pipe is *elastic*.

(where, t = Thickness of pipewall)

where, L = Length of pipe,

V = Velocity of flow,

K = Bulk modulus of water, and

E = Modulus of elasticity for pipe material.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The pipe running partially/completely full behaves like an open channel.
2. In a laminar flow, Reynold's number is
   (*a*) less than 2000
   (*b*) more than 2000
   (*c*) more than 2000 but less than 4000
   (*d*) none of the above.
3. In a turbulent flow, Reynold's number is
   (*a*) less than 4000
   (*b*) more than 4000
   (*c*) between 2000 and 4000
   (*d*) none of the above.
4. In case of a laminar flow, the loss of pressure head is
   (*a*) proportional to $(\text{velocity})^2$
   (*b*) proportional to velocity
   (*c*) proportional to $(\text{velocity})^{1/2}$
   (*d*) none of the above.
5. In case of a turbulent flow, the loss of head is approximately proportional to
   (*a*) velocity (*b*) $(\text{velocity})^{1/2}$
   (*c*) $(\text{velocity})^{3/4}$ (*d*) $(\text{velocity})^2$
6. Darcy-Weisbach equation is used to find loss of head due to :
   (*a*) sudden enlargement
   (*b*) sudden contraction
   (*c*) friction
   (*d*) none of the above.
7. Chezy's formula is given as
   (*a*) $V = C\sqrt{m^2 i}$ (*b*) $V = C^2\sqrt{mi^2}$
   (*c*) $V = C\sqrt{mi}$ (*d*) $V = C\sqrt{m^2 i^3}$
8. Loss of head due to sudden enlargement is given as
   (*a*) $\frac{(V_1 - V_2)^3}{2g}$ (*b*) $\frac{(V_1 - V_2)^2}{2g}$
   (*c*) $\frac{V_1^2 - V_2^2}{2g}$ (*c*) $\frac{\sqrt{V_1 - V_2}}{2g}$
9. Loss of head due to sudden contraction is given as
   (*a*) $\frac{V^2}{g}\left(\frac{1}{C_c} - 1\right)^2$ (*b*) $\frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$
   (*c*) $\frac{V_2}{g^2}\left(\frac{1}{C_c} - 1\right)^2$ (*d*) $\frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)$.
10. Loss of head due to an obstruction is given as
    (*a*) $\left[\frac{A}{A-a} - 1\right]^2 \frac{V^2}{2g}$
    (*b*) $\left[\frac{A}{C_c(A-a)} - 1\right] \frac{V^2}{2g}$
    (*c*) $\left[\frac{A}{C_c a} - 1\right]^2 \frac{V^2}{2g}$
    (*d*) $\left[\frac{A^2}{A-a} - 1\right]^2 \frac{V^2}{2g}$.