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3

HYDROSTATIC FORCES ON SURFACES

3.1. Introduction. 3.2. Total pressure and centre of pressure. 3.3. Horizontally immersed surface. 3.4. Vertically immersed surface. 3.5. Inclined immersed surface. 3.6. Curved immersed surface 3.7. Dams. 3.8. Possibilities of dam failure. Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

Introduction

In chapter 2, we have studied that a liquid, at rest, exerts some pressure on all sides of the ...iner. The intensity of pressure (p) was related to specific weight w of the liquid and vertical ...h of the point by eqn. $p = wh$. In this chapter, we shall discuss the total pressure on a surface and ...osition. The term '*hydrostatics*' means the study of pressure, exerted by a liquid *at rest*. The ...ction of such a pressure is *always perpendicular to the surface, on which it acts.*

Total Pressure and Centre of Pressure

Total pressure. It is defined as the *force exerted by static fluid on a surface (either plane or ...ed) when the fluid comes in contact with the surface. This force is always at right angle (or ...al) to the surface.*

Centre of pressure. It is defined as the *point of application of the total pressure on the surface.*

Now we shall discuss the total pressure exerted by a liquid on the immersed surface. The ...ersed surfaces may be:

1. Horizontal plane surface
2. Vertical plane surface
3. Inclined plane surface
4. Curved surface.

Horizontally Immersed Surface

Total Pressure (P):

Refer Fig. 3.1. Consider a plane horizontal surface im...ed in a liquid.

Let, A = Area of the immersed surface,

$\bar{x}$ = Depth of horizontal surface from the liquid surface, and

w = Specific weight of the liquid.

The total pressure on the surface,

P = Weight of the liquid above the immersed surface

= Specific weight of liquid × volume of liquid

= Specific weight of liquid × area of surface × depth of liquid

$= wA\bar{x}$

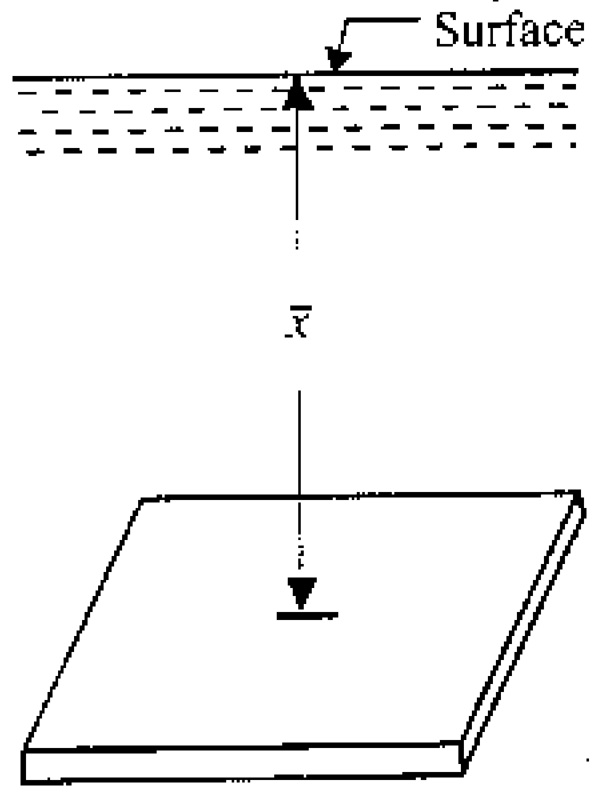


Fig. 3.1. Horizontally immersed surface.

3.4 Vertically Immersed Surface

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.2.

Let, A = Total area of the surface,

G = Centre of the area of the surface,

$\bar{x}$ = Depth of centre of area,

OO = Free surface of liquid, and

$\bar{h}$ = Distance of centre of pressure from free surface of liquid.

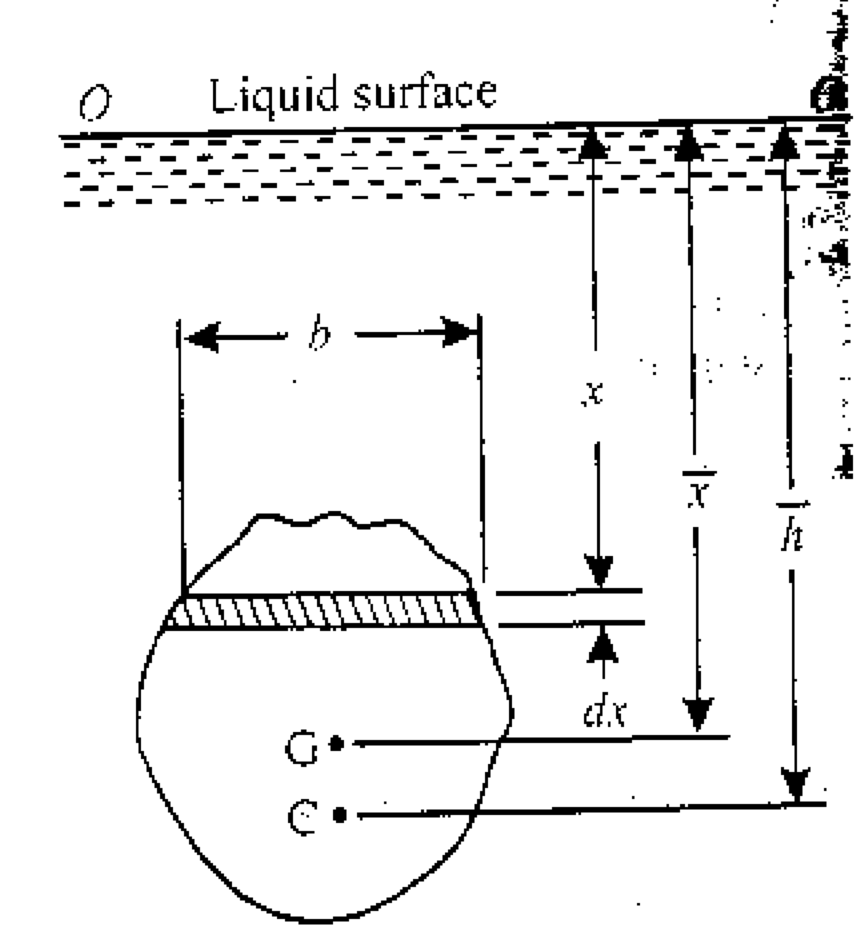


Fig. 3.2. Vertically immersed surface.

(a) Total pressure (P):

Consider a thin horizontal strip of the surface of thickness dx and breadth b. Let the depth of the strip be x. Let the intensity of pressure on strip be p; this may be taken as uniform as the strip is extremely small. Then $p = wx$

where, w = specific weight of the liquid.

Total pressure on the strip $= p.b.dx. = wx \, . \, bdx$

Total pressure on the whole area, $P = \int wx \, . \, bdx = w \int bdx \, . \, x$

But $\int bdx \, . \, x$ = moment of the surface area about the liquid level $= A\bar{x}$

$$\therefore \quad P = wA\bar{x} \qquad \text{...[same as in Art. 3.3]}$$

or, the total pressure on a surface is equal to the *area multiplied by the intensity of pressure at the centre of area of the figure.*

The eqn., $P = wA\bar{x}$ holds good for all surfaces whether flat or curved.

(b) Centre of pressure ($\bar{h}$):

The intensity of pressure on an immersed surface is not uniform, but *increases with depth.* As the pressure is greater over the lower portion of the figure, therefore the resultant pressure, on any immersed surface will act at some point, below the centre of gravity of the immersed surface and towards the lower edge of the figure. *The point through which this resultant pressure acts is known as '**centre of pressure**'* and is always expressed in terms of *depth* from the liquid surface.

Referring to Fig. 3.2, let C be the centre of pressure of the immersed figure. Then the resultant pressure P will act through the point.

Let, $\bar{h}$ = Depth of centre of pressure below free liquid surface, and

I_0 = Moment of inertia of the surface about OO.

Consider the horizontal strip of thickness dx. Total pressure on strip $= w.x.b.dx$

Moment of this pressure about free surface $OO = (w.x. \, b.dx) \, x = w.x^2.b.dx$

Total moment of all such pressures for whole area, $M = \int w.x^2.b.dx. = w \int x^2.b.dx$

But $\int x^2.b.dx = I_0$ = moment of inertia of the surface about the free surface OO (or second moment of area)

$$M = wI_0 \qquad \text{...(i)}$$

The sum of the moments of the pressure is also equal to $P \times \bar{h}$ ...(ii)

Now equating eqns. (i) and (ii), we get

$$P \times \bar{h} = wI_0 .$$

$$wA\bar{x} \times \bar{h} = wI_0 \quad (\because P = wA\bar{x})$$

$$\bar{h} = \frac{I_0}{A\bar{x}} \qquad ...(iii)$$

Also, $I = I_G + Ah^2$ (Theorem of parallel axis)

where, I_G = Moment of inertia of the figure about horizontal axis through its centre of gravity, and

h = Distance between the free liquid surface and the centre of gravity of the figure ($\bar{x}$ in this case)

Thus rearranging equation (*iii*), we have

Table 3.1

The centre of gravity (G) and moment of inertia (I) of some important geometrical figures:

S.No.	*Name of figure*	*C.G. from the base*	*Area*	*I about an axis passing through C.G. and parallel to the base*	*I about base*
1.	Triangle Fig. 3.3	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
2.	Rectangle Fig. 3.4	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
3.	Circle Fig. 3.5	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4.	Trapezium Fig. 3.6	$x = \left[\frac{2a+b}{a+b}\right]\frac{h}{3}$	$\left(\frac{a+b}{2}\right)h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^2$	—

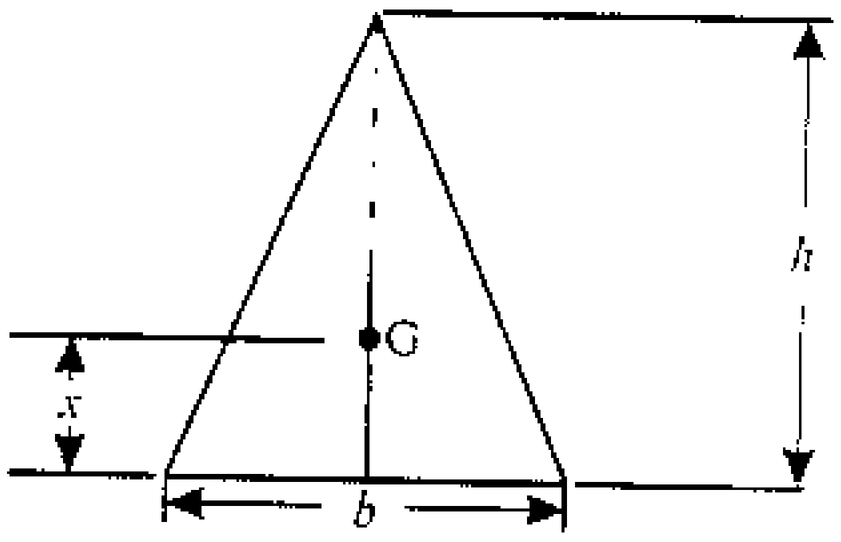


Fig. 3.3

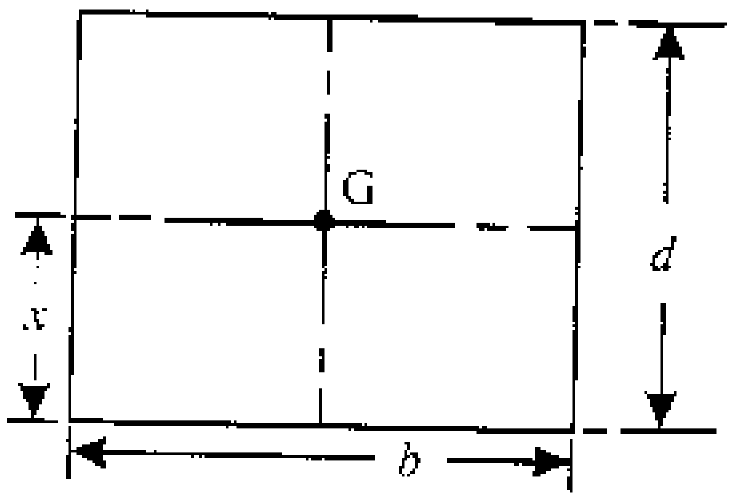


Fig. 3.4

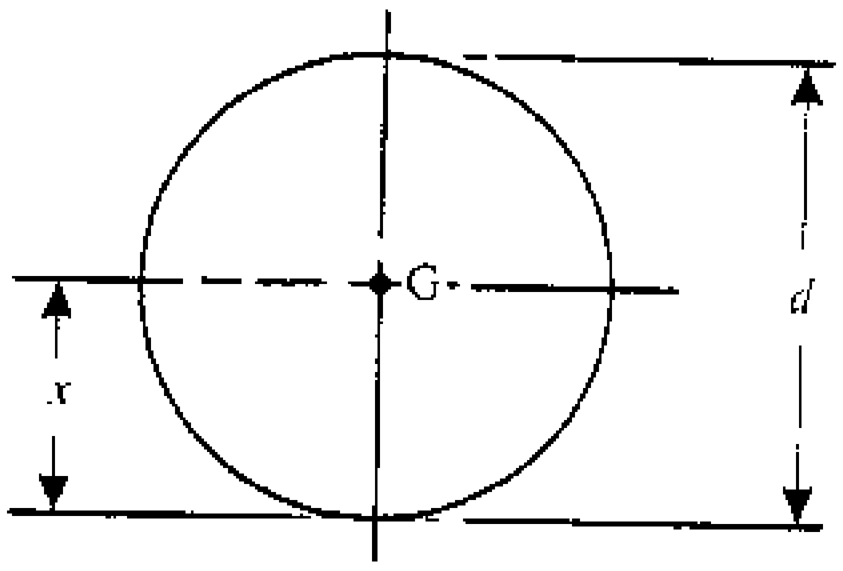


Fig. 3.5

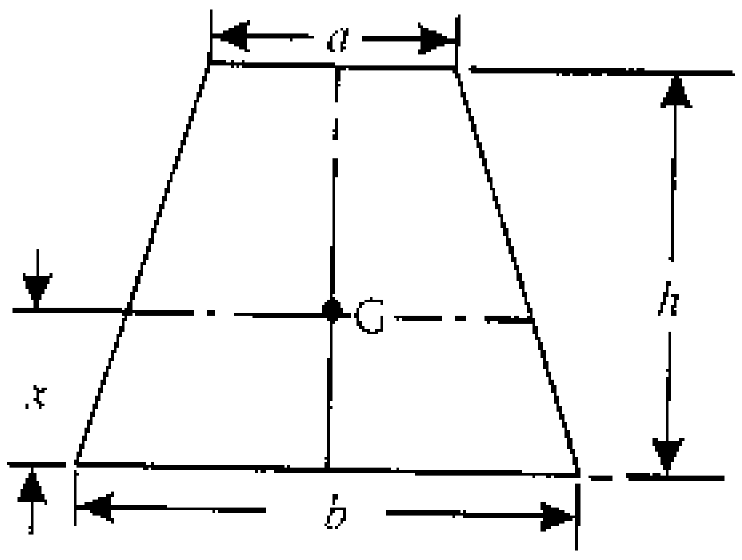


Fig. 3.6

$$\bar{h} = \frac{I_G + A\bar{x}^2}{A\bar{x}} = \frac{I_G}{A\bar{x}} + \bar{x}$$

Hence, *centre of pressure,*

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} \qquad ...(3.2)$$

Example 3.1. *Fig. 3.7 shows a circular plate* of diameter *1.2 m placed vertically in water in such a way that the centre of the place is 2.5 m below the free* surface of water. Determine: (*i*) *Total pressure on the plate.* (*ii*) *Position of centre of pressure.*

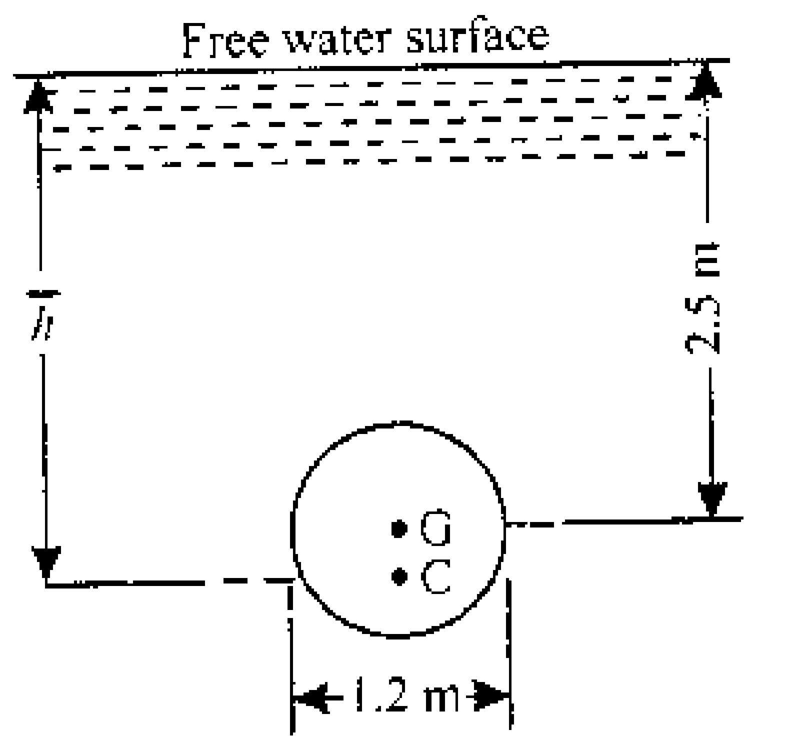


Fig. 3.7

Solution. Diameter of the plate, $d = 1.2$ m

Area,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 1.2^2 = 1.13 \text{ m}^2$$

$$\bar{x} = 2.5 \text{ m}$$

(*i*) **Total pressure, P:**

Using the relation

$$P = wA\bar{x} = 9.81 \times 1.13 \times 2.5 = \mathbf{27.7\ kN\ (Ans.)}$$

(*ii*) **Position of centre of pressure, $\bar{h}$:**

Using the relation,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where

$$I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 1.2^4 = 0.1018 \text{ m}^4$$

$$\bar{h} = \frac{0.1018}{1.13 \times 2.5} + 2.5 = 2.536 \text{ m}$$

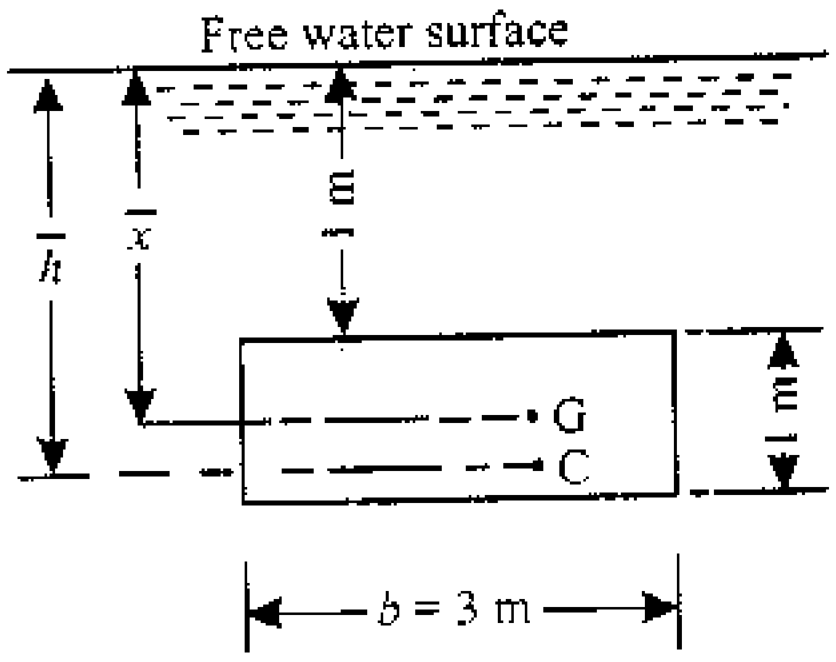
i.e. $\bar{h}$ = **2.536 m (Ans.)**

Example 3.2. *A rectangular plate 3 metres long and 1 metre wide is immersed vertically in water in such a way that its 3 metres side is parallel to the water surface and is 1 metre below it. Find:* (*i*) *Total pressure on the plate and* (*ii*) *Position of centre of pressure.*

Solution. Width of the plane surface, $b = 3$ m

Depth of the plane surface, $d = 1$ m

Area of the plane surface,



$$A = b \times d = 3 \times 1 = 3 \text{ m}^2$$

$$\bar{x} = 1 + \frac{1}{2} = 1.5 \text{ m}$$

(*i*) **Total pressure P:**

Using the relation:

$$P = wA\bar{x} = 9.81 \times 3 \times 1.5$$

$$= \mathbf{44.14\ kN\ (Ans.)}$$

(*ii*) **Centre of pressure, $\bar{h}$:**

Using the relation: $\bar{h} = \dfrac{I_G}{A\bar{x}} + \bar{x}$

But

$$I_G = \frac{bd^3}{12} = \frac{3 \times 1^3}{12} = 0.25 \text{ m}^4$$

$$\therefore \quad \bar{h} = \frac{0.25}{3 \times 1.5} + 1.5 = 1.556 \text{ m}$$

i.e. $\bar{h}$ = **1.556 m (Ans.)**

Example 3.3. *An isosceles triangular plate of base 3 m and altitude 3 m is immersed vertically in an oil of specific gravity 0.8. The base of the plate coincides with the free surface of oil. Determine:*

(*i*) *Total pressure on the plate;* (*ii*) *Centre of pressure.*

Solution. Base of the plate, $b = 3$ m

Height of the plate, $h = 3$ m

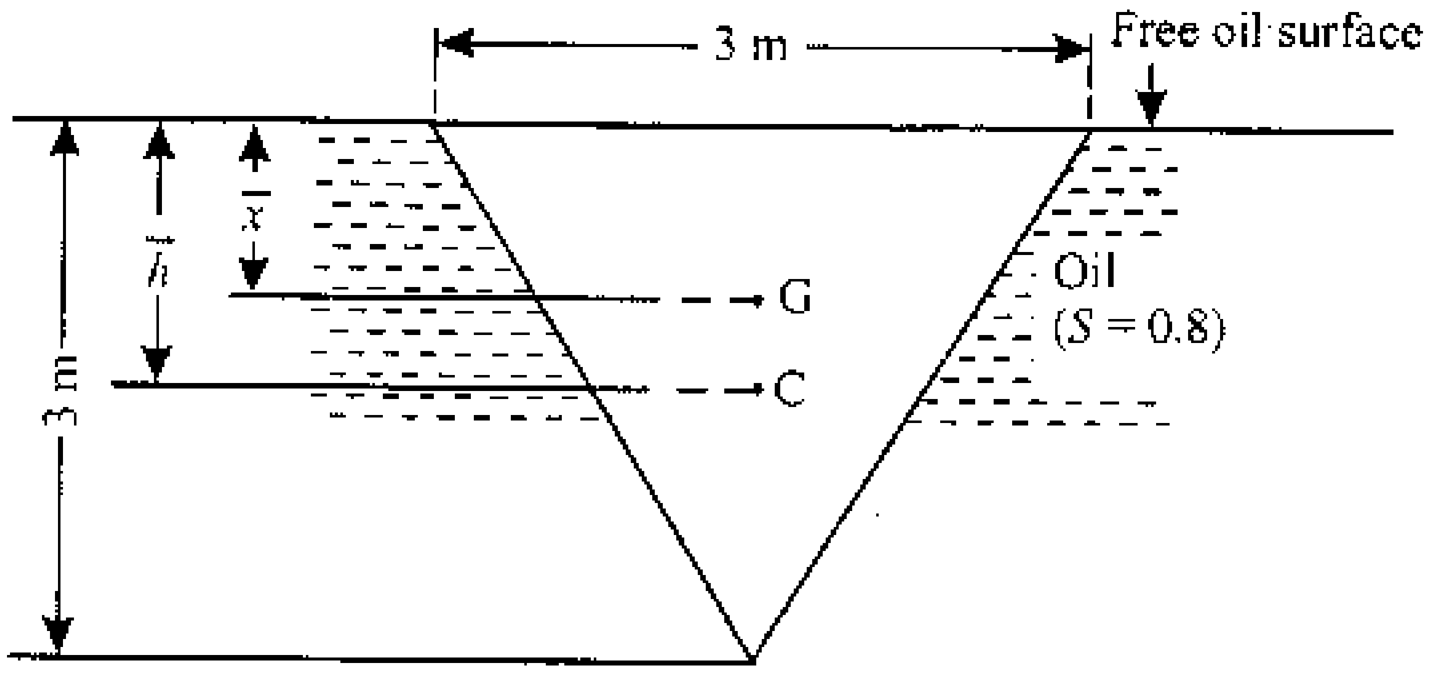


Fig. 3.9

Area, $$A = \frac{b \times h}{2} = \frac{3 \times 3}{2} = 4.5 \text{ m}^2$$

Specific gravity of oil, $S = 0.8$

The distance of C.G. from the free surface of oil,

$$\bar{x} = \frac{1}{3}h = \frac{1}{3} \times 3 = 1 \text{ m}$$

(*i*) **Total pressure on the plate, P:**

We know that $P = wA\bar{x}$

$= (0.8 \times 9.81) \times 4.5 \times 1$

P = **35.3 kN (Ans.)**

(*ii*) **Centre of pressure, $\bar{h}$:**

Centre of pressure is given by the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(bh^3/36)}{A\bar{x}} + \bar{x}$$

$$= \frac{(3 \times 3^3/36)}{4.5 \times 1} + 1$$

$\bar{h}$ = **1.5 m (Ans.)**

Example 3.4. *A circular opening, 2.5 m diameter, in a vertical side of tank is closed by a disc of 2.5 m diameter which can rotate about a horizontal diameter. Determine:*

(*i*) *The force on the disc;*

(*ii*) *The torque required to maintain the disc in equilibrium in vertical position when the head of water above horizontal diameter is 3.5 m.*

Solution. Diameter of the opening, d = 2,5 m

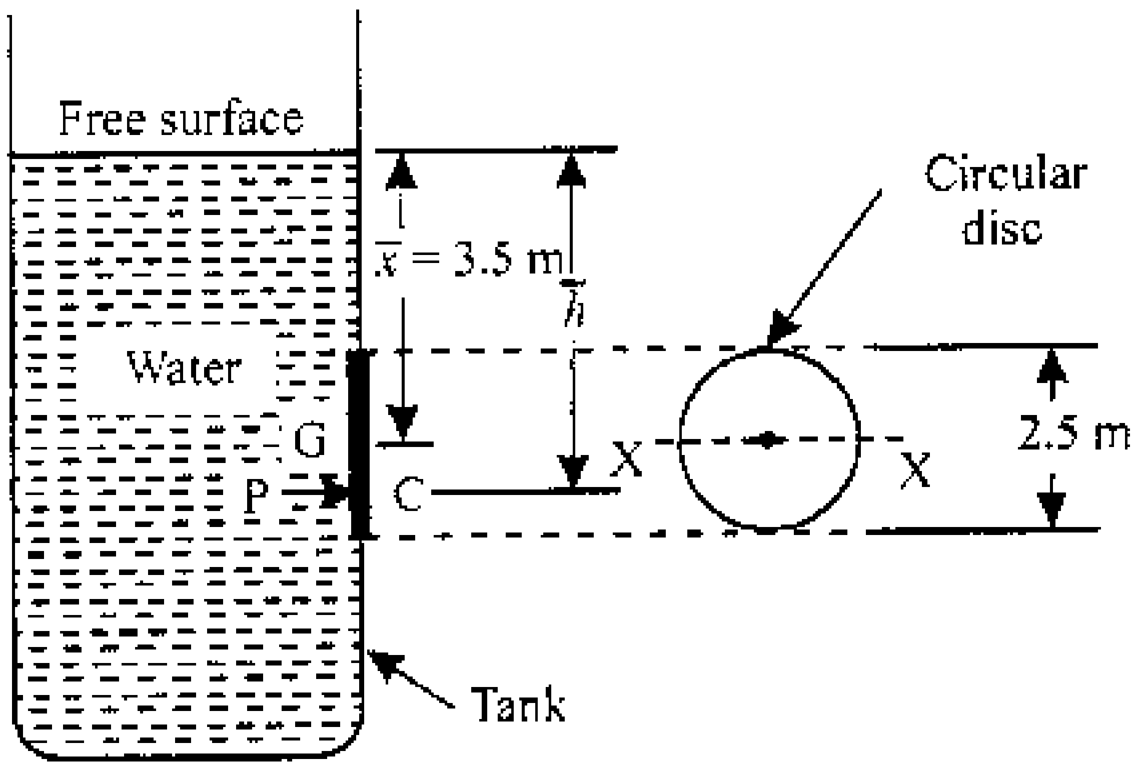


Fig. 3.10

∴ Area of the opening,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 2.5^2 = 4.91 \text{ m}^2$$

Depth of C.G., $\bar{x} = 3.5$ m

(*i*) **Force on the disc, P:**

Using the relation:

$$P = wA\bar{x} = 9.81 \times 4.91 \times 3.5$$
$$= \mathbf{168.6\ kN\ (Ans.)}$$

(*ii*) **Torque required, T:**

In order to determine the torque (T) required to maintain the disc in equilibrium, let us first calculate the point of application of force acting on the disc, *i.e.* centre of pressure of the force P. The depth of centre of pressure ($\bar{h}$) is given by the relation,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(\pi/64 \times d^4)}{(\pi/4 \times d^2)\bar{x}} + \bar{x} \qquad \left[\because I_G = \frac{\pi}{64} \times d^4\right]$$

$$= \frac{(\pi/64 \times 2.5^4)}{(\pi/4 \times 2.5^2) \times 3.5} + 3.5 = 3.61 \text{ m}$$

i.e., the force P is acting at a distance of 3.61 m from the free surface. Moment of this force about horizontal diameter $X - X$

$$= P(\bar{h} - \bar{x}) = 168.6\,(3.61 - 3.5)$$
$$= 18.55 \text{ kNm.} \quad (anticlockwise)$$

Hence a torque (T) of **18.55 kNm** must be applied on the disc in the **clockwise direction** to maintain the disc in equilibrium position. **(Ans.)**

Example 3.5. *A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2.4 m long and the tank contains a liquid of specific gravity 1.2. The centre of aperture is 1.8 m below the free surface. Calculate.*

(*i*) *The thrust exerted on the plate by the liquid;*

(*ii*) *The position of its centre of pressure* **(Punjab University)**

Solution. Refer Fig. 3.11

Diagonal of aperture $PR = QS = 2.4$ *m*

Area of square aperture, A = area of $\Delta\, PQR$ + area of ΔPSR.

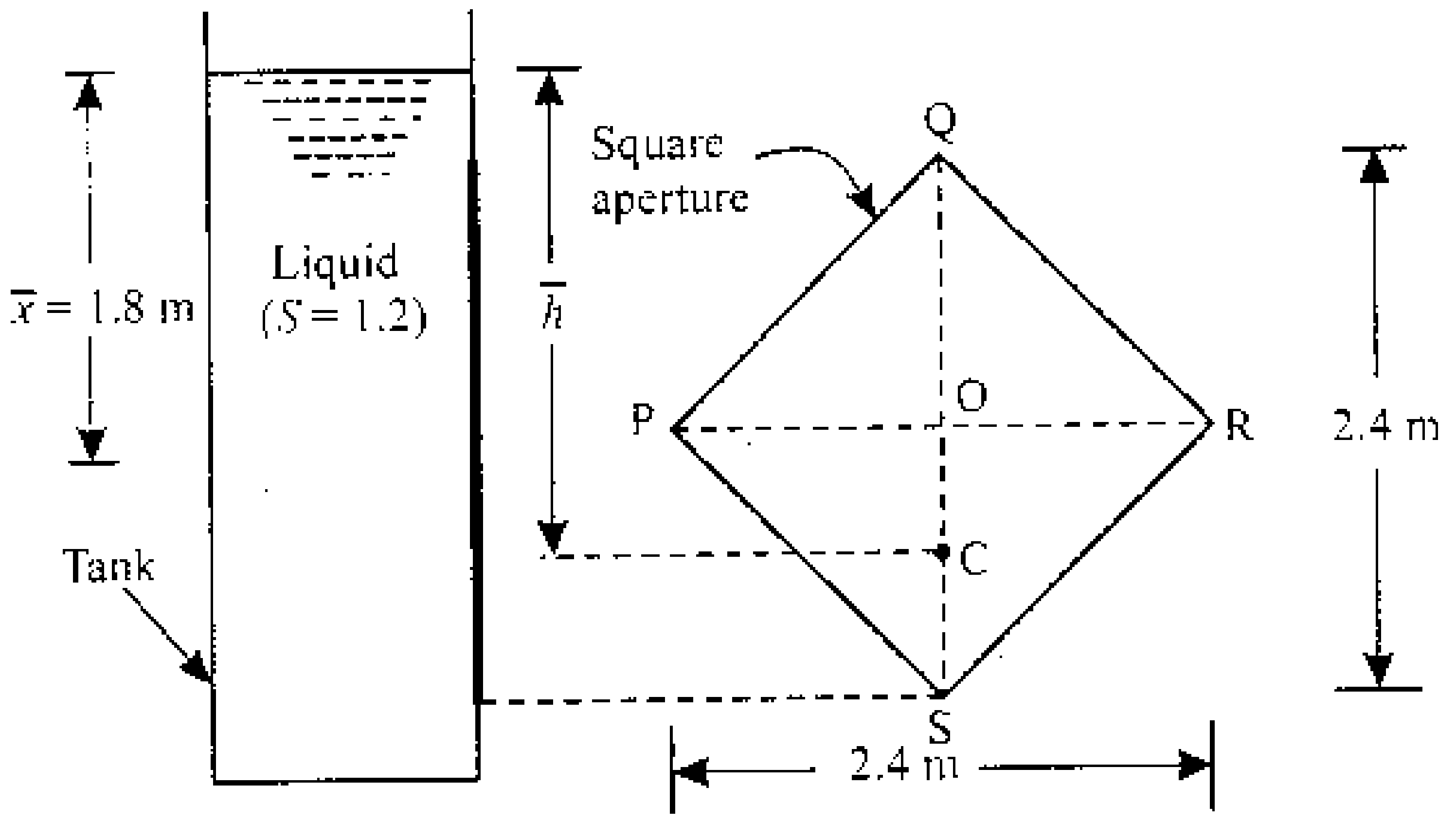


Fig. 3.11

$$= \frac{1}{2} PR \times OQ + \frac{1}{2} PR \times OS$$

$$= \frac{1}{2} \times 2.4 \times \left(\frac{2.4}{2}\right) + \frac{1}{2} \times 2.4 \times \left(\frac{2.4}{2}\right) = 2.88 \text{ m}^2$$

Depth of centre of aperture plate from free liquid surface, $\bar{x} = 1.8$ m

(*i*) **Thrust exerted on the plate *P*:**

Pressure force or thrust on the plate,

$$p = wA\bar{x} = (1.2 \times 9.81) \times 2.88 \times 1.8 = \textbf{61.026 kN (Ans.)}$$

(*ii*) **The position of its centre of pressure, $\bar{h}$:**

Centre of pressure is given by the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where, I_G = M. O. I of PQRS about diagonal PR.

= M.O.I. of Δ PQR + M.O.I of PSR ...about PR

$$= \frac{2.4 \times (1.2)^3}{12} + \frac{2.4 \times (1.2)^3}{12} = 0.6912 \text{ m}^4 \qquad (\because OQ = OS)$$

[∵ The M.O.I. of a triangle about its base equals $\frac{\text{base} \times (\text{height})^3}{12}$]

$$\therefore \quad \bar{h} = \frac{0.6912}{2.88 \times 1.8} + 1.8 = \textbf{1.933 m (Ans.)}$$

Example 3.6. *A trapezoidal plate of parallel sides l and 2l and height h immersed vertically in water with its side of length l horizontal and topmost. The top edge is at a depth h below the water surface. Determine:*

(*i*) *The total force on one side of the plate.*

(*ii*) *The location of the centre of pressure.*

Solution. Refer to Fig. 3.12, the trapezium can be considered to be made of:

(*i*) A rectangle: l (width) × h (height)

(*ii*) A triangle: l *(base)* × h (height)

(*i*) **Total force on one side of the plate P:**

Refer to Fig. 3.13.

For Rectangular part:

Pressure force P_1 on the rectangular part,

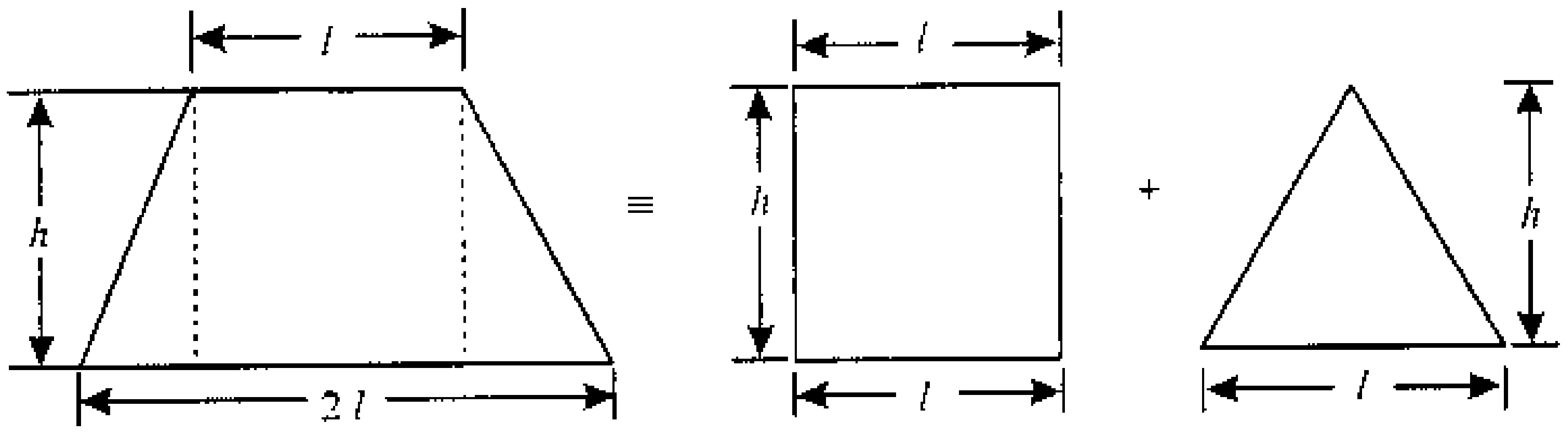


Fig. 3.12

$$P_1 = w(l \times h)\left(h + \frac{h}{2}\right) = \frac{3}{2}\, wlh^2$$

Centre of pressure of force P_1,

$$\bar{h}_1 = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(l \times h^3/12)}{(l \times h) \times \left(h + \frac{h}{2}\right)} + \left(h + \frac{h}{2}\right)$$

$$= \frac{h}{18} + \frac{3h}{2} = \frac{14}{9}h$$

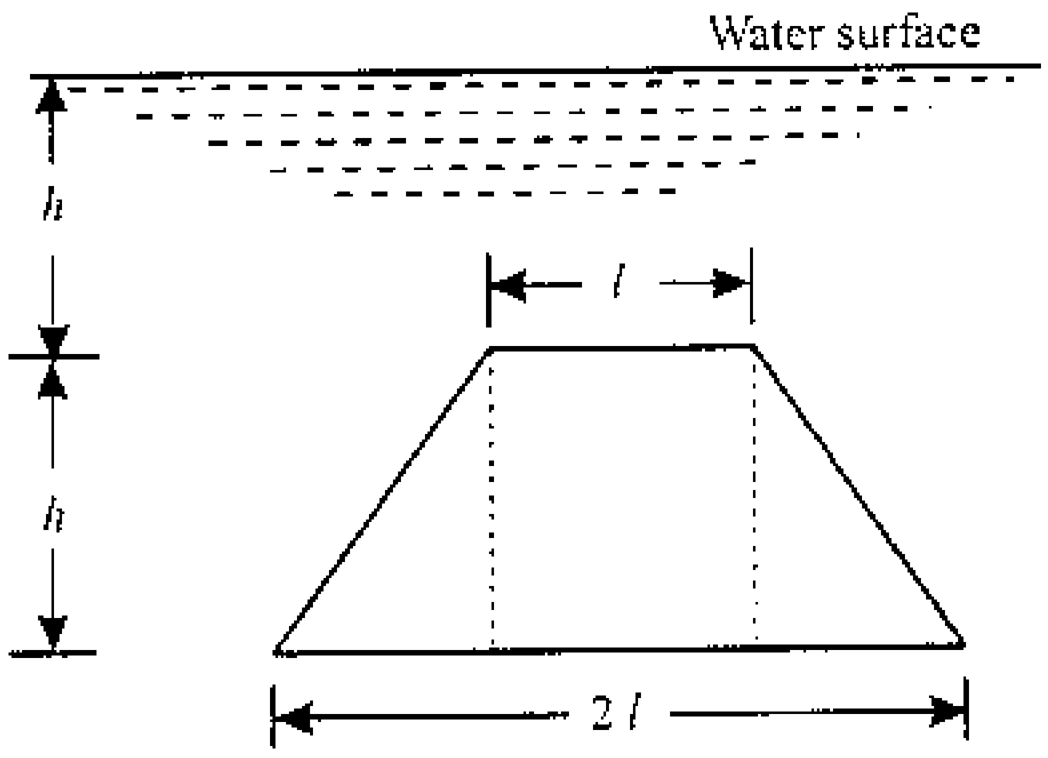


Fig. 3.13

For Triangular part:

Pressure force P_2 on the triangular part;

$$P_2 = w\left(\frac{1}{2} \times l \times h\right) \times \left(h + \frac{2}{3}h\right)$$

$$= \frac{5}{6} wlh^2$$

Centre of pressure of force P_2,

$$\bar{h}_2 = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(lh^3/36)}{\left(\frac{1}{2}l \times h\right) \times \left(h + \frac{2}{3}h\right)} + \left(h + \frac{2}{3}h\right)$$

$$= \frac{h}{30} + \frac{5h}{3} = \frac{51}{30}h$$

$\therefore$ Total force/thrust, $P = P_1 + P_2$

$$= \frac{3}{2}\, wlh^2 + \frac{5}{6}\, wlh^2 = \frac{7}{3}\mathbf{wlh^2}\ \textbf{(Ans.)}$$

(*ii*) **The location of the centre of pressure, $\bar{h}$:**

Centre of pressure of total force,

$$\bar{h} = \frac{P_1 \bar{h}_1 + P_2 \bar{h}_2}{P}$$

$$= \frac{\frac{3}{2} wlh^2 \times \frac{14}{8} h + \frac{5}{6} wlh^2 \times \frac{51}{30} h}{\frac{7}{3} wl h^2} = \frac{45}{28} h$$

i.e. $\bar{h} = \frac{45}{28}$ **h (Ans.)**

Example 3.7. *A trapezoidal 2 m wide at the bottom and 1 m deep has side slopes 1 : 1. Determine:*

(*i*) *Total pressure;*

(*ii*) *Centre of pressure on the vertical gate closing the channel when it is full of water.*

Solution. Refer Fig. 3.14

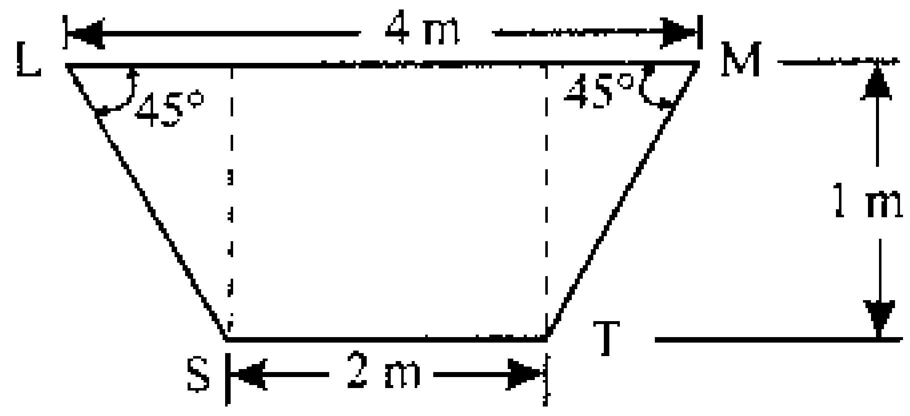


Fig. 3.14

(*i*) **Total Pressure, P :**

For rectangle:

Area, $A_1 = 2 \times 1 = 2 m^2$

$$\bar{x} = \frac{1}{2} = 0.5 m$$

$$P_1 = wA\bar{x} = 9.81 \times 2 \times 0.5 = 9.81 \text{ kN}$$

This acts at a depth $\bar{h}_1$.

But $\bar{h}_1 = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(2 \times 1^3/12)}{2 \times 0.5} + 0.5 = 0.6 \text{ m}$ ... from the top

For triangles:

Area, $A_2 = 2 \times \frac{1}{2} \times 1 \times 1 = 1 \text{ m}^2$ (there are two triangles); $\bar{x} = \frac{1}{3}$ m

$$P_2 = wA\bar{x} = 9.81 \times 1 \times \frac{1}{3} = 3.27 \text{ kN}$$

This acts at a depth of $\bar{h}_2$.

But $\bar{h}_2 = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(2 \times 1^3/36)}{1 \times 1/3} + \frac{1}{3} = 0.5 \text{ m}$ ...from the top.

i.e. $\bar{h}_2 = 0.5 m$

Total pressure,

$P = P_1 + P_2 = 9.81 + 3.27 =$ **13.08 kN (Ans.)**

(*ii*) **Centre of pressure , $\bar{h}$:**

Taking moments about the top, we get: $P \times \bar{h} = P_1 \times \bar{h}_1 + P_2 \times \bar{h}_2$

or $\bar{h} = \frac{P_1 \bar{h}_1 + P_2 \bar{h}_2}{P} = \frac{9.81 \times 0.66 + 3.27 \times 0.5}{13.08} =$ **0.62 m (Ans.)**

Example 3.8. *An isosceles triangle of base 3 metres and altitude 6 metres, is immersed vertically in water, with its axis of symmetry horizontal as shown in Fig. 3.15. If the head of water on it is [illegible] metres, determine:*

(*i*) *Total pressure on the plate, and* (*ii*) *The position of the centre of pressure.*

Solution. Area of the triangle

$$A = \frac{1}{2} \times 3 \times 6 = 9\,\text{m}^2$$

Depth of C.G. of the plate from the water surface,

$$\bar{x} = 9\,\text{m}$$

(*i*) **Total pressure, *P*:**

We know that,

$$P = wA\bar{x} = 9.81 \times 9 \times 9 = \mathbf{794.6\ kN\ (Ans.)}$$

(*ii*) **Centre of pressure, $\bar{h}$:**

Using the relation: $\bar{h} = \dfrac{I_G}{A\bar{x}} + \bar{x}$

But I_G= moment of inertia of ΔABD about AD + moment of inertia of $\Delta\, ACD$ about AD

$$= \frac{6 \times 1.5^3}{12} + \frac{6 \times 1.5^3}{12} = 3.375\ \text{m}^4$$

$$\bar{h} = \frac{3.375}{9 \times 9} + 9 = \mathbf{9.04\ m\ (Ans.)}$$

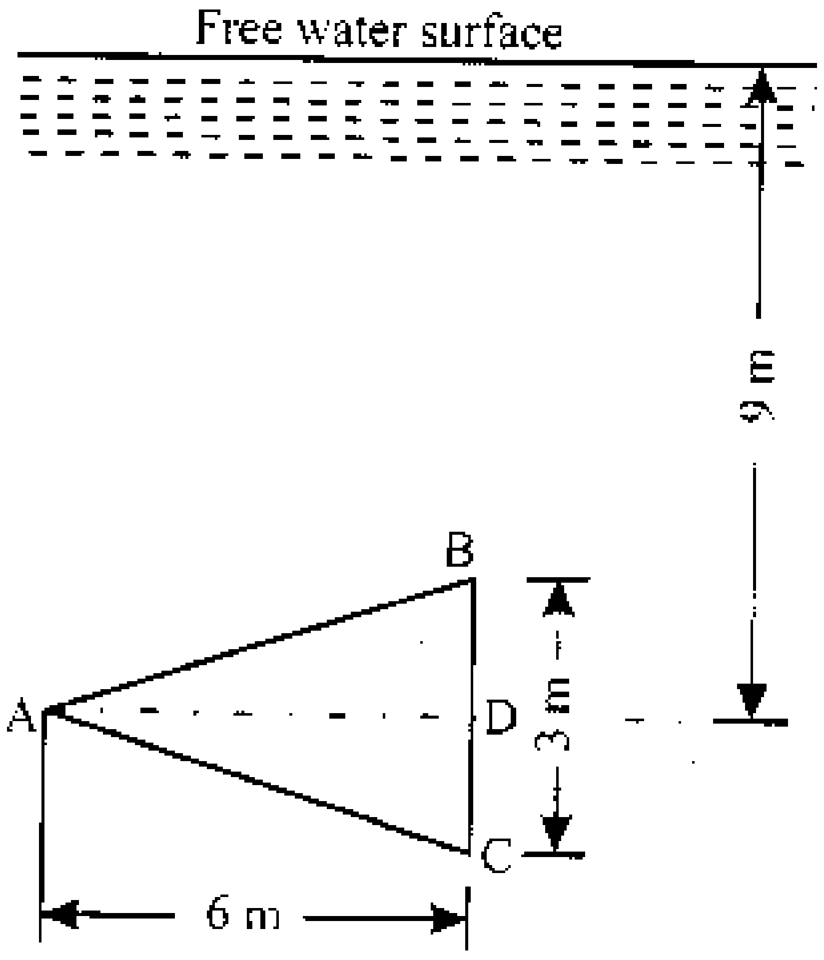


Fig. 3.15

Example 3.9. *A circular lamina of radius R is kept immersed in a liquid such that its top most point A is on the free surface. Determine the depth and width of the horizontal chord BC so that the total thrust due to hydrostatic pressure on the triangle ABC is maximum.* **(AMIE Winter, 2000)**

Solution. Refer Fig. 3.16.

The total thrust/pressure on the submerged triangle ABC is,

$$F = P = wA\,\bar{x} = w \times \left(\frac{1}{2} \times b \times h\right) \times \frac{2h}{3} = \frac{1}{3}\,wbh^2$$

But, $h = R + \sqrt{R^2 - b^2}$ (O is the centre of the circle)

$$\therefore \quad F = \frac{1}{3}wb\left[R + \sqrt{R^2 - b^2}\,\right]^2$$

For F to be maximum, $\dfrac{dF}{db} = 0$

i.e.

$$\frac{d}{db}\left[b(R + \sqrt{R^2 - b^2})^2\right] = 0$$

$$b \times 2\,(R + \sqrt{R^2 - b^2}\,) \times \frac{1}{2}(R^2 - b^2)^{-1/2}(-2b) + (R + \sqrt{R^2 - b^2}\,)^2 = 0$$

$$\frac{-2b^2}{\sqrt{R^2 - b^2}} + R + \sqrt{R^2 - b^2} = 0$$

or

$$-\,2b^2 + R\sqrt{(R^2 - b^2)} + R^2 - b^2 = 0$$

or

$$R\sqrt{R^2 - b^2} + R^2 - 3b^2 = 0$$

or

$$R\sqrt{R^2 - b^2} = 3b^2 - R^2$$

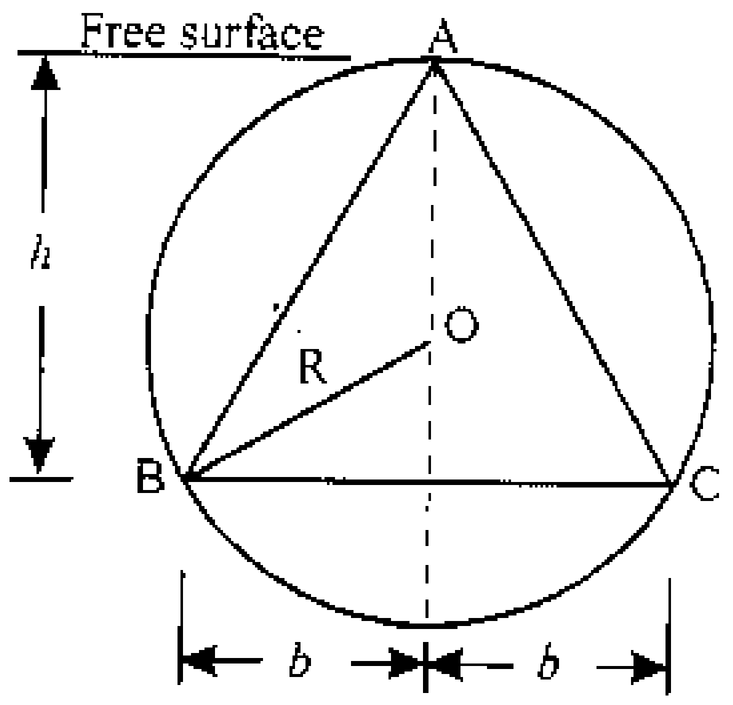


Fig. 3.16

Squaring both sides, we get

$$R^2 (R^2 - b^2) = 9b^4 + R^4 - 6R^2b^2$$

$$R^4 - R^2b^2 = 9b^4 + R^4 - 6R^2b^2$$

$$9b^4 = 5R^2b^2$$

or $$9b^2 = 5R^2$$

or $$b = \sqrt{\frac{5}{9}}\, R = \frac{\sqrt{5}}{3} R$$

and $$h = R + \sqrt{R^2 - \frac{5}{9}R^2} = \frac{5}{3} R$$

Hence, for maximum thrust, the depth and width of the chord are:

Depth, $$\boldsymbol{h = \frac{5}{3} R}\text{, and}$$

width, $$\boldsymbol{2b = \frac{2}{3}\sqrt{5}\, R} \textbf{ (Ans.)}$$

Example 3.10. *Determine the total force and location of centre of pressure for plate LMSUT immersed vertically as shown in Fig.* 3.17.

Solution. Area LMST, $A_1 = 2 \times 2 = 4 \text{m}^2$

Area TSU, $A_2 = \frac{1}{2} \times 2 \times 2 = 2 \text{ m}^2$

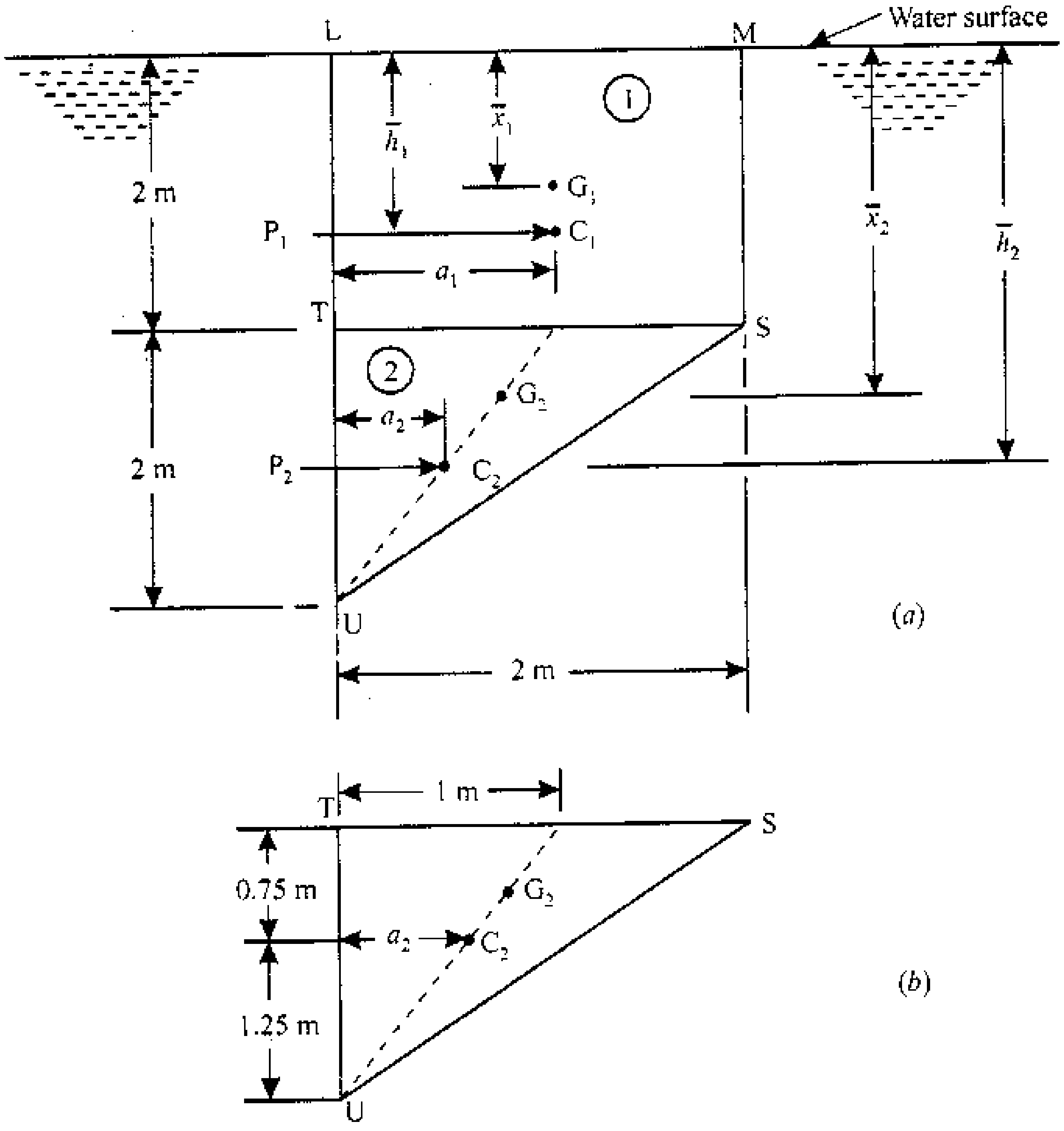


Fig. 3.17

Distance of centroid G_1 from water surface, $\bar{x}_1 = \frac{2}{2} = 1$ m

Distance of centroid G_2 from water surface, $\bar{x}_2 = 2 + \frac{2}{3} = 2.667$ m

Total pressure on area LMST, $P_1 = wA_1\bar{x}_1 = 9.81 \times 4 \times 1 = 39.24$ kN

Total pressure on area TSU, $P_2 = wA_2\bar{x}_2 = 9.81 \times 2 \times 2.667 = 52.33$ kN

Total pressure $P = P_1 + P_2 = 39.24 + 52.33 = 91.57$ kN

Distance of centre of pressure (C_1) of area *LMST* from free water surface,

$$\bar{h}_1 = \frac{I_{G_1}}{A_1\bar{x}_1} + \bar{x}_1 = \frac{\frac{2 \times 2^3}{12}}{2 \times 2 \times 1} + 1 = 1.333 \text{ m}$$

Distance of centre of pressure (C_2) of area *TSU* from the free water surface,

$$\bar{h}_2 = \frac{I_{G_2}}{A_2\bar{x}_2} + \bar{x}_2 = \frac{\frac{2 \times 2^3}{36}}{2 \times 2.667} + 2.667 = 2.75 \text{ m}$$

The depth ($\bar{h}$) at which the resultant force will act can be determined by taking moments of forces P_1 and P_2 about water surface.

i.e. $$P_1 \times \bar{h}_1 + P_2 \times \bar{h}_2 = P \times \bar{h}$$

$$39.24 \times 1.333 + 52.33 \times 2.75 = 91.57 \times \bar{h}$$

$$\therefore \quad \bar{h} = \frac{39.24 \times 1.333 + 52.33 \times 2.75}{91.57} = 2.14 \text{ m}$$ **below the water surface (Ans.)**

The horizontal location of centre of pressure can be obtained by taking moments of P_1 and P_2 about *LTU*. The force P_1 acts at 1 m from line *LTU*. The distance a_2 where force P_2 acts can be obtained as under:

$$\frac{1}{2} = \frac{a_2}{1.25} \quad \text{[from similarity of triangles (Fig. 3.17) (b)]}$$

or $$a_2 = 0.625 \text{ m}$$

$$\therefore \quad P_1 . a_1 + P_2 . a_2 = P . \bar{a}$$

$$39.24 \times 1 + 52.33 \times 0.625 = 91.57 \times \bar{a}$$

$$\therefore \quad \bar{a} = \frac{39.24 \times 1 + 52.33 \times 0.625}{91.57} = 0.786 \text{ m}$$

Hence co-ordinates of centre of pressure are 2.14 m below water surface and 0.786 m from *LTU*. **(Ans.)**

Example 3.11. *A sliding gate 3 m wide and 1.5 m high lies a vertical plane and has a co-efficient of friction of 0.2 between itself and guides. If the gate weighs 30 kN, find the vertical force required to raise the gate if its upper edge is at a depth of 9 m from free surface of water.*

Solution. Width of the gate, $b = 3$ m

Depth/height of the gate,

$$d = 1.5 \text{ m}$$

Area of the gate, $A = b \times d = 3 \times 1.5 = 4.5 \text{ m}^2$

Weight of the gate, $W = 30$ kN

Co-efficient of friction, $\mu = 0.2$

Vertical force required to raise the gate:

Depth of *c.g.* of the gate from water surface,

$$\bar{x} = 9 + \frac{1.5}{2} = 9.75 \text{ m}$$

Pressure force on the gate,

$$P = wA\bar{x} = 9.81 \times 4.5 \times 9.75 = 430.4 \text{ kN}$$

Force required to raise the gate

= frictional force + weight of the gate

$= \mu P + W$

$= 0.2 \times 430.4 + 30 =$ **116.08 kN (Ans.)**

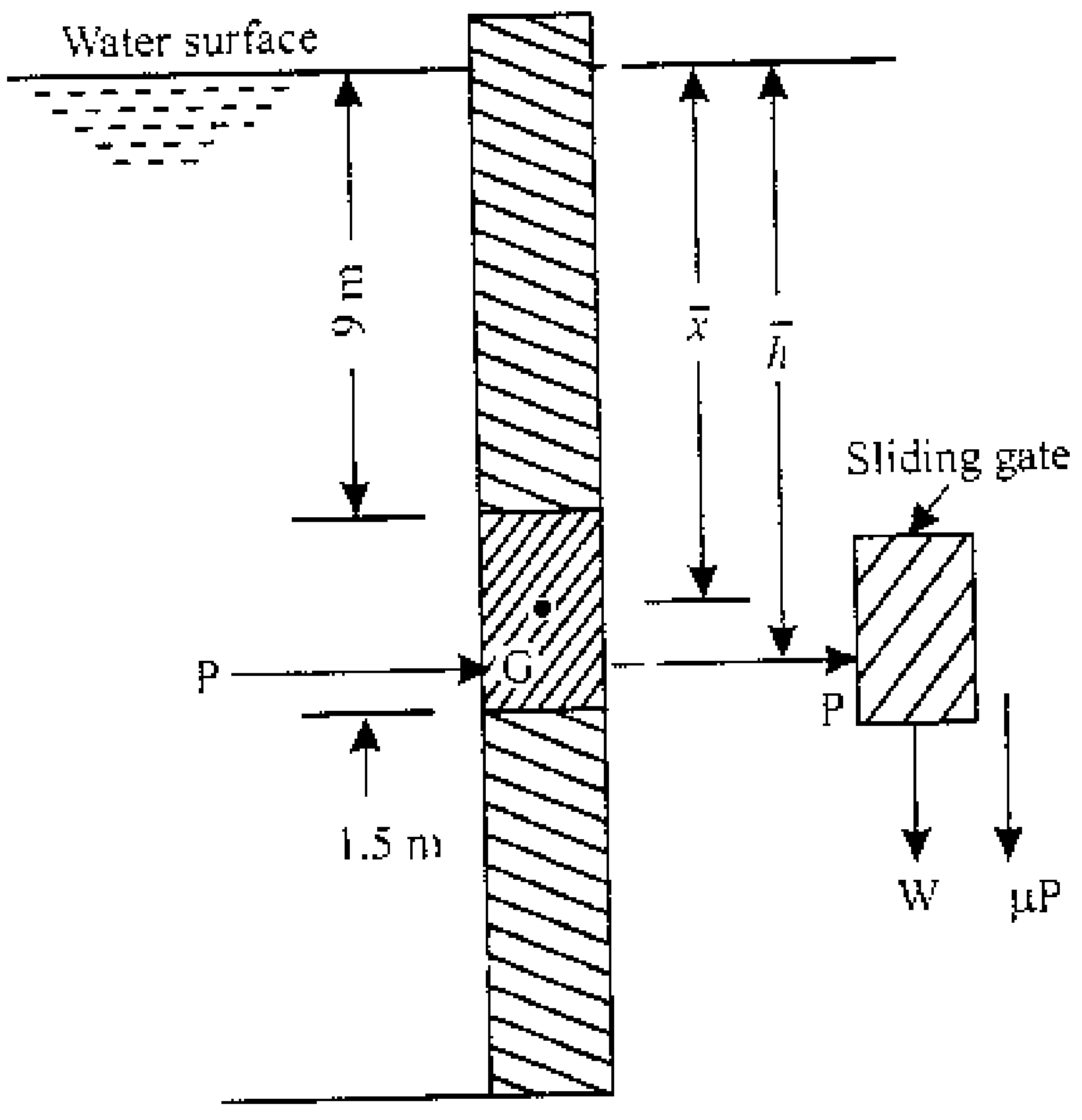


Fig. 3.18

Example 3.12. *The hydrostatic water pressure acts only on one side and to a depth of 12 m from the top of a dock gate which is reinforced with three horizontal beams.*

(*i*) *Calculate the load taken by each beam*

(*ii*) *Locate the position of beams in order that each carries an equal load.*

Solution. Refer Fig. 3.19. Consider an elementary strip of thickness dh at a depth h. Then for a unit width of the gate, we have

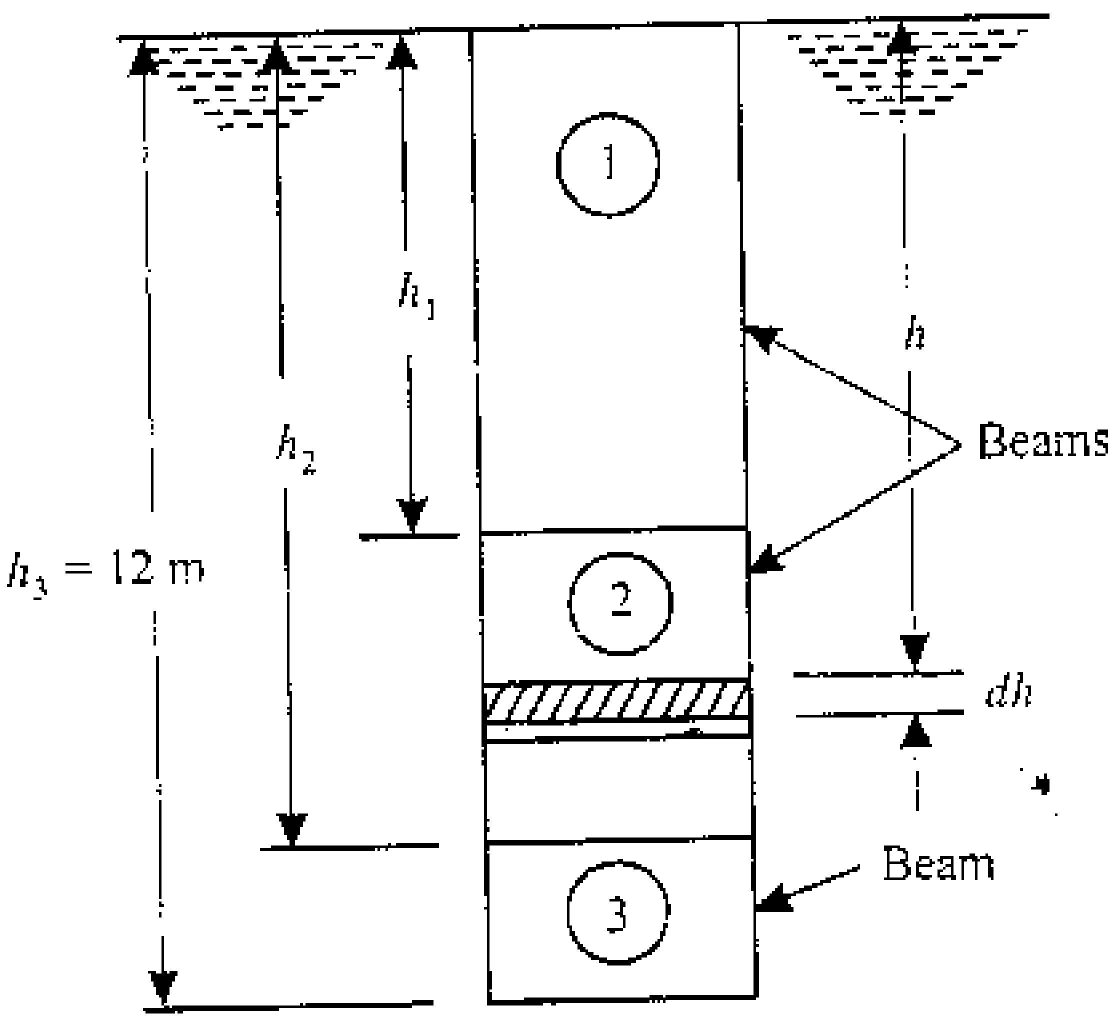


Fig. 3.19

Pressure/force on the element,

$$dP = w \times (dh \times 1) \times h = wh\, dh$$

Pressure on section 1,

$$P_1 = \int_0^{h_1} wh\, dh = \frac{w}{2} h_1^2$$

Pressure on section 2,

$$P_2 = \int_{h_1}^{h_2} wh\, dh = \frac{w}{2}\left(h_1^2 - h_1^2\right)$$

Pressure on section 3,

$$P_3 = \int_{h_2}^{h_3} wh\, dh = \frac{w}{2}\left(h_3^2 - h_2^2\right)$$

Total pressure on the gate,

$$P = \int_0^{h_3} wh\, dh = \frac{w}{2} h_3^2$$

Load carried by each section is same and it equals $\frac{1}{3}rd$ of total pressure/force on the gate.

Thus, $$\frac{w}{2} h_1^2 = \frac{w}{2}(h_2^2 - h_1^2) = \frac{w}{2}(h_3^2 - h_2^2) = \frac{w}{6} h_3^2$$

$\therefore$ $$h_1^2 = h_2^2 - h_1^2 = 144 - h_2^2 = \frac{144}{3} = 48$$

Solving the above equations, we get

$$h_1 = 6.93 \text{ m};\ h_2 = 9.8 \text{ m}$$

(*i*) Load taken by each beam:

Load taken by each beam $= \frac{w}{2} h_1^2 = \frac{9810}{2} \times 6.93^2 =$ **235562 (Ans.)**

(ii) **Centres of pressure, $\bar{h}_1$, $\bar{h}_2$, $\bar{h}_3$:**

$$\bar{h}_1 = \frac{2}{3} h_1 = \frac{2}{3} \times 6.93 = \mathbf{4.62\,m\,(Ans.)}$$

In order to obtain the centre of pressure for the section 2, taking moments of relevant forces about F.W.S., we get

$$\frac{w}{2}(h_2^2 - h_1^2) \times \bar{h}_2 = \left(\frac{w}{2}h_2^2 \times \frac{2}{3}h_2\right) - \left(\frac{w}{2}h_1^2 \times \frac{2}{3}h_1\right)$$

or,

$$\bar{h}_2 = \frac{2}{3}\left[\frac{h_2^3 - h_1^3}{h_2^2 - h_1^2}\right] = \frac{2}{3}\left[\frac{(9.8)^3 - (6.93)^3}{(9.8)^2 - (6.93)^2}\right]$$

$$= \frac{2}{3}\left[\frac{608.38}{48.015}\right] = \mathbf{8.45\,m\ (Ans.)}$$

Similarly for the bottom portion, the centre of pressure from the F.W.S.,

$$\bar{h}_3 = \frac{2}{3}\left[\frac{h_3^3 - h_2^3}{h_3^2 - h_2^2}\right] = \frac{2}{3}\left[\frac{(12)^3 - (9.8)^3}{(12)^2 - (9.8)^2}\right]$$

$$= \frac{2}{3}\left[\frac{786.81}{47.96}\right] = \mathbf{10.94\ m\ (Ans.)}$$

Example 3.13. *Fig. 3.20 shows a tank containing water and liquid (sp. gravity = 0.9) upto height 0.25 m and 0.5 m respectively. Calculate:*

(i) *Total pressure on the side of the tank;*

(ii) *The position of centre of pressure from one side of the tank, which is 1.5 m wide.* **(U.P.S.C.)**

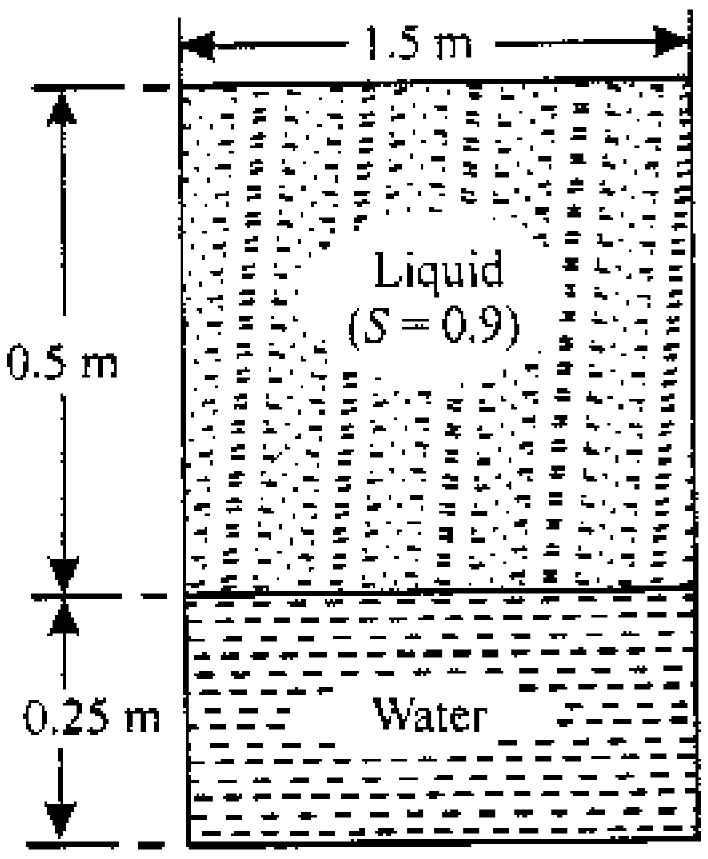


Fig. 3.20

Solution. Depth of water = 0.25 m

Depth of liquid = 0.5 m

Sp. gravity of liquid, $S = 0.9$

Width of the tank = 1.5 m

(i) **Total pressure on one side of the tank, *P*:**

Total pressure (P) is calculated by drawing pressure diagram, which is shown in Fig. 3.21.

Intensity of pressure on top, $p_L = 0$

Intensity of pressure on T (or TS),

$$p_T = w_1 h_1 = (0.9 \times 9.81) \times 0.5 = 4.41\ \text{kN/m}^2$$

Intensity of pressure on the base (or MN),

$$p_M = w_1 h_1 + w_2 h_2 = 4.41 + 9.81 \times 0.25$$

$$= 4.41 + 2.45 = 6.86\ \text{kN/m}^2$$

Now, force P_1 = area of the ΔLTS × width of the tank

$$= \frac{1}{2} \times LT \times TS \times 1.5 = \frac{1}{2} \times 0.5 \times 4.41 \times 1.5 = 1.65\ \text{kN}$$

Force, P_2 = area of rectangle $MTSU$ × width of the tank

$= MT \times TS \times 1.5$

$= 0.25 \times 4.41 \times 1.5 = 1.65$ kN

P_3 = area of $\Delta\ SUN$ × width of the tank

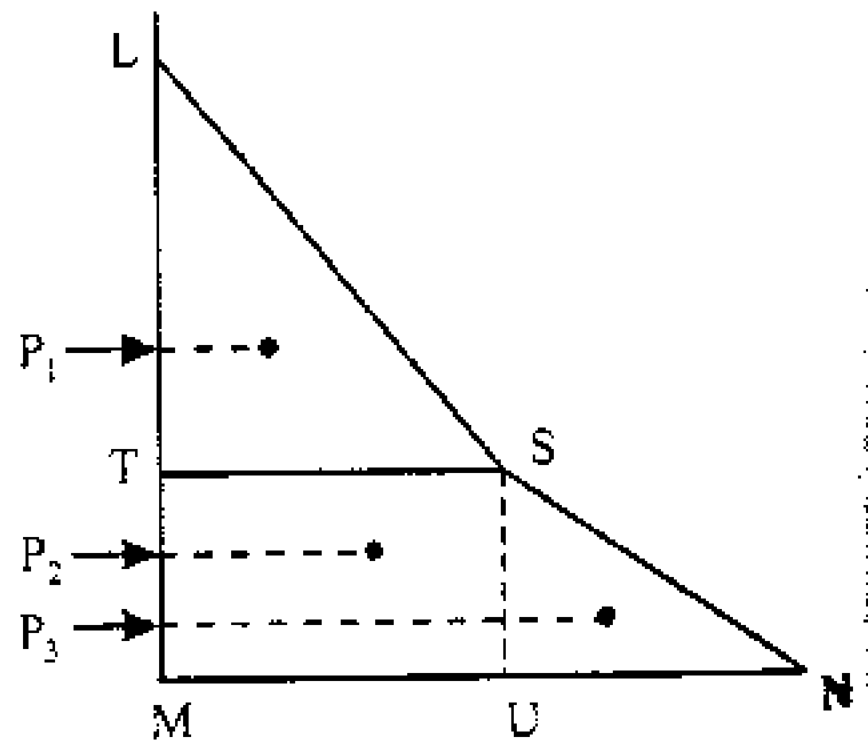


Fig. 3.21. Pressure diagram

$$= \frac{1}{2} \times SU \times UN \times 1.5 = \frac{1}{2} \times 0.25 \times 2.45 \times 1.5 = 0.46 \text{ kN}$$

Total pressure,

$$P = P_1 + P_2 + P_3 = 1.65 + 1.65 + 0.46 = \mathbf{3.76 \text{ kN (Ans.)}}$$

(*ii*) **Centre of pressure, $\bar{h}$:**

Taking moments of all the forces about L, we get

$$P \times \bar{h} = P_1 \times \frac{2}{3} LT + P_2 \times \left(LT + \frac{1}{2} TM \right) + P_3 \times \left(LT + \frac{2}{3} MT \right)$$

$$3.76 \times \bar{h} = 1.65 \times \frac{2}{3} \times 0.5 + 1.65 \left(0.5 + \frac{1}{2} \times 0.25 \right) + 0.46 \left(0.5 + \frac{2}{3} \times 0.25 \right)$$

$$= 0.55 + 1.03 + 0.306$$

$$\bar{h} = \mathbf{0.5016 \text{ m from the top (Ans.)}}$$

Example 3.14. *An opening in a dam is covered by the use of a vertical sluice gate. The opening is 2 m wide and 1.2 m high. On the upstream of the gate the liquid of specific gravity 1.45, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find:*

(*i*) *The resultant force acting on the gate and position of centre of pressure;*

(*ii*) *The force acting horizontally at the top of the gate which is capable of opening it.*

Assume that the gate is hinged at the bottom. **[AMIE]**

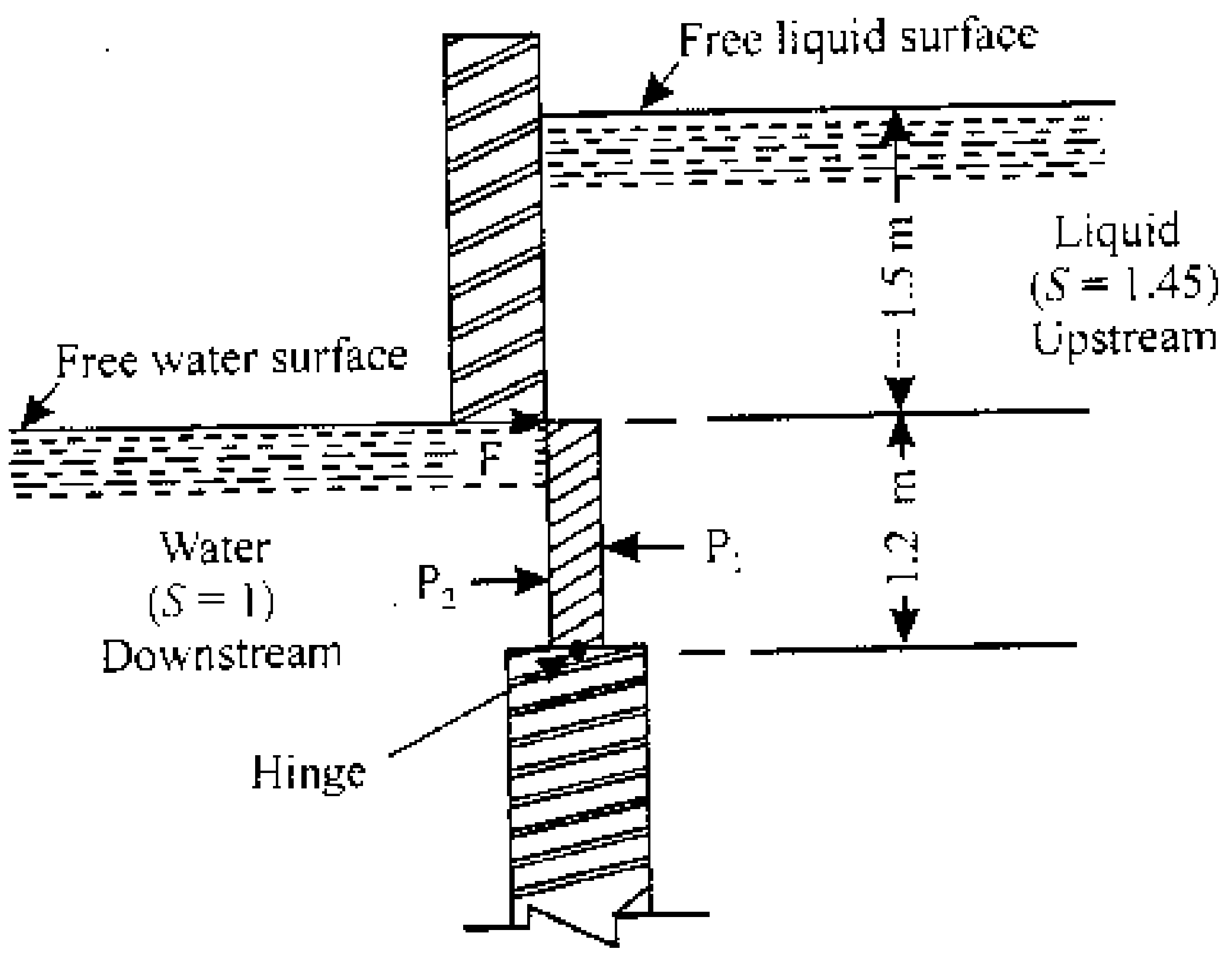


Fig. 3.22

Solution. Width of the gate, $b = 2$ m

Depth of the gate, $d = 1.2$ m

Area, $A = b \times d = 2 \times 1.2 = 2.4 \text{ m}^2$

Specific gravity of liquid = 1.45

Let, P_1 = Force exerted by the liquid of sp. gravity 1.45 on the gate, and

P_2 = Force exerted by water on the gate.

(*i*) **Resultant force, P:**

Position of centre of pressure of resultant force:

We know that, $P_1 = wA\bar{x}_1$

where, $w = 9.81 \times 1.45 = 14.22 \text{ kN/m}^3$,

$A = 2 \times 1.2 = 2.4 \text{ m}^2$

$\bar{x}_1 = 1.5 + \frac{1.2}{2} = 2.1$ m,

$P_1 = 14.22 \times 2.4 \times 2.1 = 71.67$ kN.

Similarly, $P_2 = wA\bar{x}_2$

where, $w = 9.81 \text{ kN/m}^3$,

$A = 2.4 \text{ m}^2$,

$$\bar{x}_2 = \frac{1.2}{2} = 0.6 \text{ m}$$

$$P_2 = 9.81 \times 2.4 \times 0.6 = 14.13 \text{ kN.}$$

Resultant force, $P = P_1 - P_2 = 71.67 - 14.13$

$= \mathbf{57.54\ kN}$ **(Ans.)**

The force P_1 acts at a depth of $\bar{h}_1$ from free liquid surface, which is given by,

$$\bar{h}_1 = \frac{I_G}{A\bar{x}_1} + \bar{x}_1$$

where, $I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 \text{m}^4$

$$A = 2.4 \text{ m}^2,\ \bar{x} = 1.5 + \frac{1.2}{2} = 2.1 \text{ m}$$

$$\bar{h}_1 = \frac{0.288}{2.4 \times 2.1} + 2.1 = 2.157 \text{ m}$$

$\therefore$ Distance of P_1 from the hinge $= (1.5 + 1.2) - \bar{h}_1 = 2.7 - 2.157 = 0.543$ m

Similarly the force P_2 acting at a depth of $\bar{h}_2$ from the liquid surface is given by,

$$\bar{h}_2 = \frac{I_G}{A\bar{x}_2} + \bar{x}_2$$

where, $I_G = 0.288 \text{ m}^4$ (as above); $\bar{x}_2 = \frac{1.2}{2} = 0.6 \text{m}$; $A = 2.4 \text{ m}^2$

$$\therefore \quad \bar{h}_2 = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.8 \text{ m}$$

$\therefore$ Distance of P_2 from the hinge $= 1.2 - 0.8 = 0.4$ m

Now the resultant force will act at a distance given by

$$\frac{71.67 \times 0.543 - 14.13 \times 0.4}{57.54} = \mathbf{0.578 \text{ m}} \textbf{ above the hinge (Ans.)}$$

(*ii*) **Force required to open the gate, *F*:**

Taking moments of P_1, P_2 and F about the hinge, we get

$$F \times 1.2 + P_2 \times 0.4 = P_1 \times 0.543$$

or, $F \times 1.2 + 14.13 \times 0.4 = 71.67 \times 0.543$

or, $F = \frac{71.67 \times 0.543 - 14.13 \times 0.4}{1.2} = \mathbf{27.72\ kN}$ **(Ans.)**

Example 3.15. *For the system shown in Fig. 3.23 calculate the height H of the oil at which the rectangular hinged gate will just begin to rotate anticlockwise.*

Solution.

Refer Fig. 3.23.

Height H of the oil:

- *Force due to oil,* $P_1 = w_0 A \bar{x}$

$$= (0.8 \times 9.81) \times (0.6 \times 1.5) \times \left[(H - 1.5) + \frac{1.5}{2}\right]$$

$$= 7.063\,(H - 0.75) = 7.063\,H - 5.297$$

Centre of pressure of force P_1,

$$\bar{h}_1 = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{0.6 \times (1.5)^3 / 12}{(0.6 \times 1.5) \times (H - 0.75)} + (H - 0.75)$$

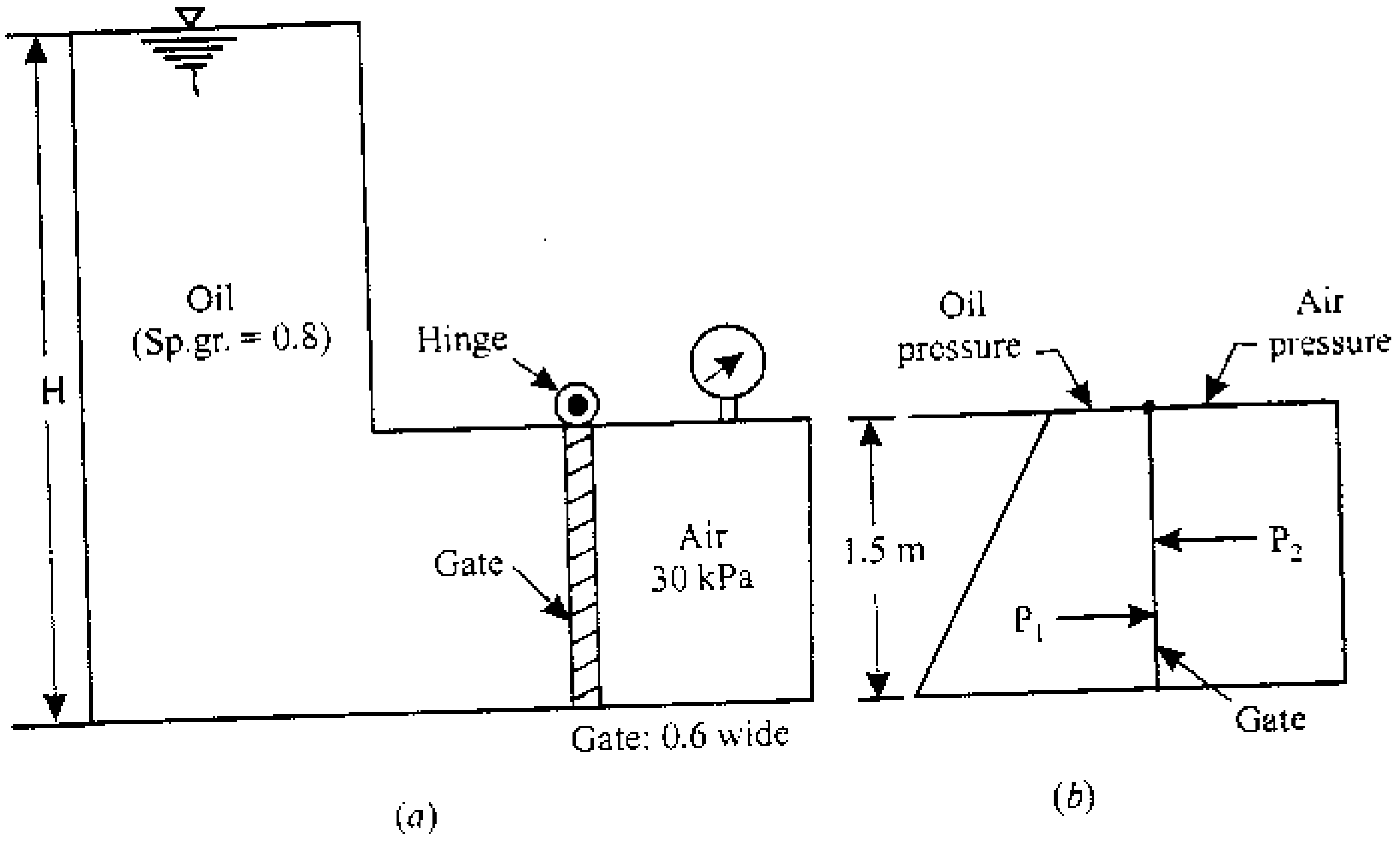


Fig. 3.23

$$= \frac{0.1875}{(H - 0.75)} + (H - 0.75)$$

- *Force due to air pressure*, $P_2 = p.A$

$$= 30 \times (0.6 \times 1.5) = 27\text{kN}$$

Centre of pressure of this force (below the oil surface),

$$\bar{h}_2 = (H - 0.75)$$

Taking moment about the hinge, we get.

$$P_1 \times \left[\bar{h}_1 - (H - 1.5)\right] = P_2 \times \left[\bar{h}_2 - (H - 1.5)\right]$$

$$(7.063\,H - 5.297)\left\{\frac{0.1875}{(H - 0.75)} + (H - 0.75) - (H - 1.5)\right\}$$

$$= 27 \times \{(H - 0.75) - (H - 1.5)\}$$

$$(7.063\,H - 5.297)\left\{\frac{0.1875}{(H - 0.75)} + 0.75\right\} = 27 \times 0.75$$

On solving by trial and error, we get

H = 4.324 m (Ans.)

Example 3.16. *A tank of 1m length and of cross-section shown in fig. 3.24 contains water. The tank is made of 4 mm steel plates.*

(*i*) *What is the force on the bottom due to water?*

(*ii*) *What are the longitudinal tensile stresses in the side walls AB if (a) the tank is suspended from the top and (b) it is supported at the bottom?*

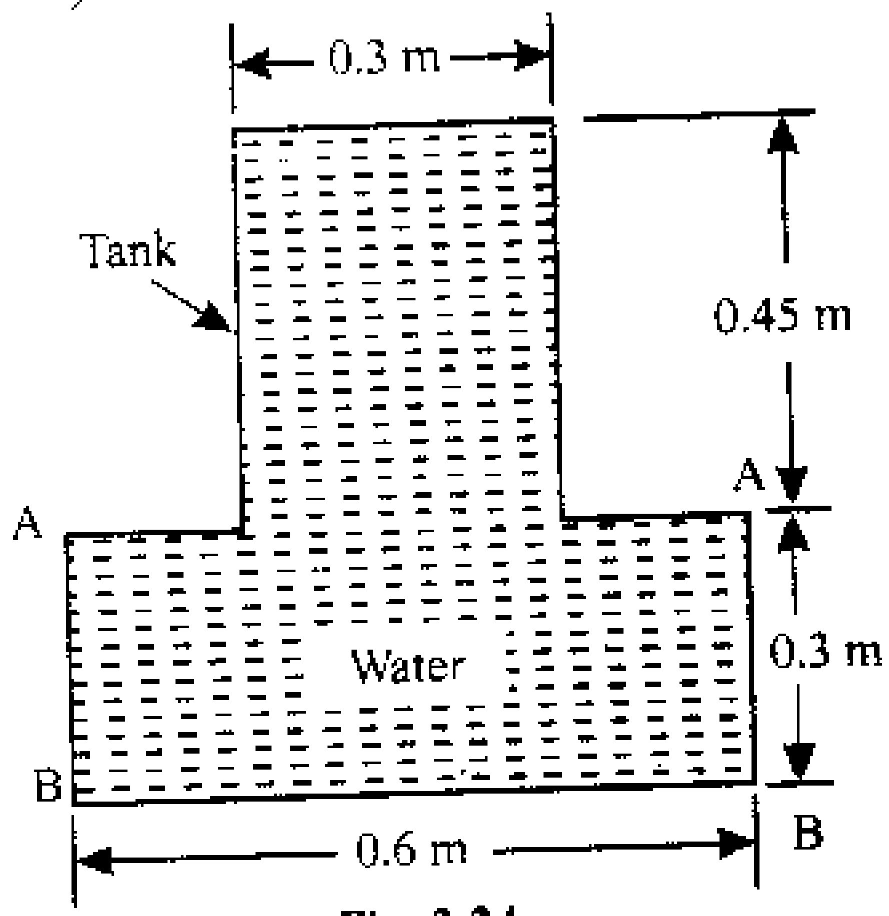


Fig. 3.24

Solution.

Refer Fig. 3.24

(*i*) **Force on the bottom:**

Force on the bottom due to water,

$$P_{bottom} = wA\bar{x}$$

$$= 9.81 \times (0.6 \times 1.0) \times 0.75 = \mathbf{4.414\ kN\ (Ans.)}$$

(*ii*) **Longitudinal tensile stresses:**

Force on the surface *AA*,

$$P_{AA} = 9.81 \times (0.3 \times 1.0) \times 0.45 = 1.324 \text{ kN}$$

(*a*) *When suspended from the top* the stress on the side walls,

$$\sigma = \frac{4.414}{(0.6 + 0.6 + 1.0 + 1.0) \times \dfrac{4}{1000}} = \mathbf{344.8\ kN/m^2\ (Ans.)}$$

(*b*) *When supported from bottom* the stress on the side walls,

$$\sigma = \frac{1.324}{(0.6 + 0.6 + 1.0 + 1.0) \times \dfrac{4}{1000}} = \mathbf{103.4\ kN/m^2\ (Ans.)}$$

Example 3.17. *A vertical square 1.2 m × 1.2 m is submerged in the water with upper edge 0.6 m below the water surface. Locate the horizontal line on the surface of the square such that the force on the upper portion equals the force on the lower portion.*

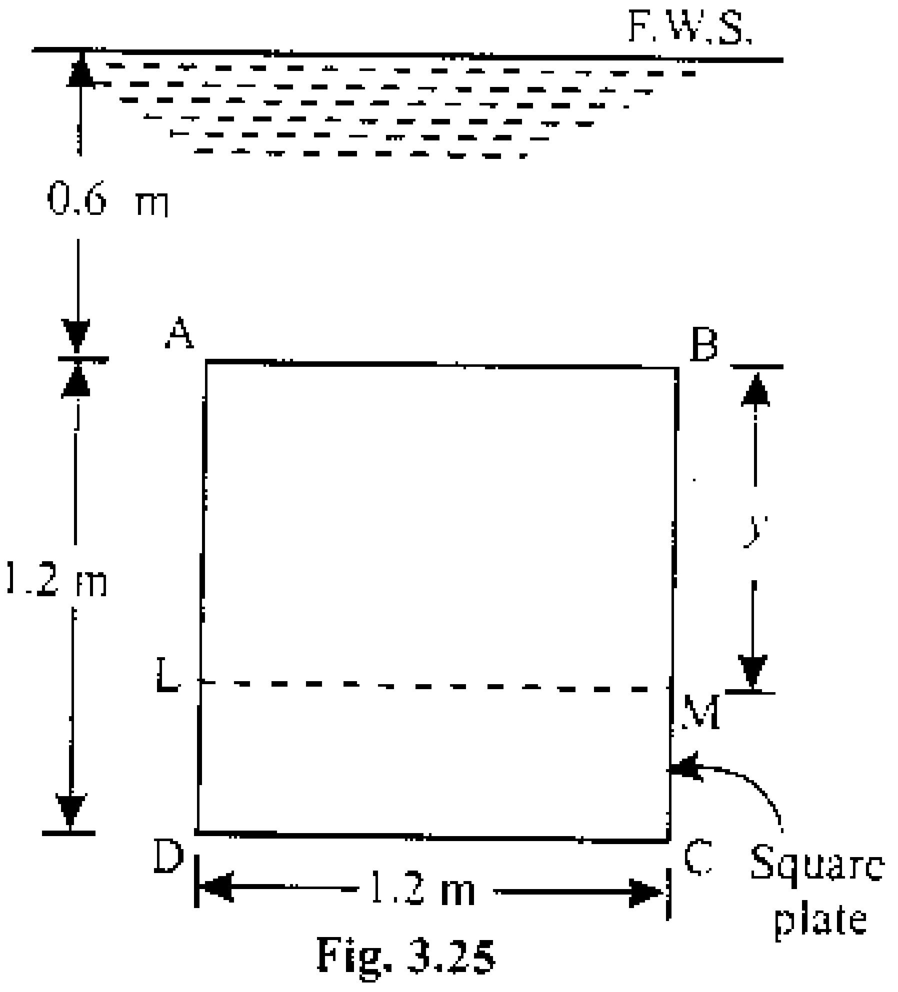


Fig. 3.25

Solution. Refer Fig. 3.25. ABCD is the square plate submerged vertically in water with upper edge AB at a depth of 0.6 m below the free water surface (*F. W. S.*)

Let LM be the line such that force on ALMB equals the force on LDCM, and evidently the force on each portion equals half the total force on the entire plate ABCD.

Total pressure on the plate ABCD

$$= wA\bar{x}$$

$$= w \times (1.2 \times 1.2) \times \left(0.6 + \frac{1.2}{2}\right)$$

$$= 1.728\, w$$

Total pressure on the position ALMB

$$= w \times (1.2 \times y) \times \left(0.6 + \frac{y}{2}\right) = 1.2\, wy\left(0.6 + \frac{y}{2}\right)$$

Now, pressure force on ALMB $= \dfrac{1}{2} \times$ pressure force on ABCD

$$1.2\, wy\left(0.6 + \frac{y}{2}\right) = \frac{1}{2} \times 1.728\, w$$

or, $\quad 0.6y + \dfrac{y^2}{2} = 0.72$

or, $\quad y^2 - 1.2y - 1.44 = 0$

or, $\quad y = \dfrac{-1.2 \pm \sqrt{(1.2)^2 + 4 \times 1.44}}{2} = \dfrac{-1.2 \pm 2.683}{2}$

$$= 0.7415 \text{ m or } -1.9415 \text{ m.}$$

i.e., $\quad$ y = **0.7415 m (Ans.)**

Example 3.18. *A rectangular vertical door, 2.4 m (height) × 1.2 m (wide), is fastened by two hinges situated 18 cm below the top and 18 cm above the bottom on one vertical edge, and by one clamp at the centre of other vertical edge. The door is subjected to water pressure on one side and the depth of water above the top of door is 1.2 m. Calculate the reactions at the hinges and at the clamp.*

Solution. Refer to Fig. 3.26.

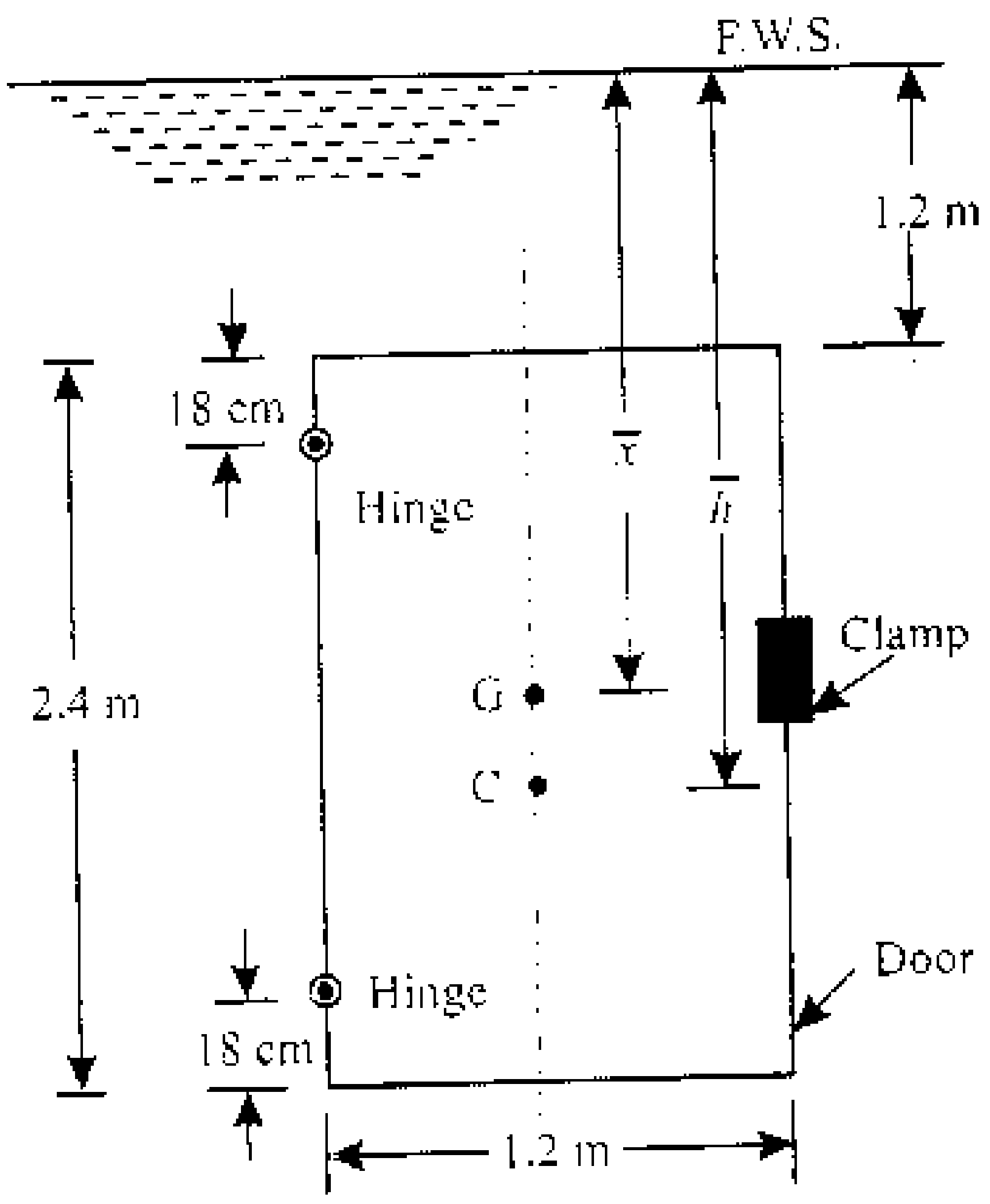


Fig. 3.26

Depth of centroid of the door, $\bar{x} = 1.2 + \dfrac{2.4}{2} = 2.4$ m

Area of the door, $A = 2.4 \times 1.2 = 2.88 \text{ m}^2$

Total pressure on the door,

$$P = wA\bar{x} = 9.81 \times 2.88 \times 2.4 = 67.8 \text{ kN}$$

Depth of centre of pressure,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

$$\frac{(1.2 \times 2.4^3/12)}{2.88 \times 2.4} + 2.4 = 2.6 \text{m}$$

i.e., $(2.4 + 1.2) - 2.6 = 1$ m from the base

Let, R_{th} = Reaction at the top hinge,

R_{bh} = Reaction at the bottom hinge, and

R_{cl} = Reaction at the clamp.

The symmetry of the arrangement suggests that half of the total pressure force is taken by the two hinges and the other half by the clamp.

∴ *Reaction at the clamp.* $\mathbf{R_{cl}} = \dfrac{67.8}{2}$ = **33.9 kN (Ans.)**

Taking moments of all forces about the horizontal axis through the bottom hinge, we get

$$P \times (1 - 0.18) = R_{cl} \times \left(\frac{2.4}{2} - 0.18\right) + R_{th} \times (2.4 - 0.18 - 0.18)$$

$$67.8 \times 0.82 = 33.9 \times 1.02 + R_{th} \times 2.04$$

or $\mathbf{R_{th} = 10.3}$ **kN (Ans.)**

∴ Reaction at the bottom hinge, $\mathbf{R_{bh}} = 33.9 - R_{th} = 33.9 - 10.3 =$ **23.6 kN (Ans.)**

3.5 Inclined Immersed Surface

Refer Fig. 3.27. Consider a plane inclined surface, immersed in a liquid.

Let, A = Area of the surface,

$\bar{x}$ = Depth of centre of gravity of immersed surface from the free liquid surface,

θ = Angle at which the immersed surface is inclined with the liquid surface, and

w = Specific weight of the liquid.

(*a*) **Total pressure (*P*):**

Consider a strip of thickness dx, width b at a distance l from O (A point, on the liquid surface, where the immersed surface will meet, if produced).

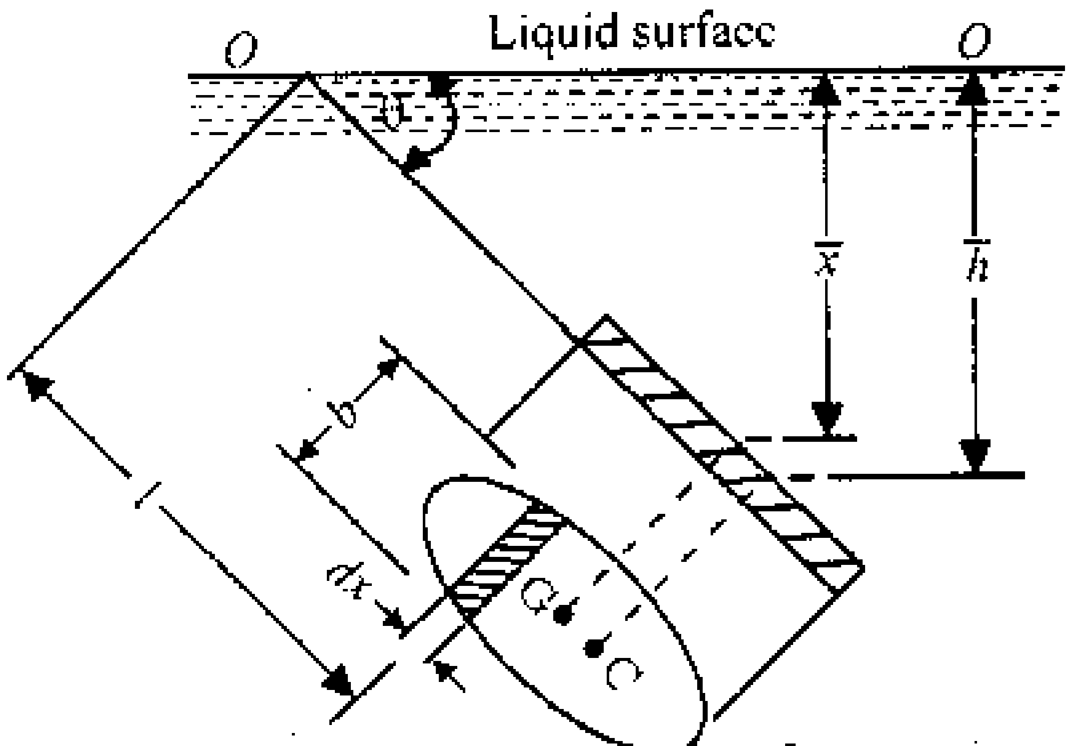


Fig. 3.27. Inclined immersed surface.

The intensity of pressure on the strip

$= wl \sin\theta$

Area of the strip $= b.dx$

Pressure on the strip

$=$ intensity of pressure $\times$ *area* $= wl \sin\theta \,.\, b.\, dx$

Now total pressure on the surface,

$$P = \int wl \sin\theta \,.\, b \,.\, dx = w \sin\theta \int l \,.\, b \,.\, dx$$

But $\int l \,.\, b \,.\, dx =$ moment of surface area about 00

$$= \frac{A\bar{x}}{\sin\theta},$$

$\therefore P = w \sin\theta \,.\, \dfrac{A\bar{x}}{\sin\theta} = wA\bar{x}$ (same as in Art. 3.3 and 3.4)

(*b*) **Centre of pressure ($\bar{h}$):**

Referring to Fig 3.27, let C be the centre of pressure of the inclined surface.

Let, $\bar{h}$ = Depth of centre of pressure below free liquid surface.

I_G = Moment of inertia of the immersed surface about OO,

$\bar{x}$ = Depth of centre of gravity of the surface from the liquid surface,

θ = Angle at which the immersed surface is inclined with the liquid surface, and

A = Area of the surface.

Consider a strip of thickness of dx, width b and at distance l from OO.

The intensity of pressure on the strip $= wl\sin\theta$

Area of strip $= b \,.\, dx$

$\therefore$ Pressure on the strip = intensity of pressure $\times$ area $= wl \sin\theta\, b \,.\, dx$

Moment of the pressure about $OO = (wl \sin\theta \,.\, b.dx)\, l = wl^2 \sin\theta \,.\, b \,.\, dx$

Now sum of moments of all such pressures about O,

$$M = \int wl^2 \sin\theta \,.\, b\, dx = w \sin\theta \int l^2 \,.\, b \,.\, dx$$

But $\int l^2 \,.\, b \,.\, dx = I_0 =$ moment of inertia of the surface about the point 0 (or the second moment of area)

$$M = w \sin\theta \,.\, I_0 \qquad ...(i)$$

The sum of moments of all such pressures about O is also equal to $\dfrac{P\bar{h}}{\sin\theta}$ ...(*ii*)

where P is the total pressure on the surface.

Equating eqns. (*i*) and (*ii*), we get

$$\frac{P\bar{h}}{\sin\theta} = w \sin\theta \,.\, I_0$$

$$\frac{wA\bar{x}\,\bar{h}}{\sin\theta} = w \sin\theta \,.\, I_0 \qquad (\because P = wA\bar{x})$$

$$\bar{h} = \frac{I_0 \sin^2\theta}{A\bar{x}} \qquad ...(iii)$$

Also $I_0 = I_G + Ah^2$ ...Theorem of parallel axes.

Where I_G = moment of inertia of figure about horizontal axis through its centre of gravity.

h = distance between 0 and the centre of gravity of the figure $= l\left(=\frac{\bar{x}}{\sin\theta}\right)$ in this case.

Rearranging equation (*iii*)

$$\bar{h} = \frac{\sin^2\theta}{A\bar{x}}(I_G + Al^2) = \frac{\sin^2\theta}{A\bar{x}}\left[I_G + A\left(\frac{\bar{x}}{\sin\theta}\right)^2\right] = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

Hence, *centre of pressure* $\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$ ...(3.3)

It will be noticed that if $\theta = 90°$ eqn (3.3) becomes the same as equation (3.2).

Example 3.19. *A 1m wide and 1.5 m deep rectangular plane surface lies in water in such a way that its plane makes an angle of 30° with the free water surface. Determine the total pressure and position of centre of pressure when the upper edge is 0.75 m below the free water surface.*

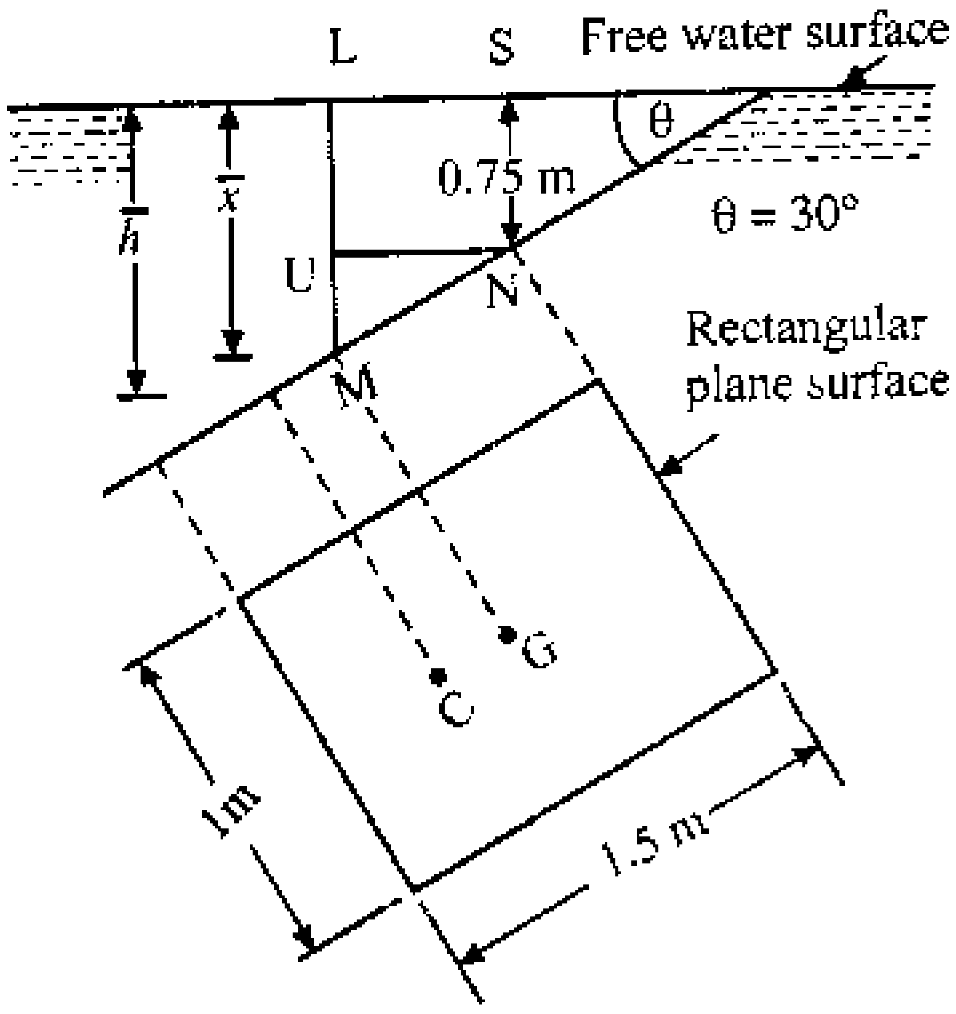


Fig. 3.28

Solution. Width of the plane surface = 1m

Depth of the plane surface = 1.5 m

Inclination, $\theta = 30°$

Distance of upper edge from free water surface = 0.75 m

(*i*) **Total pressure, P:**

Using the relation, $P = wA\bar{x}$

where, $w = 9.81$ kN/m³,

Area, $A = 1.5 \times 1 = 1.5$ m²,

$\bar{x} = LU + UM = 0.75 + MN \sin 30°$

$= 0.75 + \frac{1.5}{2} \times 0.5 = 1.125$ m

$P = 9.81 \times 1.5 \times 1.125$ m

$=$ **16.55 kN (Ans.)**

(*ii*) **Centre of pressure, $\bar{h}$:**

Using the relation, $\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$

where, $I_G = \frac{1 \times 1.5^3}{12} = 0.281$ m⁴

$\therefore \quad \bar{h} = \frac{0.281 \times (0.5)^2}{1.5 \times 1.125} + 1.125 =$ **1.166 m (Ans.)**

Example 3.20. *A circular plate 1.5 m diameter is submerged in water, with its greatest and least depths below the surface being 2 m and 0.75 m respectively. Determine:*

(*i*) *The total pressure on one face of the plate, and*

(*ii*) *The position of the centre of pressure.*

Solution. Diameter of the plate, = 1.5 m

Area of the plate,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 1.5^2 = 1.767 \text{ m}^2$$

Refer Fig. 3.29

Distance $SN = 0.75$ m, $UM = 2$m

Distance of *c.g.* from free surface,

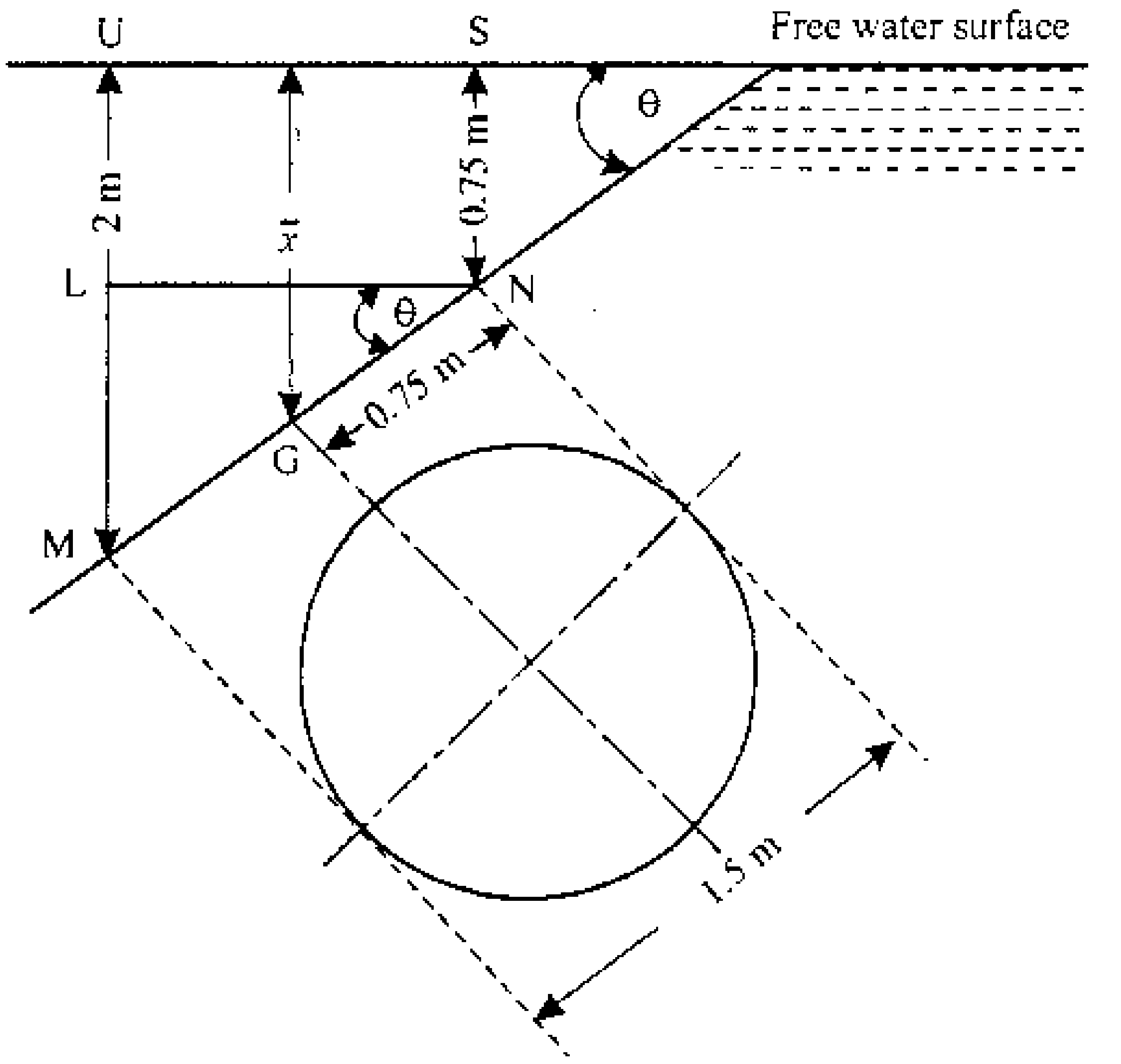


Fig. 3.29

$$\bar{x} = SN + GN \sin\theta$$

$$= 0.75 + 0.75 \sin\theta$$

$$\text{But } \sin\theta = \frac{LM}{MN} = \frac{UM - UL}{MN}$$

$$= \frac{2 - 0.75}{1.5} = 0.8333$$

$$\therefore \quad \bar{x} = 0.75 + 0.75 \times 0.8333 = 1.375 \text{ m}$$

(*i*) **Total pressure, P:**

We know that,

$$P = wA\bar{x} = 9.81 \times 1.767 \times 1.375$$

$$= \mathbf{23.83 \text{ kN (Ans.)}}$$

(*ii*) **Centre of pressure, $\bar{h}$:**

Using the relation,

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

$$= \frac{\pi/64 \times 1.5^4 \times (0.8333)^2}{1.767 \times 1.375} + 1.375 = 1.446$$

i.e., $\bar{h}$ = **1.446 m (Ans.)**

Example 3.21. *An annular plate 2m external diameter and 1m internal diameter with its great- and least depths below the surface being 1.5 m and 0.75 m respectively. Calculate the magni-, direction and location of the force acting upon one side of the plate due to water pressure.*

Solution. Refer 3.30. From the geometry of the figure, we have.

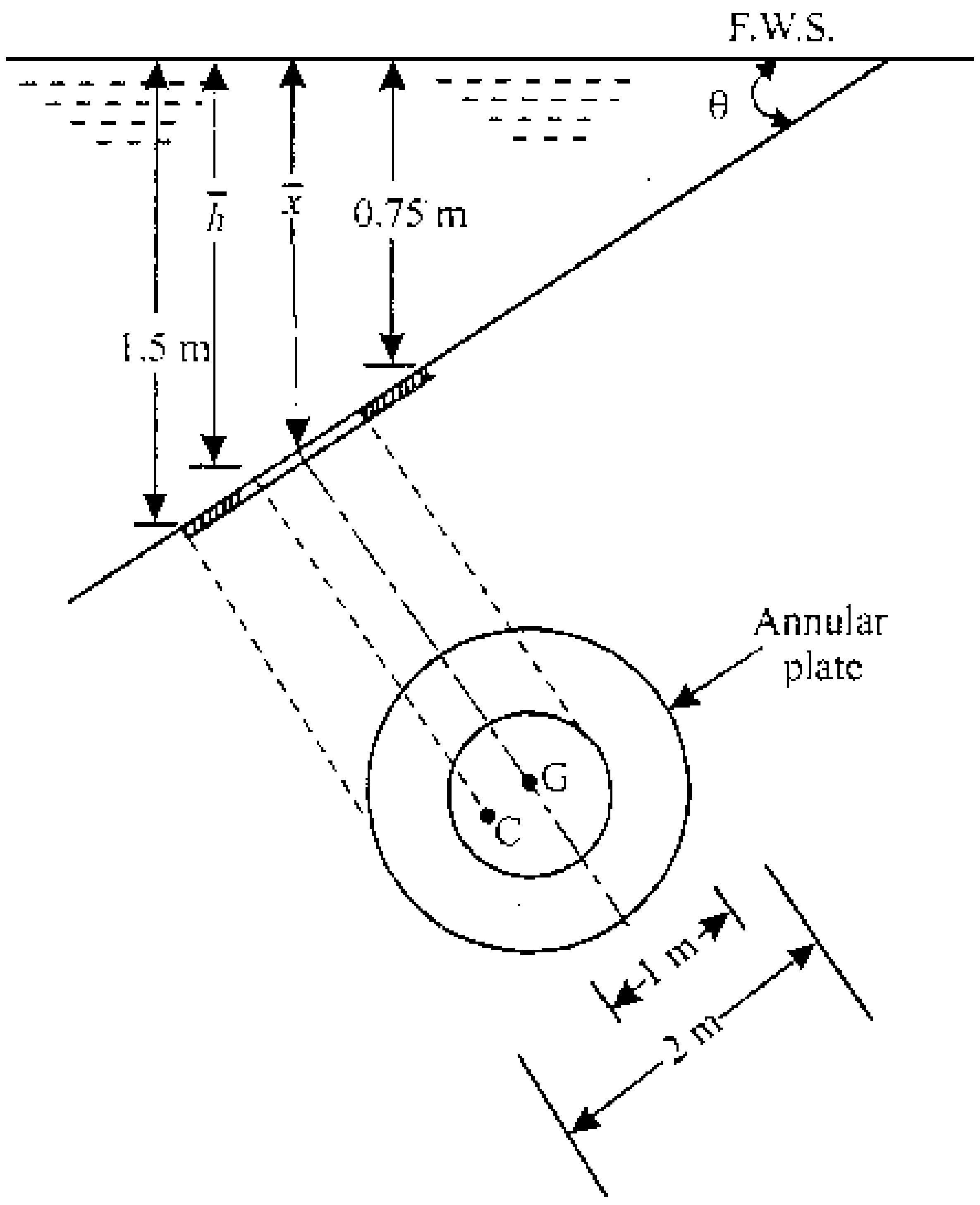


Fig. 3.30

$$\sin\theta = \frac{1.5 - 0.75}{2} = 0.375$$

$\therefore$ $\theta = \sin^{-1}(0.375) = 22°$

Area of the plate, $A = \frac{\pi}{4}(2^2 - 1^2) = 2.356\ \text{m}^2$

Depth of centroid, $\bar{x} = \frac{1.5 + 0.75}{2} = 1.125\ \text{m}$

Total pressure force,

$P = wA\bar{x}$ = 9.81× 2.356 × 1.125 = **26 kN (Ans.)**

This force acts perpendicular to the plate so it is acting in a direction which is 90° – 22° = **68°** **the vertical** (Ans.)

Depth of centre of pressure,

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

$$= \frac{\frac{\pi}{64}(2^4 - 1^4) \times (\sin 22°)^2}{\frac{\pi}{4}(2^2 - 1^2) \times 1.125} + 1.125$$

$$= \frac{0.1033}{2.651} + 1.125 = \mathbf{1.164\ m\ (Ans.)}$$

Example 3.22. *A triangular plate of 1 metre base and 1.5 metre altitude is immersed in water. The plane of the plate is inclined at 30° with free water surface and the base is parallel to and at a depth of 2 metres from water surface. Find the total pressure on the plate and the position of centre of pressure.*

Solution. Refer Fig. 3.31.

Area of the plate,

$$A = \frac{1}{2} \times 1 \times 1.5 = 0.75\ m^2$$

Inclination of the plate, $\theta = 30°$

Total pressure on the plate, *P*:

The depth of *c.g.* of the plate from water surface,

$$\bar{x} = 2 + \frac{1.5}{3} \sin 30°$$

$$= 2 + 0.5 \times 0.5 = 2.25\ m$$

Using the relation,

$$P = wA\bar{x} = 9.81 \times 0.75 \times 2.25$$

$$= \mathbf{16.55\ kN\ (Ans.)}$$

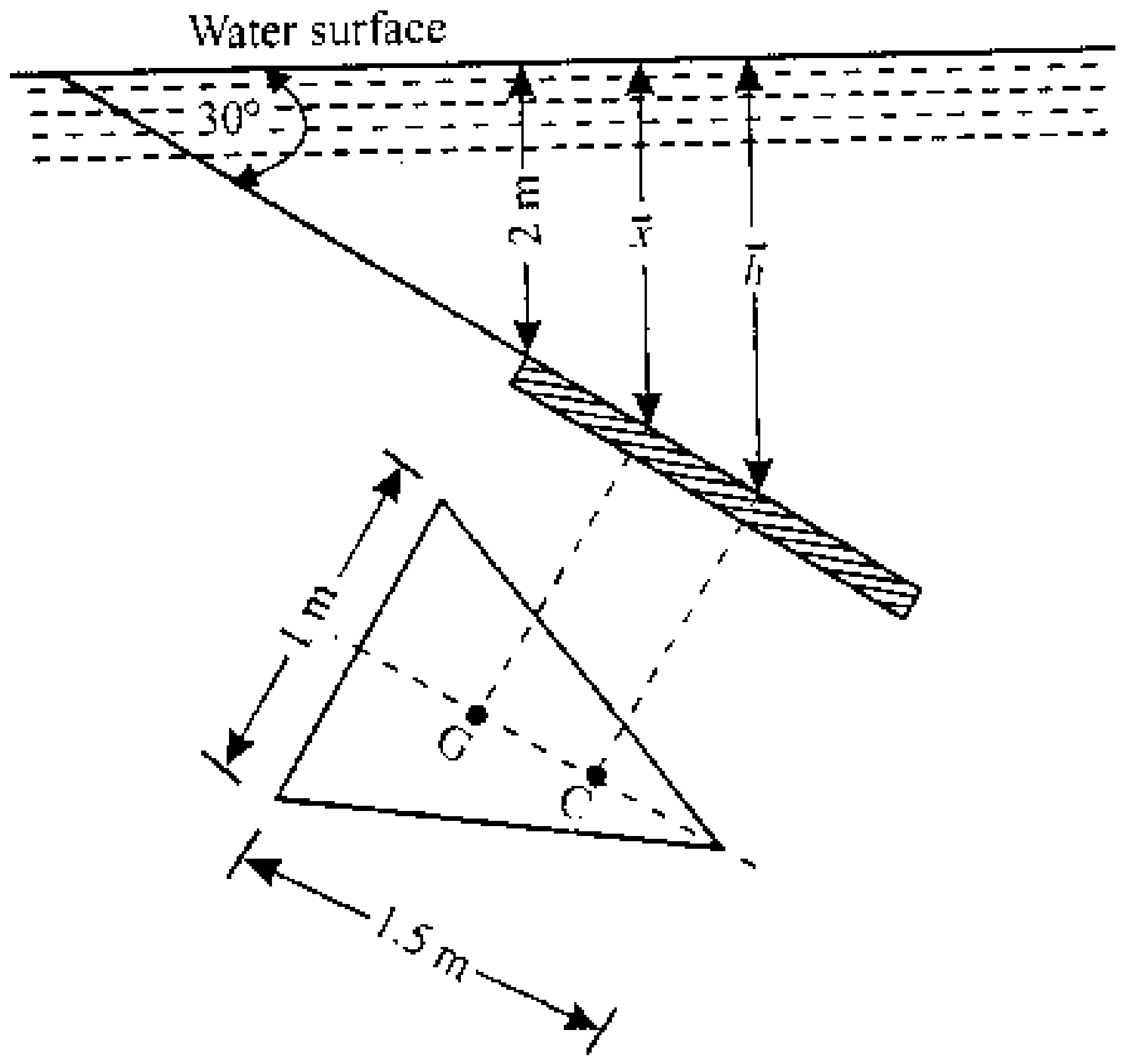


Fig. 3.31

Depth of centre of pressure, $\bar{h}$:

Moment of inertia of a triangular section about its *c.g.*,

$$I_G = \frac{1 \times 1.5^3}{36} = 0.09375\ m^4$$

Using the relation,

$$\bar{h} = \frac{I_G \sin^2 \theta}{A\bar{x}} + \bar{x} = \frac{0.09375 \sin^2 30°}{0.75 \times 2.25} + 2.25$$

$$= \mathbf{2.264\ m\ (Ans.)}$$

Example 3.23. *A trapezoidal plate measuring 1 m at the top edge and 1.5 m at the bottom edge is immersed in water with the plan making an angle of 30° to the free surface of water. The top and the bottom edges lie at 0.5 m and 1 m respectively from the surface. Determine the hydrostatic force on the plate.*

Solution.

Refer Fig. 3.32, *Given*: $a = 1$ m; $b = 1.5$ m; $h = AB = \frac{1.0 - 0.5}{\sin 30°} = 1$ m

Distance of centroid of a trapezium plate from its base,

$$h_G = \frac{h}{3}\left(\frac{2a + b}{a + b}\right)$$

$$= \frac{1}{3}\left(\frac{2 \times 1 + 1.5}{1 + 1.5}\right) = 0.467\ m$$

Depth of centroid from the free water, surface,

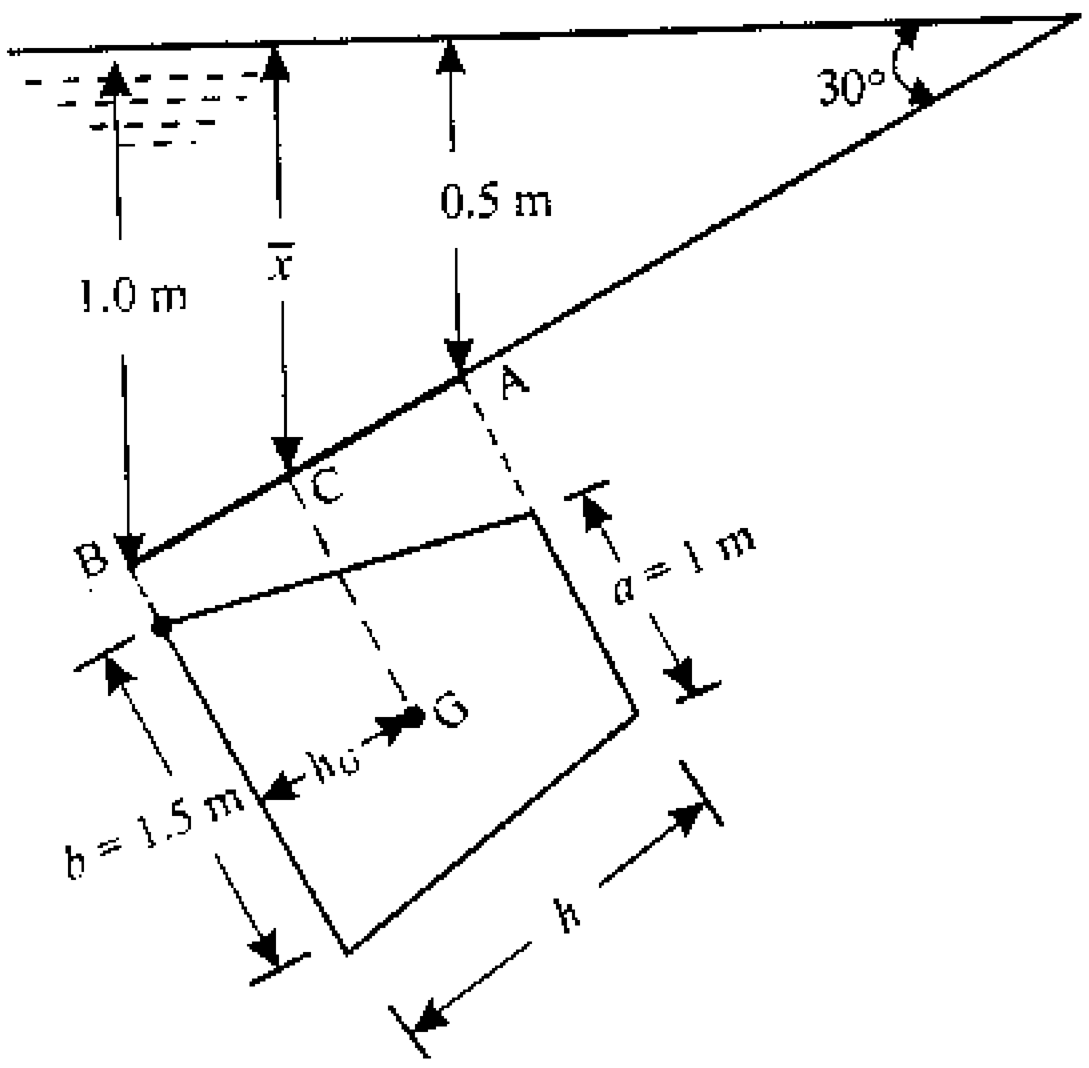


Fig. 3.32

$$\bar{x} = 1.0 - BC \sin 30° = 1.0 - 0.467 \times \sin 30° = 0.766\,5 \text{ m}$$

Area of trapezium, $A = \left(\frac{a+b}{2}\right) h = \left(\frac{1+1.5}{2}\right) \times 1 = 1.25 \text{ m}^2$

∴ *Hydrostatic force* $P = wA\bar{x} = 9.81 \times 1.25 \times 0.7665 =$ **9.399 kN (Ans.)**

Example 3.24. *An inclined rectangular sluice gate AB 1.2 m by 5 m size as shown in Fig. 3.33 is installed to control the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it.*

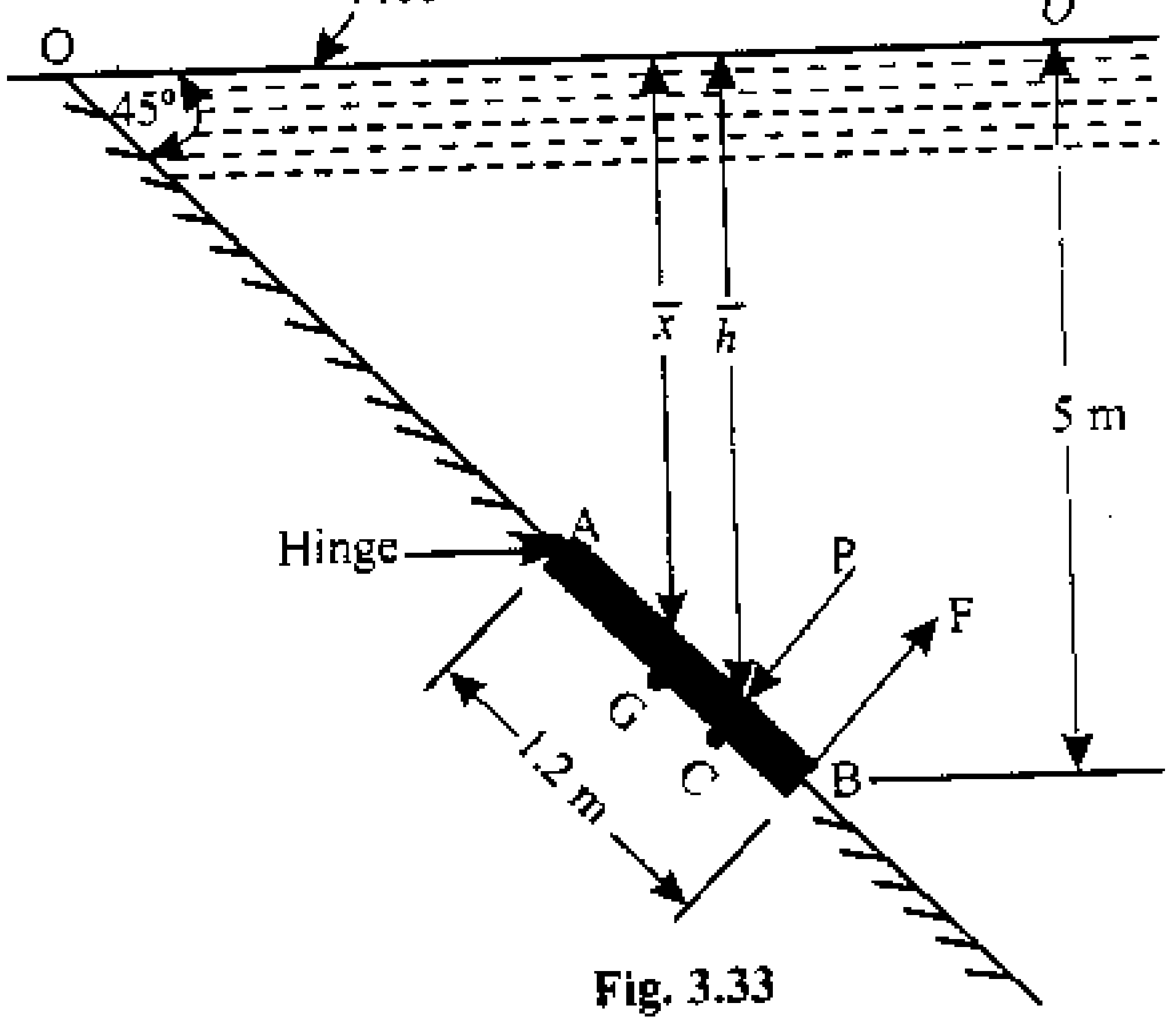


Fig. 3.33

Solution. Size of the gate = 1.2 m × 5m

Area of the gate = 1.2 × 5 = 6 m²

Refer Fig. 3.33.

Depth of c.g. of the gate from free water surface,

$$\bar{x} = 5 - BG \sin 45°$$
$$= 5 - 0.6 \times 0.707 = 4.576 \text{ m}$$

The total pressure force (P) acting on the gate,

$$P = wA\bar{x}$$
$$= 9.81 \times 6 \times 4.576 = 269.3 \text{ kN}$$

This force acts at a depth h, given by the relation,

$$\bar{h} = \frac{I_G \sin^2 \theta}{A\bar{x}} + \bar{x}$$

where, $I_G = M.O.I.$ of gate $= \frac{bd^3}{12} = \frac{5 \times 1.2^3}{12} = 0.72 \text{ m}^4, \theta = 45°$

$$\bar{h} = \frac{0.72 \times \sin^2 45°}{6 \times 4.576} + 4.576 = 4.589 \text{ m}$$

From Fig. 3.33, we have $\frac{\bar{h}}{OC} = \sin 45°$

Distance $OC = \frac{\bar{h}}{\sin 45°} = \frac{4.589}{0.707} = 6.49$ m; Distance $OB = \frac{5}{\sin 45°} = 7.072$ m

$\therefore$ Distance $BC = OB - OC = 7.072 - 6.49 = 0.582$ m

Distance $AC = AB - BC = 1.2 - 0.582 = 0.618$ m

Taking moments about the hinge A, we get $F \times AB = P \times AC$

$$F = \frac{P \times AC}{AB} = \frac{269.3 \times 0.618}{1.2} = \textbf{138.69 kN (Ans.)}$$

Example 3.25. *A 6 m × 2 m rectangular gate is hinged at the base and is inclined at an angle of 60° with the horizontal. The upper end of the gate is kept in position by a weight of 60 kN acting at angle of 90° as shown in Fig. 3.34. Neglecting the weight of the gate, find the level of water when the gate begins to fall.*

Solution. Length of the gate, $l = 6$ m

Width of the gate, $b = 2$ m

Inclination, $\theta = 60°$

Weight, $W = 60$ kN

Level of water when the gate begins to fall:

Refer Fig. 3.34

Let h = Height of free water surface from the bottom when the gate just begins to fall.

Then length of gate in the shape of plate, submerged in water,

$$AD = \frac{AC}{\sin\theta} = \frac{h}{\sin 60°} = \frac{h}{0.866}$$
$$= 1.1547\ h$$

$\therefore$ Area of the gate immersed in water,

$$A = AD \times \text{width}$$
$$= 1.1547\ h \times 2 = 2.309\ h\ \text{m}^2$$

Also depth of *c.g.* of the immersed area,

$$\bar{x} = \frac{h}{2} = 0.5\ h$$

Total pressure on the gate,

$$P = wA\bar{x} = 9.81 \times 2.309\ h \times 0.5\ h$$
$$= 11.326\ h^2\ \text{kN}$$

The centre of pressure of the immersed surface ($\bar{h}$) is given by

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

where, I_G = moment of inertia of the *immersed area*

$$= \frac{b \times AD^3}{12} = \frac{2}{12}(1.1547\ h)^3$$
$$= 0.2566\ h^3$$

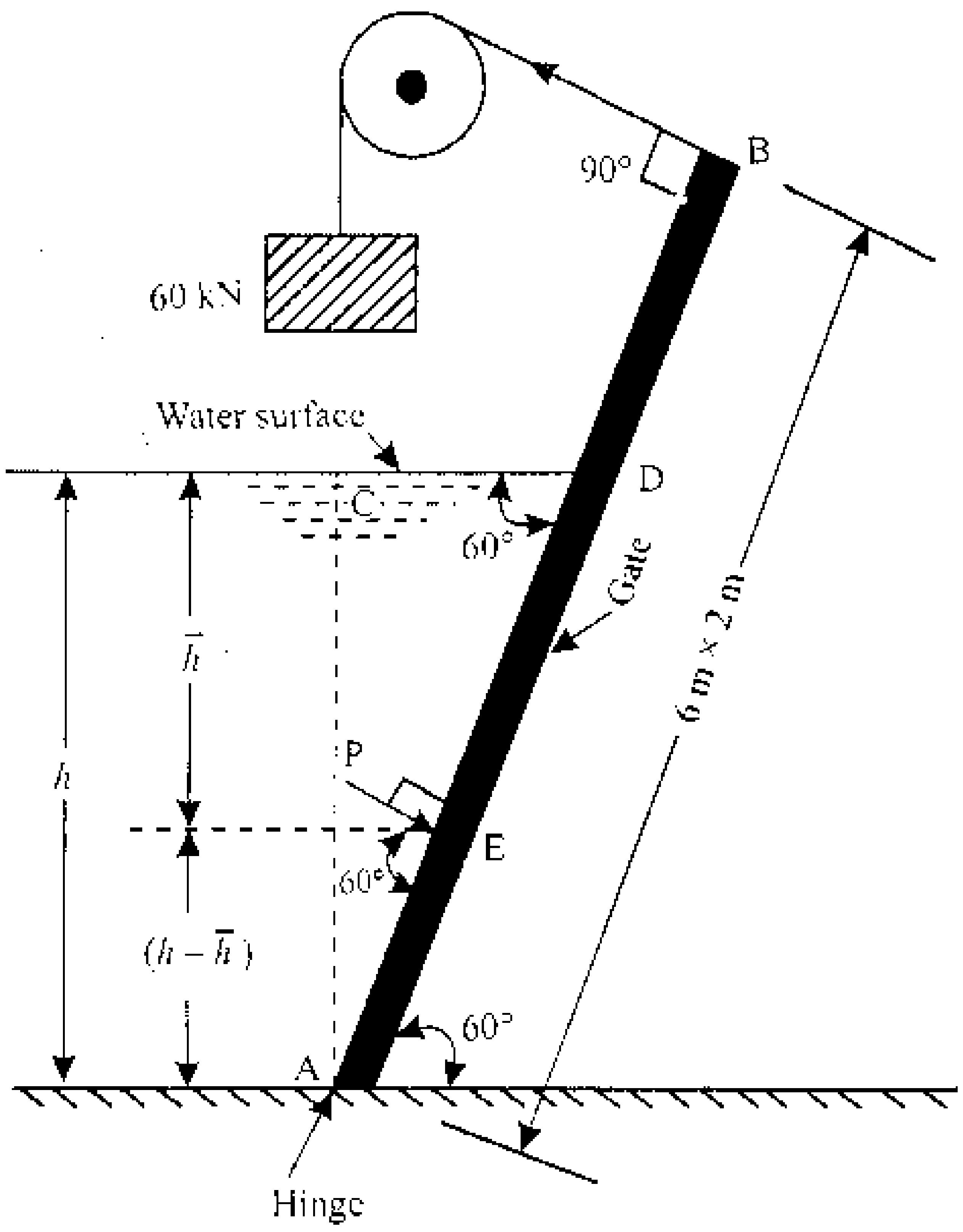


Fig. 3.34

$$\bar{h} = \frac{0.2566\,h^3 \times (\sin 60°)^2}{2.309\,h \times 0.5\,h} + 0.5\,h = 0.667\,h \text{ metres.}$$

Distance of centre of pressure from the hinge (or pivot) along the length of the gate,

$$AE = \frac{h - \bar{h}}{\sin 60°} = \frac{h - 0.667\,h}{0.866} = 0.384\,h$$

Taking moments about the hinge, we get

$$P \times AE = 60 \times AB$$

$$11.326\,h^2 \times 0.384\,h = 60 \times 6$$

or $$h^3 = \frac{60 \times 6}{11.326 \times 0.384} = 82.774$$

or $$h = \mathbf{4.36\ m\ (Ans.)}$$

Example 3.26. *Fig. 3.35 shows a circular opening in the sloping wall of the reservoir closed by disc valve 0.9 m diameter. The disc is hinged at H and a balance weight W is just sufficient to hold the valve closed when the reservoir is empty. How much additional weight should be placed on the arm, 1.2 m from the hinge in order that the valve shall remain closed until the water level is 0.72 m above the centre of the valve.*

Solution. Dia. of the valve, $d = 0.9$ m

Area of the valve $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.9^2 = 0.636\ \text{m}^2$

Inclination, $\theta = 60°$

Distance of *c.g.* of the valve from free water surface, $\bar{x} = 0.72$ m

Additional weight, W′:

Total pressure on the valve, $P = wA\bar{x} = 9.81 \times 0.636 \times 0.72 = 4.49$ kN

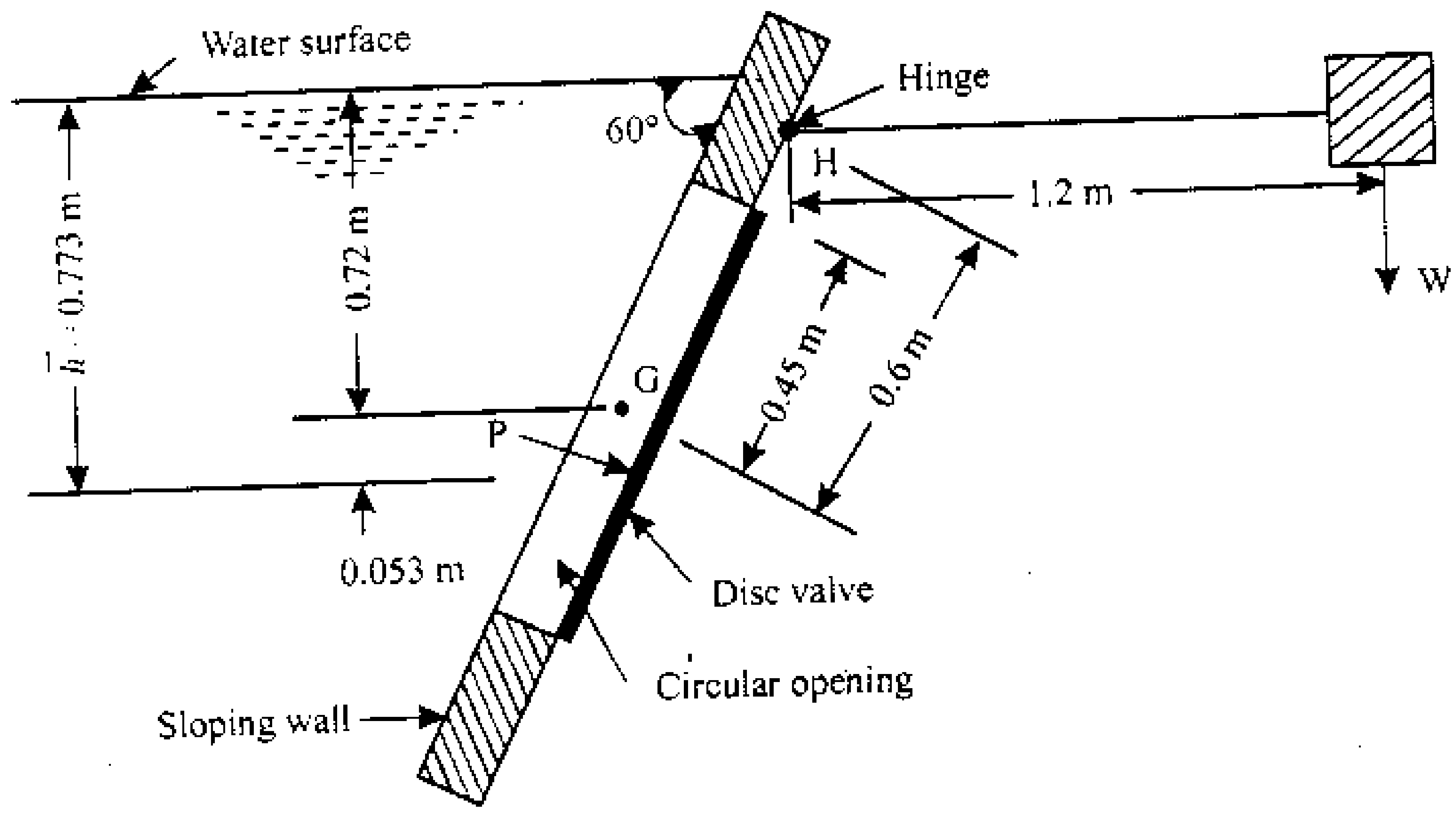


Fig. 3.35

Distance of centre of pressure ($\bar{h}$) is given by

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x} = \frac{\pi/64 \times 0.9^4 \times (\sin 60°)^2}{0.636 \times 0.72} + 0.72 = 0.773 \text{ m (from free water surface)}$$

$$= 0.053 \text{ m below the centroid } G.$$

Taking moments of all the forces about the hinge, we have

$$P\left(\frac{0.053}{\sin 60^\circ} + 0.6\right) = W' \times 1.2$$

or, $4.49\,(0.0612 + 0.6) = W' \times 1.2$ or $W' =$ **2.47 kN (Ans.)**

Example 3.27. *Fig. 3.36 shows a gate supporting water. Taking the width of the gate as unity find: (i) Depth of water (h) so that the gate tips about the hinge.*

(ii) Reaction at the hinge.

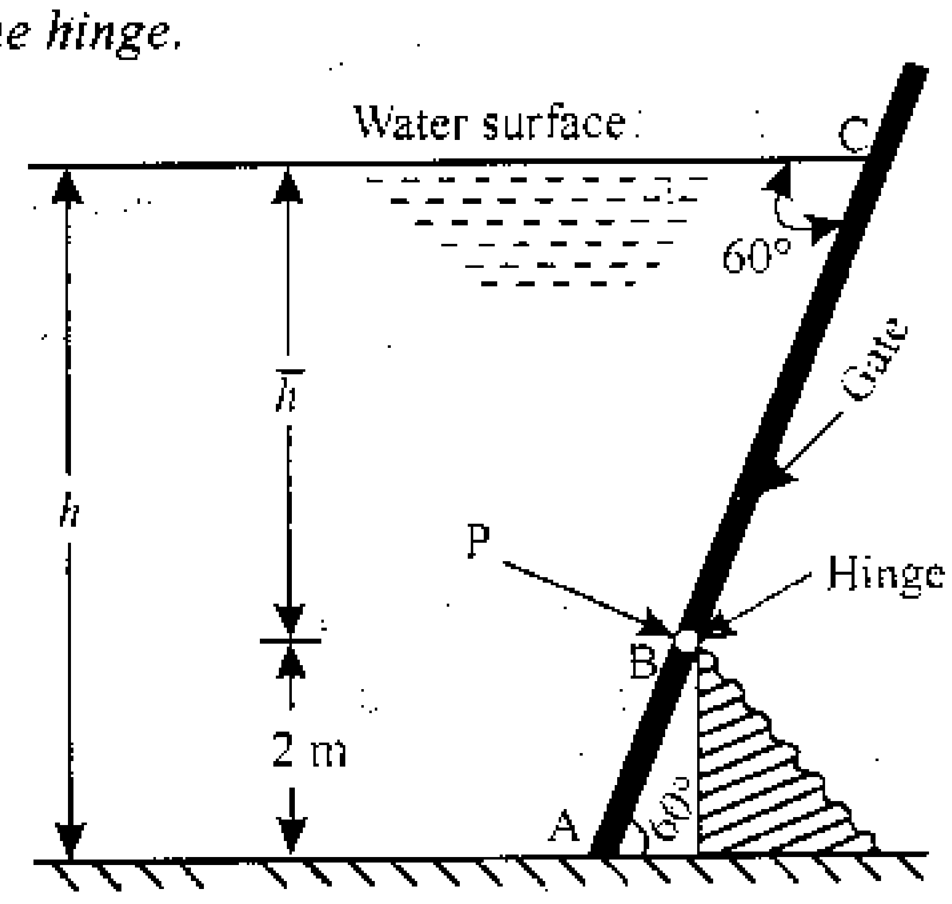


Fig. 3.36

Solution. Inclination of the gate, $\theta = 60^\circ$

(*i*) **Depth of water, h:**

As the depth of water increases, the total pressure P on the gate moves upwards, and just before tipping, P acts at the hinge.

$\therefore$ Depth of centre of pressure

$$\bar{h} = (h - 2)\text{ m}$$

But $\bar{h}$ is also given by

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

Taking width of gate unity, we have

Area, $\quad A = AC \times 1 = \dfrac{h}{\sin 60^\circ} \times 1$

$$= 1.1547\,h \qquad \left(\because AC = \frac{h}{\sin 60^\circ}\right)$$

Distance of *c.g.* from water surface, $\bar{x} = \dfrac{h}{2} = 0.5\,h$. Moment of inertia,

$$I_G = \frac{1 \times AC^3}{12} = \frac{1}{12} \times (h/\sin 60^\circ)^3 = 0.1283\,h^3$$

$$\therefore \quad \bar{h} = \frac{0.1283\,h^3 \times (\sin 60^\circ)^2}{1.1547\,h \times 0.5\,h} + 0.5\,h = 0.667\,h$$

Equating the two values of h, we get, $h - 2 = 0.667\,h$ or $0.333\,h = 2$

or $\quad h =$ **6 m (Ans.)**

Example 3.28. *A gate supporting water takes the form of an inclined shield which swings around a hinged axis O (Fig. 3.37).*

Determine: (i) The position x of the hinge at which a water level of h = 5.1 m on the left would cause the gate to tip over the hinge.

(ii) the magnitude of hydrostatic free on the gate just before it opens (tips about the hinge) automatically. Neglect frictional effects.

Solution. Refer Fig. 3.37. **The position x of the hinge:**

The gate would tip about the hinge point O when the line of action of the resultant pressure force lies from O to B anywhere on the gate; the limiting condition being the situation when the resultant force passes through the hinged point O. The resultant also passes through the centroid of the pressure diagram, and the centroid lies at a distance $\frac{1}{3} \times AB$ form the bottom point A.

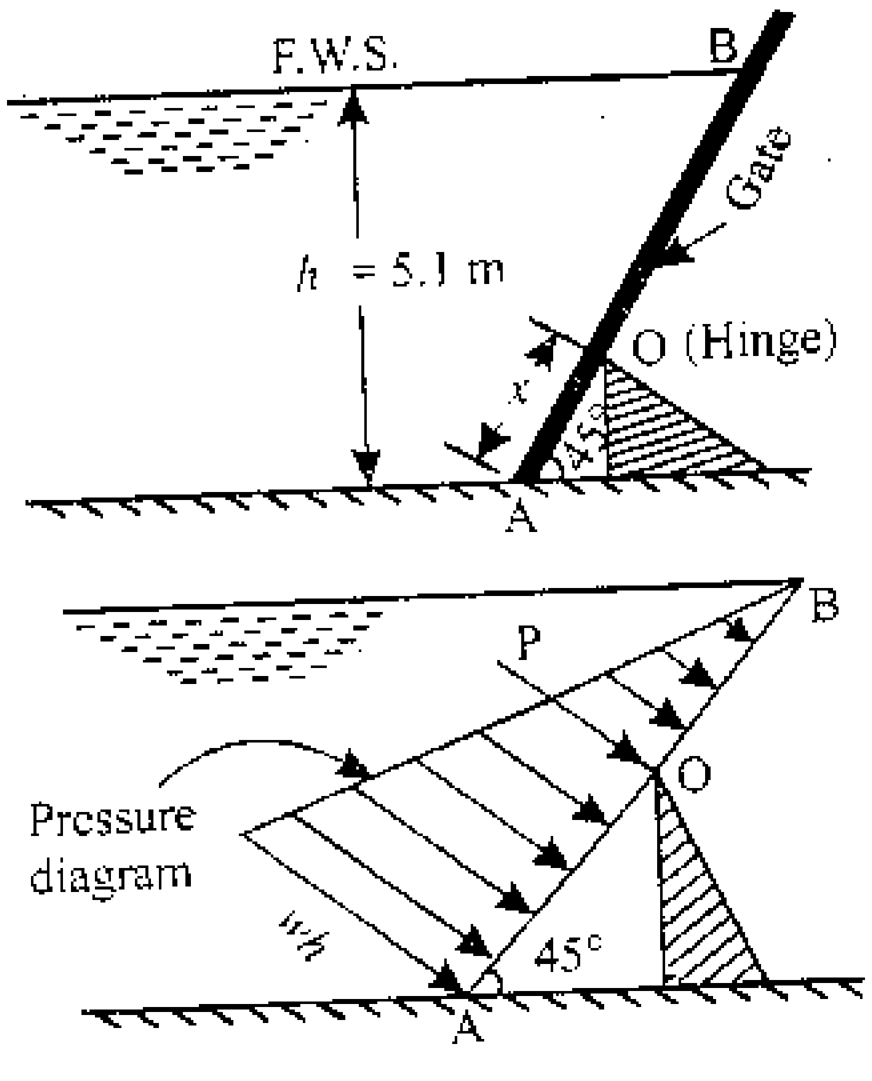


Fig. 3.37

$\therefore \quad x = \frac{1}{3} \times AB$

or AB (length of the gate) = $3x$

Depth of water, $h = 3x \times \sin 45°$

i.e. $\quad 5.1 = 3x \times \sin 45°$

$\therefore \quad x = \frac{5.1}{3 \times \sin 45°} = \mathbf{2.4\ m\ (Ans.)}$

(*i*) **The magnitude of hydrostatic force *P*:**

P = Area of pressure diagram × width of gate

$= \left(\frac{1}{2} \times AB \times wh\right) \times 1$ ... considering unit width

$= \left[\frac{1}{2} \times (3 \times 2.4) \times 9.81 \times 5.1\right] \times 1 \ (\because AB = 3x)$

= 180.11 kN (Ans.)

Example 3.29. *A 3.6 m square gate provided in an oil tank is hinged at its top edge (Fig. 3.38). The tank contains gasoline (sp. gr = 0.7) upto a height of 1.8 m above the top edge of the plate. The space above the oil is subjected to a negative pressure of 8250 N/m². Determine the necessary vertical pull to be applied at the lower edge to open the gate.* **(GATE)**

Solution. Refer Fig. 3.38

Head of oil equivalent to negative pressure 8250 N/m²,

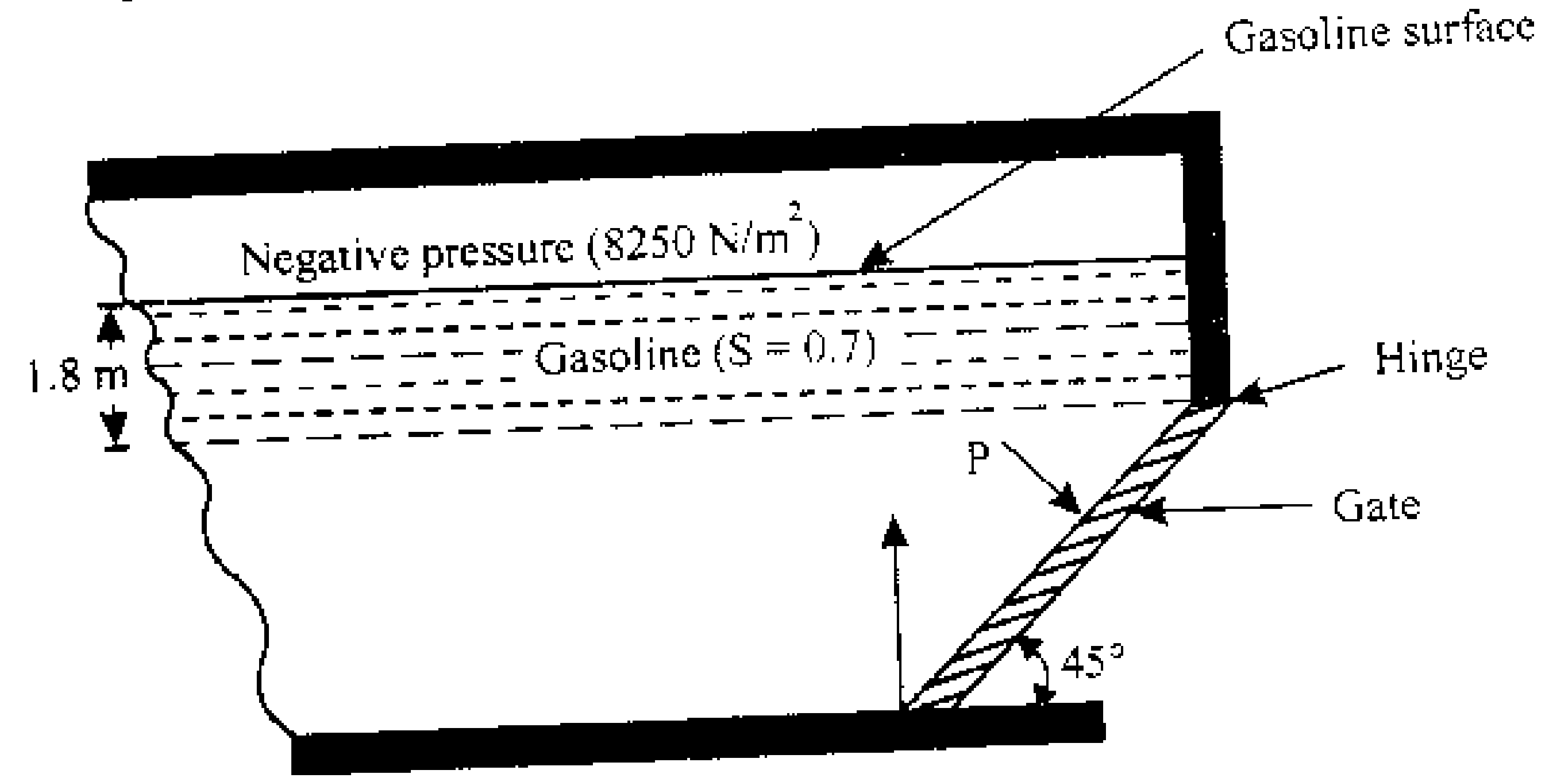


Fig. 3.38

$$h = \frac{p}{w} = \frac{8250}{0.7 \times 9810} = 1.2 \text{ m}$$

This negative pressure will reduce the oil head above the top edge of the gate from 1.8 – 1.2 = 0.6 m of oil. Calculations for the magnitude and location of the pressure force are thus to be made corresponding to 0.6 m of oil.

Now, $\quad \bar{x} = 0.6 + \frac{3.6}{2} \sin 45° = 1.873$ m

Area, $\quad A = 3.6 \times 3.6 = 12.96$ m²

Pressure $P = wA\bar{x} = 0.7 \times 9810 \times 12.96 \times 1.873 = 166690$ N

Centre of pressure, $\bar{h} = \dfrac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$

$$= \frac{\frac{1}{12} \times 3.6 \times (3.6)^3 \times (\sin 45°)^2}{12.96 \times 1.873} + 1.873 = 2.16 \text{ m}$$

∴ Vertical distance of centre of pressure below top edge of the gate

$$= 2.16 - 0.6 = 1.56 \text{ m}$$

Taking moments about the hinge,

$$F \sin 45° \times 3.6 = P \times \frac{1.56}{\sin 45°}$$

Hence, vertical force, $F = \dfrac{P \times 1.56}{3.6 \times (\sin 45°)^2} = \dfrac{166690 \times 1.56}{3.6 \times (\sin 45°)^2}$ = **144465 N (Ans.)**

Example 3.30. *There is an opening in a container shown in Fig. 3.39. Find the force F and the reaction at the hinge.*

Solution. Gauge pressure = 23.5 kN/m²

$$= \frac{23.5}{9.81 \times 0.8} = 3 \text{ m of oil} \qquad \left(\because h = \frac{p}{w}\right)$$

The free liquid surface may be considered as 3 m above the hinge A (Fig. 3.40)

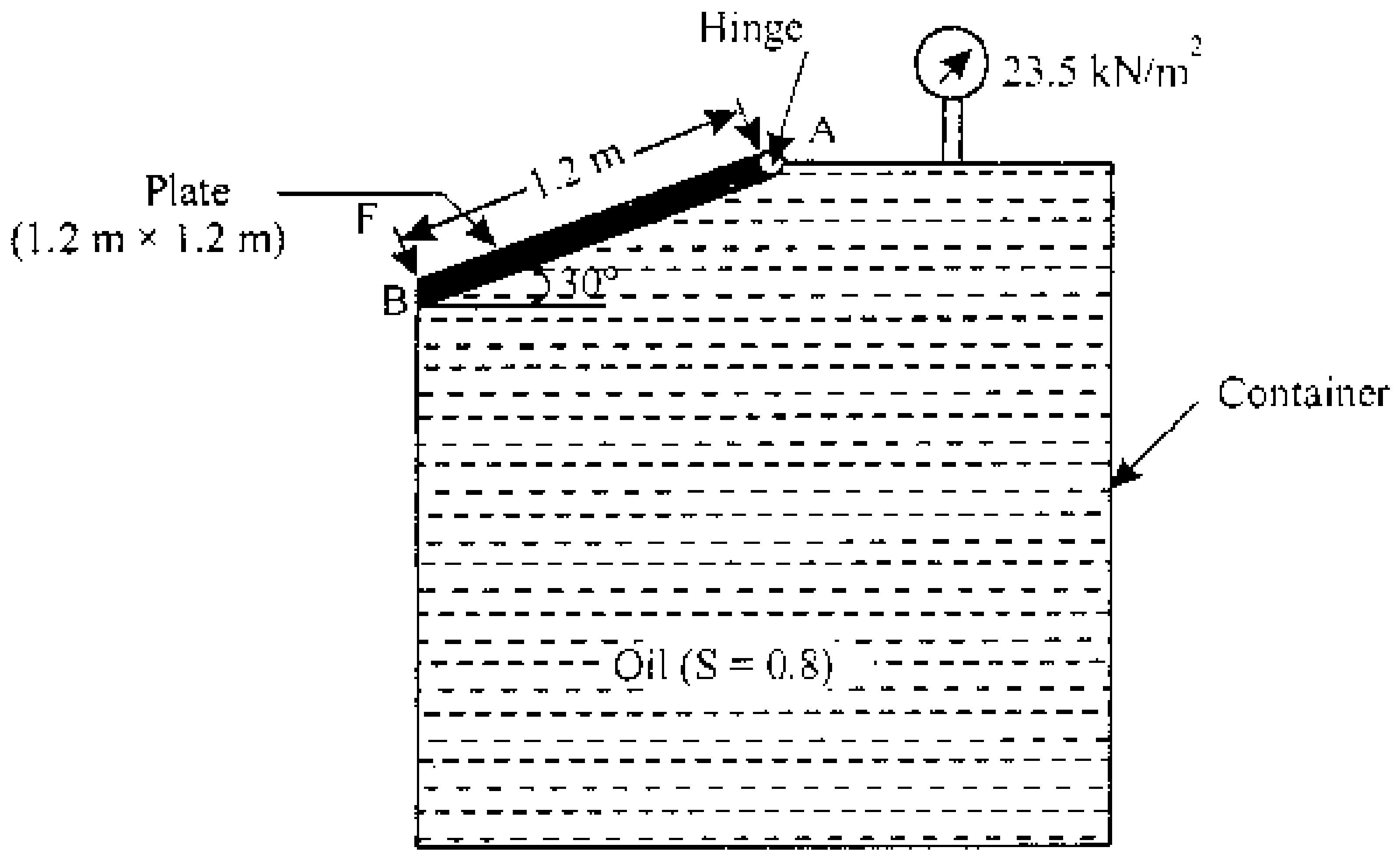


Fig. 3.39

Now, distance of centroid G of the plate from the oil surface,

$$\bar{x} = 3 + 0.6 \sin 30° = 3.3 \text{ m}$$

Total pressure on the plate,

$$P = wA\bar{x}$$
$$= (9.81 \times 0.8) \times (1.2 \times 1.2) \times 3.3$$
$$= 37.29 \text{ kN}$$

Distance of centre of pressure,

($\bar{h}$) is given by:

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x} = \frac{\frac{1.2 \times 1.2^3}{12} \times (\sin 30°)^2}{(1.2 \times 1.2) \times 3.3} + 3.3 = 3.309 \text{ m}$$

Taking moments about the hinge A, we get

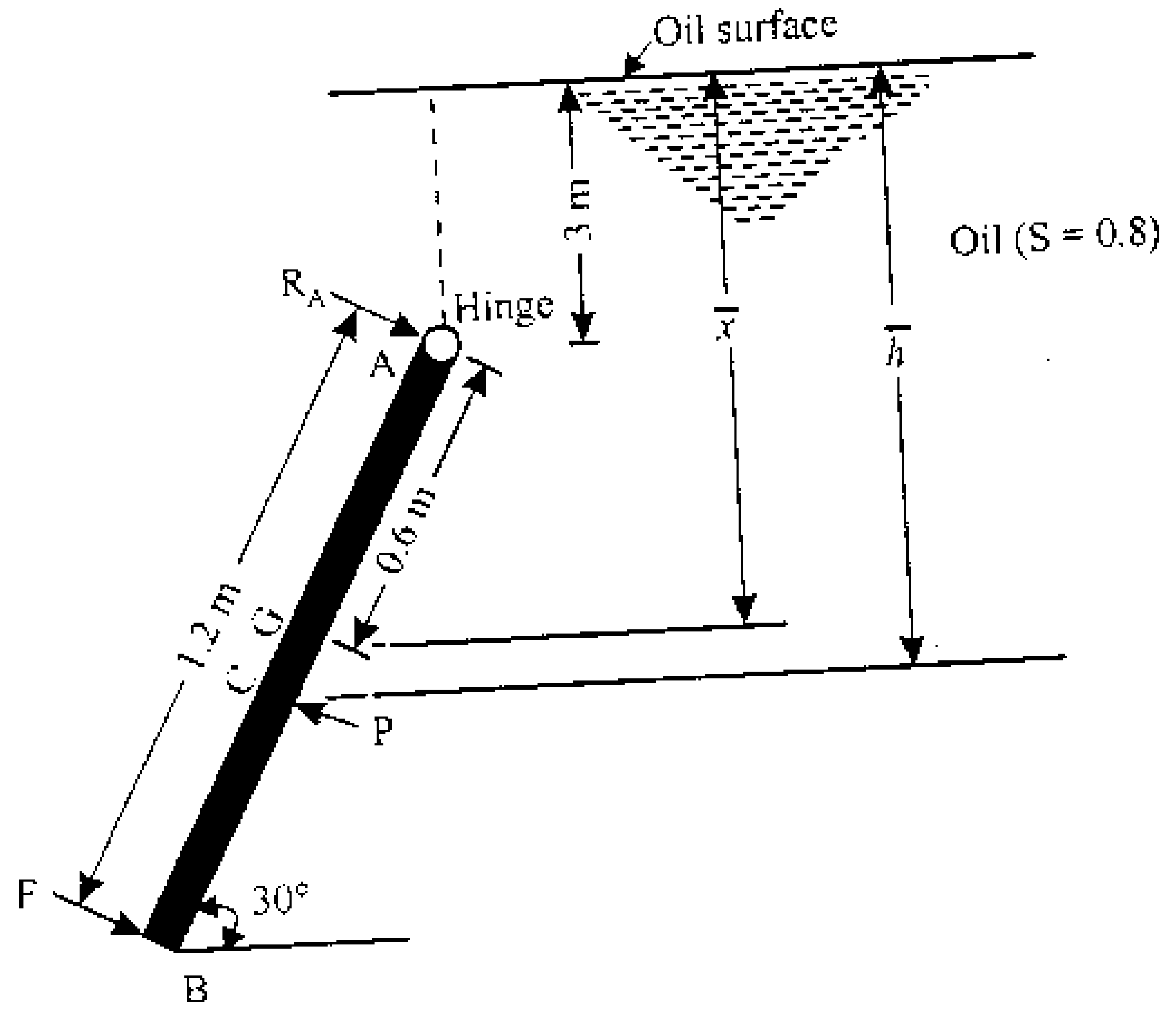


Fig. 3.40. Free body diagram.

$$F \times 1.2 = P \times CA = P \times \left[\frac{\bar{h} - 3}{\sin 30^\circ}\right]$$

or

$$F \times 1.2 = 37.29 \times \frac{(3.309 - 3)}{\sin 30^\circ}$$

$$= 23.045$$

or

$$F = 19.2 \text{ kN}$$

Let R_A = reaction at the hinge.

Then $R_A + F = P$

or $R_A = P - F$

$= 37.29 - 19.2$

$= \mathbf{18.09\ kN}$ **(Ans.)**

Example 3.31. *Fig. 3.41. shows a rectangular sluice gate AB, 3 m wide and 4.5 m long hinged A. It is kept closed by a weight fixed to the gate. The total weight of the gate and weight fixed to e gate is 515 kN. The centre of gravity of the weight and gate is at G. Find the height of the water which will first cause the gate to open.*

Solution. Width of gate, $b = 3$ m, length of gate; $l = 4.5$ m

Area, $A = 3 \times 4.5 = 13.5 \text{ m}^2$

Weight of gate and $W = 515$ kN; angle of inclination, $\theta = 45^\circ$

Height of water, h: $\bar{x} = h - LS = h - (AS - AL) = h - (AB \sin\theta - LG \tan\theta)$

$= h - (4.5 \sin 45^\circ - 0.9 \tan 45^\circ)$

$= h - (3.18 - 0.9) = (h - 2.28)$ m

The total pressure (P) is given by,

$$P = wA\bar{x} = 9.81 \times 13.5 \times (h - 2.28) = 132.43\,(h - 2.28)$$

The total pressure is acting at centre of pressure at C as shown in the Fig. 3.41. The depth of C from the free surface is given by

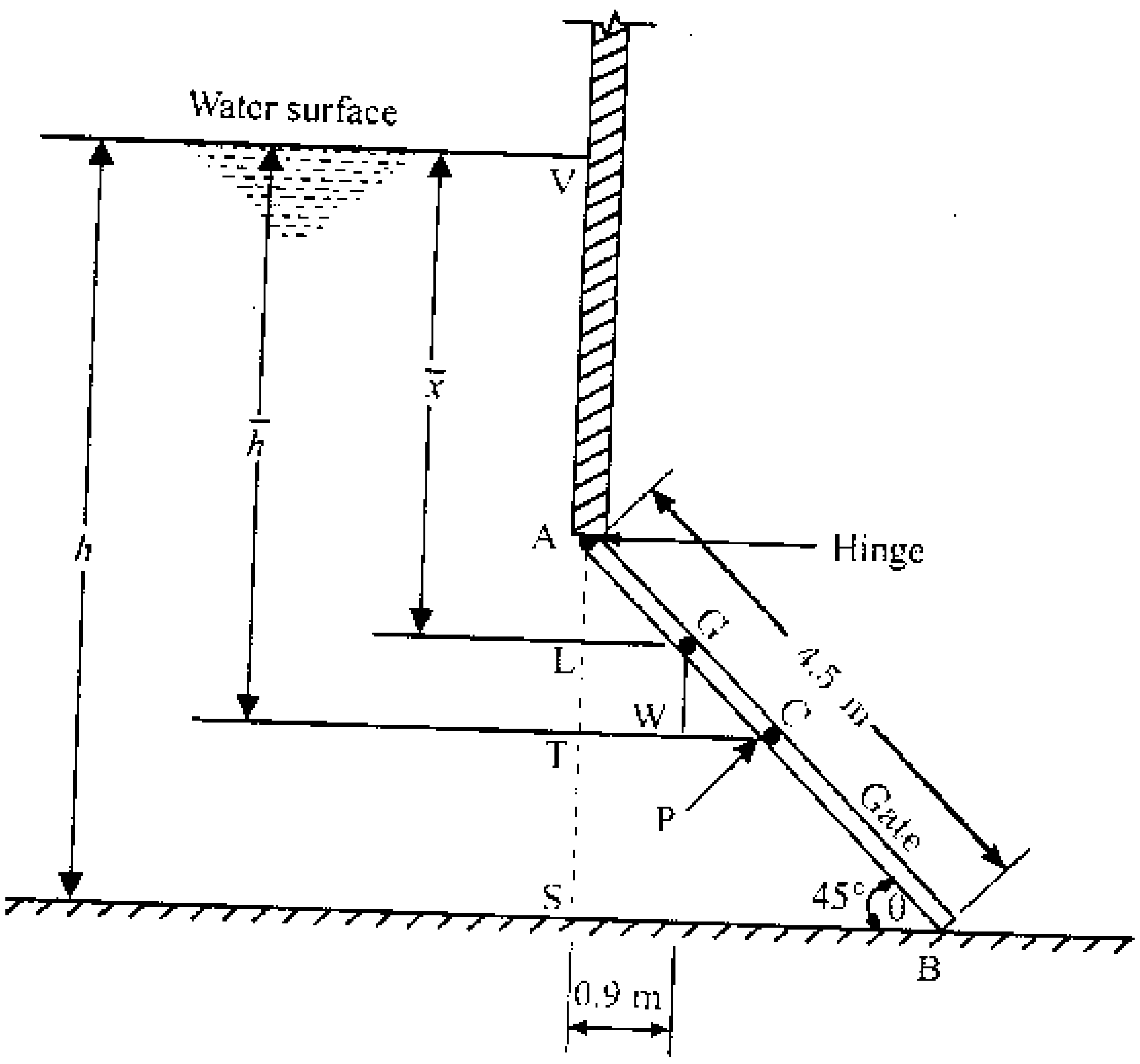


Fig. 3.41

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x} = \frac{\frac{3 \times 4.5^3}{12} \times (\sin 45°)^2}{13.5 \times (h - 2.28)} + (h - 2.28)$$

or $$\bar{h} = \frac{0.843}{(h - 2.28)} + (h - 2.28)$$

Now taking moments about hinge A, we get

$$515 \times LG = P \times AC$$

or $$515 \times 0.9 = 132.43\,(h - 2.28) \times \frac{AT}{\sin 45°}$$

$\therefore$ $$AT = \frac{515 \times 0.9 \times \sin 45°}{132.43(h - 2.28)} = \frac{2.47}{(h - 2.28)} \qquad \ldots(i)$$

But $$AT = \bar{h} - VA$$

or $$AT = \frac{0.843}{(h - 2.28)} + (h - 2.28) - VA \qquad \ldots(ii)$$

But $$VA = VS - AS = h - 4.5 \sin 45°$$
$$= h - 3.18$$

Substituting this value in (ii), we get

$$AT = \frac{0.843}{(h-2.28)} + (h-2.28) - (h-3.18) = \frac{0.843}{(h-2.28)} + 3.18 - 2.28$$

or
$$AT = \frac{0.843}{h-2.28} + 0.9 \quad \text{...(iii)}$$

Equating the values of AT from (*i*) and (*iii*), we get

$$\frac{2.47}{h-2.28} = \frac{0.843}{h-2.28} + 0.9$$

$$2.47 = 0.843 + 0.9\,(h-2.28) = 0.843 + 0.9\,h - 2.052$$

or
$$0.9\,h = 2.47 - 0.843 + 2.052 = 3.679$$

$$h = \frac{3.679}{0.9} = \mathbf{4.08\ m\ (Ans.)}$$

3.6. Curved Immersed Surface

Consider a curved surface LM submerged in a static fluid as shown in Fig. 3.42. At any point on the curved surface, the pressure acts normal to the surface. Thus if dA is the area of a small element of the curved surface lying at a vertical depth of h from surface of the liquid, then the total pressure on the elemental area is

$$dp = p \times dA = (wh) \times dA \quad \text{...(3.4)}$$

This force dP acts normal to the surface. Further integration of eqn. (3.4) would provide the total pressure on the curved surface and hence

$$P = \int w\,h\,dA \quad \text{...(3.5)}$$

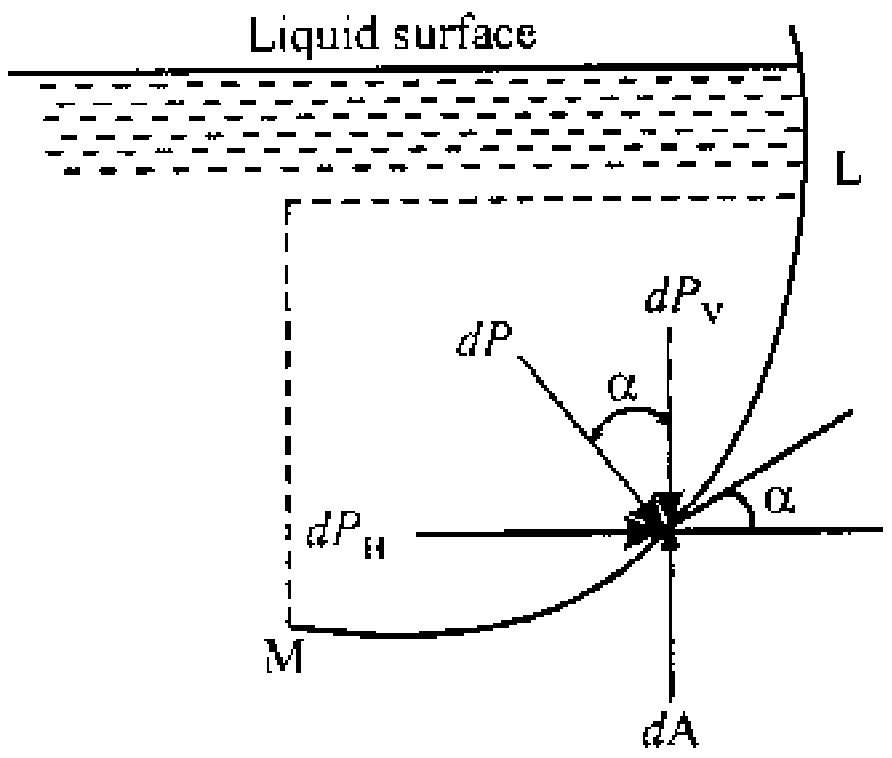


Fig. 3.42. Curved immersed surface.

But, is case of curved surface the direction of the total pressures on the elementary areas are not in the same direction, but varies from point to point. Thus the integration of eqn. (3.5) for curved surface is impossible. The problem, however, can be solved by resolving the force P into horizontal and vertical components P_H and P_V. Then total force on the curved surface is,

$$P = \sqrt{P_H^2 + P_V^2} \quad \text{...(3.6)}$$

The direction of the resultant force P with the horizontal is given by: $\tan\theta = \dfrac{P_V}{P_H}$

or
$$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) \quad \text{...(3.7)}$$

Here, P_H = total pressure force on the projected area of the curved surface on vertical plane, and,

P_V = weight of the liquid supported by the curved surface upto free surface of liquid.

Example 3.32. *The profile of a vessel is quadrant of a circle of radius R. Determine the horizontal and vertical components of the total pressure force, from the first principles.*

Solution. Consider an elementary strip of radius R at depth h and subtending an angle as shown in Fig. 3.43.

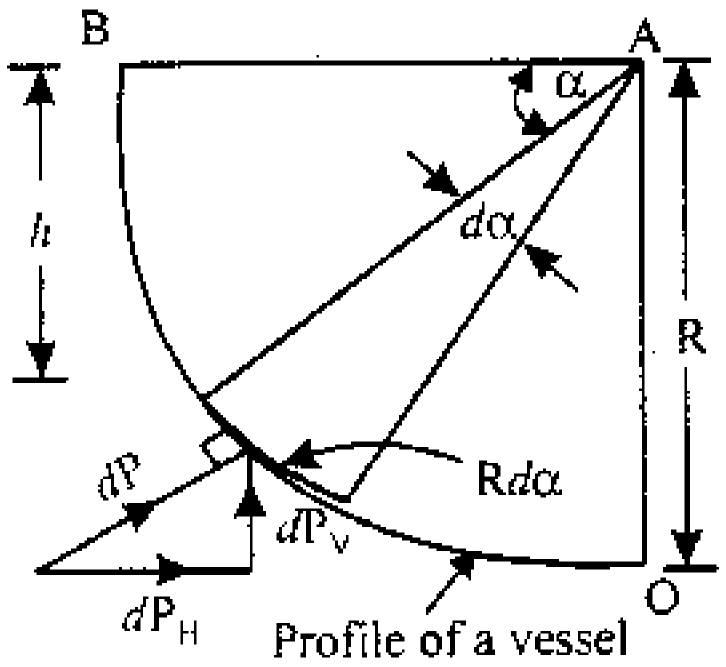


Fig. 3.43

Let the vessel has a unit depth perpendicular to the plane of paper. Then Area of the element, $dA = Rd\alpha$ × unit depth =

Depth $h = R \sin\alpha$

Intensity of pressure, $p = wh = w\,R \sin\alpha$

Pressure force, $dp = p \times dA = wR\sin\alpha \times R d\alpha$

$= wR^2 \sin\alpha\, d\alpha$

Vertical component of dP,

$$dP_V = wR^2 \sin\alpha\, d\alpha \times \sin\alpha = w\,R^2 \sin^2\alpha\, d\alpha$$

Horizontal component of dP,

$$dP_H = wR^2 \sin\alpha\, d\alpha \times \cos\alpha = wR^2 \sin\alpha \cos\alpha\, d\alpha$$

∴ Total vertical pressure force,

$$P_V = \int_0^{\pi/2} wR^2 \sin^2\alpha\, d\alpha$$

$$= \frac{wR^2}{2}\left[\int_0^{\pi/2}\left(\frac{1-\cos 2\alpha}{2}\right) d\alpha\right]$$

$$= \frac{wR^2}{2}\left[|\alpha|_0^{\pi/2} - \left|\frac{\sin 2\alpha}{2}\right|_0^{\pi/2}\right] = \frac{wR^2\pi}{4}$$

$$= w\left(\frac{\pi R^2}{4} \times \text{unit length}\right)$$

= specific weight × (volume of liquid contained in curved surface).

- Thus the *vertical component of pressure force on a curved surface equals the weight of the volume liquid extending vertically from the curved surface to the free surface of liquid.* **(Ans.)**

Total *horizontal* pressure force,

$$P_H = \int_0^{\pi/2} wR^2 \sin\alpha \cos\alpha\, d\alpha$$

$$= \frac{wR^2}{2}\int_0^{\pi/2} 2\sin\alpha \cos\alpha\, d\alpha$$

$$= \frac{wR^2}{2}\int_0^{\pi/2} \sin 2\alpha\, d\alpha = \frac{wR^2}{2}\left[-\frac{\cos 2\alpha}{2}\right]_0^{\pi/2} = \frac{wR^2}{2}$$

$$= w\,(R \times \text{unit length}) \times \frac{R}{2} \equiv w A \bar{x}$$

- *Thus the horizontal component of pressure force on a curved surface equals the force on projected area of curved surface on a vertical plane.*

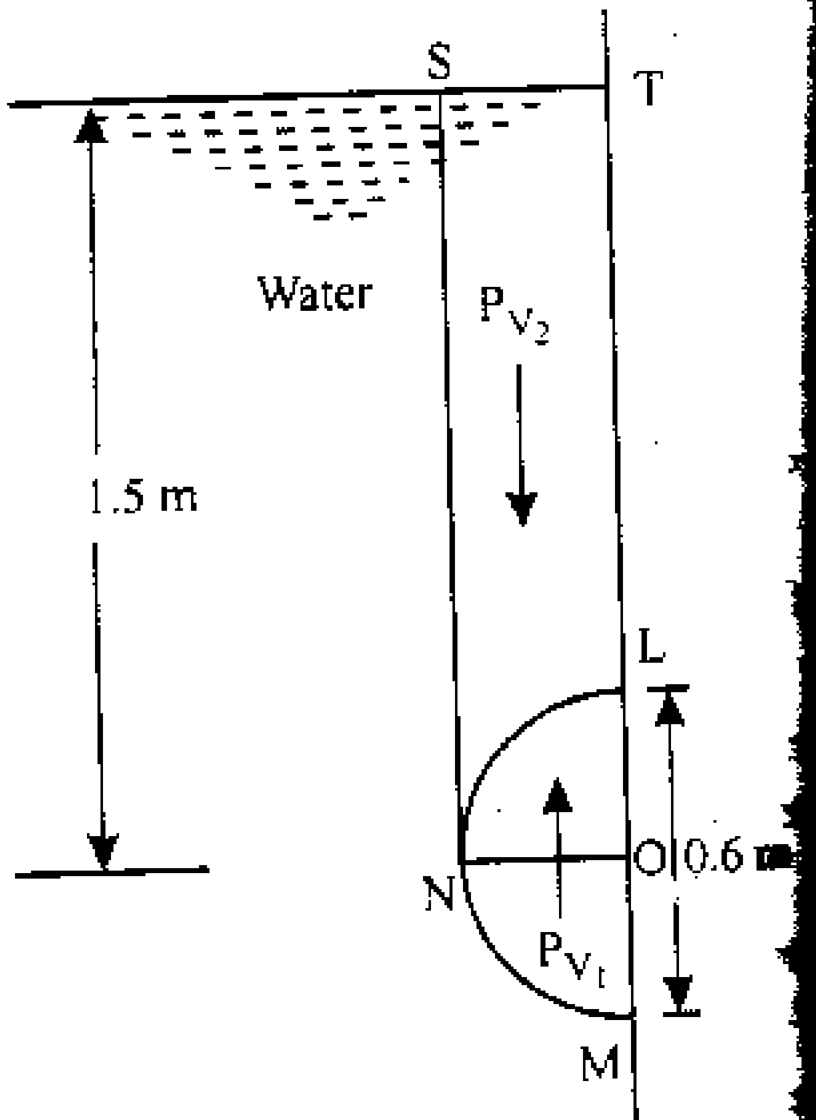


Fig. 3.44

Example 3.33. *A hemisphere projection of diameter 0.6 m exists on one of the vertical sides of a tank. If the tank contains water to an elevation of 1.5 m above the centre of the hemisphere, calculate the vertical and horizontal forces acting on the projection.*

Solution. Refer Fig. 3.44.

Vertical force, $P_V = P_{V_1} - P_{V_2}$

= Weight volume of water MNST – weight of volume of water LNST

= Weight of water contained by the hemisphere LNM

$$= w \times \frac{1}{2}\left(\frac{4}{3}\pi R^3\right)$$

$$= 9.81 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times (0.3)^3 = \mathbf{0.555\ kN\ (Ans.)}$$

Horizontal force, $P_H = wA\bar{x}$

$$= 9.81 \times \pi \times (0.3)^2 \times 1.5 = \mathbf{4.16\ kN\ (Ans.)}$$

Example 3.34. *Fig. 3.45. shows a curved surface LM, which is in the form of a quadrant of a circle of radius 3 m, immersed in the water. If the width of the gate is unity, calculate the horizontal and vertical components of the total force acting on the curved surface.*

Solution. Radius of the gate = 3 m

Width of the gate = 1m

Refer Fig. 3.45

Distance $LO = OM = 3$ m

Horizontal component of total force, P_H:

Horizontal force (P_H) exerted by water on gate is given by,

P_H = Total pressure force on the projected area of curved surface LM on vertical plane

= Total pressure force on OM

(projected area of curved surface on vertical plane $= OM \times 1) = wA\bar{x}$

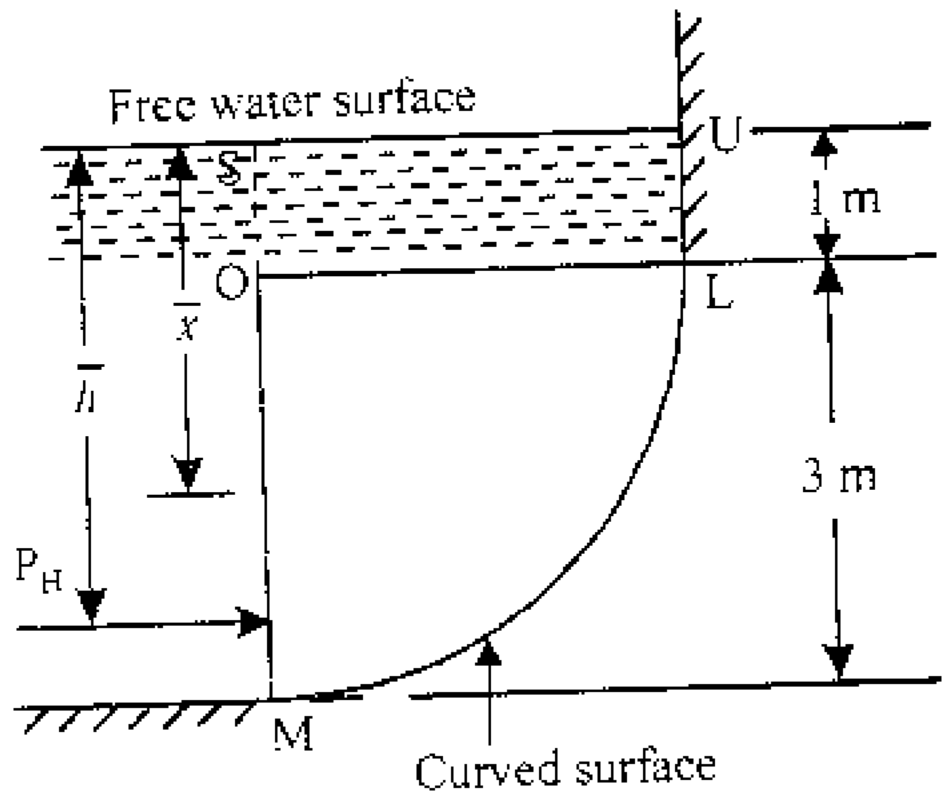


Fig. 3.45. Curved surface (gate).

But, $A = OM \times 1 = 3 \times 1 = 3\text{m}^2$ and $\bar{x} = 1 + \frac{3}{2} = 2.5$ m

$$P_H = 9.81 \times (3 \times 1) \times 2.5 = \mathbf{73.57\ kN\ (Ans.)}$$

The point of application of P_H is given by $\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$

where, $I_G = M.O.I.$ of OM about its $c.g.$ $= \frac{bd^3}{12} = \frac{1 \times 3^3}{12} = 2.25\ \text{m}^4$

$$\therefore \quad \bar{h} = \frac{2.25}{(3 \times 1) \times 2.5} + 2.5 = \mathbf{2.8\ m\ from\ water\ surface\ (Ans.)}$$

Vertical component of total force, P_V:

Vertical force (P_V) exerted by water is given by,

P_V = weight of water supported by LM upto free surface

= weight of portion $ULMOS$ = weight of $ULOS$ + weight of water in LOM

= w (volume of $ULOS$ + volume of LOM)

$$= 9.81\left[UL \times LO + \frac{\pi \times (LO)^2}{4} \times 1\right] = 9.81\left[1 \times 3 + \frac{\pi \times 3^2}{4} \times 1\right]$$

$$= 9.81\ (3 + 7.068)\ \text{kN} = \mathbf{98.77\ kN\ (Ans.)}$$

Example 3.35. *Fig. 3.46 shows a gate having a quadrant shape of radius of 1 m subjected to water pressure. Find the resultant force and its inclination with the horizontal. Take the length of the gate as 2 m.*

Solution. Radius of the gate, $r = 1$m

Length of the gate = 2 m

Horizontal force, P_H:

P_H = force on the projected area of the curved surface on vertical plane

= force on $MO = wA\bar{x}$

where, $w = 9.81\ kN/m^3$

A = area of MO (projected area) $= 1 \times 2 = 2\ m^2$

$$x = \frac{1}{2} = 0.5\,m$$

$$P_H = 9.81 \times 2 \times 0.5 = 9.81\ kN$$

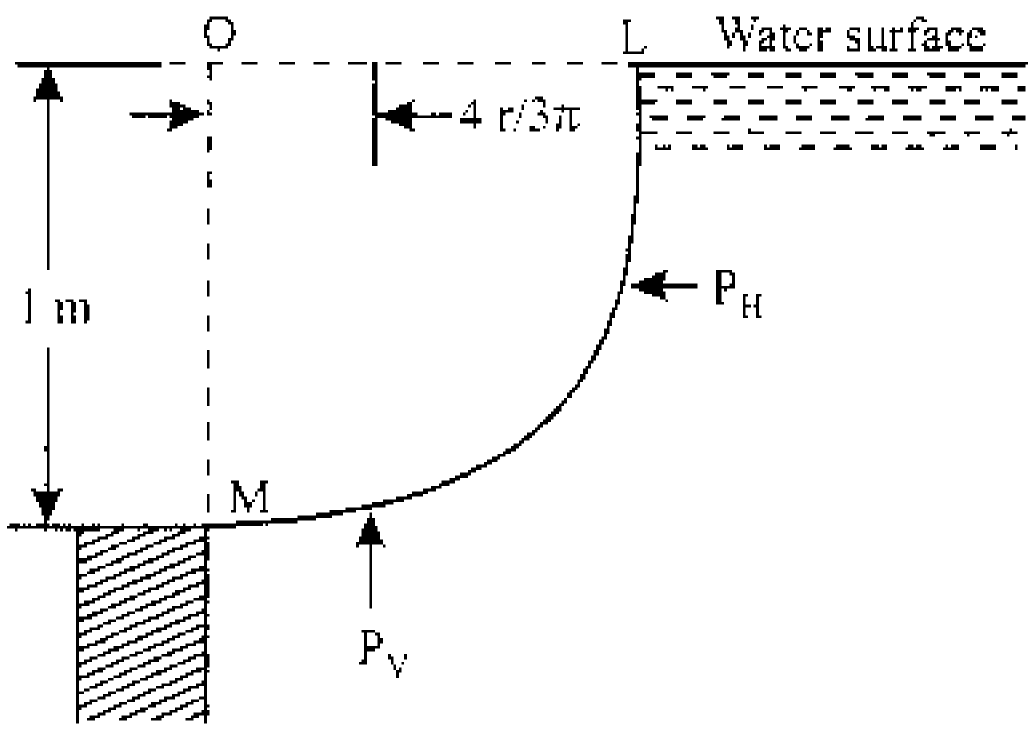


Fig. 3.46

Vertical force, P_V:

P_V = weight of water (imagined) supported by LM

$$= w \times \text{area of } LOM \times 2.0 = w \times \frac{\pi \times r^2}{4} \times 2$$

$$= 9.81 \times \frac{\pi}{4} \times 1^2 \times 2 = 15.4\ kN$$

(*i*) **Resultant force *P*:**

$$P = \sqrt{P_H^2 + P_V^2} = \sqrt{9.81^2 + 15.4^2} = \mathbf{18.26\ kN\ (Ans.)}$$

(*ii*) **The angle made by the resultant force with the horizontal, θ:**

We know that,

$$\tan\theta = \frac{P_V}{P_H} = \frac{15.4}{9.81} = 1.569 \quad \text{or} \quad \theta = \mathbf{57.48°\ (Ans.)}$$

Example 3.36. *A liquid of specific gravity 0.9 is filled in a container, shown in Fig. 3.47, upto a depth of 2.4 m. Determine the magnitude and direction of hydrostatic pressure force per unit length of container exerted on its vertical face MN and curved corner NQ.*

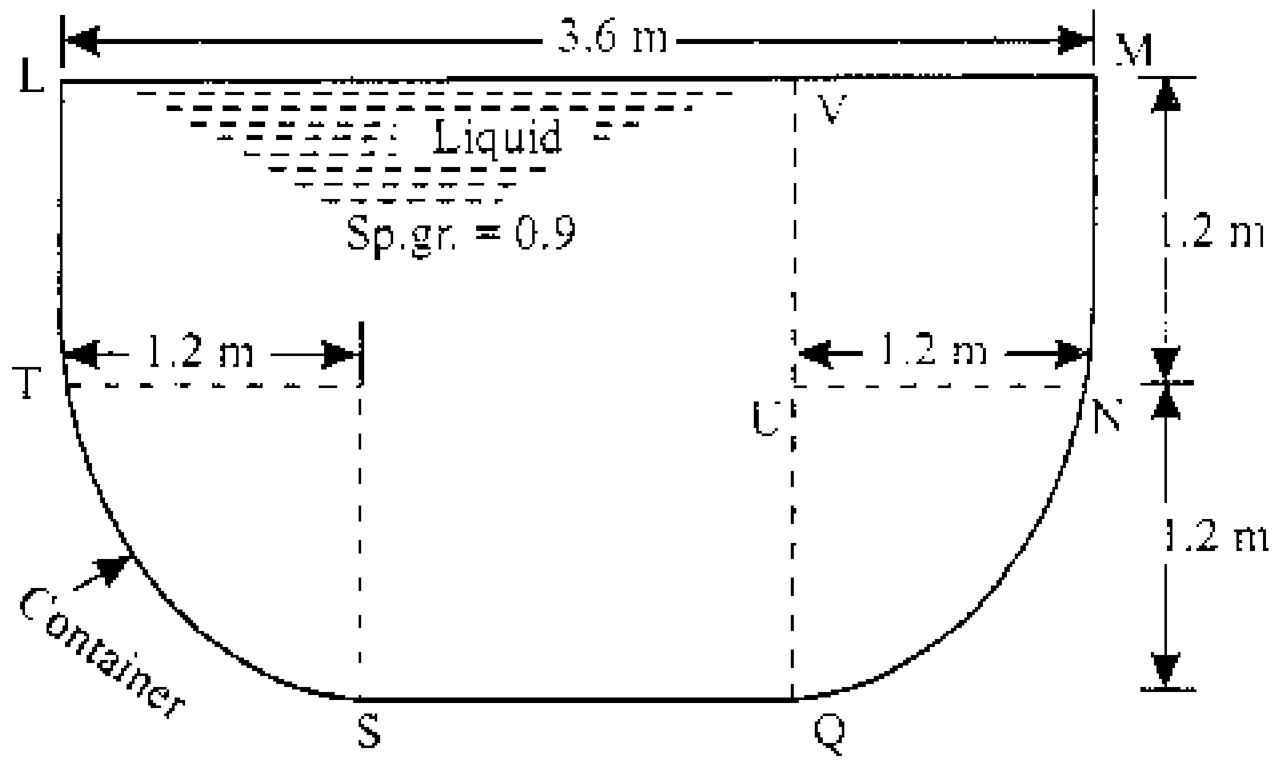


Fig. 3.47

Solution. Refer Fig 3.47

Vertical face MN:

$$P = wA\bar{x} = w \times (\text{MN} \times \text{unit length}) \times \frac{MN}{2}$$

$$= (9.81 \times 0.9) \times (1.2 \times 1) \times \frac{1.2}{2} = \mathbf{6.357\ kN\ (Ans.)}$$

This force acts horizontally towards right and its point of application is given by,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{1 \times \frac{1.2^3}{12}}{(1.2 \times 1) \times \frac{1.2}{2}} + \frac{1.2}{2} = \mathbf{0.8\ m\ (Ans.)}$$

Curved surface NQ:

Horizontal component of hydrostatic pressure force on the curved corner NQ,

P_H = Specific weight × vertical projected area × depth of centre of vertical projection

$$= w \times (\text{QU} \times \text{unit length}) \times (\text{VU} + \frac{UQ}{2})$$

$$= (9.81 \times 0.9) \times (1.2 \times 1) \times (1.2 + \frac{1.2}{2}) = 19.07 \text{ kN}$$

Vertical component of hydrostatic pressure force on the curved corner,

P_V = Weight of liquid contained in portion MNQUV

= Specific weight [Volume of liquid in portion MNUV + volume of liquid in portion NQU]

$$= w\,[\text{MN} \times \text{NU} \times \text{unit length} + \frac{1}{4}\pi \times (NU)^2 \times \text{unit length}]$$

$$= (9.81 \times 0.9)\,[1.2 \times 1.2 \times 1 + \frac{1}{4} \times \pi \times (1.2)^2 \times 1] = 22.7 \text{ kN}$$

Resultant pressure force, $P = \sqrt{P_H^2 + P_V^2} = \sqrt{(19.07)^2 + (22.7)^2} = \mathbf{29.65\ kN\ (Ans.)}$

The angle made by the resultant with the horizontal,

$$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) = \tan^{-1}\left(\frac{22.7}{19.07}\right) = \mathbf{49.97°\ (Ans.)}$$

Example 3.37. *A cylinder 2.2 m in diameter and 3.3 m long supported as shown in Fig. 3.48, retains water on one side. If the cylinder weighs 165 kN calculate the vertical reaction at L and horizontal reaction at M.*

Neglect the frictional effects.

Solution.

Radius of cylinder

$$= \frac{2.2}{2} = 1.1 \text{ m}$$

Length of cylinder = 3.3 m

Weight of cylinder = 165 kN

- The horizontal component of the resultant hydrostatic force acting on the gate is the horizontal force on the projected area of the curved surface on a vertical plane.

i.e. P_H = Hydrostatic pressure force on the curved area LSN projected on the vertical plane LON,

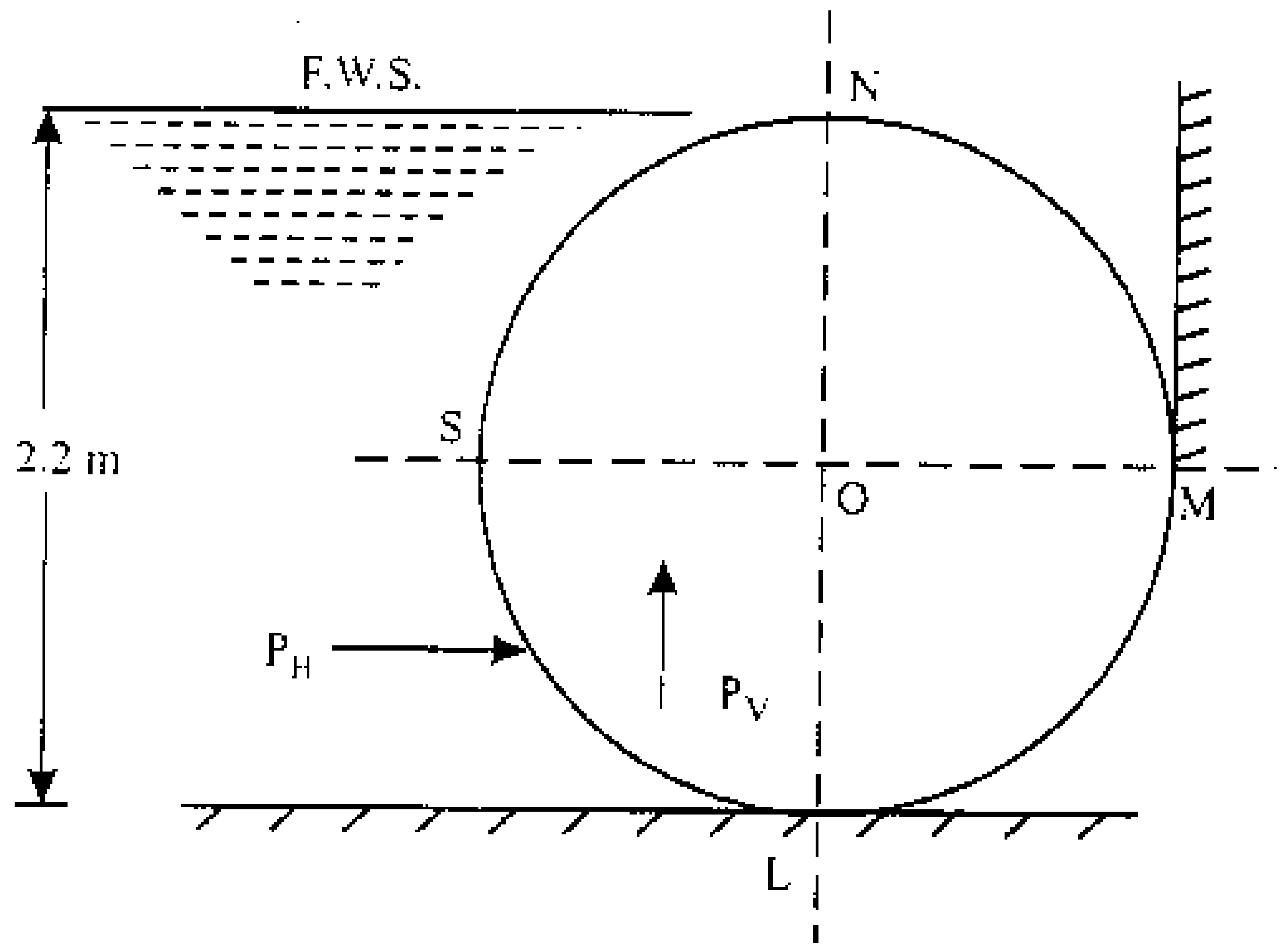


Fig. 3.48

$$= wA\bar{x}$$

$$= 9.81 \times (2.2 \times 3.3) \times \frac{2.2}{2} = 78.34 \text{ kN}$$

∴ *Horizontal reaction at M* = **78.34 kN (Ans.)**

• The *vertical component* of the resultant hydrostatic force is the weight of water supported by the curved surface LSN which represents a semicircle.

$$P_V = w \times \text{volume of surface LSN}$$

$$= w \times \left(\frac{\pi}{2} \times (\text{radius})^2 \times \text{length}\right)$$

$$= 9.81 \times \left[\frac{\pi}{2} \times (1.1)^2 \times 3.3\right] = 61.53 \text{ kN}$$

P_V is acting in the *upward direction*.

∴ For equilibrium of cylinder,

Vertical reaction at L = weight of cylinder $- P_V$

$= 165 - 61.53 =$ **103.47 kN (Ans.)**

Example 3.38. *Fig. 3.49 shows a radial gate. If it is 3 m long, find the magnitude and direction of the resultant force acting on it.*

Solution. Length of radial gate = 3 m

Refer Fig. 3.49.

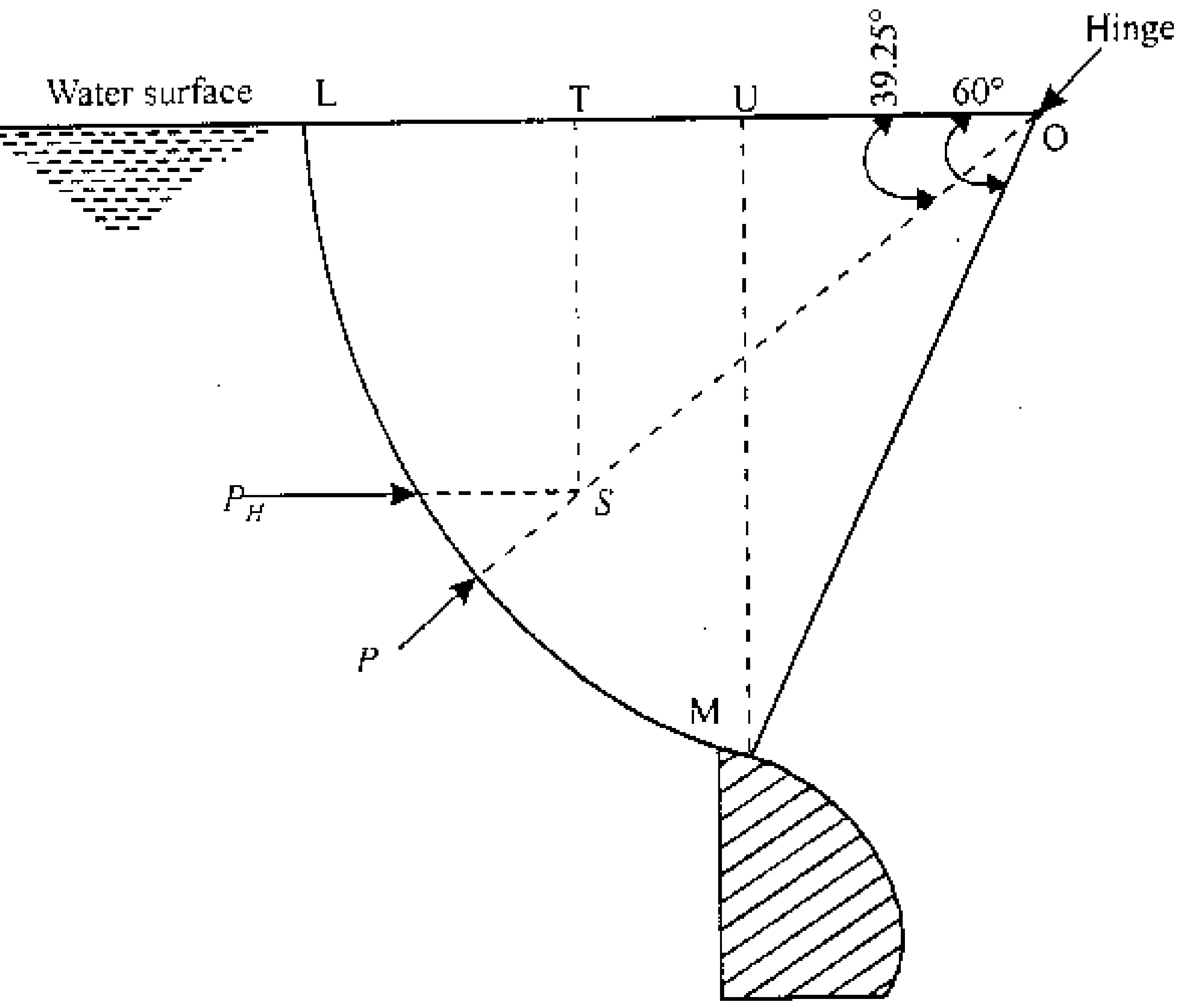


Fig. 3.49

$$MU = 3 \sin 60° = 2.6 \text{ m}$$

Horizontal force on the curved surface,

$$P_H = wA\bar{x}$$

$$= 9.81 \times (2.6 \times 3) \times \frac{2.6}{2}$$

$$= 99.47 \text{ kN}$$

It will act at $\frac{2.6}{3}$ or 0.867 m above *M*.

Vertical force, P_V = weight of water displaced

= weight of volume equal to $LMU \times 3$.

Now, Area LMU = area LOM – area MUO

$$= \pi R^2 \times \frac{60^0}{360^0} - \frac{1}{2} \times 2.6 \times 3 \cos 60°$$

$$= \pi \times 3^2 \times 1/6 - \frac{1}{2} \times 2.6 \times 3 \times 0.5 = 4.712 - 1.95 = 2.762 \text{ m}^2$$

$$P_V = 2.762 \times 3 \times 9.81 = 81.28 \text{ kN};$$

$$P = \sqrt{P_H^2 + P_V^2} = \sqrt{99.47^2 + 81.28^2} = 128.45 \text{ kN}$$

Hence magnitude of resultant force = **128.45 kN (Ans.)**

Let θ = Inclination of P with horizontal.

Then, $\tan\theta = \frac{P_V}{P_H} = \frac{81.28}{99.47} = 0.817$ or θ = **39.25° (Ans.)**

and *P must pass through* 0.

As P_H acts at (2.6 – 0.867) = 1.733 m below water surface,

$$OT = \frac{ST}{\tan 39.25°} = \frac{1.733}{0.817} = 2.12 \text{ m, and}$$

$$UT = OT - OU = 2.12 - 3 \cos 60° = 0.62 \text{ m}$$

Hence point of application of P is *0.62 m to the left of MU and 1.733 m below water surface.* **(Ans.)**

Example 3.39. *A cylinder having 3 m diameter and 1.5 m length is resting on the floor. On one side, water is filled upto half the depth while on the other side oil of relative density 0.8 filled upto the top (Fig 3.50). If the weight of the cylinder is 33.75 kN, determine the magnitudes of the horizontal and vertical components of the force which will keep the cylinder just touching the floor.*

Solution. *Given:* Diameter of the cylinder, d = 3 m; Length of the cylinder, l = 1.5 m

Weight of the cylinder; W = 30 kN; Relative density of the oil = 0.8

Specific weight of the oil, $w_{oil} = 9.81 \times 0.8 = 7.85 \text{ kN/m}^3$

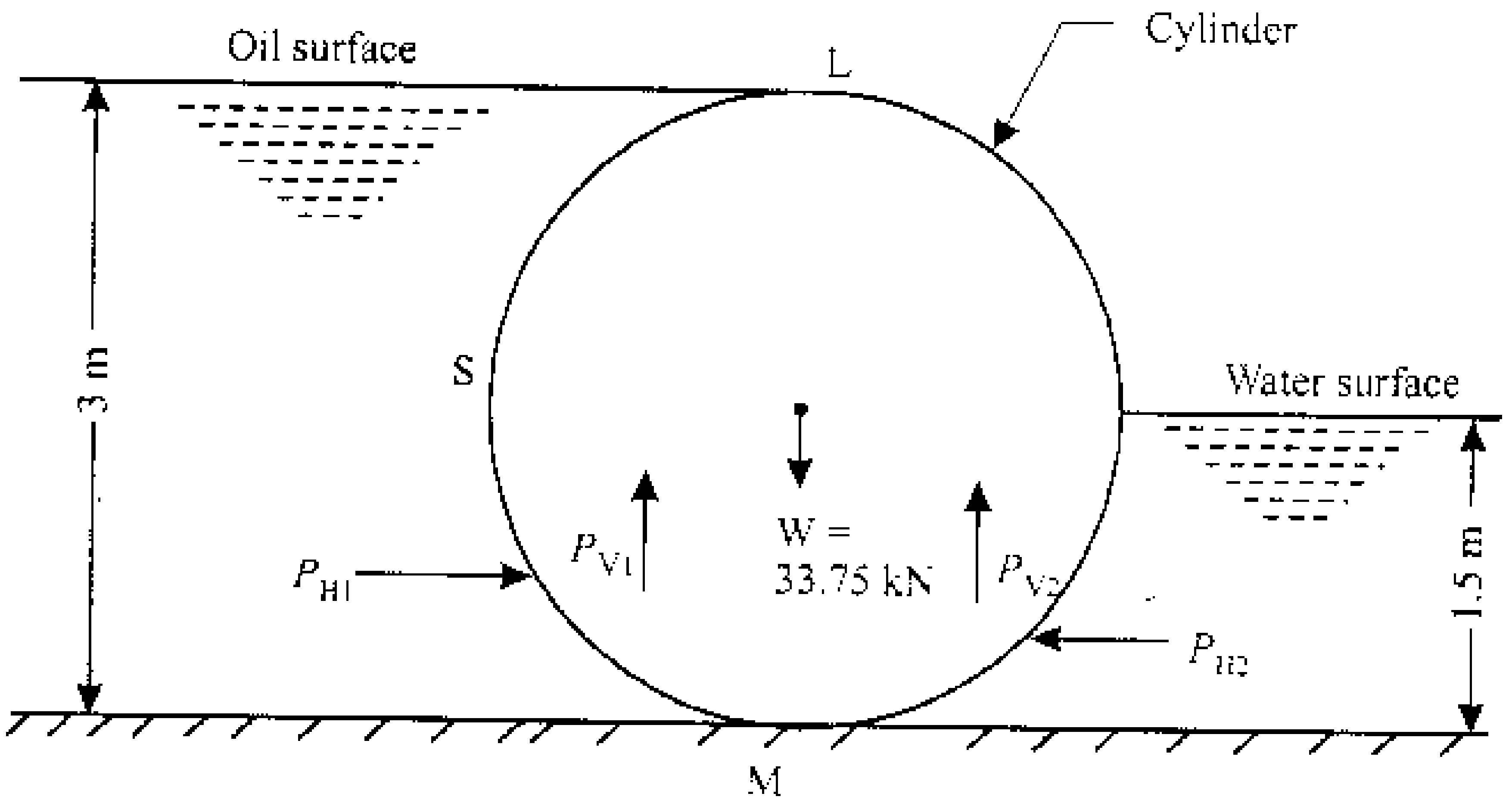


Fig. 3.50

Horizontal components: Horizontal force, $P_{H1} = 7.85 \times (3 \times 1.5) \times \frac{3}{2} = 52.98$ kN

This will act at $\frac{3}{3}$ or 1 m from bed.

Horizontal force, $P_{H2} = 9.81 \times (1.5 \times 1.5) \times \frac{1.5}{2} = 16.55$ kN

This will act at $\frac{1.5}{3}$ or 0.5 m from bottom.

Hence (52.98 – 16.55) = **36.43 kN** *force acting towards right is required to hold the cylinder stationary.* **(Ans.)**

If it acts at a distance y, then taking moments about the bed, we get

$$P_{H1} \times 1 - P_{H2} \times 0.5 = (P_{H1} - P_{H2}) \times y$$

$$52.98 - 16.55 \times 0.5 = (52.98 - 16.55) \times h_1$$

$$\therefore \quad y = \frac{52.98 - 16.55 \times 0.5}{52.98 - 16.55} = \frac{44.7}{36.43} = \mathbf{1.227\ m\ (Ans.)}$$

Vertical components:

$$P_{V1} = 7.85 \times \frac{\pi \times 1.5^2}{2} \times 1.5 = 41.61 \text{ kN}$$

It will act at $\frac{4 \times 1.5}{3\pi} = 0.636$ m to left of LM; $P_{V2} = 9.81 \times \frac{\pi \times 1.5^2}{4} \times 1.5 = 26$ kN

It will act at 0.636 m right of LM.

Since vertical forces must balance, therefore,

External force required = 41.61 + 26 – 33.75 = **33.86 kN (Ans.)**

This external force is required in *vertically downward* direction. To find out its line of action, taking moments about the vertical line along which P_{V2} acts, we get

$$W \times 0.636 + 33.86 \times x = P_{V1} \times (0.636 + 0.636)$$

$$33.75 \times 0.636 + 33.86\,x = 41.61 \times 1.272$$

$$x = \mathbf{0.929\ m\ (Ans.)}$$

Example 3.40. *A tank is filled with water under pressure and the pressure gauge fitted at the top indicates a pressure of 18 kPa. The tank measures 3 m perpendicular to the plane of the paper, and the curved surface LM of the top is quarter of a circular cylinder of radius 2.4 m. Determine:*

(*i*) *Horizontal and vertical components of water pressure on the curved surface LM, and*

(*ii*) *Magnitude and direction of the resultant force.*

Solution. Refer 3.51.

Pressure indicated by pressure gauge $p = 18$ kP $= 18 \times 10^3$ N/m^2

$\therefore$ The water head equivalent,

$$h = \frac{p}{w} = \frac{18 \times 10^3}{9810} = 1.835 \text{ m}$$

Hence the free water surface can be imagined to be 1.835 m above the top of the tank.

P_H (*Horizontal component*) = Hydrostatic pressure force on vertical projection MN or the curved surface LM

$$= wA\bar{x}$$

$$= 9.81 \times (2.4 \times 3) \times \left(1.835 + \frac{2.4}{2}\right)$$

$$= \mathbf{214.37\ kN \rightarrow (Ans.)}$$

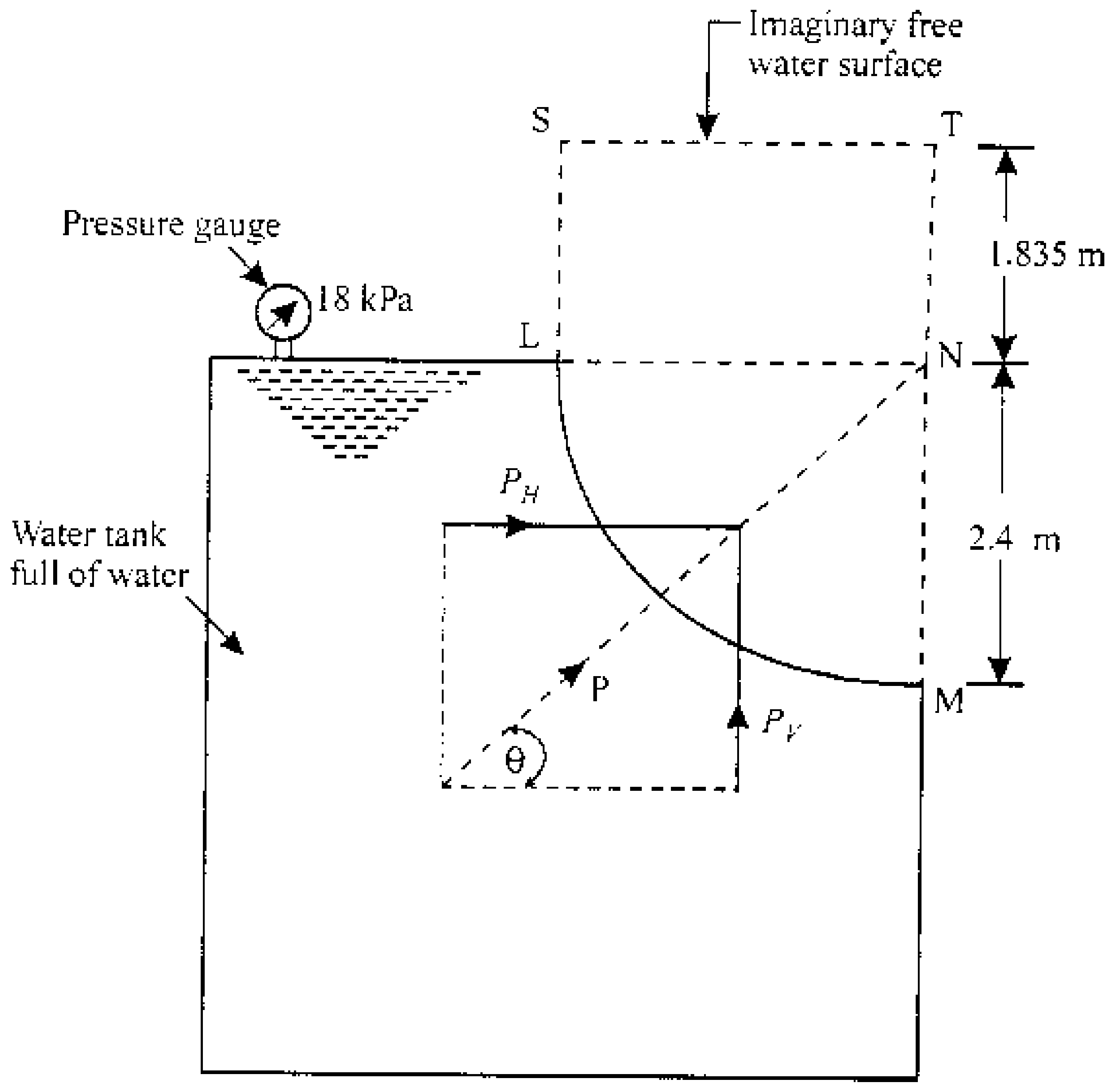


Fig. 3.51

P_V (*Vertical component*) = Weight of volume of water above *LM* upto imaginary water surface *i.e., of volume* SLMNT

$$= \left\{1.835 \times 2.4 + \frac{1}{4} \times \pi \times 2.4^2\right\} \times 3 \times 9.81 = \mathbf{262.75 kN} \uparrow \textbf{(Ans.)}$$

The resultant force $\quad \boldsymbol{P} = \sqrt{(P_H)^2 + (P_V)^2} = \sqrt{(214.37)^2 + (262.75)^2} = \mathbf{339.1\ kN\ (Ans.)}$

The inclination of P with the horizontal,

$$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) = \tan^{-1}\left(\frac{262.75}{214.37}\right) \simeq \mathbf{50.8° \ (Ans.)}$$

Example 3.41. *In the fig. 3.52. is shown the cross-section of the tank full of water under pressure. The length of the tank is* 3 m. *An empty cylinder lies along the length of the tank on one of its corners as shown. Find the horizontal and vertical components of the force acting on the curved surface LMN of the cylinder.*

Solution. Length of the tank = 3 m

Radius, r = 1.5 m

Pressure, p = 30 kN/m^2

Pressure head, $h \qquad p = \frac{p}{w} = \frac{30}{9.81} = 3\,\text{m}$

Free water surface will be at a height of 3 from the top of the tank; equivalent free water surface is shown in Fig. 3.52.

(*i*) **Horizontal component of force, P_H:**

$$P_H = wA\bar{x}$$

where, w = specific weight of water (= 9.81 kN/m³)

A = area projected on vertical plane
$= 2.25 \times 3 = 6.75 \text{ m}^2$

$$\bar{x} = 3 + \frac{2.25}{2} = 4.125 \text{ m}$$

$$P_H = 9.81 \times 6.75 \times 4.125$$

$= $ **273.15 kN (Ans.)**

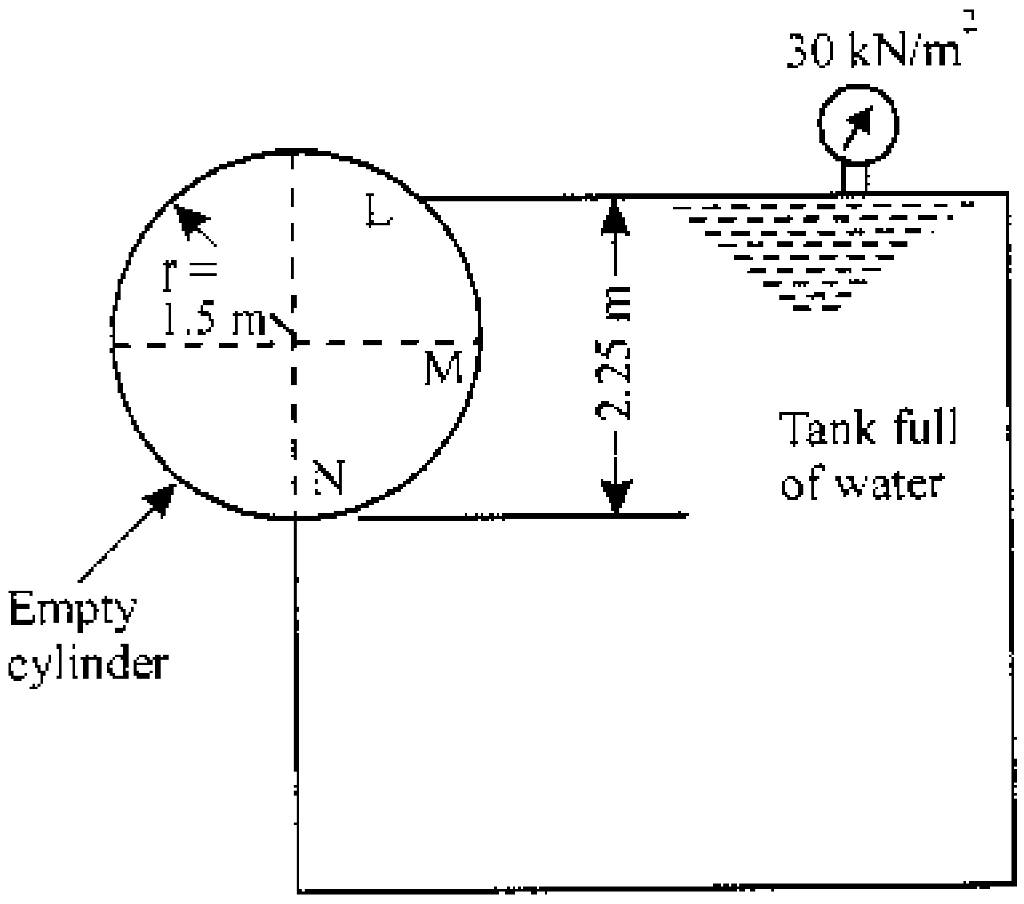


Fig. 3.52

(*ii*) **Vertical component of force, P_V:**

P_V = Weight of water enclosed or supported actually or imaginary by curved surface *LMN*

= Weight of water in the portion *NOTULMN*

= Weight of water in *NOTZMN* – weight of water in *LUZM*.

But weight of water in *NOTZMN* = weight of water in NOM + weight of water in *OMZTO*

$$= w\left(\frac{\pi r^2}{4} + OM \times MZ\right) \times 3$$

$$= 9.81\left(\frac{\pi \times 1.5^2}{4} + 1.5 \times 3.75\right) \times 3 = 217.5 \text{ kN}$$

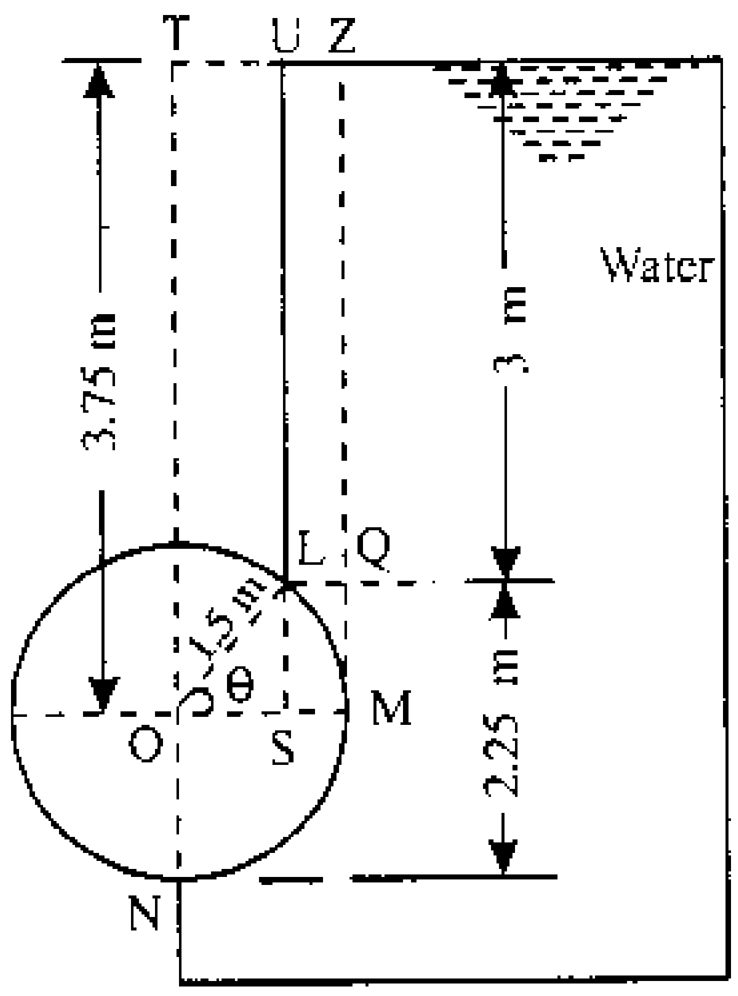


Fig. 3.53

Weight of water in *LUZM* = w (area of *LUZM*) × 3

= 9.81 [area of *LUZQ* + *LQMS* – *LSM*] × 3

In ΔLSO, $\sin\theta = \frac{LS}{OL} = \frac{0.75}{1.5} = 0.5$, $\therefore \theta = 30°$

$$MS = MO - SO = 1.5 - OL\cos\theta$$

$$= 1.5 - 1.5 \times \cos 30° = 0.2 \text{ m}$$

Area $LSM = LMO - LSO$

$$= \pi r^2 \times \frac{30°}{360°} - \frac{1}{2} \times OS \times LS$$

$$= \pi \times 1.5^2 \times \frac{1}{12} - \frac{1}{2} \times (1.5 \times \cos 30°) \times (1.5 \sin 30°)$$

$$= 0.589 - 0.487 = 0.102 \text{ m}$$

$\therefore$ Weight of water in *LUZM*

$= 9.81[\,LQ \times ZQ + LQ \times QM - 0.102\,] \times 3$

$= 9.81[\,0.2 \times 3 + 0.2 \times 1.5 \sin 30° - 0.102\,] \times 3$ ($\because LQ = MS$)

$= 9.81\,(0.6 + 0.15 - 0.102) \times 3$

$= 19.07$ kN

$P_V = 217.5 - 19.07 =$ **198.43 kN (Ans.)**

Example 3.42. *A cylindrical tank of 1.5 m diameter and height 0.75 m has a hemispherical dome. The tank contains oil of relative density 0.84 [Fig. 3.54]. The dame is joined to the cylinder portion by four equally spaced bolts. If the pressure gauge at a point M, 0.3 m, above the base of the tanks, reads 50 kPa determine the force on each bolt.*

Solution. Equivalent of pressure p_L in terms of oil column,

$$p_L = w_0\, h_L$$

$$50 = (0.84 \times 9.81) h_L$$

$$\therefore \quad h_L = \frac{50}{0.84 \times 9.81} = 6.07 \text{ m of oil}$$

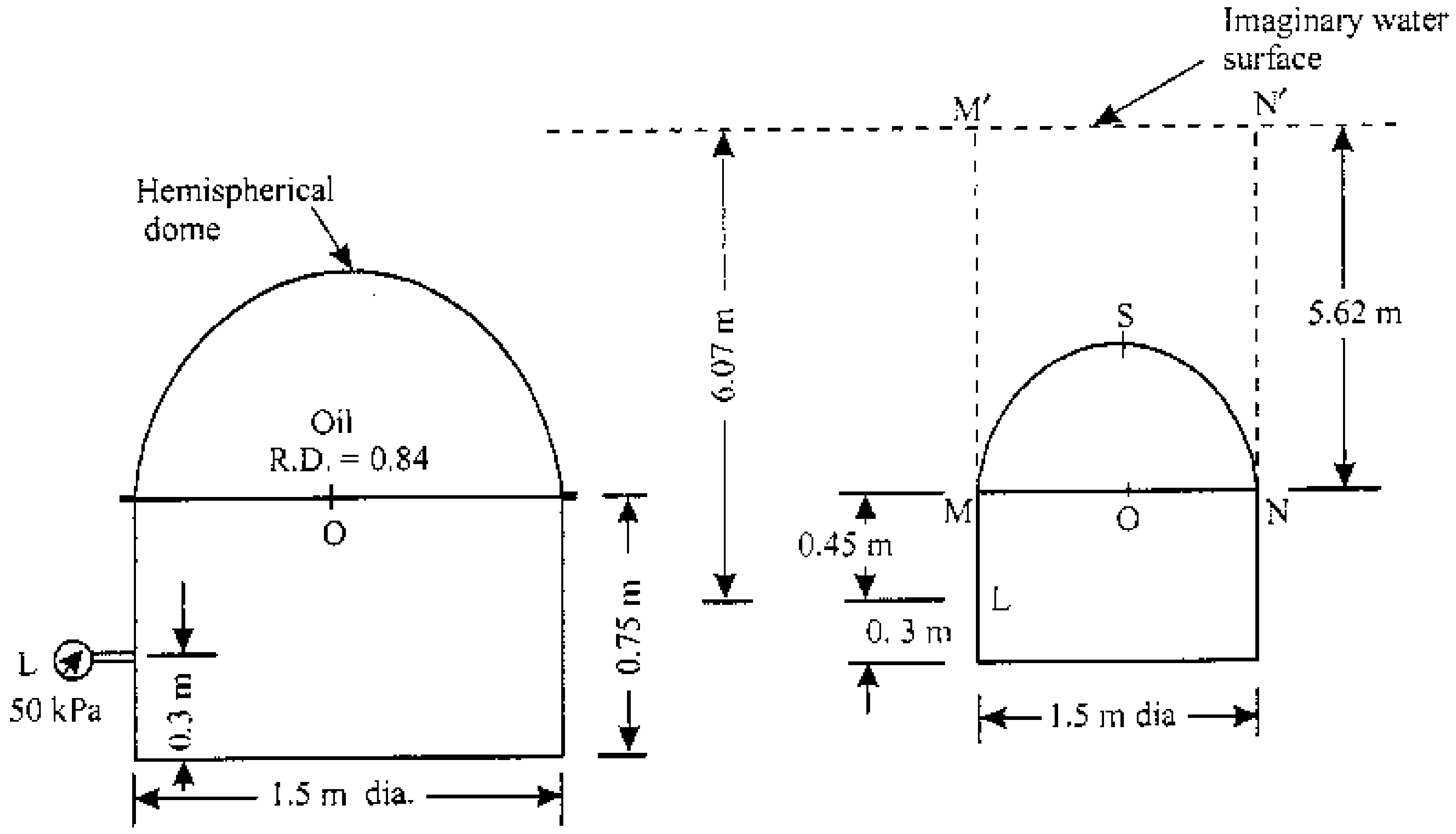


Fig. 3.54 **Fig. 3.55**

The imaginary oil surface at an elevation of h_L = 6.07 m is now considered (Fig. 3.55).

Above the base plane MN of the dome the elevation of the imaginary oil surface is

$$= 6.07 - 0.45 = 5.62 \text{ m}$$

By symmetry there is no horizontal force on the done.

The vertical force P_V = Weight of oil above the dome surface upto the imaginary oil surface

= Weight of volume MSNN′M′

$$= 0.84 \times 9.81 \left[\left\{ \frac{\pi \times (1.5)^2}{4} \times 5.62 \right\} - \left\{ \frac{1}{2} \times \frac{4}{3} \pi \,(0.75)^3 \right\} \right]$$

$$= 8.24\,(9.931 - 0.884) = 74.55 \text{ kN}$$

This force is shared by four bolts.

$\therefore$ Tensile force on each bolt = $\frac{74.55}{4}$ = **18.64 kN (Ans)**

7. Dams

A dam is a massive structure, built up mostly with R.C.C. or stone or earth, across a river or a eam for the purpose of impounding or storing water. Its cross-section may be triangular, rectangu-r or trapezoidal. That side of the dam to which the water from the river or the stream approaches is wn *upstream* and the other, *downstream*. A dam which resists the water pressure by its own ight only, is termed as a *gravity dam* (viz Bhakra dam).

Fig 3.56 shows the trapezoidal cross-section of the dam with a vertical face and a straight slope or batter for the back.

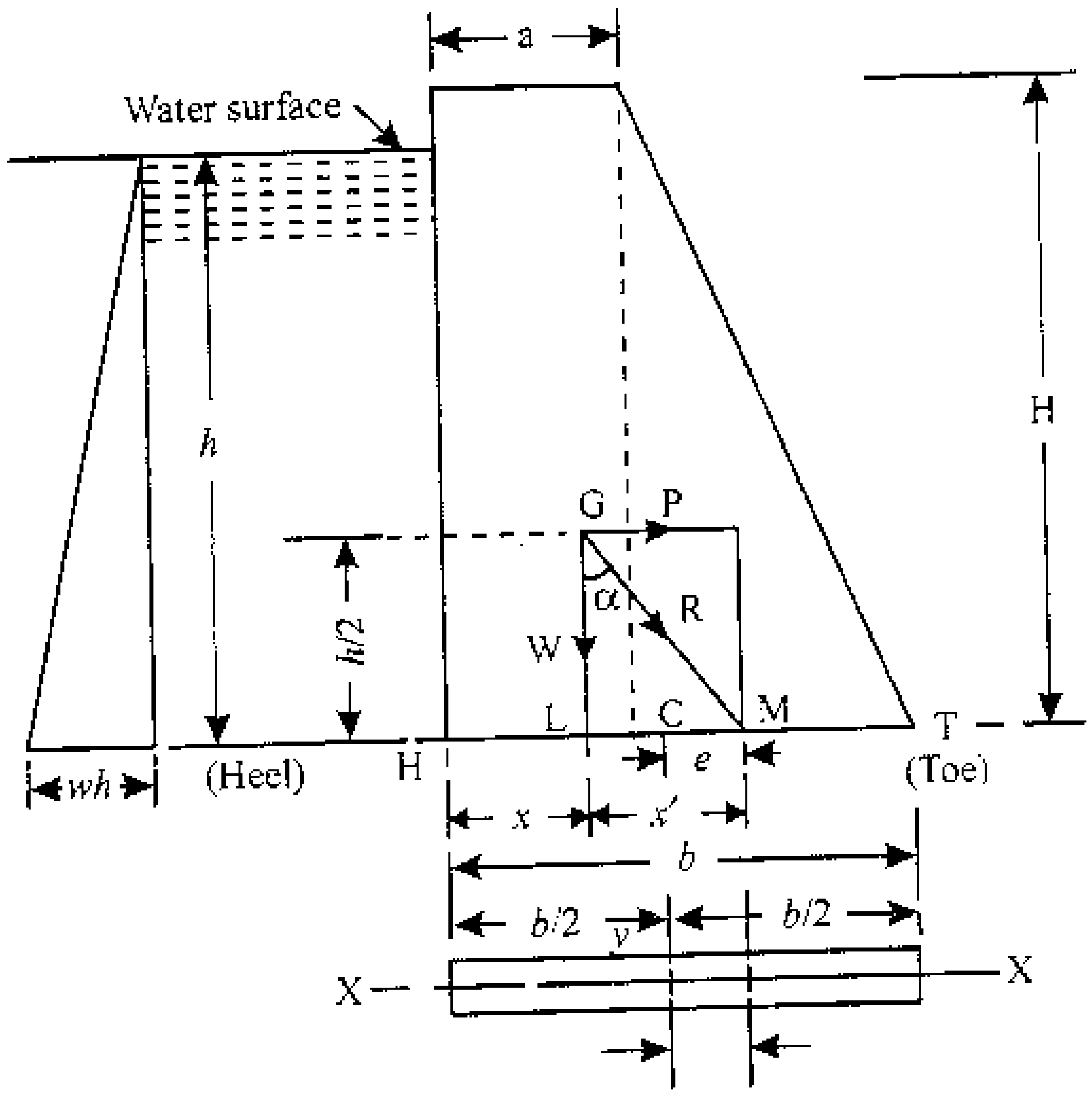


Fig. 3.56

Let, a = Top width of the dam,

b = Base width of the dam,

H = Height of the dam, and

h = Height of water column.

Consider 1 m length of the dam.

Weight of masonry = area × length × density of masonry

$$\therefore \quad W = \left(\frac{a+b}{2}\right) \times H \times 1 \times \text{density of masonry} \qquad \text{...(3.8)}$$

Let the *c.g.* of the section be at a distance x from the vertical face. Now dividing the trapezium into a rectangle and a triangle and taking moments about the vertical face, we get

$$a \times h \times \frac{a}{2} + \frac{1}{2}(b-a) \times h\left[a + \left(\frac{b-a}{3}\right)\right]$$

$$= \left[a \times h + \left(\frac{b-a}{2}\right) \times h\right]\bar{x}$$

$$\therefore \quad \bar{x} = \frac{a \times h \times \frac{a}{2} + \frac{1}{2}(b-a) \times h\left[a + \frac{b-a}{3}\right]}{a \times h + \left(\frac{b-a}{2}\right) \times h}$$

$$\text{or} \quad \bar{x} = \frac{a^2 + ab + b^2}{3(a+b)} \qquad \text{...(3.9)}$$

Total water pressure (P) = area × average pressure

$\therefore \quad P = (l \times h) \times \frac{wh}{2} = \frac{wh^2}{2}$

This pressure acts at $h/3$ from the base of dam. Let the resultant R of P and W cuts the base of the dam at the point M.

Then, from triangle GLM, we get $\tan\alpha = \frac{LM}{GL} = \frac{P}{W}$

i.e., $\frac{x'}{h/3} = \frac{P}{W}$ or $x' = \frac{P}{W} \cdot h/3$ ...(3.10)

The eccentricity of the resultant force, $e = (x + x') - b/2$ ...(3.11)

It e is +ve maximum stresses will develop towards the toe (T) and if it is –ve, maximum stresses will develop towards heel (H)

Stresses at the base:

Direct stress, $\sigma_d = \frac{\text{weight of masonry}}{\text{area at the base}} = \frac{W}{b \times 1} = \frac{W}{b}$ (compressive) ...(3.12)

Bending stress,

$$\sigma_b = \pm \frac{My}{I} = \frac{(W.e) \times b/2}{1/12 \times l \times b^3}$$ [Bending will take place about $Y - Y$ axis]

$$= \pm \frac{6\,W.e}{b^2}$$ (–*ve* sign stands for tensile stress here) ...(3.13)

Maximum intensity of stress, $\sigma_{max} = \sigma_d - \sigma_b$

$$= + \frac{W}{b} + \frac{6We}{b^2} = + \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$ (Compressive) ...(3.14)

Minimum intensity of stress,

$$\sigma_{min} = \sigma_d + (-\sigma_b) = \frac{W}{b} - \frac{6We}{b^2} = \frac{W}{b}\left(1 - \frac{6We}{b}\right)$$ (Compressive) ...(3.15)

It may be noted that σ_{min} may be tensile or compressive.

Possibilities of Dam Failure

The following are the *possibilities of dam failure*:

(*i*) Failure due to sliding along its base.
(*ii*) Failure due to tension or compression.
(*iii*) Failure due to shear at the weakest section.
(*iv*) Failure due to overturning.

(*i*) Failure due to sliding along its base:

The sliding of the dam is caused by the horizontal water pressure, P. The foundation offers frictional resistance which resists sliding. *The dam will be stable against sliding if the frictional resistance is more than the sliding (or driving) force P.*

Now, frictional force, $F = \mu W$

where μ is the co-efficient of friction between two adjacent surfaces along which sliding is likely to take place.

Sliding or driving force $= P$

Factor of safety against sliding $= \frac{\mu W}{P}$ (This value *must be greater than unity*).

(*ii*) Failure due to tension or compression:

The dam will be stable if no tensile stress across the cross-section is produced. It means σ_d should be more than or equal to σ_b. In certain cases where tension cannot be avoided it should not increase more than 0.4 N/mm^2.

i.e., $\sigma_d \geq \sigma_b$

$$\frac{W}{b} \geq \frac{6W.e}{b^2} \quad \text{or} \quad 1 \geq \frac{6 \times e}{b} \quad \text{or} \quad e \leq \frac{b}{6} \qquad ...(3.16)$$

Now, when $e = \dfrac{b}{6}$

$$HM = x + x' = \frac{b}{2} + e = \frac{b}{2} + \frac{b}{6} = \frac{2b}{3} \qquad ...(3.17)$$

Therefore the resultant must always be in the *middle third* of the base.

(*iii*) **Failure due to shear at the weakest section:**

If, A' = The least cross-sectional area of the dam at any section, and

$\sigma_{s(max)}$ = Maximum safe shear stress of the dam material,

Then, resistance against shear = $\sigma_{s(max)} \times A'$

Factor of safety against shear = $\dfrac{\sigma_{s(max)} \times A'}{P'}$ (It must be *greater than unity*)

where P' = total liquid pressure due to water column *above the section.*

(*iv*) **Failure due to overturning:**

Referring to Fig. 3.56 we find that water pressure tends to overturn the wall about the toe T whereas W tends to counteract the turning effect.

Taking moments of P and W about toe T, we get overturning moment = $P \times h/3$.

and resisting moment = $W(b - x)$

$$\text{Factor of safety against overturning} = \frac{W(b-x)}{P \times (h/3)}$$

$$= \frac{3W(b-x)}{Ph} \qquad \text{(it must be } \textit{greater than unity}\text{)}$$

Example 3.43. *A concrete dam of trapezoidal section having water on vertical face is 12 m high. The base of the dam is 8 metres wide and top 2 metres wide. Find the resultant thrust on the base per metre length of dam, and the point where it intersects the base. Take the specific weight of masonry as 240 kN/m³ and water level coinciding with the top of the dam.*

Solution. Refer Fig. 3.57

Top width = 2 m, base width = 8 m, height = 12 m

Consider 1 m length of dam.

Weight of masonry,

$$W = \left(\frac{8+2}{2}\right) \times 12 \times 1 \times 24 = 1440 \text{ kN}$$

Water pressure,

$$P = \frac{wh^2}{2} = \frac{9.81 \times 12^2}{2} = 706 \text{ kN}$$

Resultant thrust, $R = \sqrt{P^2 + W^2}$

$$= \sqrt{(706)^2 + (1440)^2} = \mathbf{1604\ kN\ (Ans.)}$$

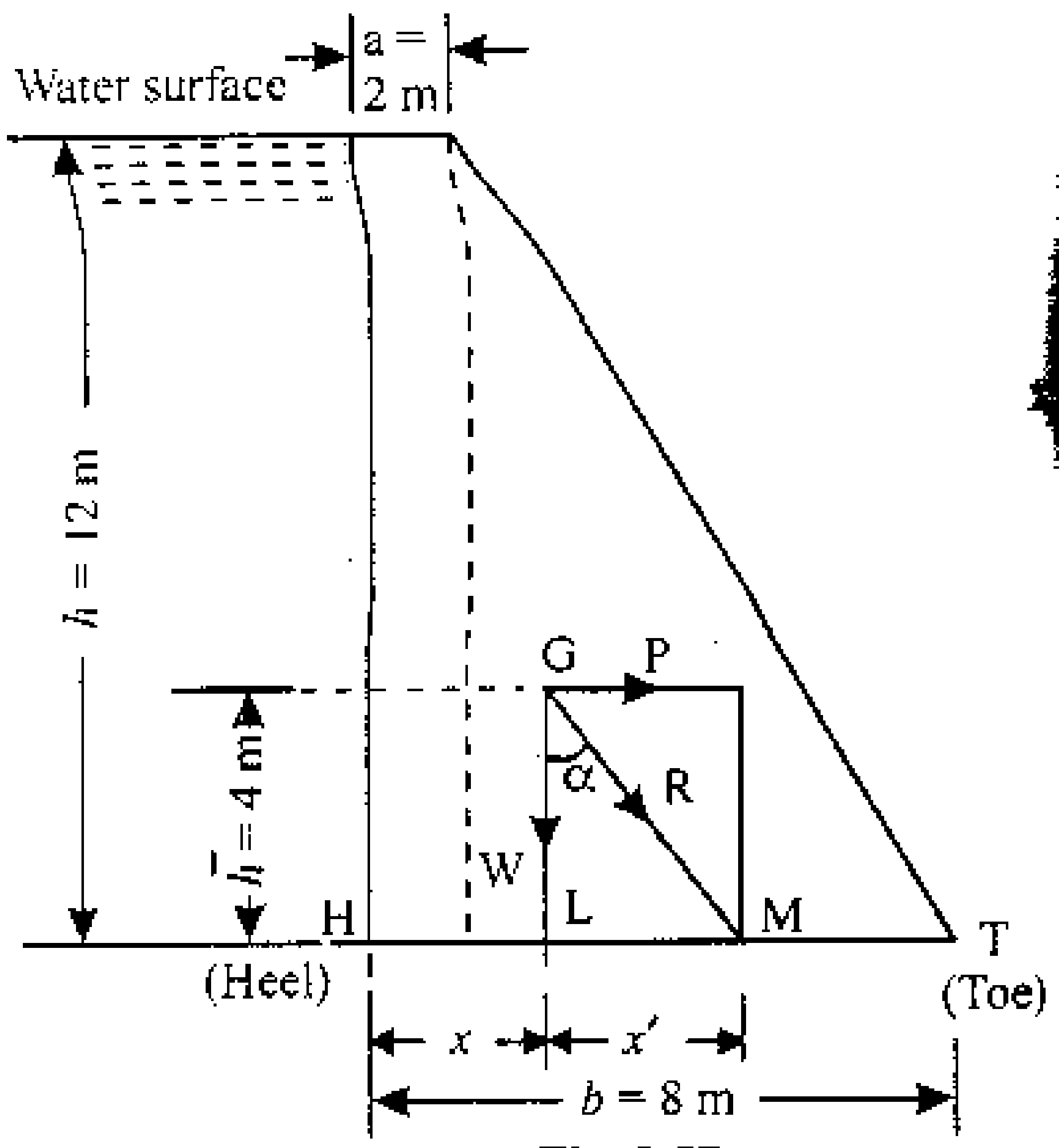


Fig. 3.57

The point, where the resultant thrust cuts the base:

Let x metres be the distance of $c.g.$ from the vertical face. Dividing the trapezium into rectangle and triangle and taking moments about the vertical face, we have

$$12 \times 2 \times \frac{2}{2} + \frac{1}{2} \times 12 \times 6 \left(2 + \frac{6}{3}\right)$$

$$= (12 \times 2 + \frac{1}{2} \times 12 \times 6) \times x$$

$$= 24 + 36 \times 4 = 60\,x$$

or $$x = \frac{24 + 36 \times 4}{60} = 2.8 \text{ m}$$

The value of 'x' can also be found by using the relation

$$x = \frac{a^2 + ab + b^2}{3(a + b)} = \frac{2^2 + 2 \times 8 + 8^2}{3(2 + 8)} = 2.8 \text{ m}$$

From Fig. 3.57, $\tan\alpha = \dfrac{P}{W} = \dfrac{706}{1440}$

Also, $$\tan\alpha = \frac{x'}{4} = \frac{706}{1440} \text{ or } x' = \frac{706 \times 4}{1440} = 1.96 \text{ m}$$

Now, the point where the resultant thrust cuts the base

$$= x + x' = 2.8 + 1.96 = \mathbf{4.76 \text{ m from H (heel) (Ans.)}}$$

Example 3.44. *A masonry dam trapezoidal in cross-section is 4 m wide at the top, 8 m wide at the base and 10 m high. It retains water level with top against a vertical face. Obtain stress distribution at the base if specific gravity of masonry is 2.5.*

Solution. Refer Fig. 3.58

Top width = 4 m; base width = 8 m; height = 10 m

Density of masonry = 2.5 × 9.81 = 24.5 kN/m³

Consider 1 m length of the dam.

Weight of masonry acting through $c.g.$,

$$W = \left(\frac{8 + 4}{2}\right) \times 10 \times 1 \times 24.5 = 1470 \text{ kN}$$

Water pressure,

$$P = \frac{wh^2}{2} = \frac{9.81 \times 10^2}{2} = 490.5 \text{ kN}$$

Let x metres be the distance of $c.g.$ from the vertical face. Dividing the trapezium into a rectangle and a triangle and taking moments about the vertical face, we have

$$10 \times 4 \times \frac{4}{2} + \frac{1}{2} \times 10 \times 4 \left(4 + \frac{4}{3}\right)$$

$$= \left(10 \times 4 + \frac{1}{2} \times 10 \times 4\right) \times x$$

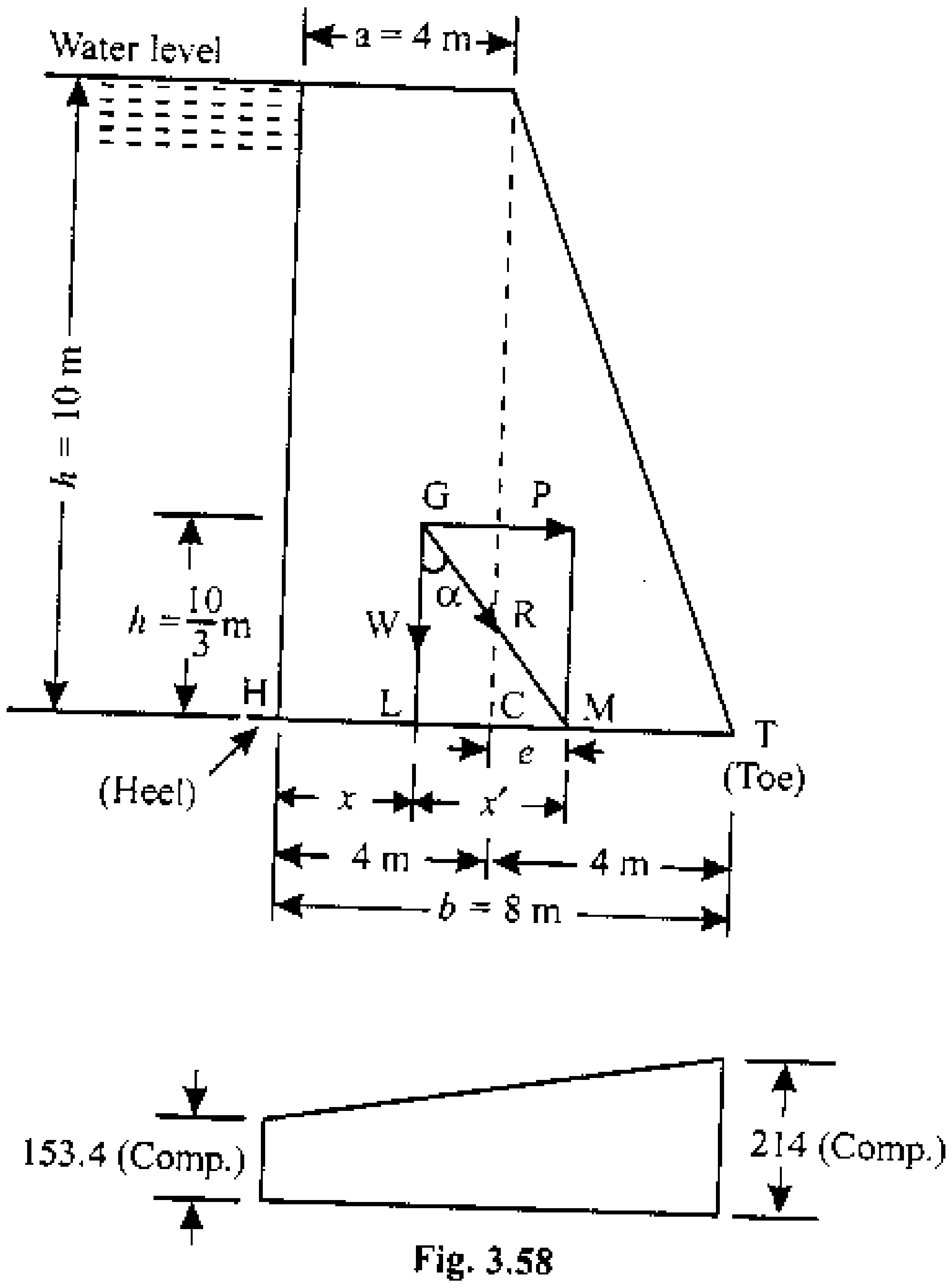


Fig. 3.58

or, $$80 + 20 \times 5.333 = 60x$$

$$x = \frac{80 + 20 \times 5.33}{60} = 3.11 \text{ m}$$

From Fig. 3.58.,

$$\tan\alpha = \frac{P}{W} = \frac{490.5}{1470}$$

Also, $$\tan\alpha = \frac{x'}{10/3}$$

$\therefore$ $$\frac{x'}{10/3} = \frac{490.5}{1470} = \frac{1}{3}$$

or, $$x' = \frac{1}{3} \times \frac{10}{3} = \frac{10}{9} = 1.11 \text{ m}$$

Now, $$x + x' = 3.11 + 1.11$$

$= 4.22$ *m* which is well *within two third base width.*

Eccentricity of resultant thrust,

$$e = (x + x') - 4 = 4.22 - 4 = 0.22 \text{ m}$$

Bending stress,

$$\sigma_b = \frac{M}{I} y = \frac{(W.e).y}{I}$$

$$= \frac{1470 \times 0.22 \times 8/2}{\frac{1 \times 8^3}{12}} = \frac{1470 \times 0.22 \times 4 \times 12}{8^3} = \pm\, 30.3 \text{ kN/m}^2$$

Direct stress,

$$\sigma_d = \frac{\text{weight of masonry}}{\text{area of base}} = \frac{1470}{8 \times 1} = 183.7 \text{ kN/m}^2$$

$\sigma_{max} = \sigma_d + \sigma_b = 183.7 + 30.3 =$ **214 kN/m² (comp.) (Ans.)**

$\sigma_{min} = \sigma_d - \sigma_b = 183.7 - 30.3 =$ **153.4 kN/m² (comp.) (Ans.)**

Example 3.45. *A masonry weir is of trapezoidal cross-section with a top width of 2.0 m and of height 5m with upstream slope of 1 vertical in 0.1 horizontal and a downstream slope of 1 vertical in 0.75 horizontal. If the weir has water stored upto its crest on the upstream side and has a tail water of 2 m depth on the downstream, calculate, per unit length of weir,*

(*i*) *the resultant force on the base of the weir.*

(*ii*) *the minimum and maximum stresses on the base of the weir.*

Assume specific weight of masonry as 22 kN/m³ and neglect uplift forces.

Solution. Refer Fig. 3.59.

Consider 1 *m* length of the weir.

(*i*) **Resultant force on the base:**

The forces acting on the weir are:

(*i*) Weight of masonry, $W = W_1 + W_2 + W_3$

(*ii*) Vertical force due to water on the upstream slope, P_{V1}

(*iii*) Vertical force due to tail water on the downstream slope, P_{V2}

(*iv*) Horizontal force due to upstream side, P_{H1}

(*v*) Horizontal water force on the downstream side, P_{H2}

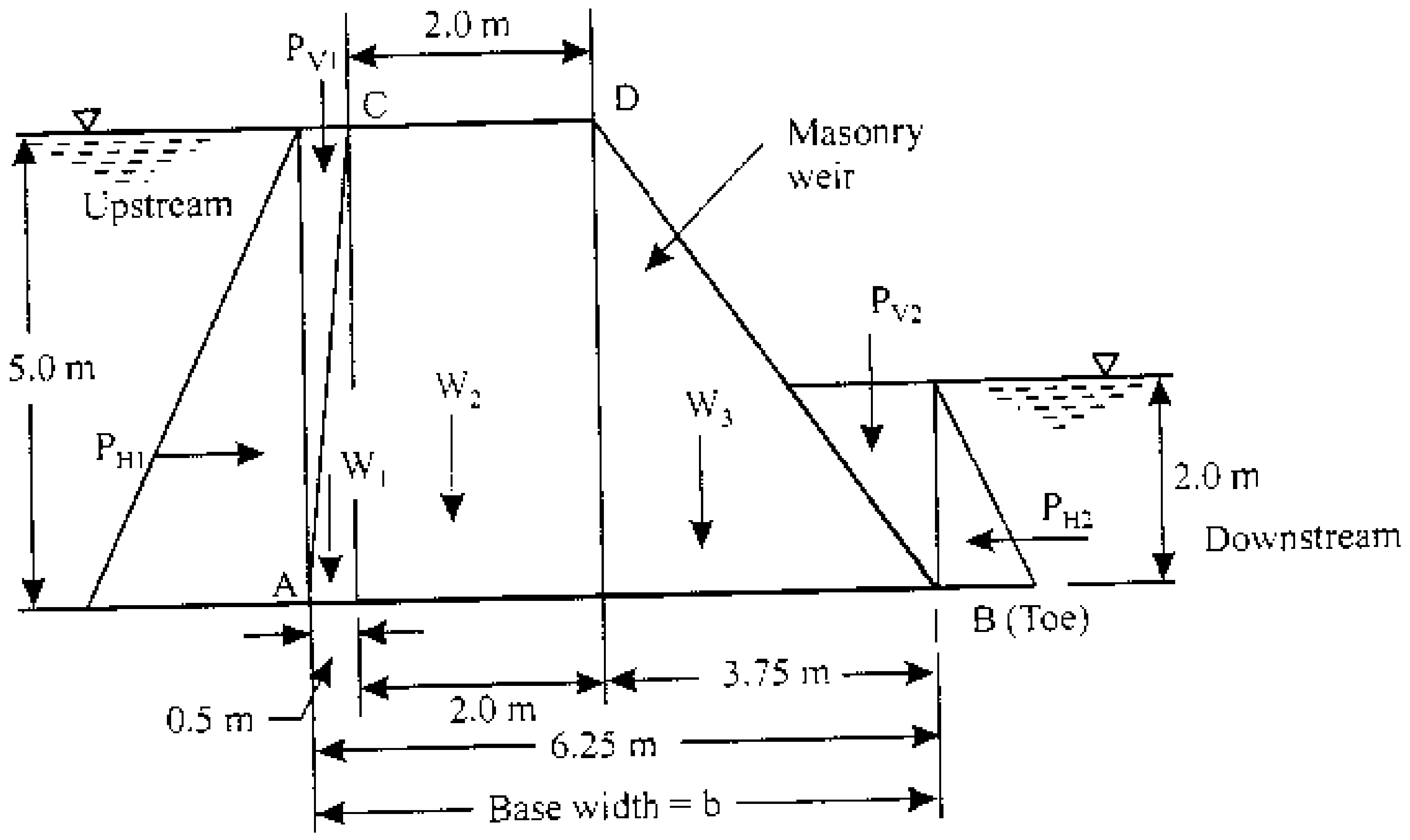


Fig. 3.59

The magnitudes of these forces, their distances from the toe of the weir (edge B) and the moments of these forces about B are tabulated in the table below:

Force	*Description Magnitude, kN*	*Magnitude, kN* Horiz. force	*Magnitude, kN* Vert. force	*Lever arm about B (m)*	*Moment (Clockwise)*	*Moment (Anticlockwise)*
W_1	$(0.5 \times 5) \times \frac{1}{2} \times 1 \times 22$		27.5	5.917		162.7
W_2	$(2.0 \times 5.0) \times 10 \times 22$		220.0	4.75		1045.0
W_3	$(3.75 \times 5) \times \frac{1}{2} \times 1 \times 22$		206.25	2.50		515.6
P_{V1}	$(0.5 \times 5) \times \frac{1}{2} \times 1 \times 9.81$		12.26	6.083		74.6
P_{V2}	$(1.5 \times 2.0) \times \frac{1}{2} \times 1 \times 9.81$		14.71	0.50		7.36
P_{H1}	$(5 \times 1) \times \frac{5}{2} \times 9.81$	122.62		1.66	204.4	
P_{H2}	$(2 \times 1) \times \frac{2}{2} \times 9.81$	– 19.62		0.66		13.08
Sum		**103**	**480.72**		**204.4**	**1818.34**

Sum of vertical forces, $\Sigma V = 480.72$ kN

Sum of horizontal forces, $\Sigma H = 103$ kN

Resultant,
$$R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$
$$= \sqrt{(480.72)^2 + (103)^2} = 491.63 \text{ kN (Ans.)}$$

If θ is the *inclination* of the resultant to horizontal, then

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{480.72}{103} = 4.667$$

or, $$\theta = \tan^{-1}(4.667) = \mathbf{77.9°}\ \textbf{(Ans.)}$$

(*ii*) **The minimum and maximum stresses; σ_1, σ_2:**

$$\Sigma M = 1818.34 - 204.4 = 1613.94 \text{ kNm}$$

x = Distance of point of action of the resultant from B

$$= \frac{\Sigma M}{\Sigma V} = \frac{1613.94}{480.72} = 3.357 \text{ m}$$

As b = base width = 6.25 m, the

$$\text{Eccentricity, } e = x - \frac{b}{2} = 3.357 - \frac{6.25}{2} = 0.232 \text{ m}$$

Maximum and minimum stresses,

$$\sigma_{1,2} = \frac{\Sigma V}{b}\left(1 \pm \frac{6e}{b}\right)$$

$$= \frac{480.72}{6.25}\left(1 \pm \frac{6 \times 0.232}{6.25}\right)$$

∴ $$\sigma_1 = \mathbf{94.04\ kN/m^2}\ \textbf{(Ans.)}$$

$$\sigma_2 = \mathbf{59.78\ kN/m^2}\ \textbf{(Ans.)}$$

Example 3.46. *The curved face of a dam is shaped according to the relation* $y = \frac{x^2}{12.25}$ *as shown in the Fig. 2.60. If the width of the dam is unity and height of water retained by the dam is 12 m determine the magnitude and direction of the resultant water pressure acting on the curved face of the dam.*

Solution. Profile of the curved face of the dam is,

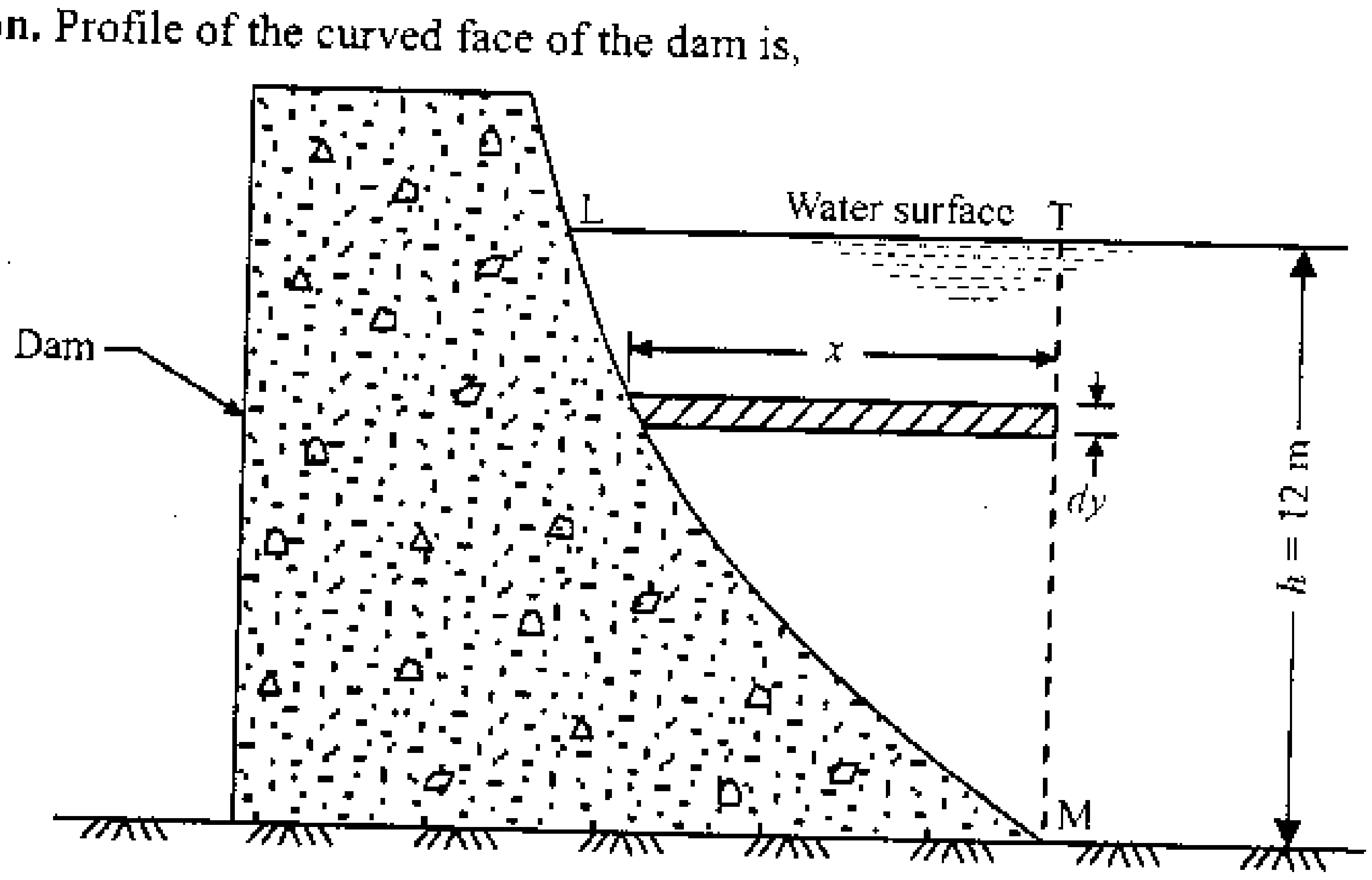


Fig. 3.60

$$y = \frac{x^2}{12.25}$$

or $$x^2 = 12.25\, y$$

$$x = 3.5\sqrt{y}$$

Height of water,

$h = 12$ m

Width, $b = 1$m

Magnitude and direction of resultant water pressure:

Horizontal component, P_H = Pressure due to water on curved area projected on vertical plane

= Pressure on area $MT = wA\bar{x}$

where, $A = MT \times 1 = 12 \times 1 = 12$ m^2

$x = 12/2 = 6$ m

$\therefore$ $P_H = 9.81 \times 12 \times 6 = 706.3$ kN

Vertical component, P_V = weight of water supported by the curve LM

= weight of water in portion LMT

$= w \times$ (Area of LMT) $\times$ width of dam $= w\left[\int_0^{12} x \, . \, dy\right] \times 1.0$

where, $x \, . \, dy$ = area of strip $\quad (\because x = 3.5\sqrt{y})$

$$= 9.81 \int_0^{12} 3.5\sqrt{y}\, dy$$

$$= 9.81 \times 3.5\left[\frac{y^{3/2}}{3/2}\right]_0^{12} = 34.33 \times 2/3\ [(12)^{3/2}] = 951.4 \text{ kN}$$

Resultant water pressure on the dam,

$$P = \sqrt{P_H^2 + P_V^2} = \sqrt{(706.3)^2 + (951.4)^2}$$

or P = **1184.9 kN (Ans.)**

Direction of the resultant is given by

$$\tan\alpha = \frac{P_V}{P_H} = \frac{951.4}{706.3} = 1.347$$

$\therefore$ α = **53.4° (Ans.)**

Example 3.47. *Fig. 3.61 shows a gate whose profile is given by* $x = \sqrt{y}$. *It holds water to a depth of 1 m behind it. If the width of the gate is 5 m, determine the moment M required to hold the gate in place.*

Solution. Profile of the gate: $x = \sqrt{y}$

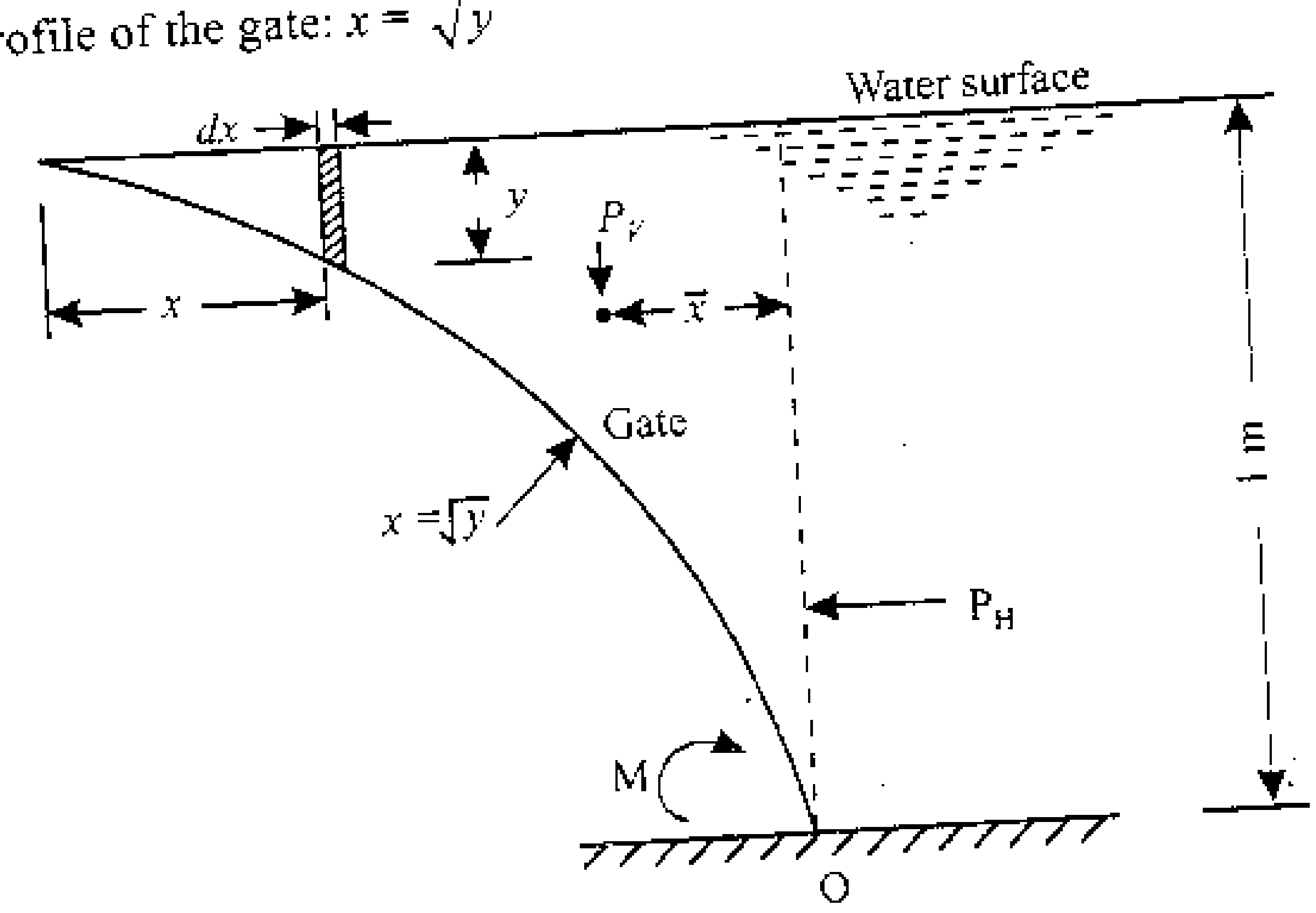


Fig. 3.61

Horizontal force, $P_H = wA\,\bar{x} = 9.81 \times (1 \times 5) \times \frac{1}{2} = 24.25$ kN

It will act at $\frac{1}{3}$ m or 0.333 m from the bottom.

P_V = weight of liquid above the gate

$$= 5 \times w \int y\,dx$$

$$= 5w \int_0^1 x^2\,dx \quad \left[\because x = \sqrt{y} \text{ or } y = x^2\right]$$

$$= 5 \times 9.81 \left|\frac{x^3}{3}\right|_0^1$$

$$= 5 \times 9.81 \times \frac{1}{3} = 16.35 \text{ kN}$$

The vertical line along which P_V will act is obtained by taking moments of elementary force $5\,wy.dx$ about Y-axis and equating it to $P_V \times \bar{x}$.

i.e.
$$P_V \times \bar{x} = \int_0^1 5\,wy\,(1-x)dx = \int_0^1 5wx^2(1-x)dx$$

$$= 5w\left|\frac{x^3}{3} - \frac{x^4}{4}\right|_0^1 = 5w\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{5w}{12}$$

$\therefore$
$$\bar{x} = \frac{5w}{12 \times P_V} = \frac{5 \times 9.81}{12 \times 16.35} = 0.25 \text{ m}$$

Moment M:

Moment of P_H and P_V about Z- axis passing through 0 is

$$M = P_H \times 0.333 - P_V \times 0.25$$

$$= 24.52 \times 0.333 + 16.35 \times 0.25 = 12.25 \text{ kNm}$$

i.e. M = **12.25 kNm (Ans.)**

Example 3.48. *A dam has a parabolic shape* $y = y_0\left(\frac{x}{x_0}\right)^2$ *as shown in Fig 3.62 having* x_0 = *6 m and* y_0 = *9 m. The fluid is water with density = 1000 kg/m*^{3}*. Compute the horizontal, vertical and the resultant thrust exerted by water per metre length of the dam.*

Solution. *Given*: Width of the dam = 1 m;

Equation of the curve: $y = y_0\left(\frac{x}{x_0}\right)^2$; x_0 = 6m; y_0 = 9m

Density of water ρ = 1000 kg/m^3

Horizontal, vertical and resultant thrust exerted by water:

Equation of the curve OL : $y = y_0\left(\frac{x}{x_0}\right)^2$

$$= 9\left(\frac{x}{6}\right)^2 = 9 \times \frac{x^2}{36} = \frac{x^2}{4}$$

or
$$x^2 = 4y \text{ or } x = 2\sqrt{y}$$

Horizontal thrust:

Horizontal thrust exerted by water,

F_x = force exerted by water on vertical surface OM; the surface obtained by projecting the curved face on vertical plane.

$$= \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times (9 \times 1) \times \frac{9}{2} = 397305 \text{ N}$$

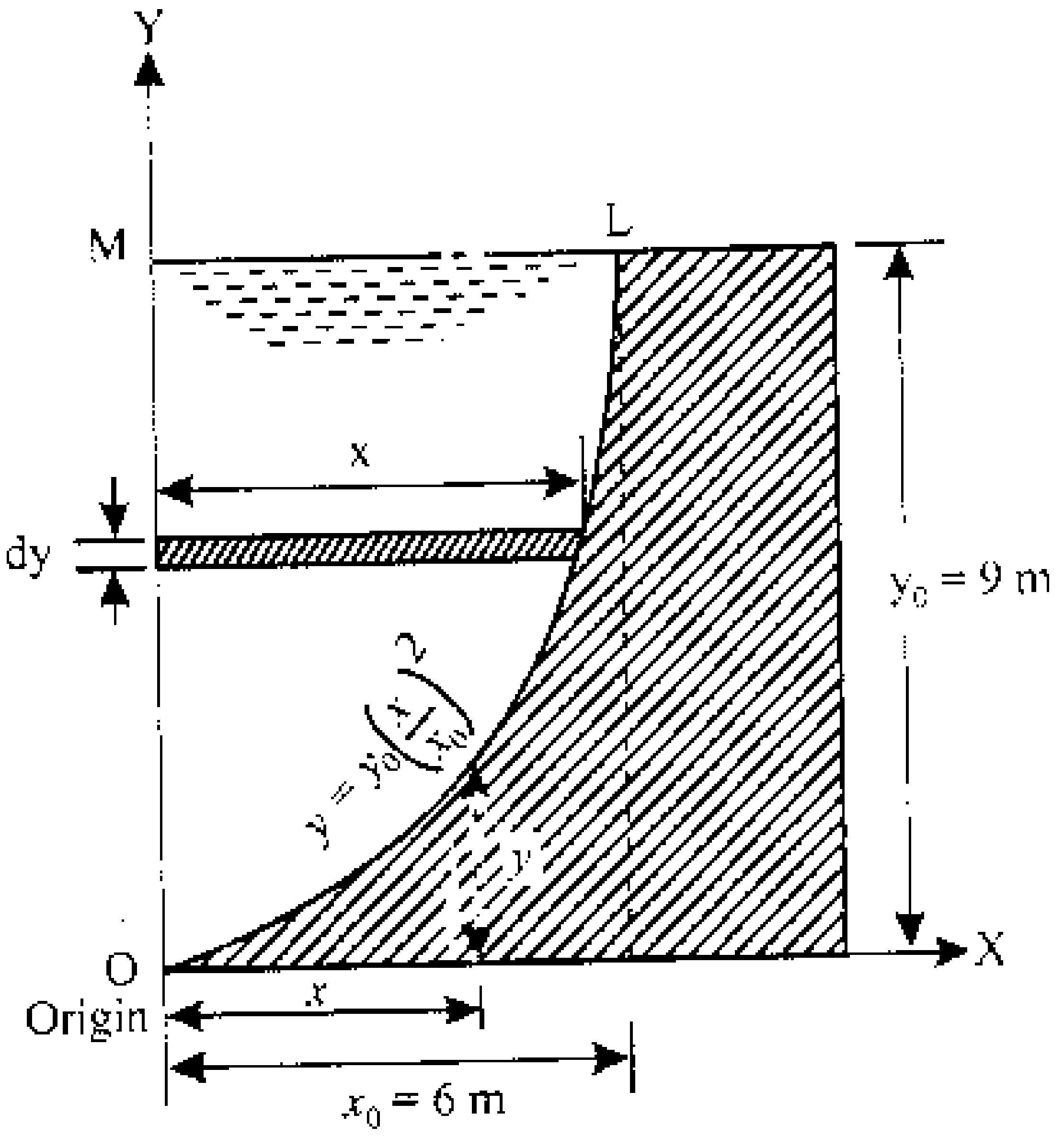
or **397.3 kN (Ans.)**

Vertical thrust:

Vertical thrust exerted by water,

F_y = weight of water supported by curved face OL upto free water surface

= weight of water in portion OLM × width dam



$$\rho g \times \text{area of OLM} \times \text{width of dam}$$

$$= 1000 \times 9.81 \times \left[\int_0^9 x \times dy\right] \times 1.0$$

$$= 1000 \times 9.81 \times \left[\int_0^9 2\sqrt{y} \times dy\right] \times 1.0$$

$$= 19620 \times \left[\frac{y^{3/2}}{3/2}\right]_0^9 = 19620 \times \frac{2}{3} \times (9)^{3/2}$$

= 353160 N or **353.16 kN (Ans.)**

Resultant thrust:

Resultant thrust exerted by water,

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(397.3)^2 + (353.16)^2} = \mathbf{531.57\ kN\ (Ans.)}$$

Direction of the resultant is given by,

$$\tan\theta = \frac{F_y}{F_x} = \frac{353.16}{397.3} = 0.889$$

$$\therefore \quad \theta = \tan^{-1}(0.889) = \mathbf{41.63^\circ\ (Ans.)}$$

Lock Gates

Lock gates *are provided in navigation chambers to change the water level in a canal or river navigation.* There are two sets of gates G_1 and G_2, one set on either side of the chamber. The working of the gates (Fig.3.63) is as follows:

Suppose the ship is at position 1 (on the left hand side of the chamber) and it is to be transferred to position 2 (on the right hand side). To do so the following *procedure* is adopted:

(i) Open the sluice S_1 on the upstream gate G_1 and fill the chamber upto level *L–L*.

(ii) Open the lock gate G_1 on the upstream and permit the ship to enter the chamber.

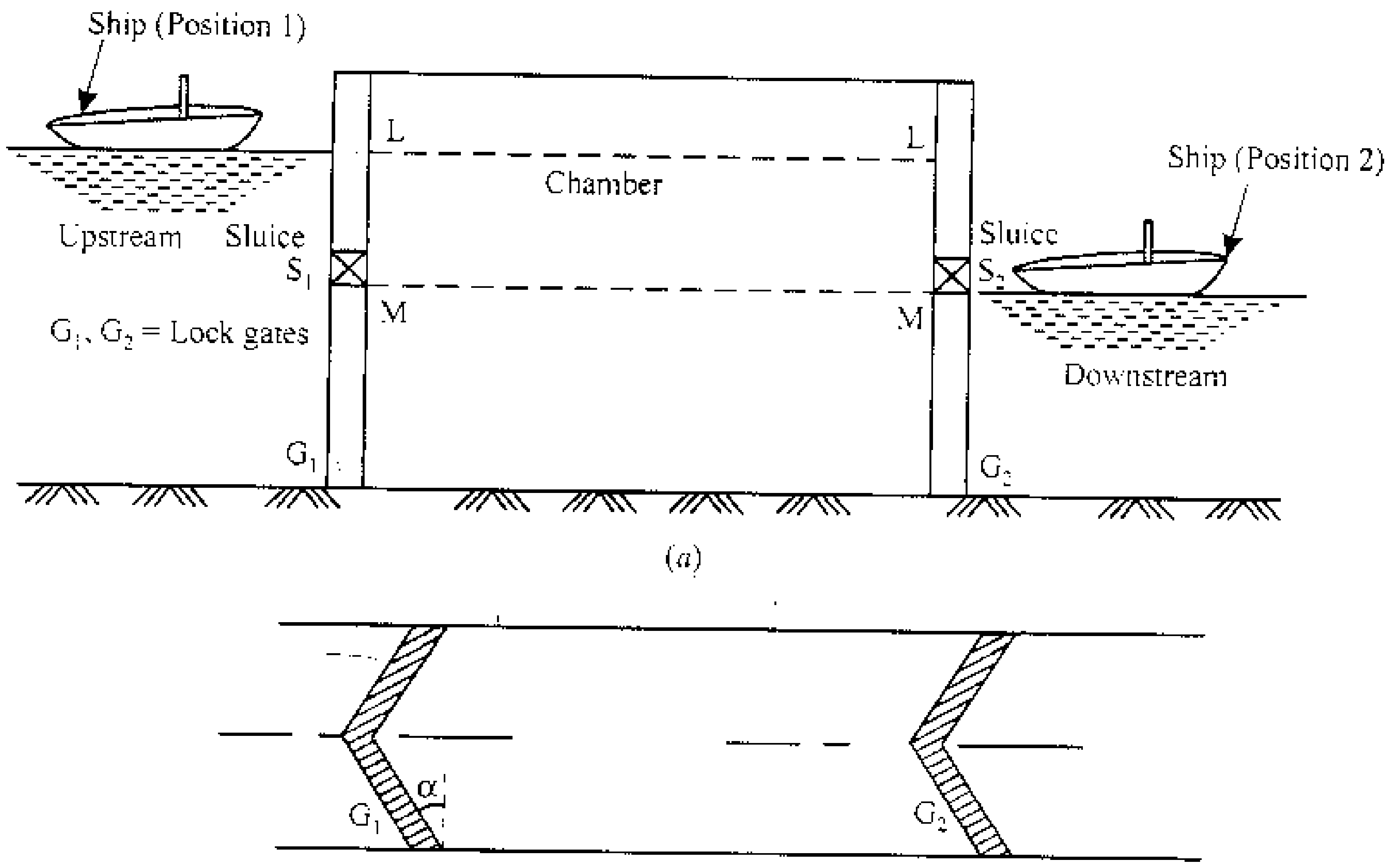
(iii) Close the gate G_1.

(iv) Open the sluice S_2 and allow the water to fall to level *MM*.

(v) Open the downstream gate G_2 and permit the ship to leave the chamber.

(vi) Reverse the procedure in case the ship is to be transferred from position 2 to position 1.

Total pressure on the gates and reactions at top and bottom hinges:



(*a*)

(*b*)

Fig. 3.63

Fig. 3.64 shows plan and elevation of a pair of lock gates. Let *AB* and *BC* be two lock gates, eac carried on two hinges fixed on their top and bottom at both *A* and *C*. Due to action of water, the gates are tightly closed to one another at *B*.

Consider the gate AB:

Let,
P = Resultant force due to water acting at right angle to the gate,
N = Reaction force supplied by gate *BC* to gate *AB* and acting perpendicular t the contact surfaces,
R = Resultant reaction of the top and bottom hinges (assumed to lie in the same horizontal plane in which P and N lies), and
α = Angle of inclination of gate to normal side of lock.

As the gate is in equilibrium under the forces, P, N and R, they will all intersect at one point. L P and N intersect at D; then R must pass through this point. Then triangle ABD will be isosceles, $\angle DBA$ and $\angle DAB$ equal α.

Resolving the forces in a direction parallel to gate (AB),

$$R \cos \alpha = N \cos \alpha \qquad \therefore R = N \qquad \text{...(3.1}$$

Resolving normal to the gate (AB), $P = R \sin \alpha + N \sin \alpha = (R + N) \sin \alpha = 2R \sin \alpha$

$$\therefore \qquad R = \frac{P}{2 \sin \alpha} \qquad \text{...(3.1}$$

(Also, inclination of R to centre line of gate = α)

Consider water pressure on the gate.

Let,
H_1 = Height of water to left of gate (*i.e.* upstream side),
H_2 = Height of water to right of gate (*i.e.* downstream side),
H = Height of top hinge from the bottom of gate,
P_1 = Total pressure of water to left of gate,
P_2 = Total pressure of water to right of gate.

R_t = Reaction of top hinge, and

R_b = Reaction of bottom hinge.

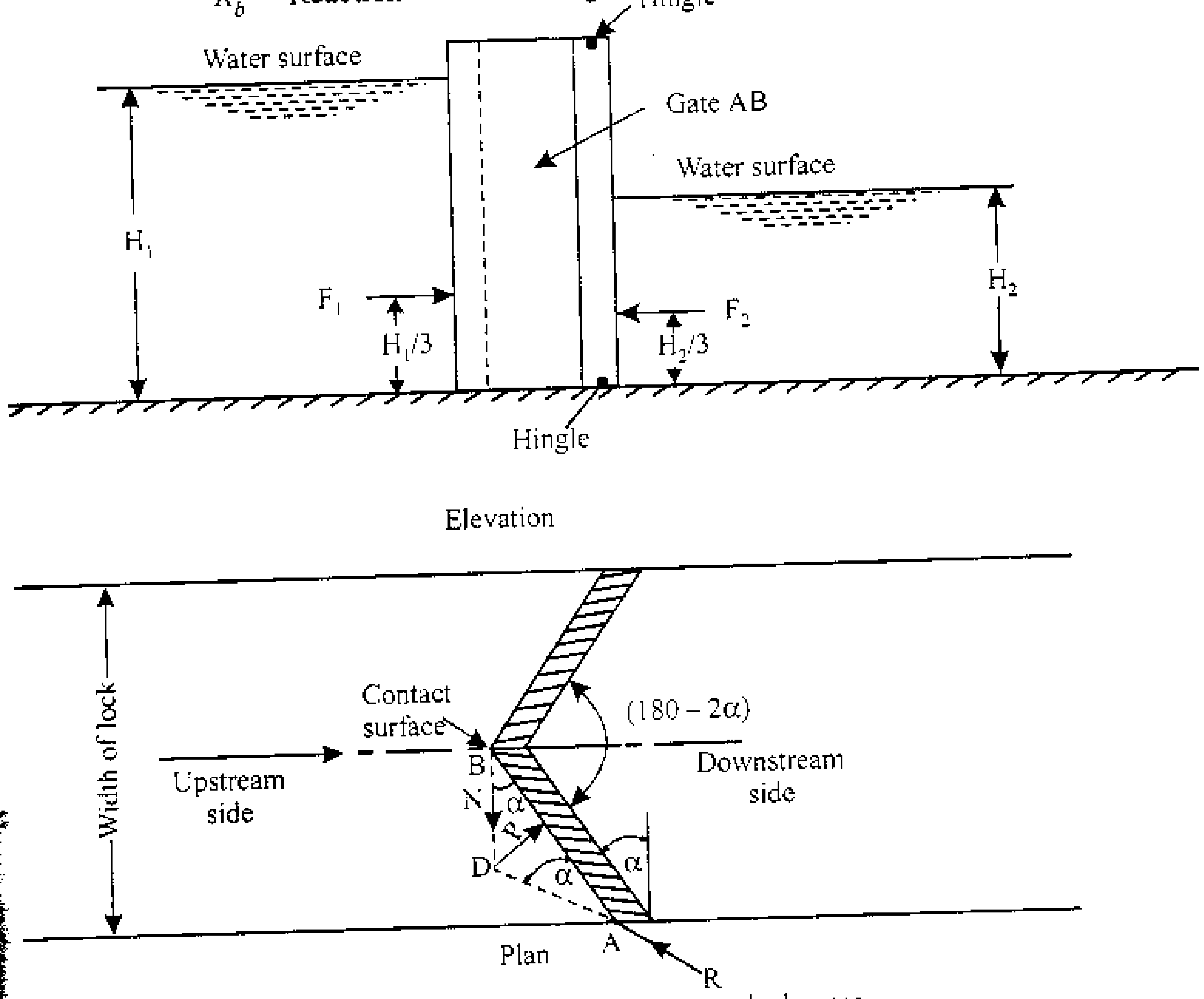


Fig. 3.64. Resultant pressure on lock gates.

Then, $$R_t + R_b = R$$

Also, $P_1 = \dfrac{wH_1}{2} \times$ wetted area of the gate (It will act at the centre of pressure which is $\dfrac{H_1}{3}$ from bottom).

and $P_2 = \dfrac{wH_2}{2} \times$ wetted area of the gate (It will act at $\dfrac{H_2}{3}$ from the bottom).

Then, $$P = P_1 - P_2$$

It may be noted that only half the water pressure may be taken as acting on the hinge edge of the gate; the remaining half will be taken by the reaction of the gate BC.

Taking moments about the lower hinge, we have

$$R_t \sin\alpha \times H = \left(\frac{P_1}{2} \times \frac{H_1}{3}\right) - \left(\frac{P_2}{2} \times \frac{H_2}{3}\right) \quad ...(i)$$

Resolving the forces *horizontally*, we get

$$R_t \sin\alpha + R_b \sin\alpha = \frac{P_1}{2} - \frac{P_2}{2} \quad ...(ii)$$

Then, form eqns. (*i*) and (*ii*), R_t and R_b may be found.

Example 3.49. *Each gate of a lock is 6 m high and 5 m wide, supported on one side by two hinges, each 0.5m from the top and from bottom. The angle between the gates in closed position is 120°. If the water levels are 5m and 1.25 m on the upstream and downstream sides respectively, find:*

(*i*) *The magnitude and position of the resultant water pressure on each gate,*

(*ii*) *The magnitude of reaction, and between the gates*

(*iii*) *The magnitudes of the reactions at the hinges.*

Assume the reaction between the gates to be in the same horizontal plane as that of the resultant water pressure. **[Jadavpur University]**

Solution. Height of each gate = 6 m; width of each gate = 5 m

Height of water on upstream side, $H_1 = 5$ m

Height of water on downstream side, $H_2 = 1.25$ m

Angle between the gates = 120°, $\therefore\ \alpha = \dfrac{180 - 120}{2} = 30°$

(*i*) **The magnitude and position of the resultant pressure:**

Upstream side: Wetted area of gate, $A_1 = 5 \times 5 = 25$ m²

Total pressure on each gate, $P_1 = wA_1\bar{x}_1 = 9.81 \times 25 \times \dfrac{5}{2} = 613.12$ kN

Position of centre of pressure $\bar{h}_1 = \dfrac{H_1}{3}$ from the bottom $= \dfrac{5}{3}$ m or 1.667 m from the bottom

Downstream side: Wetted area of gate $A_2 = 5 \times 1.25 = 6.25$ m²

Total pressure on each gate, $P_2 = wA_2\bar{x}_2 = 9.81 \times 6.25 \times \dfrac{1.25}{2} = 38.32$ kN

Position of centre of pressure $\bar{h}_2 = \dfrac{1.25}{3} = 0.417$ m from the bottom.

Now, resultant water pressure on each gate,

$$\mathbf{P} = P_1 - P_2 = 613.12 - 38.32 = \mathbf{574.8\ kN\ (Ans.)}$$

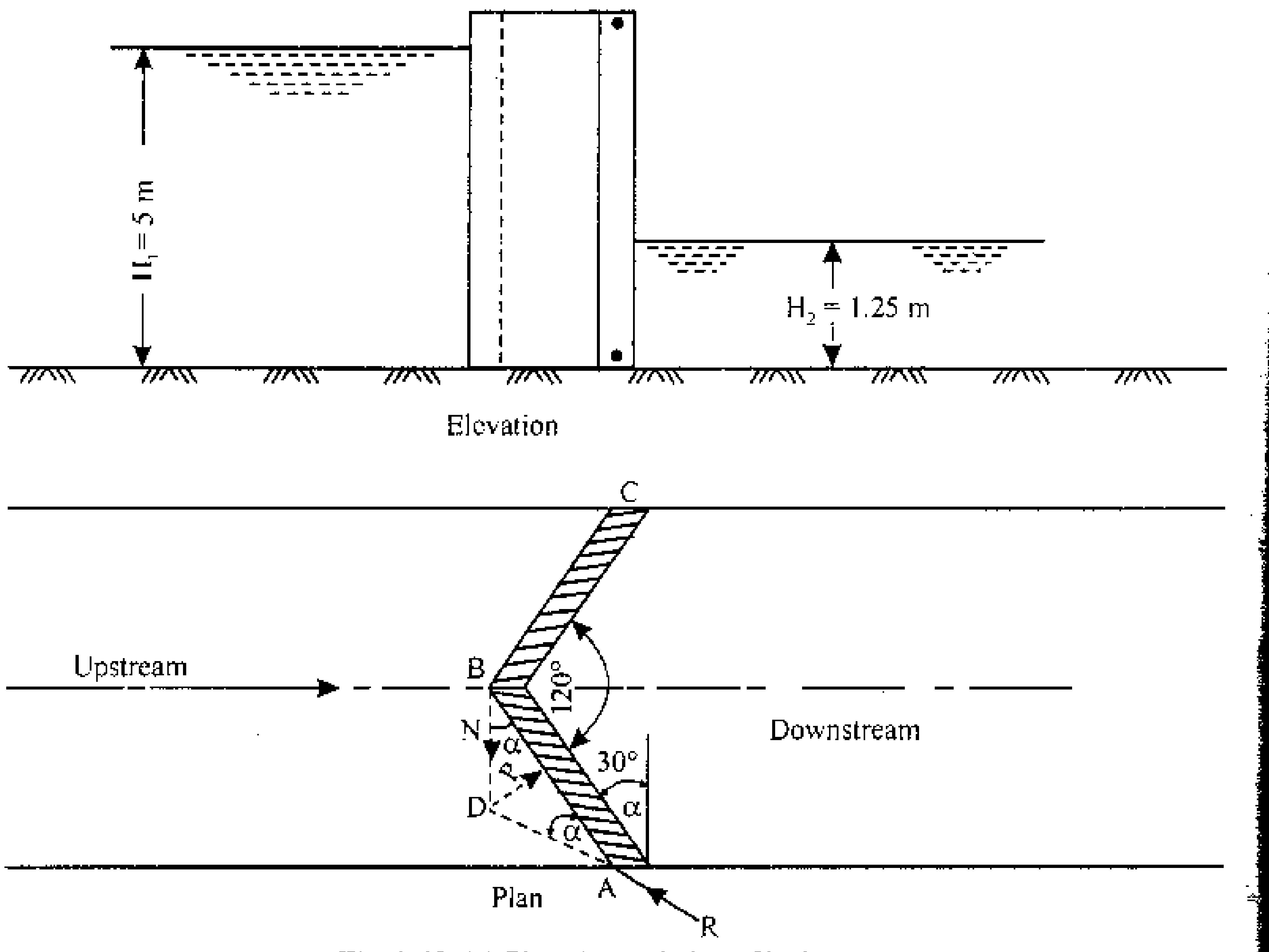


Fig. 3.65. (*a*) Elevation and plan of lock gates.

Let $\bar{h}$ is height of P from the bottom, then taking moments of P_1, P_2 and P about the bottom, we get:

$P \times \bar{h} = P_1 \times \bar{h}_1 - P_2 \times \bar{h}_2$ or $574.8 \times \bar{h} = 613.12 \times 1.667 - 38.32 \times 0.417 = 1006.09$

$\therefore$ $\bar{h} =$ **1.75 m from the bottom (Ans.)**

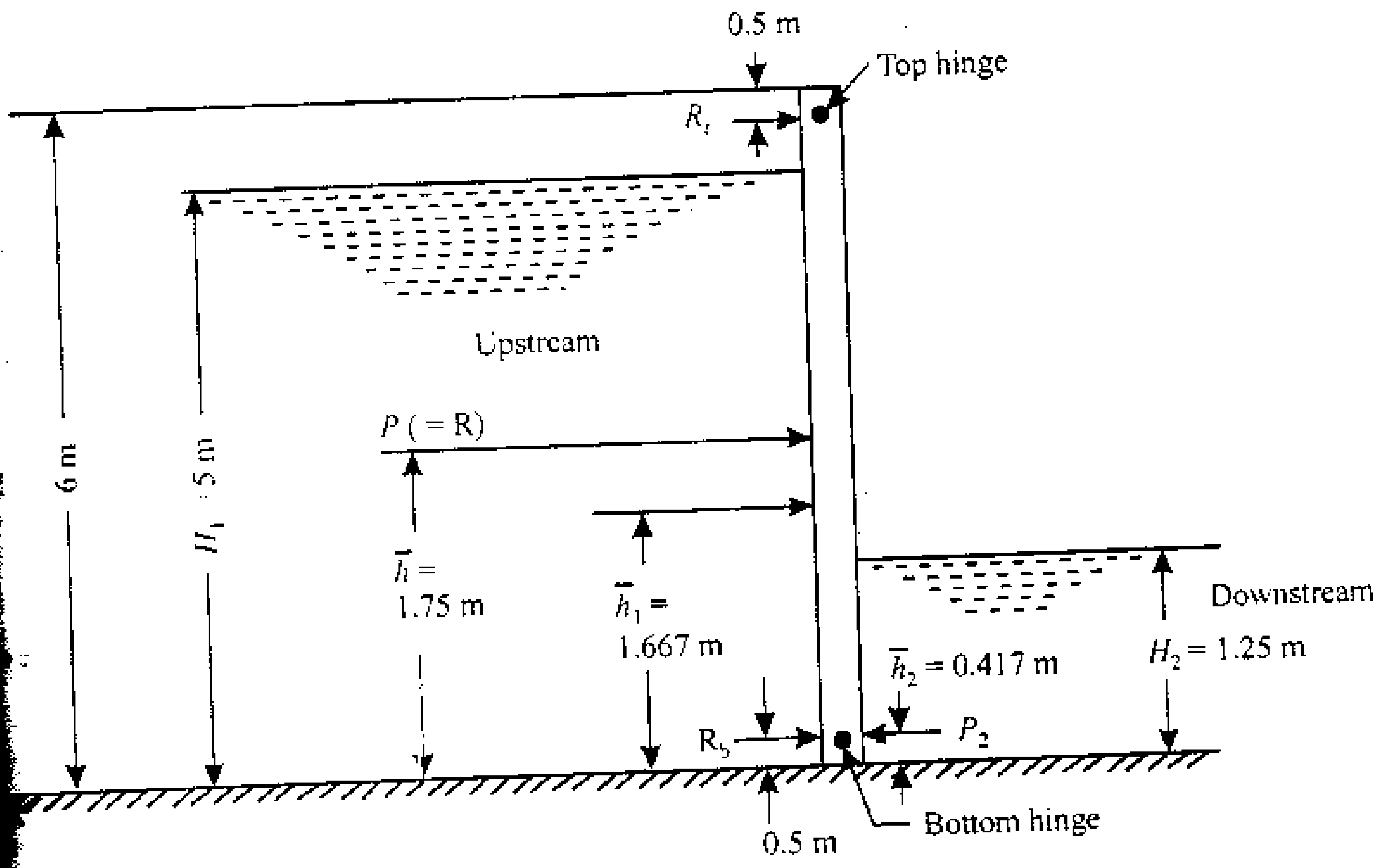


Fig. 3.65. (*b*). Resultant pressure and reactions at hinges of lock gates.

(*ii*) The magnitude of reaction between the gates, N:

Refer Fig. 3.65 (*a*)

Resolving the forces at D in a direction parallel to gate (*i.e.* along AB), we have

$$N \cos \alpha = R \cos \alpha \quad \text{or} \quad N = R$$

Resolving normally to gate (*i.e.* normal to AB), we have

$$P = R \sin \alpha + N \sin \alpha = (R + N) \sin \alpha = 2N \sin \alpha$$

$$\therefore \quad N (= R) = \frac{P}{2 \sin \alpha} = \frac{574.8}{2 \sin 30°} = \textbf{574.8 kN (Ans.)}$$

(*iii*) The magnitudes of the reactions at the hinges:

Refer Fig. 3.65 (*b*)

Let, R_t = Reaction at the top hinge, and

R_b = Reaction at the bottom hinge.

Then, $R_t + R_b = R = 574.8$ kN

Taking moments of hinge reactions R_t, R_b and $P(= R)$ about the bottom hinge, we have

$$R_t (6 - 1) + R_b \times 0 = R \times (1.75 - 0.5)$$

$$5R_t = 574.8 \times (1.75 - 0.5)$$

$\therefore$ $\mathbf{R_t}$ = **143.7 kN (Ans.)**

and $\mathbf{R_b}$ = 574.8 − 143.7 = **431.1 kN (Ans.)**

HIGHLIGHTS

1. The term hydrostatics means the study of pressure, exerted by a fluid at rest.
2. Total pressure (P) is the force exerted by a fluid on a surface (either plane or curved) when the fluid comes in contact with the surface.

   For vertically immersed surface, $P = wA\bar{x}$

   For inclined immersed surface, $P = wA\bar{x}$

   where A = area of immersed surface, and $\bar{x}$ = depth of centre of gravity of immersed surface from the free liquid surface.
3. *Centre of pressure* ($\bar{h}$) is the point through which the resultant pressure acts and is always expressed in terms of depth from the liquid surface.

   For vertically immersed surface, $\bar{h} = \dfrac{I_G}{A\bar{x}} + \bar{x}$

   For inclined immersed surface, $\bar{h} = \dfrac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$

   where I_G stands for moment of inertia of figure about horizontal axis through its centre of gravity.
4. The total force on a curved surface is given by

   $$P = \sqrt{P_H^2 + P_V^2}$$

   where, P_H = Horizontal force on curved surface
   = Total pressure force on the projected area of the curved surface on the vertical plane = $wA\bar{x}$, and
   P_V = Vertical force on submerged curved surface
   = Weight of liquid actually or imaginary supported by curved surface.

   The direction of the resultant force P with the horizontal is given by

   $$\tan\theta = \frac{P_V}{P_H} \text{ or } \theta = \tan^{-1}\frac{P_V}{P_H}$$
5. Resultant force on a sluice gate $P = P_1 - P_2$

   where, P_1 = Pressure force on the upstream side of the sluice gate, and
   P_2 = Pressure force on the downstream side of the sluice gate.
6. For a lock gate, the reaction between two gates is equal to the reaction at the hinge,

   i.e., $N = R$

   Also reaction between the two gates, $N = \dfrac{P}{2\sin\alpha}$

   where, P = Resultant water pressure on the lock gate = $P_1 - P_2$, and
   α = Inclination of the gate to normal side of lock.

OBJECTIVE TYPE QUESTIONS

se the correct Answer:

The intensity of pressure p is related to specific weight w of the liquid and vertical depth h of the point by the equation

(*a*) $p = wh$ (*b*) $h = pw$

(*c*) $p = wh^2$ (*d*) $p = wh^3$.

The point of application of the total pressure on the surface is

(*a*) centroid of the surface

(*b*) centre of pressure

(*c*) either of the above

(*d*) none of the above.

If A is the area of the immersed surface, w is the specific weight of the liquid and $\bar{x}$ is the depth of horizontal surface from the liquid surface then the total pressure P on the surface is given by

(*a*) $p = wA^2\bar{x}$ (*b*) $p = w^2A\bar{x}$

(*c*) $p = wA\bar{x}$ (*d*) $p = wA\bar{x}^2$

Centre of pressure $(\bar{h})$ in case of an inclined immersed surface is given by

(*a*) $\bar{h} = \dfrac{I_G \sin\theta}{A\bar{x}} + \bar{x}$ (*b*) $\bar{h} = \dfrac{I_G \sin\theta}{A^2\bar{x}} + \bar{x}$

(*c*) $\bar{h} = \dfrac{I_G^2 \sin\theta}{A\bar{x}} + \bar{x}$ (*d*) $\bar{h} = \dfrac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$

The side of the dam to which the water from the river or the stream approaches is known as

(*a*) downstream

(*b*) upstream

(*c*) either of the above

(*d*) none of the above.

6. Which of the following is a possibility of dam failure?

(*a*) Failure due to sliding along its base

(*b*) Failure due to tension or compression

(*c*) Failure due to shear at the weakest section

(*d*) Failure due to overturning

(*e*) All of the above.

7. Lock gates are provided to

(*a*) change the water level in a canal or river for irrigation

(*b*) store water for irrigation purpose

(*c*) either of the above

(*d*) none of the above.

8. Total force on a curved surface is given by

(*a*) $P = (P_H^2 + P_V^2)^{3/2}$ (*b*) $P = \sqrt{P_H^2 + P_V^2}$

(*c*) $P = \left(P_H^2 + P_V^2\right)^{5/2}$ (*d*) $P = P_H + P_V$

9. Resultant pressure on a sluice gate is given by

(*a*) $P = P_1 - P_2$ (*b*) $P = P_1 + P_2$

(*c*) $P = \sqrt{P_1^2 + P_2^2}$ (*d*) $P = (P_1^2 + P_2^2)^{3/2}$.

10. The term........ means the study of pressure exerted by a fluid at rest.

(*a*) hydrostatics (*b*) fluid mechanics

(*c*) continuum (*d*) kinetics,

Answers

1. (*a*)	2. (*b*)	3. (*c*)	4. (*d*)	5. (*b*)
6. (*e*)	7. (*a*)	8. (*b*)	9. (*a*)	10. (*a*).

THEORETICAL QUESTIONS

Define the following terms:

(*i*) Total pressure, and

(*ii*) Centre of pressure.

Derive expressions for total pressure and centre of pressure for a vertically immersed surface.

Derive an expression for the depth of centre of pressure from free surface of liquid of an inclined plane surface submerged in the liquid.

4. Derive an expression for the reaction between the gates as

$$R = \frac{P}{2\sin\alpha}$$

where, P = resultant force due to water acting at right angles to the gate, and

α = angle of inclination of gate to normal side of lock.