CHAPTER

6

FLUID DYNAMICS

6.1. Introduction. 6.2. Different types of heads (or energies) of a liquid in motion. 6.3. Bernoulli's equation. 6.4. Euler's equation for motion. 6.5. Bernoulli's equation for real fluids. 6.6. Practical applications of Bernoulli's equation — Venturimeter-Orificemeter—Pitot tube. 6.7. Free liquid jet. 6.8. Impulse momentum equation. 6.9. Kinetic energy and momentum correction factors (Coriolis co-efficients. 6.10. Moment of momentum equation. 6.11. Vortex motion—Forced vortex flow-free vortex flow-equation of motion for vortex flow-equation of forced vortex-rotation of liquid in a closed cylindrical vessel.—Highlights—Objective Type Question—Theoretical Questions —Unsolved Examples.

1. Introduction

When the fluids are at rest, the only fluid property of significance is the specific weight of the ids. On the other hand, when a fluid is in motion various other fluid properties become significant, ch the nature of flow of a real fluid is complex. *The science which deals with the geometry of ion of fluids without reference to the forces causing the motion is known as* **"hydrokinematics"** simply *kinematics*). Thus, kinematics involves merely the description of the motion of fluids in s of space-time relationship. *The science which deals with the action of the forces in producing hanging motion of fluids is known as* **"hydrokinetics"** (or simply *kinetics*). Thus, the study of s in motion involves the consideration of both the kinematics and kinetics. The dynamic tion of fluid motion is obtained by applying Newton's second law of motion to a fluid element idered as a free body. *The fluid is assumed to be incompressible and non-viscous.*

In fluid mechanics the basic equations are: (*i*) *Continuity equation*, (*ii*) *Energy equation*, and (*iii*) *lse-momentum equation.* In this chapter energy equation and impulse-momentum equations will scussed (Continuity equation has already been discussed in the chapter 5).

Different Types of Heads (or Energies) of a Liquid in Motion

There are three types of energies or heads of flowing liquids:

1. **Potential head or potential energy:**

   This is due to configuration or position above some suitable datum line. It is denoted by z.

2. **Velocity head or kinetic energy:**

   This is due to velocity of flowing liquid and is measured as $\frac{V^2}{2g}$ where V is the velocity of flow and 'g' is the acceleration due to gravity ($g = 9.81$)

3. **Pressure head or pressure energy:**

   This is due to the pressure of liquid and reckoned as $\frac{p}{w}$ where p is the pressure and w is the weight density of the liquid.

Total head/energy:

Total head of a liquid particle in motion is the sum of its potential head, kinetic head and pressure head. Mathematically,

Total head, $$H = z + \frac{V^2}{2g} + \frac{p}{w} \text{ m of liquid} \quad \text{...[6.1 (a)]}$$

Total energy, $$E = z + \frac{V^2}{2g} + \frac{p}{w} \text{ Nm/kg of liquid} \quad \text{...[6.1 (b)]}$$

Example 6.1. *In a pipe of 90* mm *diameter water is flowing with a mean velocity of 2 m/s and at a gauge pressure of 350 kN/m². Determine the total head, if the pipe is 8 metres above the datum line. Neglect friction.*

Solution. Diameter of the pipe = 90 mm

Pressure, $p = 350$ kN/m^2

Velocity of water, $V = 2$ m/s

Datum head, $z = 8$ m

Specific weight of water, $w = 9.81$ kN/m^3

Total head of water, H:

$$H = z + \frac{V^2}{2g} + \frac{p}{w}$$

$$= 8 + \frac{2^2}{2 \times 9.81} + \frac{350}{9.81} = 43.88 \text{ m}$$

H **= 43.88 m (Ans.)**

6.3. Bernoulli's Equation

Bernoulli's equation states as follow:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line." Mathematically,

$$\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$$

where, $\frac{p}{w}$ = Pressure energy,

$\frac{V^2}{2g}$ = Kinetic energy, and

z = Datum (or elevation) energy.

Proof.

Consider an ideal incompressible liquid through a non-uniform pipe as shown in Fig 6.1. Let consider two sections LL and MM and assume that the pipe is running full and there is continuity flow between the two sections;

Let, p_1 = Pressure at *LL*,

V_1 = Velocity of liquid at *LL*,

z_1 = Height of *LL* above the datum,

A_1 = Area of pipe at *LL*, and

p_2, V_2, z_2, A_2 = Corresponding values at *MM*.

Let the liquid between the two sections LL and MM move to $L'L'$ and $M'M'$ through very small s dl_1 and dl_2 as shown in Fig. 6.1. This movement of liquid between LL and MM is equivalent movement of the liquid between LL and $L'L'$ and MM and $M'M'$, the remaining liquid between d MM being unaffected.

Let, W = Weight of liquid between LL and $L'L'$.

As the flow is continuous,

$$\therefore \quad W = wA_1 \cdot dl_1 = wA_2 \cdot dl_2$$

$$A_1 \cdot dl_1 = \frac{W}{w} \quad \text{...}(i)$$

Similarly,

$$A_2 \cdot dl_2 = \frac{W}{w} \quad \text{...}(ii)$$

$$\therefore \quad A_1 \cdot dl_1 = A_2 . dl_2$$

Work done by pressure at LL, in moving the liquid to $L'L'$

$$= \text{Force} \times \text{distance} = p_1 \cdot A_1 \cdot dl_1$$

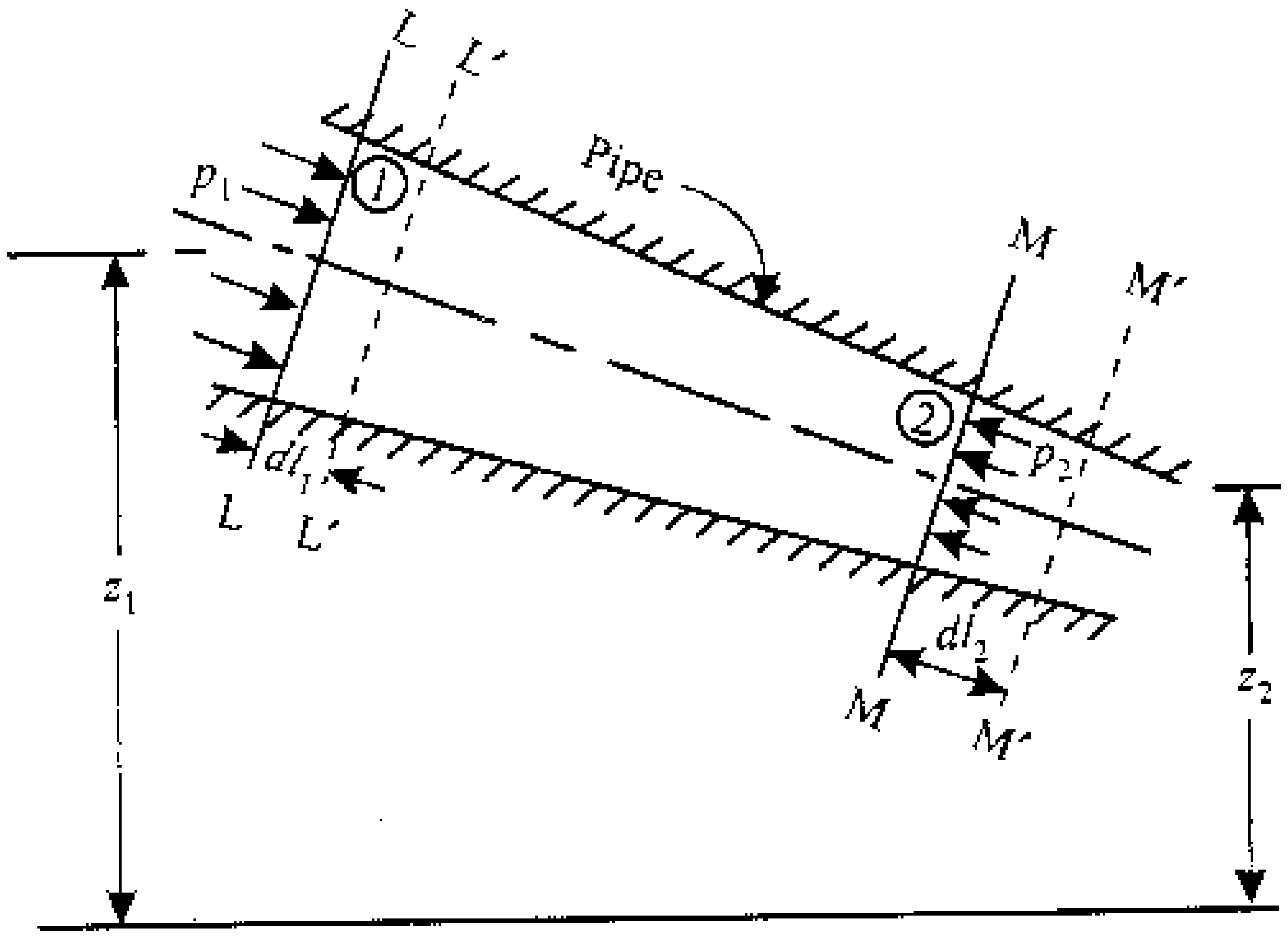

Fig. 6.1. Bernoulli's equation.

Similarly, work done by the pressure at MM in moving the liquid to $M'M' = -p_2 . A_2 \cdot dl_2$

(– ve sign indicates that direction of p_2 is *opposite* to that of p_1)

$\therefore$ Total work done by the pressure

$$\begin{aligned} &= p_1 \cdot A_1\, dl_1 - p_2\, A_2\, dl_2 \\ &= p_1 \cdot A_1\, dl_1 - p_2\, A_1\, dl_1 \qquad (\because A_1 dl_1 = A_2 dl_2) \\ &= A_1 \cdot dl_1 (p_1 - p_2) \\ &= \frac{W}{w}(p_1 - p_2) \qquad \left(\because A_1 \cdot dl_1 = \frac{W}{w}\right) \end{aligned}$$

Loss of potential energy = $W(z_1 - z_2)$

Gain in kinetic energy $= W\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = \frac{W}{2g}(V_2^2 - V_1^2)$

Also, loss of potential energy + work done by pressure = gain in kinetic energy

$$\therefore \quad W(z_1 - z_2) + \frac{W}{w}(p_1 - p_2) = \frac{W}{2g}(V_2^2 - V_1^2)$$

or $$(z_1 - z_2) + \left(\frac{p_1}{w} - \frac{p_2}{w}\right) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

or $$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \qquad ...(6.2)$$

which proves *Bernoulli's equation.*

Assumptions:

It may be mentioned that the following *assumptions* are made in the derivation of Bernoulli's equation:

1. The liquid is ideal and incompressible.
2. The flow is steady and continuous.
3. The flow is along the streamline, *i.e.*, it is one-dimensional.
4. The velocity is uniform over the section and is equal to the mean velocity.
5. The only forces acting on the fluid are the *gravity forces* and the *pressure forces.*

6.4. Euler's Equation for Motion

Consider steady flow of an ideal fluid along the stream tube. Separate out a small element of fluid of cross-sectional area dA and length ds from stream tube as a free body from the moving fluid.

Fig. 6.2. shows such a small element LM of fluid of cross-section area dA and length ds.

Let, p = Pressure on the element at L.

$p + dp$ = Pressure on the element at M, and

V = Velocity of the fluid element.

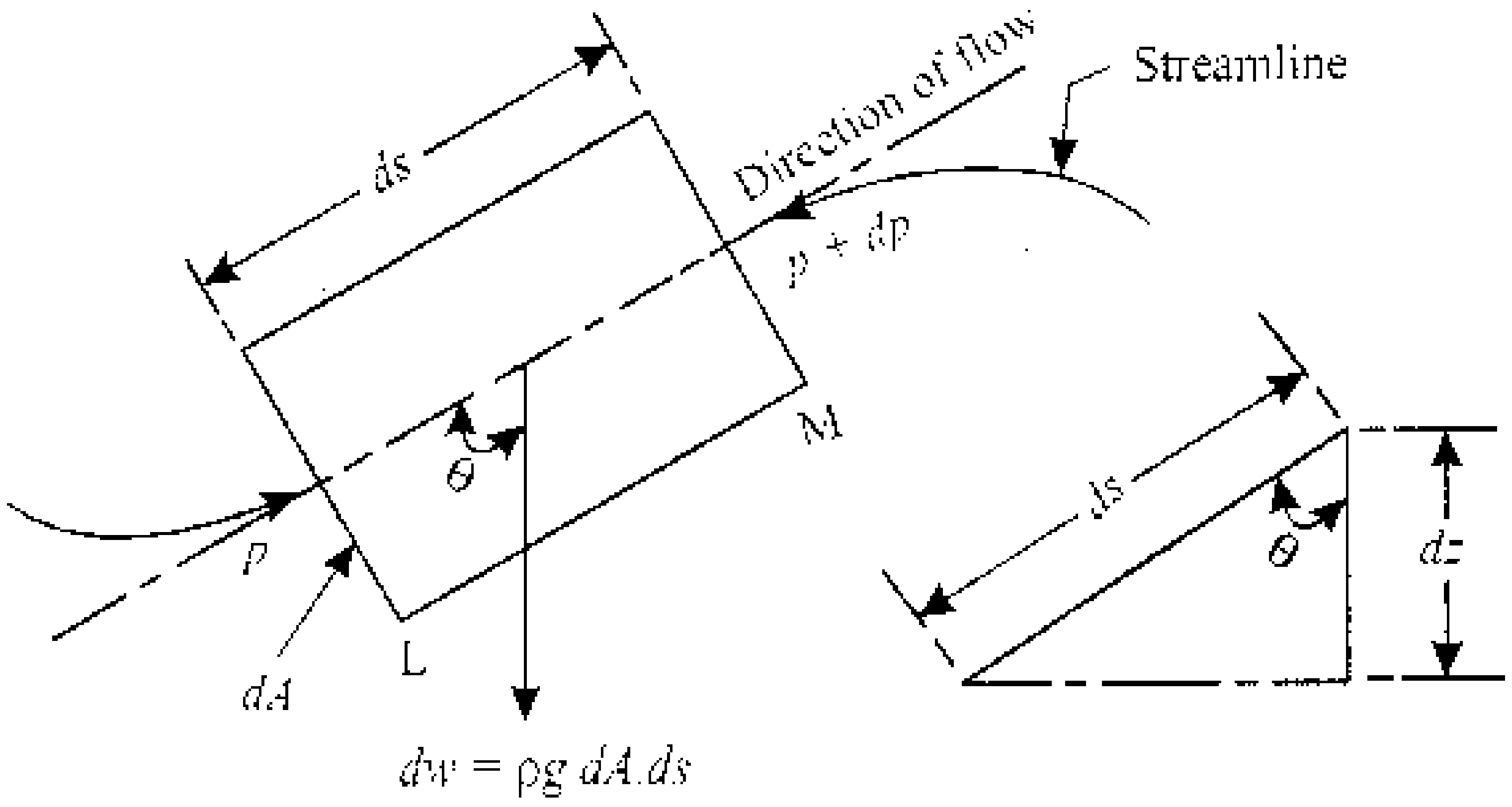

Fig. 6.2. Forces on a fluid element (Euler's equation).

The external forces tending to accelerate the fluid element in the direction of streamline are as follows:

1. Net pressure force in the direction of flow is,

$$p.dA - (p - dp)\, dA = -\, dp\,.\, dA \qquad ...(i)$$

2. Component of the weight of the fluid element in the direction of flow is

$$= -\,\rho.g.dA.ds.\cos\theta$$

$$= -\,\rho.g.dA.ds\left(\frac{dz}{ds}\right) \qquad \left(\because \cos\theta = \frac{dz}{ds}\right)$$

$$= -\,\rho.g.dA.dz \qquad ...(ii)$$

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Mass of the fluid element

$$= \rho.dA.ds \qquad \text{...}(iii)$$

The acceleration of the fluid element

$$a = \frac{dV}{dt} = \frac{dV}{ds} \times \frac{ds}{dt} = V.\frac{dV}{ds} \qquad \text{...}(iv)$$

Now, according to Newton's second law of motion, Force = mass × acceleration

$$\therefore \qquad -dp.dA - \rho.g.dA.\,dz = \rho.dA.\,ds \times V.\,\frac{dV}{ds}$$

Dividing both sides by $\rho.dA$, we get

$$\frac{-dp}{\rho} - g.dz = V.\,dV$$

or

$$\frac{dp}{\rho} + V.dV + g.dz = 0 \qquad \text{...(6.3)}$$

This is the required **Euler's equation** for motion, and is in the form of *differential equation.*

Integrating the above eqn., we get

$$\frac{1}{\rho}\int dp + \int V.dV + \int g.dz = \text{constant}$$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Dividing by g, we get

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

or

$$\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$$

or, in other words,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

which proves *Bernoulli's equation.*

Euler's Equation in Cartesian Coordinates:

Consider an infinitely small mass of fluid enclosed in an elementary parallelopiped of sides dx, dy and dz as shown in Fig. 6.3. The motion of the fluid element is influenced by the following forces:

(*i*) **Normal forces due to pressure:**

The intensities of hydrostatic pressure acting normal to each face of the parallelepiped are shown Fig. 6.3.

The net pressure force in the X-direction

$$= p.dy.dz - \left(p + \frac{\partial p}{\partial x}dx\right)dy\,dz$$

$$= -\frac{\partial p}{\partial x}\,dx.dy.dz$$

(*ii*) **Gravity or body force:**

Let B be the body force *per unit mass of fluid* having components B_x, B_y and B_z in the X, Y and directions respectively.

Then, the body force acting on the parallelopiped in the direction of X-coordinate is $= B_x.\rho.dx.dy.dz$

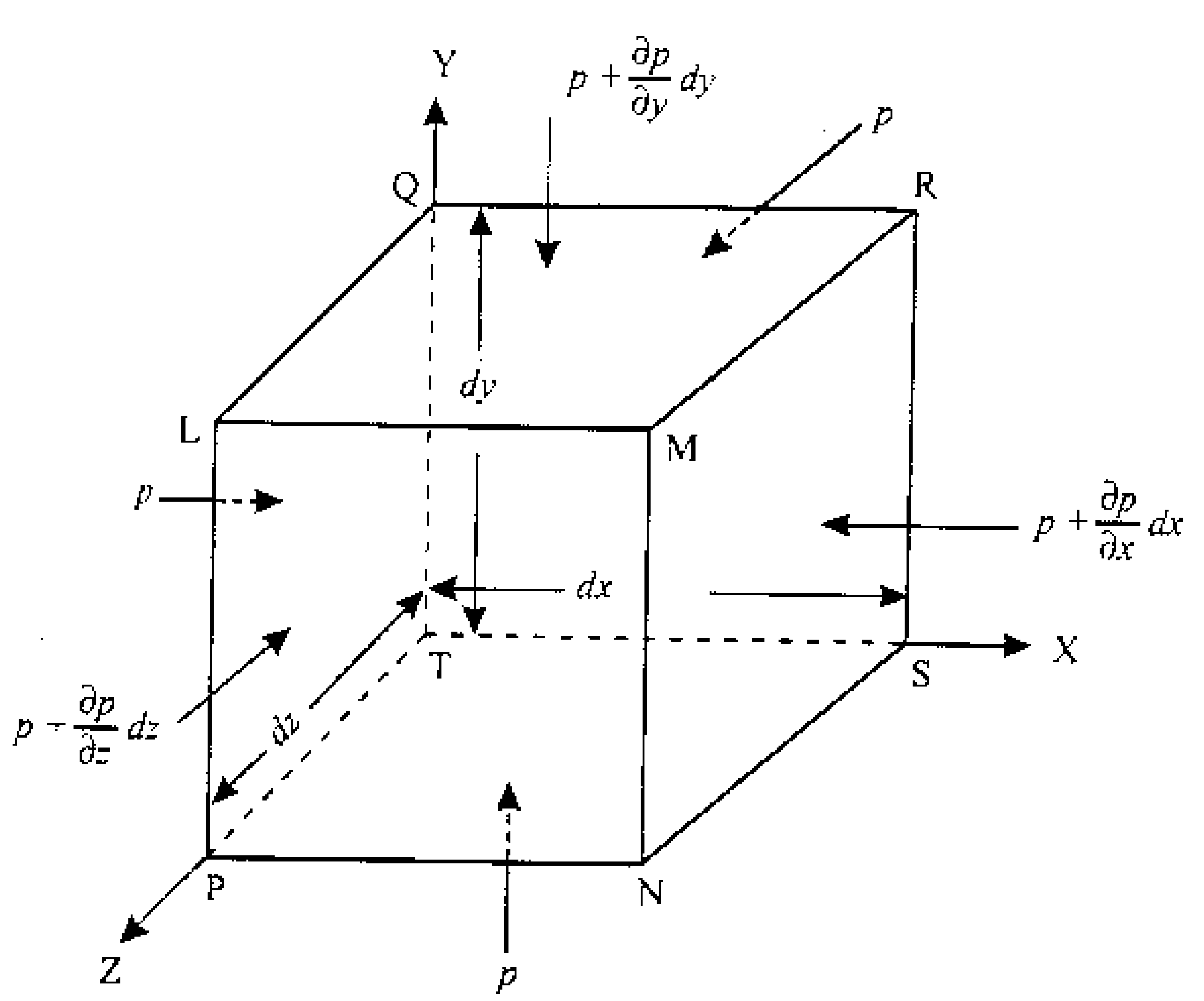

Fig. 6.3. Normal surface forces on a non-viscous fluid element.

(*iii*) **Inertia forces:**

The inertia force acting on the fluid mass, along the X-coordinate is given by,

$$\text{Mass} \times \text{acceleration} = \rho.\, dx\,.\, dy.\, dz.\, \frac{du}{dt}$$

As per Newton's second law of motion summation of forces acting in the fluid element in any direction equals the resulting inertia forces in that direction. Thus, along X-direction:

$$B_x.\rho.\, dx.dy.dz - \frac{\partial p}{\partial x}\, dx.dy.dz. = \rho.dx.dy.dz.\frac{du}{dt}$$

Dividing both sides by $\rho.dx.dy.dz$, we have

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt} \qquad ...(i)$$

In this equation each term has dimensions of force per unit mass or acceleration. Obviously the total acceleration in a given direction is prescribed by the algebraic sum of the body force and the pressure gradient in that direction since the velocity components are functions of position and time, *i.e.*, $u = f(x, y, z, t)$, therefore, the total derivative of velocity u in the X-direction can be written as:

$$du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

or

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}.\frac{dx}{dt} + \frac{\partial u}{\partial y}.\frac{dy}{dt} + \frac{\partial u}{\partial z}.\frac{dz}{dt}$$

Substituting, $\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$ and $\frac{dz}{dt} = w$; we have

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad ...(ii)$$

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Combining eqns. (*i*) and (*ii*), we get the force components as,

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \quad ...(iii)$$

Similarly,
$$B_y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \quad ...(iv)$$

and,
$$B_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \quad ...(v)$$

For steady flow:
$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$$

Thus, the Euler's equation for a steady three-dimensional flow can be written as:

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \quad ...(vi)$$

$$B_y - \frac{1}{\rho}\frac{\partial p}{\partial y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \quad ...(vii)$$

$$B_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \quad ...(viii)$$

In Euler's equation each term represents force per unit mass. Thus, if each equation is multiplied by the respective projections of the elementary displacement, the resulting equation would represent energy. Thus, in order to get total energy in the three-dimensional-steady-incompressible flow, the energy terms can be combined as follows:

$$B_x\,dx - \frac{1}{\rho}\frac{\partial p}{\partial x}dx = u\frac{\partial u}{\partial x}dx + v\frac{\partial u}{\partial y}dx + w\frac{\partial u}{\partial z}dx \quad ...(ix)$$

$$B_y\,dy - \frac{1}{\rho}\frac{\partial p}{\partial y}dy = u\frac{\partial v}{\partial x}dy + v\frac{\partial v}{\partial y}dy + w\frac{\partial v}{\partial z}dy \quad ...(x)$$

$$B_z dz - \frac{1}{\rho}\frac{\partial p}{\partial y}dz = u\frac{\partial w}{\partial x}dz + v\frac{\partial w}{\partial y}dz + w\frac{\partial w}{\partial z}dz \quad ...(xi)$$

From the equation of a streamline in a three-dimensional flow, we have:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$udy = vdx;\ vdz = wdy;\ udz = wdx$$

Substituting these values in eqns (*ix*), (*x*) and (*xi*), we get

$$B_x dx - \frac{1}{\rho}\frac{\partial p}{\partial x}dx = u\frac{\partial u}{\partial x}dx + u\frac{\partial u}{\partial y}dy + u\frac{\partial u}{\partial z}dz \quad ...(xii)$$

$$B_y dy - \frac{1}{\rho}\frac{\partial p}{\partial y}dy = v\frac{\partial v}{\partial x}dx + v\frac{\partial v}{\partial y}dy + v\frac{\partial v}{\partial z}dz \quad ...(xiii)$$

$$B_z dz - \frac{1}{\rho}\frac{\partial p}{\partial z}dz = w\frac{\partial w}{\partial x}dx + w\frac{\partial w}{\partial y}dy + w\frac{\partial w}{\partial z}dz \quad ...(xiv)$$

Acceleration terms are of form $u\frac{\partial u}{\partial x}$ which can be replaced by $\frac{1}{2}\frac{\partial(u^2)}{\partial x}$. Thus,

$$B_x\,dx - \frac{1}{\rho}\frac{\partial p}{\partial x}dx = \frac{1}{2}\left[\frac{\partial}{\partial x}(u^2)dx + \frac{\partial}{\partial y}(u^2)\,dy + \frac{\partial}{\partial z}(z^2)dz\right] = \frac{1}{2}d(u^2) \quad ...(xv)$$

Similarly,
$$B_y\,dy - \frac{1}{\rho}\frac{\partial p}{\partial y}dy = \frac{1}{2}d(v^2) \qquad ...(xvi)$$

and,
$$B_z\,dz - \frac{1}{\rho}\frac{\partial p}{\partial y}dz = \frac{1}{2}d(w^2) \qquad ...(xvii)$$

Adding eqns (*xv*), (*xvi*) and (*xvii*), we get

$$B_x dx + B_y dy + B_z dz - \frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right)$$
$$= \frac{1}{2}\left[d(u^2) + d(v^2) + d(w^2)\right]$$

or,
$$B_x dx + B_y dy + B_z dz - \frac{1}{\rho}dp = \frac{1}{2}d(V^2) \qquad ...(xviii)$$

where, V = Total velocity vector.

When gravity is the only body force acting on the third element, then

$$B_x = 0,\ B_z = 0 \text{ and } B_y = -g$$

$B_y = -g$ since the *gravitational force* acts in the *downward* direction which is negative 'with' respect to Y, which is positive upward. Inserting these values in (*xviii*), we get

$$-g - \frac{1}{\rho}dp = \frac{1}{2}d(V^2)$$

or,
$$-g - \frac{1}{\rho}dp = VdV$$

or, $\frac{dp}{\rho} + VdV + g = 0$ which is the same as Euler's equation (6.3).

Hydrostatic equation from Euler's equation:

If the fluid is at rest then the velocity terms in the Euler's eqns, (*vi*), (*vii*) and (*viii*), vanish and we have

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0;\ B_y - \frac{1}{\rho}\frac{\partial p}{\partial y} = 0;\ B_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = 0$$

Further if gravity is the only body force, then

$$B_x = 0;\ B_y = -g;\ B_z = 0$$

$$\therefore \quad \frac{1}{\rho}\frac{\partial p}{\partial x} = 0;\ \frac{1}{\rho}\frac{\partial p}{\partial z} = 0 \qquad ...(xix)$$

and
$$-g - \frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \qquad ...(xx)$$

Eqn. (*xix*), signifies that fluid pressure *p* is independent of *x* and *z*. In that case $\frac{\partial p}{\partial y} = \frac{dp}{dy}$ and

$$-g - \frac{1}{\rho}\frac{dp}{dy} = 0$$

or
$$dp = -\rho g dy \quad \text{or} \quad dp = -w dy$$

Integrating both sides, we get,

$$\int_1^2 dp = -w\int_1^2 dy$$

or $$(p_2 - p_1) = -w(y_2 - y_1)$$

or $$dp = w dy$$

which represents the hydrostatic equation. Thus, hydrostatic equation is merely a *corollary of Euler's equation.*

Example 6.2. *A discharge through a 24 cm diameter horizontal pipe increases linearly from 30 to 120 litres/sec. of water in 4 seconds.*

(*i*) *What pressure gradient must exist to produce this acceleration?*

(*ii*) *What is the difference in pressure intensity that exists between two sections that lie 9 m apart?*

Solution. The Euler's equation for one-dimensional flow along the pipe axis may be written as:

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \qquad ...(i)$$

As the pipe is of uniform cross-sectional area, the velocity remains constant along the flow direction and therefore,

$$\frac{\partial u}{\partial x} = 0$$

Further, since the pipe has been laid horizontally, therefore, the body forces per units volume in the direction X = 0

Thus, the eqn. (*i*) reduces to

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t}$$

The change in velocity when the flow changes from 30 to 120 litres/sec

$$\partial u = (u_2 - u_1) = \frac{120 \times 10^{-3}}{\frac{\pi}{4} \times (0.24)^2} - \frac{30 \times 10^{-3}}{\frac{\pi}{4} \times (0.24)^2} = 1.989 \text{ m/s}$$

This change takes place in 4 sec,

$$\therefore \qquad \frac{\partial u}{\partial t} = \frac{1.989}{4} = 0.497 \text{ m/s}^2$$

(*i*) **Pressure gradient, $\frac{\partial p}{\partial x}$:**

$$\frac{\partial p}{\partial x} = -\rho\frac{\partial u}{\partial t} = -1000 \times 0.497 = \mathbf{-497\ N/m^2/m}\ \textbf{(Ans.)}$$

(*ii*) **Difference in pressure intensity between the sections:**

Difference in pressure intensity between two sections that lie 9 m apart

$$= \frac{\partial p}{\partial x} \times 9 = -497 \times 9 = \mathbf{-4473\ N/m^2}\ \textbf{(Ans.)}$$

Example 6.3. *Brine of specific gravity 1.15 is draining from the bottom of a large open tank through a 80 mm pipe. The drain pipe ends at a point 10 m below the surface of the brine in the tank. Considering a streamline starting at the surface of the brine in the tank and passing through the centre of the drain line to the point of discharge and assuming the friction is negligible, calculate the velocity of flow along the streamline at the point of discharge from the pipe.*

(AMIE Summer, 2000)

Solution. Refer Fig. 6.4

Section 1 – The surface of brine in the tank

Section 2 – The point of discharge.

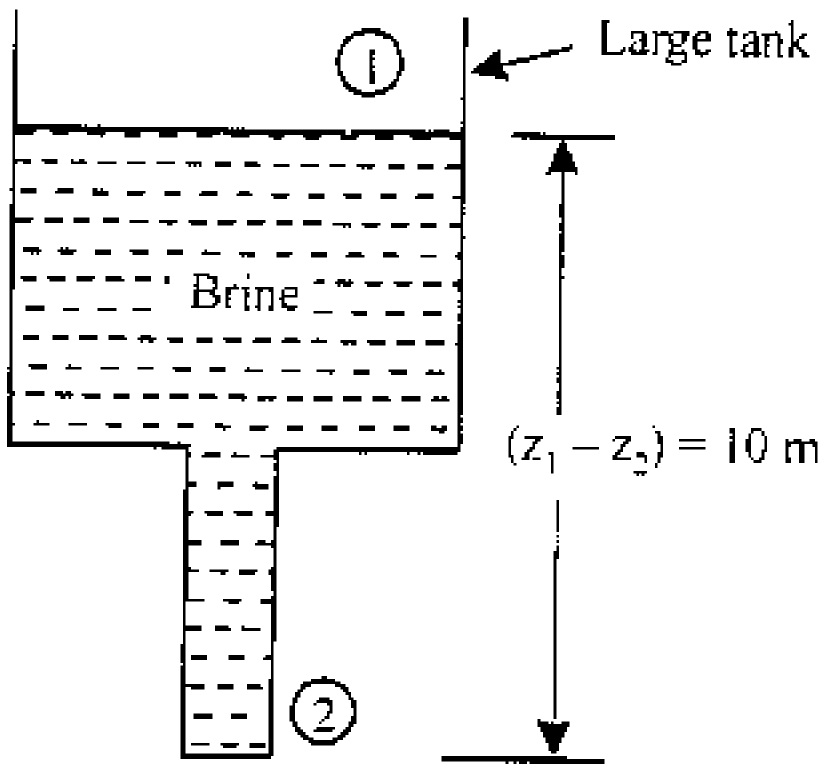

Fig. 6.4

Applying Bernoulli's equation between point 1 and 2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

Here $p_1 = p_2 = p_{atm}$ (atmospheric pressure),

$V_1 = 0$ and $(z_1 - z_2) = 10$ m

$$\therefore \quad V_2^2 = 2g(z_1 - z_2) = 2g \times 10 = 2 \times 9.81 \times 10 = 196.2$$

or $V_2 \simeq$ **14 m/s (Ans.)**

Example 6.4. *A pipeline (Fig. 6.5) is 15 cm in diameter and it is at an elevation of 100 m at section A. At section B it is an elevation of 107 m and has diameter of 30 cm. When a discharge of 50 litre/sec of water is passed through this pipeline, pressure at A is 35 kPa. The energy loss in pipe is 2m of water. Calculate pressure at B if flow is from A and B.* ***(AMIE Summer, 1997)***

Solution. *Given.* D_A = 15 cm = 0.15 m; D_B = 30 cm = 0.3 m;

p_A = 35 kPa; Q = 50 litre/sec = 0.05 m³/s;

h_f = 2 m of water; Direction of flow: from A to B

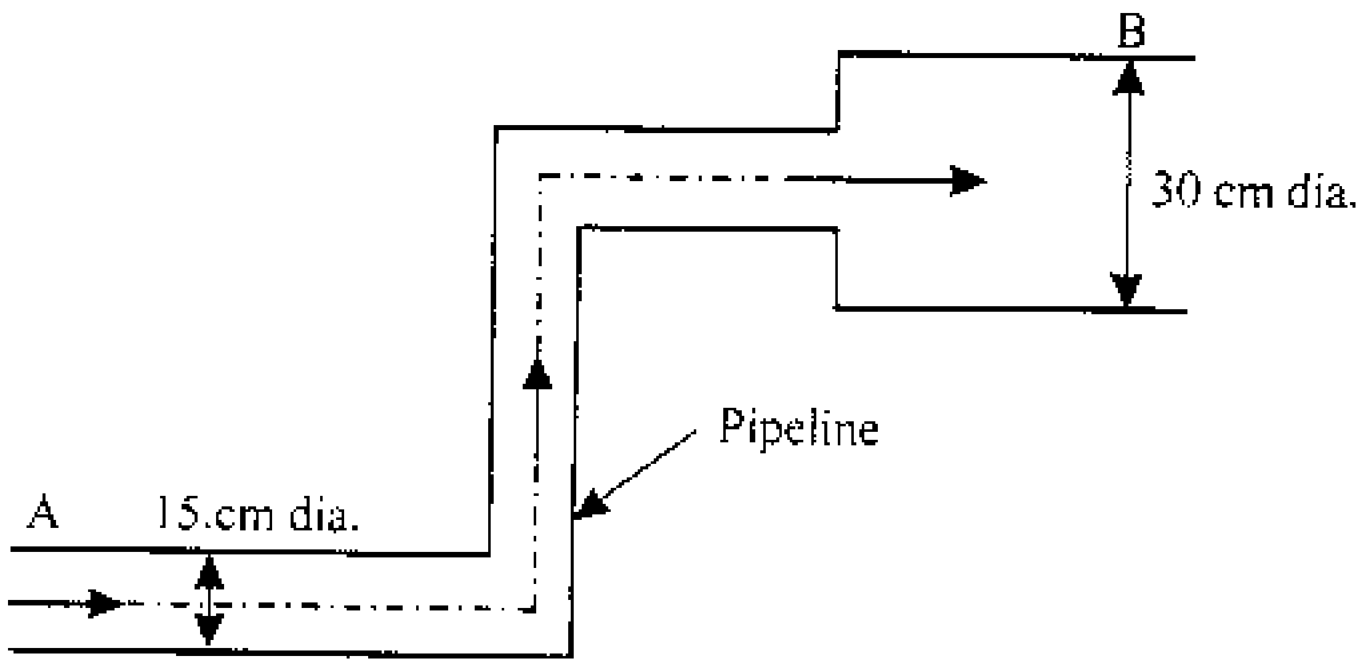

Fig. 6.5

Pressure at B, p_B:

$$V_A = \frac{Q}{\frac{\pi}{4} \times D_A^2} = \frac{0.05}{\frac{\pi}{4} \times 0.15^2} = 2.829 \text{ m/s}$$

$$V_B = \frac{Q}{\frac{\pi}{4} \times D_B^2} = \frac{0.05}{\frac{\pi}{4} \times (0.3)^2} = 0.707 \text{ m/s}$$

Applying Bernoulli's equation between section A and B, we get

$$\frac{p_A}{w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{w} + \frac{V_B^2}{2g} + z_B + h_f$$

or

$$\frac{p_B}{w} = \frac{p_A}{w} + \left(\frac{V_A^2 - V_B^2}{2g}\right) + (z_A - z_B) - h_f$$

or

$$p_B = p_A + w\left[\left(\frac{V_A^2 - V_B^2}{2g}\right) + (z_A - z_B) - h_f\right]$$

$$= 35 + \frac{(1000 \times 9.81)}{1000}\left[\left(\frac{2.829^2 - 0.707^2}{2 \times 9.81}\right) + (100 - 107) - 2\right]$$

$$35 + 9.81\ (0.3824 - 7 - 2) = -\ 49.54\ \text{kPa}.$$

i.e., p_B **= – 49.54 kPa.** This shows that the given pressure at A, 35 kPa is *gauge pressure* and hence there is *vacuum at point B.* (**Ans.**)

Example 6.5. *An open circuit wind tunnel draws in air from the atmosphere through a well contoured nozzle. In the test section, where the flow is straight and nearly uniform, a static pressure tap is drilled into the tunnel wall. A manometer connected to the tap shows that the static pressure within the tunnel is 45 mm of water below atmospheric. Assume that air is incompressible and at 25°C, pressure is 100 kPa (absolute). Calculate the velocity in the wind tunnel section (Refer to Fig. 6.6). Density of water is 999 kg/m³ and characteristic gas constant for air is 287 J/kg K.* **(GATE)**

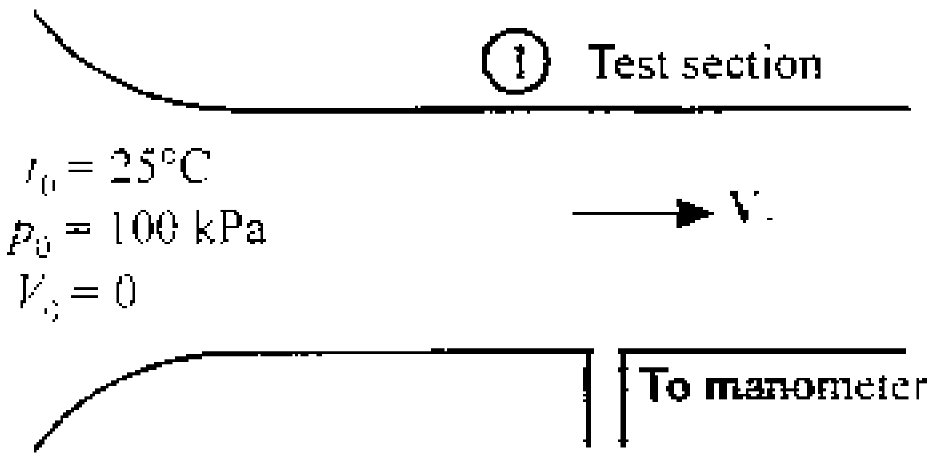

Fig. 6.6

Solution. *Given*: $T_0 = 25 + 273 = 298$ K;

$p_0 = 100$ kPa (abs.); $V_0 = 0$;

Velocity in the wind tunnel section V_1:

As per the problem, air is assumed as incompressible (*i.e.*, $\rho_0 = \rho_1 = \rho$). Velocity at test section can be found by using the equation:

$$\frac{p_0}{w} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{w} + \frac{V_1^2}{2g} + z_1$$

where, $z_0 = z_1$; $V_0 = 0$, $p_0 = 100$ kPa (abs.)

$p_1 = 45$ mm of water below atmosphere

$$= 999 \times 9.81 \times \frac{45}{1000}\ \text{Pa}$$

$$= 999 \times 9.81 \times \frac{45}{1000} \times 10^{-3}\ \text{kPa} = 0.44\ \text{kPa}$$

$$= 999 \times 9.81 \times \frac{45}{1000} \times 10^{-3}\ \text{kPa} = 0.44\ \text{kPa below atmosphere}$$

$\therefore$ $p_1(\text{absolute}) = P_{\text{atm}}\ (\text{in kPa}) - 0.44$ kPa

$= 100 - 0.44 = 99.56$ kPa

Also $pV = mRT = \rho RT$ $\left(\text{where } \rho = \frac{m}{V}\right)$

or $$\rho = \frac{p}{RT} = \frac{100 \times 10^3}{287 \times 298} = 1.169\ \text{kg/m}^3$$

$\therefore$ $w = \rho g = 1.169 \times 9.81 = 11.468\ \text{N/m}^3$

Substituting these values in (*i*), we get

$$\frac{100 \times 10^3}{11.468} = \frac{99.56 \times 10^3}{11.468} + \frac{V_1^2}{2 \times 9.81}$$

$$8719.9 = 8681.5 + \frac{V_1^2}{2 \times 9.81}$$

$\therefore$ $V_1 = \sqrt{(8719.9 - 8681.5) \times 2 \times 9.81}$ **= 27.45 m/s (Ans.)**

Example 6.6. *Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section.*

Frictional effects may be neglected. Assume density of water to be 999 kg/m³.

(AMIE Summer, 2001)

Solution. Refer to Fig. 6.7. $D_1 = 0.3$ m; $D_2 = 0.15$ m; $z_1 = 0$; $z_2 = 10$ m; $p_1 = 260$ kPa, $V_1 = 3$ m/s; $\rho = 999$ kg/m³.

From continuity equation, $A_1 V_1 = A_2 V_2$,

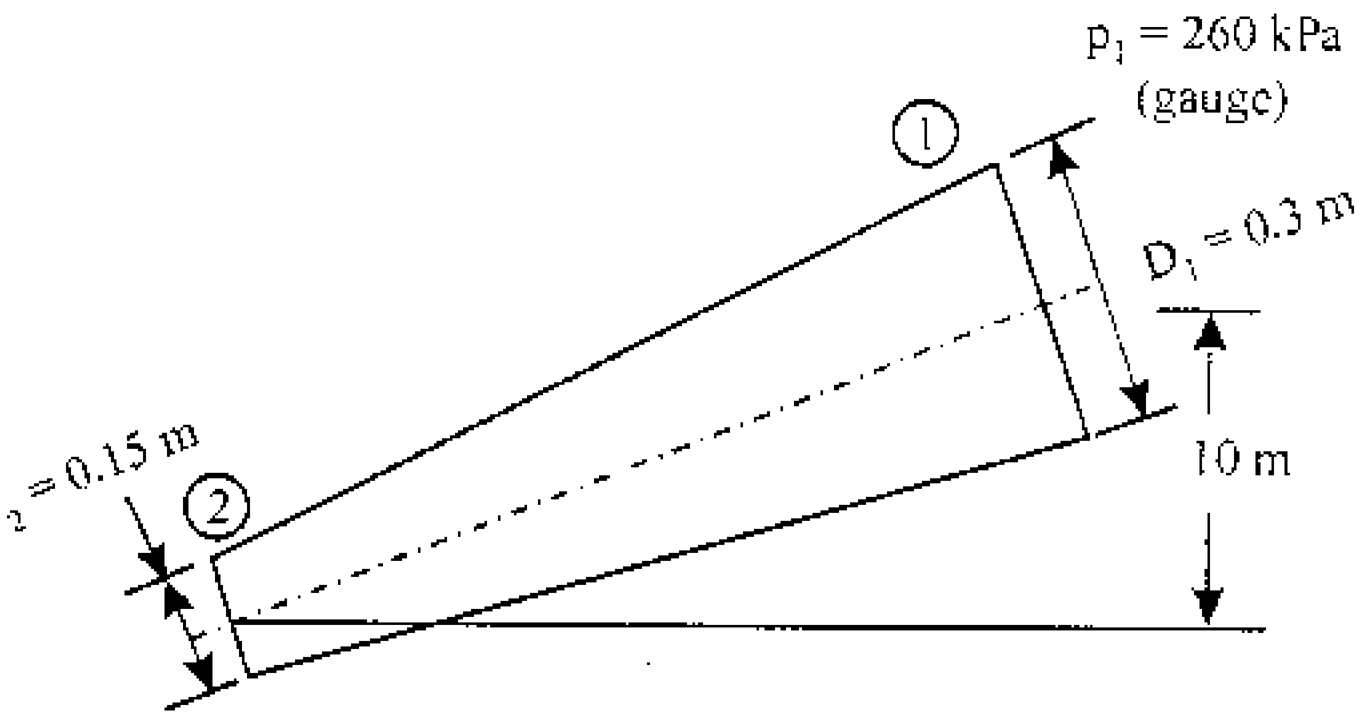

Fig. 6.7

$$V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{\frac{\pi}{4} D_1^2}{\frac{\pi}{4} D_2^2}\right) \times V_1$$

$$= \left(\frac{D_1}{D_2}\right)^2 \times V_1 = \left(\frac{0.3}{0.15}\right)^2 \times 3 = 12 \text{ m/s}$$

Weight density of water,

$$w = \rho g = 999 \times 9.81 = 9800.19 \text{ N/m}^3$$

From Bernoulli's equation between sections 1 and 2 (neglecting friction effects as given), we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{260 \times 1000}{9800.19} + \frac{(3)^2}{2 \times 9.81} + 10$$

$$= \frac{p_2}{9800.19} + \frac{(12)^2}{2 \times 9.81} + 0$$

$$26.53 + 0.459 - 10 = \frac{p_2}{9800.19} + 7.34$$

or $\quad p_2 = 290566$ N/m² = **290.56 kPa (Ans.)**

Example 6.7. *The water is flowing through a tapering pipe having diameters 300 mm and 150 mm at sections 1 and 2 respectively. The discharge through the pipe is 40 litres/sec. The section 1 is 10 m above datum and section 2 is 6 m above datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kN/m².*

Solution. A

Diameter, D_1

∴ Area,

Pressure,

Height of upp

At Section 2:

Diameter, D_2

∴ Area,

Height of low

Rate of flow (

Intensity of pre

Now,

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Applying Berno

and,

Solution. At Section 1:

Diameter, $D_1 = 300$ mm $= 0.3$ m

$\therefore$ Area, $\quad A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707\,\text{m}^2$

Pressure, $\quad p_1 = 400$ kN/m^2

Height of upper end above the datum, $z_1 = 10$ m

At Section 2:

Diameter, $D_2 = 150$ mm $= 0.15$ m

$\therefore$ Area, $\quad A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767\,\text{m}^2$

Height of lower end above the datum, $z_2 = 6$ m

Rate of flow (*i.e.*, discharge),

$$Q = 40 \text{ litres/sec} = \frac{40 \times 10^3}{10^6}$$

$$= 0.04\ \text{m}^3/\text{s}$$

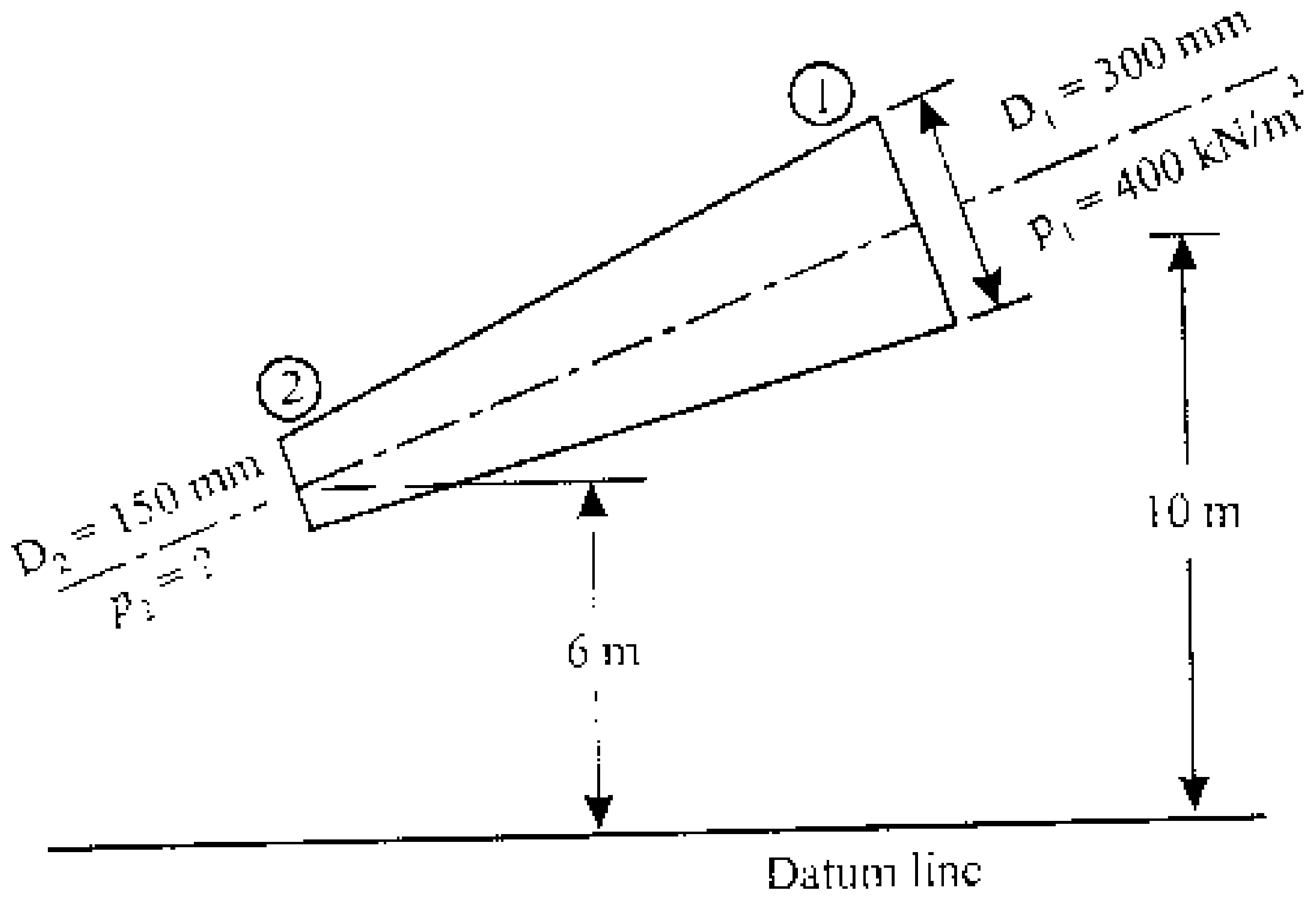

Fig. 6.8

Intensity of pressure at section 2, p_2:

Now, $\quad Q = A_1 V_1 = A_2 V_2$

$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.04}{0.0707} = 0.566\,\text{m/s}$

and $\quad V_2 = \frac{Q}{A_2} = \frac{0.04}{0.01767} = 2.264$ m/s

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

and, $$\frac{p_2}{w} = \frac{p_1}{w} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) + (z_1 - z_2)$$

$$= \frac{400}{9.81} + \frac{1}{2 \times 9.81}(0.566^2 - 2.264^2) + (10 - 6)$$

($\because w = 9.81$ kN/m^3)

$$= 40.77 - 0245 + 4 = 44.525 \text{ m}$$

$$\therefore \quad p_2 = 44.525 \times w = 44.525 \times 9.81 = \mathbf{436.8\ kN/m^2\ (Ans.)}$$

Example 6.8. *A pipe 200 m long slopes down at 1 in 100 and tapers from 600 mm diameter at the higher end to 300 mm diameter at the lower end, and carries 100 litres/sec of oil (sp. gravity 0.8). If the pressure gauge at the higher end reads 60 kN/m², determine:*

(*i*) *Velocities at the two ends;*

(*ii*) *Pressure at the lower end.*

Neglect all losses.

Solution. Length of the pipe, $l = 200$ m; diameter of the pipe at the higher end, $D_1 = 600$ mm = 0.6 m.

$$\therefore \text{ Area, } A_1 = \frac{\pi}{4} \times 0.6^2 = 0.283 \text{ m}^2$$

Diameter of the pipe at the lower end,

$$D_2 = 300 \text{ mm} = 0.3 \text{ m}$$

$$\therefore \text{ Area, } A_2 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

Height of the higher end, above datum,

$$z_1 = \frac{1}{100} \times 200 = 2 \text{ m}$$

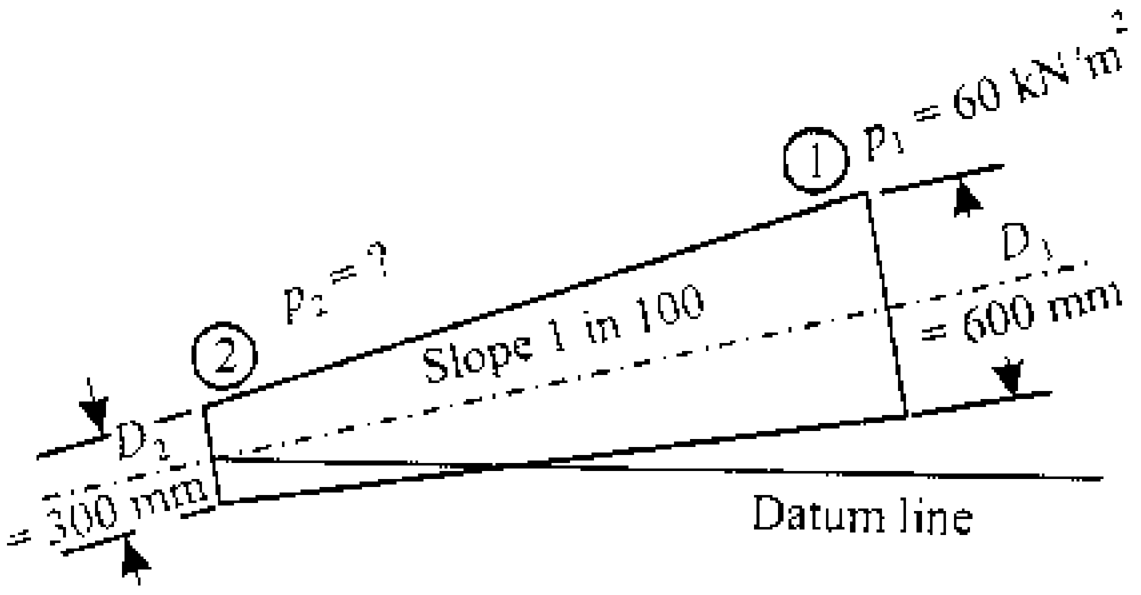

Fig. 6.9

Height of the lower end, above datum $z_2 = 0$

Rate of oil flow, $Q = 100$ litres/sec = 0.1 m³/s

Pressure at the higher end, $p_1 = 60$ kN/m²

(*i*) **Velocities, V_1, V_2:**

Now, $Q = A_1 V_1 = A_2 V_2$

Where, V_1 and V_2 are the velocities at the higher and lower ends respectively.

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.1}{0.283} = \mathbf{0.353\,m/s\ (Ans.)}$$

$$\text{and} \quad V_2 = \frac{Q}{A_2} = \frac{0.1}{0.0707} = \mathbf{1.414\,m/s\ (Ans.)}$$

(*ii*) **Pressure at the lower end p_2:**

Using Bernoulli's equation for both ends of pipe, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{60}{0.8 \times 9.81} + \frac{0.353^2}{2 \times 9.81} + 2$$

$$= \frac{p_2}{0.8 \times 9.81} + \frac{1.414^2}{2 \times 9.81} + 0$$

$$7.64 + 0.00635 + 2 = \frac{p_2}{0.8 \times 9.81} + 0.102$$

$$\therefore \quad \frac{p_2}{0.8 \times 9.81} = 9.54 \text{ m}$$

$$\text{or} \quad p_2 = \mathbf{74.8\ kN/m^2\ (Ans.)}$$

Example 6.9. *A 6m long pipe is inclined at an angle of 20° with the horizontal. The smaller section of the pipe which is at lower level is of 100 mm diameter and the larger section of the pipe is of 300 mm diameter as shown in Fig. 6.10. If the pipe is uniformly tapering and the velocity of water at the smaller section is 1.8 m/s. Determine the difference of pressures between the two sections.*

Solution. Length of the pipe, $l = 6$ m

Angle of inclination, $\theta = 20°$

At Section 1:

Diameter, $D_1 = 100$ mm $= 0.1$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} \times 0.1^2 = 0.00785$ m^2

Velocity, $V_1 = 1.8$ m/s

Datum, $z_1 = 0$

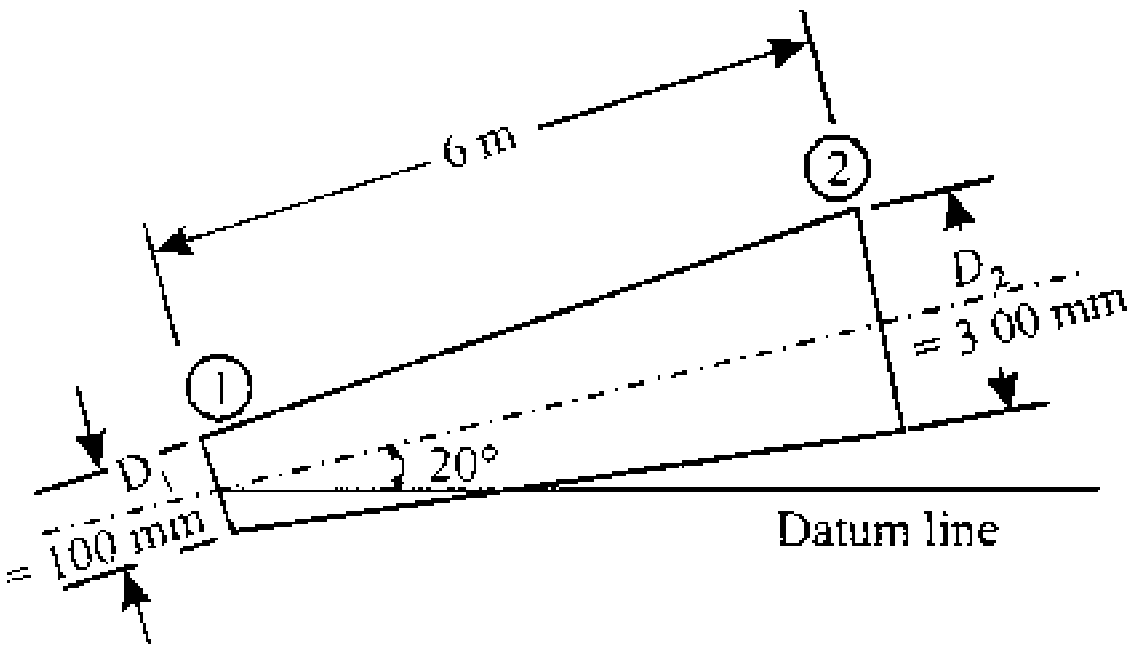

Fig. 6.10

At Section 2:

Diameter, $D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times 0.3^2 = 0.0707$ m^2

Datum, $z_2 = 0 + 6 \sin\theta = 6 \sin 20° = 6 \times 0.342 = 2.05$ m

Let, p_1 = Pressure at section 1 in kN/m^2, and

p_2 = Pressure at section 2 in kN/m^2.

Difference of pressures, $(p_1 - p_2)$:

From the equation of continuity, we know that

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{0.00785 \times 1.8}{0.0707} = 0.2 \text{ m/s}.$$

Applying Bernoulli's equation to both sections of the pipe, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

or

$$\left(\frac{p_1}{w} - \frac{p_2}{w}\right) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) + (z_2 - z_1)$$

$$= \frac{1}{2g}(V_2^2 - V_1^2) + (z_2 - z_1)$$

$$= \frac{1}{2 \times 9.81}(0.2^2 - 1.8^2) + (2.05 - 0) = 1.88$$

$$\therefore \quad (p_1 - p_2) = w \times 1.88 = 9.81 \times 1.88 = \mathbf{18.44\ kN/m^2\ (Ans.)}$$

[$\because$ w (for water) = 9.81 kN/m^3]

Example 6.10. *Water is flowing through a pipe having diameters 600 mm and 400 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 350 kN/m^2 and the pressure at the upper end is 100 kN/m^2. Determine the difference in datum head if the rate of flow through the pipe is 60 litres/sec.*

Solution. At section 1:

Diameter $D_1 = 600$ mm $= 0.6$ m

$\therefore$ Area $\quad A_1 = \frac{\pi}{4} \times 0.6^2 = 0.283$ m^2

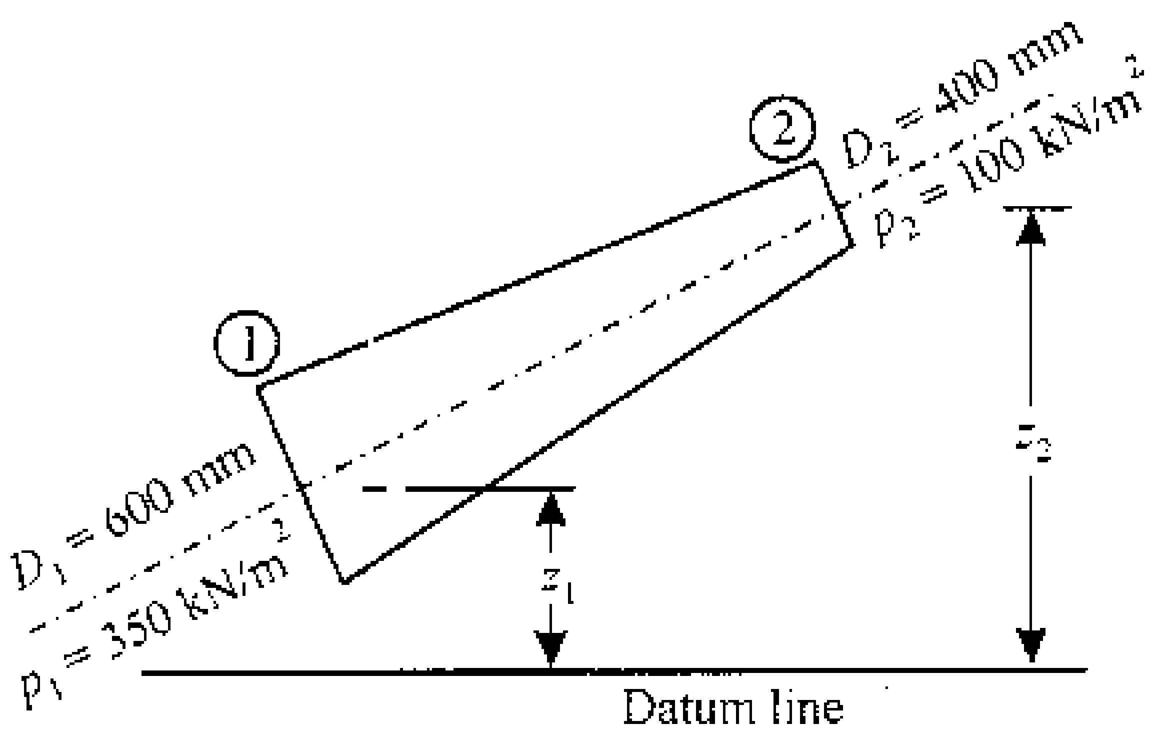

Fig. 6.11

Pressure p_1 = 350 kN/m^2

At Section 2:

Diameter D_2 = 400 mm = 0.4 m

Area $A_2 = \frac{\pi}{4} \times 0.4^2 = 0.1257\ \text{m}^2$

Pressure p_2 = 100 kN/m^2

Rate of flow,

$$Q = 60\ \text{litres/sec} = \frac{60}{1000} = 0.06\ \text{m}^3/\text{sec}.$$

Now, $Q = A_1V_1 = A_2V_2$

[Where V_1 and V_2 are the velocities at sections 1 and 2 respectively.]

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.06}{0.283} = 0.212\ \text{m/s}$$

$$\text{and,} \quad V_2 = \frac{Q}{A_2} = \frac{0.06}{0.1257} = 0.477\ \text{m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{350}{9.81} + \frac{0.212^2}{2 \times 9.81} + z_1 = \frac{100}{9.81} + \frac{0.477^2}{2 \times 9.81} + z_2$$

$$35.67 + 0.0023 + z_1 = 10.19 + 0.0116 + z_2$$

$$\mathbf{z_2 - z_1 = 25.47\ m\ (Ans.)}$$

Example 6.11. *Gasoline (sp. gr. 0.8) is flowing upwards a vertical pipeline which tapers from 300 mm to 150 mm diameter. A gasoline mercury differential manometer is connected between 300 mm 150 mm pipe section to measure the rate of flow. The distance between the manometer tappings is 1 metre and gauge reading is 500 mm of mercury. Find:*

(*i*) *Differential gauge reading in terms of gasoline head;*

(*ii*) *Rate of flow.*

Neglect friction and other losses between tappings.

[Allahabad University]

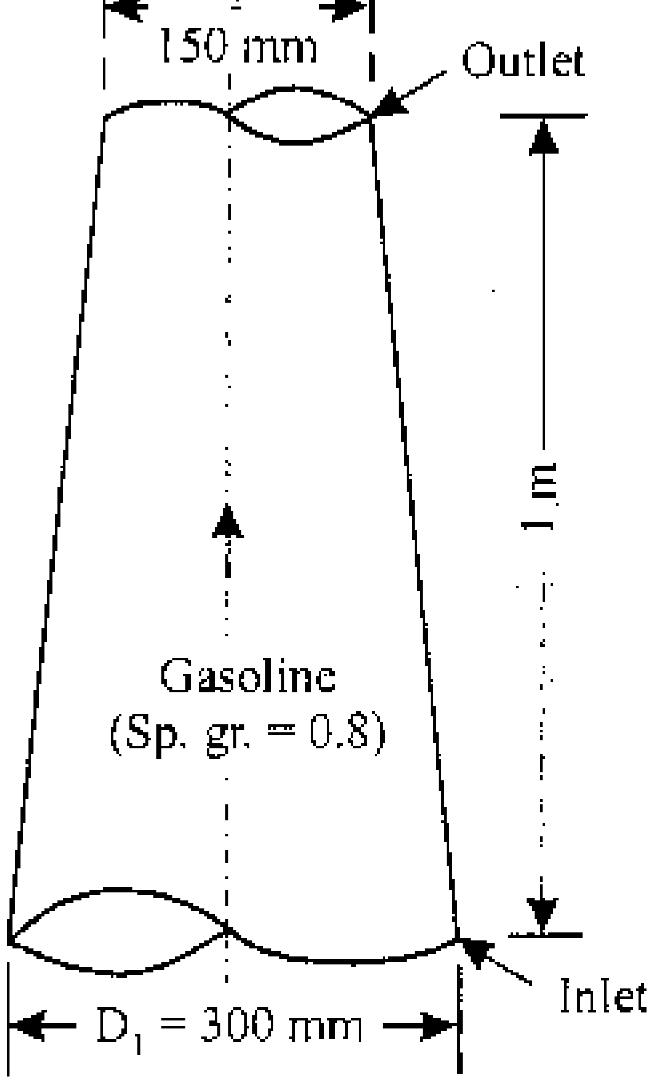

Fig. 6.12

Solution. Sp. gravity of gasoline = 0.8

At Inlet:

Diameter D_1 = 300 mm = 0.3 m

$$\therefore \quad \text{Area } A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707\ \text{m}^2$$

At Outlet:

Diameter D_2 = 150 mm = 0.15 m

$$\therefore \quad \text{Area } A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767\ \text{m}^2$$

Length of the pipe = 1m

Let datum of the pipe at inlet, $z_1 = 0$

∴ Datum of th

Gauge reading, *h*

(*i*) **Differential**

The gauge re

(*ii*) **Rate of flo**

Let,

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Example 6.12. *Th*

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pump may deliver. Ta

Solution. Refer Fi

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Taking point 1 as

∴ Datum of the pipe a at outlet, $z_2 = 0 + 1 = 1$ m

Gauge reading, h = 500 mm of mercury = 0.5 m of mercury.

(*i*) **Differential gauge reading in terms of gasoline head:**

The gauge reading = 0.5 m of mercury

$$= \frac{13.6 - 0.8}{0.8} \times 0.5 \text{ of gasoline}$$

$$= \textbf{8 m of gasoline (Ans.)}$$

(*ii*) **Rate of flow, Q:**

Let, V_1 = Velocity of gasoline at the inlet, and
V_2 = Velocity of gasoline at the outlet.

We know that, as per equation of continuity

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{0.0707 \times V_1}{0.01767} = 4V_1$$

Now, using Bernoulli's equation for the inlet and outlet of the pipe, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\left\{\frac{p_1}{w} - \frac{p_2}{w}\right\} + \left\{\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right\} + (z_1 - z_2) = 0$$

$$8 + \left[\frac{V_1^2}{2g} - \frac{(4V_1)^2}{2g}\right] + [0 - 1] = 0$$

or
$$8 - \frac{15V_1^2}{2g} - 1 = 0$$

or
$$\frac{15V_1^2}{2g} = 7$$

$$\therefore \quad V_1 = \left[\frac{7 \times 2 \times 9.81}{15}\right]^{1/2} = 3.026 \text{ m/s}.$$

∴ Rate of flow, $Q = A_1V_1 = 0.0707 \times 3.026 =$ **0.2139 m³/s (Ans.)**

Example 6.12. *The suction pipe of a pump rises at a slope of 3 vertical in 4 along the pipe which 12 cm in diameter. The pipe is 7.2 m long; its lower end being just below the water surface in the servoir. For design reasons, it is underable that pressure at inlet to the pump shall fall to more than 5 kPa below atmosphere pressure. Neglecting friction determine the maximum discharge that the p may deliver. Take atmospheric pressure as 101.32 kPa.* **(Banglore University)**

Solution. Refer Fig. 6.13. *Given:* d = 12 cm = 0.12 m; l = 7.2 m; p_{atm} = 101. 32 kPa = 101.32 kN/m².

Applying Bernoulli's equation at point 1 (F.W.S) and point 2 (suction point to pump), we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \qquad ...(i)$$

ty V_1 on the free water surface (F.W.S) = 0 (sump being very large)

$$p_1 = p_{\text{atm}} = 101.32 \text{ kN/m}^2, \; p_2 = 101{\cdot}32 - 75 = 26.32 \text{ kN/m}^2$$

Taking point 1 as the datum head, we have,

$$z_1 = 0;\ z_2 = 7.2 \times \frac{3}{4} = 5.4 \text{ m}$$

Inserting the various values in the eqn (*i*), we have

$$\frac{101.32}{9.81} + 0 + 0 = \frac{26.32}{9.81} + \frac{V_2^2}{2g} + 5.4$$

or $\quad V_2 = 6.64$ m/s

∴ Discharge that the pump may deliver,

$$Q = A_2 \times V_2 = \frac{\pi}{4} \times (0.12)^2 \times 6.64 = \mathbf{0.075\ m^3/s\ (Ans.)}$$

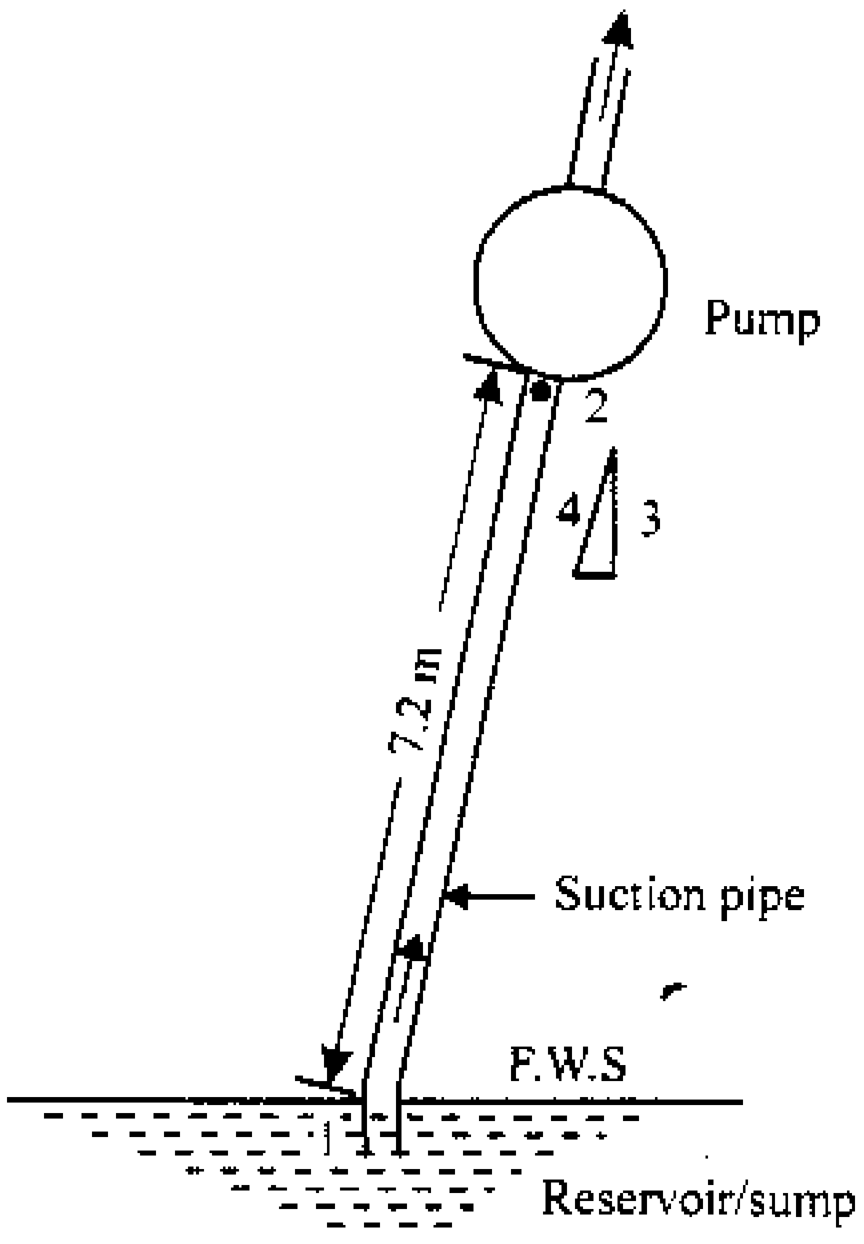


Fig. 6.13

6.5. Bernoulli's Equation for Real Fluid

Bernoulli's equation earlier derived was based on the assumption that fluid is non-viscous and therefore frictionless. Practically, all fluids are real (and not ideal) and therefore are viscous and as such there are always some losses in fluid flows. These losses have, therefore, to be taken into consideration in the application of Bernoulli's equation which gets modified (between sections 1 and 2) for *real fluids* as follows:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L \qquad \text{...(6.4)}$$

where, h_L = loss of energy between sections 1and 2.

Example 6.13. *The following data relate to a conical tube of length 3.0 m fixed vertically with its smaller end upwards and carrying fluid in the downward direction.*

The velocity of flow at the smaller end = 10 m/s.

The velocity of flow at the larger end = 4 m/s.

The loss of head in the tube $= \dfrac{0.4\,(V_1 - V_2)^2}{2g}$

where V_1 *and* V_2 *are velocities at the smaller and larger ends respectively.*

Pressure head at the smaller end = 4 m of liquid.

Determine the pressure head at the larger end.

Solution. Length of tube, l = 3.0 m

Velocity, V_1 = 10 m/s.

Pressure head, $\dfrac{p_1}{w} = 4$ m of liquid.

Velocity, V_2 = 4 m/s.

Loss of head,

$$h_L = \frac{0.4(V_1 - V_2)^2}{2g} = \frac{0.4(10-4)^2}{2 \times 9.81} = 0.73 \text{ m}$$

Pressure head at the larger end, $\dfrac{p_2}{w}$ **:**

Applying Bernoulli's equation at sections (1) and (2), we get

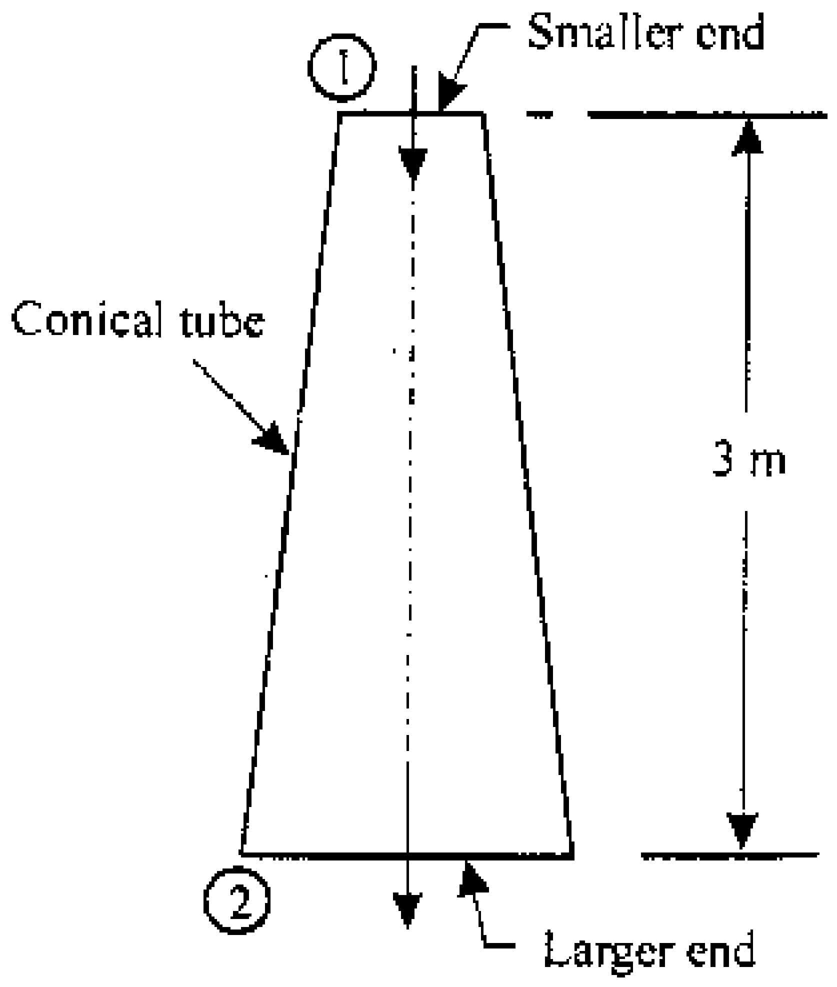


Fig. 6.14

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L$$

Let the datum line passes through section (2).

Then, $z_2 = 0,\ z_1 = 3.0$ m

$$\therefore \quad 4 + \frac{10^2}{2g} + 3.0 = \frac{p_2}{w} + \frac{4^2}{2g} + 0 + 0.73$$

or $$(4 + 5.09 + 3.0) = \frac{p_2}{w} + 0.815 + 0.73$$

or $$12.09 = \frac{p_2}{w} + 1.54$$

$$\therefore \quad \frac{p_2}{w} = \textbf{10.55 m of liquid (Ans.)}$$

Example 6.14. *In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at section 1 which was at elevation 10 m. At another section 2 at elevation 12 m, the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water at 20°C and 760 mm Hg is 998 kg/m³.* **(AMIE Winter, 1998)**

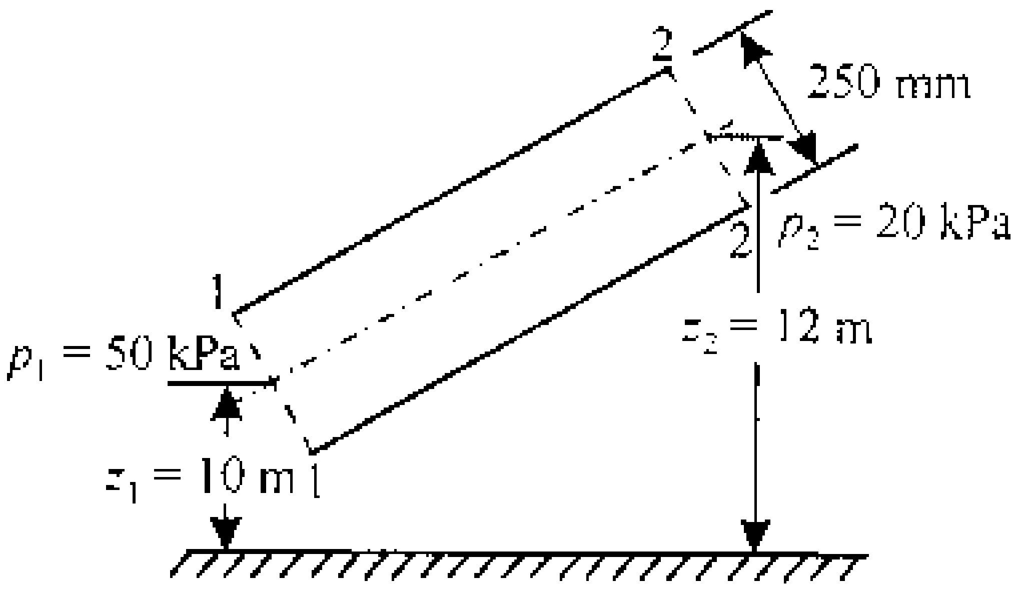


Fig. 6.15

Solution. *Given:* $D = 250$ mm $= 0.25$ m, $p_1 = 50$ kPa $= 50 \times 10^3$ N/m²; $z_1 = 10$ m; $z_2 = 12$ m; $p_2 = 20$ kPa $= 20 \times 10^3$ N/m²,

$V_1 = V_2 = 1.25$ m/s, $\rho = 998$ kg/m³.

Refer Fig. 6.15

Loss of head h_L**:**

Total energy at *section 1–1*,

$$E_1 = \frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{50 \times 10^3}{998 \times 9.81} + \frac{1.25^2}{2 \times 9.81} + 10 = 15.87 \text{ m}$$

Total energy of *section 2–2*,

$$E_2 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 = \frac{20 \times 10^3}{998 \times 9.81} + \frac{1.25^2}{2 \times 9.81} + 12 = 14.122 \text{ m}$$

$\therefore$ Loss of head, $h_L = E_1 - E_2 = 15.187 - 14.122 =$ **1.065 m (Ans.)**

Direction of flow:

Since $E_1 > E_2$ *direction of flow is from* **section 1-1 to section 2-2. (Ans.)**

Example 6.15. *A pipe line carrying oil (sp. gr. 0.8) changes in diameter from 300 mm at position 1 to 600 mm diameter at position 2 which is 5 metres at a higher level. If the pressures at positions 1 and 2 are 100 kN/m² and 60 kN/m² respectively and the discharge is 300 litres/sec. Determine:*

(*i*) *Loss of head, and*

(*ii*) *Direction of flow,*

Solution. Discharge, $Q = 300$ litres/sec

$$= \frac{300}{1000} = 0.3 \text{ m}^3/\text{s}.$$

Sp. gr. of oil = 0.8

∴ Weight of oil,

$$w = 0.8 \times 9.81 = 7.85 \text{ kN/m}^3$$

At position '1':

Diameter of pipe,

$$D_1 = 300 \text{ mm} = 0.3 \text{ m}$$

∴ Area of pipe, $A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$

Pressure, $p_1 = 100 \text{ kN/m}^2$

If the datum line passes through section 1 (Fig. 6.16) then datum, $z_1 = 0$

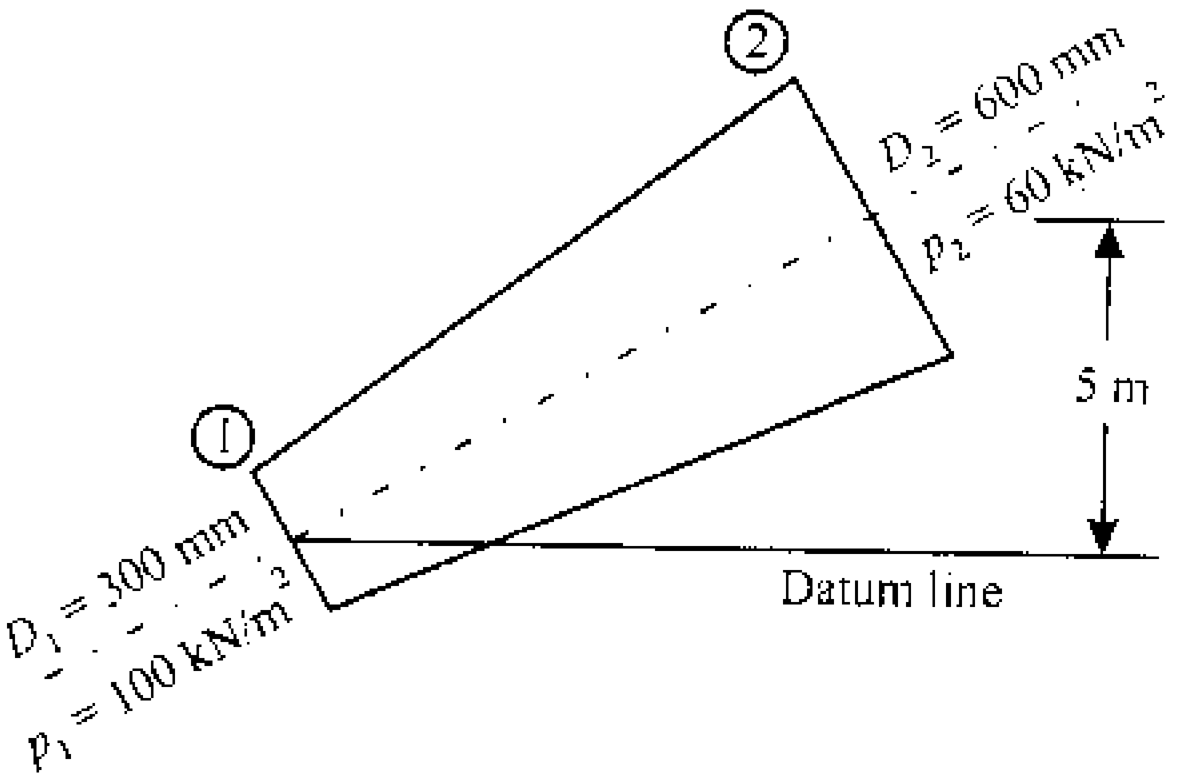


Fig. 6.16

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.3}{0.0707} = 4.24 \text{ m/s}$

At position '2':

Diameter of pipe, $D_2 = 600 \text{ mm} = 0.6 \text{ m}$

∴ Area of pipe, $A_2 = \frac{\pi}{4} \times 0.6^2 = 0.2828 \text{ m}^2$

Pressure, $p_2 = 60 \text{ kN/m}^2$

Datum, $z_2 = 5\text{m}$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.3}{0.2828} = 1.06 \text{ m/s}$.

(*i*) **Loss of head, h_L:**

Total energy at *position 1*,

$$E_1 = \frac{p_1}{w} + \frac{V_1^2}{2g} + z_1$$

$$= \frac{100}{7.85} + \frac{(4.24)^2}{2 \times 9.81} + 0 = 12.74 + 0.92 = 13.66 \text{ m}$$

Total energy at *position 2*,

$$E_2 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$= \frac{60}{7.85} + \frac{(1.06)^2}{2 \times 9.81} + 5 = 7.64 + 0.06 = 7.7 \text{ m}$$

$\therefore$ Loss of head, $h_L = E_1 - E_2$

$$= 13.66 - 7.7 = 5.96 \text{ m}$$

i.e., h_L = **5.96 m (Ans.)**

(*ii*) Direction of Flow:

Since $E_1 > E_2$ therefore flow takes place **from 1 to 2. (Ans.)**

Example 6.16. *A conical tube is fixed vertically with its smaller end upwards and it forms a part* *peline. The velocity at the smaller end is 4.5 m/s and at the large end 1.5 m/s. Length of conical* **is** *1.5 m. The pressure at the upper end is equivalent to a head of 10 m of water.*

(*i*) *Neglecting friction, determine the pressure at the lower end of the tube.*

If head loss in the tube is $\frac{0.3\,(V_1 - V_2)^2}{2g}$, *where* V_1 *is the velocity at the smaller end and* V_2 *is the velocity at the larger end, determine the pressure at the lower end (larger cross-section).* **(AMIE Summer, 1998)**

Solution. *Given*: $V_1 = 4.5$ m/s; $V_2 = 1.5$ m/s; $L = z_1 - z_2 = 1.5$ m;

$$\frac{p_1}{w} = 10 \text{ m of water}; \quad h_f = \frac{0.3(V_1 - V_2)^2}{2g}$$

ressure at the lower end, p_2:

Neglecting friction:

Applying Bernoulli's equation between points 1 and 2, we get

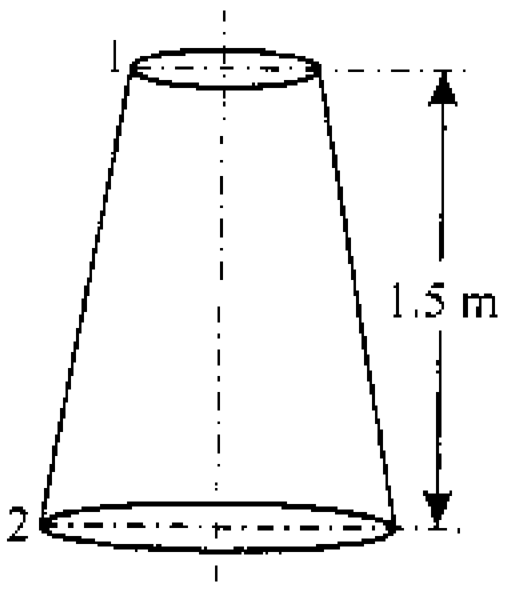


Fig. 6.17

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

or
$$\frac{p_2}{w} = \frac{p_1}{w} + \frac{1}{2g}(V_1^2 - V_2^2) + (z_1 - z_2)$$

$$= 10 + \frac{1}{2 \times 9.81}(4.5^2 - 1.5^2) + 1.5$$

$$= 10 - 0.917 + 1.5 \simeq 12.42 \text{ m of water}$$

or
$$p_2 = 12.42 \times 9810 \text{ N/m}^2 = 12.42 \times 9810 \times 10^{-5} \text{ bar}$$

$$= \mathbf{1.218 \text{ bar (Ans.)}}$$

Considering loss of head (h_f) in the tube:

$$h_f = \frac{0.3(V_1 - V_2)^2}{2g}$$

Applying Bernoulli's equation between points 1 and 2, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

or
$$\frac{p_2}{w} = \frac{p_1}{w} + \frac{1}{2g}(V_1^2 - V_2^2) + (z_1 - z_2) - h_f$$

$$= 10 + \frac{4.5^2 - 1.5^2}{2 \times 9.81} + 1.5 - \frac{0.3(4.5 - 1.5)^2}{2 \times 9.81}$$

$$= 10 - 0.917 + 1.5 - 0.138 = 12.279 \text{ m of water}$$

or $$p_2 = 12.279 \times 9810 \times 10^{-5} \text{ bar} = \mathbf{1.204 \text{ bar (Ans.)}}$$

Example 6.17. *A drainage pump has tapered suction pipe. The pipe is running full of water. The pipe diameters at the inlet and at the upper end are 1 m and 0.5 m respectively. The free water surface is 2 m above the centre of the inlet and centre of upper end is 3 m above the top of free water surface. The pressure at the tip end of the pipe is 25 cm of mercury and it is known that loss of head by friction between top and the bottom section is one tenth of the velocity head at the top section. Compute the discharge in litre/sec. Neglect loss of head at the entrance of the tapered pipe.*

(AMIE Summer, 1999)

Solution. *Given*: D_1 =1m; D_2 = 1.5 m;

$$p_1 = 76 \text{ cm of Hg} = \frac{76}{100} \times 13.6 = 10.336 \text{ of water;}$$

$$p_2 = 25 \text{ cm of Hg} = \frac{25}{100} \times 13.6 = 3.4 \text{ m of water; } h_f = \frac{1}{10}\frac{V_2^2}{2g}.$$

Discharge, Q:

Refer Fig. 6.18. Applying continuity equation for the flow through pipe, we get

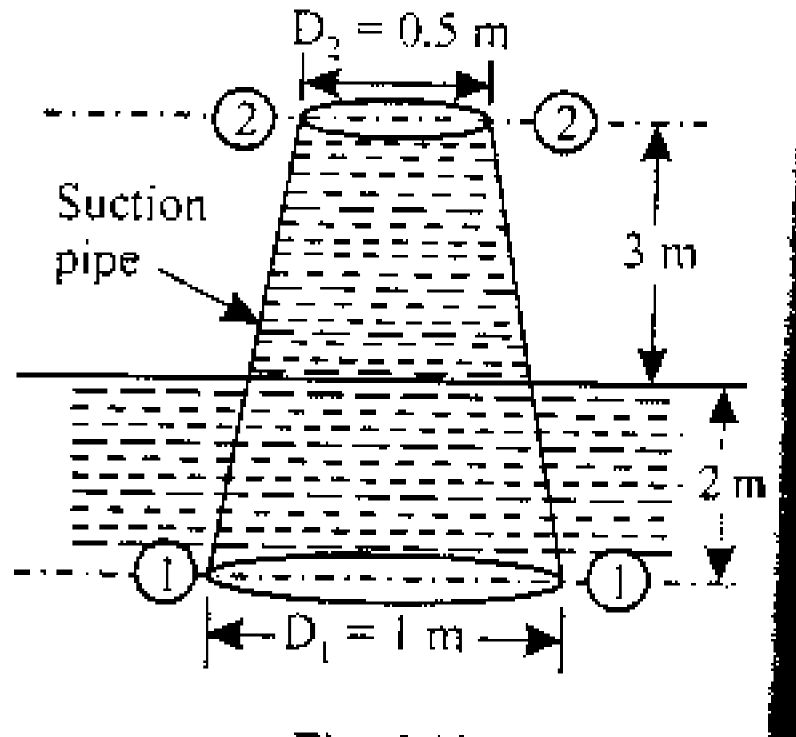


Fig. 6.18

$$A_1V_1 = A_2V_2$$

$$\frac{\pi}{4}D_1^2 V_1 = \frac{\pi}{4}D_2^2 V_2$$

or $$D_1^2 V_1 = D_2^2 V_2$$

or $$1^2 \times V_1 = (0.5)^2 V_2$$

or $$V_2 = 4V_1$$

Now, applying Bernoulli's equation at 1-1 and 2-2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$10.336 + \frac{V_1^2}{g} + 0 = 3.4 + \frac{16V_1^2}{2g} + 5 + \frac{1}{10} \times \frac{16V_1^2}{2g}$$

$$\frac{16V_1^2}{2g} + \frac{1.6V_1^2}{2g} - \frac{V_1^2}{2g} = 10.336 - 3.4 - 5 = 1.936$$

or $$16.6\ V_1^2 = 2 \times 9.81 \times 1.936 = 37.98$$

∴ $$V_1 = 1.513 \text{ m/s}$$

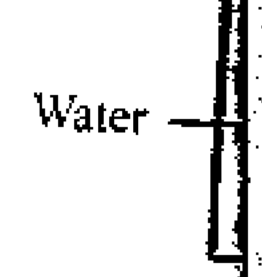
Discharge $Q = A_1V_1 = \frac{\pi}{4} \times 1^2 \times 1.513 = 1.188$ m³/s = **1188 litres/sec (Ans.)**

Example 6.18. *The closed tank of a fire engine is partly filled with water. the air space abo being under pressure. A 6 cm bore connected to the tank discharges on the roof of a building 2.5 above the level of water in the tank. The friction losses are 45 cm of water.*

Determine the air pressure which must be maintained in the tank to deliver 20 litres/sec on the roof.

(Madras University)

Solution. Refer to Fig. 6.19 *Given*: Diameter of hose pipe d = 6 cm = 0.06 m; Friction, h_f = 45 cm or 0.45 m of water.



Discharge, Q =

Velocity of wa

Applying Bern

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or,

∴

Example 6.19. *A*
gr. = 0.8) from the tan
surface in the tank is a
at an elevation of 5.7
(*i*) *The discharg*
(*ii*) *The pressure*
The losses in the p
the outlet;

Solution. Conside
in Fig. 6.20. The velocit
2, we get

or,

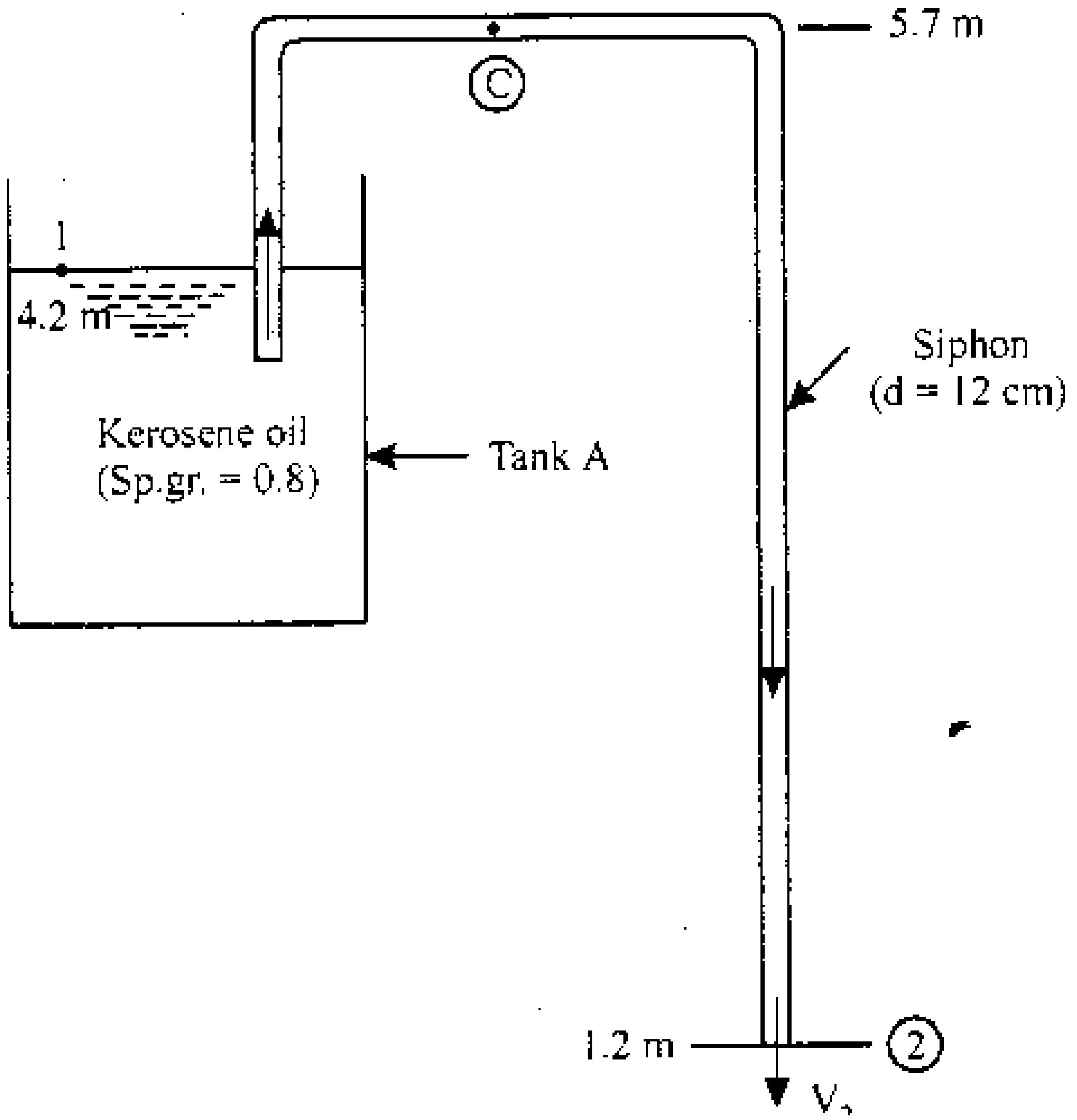

Fig. 6.20

(*i*) **The discharge in the pipe Q:**

$$Q = A_2V_2 = \frac{\pi}{4} \times (0.12)^2 \times 5.05 = \mathbf{0.057\ m^3/s\ (Ans.)}$$

(*ii*) **The pressure at point C:**

Apply Bernoulli's equation at points 1 and *C*, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_C}{w} + \frac{V_C^2}{2g} + z_C + h_{f(1-C)}$$

$$0 + 0 + 4.2 = \frac{p_C}{w} + \frac{(5.05)^2}{2 \times 9.81} + 5.7 + 0.5$$

or $$\frac{p_C}{w} = -3.3\,\text{m}$$

or $$p_C = (0.8 \times 9.81) \times (-3.3) = -25.9\ \text{kN/m}^2 \text{ or } \mathbf{-25.9\ kPa\ (gauge)\ (Ans.)}$$

Example 6.20. *The outlet at the bottom of a tank is so formed that velocity of water at point (see Fig. 6.21) is 2.2 times the mean velocity within the outlet pipe. What is the greatest length of pipe which may be used without producing cavitation? Neglect all other losses.*

Take atmospheric pressure = 96.24 kPa (abs.) and vapour pressure = 3.9 kPa (abs.)

Solution. *Given*: $V_A = 2.2\ V_2$; $p_1 = p_2 = p_{atm.} = 96.24$ kPa = 96.24 kN/m^2

Vapour pressure, $p_A = 3.9$ kPa = 3.9 kN/m^2 (abs.)

Applying Bernoulli's equation to points 1 and *A*, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_A}{w} + \frac{V_A^2}{2g} + z_A$$

$$\frac{96.24}{9.81} + 0 + 1.7 = \frac{3.9}{9.81} + \frac{V_A^2}{2 \times 9.81} + 0$$

∴ $V_A = 14.76$ m/s

$$V_2 = \frac{V_A}{2.2} = \frac{14.76}{2.2} = 6.71 \text{ m/s}$$

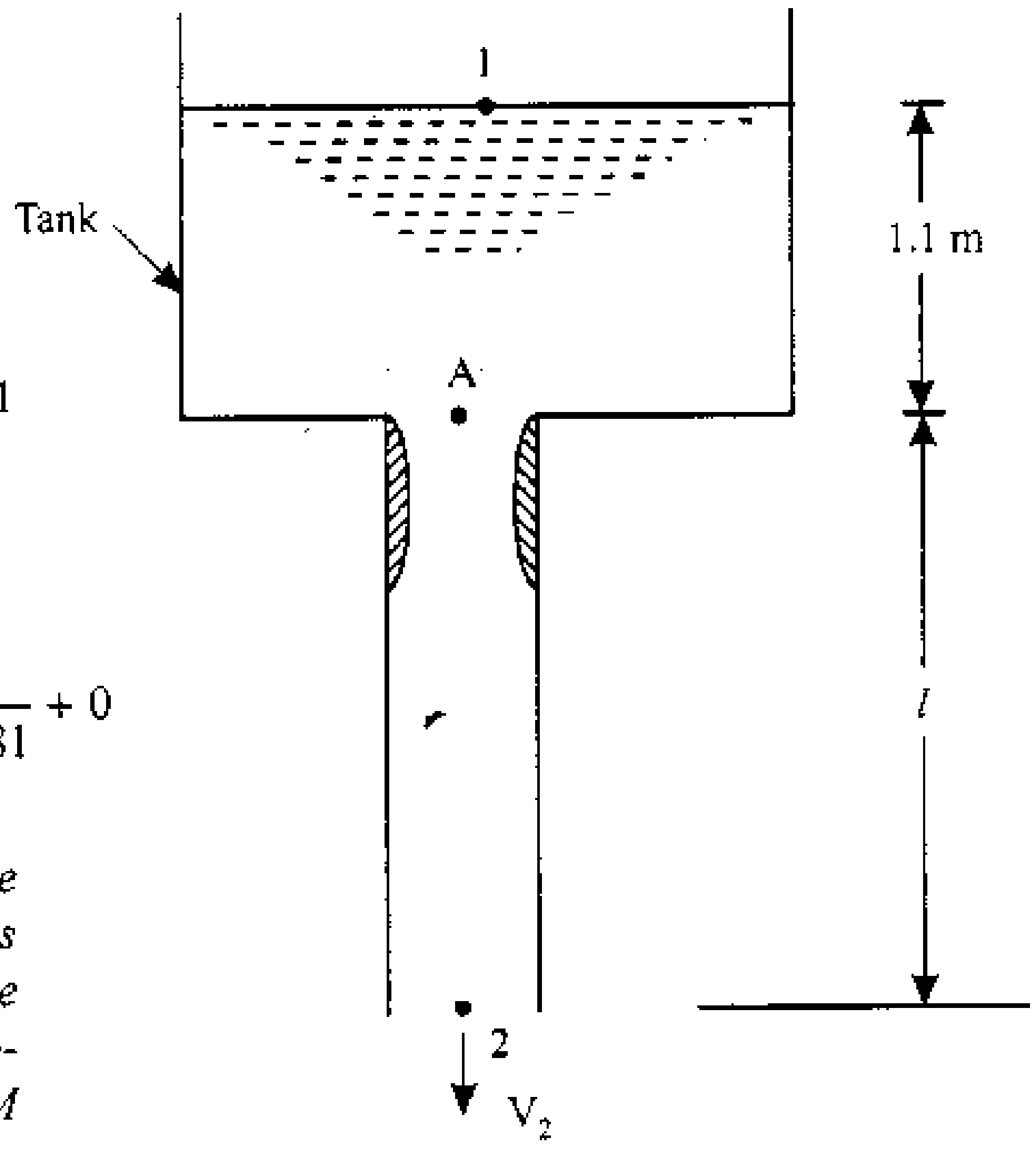

Fig. 6.21

Applying Bernoulli's equations to point 1

we get

$$+ \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{96.24}{9.81} + 0 + (l + 1.1) = \frac{96.24}{9.81} + \frac{6.71^2}{2 \times 9.81} + 0$$

∴ $l = 1.195$ m **(Ans.)**

ample 6.21. *A turbine has a supply line eter 45 and a tapering draft tube as in Fig. 6.22. When the flow in the pipe m³/s the pressure head at point L up- of the turbine is 35 m and at a point M aft tube, where the diameter is 65 cm, ssure head is – 4.1 m. Point M is 2.2 m e point L. Determine the power output of the turbine by assuming 92% efficiency.*

tion. $V_L = \frac{Q}{A_L} = \frac{0.6}{\frac{\pi}{4} \times (0.45)^2} = 3.77$ m/s

$$V_M = \frac{Q}{A_M} = \frac{0.6}{\frac{\pi}{4} \times (0.65)^2} = 1.81 \text{ m/s}$$

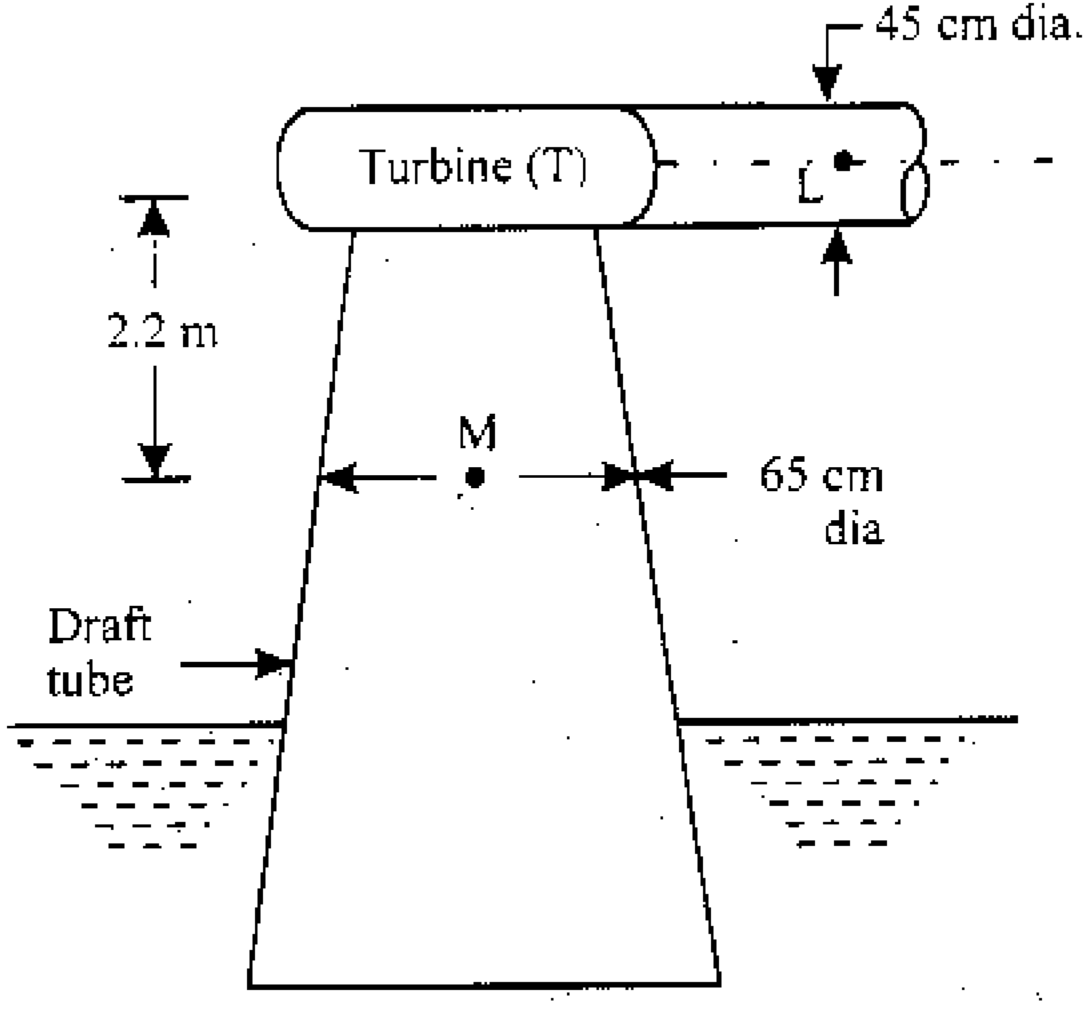

Fig. 6.22

plying Bernoulli's equation to points and M:

$$\frac{P_L}{w} + \frac{V_L^2}{2g} + z_L = \frac{P_M}{w} + \frac{V_M^2}{2g} + z_M + H_T$$

$$35 + \frac{(3.77)^2}{2 \times 9.81} + 2.2 = -4.1 + \frac{(1.81)^2}{2 \times 9.81} + O + H_T$$

$$\therefore \qquad H_T = 41.86 \text{ m}$$

Power output of the turbine,

$$P = wQH_T \times \eta$$

$$= 9.81 \times 0.6 \times 41.86 \times 0.92 = \mathbf{226.68\ kW\ (Ans.)}$$

Example 6.22. *Fig 6.23 shows a pipe connecting a reservoir to a turbine which discharg water to the tail race through another pipe. The head loss between the reservoir and the turbine is times the kinetic head in the pipe and that from the turbine to the tail race is 0.4 times the kinetic he in the pipe. The rate of flow is 1.2 m³/s and the pipe diameter in both cases is 1.1 m. Determine:*

(*i*) *The pressure at the inlet and exit of the turbine.*

(*ii*) *The power generated by the turbine.*

Solution. *Given*: $d = 1.1$ m; $Q = 1.2$ m³/s, $h_{f(1-2)} = 8 \times \frac{V^2}{2g}$; $h_{f(3-4)} = 0.4 \times \frac{V^2}{2g}$

(*i*) **The pressure at the inlet and exit of the turbine; p_2, p_3:**

Flow velocity in the pipe, $V = \dfrac{Q}{\frac{\pi}{4}d^2} = \dfrac{1.2}{\frac{\pi}{4} \times (1.1)^2} = 1.263$ m/s

Since the pipes before and after the turbine and of equal diameter,

$$V_2 = V_3 = 1.263 \text{ m/s}$$

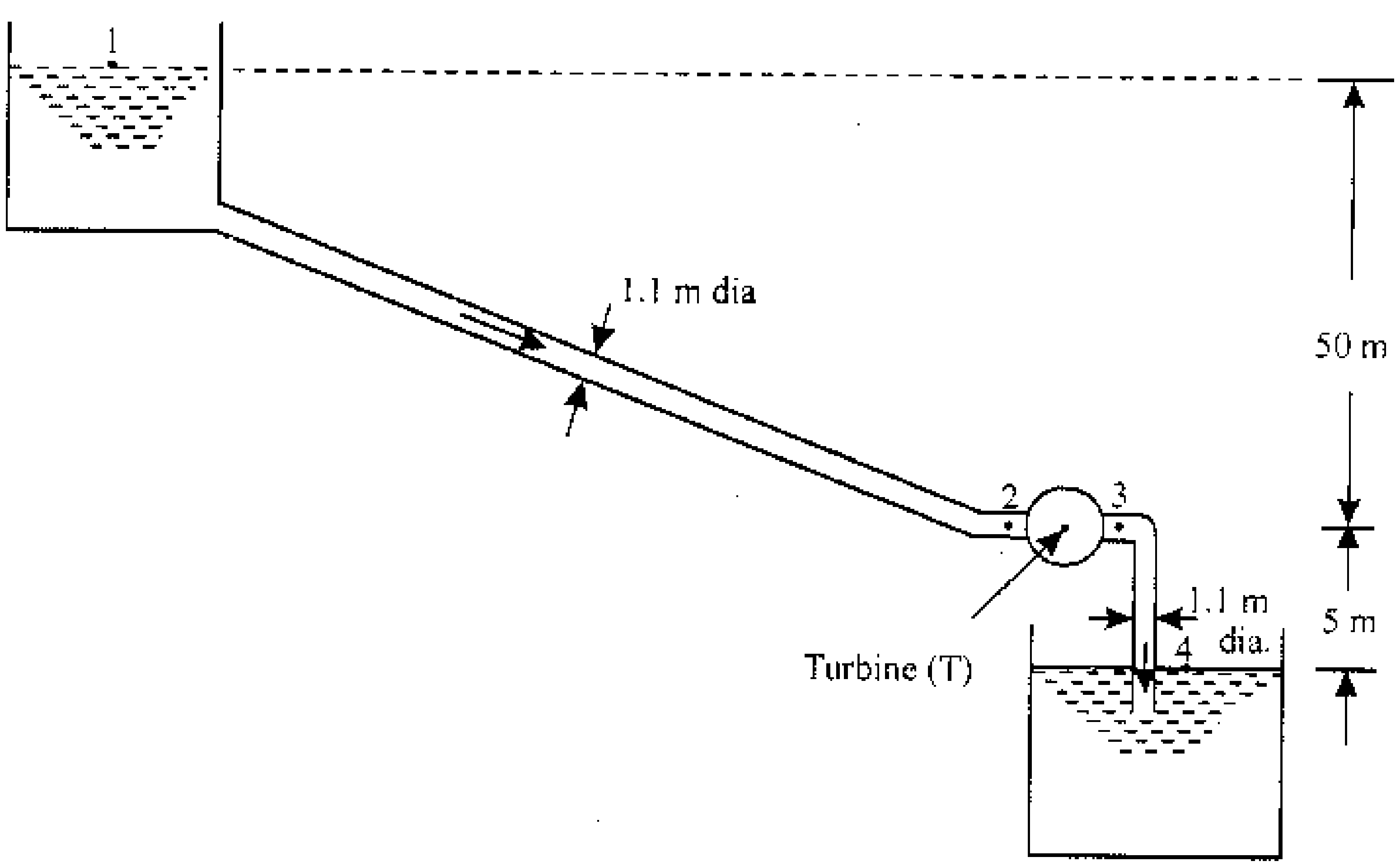

Fig. 6.23

Further, $V_1 = V_4 = 0$; $p_1 = p_4 = 0$ (atmospheric pressure)

Applying Bernoulli's equation between point 1 and 2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + 8 \times \frac{V_2^2}{2g}$$

$$0 + 0 + 50 = \frac{p_2}{w} + \frac{(1.263)^2}{2 \times 9.81} + 0 + 8 \times \frac{(1.263)^2}{2 \times 9.81}$$

or $\frac{p_2}{w}$ = **49.27 m of water = 483.34 kN/m² or kPa (Ans.)**

Applying Bernoulli's equation between point 3 and 4, we have

$$\frac{p_3}{w} + \frac{V_3^2}{2g} + z_3 = \frac{p_4}{w} + \frac{V_4^2}{2g} + z_4 + 0.4 \times \frac{V_3^2}{2g}$$

$$\frac{p_3}{w} + \frac{(1.263)^2}{2 \times 9.81} + 5 = 0 + 0 + 0 + 0.4 \times \frac{(1.263)^2}{2 \times 9.81}$$

or $\frac{p_3}{w}$ = **– 5.049 m of water = – 49.53 kN/m² or kPa (Ans.)**

The power generated by the turbine, P:

Applying Bernoulli's equation between points 2 and 3, we get

$$\frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{w} + \frac{V_3^2}{w} + z_3 + H_T$$

where, H_T = energy developed by the turbine per unit weight of liquid = Nm/N or m of liquid

$$49.27 = -5.049 + H_T \quad (\because V_2 = V_3 \text{ and } z_2 = z_3)$$

$$\therefore \quad H_T = 54.32 \text{ m of water.}$$

Hence, power generated by the turbine,

$$P = wQH_T = 9.81 \times 1.2 \times 54.32 = \mathbf{639.46 \text{ kW (Ans.)}}$$

mple 6.23. *In the Fig. 6.24 is shown a turbine with inlet pipe and a draft tube. If the y of turbine is 80 per cent and discharge is 1000 litres/s, find:*

The power developed by the turbine, and

The reading of the gauge G. **(Panjab University)**

tion. Efficiency of the turbine, η = 80%

harge through the turbine, Q = 1000 litres/sec = 1m³/s

eter of the inlet pipe = 0.4 m

Area of the inlet pipe,

$$A = \pi/4 \times 0.4^2 = 0.1257 \text{ m}^2$$

ocity of water through the pipe,

$$V = \frac{Q}{A} = \frac{1}{0.1257} = 7.96 \text{ m/s}$$

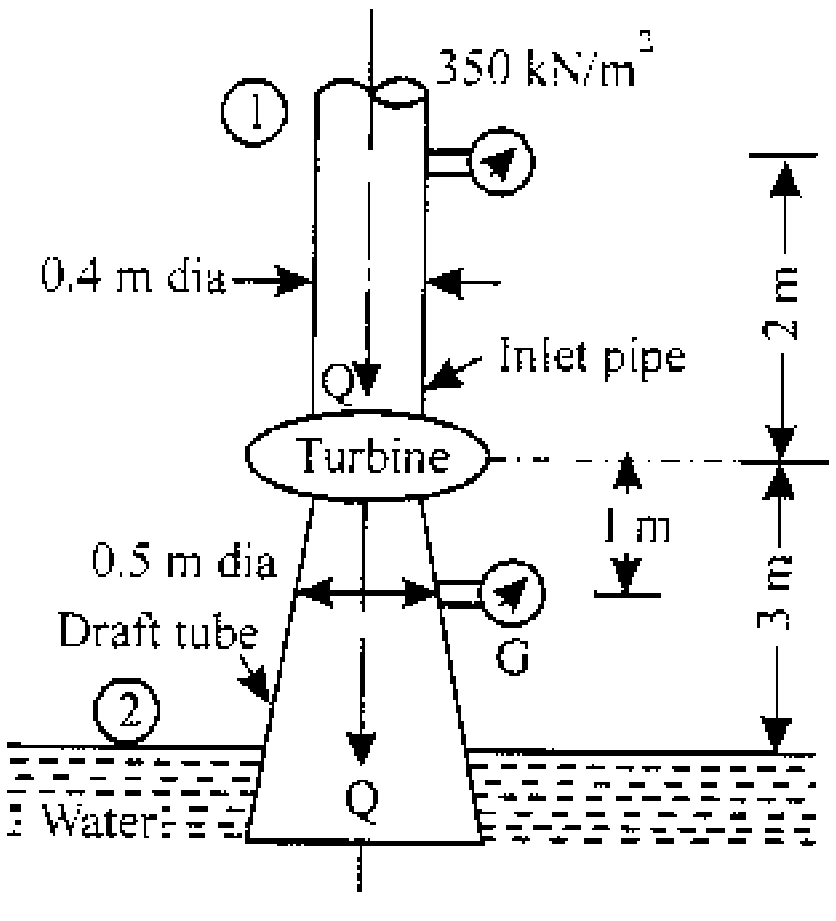

Fig. 6.24

Power developed by the turbine, P:

Applying Bernoulli's equation at 1 and 2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{losses} + H_T$$

here, H_T = Energy developed by the turbine per it weight of liquid = Nm/N or m of liquid

$$\frac{350}{9.81} + \frac{7.96^2}{2 \times 9.81} + 5 = 0 - 0 + 0 + H_T$$

(neglecting losses)

$$(\because V = V_1 = 7.96 \text{ m/s})$$

$$H_T = 35.68 + 3.23 + 5 = 43.91 \text{ m}$$

(where w = 9.81 kN/m³)

wer developed, $P = wQH_T \times \eta \; kW$ (where w = 9.81 kN/m³)

$$= 9.81 \times 1 \times 43.91 \times 0.8 = 3446 \text{ kW}$$

P = 344.6 kW (Ans.)

(*ii*) **Reading of the gauge G, p_G:**

Applying Bernoulli's equation at 1 and *G*, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_G}{w} + \frac{V_G^2}{2g} + z_G + H_T$$

$$\frac{350}{9.81} + \frac{7.96^2}{2 \times 9.81} + 3 = \frac{p_G}{w} + \frac{V_G^2}{2g} + 0 + 43.91 \qquad \ldots(i)$$

But, $$V_G = \frac{Q}{A_G} = \frac{1}{\pi/4 \times 0.5^2} = 5.09 \text{ m/s}$$

Substituting the value of V_G in (*i*) and rearranging, we get

$$\frac{p_G}{w} = \frac{350}{9.81} + \frac{7.96^2}{2 \times 9.81} + 3 - \frac{5.09^2}{2 \times 9.81} - 43.91$$

$$= 35.68 + 3.23 + 3 - 1.32 - 43.91 = -3.32 \text{ m of water}$$

$$\therefore \quad p_G = 9.81 \times (-3.32) = \mathbf{-32.57 \text{ kN/m}^2 \text{ (Ans.)}}$$

Example 6.24. *Fig. 6.25 shows a pump P pumping 72 litres/sec of water from a tank.*

(*i*) *What will be the pressures at points L and M when the pump delivers 12 kW of power to the flow? Assume the losses in the system to be negligible.*

(*ii*) *What will be the pressure at M when the loss in the inlet up to the pump is negligible and between the pump and the point M, a loss equal to 1.8 times the velocity head at B takes place.*

Solution. *Given*: Q = 72 litres/sec.

$= 0.072 \text{ m}^3\text{/s}$;

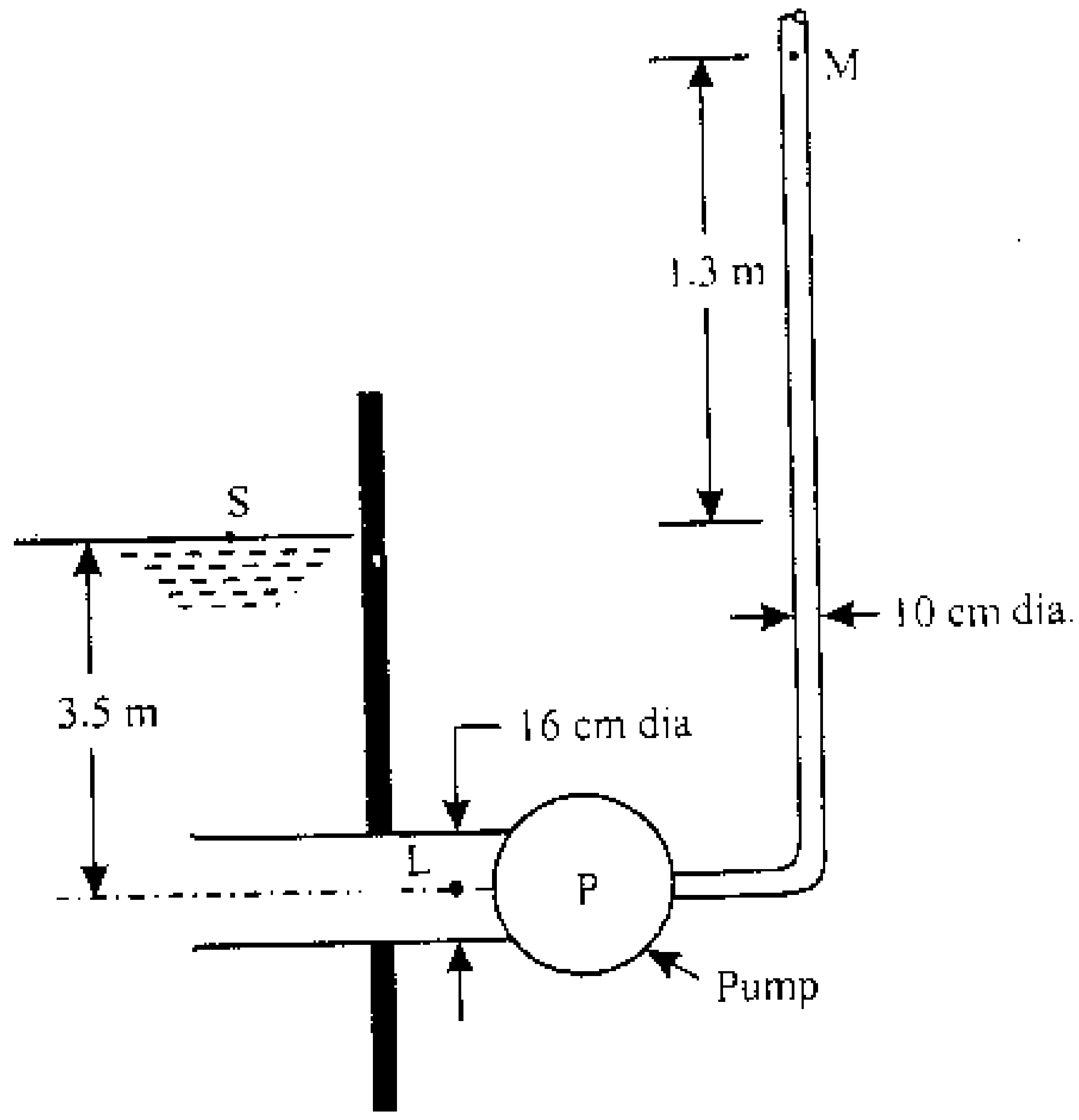

Fig. 6.25

Power delivered by the pump = 12 kW;

$$Q = 0.072 = A_L V_L = A_M V_M$$

Power delivere

or 9.81 × Q

∴ H_p = 16.9

(*i*) **Pressures**

Applying

or $\frac{p_L}{w}$ =

By applyin

or

(*ii*) **Pressure a**

By applying

losses, we

or,

or,

or,

Example 6.25.

e tank through a 8

6 cm steel pipe to

e feed tank. If the

rcent, determine:

(*i*) *Power ratin*

(*ii*) *Pressure de*

$$V_L = \frac{0.072}{A_L} = \frac{0.072}{\frac{\pi}{4} \times (0.16)^2} = 3.581 \text{ m/s}$$

$$V_M = \frac{0.072}{A_M} = \frac{0.072}{\frac{\pi}{4} \times (0.1)^2} = 9.167 \text{ m/s}$$

er delivered by the pump P

$$= wQH_p = 12 \text{ kW}$$

$$9.81 \times 0.072 \times H_p = 12$$

$H_p = 16.99$ m = Head delivered by the pump.

Pressures at point L and M:

Applying Bernoulli's equation to points S and L, we have,

$$\frac{p_S}{w} + \frac{V_S^2}{2g} + z_S = \frac{p_L}{w} + \frac{V_L^2}{2g} + z_L$$

$$0 + 0 - 3.5 = \frac{p_L}{w} + \frac{(3.581)^2}{2 \times 9.81} + 0$$

or $\frac{p_L}{w} = 2.846$ m or $p_L = 9.81 \times 2.846 = 27.92$ kN/m^2 or **27.92 kPa (Ans.)**

By applying Bernoulli's equation between S and L with level at L as datum, we get

$$\frac{p_S}{w} + \frac{V_S^2}{2g} + z_S + H_P = \frac{p_M}{w} + \frac{V_M^2}{2g} + z_M$$

$$0 + 0 + 3.5 - 16.99 = \frac{p_M}{w} + \frac{(9.167)^2}{2 \times 9.81} + (3.5 + 1.3)$$

or $\frac{p_M}{w} = 11.407$ m or $p_M = 9.81 \times 11.407 =$ **111.9 kN/m^2 or kPa (Ans.)**

Pressure at M when losses are considered:

By applying Bernoulli's equation between S and M with level at L as datum, *considering losses*, we have

$$\frac{p_S}{w} + \frac{V_S^2}{2g} + z_S + H_P = \frac{p_M}{w} + \frac{V_M^2}{2g} + z_M + \text{losses} \left(i.e., 1.8 \times \frac{V_M^2}{2g} \right)$$

or, $$0 + 0 + 3.5 + 16.99 = \frac{p_M}{w} + \frac{(9{\cdot}167)^2}{2 \times 9.81} + (3.5 + 1.3) + 1{\cdot}8 \times \frac{(9{\cdot}167)^2}{2 \times 9{\cdot}81}$$

or, $$3.5 + 16.99 = \frac{p_M}{w} + 16.79$$

or, $\frac{p_M}{w} = 3.7$ m or $p_M = 9.81 \times 3.7 =$ **36.3 kN/m^2 or kPa (Ans.)**

mple 6.25. *Fig. 6.26 shows a pump drawing a solution (specific gravity =1.8) from a stor- through a 8 cm steel pipe in which the flow velocity is 0.9 m/s. The pump discharges through el pipe to an overhead tank, the end of discharge is 12 m above the level of the solution in ank. If the friction losses in the entire piping system are 5.5 m and pump efficiency is 65 determine:*

Power rating of the pump.

Pressure developed by the pump.

Solution. *Given*: $d_2 = 8$ cm or 0.08 m; $d_3 = 6$ cm or 0.06 m; $V_2 = 0.9$ m/s, $\eta_{pump} = 65\%$

(*i*) **Power rating of the pump:**

From continuity equation, we have

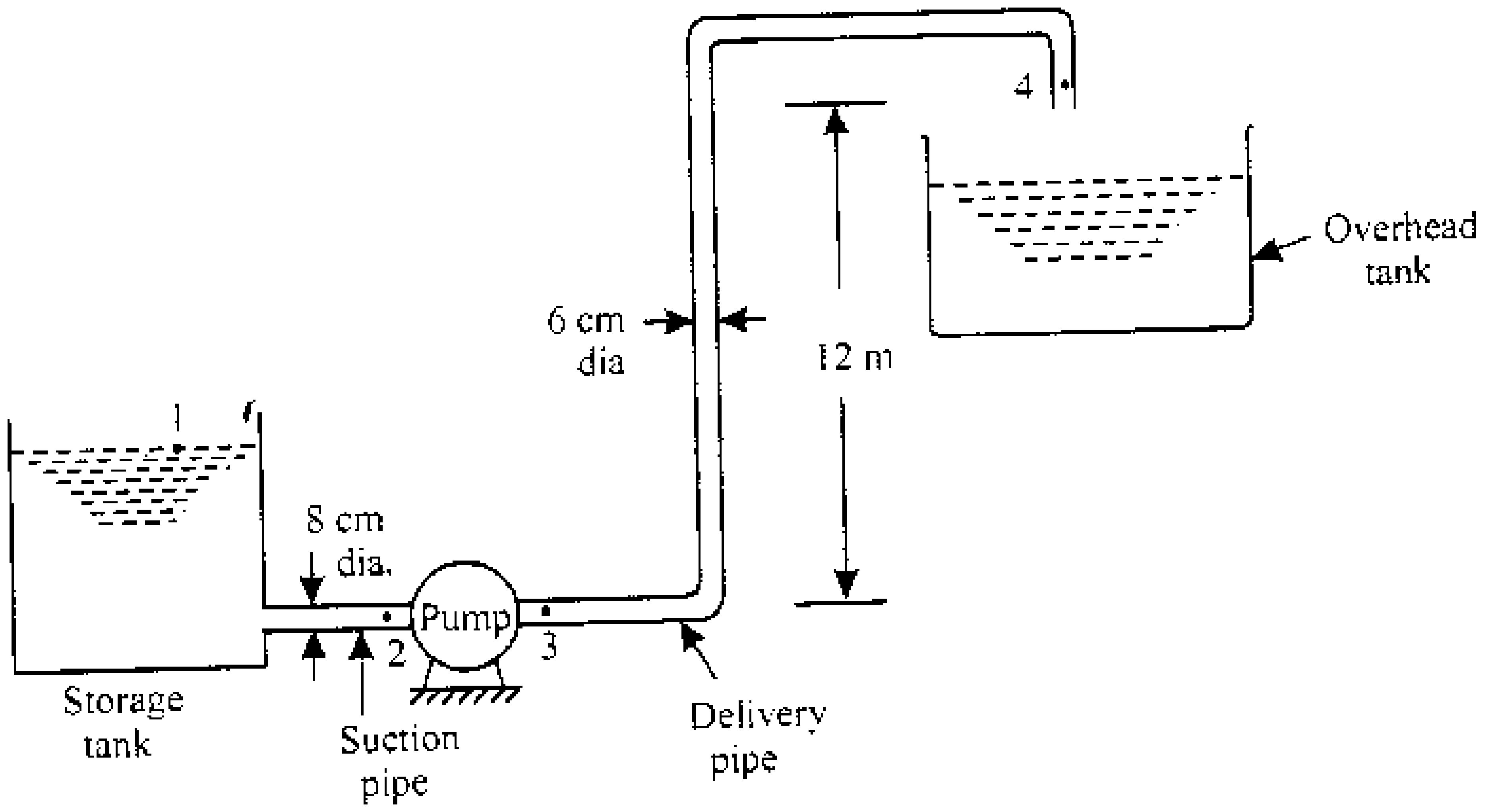

Fig. 6.26

$$A_2V_2 = A_3V_3$$

or $$V_3(=V_4) = \frac{A_2V_2}{A_3} = \frac{\frac{\pi}{4}\times(0.08)^2\times 0.9}{\frac{\pi}{4}\times(0.06)^2} = 1.6 \text{ m/s}$$

Applying Bernoulli's equation between points 1 and 4, we get

$$\frac{p_1}{w}+\frac{V_1^2}{2g}+z_1+H_P = \frac{p_4}{w}+\frac{V_4^2}{2g}+z_4+\text{Losses}$$

(where H_P = Energy added by the pump per unit weight of liquid in Nm/N or m of the liqu pumped)

$$0+0+0+H_P = 0+\frac{(1.6)^2}{2\times 9.81}+12+5.5$$

or $$H_P = 17.63 \text{ m of liquid}$$

$$\therefore \text{ Power rating of the pump} = \frac{wQH_P}{\eta_{pump}} = \frac{(9.81\times 1.8)\times\left(\frac{\pi}{4}\times 0.08^2\times 0.9\right)\times 17.63}{0.65}$$

= **2.167 kW (Ans.)**

(*ii*) **Pressure developed by the pump, $(p_3 - p_2)$:**

Applying Bernoulli's equation between points 2 and 3, we have

$$\frac{p_2}{w}+\frac{V_2^2}{2g}+z_2+H_P = \frac{p_3}{w}+\frac{V_3^2}{2g}+z_3$$

$$\left(\frac{p_3-p_2}{w}\right) = \left(\frac{V_2^2-V_3^2}{2g}\right)+H_P \qquad \ldots(\because z_2 =$$

$$= \frac{(0.9)^2-(1.6)^2}{2\times 9.81}+17.63 = 17.54 \text{ m}$$

or $p_3 - p_2 = 17.54\times(9.81\times 1.8) =$ **309.72 kN/m² or kPa (Ans.)**

Example
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(*ii*) **The el**

Also,

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Hence.

Example 6.26. *A pump is 2.2 m above the water level in the sump and has a pressure of –20 cm ercury at the suction side. The suction pipe is of 20 cm diameter and the delivery pipe is short cm diameter pipe ending in a nozzle of 8 cm diameter. If the nozzle is directed vertically up-ds at an elevation of 4.2 m above the water sump level. Determine:*

(*i*) *The discharge.*

(*ii*) *The power input into the flow by the pump.*

(*iii*) *The elevation, above the water sump level, to which the jet would reach.*

Neglect all losses.

Solution. (*i*) **The discharge, Q:**

Applying Bernoulli's equation, to points 1 and 2 (Fig 6.27), we get

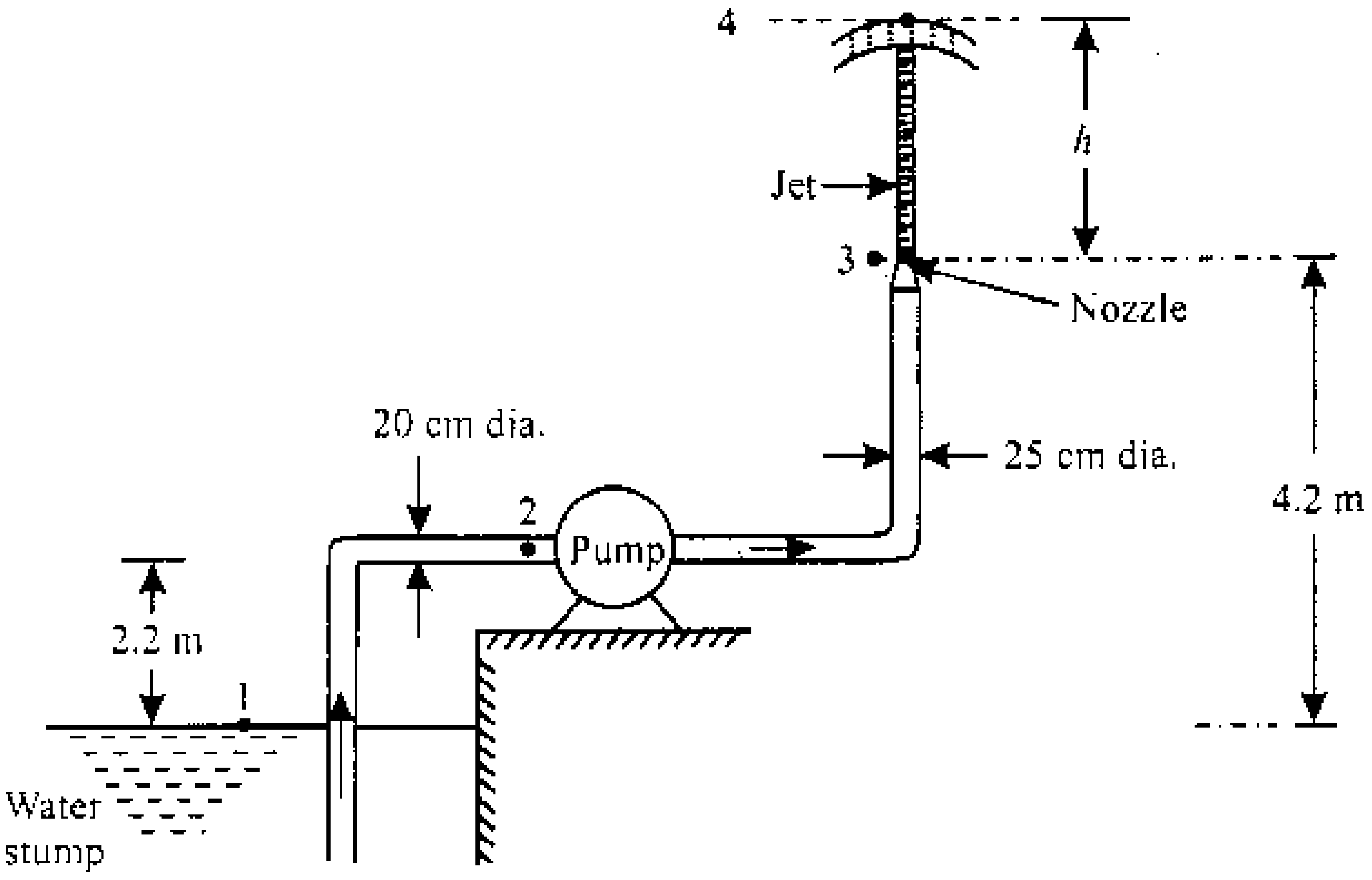

Fig. 6.27

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$0 + 0 + 0 = (-0.2 \times 13.6) + \frac{V_2^2}{2g} + 2.2$$

or $\qquad V_2 = 3.194$ m/s

Discharge, $\qquad Q = \frac{\pi}{4} \times 0.21^2 \times 3.194 = \mathbf{0.1\ m^3/s}$ **(Ans.)**

The elevation, to which the jet will reach, h:

Also, $\qquad Q = A_2V_2 = A_3V_3$

$\therefore \qquad \frac{\pi}{4} \times (0.2)^2 \times 3.194 = \frac{\pi}{4} \times (0.08)^2 \times V_3$

or $\qquad V_3 = 19.962$ m/s

or $\qquad \frac{V_3^2}{2g} = \frac{(19.962)^2}{2 \times 9.81} = 20.31$ m

Hence, the height to which the jet will reach, h = **20.31 m (Ans.)**

(*iii*) **The power input to the flow by the pump, P:**

The elevation of point 4, the summit of the jet, is

$$= 4.2 + 20.31 = 24.51 \text{ m}$$

Applying Bernoulli's equation to points 1 and 3, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 + H_P = \frac{p_3}{w} + \frac{V_3^2}{2g} + z_3$$

$$0 + 0 + 0 + H_P = 0 + 20.31 + 4.2$$

or $$H_P = 24.51 \text{m}$$

Power delivered by the pump, $p = wQH_p$

$$= 9.81 \times 0.1 \times 24.51 = \mathbf{24.04 \text{ kW}}$$ **(Ans.)**

Example 6.27. *Fig 6.28 shows a pump employed for lifting water from a sump. If it is requ to pump 60 litres/sec of water through a 0.1 m diameter pipe from the sump to a point 10 m ab determine the power required. Also determine pressure intensities at L and M.* **(Delhi Univers**

Assume an overall efficiency of 70 percent.

Solution. Quantity of water to be pumped , Q = 60 litres/sec.

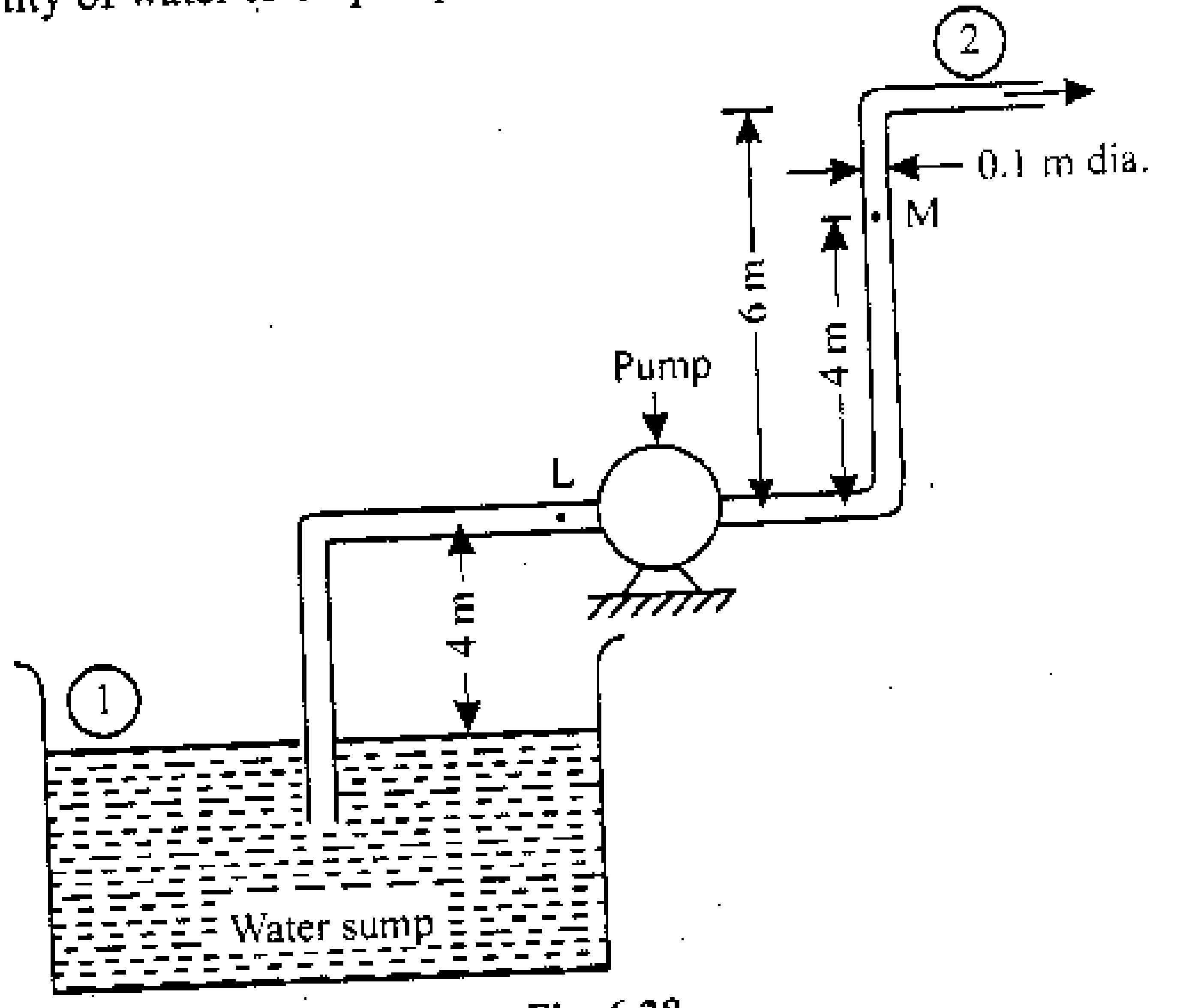

Fig. 6.28

$$= \frac{60}{1000} = 0.06 \text{ m}^3/\text{s}$$

Dia of the pipe, $d = 0.1$ m

∴ Area of the pipe, $A = \pi/4 \times 0.1^2 = 0.00785 \text{ m}^2$

Overall efficiency, $\eta_0 = 70\%$

Power required, P:

As per continuity equation,

$$Q = AV$$

[where, V = velocity of water in the pipe]

∴ $$0.06 = 0.00785 \text{ V}$$

(*iii*) **The power input to the flow by the pump, P:**

The elevation of point 4, the summit of the jet, is

$$= 4.2 + 20.31 = 24.51 \text{ m}$$

Applying Bernoulli's equation to points 1 and 3, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 + H_P = \frac{p_3}{w} + \frac{V_3^2}{2g} + z_3$$

$$0 + 0 + 0 + H_P = 0 + 20.31 + 4.2$$

or $$H_P = 24.51 \text{m}$$

Power delivered by the pump, $p = wQH_p$

$$= 9.81 \times 0.1 \times 24.51 = \mathbf{24.04 \ kW \ (Ans.)}$$

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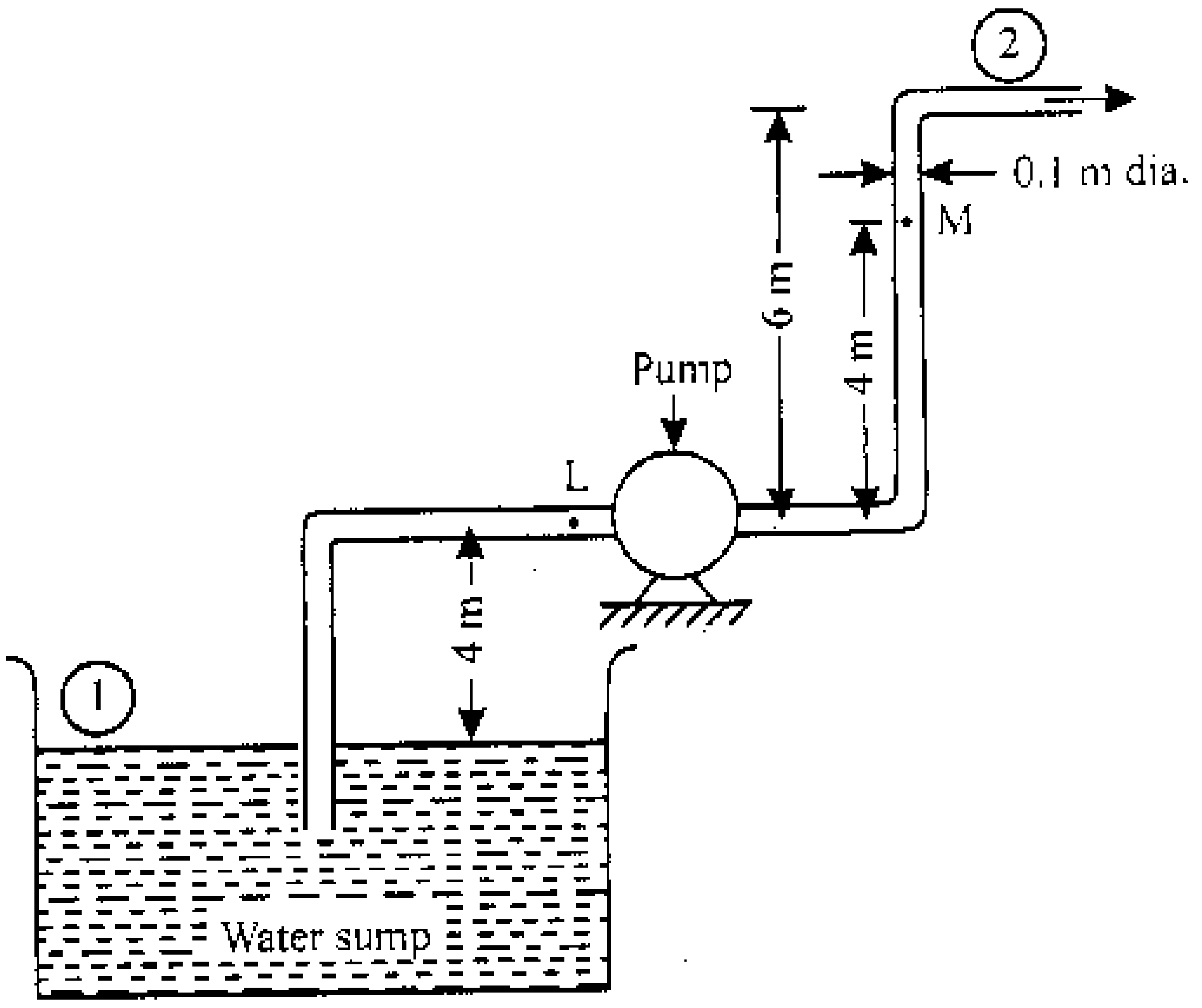


Fig. 6.28

$$= \frac{60}{1000} = 0.06 \text{ m}^3/\text{s}$$

Dia of the pipe, $d = 0.1$ m

∴ Area of the pipe,

$$A = \pi/4 \times 0.1^2 = 0.00785 \text{ m}^2$$

Overall efficiency,

$$\eta_0 = 70\%$$

Power required, P:

As per continuity equation,

$$Q = AV$$

[where, V = velocity of water in the pipe]

∴ $$0.06 = 0.00785 \text{ V}$$

or $$V = \frac{0.06}{0.00785} = 7.64 \text{ m/s}$$

Applying Bernoulli's equation at 1 points and 2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 + H_p = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

(where H_p = energy added by the pump per unit weight of liquid in Nm/N or m of the liquid ped)

$$0 + 0 + 0 + H_p = 0 + \frac{(7.64)^2}{2 \times 9.81} + 10 \qquad (\because V = V_2 = 7.64 \text{ m/s})$$

$\therefore$ $H_p = 12.97$ m of water

$\therefore$ Power required to run the pump,

$$P = \frac{wQH_P}{\eta_0} = \frac{9.81 \times 0.06 \times 12.97}{0.7} \text{ kW} \qquad (\because w = 9.81 \text{ kN/m}^3)$$

i.e., $\mathbf{P = 10.9}$ **kW (Ans.)**

Pressure intensities at *L* and *M*:

Applying Bernoulli's equation at 1 and *L*, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_L}{w} + \frac{V_2^2}{2g} + z_2$$

$$0 + 0 + 0 = \frac{p_L}{w} + \frac{7.64^2}{2 \times 9.81} + 4$$

$$\therefore \quad \frac{p_L}{w} = -\frac{7.64^2}{2 \times 9.81} - 4 = -6.97 \text{ m} \qquad (\because V_L = V_2 = 7.64 \text{ m/s})$$

$$P_L = 9.81 \times (-6.97) = -68.4 \text{ kN/m}^2 \textbf{ (Ans.)}$$

pplying Bernoulli's equation at *l* and *M*, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 + H_P = \frac{p_M}{w} + \frac{V_M^2}{2g} + z_M$$

$$0 + 0 + 0 + 12.97 = \frac{p_M}{w} + \frac{7.64^2}{2 \times 9.81} + 8$$

$$\therefore \quad \frac{p_M}{w} = 12.97 - \frac{7.64^2}{2 \times 9.81} - 8 = 12.97 - 2.97 - 8 = 2 \text{ m}$$

$$p_M = 9.81 \times 2 = \mathbf{19.62} \textbf{ kN/m}^2 \textbf{ (Ans.)}$$

Practical Applications of Bernoulli's Equation

Although Bernoulli's equation is applicable in all problems of incompressible flow where there lvement of energy considerations but here we shall discuss its applications in the following *ring devices:*

1. Venturimeter	2. Orificemeter
3. Rotameter and elbow meter	4. Pitot tube.

6.6.1. Venturimeter

A venturimeter is one of the most important practical applications of Bernoulli's theorem. *It is an ment used to measure the rate of discharge in a pipeline and is often fixed permanently at sections of the pipeline to know the discharges there.*

A venturimeter has been named after the 18th century Italian *engineer Venturi.*

Types of venturimeters:

Venturimeters may be *classified* as follows:

1. Horizontal venturimeters
2. Vertical venturimeters
3. Inclined venturimeters.

6.6.1.1. Horizontal venturimeters

A venturimeter consists of the following *three* parts:

(*i*) A short converging part,

(*ii*) Throat, and

(*iii*) Diverging part.

Expression for rate of flow:

Fig 6.29 shows a venturimeter fitted in horizontal pipe through which a fluid is flowing.

Let, D_1 = Diameter at inlet or at section 1,

A_1 = Area at inlet $\left(= \frac{\pi}{4} d_1^2\right)$

p_1 = Pressure at section 1,

V_1 = Velocity of fluid at section 1,

and D_2, A_2, p_2, and V_2 are the corresponding values at section 2.

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \quad \text{...(i)}$$

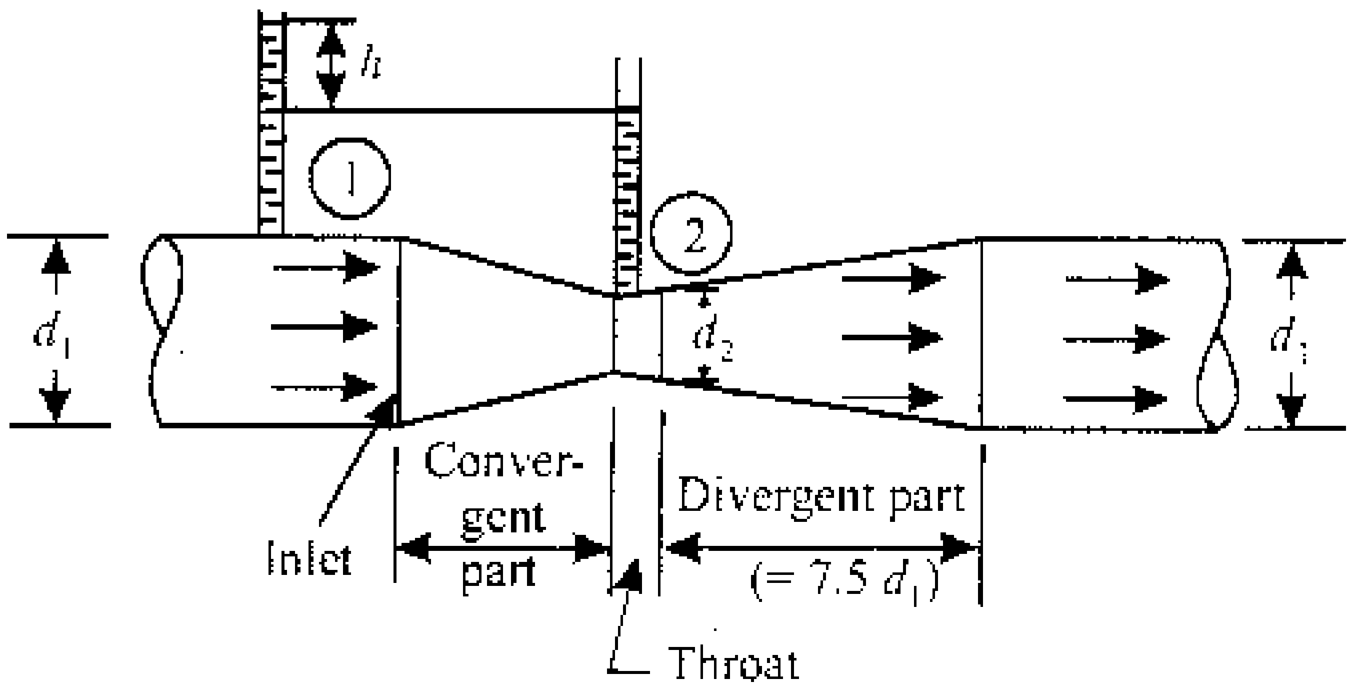
Here, $z_1 = z_2$ ... since the pipe is horizontal.

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

or,

$$\frac{p_1 - p_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{...(ii)}$$

But, $\frac{p_1 - p_2}{w}$ = difference of pressure heads at sections 1 and 2 and is equal to h.



$\left(\text{Throat ratio } \frac{d_2}{d_1} \text{ varies } \frac{1}{4} \text{ to } \frac{3}{4}\right)$

Fig. 6.29. Venturimeter.

i.e.,
$$\frac{p_1 - p_2}{w} = h$$

Substituting this value of $\frac{p_1 - p_2}{w}$ in eqn. (*ii*), we get

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad \text{...(iii)}$$

Applying continuity equation at sections 1 and 2, we have

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_1 = \frac{A_2 V_2}{A_1}$$

Substituting the value of V_1 in eqn. (*iii*) we get

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{A_2 V_2}{A_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right)$$

$$h = \frac{V_2^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_1^2}\right) \quad \text{or} \quad V_2^2 = 2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)$$

$$V_2 = \sqrt{2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)} = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Discharge,
$$Q = A_2 V_2 = A_2 \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \qquad \text{...(6.5)}$$

$$Q = C\sqrt{h}$$

ere. C = constant of venturimeter

$$= \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g}$$

(6.5) gives the discharge under ideal conditions and is called *theoretical discharge.* Actual er (Q_{act}) which is less than the theoretical discharge ($Q_{th.}$) is given by:

$$Q_{act.} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \qquad \text{...(6.6)}$$

ere, C_d = *Co-efficient of venturimeter* (or co-efficient of discharge) and its value is *less than* varies between 0.96 and 0.98)

Due to variation of C_d venturimeters are not suitable for *very low velocities.*

Value of 'h' by differential U-tube manometer:

Case. I. *Differential manometer containing a liquid heavier than the liquid flowing through pipe*

Let,
S_{hl} = Sp. gravity of heavier liquid,
S_p = Sp. gravity of liquid flowing through pipe, and
y = Difference of the heavier liquid column in U-tube

Then $$h = y\left[\frac{S_{hl}}{S_p} - 1\right] \qquad \text{...(6.7)}$$

Case. II. *Differential manometer containing a liquid lighter than the liquid flowing through the pipe.*

Let, S_{ll} = sp. gravity of lighter liquid,

S_p = sp. gravity of liquid flowing through pipe, and

y = difference of lighter liquid column in U-tube.

Then, $$h = y\left[1 - \frac{S_{ll}}{S_p}\right] \qquad \text{...(6.8)}$$

Example 6.28. *A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is used to measure the flow of water. The pressure at inlet is 0.18 N/mm² and the vacuum pressure at the throat is 280 mm of mercury. Find the rate of flow. The value of C_d may be taken as 0.98.*

Solution. Inlet diameter of venturimeter, D_1 = 200 mm = 0.2 m

$\therefore$ Area of inlet, $A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Throat diameter, D_2 = 100 mm = 0.1 m

$\therefore$ Area of throat, $A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Pressure at inlet, $p_1 = 0.18 \text{ N/mm}^2 = 180 \text{ kN/m}^2$

$\therefore$ $\frac{p_1}{w} = \frac{180}{9.81} = 18.3 \text{ m}$

Vacuum pressure at the throat,

$$\frac{p_2}{w} = -280 \text{ mm of mercury}$$
$$= -0.28 \text{ m of mercury} = -0.28 \times 13.6 = -3.8 \text{ m of water}$$

Co-efficient of discharge, $C_d = 0.98$

$\therefore$ Differential head, $h = \frac{p_1}{w} - \frac{p_2}{w} = 18.3 - (-3.8) = 22.1 \text{ m}$

Rate of flow, Q:

Using the relation,

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}, \text{ we have}$$

$$= 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - (0.00785)^2}} \times \sqrt{2 \times 9.81 \times 22.1}$$

$$= \frac{0.000241}{0.0304} \times 20.82$$

or Q = **0.165 m³/s (Ans.)**

Example 6.29. *A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is employed to measure the flow of water. The reading of the differential manometer connected to the inlet is 180 mm of mercury. If the co-efficient of discharge is 0.98 determine the rate of flow.*

Solution. Inlet diameter of venturimeter, D_1 = 200 mm = 0.2 m

$\therefore$ Area at inlet $A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Throat diameter, D_2 = 100 mm = 0.1 m

$\therefore$ Area of throat $\quad A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Reading of differential manometer, y = 180 mm (= 0.18 m) of mercury

Co-efficient of discharge, C_d = 0.98

Rate of flow, Q:

To find difference of pressure head (h) using the relation,

$$h = \left[\frac{S_{hl}}{S_p} - 1\right], \text{ we have}$$

where, $\quad S_{hl}$ = Sp.gr. of mercury (heavy liquid) = 13.6, and

S_p = Sp. gr. of liquid through the pipe *i.e.*, water = 1

$$h = 0.18\left[\frac{13.6}{1} - 1\right] = 2.268 \text{ m}$$

To find Q, using the relation,

$$Q = C_d\,\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}\text{ , we get}$$

$$Q = 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - (0.00785)^2}} \times \sqrt{2 \times 9.81 \times 2.268}$$

or $\quad Q = \frac{0.000241}{0.0304} \times 6.67 = \mathbf{0.0528\ m^3/s\ (Ans.)}$

Example 6.30. *A horizontal venturimeter with inlet and throat diameters 300 mm and 100 mm respectively is used to measure the flow of water. The pressure intensity at inlet is 130 kN/m² while the vacuum pressure head at the throat is 350 mm of mercury. Assuming that 3 per cent of head is lost in between the inlet and throat, find:*

(*i*) *The value of C_d (co-efficient of discharge) for the venturimeter, and*

(*ii*) *Rate of flow.*

Solution. Inlet diameter of the venturimeter, D_1 = 300 mm = 0.3 m

$\therefore$ Area at inlet. $\quad A_1 = \frac{\pi}{4} \times 0.3^2 = 0.07 \text{ m}^2$

Throat diameter, $\quad D_2$ = 100 = 0.1

$\therefore$ Area of throat, $\quad A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Pressure at inlet, $\quad p_1 = 130 \text{ kN/m}^2$

$\therefore$ Pressure head, $\quad \frac{p_1}{w} = \frac{130}{9.81} = 13.25 \text{ m}$

Similarly, pressure head at throat,

$$\frac{p_2}{w} = -\ 350 \text{ mm of mercury} = -\ 0.35 \times 13.6 \text{ m of water} = -\ 4.76 \text{ m}$$

(*i*) **Co-efficient of discharge, C_d:**

Differential head, $\quad h = \frac{p_1}{w} - \frac{p_2}{w} = 13.25 - (-\ 4.76) = 18.01 \text{ m}$

Head lost, $h_f = 3\%$ of $h = \frac{3}{100} \times 18.01 = 0.54$ m

$$\therefore \quad C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{18.01 - 0.54}{18.01}} = 0.985$$

(*ii*) **Rate of flow, Q:**

Using the relation,

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}\text{, we have}$$

$$Q = 0.985 \times \frac{0.07 \times 0.00785}{\sqrt{0.07^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 18.01}$$

$$= \frac{0.000541}{0.0956} \times 18.79 = \mathbf{0.146\ m^3/s\ (Ans.)}$$

Example 6.31. *A venturimeter (throat diameter =* 10.5 *cm) is fitted to a water pipeline (internal diameter =* 21.0 *cm) in order to monitor flow rate. To improve accuracy of measurement, pressure difference across the venturimeter is measured with the help of an inclined tube manometer, the angle of inclination being* 30° *(Fig.* 6.30*). For a manometer reading of* 9.5 *cm of mercury, find the flow rate. Discharge co-efficient of venturimeter is* 0.984.

(GATE)

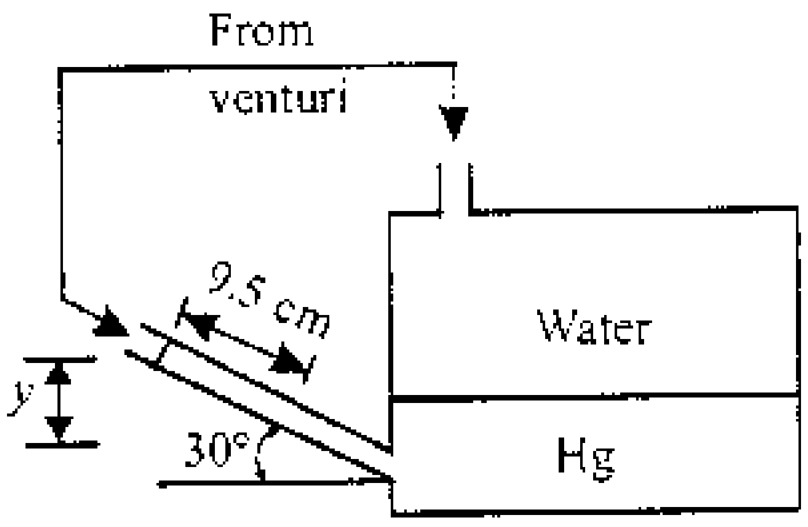


Fig. 6.30

Solution. Internal dia., $D_1 = 21.0$ cm $= 0.21$ m;

Area of inlet, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.21)^2 = 0.0346\ \text{m}^2$

Throat dia, $D_2 = 10.5$ cm $= 0.105$ m

$\therefore$ Area at throat, $A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times (0.105)^2 = 0.00866\ \text{m}^2$

Discharge co-efficient of venturimeter, $C_d = 0.984$

Pressure head, $h = y\left[\frac{S_{Hg}}{S_{water}} - 1\right] = (9.5 \sin 30°)\left[\frac{13.6}{1} - 1\right]$

$= 59.85$ cm $= 0.5985$ m

Discharge (Q) through a venturi-meter is given by:

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$$= 0.984 \times \frac{0.0346 \times 0.00866}{\sqrt{(0.0346)^2 - (0.00866)^2}} \times \sqrt{2 \times 9.81 \times 0.5985}$$

$$= 0.984 \times 0.008945 \times 3.427 = \mathbf{0.0302\ m^3/s\ (Ans.)}$$

Example 6.32. *Water at the rate of* 30 *litres/sec is flowing through a* 0.2 *m. I.D. pipe. A venturimeter of throat diameter* 0.1 *m is fitted in the pipeline. A differential manometer in the pipeline has an indicator liquid M and the manometer reading is* 1.16 *m. What is the relative density of the manometer liquid M? Venturi co-efficient =* 0.96; *density of water =* 998 *kg/m*^{3}.

(AMIE Summer, 200

Solution. *Given:* $Q = 30$ litres/sec $= 30 \times 10^{-3}\ \text{m}^3/\text{s} = 0.03\ \text{m}^3/\text{s} = D_1 = 0.2$ m; $D_2 = 0.1$ m; $C_d = 0.96$, $\rho_w = 998\ \text{kg/m}^3$; $y = 1.16$ m

Assume venturimeter to be *horizontal.* The flow rate is given by,

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \qquad \ldots(i)$$

Here, $A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.03141\ \text{m}^2$, and

$$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.1^2 = 0.007854\ \text{m}^2$$

Substituting the various values in (i), we get

$$0.03 = 0.96 \times \frac{0.03141 \times 0.007854}{\sqrt{0.03141^2 - 0.007854^2}} \times \sqrt{2 \times 9.81} \times \sqrt{h}$$

or $$0.03 = 0.96 \times 0.008112 \times 4.43 \times \sqrt{h}$$

or $$h = \left(\frac{0.03}{0.96 \times 0.008112 \times 4.43}\right)^2 = 0.756\ \text{m}$$

Also, $$h = y\left(\frac{S_{hl}}{S_{ll}} - 1\right) \qquad \text{[Eqn. (6.7)]}$$

$$0.756 = 1.16\left(\frac{S_{hl}}{0.998} - 1\right)$$

$$\therefore \quad S_{hl} = \left(\frac{0.756}{1.16} + 1\right) \times 0.998 = 1.648$$

Hence specific gravity/relative density of the manometer fluid $M =$ **1.648 (Ans.)**

Example 6.33. *A venturimeter is installed in a pipeline carrying water and is 30 cm in diameter. The throat diameter is 12.5 cm. The pressure in pipeline is 140 kN/m², and the vacuum in the throat is 37.5 cm of mercury. Four percent of the differential head is lost between the gauges. Working from first principles find the flow rate in the pipeline in l/s assuming the venturimeter to be horizontal.*

(AMIE Summer, 2000)

Solution. Refer to Fig. 6.29. *Given:* $D_1 = 30$ cm $= 0.3$ m; $D_2 = 12.\ 5$ cm; $= 0.125$ m; $p_1 = 140$ kN/m² $p_2 = -\ 37.5$ cm of mercury

$$= -\frac{37.5 \times 13.6}{100} = -\ 5.1 \text{ m of water; } h_f = 4\% \text{ of differential head.}$$

Flow rate in pipeline, Q:

$$\frac{p_1}{w} = \frac{140 \times 10^3}{9810} = 14.27 \text{ m of water}$$

$$\frac{p_2}{w} = -\ 5.1 \text{ m of water (Calculated above)}$$

$$h_f = 4\% \text{ of differential head}$$

$$= \frac{4}{100}\left(\frac{p_1}{w} - \frac{p_2}{w}\right) = \frac{4}{100}\ [(14.27 - (-\ 5.1)] = 0.775 \text{ m of water.}$$

Applying Bernoulli's equation to the entrance (1) and throat (2) of the venturimeter, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

or, $$\frac{V_1^2 - V_2^2}{2g} = \left(\frac{p_2}{w} - \frac{p_1}{w}\right) + h_f \qquad (\because z_1 = z_2)$$

or, $$\frac{V_1^2 - V_2^2}{2g} = -5.1 - 14.27 + 0.775 = -18.59$$

$$\frac{V_1^2}{2g}\left[1 - \left(\frac{V_2}{V_1}\right)^2\right] = -18.59$$

Also, $$A_1V_1 = A_2V_2$$

or, $$\frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{\frac{\pi}{4}D_1^2}{\frac{\pi}{4}D_2^2}\right) = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{0.3}{0.125}\right)^2 = 5.76$$

or, $$\frac{V_1^2}{2}[1 - (5.76)^2] = -18.59$$

or, $$\frac{V_1^2}{2} \times (-32.18) = 2 \times 9.81 \times (-18.59)$$

or, $$V_1 = \left(\frac{2 \times 9.81 \times 18.59}{32.18}\right)^{1/2} = 3.367 \text{ m/s}$$

Hence discharge, $$Q = A_1V_1 = \frac{\pi}{4} \times (0.3)^2 \times 3.367 \times 10^3 \ l/s$$

$$\simeq 238 \ l/s \ \textbf{(Ans.)}$$

6.6.1.2. Vertical and inclined venturimeters

Vertical or inclined venturimeters are employed for measuring discharge on pipelines which are *not horizontal*. The same formula for discharge as used for horizontal venturimeter holds good in these cases as well.

Here, $$h = \left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2)$$

[In horizontal venturimeters $z_1 - z_2 = 0$ as $z_1 = z_2$]

Vertical Venlurimeters

Example 6.34. *A 200 mm × 100 mm venturimeter is provided in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 220 mm. Find the rate of flow. Assume $C_d = 0.98$.*

Solution. Diameter at the inlet, $D_1 = 200$ mm = 0.2 m

∴ Area of inlet, $A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$

Diameter at the throat, $D_2 = 100$ mm = 0.1 m

∴ Area at the throat, $A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Sp. gravity of heavy liquid (in the manometer), $S_{hl} = 13.6$

Sp. gravity of liquid flowing through pipe, $S_p = 1.0$

Co-efficient of discharge, $C_d = 0.98$

Reading of the differential manometer, $y = 220$ mm = 0.22 m

Rate of flow, Q:

Differential head,

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = y\left|\frac{S_{hl}}{S_p} - 1\right|$$

$$= 0.22\left(\frac{13.6}{1.0} - 1.0\right) = 2.77 \text{ m}$$

Using the relation, $Q = C_d \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$, we have

$$Q = 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 2.77}$$

$$= \frac{0.000241}{0.0304} \times 7.34 = \mathbf{0.0584 \text{ m}^3/\text{s (Ans.)}}$$

Example 6.35. *A 300 mm × 150 mm venturimeter is provided in a vertical pipeline carrying oil of specific gravity 0.9, flow being upward. The difference in elevation of the throat section and entrance section of the venturimeter is 300 mm. The differential U-tube mercury manometer shows a gauge deflection of 250 mm. Calculate:*

(*i*) *The discharge of oil, and*

(*ii*) *The pressure difference between the entrance section and the throat section.*

Take the co-efficient of meter as 0.98 and specific gravity of mercury as 13.6. **[AMIE]**

Solution. Diameter at inlet, D_1 = 300 mm = 0.3 m

∴ Area of inlet, $A_1 = \frac{\pi}{4} \times 0.3^2 = 0.07 \text{m}^2$

Diameter at throat, D_2 = 150 mm = 0.15 m

∴ Area at throat,

$$A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

Specific gravity of heavy liquid (mercury) in U-tube manometer, S_{hl} = 13.6

Specific gravity of liquid (oil) flowing through pipe, S_p = 0.9

Reading of differential manometer, y = 250 mm = 0.25 m

The differential 'h' is given by:

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right)$$

$$= y\left[\frac{S_{hl}}{S_p} - 1\right] = 0.25\left[\frac{13.6}{0.9} - 1\right]$$

= 3.53 m of oil

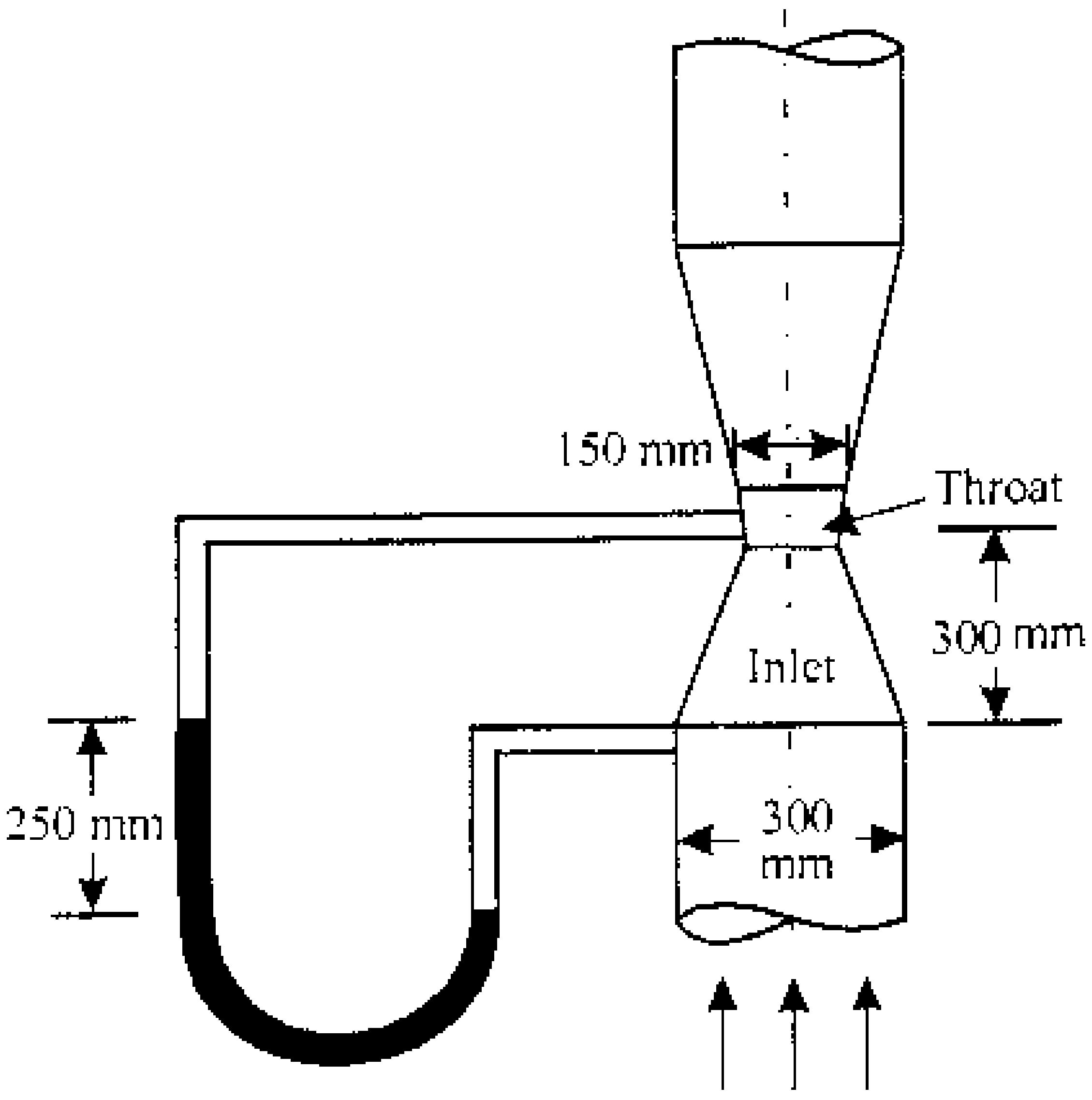


Fig. 6.31

(*i*) **Discharge of oil, Q:**

Using the relation, $Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$, we have

$$Q = 0.98 \times \frac{0.07 \times 0.01767}{\sqrt{0.07^2 - 0.01767^2}} \times \sqrt{2 \times 9.81 \times 3.53}$$

$$= \frac{0.001212}{0.0677} \times 8.32 = \mathbf{0.1489\ m^3/s.\ (Ans.)}$$

(*ii*) **Pressure difference between entrance and throat sections, $p_1 - p_2$:**

We know that, $$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = 3.53$$

or $$\left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2) = 3.53$$

But $z_2 - z_1 = 300$ mm or 0.3 m ... (*Given*)

$$\therefore \quad \left(\frac{p_1}{w} - \frac{p_2}{w}\right) - 0.3 = 3.53 \quad \text{or} \quad \frac{p_1 - p_2}{w} = 3.83$$

or $$p_1 - p_2 = (9.81 \times 0.9) \times 3.83 = \mathbf{33.8\ kN/m^2\ (Ans.)}$$

Example 6.36. *A vertical venturimeter carries a liquid of relative density 0.8 and has inlet and throat diameters of 150 mm and 75 mm respectively. The pressure connection at the throat is 150 mm above that at the inlet. It the actual rate of flow is* 40 *litres/sec and the* $C_d = 0.96$, *calculate the pressure difference between inlet and throat in N/m².* **(AMIE Summer, 1999)**

Solution. *Given*: Sp. gravity = 0.8, $D_1 = 150$ mm = 0.15 m; $D_2 = 75$ mm = 0.075 m; $z_2 - z_1 = 150$ mm = 0.15 m, $Q_{act} = 40$ litres/sec = 0.04 m³/s, $C_d = 0.96$.

Pressure difference $(p_1 - p_2)$:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.15^2 = 0.01767\,\text{m}^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.075)^2 = 0.00442\ \text{m}^2$$

$$Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}\text{, we get}$$

$$0.04 = 0.96 \times \frac{0.01767 \times 0.00442}{\sqrt{0.01767^2 - 0.00442^2}} \times \sqrt{2 \times 9.81} \times \sqrt{h}$$

or $$0.04 = 0.96 \times 0.004565 \times 4.429\sqrt{h}$$

$$\therefore \quad h = \left(\frac{0.04}{0.96 \times 0.004565 \times 4.429}\right)^2 = 4.247\ \text{m}$$

Also $$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right)$$

or, $$4.247 = \left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2)$$

$$= \left(\frac{p_1 - p_2}{\rho g}\right) - 0.15 \qquad (\because z_2 - z_1 = 0.15\ \text{m})$$

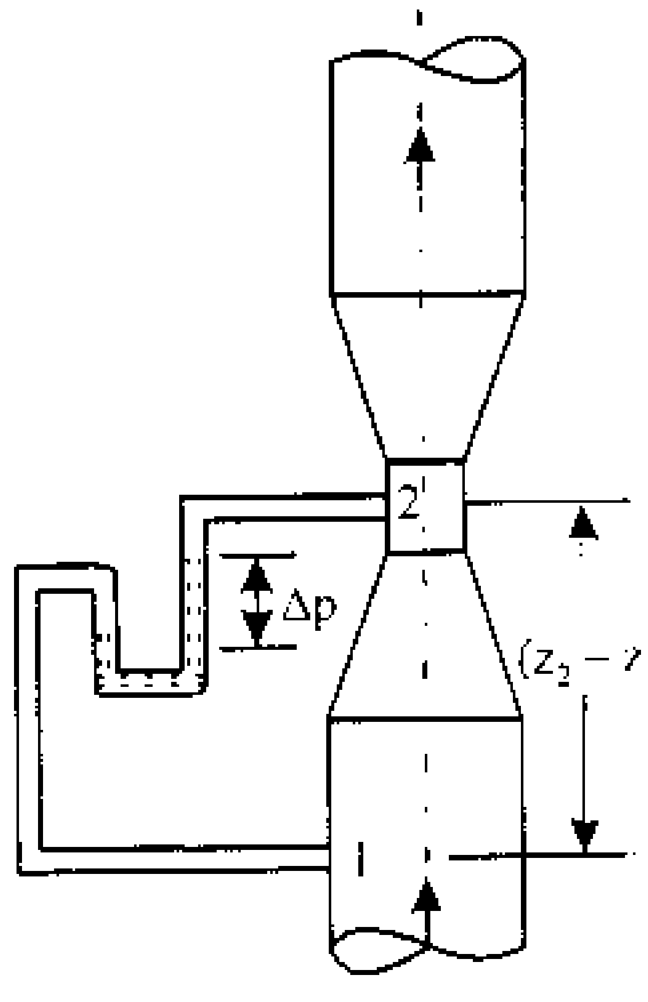


Fig. 6.32. Vertical ventrerimeter

or, $(p_1 - p_2) = \rho g\,(4.247 + 0.15)$

$= (0.8 \times 1000 \times 9.81)\,(4.247 + 0.15)\ \text{N/m}^2$

$= \mathbf{34.51\ kN/m^2}$ **(Ans.)**

Inclined Venturimeters

Example 6.37. *Determine the rate of flow of water through a pipe 300 mm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 150 mm. The difference of pressure between the main and throat is measured by a liquid of sp. gravity 0.7 in an inverted U- tube which gives a reading of 260 mm. The loss of head between the main and throat is 0.3 times the kinetic head of the pipe.*

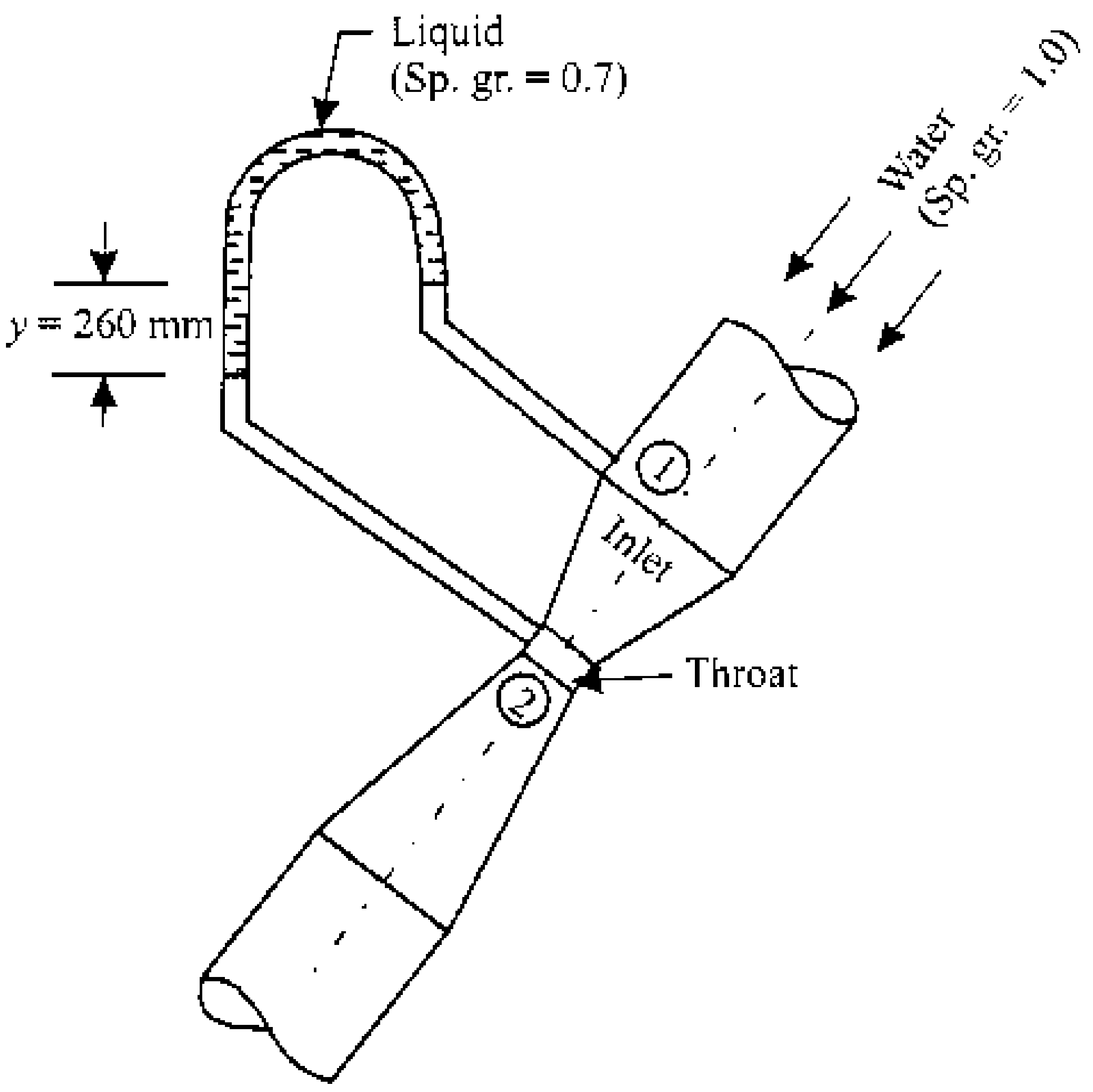


Fig. 6.33

Solution. Diameter at inlet,

$D_1 = 300\ \text{mm} = 0.3\ \text{m}$

$\therefore$ Area of inlet,

$$A_1 = \frac{\pi}{4} \times 0.3^2 = 0.07\ \text{m}^2$$

Throat diameter, $D_2 = 150\ \text{mm} = 0.15\ \text{m}$

$\therefore$ Area at throat,

$$A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767\ \text{m}^2$$

Specific gravity of lighter liquid (U-tube), $S_{ll} = 0.7$

Specific gravity of liquid (water) flowing through pipe, $S_p = 1.0$

Reading of differential manometer,

$y = 260\ \text{mm} = 0.26\ \text{m}$

Difference of pressure head, h is given by:

$$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = h$$

Also, $$h = y\left(1 - \frac{S_{ll}}{S_p}\right) = 0.26\left(1 - \frac{0.7}{1.0}\right)$$

$$= 0.078\ \text{m of water}$$

Loss of head, $h_L = 0.3 \times$ kinetic head of pipe ...(*Given*)

Now, applying Bernoulli's equation at sections '1' and '2', we get

$$\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{w} + z_2 + \frac{V_2^2}{2g} + h_L$$

$$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = h_L$$

But, $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = 0.078\ \text{m of water}$$ ... (as above)

and $$h_L = 0.3 \times \frac{V_1^2}{2g}$$ ...(*Given*)

$$\therefore \quad 0.078 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0.3 \times \frac{V_1^2}{2g}$$

or
$$0.078 + 0.7 \times \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0 \qquad \ldots(i)$$

Applying continuity equation at sections '1' and '2', we get

$$A_1 V_1 = A_2 V_2$$

$$\therefore \quad V_1 = \frac{A_2 V_2}{A_1} = \frac{\pi/4 \times 0.15^2}{\pi/4 \times 0.30^2} \times V_2 = \frac{V_2}{4}$$

Substituting this value of V_1 in eqn. (*i*), we get

$$0.078 + 0.7 \times \frac{(V_2/4)^2}{2g} - \frac{V_2^2}{2g} = 0$$

or
$$0.078 + \frac{V_2^2}{2g}\left(\frac{0.7}{16} - 1\right) = 0$$

or
$$\frac{V_2^2}{2g} \times (-0.956) = -0.078$$

or
$$V_2^2 = \frac{0.078 \times 2 \times 9.81}{0.956} = 1.6 \text{ m}^2 \text{ or } V_2 = 1.26 \text{ m/s}$$

$\therefore$ Rate of flow $Q = A_2 V_2 = 0.01767 \times 1.26 =$ **0.0222 m³/s. (Ans.)**

Example 6.38. *The following data relate to an inclined venturimeter:*

Diameter of the pipeline = 400 mm

Inclination of the pipeline with the horizontal = 30°

Throat diameter = 200 mm

The distance between the mouth and throat of the meter = 600 mm

Sp. gravity of oil flowing through the pipeline = 0.7

S.p. gravity of heavy liquid (U-tube) = 13.6

Reading of the differential manometer = 50 mm

The co-efficient of the meter = 0.98

Determine the rate of flow in the pipeline. **(Punjab University)**

Solution. Diameter at inlet, $D_1 = 400$ mm $= 0.4$ m

$\therefore$ Area of inlet, $\quad A_1 = \frac{\pi}{4} \times 0.4^2 = 0.1257 \text{ m}^2$

Throat diameter, $\quad D_2 = 200$ mm $= 0.2$ m

$\therefore$ Area at throat, $\quad A_2 = \frac{\pi}{4} \times 0.02^2 = 0.0314 \text{ m}^2$

Reading of the differential manometer (U-tube), $y = 50$ mm $= 0.05$ m

Difference of pressure head h is given by:

$$h = y\left[\frac{S_{hl}}{S_p} - 1\right]$$

where, S_{hl} = sp. gravity of heavy liquid (*i.e.*, mercury) in U-tube = 13.6

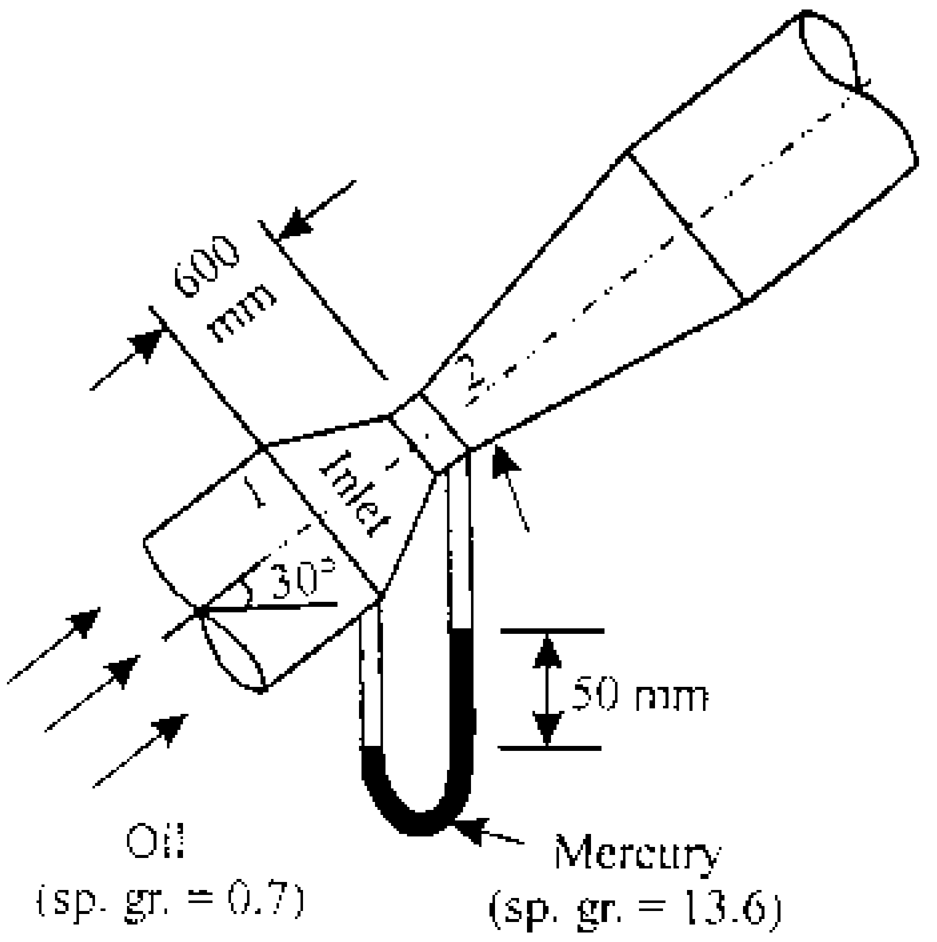


Fig. 6.34

S_p = sp. gravity of liquid (*i.e.*, oil) flowing through the pipe = 0.7

$$\therefore \qquad h = 0.05\left(\frac{13.6}{0.7} - 1\right) = 0.92 \text{ m of oil}$$

Now, applying Bernoulli's equation at sections '1' and '2', we get,

$$\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{w} + z_2 + \frac{V_2^2}{2g} \qquad ...(i)$$

or $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0$$

But $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = h$$

or $$\left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2) = h$$

It may be noted that *differential gauge reading will include in itself the difference of pressure head and the difference of datum head.*

Thus, eqn. (*i*) reduces to:

$$h + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0 \qquad ...(ii)$$

Applying continuity equation at sections '1' and '2' we get

$$A_1V_1 = A_2V_2$$

or $$V_1 = \frac{A_2V_2}{A_1} = \frac{\pi/4 \times 0.2^2}{\pi/4 \times 0.4^2} \times V_2 = \frac{V_2}{4}$$

Substituting the value of V_1 and h in eqn. (*ii*) we get,

$$0.92 + \frac{V_2^2}{16 \times 2g} - \frac{V_2^2}{2g} = 0$$

or $$\frac{V_2^2}{2g}\left(1 - \frac{1}{16}\right) = 0.92 \quad \text{or} \quad V_2^2 \times \frac{15}{16} = 0.92$$

or $$V_2^2 = \frac{0.92 \times 2 \times 9.81 \times 16}{15} = 19.25 \quad \text{or} \quad V_2 = 4.38 \text{ m/s}$$

Rate of flow of oil, $Q = A_2V_2 = 0.0314 \times 4.38 =$ **0.1375 m³/s (Ans.)**

6.6.2. Orificemeter

Orificemeter or orifice plate is a device (*cheaper* than a venturimeter) employed for *measuring the discharge of fluid through a pipe.* It also works on the same principle of a venturimeter.

It consists of a flat circular plate having a circular sharp edged hole (called orifice) concentric with the pipe. The diameter of the orifice may vary from 0.4 to 0.8 times the diameter of the pipe but its value is generally chosen as 0.5. A differential manometer is connected at section (1) which is at a distance of 1.5 to 2 times the pipe diameter upstream from the orifice plate, and at section (2) which is at a distance of about half the diameter of the orifice from the orifice plate on the downstream side.

Let,
A_1 = area of pipe at section (1),
V_1 = velocity at section (1),
p_1 = pressure at section (1), and
A_2 V_2 and p_2 are corresponding values at section (2).

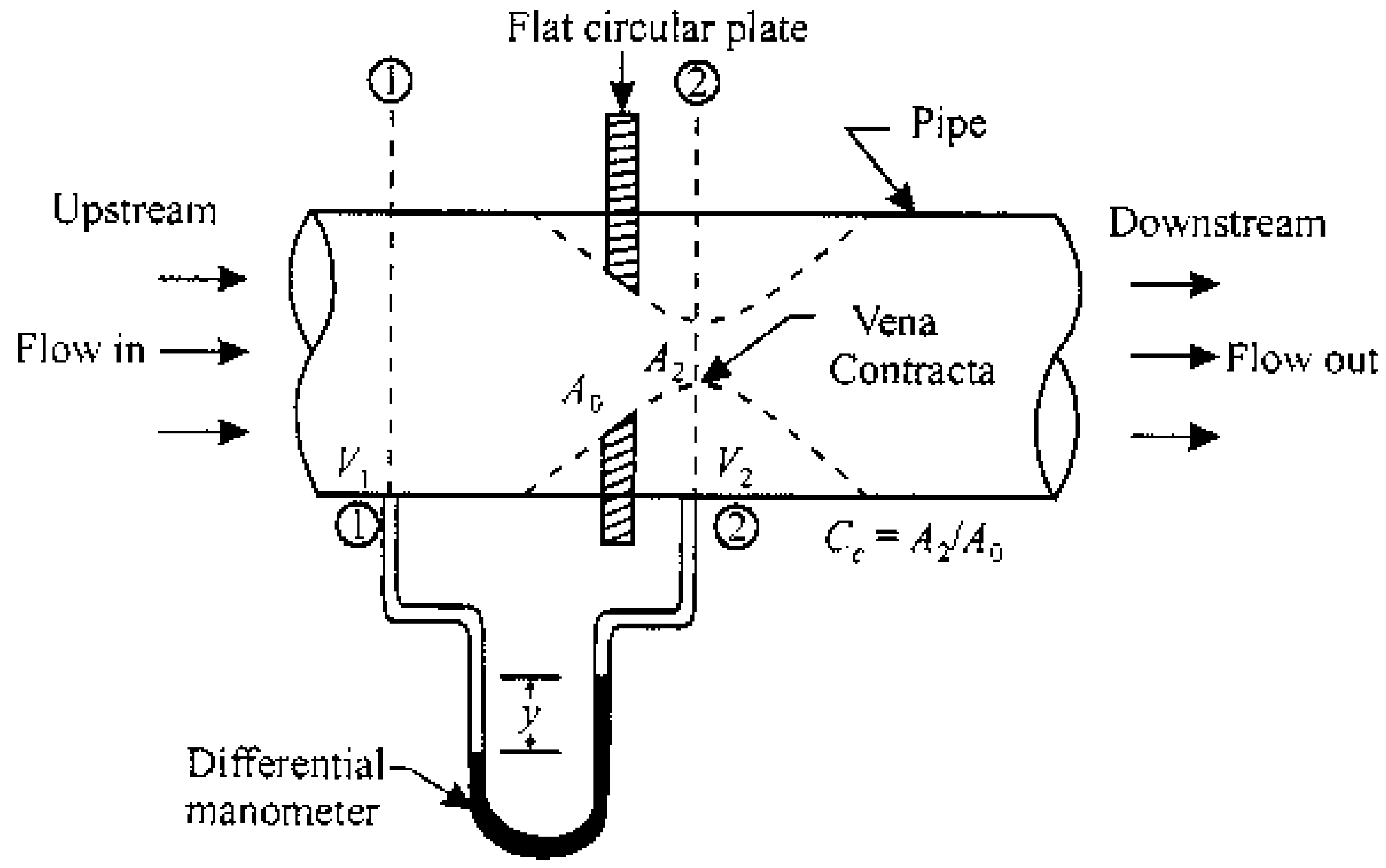


Fig. 6.35. Orficemeter

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

or
$$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

or
$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\left[\because h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \text{differential head}\right]$$

or
$$\frac{V_2^2}{2g} = h + \frac{V_1^2}{2g} \qquad \text{...}(i)$$

or
$$V_2 = \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)} = \sqrt{2gh + V_1^2}$$

Now, section (2) is at *vena contract* and A_2 represents the area at vena contracta. If A_0 is the area of orifice then, we have

$$C_c = \frac{A_2}{A_0}$$

(where C_c = co-efficient of contraction)

$$\therefore \qquad A_2 = A_0\, C_0 \qquad \text{...}(ii)$$

Using continuity equation, we get

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_1 = \frac{A_2 V_2}{A_1}$$

or
$$V_1 = \frac{A_0\, C_c V_2}{A_1} \qquad \text{...}(iii)$$

Substituting the value of V_1 in eqn. (*i*), we get

$$V_2 = \sqrt{2gh + \frac{A_0^2 . C_c^2 . V_2^2}{A_1^2}}$$

or $$V_2^2 = 2gh + \left(\frac{A_0}{A_1}\right)^2 \cdot C_2^2 \cdot V_2^2$$

or $$V_2^2 \left[1 - \left(\frac{A_0}{A_1}\right)^2 C_2^2\right] = 2gh$$

$\therefore$ $$V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}$$

$\therefore$ The discharge, $Q = A_2 V_2 = A_0 \cdot C_c \cdot V_2$

[$\because$ $A_2 = A_0 \cdot C_c$ ... as above (eqn. (*ii*)]

$$= A_0 C_c \frac{\sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}} \qquad ...(iv)$$

The above expression is simplified by using,

$$C_d = C_c \frac{\sqrt{1 - (A_0 / A_1)^2}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}$$

(where, C_d = co-efficient of discharge)

$$C_c = C_d \frac{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}{\sqrt{1 - (A_0 / A_1)^2}}$$

Substituting this value of C_c in eqn. (*iv*), we get

$$Q = A_0 \cdot C_d \frac{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}{\sqrt{1 - (A_0 / A_1)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2 C_c^2}}$$

$$= \frac{C_d \cdot A_0 \sqrt{2gh}}{\sqrt{1 - (A_0 / A_1)^2}} = \frac{C_d \cdot A_0 \cdot A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

i.e., $$Q = C_d \frac{A_0 \cdot A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}} \qquad ...(6.9)$$

It may be noted that C_d (co-efficient of discharge) of an orifice is *much smaller than that of a venturimeter.*

Difference between a venturimeter and an orificemeter:

A **venturimeter** is *a device which is inserted into pipeline to measure incompressible fluid flow rates*. It consists of a convergent section which reduces the diameter to between one half to one-fourth of the pipe diameters. This is followed by a divergent section. The pressure difference between the position just before the venturi and at the throat of the venturi is measured by a differential manometer. The *working of the venturi is based on the Bernoulli's principle,* that is *when the velocity head increases in an accelerated flow, there is a corresponding reduction in the piezometric head.*

The **orificemeter** *is opening, usually round, located in the side wall of the tank or reservoir, for measuring the flow of a liquid.* The main feature of the orificemeter is that *most of the potential energy of the liquid is converted into kinetic energy of the free jet issuing through the orifice.*

The main points of *difference* between a venturimeter and orificemeter are:

1. The venturimeter can be used for *measuring the flow rates of all incompressible flows.* (gases with low pressure variations, as well as liquids), whereas orifice meters are generally used for measuring the *flow rates of liquids.*

2. Venturimeter is installed *in pipeline only*, and the accelerated flow through the apparatus, is subsequently decelerated to the original velocity at the outlet of the venturimeter. The flow continues through the pipeline. In the orificemeter the entire potential energy of the fluid is converted to kinetic energy, and the jet *discharges freely into the open atmosphere*.
3. In venturimeter, the flow velocity is *measured by noting the pressure difference between the inlet and the throat of the venturimeter*, whereas in the orificemeter the discharge velocity is measured by using *Pitot tube* or by *trajectory method*.

Example 6.39. *The following data relate to an orificemeter:*

Diameter of the pipe = 240 mm

Diameter of the orifice = 120 mm

Sp. gravity of oil = 0.88

Reading of differential manometer = 400 mm of mercury

Co-efficient of discharge of the meter = 0.65.

Determine the rate of flow of oil.

Solution. Diameter of the pipe D_1 = 240 mm = 0.24 m

∴ Area of the pipe, $A_1 = \frac{\pi}{4} \times 0.24^2 = 0.0452 \text{ m}^2$

Diameter of the orifice, $D_0 = 120 \text{ mm} = 0.12 \text{ m}$

∴ Area of the orifice, $A_0 = \frac{\pi}{4} \times 0.12^2 = 0.0113 \text{ m}^2$

Co-efficient of discharge, $C_d = 0.65$

Sp. gravity of oil, $S_0 = 0.88$

Reading of differential manometer, y = 400 mm of mercury = 0.4 m of mercury

∴ Differential head, $h = y\left[\frac{S_{hl}}{S_o} - 1\right]$

[where S_{hl} = sp. gravity of heavier liquid = 13.6 (for mercury)]

$$= 0.4\left[\frac{13.6}{0.88} - 1\right] = 5.78 \text{ m of oil}$$

Discharge Q:

Using the relation, $Q = C_d \frac{A_0 \cdot A_1 \cdot \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$, we have

$$Q = 0.65 \times \frac{0.0113 \times 0.0452 \times \sqrt{2 \times 9.81 \times 5.78}}{\sqrt{(0.0452)^2 - (0.0113)^2}}$$

$$= \frac{0.000353}{0.0437} = \mathbf{0.08 \, m^3/s} \text{ (Ans.)}$$

Example 6.40. *Water flows at the rate of 0.015 m³/s through a 100 mm diameter orifice used in a 200 mm pipe. What is the difference of pressure head between the upstream section, and the vena contracta section? Take co-efficient of contraction C_c = 0.60 and C_v = 1.0.* **(AMIE Winter, 2001)**

Solution. *Given*: $Q = 0.015 \text{ m}^3/\text{s}$; D_0 = 100 mm = 0.1 m; D_1 = 200 mm = 0.2 m; $C_c = 0.60$; C_v = 1.0

Difference in pressure head *h*: Refer to Fig. 6.35.

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.03142 \text{ m}^2$$

$$A_0 = \frac{\pi}{4}D_0^2 = \frac{\pi}{4} \times 0.1^2 = 0.007854 \text{ m}^2$$

$$C_d = C_c \times C_v = 0.60 \times 1.0 = 0.6$$

Using the relation: $Q = C_d \dfrac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$

or $$0.015 = 0.6 \times \frac{0.007854 \times 0.03142 \sqrt{2 \times 9.81 \times h}}{\sqrt{(0.03142)^2 - (0.007854)^2}} \quad \text{...[Eqn. (6.9)]}$$

or $$0.015 = 0.6 \times \frac{0.001093\sqrt{h}}{0.03042}$$

or $$h = \left(\frac{0.015 \times 0.03042}{0.6 \times 0.001093}\right)^2 = \mathbf{0.484 \text{ m of water (Ans.)}}$$

Example 6.41. (*a*) *Derive an expression for the volumetric flow rate of a fluid flowing through orificemeter. Write down the advantage and disadvantages of using orificemeter over a venturimeter.*

(*b*) *Water is flowing through a pipeline of 50 cm ID at 30°C. An orifice is placed in the pipeline measure the flow rate. Orifice diameter is 20 cm. If the manometer reads 30 cm of Hg, calculate e water flow rate and velocity of the fluid through the pipe.*

ρ_{water} *at 30°C = 987 kg/m³*

ρ_{Hg} *= 13600 kg/m³*

Orifice co-efficient. = 0.6

Sol. (*a*) Refer Article 6.6.2.

Advantage of orificemeter over venturimeter is that its *length is short and hence it can be used a wide variety of application. Venturimeter has excessive length.*

The **disadvantage of** orificemeter is that a *sizeable pressure loss is increased because of the separation downstream of the plate. In a venturimeter the expanding section keeps boundary er separation to a minimum, resulting in good pressure recovery across the meter.*

(*b*) *Given*: $D_1 = 50$ cm = 0.5 m; $D_0 = 20$ cm = 0.2; $y = 30$ cm of Hg = 0.3 m of Hg

ρ_{water} at 30°C = 981 kg/m³, $\rho_{Hg} = 13600$ kg/m³;

$C_0 = 0.6$.

Water flow rate, Q:

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4} \times 0.5^2 = 0.1963 \text{ m}^2$$

$$A_0 = \frac{\pi}{4}D_0^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$h = y\left(\frac{\rho_{Hg}}{\rho_{H_2O}} - 1\right) = 0.3\left(\frac{13600}{987} - 1\right) = 3.834 \text{ m}$$

Using the relation; $Q = C_d \dfrac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$, we get

$$Q = 0.6 \times \frac{0.0314 \times 0.1963 \times \sqrt{2 \times 9.81 \times 3.834}}{\sqrt{(0.1963)^2 - (0.0314)^2}}$$

$$= 0.6 \times \frac{0.05346}{0.1938} = \mathbf{0.1655\ m^3/s\ (Ans.)}$$

Velocity of water through pipe,

$$V_1 = \frac{Q}{A_1} = \frac{0.1655}{0.1963} = \mathbf{0.843\ m/s\ (Ans.)}$$

6.6.3 Rotameter and Elbow meter

6.6.3.1. Rotameter. Refer Fig. 6.36.

Construction: It consists of a tapered metering glass tube, inside of which is located a rotor or active element (float) of the meter. The tube is provided with inlet and outlet connections. The specific gravity of the float or bob material is higher than that of the fluid to be metered. On a part of the float spherical slots are cut which cause it (float) to rotate slowly about the axis of the tube and keep it centred. Owing to this spinning accumulation of any sediment on the top or sides of float is checked. However, the stability of the bob may also be ensured by using a guide along which the float would slide.

Working. When the rate of flow increases the float rises in the tube and consequently there is an increase in the annular area between the float and the tube. Thus, the float rides higher or lower depending on the rate of flow.

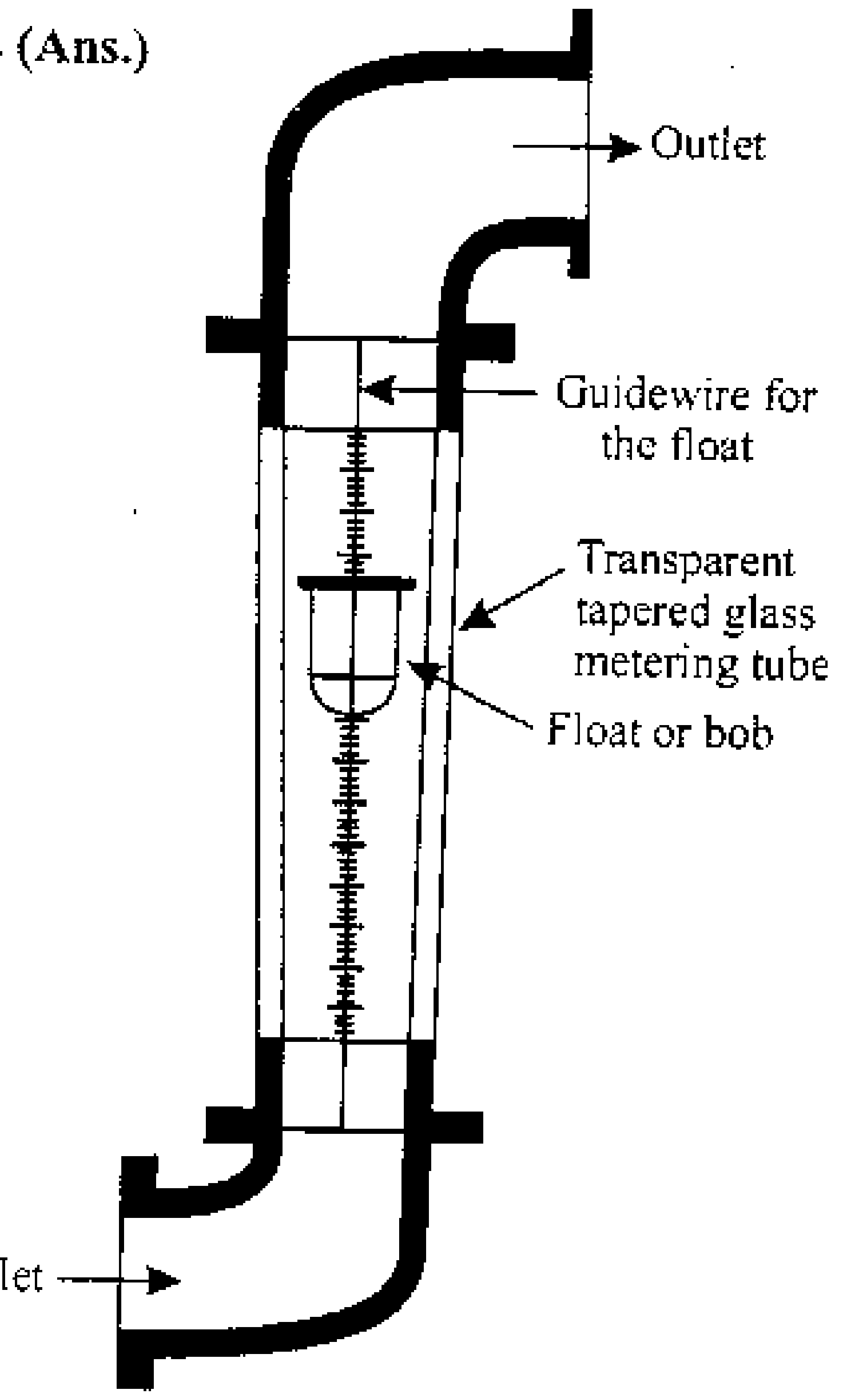


Fig. 6.36. Rotameter.

The discharge through a rotameter is given by,

$$Q = C_d\, A_{ann.}\left[2gV_{fl}\left(\rho_{fl} - \rho_f\right)/A_f\rho_f\right]^{1/2} \qquad \ldots (6.10)$$

where,
Q = Volume flow rate,
C_d = Co-efficient of discharge,
$A_{ann.}$ = Annular area between float and tube,
V_{fl} = Volume of float,
ρ_{fl} = Density of float material,
ρ_f = Density of fluid, and
A_f = Maximum cross-sectional area of the fluid.

As the flow area $A_{ann.}$ is a function of height of float in the tube, the flow rate scale can be engraved on the tube corresponding to a particular float.

Advantages :

1. Simpler in operation.
2. Handling and installation easy.
3. Wide variety of corrosive fluids can be handled.
4. Low cost, relatively.

Limitations :

1. Mounted vertically, limited to small pipe sizes and capacities.
2. Less accurate, compared to venturimeter and orificemeter.

6.6.3.2. Elbow meter

When liquid flows around a pipe bend. There is an increase in pressure with radius, i.e. the *pressure at the outer wall of the bend is more than that at the inner wall.* This difference of pressure which exists between the outside and inside of the bend is used for the measurement of discharge in a pipeline.

As shown in Fig. 6.37. the pipe bend is provided with two pressure tappings; one each at the inner and outer walls of the bend. These tappings are connected to the limbs of U-tube manometer.

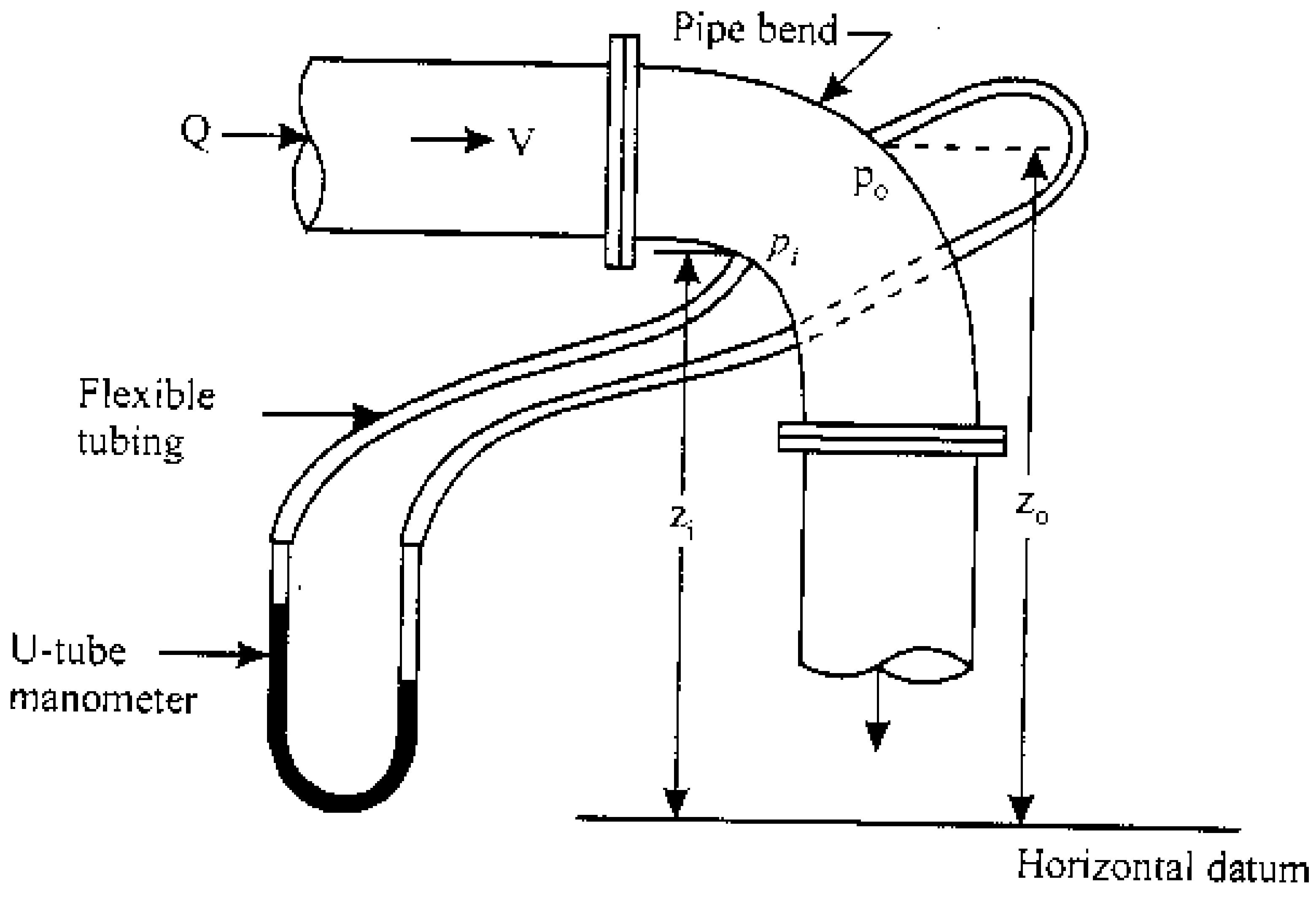


Fig. 6.37. Elbow meter.

As per literature, the following relation between velocity and pressure difference is available:

$$K\frac{V^2}{2g} = \left(\frac{p_0}{w} + z_0\right) - \left(\frac{p_i}{w} + z_i\right) \quad ...(6.11)$$

where,
K = Constant (depends upon the shape and size of the bend), *ranges from* 1.3 *to* 3.2,and
V = Velocity of flow.

Suffices 0 and i represent the conditions at the outer and inner walls of the pipe bend,

or $$V = \frac{1}{\sqrt{K}}\sqrt{2g}\sqrt{\left(\frac{p_0}{w} + z_0\right) - \left(\frac{p_i}{w} + z_i\right)} \quad ...[6.11\,(a)]$$

∴ Discharge, $$Q = AV = C_d\, A\,\sqrt{2g}\sqrt{\left(\frac{p_0}{w} + z_0\right) - \left(\frac{p_i}{w} + z_i\right)} \quad ...(6.12)$$

where, $C_d = \frac{1}{\sqrt{k}}$ = Co-efficent of discharge, and
A = Cross-sectional area of the pipe.
(C_d varies between 0.56 and 0.88)

The following empirical relation has been suggested:

$$C_d = \sqrt{\frac{R_b}{D}},$$

where, R_b = Radius of pipe bend, and
D = Diameter of the pipe.

- An elbowmeter can be conveniently used for the measurement of discharge in pipes which are fitted with elbows and bends.
- Its accuracy, with proper calibration, approaches that of a venturimeter or nozzle.

6.6.4. Pitot Tube

Pitot tube is *one of the most accurate devices for velocity measurement.* It works on the *principle that if the velocity of flow at point becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure.*

It consists of a glass tube in the form of a 90° bend of short length open at both its ends. It is placed in the flow with its bent leg directed upstream so that a *stagnation point* is created immediately in front of the opening (Fig. 6.38). The kinetic energy at this point gets converted into pressure energy causing the liquid to rise in the vertical limb, to a height equal to the stagnation pressure.

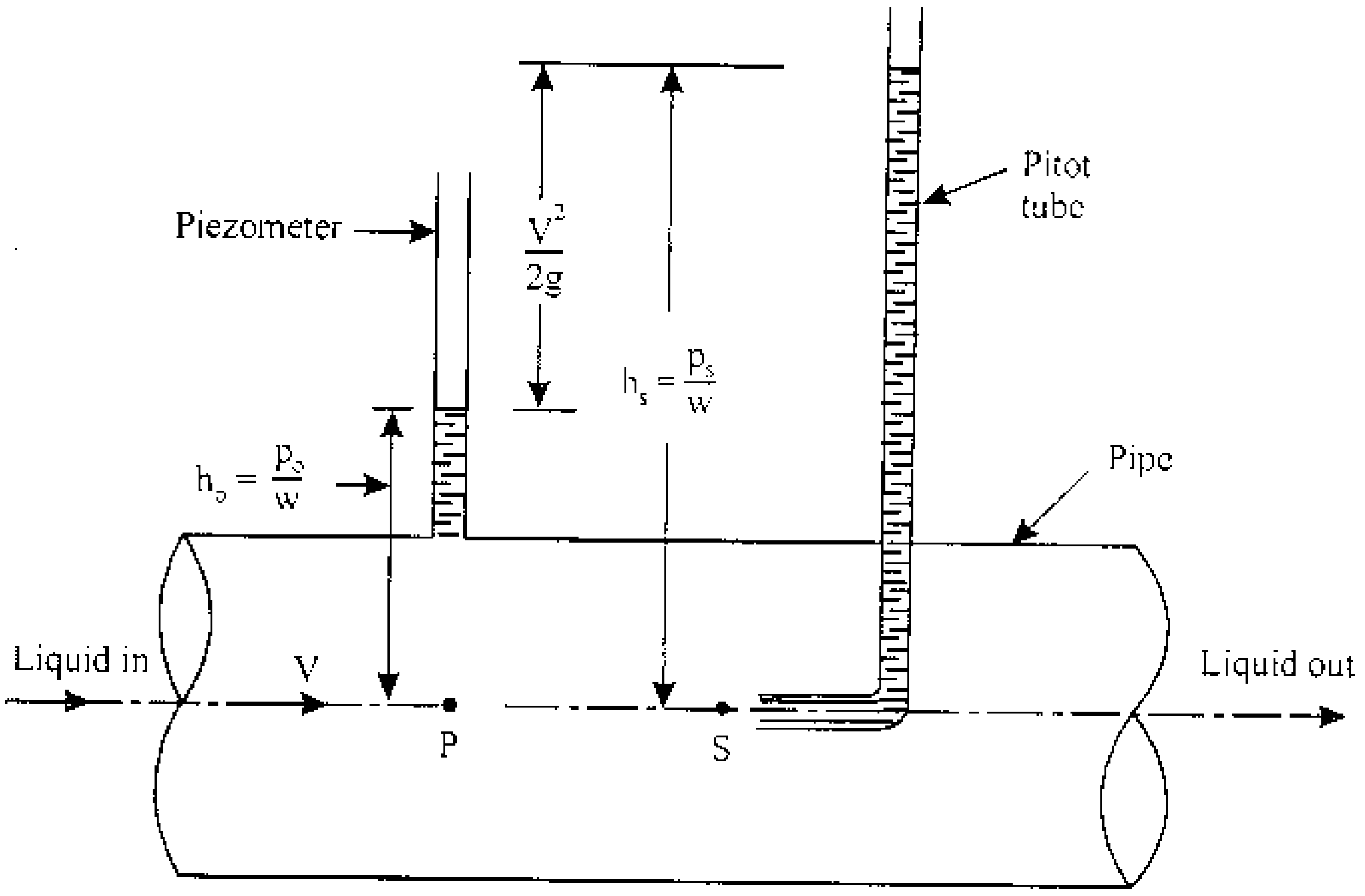


Fig. 6.38. Pitot tube.

Applying Bernoulli's equation between stagnation point (S) and point (P) in the undisturbed flow at the same horizontal plane, we get:

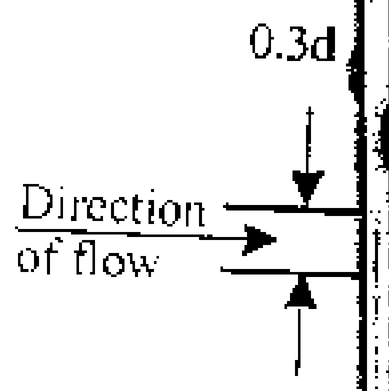
$$\frac{p_0}{w} + \frac{V^2}{2g} = \frac{p_s}{w} \quad \text{or} \quad h_0 + \frac{V^2}{2g} = h_s$$

or,
$$V = \sqrt{2g\,(h_s - h_0)} \quad \text{or} \quad \sqrt{2g\,\Delta h} \qquad \ldots(1)$$

where,
p_0 = Pressure at point 'P', *i.e.* static pressure,
V = Velocity at point 'P', *i.e.* free flow velocity,
p_s = Stagnation pressure at point 'S', and
Δh = Dynamic pressure
= Difference between stagnation pressure head (h_s) and static pressure head (h_0).

The height of liquid rise in the Pitot tube indicates the stagnation head. The static pressure head may be measured separately with a piezometer (Fig. 6.38).

Both the static pressure as well as stagnation pressure can be measured in a device known as **Pitot-static tube.** (Fig. 6.39).



It consists of two co
The outer tube has addit
the liquid level in it giv
differential manometer
pressure head.

If y is the manometri

where, S_m = Specifi
S = Specifi

When a Pitot-tube is
and causes the static press
tends to produce an exces
fied to give the actual vel

where, C = *a* connect

The most commonly
signed that the effect of ste

Example 6.42. *A sub*
below the surface of wate
nected to the two limbs of a
Find the speed of the sub

Take the specific-gravi

Solution. Reading of th
Sp. gravity of mercury,
Sp. gravity of sea water,

To find the head, (h), using the relation: $h = y\left(\frac{S_{hl}}{S_l} - 1\right)$, we have

$$h = 0.2\left(\frac{13.6}{1.025} - 1\right) = 2.45$$

∴ Velocity of the submarine

$$V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.45} = \textbf{6.93 m/s or 24.9 km/h (Ans.)}$$

Example 6.43. *Petroleum oil (sp. gr. = 0.9 and viscosity = 13 cP) flows isothermally through a horizontal 5 cm pipe. A Pitot tube is inserted at the centre of a pipe and its leads are filled with the same oil and attached to a U-tube containing water. The reading on the manometer is 10 cm. Calculate the volumetric flow of oil in m³/s. The co-efficient of Pitot tube is 0.98.* **(AMIE Summer, 2002)**

Solution. *Given*: Sp gr. of oil = 0.9; μ = 13 cP $= \frac{13}{100} \times 0.1$ Ns/m² = 0.013 Ns/m²;

y = 10 cm of Hg = 0.1 m of Hg., D = 5 cm = 0.05 m;

Co-efficient of Pitot tube, C_v = 0.98

Volumetric flow of oil:

Differential head, $$h = y\left(\frac{S_{Hg}}{S_{Oil}} - 1\right) = 0.1\left(\frac{13.6}{0.9} - 1\right) = 1.411$$

∴ Actual velocity of flow, $V = C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.411} = \textbf{5.156 m/s}$

Volumetric flow of oil = A × V = $\frac{\pi}{4} \times 0.05^2 \times 5.156$ = **0.01 m³/s (Ans.)**

Example 6.44. *For the flow situation shown in Fig. 6.40 determine the ratio $\frac{h_1}{h_2}$ if the area ratio $\frac{A_1}{A_2} = 1.8$.*

Neglect loses due to friction.

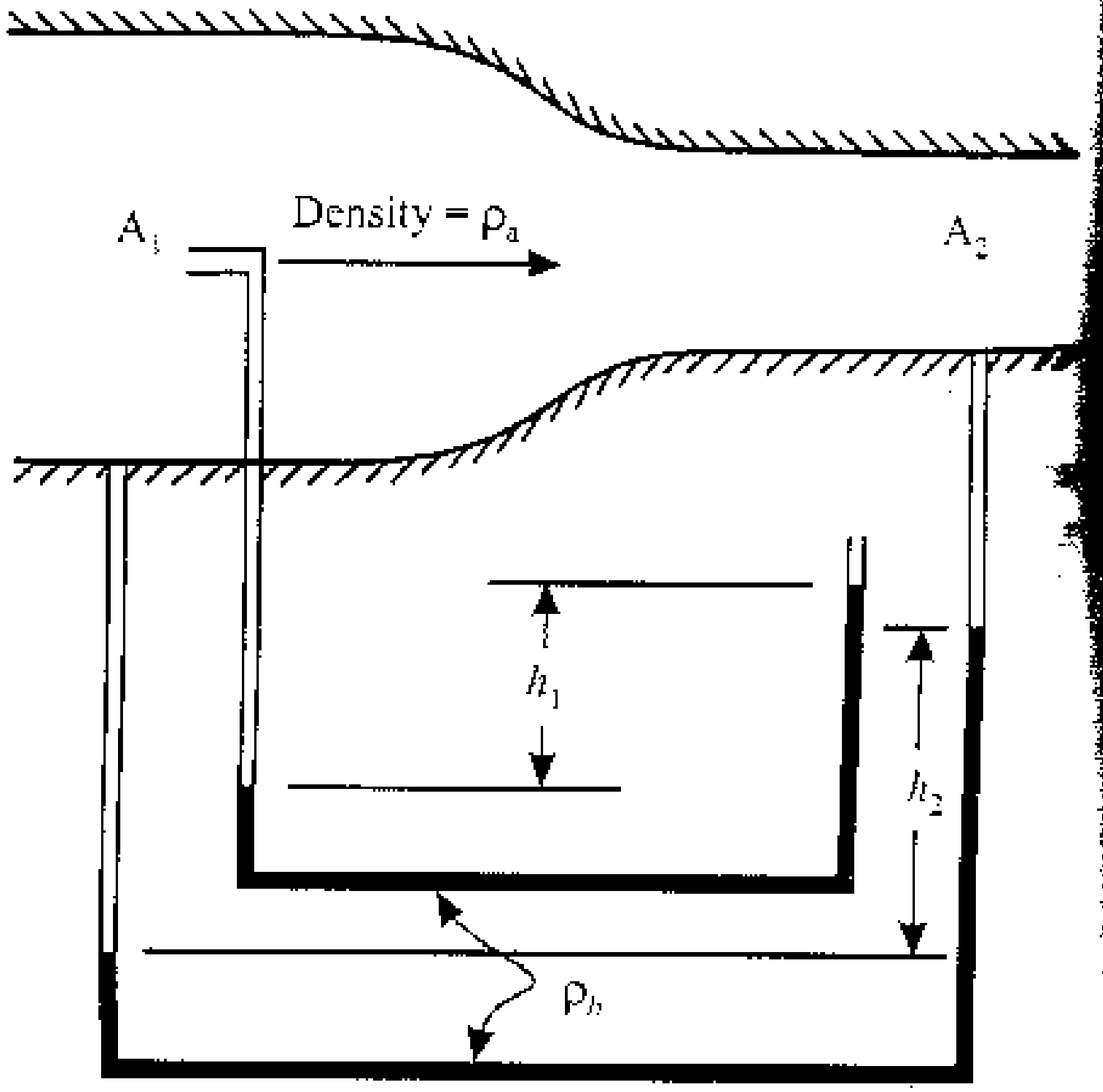


Fig. 6.40

Solution. Refer Fig. 6.40.

For Pitot tube :

$p_s + \rho_a g h_1 = p_2 + \rho_b g h_1$ ...(*i*)

For priezometric tubes:

$p_1 + \rho_a g h_2 = p_2 + \rho_b g h_2$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$$(p_s - p_1) + \rho_a g(h_1 - h_2) = \rho_b g(h_1 - h_2)$$

$$(p_s - p_1) = (h_1 - h_2)(\rho_b - \rho_a)g$$

$$\frac{p_s - p_1}{\rho_a g} = (h_1 - h_2)\left(\frac{\rho_b}{\rho_a} - 1\right) \quad ...(iii)$$

.....Dividing by $\rho_a g$.

From piezometric tappings,

$$(p_1 - p_2) = h_2 g(\rho_b - \rho_a)$$

$$\frac{p_1 - p_2}{\rho_a g} = h_2\left(\frac{\rho_b}{\rho_a} - 1\right)$$

...Dividing by

(It may be noted that horizontal component of velocity U is $U \cos\theta$ which *remains constant* whereas the vertical component $U \sin\theta$ is *affected by gravity*.)

From eqn. (*i*) we have, $t = \dfrac{x}{U\cos\theta}$

Substituting the value of t in eqn. (*ii*), we get

$$y = U\sin\theta \times \frac{x}{U\cos\theta} - \frac{1}{2}g \times \frac{x^2}{U^2\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2}{2U^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2U^2} \qquad \left(\because \frac{1}{\cos^2\theta} = \sec^2\theta\right) \qquad ...(6.13)$$

This is the equation of *a parabola.*

(*i*) **Maximum height attained by the jet, h:**

Using the relation:

$$V_2^2 - V_1^2 = -2gh \quad (-\,ve \text{ sign is used as the particle is moving } upward)$$

where, V_1 = Initial vertical component = $U\sin\theta$, and

$V_2 = 0$ at the highest point.

$$\therefore \quad 0 - (U\sin\theta)^2 = -2gh$$

or
$$h = \frac{U^2\sin^2\theta}{2g} \qquad ...(6.14)$$

(*ii*) **Time of flight, T:**

Time of flight is the time taken by the fluid particle in reaching from A to B (Fig. 6.41). From eqn. (*ii*), we have

$$y = U\sin\theta \times t - \frac{1}{2}gt^2$$

When the particle reaches the point B, $y = 0$, $t = T$

Putting these values in the above equation, we get

$$O = U\sin\theta \times T - \frac{1}{2}g \times T^2$$

or
$$T = \frac{2U\sin\theta}{g} \qquad ...(6.15)$$

Time taken to reach the highest point, $T' = \dfrac{T}{2} = \dfrac{2U\sin\theta}{2g} = \dfrac{U\sin\theta}{g}$

i.e.,
$$T' = \frac{U\sin\theta}{g} \qquad ...(6.16)$$

(*iii*) **Horizontal range of the jet, *r*:**

The range (r) of the jet is the total *horizontal distance travelled by the fluid particle.*

Then r, (*i.e.*, distance AB) = velocity component in direction × time taken by the particle to reach from A to B

$$= U\cos\theta \times T = U\cos\theta \times \frac{2U\sin\theta}{g}$$

$$= \frac{U^2 \times 2\sin\theta \times \cos\theta}{g} = \frac{U^2 \sin 2\theta}{g}$$

i.e., $$r = \frac{U^2 \sin 2\theta}{g} \qquad \text{...(6.17)}$$

The range will be maximum, when $\sin 2\theta = 1$

i.e., $$2\theta = 90° \text{ or } \theta = 45°$$

Then maximum range, $$r_{\max} = \frac{U^2 \sin(2 \times 45°)}{g} = \frac{U^2}{g}$$

i.e., $$r_{\max} = \frac{U^2}{g}$$

Example 6.45. *A nozzle is situated at a distance of 1.2 m above the ground level and is inclined* *0° to the horizontal. The diameter of the nozzle is 40 mm and the jet of water from the nozzle* *es the ground at a horizontal distance of 5 m. Find the flow rate.*

Solution. Distance of nozzle above the ground = 1.2 m

Angle of inclination, $\theta = 60°$

Diameter of the nozzle,

$d = 40 \text{ mm} = 0.04 \text{ m}$

∴ Area of nozzle,

$$A = \frac{\pi}{4} \times 0.04^2 = 0.001256 \text{ m}^2$$

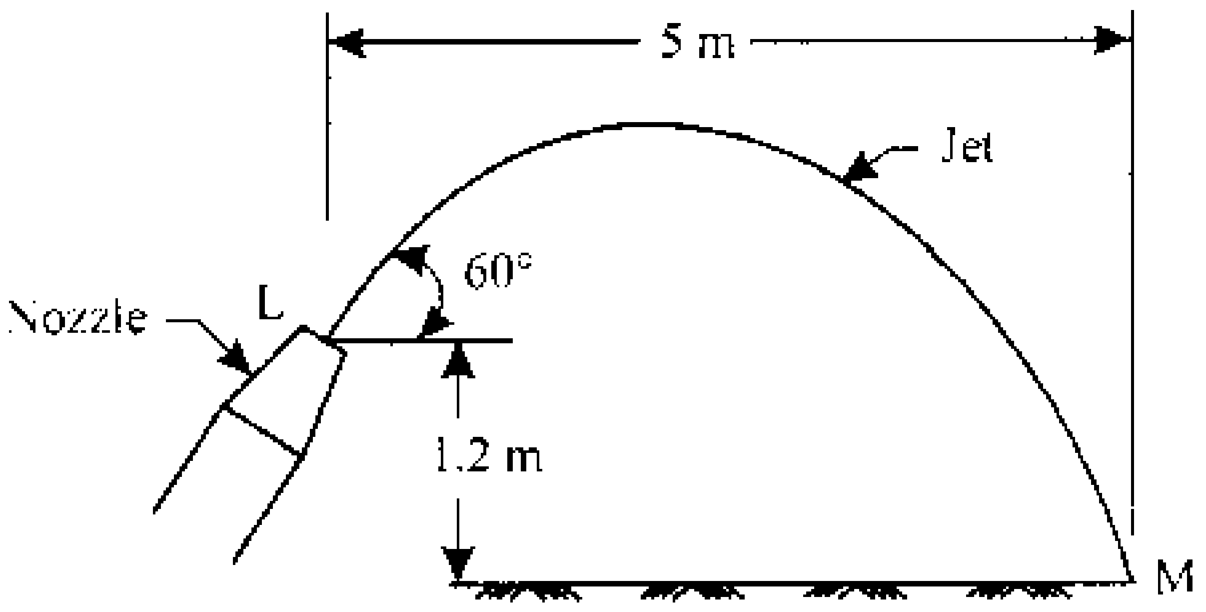


Fig. 6.42

The horizontal distance, $x = 5$ m

The co-ordinates of the point M, which [illegible] e centre line of the jet of water and is [illegible] d on the ground, with respect to L [illegible] n) are: $x = 5$ m, $y = -1.2$ m (∵ Point M is vertically down by 1.2 m)

e equation of the jet is given by :

$$y = x\tan\theta - \frac{gx^2}{2U^2\cos^2\theta} \qquad \text{...}(i)$$

here, U = Velocity of the jet.

w rate, Q:

he eqn. (*i*) can be written as:

$$y = x\tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

$$-1.2 = 5\tan 60° - \frac{9.81 \times 5^2}{2U^2}(1 + \tan^2 60°)$$

$$(\because \sec^2\theta = 1 + \tan^2\theta)$$

$$-1.2 = 5 \times 1.732 - \frac{122.62}{U^2}(1+3)$$

$$-1.2 = 8.66 - \frac{498.48}{U^2}$$

$$U^2 = \frac{498.48}{(8.66 + 1.2)} = 49.74 \quad \text{or} \quad U = 7.05 \text{ m/s}$$

ence, flow rate, $Q = A \times U = 0.001256 \times 7.05 =$ **0.00885 m³/s (Ans.)**

Example 6.46. *It is required to place an orifice in the side of a tank at such an elevation that the jet will attain a maximum horizontal distance from the tank at the level of its base. What is the proper distance from the orifice to the free surface when the depth of liquid in the tank is maintained at 1.2 m?*

Solution. Depth of liquid in the tank = 1.2 m

$$x = \sqrt{2gh} \times t \qquad \text{...(i)}$$

and, $$y = -\frac{1}{2} gt^2 \qquad \text{...(ii)}$$

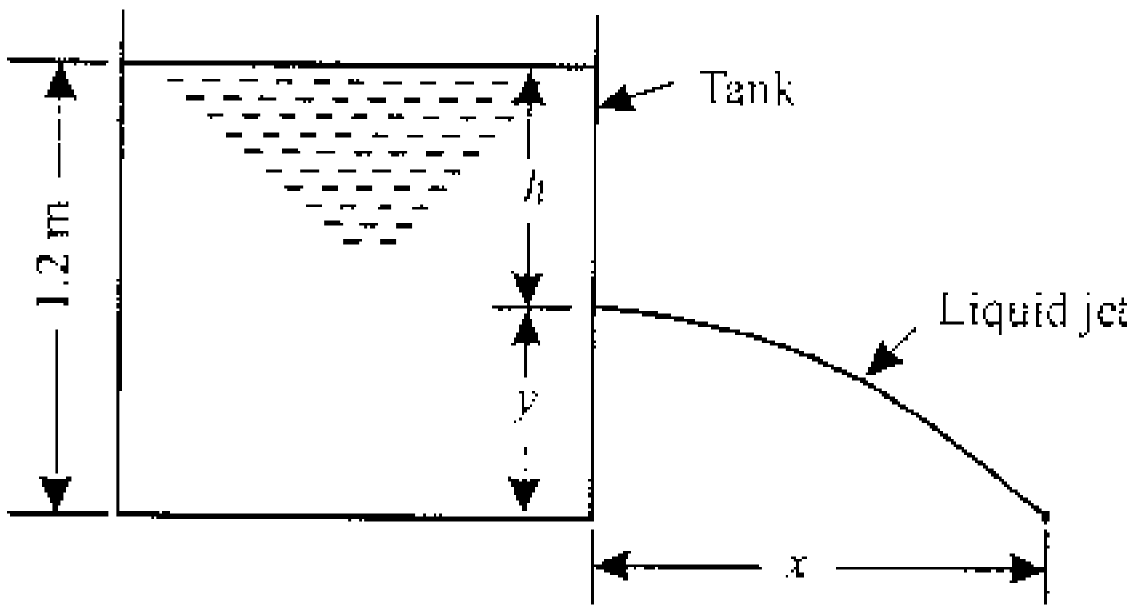


Fig. 6.43

Eliminating t, we get

$$y = -\frac{1}{2} g \times \left(\frac{x}{\sqrt{2gh}}\right)^2$$

$$= -\frac{1}{2} \times g \times \frac{x^2}{2gh}$$

or, $$y = -\frac{x^2}{4h}$$

Also, $1.2 = h + y$ or $y = 1.2 - h$

$$\therefore \qquad (1.2 - h) = -\frac{x^2}{4h}$$

or, $$x^2 = -4h\,(1.2 - h) = -4.8\,h + 4h^2$$

For horizontal distance x to be maximum $\frac{dx}{dh} = 0$

$$\therefore \qquad 2x\frac{dx}{dh} = -4.8 + 8h = 0 \quad \text{or} \quad h = 0.6 \text{ m}$$

Thus, the orifice should be located at a distance of **0.6 m below the free surface. (Ans.)**

Example 6.47. *Ten nozzles each 25 mm in diameter, all inclined at an angle of 45° with the horizontal are used in an ornamental fountain. The jet issuing from the nozzle falls into a basin at a point 1.5 m vertically beneath the nozzle and 4.5 m horizontally from it. The velocity co-efficient of nozzle is 0.97. Determine:*

(*i*) *Pressure head at the nozzle, and*

(*ii*) *Total discharge from the nozzles.*

Solution. Diameter of each nozzle,

$$d = 25 \text{ mm} = 0.025 \text{ m}$$

Angle of inclination, $\theta = 45°$

Velocity co-efficient of nozzle,

$$C_v = 0.97$$

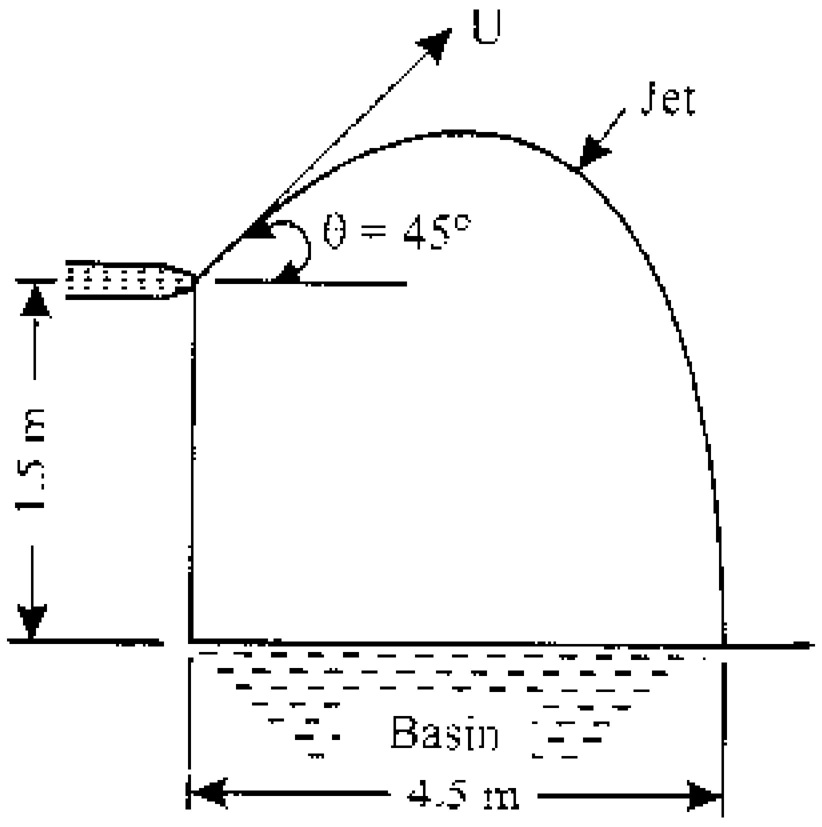


Fig. 6.44

(*i*) **Pressure head at the nozzle, H:**

Refer Fig. 6.44

Horizontal distance traversed,

$$x = 4.5 \text{ m}$$

Vertical distance traversed, $y = -1.5$ m

For horizontal motion:

$$x = U \cos\theta \times t$$

or, $$4.5 = U \cos 45° \times t = 0.707\, Ut \qquad \text{...(i)}$$

For ve

or

From (*i*

or

From eq

Also

or

i.e., Pres

(*ii*) **Total dis**

Total dis

i.e., Total

Example 6.4
from a nozzle 30
above the ground,
can stand and still

Solution *Give*

Refer Fig. 6.4

A

Ground

Greatest horiz

Now, $m = \rho AU$

For vertical motion:

$$y = U\sin\theta . t - \frac{1}{2}gt^2$$

or $$-1.5 = 0.707\,Ut - \frac{1}{2}gt^2 \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get

$$-1.5 = 4.5 - \frac{1}{2}\times 9.81\times t^2 \quad \text{or} \quad 4.905\,t^2 = 6$$

or $$t = \left(\frac{6}{4.905}\right)^{1/2} = 1.106\,\text{s}$$

From eqn. (*i*), we have

$$U = \frac{4.5}{0.707\times 1.106} = 5.75\ \text{m/s}$$

Also $$U = C_v\times\sqrt{2gh} \quad \text{or} \quad 5.75 = 0.97\times\sqrt{2\times 9.81\times H}$$

or $$5.93 = \sqrt{2\times 9.81\times H} \quad \text{or} \quad H = \frac{5.93^2}{2\times 9.81} = 1.79\ \text{m}$$

i.e., Pressure head at the nozzle = **1.79 m (Ans.)**

(*ii*) **Total discharge from the nozzles:**

Total discharge, $$Q = (\pi/4\times d^2\times U)\times\text{number of nozzles}$$

$$= \pi/4\times 0.025^2\times 5.75\times 10 = 0.0282\ \text{m}^3/\text{s}$$

i.e., Total discharge through nozzles = **0.0282 m³/s. (Ans.)**

Example 6.48. *A fireman must reach a window 40 m above the ground with a water jet, issued a nozzle 30 mm in diameter and discharging 30 kg/s. Assuming the nozzle height to be 2 m above the ground, determine the greatest horizontal distance from the building where the fireman stand and still reach the jet into the window.* **(AMIE Summer, 2000)**

Solution *Given*: D = 30 mm = 0.03 m; m = 30 kg/s

Refer Fig. 6.45

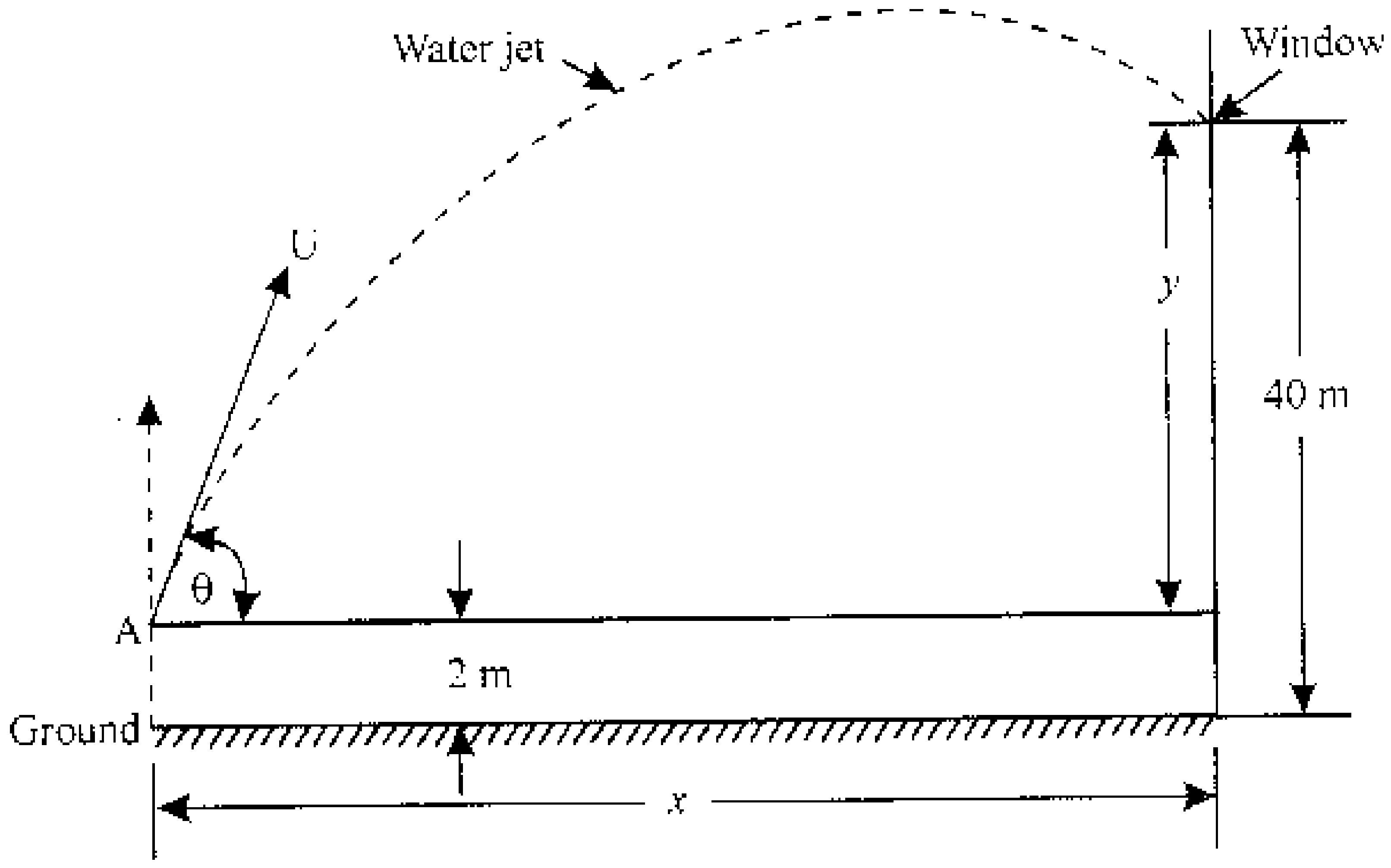


Fig. 6.45

Greatest horizontal distance, *x*:

Now, $m = \rho AU$ (where, U = velocity of water jet)

$$\therefore \qquad U = \frac{m}{\rho A} = \frac{30}{1000 \times \frac{\pi}{4} \times (0.03)^2} = 42.44 \text{ m/s}$$

Let, θ = Angle of inclination of the nozzle.

Then, $x = U\cos\theta \times t \quad \text{or} \quad t = \frac{x}{U\cos\theta}$

$$y = U\sin\theta \times t - \frac{1}{2}gt^2$$

$$= U\sin\theta \times \frac{x}{U\cos\theta} - \frac{1}{2}g\left(\frac{x}{U\cos\theta}\right)^2$$

$$= x\tan\theta - \frac{1}{2}g\frac{x^2}{U^2\cos^2\theta} = x\tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

or $$x\tan\theta - \frac{gx^2}{2U^2}\sec^2\theta - y = 0 \qquad \text{...}(i)$$

The maximum value of x is obtained by differentiating (i) w.r.t. θ, and putting $\frac{dx}{d\theta} = 0$

$$\therefore \quad \left[x\sec^2\theta + \tan\theta \times \frac{dx}{dt}\right] - \frac{g}{2U^2}\left[x^2 \times 2\sec\theta \times \sec\theta\tan\theta + \sec^2\theta \times 2x \times \frac{dx}{d\theta}\right] = 0$$

Putting $\frac{dx}{d\theta} = 0$, we get

$$x\sec^2\theta - \frac{g}{2U^2}(2x^2\sec^2\theta.\tan\theta) = 0$$

or $$x\sec^2\theta = \frac{g}{2U^2} \times 2x^2 \sec^2\theta.\tan\theta$$

or $$x = \frac{U^2}{g\tan\theta}$$

Also $y = 40 - 2 = 38$ m

Substituting for x and y in eqn. (i), we get

$$\frac{U^2}{g\tan\theta} \times \tan\theta - \frac{g}{2U^2} \times \left(\frac{U^2}{g\tan\theta}\right)\sec^2\theta - 38 = 0$$

$$\frac{U^2}{g} - \frac{U^2}{2g\sin^2\theta} - 38 = 0$$

Substituting for U = 42.44 m/s, we get

$$\frac{(42.44)^2}{9.81} - \frac{(42.44)^2}{2 \times 9.81 \times \sin^2\theta} - 38 = 0$$

$$183.6 - \frac{91.8}{\sin^2\theta} - 38 = 0$$

$$\sin^2\theta = \frac{91.8}{(183.6 - 38)} = 0.6305$$

or $\sin\theta = 0.794 \quad \text{or} \quad \theta = \sin^{-1}(0.794) = 52.56°$

Hence, $$x = \frac{U^2}{g \tan\theta} = \frac{(42.44)^2}{9.81 \times \tan(52.56°)} = \mathbf{140.58\ m\ (Ans.)}$$

Example 6.49. *The nozzle shown in Fig 6.46. has a jet diameter of 25 mm. The pressures on the water surface on the two sides of the arrangement are* $p_1 = 170$ *kN/m*^{2} *(gauge)* and $p_2 = 300$ *mm of Hg. Determine:*

(*i*) *The discharge through the nozzle;*

(*ii*) *The maximum height of the free jet above the nozzle.* **[IIT Delhi]**

Solution. Diameter of the jet = 25 mm = 0.025 m

Pressure $p_1 = 170$ kN/m^2

Pressure $p_2 = 300$ mm or 0.3 m of Hg

$= 0.3 \times 13.6 = 4.08$ m of water.

(*i*) **Discharge through the nozzle, Q:**

Applying Bernoulli's equation to 1 (water surface) and 2 (the jet as it emerges from the nozzle), we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{170}{9.81} + 0 + 3 = 4.08 + \frac{V_2^2}{2g} + 0$$

$$17.33 + 3 = 4.08 + \frac{V_2^2}{2g}$$

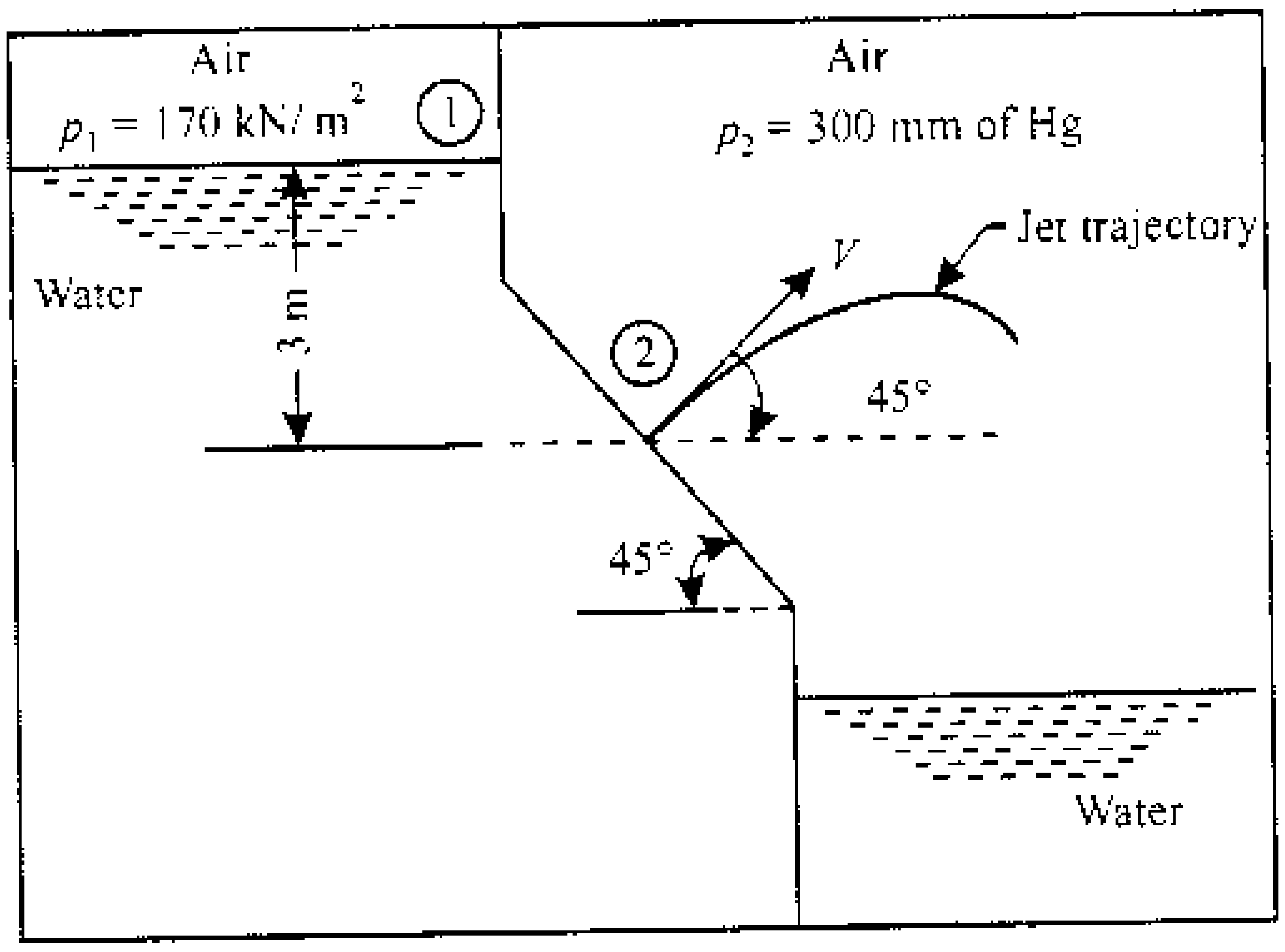


Fig. 6.46

or $$V_2^2 = [(17.33 + 3) - 4.08] \times 2g = 16.25 \times 2 \times 9.81$$

∴ $$V_2 = 17.85 \text{ m/s} \qquad (V_2 = V)$$

Hence $$Q = A \times V_2$$
$$= \pi/4 \times 0.025^2 \times 17.85 = 0.00876 \text{ m}^3\text{/s or } \mathbf{8.76\ litres/sec.\ (Ans.)}$$

(*ii*) **Maximum height of the free jet above the nozzle:**

Vertical component of the jet velocity = $V \sin 45° = 17.85 \sin 45° = 12.62$ m/s

∴ Maximum height to which jet will rise,

$$h = \frac{12.62^2}{2 \times 9.81} = \mathbf{8.12\,m\ (Ans.)}$$

Example 6.50. *A vertical jet of water 75 mm in diameter leaving the nozzle with a 9.2 m/s velocity strikes a horizontal and movable disc weighing 170 N (Fig.6.47). The jet is then deflected horizontally. Determine the vertical distance y above the nozzle tip at which the disc will be held in equilibrium.* **[Roorkee University]**

Solution. Diameter of the jet, $d = 75$ mm $= 0.075$ m

Velocity of the jet at nozzle exit; $V = 9.2$ m/s

Weight of the disc, $W = 170$ N

Vertical Distance, *y*:

Let v = jet velocity at an elevation y.

Applying Bernoulli's theorem between the jet at nozzle exit and the jet at an elevation y, we get

$$\frac{V^2}{2g} = \frac{v^2}{2g} + y$$

or $$V^2 = v^2 + 2gy \qquad \text{...}(i)$$

Also the momentum equation is written as

$$\frac{wQ}{g} . v = 170$$

where, $Q = \pi/4 \times d^2 \times V = \pi/4 \times 0.075^2 \times 9.2$

$= 0.0406$ m³/s, and

$w = 9810$ N/m³

$$\therefore \qquad \frac{9810 \times 0.0406 \times v}{9.81} = 170$$

or $$v = \frac{170 \times 9.81}{9810 \times 0.0406} = 4.187 \text{ m/s}$$

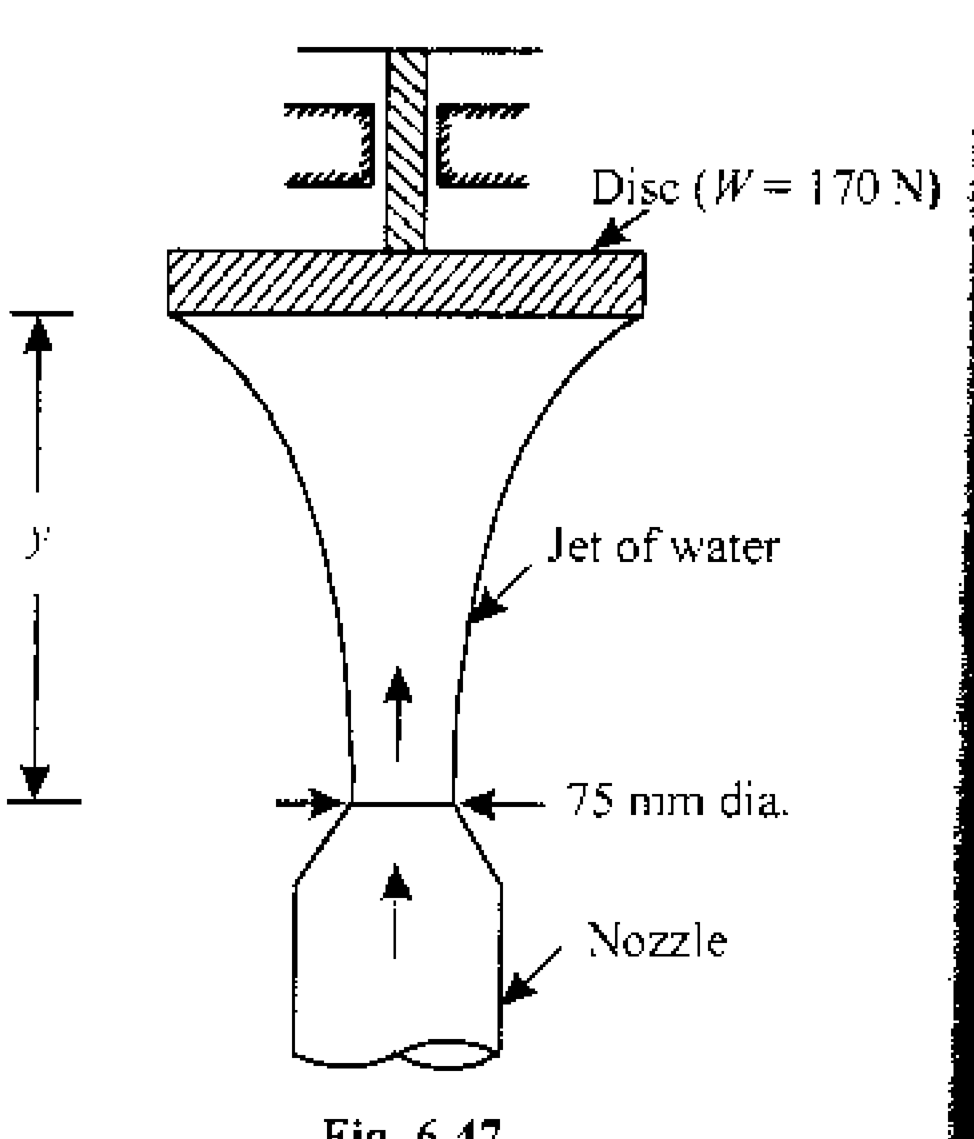


Fig. 6.47

Substituting this value of v in (i), we get

$$9.2^2 = (4.187)^2 + 2 \times 9.81 \times y$$

or $$84.64 = 17.53 + 19.62\,y$$

or $$y = \frac{84.64 - 17.53}{19.62} = \mathbf{3.43\,m\ (Ans.)}$$

6.8. Impulse-Momentum Equation

The **impulse-momentum equation** is one of the basic tools (other being continuity and Bernoulli's equations) for the solution of flow problems. Its application leads to the solution of problems in fluid mechanics which cannot be solved by energy principles alone. Sometimes it is used in conjunction with the energy equation to obtain complete solution of engineering problems.

The momentum equation is based on the *law of conservation of momentum* or *momentum principle* which states as follows:

"The net force acting on a mass of fluid is equal to change in momentum of flow per unit time in that direction"

As per Newton's second law of motion,

$$F = ma$$

where,
m = Mass of fluid,
F = Force acting on the fluid, and
a = Acceleration (acting in the same direction as F)

But acceleration, $a = \dfrac{dv}{dt}$

$$\therefore \quad F = m.\frac{dv}{dt} = \frac{d(mv)}{dt} \quad ...(6.18)$$

('m' is taken inside the differential, being constant)

This equation is known as *momentum principle.* It can also be written as:

$$F.dt = d\,(mv) \quad ...(6.19)$$

This equation is known as **Impulse-momentum equation.** It may be stated as follows:

"The impulse of a force F acting on a fluid mass 'm' in a short interval of time dt is equal to the ge of momentum d(mv) in direction of force".

The impulse-momentum equations are often called simply *momentum equations*.

Applications of impulse-momentum equation:

The impulse-momentum equation is used in the following types of problems:

1. *To determine the resultant force acting on the boundary of flow passage by a stream of fluid as the stream changes its direction, magnitude or both.* Problems of this type are:
   (*i*) Pipe bends, (*ii*) Reducers (*iii*) Moving vanes, (*iv*) Jet propulsion, etc.
2. To determine the *characteristic of flow when there is an abrupt change of flow section.* Problems of this type are:
   (*i*) Sudden enlargement in a pipe. (*ii*) Hydraulic jump in a channel, etc.

Steady flow momentum equation:

The entire flow space may be considered to be made up of innumerable stream tubes. Let us ider one such stream tube lying in the X-Y plane (Fig 6.48) and having steady flow of fluid. Flow be assumed to the uniform and normal to the inlet and outlet areas.

Let, V_1, ρ_1 = Average velocity and density (of fluid mass) respectively at the entrance,

V_2, ρ_2 = Average velocity and density respectively at the exit.

Further let the mass of fluid in the re- 1 2 3 4 shifts to new position 1′2′3′4′ to the effect of external forces on the n after a short interval. Due to gradual se in the flow area in the direction of velocity of fluid mass and hence the ntum is gradually reduced. Since the 1′2′3 4 is common to both the regions 3 4 and 1′2′3′4′ therefore, it will not ence any change in momentum. Ob- y, then the changes in momentum of fluid masses in the sections 1 2 2′1′ 3 3′ 4′ will have to be considered.

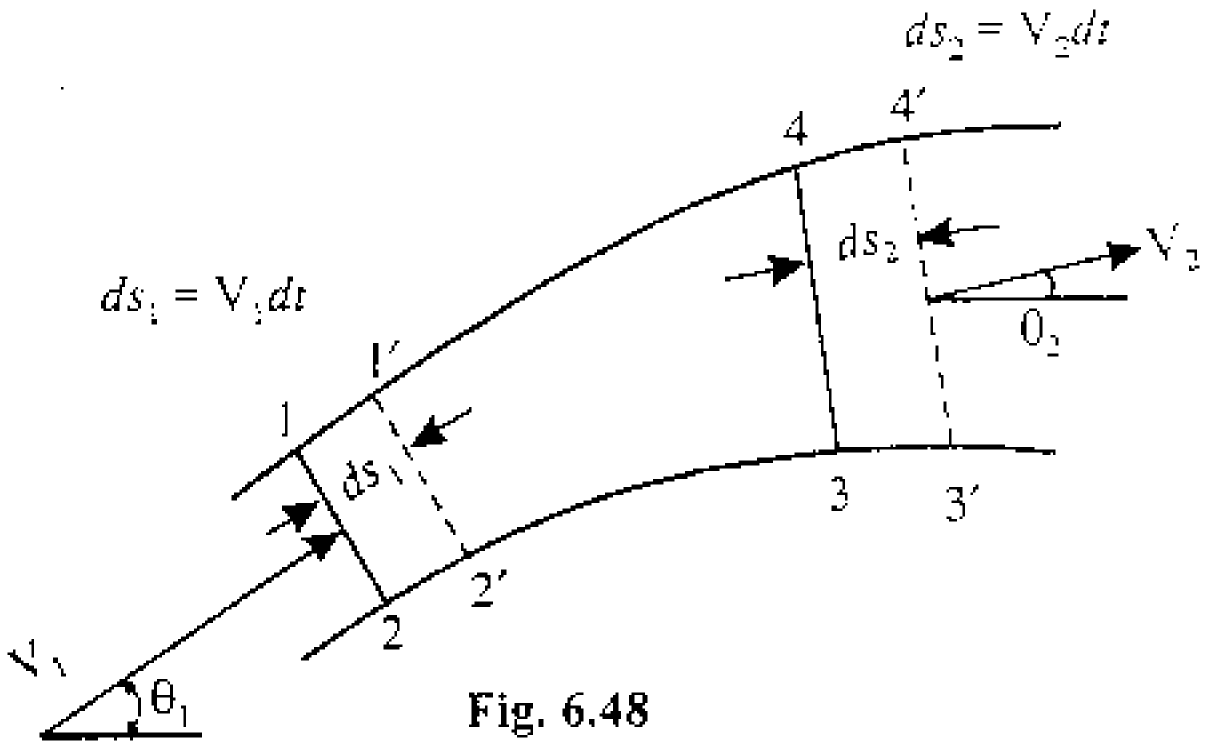


Fig. 6.48

ccording to the *principle of mass conservation,* Fluid mass with the region 12 2′1′ = fluid mass within the region 4 3 3′4′

$$\rho_1 A_1\, ds_1 = \rho_2 A_2 ds_2 \qquad \text{...(6.20)}$$

∴ Momentum of fluid mass contained in the region 1 2 2′1′

$$= (\rho_1 A_1\, ds_1)\, V_1 = (\rho_1 A_1\, V_1 .\, dt)\, V_1$$

Momentum of fluid mass contained in the region 4 3 3′4′

$$= (\rho_2 A_2\, ds_2)\, V_2 = (\rho_2 A_2\, V_2 .\, dt)\, V_2$$

∴ Change in momentum = $(\rho_2 A_2\, V_2 . dt)\, V_2 - (\rho_1 A_1\, V_1 .\, dt)\, V_1$

But, $\rho_1 = \rho_2 = \rho$ ...for steady incompressible flow

and, $A_1 V_1 = A_2 V_2 = Q$ ...from continuity considerations

∴ Change in momentum = $\rho Q (V_2 - V_1)\, dt$

Using impulse-momentum principle, we have

$$Fdt = \rho Q\,(V_2 - V_1) dt \qquad \text{...(6.21)}$$

or

$$F = \frac{wQ}{g}(V_2 - V_1) \qquad \text{...(6.22)}$$

The quantity $\frac{wQ}{g} = \rho Q$ is the mass flow per second and is called **mass flux.**

Resolving V_1 and V_2 along X-axis and Y-axis, we get

Components along *X-axis*: $V_1 \cos\theta_1$ and $V_2 \cos\theta_2$

Components along Y-$\sin\theta_1$ and $V_2 \sin\theta_2$

(where θ_1 and θ_2 are the inclinations with the horizontal of the centre line of the pipe at 1-2 and 3-4).

∴ Components of force F along X-axis and Y-axis are:

$$F_x = \frac{wQ}{g}(V_2 \cos\theta_2 - V_1 \cos\theta_1)$$

$$F_y = \frac{WQ}{g}(V_2 \sin\theta_2 - V_1 \sin\theta_1) \qquad \text{...(6.23)}$$

Eqn. (6.23) represents the components of the force exerted by the *pipe bend on the fluid mass.* Usually, we are interested in the forces by the fluid on the pipe bend. Since action and reaction are equal and opposite (Newton's third law of motion), the fluid mass would exert the same force on the pipe bend but in *opposite direction* and as such the force components exerted by the fluid on the pipe bend are given as follows:

$$\left.\begin{aligned} F_x &= \frac{wQ}{g}(V_1 \cos\theta_1 - V_2 \cos\theta_2) \\ F_y &= \frac{wQ}{g}(V_1 \sin\theta_1 - V_2 \sin\theta_2) \end{aligned}\right\} \qquad \text{...(6.24)}$$

Since the *dynamic* forces (eqn. 6.23) must be supplemented by the *static pressure forces* acting over the inlet and outlet sections, therefore, we have:

$$\left.\begin{aligned} F_x &= \frac{wQ}{g}\,(V_1 \cos\theta_1 - V_2 \cos\theta_2) + p_1 A_1 \cos\theta_1 - p_2 A_2 \cos\theta_2 \\ F_y &= \frac{wQ}{g}\,(V_1 \sin\theta_1 - V_2 \sin\theta_2) + p_1 A_1 \sin\theta_1 - p_2 A_2 \sin\theta_2 \end{aligned}\right\} \qquad \text{...(6.25)}$$

The magnitude of the resultant force acting on the pipe bend

$$F_R = \sqrt{F_x^2 + F_y^2} \qquad \text{...(6.2}$$

or $$\frac{p_2}{w} = 2586 + 5.1 - 20.4 = 2570.7$$

or $$p_2 = 2570.7 \times 0.0116 = 29.82 \text{ kN/m}^2$$

Magnitude and direction of force (resultant) F_R:

Force along X-axis:

$$F_x = \frac{wQ}{g}(V_{1x} - V_{2x}) + (p_1A_1)_x + (p_2A_2)_x$$

where, $V_{1x} = 10$ m/s; $V_{2x} = V_2 \cos 45° = 20 \times 0.707 = 14.14$ m/s

$(p_1A_1)_x = p_1A_1 = 30 \times 1 = 30$ kN; $(p_2A_2)_x = -p_2A_2 \cos 45° = -\,29.82 \times 0.5 \times 0.707 = -\,10.54$ kN

$$\therefore \quad F_x = \frac{0.0116}{9.81} \times 10\,(10 - 14.14) + 30 - 10.54 = 19.41 \text{ kN } (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}(V_{1y} - V_{2y}) + (p_1A_1)_y + (p_2A_2)_y$$

where, $V_{1y} = 0$; $V_{2y} = V_2 \sin 45° = 20 \times 0.707 = 14.14$ m/s

$(p_1A_1)_y = 0$; $(p_2A_2)_y = -p_2A_2 \sin 45° = -\,29.82 \times 0.5 \times 0.707 = -\,10.54$ kN

$$\therefore \quad F_y = \frac{0.0116 \times 10}{9.81}(0 - 14.14) + 0 - 10.54 = -\,10.71 \text{ kN } (\downarrow)$$

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19.41)^2 + (10.71)^2} = 22.17$ **kN (Ans.)**

The direction of F_R with X-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{10.71}{19.41} = 0.5518$$

or $$\theta = \tan^{-1} 0.5518 = 28.88° \text{ or } \mathbf{28°53'}$$ **(Ans.)**

Example 6.52. *250 litres/sec. of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135°, find the magnitude and direction of the resultant force on the bend. The pressure of the water flowing is 400 kN/m². Take specific weight of water as 9.81 kN/m³.* **[AMIE]**

Solution. Diameter of the bend at inlet, $D_1 = 300$ mm $= 0.3$ m

Diameter of the bend at outlet, $D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area $A_1 = A_2 = \pi/4 \times 0.3^2 = 0.07068$ m²

Discharge $Q = 250$ litres/sec $= 0.25$ m³/s.

Pressure, $p_1 = p_2 = 400$ kN/m²

Velocity at section 1-1, $V_1 = \dfrac{Q}{A_1} = \dfrac{0.25}{0.07068} = 3.54$ m/s

Velocity at section 2-2, $V_2 = V_1 = 3.54$ m/s $\qquad (\because A_1 = A_2)$

Force along X-axis:

$$F_x = \frac{wQ}{g}[V_1 - (-V_2 \cos 45°)] + p_1A_1 + p_2A_2 \cos 45°$$

$$= \frac{9.81 \times 0.25}{9.81}[3.54 - (-\,3.54 \times 0.707)]$$

$$+ (400 \times 0.07068) + (400 \times 0.07068 \times 0.$$

$$= 0.25 \times (3.54 - 3.54 \times 0.707) + 28.27 + 19.98$$

$$= 49.76 \text{ kN } (\rightarrow)$$

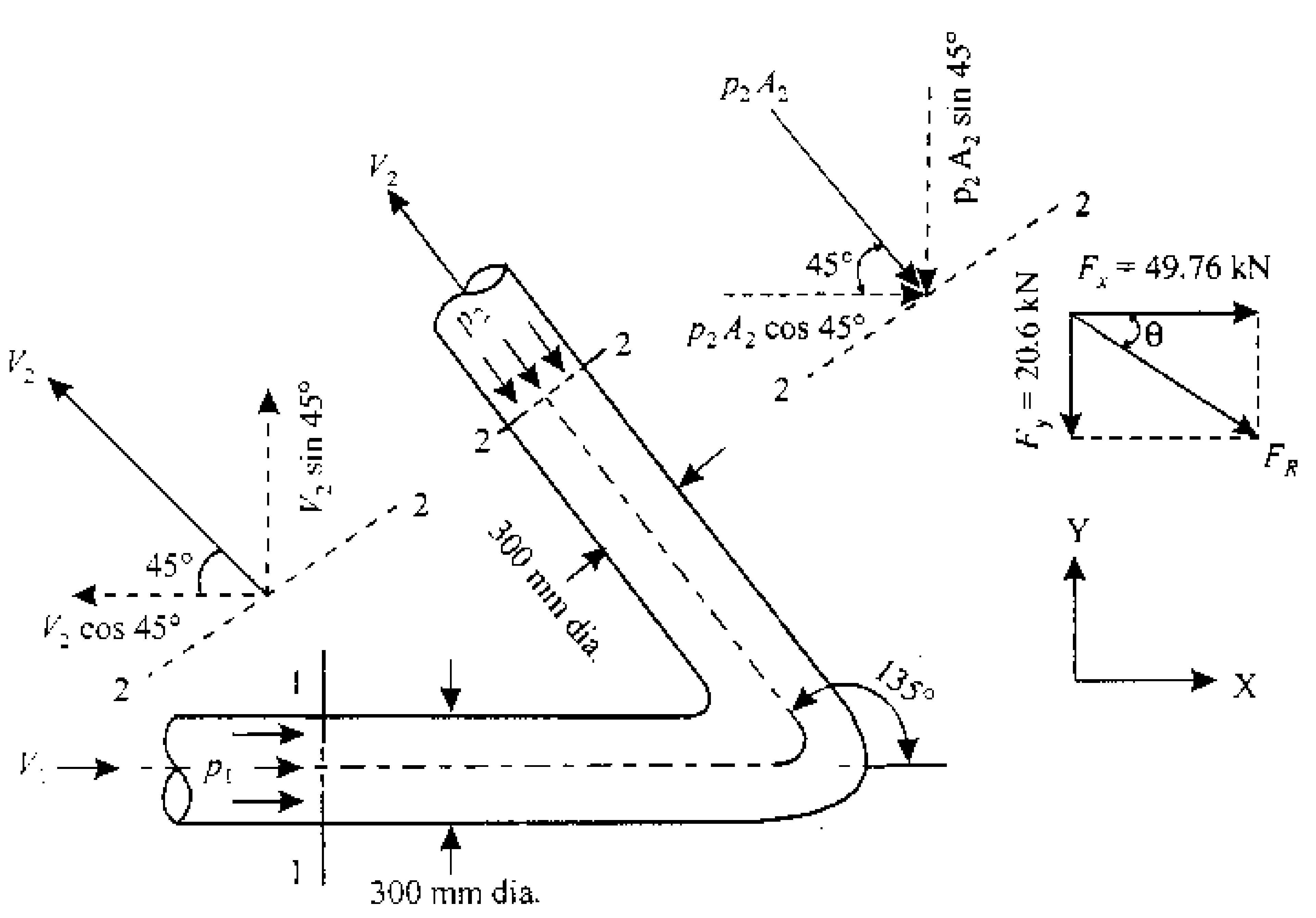


Fig. 6.50

Force along Y-axis:

$$F_y = \frac{wQ}{g}\ [\,0 - V_2 \sin 45°] - p_2 A_2 \sin 45°$$

$$= \frac{9.81 \times 0.25}{9.81}(0 - 3.54 \times 0.707) - 400 \times 0.07068 \times 0.707$$

$$= -\,0.625 - 19.98 = -\,20.6 \text{ kN}(\downarrow)$$

The *magnitude of the resultant force,*

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{49.76^2 + 20.6^2} = \mathbf{53.85\ kN\ (Ans.)}$$

The *direction of* F_R with X- axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{20.6}{49.76} = 0.414$$

$$\therefore \qquad \theta = \tan^{-1} 0.414 = \mathbf{22.5°\ (Ans.)}$$

Example 6.53. *360 litres per second of water is flowing in a pipe. The pipe is bent by 120°. The bend measures 360 mm × 240 mm and volume of the bend is 0.14 m³. The pressure at the nce is 73 kN/m² and the exit is 2.4 m above the entrance section.*

Find the force exerted on the bend.

Solution. Discharge through the pipe, Q = 360 litres/sec = 0.36 m³/s

Volume of bend = 0.14 m³

Diameter of the bend at 1-1, D_1 = 360 mm = 0.36 m

$$\therefore \text{ Area} \qquad A_1 = \frac{\pi}{4} \times 0.36^2 = 0.1018 \text{ m}^2$$

Diameter of the bend at 2-2, D_2 = 240 mm = 0.24 m

$$\text{Area,} \qquad A_2 = \frac{\pi}{4} \times 0.24^2 = 0.04524 \text{ m}^2$$

Velocity at section 1-1, $V_1 = \dfrac{Q}{A_1} = \dfrac{0.36}{0.1018} = 3.54$ m/s

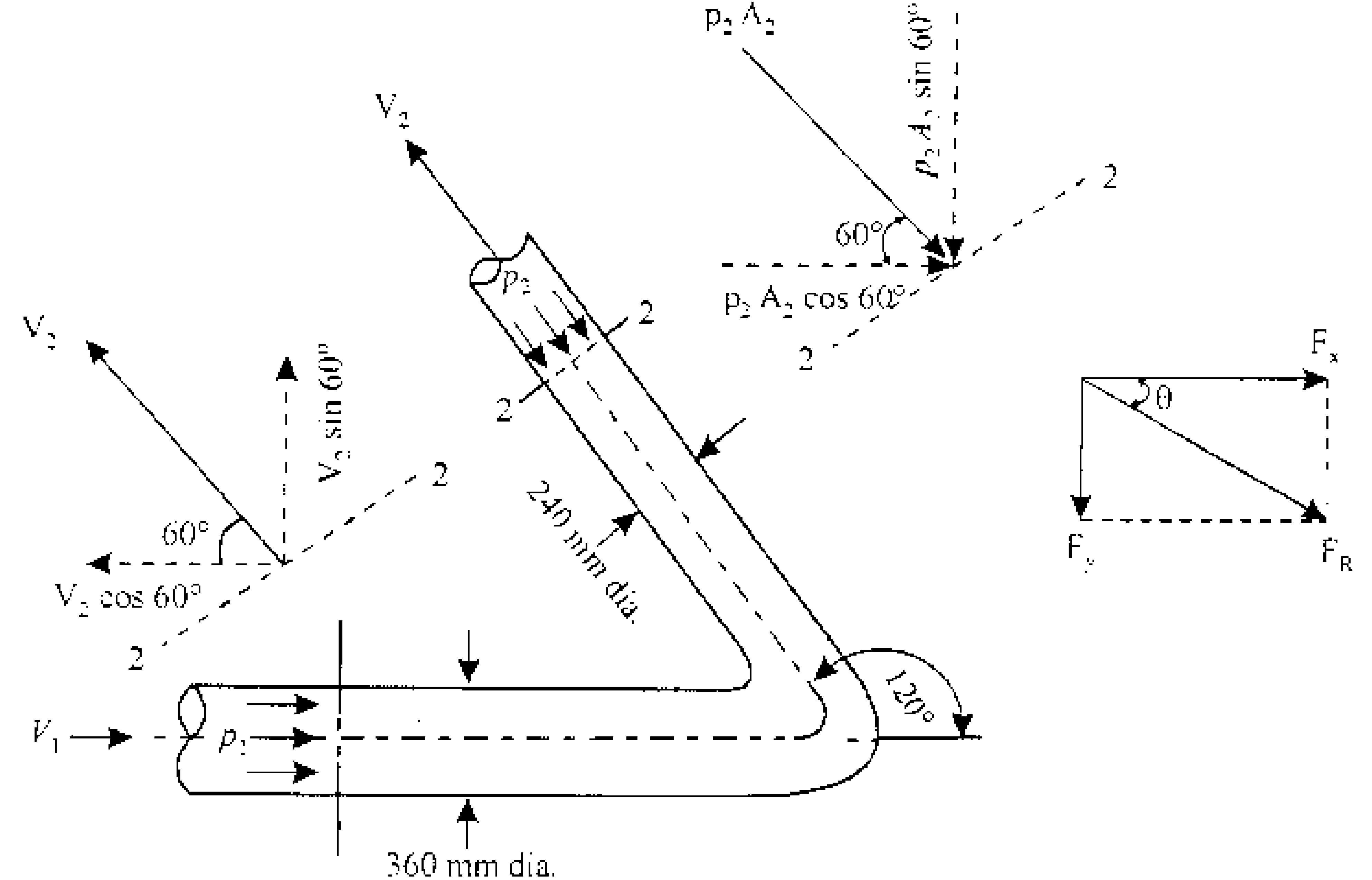


Fig. 6.51

Velocity at section 2-2, $V_2 = \dfrac{Q}{A_2} = \dfrac{0.36}{0.04524} = 7.96$ m/s

Considering a horizontal line through the section 1-1 as datum for elevation head and applying Bernoulli's equation to the sections 1-1 and 2-2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{72}{9.81} + \frac{3.54^2}{2 \times 9.81} + 0 = \frac{p_2}{w} + \frac{7.96^2}{2 \times 9.81} + 2.4 \qquad (\because p_1 = 72 \text{ kN/m}^2 \ldots \textit{Given})$$

$$7.34 + 0.64 = \frac{p_2}{w} + 3.23 + 2.4$$

$$\therefore \qquad \frac{p_2}{w} = 2.35 \text{ or } p_2 = 2.35 \times 9.81 = 23.05 \text{ kN/m}^2$$

Force along the X-axis:

$$F_x = \frac{wQ}{g}\,[V_1 - (-V_2 \cos 60°)] + p_1 A_1 + p_2A_2 \cos 60°$$

$$= \frac{9.81 \times 0.36}{9.81}\,[3.54 - (-7.96 \times 0.5)] - 72 \times 0.1018 - 23.05 \times 0.04524 \times \mathbf{0.5}$$

$$= 0.36\,(3.54 + 3.98) + 7.33 + 0.52 = 10.55 \text{ kN } (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}\,[0 - V_2 \sin 60°] - p_2 A_2 \sin 60° - \text{weight of water in the bend}$$

$$= \frac{9.81 \times 0.36}{9.81}\,(0 - 7.96 \times 0.866) - 23.05 \times 0.04524 \times 0.866 - 0.14 \times 9.8$$

$$= 0.36\,(-6.89) - 0.9 - 1.37 = -4.75 \text{ kN } (\downarrow)$$

Magnitude of the resultant force acting on the bend,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{10.55^2 + 4.75^2} = \mathbf{11.57\ kN\ (Ans.)}$$

Direction of the resultant force with the X-axis,

$$\tan\theta = \frac{F_y}{F_x} = \frac{4.75}{10.55} = 0.4502 \text{ or } \theta = \mathbf{24.24° \ (Ans.)}$$

Example 6.54. *Fig. 6.52 shows a 90° reducer-bend through which water flows.* ***The pressure at the inlet is*** *210 kN/m² (gauge) where the cross-sectional area is 0.01 m². At the exit section, the area is 0.0025 m² and the velocity is 16 m/s. The pressure at the exit is atmospheric. Determine the magnitude and direction of the resultant force on the bend.*

Solution. Area at section 1-1, $A_1 = 0.01$ m²

Area at section 2-2, $A_2 = 0.0025$ m²

Velocity at the exit, $V_2 = 16$ m/s.

Discharge, $Q = A_2 V_2 = 0.0025 \times 16 = 0.04$ m³/s.

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.04}{0.01} = 4 \text{ m/s}$$

Assume the bend is horizontal and in *XY* plane.

Force along X-axis:

$$F_x = \frac{wQ}{g}(V_1 - 0) + p_1 A_1$$

$$= \frac{9.81 \times 0.04}{9.81}(4 - 0) + 210 \times 0.01 = 0.16 + 2.1 = 2.26 \text{ kN } (\rightarrow)$$

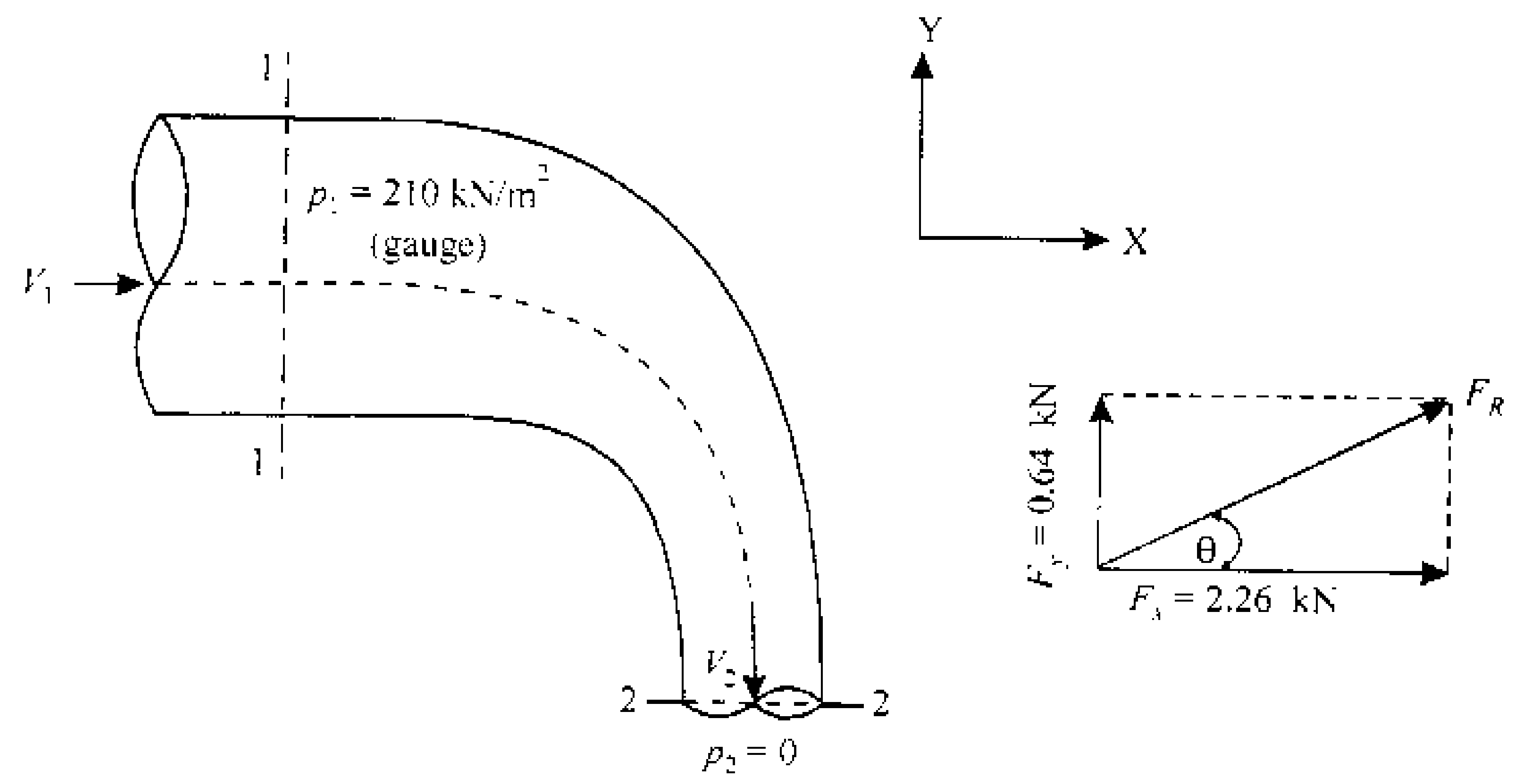


Fig. 6.52

Force along Y-axis:

$$F_y = \frac{wQ}{g}[0 - (-V_2)] + p_2 A_2$$

$$= \frac{9.81 \times 0.04}{9.81}(0 + 16) + 0 = 0.64 \text{ kN } (\uparrow)$$

Magnitude of the resultant force acting on the bend,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{2.26^2 + 0.64^2} = \mathbf{2.35\ kN\ (Ans.)}$$

Direction of the resultant force with the X-axis,

$$\tan\theta = \frac{F_y}{F_x} = \frac{0.64}{2.26} = 0.2832 \quad \therefore\ \theta = \tan 0.2832 = 15.8°$$

$$\therefore \qquad \theta = \mathbf{15.8°\ (Ans.)}$$

Example 6.55. *Water enters a reducing pipe horizontally and comes out vertically in the downward direction. If the inlet velocity is 5 m/s and pressure is 80 kPa* (*gauge*) and the diameters at the entrance and exit sections are *30 cm and 20 cm respectively, calculate the components of the reaction acting on the pipe.* **(AMIE)**

Solution. *Given*: D_1 = 30 cm = 0.3 m; D_2 = 20 cm = 0.2 m; V_1 = 5 m/s; p_1 = 80 kPa = 80 kN/m^2.

Components of the reaction acting on the pipe: Refer to Fig. 6.53.

From continuity equation, we have

$$Q = A_1V_1 = \pi/4 \times 0.3^2 \times 5 = 0.3534\ \text{m}^3/\text{s}$$

$$A_1V_1 = A_2V_2 \quad \text{or} \quad \frac{\pi}{4} \times 0.3^2 \times 5 = \frac{\pi}{4} \times 0.2^2 \times V_2$$

$$\therefore \qquad V_2 = 11.25\ \text{m/s}$$

Assume the bend is horizontal and in XY plane.

Applying Bernoulli's equation between sections (1) and (2), we have

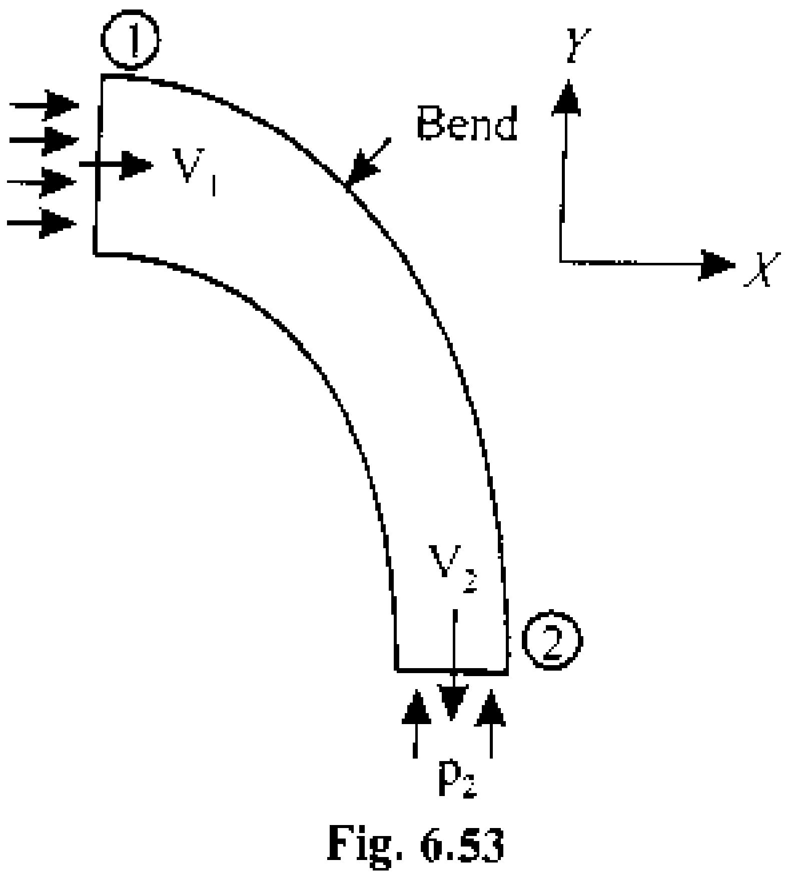


Fig. 6.53

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

or,

$$\frac{p_2}{w} = \frac{p_1}{w} + \left(\frac{V_1^2 - V_2^2}{2g}\right)$$

$(\because z_1 = z_2)$

$$= \frac{80}{9.81} + \left(\frac{5^2 - 11.25^2}{2 \times 9.81}\right) = 2.978\ \text{m}$$

or,

$$p_2 = 9.81 \times 2.978 = 29.22\ \text{kN/m}^2$$

Force along X-axis:

$$F_x = \frac{wQ}{g}(V_1 - 0) + p_1A_1$$

$$= \frac{9.81 \times 0.3534}{9.81}(5 - 0) + 80 \times \frac{\pi}{4} \times 0.3^2 = \mathbf{7.42\ kN\ (Ans.)}$$

$$F_y = \frac{wQ}{g}[0 - (-V_2)] + p_2A_2 = \frac{wQV_2}{g} + p_2A_2$$

$$= \frac{9.81 \times 0.3534 \times 11.25}{9.81} + 29.22 \times \frac{\pi}{4} \times 0.2^2 = \mathbf{4.89\ kN\ (Ans.)}$$

Example 6.56. *The angle of a reducing bend is 60°* (*that is deviation from initial direction to final direction*). *Its initial diameter is 300 mm and final diameter 150 mm and is fitted in a pipeline, carrying a discharge of 360 litres/sec. The pressure at the commencement of the bend is 2.943 bar. The friction loss in the pipe bend may be assumed as 10 percent of kinetic energy at exit of the bend. Determine the force exerted by the reducing bend.* **[UPSC Exams.]**

Solution. Diameter at the inlet, D_1 = 300 mm = 0.3 m

∴ Area, $A_1 = \pi/4 \times 0.3^2 = 0.07068\ \text{m}^2$

Diameter at the outlet, $D_2 = 150$ mm $= 0.15$ m

∴ Area, $A_2 = \pi/4 \times 0.15^2 = 0.01767\ \text{m}^2$

Discharge through the bend $Q = 3600$ litres/sec $= 0.36\ \text{m}^3/\text{s}$

Pressure at the inlet, $p_1 = 2.943$ bar $= 294.3\ \text{kN/m}^2$

[∵ 1 bar $= 10^5\ \text{N/m}^2 = 10^2\ \text{kN/m}^2$

∴ Velocity at the inlet, $V_1 = \dfrac{Q}{A_1} = \dfrac{0.36}{0.07068} = 5.09$ m/s

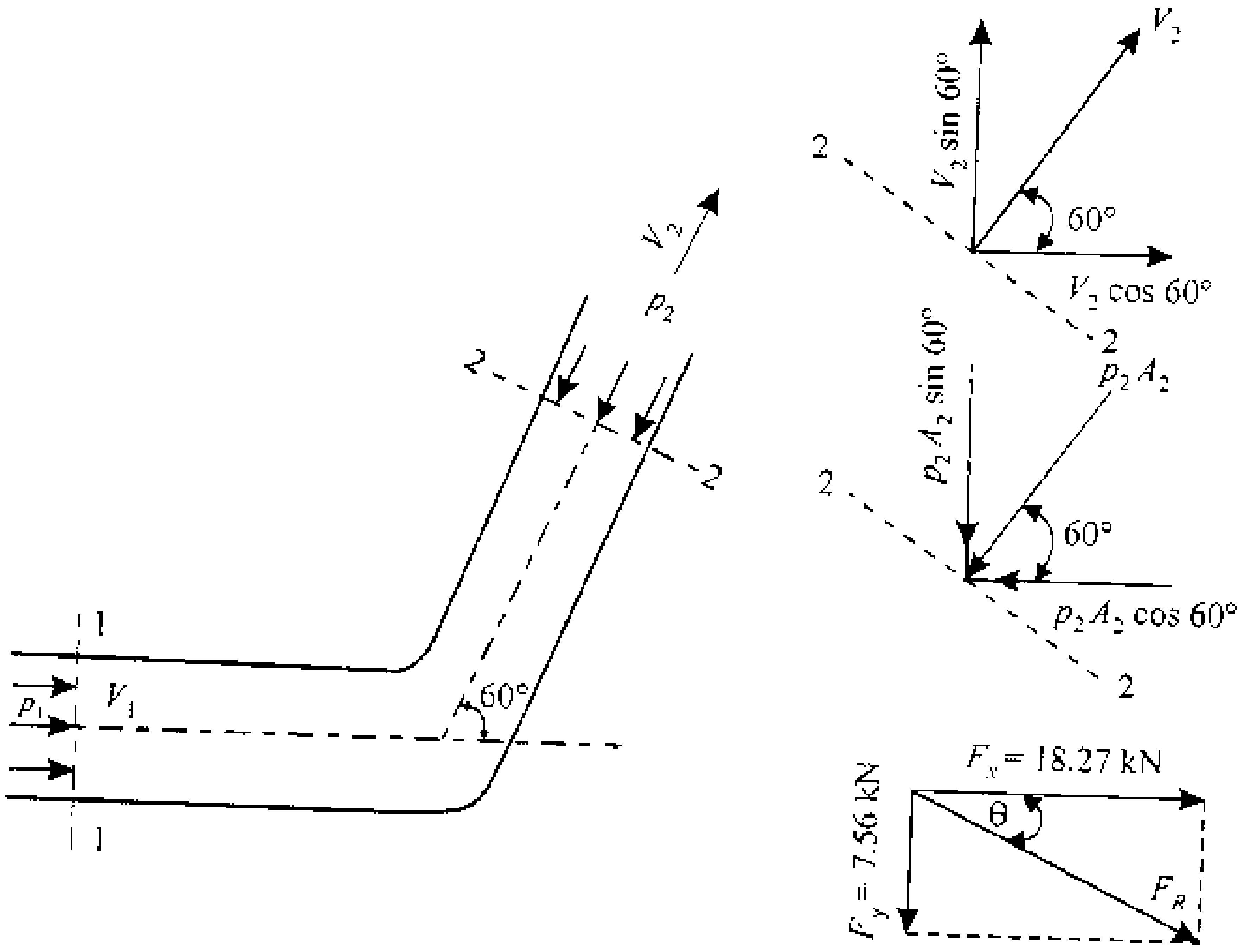


Fig. 6.54

Velocity at the outlet, $V_2 = \dfrac{Q}{A_2} = \dfrac{0.36}{0.01767} = 20.37$ m/s

Friction loss in the pipe $= 0.1 \times \dfrac{V_2^2}{2g}$ ...(*Given*)

Applying Bernoulli's equation at section 1-1 and 2-2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + 0.1 \times \frac{V_2^2}{2g}$$

(∴ $z_1 = z_2$ since the bend lies in the horizontal plane)

$$\frac{294.3}{9.81} + \frac{5.09^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{20.37^2}{2 \times 9.81} + 0.1 \times \frac{20.37^2}{2 \times 9.81}$$

$$30 + 1.32 = \frac{p_2}{w} + 21.15 + 2.11$$

$\therefore \quad \frac{p_2}{w} = 8.06 \quad$ or $\quad p_2 = 9.81 \times 8.06 = 79.07$ kN/m^2

Force along X-axis:

$$F_x = \frac{wQ}{g}[V_1 - V_2 \cos 60°] + p_1 A_1 - p_2 A_2 \cos 60°$$

$$= \frac{9.81 \times 0.36}{9.81}(5.09 - 20.37 \times 0.5) + 294.3 \times 0.07068 - 79.07 \times 0.01767 \times 0.5$$

$$= -1.8342 + 20.8 - 0.698 = 18.27 \text{ kN } (\rightarrow)$$

Force along the Y-axis:

$$F_y = \frac{wQ}{g}(0 - V_2 \sin 60°) - p_2 A_2 \sin 60°$$

$$= \frac{9.81 \times 0.36}{9.81}(0 - 20.37 \sin 60°) - 79.07 \times 0.01767 \times \sin 60°$$

$$= -6.35 - 1.21 = -7.56 \text{ kN or } 7.56 \text{ kN } (\downarrow)$$

Magnitude of the resultant force acting on the bend,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{18.27^2 + 7.56^2} = \mathbf{19.77\ kN}$$

Direction of the resultant force with the X-axis,

$$\tan\theta = \frac{F_y}{F_x} = \frac{7.56}{18.27} = 0.4138$$

$$\theta = \tan^{-1} 0.4138 = \mathbf{22.48°}$$

An equal and opposite force will be exerted by the reducing bend. (**Ans.**)

Example 6.57. *A 0.4 m × 0.3 m, 90° vertical bend carries 0.5 m³/s oil of specific gravity 0.85 with a pressure of 118 kN/m² at inlet to the bend. The volume of the bend is 0.1 m³. Find the magnitude and direction of the force on the bend. Neglect friction and assume both inlet and outlet sections to be at same horizontal level. Also assume that water enters the bend at 45° to the horizontal.* **(AMIE Summer, 2000)**

Solution. *Given*: $D_1 = 0.4$ m, $\therefore A_1 = \frac{\pi}{4} \times 0.4^2 = 0.12566$ m^2; $D_2 = 0.3$ m;

$\therefore A_2 = \frac{\pi}{4} \times 0.3^2 = 0.07068$ m^2; $Q = 0.5$ m^3/s; $S_{oil} = 0.85$; $p_1 = 118$ kN/m^2; Volume of bend $= 0.1$ m^3

Refer to Fig. 6.55.

$$V_1 = \frac{Q}{A_1} = \frac{0.5}{0.12566} = 3.98 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.5}{0.07068} = 7.074 \text{ m/s}$$

Applying Bernoulli's equation between sections (1) and (2), we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{losses}$$

Since $z_1 = z_2$ and losses are negligible (Given)

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

Substituting the values, we get

$$\frac{118\times10^3}{(1000\times0.85\times9.81)}+\frac{(3.98)^2}{2\times9.81}=\frac{p_2}{(1000\times0.85\times9.81)}+\frac{(7.074)^2}{2\times9.81}$$

or
$$\frac{118\times10^3}{850}+\frac{(3.98)^2}{2}=\frac{p_2}{850}+\frac{(7.074)^2}{2}$$

or
$$p_2=850\left[\frac{118\times10^3}{850}+\frac{(3.98)^2-(7.074)^2}{2}\right]=103464\ \text{N/m}^2$$

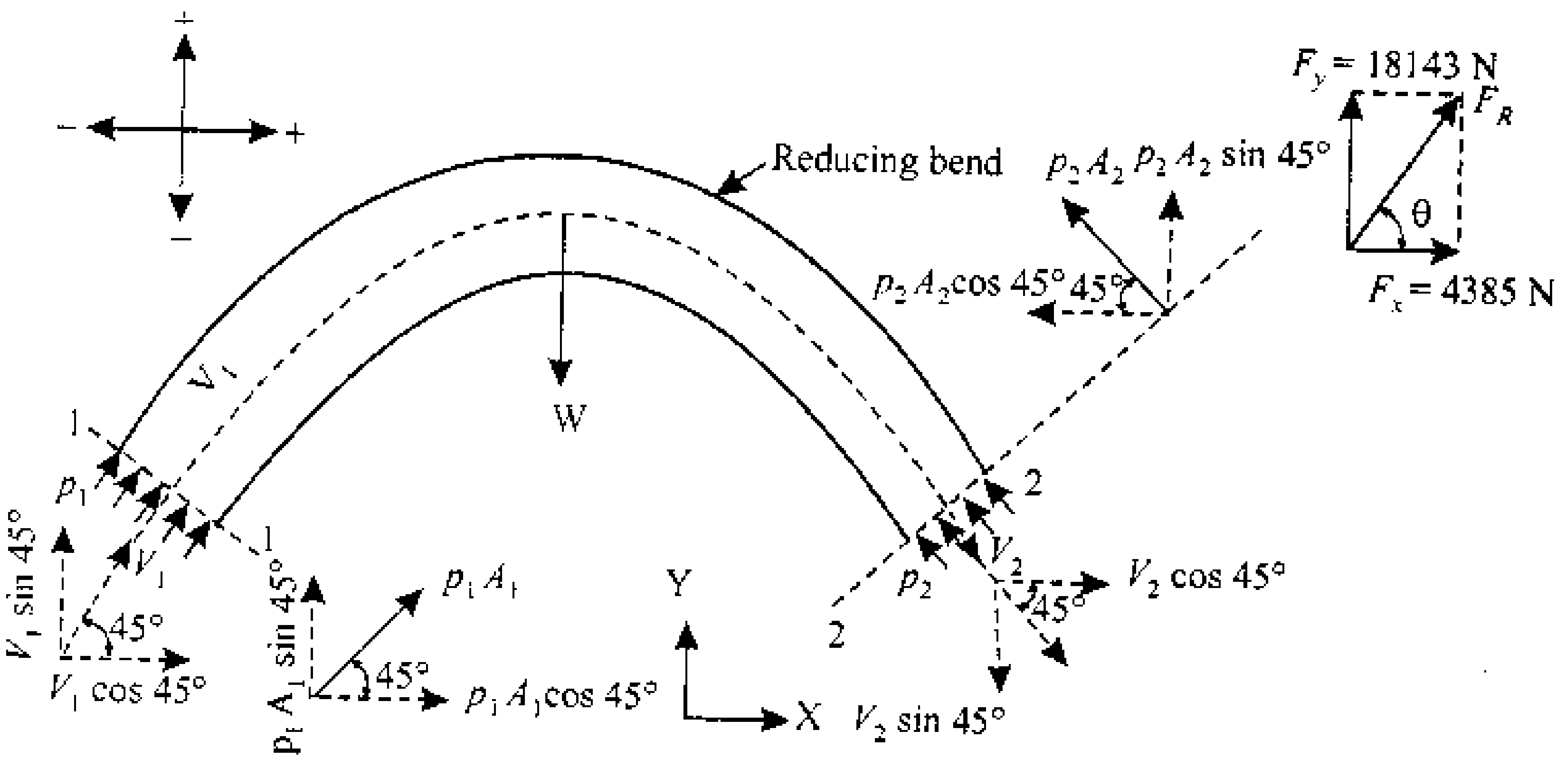


Fig. 6.55

Magnitude and direction of the force (resultant), F_R:

Force along X-axis:

$$F_x=\frac{wQ}{g}\left[V_{1x}-V_{2x}\right]+(p_1A_1)_x+(p_2A_2)_x$$

where, $V_{1x}=V_1\cos45°=3.98\cos45°=2.814$ m/s, $V_{2x}\cos45°=7.074\times\cos45°=5.0$ m/s

$(p_1A_1)_x=p_1A_1\cos45°=118\times10^3\times0.12566\cos45°=10484.89$ N,

$(p_2A_2)_x=-p_2A_2\cos45°=-103464\times0.07068\cos45°=-5170.96$ N

$$\therefore\quad F_x=\frac{(1000\times0.85\times9.81)0.5}{9.81}[2.814-5.0]+10484.89+(-5170.96)\simeq4385\ \text{N}\ (\rightarrow)$$

Force along Y-axis:

$$F_y=\frac{wQ}{g}\left[V_{1y}-V_{2y}\right]+(p_1A_1)_y+(p_2A_2)_y-W$$

$V_{1y}=V_1\sin45°=3.98\sin45°=2.814$ m/s, $V_{2y}=-V_2\sin45°$
$=-7.074\times\sin45°=-5.0$ m/s

$(p_1A_1)_y=p_1A_1\sin45°=118\times10^3\times0.12566\sin45°=10484.89$ N

$(p_2A_2)_y=p_2A_2\sin45°=103464\times0.07068\times\sin45°=5170.96$ N

$W=0.1(0.85\times1000)\times9.81=833.85$ N

$$\therefore\ F_y=\frac{(1000\times0.85\times9.81)0.5}{9.81}[2.814-(-5.0)]+10484.89+5170.96-833.85=18143\ \text{N}\ (\uparrow)$$

∴ Resultant force on the bend

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(4385)^2 + (18143)^2} = \mathbf{18665\ (Ans.)}$$

Inclination of F_x to the X-direction is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{18143}{4385}\right) = \mathbf{76.4^\circ\ (Ans.)}$$

Example 6.58. *The following data refer to the Y-fitting shown in Fig. 6.56.*

Reading of the pressure gauge at section 1-1 = 30 kN/m^2.

Discharge in at the section 1-1 = 15 *litres/sec.*

Discharge out from the section 3-3 = 5 *litres/sec.*

Assuming one-dimensional flow, neglecting elevation head and energy loss while making the energy balance, determine:

(*i*) *The pressures at the sections* 2-2 *and* 3-3;

(*ii*) *The force needed to hold the fitting in position.* **(Roorkee University)**

Solution. Refer to Fig. 6.56.

Given: D_1 = 100 mm = 0.1 m. D_2 = 80 m, = 0.08 m; D_3 = 60 mm = 0.06 m

∴ Area $A_1 = (\pi/4) \times 0.1^2 = 0.007854$ m^2, area $A_2 = (\pi/4) \times 0.08^2 = 0.005026$ m^2, and

area $A_3 = (\pi/4) \times 0.06^2 = 0.002827$ m^2

Pressure at this section 1-1, p_1 = 30 kN/m^2

Now, $Q_1 = Q_2 - Q_3$ or $15 = Q_2 + 5$

∴ Q_2 = 10 litres/sec.

Velocity at the section 1-1, $V_1 = \dfrac{Q_1}{A_1} = \dfrac{15 \times 10^{-3}}{0.007854} = 1.91$ m/s

Velocity at the section 2-2, $V_2 = \dfrac{Q_2}{A_2} = \dfrac{10 \times 10^{-3}}{0.005026} = 1.99$ m/s

Velocity at the section 3-3, $V_3 = \dfrac{Q_3}{A_3} = \dfrac{5 \times 10^{-3}}{0.002827} = 1.77$ m/s

(*i*) **Pressures at section 2-2 and 3-3; p_2, p_3:**

Applying Bernoulli's equation between sections 1-1 and 2-2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{p_2}{w} - \frac{p_1}{w} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$ (Neglecting elevation datum)

or $$\frac{p_2 - p_1}{w} = \frac{V_1^2 - V_2^2}{2g}$$

$$= \frac{1.91^2 - 1.99^2}{2 \times 9.81} = -0.0159$$

or $p_2 - p_1 = -w \times 0.0159 = -9.81 \times 0.0159 = -0.156$ kN/m^2

(∵ w = 9.81 kN/m^3 for water)

or $\mathbf{p_2} = -0.156 + p_1 = -0.156 + 30 = \mathbf{29.84}$ **kN/m^2 (Ans.)**

Similarly, for sections 1-1 and 3-3; we get

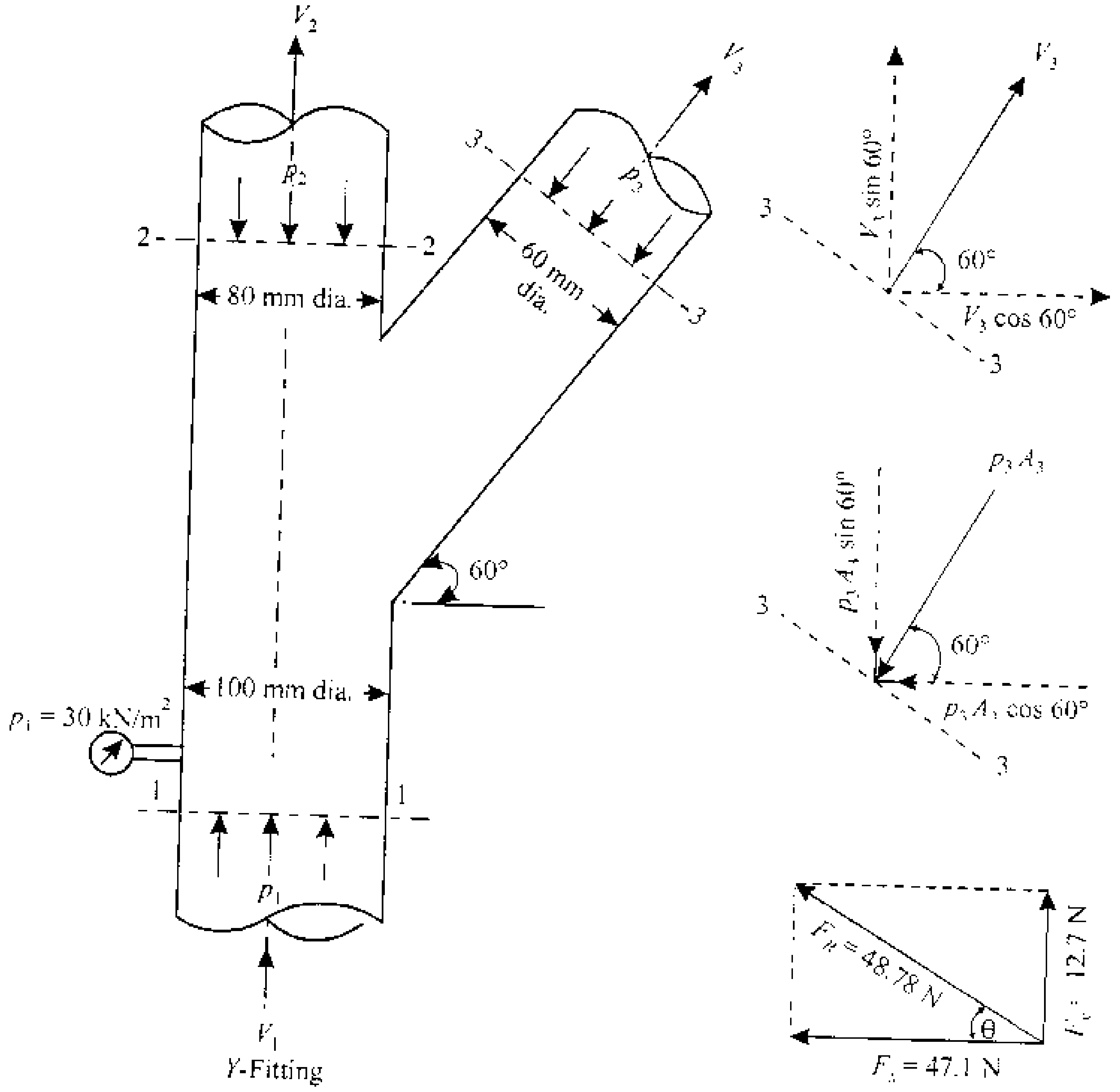


Fig.6.56

$$\frac{p_3 - p_1}{w} = \frac{V_1^2 - V_3^2}{2g} = \frac{1.91^2 - 1.77^2}{2 \times 9.81} = 0.02626$$

or $\quad p_3 - p_1 = 9.81 \times 0.2626 = 0.26$

∴ $\quad$ **$p_3 = 0.26 + p_1 = 0.26 + 30 = 30.26$ kN/m² (Ans.)**

(*ii*) **Force needed to hold the fitting in position :**

Force along X-axis:

$$F_x = \frac{wQ_3}{g}(0 - V_3 \cos 60°) - p_3 A_3 \cos 60°$$

(The velocities V_1 and V_2 and pressure p_1 and p_2 have no components in X-direction)

$$= \frac{9.81 \times 5 \times 10^{-3}}{9.81}(0 - 1.77 \times 0.5) - 30{\cdot}26 \times 0.002827 \times 0.5$$

$$= -0.004425 - 0.0427 = -0.00471 \text{ kN or } -47.1 \text{ N}$$

or $\quad F_x = 47.1$ N (←)

Force along Y-axis:

$$F_y = \frac{wQ_1}{g}V_1 - \frac{wQ_2}{g}V_2 - \frac{wQ_3}{g}V_3 \sin 60° + p_1 A_1 - p_2 A_2 - p_3 A_3 \sin 60°$$

$$= \frac{w}{g}(Q_1 V_1 - Q_2 V_2 - Q_3 V_3 \sin 60°) + p_1 A_1 - p_2 A_2 - p_3 A_3 \sin 60°$$

$$= \frac{9.81}{9.81}\ (15 \times 10^{-3} \times 1.91 - 10 \times 10^{-3} \times 1.99 - 5 \times 10^{-3} \times 1.77 \times 0.866)$$

$$+ 30 \times 0.007854 - 29.84 \times 0.005026 - 30.26 \times 0.002827 \times 0.866$$

$$= (0.02865 - 0.0199 - 0.007664) + 0.2356 - 0.1499 - 0.0741$$

$$= 0.0127 \text{ kN or } 12.7 \text{ N } (\uparrow)$$

The magnitude of resultant force acting on the fitting is

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{47.1^2 + 12.7^2} = \mathbf{48.78\,N\ (Ans.)}$$

and the *direction* of the resultant force with X-axis (Fig. 6.56) is

$$\tan\theta = \frac{F_y}{F_x} = \frac{12.7}{47.1} = 0.2696$$

$$\therefore \qquad \theta = \mathbf{15.09° \ (Ans.)}$$

An equal and opposite force will be needed to hold the fitting in position. **(Ans.)**

Example 6.59. *At inlet to a horizontal pipe of radius a, fitted at side of a vertical tank, the velocity distribution is uniform with magnitude V_0. But at the outlet section, where the flow is fully developed, the velocity distribution is given by,*

$$u = 2V_0\left(1 - \frac{r^2}{a^2}\right)$$

where u is the velocity at any radius r from the axis of the pipe. Determine the horizontal force required to hold the pipe in position. **(AMIE Summer, 2000)**

Solution. For uniform velocity *at inlet,*

Momentum = $\rho A V_0^2 = \rho\pi a^2 V_0^2$ (where, a = radius of the pipe)

At outlet, momentum $= \int \rho u^2\, dA$

$$= \rho \int_0^a \left\{2V_0\left(1 - \frac{r^2}{a^2}\right)\right\}^2 2\pi r \,.\, dr$$

$$= 8\pi\, \rho \frac{V_0^2}{a^4} \int_0^a (a^2 - r^2)^2\, r \,.\, dr$$

$$= 8\pi\, \rho \frac{V_0^2}{a^4} \int_0^a (a^4 r - 2a^2 r^3 + r^5)\, dr$$

$$= 8\pi\, \rho \frac{V_0^2}{a^4}\left[a^4 \times \frac{r^2}{2} - 2a^2 \times \frac{r^4}{4} + \frac{r^6}{6}\right]_0^a$$

$$= 8\pi\, \rho \frac{V_0^2}{a^4}\left(\frac{a^6}{2} - \frac{a^6}{2} + \frac{a^6}{6}\right)$$

$$= \frac{4}{3}\pi\, \rho\, V_0^2\, a^2$$

- Assuming that the *pressures at inlet and outlet are same,* the force required is equal to change in momentum, or

Force, $\boldsymbol{F}$ = Momentum at outlet – Momentum at inlet

$$= \frac{4}{3}\pi\rho V_0^2\, a^2 - \rho\pi a^2\, V_0^2 = \pi\rho a^2\, V_0^2\left(\frac{4}{3} - 1\right)$$

$$F + p_1A_1 - p_2A_2 = \rho Q(V_2 - V_1)$$

or $$F = \rho Q(V_2 - V_1) + (p_2 - p_1)A_2$$

($\because$ A_1 = A$_2$ and pressure p_1 is everywhere the same except in the jet)

or $$F = \frac{11.76}{9.81} \times 100\,(1000 - 200) + (93.1 - 98.1)\,10^3 \times \frac{\pi}{4} \times 2^2\ \text{N}$$

or $$F = 80.2 \times 10^3\ \text{N} = 80.2\ \text{kN}$$

Thrust on the rocket is equal and opposite to F = **80.2 kN. (Ans.)**

(*ii*) **Energy supplied by the rocket:**

Absolute velocity of the jet = 1000 – 200 = 800 m/s

Energy per unit weight of air at section (2)

$$= \frac{p}{w} + \frac{V^2}{2g} = \frac{93.1 \times 10^3}{11.76} + \frac{800^2}{2 \times 9.81}$$

$$= 7.92 \times 10^3 + 32.62 \times 10^3 = 40.54 \times 10^3\ \text{Nm/N of air}$$

Energy per unit weight of air at section (1)

$$= \frac{p}{w} = \frac{98.1 \times 10^3}{11.76} = 8.34 \times 10^3\ \text{Nm/N of air}$$

(Absolute velocity being zero)

$\therefore$ Energy supplied by the rocket per unit weight of air

$$= (40.54 - 8.34) \times 10^3\ \text{Nm} = \mathbf{32.2\ kNm/N\ (Ans.)}$$

(*iii*) **Energy supplied per second by the rocket:**

Energy supplied per second by the rocket

$$= wQ \times 32.2 \times 10^3 = 11.76 \times 100 \times 32.2 \times 10^3 = \mathbf{37.86 \times 10^6\ Nm\ (Ans.)}$$

(*iv*) **Power developed by the rocket:**

Power developed by the rocket = Energy supplied per second

$$= 37.86 \times 10^6\ \text{Nm/s} = \mathbf{37.86 \times 10^3\ kW\ (Ans.)}$$

6.9. Kinetic Energy and Momentum Correction Factors (Coriolis Co-efficients)

While deriving Bernoulli's equation, it is assumed that the velocity distribution across a single stream tube is uniform. But if there is an appreciable variation in the velocity distribution (on account of viscous and boundary resistance) correction factors α and β have to be applied to obtain the exact amount of kinetic energy or momentum available at a given cross-section.

Kinetic energy correction factor (α):

'Kinetic energy correction factor' is defined as the ratio of the kinetic energy of flow per second based on actual velocity across a section to the kinetic energy of flow per second based on average velocity across the same section. It is denoted by α. Mathematically,

$$\alpha = \frac{\text{Kinetic energy per second based on actual velocity}}{\text{Kinetic energy per second based on average velocity}} \qquad ...(6.27)$$

Refer to Fig. 6.59.

Let, $\bar{u}$ = Average velocity at the section LL,

u = Local or point or actual velocity,

dA = Elementary area, and

A = Area of cross-section.

For the velocity variation across the section LL of the stream tube the total K.E. for the entire section is given as:

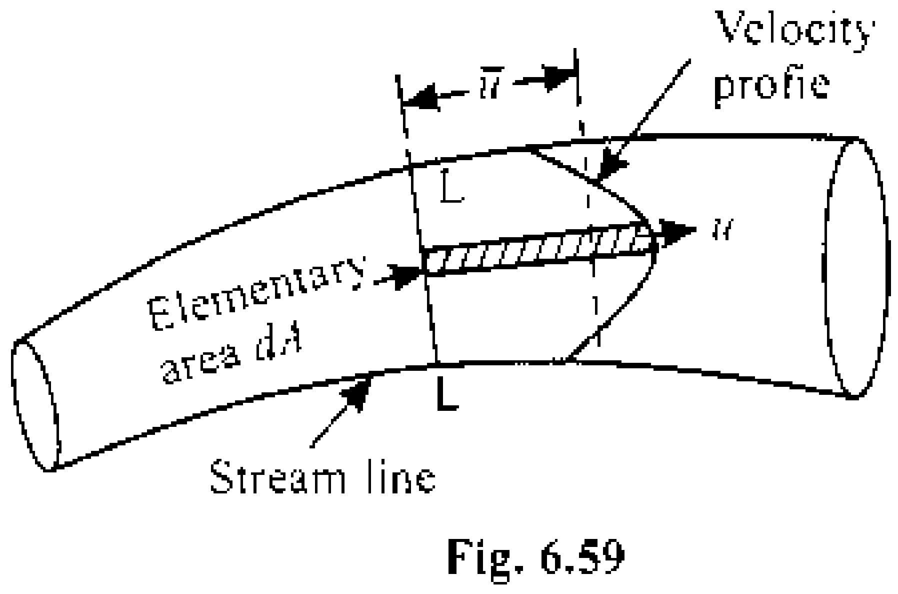


Fig. 6.59

$$K.E. = \frac{1}{2}m\bar{u}^2 = \frac{1}{2}(\rho A\bar{u})\bar{u}^2 = \frac{1}{2}\rho A\bar{u}^3 \quad ...(i)$$

True K.E. for the entire cross-section

$$= \int \frac{1}{2}dm.u^2 = \int \frac{1}{2}(\rho.dA.u)u^2 = \frac{\rho}{2}\int_A u^3\,dA \quad ...(ii)$$

$$\alpha = \frac{\frac{\rho}{2}\int_A u^3\,dA}{\frac{\rho}{2}A\bar{u}^3} = \frac{1}{A}\int\left(\frac{u}{\bar{u}}\right)^3 dA \quad ...(6.28)$$

$\alpha = 1$ *for uniform velocity distribution and tends to become greater than* 1 *as the distribution of velocity becomes less and less uniform.*

$\alpha = 1.02$ to 1.15 for *turbulent flows.*

$\alpha = 2$ for *laminar flow.*

It may be noted that in most of the fluid mechanics computations, α is taken as 1 without introducing much error, since the *velocity is a small percentage of the total head.*

Momentum correction factor (β):

'*Momentum correction factor' is defined as the ratio of momentum of the flow per second based on actual velocity to the momentum of the flow per second based on average velocity across a section.* It is denoted by β. Mathematically,

$$\beta = \frac{\text{momentum per second based on actual velocity}}{\text{momentum per second based on average velocity}} \quad ...(6.29)$$

Refer to Fig. 6.59.

The momentum of fluid mass m is

$$= m\bar{u} = (\rho A\bar{u})\bar{u} = \rho A\bar{u}^2 \quad ...(iii)$$

The true momentum at the section LL is

$$\int_{LL} dm.u = \int_{LL} (\rho dA.u)u = \int_A \rho u^2 dA \quad ...(iv)$$

$$\beta = \frac{\int_A \rho u^2 dA}{\rho A\bar{u}^2} = \frac{1}{A}\int_A \left(\frac{u}{\bar{u}}\right)^2 dA \quad ...(6.30)$$

$\beta = 1$ for *uniform flow,*

$\beta = 1.01$ to 1.07 for *turbulent flow in pipes,* and

$\beta = \frac{4}{3} = 1.33$ for *laminar flow in pipes.*

The value of β may be greater for open channel flow.

In most cases, β is taken as 1.

Note: Since majority of the flow situations are turbulent in character, the usual practice is to assign *unit value to* α and β.

Example 6.61. *The velocity distribution for turbulent flow in pipe is given approximately by Prandtl's one-seventh power law.*

$$u = U_m\left(\frac{y}{r_0}\right)^{1/7}$$

where u is the local velocity of flow at a distance y from the pipe wall, U_m is the maximum velocity at the centre line of the pipe and r_0 is the pipe radius. Find the following:

(*i*) *Average velocity,*

(*ii*) *Kinetic energy correction factor, and*

(*iii*) *Momentum correction factor.* **[Panjab University]**

Solution. (*i*) **Average velocity:**

Refer to Fig. 6.60. Consider an elementary area dA in the form of a ring at a radius $(r_0 - y)$ and of thickness dy then,

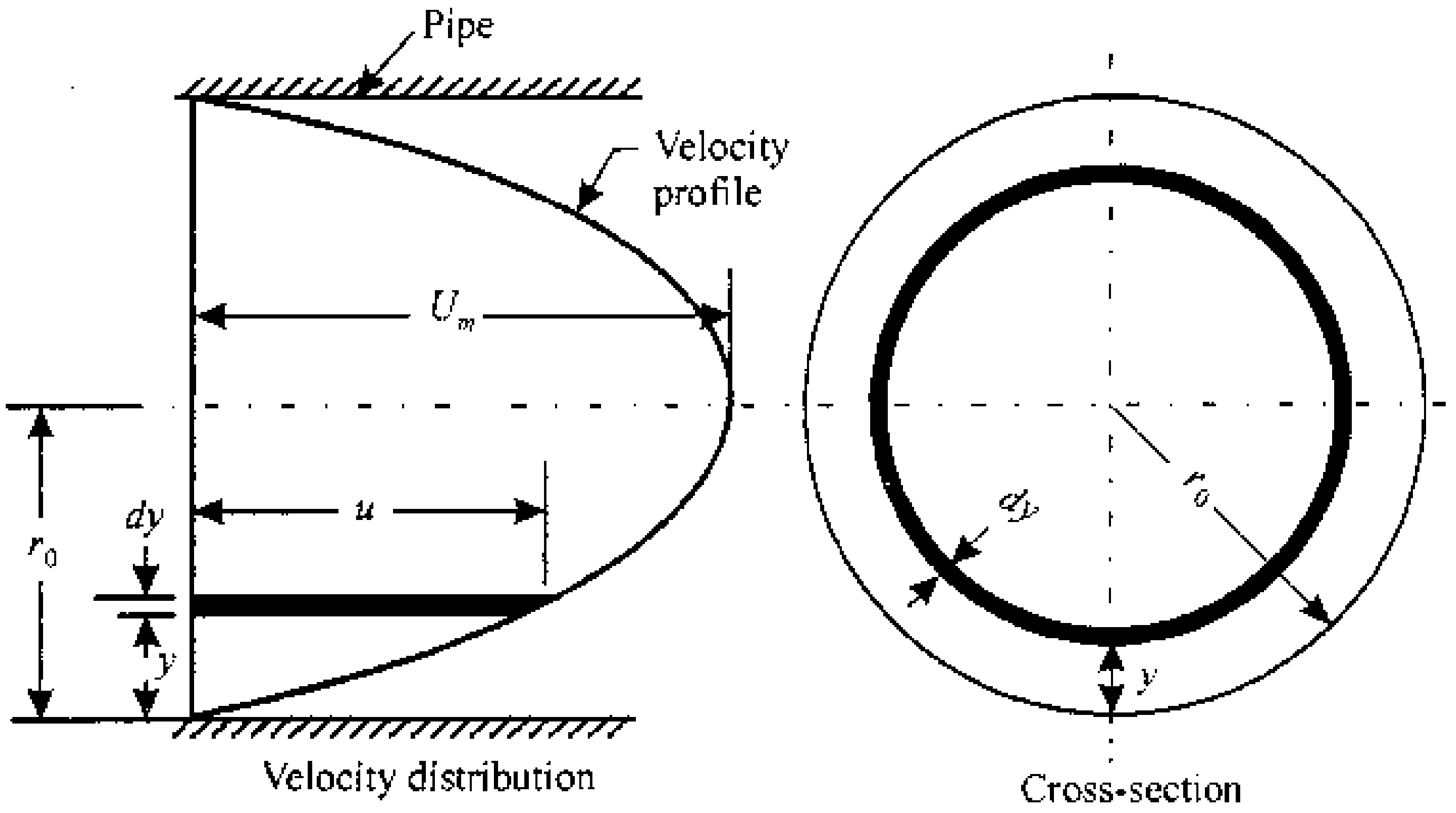


Fig. 6.60. Velocity distribution and cross-section of a circular pipe.

$$dA = 2\pi(r_0 - y)\, dy$$

Rate of fluid flowing through the *ring*

$$= dQ = \text{area of ring element} \times \text{local velocity}$$

$$= 2\pi(r_0 - y)\, dy\, u$$

$\therefore$ Total flow,

$$Q = \int_0^{r_0} 2\pi u (r_0 - y) dy$$

$$= \int_0^{r_0} 2\pi U_m \left(\frac{y}{r_0}\right)^{1/7} \cdot (r_0 - y) dy$$

$$= \frac{2\pi U_m}{(r_0)^{1/7}} \int_0^{r_0} (y)^{1/7} (r_0 - y) dy$$

$$= \frac{2\pi U_m}{(r_0)^{1/7}} \int_0^{r_0} (r_0 y^{1/7} - y^{8/7}) dy$$

$$= \frac{2\pi U_m}{(r_0)^{1/7}} \left[\frac{7}{8} r_0 \cdot y^{8/7} - \frac{7}{15} y^{15/7}\right]_0^{r_0}$$

$$= \frac{2\pi U_m}{(r_0)^{1/7}} \left[\frac{7}{8} r_0 \cdot (r_0)^{8/7} - \frac{7}{15} (r_0)^{15/7}\right]$$

$$= \frac{2\pi U_m}{(r_0)^{1/7}} \left[\frac{7}{8} (r_0)^{15/7} - \frac{7}{15} (r_0)^{15/7}\right]$$

$$= \frac{2\pi U_m}{(r_0)^{1/7}} \times (r_0)^{15/7}\left[\frac{7}{8}-\frac{7}{15}\right]$$

$$= 2\pi U_m\left(\frac{49}{120}r_0^2\right) \qquad ...(i)$$

If $\bar{u}$ is the average velocity, then $Q = A\bar{u} = \pi r_0^2\,\bar{u}$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\pi r_0^2\,\bar{u} = 2\pi U_m\left(\frac{49}{120}r_0^2\right)$$

$$\therefore \qquad \bar{u} = \frac{2\pi U_m\left(\frac{49}{120}r_0^2\right)}{\pi r_0^2} = \frac{49}{60}U_m$$

i.e., $\bar{u} = \frac{49}{60}U_m$ **(Ans.)**

(*ii*) **Kinetic energy correction factor, α:**

$$\alpha = \frac{1}{A\bar{u}^3}\int_0^{r_0} u^3\,dA \qquad \text{[Eqn. (6.28)]}$$

$$= \frac{1}{A\bar{u}^3}\int_0^{r_0} U_m^3\left(\frac{y}{r_0}\right)^{3/7} 2\pi(r_0-y)\,dy$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3(r_0)^{3/7}}\int_0^{r_0} y^{3/7}(r_0-y)\,dy$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3(r_0)^{3/7}}\int_0^{r_0}(r_0\,y^{3/7}-y^{10/7})\,dy$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3(r_0)^{3/7}}\left[r_0\times\frac{7}{10}y^{10/7}-\frac{7}{17}y^{17/7}\right]_0^{r_0}$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3(r_0)^{3/7}}\left[\frac{7}{10}(r_0)^{17/7}-\frac{7}{17}(r_0)^{17/7}\right]$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3(r_0)^{3/7}}\left[\frac{49}{170}(r_0)^{17/7}\right]$$

Substituting the values $A = \pi r_0^2$ and $\bar{u} = \frac{49}{60}U_m$, we get

$$\alpha = \frac{2\pi U_m^3}{\pi r_0^2\left(\frac{49}{60}U_m\right)^3(r_0)^{3/7}}\left[\frac{49}{170}(r_0)^{17/7}\right] = \mathbf{1.06\ (Ans.)}$$

(*iii*) **Momentum correction factor, β:**

$$\beta = \frac{1}{A\bar{u}^2}\int u^2\,dA \qquad \text{[Eqn. (6.29)]}$$

$$= \frac{1}{A\bar{u}^2}\int_0^{r_0} U_m^2\left(\frac{y}{r_0}\right)^{2/7} 2\pi(r_0 - y)\,dy$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2 (r_0)^{2/7}}\int_0^{r_0} y^{2/7}(r_0 - y)\,dy$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2 (r_0)^{2/7}}\int_0^{r_0} (r_0 y^{2/7} - y^{9/7})\,dy$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2 (r_0)^{2/7}}\left[r_0 \times \frac{7}{9}y^{9/7} - \frac{7}{16}y^{16/7}\right]_0^{r_0}$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2 (r_0)^{2/7}}\left[\frac{7}{9}(r_0)^{16/7} - \frac{7}{16}(r_0)^{16/7}\right]$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2 (r_0)^{2/7}}\left[\frac{49}{144}(r_0)^{16/7}\right]$$

Substituting the values $A = \pi r_0^2$ and $\bar{u} = \frac{49}{60}U_m$, we get

$$\beta = \frac{2\pi U_m^2}{\pi r_0^2\left(\frac{49}{60}U_m\right)^2 (r_0)^{2/7}}\left[\frac{49}{144}(r_0)^{16/7}\right] = \mathbf{1.02\ (Ans.)}$$

Example 6.62. *In a circular pipe the velocity profile is given as*

$$u = U_m\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where u is the velocity at any radius r, U_m is the velocity at the pipe axis, and R is the radius of the pipe. Find:

(*i*) *Average velocity,*

(*ii*) *Energy correction factor, and*

(*iii*) *Momentum correction factor.* **[Panjab University]**

Solution. Refer to Fig. 6.61. Consider an elementary area dA in the form of a ring at a radius r and of thickness dr, then $dA = 2\pi\, r.dr$

Flow rate through the ring = dQ = Elemental area × local velocity = $2\pi r.dr.u$

∴ Total flow

$$Q = \int_0^R 2\pi r.u.dr$$

$$= \int_0^R 2\pi\, U_m\left(1 - \frac{r^2}{R^2}\right) r\,dr$$

$$= 2\pi U_m \int_0^R \left(r - \frac{r^3}{R^2}\right)dr = 2\pi U_m\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$= 2\pi U_m\left(\frac{R^2}{2} - \frac{R^2}{4}\right) = 2\pi U_m\left(\frac{R^2}{4}\right)$$

i.e.,

$$Q = 2\pi U_m \left(\frac{R^2}{4}\right) \qquad \ldots(i)$$

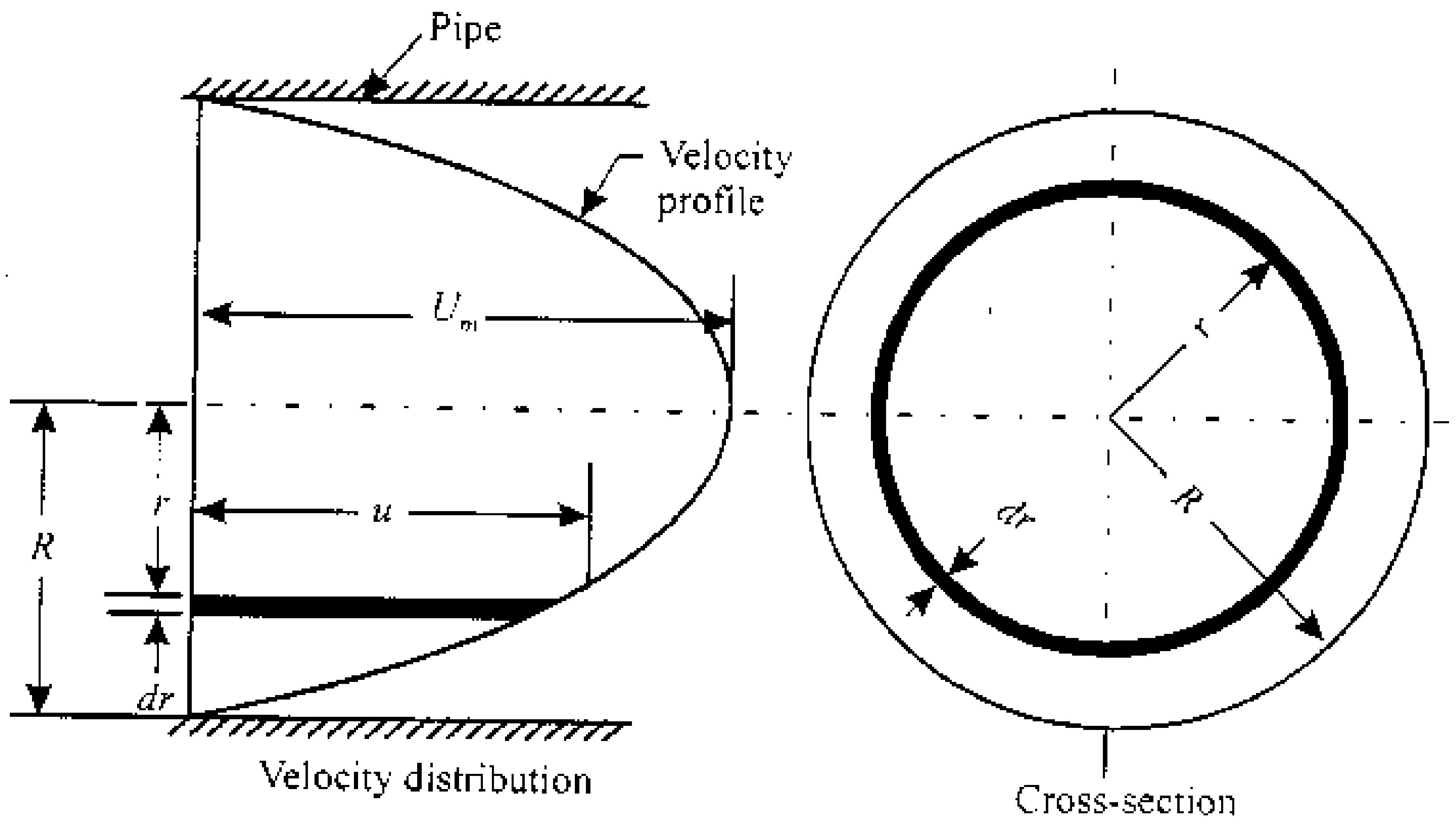


Fig. 6.61. Velocity distribution and cross-section of a pipe.

(*ii*) **Average velocity, $\bar{u}$:**

If $\bar{u}$ is the average flow velocity, then

$$Q = A\bar{u} = \pi R^2 \bar{u} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get

$$\pi R^2 \bar{u} = 2\pi U_m \left(\frac{R^2}{4}\right)$$

$$\therefore \qquad \bar{u} = \frac{2\pi U_m \left(\dfrac{R^2}{4}\right)}{\pi R^2} = \frac{U_m}{2} \quad \textbf{(Ans.)}$$

(*iii*) **Kinetic energy correction factor, α:**

$$\alpha = \frac{1}{A\bar{u}^3} \int_0^R u^3 \, dA \qquad \ldots\text{[Eqn. 6.28]}$$

$$= \frac{1}{A\bar{u}^3} \int_0^R U_m^3 \left[1 - \left(\frac{r}{R}\right)^2\right]^3 2\pi \, r dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3} \int_0^R \left(1 - \frac{3r^2}{R^2} + \frac{3r^4}{R^4} - \frac{r^6}{R^6}\right) r \, dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3} \int_0^R \left(r - \frac{3r^3}{R^2} + \frac{3r^5}{R^4} - \frac{r^7}{R^6}\right) dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3} \left[\frac{r^2}{2} - \frac{3r^4}{4R^2} + \frac{3r^6}{6R^4} - \frac{r^8}{8R^6}\right]_0^R$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3} \left[\frac{R^2}{2} - \frac{3}{4}R^2 + \frac{R^2}{2} - \frac{R^2}{8}\right]$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\left(\frac{R^2}{8}\right)$$

Substituting the values $A = \pi R^2$ and $\bar{u} = \frac{U_m}{2}$, we get

$$\alpha = \frac{2\pi U_m^3}{\pi R^2 \times \left(\frac{U_m}{2}\right)^3} \times \left(\frac{R^2}{8}\right) = \mathbf{2}\ \textbf{(Ans.)}$$

(*iv*) **Momentum correction factor, β:**

$$\beta = \frac{1}{A\bar{u}^2}\int u^2\, dA \qquad \text{...[Eqn. 6.29]}$$

$$= \frac{1}{A\bar{u}^2}\int_0^R U_m^2\left[1-\left(\frac{r}{R}\right)^2\right]^2 2\pi r\, dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R\left(1 - 2\times\frac{r^2}{R^2} \div \frac{r^4}{R^4}\right) r\, dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R\left(r - 2\times\frac{r^3}{R^2} + \frac{r^5}{R^4}\right) dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left[\frac{r^2}{2} - 2\times\frac{r^4}{4R^2} + \frac{1}{6}\times\frac{r^6}{R^4}\right]_0^R$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left[\frac{R^2}{2} - \frac{R^2}{2} + \frac{R^2}{6}\right]$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left(\frac{R^2}{6}\right)$$

Substituting the values $A = \pi R^2$ and $\bar{u} = \frac{U_m}{2}$, we get

$$\beta = \frac{2\pi U_m^2}{\pi R^2 \times \left(\frac{U_m}{2}\right)^2}\left(\frac{R^2}{6}\right) = 1.33\ \text{(Ans.)}$$

6.10. Moment of Momentum Equation

Moment of momentum equation is derived from *moment of momentum principle* which states as follows:

"*The resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum*".

When the moment of momentum of flow leaving a control volume is different from that entering it, the result is a torque acting over the control volume.

Let, Q = Steady rate of flow of fluid,

ρ = Density of fluid,

V_1 = Velocity of fluid at section 1,

r_1 = Radius of curvature at section 1, and

V_2 and r_2 = Velocity and radius of curvature at section 2.

Momentum of fluid at section 1= mass × velocity = $\rho Q \times V_1$

∴ Moment of momentum per second of fluid at section 1 = $\rho Q \times V_1 \times r_1$

Similarly, moment of momentum per second of fluid at section 2 = $\rho Q \times V_2 \times r_2$

∴ Rate of change of moment of momentum = $\rho Q\, V_2 r_2 - \rho Q V_1 r_1 = \rho Q(V_2 r_2 - V_1 r_1)$

According to the moment of momentum principle,

Resultant torque = Rate of change of moment of momentum

$$T = \mathrm{r}Q\,(V_2\, r_2\, V_1 r_1) \qquad \text{...(6.31)}$$

Eqn. (6.31) is known as *moment of momentum equation.* This equation is used

(*i*) to find torque exerted by water on sprinkler, and

(*ii*) to analyse flow problems in turbines and centrifugal pumps.

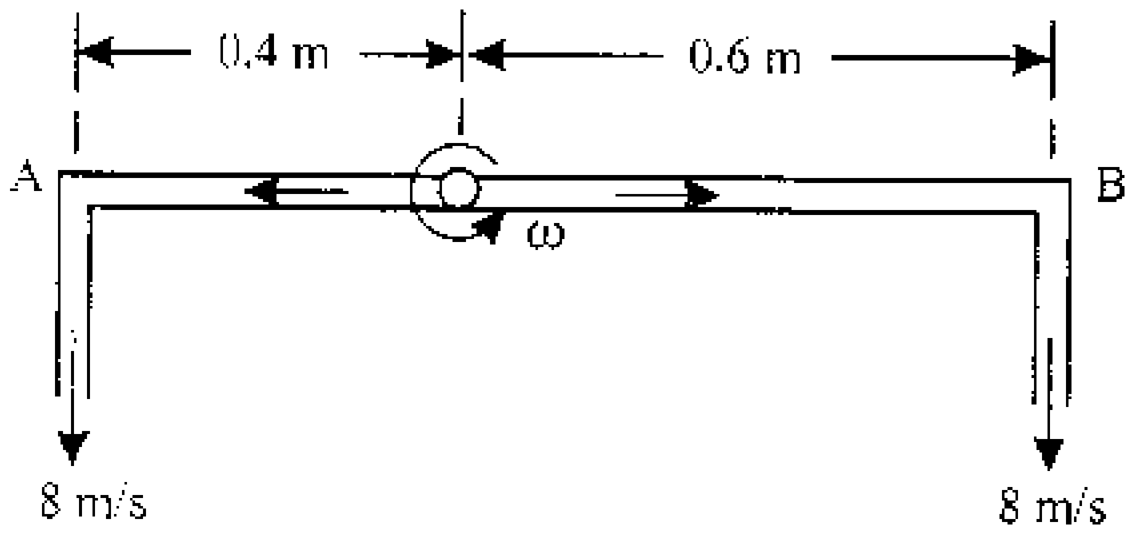


Fig. 6.62

Example 6.63. *Fig. 6.62 show an unsym-ical sprinkler. It has a frictionless shaft equal flow through each nozzle with a ve- of 8 m/s relative to the nozzle. Find the of rotation in r.p.m.*

Solution. Refer to Fig. 6.62.

$$r_A = 0.4 \text{ m}, r_B = 0.6 \text{ m}$$

Velocity relative to the nozzle

$$V_A\,(= V_B) = 8 \text{ m/s}$$

Let, ω = Angular velocity of the sprinkler.

Absolute velocity, $V_1 = V_A + \omega\, r_A = 8 - \omega \times 0.4 = 8 + 0.4\,\omega$

Absolute velocity, $V_2 = V_B - \omega r_B = 8 - \omega \times 0.6 = 8 - 0.6\,\omega$

Speed of rotaiton, N (r.p.m):

The moment of momentum of the fluid entering sprinkler is given zero and also there is no al torque applied on the sprinkler. Hence resultant torque is zero, *i.e.*

$$T = 0$$

$$\therefore \quad \rho Q\,(V_2 r_2 - V_1\, r_1) = 0$$

$$V_2 r_2 - V_1 r_1 = 0 \qquad (\because \rho Q \neq 0)$$

$$(8 - 0.6\,\omega) \times 0.6 = (8 + 0.4\,\omega) \times 0.4$$

$$4.8 - 0.36\,\omega = 3.2 - 0.16\,\omega$$

$$0.52\,\omega = 1.6$$

$$\omega = 3.077 \text{ rad/s}$$

But
$$\omega = \frac{2\pi N}{60}$$

$$\therefore \quad N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.077}{2\pi} = \mathbf{29.4 \text{ r.p.m. (Ans.)}}$$

Example 6.64. *A lawn sprinkler shown in Fig 6.63 has 12 mm diamete nozzle at the end of a g arm and discharge water with a velocity of 15 m/s. Determine:*

(*i*) *Torque required to hold the rotating arm stationary, and*

(*ii*) *Constant speed of rotation of the arm, if free to rotate.*

Solution. Diameter of each nozzle = 12 mm = 0.012 m

∴ Area of each nozzle

$$= \frac{\pi}{4} \times 0.012^2 = 0.000113 \, m^2$$

Velocity of flow, $V_A (= V_B) = 15$ m/s

∴ Discharge through each nozzle,

$$Q = \text{area} \times \text{velocity}$$

$$= 0.000113 \times 15 = 0.001695 \, m^3/s$$

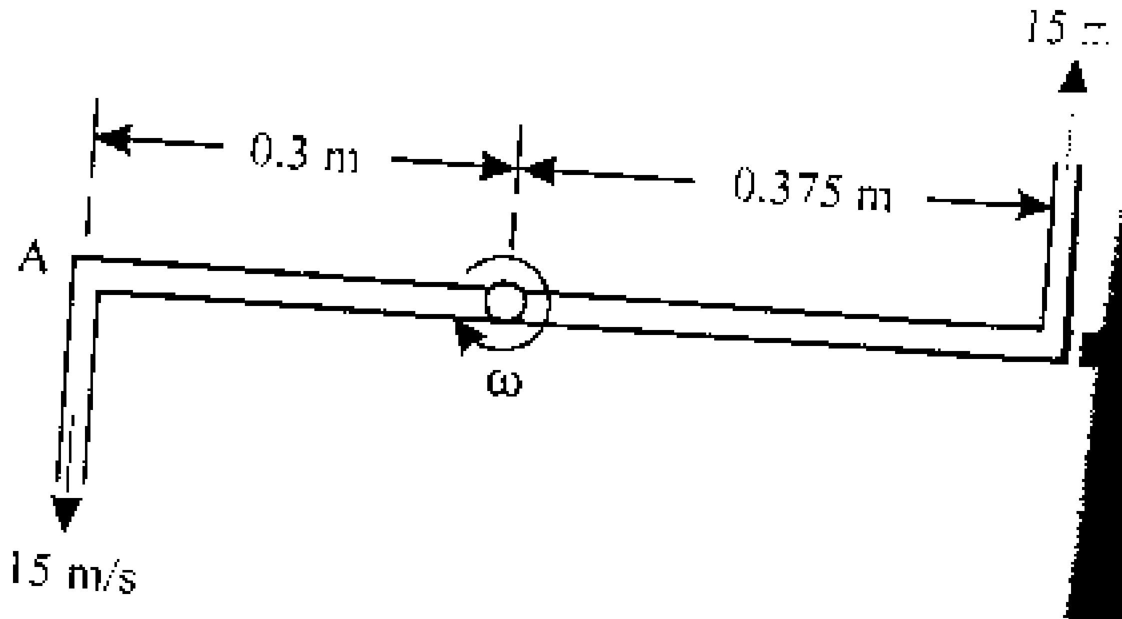


Fig. 6.63

(*i*) **Torque required to hold the rotating arm stationary:**

Torque exerted by water coming thorugh nozzle A on the sprinkler

$$= \rho Q V_A \times r_A = \frac{9810}{9.81} \times 0.001695 \times 15 \times 0.3 = 7.627 \text{ Nm}$$

Torque exerted by water coming through nozzle B on the sprinkler

$$= \rho Q V_B \times r_B = \frac{9810}{9.81} \times 0.001695 \times 15 \times 0.375 = 9.534 \text{ Nm}$$

∴ Total torque exerted by water on sprinkler

$$= 7.627 + 9.534 = 17.161 \text{ Nm}$$

∴ Torque required to hold the rotating arm stationary

= Torque exerted by water on sprinkler

= **17.161 Nm (Ans.)**

(*ii*) **Constant speed of rotation of the arm, if free to rotate, N (r.p.m.):**

Let, ω = angular speed of rotation of the sprinkler.

Then, absoute velocities of flow of water at the nozzles A and B are,

$$V_1 = 15 - 0.3\,\omega$$

and

$$V_2 = 15 - 0.375\,\omega$$

Torque exerted by water coming out at A, on spirinker

$$= \rho Q V_1 \times r_A = \frac{9810}{9.81} \times 0.001695 \times (15 - 0.3\omega) \times 0.3$$

$$= 0.5085\,(15 - 0.3\,\omega)$$

Torque exerted by water, coming out at B, on sprinkler

$$= PQV_2 \times r_B = \frac{9810}{9{\cdot}81} \times 0{\cdot}001695 \times (15 - 0{\cdot}375\omega) \times 0{\cdot}375$$

$$= 0{\cdot}6356\,(15 - 0{\cdot}375\,\omega)$$

∴ Total torque exerted by water

$$= 0.5085\,(15 - 0.3\,\omega) + 0.6356\,(15 - 0.375\,\omega)$$

Since moment of momentum of the flow entering is zero and no external torque is appli sprinkler, so the resultant torque on the sprinkler must be zero.

∴ $$0.5085\,(15 - 0.3\,\omega) + 0.6356\,(15 - 0.375\,\omega) = 0$$

$$7.627 - 0.1526\,\omega + 9.534 - 0.238\,\omega = 0$$

$$17.161 - 0.3906\,\omega = 0$$

or $$\omega = \frac{17.161}{0.3906} = 43.93 \text{ rad/sec}$$

Also $\omega = \frac{2\pi N}{60} = 43.93$

$\therefore$ $N = \frac{60 \times 43.93}{2\pi}$ **= 419.5 r.p.m. (Ans.)**

11. Vortex Motion

The **vortex motion** is *defined as a motion in which the whole fluid mass rotates about an axis.* A s of fluid in rotation about a fixed axis is called ***vortex.***

A vortex motion is chracterised by a flow pattern wherein the streamlines are curved. When fluid s between curved streamlines, centrifugal forces are set up and these are conuter-balanced by pressure force acting in the radial direction.

The vortex flow is of the following types:

1. Forced vortex flow, and
2. Free vortex flow.

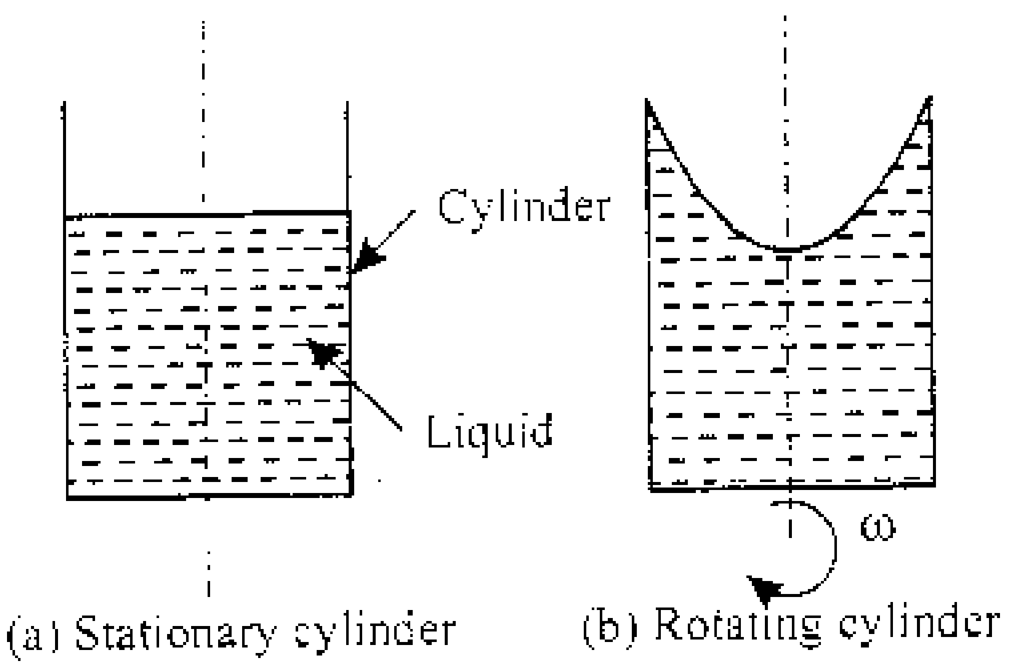
6.11.1 Forced Vortex Flow

Forced vortex flow is one in which the fluid *mass is made to rotate by means of some external cy.* The external agency is generally the me-ical power which imparts a constant torque on fluid mass. Then, in such a flow there is always nditure of energy. The forced vortex motion is called *flywheel vortex* or *rotational vortex.*

In this type of flow, the fluid mass rotates at a stant angular velocity ω. The tangential velocity y fluid particle is given by

$$v = \omega r \qquad ...(6.32)$$

(where r = radius of the fluid particle from the s of rotation)



(a) Stationary cylinder (b) Rotating cylinder

Fig. 6.64. Forced vortex flow.

$\therefore$ Angular velocity, $\omega = \frac{v}{r}$ = constant ...[6.32 (*a*)]

Example:

1. Rotation of water through the runner of a turbine.
2. Rotation of liquid inside the impeller of a centrifugal pump.
3. Rotation of liquid in a vertical cylinder, (Fig. 6.64)

6.11.2 Free Vortex Flow

Free *vortex flow* is one in which the *fluid mass rotates without any external impressed contact e.* The whole fluid mass rotates either due to fluid pressure itself or the gravity or due to rotation viously imparted. The free vortex motion is also called *potential vortex* or *irrotational vortex.*

Example:

1. Flow around a circular bend.
2. A whirlpool in a river.
3. Flow of liquid in a centrifugal pump casing after it has left the impeller.
4. Flow of water in a turbine causing before it enters the guide vanes.
5. Flow of liquid through a hole/outlet provided at the bottom of a shallow vessel (*e.g.*, wash sin, bath tub, etc.)

In free vortex the relation between velocity and radius is obtained by putting the value of external que equal to zero, or, the time rate change of angular momentum (*i.e.*, moment of momentum) must zero.

Let us consider a particle of mass m at a radius distance r from the axis of rotation, haivng a tangential velocity, v. Then

Moment of momentum = $(m \times v) \times r = mvr$

$\therefore$ Time rate of change of momentum

$$= \frac{\partial}{\partial t}(mvr)$$

But for the vortex, $= \dfrac{\partial}{\partial t}(mvr) = 0$

Integrating, we get mvr = constant

Since m is constant, vr = constant = C ...(6.33)

Where C is a constant and is known as strength of vortex.

$$\therefore \qquad v = \frac{C}{r} \qquad ...(6.34)$$

$$\text{or} \qquad v \propto \frac{1}{r} \qquad ...(6.34\ a)$$

i.e. tangentital velocity is *inversely proportional* to distance r.

6.11.3. Equation of Motion for Vortex Flow.

Refer to Fig. 6.65. ABCD is fluid element rotating at a uniform velocity in a horizontal plane about an axis perpendicular to the plane of paper and passing through 0.

Let, r = Radius of the element from O,

Δr = Radial thickness of the element,

ΔA = Area of cross-section of element, and

$\Delta\theta$ = Angle subtended by the element at O.

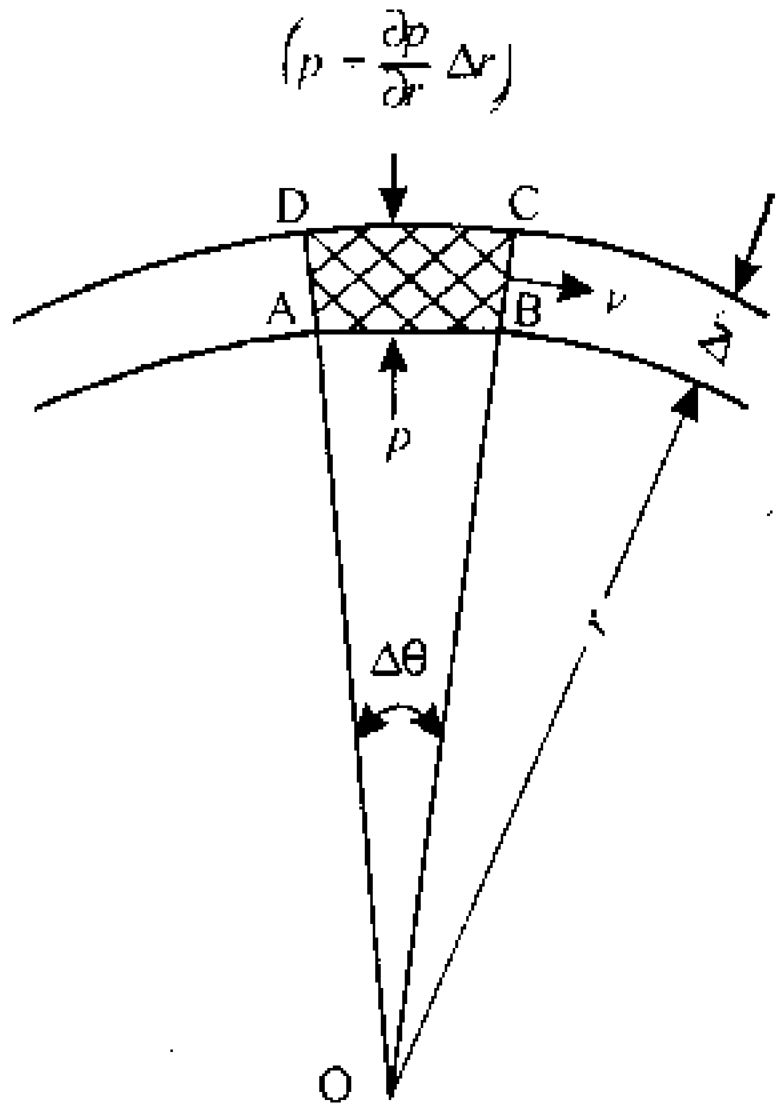


Fig. 6.65. Flow in a circular path.

The various forces acting on the element are:

1. Centrifugal force, $\dfrac{mv^2}{r}$ acting away from the centre, O
2. Pressure force $p\Delta A$ on the face AB
3. Pressure force $\left(p + \dfrac{\partial p}{\partial r}\Delta r\right)\Delta A$ on the face CD.

Equating the forces in the radial direction, we get

$$\left(p + \frac{\partial p}{\partial r} \cdot \Delta r\right)\Delta A - p\Delta A = \frac{mv^2}{r}$$

But, m = mass density × volume = $\rho \times \Delta A \times \Delta r$

$$\therefore \qquad \left(p + \frac{\partial p}{\partial r}\Delta r\right)\Delta A - p\Delta A = \rho\Delta A\,\Delta r\frac{v^2}{r}$$

$$\text{or} \qquad p\Delta A + \frac{\partial p}{\partial r}\Delta r\,\Delta A - p\Delta A = \rho\Delta A\,\Delta r\frac{v^2}{r}$$

$$\text{or} \qquad \frac{\partial p}{\partial r}\Delta r\,\Delta A = \rho\Delta A\,\Delta r\frac{v^2}{r}$$

$$\text{or} \qquad \frac{\partial p}{\partial r} = \frac{\rho v^2}{r} \qquad ...(6.35)$$

The expression $\dfrac{\partial p}{\partial r}$ is called *pressure gradient* in the radial direction.

$\left(\text{Since } \dfrac{\partial p}{\partial r} \text{ is } +ve, \text{ therefore, pressure increases with the increase of radius } r.\right)$

In the vertical plane, the variation of pressure is given by the hydrostatic law, *i.e.*,

$$\frac{\partial p}{\partial z} = -\rho g \qquad \text{...(6.36)}$$

As the pressure is a function of r and z, therefore total derivative of p,

$$\partial p = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} dz$$

Substituting the values of $\frac{\partial p}{\partial r}$ and $\frac{\partial p}{\partial z}$ from eqns. (6.35) and (6.36) respectively, we get

$$dp = \frac{\rho v^2}{r} dr - \rho g\ dz \qquad \text{...(6.37)}$$

Eqn. (6.37) gives the *variation of pressure of a rotating fluid in any plane.*

6.11.4. Equation of Forced Vortex Flow

In case of forced vortex flow,

$$v = \omega r \qquad \text{...[Eqn. 6.32]}$$

(where ω = constant angular velocity)

Putting the value of v in eqn. (6.37), we get

$$dp = \frac{\rho \omega^2 r^2}{r} dr - \rho g\ dz$$

or

$$dp = \rho\omega^2 r dr - \rho g dz$$

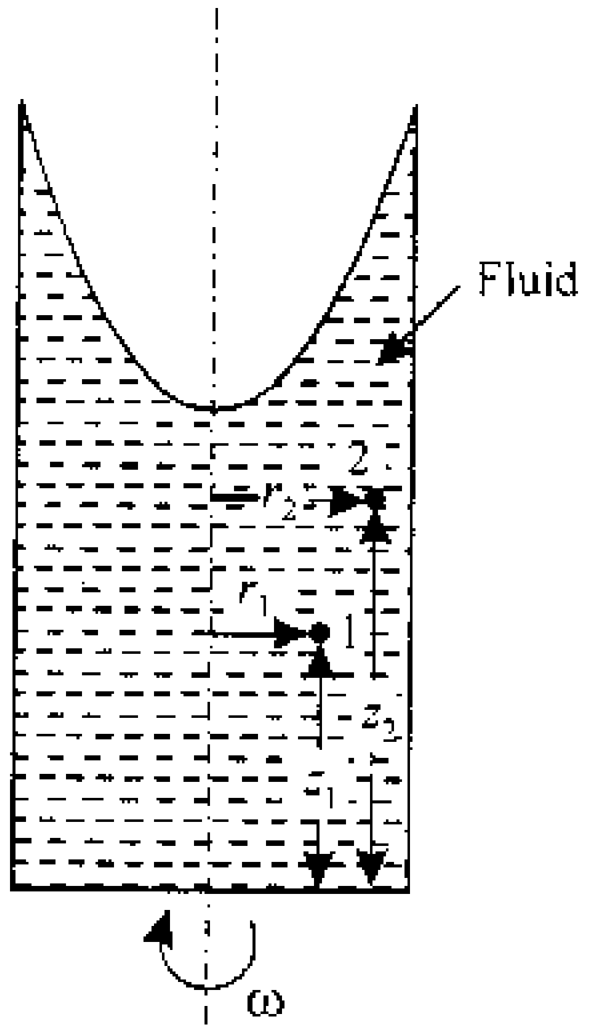


Fig. 6.66. Forced vortex flow.

Considering points 1 and 2 in the fluid having forced vortex flow [illegible]g. 6.66) and integrating the above eqn for these points, we get

$$\int_1^2 dp = \int_1^2 \rho\omega^2 r\, dr - \int_1^2 \rho g\, dz$$

or

$$[p]_1^2 = \rho\omega^2 \left[\frac{r^2}{2}\right]_1^2 - \rho g [z]_1^2$$

or

$$(p_2 - p_1) = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

$$= \frac{\rho}{2}(\omega^2 r_2^2 - \omega^2 r_1^2) - \rho g(z_2 - z_1)$$

$$= \frac{\rho}{2}(v_2^2 - v_1^2) - \rho g(z_2 - z_1) \qquad [\because v_1 = \omega_1 r_1 \text{ and } v_2 = \omega_2 r_2]$$

— When the points 1 and 2 lie on the free surface of the liquid, then $p_1 = p_2$ and the above equation becomes

$$0 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) - \rho g(z_2 - z_1)$$

or

$$g(z_2 - z_1) = \left(\frac{v_2^2 - v_1^2}{2}\right)$$

or

$$z_2 - z_1 = \frac{v_2^2 - v_1^2}{2g}$$

— When the point 1 lies on the axis of rotation, then

$v_1 = \omega r_1 = \omega \times 0 = 0$; the above eqn. reduces to

$$z_2 - z_1 = \frac{v_2^2}{2g}$$

If $\qquad z_2 - z_1 = z \qquad$ (say), then we have

$$z = \frac{v_2^2}{2g} = \frac{\omega^2 r_2^2}{2g} \qquad \ldots(6.38)$$

Thus, z varies with square of r. Hence eqn. (6.38) is an equation of *parabola* which means that the free surface of the liquid is a paraboloid.

Example 6.65. *Prove that in case of force vortex, the rise of liquid level at the ends is equal to the fall of liquid level at the axis of rotation.*

Solution. Refer to Fig. 6.67.

Let, R = Radius of the cylinder, and OO = Initial liquid level when the cylinder is stationary.

Let the cylinder is rotated at constant angular velocity ω. The liquid will rise at the ends and will fall at the centre.

Let, $\quad y_r$ = Rise of liquid at the ends (from OO), and

y_f = Fall of liquid at the centre (from OO)

Now, initial height of liquid = $(h + y_f)$

$\therefore$ Volume of liquid in cylinder

$$= \pi R^2 (h + y_f) \qquad \ldots(i)$$

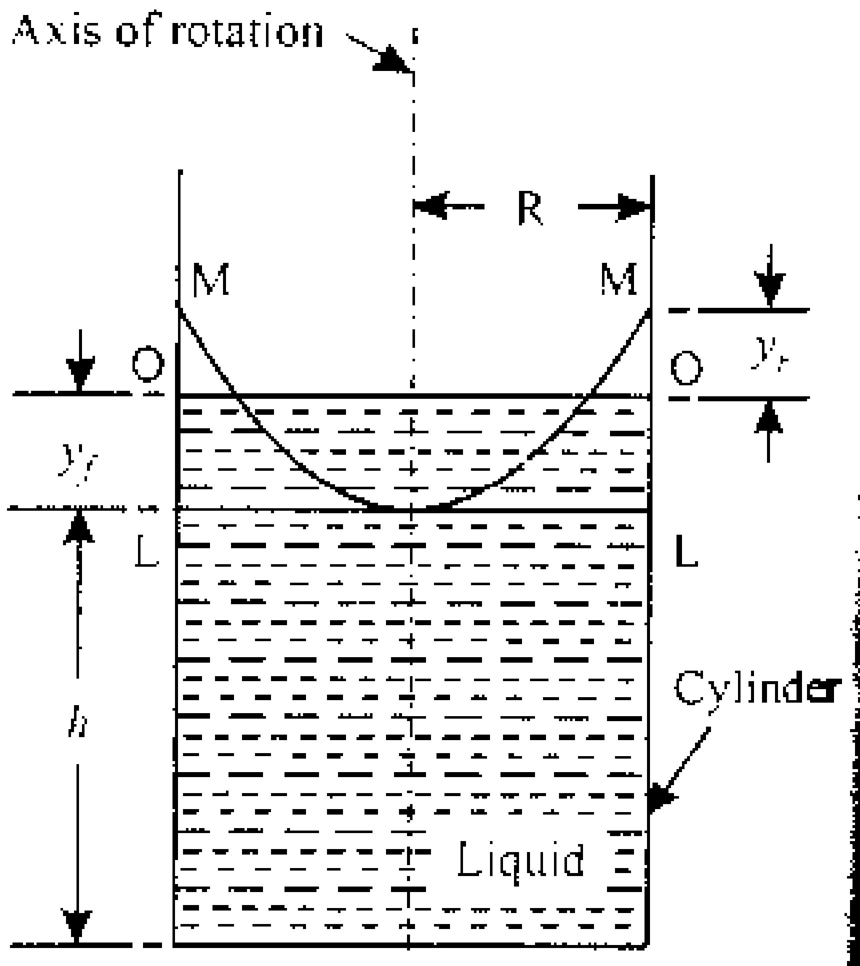


Fig. 6.67

Volume of liquid = [volume of cylinder up to level MM] − [volume of paraboloid]

$$= (\pi R^2 \times \text{liquid height up to level } MM) - \left(\frac{1}{2} \times \pi R^2 \times \text{height of paraboloid}\right)$$

$$= \pi R^2 \times (h + y_f + y_r) - \frac{\pi R^2}{2}(y_f + y_r)$$

$$= \pi R^2 h + \pi R^2 (y_f + y_r) - \frac{\pi R^2}{2}(y_f + y_r)$$

$$= \pi R^2 h + \frac{\pi R^2}{2}(y_f + y_r) \qquad \ldots($$

Equating (i) and (ii), we get

$$\pi R^2 (h + y_f) = \pi R^2 h + \frac{\pi R^2}{2}(y_f + y_r)$$

$$\pi R^2 h + \pi R^2 y_f = \pi R^2 h + \frac{\pi R^2}{2} y_f + \frac{\pi R^2}{2} y_r$$

or $$\pi R^2 y_f - \frac{\pi R^2}{2} y_f = \frac{\pi R^2}{2} y_r$$

or $$\frac{\pi R^2}{2} y_f = \frac{\pi R^2}{2} y_r$$

or $$y_f = y_r$$

i.e., Fall of liquid at centre = Rise of liquid at the ends .....**Proved.**

Example 6.66. *A cylindrical tank 0.9 m in diameter and 2 m high open at top is filled* ***with water*** *a depth of 1.5 m. It is rotated about its vertical axis at N. r.p.m. Determine the value of N* ***which will*** *se water level even with the brim.* **(AMIE Winter, 2001)**

Solution. Refer to Fig. 6.68. *Given*: Radius, $R = \frac{0.9}{2} = 0.45$ m ; Length, = 2m; Initial height of r = 1.5 m.

Speed which will raise water level even with n, N:

When the vessel is rotated, praboloid is formed.

Volume of air before rotation = Volume of air r rotation

$$\pi R^2 \times 2 - \pi R^2 \times 1.5 = \frac{1}{2}\pi R^2 z$$

or $$z = 1.0 \text{ m}$$

Using the relation:

$$z = \frac{\omega^2 r^2}{2g}, \text{ we get}$$

$$1.0 = \frac{\omega^2 R^2}{2 \times 9.81} \quad (\text{Here, } r = R)$$

$$\omega = \sqrt{\frac{1.0 \times 2 \times 9.81}{(0.45)^2}} = 9.843$$

But $$\omega = \frac{2\pi N}{60}$$

∴ $$N = \frac{9.843 \times 60}{2\pi} = 93.99 \text{ r.p.m. (Ans.)}$$

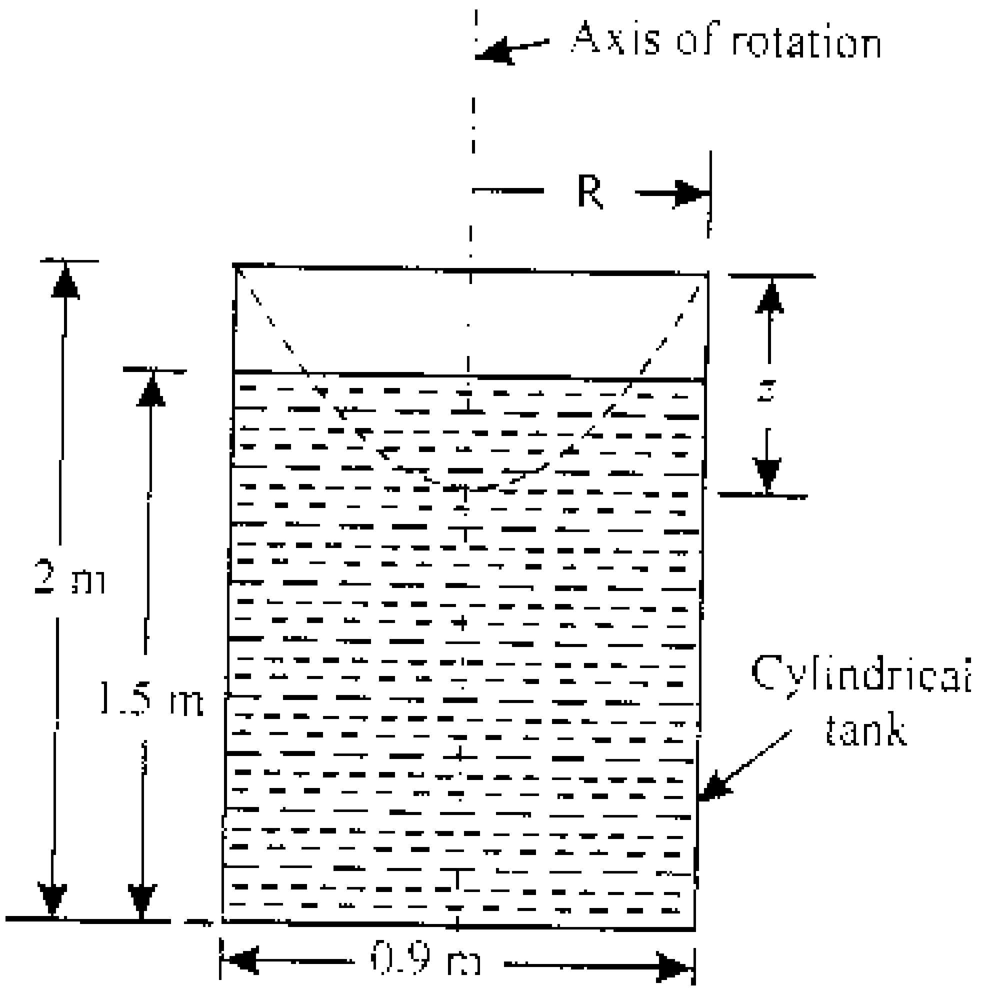


Fig. 6.68

xample. 6.67. *Find the maximum speed of an open circular cylinder, having 180 mm diameter, mm length and containing water up to a height of 960 mm, at which it should be rotated about tical its vertical axis so that no water spills.* **(M.U.)**

lution. Given: Radius of the cylinder, $R = \frac{180}{2} = 90$ mm = 0.09 m

Length of the cylinder, $l = 1200$ mm = 1.2 m

tial height of water, $h = 960$ mm = 0.96 m

aximum speed of rotation, N:

Let, ω = Agular velocity of the cylinder when the water is about to spin.

e know, rise of liquid at the ends = Fall of liquid at centre

t, Rise of liquid at the ends = Length of cylinder − initial height

$$= 1.2 - 0.96 = 0.24 \text{ m}$$

∴ Fall of liquid at centre = 0.24 m

∴ Height of parabola = 0.24 + 0.24 = 0.48 m

∴ $$z = 0.48 \text{ m}$$

sing the relation:

$$z = \frac{\omega^2 R^2}{2g}, \text{ we get}$$

$$0.48 = \frac{\omega^2 \times (0.09)^2}{2 \times 9.81}$$

or $$\omega^2 = \frac{0.48 \times 2 \times 9.81}{(0.09)^2} = 1162.67 \text{ or } \omega = 34.09 \text{ rad/s.}$$

But $$\omega = \frac{2\pi N}{60} \quad \therefore \; 34.09 = \frac{2\pi N}{60}$$

or $$N = \frac{34.09 \times 60}{2\pi} = \textbf{325.5 r.p.m (Ans.)}$$

Example 6.68. *A 0.225m diameter cylinder is 1.5 m long and contains water up to a heigh 1.05 m. Estimate the speed at which the cylinder may be rotated about its vertical axis so that axial depth becomes zero.*

Solution. Radius of the cylinder,

$$r = \frac{0.225}{2} = 0.1125 \text{ m}$$

Length of the cylinder, l = 1.5 m

Initial height of water = 1.05 m

When axial depth is zero,

depth of paraboloid = 1.5 m

Speed of rotation N:

Using the relation:

$$z = \frac{\omega^2 R^2}{2g}, \text{ we get}$$

$$1.5 = \frac{\omega^2 \times 0.1125^2}{2 \times 9.81}$$

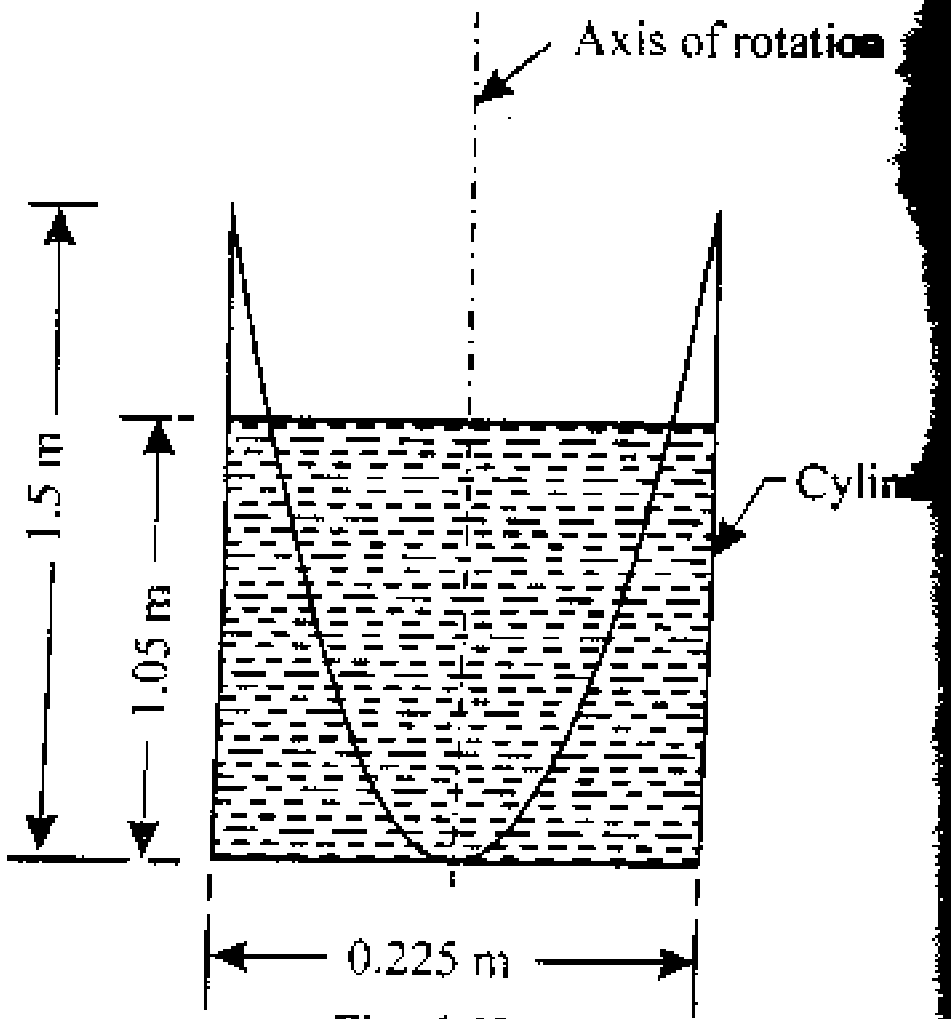


Fig. 6.69

or $$\omega^2 = \frac{1.5 \times 2 \times 9.81}{0.1125^2} = 2325.33$$

or $$\omega = 48.22 \text{ rad/s}$$

But $$\omega = \frac{2\pi N}{60}$$

$\therefore$ $$48.22 = \frac{2\pi N}{60}$$

or $$N = \frac{48.22 \times 60}{2\pi} = \textbf{460.46 r.p.m. (Ans.)}$$

Example 6.69. *For the example 6.68 find the difference in total pressure force due to rotatio*

(*i*) *At the bottom of cylinder, and*

(*ii*) *On the sides of the cylinder.*

Solution. *Given*: Same as in example 6.68.

(*i*) **Difference in total pressure force at the bottom of the cylinder:**

Total pressure force at the bottom **before rotation**

$$F_{\text{before rot.}} = wA\bar{h}$$

where, $w = 9810 \text{ N/m}^3$,

A = area of bottom = $\pi R^2 = \pi \times 0{\cdot}1125^2 = 0.03976 \text{ m}^2$

$$= \pi D \times \text{height of water} = \pi \times 0.225 \times 1.5 = 1.06 \text{ m}^2$$

$$\bar{h} = \frac{1}{2} \times \text{height of water} = \frac{1}{2} \times 1.5 = 0.75 \text{ m}$$

$$\therefore \quad F_{\text{after rot.}} = 9810 \times 1.06 \times 0.75 = 7798.95 \text{ N}$$

∴ Difference in pressure force on the sides

$$= F_{\text{after rot.}} - F_{\text{before rot.}}$$

$$= 7798.95 - 3822.5 = \mathbf{3976{\cdot}45 \text{ N (Ans.)}}$$

Example 6.70. *An open cylindrical vessel 180 mm in diameter and 450 mm deep is filled with up to the top. Estimate the volume of water left in the vessel when it is rotated about its vertical*

(*i*) *With a speed of 200 r.p.m., and*

(*ii*) *With a speed of 400 r.p.m.*

Solution. Radius of the vessel, $R = \frac{180}{2} = 90 \text{ mm} = 0.09 \text{ m}$

Initial height of water = 450 mm = 0.45 m

∴ Initial volume of water = $\pi \times 0.09^2 \times 0.45 = 0.01145 \text{ m}^2$

(*i*) **Volume of water left at a speed of 200 r.p.m.:**

Angular, speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94 \text{ rad./s}$

Height of parabola is given by

$$z = \frac{\omega^2 R^2}{2g} = \frac{(20.94)^2 \times 0.09^2}{2 \times 9.81} = 0.181 \text{ m}$$

Since the vessel is initially full of water, water will be spilled when it is rotated.

Volume of water spilled = Volume of paraboloid.

But, volume of paraboloid = $\frac{1}{2}$ (Area of cross-section × height of parabola)

$$= \frac{1}{2} \times \pi R^2 \times z = \frac{1}{2} \times \pi \times 0.09^2 \times 0.181 = 0.002303 \text{ m}^3$$

∴ Volume of water left = Initial volume – volume of water spilled

$$= 0.01145 - 0.00203 = \mathbf{0.009147 \text{ m}^3 \text{ (Ans.)}}$$

(*ii*) **Volume of water left at a speed of 400 r.p.m.:**

Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 400}{60} = 41.88 \text{ rad/s}$

Height of the parabola, $z = \frac{\omega^2 R^2}{2g} = \frac{(41.88)^2 \times 0.09^2}{2 \times 9.81} = 0.724 \text{ m}$

Since the height of parabola is more than the height of the cylinder, therefore, the sh the imaginary parabola will be as shown in Fig. 6.71

Let, r = Radius of the parabola at the bottom of the vessel

$= 0.724 - 0.45 = 0.274$ m

Volume of water left in the vessel

= Volume of water in the portions LMN and PQS

= Initial volume of water–volume of paroboloid LOS + volume of paraboloid

Now, volume of paraboloid LOS

$$= \frac{1}{2} (\pi R^2 \times \text{height of parabola})$$

$$= \frac{1}{2} \times \pi \times 0.09^2 \times 0.724 = 0.00921 \text{ m}^3$$

For the imaginary parabola (*NOP*),

$\omega = 41.88$ r.p.m.

$z = 0.274$ m

r = Radius at the bottom of the vessel

Using the relation:

$$z = \frac{\omega^2 r^2}{2g}, \text{ we get}$$

$$0.274 = \frac{41.88^2 \times r^2}{2 \times 9.81}$$

or $$r^2 = \frac{0.274 \times 2 \times 9.81}{41.88^2} = 0.003065 \text{ m}^2$$

$\therefore$ $r = 0.0554$ m

$\therefore$ Volume of paraboloid *NOP*

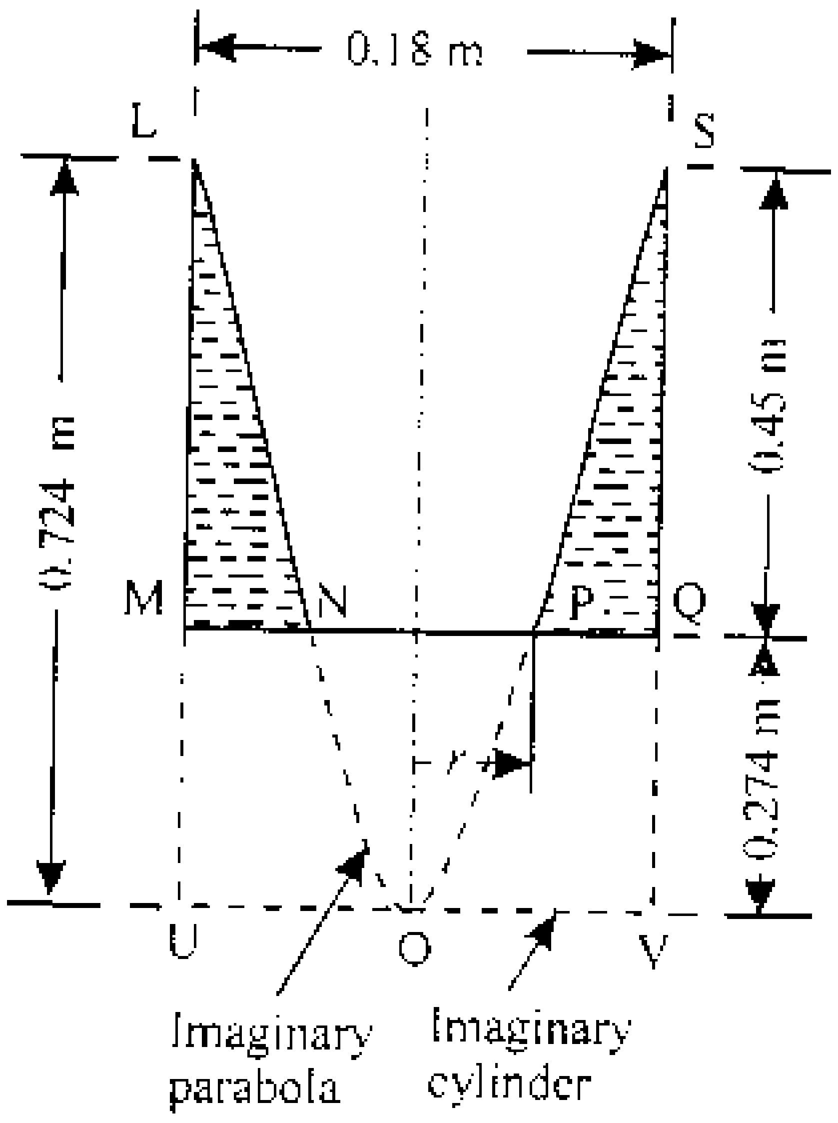


Fig. 6.71

$$= \frac{1}{2}(\text{area at the top of the imaginary parabola} \times \text{height of parabola})$$

$$= \frac{1}{2} \times \pi r^2 \times 0.274$$

$$= \frac{1}{2} \times \pi \times 0.0554^2 \times 0.274 = 0.00132 \text{ m}^3$$

$\therefore$ Volume of water left

$= 0.01145 - 0.00921 + 0.00132$

= 0.00356 m^3 (Ans.)

1.5 Rotation of Liquid in a Closed Cylindrical Vessel

en a cylindrical vessel sealed at the top and filled with some liquid is rotated about its vertical ical axis, the shape of paraboloid formed due to rotation of the vessel will be as shown in Fig r different speeds of rotation.

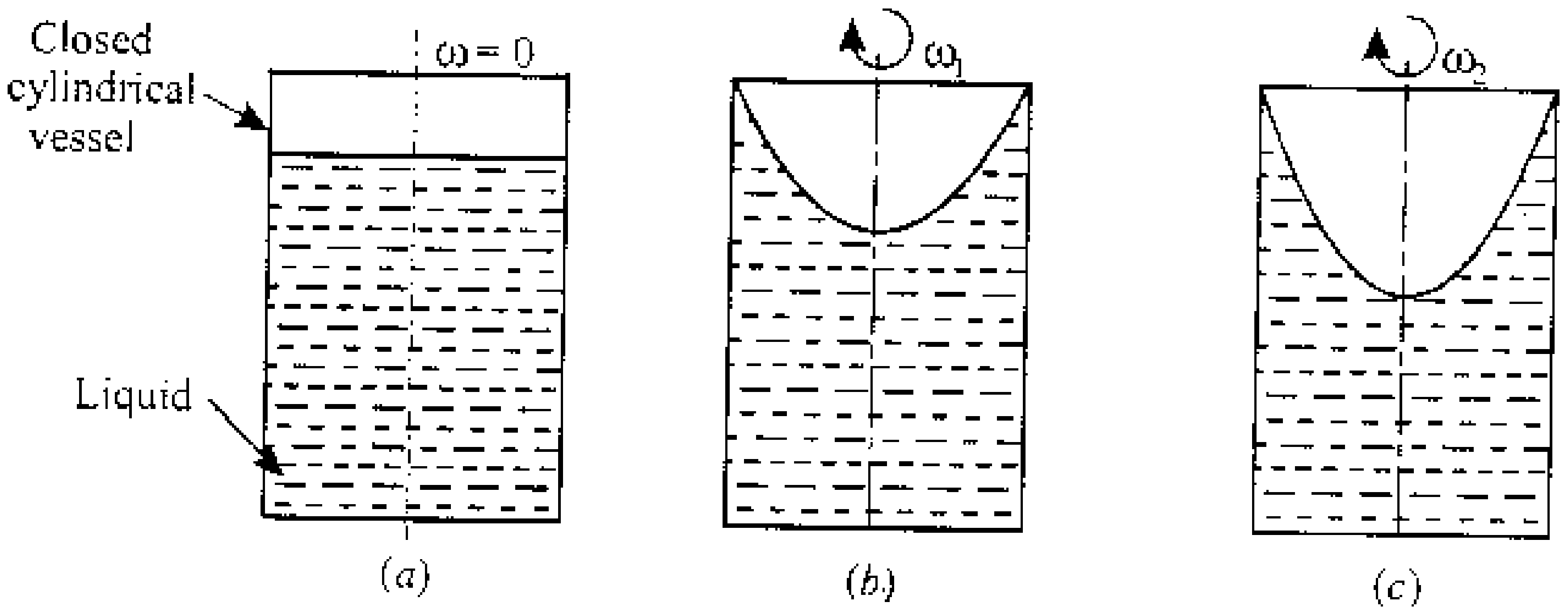


Fig. 6.72. Rotation of liquid in a closed cylindrical vessel.

Fig. 6.72 (*a*) shows the cylindrical vessel when it is stationary (*i.e.*, it is not rotated, $\omega = 0$)

Fig. 6.72 (*b*) shows the shape of the paraboloid formed when the speed of rotation is ω_1.

Fig. 6.72 (*c*) shows the shape of the paraboloid formed when the speed of rotation is ω_2 $(\omega_2 > \omega_1)$. In this case the following are unknown:

1. Radius of the parabola at the top of the vessel,
2. Height of the parabola formed corresponding to the angular speed, ω_2.

To solve these, two unknown equations are required:

(*i*) One equation is : $z = \dfrac{\omega^2 r^2}{2g}$ ...(i

(*ii*) Second equation is from the fact that for closed Vessel:

Volume of air before rotation = Volume of air after rotation

Volume of air before rotation = Volume of closed vessel – volume of liquid in the vessel

Volume of air after rotation = Volume of paraboloid formed

$$= \frac{1}{2}\pi r^2 \times z\,.$$

Example 6.71. *A cylindrical vessel closed at the top and bottom is 0.24 m in diameter, 1.44 long and contains water up to height of 0.96 m.*

(*i*) *Find the height of paraboloid formed, if it is rotated at 480 r.p.m; about its vertical axis.*

(*ii*) *Find the speed of rotation of the vessel, when axial depth of water is zero.*

Solution. Radius of the vessel,

$$R = \frac{0.24}{2} = 0.12\,\text{m}$$

Length of the vessel, $L = 1.44$ m

Initial height of water = 0.96 m

(*i*) **Height of the paraboloid, *z*:**

Speed, $N = 480$ r.p.m.

$$\therefore \quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 480}{60} = 50.26 \text{ rad/s}$$

When the vessel is rotated, paraboloid is formed (Fig. 6.73)

Let, r = Radius of paraboloid at the top of the vessel, and

z = Height of the paraboloid.

As the vessel is closed one, therefore,

Volume of air before rotation = Volume of air after rotation

or $$\pi R^2 L - \pi R^2 \times 0.96 = \frac{1}{2}\pi r^2 z$$

or $$\pi R^2 (1.44 - 0.96) = \frac{1}{2}\pi r^2 z$$

or $r^2 z = 2 \times 0.12^2 (1.44 - 0.96) = 0.0138$ ...(*i*)

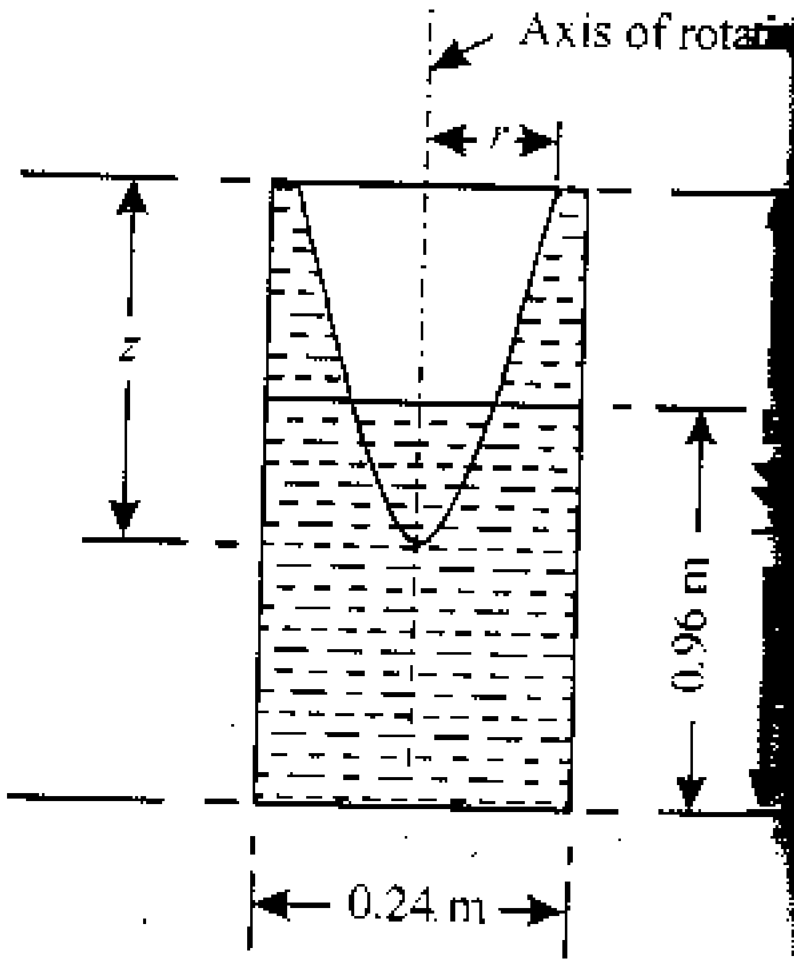


Fig. 6.73

Using the relation:

$$z = \frac{\omega^2 r^2}{2g}, \text{ we get}$$

$$z = \frac{50.26^2 \times r^2}{2 \times 9.81} = 128.75\, r^2$$

$$\therefore \quad r^2 = \frac{z}{128.75}$$

Substituting this value of r^2 in (*i*), we get

$$= \frac{z}{128.75} \times z = 0.0138$$

$\therefore$ $$z^2 = 0.0138 \times 128.75 = 1.777$$

or $$\mathbf{z = 1.333\ m\ (Ans.)}$$

(*ii*) **Speed of rotation, N:**

Let ω is the angular velocity, when axial depth is zero.

When axial depth is zero:

The height of paraboloid at the top = r

Using the relation:

$$z = \frac{\omega^2 r^2}{2g}, \text{ we get}$$

$$1.44 = \frac{\omega^2 r^2}{2 \times 9.81}$$

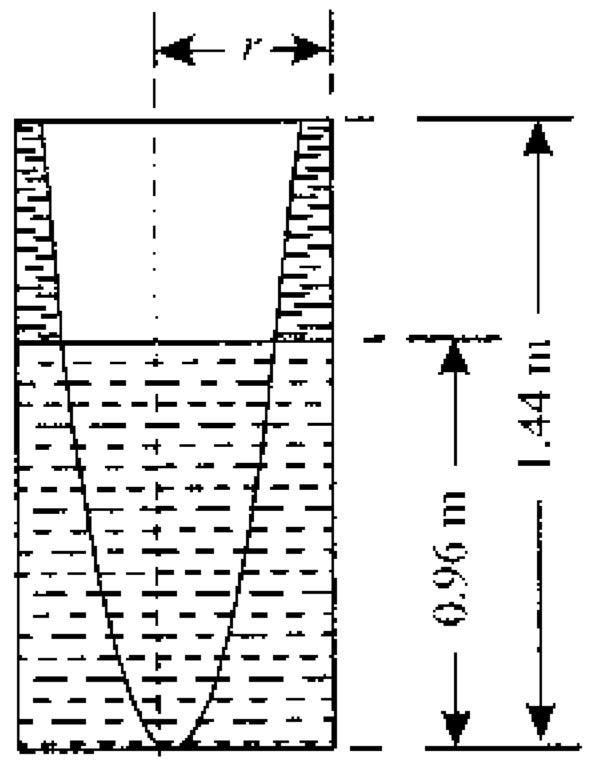


Fig. 6.74

$\therefore$ $$\omega^2 r^2 = 1.44 \times 2 \times 9.81 = 28.25 \quad \ldots(i)$$

Volume of air before rotation = volume of air after rotation

$\therefore$ $$\pi R^2 (1.44 - 0.96) = \text{Volume of paraboloid}$$

$$= \frac{1}{2}\pi r^2 z = \frac{\pi r^2}{2} \times 1.44$$

or $$\pi \times 0.12^2 \times 0.48 = 0.72\ \pi r^2$$

$\therefore$ $$r^2 = \frac{0.12^2 \times 0.48}{0.72} = 0.0096$$

Substituting the value r^2 in (*i*), we get

$$\omega^2 \times 0.0096 = 28.25$$

$\therefore$ $$\omega = \left(\frac{28.25}{0.0096}\right)^{1/2} = 54.25 \text{ rad/s}$$

But $$\omega = \frac{2\pi N}{60} \quad \therefore\ 54.25 = \frac{2\pi N}{60}$$

or $$\mathbf{N} = \frac{54.25 \times 60}{2\pi} = \mathbf{518\ r.p.m.\ (Ans.)}$$

Example 6.72. *A vessel, cylindrical in shape and closed at the top and bottom is* 0.24 *m in* *eter,* 1.44 *m long and contains water up to a height of* 0.96 *m. If is rotated at* 700 *r.p.m. what is* *area uncovered at the bottom of the tank?*

Solution. Radius of vessel, $R = \frac{0.24}{2} = 0.12$ m

Length of the vessel, $L = 1.44$ m

Initial height of water = 0.96 m

Speed, $N = 700$ r.p.m.

$\therefore$ Angular speed, $$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 700}{40} = 73.3 \text{ rad/s}$$

Area uncovered at the bottom of the tank:

If the tank is not closed at the top and also is very long, then the height of parabola corresponding = 73.3 rad/s will be

$$= \frac{\omega^2 R^2}{2g} = \frac{73.3^2 \times 0.12^2}{2 \times 9.81} = 3.943 \text{ m}$$

From Fig. 6.75, $y_1 + 1.44 + y_2 = 3.943$

or
$$y_1 + y_2 = 2.503 \text{ m} \qquad \ldots(i)$$

For the parabola GOH, we have

$$(1.44 + y_1) = \frac{\omega^2 r_1^2}{2g} = \frac{73.3^2 \times r_1^2}{2 \times 9.81} = 273.85\, r_1^2 \qquad \ldots(ii)$$

For the parabola IOJ, we have

$$y_1 = \frac{\omega^2 r_2^2}{2g} = \frac{73.3^2 \times r_2^2}{2 \times 9.81} = 273.85\, r_2^2 \qquad \ldots(iii)$$

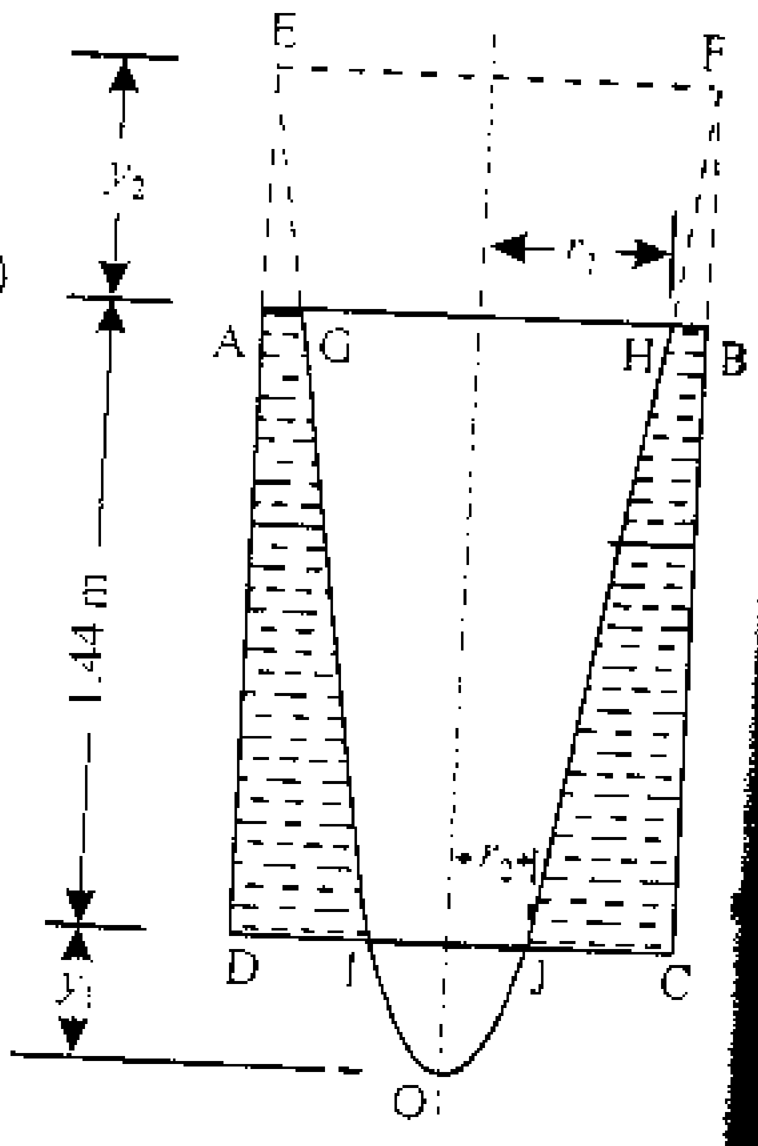


Fig. 6.75

Now, volume of air before rotation

= Volume of air after rotation

Volume of air before rotation

$$= \pi R^2 (1.44 - 0.96) = \pi \times 0.12^2 \times 0.48$$
$$= 0.0217 \text{ m}^3 \qquad \ldots(iv)$$

Volume of air after rotation = Volume of paraboloid GOH – volume of paraboloid IOJ

$$= \frac{1}{2} \times \pi r_1^2 \times (1.44 + y_1) - \frac{1}{2} \pi r_2^2 \times y_1 \qquad \ldots(v)$$

Equating (iv) and (v), we get

$$0.0217 = \frac{\pi}{2}[r_1^2 (1.44 + y_1) - r_2^2 y_1] \qquad \ldots(vi)$$

But from (ii), we have $r_1^2 = \left(\frac{1.44 + y_1}{273.85}\right)$

Substituting this value of r_1^2 in (vi), we get

or
$$0.0217 = \frac{\pi}{2}\left[\left(\frac{1.44 + y_1}{273.85}\right)(1.44 + y_1) - r_2^2 y_1\right]$$

Now, substituting the value of y_1 from (iii) in the above eqn., we get

$$0.0217 = \frac{\pi}{2}\left[\left(\frac{1.44 + 273.85\, r_2^2}{273.85}\right)(1.44 + 273.85 r_2^2) - r_2^2 \times 273.85 r_2^2\right]$$

or
$$\frac{0.0217 \times 2 \times 273.85}{\pi} = (1.44 + 273.85 r_2^2)^2 - (273.85)^2 r_2^4$$

$$\left[\text{Multiplying both sides by } 273.85 \times \frac{2}{\pi}\right]$$

or
$$3.783 = (2.074 + 788.69 r_2^2 + 74994\, r_2^4) - 74994 r_2^4$$

or
$$788.69\, r_2^2 = 1.709$$

or
$$r_2^2 = 0.002167 \text{ m}^2$$

∴ Area uncovered at the base

$$= \pi r_2^2 = \pi \times 0.002167 = \mathbf{0.0068\ m^2\ (Ans.)}$$

Example 6.73. *A vessel, cylindrical in shape and closed at the top and bottom, is 0.45 m in diameter and 1.5 m long. It contains water up to a depth of 1.2 m. The air above the water surface is*

$= \text{pressure head due to air} + OJ$

$= 9.17 + (HJ - HO)$ $\quad (\because OJ = HJ - H$

$= 9.17 + (1.5 - 1.224)$ $\quad \left\{ \begin{aligned} HJ &= 1.5 \text{ m} \\ HO &= y_2 = 1.224 \text{ m} \end{aligned} \right\}$

= 9.446 m of water (Ans.)

(*ii*) *At the edge*:

The pressure head at the edge M

$= \text{pressure head due to air} + \text{height of water above } M$

$= 9.17 + AM$

$= 9.17 + (AK + KM) = 9.17 + (y_1 + KM)$

$= 9.17 + (y_1 + OJ)$

$= 9.17 + 2.545 + 0.276$ $\quad (\because y_1 = 2.545$

= 11.99 m of water (Ans.) $\quad \left[\begin{aligned} OJ &= HJ - HO \\ &= 1.5 - 1.224 = 0.276 \text{ m} \end{aligned} \right.$

Example 6.74. *A vessel cylindrical in shape and closed at the top and bottom is of radius R a height H. The vessel is completely filled with water. If it is rotated about its vertical axis with a spe ω radians/sec, what is the total pressure force exerted by water on the top and bottom of the vesse*

Solution. Radius of the vessel = R

Height of the vessel = H

Angular speed = ω

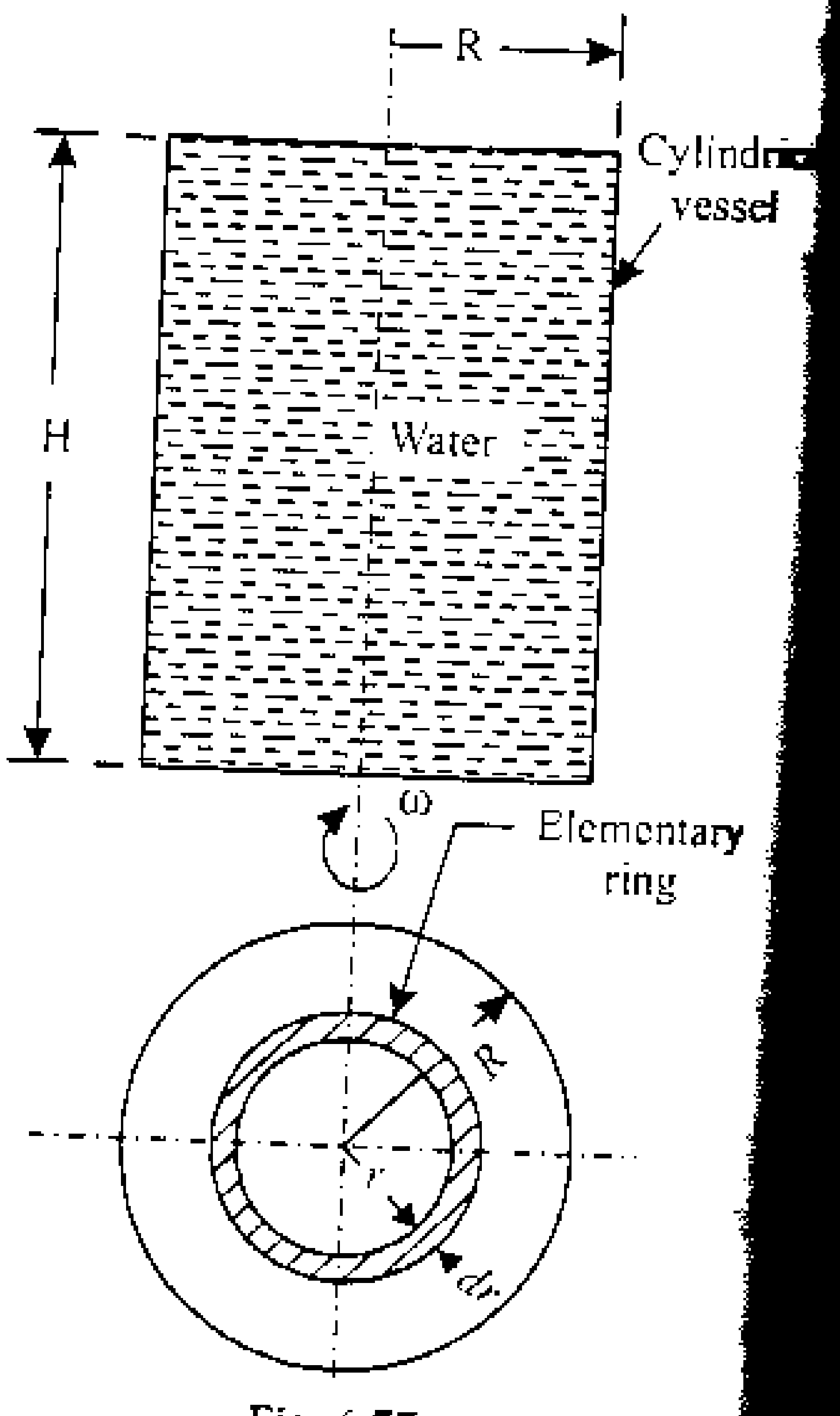


Fig. 6.77

As the vessel is closed and completely filled with water, and when it is rotated, the water will exert force on the complete top and bottom of the vessel.

Total pressure force exerted on the top of the vessel, F_{top}:

As the top of the vessel is in contact with water and is in horizontal plane, the pressure variation at any radius in horizontal plane is given as

$$\frac{\partial p}{\partial r} = \frac{\rho v^2}{r} \qquad \text{[Eqn. (6.35)]}$$

$$= \frac{\rho \omega^2 r^2}{r} = \rho \omega^2 r \qquad [\because v = \omega r)$$

Integrating both sides, we get

$$\int dp = \int \rho \omega^2 r dr$$

or $$p = \frac{\rho}{2} \omega^2 r^2$$

Refer to Fig. 6.77. Consider an elementary ring of radius r and width dr on the top of the vessel.

Area of the elementary ring = $2\pi r dr$

Force on the elementary ring = Intensity of pressure × area of ring

$= p \times 2\pi r \, dr$

$= \frac{\rho}{2} \omega^2 r^2 \times 2\pi r . dr \qquad \left(\because p = \frac{\rho}{2} \omega^2 r\right.$

∴ Total force on the top of the vessel,

$$F_{top} = \int_0^R \frac{\rho}{2}\omega^2 r^2 \times 2\pi r\, dr$$

$$= \frac{\rho}{2}\omega^2 \times 2\pi \int_0^R r^3 dr$$

$$= \frac{\rho}{2}\omega^2 \times 2\pi \left[\frac{r^4}{4}\right]_0^R = \frac{\rho}{2}\omega^2 \times 2\pi \times \frac{R^4}{4}$$

$$= \frac{\rho}{4}\omega^2 \times \pi R^4$$

i.e., $$F_{top} = \frac{\rho}{4}\omega^2 \pi R^4 \quad \text{...(6.39)}$$

Total pressure force exerted on the bottom of the vessel, F_{bottom}:

F_{bottom} = Total force on the top of the vessel + Weight of water in cylinder

$$= \frac{\rho}{4}\omega^2 \pi R^4 + \omega \times \pi R^2 \times H$$

i.e., $$F_{bottom} = \frac{\rho}{4}\omega\pi R^4 + \omega\pi R^2 H \quad \text{...(6.40)}$$

Example 6.75. *A vessel cylindrical in shape and closed at the top and bottom is 0.3 m in diam-* *and 0.225 m in height. The vessel is completely filled with water. If it is rotated about its vertical* *with a speed of 300 r.p.m. what is the total pressure force exerted by water on the top and bottom* *essel ?*

Solution. Radius of the vessel, $R = \frac{0.3}{2} = 0.15\text{ m}$

Height of the vessel, $H = 0.225$ m.

Speed, $N = 300$ r.p.m.

∴ Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.41$ rad/sec

Total pressure force exerted by water on the top of the vessel, F_{top}:

Using the relation: $$F_{top} = \frac{\rho}{4} \times \omega^2 \times \pi R^4 \quad \text{[Eqn. (6.39)]}$$

$$= \frac{w}{g \times 4}\omega^2 \pi R^4 \qquad \left[\because \rho = \frac{w}{g},\ w = 9.81\text{ kN/m}^3\right]$$

$$= \frac{9.81}{9.81 \times 4} \times (31.41)^2 \times \pi \times (0.15)^4 = 0.392\text{ kN}$$

i.e., $\mathbf{F_{top} = 0.392}$ **kN(Ans.)**

Total pressure force by water on the bottom of the vessel, F_{bottom}:

$$F_{bottom} = F_{top} + w \times \pi R^2 \times H$$

$$= 0.392 + 9.81 \times \pi \times 0.15^2 \times 0.225 = 0.548\text{ kN}$$

∴ $\mathbf{F_{bottom} = 0.548}$ **kN (Ans.)**

Example 6.76. *A closed vertical cylinder 0.4 m in diameter and 0.4 m in height is completely* *with oil of specific gravity 0.80. If the cylinder is rotated about its vertical axis at 200 rpm,* *late the thrust of oil on top and bottom covers of the cylinder.* **(AMIE Summer, 2001)**

Solution. *Given*: $R = \frac{0.4}{2} = 0.2\,\text{m};\ H = 0.4\ \text{m};\ S = 0.8;\ N = 200$ r.p.m.

F_{top}:

Using the relation: $F_{top} = \frac{\rho}{4}\omega^2 \times \pi R^4$ [Eqn. (6.39)]

$$= \frac{0.8 \times 1000}{4} \times \left(\frac{2\pi \times 200}{60}\right)^2 \times \pi \times (0.2)^4 = \mathbf{440.98\ N\ (Ans.)}$$

Again, using the relation: $\mathbf{F}_{\mathbf{bottom}} = F_{top} + w \times \pi R^2 \times H$

$$= 440.98 + (0.8 \times 1000 \times 9.81) \times \pi \times 0.2^2 \times 0.4 = \mathbf{835.46\ N\ (Ans.)}$$

Example 6.77. *A hollow sphere of radius R, completely filled with the liquid is rotated about its vertical axis at an angular speed ω. Locate the circulat line maximum pressure with respect to the centre of the sphere.* **(AMIE Winter, 2001)**

Solution. The circular line of maximum pressure will be a horizontal circle on the internal surface of the circle *aa*. Let its location be at a distance *h* below the centre of the sphere. All points on the circle *aa* will subjected to a centrifugal pressure $\frac{1}{2}\rho\,\omega^2 r^2$, and a hydrostatic pressure $\rho g\,(R + h)$.

The total pressure on any point,

$$p = \frac{1}{2}\rho\omega^2 r^2 + \rho g(R + h)$$

$$= \frac{1}{2}\rho\omega^2 r^2 + \rho g\left\{R + \sqrt{R^2 - r^2}\right\} \quad (\because h = \sqrt{R^2 - r^2})$$

For p to be maximum,

$$\frac{dp}{dr} = 0 = \frac{1}{2}\rho\omega^2 \times 2r + \rho g\left\{\frac{1}{2}(R - r^2)^{-1/2} \times (-2r)\right\}$$

$$\text{or} \quad \frac{\omega^2}{g} = \frac{1}{\sqrt{R^2 - r^2}} \quad \text{or} \quad (R^2 - r^2) = \left(\frac{g}{\omega^2}\right)^2$$

$$\text{or} \quad h^2 = R^2 - r^2 = \left(\frac{g}{\omega^2}\right)^2 \quad \text{or} \quad h = \frac{g}{\omega^2}$$

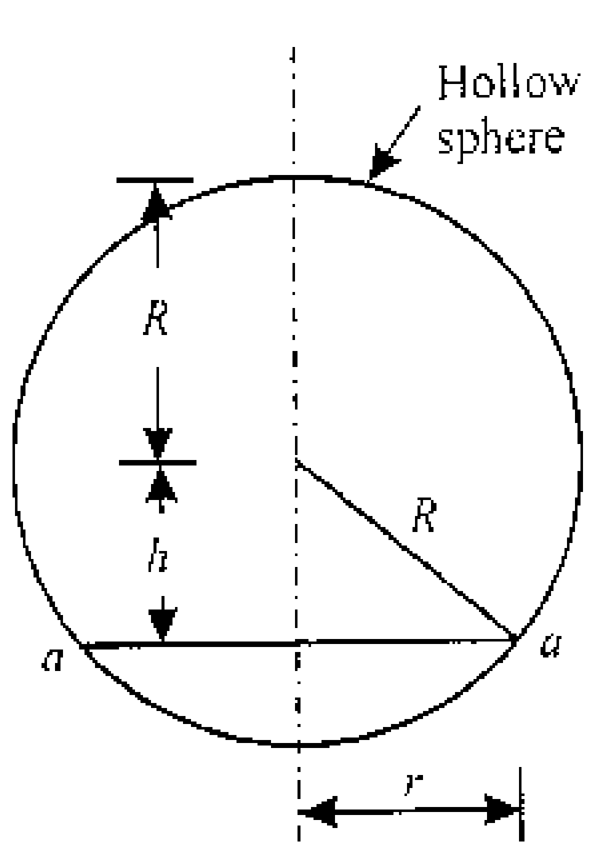


Fig. 6.78

Thus the circular line of maximum pressure is a horizontal circle, at a distance $h = \frac{g}{\omega^2}$ below the centre of the sphere. (**Ans.**)

6.11.6. Equation of Free Vortex Flow

In the case of free vortex flow, from eqn. (6.34), we have

$$v = \frac{C}{r}$$

Substituting the value of v in eqn. (6.37), we get

$$dp = \frac{\rho v^2}{r}dr - \rho g\,dz$$

$$= \rho \times \frac{C^2}{r^2 \times r}dr - \rho g dz$$

$$= \frac{\rho C^2}{r^3}dr - \rho g\,dz$$

Refer to Fig. 6.79. Consider two points 1and 2 in the fluid having radii r_1 and r_2 respectively from the central axis; their heights being z_1 and z_2 from bottom of the vessel.

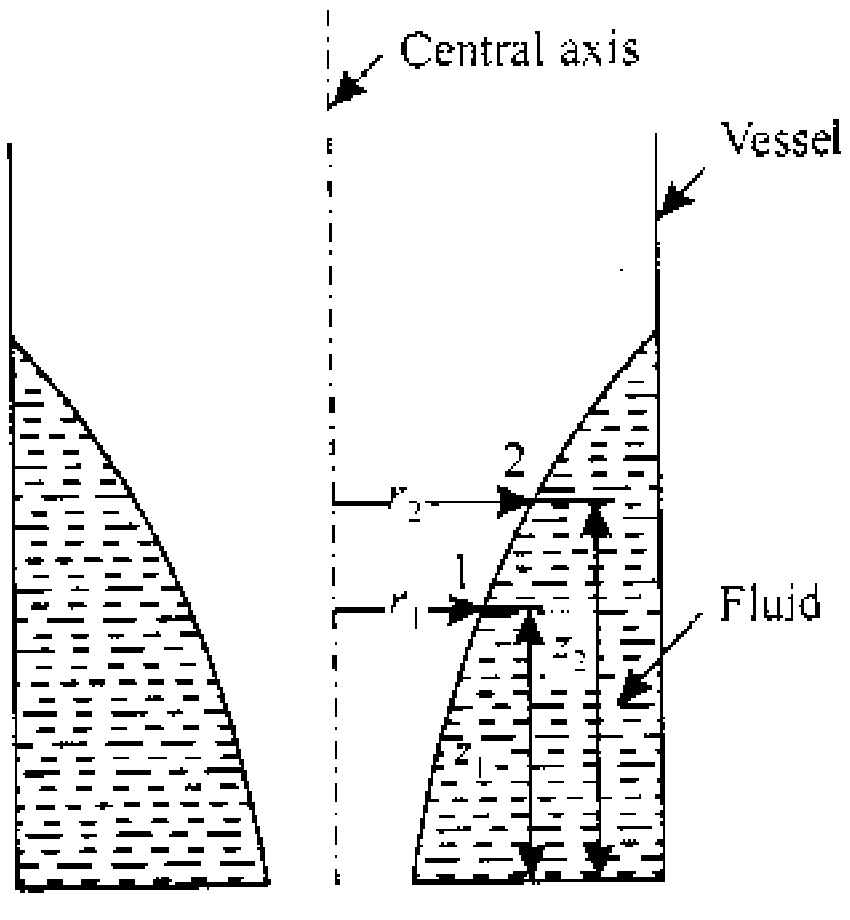


Fig. 6.79

Integrating the above equation for the points 1 and 2, we get

$$\int_1^2 dp = \int_1^2 \frac{\rho C^2}{r^3} dr - \int_1^2 \rho g \, dz$$

or

$$p_2 - p_1 = \rho C^2 \int_1^2 \frac{dr}{r^3} - \rho g \int_1^2 dz$$

$$= \rho C^2 \left[\frac{r^{-3+1}}{-3+1} \right]_1^2 - \rho g (z_2 - z_1)$$

$$= \rho C^2 \left[-\frac{1}{2r^2} \right]_1^2 - \rho g (z_2 - z_1)$$

$$= -\frac{\rho C^2}{2} \left[\frac{1}{r_2^2} - \frac{1}{r_1^2} \right] - \rho g (z_2 - z_1)$$

$$= -\frac{\rho}{2} [v_2^2 - v_1^2] - \rho g (z_2 - z_1) \quad \left(\because v_2 = \frac{C}{r_2}, v_1 = \frac{C}{r_1} \right)$$

$$= \frac{\rho}{2} (v_1^2 - v_2^2) - \rho g (z_2 - z_1)$$

Dividing both sides by ρg, we get

$$\frac{p_2 - p_1}{\rho g} = \frac{v_1^2 - v_2^2}{2g} - (z_2 - z_1)$$

or

$$\left(\frac{p_2}{\rho g} - \frac{p_1}{\rho g} \right) = \left(\frac{v_1^2}{2g} - \frac{v_2^2}{2g} \right) + (z_1 - z_2)$$

or

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \qquad ...(6.41)$$

Eqn. (6.41) is the Bernoulli's equation. Hence *Bernoulli's equation is applicable in the case of vortex flow.*

Example 6.78. *In a free cylindrical vortex flow, at a point in the fluid at a radius of* 300 *mm and a* ht *of* 150 *mm, the velocity and pressure are* 15 *m/s and* 120 *kN/m² respectively. If the fluid is air* g *weight density of* 0.012 *kN/m³, find the pressure at a radius of* 600 *mm and at a height of* 300 *mm.*

Solution. *At point* 1:

Radius, $r_1 = 300 \text{ mm} = 0.3 \text{ m}$

Height, $z_1 = 150 \text{ mm} = 0.15 \text{ m}$

Velocity, $v_1 = 15 \text{ m/s}$

Pressure, $p_1 = 120 \text{ kN/m}^2$

At point 2:

Radius, $r_2 = 600 \text{ mm} = 0.6 \text{ m}$

Height, $z_2 = 300 \text{ mm} = 0.3 \text{ m}$

Density of air, $w = 0.012 \text{ kN/m}^2$

Pressure, p_2:

We know, for free vortex flow

$$v \times r = \text{constant} \qquad \text{[Eqn. (6.33)]}$$

∴

$$v_1 r_1 = v_2 r_2$$

or

$$v_2 = \frac{v_1 r_1}{r_2} = \frac{15 \times 0.3}{0.6} = 7.5 \text{ m/s}$$

Using the equation:

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

or
$$\frac{120}{0.012} + \frac{15^2}{2 \times 9.81} + 0.15 = \frac{p_2}{w} + \frac{7.5^2}{2 \times 9.81} + 0.3$$

or
$$10000 + 11.47 + 0.15 = \frac{p_2}{w} + 2.86 + 0.3$$

or
$$10011.62 = \frac{p_2}{w} + 3.16$$

or
$$\frac{p_2}{w} = 10008.46$$

or
$$p_2 = 0.012 \times 10008.46 = \mathbf{120.1\ kN/m^2}\ \textbf{(Ans.)}$$

Example 6.79. *Two statonary, horizontal flat plates with an external diameter of 400 mm are placed 10 mm apart. A vertical pipe 50 mm in diameter delivers 0.005 m³/s of water to the centre of the plates. The water is discharged to the periphery of the plates at atmospheric pressure of 98 kN/m². Assuming radial flow and neglecting losses, determine the absolute pressure at the entrance of the flow.* **[UPSC Exams.]**

Solution. Diamete of annular space, $d_i = 50$ mm $= 0.05$ m

External diameter of the plate, $d_0 = 400$ mm $= 0.4$ m

Distance between the plates, $t = 10$ mm $= 0.01$ m

Atmospheric pressure, $p_0 = 98$ kN/m²

Discharge, $Q = 0.005$ m³/s

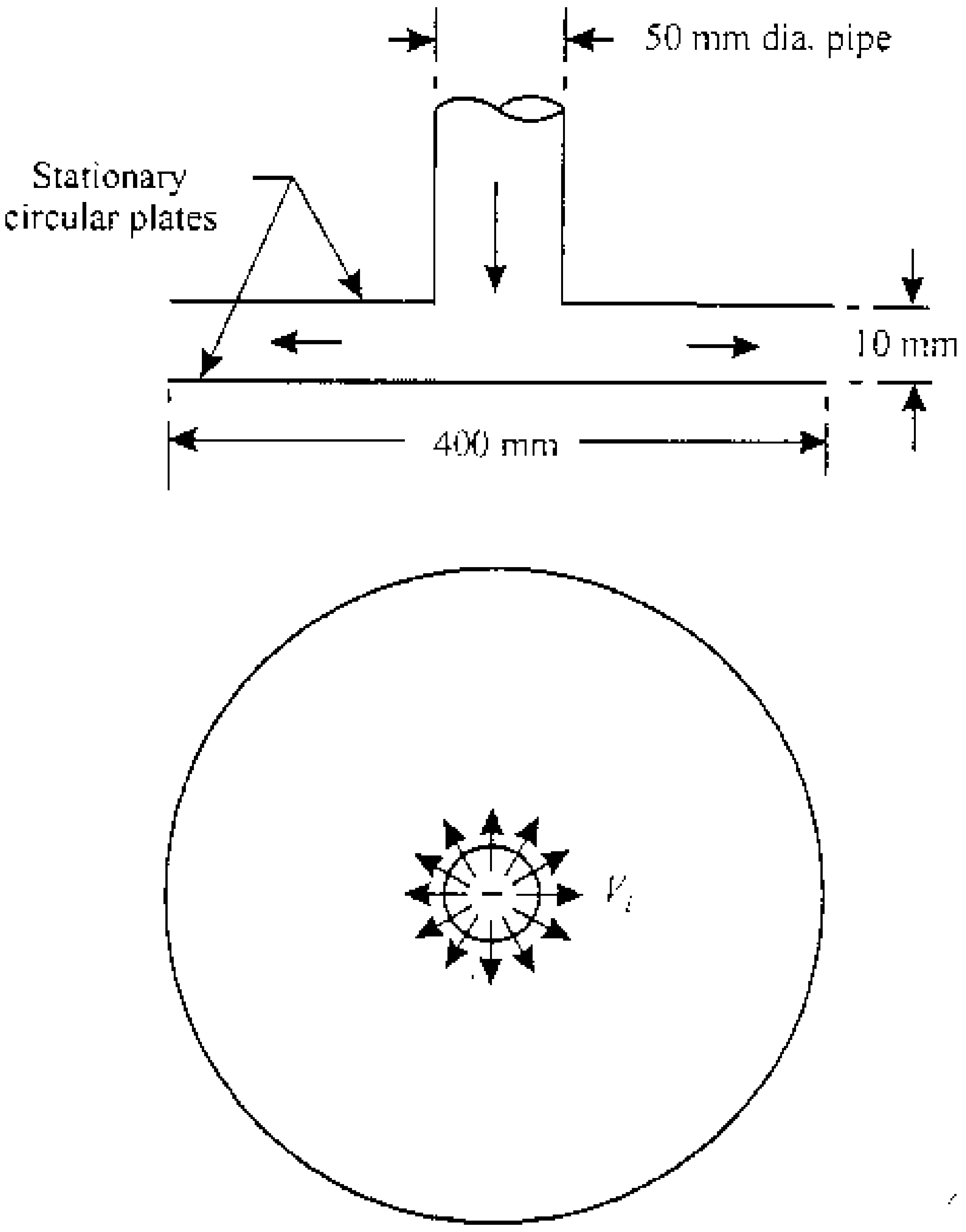


Fig. 6.80

Absolute pressure at the entrance of the flow, p_i :

Using the continuity equation, the velocity at the entrance to the annular space,

$$V_i = \frac{Q}{\pi d_i t} = \frac{0.005}{\pi \times 0.05 \times 0.01} = 3.18 \text{ m/s}$$

Velocity at the periphery of plates,

$$V_0 = \frac{Q}{\pi d_0 . t} = \frac{0.005}{\pi \times 0.4 \times 0.01} = 0.398 \text{ m/s}$$

Applying the Bernoulli's equation between the inlet to and exit from plates, we get

$$\frac{p_i}{w} + \frac{V_i^2}{2g} + z_i = \frac{p_0}{w} + \frac{V_0^2}{2g} + z_0$$

$$\frac{p_i}{w} + \frac{3.18^2}{2 \times 9.81} = \frac{98 \times 10^3}{9810} + \frac{0.398^2}{2 \times 9.81} \qquad (\because z_i = z_0)$$

or $$\frac{p_i}{w} + 0.515 = 9.989 + 0.00807$$

or $$\frac{p_i}{w} = 9.482 \text{ or } p_i = 9810 \times 9.482 = 93018 \text{ N/m}^2 \text{ or } 93 \text{ kN/m}^2$$

Hence **$p_i = 93$ kN/m² (Ans.)**

6.12. Liquids in Relative Equilibrium

When a tank filled with a liquid is made to move with a constant acceleration, initially the fluid particles will move relative to each other and to the boundries of the tank but, after a certain duration of time there will not be any relative movement between the fluid particles and boundaries of the container and the *whole fluid mass moves as a single unit* (A similar situation arises when the fluid mass is made to rotate with a uniform velocity). When such motion occurs, the fluids are said to be in **"relative equilibrium"**. Under such circumstances, since there is relative motion, the fluid is *not subjected to shearing forces*. Furthermore, the *fluid pressure acts normal to the surface in contact with it.*

Analysis of the fluid masses subjected to acceleration or deceleration can be made by using the principles of hydrostatics and giving due considerations to the effects of accelerating or decelerating forces.

6.12.1 Liquid in a Container Subjected to Uniform Acceleration in the Horizontal Direction

Consider a tank filled with liquid and being *accelerated horizontally to the right with uniform acceleration a_x.*

After sloshing of the liquid particles for some time the motion of the liquid stabilizes and the

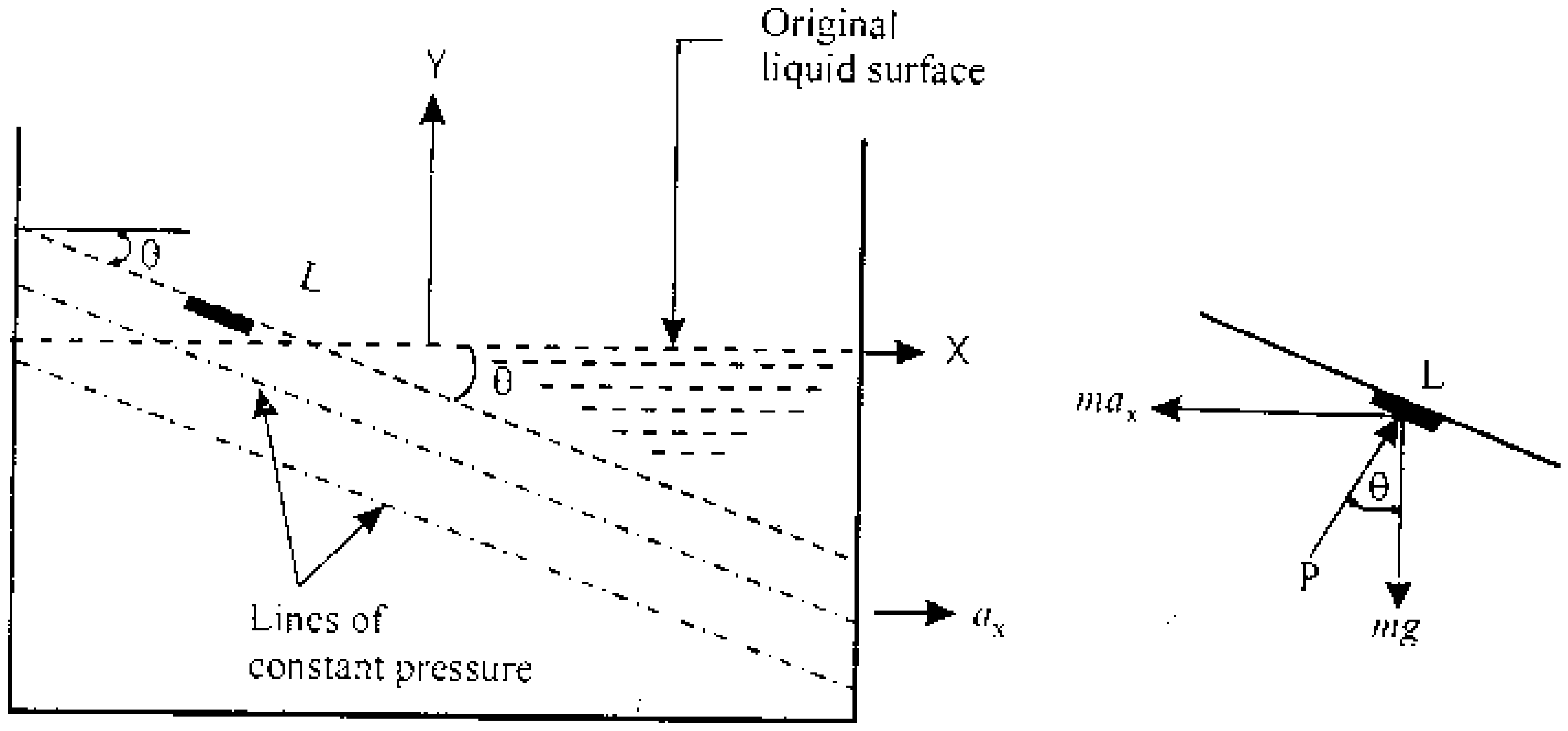


Fig. 6.81. Liquid under constant linear acceleration in horizontal direction.

liquid moves as a solid mass under the action of accelerating force. The final position of the liquid in the tank is as shown in Fig. 6.81, slope being upwards in the direction opposite to that of horizontal acceleration.

Now, let us consider the equilibrium of a fluid particle L lying on the free surface. The pressure force P exerted by the surrounding fluid on particle L is normal to the free surface. The particle is subjected to the following forces:

(*i*) Normal pressure force P,

(*ii*) Weight mg acting vertically downwards, and

(*iii*) The accelerating force ma_x acting in horiznotal direction.

Resolving horizontally and vertically respectively, we have,

$$P \sin\theta = ma_x \qquad ...(i)$$

$$P \cos\theta = mg \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\tan\theta = \frac{a_x}{g} \qquad ...(6.42)$$

Since the term $\frac{a_x}{g}$ is constant at all points on the free liquid surface, hence tan θ is constant and consequently the free surface is a straight plane inclined at θ (downward) along the direction of acceleration (See fig. 6.81).

Considering the equilibrium of a fluid element at depth h from the free surface we have:

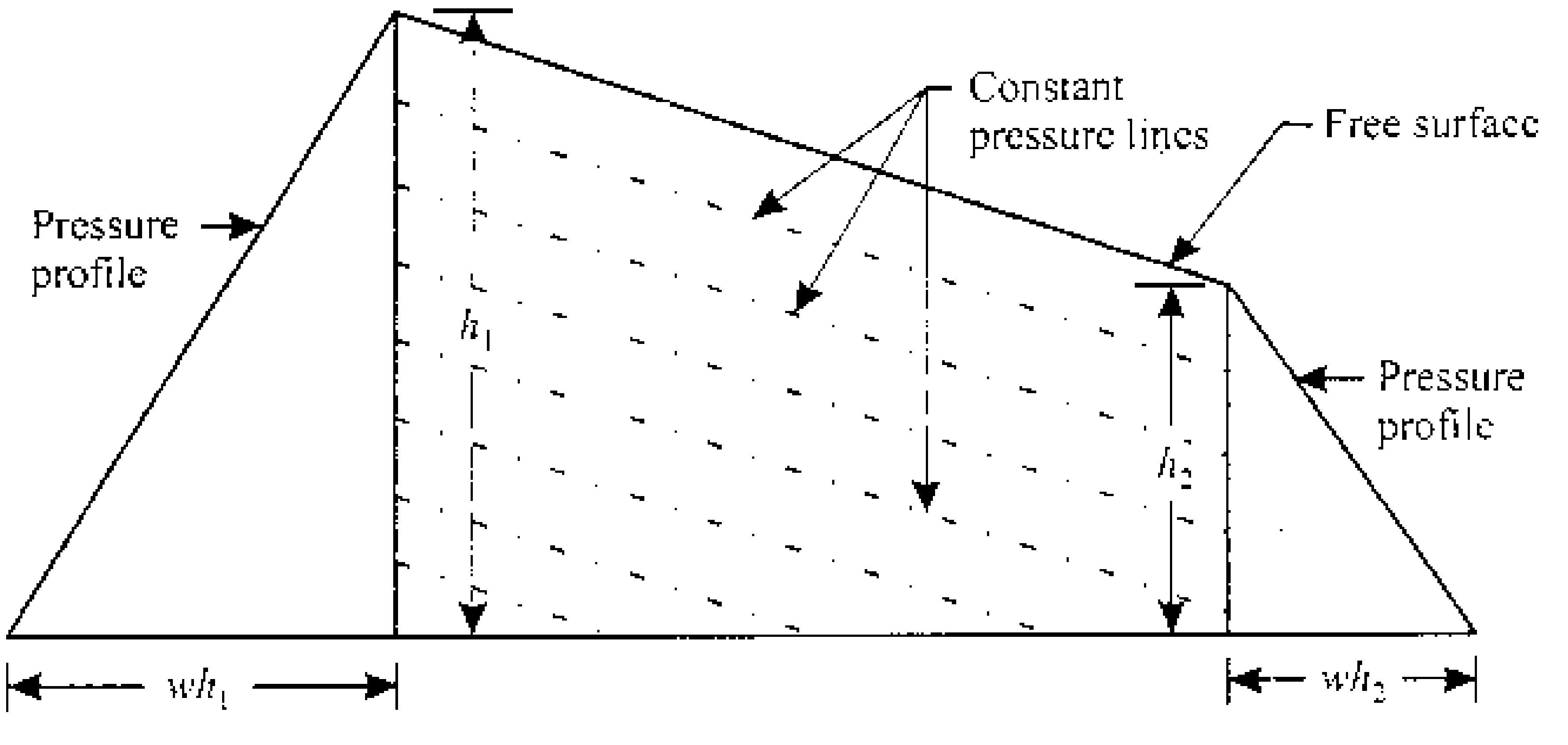


Fig. 6.82. Pressure distribution for horizontally accelerated fluid.

$$pdA = p_{\text{atm.}}dA + whdA$$

where, p_{atm} = Atmospheric pressure, and

dA = Cross-sectional area of an elementary prism.

or $$p = p_{\text{atm}} + wh;\ p = wh \text{ (gauge)} \qquad ...(6.4$$

This means that *pressure at any point in a liquid subjected to constant horizontal accelera* ***equals the head above that point***. Thus, *lines of constant pressure will be parallel to the free* ***li*** *surface*. Fig. 6.82 shows the constant pressure lines and the variation in liquid pressure on near front of the tank. With the decrease in depth in the direction of acceleration, the pressure along the bottom of the tank also decreases.

- If the tank is completely filled with liquid and is closed at the top, the pressure builds up at the rear and is greater than that at the point (there being no preliminary adjustment in the surface elevation). The slope of the constant pressure lines is, however, still governed by the relation: $\tan\theta = \frac{a_x}{g}$.

- It may be noted that *so long as the container provides a continuous connection in the* ***liquid*** *mass, its shape does not matter.*

Note. The fuel tank of an aeroplane duing take off is *an example of liquid in a container subjected to uniform acceleration in the horizontal direction.*

Example 6.80. *An open tank 6 m long, 2.4 m deep and 3.6 m wide contains oil of specific gravity* 0.85 *to a depth of* 1.2 *m. If the tank is accelerated along its length on a horizontal track at a constant acceleration* 3.2 *m/s*^{2}. *Determine*:

(*i*) *The new position of the oil surface.*

(*ii*) *Pressures at the bottom of the tank at the front and rear edges.*

(*iii*) *The amount of spill if the tank is given an horizontal acceleration of* 4.8 *m/s*^{2} *instead of* 3.2 *m/s*^{2}.

Solution. *Given*: Tank dimensions: 6*m* (length) × 3.6 *m* (width) × 2.4 *m* (depth),

Sp. gr. of oil = 0.85; $a_x = 3.2\ m/s^2$.

Refer to Fig. 6.83.

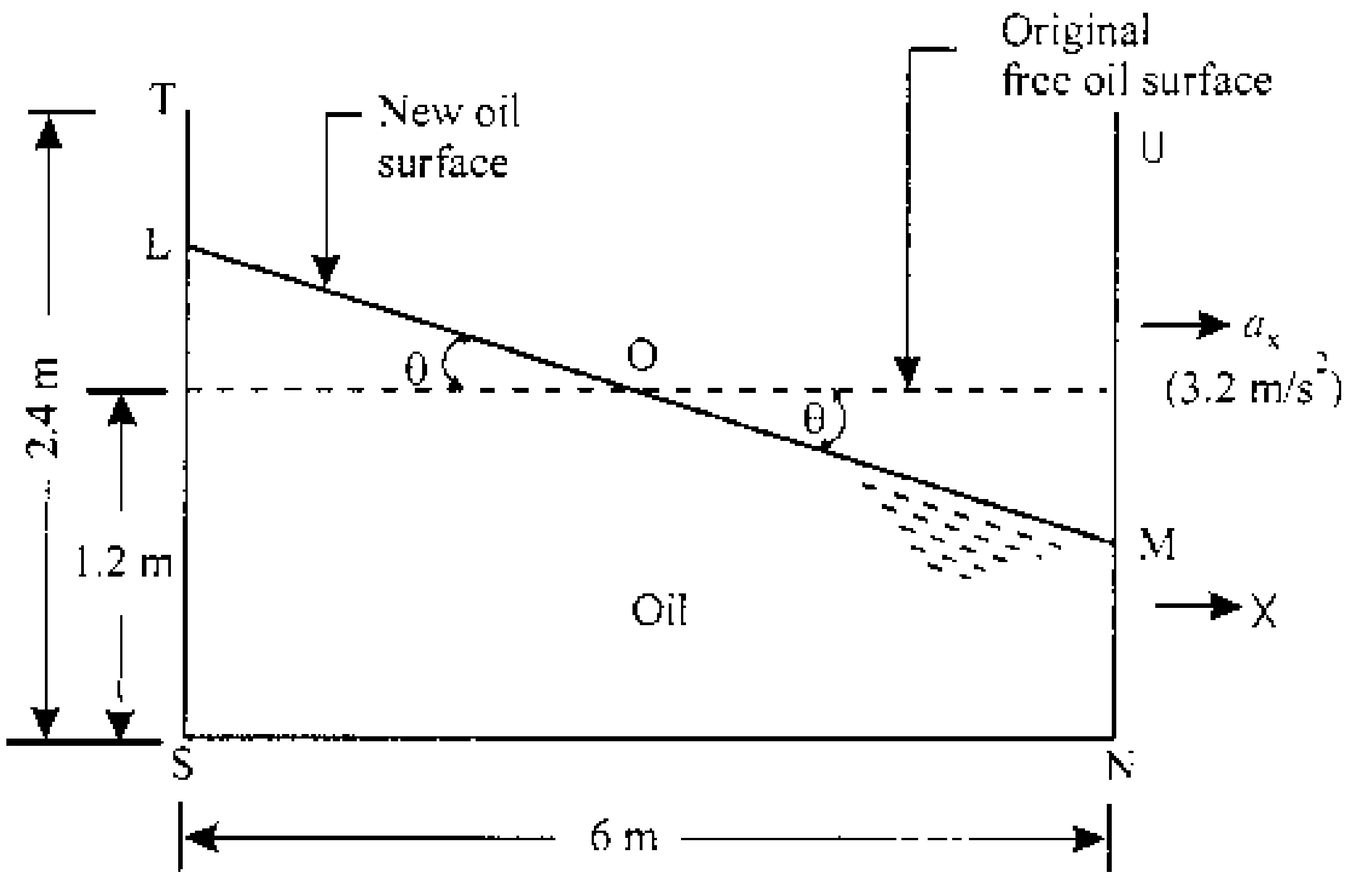


Fig. 6.83

(*i*) **New position of the oil surface, θ:**

Inclination of the new oil surface (θ) is given by

$$\tan\theta = \frac{a_x}{g} = \frac{3.2}{9.81} = 0.3262$$

$$\therefore \quad \theta = \tan^{-1}(0.3262) = \mathbf{18.07°\ (Ans.)}$$

(*ii*) **Pressures at the bottom of the tank at the front and rear edges,**

The depth of oil at the front edge N,

$$h_N = 1.2 - \frac{6}{2} \times \tan\theta$$

$$= 1.2 - 3 \times 0.3262 = 0.221 \text{ m}$$

The depth of oil at the rear edges,

$$h_S = 1.2 + \frac{6}{2}\tan\theta$$

$$= 1.2 + 3 \times 0.3262 = 2.179 \text{ m}$$

∴ Pressure at N, $p_N = wh_N = (9.81 \times 0.85) \times 0.221 =$ **1.843 kN/m^2 (Ans.)**

and, Pressure at S, $p_S = wh_S = (9.81 \times 0.85) \times 2.179 =$ **18.169 kN/m^2 (Ans.)**

(*iii*) **The amount of spill with an acceleration of 4.8 m/s²:**

Refer to Fig. [6.84 (*i*)]

Inclination of the oil surface,

$$\tan\theta' = \frac{a_x}{g} = \frac{4.8}{9.81} = 0.4893$$

$$\left(\theta' = \tan^{-1}(0.4893) = 26.07°\right)$$

If there were no spill, the oil surface would swing about an axis at O. Piezometric head at N,

$$h_S' = 1.2 + \frac{6}{2}\tan\theta' = 1.2 + 3 \times 0.4893 = 2.668\,\text{m}$$

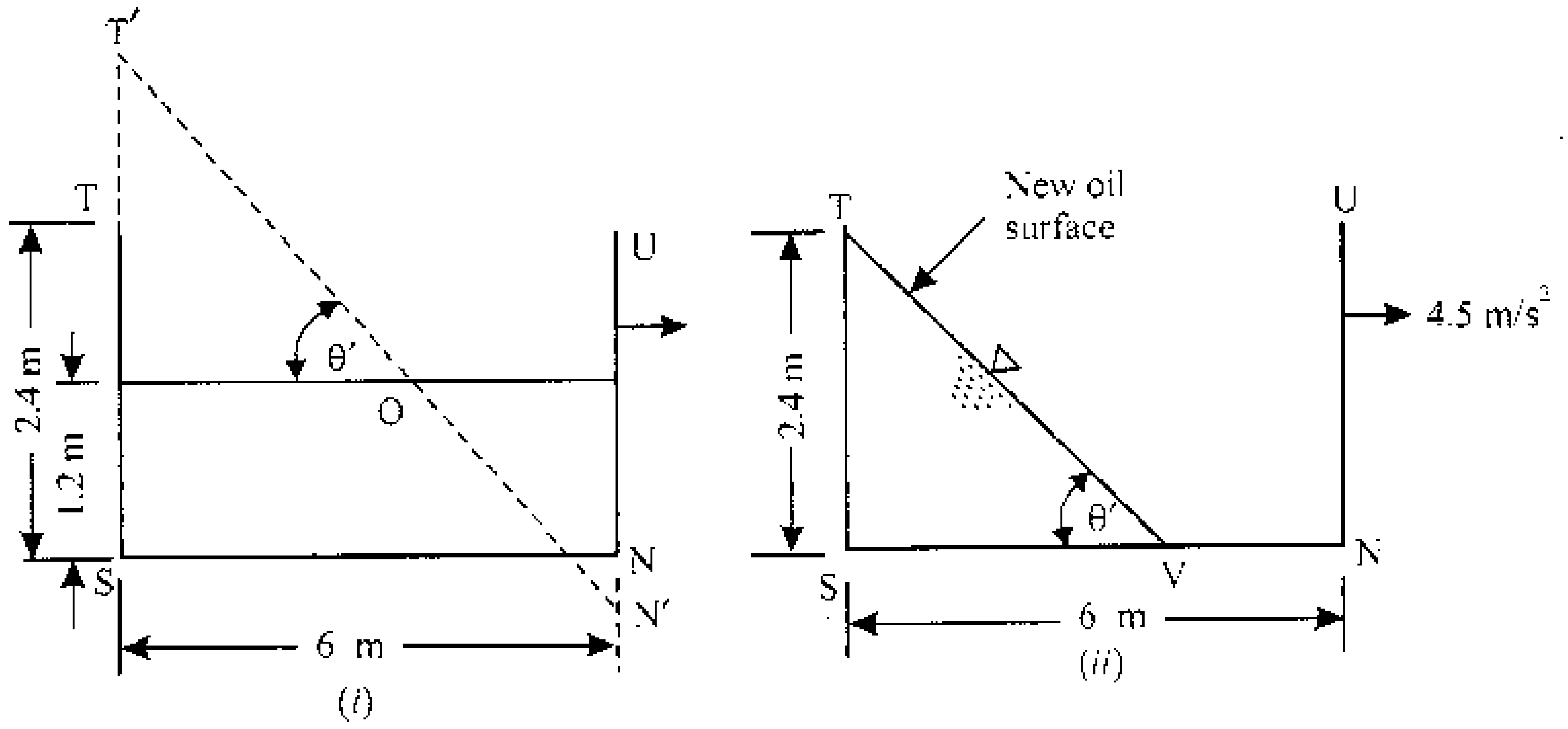


Fig. 6.84

Since this is larger than the depth of the tank there will be a spill of the oil. The new oil surface will have a depth of h_S = depth of tank = 2.4 m at S and a slope of θ'.

X-intercept of the surface at the bottom (SV) can be found from ΔTSV as follows.

$$\frac{TS}{SV} = \tan\theta'$$

or $$SV = \frac{TS}{\tan\theta'} = \frac{2.4}{0.4893} = 4.9\,\text{m}$$

TV is the new oil surface [Fig. 6.84 (*ii*)].

Volume of oil = ΔTSV × width of tank

$$= \left(\frac{1}{2} \times 2.4 \times 4.9\right) \times 3.6 = 21.17\ \text{m}^3$$

Original volume of oil = 6 × 3.6 × 1.2 = 25.92 m³

∴ *Spill of oil* = 25.92 – 21.17 = **4.75 m³ (Ans.)**

Example 6.81. *A spherical tank of raidus* 1.5 *m radius is half-filled with oil of specific gravity* 0.9. *If the tank is given a horizontal acceleration of 11 m/s², calculate:*

(*i*) *The inclination of the oil surface to the horizontal.*

(*ii*) *Maximum pressure on the tank.*

Solution. *Given*: Radius of th tank, r = 1.5 m; Sp. gr. of oil = 0.9; a_x = 11 m/s².

(*i*) **The inclination of the oil surface to the horizontal θ:**

Refer to Fig. 6.85: $\tan\theta = \frac{a_x}{g} = \frac{11}{9.81} = 1.1213$

$\therefore \quad \theta = \tan^{-1}(1.1213) = \mathbf{48.3°}$ **(Ans.)**

(LM is the original oil surface and RS is the new oil surface. The surface tilts around 0).

(*ii*) **Maximum pressure on the tank:**

The maximum pressure acts on the boundary point where the *depth (measured normal to the free surface) is maximum.*

In this case maximum depth is OT = r = 1.5 *m*

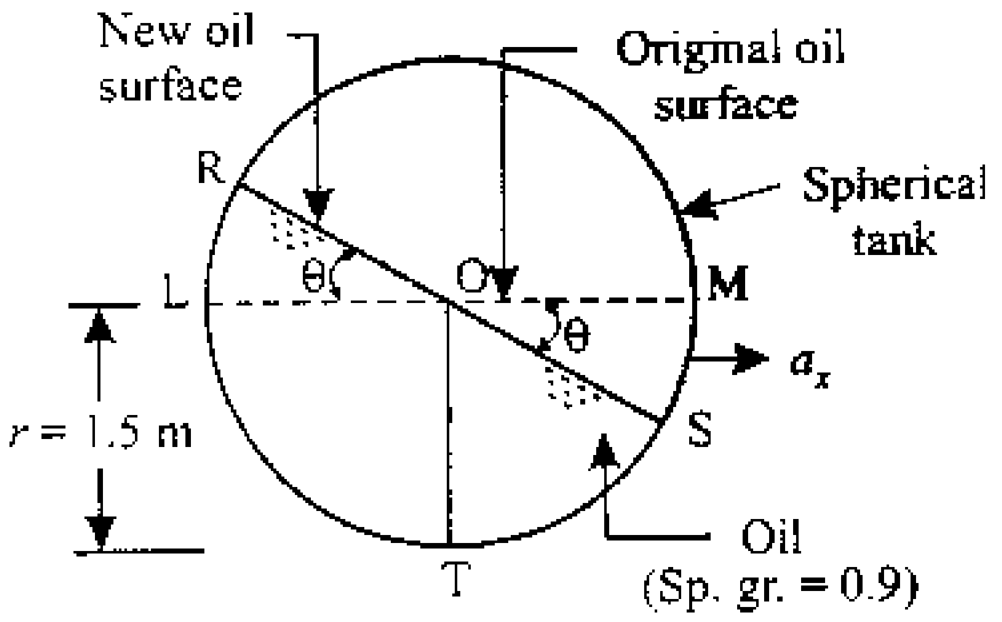


Fig. 6.85

Hence $\left(\dfrac{p}{w}\right)_{\max} = 1.5$

or $p_{\max} = w \times 1.5 = (9.81 \times 0.9) \times 1.5 = \mathbf{13.24\ kN/m^2}$ **(Ans.)**

Example 6.82. *A closed tank 5m long, 1.8 m wide and 1.6 m deep initially contains water to a depth of* 1.1 *m. The top has an opening in the front part to have air space at atmospheric pressure. If the tank is given at a acceleration at a constant value of* 2.5 *m/s² along its length, calculate the total pressure force on the top of the tank.*

Solution. *Given:* Dimensions of the closed tank = 5 m × 1.8 m × 1.6 m; $a_x = 2.5$ m/s²

In the Fig. 6.86. EF is the original water surface.

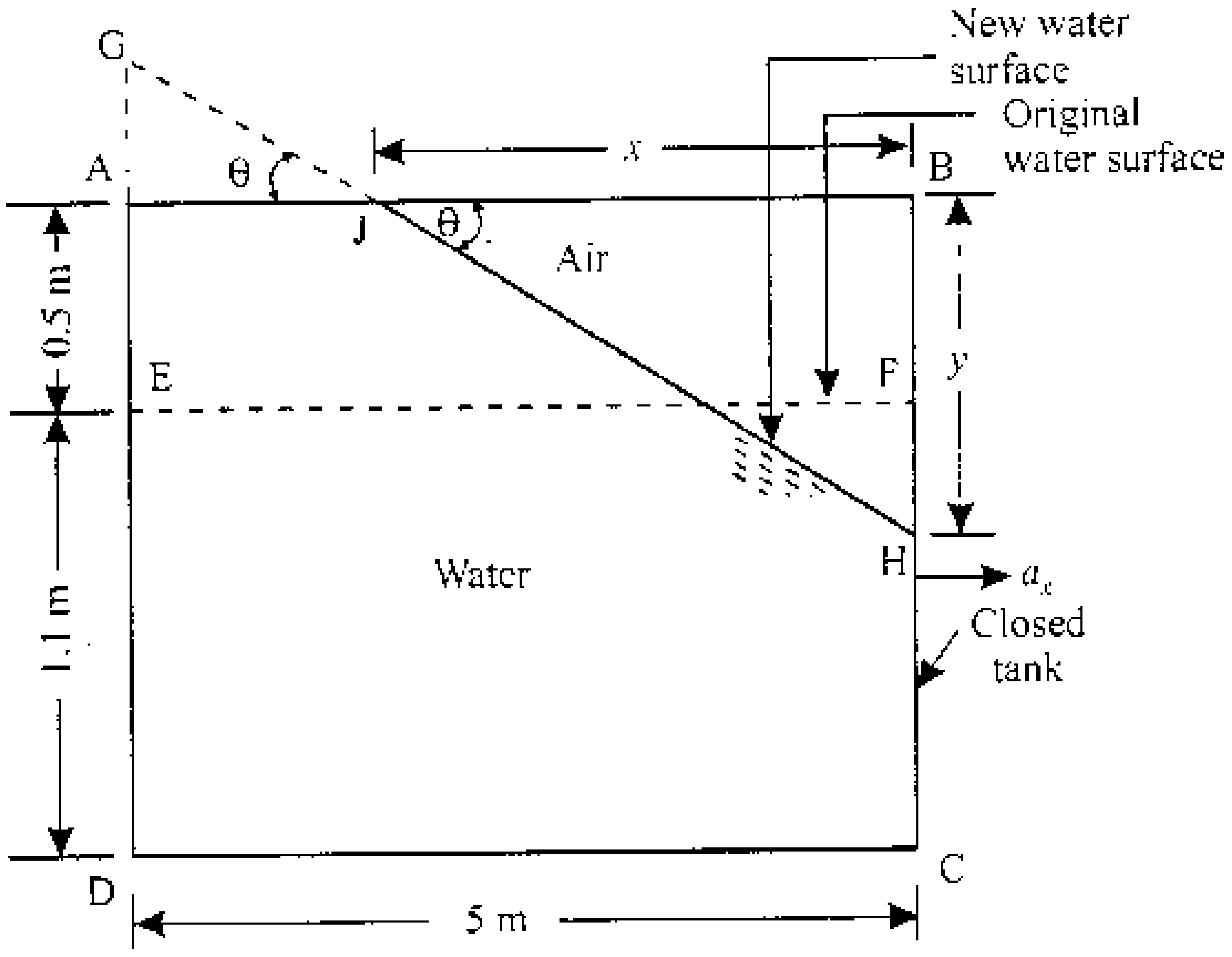


Fig. 6.86

After the acceleration of $a_x = 2.5$ m/s², the water surface slope is

$$\tan\theta = \frac{a_x}{g} = \frac{2.5}{9.81} = 0.2548$$

or $\theta = \tan^{-1}(0.2548) = 14.29°$

Since there is no spill of water, the air space will remain same as at start.

Air space volume, $V_{air} = 0.5 \times 5 \times 1.8 = 4.5\ \text{m}^3$

Let JH be the new water surface at an inclination of θ to the horizontal.

If JB = x and BH = y, b = breadth of the tank, then

$$y = x \tan\theta$$

and, $$V_{air} = \frac{1}{2} \times x \times y \times b = \frac{1}{2} \times x \times \tan\theta \times b = \frac{1}{2}\, x^2\, B \tan\theta$$

or, $$4.5 = \frac{1}{2} x^2 \times 1.8 \times 0.2548$$

$\therefore$ $$x = 4.43 \text{ m, and } y = 4.43 \times 0.2548 = 1.13 \text{ m}$$

Hence CH = Depth of water in the front = 1.6 – 1.13 = 0.47 m

$$AJ = 5 - x = 5 - 4.43 = 0.57 \text{ m}$$

$$AG = AJ \tan\theta = 0.57 \times 0.2548 = 0.145 \text{ m}$$

The pressure profile on the top is represented by the ΔAGJ extending over the width. Pressure force on the top,

$$P_{top} = \left(\frac{1}{2} \times AG \times AJ \times \text{breadth}\right) \times w$$

$$= \frac{1}{2} \times 0.145 \times 0.57 \times 1.8 \times 9.81 = \mathbf{0.73\ kN\ (Ans.)}$$

The force acts vertically upwards at $\frac{AJ}{3} = \frac{0.57}{3} = 0.19$ m *from A at the mid-width section.*

Note: In this case free surface does not tilt at the mid-length. As there is no spill the volume of *water and air volume are conserved.*

Example 6.83. *A closed tank 12 m long, 3.6 m high and 2.4 m wide contains oil of specific gravity 0.85 and is given a horizontal acceleration of 0.28 g to the right in the direction of 12 m side.*

(*i*) Calculate: *The pressure difference between (a) a point on the top rear edge and a point on the front edge. (b) a point on the bottom front edge and a point on the top front edge.*

(*ii*) *Sketch the lines of equal pressure.*

Solution. *Given*: Dimensions of the closed tank = 12 m × 2.4 m × 3.6 m;

Specific gravity of oil = 0.85; Horizontal acceleration, $a_x = 0.28$ g

Refer to Fig. 6.87. At an acceleration of a_x let UV be the hydraulic gradient line. Its inclination is given by,

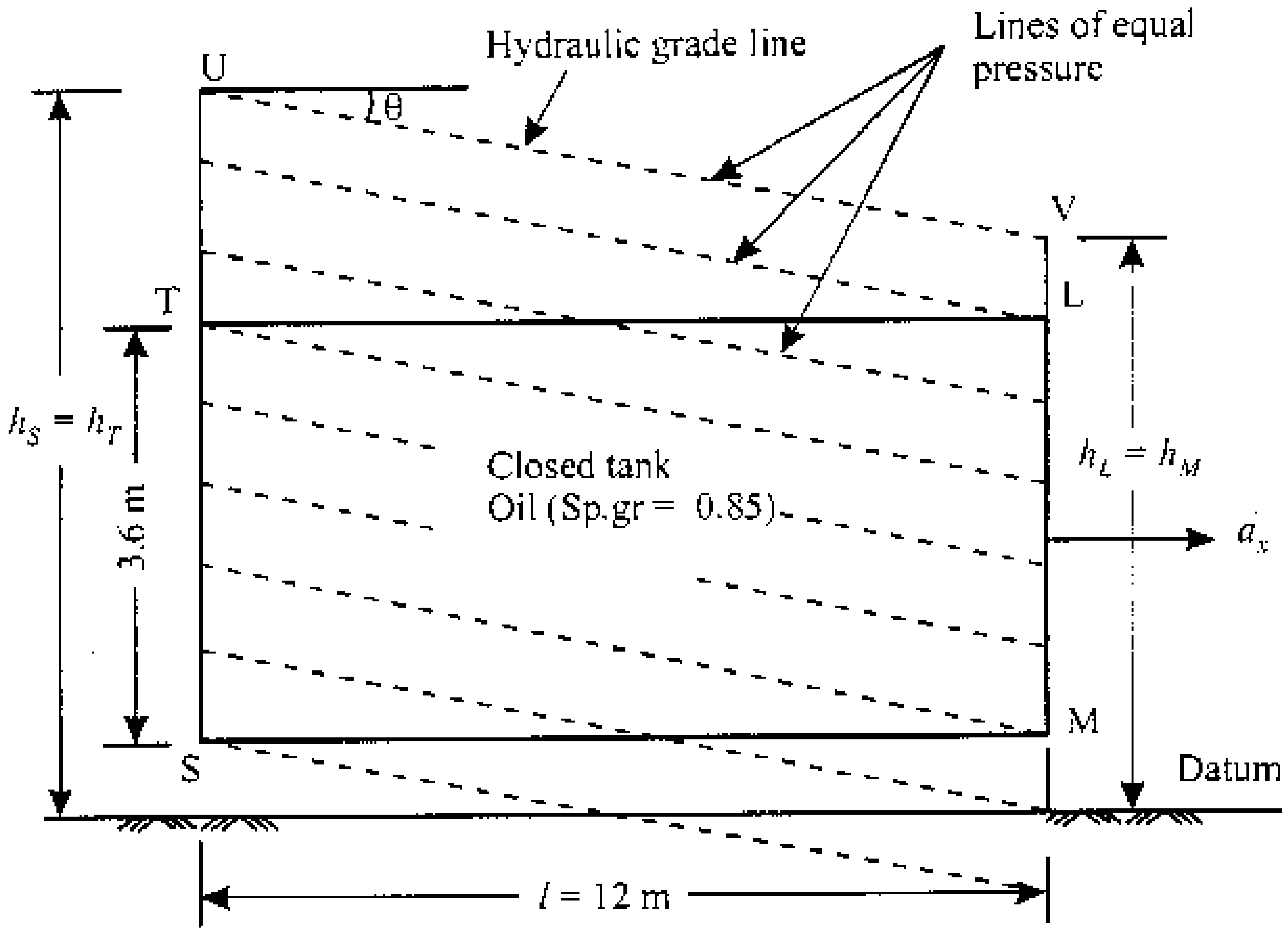


Fig. 6.87

$$\tan\theta = \frac{h_T - h_L}{l} = \frac{a_x}{g} = \frac{0.28\,g}{g} = 0.28$$

or $$\theta = \tan^{-1}(0.2) = 15.64°$$

(*a*) **$p_L - p_T$:**

$$h_T - h_L = l \times \tan\theta = 12 \times 0.28 = 3.36 \text{ m}$$

But $$h_T - h_L = \left(\frac{p_T}{w} + z_T\right) - \left(\frac{p_L}{w} + z_L\right)$$

Here, $$z_T = z_L$$

$$\therefore \quad p_T - p_L = w \times 3.36 = (9.81 \times 0.85) \times 3.36 = \mathbf{28.02\ kN/m^2\ (Ans.)}$$

(*b*) **$p_M - p_L$:**

Along ML the hydraulic gradient line is constant.

Hence, $$h_M = h_L$$

$$\left(\frac{p_M}{w} + z_M\right) - \left(\frac{p_L}{w} + z_L\right) = 0$$

or $$p_M - p_L = w\,(z_L - z_M)$$

$$= (9.81 \times 0.85) \times 3.6 = \mathbf{30.02\ kN/m^2\ (Ans.)}$$

(*ii*) Since the pressure distribution is hydrostatic in any vertical direction and the hydraulic gradient line is inclined at θ to horiznotal (line UV) the lines of equal pressure will be parallel to UV, as shown in Fig. 6.87.

Example 6.84. *A tank LMNSTU shown in Fig. 6.88. is filled with water. A small opening at T keeps the pressure at T atmospheric.*

(i) . Calculate the acceleration a_x required to cause onset of cavitation at L.

(ii) What will be the pressure at that acceleration at points M, N, S and U?

Assume local atmospheric pressure head = 10 m of water and vapour pressure head = 0.48 m (abs.) of water.

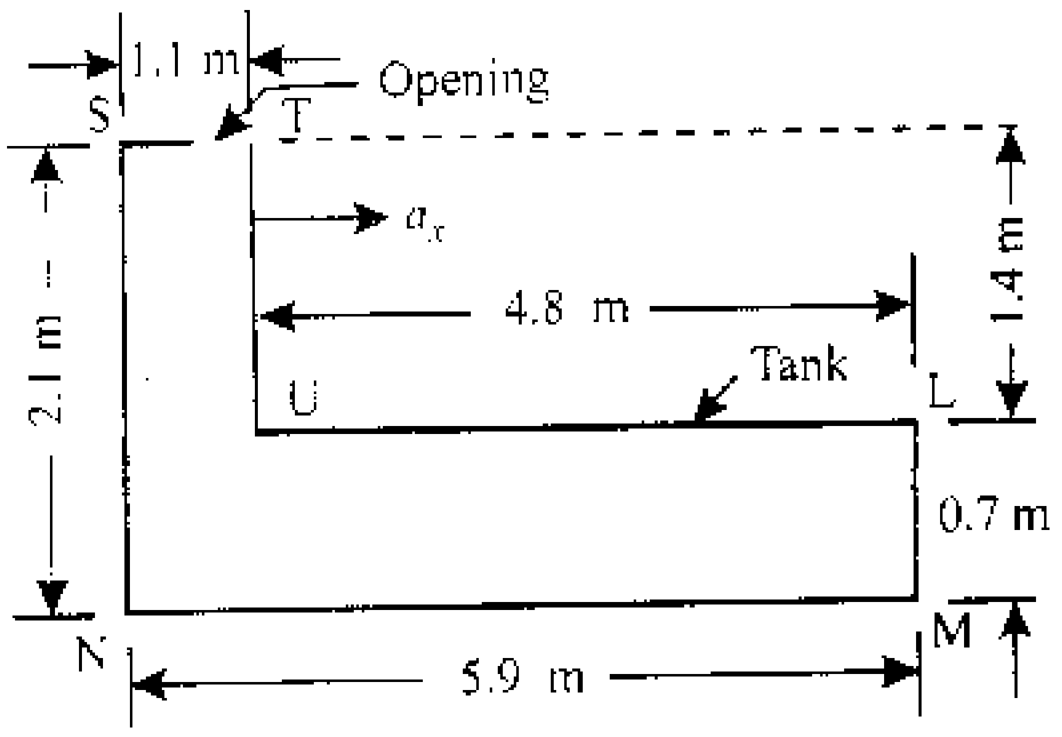


Fig. 6.88

Solution. (*i*) **Acceleration a_x:**

At the acceleration a_x the hydraulic gradient line will be inclined at θ, given by

$$\tan\theta = \frac{a_x}{g}$$

Since p_T = pressure at T = atmspheric pressure, the hydraulic gradient line will pass through T as shown in Fig. 6.89 by the line VTW

Then above an arbitrary datum:

$h_T = h_U$, and

$h_L = h_M$

Also $\quad h_T - h_L = LU \tan\theta = 4.8 \tan\theta$

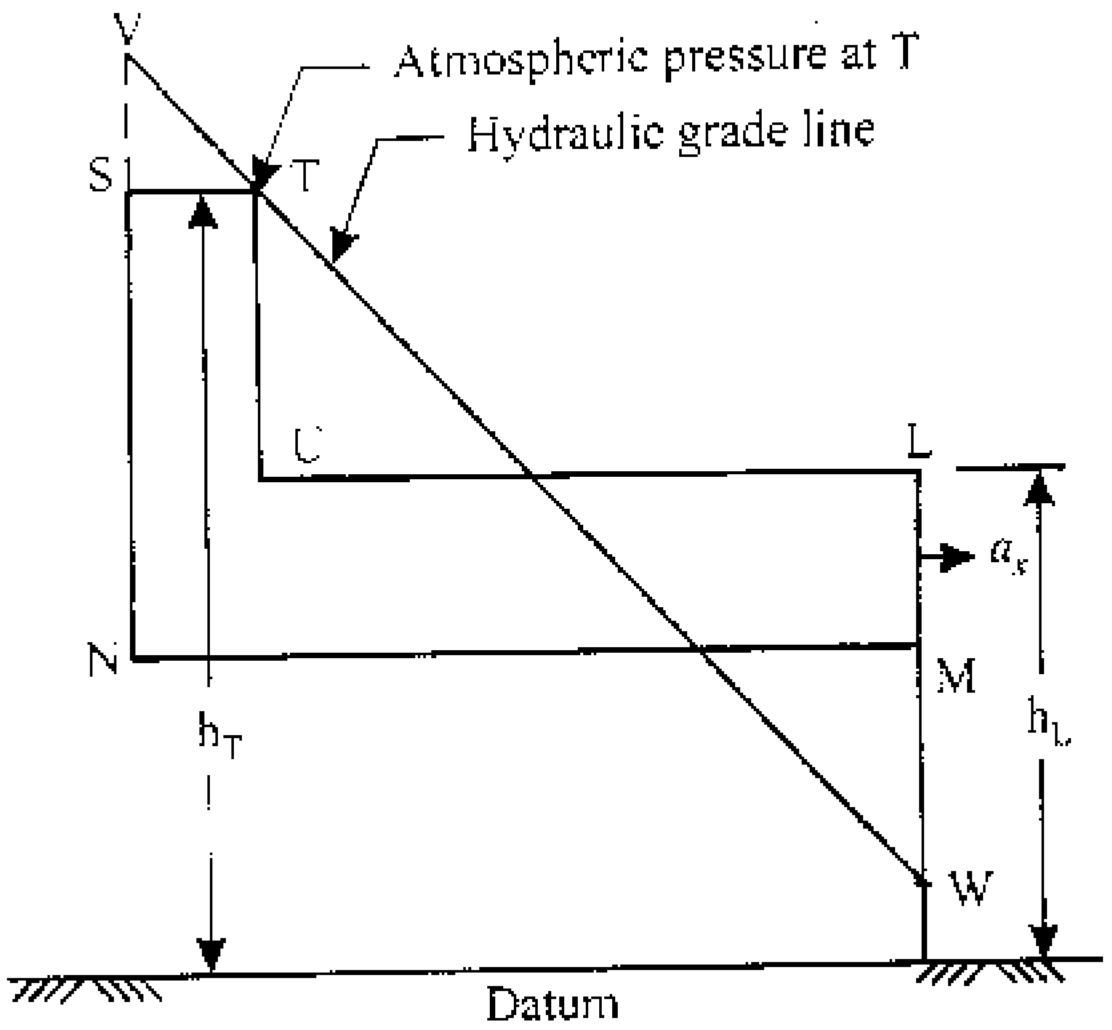


Fig. 6.89

At the onset of cavitation at L,

$$p_L = p_v = \text{Vapour pressure.}$$

Considering the absolute pressures,

$$h_T - h_L = \left(\frac{p_T}{w} + z_T\right) - \left(\frac{p_L}{w} + z_L\right)$$

$$= \left(\frac{p_T}{w} - \frac{p_L}{w}\right) + (z_T - z_L)$$

$$= (10 - 0.48) + (1.4) = 10.92 \text{ m}$$

But $\quad h_T - h_L = 4.8 \tan\theta$

Hence, $\quad \tan\theta = \dfrac{a_x}{g} = \dfrac{h_T - h_L}{4.8} = \dfrac{10.92}{4.8} = 2.275$

or $\quad a_x = 9.81 \times 2.275 = \mathbf{22.32\ m/s^2}$ **(Ans.)**

(*ii*) **Pressures at point M, N, S and U:**

Pressure at M, p_M:

$$h_M - h_L = \left(\frac{p_M}{w} + z_M\right) - \left(\frac{p_L}{w} + z_L\right) = 0$$

$$\frac{p_M}{w} = \frac{p_L}{w} + (z_L - z_m) = 0.48 + 0.7 = 1.18 \text{ m}$$

$\therefore \quad p_M = 9.81 \times 1.18 = \mathbf{11.576\ kN/m^2}$ **(abs.) (Ans.)**

Pressure at U, p_U:

$$h_T - h_U = \left(\frac{p_T}{w} + z_T\right) - \left(\frac{p_U}{w} + z_U\right) = 0$$

$$\frac{p_U}{w} = \frac{p_T}{w} + (z_T - z_U) = 10 + 1.4 = 11.4 \text{ m}$$

$\therefore \quad p_U = 9.81 \times 11.4 = \mathbf{111.834\ kN/m^2}$ **(abs.) (Ans.)**

Pressure at S, p_S:

$$h_S - h_T = 1.1 \tan\theta = 1.1 \times 2.275 = 2.5 \text{ m}$$

Also, $\quad h_S - h_T = \left(\dfrac{p_S}{w} + z_S\right) - \left(\dfrac{p_T}{w} + z_T\right)$

$$\frac{p_S}{w} = \frac{p_T}{w} + (z_T - z_S) + 2.5$$

$$= 10 + 0 + 2.5 = 12.5 \text{ m}$$

$\therefore \quad p_S = 9.81 \times 12.5 = \mathbf{122.625\ kN/m^2}$ **(Ans.)**

Pressure at N, p_N:

$$h_S - h_N = \left(\frac{p_S}{w} + z_S\right) - \left(\frac{p_N}{w} + z_N\right) = 0$$

$$\frac{p_N}{w} = \frac{p_S}{w} + (z_S - z_N)$$

$$= 12.5 + 2.1 = 14.6 \text{ m}$$

$\therefore \quad p_N = 9.81 \times 14.6 = \mathbf{143.226\ kN/m^2}$ **(abs.) (Ans.)**

Example 6.85. *A closed oil tanker 3.5 m long, 1.8 m wide and 2 deep contains 1.6 m depth of oil of specific gravity 0.8.* Calculate:

(*i*) *The acceleration which may be imparted to the tank in* ***the direction of its length so that bottom*** *front end of the tank is just exposed.*

(*ii*) *The net horizontal force acting on the tanker side and show that* ***this equals the force necessary*** *to accelerate the liquid mass in the tanker.*

Take specific weight of water = 9.81 kN/m³ ***(Roorkee University)***

Solution. *Given*: Dimensions of the tank: 3.5 m (length) × 1.8 m (width) × 2 m **(depth)**;

Depth of oil = 1.6 m, sp. gr. of oil = 0.8

(*i*) **Acceleration a_x:**

The following point are worth noting:

- Since the tank is closed, therefore, the liquid cannot spill from it under any acceleration imparted to it. The *quantity of oil inside the tank remains the same.*
- The oil surface which was initially horizontal (indicated by TU) assumes the profile MVS when the front bottom end M is just exposed.

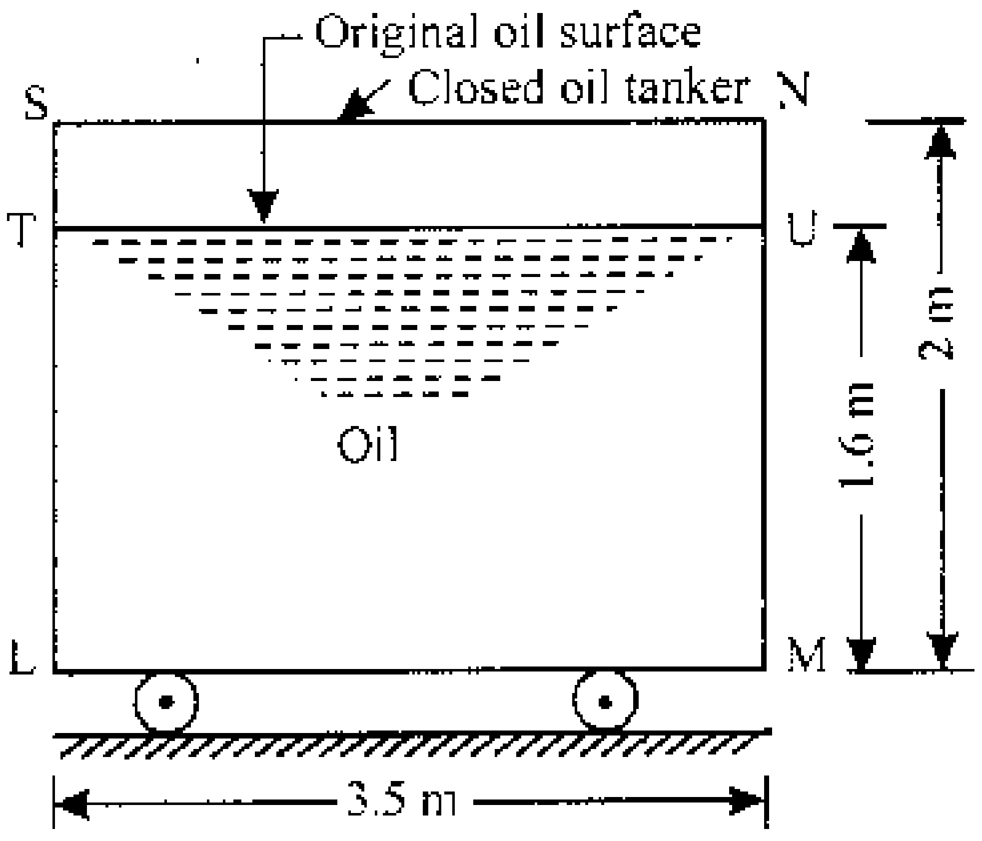


Fig. 6.90

Equating volumes of oil before and after the motion,

Volume of rectangel LMUT = Volume of trapezium LMVS

$$3.5 \times 1.6 \times 1.8 = \frac{3.5 + (3.5 - x)}{2} \times 2 \times 1.8$$

$$\therefore \qquad x = 1.4 \text{ m}$$

$$\tan\theta = \frac{MN}{NV} = \frac{2}{1.4} = 1.428$$

Also, $$\tan\theta = \frac{a_x}{g} = 1.428$$

$$\therefore \qquad a_x = 9.81 \times 1.428 = \mathbf{14.01\ m/s^2\ (Ans.)}$$

When the free surface MV is extended, it meets **LS** produced at W.

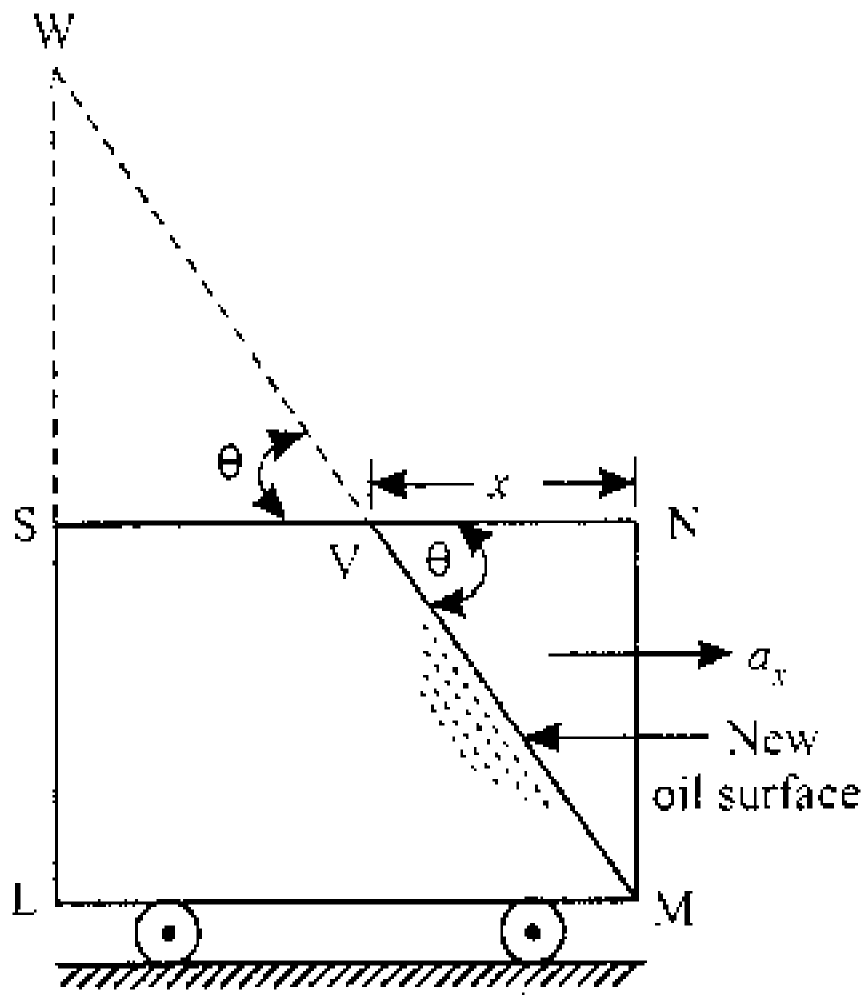


Fig. 6.91

$$\frac{SW}{SV} = \tan\theta = 1.428$$

$$\therefore \qquad SW = (3.5 - 1.4) \times 1.428 = 3 \text{ m}$$

This represents an imaginary column of oil above S.

Now, $$p_S = w \times SW = (9.81 \times 0.8) \times 3 = 23.544 \text{ kN/m}^2$$

and, $$p_L = w \times LW = (9.81 \times 0.8) \times (3 + 2) = 39.24 \text{ kN/m}^2$$

∴ Pressure force on the trailing/rear face *LS*,

$$P_{LS} = p_{avg.} \times \text{area}$$

$$= \left(\frac{23.544 + 39.24}{2}\right) \times (2 \times 1.8) = \mathbf{113\ kN}$$

$$\left[\text{Alternatively: } P_{LS} = wA\bar{x} = (9.81 \times 0.8) \times (2 \times 1.8) \times \left(3 + \frac{2}{2}\right) = 113 \text{ kN}\right]$$

The force needed to accelerate the liquid mass in the tank,

$$F = \text{Mass of oil} \times \text{uniform linear acceleration}$$

$$= (0.8 \times 1000) \times 3.5 \times 1.6 \times 1.8 \times 14.01 \times 10^{-3}\ \text{kN} = \mathbf{113\ kN\ (Ans.)}$$

Obviously the difference between the force on the two ends of the tanker is *equal to* the force necessary to accelerate the liquid mass in the tanker.

6.12.2 Liquid in a Container Subjected to Uniform Acceleration in the Vertical Direction.

Consider a tank containing liquid and moving vertically *upwards* with uniform acceleration a_y (Fig 6.92). The liquid in the tank will have a free horizontal surface *but pressure intensity at any point in the liquid will be different* from what it would be when in a state of absolute rest:

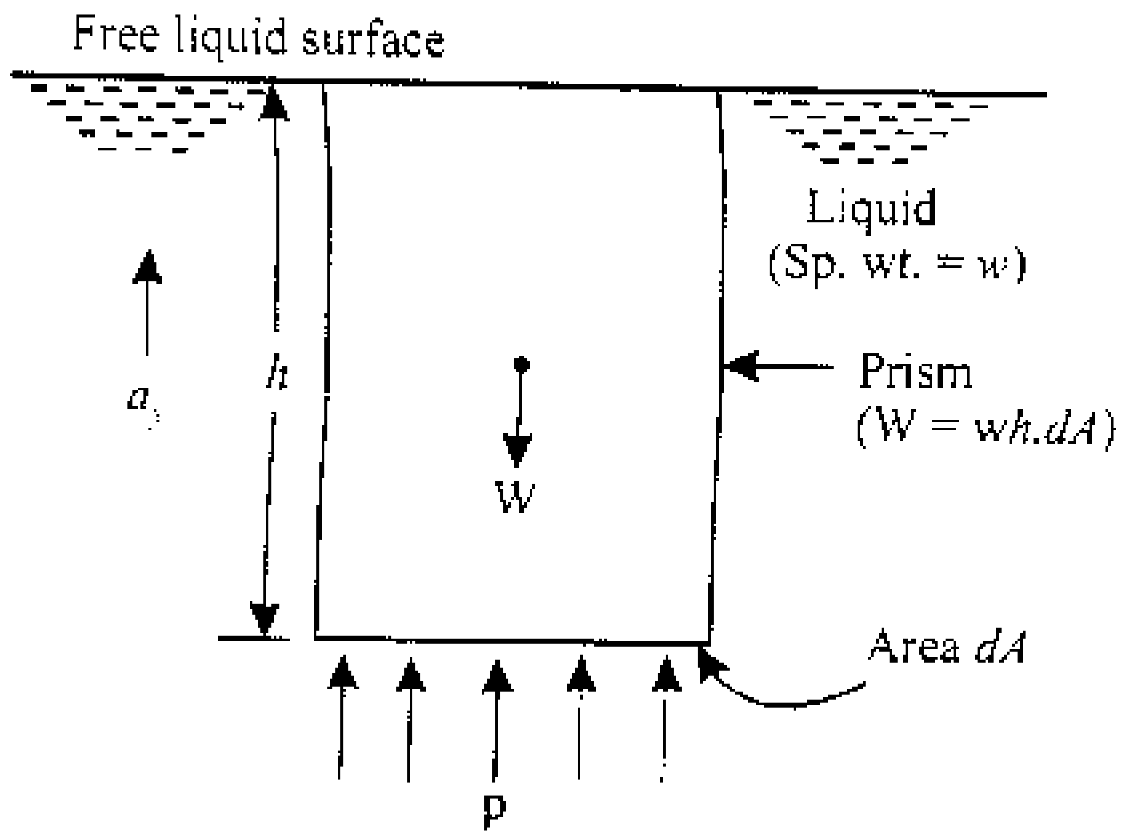


Fig. 6.92

Applying Newton's second law of motion, we have:

$$\Sigma F_y = m \times a_y$$

The force ΣF_y = Pressure force acting upwards – weight of prism acting downwards

$$= [p \text{ (intensity of pressure)} \times dA \text{ (area)}] - (w \times \text{volume of prism})$$

$$= p \times dA - w \times h \times dA$$

Also,

$$m = \frac{w}{g} \times \text{volume of elementary prism}$$

$$= \frac{w}{g} \times (h \times dA)$$

$$\therefore \quad p \times dA - w \times h \times dA = \frac{w}{g}(h \times dA) \times a_y$$

or

$$p = wh\left(1 + \frac{a_y}{g}\right) \qquad \text{...(6.44)}$$

This equation (6.44) reveals the following:

- The free liquid surface remains *horizontal*.
- The pressure variation in the vertical direction is *linear*.
- The pressure intensity at any point is *more* than the static pressure wh by an amount $wh\left(\frac{a_y}{g}\right)$ as shown in Fig. 6.93 (*a*).

If the liquid mass is uniformly accelerated vertically *downward* direction, a_y shall be negative and then eqn. (6.43) reduces to

$$p = wh\left(1 - \frac{a_y}{g}\right) \qquad \text{...(6.44)}$$

i.e., Intensity of pressure at any point is *less* than static pressure wh by an amount

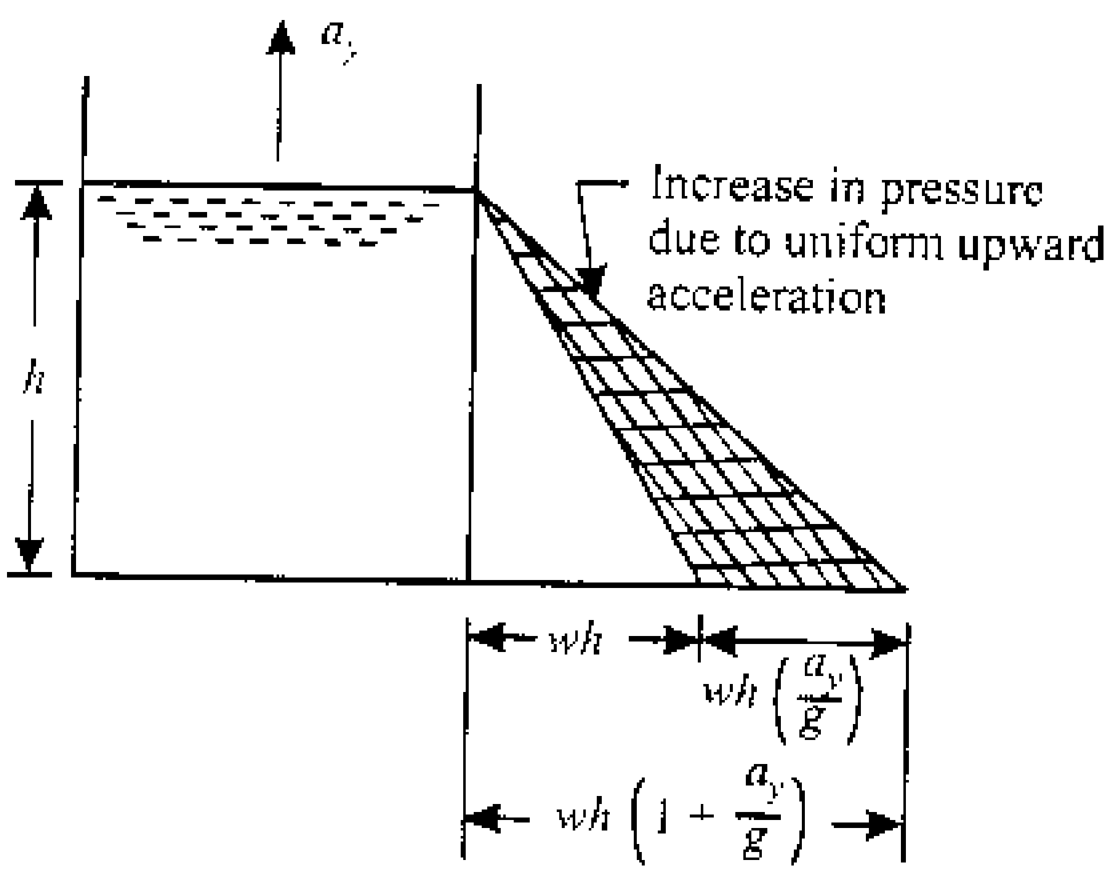


Fig. 6.93. (*a*)

(ii) The pressure at the bottom when $a_y = g$:

When the tank is lowered vertically at an acceleration, then

$$p = wh\left(1 - \frac{a_y}{g}\right) = wh\left(1 - \frac{g}{g}\right) = 0$$

i.e., the liquid remains at the atmospheric pressure throughout and there is no force on the base or on the tank walls. **(Ans.)**

6.12.3. Liquid in Container Subjected to Uniform Acceleation Along Inclined Plane.

Fig. 6.94. shows a tank filled with a liquid being accelerated *up an inclined plane* with uniform acceleration a.

Horizontal component of acceleration, $a_x = a \cos \alpha$

Vertical component of acceleration, $a_y = a \sin \alpha$

A particle L of mass m lying on the liquid surface is *in equilibrium under the action of following forces:*

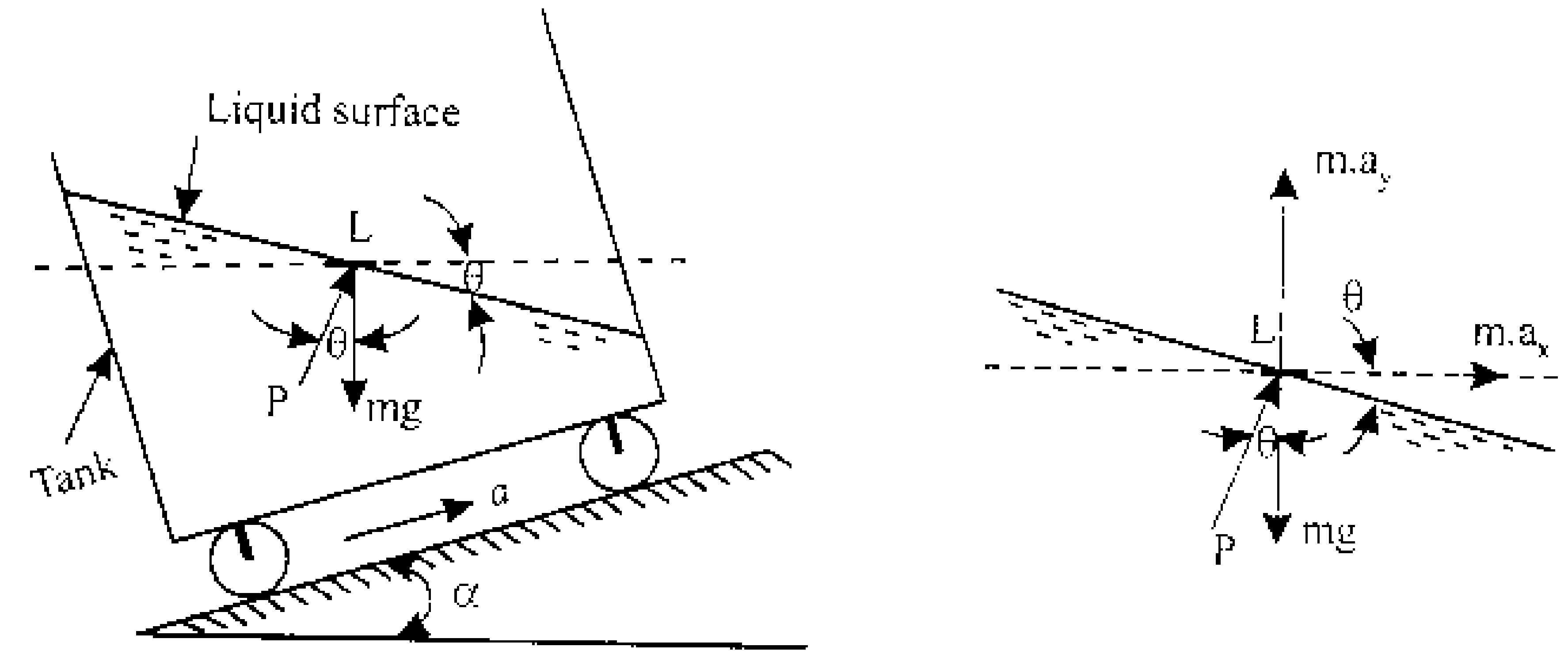


Fig. 6.94. Acceleration of fluid mass along an upward slope.

(i) Weight mg acting vertically downward,

(ii) Pressure force P acting *normal* to the surface of the fluid element, and

(iii) Accelerating force, $(m.a)$ having component $m.a_x$ in the horizontal direction and a component $m.a_y$ in the vertical direction.

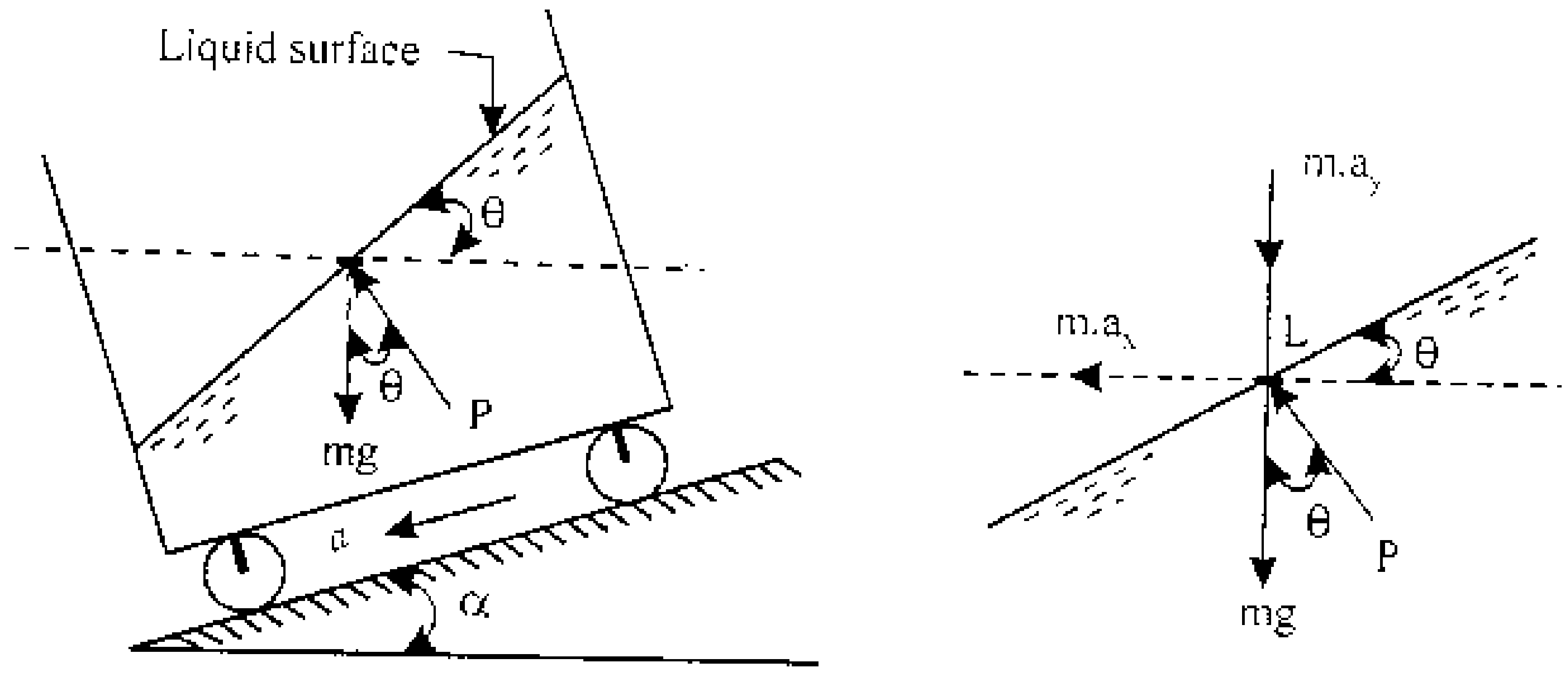


Fig. 6.95. Acceleration of fluid mass along a downward slope.

Resolving *horizontally* and *vertically* respectively, we get

$$P \sin \theta = m.a_x \qquad ...(i)$$

$$P \cos \theta = m.a_y + mg \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\tan\theta = \frac{a_x}{a_y + g} \qquad \text{...(6.45)}$$

When the fluid mass is subjected to acceleration *down the slope,* we have

$$P \sin\theta = m \,.\, a_x \qquad \text{...(iii)}$$

$$P \cos\theta = mg - m.a_y \qquad \text{...(iv)}$$

Dividing (*iii*) by (*iv*), we get

$$\tan\theta = \frac{a_x}{g - a_y} \qquad \text{...(6.46)}$$

Example 6.87. *An open rectangular tank 6 m long and 2.4 m wide is filled with water to a depth of 1.8 m. Find the slope of water surface when the tank moves with an acceleration of 2.5 m/s²*

(*i*) *up a 30° inclined plane* (*ii*) *down a 30° inclined plane.*

Solution. *Given*: Dimensions of the tank: 6 *m* (length) × 2.4 m (width);

acceleration, $a = 2.5$ m/s²; Inclination, $\alpha = 30°$.

Horizontal and vertical components of acceleration are:

$$a_x = a \cos\alpha = 2.5 \times \cos 30° = 2.165 \text{ m/s}^2$$

$$a_y = a \sin\alpha = 2.5 \times \sin 30° = 1.25 \text{ m/s}^2$$

Let, θ be the slope of the free liquid surface.

(*i*) *When the tank moves with acceleration up the inclined plane*:

$$\tan\theta = \frac{a_x}{a_y + g} = \frac{2.165}{1.25 + 9.81} = 0.196$$

$$\theta = \tan^{-1}(0.196) = \mathbf{11.1° \ (Ans.)}$$

(*ii*) *When the tank moves with an acceleration down the inclined plane*

$$\tan\theta = \frac{a_x}{g - a_y} = \frac{2.165}{9.81 - 1.25} = 0.253$$

$$\theta = \tan^{-1}(0.253) = \mathbf{14.2° \ (Ans.)}$$

HIGHLIGHTS

1. The science which deals with the geometry of motion of fluids *without reference* to the forces causing the motion is known as *hydrokinematics.*
2. The science which *deals with* the action of forces in producing or changing motion of fluids is known as *hydrokinetics* (or simply kinetics).
3. Different types of heads are:
   (*i*) Potential head (or potential energy)
   (*ii*) Velocity head (or kinetic energy)
   (*iii*) Pressure head (or pressure energy).
4. *Bernoulli's equation* states as follows:

   "In an ideal, incompressible fluid when the flow is steady and continuous, then sum of pressure energy, potential (or datum) energy and kinetic energy is constant along a stream line." Mathematically,

$$\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$$

where $\frac{p}{w}$ = pressure energy or head

$\frac{V^2}{2g}$ = kinetic energy or head, and

z = datum (or elevation) energy or head.

5. Euler's equation for motion is given as:

$$\frac{dp}{\rho} + v.dv + g.dz = 0 \qquad \text{...differential form}$$

6. Bernoulli's equation for real fluid is given as:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L$$

where, h_L = loss of energy between sections 1 and 2.

7. Practical applications of Bernoulli's equation are:

(*i*) Venturimeter; (*ii*) Orificemeter; (*iii*) Rotometer and elbow meter

(*iv*) Pitot tube.

In case of a *venturimeter*, the actual discharge (Q_{act}) is given as:

$$Q_{act.} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

where, C_d = Co-efficient of discharge (varies between 0.96 to 0.98).

A_1 = Area at inlet,

A_2 = Area at outlet, and

h = Difference of pressure head at sections 1 and 2.

8. *Free liquid jet*:

A jet of liquid from the nozzle in atmosphere is called a free *liquid jet*. The parabolic path traversed by the liquid jet under the action of gravity is known as *trajectory*.

(*i*) Equation of the jet:

$$y = x \tan\theta - \frac{gx^2 \sec^2\theta}{2U^2}$$

where, x, y = Co-ordinates of any point on jet with respect to the nozzle.

U = Velocity of the jet of water issuing form the nozzle, and

θ = Inclination of the jet issuing from nozzle with horizontal.

(*ii*) Maximum height attained by the jet $= \frac{U^2 \sin^2\theta}{2g}$

(*iii*) Time of flight, $T = \frac{2U\sin\theta}{g}$

(*iv*) Time taken to reach the highest point; $T' = \frac{U\sin\theta}{g}$

(*v*) Horizontal range of the jet, $r = \frac{U^2 \sin 2\theta}{g}$

(*vi*) Maximum range, $r_{\max} = \frac{U^2}{2g}$

(The range will be maximum when $\theta = 45°$)

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Volume of air before rotation = volume of air after rotation.

— If a closed cylindrical vessel completely filled with water is rotated about its vertical axis, the total pressure force acting on the top and bottom are:

$$F_{top} = \frac{\rho}{4}\omega^2 \pi R^4$$

and $$F_{bottom} = F_{top} + \text{weight of water in cylinder}$$

$$= F_{top} + w \times \pi R^2 \times H$$

where, ω = angular velocity

R = radius of the vessel,

H = height of the vessel, and

ρ = density of fluid $\left(= \frac{w}{g}\right)$.

Free vortex flow:

When no external torque is required to rotate the fluid mass, type of flow is called free vortex flow. In case of free vortex flow:

$$v \times r = \text{constant} \qquad ...(i)$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \qquad ...(ii)$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer:

1. Velocity head is given by
   - (a) $\frac{V}{g}$ (b) $\frac{V^2}{2g}$
   - (c) $\frac{V^3}{2g}$ (d) $\frac{V^2}{2g^2}$.

2. Bernoulli's equation, mathematically is written as
   - (a) $\frac{p}{w} + \frac{V}{2g} + z = \text{constant}$
   - (b) $\frac{p}{w^2} + \frac{V^2}{2g} + z = \text{constant}$
   - (c) $\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$
   - (d) $\frac{p^2}{w} + \frac{V^2}{2g} + z = \text{constant}$

3. Which of the following assumptions is made in the derivation of Bernoulli's equation?
   - (a) The liquid is ideal and incompressible
   - (b) The flow is steady and continuous
   - (c) The flow is one-dimensional
   - (d) The velocity is uniform over the section and is equal to mean velocity
   - (e) All of the above.

4. Euler's equation (in differential form) is written as:
   - (a) $\frac{dp}{\rho} + v^2.dv + g.dz = 0$
   - (b) $\frac{dp}{\rho} + v.dv + g.dz = 0$
   - (c) $\frac{dp}{\rho} + v.dv + g^2.dz = 0$
   - (d) $\frac{dp}{\rho^2} + v.dv + g.dz = 0$

5. In which of the following measuring devices Bernoulli's equation is used:
   - (a) Venturimeter (b) Orificemeter
   - (c) Pitot tube (d) All the above.

6. The co-efficient of discharge of an orificemeter is ......that of a venturimeter.
   - (a) equal to (b) much smaller than
   - (c) much more than (d) any of these

7. Which of the following equations is known as momentum principle:
   - (a) $F = \frac{d(m^2 v)}{dt}$ (b) $F = \frac{dv}{dt}$