CHAPTER

16

FLOW IN OPEN CHANNELS

A. UNIFORM FLOW

16.1. Introduction

16.1.1. Definition of an Open Channel. An '*open channel*' may be defined *as a passage in which liquid flows with its upper surface exposed to atmosphere.* In open channels the flow is due to *gravity*; thus the flow conditions are greatly influenced by the *slope* of the channel.

16.1.2. Comparison between Open Channel and Pipe Flow

The important points of *difference* between the two types of flows are given below:

S. No.	Aspects	Open channel flow	Pipe flow
1.	*Cause of flow*	Gravity force (provided by sloping bottom)	The pipe runs full and the flow, in general, takes place at *the expense of hydraulic pressure*; the pressure continously decreases in the direction of flow.
2.	*Geometry of cross-section*	Open channels may have any shape: triangular, rectangular, trapezoidal, parabolic, circular etc.	Pipes...generally round in cross-section ...cross-section of flow is fixed, since the flowing liquid entirely fills the pipe section.
3.	*Surface roughness*	Varies between wide limits; the hydraulic roughness varies with depth of flow.	Roughness co-efficient varies from a low value to a very high value, depending upon the material of the pipe.
4.	*Piezometric head*	$(z + y)$, where y is the depth of flow; H.G.L. coincides with the water surface.	$\left(z + \frac{p}{w}\right)$, where p is the pressure in the pipe. H.G.L. does not coincide with water surface.
5.	*Velocity distribution*	The maximum velocity occurs at a little distance below the water surface. The shape of the velocity profile is dependent on the channel roughness.	The velocity distribution is symmetrical about the pipe axis, maximum velocity occuring at the pipe centre and the velocity at the pipe wall reducing to zero.

16.1.3. Types of Channels

The various types of channels are:

1. **Natural channel.** It is the one which has irregular sections of varying shapes, developed in a natural way.

   Examples: Rivers, streams etc.

2. **Artificial channel.** It is the one which is built artificially for carrying water for various purposes. They have the cross-sections with regular geometrical shapes (which usually remain same throughout the length of the channel).

   Examples: Rectangular channel, trapezoidal channel, parabolic channel etc.

3. **Open channel.** A channel without any cover at the top is known as an *open channel*.

   Examples: Irrigation canals, rivers, streams, flumes and water falls.

4. **Covered or closed channels.** The channel having a cover at the top is known as a *covered* or *closed channel*.

   Examples: Partly filled conduits carrying public water supply such as seweage lines, underground drains, tunnels etc. not running full of water.

5. **Prismatic channel.** A channel with constant bed slope and the same cross-section along its length is known as a *prismatic channel*.

   The prismatic channels can be further subdivided as:

   (*i*) ***Exponential channel.*** It is the one in which area of cross-section of flow is directly proportional to any power of depth of flow in channel.

   Examples: Rectangular, triangular and parabolic channels.

   (*ii*) ***Non-exponential channel.*** Trapezoidal and circular channels are non-exponential channels.

16.2. Types of Flow in Channels

The flow in channels is classified into the following types, *depending upon the change in the depth of flow with respect to space and time.*

1. Steady flow and unsteady flow
2. Uniform flow and non-uniform (or varied) flow
3. Laminar flow and turbulent flow
4. Subcritical flow, critical flow and supercritical flow.

16.2.1. Steady Flow and Unsteady Flow

- When the flow characteristics (such as depth of flow, flow velocity and the flow rate at any cross-section) do not change with respect to time, the flow in a channel is said to be *steady*.

  Mathematically, $\frac{\partial y}{\partial t} = 0, \frac{\partial V}{\partial t} = 0, \text{ or } \frac{\partial Q}{\partial t} = 0$

  where y, V and Q are depth of flow, velocity and rate of flow respectively.

- The flow is said to be *unsteady flow* when these flow parameters vary with time.

  Mathematically, $\frac{\partial y}{\partial t} \neq 0; \frac{\partial V}{\partial t} \neq 0 \text{ or } \frac{\partial Q}{\partial t} \neq 0.$

16.2.2. Uniform and Non-uniform (or varied) Flow

- Flow in a channel is said to be *uniform* if the depth, slope, cross-section and velocity *remain constant* over a given length of the channel.

  Mathematically, $\frac{\partial y}{\partial l} = 0, \frac{\partial V}{\partial l} = 0$

Uniform flows are possible only in prismatic channels only. A uniform flow may be either steady or unsteady, depending upon whether or not the discharge varies with time; *unsteady uniform flow is rare in practice.*

- Flow in a channel is said to be *non-uniform* (*or varied*) when the channel depth *varies* continuously from one section to another.

Mathematically, $\frac{\partial y}{\partial l} \neq 0, \frac{\partial V}{\partial l} \neq 0$

Varied flow may be further classified as:

(*i*) *Rapidly varied flow* (*R.V.F.*). In this type of flow depth of flow *changes abruptly over* a *comparatively small length of channel.*

Examples: Hydraulic jump and the hydraulic drop.

(*ii*) *Gradually varied flow* (*G.V.F.*). In this case the change in depth of flow takes place gradually in a long length of the channel.

16.2.3. Laminar Flow and Turbulent Flow

The flow in the open channel may be characterised as laminar or turbulent depending upon the value of Reynolds number, defined as

$$Re = \frac{\rho VR}{\mu} \quad ...(16\cdot1)$$

where, V = Average velocity of flow in the channel, and

R = Hydraulic radius (defined as the ratio of area of flow to wetted perimeter)

When $Re < 500$ ...flow is *laminar*

$Re > 2000$ ...flow is *turbulent*

$500 < Re < 2000$ ...flow is *transitional.*

16.2.4. Subscritical flow, Critical Flow and Supercritical Flow

Since gravitational force is a predominant force in the case of channel flow, therefore Froude number, $Fr = \frac{V}{\sqrt{gD}}$ (where V and D are the mean velocity of flow and hydraulic depth of the channel respectively) is an important parameter for analysing open channel flows. Depending upon Froude number the channel flow may be characterised as:

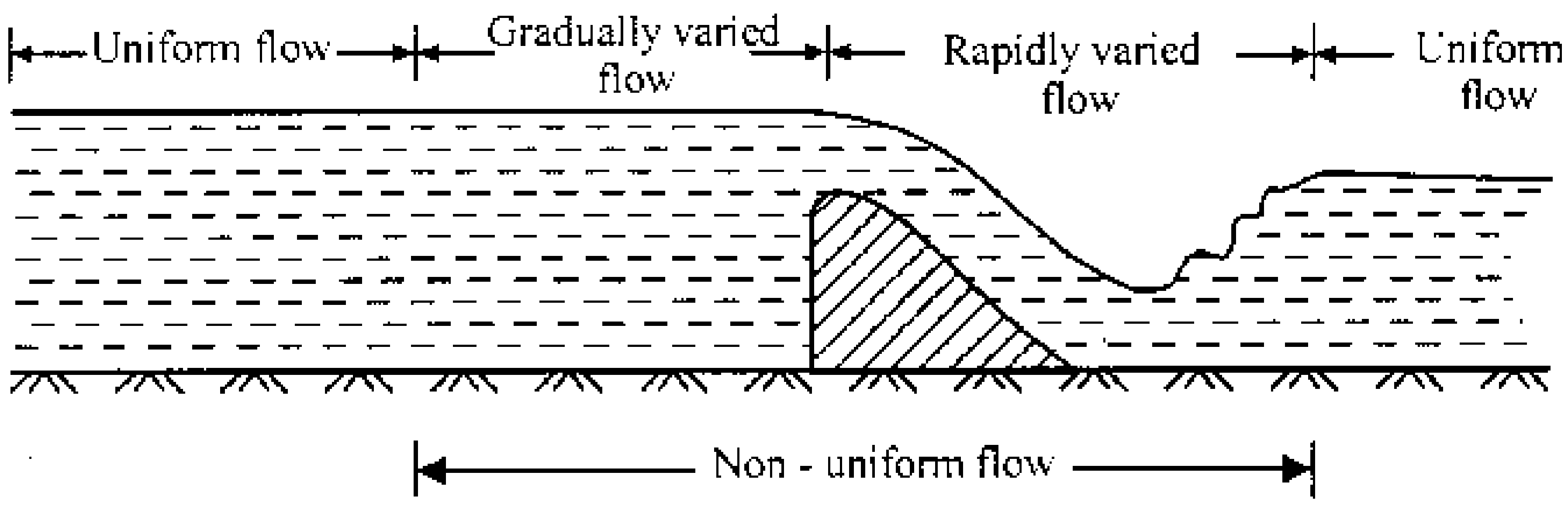


Fig. 16.1. Uniform and non-uniform flow.

(*i*) When $Fr < 1$ (or $V < \sqrt{gD}$): The flow is described as *subcritical* (or *tranquil* or streaming)

(*ii*) When $Fr = 1$: The flow is said to be in a *critical* state.

(*iii*) When $Fr > 1$: The flow is said to be *supercritical* (or *rapid* or *shooting* or *torrential*)

Some of the types of channel flow are shown in Fig. 16·1

16.3. Definitions

1. **Depth of flow (y).** It is the vertical distance of the lowest point of a channel section (bed of the channel) from the free *surface*.
2. **Depth of flow section.** It is the depth of flow normal to the bed of the channel.

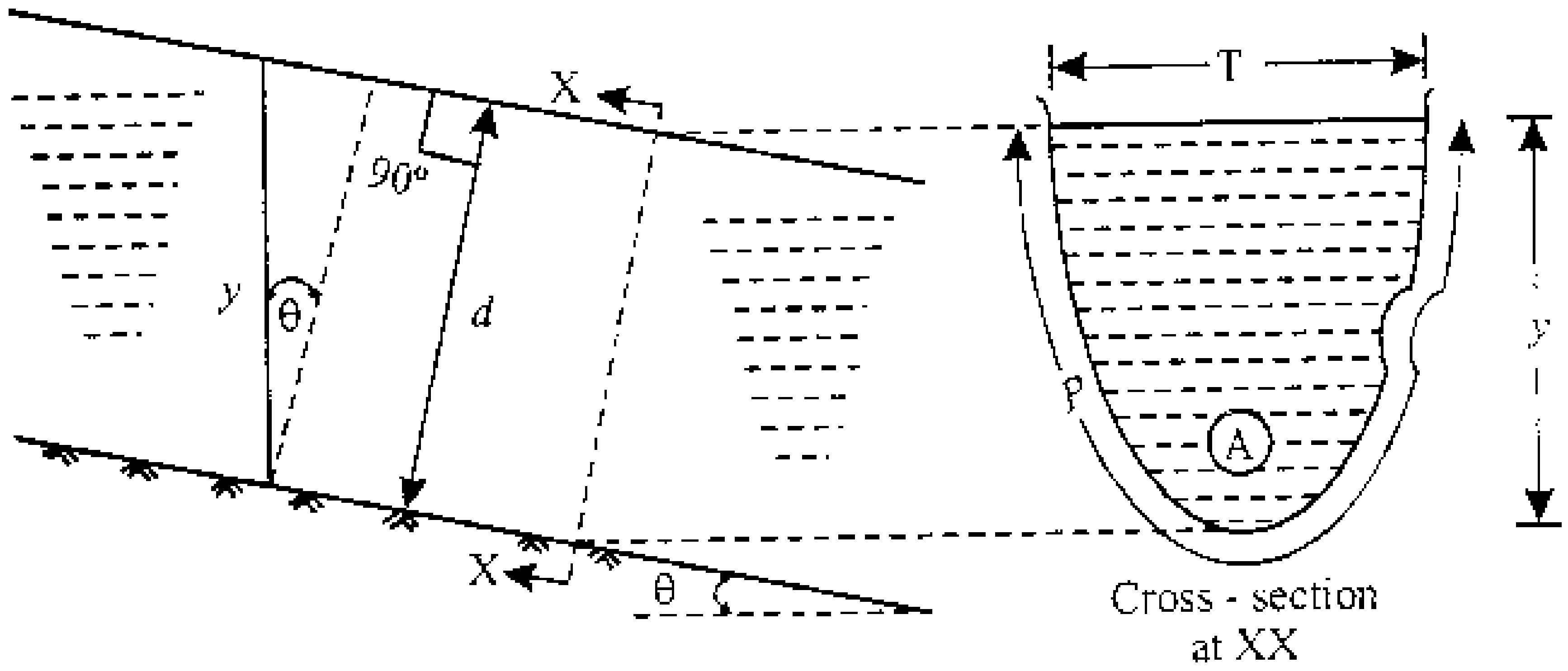


Fig. 16.2. Terms related to flow through open channel.

$$d = y \cos \theta \qquad ...(16\cdot2)$$

where, θ = The angle which the channel bed makes with the horizontal.

Since the slopes of the channels are very small

$$\cos \theta \simeq 1 \text{ and } d = y.$$

The depth of flow and depth of flow section are assumed equal, unless mentioned otherwise.

3. **Top width (T).** It is the width of the channel section at the free surface (*i.e.* the width of the liquid surface exposed to the atmospheric pressure.
4. **Wetted area (A).** It is the cross-sectional area of the flow section of the channel.
5. **Wetted perimeter (P).** It is the length of the channel boundary in contact with the flowing water at any section.
6. **Hydraulic radius (R).** It is ratio of the cross-sectional area of flow to wetted perimeter. It is also called *hydraulic mean depth*.

i.e.
$$R = \frac{A}{P} \qquad ...(16\cdot3)$$

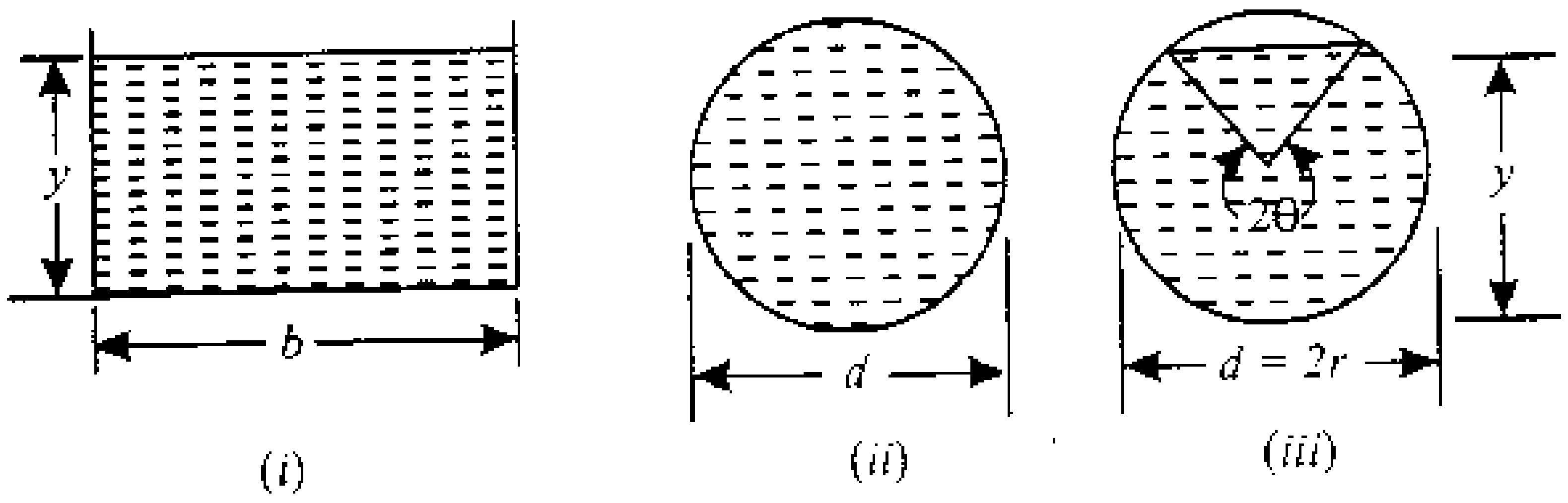


Fig. 16.3.

Examples: (*i*) Rectangular open channel:

$$R = \frac{A}{P} = \frac{b \times y}{b + 2y} \qquad ...(16\cdot4)$$

(*ii*) Pipe running full:

$$R = \frac{A}{P} = \frac{(\pi/4) \times d^2}{\pi d} = \frac{d}{4} \qquad ...(16\cdot5)$$

(*iii*) Pipe not running full:

$$R = \frac{A}{P} = \frac{\frac{r^2}{2}(2\theta - \sin 2\theta)}{2r\,\theta} \qquad \text{...(16·6)}$$

7. **Hydraulic depth (D).** It is the ratio of the wetted area A to the top width T.

i.e. $$D = \frac{A}{T} \qquad \text{...(16·7)}$$

16.4. Open Channel Formulae for Uniform Flow

For uniform flow in open channels, the following formulae will be discussed:

1. Chezy's formula
2. Manning's formula.

16.4.1. Chezy's Formula

Consider a longitudinal section of an open channel in which the flow is steady and uniform, as shown in Fig. 16·4. The forces acting on the free body of water between sections 1-1 and 2-2 in the direction of flow are as follows:

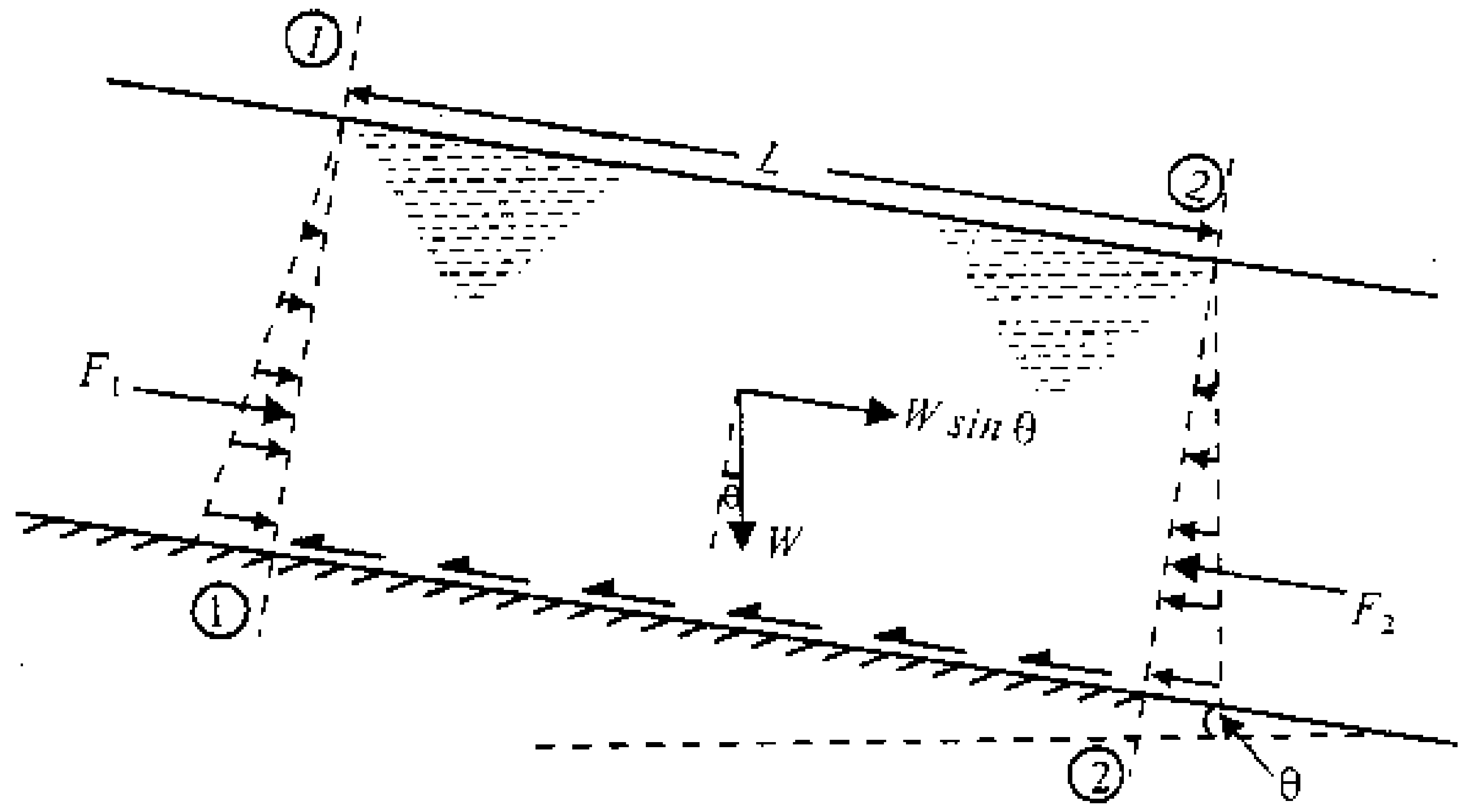


Fig. 16.4. Uniform flow in open channel.

(*i*) Pressure forces F_1 and F_2 acting on the two ends of the body; these forces balance each other since the depth of channel remains constant.

(*ii*) The component of weight of the water in the direction of flow, which is

$$= W \sin\theta = wAL \sin\theta$$

where, w = Specific weight of water,

A = Wetted cross-sectional area of channel,

L = Length of the channel considered, and

θ = Angle of inclination of channel bottom with the horizontal.

(*iii*) Frictional resistance offered by the sides of the channel which is = $\tau_0 PL$, where P is the wetted perimeter of the channel and τ_0 is the average shear stress at the channel boundary.

As the flow is steady and uniform, it is neither accelerating nor decelerating; the liquid mass is in equilibrium and the frictional resistance to flow equals the weight of liquid mass acting along the line of fluid motion. Thus

$$wAL \sin\theta = \tau_0 PL$$

Since frictional resistance τ_0 varies with (velocity)^2, τ_0 may be expressed as fV^2 where f is a *non-dimensional factor* whose value depends upon the *material* and *nature of flow surface*.

$\therefore$ $$wAL \sin\theta = fV^2 PL$$

or, $$V^2 = \frac{wAL \sin\theta}{fPL} \quad \text{or} \quad V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P}\sin\theta}$$

Since $\frac{A}{P}$ is the hydraulic radius (or hydraulic mean depth) and θ is the slope of the channel bed (S), we may write

$$V = C\sqrt{RS} \qquad \text{...(16·8)}$$

where $C = \sqrt{\frac{w}{f}}$ (*a variable which depends on the roughness of the channel surface and the flow Reynolds number*).

Eqn. (16·8) is known as ***Chezy's formula*** (named after the French engineer Antoine Chezy who developed this formula in 1775). The term C is known as *Chezy's constant*.

Discharge through the channel, Q = Area × velocity

$$= AC\sqrt{RS}$$

which can be written as:

$$Q = K\sqrt{S} \qquad \text{...(16·9)}$$

where, $$K = AC\sqrt{R}$$

The factor K is called the *conveyance* of the channel section, and is a *measure of the carrying capacity of the channel*. For a channel of constant slope, the conveyance is directly proportional to discharge Q.

Empirical relations for the Chezy's constant C:

Although Chezy's equation is quite simple, the selection of a correct value of C is rather difficult. Some of the important formulae developed for Chezy's constant C are:

(*a*) Bazin's formula:

A French hydraulician H. Bazin's (1897) proposed the following empirical formula for Chezy's constant:

$$C = \frac{157\cdot 6}{181 + \frac{K}{\sqrt{R}}} \qquad \text{...(16.10)}$$

where R is the hydraulic radius and K is the Bazin's constant whose value depends on surface roughness. Some typical values of K are:

S.No.	*Surface of channel*	*Bazin's constant (K)*
1.	Smooth cement plaster or planed wood	0·11
2.	Concrete, brick, or unplaned wood	0·21
3.	Smooth rubble masonry or poor brickwork	0·83
4.	Earth channels in very good condition	1·54
5.	Earth channels in rough condition	3·17
6.	Dredged earth channels, average condition	2·36

(b) Kutter's formula:

Two Swiss engineers Ganguillet and Kutter proposed the following empirical formula (1869) for the determination of Chezy's constant C.

$$C = \frac{23 + \frac{0 \cdot 00155}{S} + \frac{1}{N}}{1 + \left(23 + \frac{0 \cdot 00155}{S}\right)\frac{N}{\sqrt{R}}} \quad ...(16.11)$$

where N is the Kutter's constant whose value depends upon the type of the channel surface. Some typical values of N are given below:

S.No.	*Surface of channel*	*N (Kutter's/Manning's constant)*
1.	Smooth cement plaster or planed wood	0·010
2.	Very smooth concrete and planed timber	0·011
3.	Smooth concrete	0·012
4.	Glazed brickwork	0·013
5.	Vitrified clay	0·014
6.	Brick surface lined with cement mortar	0·015
7.	Earth channels in best condition	0·017
8.	Straight unlined earth channels in good condition	0·020
9.	Rivers and earth channels in fair condition	0·025
10.	Canal and river of rough surface with weeds	0·030

(c) Manning's formula:

Rober Manning (an Irish engineer) gave the following empirical relation for determination of Chezy's constant C (1889), which is simplest of all used for uniform open channel flow:

$$C = \frac{1}{N} \cdot R^{1/6}$$

where N is the Manning's constant (also known as *rugosity co-efficient*—a term generally used by British engineers) whose value depends on the channel surface.

Example 16.1. *Find the rate of flow and conveyance for a rectangular channel 7·5 m wide for uniform flow at a depth of 2·25 m. The channel is having bed slope as 1 in 1000. Take Chezy's constant C = 55.*

Also state whether the flow is tranquil or rapid.

Solution. Width of the rectangular channel, $b = 7 \cdot 5$ m

Depth of flow, $y = 2 \cdot 25$ m

$\therefore$ Area of flow, $A = b \times y = 7 \cdot 5 \times 2 \cdot 25 = 16 \cdot 875 \text{ m}^2$

Bed slope, $S = \frac{1}{1000}$

Chezy's constant, $C = 55$

Wetted perimeter, $P = b + 2y = 7 \cdot 5 + 2 \times 2 \cdot 25 = 12 \cdot 0$ m

$\therefore$ Hydraulic radius (or hydraulic mean depth), $R = \frac{A}{P} = \frac{16 \cdot 875}{12 \cdot 0} = 1 \cdot 406$ m

Rate of flow, Q:

Using Chezy's formula, average velocity,

$$V = C\sqrt{RS} = 55\sqrt{1.406 \times \frac{1}{1000}} = 2.06 \text{ m/s}$$

$\therefore$ *Discharge,* $Q = AV = 16.875 \times 2.06 =$ **34.76 m³/s (Ans.)**

Conveyance, K:

$$K = AC\sqrt{R} = 16.875 \times 55 \times \sqrt{1.406} = \mathbf{1100.5} \text{ (Ans.)}$$

State of flow (tranquil or rapid):

Froude number, $$Fr = \frac{V}{\sqrt{gy}} = \frac{2.06}{\sqrt{9.81 \times 2.25}} = 0.438$$

Since $Fr < 1.0$, the flow in the channel is **tranquil** in nature. **(Ans.)**

Example 16.2. *A triangular gutter, whose sides include an angle of 60°, conveys water at a uniform depth of 250 mm. If the discharge is 0.04 m³/s, determine the gradient of the trough. Use the Chezy's formula assuming that C = 52.* **[Panjab University]**

Solution. Depth of flow = 250 mm = 0.25 m.

Discharge through the gutter, $Q = 0.04$ m³/s

Chezy's constant, $C = 52$

Bed slope, S:

Refer Fig. 16.5. From ΔACO,

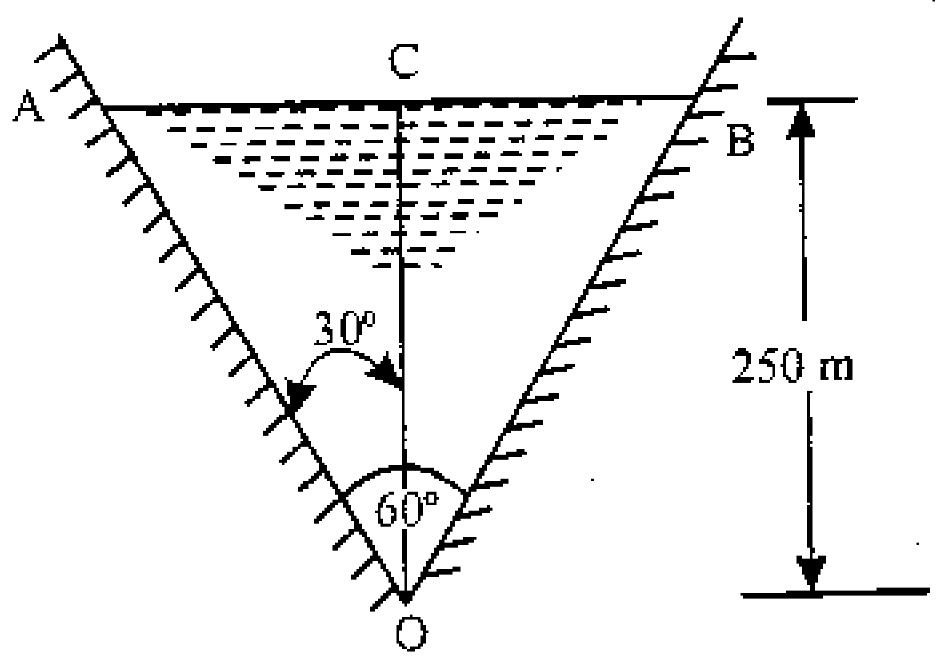


Fig. 16.5

$$\frac{CO}{AO} = \cos 30° \quad \text{or} \quad AO = \frac{CO}{\cos 30°}$$

$$= \frac{0.25}{\cos 30°} = 0.288 \text{ m}$$

i.e. $AO = BO = 0.288$ m

Further $$\frac{AC}{CO} = \tan 30°$$

or, $AC = CO \tan 30° = 0.25 \times 0.577 = 0.144$ m

or, $AB = 2AC = 0.288$ m

$\therefore$ Area of flow, $$A = \frac{1}{2} \times AB \times CO = \frac{1}{2} \times 0.288 \times 0.25 = 0.036 \text{ m}^2$$

Wetted perimeter, $P = AO + BO = 0.288 + 0.288 = 0.576$ m

Hydraulic radius, $$R = \frac{A}{P} = \frac{0.036}{0.576} = 0.0625 \text{ m}$$

Using Chezy's formula, we have

$$Q = AV = AC\sqrt{RS} \quad \text{or} \quad 0.04 = 0.036 \times 52 \times \sqrt{0.0625 \times S}$$

or, $$\sqrt{0.0625 \times S} = \frac{0.04}{0.036 \times 52} = 0.02137$$

or, $$S = \frac{0.02137^2}{0.0625} = \frac{1}{137} \quad \text{(Squaring both sides)}$$

Hence, *gradient of the trough (or bed slope) is* **1 in 137 (Ans.)**

Example 16.3. *Find the discharge of water through the channel shown in Fig. 16·6. Take the value of Chezy's constant = 60 and slope of the bed as 1 in 950.* **[Panjab University]**

Solution. Chezy's constant, $C = 60$

Bed slope, $S = \frac{1}{950}$

Discharge, Q:

Refer to Fig. 16·6.

Area of flow, A = Area $ABCD$ + area DEC

$$= 1 \cdot 2 \times 0 \cdot 6 + \frac{\pi \times 0 \cdot 6^2}{2} = 1 \cdot 285 \text{ m}^2$$

Wetted perimeter, $P = AD + DEC + CB$

$$= 0 \cdot 6 + \pi \times 0 \cdot 6 + 0 \cdot 6 = 3 \cdot 085 \text{ m}$$

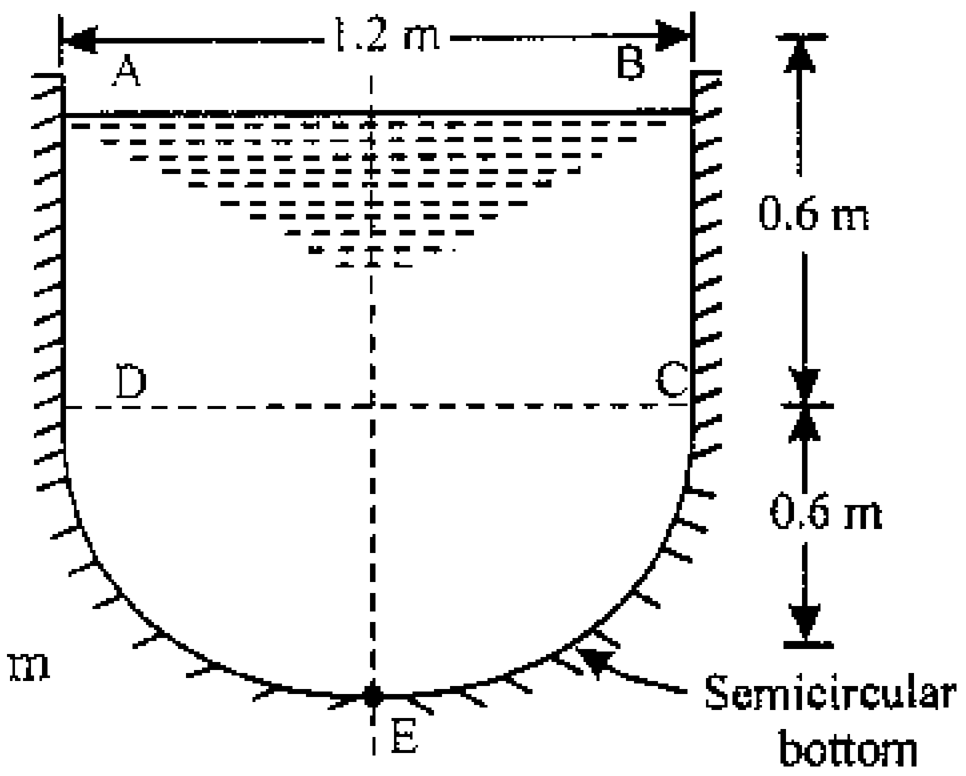


Fig. 16.6

∴ Hydraulic mean radius,

$$R = \frac{A}{P} = \frac{1 \cdot 285}{3 \cdot 085} = 0 \cdot 416 \text{ m}$$

Using Chezy's formula, we have

$$Q = AV = AC\sqrt{RS}$$

$$= 1 \cdot 285 \times 60 \times \sqrt{0 \cdot 416 \times \frac{1}{950}} = \mathbf{1 \cdot 613 \ m^3/s} \ \textbf{(Ans.)}$$

Example 16.4. *A canal of trapezoidal section has bed width of 8 m and bed slope of 1 in 4000. If the depth of flow is 2·4 m and side slopes of the channel are 1 horizontal to 3 vertical, determine the average flow velocity and the discharge carried by the channel. Also compute the average shear stress at the channel boundary.*

Take value of Chezy's constant = 55.

Solution. Width of the channel bed, $b = 8$ m

Bed slope, $S = \frac{1}{4000}$

Side slopes = 1 horizontal to 3 vertical

Depth of flow, $y = 2{\cdot}4$ m

Chezy's constant, $C = 55$

Horizontal distance $EA = BF = ny$

where, n = side slope (1 vertical to n horizontal)

Top width $CD = AB + 2BF = b + 2ny$

Slant side $AD = BC = \sqrt{y^2 + n^2 y^2}$

$$= y\sqrt{n^2 + 1}$$

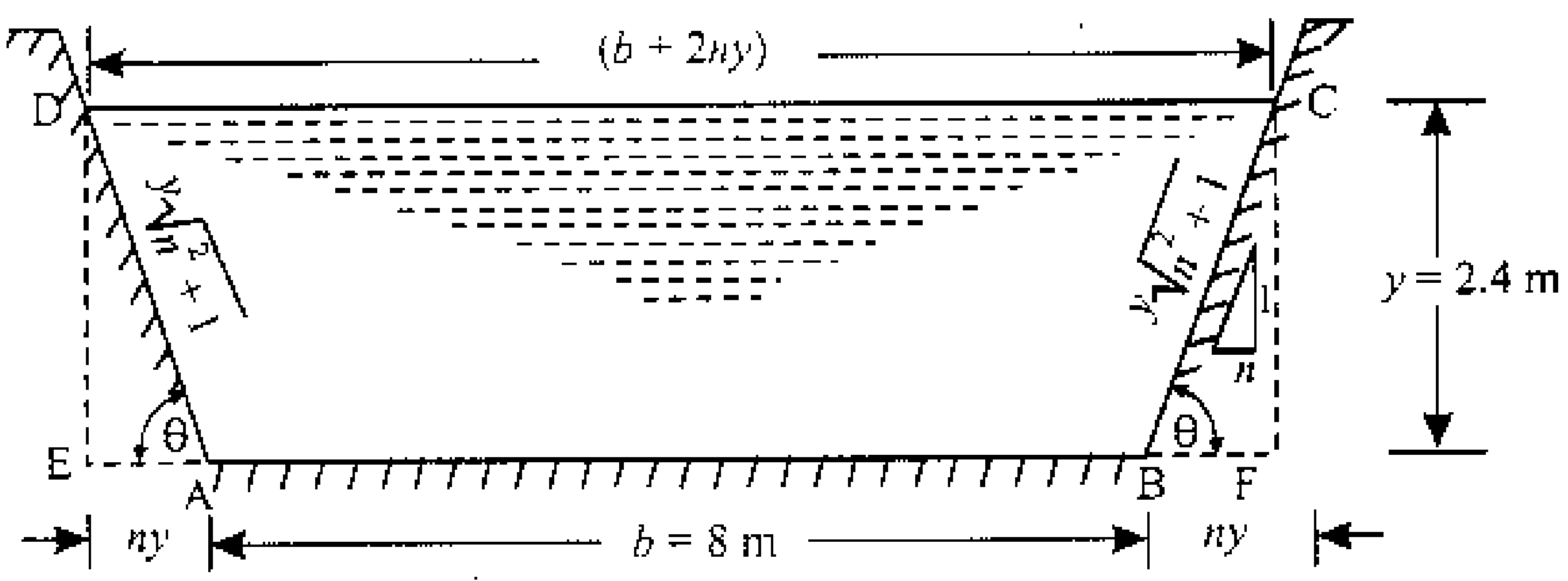


Fig. 16.7

∴ Wetted perimeter, $P = AB + AD + BC$

$$= 8 + 2y\sqrt{n^2+1} = 8 + 2 \times 2\cdot4\sqrt{\left(\frac{1}{3}\right)^2 + 1} = 13\cdot06 \text{ m} \quad (\because n = 1/3)$$

$$\text{Area of flow} = \left(\frac{\text{Top width + bottom width}}{2}\right) \times \text{height} = \frac{(b+2ny)+b}{2} \times y = y\,(b+ny)$$

$$= 2\cdot4\left(8 + \frac{1}{3} \times 2\cdot4\right) = 21\cdot12 \text{ m}^2$$

Hydraulic radius, $R = \dfrac{A}{P} = \dfrac{21\cdot12}{13\cdot06} = 1\cdot617$ m

Average flow velocity:

Average flow velocity, $V = C\sqrt{RS}$

$$= 55\sqrt{1\cdot617 \times \frac{1}{4000}} = \mathbf{1\cdot106\ m/s\ (Ans.)}$$

Discharge, Q:

Discharge through the channel, $Q = AV = 21\cdot12 \times 1\cdot106 =$ **23.36 m³/s (Ans.)**

Shear stress at channel boundary, τ_0:

Under equilibrium conditions, the frictional resistance to flow equals the weight of liquid mass acting along the line of fluid motion,

i.e. $\tau_0 LP = wAL \sin\theta$

∴ *Shear stress at the channel boundary,*

$$\tau_0 = \frac{wAL \sin\theta}{LP} = w\frac{A}{P}\sin\theta = w \times R \times S \qquad (\because S = \sin\theta)$$

$$= 9810 \times 1\cdot617 \times \frac{1}{4000} = \mathbf{3\cdot96\ N/m^2\ (Ans.)}$$

16.5. Most Economical Section of a Channel

The *most economical section* (also called the *best section or most efficient section*) *is one which gives the maximum discharge for a given amount of excavation.*

From continuity equation it is evident that discharge is maximum when velocity is maximum, the area of cross-section of channel remaining constant. From Chezy's formula and Manning's formula it can be seen that for a given value of slope and surface roughness the velocity of flow will be maximum if hydraulic radius $R \left(= \dfrac{A}{P}\right)$ is maximum. Further the area being constant hydraulic radius is maximum if the *wetted perimeter is minimum*; this condition is used to determine the dimensions of economical sections of different forms of channels. *The best form of channel which complies with this condition is one which has a semi-circular cross-section.*

16.5.1. Most Economical Rectangular Channel Section

Fig. 16·8 shows the cross-section of a rectangular channel. Let b and y be the base width and depth of flow respectively.

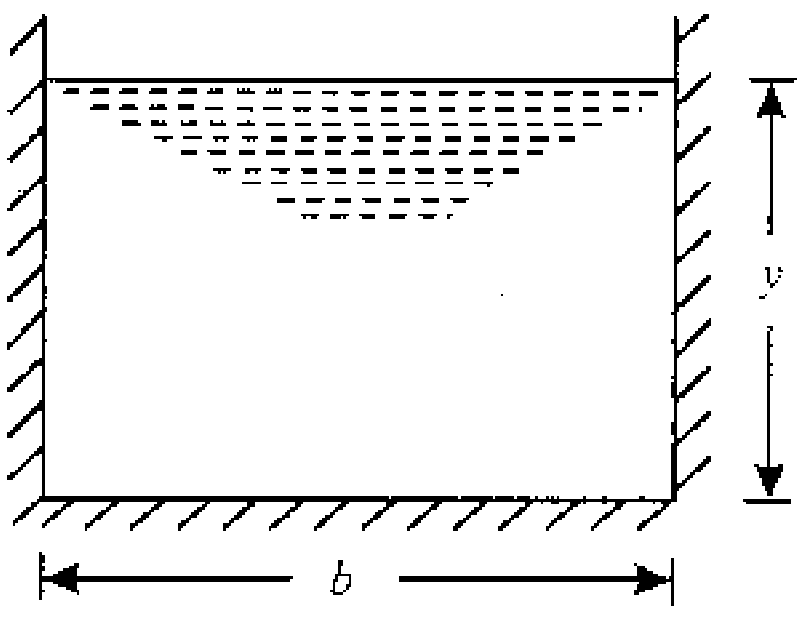


Fig. 16.8. Rectangular channel.

Area of flow, $A = b \times y$, ...(*i*)

Wetted perimeter, $P = b + 2y$ ...(*ii*)

Substituting the value of $b \left(= \frac{A}{y}\right)$ from eqn. (*i*) in eqn. (*ii*), we get

$$P = \frac{A}{y} + 2y$$

For the section to be most economical/efficient, the wetted perimeter P must be a *minimum*.

i.e. $$\frac{dP}{dy} = 0 \quad \text{or} \quad \frac{d}{dy}\left[\frac{A}{y} + 2y\right] = 0$$

or, $$-\frac{A}{y^2} + 2 = 0 \quad \text{or} \quad A = 2y^2 \quad \text{or} \quad b \times y = 2y^2 \qquad [\because A = b \times y]$$

or, $$b = 2y \quad \text{or} \quad y = b/2 \qquad ...(16 \cdot 12)$$

Hydraulic radius, *R*:

Hydraulic radius, $$R = \frac{A}{P} = \frac{b \times y}{b + 2y}$$

$$= \frac{2y \times y}{2y + 2y} = \frac{2y^2}{4y} = \frac{y}{2} \qquad (\because b = 2y)$$

i.e. $$R = \frac{y}{2} \qquad ...(16 \cdot 13)$$

Thus the rectangular channel section will be most economical when:

(*i*) *The depth of flow is equal to half the base width* $\left(y = \frac{b}{2}\right)$, or

(*ii*) *Hydraulic radius is equal to half the depth of flow* $\left(R = \frac{y}{2}\right)$

Example 16.5. *A rectangular channel is to be dug in the rocky portion of a soil. Find its most economical cross-section if it is to convey 12 m³/s of water with an average velocity of 3 m/s. Take Chezy's constant C = 50.*

Solution. Discharge, $Q = 12 \text{ m}^3/\text{s}$

Average velocity, $V = 3 \text{ m/s}$

Chezy's constant, $C = 50$

The geometric relations for *optimum discharge* through a rectangular channel are:

$$b = 2y \quad \text{and} \quad R = \frac{y}{2}, \text{ then area } A = b \times y = 2y^2$$

where b, y and R are base width of the channel, depth of flow and hydraulic radius respectively.

Now, $Q = A \times V = 2y^2 \times V$ or $12 = 2y^2 \times 3$ or $y = 1 \cdot 414$ m

i.e. Flow depth, $y = 1 \cdot 414$ m

$\therefore$ Base width of the channel, $b = 2y = 2 \times 1 \cdot 414 = 2 \cdot 828$ m

Hydraulic radius, $R = \frac{y}{2} = \frac{1 \cdot 414}{2} = 0 \cdot 707$ m

Also $V = C\sqrt{RS}$ ...Chezy's formula or $$S = \frac{V^2}{C^2 R} = \frac{3^2}{50^2 \times 0 \cdot 707} = \frac{1}{196}$$

(where S = slope bed)

Hence, $b = 2{\cdot}828$ m, $y = 1{\cdot}414$ m, $S = \frac{1}{196}$ **(Ans.)**

Example 16.6. *Determine the most economical section of a rectangular channel carrying water at the rate of 0·5 m³/s; the bed slope of the channels being 1 in 2000. Take Chezy's constant C = 50.*

Solution. Discharge, $Q = 0{\cdot}5 \text{ m}^3/\text{s}$

Bed slope, $S = \frac{1}{2000}$

Chezy's constant, $C = 50$

Most economical section:

The rectangular channel section will be most economical when

(*i*) Base width, $b = 2y$

(*ii*) Hydraulic radius, $R = \frac{y}{2}$ (where y = depth of flow)

Area of flow, $A = b \times y = 2y \times y = 2y^2$

Now, discharge $Q = AC\sqrt{RS}$ ...Chezy's formula

$$0 \cdot 5 = 2y^2 \times 50\sqrt{\frac{y}{2} \times \frac{1}{2000}}$$

$$= 100\sqrt{\frac{1}{4000}} \times y^{5/2} = 1 \cdot 581\, y^{5/2}$$

$\therefore$ $$y^{5/2} = \frac{0 \cdot 5}{1 \cdot 581} = 0 \cdot 316 \quad \text{or} \quad y = (0{\cdot}316)^{2/5} = 0{\cdot}63 \text{ m}$$

and, $$b = 2y = 2 \times 0{\cdot}63 = 1{\cdot}26 \text{ m}$$

Hence, $\boldsymbol{b = 1{\cdot}26}$ **m and** $\boldsymbol{y = 0{\cdot}63}$ **m (Ans.)**

Example 16.7. *A rectangular channel 4 m wide has depth of water 1·5 m. The slope of the bed of the channel is 1 in 1000 and value of Chezy's constant C = 55. It is desired to increase the discharge to a maximum by changing the dimensions of the section for constant area of cross-section, slope of the bed and roughness of the channel. Find the new dimensions of the channel and increase in discharge.* [AMIE]

Solution. Base width of the channel, $b = 4$ m

Depth of flow, $y = 1{\cdot}5$ m

Bed slope, $S = \frac{1}{1000}$

Chezy's constant, $C = 55$

New dimensions of the channel and increase in discharge:

Area of flow, $A = b \times y = 4 \times 1{\cdot}5 = 6 \text{ m}^2$

Wetted perimeter, $P = b + 2y = 4 + 2 \times 1{\cdot}5 = 7{\cdot}0$ m

Hydraulic radius, $R = \frac{A}{P} = \frac{6}{7} = 0 \cdot 857$

Discharge, $$Q = AC\sqrt{RS} = 6 \times 55\sqrt{0 \cdot 857 \times \frac{1}{1000}} = 9 \cdot 66 \text{ m}^3/\text{s}$$

For determining maximum discharge, for given area of cross-section, slope of the bed and roughness of the channel, we follow the procedure given below:

Let b' = New base width of the channel, and

y' = New depth of flow,

Then, area $A = b' \times y'$, where $A = 6{\cdot}0\ m^2$ (= constant)

$\therefore\quad b' \times y' = 6$

Also for maximum discharge, $b' = 2y'$

$\therefore\quad 2y' \times y' = 6 \quad \text{or} \quad y'^2 = 3 \quad \text{or} \quad y' = \sqrt{3} = 1{\cdot}732$ m,

and, $\quad b' = 2y' = 2 \times 1{\cdot}732 = 3{\cdot}464$ m

Hence *new dimensions* of the channel are: $\boldsymbol{b' = 3{\cdot}464}$ **m** and $\boldsymbol{y' = 1{\cdot}732}$ **m (Ans.)**

Wetted perimeter, $P' = b' + 2y' = 3{\cdot}464 + 2 \times 1{\cdot}732 = 6{\cdot}928$ m

$$\therefore \quad \text{Hydraulic radius, } R' = \frac{A}{P'} = \frac{6}{6\cdot928} = 0\cdot866 \text{ m}$$

$$\left[\text{Alternatively } R' = \frac{y'}{2} = \frac{1\cdot732}{2} = 0\cdot866 \text{ m (maximum discharge conditional)}\right]$$

$$\text{Maximum discharge, } Q' = AC\sqrt{R'S} = 6 \times 55 \times \sqrt{0\cdot866 \times \frac{1}{1000}} = 9\cdot71 \text{ m}^3/\text{s}$$

$\therefore$ *Increase in discharge* $= Q' - Q = 9{\cdot}71 - 9{\cdot}66 =$ **0.05 m³/s (Ans.)**

16.5.2. Most Economical Trapezoidal Channel Section

Fig 16·9 shows the cross-section of a trapezoidal channel.

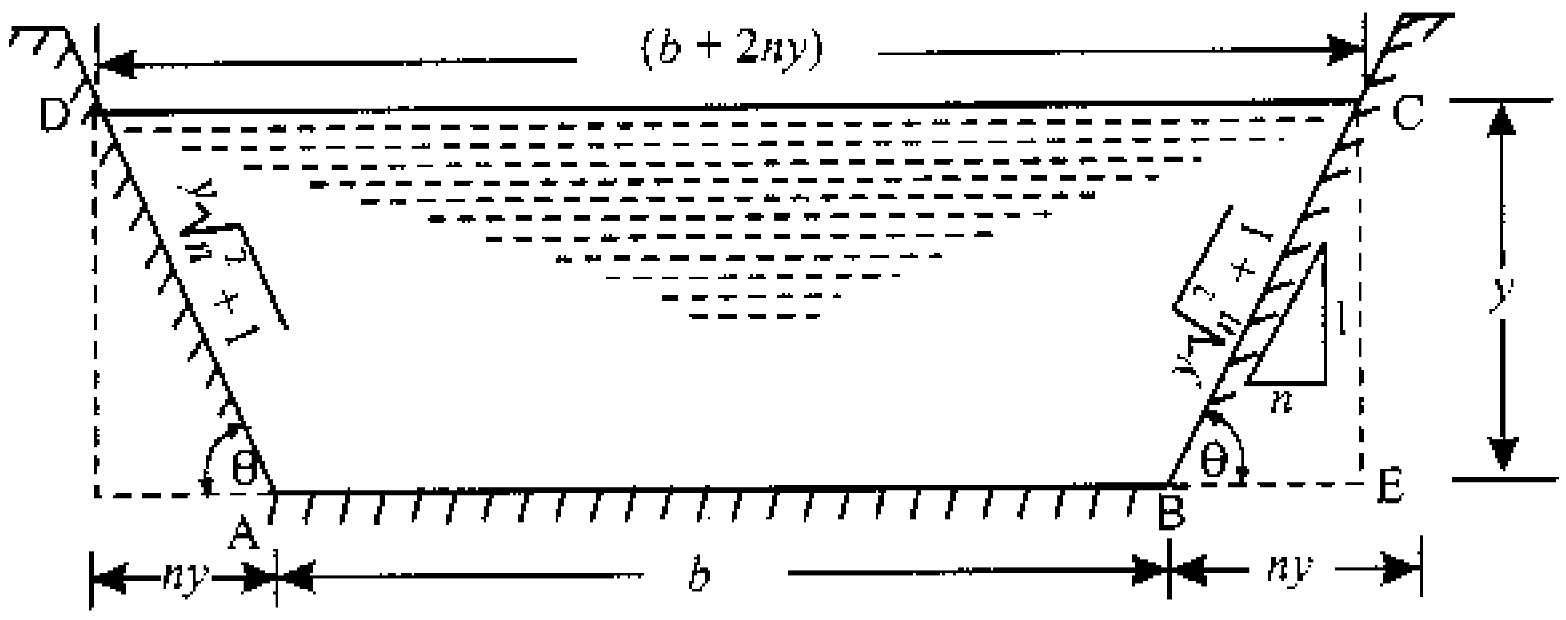


Fig. 16.9. Trapezoidal channel.

Let $\quad b$ = Base width of the channel,

y = Depth of flow, and

θ = Angle made by the sides with horizontal.

Side slope = 1 vertical to n horizontal.

$$\text{Area of flow, } A = \left(\frac{AB + CD}{2}\right) \times y = \frac{b + (b + 2ny)}{2} \times y = (b + ny)\, y \qquad \ldots(i)$$

$$\therefore \quad \frac{A}{y} = b + ny$$

$$\text{or,} \quad b = \frac{A}{y} - ny \qquad \ldots(ii)$$

Wetted perimeter, $P = AD + AB + BC = AB + 2BC$ $\quad (\because AD = BC)$

$$= b + 2\sqrt{BE^2 + CE^2}$$

$$= b + 2\sqrt{n^2y^2 + y^2}$$

or, $$P = b + 2y\sqrt{n^2 + 1} \quad \text{...(iii)}$$

Substituting the value of b from eqn. (*ii*) in eqn. (*iii*), we get

$$P = \frac{A}{y} - ny + 2y\sqrt{n^2 + 1} \quad \text{...(iv)}$$

The section of the channel will be *most economical* when its wetted perimeter (P) is *minimum, i.e.*

$$\frac{dP}{dy} = 0$$

or, $$\frac{d}{dy}\left[\frac{A}{y} - ny + 2y\sqrt{n^2 + 1}\right] = 0$$

or, $$-\frac{A}{y^2} - n + 2\sqrt{n^2 + 1} = 0 \quad (\because n \text{ is constant})$$

or, $$\frac{A}{y^2} + n = 2\sqrt{n^2 + 1}$$

Substituting the value of A from eqn. (*i*), in the above equation, we get

$$\frac{(b + ny)\,y}{y^2} + n = 2\sqrt{n^2 + 1}$$

or, $$\frac{(b + ny)}{y} + n = 2\sqrt{n^2 + 1}$$

or, $$\frac{b + ny + ny}{y} + 2\sqrt{n^2 + 1} \quad \text{or} \quad \frac{b + 2ny}{y} = 2\sqrt{n^2 + 1}$$

or, $$\frac{b + 2ny}{2} = y\sqrt{n^2 + 1} \quad \text{...(16·14)}$$

[*i.e. Half of top width = One of the sloping sides* ... Fig 16·9]

Hydraulic radius, R:

Hydraulic radius, $R = \dfrac{A}{P}$

$$A = (b + ny) \times y \quad \text{[From eqn. (i)]}$$

$$P = b + 2y\sqrt{n^2 + 1} \quad \text{[From eqn. (iii)]}$$

But, $2y\sqrt{n^2 + 1} = b + 2ny$ $\quad$ [From eqn. (16·14)]

$\therefore P = b + (b + 2ny) = 2(b + ny)$

$$\therefore \text{Hydraulic radius, } R = \frac{(b + ny)\,y}{2\,(b + ny)} = \frac{y}{2} \quad \text{...(16·15)}$$

i.e.. The hydraulic radius equals half the flow depth.

Fig. 16·10 shows a trapezoidal channel of most economical section.

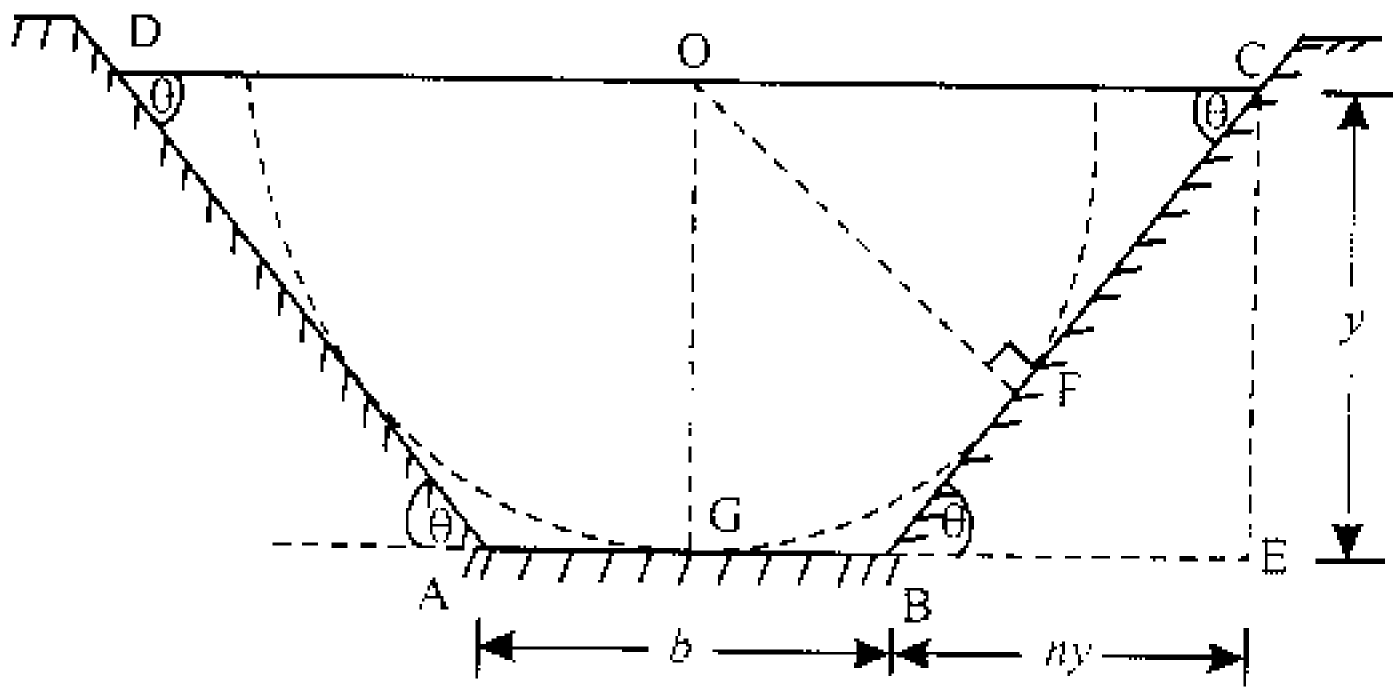


Fig. 16.10. Most economical section of a trapezoidal channel.

Let, θ = Angle made by the sloping side with the horizontal,

O = Centre of the top width DC, and

OF = A perpendicular to the sloping side BC.

The $\Delta\ OCF$ is then a right angled triangle with $\angle OCF = \theta$

$$\therefore \quad \sin\theta = \frac{OF}{OC} \quad \text{or} \quad OF = OC\sin\theta \qquad \text{...(iv)}$$

Also, from $\Delta\ BCE$, $\sin\theta = \dfrac{CE}{BC} = \dfrac{y}{\sqrt{y^2 + n^2y^2}} = \dfrac{y}{y\sqrt{n^2+1}} = \dfrac{1}{\sqrt{n^2+1}}$

Substituting the value of $\sin\theta$ in eqn. (*iv*), we have

$$OF = OC \times \frac{1}{\sqrt{n^2+1}} = y\sqrt{n^2+1} \times \frac{1}{\sqrt{n^2+1}} = y, \text{ depth of flow}$$

$$\left[\because OC = \text{Half of top width} = \frac{b+2ny}{2} = y\sqrt{n^2+1} \text{ ...Eqn. (16·14)}\right]$$

Thus a circle with centre O and radius equal to the depth of flow will be *tangential* to the three sides of a most economical trapezoidal section; this condition stipulates that the most economical section of a trapezoidal channel will be a *half-hexagon*.

Hence *conditions for most economical trapezoidal section are:*

1. $\dfrac{b+2ny}{2} = y\sqrt{n^2+1}$ (*i.e.* Half of top width = One of the sloping sides)
2. Hydraulic radius, $R = \dfrac{y}{2}$
3. A semicircle drawn from O with radius equal to depth of flow will touch the three sides of the trapezoidal channel.

Best side slope for most economical trapezoidal section:

Side slope will be the best when the section is most-economical or when the wetted perimeter is minimum. For that $\dfrac{dP}{dn} = 0$

$$\therefore \quad \frac{d}{dn}\left[\frac{A}{y} - ny + 2y\sqrt{n^2+1}\right] = 0$$

$$\left[\because P = \frac{A}{y} - ny + 2y\sqrt{n^2+1} \text{ ...From eqn. (iv)}\right]$$

or, $$-y + 2y \times \frac{1}{2}(n^2+1)^{\frac{1}{2}-1} \times 2n = 0$$

$$-y + 2ny \times \frac{1}{\sqrt{n^2+1}} = 0$$

Cancelling y and rearranging, we have

$$2n = \sqrt{n^2+1}$$

Squaring both sides, we have

$$4n^2 = n^2 + 1 \quad \text{or} \quad 3n^2 = 1$$

or, $$n = \frac{1}{\sqrt{3}} \qquad ...(16{\cdot}16)$$

If the sloping side makes an angle θ with the horizontal, then

$$\tan\theta = \frac{1}{n} = \sqrt{3} = \tan 60°$$

$\therefore$ $$\theta = 60° \qquad ...(16{\cdot}17)$$

Hence *best side slope is at 60° to the horizontal.*

For the most economical section,

Half of top width = Length of the sloping side

$$\frac{b + 2ny}{2} = y\sqrt{n^2+1}$$

Substituting the value of side slope $n = \frac{1}{\sqrt{3}}$ in the above eqn. we get

$$\frac{b + 2 \times \frac{1}{\sqrt{3}}y}{2} = y\sqrt{(1/\sqrt{3})^2 + 1} = \frac{2y}{\sqrt{3}}$$

or, $$\frac{\sqrt{3}b + 2y}{2 \times \sqrt{3}} = \frac{2y}{\sqrt{3}}$$

or, $$\sqrt{3}b + 2y = 4y \quad \text{or} \quad b = \frac{2y}{\sqrt{3}} \qquad ...([illegible])$$

Now, wetted perimeter, $$P = b + 2y\sqrt{n^2+1}$$

$$= \frac{2y}{\sqrt{3}} + 2y\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} \qquad \left(\because b = \frac{2y}{\sqrt{3}} \text{ and } n = \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2y}{\sqrt{3}} + 2y \times \frac{2}{\sqrt{3}} = \frac{6y}{\sqrt{3}} = 3 \times \frac{2y}{\sqrt{3}} = 3b$$

i.e. $$P = 3b \qquad \left(\because b = \frac{2y}{\sqrt{3}}\right)$$

Thus for a *side slope of 60°, the length of sloping side is equal to the base width of the trapezoidal section.*

Example 16.8. *A trapezoidal channel has side slopes of 3 horizontal to 4 vertical and the slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at 0·5 m³/s. Take Chezy's constant as 80.* **[Panjab University]**

Solution. Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{3}{4}$

Bed slope, $S = \frac{1}{2000}$

Discharge $Q = 0{\cdot}5 \text{ m}^3/\text{s}$

Chezy's constant, $C = 80$

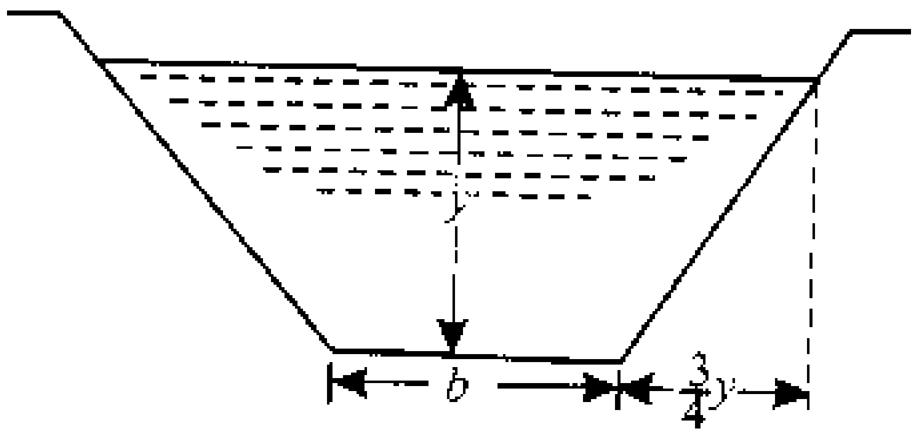


Fig. 16.11

Optimum dimensions of the channel:

For the most economical section, using eqn. (16·14), we have

$$\frac{b + 2ny}{2} = y\sqrt{n^2 + 1}$$

[where, b = base width of the channel section, and y = depth of flow.]

or, $$\frac{b \times 2 \times \frac{3}{4}y}{2} = y\sqrt{\left(\frac{3}{4}\right)^2 + 1} = \frac{5}{4}y$$

or, $$\frac{b + 1{\cdot}5y}{2} = 1{\cdot}25y \quad \text{or} \quad b = 2 \times 1{\cdot}25y - 1{\cdot}5y = y$$

i.e. $$b = y \qquad ...(i)$$

Also discharge, $Q = AC\sqrt{RS}$

[where, R = hydraulic radius, and S = bed slope]

$$0{\cdot}5 = A \times 80\sqrt{\frac{y}{2} \times \frac{1}{2000}} \qquad \left(\because R = \frac{y}{2}\right)$$

But area, $$A = (b + ny) \times y$$

$$= \left(y + \frac{3}{4}y\right) \times y = 1{\cdot}75y^2 \qquad [\because b = y \text{ ...eqn. } (i)]$$

$\therefore$ $$0{\cdot}5 = 1{\cdot}75y^2 \times 80\sqrt{\frac{y}{2} \times \frac{1}{2000}} = 2{\cdot}2136y^{5/2}$$

$\therefore$ $$y = \left(\frac{0{\cdot}5}{2{\cdot}2136}\right)^{2/5} = \mathbf{0{\cdot}55 \text{ m}} \text{ (Ans.)}$$

$$b = y = \mathbf{0.55 \text{ m}} \text{ (Ans.)}$$

$\therefore$ Optimum dimensions of the channel are:

Width (b) = *Depth of flow* (y) = **0.55 m (Ans.)**

Example 16.9. *A trapezoidal channel having the side slope equal to 60° with the horizontal and laid on a slope of 1 in 750, carries a discharge of 10 m³/s. Find the width at the base and depth of flow for most economical section. Take the value of Chezy's resitance co-efficient C = 66.* **[AMIE]**

Solution. Bed slope, $S = \frac{1}{750}$

Discharge, $Q = 10 \text{ m}^3/\text{s}$

Chezy's constant, $C = 66$

Side slope with the horizontal = 60°

Dimensions for most economical section, b and y:

For a trapezoidal channel of most economical (optimum) cross-section, the geometric parameters have the following proportions:

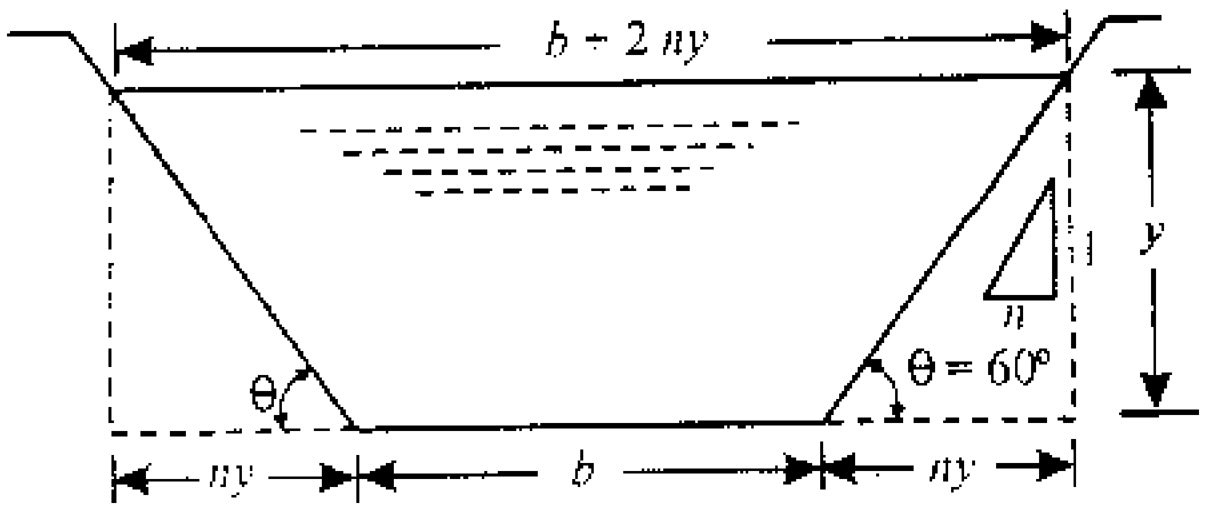


Fig. 16.12

(*i*) $\dfrac{b + 2ny}{2} = y\sqrt{n^2 + 1}$,

(*ii*) $R = \dfrac{y}{2}$

(*iii*) $\tan\theta = \tan 60° = \sqrt{3} = \dfrac{1}{n}$ or $n = \dfrac{1}{\sqrt{3}}$

Thus from (*i*) and (*iii*), we have

$$\frac{b - 2 \times \frac{1}{\sqrt{3}} y}{2} = y\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1};\ \frac{\sqrt{3}b + 2y}{2\sqrt{3}} = \frac{2y}{\sqrt{3}} \text{ or } \sqrt{3}b + 2y = 4y \text{ or } b = \frac{2}{\sqrt{3}}y$$

Area of flow, $A = (b + ny)y = \left(b + \dfrac{1}{\sqrt{3}}y\right)y = \left(\dfrac{2}{\sqrt{3}}y + \dfrac{1}{\sqrt{3}}y\right)y = \sqrt{3}y^2$

Also, discharge, $Q = AC\sqrt{RS}$

(where, R = hydraulic radius)

$$10 = \sqrt{3}\,y^2 \times 66\sqrt{\frac{y}{2} \times \frac{1}{750}} = 2\cdot95\ y^{5/2}$$

$\therefore$ *Depth of flow,* $y = \left(\dfrac{10}{2\cdot95}\right)^{2/5}$ **= 1·63 m (Ans.)**

Base width, $b = \dfrac{2}{\sqrt{3}}y = \dfrac{2}{\sqrt{3}} \times 1\cdot63$ **= 1·88 m (Ans.)**

Example 16.10. *An open channel of most economical section, having the form of a half hexagon with horizontal bottom is required to give a maximum discharge of 20·2 m³/s of water. The slope of the channel bottom is 1 in 2500. Taking Chezy's constant, C = 60 in Chezy's equation, determine the dimensions of the cross-section.*

Solution. Maximum Discharge,

$Q = 20\cdot2\ m^3/s$

Bed slope, $S = \dfrac{1}{2500}$

Chezy's constant $C = 60$

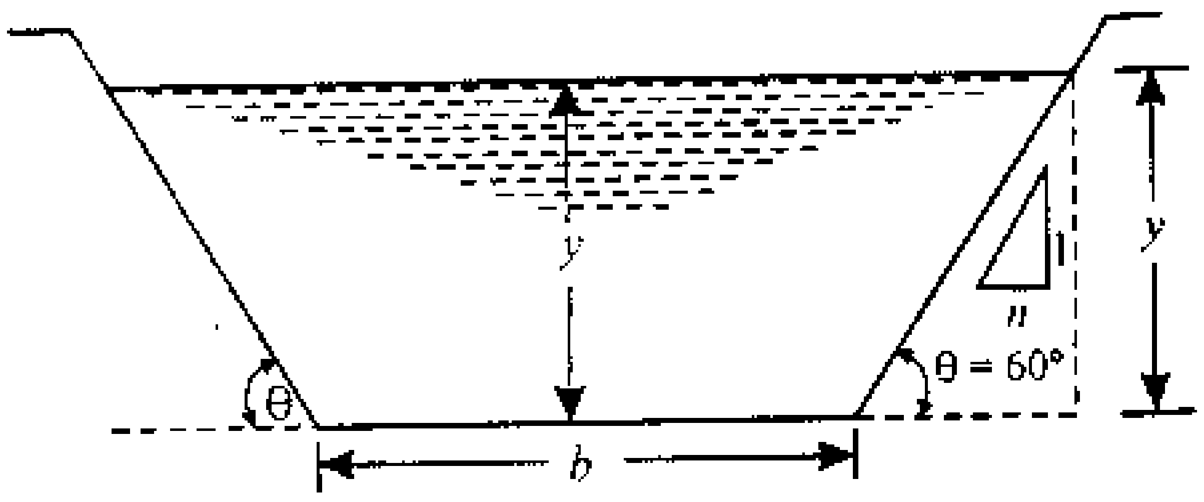


Fig. 16.13

Dimensions of the cross-section:

Since the channel has a form of a *half hexagon* (Fig. 16·13), therefore, the angle made by the sloping side with the horizontal is 60°.

$\therefore$ $\tan\theta = \tan 60° = \sqrt{3} = \dfrac{1}{n}$ or $n = \dfrac{1}{\sqrt{3}}$

For most economical section the following conditions should be satisfied:

(*i*) $\frac{b+2ny}{2} = y\sqrt{n^2+1}$, (*ii*) $R = \frac{y}{2}$, (*iii*) $n = \frac{1}{\sqrt{3}}$

Now, $\frac{b+2ny}{2} = y\sqrt{n^2+1}$

or, $\frac{b + 2\times\frac{1}{\sqrt{3}}y}{2} = y\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1}$ or $\frac{\sqrt{3}b+2y}{2\sqrt{3}} = \frac{2y}{\sqrt{3}}$ or $b = \frac{2}{\sqrt{3}}y$

Area of flow, $A = (b+ny)\,y = \left(b+\frac{1}{\sqrt{3}}y\right)y = \left(\frac{2}{\sqrt{3}}y+\frac{1}{\sqrt{3}}y\right)y = \sqrt{3}y^2$

Also, discharge, $Q = AC\sqrt{RS}$, where R is hydraulic radius;

$\therefore$ $20\cdot2 = \sqrt{3}y^2\times 60\sqrt{\frac{y}{2}\times\frac{1}{2500}} = 1\cdot47\ y^{5/2}$

or, $y = \left(\frac{20\cdot2}{1\cdot47}\right)^{2/5} = \mathbf{2\cdot852}$ **(Ans.)**

$b = \frac{2}{\sqrt{3}}y = \frac{2}{\sqrt{3}}\times 2\cdot852 = \mathbf{3\cdot29\ m}$ **(Ans.)**

Example 16.11. *A power canal of trapezoidal section has to be excavated through hard clay at least cost. Determine the dimensions of the channel, given, discharge equal to 14 m³/s, bed slope 1:2500 and Manning's N = 0·02.* **[M.U.]**

Solution. Discharge, $Q = 14$ m³/s

Bed slope, $S = \frac{1}{2500}$

Manning's $N = 0\cdot02$.

The canal can be excavated at *least cost* if the trapezoidal section is the *most economical*. The value of side slope (which is not given in this case) is given by (for most economical section),

$$n = \frac{1}{\sqrt{3}} \quad \text{...[Eqn. (16·16)]}$$

For most economical section:

$$\frac{b+2ny}{2} = y\sqrt{n^2+1} \quad \text{...[Eqn. (16·14)]}$$

(where b = base width, and y = depth of flow)

$\therefore$ $\frac{b+2\times\frac{1}{\sqrt{3}}y}{2} = y\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1} = \frac{2}{\sqrt{3}}y$ $\left(\because n = \frac{1}{\sqrt{3}}\right)$

or, $b = \frac{2}{\sqrt{3}}y\times 2 - \frac{2}{\sqrt{3}}y = \frac{2}{\sqrt{3}}y$ ...(*i*)

Area of flow, $A = (b+ny)\,y = \left(\frac{2}{\sqrt{3}}y+\frac{1}{\sqrt{3}}y\right)y = \sqrt{3}\,y^2$

Now, discharge, Q is given by,

$$Q = AC\sqrt{RS}, \quad \text{where } C = \frac{1}{N} R^{1/6}$$

or, $$Q = \sqrt{3}y^2 \times \frac{1}{N} R^{1/6} \sqrt{RS} = \sqrt{3}y^2 \times \frac{1}{N} R^{2/3} S^{1/2}$$

or, $$14 \cdot 0 = \sqrt{3}\, y^2 \times \frac{1}{0 \cdot 02} \times \left(\frac{y}{2}\right)^{2/3} \times \sqrt{\frac{1}{2500}}$$

or, $$14 \cdot 0 = 1 \cdot 732 y^2 \times y^{2/3} \times \frac{1}{(2)^{2/3}} = 1 \cdot 09\, y^{8/3}$$

or, $$y = \left(\frac{14}{1 \cdot 09}\right)^{3/8} = \mathbf{2 \cdot 6\ m\ (Ans.)}$$

$$b = \frac{2}{\sqrt{3}} y = \frac{2}{\sqrt{3}} \times 2 \cdot 6 = \mathbf{3 \cdot 0\ m\ (Ans.)}$$

Example 16.12. *Design an earthen trapezoidal channel for water having a velocity of 0·6 m/s. Side slope of the channel is 1:1·5 and quantity of water flowing is 3 m³/s. Assume C in Chezy's formula as 65.* **[Delhi University]**

Solution. Velocity of flow, $V = 0 \cdot 6$ m/s

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1 \cdot 5}{1} = 1 \cdot 5$

Discharge, $Q = 3\ \text{m}^3/\text{s}$

Chezy's constant, $C = 65$

For design, the most economical trapezoidal section is used, for which the following condition may be used:

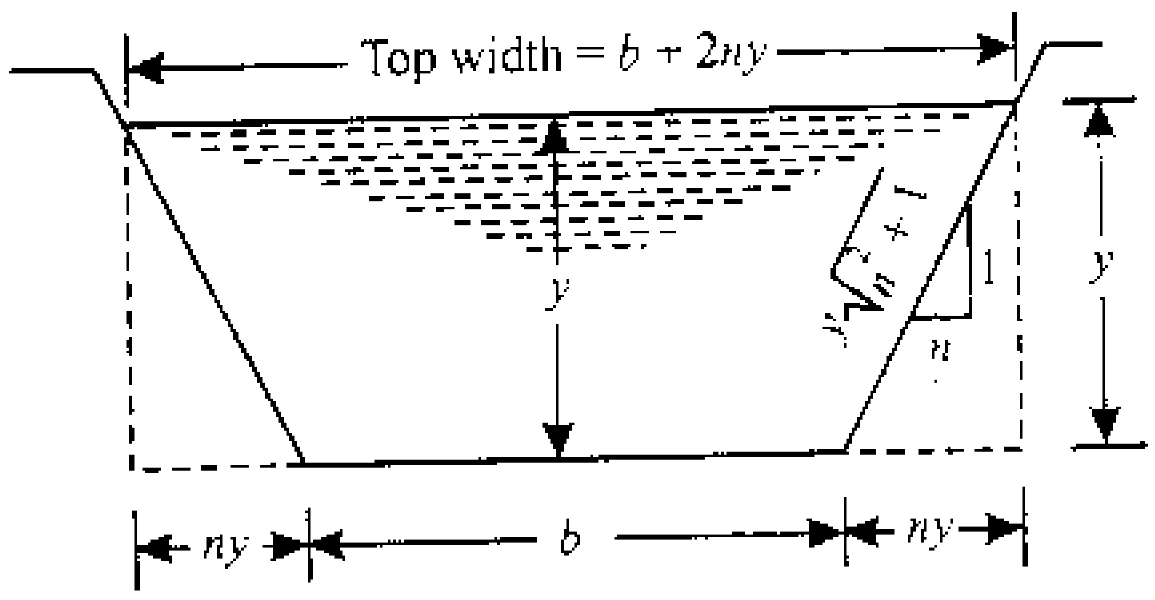


Fig. 16.14

Hydraulic radius, $$R = \frac{y}{2}$$

Area of flow, $$A = \frac{Q}{V} = \frac{3}{0 \cdot 6} = 5\ \text{m}^2$$

Wetted perimeter, $$P = b + 2y\sqrt{n^2 + 1}$$

Also, $$R = \frac{A}{P} = \frac{5}{b + 2y\sqrt{n^2+1}} = \frac{y}{2} \qquad \left(\because\ R = \frac{y}{2}\right)$$

or, $$10 = y\left[b + 2y\sqrt{1 \cdot 5^2 + 1}\right] = y\,(b + 3.6\,y) \text{ or } by + 3.6y^2$$

i.e. $$10 = by + 3 \cdot 6y^2 \qquad \ldots(i)$$

Also $A = (b + ny)\,y$

or, $$S = (b + 1 \cdot 5y)\,y \quad \text{or} \quad by + 1 \cdot 5y^2$$

or, $$by = 5 - 1 \cdot 5y^2 \qquad \ldots(ii)$$

Substituting (*ii*) in (*i*), we get

$$10 = 5 - 1 \cdot 5y^2 + 3 \cdot 6y^2 = 5 + 2 \cdot 1y^2$$

or, $$y = \left(\frac{5}{2 \cdot 1}\right)^{1/2} = 1 \cdot 543\ \text{m}$$

Substituting the value of y (= 1·543 m) in eqn. (i), we get

$$10 = b \times 1{\cdot}543 + 3{\cdot}6 \times (1{\cdot}543)^2 = 1{\cdot}543b + 8{\cdot}571$$

∴ Bottom width, $b = \dfrac{10 - 8{\cdot}571}{1{\cdot}543} = 0{\cdot}926$ m

Top width, $= b + 2ny = 0{\cdot}926 + 2 \times 1{\cdot}5 \times 1{\cdot}543 = 5{\cdot}555$ m

Velocity, $V = C\sqrt{RS}$ ... Chezy's formula

$$0{\cdot}6 = 65\sqrt{\frac{y}{2}S} \quad \text{or} \quad 0{\cdot}6 = 65\sqrt{\frac{1{\cdot}543}{2} \times S} = 57{\cdot}09\sqrt{S}$$

or, $$S = \left(\frac{0{\cdot}6}{57{\cdot}09}\right)^2 = 1{\cdot}104 \times 10^{-4} \quad \text{or} \quad \frac{1}{9054}$$

Hence specification of the trapezoidal channel would be:

Depth of flow (y) = **1.543 m**; *Slope of the bed* (S) = $\mathbf{\dfrac{1}{9054}}$

Bottom width (b) = **0.926 m**; *Top width* = **5.555 m (Ans.)**

Example 16.13. *For a trapezoidal channel with bottom width 40 m and side slopes 2H : 1V, Manning's N is 0·015 and bottom slope is 0·0002. If it carries 60 m³/s discharge, determine the normal depth.*

[Allahabad University]

Solution. Bottom width of the channel, $b = 40$ m

Side slopes = 2 H : 1 V *i.e.* $n = 2$

Manning's constant, $N = 0{\cdot}015$

Bottom/bed slope, $S = 0{\cdot}0002$

Discharge, $Q = 60$ m³/s

Normal depth, y:

Now, area $A = (b + ny)\,y = (40 + 2y) \times y$

and, perimeter, $$P = b + 2y\sqrt{n^2 + 1} = 40 + 2y\sqrt{2^2 + 1} = 40 + 2\sqrt{5}y = 40 + 4{\cdot}472y$$

∴ Hydraulic radius, $$R = \frac{A}{P} = \frac{(40 + 2y) \times y}{40 + 4{\cdot}472d}$$

Discharge, $$Q = A \times V = A \times C\sqrt{RS} \quad \text{where, Chezy's constant, } C = \frac{1}{N}R^{1/6}$$

∴ $$Q = A \times \frac{1}{N}R^{1/6}\sqrt{RS} = A \times \frac{1}{N} \times R^{2/3} \times S^{1/2}$$

or, $$60 = (40 + 2y)\,y \times \frac{1}{0{\cdot}015} \times \left[\frac{(40 + 2y) \times y}{40 + 4{\cdot}472y}\right]^{2/3} \times (0{\cdot}0002)^{1/2}$$

$$= \frac{[(40 + 2y) \times y]^{5/3}}{0{\cdot}015 \times (40 + 4{\cdot}472)^{2/3}} \times 0{\cdot}01414$$

∴ $$\frac{60 \times 0{\cdot}015 \times (40 + 4{\cdot}472y)^{2/3}}{0{\cdot}01414} = [(40 + 2y) \times y]^{5/3}$$

or, $$63{\cdot}65\,(40 + 4{\cdot}472y)^{2/3} = [(40 + 2y) \times y]^{5/3}$$

or, $$(40y + 2y^2)^{5/3} - 63{\cdot}65\,(40 + 40{\cdot}472y)^{2/3} = 0$$

By hit and trial method, we get

y = **1.31 m (Ans.)**

Example 16.14. *A trapezoidal channel with side slopes of 1 : 1 has to be designed to convey 10 m³/s at a velocity of 2 m/s, so that the amount of concrete lining for the bed and sides is minimum.*

(*i*) *Calculate the area of lining required for one metre length of the canal.*

(*ii*) *If the rugosity co-efficient N = 0·015, calculate the bed slope of the canal for uniform flow.*

[UPSC Exams.]

Solution. Side slope, $n = 1$

Discharge, $Q = 10$ m³/s

Velocity, $V = 2$ m/s

Rugosity/Manning's co-efficient, $N = 0{\cdot}015$

Area of flow, $\frac{Q}{V} = \frac{10}{2} = 5 \text{ m}^2$

Lining required for one metre length of the canal:

For minimum amount of concrete lining, the wetted perimeter must be minimum; for this condition we have (for a trapezoidal channel):

$$(i) \quad \frac{b + 2ny}{2} = y\sqrt{n^2 + 1} \quad \text{and} \quad (ii) \quad R = \frac{y}{2}$$

(where b = base width, y = depth of flow, and R = hydraulic radius)

$$\therefore \quad \frac{b + 2 \times 1 \times y}{2} = y\sqrt{1 + 1} = \sqrt{2}\, y$$

or, $\quad b + 2y = 2\sqrt{2}y \quad$ or $\quad b = 2y(\sqrt{2} - 1) = 0{\cdot}828y$

Area of flow, $\quad A = (b + ny)\, y$

or, $\quad 5 = (0{\cdot}828\, y + y)\, y = 1{\cdot}828y^2$

or, $$y = \left(\frac{5}{1{\cdot}828}\right)^{1/2} = 1{\cdot}65 \text{ m}$$

$$b = 0{\cdot}828y = 0{\cdot}828 \times 1{\cdot}65 = 1{\cdot}37 \text{ m}$$

Area of lining per metre length

$$= P \times 1 = (b + 2y\sqrt{n^2 \times 1}) \times 1 \quad (\because P = b + 2y\sqrt{n^2 + 1})$$

$$= 1{\cdot}37 + 2 \times 1{\cdot}65\sqrt{1 + 1} = \mathbf{6.04 \text{ m (Ans.)}}$$

(*ii*) **Bed slope of the canal, S:**

$$Q = AC\sqrt{RS}, \quad \text{where} \quad C = \frac{1}{N} R^{1/6}$$

or, $$Q = A \times \frac{1}{N} R^{1/6} \sqrt{RS} = A \times \frac{1}{N} R^{2/3} S^{1/2}$$

or, $$10 = 5 \times \frac{1}{0{\cdot}015} \times \left(\frac{1{\cdot}65}{2}\right)^{2/3} S^{1/2} = 293{\cdot}2\, S^{1/2}$$

$$\therefore \quad S = \left(\frac{10}{293{\cdot}2}\right)^2 = 1{\cdot}163 \times 10^{-3}$$

or, **1 in 860 (Ans.)**

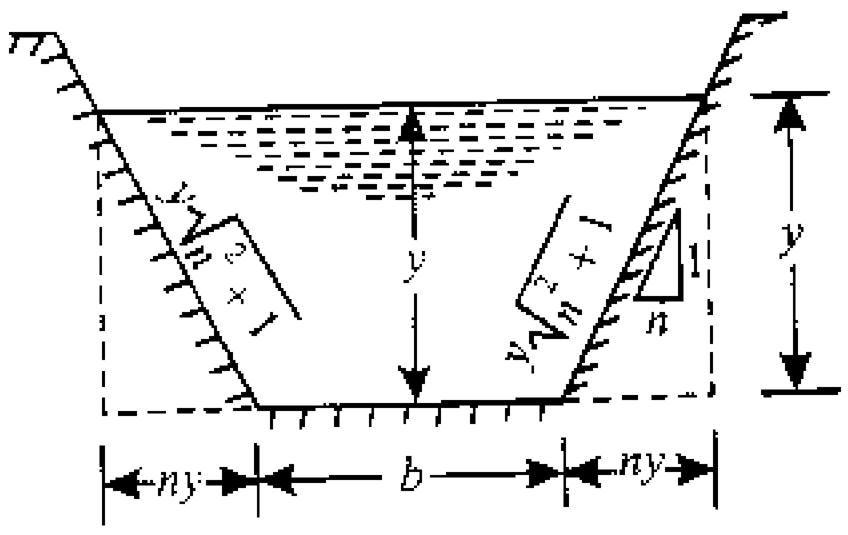


Fig. 16.15

Example 16.15. *A trapezoidal channel is required to carry 8 m^3/s of water at a velocity of 2 m/s. Find the most economical cross-section if the channel has side slopes 1 horizontal to 2 vertical. For the same discharge what saving in power would result if this trapezoidal section is replaced by a rectangular section 1·5 m deep and 4 m wide. Take Chezy's constant C = 55.*

Solution. For trapezoidal channel:

Discharge, $Q = 8\ m^3/s$

Velocity of flow, $V = 2$ m/s

∴ Area of flow, $A = Q/V = 8/2 = 4\ m^2$

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1}{2}$

Chezy's constant $C = 55$.

The trapezoidal channel section will be *most economical*, when

Half of top width = Length of one sloping side

or, $$\frac{b + 2ny}{2} = y\sqrt{n^2 + 1}$$

(where b = base width, y = depth of flow)

or, $$\frac{b + 2 \times \frac{1}{2}y}{2} = y\sqrt{\left(\frac{1}{2}\right)^2 + 1} = \frac{\sqrt{5}}{2}y$$

or, $$b + y = \sqrt{5}\,y \quad \text{or} \quad b = y(\sqrt{5} - 1) = 1 \cdot 236y \qquad ...(i)$$

Area of flow, $A = (b + ny)\,y = \left(b + \frac{1}{2}y\right)y$

or, $$4 = (1 \cdot 236y + 0 \cdot 5y)\,y = 1 \cdot 732y^2$$

or, $$y = \left(\frac{4}{1 \cdot 736}\right)^{1/2} = \mathbf{1 \cdot 52\ m\ (Ans.)}$$

$$b = 1 \cdot 236y = 1 \cdot 236 \times 1 \cdot 52 = \mathbf{1.88\ m\ (Ans.)}$$

Also hydraulic radius, R (for most economical section) $= y/2 = \frac{1 \cdot 52}{2} = 0 \cdot 76$ m

Now, velocity, $V = C\sqrt{RS}$, where S is the bed slope

∴ $$2 = 55\sqrt{0 \cdot 76 \times S} \quad \text{or} \quad S = \left(\frac{2}{55}\right)^2 \times \frac{1}{0 \cdot 76} = 1 \cdot 739 \times 10^{-3} = 1 \text{ in } 575$$

For rectangular channel:

Base width, $b = 4$ m

Depth of flow, $y = 1 \cdot 5$ m

∴ Area of flow, $A = b \times y = 4 \times 1 \cdot 5 = 6 \cdot 0\ m^2$

Wetted perimeter, $P = b - 2y = 4 + 2 \times 1 \cdot 5 = 7$ m

Hydraulic radius, $R = \frac{A}{P} = \frac{6}{7} = 0 \cdot 857$ m

Flow velocity, $V = \frac{Q}{A} = \frac{8}{6} = 1 \cdot 333$ m/s

From Chezy's formula, $V = C\sqrt{RS}$

or, $$1{\cdot}333 = 55\sqrt{0{\cdot}857 \times S}$$

or, $$S = \left(\frac{1{\cdot}333}{55}\right)^2 \times \frac{1}{0{\cdot}857} = 6{\cdot}854 \times 10^{-4} = 1 \text{ in } 1459$$

$\therefore$ Saving in head *per kilometre of channel run*

$$h = (1{\cdot}739 \times 10^{-3} - 6{\cdot}854 \times 10^{-4}) \times 1000 = 1{\cdot}054 \text{ m}$$

Hence, *saving in power* $= \dfrac{wQh}{1000}$ kW $= \dfrac{9810 \times 8 \times 1{\cdot}054}{1000} = \mathbf{82{\cdot}7\ kW}$ **(Ans.)**

Example 16.16. *A hydraulically efficient trapezoidal channel has side slopes of 1 : 1. It is required to discharge 14 m³/s with a gradient (channel slope) of 1 in 1000. If unlined, the value of Chezy's C is 45. If lined with concrete, the value is 70. If the least cost per m³ of excavation is three times the cost m² of lining, will the lined or the unlined channel be cheaper?* **[Sagar University]**

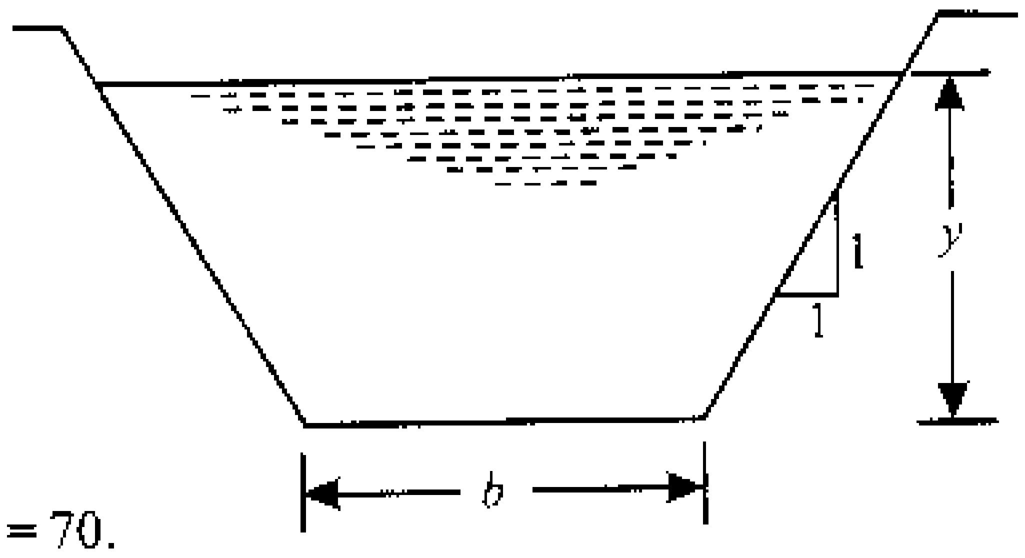


Fig. 16.16

Solution. Side slope, $n = \dfrac{1}{1} = 1$

Discharge, $Q = 14 \text{ m}^3/\text{s}$

Bed slope, $S = \dfrac{1}{1000}$

Chezy's constant C: Unlined channel = 45,
Channel lined with concrete = 70.

Hydraulically efficient trapezoidal channel must satisfy the following *conditions*:

(*i*) $\dfrac{b + 2ny}{2} = y\sqrt{n^2 + 1}$, and (*ii*) $R = \dfrac{y}{2}$

where, b = Base width of channel,
y = Depth of flow, and
R = Hydraulic radius

or, $$\frac{b + 2y}{2} = y\sqrt{1+1} = \sqrt{2}\,y \quad \text{or} \quad b + 2y = 2\sqrt{2}\,y$$

or, $$b = 2y(\sqrt{2} - 1) = 0{\cdot}828\,y$$

Unlined channel:

$$Q = AC\sqrt{RS} = (b + ny)\,y \times C\sqrt{\frac{y}{2}S}$$

$$14 = (0{\cdot}828y + y)\,y \times 45 \times \sqrt{\frac{y}{2} \times \frac{1}{1000}} = 1{\cdot}839\,y^{5/2} \quad (\because n = 1)$$

or, $$y = \left(\frac{14}{1{\cdot}839}\right)^{2/5} = 2{\cdot}25 \text{ m}$$

$$b = 0{\cdot}828\,y = 0{\cdot}828 \times 2{\cdot}25 = 1{\cdot}863 \text{ m}$$

Let, cost of lining per m^2 of the channel surface = K

Then, cost of excavation per m^3 = 3 K ...(*Given*)

Consider one metre length of channel.

Amount of excavation $= A \times 1 = (b + ny)\, y \times 1 = (1{\cdot}863 + 1 \times 2{\cdot}25) \times 2{\cdot}25 \times 1 = 9{\cdot}254\ \text{m}^2$

$\therefore$ The cost of excavation $= 3\ \text{K} \times 9{\cdot}254 = 27{\cdot}76\ \text{K}$

Lined channel:

$$Q = AC\sqrt{RS} = (b + ny)\, y \times C\sqrt{\frac{y}{2} S}$$

or,
$$14 = (0{\cdot}828y + y)\, y \times 70 \times \sqrt{\frac{y}{2} \times \frac{1}{1000}} = 2{\cdot}86\, y^{5/2}$$

or,
$$y = \left(\frac{14}{2{\cdot}86}\right)^{2/5} = 1{\cdot}88\ \text{m}$$

$$b = 0{\cdot}828y = 0{\cdot}828 \times 1{\cdot}88 = 1{\cdot}55\ \text{m}$$

Considering one metre length of channel, cost of excavation

$$= A \times 1 \times 3\text{K} = (b + ny)\, y \times 3\text{K} = (1{\cdot}55 + 1 \times 1{\cdot}88) \times 1{\cdot}88 \times 3\text{K} = 19{\cdot}34\ \text{K}$$

Cost of lining = Perimeter $(P) \times 1 \times \text{K} = (b + 2y\sqrt{n^2 + 1}) \times 1 \times \text{K}$

$$= (1{\cdot}55 + 2 \times 1{\cdot}88\sqrt{1 + 1})\, \text{K} = 6{\cdot}86\ \text{K}$$

$\therefore$ Total cost of lined channel $= 19{\cdot}34\ \text{K} + 6{\cdot}86\text{K} = 26{\cdot}2\text{K}$

Since the cost of excavation of unlined channel (27·76K) is greater than the total cost of lined channel, hence the **lined channel is cheaper. (Ans.)**

Example 16.17. *Design a concrete lined channel to carry a discharge of 500 m³/s at a slope of 1 in 4000. The side slopes of channel may be taken as 1:1. The Manning's roughness co-efficient for the lining is 0·014. Assume the permissible velocity in the section as 2·5 m/s.* **[UPSC Exams.]**

Solution. Discharge, $Q = 500\ \text{m}^3/\text{s}$

Bed slope, $S = \dfrac{1}{4000}$

Side slope, $n = \dfrac{1}{1} = 1$

Manning's roughness co-efficient, $N = 0{\cdot}0014$

Permissible velocity, $V = 2{\cdot}5\ \text{m/s}$

Base width (b) and depth of flow (y):

Area of flow, $A = \dfrac{Q}{V} = \dfrac{500}{2{\cdot}5} = 200\ \text{m}^2$

Also, $A = (b + ny)y = (b + y)y \qquad (\because n = 1)$

or, $A = (b + y)\, y = 200 \qquad \ldots(i)$

Perimeter, $P = b + 2y\sqrt{n^2 + 1} = b + 2y\sqrt{1 + 1} = b + 2\sqrt{2}\, y$

Hydraulic radius, $R = \dfrac{A}{P} = \dfrac{200}{b + 2\sqrt{2}y}$

Discharge, $Q = AC\sqrt{RS}$, where $C = \dfrac{1}{N} R^{1/6}$

or, $Q = A \times \dfrac{1}{N} R^{1/6}\sqrt{RS} = A \times \dfrac{1}{N} R^{2/3}\sqrt{S}$

Substituting the values, we get

$$500 = 200 \times \frac{1}{0.014} \times \left(\frac{200}{b + 2\sqrt{2}y}\right)^{2/3} \times \sqrt{\frac{1}{4000}} = \frac{7724.88}{(b + 2\sqrt{2}y)^{2/3}}$$

or, $$b + 2\sqrt{2}y = \left(\frac{7724.88}{500}\right)^{3/2} = 60.73 \quad ...(ii)$$

From eqn. (*i*), $(b + y) = \frac{200}{y}$ or $b = \frac{200}{y} - y$

Substituting this value of b in eqn. (*ii*), we get

$$\frac{200}{y} - y + 2\sqrt{2}y = 60.73 \text{ or } 200 - y^2 + 2\sqrt{2}y^2 = 60.73y$$

or, $$y^2(2\sqrt{2} - 1) - 60.73y + 200 = 0 \text{ or } 1.828y^2 - 60.73y + 200 = 0$$

or, $$y^2 - 33.22y + 109.41 = 0$$

or, $$y = \frac{33.22 \pm \sqrt{(33.22)^2 - 4 \times 109.41}}{2} = \frac{33.22 \pm 25.8}{2} = 29.51 \text{ m}; 3.71 \text{ m}$$

$y = 3.71$ m (rejecting the first value, being impracticable)

and, $$b = \frac{200}{y} - y = \frac{200}{3.71} - 3.71 = 50.2 \text{ m}$$

Hence, *width of the channel*, **b = 50.2 m (Ans.)**

and, *depth of flow*, **y = 3.71 m (Ans.)**

Example 16.18. *A trapezoidal canal is to carry 45 m³/s with a mean velocity of 0.6 m/s. One side of canal is vertical and the other has a slope of 2 horizontal to 1 vertical. Find the minimum hydraulic slope, if Manning's N = 0.013.* **(AMIE)**

Solution. *Given:* $Q = 45$ m³/s, $V_{mean} = 0.6$ m/s; $n = 2$; Manning's $N = 0.013$

Minimum hydraulic slope, S:

Refer Fig. 16.17.

Area of flow, $$A = \left(\frac{AB + CD}{2}\right) \times y = \left(\frac{b + (b + ny)}{2}\right) y$$

$$= \left[\frac{b + (b + 2y)}{2}\right] y = (b + y)\, y$$

But, $$A = \frac{Q}{V_{mean}} = \frac{45}{0.6} = 75 \text{ m}^2$$

$\therefore \quad (b + y)y = 75$

or, $$b = \frac{75}{y} - y \quad ...(i)$$

Wetted perimeter, $$P = b + y\sqrt{n^2 + 1} + y$$

$$= b + y\{\sqrt{n^2 + 1} + 1\}$$

or, $$P = b + y\{\sqrt{5} + 1\} \quad (\because n = 2)$$

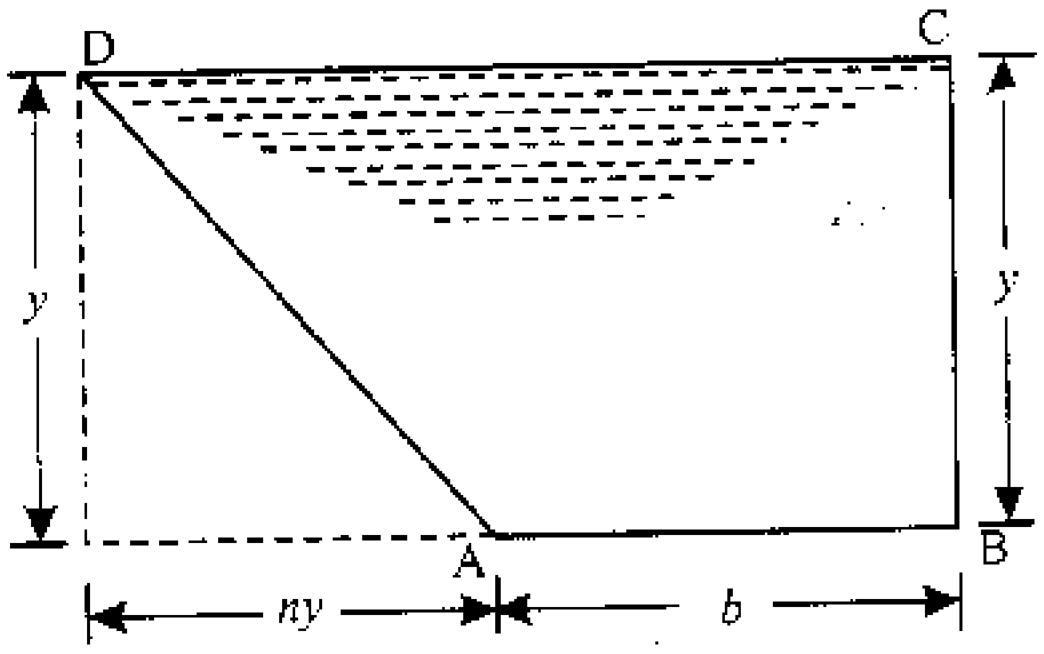


Fig. 16.17

Hydraulic radius, $R = \frac{A}{P} = \frac{75}{b + y\{\sqrt{5} + 1\}}$

$$= \frac{75}{\left(\frac{75}{y} - y\right) + y\sqrt{5} + y}$$

$$= \frac{75}{\frac{75}{y} + y\sqrt{5}}$$

Now, discharge (Q) is given by,

$$Q = AC\sqrt{RS} \quad \text{where, } C = \frac{1}{N}(R)^{\frac{1}{6}}$$

or, $$Q = A \times \frac{1}{N}(R)^{1/6}\sqrt{RS} = A \times \frac{1}{N}(R)^{\left(\frac{1}{6}+\frac{1}{2}\right)}\sqrt{S} = A \times \frac{1}{N}(R)^{2/3}\sqrt{S}$$

or, $$\sqrt{S} = \frac{QN}{A(R)^{2/3}} = \frac{45 \times 0.013}{75(R)^{2/3}} \qquad ...(i)$$

For S to be minimum, R has to be maximum.

or, $\frac{75}{\frac{75}{y} + y\sqrt{5}}$ is to be maximum

or, $\frac{75}{y} + y\sqrt{5}$ is to be minimum

or, $$\frac{d}{dy}\left[\frac{75}{y} + y\sqrt{5}\right] = 0$$

$$-\frac{75}{y^2} + \sqrt{5} = 0 \quad \text{or} \quad y^2 = \frac{75}{\sqrt{5}}$$

or, $y = 5.79$ m

Hence, for minimum slope,

$$R = \frac{75}{\frac{75}{y} + y\sqrt{5}} = \frac{75}{\frac{75}{5.79} + 5.79\sqrt{5}} = 2.896 \text{ m}$$

The minimum hydraulic slope is obtained by substituting their value of R in (*i*).

$$\therefore \quad \sqrt{S_{min}} = \frac{45 \times 0.013}{75 \times (2.896)^{2/3}} = 3.839 \times 10^{-3}$$

or, $S_{min} = (3.839 \times 10^{-3})^2 = \mathbf{1.474 \times 10^{-5}}$ **(Ans.)**

Example 16.19. *A trapezoidal channel having a cross-sectional area A_1, wetted perimeter P_1, Manning's co-efficient N, and laid to a slope S carries a discharge Q, at a depth of flow equal to y. To increase the discharge, the base width of the channel is widened by x, keeping all other parameters same. Prove that*

$$\left(\frac{Q_2}{Q_1}\right)^3 \times \left(1 + \frac{x}{P_1}\right)^2 = \left(1 + \frac{xy}{A_1}\right)^5$$

where, Q_2 is the new discharge in the channel. **[UPSC Exams.]**

Solution. The Chezy's constant (C), using Manning's formula, is given by,

$$C = \frac{1}{N}(R)^{1/6}$$

$\therefore$ Velocity of flow, $V = C\sqrt{RS} = \frac{1}{N} R^{1/6} \times (RS)^{1/2} = \frac{R^{2/3} S^{1/2}}{N}$

where, R = Hydraulic radius (or hydraulic mean depth), and
S = Slope of the channel bed.

Discharge $$Q = AV = A \times \frac{R^{2/3} S^{1/2}}{N}$$

$$= KAR^{2/3} = KA\left(\frac{A}{P}\right)^{2/3} = K\frac{A^{5/3}}{P^{2/3}} \quad \left(\because R = \frac{A}{P}\right) \quad \ldots(i)$$

where, K (a constant) $= \frac{S^{1/2}}{N}$ (S and N kept constant)

Area of cross-section of the widened canal $A_2 = (A_1 + xy)$

Wetted perimeter of the original channel, $P_2 = (P_1 + x)$

Where A_1 and P_1 are the area of cross-section and wetted perimeter respectively of the original channel.

Then from expression (i), we have

$$Q_1 = K\frac{A_1^{5/3}}{P_1^{2/3}} \quad \text{and} \quad Q_2 = K\frac{A_2^{5/3}}{P_2^{2/3}}$$

$$\therefore \quad \frac{Q_2}{Q_1} = \left(\frac{A_2}{A_1}\right)^{5/3} \times \left(\frac{P_1}{P_2}\right)^{2/3}$$

Substituting the values of A_2 and P_2 in the above equation, we have

$$\frac{Q_2}{Q_1} = \left(\frac{A_1 + xy}{A_1}\right)^{5/3} \times \left(\frac{P_1}{P_1 + x}\right)^{2/3}$$

$$= \left(1 + \frac{xy}{A_1}\right)^{5/3} \times \left(\frac{1}{1 + \frac{x}{P_1}}\right)^{2/3}$$

Taking cube on both sides, we get

$$\left(\frac{Q_2}{Q_1}\right)^3 = \left(1 + \frac{xy}{A_1}\right)^5 \times \left(\frac{1}{1 + \frac{x}{P_1}}\right)^2$$

or, $$\left(\frac{Q_2}{Q_1}\right)^3 \times \left(1 + \frac{x}{P_1}\right)^2 = \left(1 + \frac{xy}{A_1}\right)^5 \quad \text{...Proved}$$

16.5.3. Most Economical Triangular Channel Section

Fig. 16·18 shows a triangular channel. The side slopes are n (horizontal) to 1 (vertical)

Let, y = Depth of flow, and
θ = Angle made by the sides with the vertical.

From ΔODC, $\frac{CD}{DO} = \tan\theta$ or $\frac{CD}{y} = \tan\theta$

or, $CD = y\tan\theta$

Also, $\frac{DO}{CO} = \cos\theta$ or $\frac{y}{CO} = \cos\theta$

or, $CO = y\sec\theta$

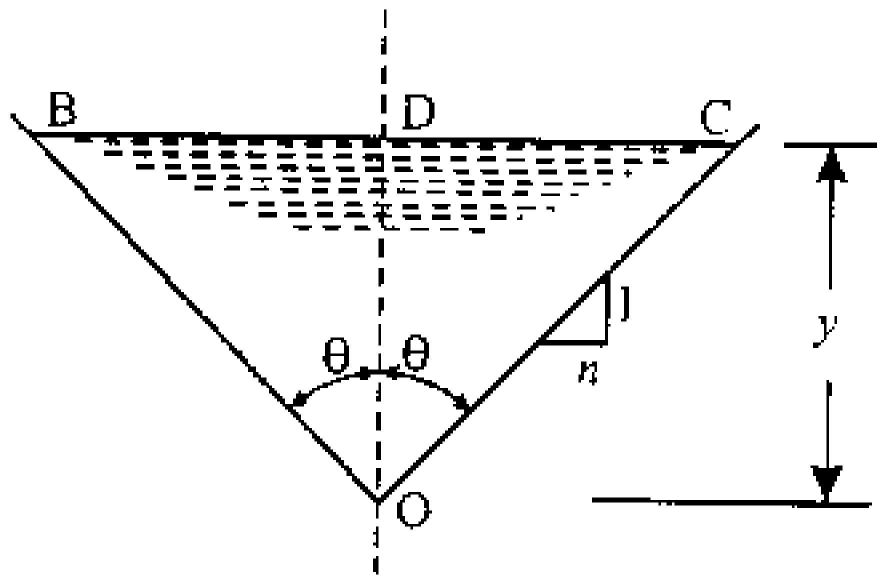


Fig. 16.18. Triangular channel.

Area of flow,

$$A = \frac{1}{2} \times BC \times DO = \frac{1}{2} \times 2CD \times DO$$

$$= \frac{1}{2} \times 2y\tan\theta \times y = y^2\tan\theta$$

i.e. $A = y^2\tan\theta$ ...(*i*)

Perimeter, $P = BO + OC = 2OC = 2y\sec\theta$ $(\because BO = OC)$ ...(*ii*)

Substituting the value of $y\left(=\sqrt{\frac{A}{\tan\theta}}\right)$ from eqn. (*i*) in eqn. (*ii*), we get

$$P = 2\sqrt{\frac{A}{\tan\theta}}\sec\theta = 2\frac{\sqrt{A}}{\sqrt{\tan\theta}}(\sec\theta) \quad ...(iii)$$

Assuming the area to be constant, eqn. (*iii*) can be differentiated with respect to θ and equated to zero for obtaining the condition for minimum *P*.

i.e. $$\frac{dP}{d\theta} = \frac{d}{d\theta}\left[2\frac{\sqrt{A}}{\sqrt{\tan\theta}}(\sec\theta)\right] = 0$$

$$= 2\sqrt{A}\left[\frac{\sqrt{\tan\theta} \times \sec\theta \cdot \tan\theta - \sec\theta \times \frac{1}{2}(\tan\theta)^{-1/2}\sec^2\theta}{\tan\theta}\right] = 0$$

$$= 2\sqrt{A}\left[\frac{\sec\theta\tan\theta}{\sqrt{\tan\theta}} - \frac{\sec^3\theta}{2(\tan\theta)^{3/2}}\right] = 0$$

or, $\sec\theta(2\tan^2\theta - \sec^2\theta) = 0$

Since $\sec\theta \neq 0$,

$\therefore$ $2\tan^2\theta - \sec^2\theta = 0$ or $2\tan^2\theta = \sec^2\theta$

or, $\sqrt{2}\tan\theta = \sec\theta$

$\therefore$ $\theta = 45°$; and $n = 1$ ...(16·18)

Hence a triangular channel section will be most economical when each of its sloping sides makes an angle of 45° with the vertical.

Hydraulic radius, $R = \frac{A}{P} = \frac{y^2\tan\theta}{2y\sec\theta}$

Substituting the value of θ from eqn. (16·18) in the above eqn., we get

$$R = \frac{y^2\tan 45°}{2y\sec 45°} = \frac{y^2}{2y \times \sqrt{2}} = \frac{y}{2\sqrt{2}} \quad ...(16·19)$$

Thus it can be seen that the *most economical triangular channel section will be a half square described on a diagonal and having equal sloping sides.*

Example 16.20. *Water flows in a channel of the shape of isosceles triangle of bed width 'a' and sides making an angle of 45° with the bed. Determine the relation between depth of flow 'd' and the bed width 'a' for maximum velocity condition and for maximum discharge condition. Use Manning's formula and note that 'd' is less than 0·5 a.* **[UPSC, CES, Exams.]**

Solution. Bed width of the channel = a

Angle of sides with the bed = 45°

Depth of flow = d ($d < 0{\cdot}5a$)

Velocity, $V = C\sqrt{RS}$

But $C = \dfrac{1}{N} R^{1/6}$, where N is Manning's constant

$$\therefore \quad V = \frac{1}{N} R^{1/6}\sqrt{RS} = \frac{1}{N} R^{2/3} S^{1/2} \qquad \ldots(i)$$

where, R = hydraulic radius, S = bed slope.

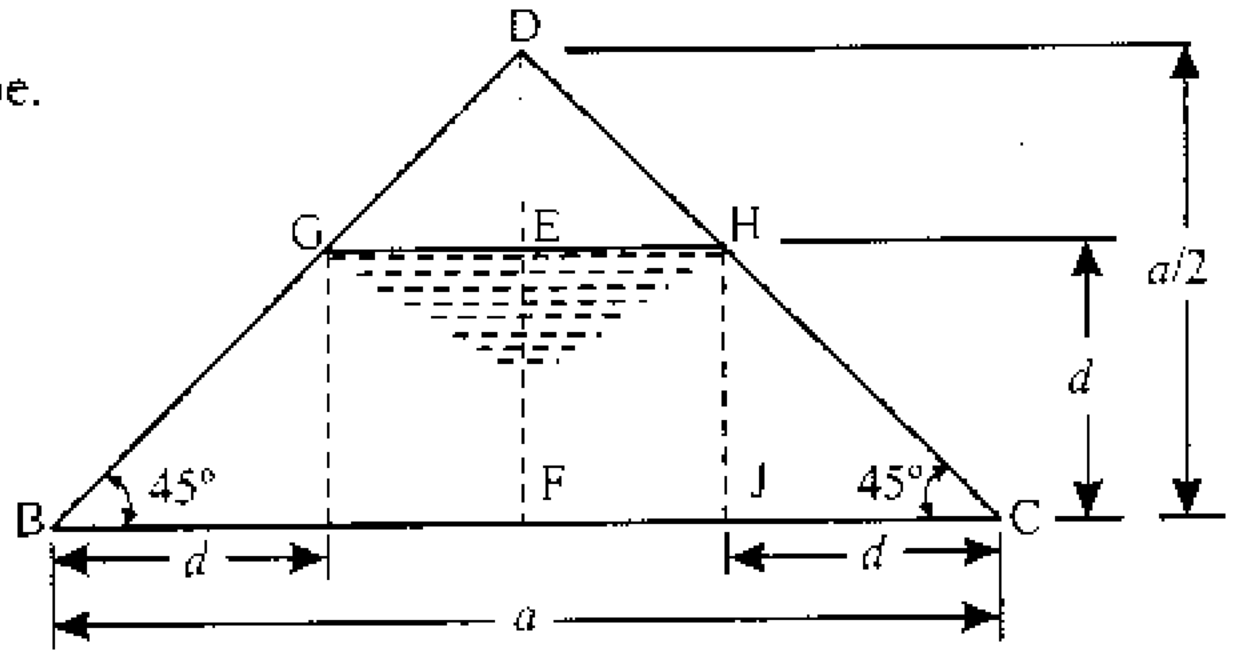


Fig. 16.19

Area of flow, $A = \left(\dfrac{GH + BC}{2}\right) \times d$

But $GH = BC - 2JC = a - 2d$

$$\left[\because \frac{HJ}{JC} = \tan 45° = 1 \quad \text{or} \quad \frac{d}{JC} = 1 \text{ or } JC = d\right]$$

$$\therefore \quad A = \left[\frac{(a-2d)+a}{2}\right] \times d = (a-d)\,d$$

Wetted perimeter, $P = BC + BG + HC = BC + 2BG$ ($\because BG = HC$)

But $BG = \sqrt{d^2 + d^2} = d\sqrt{2}$

$$\therefore \quad P = a + 2\sqrt{2}\,d$$

Hydraulic radius, $R = \dfrac{A}{P} = \dfrac{(a-d)\,d}{a + 2\sqrt{2}\,d}$

$$\therefore \quad V = \frac{1}{N}\left[\frac{(a-d)\,d}{a+2\sqrt{2}\,d}\right]^{2/3} S^{1/2}$$

For maximum velocity, $\dfrac{dV}{dd} = 0$

$$\frac{d}{dd}\left[\frac{S^{1/2}}{N}\left\{\frac{(a-d)\,d}{a+2\sqrt{2}\,d}\right\}^{2/3}\right] = 0$$

$$\frac{S^{1/2}}{N}\left[2/3\left\{\frac{(a-d)\,d}{a+2\sqrt{2}\,d}\right\}^{-\frac{1}{3}}\left\{\frac{(a+2\sqrt{2}\,d)(a-2d)-(a-d)\,d \times 2\sqrt{2}}{(a+2\sqrt{2}\,d)^2}\right\}\right] = 0$$

On simplification, we have

$$a^2 - 2ad - 2\sqrt{2}\, d^2 = 0$$

or, $$a = \frac{2d \pm \sqrt{4d^2 + 8\sqrt{2}\, d^2}}{2} = \frac{2d \pm 3 \cdot 91\, d}{2} = 2 \cdot 955\, d \quad \text{(neglecting } -ve \text{ value)}$$

or, $$\frac{d}{a} = \frac{1}{2 \cdot 955} = \mathbf{0 \cdot 338} \textbf{ (Ans.)}$$

For maximum discharge, $\frac{dQ}{dd} = 0$

$$Q = A \cdot V = (a - d)\, d \times \frac{1}{N} \left\{ \frac{(a - d)\, d}{a + 2\sqrt{2}\, d} \right\}^{2/3} S^{1/2}$$

$\therefore$ $$\frac{d}{dd} \left[(a - d)\, d \times \frac{1}{N} \left\{ \frac{(a - d)\, d}{a + 2\sqrt{2}\, d} \right\}^{2/3} S^{1/2} \right] = 0$$

or, $$\frac{d}{dd} \left[\frac{S^{1/2}}{N} \times \{(a - d)\, d\}^{5/3} \times \frac{1}{(a + 2\sqrt{2}\, d)^{2/3}} \right] = 0$$

or, $$\frac{S^{1/2}}{N} \left[\frac{(a + 2\sqrt{2}\, d)^{2/3} \times 5/3\, \{(a - d)\, d\}^{2/3} \times (a - 2d) - \{(a - d)\, d\}^{5/3} \times \left\{ \frac{2}{3} (a + 2\sqrt{2}\, d)^{-1/3} \times 2\sqrt{2} \right\}}{(a + 2\sqrt{2}\, d)^{4/3}} \right] = 0$$

On simplification, we get

$$5a^2 - 1 \cdot 5147\, ad - 22 \cdot 6274\, d^2 = 0$$

or, $$a = \frac{1 \cdot 5147\, d \pm d\sqrt{(1 \cdot 5147)^2 + 4 \times 5 \times 22 \cdot 6274}}{10}$$

$$= \frac{1 \cdot 5147\, d \pm 21 \cdot 327\, d}{10} = 2 \cdot 284\, d \quad \text{(neglecting } -ve \text{ value)}$$

or, $$\frac{d}{a} = \frac{1}{2 \cdot 284} = \mathbf{0 \cdot 4378} \textbf{ (Ans.)}$$

16.5.4. Most Economical Circular Channel Section

Circular pipes and culverts which are partly filled are treated as channels. In case of conduits the condition of area remaining constant does not hold good since both the wetted perimeter and wetted area vary with depth of flow. The most economical section (optimum section) is designed *both for conditions of maximum mean velocity and maximum flow rate.*

Velocity of flow, $V = C\sqrt{RS} = C\sqrt{\frac{A}{P} S}$ ...Chezy's formula

Discharge, $Q = AV = AC\sqrt{RS} = C\sqrt{\left(\frac{A^3}{P}\right) S}$

Thus the *flow velocity* will have a maximum value when hydraulic radius $\left(\frac{A}{P}\right)$ is maximum, and a maximum discharge is obtained when $\left(\frac{A^3}{P}\right)$ is maximum.

Fig. 16·20 shows a circular channel through which water is flowing.

Let, y = Depth of flow,

r = Radius of the channel, and

2θ = Angle subtended by water surface AB at the centre in radians.

Wetted perimeter, P = Length of arc AD

$$= \frac{2\pi r}{2\pi} \times 2\theta = 2r\theta$$

i.e. $P = 2r\theta$ ...(16·20)

Wetted area, A = Area $ADBA$

= Area of sector $OADBO$ – area of ΔOAB

$$= \frac{\pi r^2}{2\pi} \times 2\theta - \frac{1}{2} AB \times CO$$

$$= r^2\theta - \frac{1}{2} \times 2BC \times CO \quad (\because AB = 2BC)$$

$$= r^2\theta - \frac{1}{2} \times 2 \times r\sin\theta \times r\cos\theta$$

$(\because BC = r\sin\theta, CO = r\cos\theta)$

$$= r^2\theta - \frac{1}{2} r^2 \times 2\sin\theta\cos\theta$$

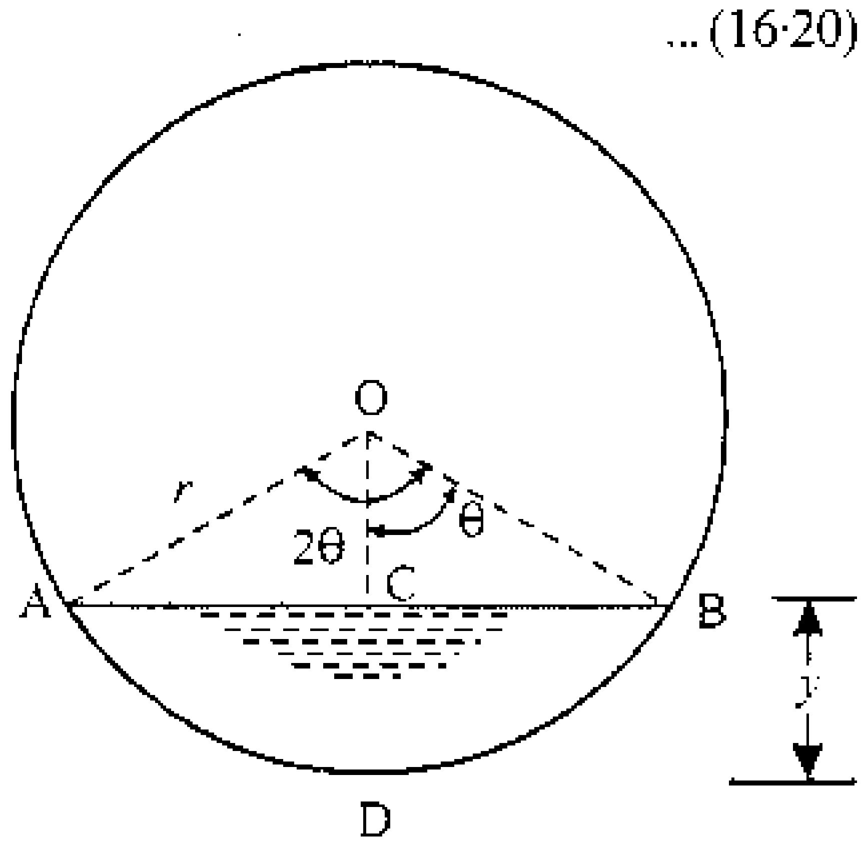


Fig. 16.20. Circular channel.

or, $$A = r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$ ...(16·21)

$(\because 2\sin\theta\cos\theta = \sin 2\theta)$

(*i*) **Condition for maximum velocity:**

The velocity will be maximum when

$$\frac{d}{d\theta}\left(\frac{A}{P}\right) = 0$$

(where A and P both are functions of θ).

or, $$\frac{P\dfrac{dA}{d\theta} - A\cdot\dfrac{dP}{d\theta}}{P^2} = 0 \quad \text{or} \quad P\frac{dA}{d\theta} - A\cdot\frac{dP}{d\theta} = 0$$ ...(*i*)

Now, $$A = r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$ [Eqn. (16·21)]

$$\frac{dA}{d\theta} = r^2\left(1 - \frac{\cos 2\theta}{2} \times 2\right) = r^2(1 - \cos 2\theta)$$

Again, $P = 2r\theta$ [Eqn. (16·20)]

$$\frac{dP}{d\theta} = 2r$$

Substituting the values of A, P, $\frac{dA}{d\theta}$ and $\frac{dP}{d\theta}$ in eqn. (*i*), we get

$$2r\theta\,[r^2(1 - \cos 2\theta)] - r^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2r = 0$$

or, $$2r^3\theta(1 - \cos 2\theta) - 2r^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

or, $$\theta (1 - \cos 2\theta) = \left(\theta - \frac{\sin 2\theta}{2}\right) = 0 \qquad \text{(cancelling } 2r^3\text{)}$$

or, $$\theta - \theta.\cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or, $$\theta.\cos 2\theta = \frac{\sin 2\theta}{2}$$

$\therefore$ $$\tan 2\theta = 2\theta$$

Solution gives : **2 θ = 257·5°** (approximately) ... by hit and trial method, or **θ = 128·75°**

Depth of flow, $y = OD - OC = r - r\cos\theta$ (Fig. 16·16)

$$= r(1 - \cos\theta) = r(1 - \cos 128{\cdot}75°) \simeq 1{\cdot}62\,r \simeq 0{\cdot}81d$$

i.e. $$\boldsymbol{y \simeq 0{\cdot}81\,d} \qquad ...(16{\cdot}22)$$

where, d = Diameter of the circular channel.

Thus *maximum velocity occurs when the depth of flow is 0·81 times the diameter of the circular channel.*

Hydraulic radius (or hydraulic mean depth) for maximum velocity,

$$R = \frac{A}{P} = \frac{r^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2r\theta} = \frac{r}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right) \qquad ...(16{\cdot}23)$$

where, $$\theta = 128{\cdot}75° = 128{\cdot}75 \times \frac{\pi}{180} = 2{\cdot}247 \text{ radians}$$

$\therefore$ $$R = \frac{r}{2 \times 2{\cdot}247}\left(2{\cdot}247 - \frac{\sin 257{\cdot}5°}{2}\right)$$

or, $$\boldsymbol{R} \simeq 0{\cdot}6086r \simeq \boldsymbol{0{\cdot}305d} \qquad ...(16.24)$$

Thus for *maximum mean velocity in a channel of circular section hydraulic radius equals* 0·305 *times the channel diameter.*

(*ii*) **Condition for maximum discharge:**

The discharge will be maximum when

$$\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0 \qquad \text{or} \qquad \frac{P \times 3A^2\frac{dA}{d\theta} - A^3\frac{dP}{d\theta}}{P^2} = 0$$

or, $$3PA^2\frac{dA}{d\theta} - A^3\frac{dP}{d\theta} = 0$$

Dividing, both sides by A^2, we get

or, $$3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0 \qquad ...(i)$$

$$P = 2r\theta \qquad \text{(Eqn. 16·20)}$$

$\therefore$ $$\frac{dP}{d\theta} = 2r$$

$$A = r^2\left(\theta - \frac{\sin 2\theta}{2}\right) \qquad \text{(Eqn. 16·21)}$$

$\therefore$ $$\frac{dA}{d\theta} = r^2(1 - \cos 2\theta)$$

Substituting the values of P, A, $\frac{dP}{d\theta}$ and $\frac{dA}{d\theta}$ in eqn. (*i*), we have

$$3 \times 2r\theta \times r^2 (1 - \cos 2\theta) - r^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2r = 0$$

or, $$6r^3\theta (1 - \cos 2\theta) - 2r^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

Dividing by $2r^3$, we get

or, $$3\theta (1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

or, $$3\theta - 3\theta \cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or, $$2\theta - 3\theta \cos 2\theta + \frac{\sin 2\theta}{2} = 0$$

or, $$4\theta - 6\theta \cos 2\theta - \sin 2\theta = 0$$

Solution gives: $2\theta = 308°$ ... by hit and trial method

$$\theta = \frac{308}{2} = \mathbf{154°}$$

Depth of flow for maximum discharge,

$$y = r(1 - \cos\theta) \qquad \text{[Fig. 16·16]}$$
$$= r(1 \cos 154°) = 1{\cdot}9r = 0{\cdot}95d$$

i.e. $$\boldsymbol{y = 0{\cdot}95d} \qquad (16{\cdot}25)$$

where, d is the diameter of the circular channel.

Thus *for maximum discharge through a circular channel, the depth of flow is equal to 0·95 times its diameter.*

Hydraulic radius for maximum discharge,

$$R = \frac{A}{P} = \frac{r^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2r\theta} = \frac{r}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

where, $$\theta = 154° = 154 \times \frac{\pi}{180} = 2{\cdot}687 \text{ radians}$$

$\therefore$ $$R = \frac{r}{2 \times 2{\cdot}687}\left(2{\cdot}687 - \frac{\sin 308°}{2}\right)$$

or, $$\boldsymbol{R} = 0{\cdot}573r = \mathbf{0{\cdot}29d} \qquad ...(16{\cdot}26)$$

Thus *for maximum discharge through a circular channel, the hydraulic radius equals 0·29 times channel diameter.*

Example 16.21. *A concrete lined circular channel of 3·6 m diameter has a bed slope of 1 in 600. Determine the velocity and flow rate for the conditions of:*

(*i*) *Maximum velocity; and* (*ii*) *Maximum discharge.*

Take Chezy's constant, C = 50.

Solution. Diameter of the circular channel, $d = 3{\cdot}6$ m

Bed slope, $S = \frac{1}{600}$

Chezy's constant, $C = 50$

Let 2θ = Total angle subtended by the water surface at the centre of the channel.

Flow velocity, V; flow rate, Q:

(*i*) *Maximum velocity condition:*

For maximum velocity condition,

$$2\theta = 257 \cdot 5^\circ = 257 \cdot 5 \times \frac{\pi}{180} = 4 \cdot 49 \text{ radians}$$

Depth of flow, $y = 0{\cdot}81d = 0{\cdot}81 \times 3{\cdot}6 = 2{\cdot}92$ m

Area of flow, $A = \frac{r^2}{2}(2\theta - \sin 2\theta)$

$$= \frac{1 \cdot 8^2}{2}(4 \cdot 49 - \sin 257 \cdot 5^\circ) = 8 \cdot 85 \text{ m}^2$$

Wetted perimeter, $P = 2\,r\theta = r \times 2\theta = 1{\cdot}8 \times 4{\cdot}49 = 8{\cdot}08$ m

Hydraulic radius, $R = \frac{A}{P} = \frac{8 \cdot 85}{8 \cdot 08} = 1 \cdot 095$

∴ Flow velocity, $V = C\sqrt{RS} = 50\sqrt{1 \cdot 095 \times \frac{1}{600}} \approx$ **2·14 m/s (Ans.)**

Flow rate, $Q = AV = 8{\cdot}85 \times 2{\cdot}14 =$ **18.94 m³/s (Ans.)**

(*ii*) *Maximum discharge condition:*

For maximum discharge condition,

$$2\theta = 308^\circ = 308 \times \frac{\pi}{180} = 5 \cdot 375 \text{ radians}$$

Depth of flow, $y = 0{\cdot}95d = 0{\cdot}95 \times 3{\cdot}6 = 3{\cdot}42$ m

Area of flow, $A = \frac{r^2}{2}(2\theta - \sin 2\theta)$

$$= \frac{1 \cdot 8^2}{2}(5 \cdot 375 - \sin 308^\circ) = 9 \cdot 984 \text{ m}^2$$

Wetted perimeter, $P = 2r\theta = r \times 2\theta = 1{\cdot}8 \times 5{\cdot}375 = 9{\cdot}675$ m

Hydraulic radius, $R = \frac{A}{P} = \frac{9 \cdot 984}{9 \cdot 675} = 1 \cdot 032$ m

∴ Flow velocity, $V = C\sqrt{RS} = 50\sqrt{1 \cdot 032 \times \frac{1}{600}} =$ **2·07 m/s (Ans.)**

Flow rate, $Q = AV = 9{\cdot}984 \times 2{\cdot}07 =$ **20.66 m³/s (Ans.)**

16.6. Open Channel Section for Constant Velocity at all Depths of Flow

It has been observed that according to Chezy's or Manning's formulae the hydraulic radius is the sole shape parameter on which the velocity of flow in a channel laid on a constant bottom slope depends. Thus, if the hydraulic radius is constant for any depth of flow the velocity of flow will be constant.

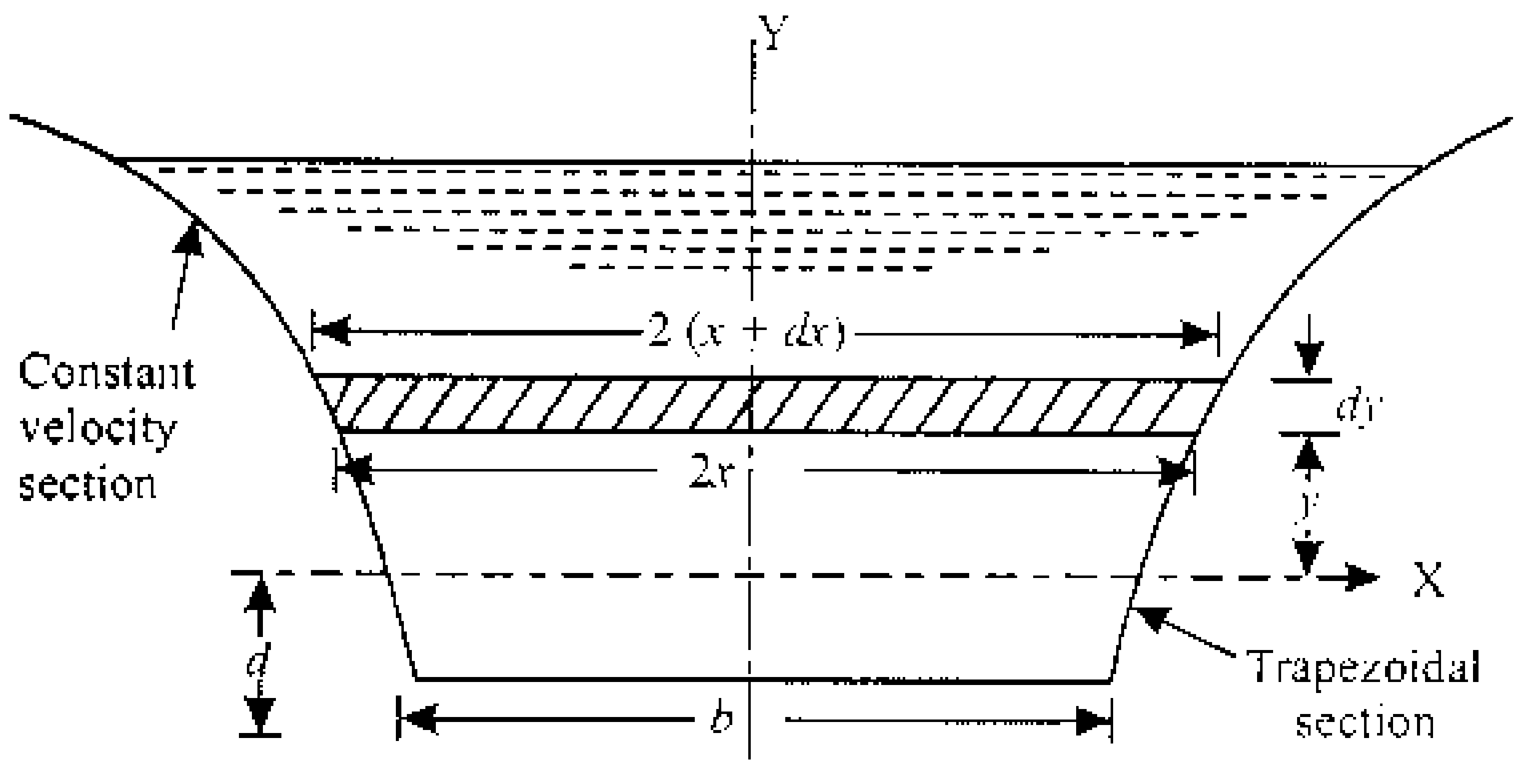


Fig. 16.21. Channel section for constant velocity at all depths.

Consider a profile of channel section (shown in Fig. 16·21) having a constant hydraulic radius R for any depth of flow. For constant velocity V, hydraulic radius has to be constant, which means that $\frac{dA}{dP}$ must remain constant at all depth of flow

or, $$\frac{dA}{dP} = R$$

where cross-sectional area, $dA = 2x\,dy$
and wetted perimeter, $dP = 2\sqrt{(dx)^2 + (dy)^2}$ } For a small portion of the channel section considered at a depth of y and dy in thickness, as shown in Fig. 16·21.

$$\therefore \quad \frac{2x\,dy}{2\sqrt{(dx)^2 + (dy)^2}} = R \quad \text{or} \quad x^2 (dy)^2 = R^2 [(dx)^2 - (dy)^2]$$

Dividing both sides by $(dx)^2$, we get

$$x^2\left(\frac{dy}{dx}\right)^2 = R^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] \quad \text{or} \quad x^2\left(\frac{dy}{dx}\right)^2 = R^2 + R^2\left(\frac{dy}{dx}\right)^2$$

or, $$\left(\frac{dy}{dx}\right)^2 (x^2 - R^2) = R^2 \quad \left(\frac{dy}{dx}\right)^2 = \frac{R^2}{x^2 - R^2}$$

or, $$\frac{dy}{dx} = \frac{R}{\sqrt{x^2 - R^2}} \quad \text{or} \quad dy = \frac{R}{\sqrt{x^2 - R^2}} \cdot dx$$

Integrating both sides, we get the following two forms:

$$y = R \cos h^{-1}\left(\frac{x}{R}\right) + C \qquad ...(16.27)$$

or, $$y = R \log_e (x + \sqrt{x^2 - R^2}) + C_1 \qquad ...(16{\cdot}27(a))$$

where, C and C_1 are the constants of integration.

However, we shall consider the first form.

The eqn. (16·27) is the equation of curves forming the sides of the section, the channel is bottomless. The value of C can be obtained if the width of the section of X-axis is known. Let the width be $2R$ at $y = 0$ *i.e.* $x = R$ at $y = 0$

$\therefore \quad 0 = R \cosh^{-1}\left(\frac{R}{R}\right) + C$ (From eqn. 16·27)

or, $\quad C = 0$

Thus eqn. (16·27) becomes

$$y = R \cosh^{-1}\left(\frac{x}{R}\right) \qquad ...(16\cdot28)$$

The channel section, below the X-axis may be of any regular shape (e.g. rectangular, trapezoidal, triangular, semicircular etc.). If the section is trapezoidal (as shown in Fig 16·17) for the section below the X-axis, then

Area, $\quad A_1 = (b + 2R)\frac{d}{2}$

Perimeter, $\quad P_1 = b + 2\sqrt{\left(R - \frac{b}{2}\right)^2 + d^2}$

It has been observed in the case of an open channel that the velocity increases with the increase in depth of flow, thereby causing damage (scouring of the bed and sides) to the channel section. On the contrary if the depth of flow decreases, the velocity decreases which may cause silting of the suspended matter in the liquid. Both these defects are removed by having constant velocity channels (where in the *large fluctuations in the velocity are avoided*).

Channel sections of constant velocity are designed particularly in the case of large sewers in which the discharge ranges froma certain minimum value that flows daily to a very large value during rainy season. In such sewers, the bottom portion (triangular or trapezoidal) is designed for the minimum discharge which flows during lean period, when the discharge increases further, the *constant-velocity section becomes effective and discharges the increased flow at the constant velocity.*

Example 16.22. *It is required to design a channel to give a constant mean velocity of flow of 1·8 m/s at all depths of flow. The lower portion of the channel to carry the minimum discharge is rectangular and has the best proportion, the bottom width being 1·5 m. Determine:*

(*i*) *The channel bed slope;*

(*ii*) *The depth of flow when the width of water surface is 9 m.*

Take Manning's N = 0·015.

Solution. Velocity of flow at all depths, $V = 1\cdot8$ m/s

Bottom width, $b = 1\cdot5$ m

Manning's $N = 0\cdot015$

Width of water surface = 9 m

The bottom portion is rectangular and has the best proportion, thus $b = 2d$, where b and d are the base width and depth of flow respectively.

or. $\quad 1\cdot5 = 2d; \quad \therefore \quad d = 0\cdot75$ m

Also for the best channel section, $R = \frac{d}{2} = \frac{0\cdot75}{2} = 0\cdot375$ m

(*i*) **The channel bed slope, S:**

Using Manning's formula, $\quad V = \frac{1}{N} R^{2/3} S^{1/2}$

(where, R = hydraulic radius)

or, $$1 \cdot 8 = \frac{1}{0 \cdot 015} \times (0 \cdot 375)^{2/3} S^{1/2}$$

or, $$S^{1/2} = \frac{1 \cdot 8 \times 0 \cdot 015}{(0 \cdot 375)^{2/3}} = 0 \cdot 05192 \text{ or } S = 0{\cdot}0027 \text{ or } 1 \text{ in } 370$$

Hence, *the channel bed slope* = **1 in 370 (Ans.)**

(*ii*) **Depth of flow:**

$$y = R \log_e [x + \sqrt{x^2 - R^2}] + C \qquad \text{[Ean. 16·27 (a)]}$$

When, $$x = \frac{1 \cdot 5}{2} = 0 \cdot 75 \text{ m}, \; y = 0$$

$\therefore$ $$C = -R \log_e [0 \cdot 75 + \sqrt{0 \cdot 75^2 - R^2}]$$

Substituting this value of C in the above eqn., we have

$$y = R \log_e [x + \sqrt{x^2 + R^2}] - R \log_e [0 \cdot 75 + \sqrt{0 \cdot 75^2 - R^2}]$$

or, $$y = R \log_e \left[\frac{x + \sqrt{x^2 - R^2}}{0 \cdot 75 + \sqrt{0 \cdot 75^2 - R^2}} \right]$$

When $x = \frac{9}{2} = 4 \cdot 5$ m (given), $R = 0{\cdot}375$ m (calculated earlier); substituting these values, we have

$$y = 0 \cdot 375 \log_e \left[\frac{4 \cdot 5 + \sqrt{4 \cdot 5^2 - 0 \cdot 375^2}}{0 \cdot 75 + \sqrt{0 \cdot 75^2 - 0 \cdot 375^2}} \right] = 0 \cdot 375 \log_e \left[\frac{4 \cdot 5 + 4 \cdot 484}{0 \cdot 75 + 0 \cdot 694} \right] = 0 \cdot 697 \text{ m}$$

$\therefore$ Total depth of flow = $d + y = 0{\cdot}75 + 0{\cdot}697$ = **1.447 m (Ans.)**

B. NON-UNIFORM FLOW

16.7. Non-uniform Flow Through Open Channels

Whereas in *uniform flow* the gravity force on the flowing liquid just *balances* the frictional force between the flowing liquid and that inside surface of the channel which is in contact with this liquid, the friction force and gravity force are *not in balance* in case of *a steady non-uniform flow*. Non-uniform flow may be caused by:

(*i*) The change in width, depth, bed slope etc. of a channel;

(*ii*) An obstruction, constructed across a channel of uniform width.

- *Waves and surges in open channel produce unsteady non-uniform flow.*

Non-uniform flow is also known as the *flow of varying depth or, the varied flow*. The varied flow may be:

(*i*) *Gradually varied flow (G.V.F.)*. In this case of flow the depth of flow increases or decreases *gradually* in the direction of flow; this change from one depth of flow to another occurs gradually in a distance of *appreciable length*.

(*ii*) *Rapidly varied flow (R.V.F.)*. In this case a sudden change of depth occurs at a particular point of a channel and the change from one depth to another takes place in a distance of *very short length*.

16.8. Specific Energy and Specific Energy Curve

The total energy of flow per *unit weight* of liquid is given by:

$$\text{Total energy} = z + y + \frac{V^2}{2g}$$

where, z = Elevation of the channel bottom above the horizontal bottom,

y = Depth of flow, and

V = Average velocity of flow.

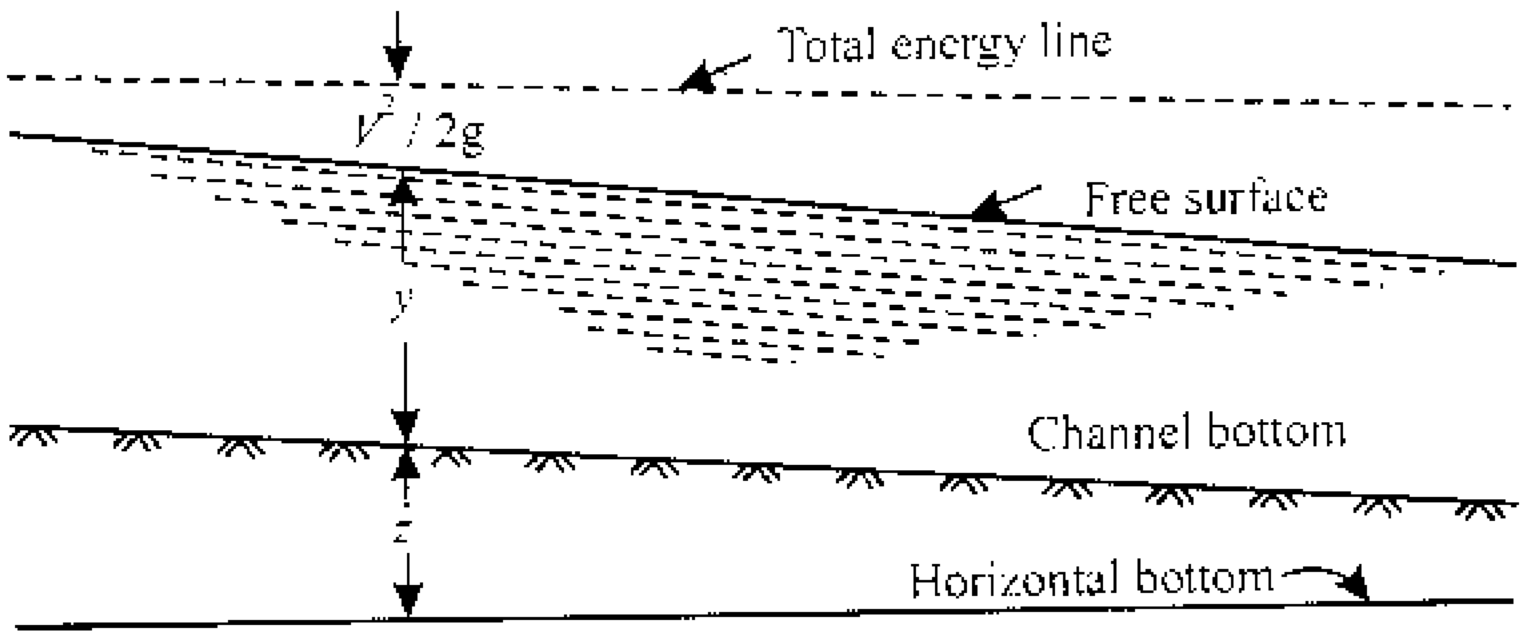


Fig. 16.22. Specific energy.

If the channel bottom itself is taken as the datum (Fig. 16·22), then total energy for unit weight of liquid,

$$E = y + \frac{V^2}{2g} \qquad ...(16·29)$$

The energy E given by eqn. (16·29) is known as *specific energy*. Thus *specific energy* is defined as the *energy per unit weight of flowing liquid above the channel bottom*. Although the total (or Bernoulli's) energy is reduced by friction, the specific energy can *increase* or *decrease from* section to section if the *bed elevation changes; however, for uniform flow the specific energy remains constant along the flow.*

It is evident from eqn. (16·29) that specific energy comprises

(*i*) Potential energy of flow (E_p), y, and

(*ii*) Kinetic energy of flow (E_k), $\frac{V^2}{2g}$.

i.e.
$$E = y + \frac{V^2}{2g} = E_p + E_k$$

For the sake of simplicity let us consider a channel of *rectangular section*.

Let, b = Width of channel,

y = Depth of flow, and

Q = Discharge through the channel.

Now, Velocity of flow, $V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{b \times y} = \frac{q}{y}$ ...(16·30)

(where q = discharge per unit width)

$\therefore$ Specific energy, $E = y + \frac{(q/y)^2}{2g}$

or,
$$E = y + \frac{q^2}{2gy^2} = E_p + E_k \qquad ...(16·31)$$

For a given channel section and discharge, eqn. (16·31) can be represented graphically as a plot of specific energy E against the depth of flow. Such a plot is called the *specific energy curve/diagram* and it consists of a family of similar curves each representing a given unit discharge.

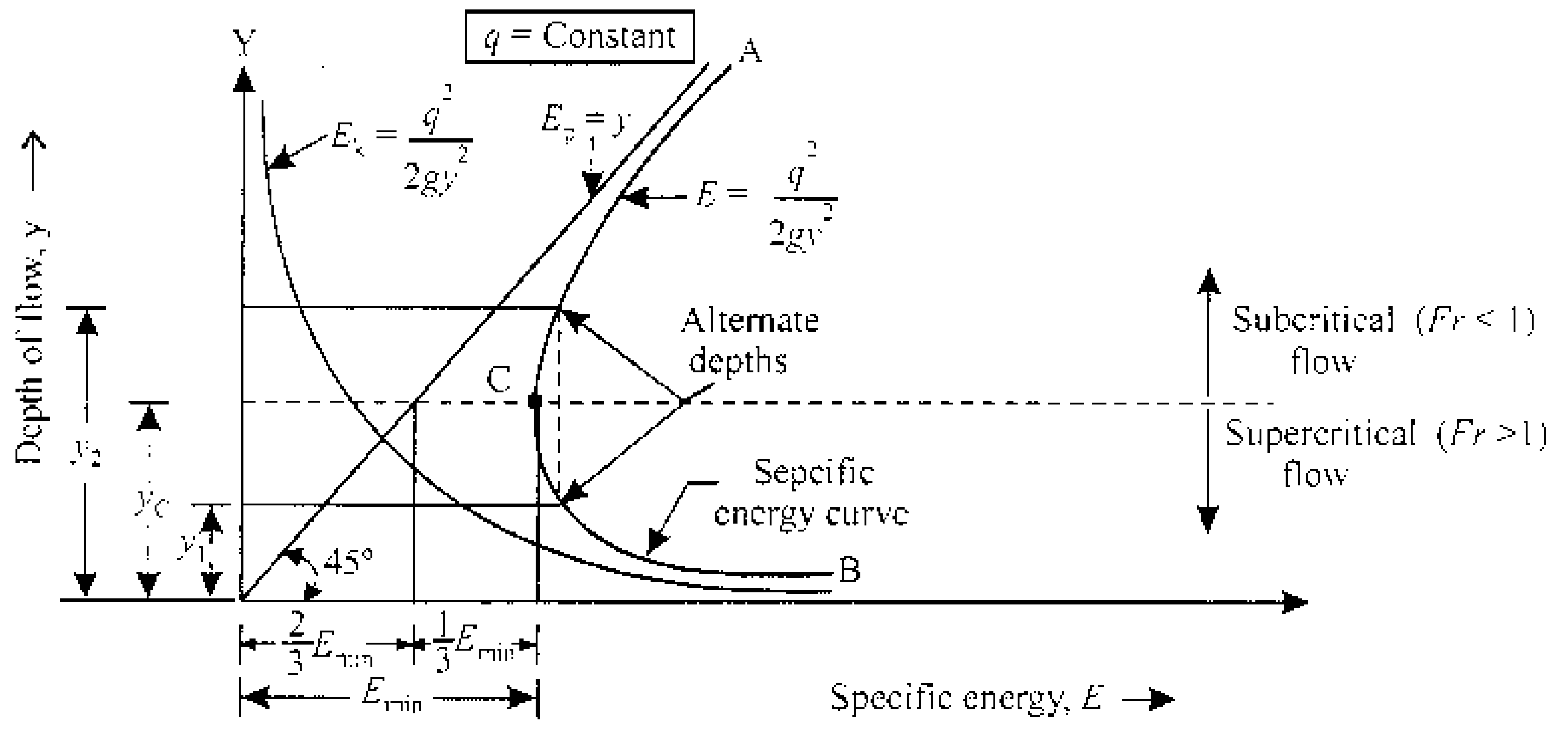


Fig. 16.23. Specific energy curve.

The specific energy plot of Fig. 16·23 entails the following information:

(*i*) The curve for potential energy (*i.e.* $E_p = y$) is a *straight line* passing through the origin, making an angle of 45° with each of the two axes (X and Y),

(*ii*) The *curve* for kinetic energy is a *parabola*. Plot for *specific energy is obtained by adding kinetic energy to potential energy.*

(*iii*) Specific energy is asymptotic to the horizontal axis for small values of y and asymptotic to 45° line for high values of y.

(*iv*) At a certain depth y_c, called the *critical depth*, the specific energy curve has a point of *minimum specific energy*, the corresponding flow velocity is called the critical velocity V_c.

(*v*) For every value of specific energy other than minimum there are two possible depths of flow (y_1 and y_2), one greater and other less than critical depth y_c; these two depths (for same specific energy) are referred to as *alternate or conjugate depths.*

Critical depth, y_c. It can be seen from the specific energy curve ACB (Fig 16·23) that, there is one point C on the curve which has a *minimum specific energy*, thereby indicating that below this value of specific energy the given discharge cannot occur. *The depth of flow at which the specific energy is minimum is called critical depth y_c.*

The mathematical expression for critical depth can be obtained by differentiating the specific energy equation, $E = y + \frac{q^2}{2gy^2}$ with respect to y and equating the derivative to zero. Thus,

$$\frac{dE}{dy} = \frac{d}{dy}\left[y + \frac{q^2}{2gy^2}\right] = 0$$

or, $$1 + \frac{q^2}{2g}\left(-\frac{2}{y^3}\right) = 0 \text{ or } 1 = \frac{2q^2}{2gy^3}$$

or, $$y = \left(\frac{q^2}{g}\right)^{1/3}$$

But when the specific energy is minimum the depth of flow is critical; denoted by y_c.

$$\therefore \qquad y_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad ...(16\cdot32)$$

Critical velocity, V_c. *The velocity* of flow at critical depth is known as *critical velocity;* denoted by V_c. Its value is obtained as follows:

$$\text{Velocity} = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{b \cdot y} = \frac{q}{y}$$

$$\therefore \qquad V_c = \frac{q}{y_c} = \frac{q}{\left(\frac{q^2}{g}\right)^{1/3}} = q^{1/3} g^{1/3} \qquad \left[\because \ y_c = \left(\frac{q^2}{g}\right)^{1/3}\right]$$

$$\text{or,} \qquad V_c^3 = qg \qquad \left(\text{where, } q = \text{discharge per unit width} = \frac{Q}{b}\right)$$

$$\text{Also} \qquad q = V_c \times y_c$$

$$\therefore \qquad V_c^3 = V_c\, y_c\, g \ \text{ or } \ V_c^2 = g y_c$$

$$\text{or,} \qquad V_c = \sqrt{g y_c} \qquad ...(16\cdot33)$$

Minimum specific energy in terms of critical depth. The specific energy is given by,

$$E = y + \frac{q^2}{2gy^2} \qquad \text{(Eqn. 16·31)}$$

The specific energy is *minimum* when *depth of flow is critical* and hence the above equation becomes:

$$E_{min} = y_c + \frac{q^2}{2gy_c^2}$$

$$\text{But,} \qquad y_c = \left(\frac{q^2}{g}\right)^{1/3} \ \text{ or } \ y_c^3 = \frac{q^2}{g} \qquad \text{(Eqn. 16·32)}$$

$$\therefore \qquad E_{min} = y_c + \frac{y_c^3}{2y_c^2} = y_c + \frac{y_c}{2} = \frac{3y_c}{2} \qquad \text{(Eqn. 16·34)}$$

$$\left(\text{or } \ y_c = \frac{2}{3} E_{min}\right)$$

Critical flow. Refer to Fig. 16·23. *A critical flow is one in which specific energy is minimum.* A flow *corresponding to critical depth is* also known as *critical flow.*

From eqn. (16·33), we have

$$V_c = \sqrt{g y_c} \ \text{ or } \ \frac{V_c}{\sqrt{g y_c}} = 1$$

$$\text{But,} \qquad \frac{V_c}{\sqrt{g y_c}} = \text{Froude number } (Fr)$$

$\therefore$ $Fr = 1$ for *critical flow.*

Subcritical flow. The flow is subcritical (or streaming or tranquil) when the depth of flow in a channel is *greater than the critical depth* (y_c). In this type of flow, $Fr < 1$.

Supercritical flow. The flow is supercritical (or shooting or torrential) when the depth of flow in a channel is *less than the critical depth* (y_c). In this case $Fr > 1$.

Condition for maximum discharge for a given value of specific energy:

The specific energy (E) at any section of a channel is given by:

$$E = y + \frac{V^2}{2g}, \text{ where } V = \frac{Q}{A} = \frac{Q}{b \times y}$$

$$\therefore \quad E = y + \frac{Q^2}{b^2 y^2} \times \frac{1}{2g} = y + \frac{Q^2}{2gb^2 y^2}$$

or, $$Q^2 = 2gb^2y^2 (E - y)$$

or, $$Q = b\sqrt{2g\,(Ey^2 - y^3)}$$

For discharge Q to be maximum the expression $(Ey^2 - y^3)$ should be *maximum.*

i.e. $$\frac{d}{dy}(Ey^2 - y^3) = 0$$

or, $$2Ey - 3y^2 = 0 \qquad (\because E \text{ is constant})$$

or, $$2Ey = 3y^2$$

or, $$y = \frac{2}{3}E \qquad ...(16{\cdot}35)$$

or, $$E = \frac{3y}{2} \qquad ...[16{\cdot}35(a)]$$

According to eqn. (16·34) specific energy is minimum when it is equal to $\frac{3}{2}$ times the value of depth of critical flow. Here according to eqn. [16·35 (a)] the specific energy is equal to $\frac{3}{2}$ times the depth of flow. Thus eqn. [16·35 (a)] represents minimum specific energy and y is the critical depth. Hence the *condition for maximum discharge for given value of specific energy* is that the *depth of flow should be critical.*

Fig. 16·24 shows the discharge curve.

Example 16.23. *A 8 m wide channel conveys 15 m³/s of water at a depth of 1·2 m. Calculate:*

(*i*) *Specific energy of the flowing water;*

(*ii*) *Critical depth, critical velocity and minimum specific energy;*

(*iii*) *Froude number and state whether flow is subcritical or supercritical.*

Solution. Width of the channel, $b = 8$ m

Discharge, $Q = 15$ m³/s

Depth of flow, $y = 1{\cdot}2$ m

(*i*) **Specific energy of the flowing water:**

Average flow velocity,

$$V = \frac{Q}{b \times y} = \frac{15}{8 \times 1{\cdot}2} = 1{\cdot}5625 \text{ m/s}$$

Discharge per unit width,

$$q = \frac{Q}{b} = \frac{15}{8} = 1{\cdot}875 \text{ m}^3\text{/s per m}$$

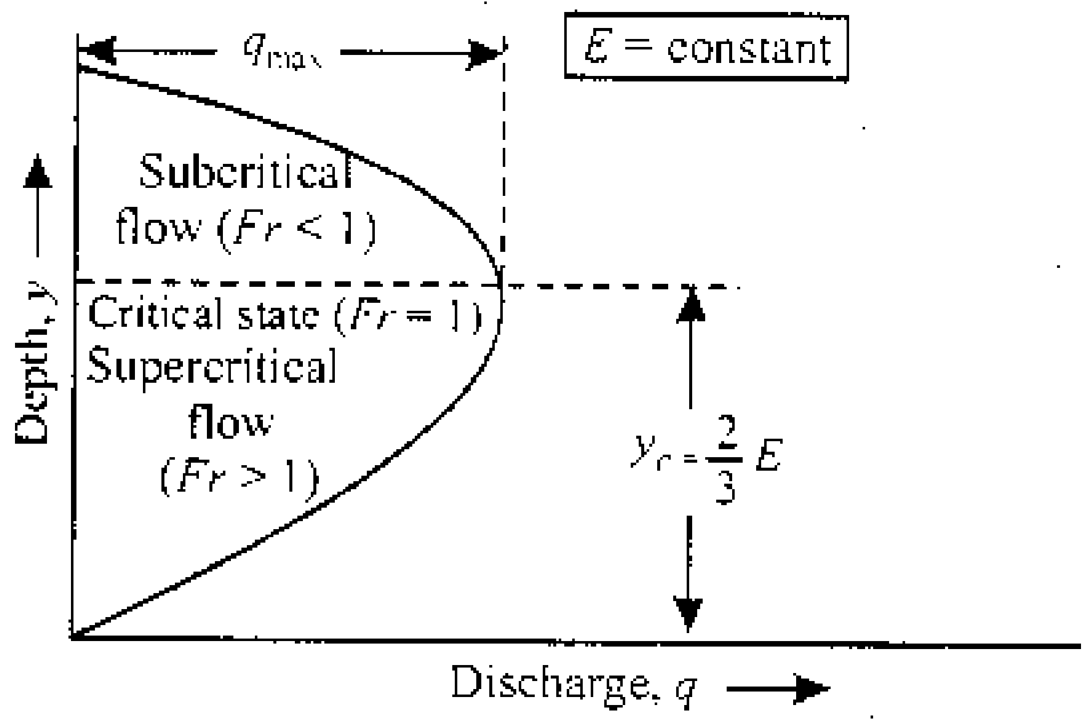


Fig. 16.24. Discharge curve.

$\therefore$ Specific energy,

$$E = y + \frac{V^2}{2g} = 1 \cdot 2 + \frac{1 \cdot 5625^2}{2 \times 9 \cdot 81} = \mathbf{1 \cdot 324\ m\ (Ans.)}$$

(*ii*) **Critical depth (y_c), critical velocity (V_c) and E_{min}:**

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1 \cdot 875^2}{9 \cdot 81}\right)^{1/3} = \mathbf{0 \cdot 71\ m/s\ (Ans.)}$

Critical velocity, $V_c = \sqrt{gy_c} = \sqrt{9 \cdot 81 \times 0 \cdot 71} = \mathbf{2 \cdot 64\ m/s\ (Ans.)}$

Minimum specific energy, $E_{min} = \frac{3}{2} y_c = \frac{3}{2} \times 0 \cdot 71 = \mathbf{1 \cdot 065\ m\ (Ans.)}$

$$\left(\text{Alternatively: } E_{min} = y_c + \frac{V_c^2}{2g} = 0 \cdot 71 + \frac{2 \cdot 64^2}{2 \times 9 \cdot 81} = 1 \cdot 065 \text{ m}\right)$$

(*iii*) **Froude number and nature of flow:**

Froude number, $Fr = \frac{V}{\sqrt{gy}} = \frac{1 \cdot 5625}{\sqrt{9 \cdot 81 \times 1 \cdot 2}} = 0 \cdot 455$

Since $Fr < 1$, the flow is *subcritical* or *tranquil state*. This is also evident from the fact that $y > y_c$ *i.e.*, $\frac{y}{y_c} > 1$.

Example 16.24. *The specific energy for a 3 m wide channel is to be 3 Nm/N. What would be the maximum possible discharge?* **[AMIE]**

Solution. Width of channel, $b = 3$ m

Specific energy, $E = 3$ Nm/N

For the given value of specific energy, the discharge will be maximum, when depth of flow is critical. From eqn. (16·35), for maximum discharge, we have

$$y_c = y = \frac{2}{3} E = \frac{2}{3} \times 3 = 2 \text{ m}$$

$\therefore$ Maximum discharge, Q_{max} = Area × velocity

$= (b \times y_c) \times V_c$ ($\because$ At critical depth, y_c, the velocity will be critical.)

But $V_c = \sqrt{gy_c} = \sqrt{9 \cdot 81 \times 2} = 4 \cdot 43$ m/s

Substituting the values, we have

$Q_{max} = 3 \times 2 \times 4{\cdot}43 = \mathbf{26.58\ m^3/s\ (Ans.)}$

Example 16.25. *Water flows at a steady and uniform depth of 2 m in an open channel of rectangular cross-section having base width equal to 5 m and laid at a slope of 1 in 1000. It is desired to obtain critical flow in the channel by providing a hump in the bed. Calculate the height of the hump and sketch the flow profile. Consider the value of Manning's rugosity co-efficient N = 0·02 for the channel surface.* **[UPSC Exams.]**

Solution. Depth of flow, $y = 2$ m

Base width of channel, $b = 5$ m

Bed slope, $S = 1$ in 1000

Manning's co-efficient, $N = 0{\cdot}02$

Height of the hump, h:

For rectangular channel : Area, $A = b \times y$, and

Perimeter, $P = b + 2y$

$\therefore$ Hydraulic radius, $R = \frac{A}{P} = \frac{b \times y}{b + 2y} = \frac{5 \times 2}{5 + 2 \times 2} = 1 \cdot 111$ m

Discharge, $Q = A \times V = A \times C\sqrt{RS}$

where, Chezy's constant $C = \frac{1}{N} R^{1/6}$

$$\therefore \quad Q = A \times \frac{1}{N} R^{1/6} \sqrt{RS} = \frac{A}{N} R^{2/3} S^{1/2}$$

$$= \frac{(5 \times 2)}{0 \cdot 02} \times (1 \cdot 111)^{2/3} \times \left(\frac{1}{1000}\right)^{1/2} = 16 \cdot 96 \text{ m}^3/\text{s} \quad \text{(substituting the values)}$$

Discharge per unit width, $q = \frac{Q}{b} = \frac{16 \cdot 95}{5} = 3 \cdot 392 \text{ m}^3/\text{s}$

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3 \cdot 392^2}{9 \cdot 81}\right)^{1/3} = 1 \cdot 055$ m

Minimum specific energy, $E_{min} = \frac{3}{2} \times 1 \cdot 055 = 1 \cdot 5825$ m

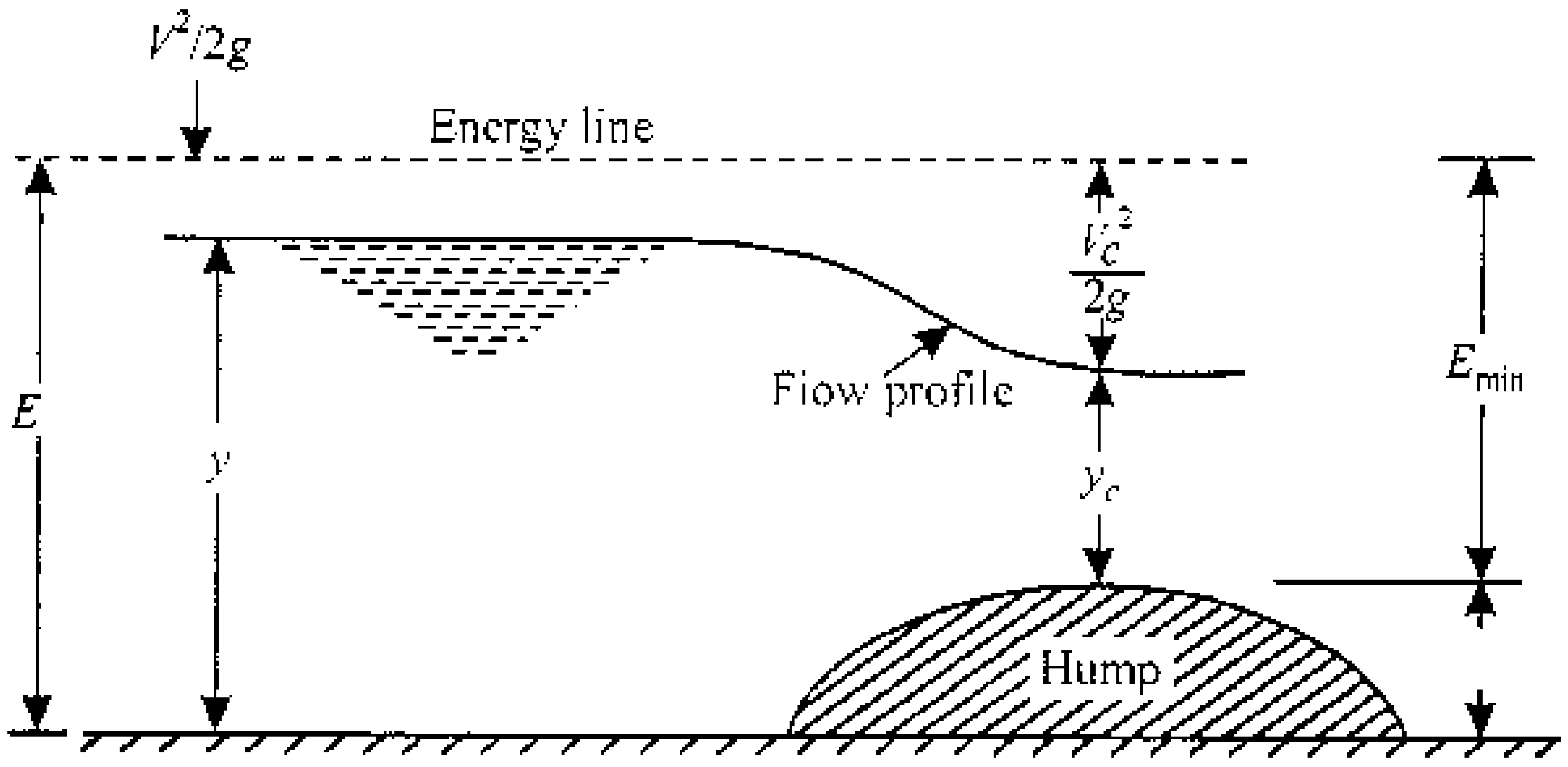


Fig. 16.25.

Specific energy in normal flow,

$$E = y + \frac{V^2}{2g} = 2 + \frac{1 \cdot 696^2}{2 \times 9 \cdot 81} = 2 \cdot 147 \text{ m}$$

where, $$V = \frac{1}{N} R^{2/3} S^{1/2} = \frac{1}{0 \cdot 02} \times (1 \cdot 111)^{2/3} \times \left(\frac{1}{1000}\right)^{1/2} = 1 \cdot 696 \text{ m/s}$$

Height of hump provided $= E - E_{min} = 2 \cdot 147 - 1 \cdot 5825 =$ **0.5645 m (Ans.)**

The flow profile has been shown in Fig. 16·25.

16.9. Hydraulic Jump or Standing Wave

In an open channel when rapidly flowing stream abruptly changes to slowly flowing stream, a distinct rise or jump in the elevation of liquid surface takes place, this *phenomenon* is *known as* **hydraulic jump** (which is analogous to shock wave in compressible fluids). The hydraulic jumps *converts kinetic energy of stream rapidly flowing into potential energy*. Due to this there is a *loss of kinetic energy*. At the place where hydraulic jump occurs *rollers of turbulent water* (eddying turbulences) form, which *cause dissipation of energy*. A hydraulic jump occurs in practice at the toe of *spillways* or *below a sluice gate where the velocity is very high*.

The hydraulic jump is also known as a **standing wave** because it is, in essence, a wave which is stationary (*i.e.*, at stand-still) at one place. Such a standing wave is shown in Fig. 16·26.

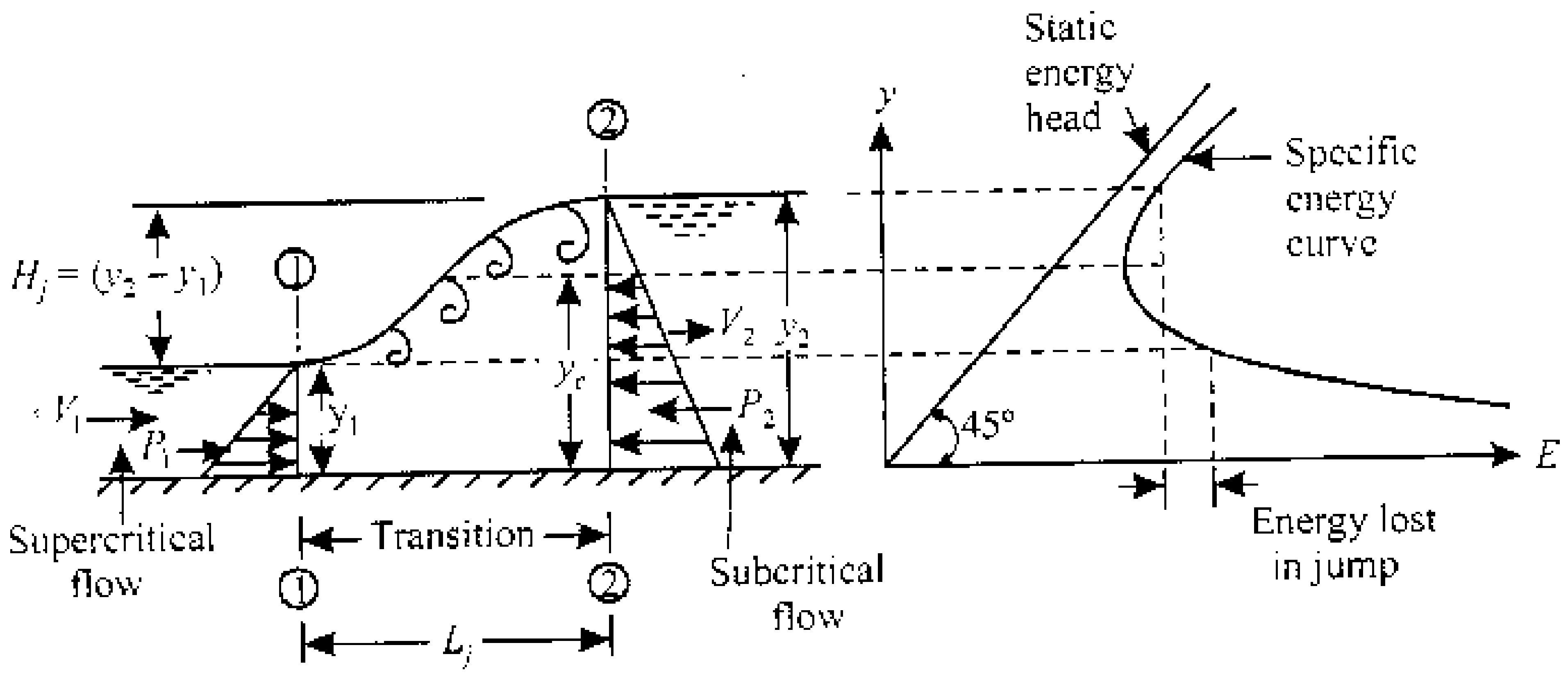


Fig. 16.26. Hydraulic jump.

Analysis of hydraulic jump:

The following *assumptions* are made in the analysis of hydraulic jump:

1. Loss of head due to friction at the walls and channel bed is negligible.
2. The flow is uniform and the pressure distribution is hydrostatic before and after the jump.
3. The channel is horizontal or it has a very small slope. The weight component in the direction of flow is neglected.
4. The momentum correction factor (β) is unity.

Height of hydraulic jump (H_j):

Refer Fig. 16·26

Discharge per unit width, $q = V_1 y_1 = V_2 y_2$ ...*Continuity equation* ...(*i*)

$$\therefore \quad V_1 = \frac{q}{y_1} \quad \text{and} \quad V_2 = \frac{q}{y_2}$$

In case of hydrostatic pressure distribution, the pressure force at any section is,

$$P = wA\bar{y}$$

where, $\bar{y}$ = Vertical depth of centroid of wetted area from the liquid surface.

$$\therefore \quad P_1 = w \times (y_1 \times 1) \times \frac{y_1}{2} = \frac{wy_1^2}{2} \quad \text{...Pressure force at section 1-1}$$

$$P_2 = w \times (y_2 \times 1) \times \frac{y_2}{2} = \frac{wy_2^2}{2} \quad \text{...Pressure force at section 2-2}$$

Net force acting on mass of water between 1-1 and 2-2

$$= P_2 - P_1 = \frac{wy_2^2}{2} - \frac{wy_1^2}{2} = \frac{w}{2}(y_2^2 - y_1^2) \quad \text{...(ii)}$$

$$[\because P_2 > P_1 \text{ as } y_2 > y_1]$$

Now, change in linear momentum = $\rho q (V_1 - V_2)$

But, according to *impulse-momentum equation*;

Net force acting on a mass of fluid = Rate of change of momentum in the same direction

$$\therefore \quad P_2 - P_1 = \rho q\,(V_1 - V_2) \quad \text{or} \quad \frac{w}{2}(y_2^2 - y_1^2) = \rho q\,(V_1 - V_2)$$

Substituting, $V_1 = \dfrac{q}{y_1}$ and $V_2 = \dfrac{q}{y_2}$, we have

$$\frac{\rho g}{2}(y_2^2 - y_1^2) = \rho q\left(\frac{q}{y_1} - \frac{q}{y_2}\right) = \rho q^2\left(\frac{1}{y_1} - \frac{1}{y_2}\right) \qquad (\because w = \rho g)$$

$$\frac{g}{2}(y_2 + y_1)(y_2 - y_1) = q^2\left(\frac{1}{y_1} - \frac{1}{y_2}\right) = q^2\left(\frac{y_2 - y_1}{y_1 y_2}\right)$$

Dividing both sides by $(y_2 - y_1)$, we get

$$\frac{g}{2}(y_2 + y_1) = \frac{q^2}{y_1 y_2} \quad \text{or} \quad (y_2 + y_1) = \frac{2q^2}{g y_1 y_2} \qquad ...(iv)$$

Multiplying both sides by y_2, we have

$$y_2^2 + y_1 y_2 = \frac{2q^2}{g y_1} \quad \text{or} \quad y_2^2 + y_1 y_2 - \frac{2q^2}{g y_1} = 0 \qquad ...(v)$$

$$\text{or,} \qquad y_2 = \frac{-y_1 \pm \sqrt{y_1^2 + 4 \times 1 \times \dfrac{2q^2}{g y_1}}}{2}$$

$$= -\frac{y_1}{2} \pm \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}$$

$$\text{or,} \qquad y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}} \quad \text{or} \quad -\frac{y_1}{2} - \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}$$

Neglecting the second root (being impossible, –*ve* depth), we have

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}} \qquad ...(16\cdot36)$$

$$= -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2 \times (V_1 y_1)^2}{g y_1}} \qquad (\because q = V_1 y_1)$$

$$\text{or,} \qquad y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2 V_1^2 y_1}{g}} \qquad ...(16.37)$$

Expression of y_2 in terms of Froude number (Fr):

Eqn. (16·37) can be written as:

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4}\left(1 + \frac{8V_1^2}{g y_1}\right)}$$

$$\text{or,} \qquad y_2 = -\frac{y_1}{2} + \frac{y_1}{2}\sqrt{1 + \frac{8V_1^2}{g y_1}} \qquad ...(vi)$$

But, $$Fr_1 = \frac{V_1}{\sqrt{gy_1}} \quad \text{or} \quad (Fr_1)^2 = \frac{V_1^2}{gy_1}$$

∴ Substituting this value in expression (*vi*), we have

$$y_2 = -\frac{y_1}{2} + \frac{y_1}{2}\sqrt{1 + 8\,(Fr_1)^2}$$

or, $$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\,(Fr_1)^2} - 1\right) \qquad ...(16\cdot38)$$

∴ Height of hydraulic jump, $H_j = y_2 - y_1$ ...(16·39)

Length of hydraulic jump (L_j). Length of hydraulic jump represents *that short distance over which the jump occurs* (Refer Fig. 16·26). For rectangular channels with horizontal floor, length of a jump has been found to vary between *5 to 7 times the height of the jump.*

i.e., $L_j = 5$ to $7\ H_j$ ...(16·40)

Loss of energy due to hydraulic jump:

The loss of energy due to hydraulic jump is equal to the difference of specific energies at the upstream (1-1) and downstream (2-2) sections.

i.e. $$E_2 = E_1 - E_2$$

$$= \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - (y_2 - y_1)$$

$$= \left(\frac{q^2}{2gy_1^2} - \frac{q^2}{2gy_2^2}\right) - (y_2 - y_1) \qquad \left(\because V_1 = \frac{q}{y_1},\ V_2 = \frac{q}{y_2}\right)$$

or, $$= \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right) - (y_2 - y_1) \qquad ...(vii)$$

or, $$E_L = \frac{q^2}{2g}\left(\frac{y_2^2 - y_1^2}{y_1^2\,y_2^2}\right) - (y_2 - y_1)$$

But, $$q^2 = gy_1y_2\left(\frac{y_2 + y_1}{2}\right) \quad \text{...(From expression } iv)$$

∴ Loss of energy, $$E_L = gy_1y_2\left(\frac{y_2 + y_1}{2}\right) \times \frac{(y_2^2 - y_1^2)}{2\,gy_1^2y_2^2} - (y_2 - y_1)$$

$$= \frac{(y_2 + y_1)\,(y_2^2 - y_1^2)}{4y_1\,y_2} - (y_2 - y_1)$$

$$= \frac{(y_2 + y_1)\,(y_2 + y_1)\,(y_2 - y_1)}{4y_1y_2} - (y_2 - y_1)$$

$$= (y_2 - y_1)\left[\frac{(y_2 + y_1)^2}{4y_1y_2} - 1\right]$$

$$= (y_2 - y_1)\left[\frac{y_2^2 + y_1^2 + 2y_1y_2 - 4y_1y_2}{4y_1y_2}\right]$$

$$= (y_2 - y_1)\left[\frac{(y_1 - y_2)^2}{4y_1 y_2}\right]$$

or, $$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2} \quad \text{...(16·41)}$$

Example 16.26. *A sluice gate discharges water into horizontal rectangular channel with a velocity of 10 m/s and depth of flow of 1 m. Determine the depth of flow of water after the jump and consequent loss in total head.* [AMIE]

Solution. Velocity of flow before hydraulic jump, $V_1 = 10$ m/s.

Depth of flow before hydraulic jump, $y_1 = 1$ m

Depth of flow after the jump, y_2:

Discharge *per unit width*, $q = V_1 \times y_1 = 10 \times 1 = 10$ m³/s per m

The depth of flow after the jump is given by,

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{gy_1}} \quad \text{...[Eqn. (16·36)]}$$

or, $$y_2 = -\frac{1}{2} + \sqrt{\frac{1^2}{4} + \frac{2 \times 10^2}{9 \cdot 81 \times 1}} = \mathbf{4 \cdot 043\ m\ (Ans.)}$$

Loss in total head, E_L:

Loss in total head is given by,

$$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2} \quad \text{...[Eqn. (16·31)]}$$

or, $$E_L = \frac{(4 \cdot 043 - 1)^3}{4 \times 1 \times 4 \cdot 043} = \mathbf{1 \cdot 742\ m\ (Ans.)}$$

Example 16.27. *A 3·6 m wide rectangular channel conveys 9·0 m³/s of water with a velocity of 6 m/s.*

(*i*) *Is there a condition for hydraulic jump to occur? If so, calculate the height, length and strength of the jump.*

(*ii*) *What is loss of energy per kg of water?*

Solution. Width of channel, $b = 3{\cdot}6$ m

Discharge, $Q = 9{\cdot}0$ m³/s

Velocity of flow before jump, $V_1 = 6$ m/s

(*i*) **Is there a condition for hydraulic jump to occur?**

Depth of water before jump, $y_1 = \frac{Q}{b \times V_1} = \frac{9 \cdot 0}{3 \cdot 6 \times 6} = 0 \cdot 4167$ m

Discharge per unit width, $q = \frac{Q}{b} = \frac{9 \cdot 0}{3 \cdot 6} = 2 \cdot 5$ m³/s per m

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2 \cdot 5^2}{9 \cdot 81}\right)^{1/3} = 0 \cdot 86$ m

Since $y_1 < y_c$, a jump would occur. (Ans.)

Froude number ahead of jump,

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{6}{\sqrt{9.81 \times 0.4167}} = 2.967$$

Depth of water downstream the jump,

$$y_2 = \frac{y_1}{2}\left[\sqrt{1 + 8(Fr_1)^2} - 1\right] \qquad \text{...(Eqn. 16·38)}$$

$$y_2 = \frac{0.4167}{2}\left[\sqrt{1 + 8 \times 2.967^2} - 1\right] = 1.5525 \text{ m}$$

$\therefore$ **Height of jump, $H_j = y_2 - y_1$ = 1·5525 – 0·4167 = 1.1358 m (Ans.)**

Length of jump, $L_j = 6(y_2 - y_1)$ = 6 × 1·1358 = 6.8148 m (Ans.)

$$\textbf{Strength of jump} = \frac{y_2}{y_1} = \frac{1.5525}{0.4167} = \mathbf{3.726} \textbf{ (Ans.)}$$

(*ii*) **Loss of energy per kg of water, E_L:**

Velocity before jump, $V_1 = 6$ m/s ...(*Given*)

$$\text{Velocity after jump, } V_2 = \frac{q}{y_2} = \frac{2.5}{1.5525} = 1.61 \text{ m/s}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.4167 + \frac{6^2}{2 \times 9.81} = 2.25 \text{ m}$$

$$E_2 = y_2 + \frac{V_2^2}{2g} = 1.5525 + \frac{1.61^2}{2 \times 9.81} = 1.68 \text{ m}$$

$\therefore$ Loss of energy in the jump, $E_L = E_1 - E_2$ = 2·25 – 1·68 = **0.57 m (Ans.)**

$$\left[\text{Alternatively, } E_L = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(1.5525 - 0.4167)^3}{4 \times 1.5525 \times 0.4167} = \mathbf{0.57 \text{ m}}\right]$$

Example 16.28. *In a rectangular channel of 0·5 m width, a hydraulic jump occurs at a point where depth of water flow is 0·15 m and Froude number is 2·5. Determine:*

(*i*) *The specific energy;* (*ii*) *The critical and subsequent depths,*
(*iii*) *Loss of head, and;* (*iv*) *Energy dissipated.*

Solution. Width of the channel, $b = 0.5$ m

Depth of flow, $y_1 = 0.15$ m

Froude number, $Fr = 2.5$

Now, $$Fr = \frac{V_1}{\sqrt{gy_1}}, \text{ where } V_1 \text{ is the upstream velocity}$$

$$\therefore \quad 2.5 = \frac{V_1}{\sqrt{9.81 \times 0.15}} \quad \text{or} \quad V_1 = 3.03 \text{ m/s}$$

Discharge per unit width, $q = V_1y_1$ = 3·03 × 0·15 = 0·4545 m³/s per m

(*i*) **Specific energy, E:**

$$E = y_1 + \frac{V_1^2}{2g} = 0.15 + \frac{3.03^2}{2 \times 9.81} = \mathbf{0.618 \text{ m}} \textbf{ (Ans.)}$$

(*ii*) **Critical depth, y_c:**

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{0.4545^2}{9.81}\right]^{1/3} = \mathbf{0.276\ m\ (Ans.)}$$

Subsequent depth, $y_2 = \frac{y_1}{2}\left[\sqrt{1 + 8\,(Fr_1)^2} - 1\right]$

or $$y_2 = \frac{0.15}{2}\left[\sqrt{1 + 8 \times 2.5^2} - 1\right] = \mathbf{0.461\ m\ (Ans.)}$$

(*iii*) **Loss head, E_L:**

$$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(0.461 - 0.15)^3}{4 \times 0.15 \times 0.461} = \mathbf{0.108\ m\ (Ans.)}$$

(*iv*) **Power dissipated, P:**

$$P = wQE_L$$

where $Q = A_1V_1 = (b \times y_1)\,V_1 = (0.5 \times 0.15) \times 3.03 = 0.227\ m^3/s$

$\therefore$ $P = 9810 \times 0.227 \times 0.108 = \mathbf{240.5\ W\ (Ans.)}$

Example 16.29. *Find in terms of specific energy E, an expression for the critical depth in a trapezoidal channel with bottom width b and side slope 1 vertical to n horizontal.* **[AMIE]**

Solution. The specific energy (E) of a channel is given as:

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} \qquad (\because V = Q/A)$$

where, y = Depth of flow,

V = Average velocity of flow,

Q = Discharge, and

A = Area of cross-section of the channel.

The condition for minimum specific energy (F_{min}) can be obtained by differentiating the specific energy equation with respect to y and equating the derivative to zero. Thus,

$$\frac{dE}{dy} = \frac{d}{dy}\left[y + \frac{Q^2}{2g\,A^2}\right] = 0$$

$$= 1 + \frac{Q^2}{2g} \times \left(-2 \times A^{-3} \cdot \frac{dA}{dy}\right) = 1 - \frac{Q^2}{gA^3} \times \frac{dA}{dy} = 0 \qquad (\because Q = \text{constant})$$

$$\therefore \quad \frac{dA}{dy} = \frac{gA^3}{Q^2} = \frac{gA_c^3}{Q^2}, \quad \text{for critical condition} \qquad ...(i)$$

In case of a trapezoidal channel,

$$A = (b + ny)\,y = by + ny^2$$

$$\frac{dA}{dy} = b + 2ny = b + 2ny_c, \quad \text{for critical condition} \qquad ...(ii)$$

From expressions (*i*) and (*ii*), we have

$$\frac{gA_c^3}{Q^2} = b + 2ny_c \quad \text{or} \quad \frac{Q^2}{g} = \frac{A_c^3}{b + 2ny_c} \qquad ...(iii)$$

The specific energy for critical conditions becomes:

$$E = y_c + \frac{Q^2}{2gA_c^2}$$

Substituting the value of $\frac{Q^2}{g}$ from expression (*iii*), we get

$$E = y_c + \frac{A_c^3}{2A_c^2 (b + 2ny_c)} = y_c + \frac{A_c}{2(b + 2ny_c)}$$

or, $$E = y_c + \frac{(b + ny_c)\, y_c}{2\,(b + 2ny_c)} \quad ...(iv)$$

or, $$E \times 2\,(b + 2ny_c) = y_c \times 2\,(b + 2ny_c) + (b + ny_c)\, y_c$$

or, $$2bE + 4nEy_c = 2by_c + 4ny_c^2 + by_c + ny_c^2$$

Rearranging the above equation, we have

$$5ny_c^2 + (3b - 4nE)\, y_c - 2bE = 0$$

or, $$y_c = \frac{-(3b - 4nE) \pm \sqrt{(3b - 4nE)^2 - 4 \times 5n \times (-2bE)}}{2 \times 5n}$$

or, $$y_c = \frac{(4nE - 3b) \pm \sqrt{(3b - 4nE)^2 + 40nbE}}{10n} \quad \textbf{(Ans.)}$$

Note : When $n = 0$, the expression (*iv*) becomes

$$E = y_c + \frac{b \cdot y_c}{2b} = y_c + \frac{y_c}{2} = \frac{3y_c}{2} \quad \text{or} \quad y_c = \frac{2}{3} E$$

which is the condition for maximum discharge for a given value of specific energy in a *rectangular channel.*

Example 16.30. (Flow in venturiflume) *A venturiflume is 1·30 m wide at entrance and 0·65 m in the throat. Neglecting hydraulic losses in the flume, calculate the flow if the depths at the entrance and throat are 0·65 m and 0·60 m respectively. A hump is now installed at the throat, of height 200 mm, so that a standing wave (hydraulic jump) is formed beyond the throat. What is the increase in the upstream depth when the same flow as before passes through the flume?*

[Roorkee University]

Solution. Width of venturiflume at entrance, $b_1 = 1{\cdot}3$ m

Width at throat, $b_2 = 0{\cdot}65$ m

Depth of flow at section 1, $y_1 = 0{\cdot}65$ m

Depth of flow at section 2, $y_2 = 0{\cdot}6$ m

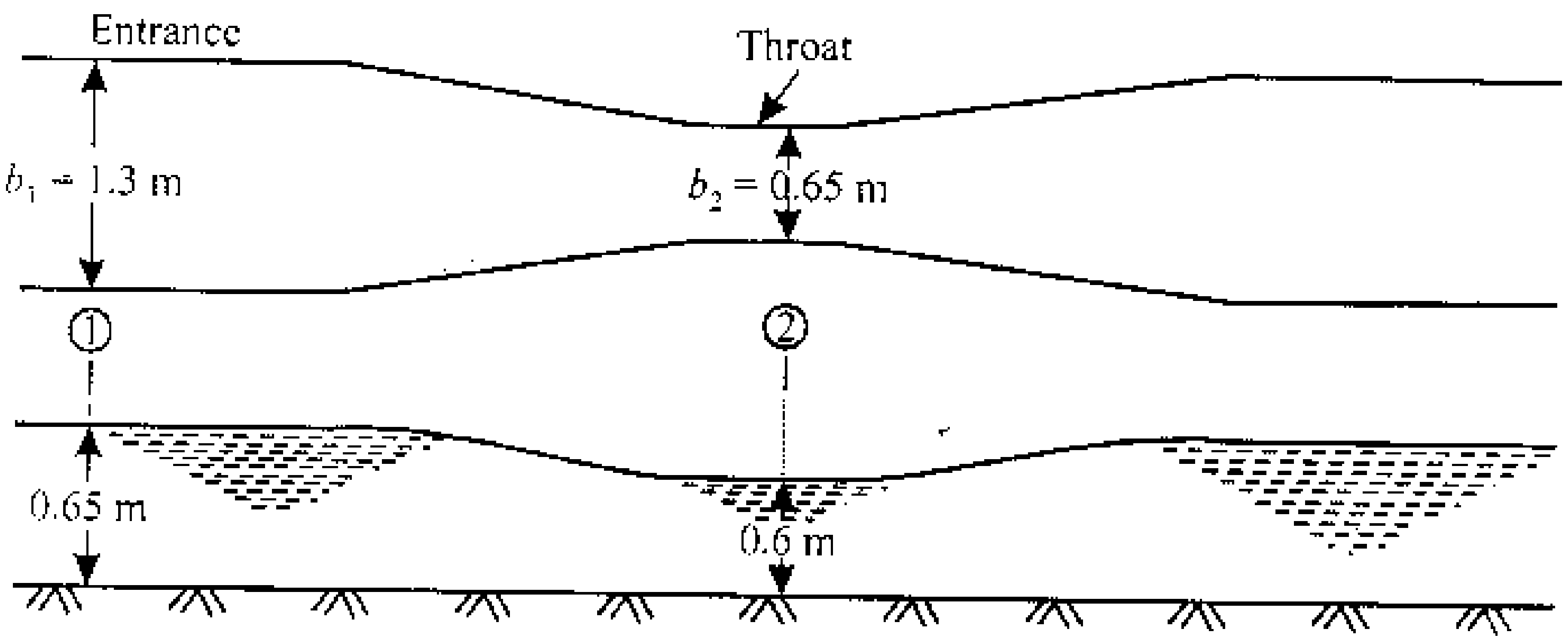


Fig. 16.27. Flow in venturiflume.

Using continuity equation, we have

Discharge, $Q = b_1 y_1 V_1 = b_2 y_2 V_2$

or, $Q = 1{\cdot}3 \times 0{\cdot}65 \times V_1 = 0{\cdot}65 \times 0{\cdot}6 V_2$, $\therefore\ V_2 = \dfrac{1{\cdot}3 \times 0{\cdot}65 V_1}{0{\cdot}65 \times 0{\cdot}6} = 2{\cdot}17 V_1$

Neglecting losses, specific energy at (1) = specific energy at (2)

i.e. $$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$0{\cdot}65 + \frac{V_1^2}{2g} = 0{\cdot}6 + \frac{(2{\cdot}17V_1)^2}{2g}$$

or, $$\frac{(2{\cdot}17V_1)^2}{2g} - \frac{V_1^2}{2g} = 0{\cdot}65 - 0{\cdot}6 = 0{\cdot}05$$

or, $$\frac{V_1^2}{2g}(2{\cdot}17^2 - 1) = 0{\cdot}05$$

or, $$V_1^2 = \frac{0{\cdot}05 \times 2g}{(2{\cdot}17^2 - 1)} = \frac{0{\cdot}05 \times 2 \times 9{\cdot}81}{2{\cdot}17^2 - 1} = 0{\cdot}2645$$

or, $V_1 = 0{\cdot}514$ m/s

The discharge, $Q = b_1 y_1 V_1 = 1{\cdot}3 \times 0{\cdot}65 \times 0{\cdot}514 = 0{\cdot}434\ \text{m}^3/\text{s}$

Critical depth in contracted portion,

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

where, $q = \dfrac{Q}{b_1} = \dfrac{0{\cdot}434}{0{\cdot}65} = 0{\cdot}67\ \text{m}^3/\text{s per m}$

$$\therefore\quad y_c = \left(\frac{0{\cdot}67^2}{9{\cdot}81}\right)^{1/3} = 0{\cdot}357\ \text{m}$$

The new specific energy corresponding to critical flow at the throat when hump of height h is installed

$$= h + y_c + \frac{V_c^2}{2g} = h + \frac{3}{2}y_c = 0{\cdot}2 + \frac{3}{2} \times 0{\cdot}357 = 0{\cdot}735\ \text{m}$$

(where h = 200 mm = 0·2 m ...(*Given*)

The upstream surface will rise till the upstream specific energy equals 0·735 m.

$$0{\cdot}735 = y_1 + \frac{V_1^2}{2g}$$

$$= y_1 + \frac{[Q/(b_1 \times y_1)]^2}{2g}$$

or, $$0{\cdot}735 = y_1 + \frac{Q^2}{2g \times (1{\cdot}3y_1)^2} = y_1 + \frac{0{\cdot}435^2}{2 \times 9{\cdot}81 \times 1{\cdot}69 y_1^2} = y_1 + \frac{0{\cdot}0057}{y_1^2}$$

i.e. $y_1^3 - 0{\cdot}735 y_1^2 + 0{\cdot}0057 = 0$

Solving by trial and error, $y_1 = 0{\cdot}72$ m

The increase in the upstream depth = 0·72 − 0·65 = 0·07 m = **70 mm (Ans.)**

Example 16.31. *A sluice across a channel 7·2 m wide discharges a stream 1·2 m deep. What is the flow rate when the depth upstream of the sluice is 8·4 m ? On the downstream side concrete blocks have been placed to create condition for hydraulic jump to occur. Calculate the force on the blocks if the downstream depth is 3·6 m.*

Solution. Refer to Fig. 16·28

Applying continuity equation at sections (1-1), (2-2) and (3-3), we have

Discharge, $Q = (b_1 \times y_1)\, V_1 = (b_2 \times y_2)\, V_2 = (b_3 \times y_3)\, V_3$

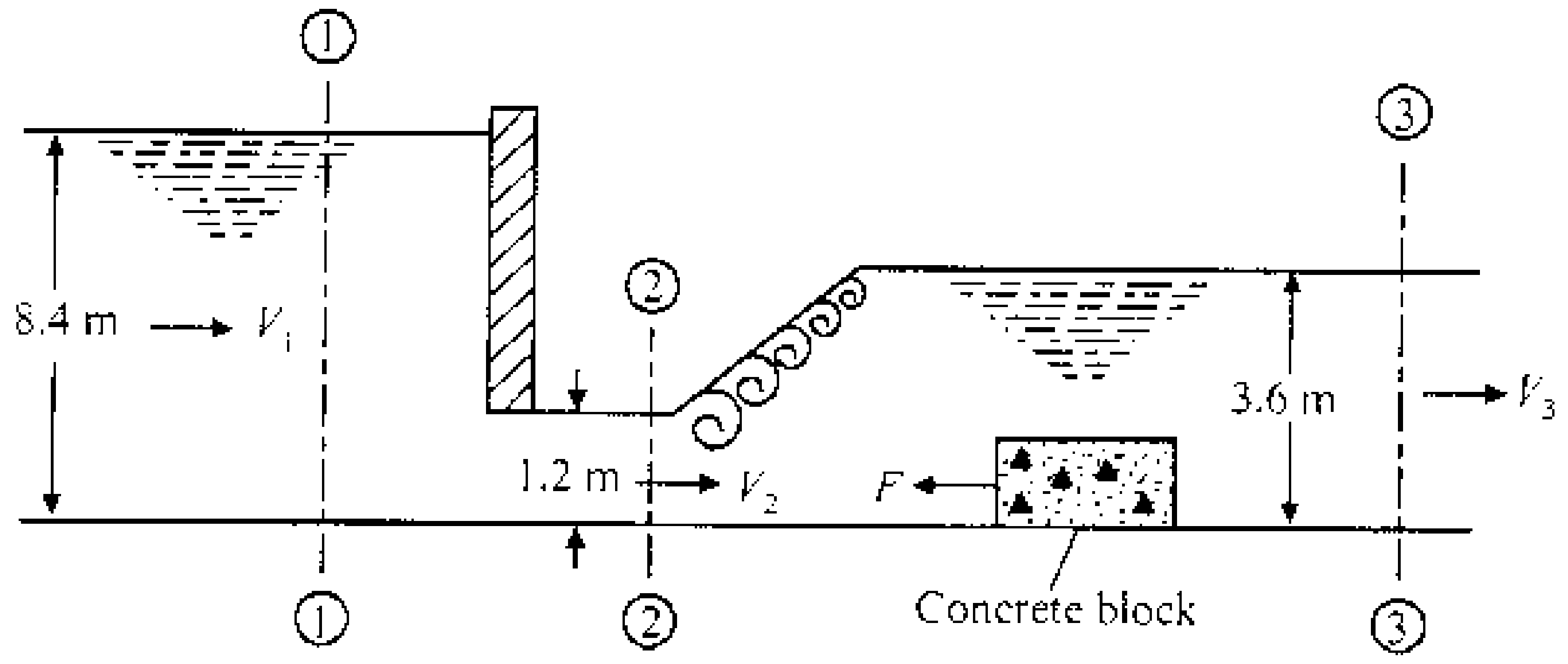


Fig. 16.28

But, $b_1 = b_2 = b_3 = 7{\cdot}2$ m, $y_1 = 8{\cdot}4$ m, $y_2 = 1{\cdot}2$ m, $y_3 = 3{\cdot}6$ m

$\therefore$ $Q = (7{\cdot}2 \times 8{\cdot}4)\, V_1 = (7{\cdot}2 \times 1{\cdot}2)\, V_2 = (7{\cdot}2 \times 3{\cdot}6)\, V_3$

From which, $V_2 = 7V_1$ and $V_3 = 2{\cdot}333\, V_1$

Neglecting frictional losses between sections (1-1) and (2-2) the specific energies at (1-1) and (2-2) are equal.

or, $$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

or, $$8\cdot4 + \frac{V_1^2}{2g} = 1\cdot2 + \frac{(7V_1)^2}{2g} = 1\cdot2 + \frac{49V_1^2}{2g} \qquad (\because V_2 = 7V_1)$$

or, $$\frac{48V_1^2}{2g} = 7\cdot2 \quad \text{or} \quad V_1^2 = \frac{7\cdot2 \times 2 \times 9\cdot81}{48} = 2\cdot943$$

$\therefore$ $V_1 = 1{\cdot}715$ m/s

Flow rate, Q:

$$Q = (b_1 y_1)\, V_1 = 7\cdot2 \times 8\cdot4 \times 1\cdot715 = \mathbf{103.72\ m^3/s\ (Ans.)}$$

Force on the blocks, F:

Applying momentum equation to sections (2-2) and (3-3), neglecting the boundary friction, we have,

$$P_2 - F - P_3 = \frac{wQ}{g}(V_3 - V_2)$$

$$w\, A_2\, \bar{y}_2 - F - w\, A_3\, \bar{y}_3 = \frac{w\,Q}{g}(V_3 - V_2)$$

$$9810 \times (7\cdot2 \times 1\cdot2) \times \frac{1\cdot2}{2} - F - 9810 \times (7\cdot2 \times 3\cdot6) \times \frac{3\cdot6}{2} = \frac{9810 \times 103\cdot72}{9\cdot81}\,(2\cdot333V_1 - 7V_1)$$

$$50855 - F - 457695 = 103720\,(-4\cdot667 \times 1\cdot715)$$

or, $F = 50855 - 457695 - 103720\,(4{\cdot}667 \times 1{\cdot}715) = 423325$ N or $423{\cdot}325$ kN

Hence, the force on the concrete blocks = **423.325 kN** which acts in a direction *opposite* to F **(Ans.)**

Example 16.32. *Uniform flow occurs at a depth of 1·5 m in a long rectangular channel 3 m wide and laid to a slope of 0·0009. If Manning's N = 0·015 calculate:*

(*i*) *Maximum height of hump on the floor to produce critical depth.*

(*ii*) *Width of contraction which will produce critical depth without increasing the upstream depth of flow.* **[IIT Madras]**

Solution. Depth of flow, $y = 1 \cdot 5$ m

Width of channel, $b = 3$ m

Bed slope, $S = 0 \cdot 0009$

Manning's $N = 0 \cdot 015$

Height of the hump, h:

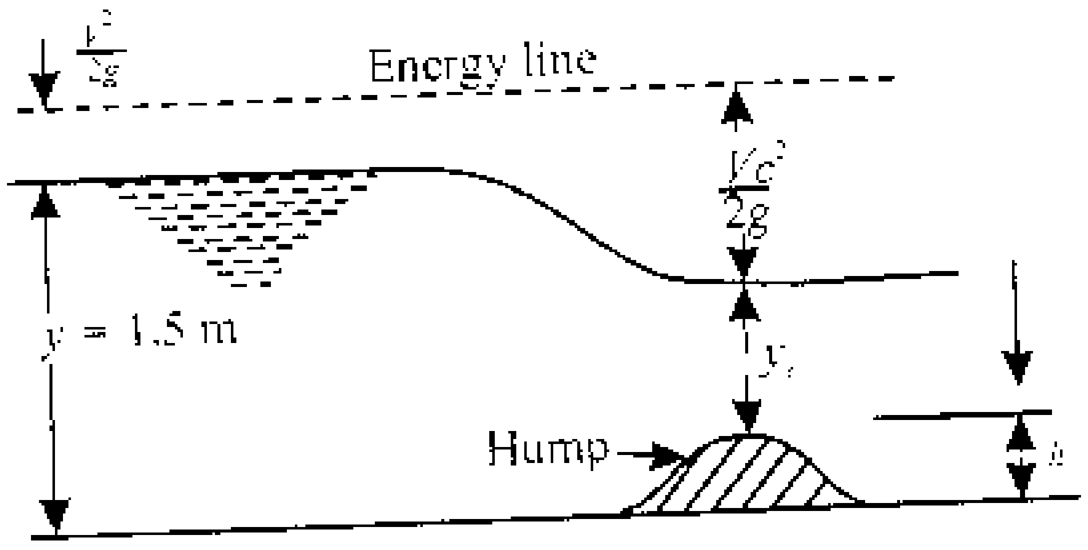


Fig. 16.29

Discharge, $Q = A \times V = A \times C\sqrt{RS}$

where, C (Chezy's constant) $= \frac{1}{N} R^{1/6}$

$$\therefore \quad Q = A \times \frac{1}{N} R^{1/6} \sqrt{RS}$$

$$= A \times \frac{1}{N} R^{2/3} S^{1/2} \quad \ldots(i)$$

(where, V = average velocity of flow, and

R = hydraulic radius)

Here, area $A = b \times y = 3 \times 1 \cdot 5 = 4 \cdot 5 \text{ m}^2$

Perimeter, $P = b + 2y = 3 + 2 \times 1 \cdot 5 = 6$ m and $R = \frac{A}{P} = \frac{4 \cdot 5}{6} = 0 \cdot 75$ m

Substituting the values in expression (*i*), we get

$$Q = 4 \cdot 5 \times \frac{1}{0 \cdot 015} \times (0 \cdot 75)^{2/3} \times (0 \cdot 0009)^{1/2} = 7 \cdot 43 \text{ m}^3\text{/s}$$

Discharge per unit width, $q = \frac{Q}{b} = \frac{7 \cdot 43}{3} = 2 \cdot 477 \text{ m}^3\text{/s per m}$

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2 \cdot 477^2}{9 \cdot 81}\right)^{1/3} = 0 \cdot 855$ m

Now, equating the specific energies upstream and at the hump, we get

$$1 \cdot 5 + \frac{V^2}{2g} = h + y_c + \frac{V_c^2}{2g} \quad \ldots(ii)$$

Here, $V = \frac{Q}{A} = \frac{7 \cdot 43}{4 \cdot 5} = 1 \cdot 65$ m/s, and

$$V_c = \sqrt{gy_c} \quad \text{or} \quad V_c^2 gy_c \quad \text{or} \quad y_c = \frac{V_c^2}{g} \quad \text{or} \quad \frac{V_c^2}{2g} = \frac{y_c}{2}$$

Substituting the values in expression (*ii*), we have

$$1 \cdot 5 + \frac{1 \cdot 65^2}{2 \times 9 \cdot 81} = h + 0 \cdot 855 + \frac{0 \cdot 855}{2} \quad \text{or} \quad 1 \cdot 6387 = h + 1 \cdot 2825$$

$\therefore$ **h = 0.3562 m (Ans.)**

(*ii*) **Width of contraction:**

Let, b_c = Width at the contracted portion to produce critical depth.

Now, Upstream specific energy = Specific energy at the contracted portion.

$$1\cdot6387 = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2} = \frac{3}{2}y_c$$

$$= \frac{3}{2}\left[\frac{q^2}{g}\right]^{1/3} = \frac{3}{2}\left[\frac{(Q/b_c)^2}{g}\right]^{1/3} = \frac{3}{2}\left[\frac{(7\cdot43)^2}{b_c^2\times9\cdot81}\right]^{1/3}$$

or, $$\left[\frac{(7\cdot43)^2}{b_c^2\times9\cdot81}\right]^{1/3} = 1\cdot6387\times\frac{2}{3} = 1\cdot0925$$

or, $$\frac{7\cdot43^2}{b_c^2\times9\cdot81} = (1\cdot0925)^3 = 1\cdot304$$

$\therefore$ $$b_c = \left(\frac{7\cdot43^2}{1\cdot304\times9\cdot81}\right)^{\frac{1}{2}} = \mathbf{2\cdot077\ m\ (Ans.)}$$

Example 16.33. *Water flows at a velocity of 1 m/s and a depth of 2 m in an open channel of rectangular cross-section, 3 m wide. At a certain section the width is reduced to 1·8 m and the bed is raised by 0·65 m. Will the upstream depth be affected? If so to what extent?* **[UPSC, CES Exams.]**

Solution. Velocity of flow, V = 1 m/s

Depth of flow, y = 2 m

Width of channel, b = 3 m

Width of contracted section, b_c = 1·8 m

At section (1): Refer to Fig. 16·30

Specific energy at the section,

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{(1)^2}{2\times9\cdot81} = 2\cdot051\text{ m}$$

Discharge, $Q = A\cdot V = (b_1\times y_1)\times V_1$

$= (3\times2)\times1 = 6\text{ m}^3/\text{s}$

Discharge per unit width,

$$q_1 = \frac{Q}{b_1} = \frac{6}{3} = 2\text{ m}^3/\text{s per m}$$

Critical depth

$$(y_c)_1 = \left(\frac{q_1^2}{g}\right)^{1/3} = \left[\frac{2^2}{9\cdot81}\right]^{1/3} = 0\cdot7415\text{ m}$$

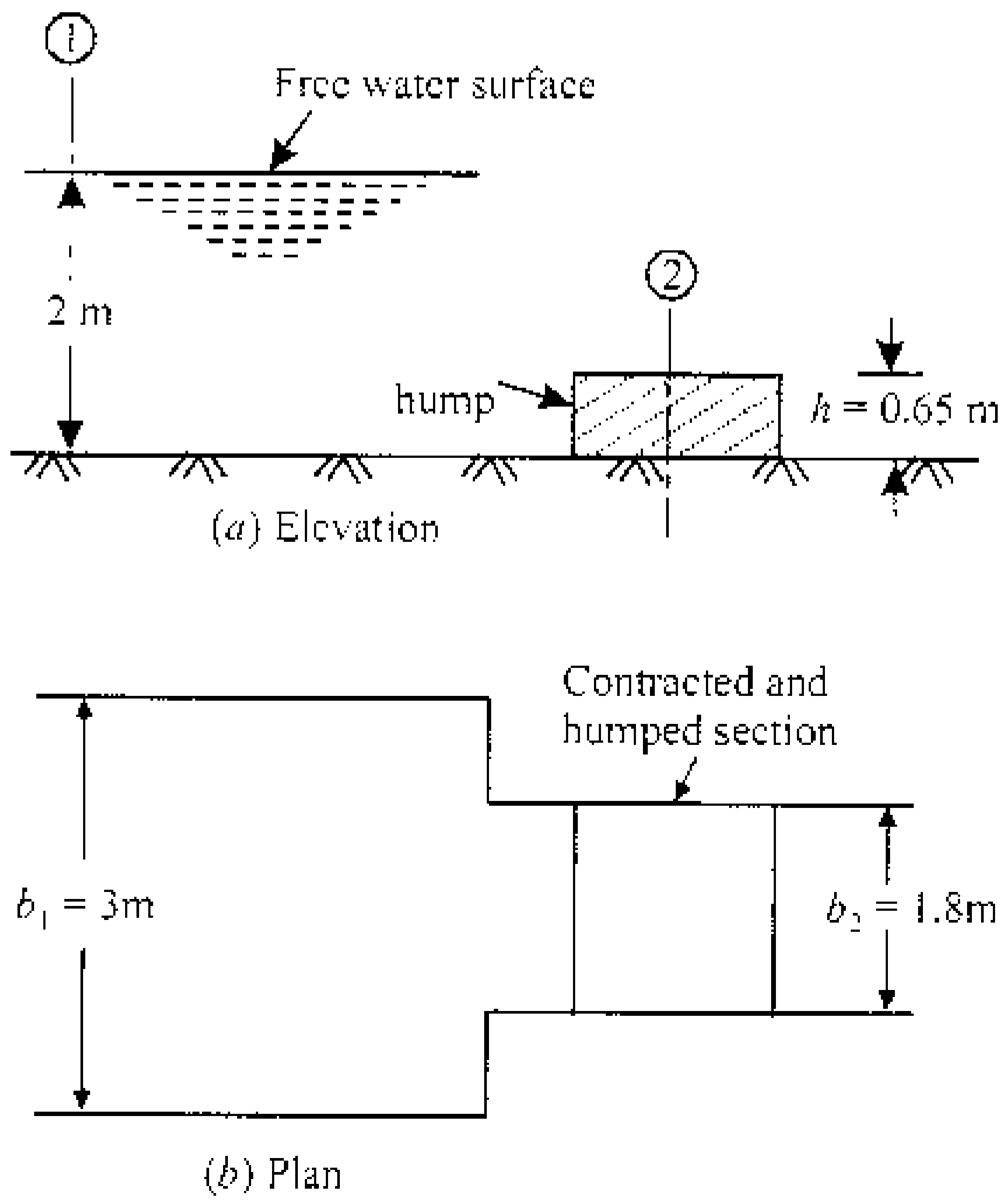


Fig. 16.30

Since $y_1 > y_c$, the flow in the channel is *sub-critical.*

Minimum specific energy at section 1,

$$(E_{min})_1 = \frac{3}{2}(y_c)_1 = \frac{3}{2} \times 0 \cdot 7415 = 1 \cdot 1122 \text{ m}$$

At the section (2) (contracted and humped section):

Discharge per unit width, $q_2 = \frac{Q}{b_2} = \frac{6}{1 \cdot 8} = 3 \cdot 333 \text{ m}^3/\text{ per m}$

Critical depth,

$$(y_c)_2 = \left(\frac{q_2^2}{g}\right)^{1/3} = \left(\frac{3 \cdot 333^2}{9 \cdot 81}\right)^{1/3} = 1 \cdot 0423 \text{ m}$$

Minimum specific energy,

$$(E_{min})_2 = \frac{3}{2}(y_c)_2 = \frac{3}{2} \times 1 \cdot 0423 = 1 \cdot 5634 \text{ m}$$

Specific energy w.r.t. channel bed at section (2),

$$E_2 = (E_{min})_2 + h = 1 \cdot 5634 + 0 \cdot 65 = 2 \cdot 2134 \text{ m}$$

Since $E_2 > E_1$ the upstream depth will be affected. The flow will be possible only when the upstream water level is increased such that

$$E_1 = E_2 \text{ or } y_1 + \frac{V_1^2}{2g} = 2 \cdot 2134 \quad \text{...(i)}$$

Also $\quad Q = b_1 y_1 \times V_1 \quad$ ...Continuity equation

$$6 = 3 \times y_1 \times V_1 \text{ or } V_1 y_1 = 2 \quad \text{...(ii)}$$

From expressions (*i*) and (*ii*), we have

$$y_1 + \frac{(2/y_1)^2}{2g} = 2 \cdot 2134$$

or, $$y_1 + \frac{4}{2g \times y_1^2} = 2 \cdot 2134$$

or, $$y_1 + \frac{0 \cdot 204}{y_1^2} = 2 \cdot 2134 \text{ or } y_1^3 - 2 \cdot 2134\, y_1^2 + 0 \cdot 204 = 0$$

Solving by trial and error, we get $y_1 = 2 \cdot 17$ m

Hence the water level on the upstream side will be headed up by,

$(2 \cdot 17 - 2) = 0 \cdot 17$ m or **170 mm (Ans.)**

Example 16.34. *A hydraulic jump occurs in a V-shaped channel having sides sloping at 45°. Derive an equation relating the two depths and the flow rate.*

If the depths before and after the jump in the above channel are 0·50 m and 1·0 m, determine:

(*i*) *The flow rate;*

(*ii*) *Froude numbers before and after the jump* **[Roorkee University]**

Solution. Let, y_1 = Depth of flow *before* hydraulic jump,

V_1 = Velocity of flow *before* hydraulic jump, and

y_2, V_2 = Depth of flow and velocity of flow respectively *after* hydraulic jump.

Refer to Fig. 16·31:

According to impulse-momentum equation;

Net force acting on a mass of fluid = Rate of change of momentum in the same direction

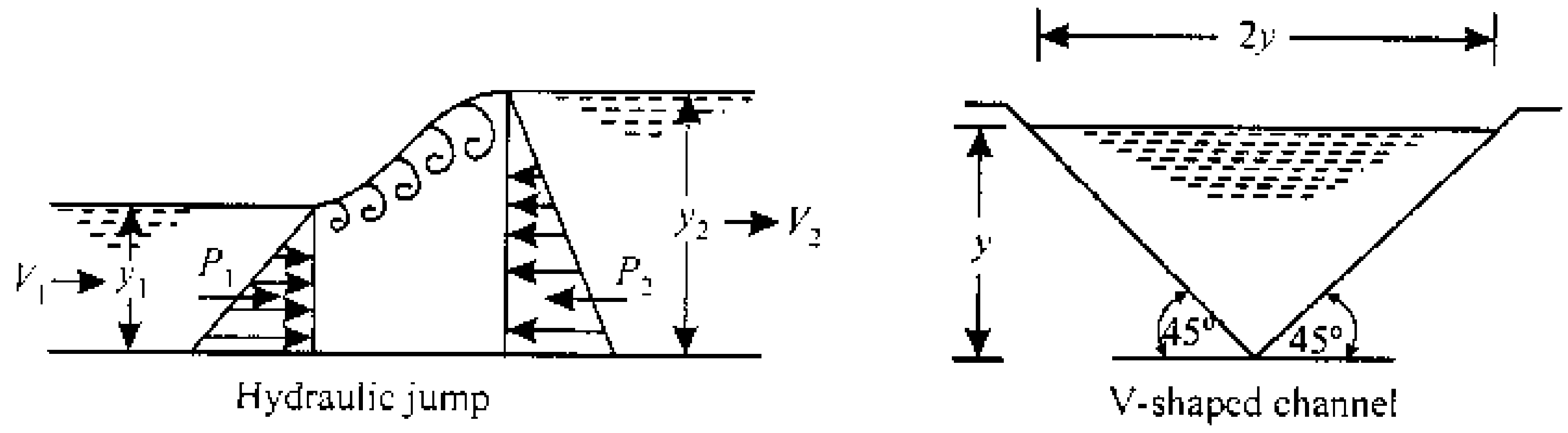


Fig. 16.31. Hydraulic jump – V-shaped channel.

$$\therefore \quad P_2 - P_1 = \frac{wQ}{g}(V_1 - V_2) \quad ...(i)$$

(where, Q = discharge or flow rate)

Area, $A = \frac{1}{2} \times 2y \times y = y^2$ and $\bar{y} = \frac{1}{3} y$

$$\therefore \quad P_1 = wA_1\bar{y}_1 = w \times y_1^2 \times \left(\frac{1}{3} y_1\right) = \frac{1}{3} wy_1^3$$

and, $P_2 = wA_2\,\bar{y}_2 = w \times y_2^2 \times \left(\frac{1}{3} y_2\right) = \frac{1}{3} wy_2^3$

From continuity equation, we have

$$Q = A_1V_1 = A_2V_2; \quad V_1 = \frac{Q}{A_1} = \frac{Q}{y_1^2} \text{ and } V_2 = \frac{Q}{A_2} = \frac{Q}{y_2^2}$$

Substituting these quantities in expression (*i*), we have

$$\frac{1}{3} wy_2^3 - \frac{1}{3} wy_1^3 = \frac{wQ}{g}\left(\frac{Q}{y_1^2} - \frac{Q}{y_2^2}\right)$$

or, $\frac{1}{3}(y_2^3 - y_1^3) = \frac{Q^2}{g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right)$. (cancelling w on both sides)

or, $\frac{1}{3}(y_2^3 - y_1^3) = \frac{Q^2}{g}\left(\frac{y_2^2 - y_1^2}{y_1^2\, y_2^2}\right)$

or, $Q^2 = \frac{g}{3} \times y_1^2\, y_2^2 \left(\frac{y_2^3 - y_1^3}{y_2^2 - y_1^2}\right)$

or, $Q = y_1\, y_2 \sqrt{\frac{g}{3}\left(\frac{y_2^3 - y_1^3}{y_2^2 - y_1^2}\right)}$

This is the required equation relating the two depths and the flow rate.

Depths: y_1 = 0·5 m, y_2 = 1·0 m ...(*Given*)

(*i*) **Flow rate, Q:**

$$Q = y_1\, y_2 \sqrt{\frac{g}{3}\left(\frac{y_2^3 - y_1^3}{y_2^2 - y_1^2}\right)}$$

or $Q = 0\cdot5 \times 1\cdot0 \sqrt{\frac{9\cdot81}{3}\left(\frac{1^3 - 0\cdot5^3}{1^2 - 0\cdot5^2}\right)} = 0\cdot5\sqrt{3\cdot27\left(\frac{1 - 0\cdot125}{1 - 0\cdot25}\right)} = \mathbf{0\cdot977\ m^3/s}$ **(Ans.)**

(*ii*) **Froude number before and after jump, Fr_1, Fr_2:**

Froude number, $Fr = \dfrac{V}{\sqrt{g\ D}}$

where, D = Hydraulic depth $= \dfrac{A}{T}$

(T = top width of the channel, A = area of cross-section of the channel)

$$V_1 = \frac{Q}{y_1^2} = \frac{0 \cdot 977}{0 \cdot 5^2} = 3 \cdot 91 \text{ m/s}$$

$$V_2 = \frac{Q}{y_2^2} = \frac{0 \cdot 977}{1^2} = 0 \cdot 977 \text{ m/s}$$

$$D_1 = \frac{A_1}{T} = \frac{y_1^2}{2y_1} = \frac{y_1}{2} = \frac{0 \cdot 5}{2} = 0 \cdot 25 \text{ m}$$

$$D_2 = \frac{A_2}{T} = \frac{y_2^2}{2y_2} = \frac{y_2}{2} = \frac{1 \cdot 0}{2} = 0 \cdot 5 \text{ m}$$

$$Fr_1 = \frac{V_1}{\sqrt{gD_1}} = \frac{3 \cdot 91}{\sqrt{9 \cdot 81 \times 0 \cdot 25}} = 2 \cdot 5 \textbf{ (Ans.)}$$

$$Fr_2 = \frac{V_2}{\sqrt{gD_2}} = \frac{0 \cdot 977}{\sqrt{9 \cdot 81 \times 0 \cdot 5}} = 0 \cdot 44 \textbf{ (Ans.)}$$

Example 16.35. (Surges in open channels). *A horizontal rectangular channel of 3 m width and 2 m water depth conveys water at 18 m³/s. If the flow rate is suddenly reduced to $\frac{2}{3}$ of its original value, compute the magnitude and speed of the upstream surge.*

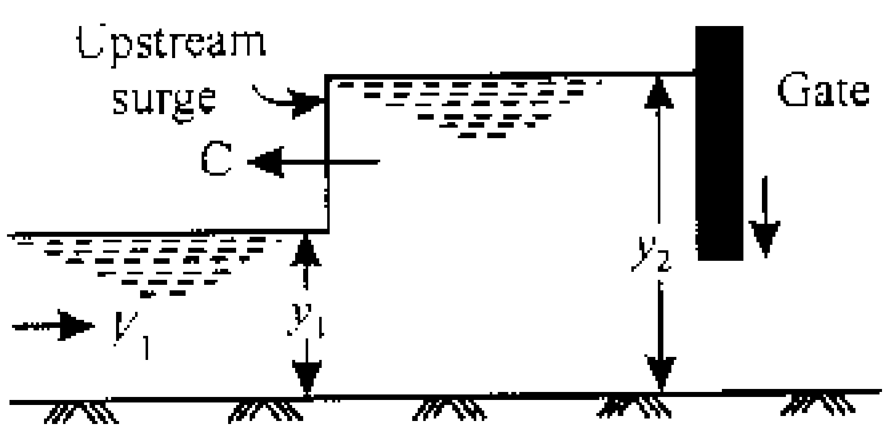
Assume that the front of the surge is rectangular and friction in the channel is neglected.

[UPSC Exams.]

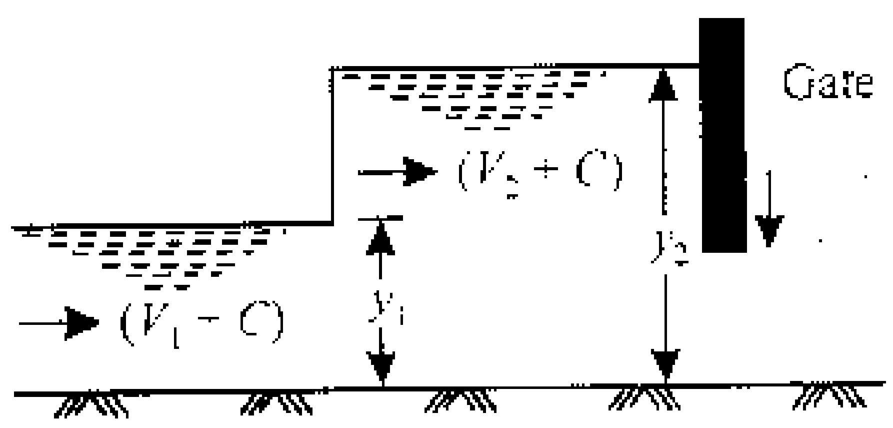
Solution. Width of channel, $b = 3$ m

Depth of water, $y_1 = 2$ m

The flow rate or discharge, $Q_1 = 18 \text{ m}^3/\text{s}$



(*a*) Observer stationary.



(*b*) Observer moving with the surge.

Fig. 16.32

In a channel, when discharge is suddenly reduced by operating a gate, an upstream surge will be developed which will move with a constant velocity C (also known as *celerity* of the wave) as shown in Fig. 16·32 (*a*). An observer standing on the canal bank will notice the surge moving upstream. This unsteady flow case can be transformed into a steady one by *superimposing* flow with velocity C in opposite direction as shown in Fig. 16·32 (*b*)

Also, $by_1 (V_1 + C) = by_2 (V_2 + C)$ ...*Continuity equation*

or. $y_1 (V_1 + C) = y_2 (V_2 + C)$

Again, $\frac{wb}{2}(y_2^2 - y_1^2) = \frac{wb}{g} y_1 (V_1 + C)(V_1 - V_2)$ ...*Momentum equation*

or, $$(y_2^2 - y_1^2) = \frac{2y_1}{g}(V_1 + C)(V_1 - V_2) \quad ...(i)$$

Now, $$V_1 = \frac{Q_1}{b \times y_1} = \frac{18}{3 \times 2} = 3 \text{ m/s}$$

$$Q_2 = \frac{2}{3} Q_1 \quad ...(Given)$$

$$\therefore \quad Q_2 = \frac{2}{3} \times 18 = 12 \text{ m}^3\text{/s}$$

$$Q_2 = (b_2 \times y_2)\, V_2 = b_2 \times V_2 y_2$$

$$\therefore \quad V_2\, y_2 = \frac{Q_2}{b_2} = \frac{12}{3} = 4 \text{ m}^2\text{/s per m}$$

Now, $V_1 y_1 = V_2 y_2 + C(y_2 - y_1)$ ...*Continuity equation*

$$3 \times 2 = 4 + C(y_2 - 2)$$

or, $$C = \frac{2}{y_2 - 2}$$

Substituting the values in expression (*i*) we have

$$(y_2^2 - 2^2) = \frac{2 \times 2}{9 \cdot 81}\left(3 + \frac{2}{y_2 - 2}\right)(3 - V_2)$$

$$(y_2^2 - 4) = 0 \cdot 41\left(3 + \frac{2}{y_2 - 2}\right)\left(3 - \frac{4}{y_2}\right) \quad \left[\because V_2 y_2 = 4 \text{ or } V_2 = 4/y_2\right]$$

Solving by trial and error, $y_2 = 2 \cdot 8$ m and $V_2 = \frac{4}{2 \cdot 8} = 1 \cdot 428$ m/s

Height of the surge $= y_2 - y_1 = 2 \cdot 8 - 2 = $ **0.8 m (Ans.)**

Velocity of the upstream surge, $C = \frac{2}{y_2 - 2} = \frac{2}{2 \cdot 8 - 2} = \mathbf{2 \cdot 5 \text{ m/s}}$ **(Ans.)**

16.10. Gradually Varied Flow

Gradually varied flow (G.V.F.) is one in *which the depth changes gradually over a long distance.* In a rapidly varied flow, the change in depth takes place in a short distance.

Gradually varied flow may be caused due to one or more of the following *factors*:

1. The change in the shape and size of the channel cross-section,
2. The change in slope of the channel,
3. The presence of obstruction (e.g., weir etc.), and
4. The change in frictional forces at the boundaries.

16.10.1. Equation of Gradually Varied Flow

The following *assumptions* are made for analysing gradually varied flow:

1. The channel is a prismatic (a channel with constant section and alignment).
2. The bed slope is small.
3. The flow is steady and hence discharge is constant.

4. The pressure distribution over the channel section is hydrostatic *i.e.* streamlines are practically straight and parallel.
5. The energy correction factor (α) is unity.
6. The roughness co-efficient is constant for the length of the channel and it does not depend on the depth of flow.
7. The Chezy and Manning correlations are equally applicable to gradually varied flow for determining the slope of energy line.

Consider a rectangular channel having gradually varied flow (Fig. 16·33), the depth of flow gradually decreasing in the direction of flow.

Let, b = Width of the channel,

Q = Discharge through the channel,

z = Height of bottom of channel above datum,

y = Depth of flow,

V = Mean velocity of flow,

$S_b = \tan i \simeq j$ = slope of the channel bed, and

$S_e = \tan j \simeq j$ = slope of energy line.

According to Bernoulli's equation, the energy equation at any section is given by,

$$E = z + y + \frac{V^2}{2g} \qquad ...(i)$$

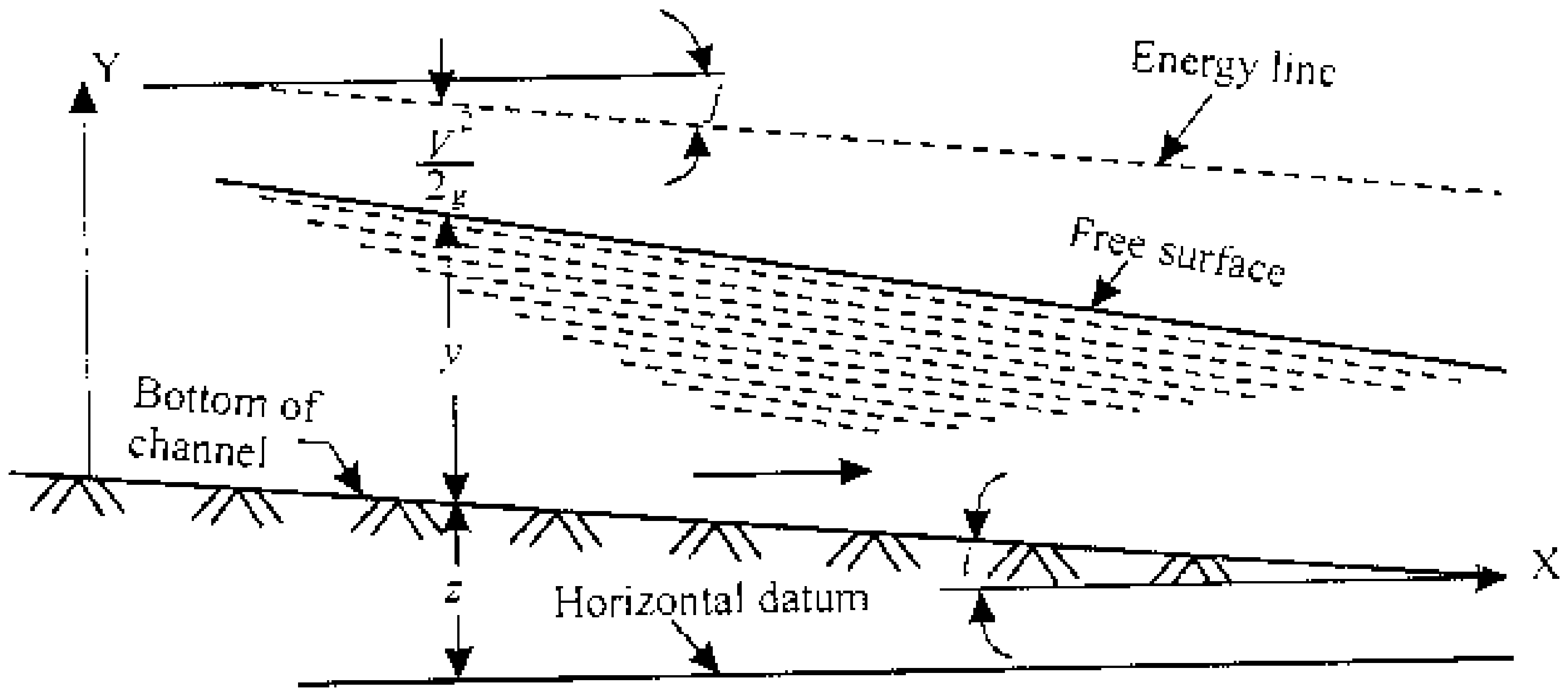


Fig. 16.33. Gradually varied flow in a channel.

Taking the bottom of the channel on the X-axis and the vertically upwards direction measured from the channel bottom, as the Y-axis, differentiation of eqn. (i), with respect to x yields

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \qquad ...(ii)$$

Now $$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dx}\left[\frac{(Q/A)^2}{2g}\right] = \frac{d}{dx}\left[\frac{(Q/b\cdot y)^2}{2g}\right] \qquad \left[\because V = \frac{Q}{A} \text{ and } A = b\cdot y\right]$$

$$= \frac{d}{dx}\left[\frac{Q^2}{b^2\cdot y^2 \times 2g}\right] = \frac{Q^2}{b^2 \times 2g}\,\frac{d}{dx}\left(\frac{1}{y^2}\right) \qquad \left[\because Q, b \text{ and } y \text{ are constant}\right]$$

$$= \frac{Q^2}{b^2 \times 2g} \frac{d}{dy}\left(\frac{1}{y^2}\right)\frac{dy}{dx}$$

$$= \frac{Q^2}{b^2 \times 2g}\left(-\frac{2}{y^3}\right)\frac{dy}{dx} = \frac{-2Q^2}{b^2 \times 2gy^3}\frac{dy}{dx}$$

or, $$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{-Q^2}{b^2 \cdot y^2 \times gy}\frac{dy}{dx} = -\frac{V^2}{gy}\frac{dy}{dx} \qquad \left(\because \frac{Q}{b \cdot y} = V\right)$$

Substituting the value of $\frac{d}{dx}\left(\frac{V^2}{2g}\right)$ in expression (*ii*), we get

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dy}{dx} - \frac{V^2}{gy}\frac{dy}{dx}$$

or, $$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dy}{dx}\left(1 - \frac{V^2}{gy}\right) \qquad ...(iii)$$

Now, $\frac{dE}{dx}$ = Slope of energy line = $-S_e$

and, $\frac{dz}{dx}$ = Slope of bed of the channel = $-S_b$

– *Ve* signs with S_e and S_b indicate that the values of E and z *decrease* with the *increase* of x.

Substituting the values of $\frac{dE}{dx}$ and $\frac{dz}{dx}$ in expression (*iii*), we get,

$$-S_e = -S_b + \frac{dy}{dx}\left(1 - \frac{V^2}{gy}\right) \quad \text{or} \quad \frac{dy}{dx} = \frac{(S_b - S_e)}{\left(1 - \frac{V^2}{gy}\right)} \qquad ...(16.42)$$

or, $$\frac{dy}{dx} = \frac{(S_b - S_e)}{\left[1 - (Fr)^2\right]} \qquad ...(16.43)\left(\because \frac{V}{\sqrt{gy}} = Fr\right)$$

$\frac{dy}{dx}$ represents the variation of depth along the bottom of the channel and is also called the *slope of the free water surface.*

(*i*) When $\frac{dy}{dx} = 0$: y is constant (or depth of water above the bottom of channel is *constant*); it means that *free water surface is parallel to the channel bed.*

(*ii*) When $\frac{dy}{dx} > 0$ $\left(or\ \frac{dy}{dx}\ is\ +ve\right)$: *It indicates that the depth of water increases in the direction of flow, the profile of water so obtained is called* **'back water curve'**.

(*iii*) When $\frac{dy}{dx} < 0$ *or* $\frac{dy}{dx}$ *is – ve* : *It indicates that the depth of water decreases* in the direction of flow. The profile of water so obtained is known as **'drop down curve'**.

16.10.2. Back Water Curve and Afflux

In an open channel when the flow is uniform, the flow has constant depth at all the sections and the surface of the free water lies parallel to bed of the channel. But when an obstruction like a dam, weir etc. comes across the channel width the water level rises and it has maximum depth from the bed at some section (Fig. 16·34). If y_1 is the depth of water at the point, where the water starts rising up and y_2 is the maximum height of rising water from the bed, then this increase in depth (*i.e.* $y_2 - y_1$) is known as **'afflux'** and the curved surface of the liquid with its *concavity* upwards, is known as **'back water curve'**.

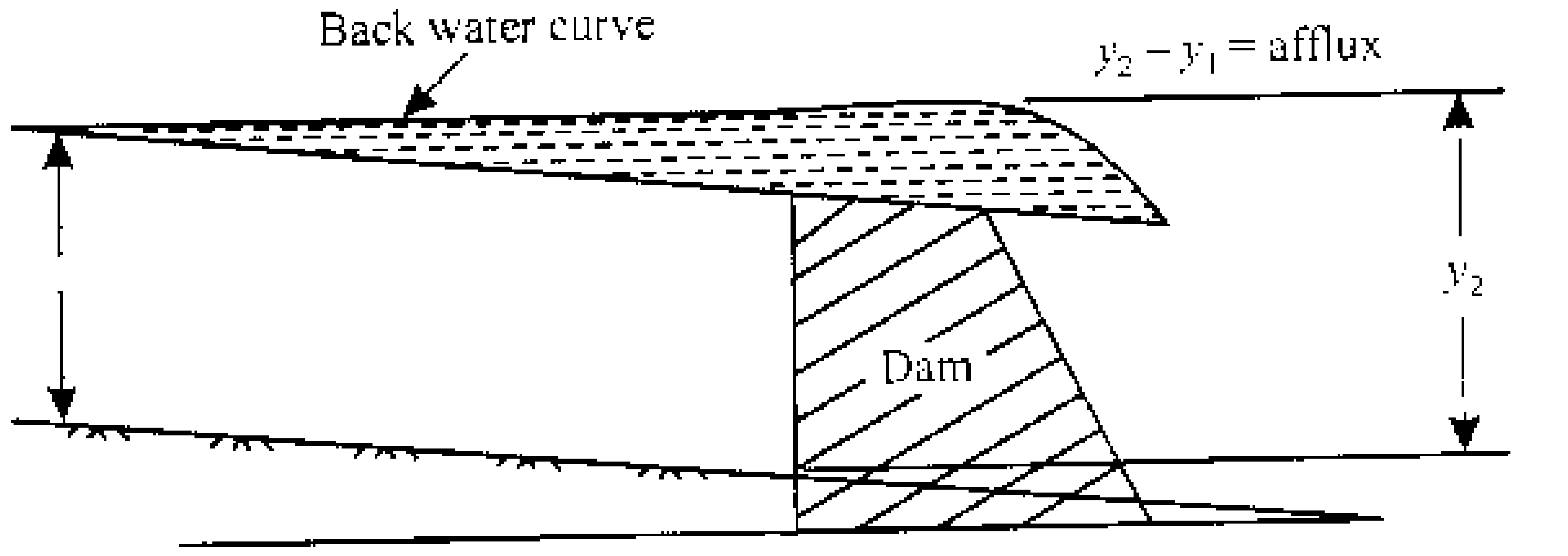


Fig. 16.34. Back water curve and afflux.

Length of back water curve:

The length of back water curve is the distance along the bed of the channel between the section where water starts rising to the section and where water has maximum depth.

Consider a channel in which a back water curve is formed as shown in Fig. 16·35. Let two sections 1-1 and 2-2 are so chosen that distance between them represents the length of backwater curve.

Let, y_1 = Depth of flow at section 1-1,

V_1 = Velocity of flow at section 1-1,

y_2 = Depth of flow at section 2-2,

V_2 = Velocity of flow at section 2-2,

S_b = Bed slope,

S_e = Energy line slope, and

l = Length of back water curve.

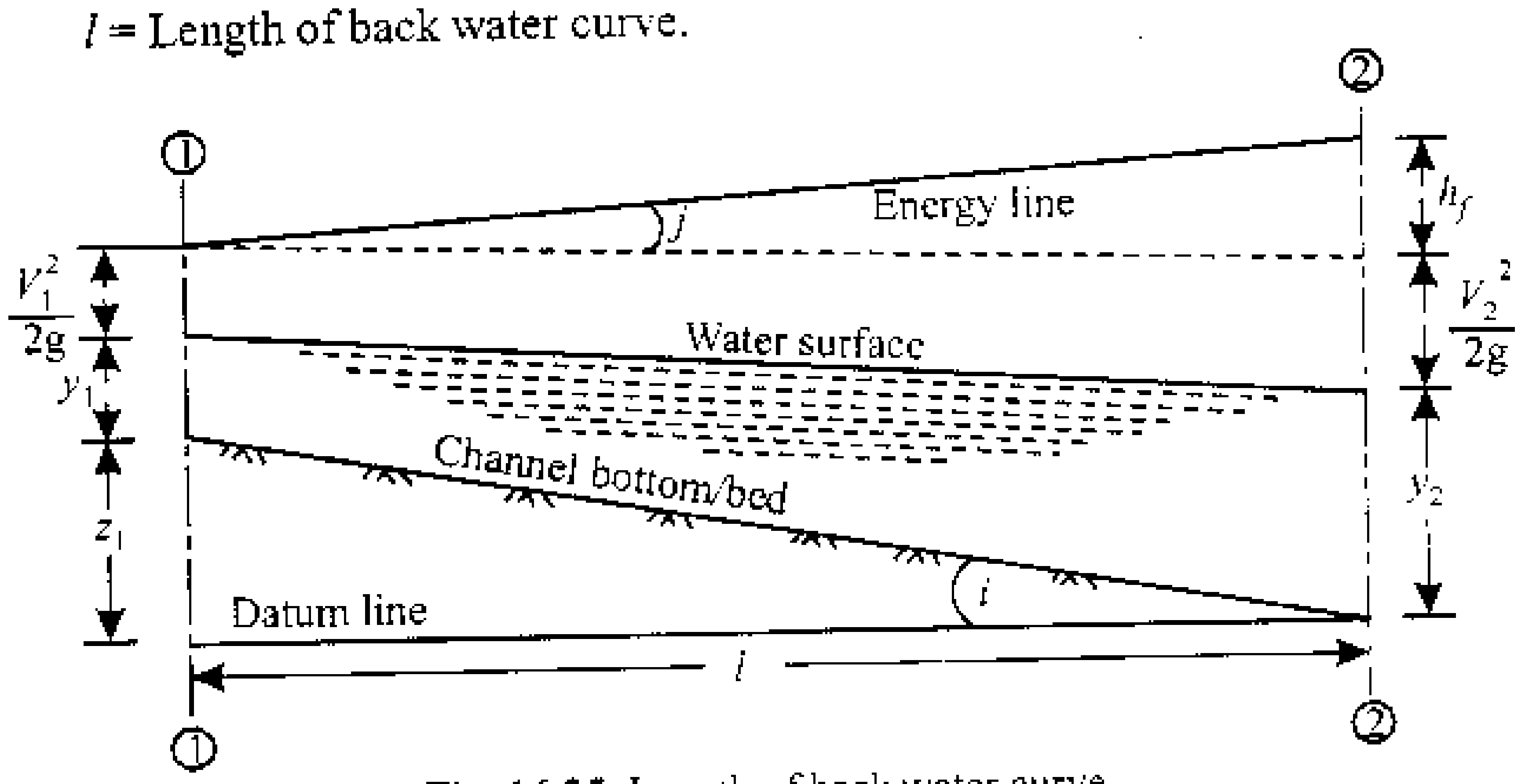


Fig. 16.35. Length of back water curve.

Applying Bernoulli's equation at the two sections with channel bed at section 2-2 as the datum for potential head, we have

$$z_1 + y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_f \qquad (\because z_2 = 0)$$

where, h_f = Loss of head due to friction = $S_e \times l$, and $z_1 = S_b \times l$

$$\therefore \qquad S_b \times l + y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + S_e \times l$$

or,
$$S_b \times l - S_e \times l = \left(y_2 + \frac{V_2^2}{2g}\right) - \left(y_1 + \frac{V_1^2}{2g}\right)$$

or,
$$l\,(S_b - S_e) = E_2 - E_1 \qquad \left(\text{where } E_2 = y_2 + \frac{V_2^2}{2g},\ E_1 = y_1 + \frac{V_1^2}{2g}\right)$$

$$\therefore \qquad l = \frac{E_2 - E_1}{S_b - S_e} \qquad \text{...(16·43)}$$

where, E_1 and E_2 represent the specific energies at the beginning and end of the backwater curve. The value of S_e (slope of energy line) is determined by Manning's formula or Chezy's formula corresponding to flow conditions at mean/average depth of flow.

Example 16.36. *In a rectangular channel 12 m wide and 3·6 m deep water is flowing with a velocity of 1·2 m/s. The bed slope of the channel is 1 in 4000. If flow of water through the channel is regulated in such a way that energy line is having a slope of 0·0004 find the rate of change of depth of water in the channel.*

Solution. Width of channel, b = 12 m

Depth of the channel, y = 3·6 m

Velocity of flow, V = 1·2 m/s

Bed slope, $S_b = \frac{1}{4000} = 0 \cdot 00025$

Slope of the energy line, S_e = 0·00004

Rate of change of depth of water, $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{S_b - S_e}{\left(1 - \frac{V^2}{gy}\right)} \qquad \text{...[Eqn. (16·42)]}$$

Substituting the values, we get,

$$\frac{dy}{dx} = \frac{0 \cdot 00025 - 0 \cdot 00004}{\left(1 - \frac{1 \cdot 2^2}{9 \cdot 81 \times 3 \cdot 6}\right)} = \frac{0 \cdot 00021}{0 \cdot 9592} = \mathbf{0 \cdot 0002189} \ \textbf{(Ans.)}$$

Example 16.37. *In a rectangular channel of width 24 m and depth of flow 6 m, the rate of flow of water is 86·4 m³/s. If the bed slope of the channel is 1 in 4000 find the slope of the free water surface. Take Chezy's constant C = 60.*

Solution. Width of the channel, b = 24 m

Depth of flow, y = 6 m

Rate of flow or discharge, Q = 86·4 m³/s

Bed slope, $S_b = \frac{1}{4000} = 0 \cdot 00025$

Chezy's constant, $C = 60$.

Slope of the free water surface, $\frac{dy}{dx}$:

Discharge, $Q = A \times V = A \times C\sqrt{RS_b}$

where, A = area of flow $= b \times y = 24 \times 6 = 144 \text{ m}^2$

Hydraulic radius $= \frac{A}{P} = \frac{144}{b + 2y} = \frac{144}{24 + 2 \times 6} = 4 \text{ m}$

The slope of the energy line (S_e) is determined from Chezy's formula.

$\therefore \quad 86 \cdot 4 = 144 \times 60\sqrt{4 \times S_e} = 17280\sqrt{S_e}$ [Art. 16·10, point 7]

or $\quad S_e = \left(\frac{86 \cdot 4}{17280}\right)^2 = 0 \cdot 000025$

Now, $\quad \frac{dy}{dx} = \frac{S_b - S_e}{1 - \frac{V^2}{gy}} = \frac{0 \cdot 00025 - 0 \cdot 000025}{1 - \frac{0 \cdot 6^2}{9 \cdot 81 \times 6}} = 0 \cdot 000226$

$$\left(V = \frac{Q}{b \times y} = \frac{86 \cdot 4}{24 \times 6} = 0 \cdot 6 \text{ m/s}\right)$$

Hence slope of the free water surface = **0.000226 (Ans.)**

Example 16.38. *A wide channel laid to a slope of 1 in 1000 carries a discharge of 3·5 m³/s per metre width at a depth of 1·6 m. Find out the value of Chezy's constant C. Consider the flow to be uniform.*

If the actual depth varies from 1·5 m at an upstream location to 1·7 m at a location 300 m downstream or in other words the flow is gradually varied, what will be the value of Chezy's co-efficient C. **[Roorkee University]**

Solution. Bed slope of channel, $S_b = \frac{1}{1000}$

Discharge, $q = 3 \cdot 5 \text{ m}^3\text{/s}$ per metre width

Depth of water, $y = 1 \cdot 6$ m

$\therefore \quad$ Velocity of flow $= \frac{q}{y} = \frac{3 \cdot 5}{1 \cdot 6} = 2 \cdot 1875 \text{ m/s}$

Hydraulic radius, $\quad R = \frac{A}{P} = \frac{b \times y}{b + 2y}$

For a wide channel, the width b of the stream is large in comparison with depth of flow y. Therefore

$$R \simeq \frac{b \times y}{b} = y = 1 \cdot 6 \text{ m}$$

(*i*) **Uniform flow:**

$$V = C\sqrt{RS_b} \quad \text{...Chezy's formula}$$

$$2 \cdot 1875 = C\sqrt{1 \cdot 6 \times \frac{1}{1000}} = 0 \cdot 04\ C$$

$$\therefore \qquad C = \frac{2 \cdot 1875}{0 \cdot 04} = \mathbf{54 \cdot 68\ (Ans.)}$$

(*ii*) **Gradually varied flow:**

Slope of the free water surface, $\frac{dy}{dl} = \frac{1 \cdot 7 - 1 \cdot 5}{300} = 0 \cdot 000667$

Average flow depth, $y = \frac{y_1 + y_2}{2} = \frac{1 \cdot 7 + 1 \cdot 5}{2} = 1 \cdot 6$ m

Velocity at average flow depth, $V = \frac{q}{y} = \frac{3 \cdot 5}{1 \cdot 6} = 2 \cdot 1875$ m

Hydraulic radius, $R = y = 1 \cdot 6$ m

The rate of change of depth is given by,

$$\frac{dy}{dl} = \frac{S_b - S_e}{1 - \frac{V^2}{gy}}$$

or, $$0 \cdot 000667 = \frac{0 \cdot 001 - S_e}{1 - \frac{2 \cdot 1875^2}{9 \cdot 81 \times 1 \cdot 6}} = \frac{0 \cdot 001 - S_e}{0 \cdot 695}$$

or, $$S_e = 0 \cdot 001 - 0 \cdot 000667 \times 0 \cdot 695 = 0 \cdot 000536$$

Now, $$V = C\sqrt{RS_e} \quad \text{...Chezy's formula}$$

$$2 \cdot 1875 = C\sqrt{1 \cdot 6 \times 0 \cdot 000536} = 0 \cdot 0293\ C$$

or, $$C = \frac{2 \cdot 1875}{0 \cdot 0293} = \mathbf{74 \cdot 65\ (Ans.)}$$

Example 16.39. *The normal depth of flow of water, in a rectangular channel 1·5 m wide, is one metre. The bed slope of the channel is 0·0006 and Manning's roughness co-efficient N 0·012. Find the critical depth.*

At a certain section of the same channel the depth is 0·92 m while at a second section the depth is 0·86 m. Find the distance between the two sections. Also find whether the section is located downstream or upstream with respect to the first section. **[UPSC Exams.]**

Solution. Width of the channel, $b = 1 \cdot 5$ m

Normal depth of water, $y_n = 1$ m

$\therefore$ Area of flow, $A = b \times y_n = 1 \cdot 5 \times 1 = 1 \cdot 5\ \text{m}^2$

Perimeter, $P = b + 2y_n = 1 \cdot 5 + 2 \times 1 = 3 \cdot 5$ m

$\therefore$ Hydraulic radius, $R = \frac{A}{P} = \frac{1 \cdot 5}{3 \cdot 5} = 0 \cdot 4286$ m

Manning's co-efficient $N = 0 \cdot 012$

Bed slope, $S_b = 0 \cdot 0006$

Critical depth:

Discharge, $Q = A \times V = A \times C\sqrt{RS_b} = A \times \frac{1}{N} R^{1/6}\sqrt{RS_b} = A \times \frac{1}{N} R^{2/3}\sqrt{S_b}$

(where, Chezy's constant, $C = \frac{1}{N} R^{1/6}$)

or, $$Q = 1 \cdot 5 \times \frac{1}{0 \cdot 012} \times (0 \cdot 4286)^{2/3} \times (0 \cdot 0006)^{1/2} = 1 \cdot 74 \text{ m}^3/\text{s}$$

Discharge per unit width, $q = \frac{Q}{b} = \frac{1 \cdot 74}{1 \cdot 5} = 1 \cdot 16 \text{ m}^3/\text{s per m}$

The critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{1 \cdot 16^2}{9 \cdot 81}\right]^{1/3}$ **= 0·516 m (Ans.)**

Specific energy at 0.92 m depth:

$$E_1 = 0 \cdot 92 + \frac{V_1^2}{2g}$$

where, $$V_1 = \frac{Q}{b \times 0 \cdot 92} = \frac{1 \cdot 74}{1 \cdot 5 \times 0 \cdot 92} = 1 \cdot 26 \text{ m/s}$$

$$\therefore \quad E_1 = 0 \cdot 92 + \frac{1 \cdot 26^2}{2 \times 9 \cdot 81} = 1 \cdot 0 \text{ m}$$

Specific energy at 0.86 m depth:

$$E_2 = 0 \cdot 86 + \frac{V_2^2}{2g}$$

where, $$V_2 = \frac{Q}{b \times 0 \cdot 86} = \frac{1 \cdot 74}{1 \cdot 5 \times 0 \cdot 86} = 1 \cdot 35 \text{ m/s}$$

$$\therefore \quad E_2 = 0 \cdot 86 + \frac{1 \cdot 35^2}{2 \times 9 \cdot 81} = 0 \cdot 953 \text{ m}$$

Slope of energy line (S_e) at the mean section:

$$y = \frac{y_1 + y_2}{2} = \frac{0 \cdot 92 + 0 \cdot 86}{2} = 0 \cdot 89 \text{ m}$$

Now $$Q = A \times \frac{1}{N} R^{2/3} (S_e)^{1/2} \quad \text{or} \quad Q^2 = A^2 \times \frac{R^{4/3}}{N^2} S_e$$

$$\therefore \quad S_e = \frac{Q^2 N^2}{A^2 R^{4/3}} = \frac{1 \cdot 74^2 \times 0 \cdot 012^2}{(1 \cdot 5 \times 0 \cdot 89)^2 \times (0 \cdot 407)^{4/3}} = 8 \cdot 11 \times 10^{-4} = 0 \cdot 000811$$

$$\left(\because R = \frac{A}{P} = \frac{1 \cdot 5 \times 0 \cdot 89}{1 \cdot 5 + 2 \times 0 \cdot 89} = 0 \cdot 407\right)$$

Distance between the two sections,

$$\Delta x = \frac{E_2 - E_1}{S_b - S_e} = \frac{0 \cdot 953 - 1 \cdot 0}{0 \cdot 0006 - 0 \cdot 000811} = \mathbf{222 \cdot 75 \text{ m (Ans.)}}$$

Slope of water surface, $\frac{dy}{dx} = \frac{S_b - S_e}{1 - \frac{V^2}{gy}}$

Average depth of flow = 0·89 m (calculated above)

Velocity at mean section, $V = \frac{Q}{1\cdot5 \times 0\cdot89} = \frac{1\cdot74}{1\cdot5 \times 0\cdot89} = 1\cdot3$ m/s

$$\therefore \quad \frac{dy}{dx} = \frac{0\cdot0006 - 0\cdot000811}{1 - \frac{1\cdot3^2}{9\cdot81 \times 0\cdot89}} = -2\cdot616 \times 10^{-4}$$

Since $\frac{dy}{dx}$ is $-ve$, therefore, the second section is **downstream (Ans.)**

Example 16.40. (Length of backwater curve). *Draw the specific energy diagram for various constant discharges and show the alternate and critical depths.*

A weir is installed across a rectangular open channel thereby raising the flow depth from 1·5 m in a normal flow to 2·5 m at the weir. The width of the channel is 10 m and it is laid to a slope of 1 in 10000. Find an approximate length of the backwater curve considering the average velocity, average depth and average slope midway between the two sections. Take the value of Manning's rugosity co-efficient equal to 0·02. **[Delhi University]**

Solution. ***Upstream section 1-1*:**

Width of the channel, $b_1 = 10$ m

Depth of flow, $y_1 = 1\cdot5$ m

$\therefore$ Area of flow, $A_1 = b_1 \times y_1 = 10 \times 1\cdot5 = 15$ m²

Wetted perimeter, $P_1 = b_1 + 2y_1 = 10 + 2 \times 1\cdot5 = 13$ m

$\therefore$ Hydraulic radius, $R_1 = \frac{A_1}{P_1} = \frac{15}{13} = 1\cdot154$ m

Chezy's constant, $C_1 = \frac{1}{N}(R)^{1/6} = \frac{1}{0\cdot02} \times (1\cdot154)^{1/6} = 51\cdot2$

(where, $N = 0\cdot02$ ... *Given*)

Velocity of flow, $V_1 = C_1\sqrt{RS_b} = 51\cdot2\sqrt{1\cdot154 \times \frac{1}{10000}} = 0\cdot55$ m/s

(where slope of the channel bed, $S_b = \frac{1}{10000}$ ...*Given*)

Specific energy, $E_1 = y_1 + \frac{V_1^2}{2g} = 1\cdot5 + \frac{0\cdot55^2}{2 \times 9\cdot81} = 1\cdot515$ m

***Downstream section 2-2*:**

Width of the channel, $b_2 = b_1 = 10$ m

Depth of flow, $y_2 = 2\cdot5$ m

Area of flow, $A_2 = b_2 \times y_2 = 10 \times 2\cdot5 = 25$ m²

Wetted perimeter, $P_2 = b_2 + 2y_2 = 10 + 2 \times 2\cdot5 = 15$ m

$\therefore$ Hydraulic radius, $R_2 = \frac{A_2}{P_2} = \frac{25}{15} = 1\cdot667$ m

Also, $A_1V_1 = A_2V_2$ ...*Continuity equation*

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{15 \times 0\cdot55}{25} = 0\cdot33 \text{ m/s}$$

Specific energy, $E_2 = y_2 + \frac{V_2^2}{2g} = 2\cdot5 + \frac{0\cdot33^2}{2\times9\cdot81} = 2\cdot505$ m

The value of S_e (slope of energy line) is calculated by Chezy's formula corresponding to flow *conditions at the average depth of flow.*

Average depth of flow, $y = \frac{y_1 + y_2}{2} = \frac{1\cdot5 + 2\cdot5}{2} = 2$ m

At the average depth of flow,

Area of flow, $A = b \times y = 10 \times 2 = 20$ m^2 $(\because b_1 = b_2 = b = 10 \text{ m})$

Wetted perimeter, $P = b + 2y = 10 + 2 \times 2 = 14$ m

$\therefore$ Hydraulic radius, $R = \frac{A}{P} = \frac{20}{14} = 1\cdot428$ m

Again $AV = A_1V_1$

$\therefore$ Velocity of flow, $V = \frac{A_1V_1}{A} = \frac{15 \times 0\cdot55}{20} = 0\cdot4125$ m/s

Chezy's constant, $C = \frac{1}{N}(R)^{1/6} = \frac{1}{0\cdot02} \times (1\cdot428)^{1/6} = 53\cdot06$

Velocity $V = C\sqrt{R \times S_e}$

or, $0\cdot4125 = 53\cdot06\sqrt{1\cdot428 \times S_e} = 63\cdot4\sqrt{S_e}$

or, $S_e = \left(\frac{0\cdot4125}{63\cdot4}\right)^2 = 0\cdot0000423$

Length of back water curve,

$$l = \frac{E_2 - E_1}{S_b - S_e}, \text{ where } S_b \text{ is the slope of channel bed}$$

$$= \frac{2\cdot505 - 1\cdot515}{0\cdot0001 - 0\cdot0000423} = 17157 \text{ m or } \mathbf{17.157 \text{ km (Ans.)}}$$

Example 16.41. (Back water curve). *A river 45 m wide has a normal depth of flow of 3 m and an average bed slope of 1 in 10000. A weir is built across the river raising the water surface level at the weir site to 5 m above the bottom of the river. Assuming that the back water curve is an arc of circle, calculate the approximate length of the backwater curve. Consider that the river is prismatic. Take the value of N in Manning's formula as 0·025.* **[UPSC Exams.]**

Solution. Width of the bed, $b = 45$ m

Depth of flow (normal), $y_n = 3$ m

Average bed slope, $S_b = \frac{1}{10000} = 0\cdot0001$

Depth of flow at weir site, $y = 5$ m

Manning's co-efficient, $N = 0\cdot025$

Afflux, $h = y - y_n = 5 - 3 = 2$ m

Length of back water curve, *l*:

Length of backwater curve, *by circular arc method* is given as

$$l = \frac{2h}{dy/dx} \quad ...(i)$$

Area of flow, $A = 45 \times 3 = 135 \text{ m}^2$

Perimeter, $P = 45 + 2 \times 3 = 51 \text{ m}$

Hydraulic radius, $$R = \frac{A}{P} = \frac{135}{51} = 2 \cdot 65 \text{ m}$$

Discharge, $$Q = A \times V = A \times C\sqrt{RS_b} = A \times \frac{1}{N} R^{1/6} \sqrt{RS_b} = A \times \frac{1}{N} R^{2/3} S_b^{1/2}$$

(where, Chezy's constant, $C = \frac{1}{N} \times R^{1/6}$)

or, $$Q = 135 \times \frac{1}{0 \cdot 025} \times (2 \cdot 65)^{2/3} \times (0 \cdot 0001)^{1/2} = 103 \cdot 4 \text{ m}^3/\text{s}$$

At the weir site:

$$y = 5 \text{ m}, V = \frac{Q}{45 \times 5} = \frac{103 \cdot 4}{45 \times 5} = 0 \cdot 46 \text{ m/s}$$

Hydraulic radius, $$R = \frac{A}{P} = \frac{45 \times 5}{45 + 2 \times 5} = 4 \cdot 09 \text{ m}$$

Slope of water surface at the weir,

$$\frac{dy}{dx} = \frac{S_b - S_e}{1 - \frac{V^2}{gy}}$$

where, S_e is the slope of the total energy line at the weir, V and y are the velocity and depth of flow respectively at the weir.

$$S_e = \frac{Q^2 N^2}{A^2 R^{4/3}} \qquad \text{(Refer to example 16·39)}$$

$$= \frac{V^2 N^2}{R^{4/3}} = \frac{0 \cdot 46^2 \times 0 \cdot 025^2}{(4 \cdot 09)^{4/3}} = 2 \cdot 02 \times 10^{-5} = 0 \cdot 0000202$$

$$\frac{V^2}{gy} = \frac{0 \cdot 46^2}{9 \cdot 81 \times 5} = 0 \cdot 0043$$

$$\therefore \quad \frac{dy}{dx} = \frac{0 \cdot 0001 - 0 \cdot 0000202}{1 - 0 \cdot 0043} = 0 \cdot 00008$$

Substituting the value of $\frac{dy}{dx}$ in expression (*i*), we have

$$l = \frac{2 \times 2}{0 \cdot 00008} = 50000 \text{ m} \quad \text{or} \quad \mathbf{50 \text{ km (Ans.)}}$$

16.11. Measurement of Flow of Irregular Channels

The term "*irregular channels*" includes *large rivers* and *small streams*. In case of a small stream; it is possible to obtain the quantity of flow *by fitting a notch or weir across the stream*; the *discharge may then be calculated by measuring the head over the notch*. However, this method cannot be employed for large rivers on account of the expense and the obstruction which may be caused to navigation. In order to obtain the discharge through a large river (or irregular channel) we require: (*i*) Area of flow and (*ii*) Mean velocity of flow. By knowing this data discharge is calculated as follows:

Discharge = Area of flow × mean velocity of flow.

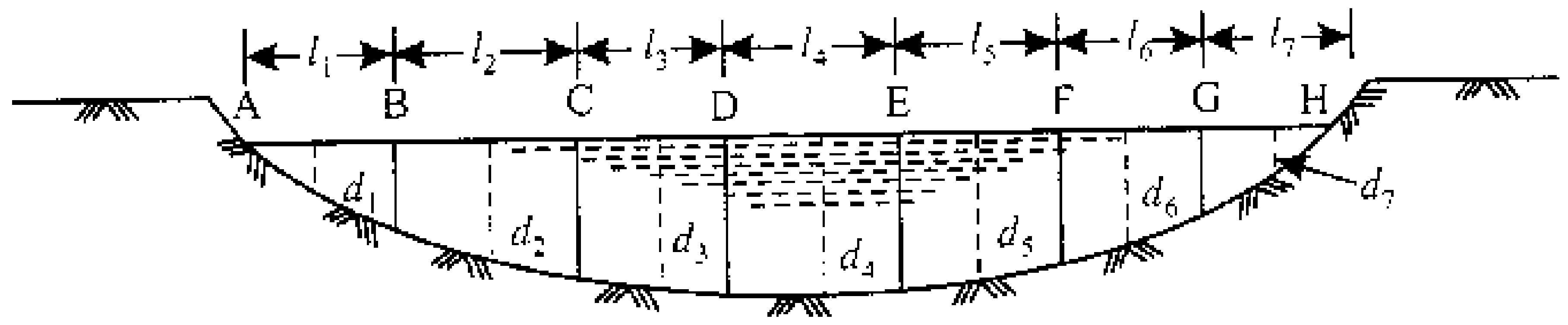


Fig. 16.36. Cross-section of river with unequal segments (Segments method).

16.11.1. Area of Flow

The area of flow may be *calculated* by the several methods but the following are important ones:

1. Simple segments method. 2. Simpson's rule.

1. **Simple segments method :**

In this method, the cross-section of the river is divided into a number of segments *AB*, *BC*, *CD*, etc. as shown in Fig. 16·36.

Let, l_1, l_2, l_3 ... = Lengths of the segments *AB*, *BC*, *CD* ... respectively, and

d_1, d_2, d_3 ... = Mean depths of the respective segment.

Then, area of flow, A = Area of segment *AB* + area of segment *BC* + area of segment *CD* + ...

$$= l_1 d_1 + l_2 d_2 + l_3 d_3 + \dots$$

2. **Simpson's rule:**

A greater accuracy in the computation of discharge may be obtained by using Simpson's rule. In this method the whole river width is divided into an *even number of equal segments* so that there are *odd* number of depths taken at the end of each segment as shown in Fig. 16·28. Then area of flow,

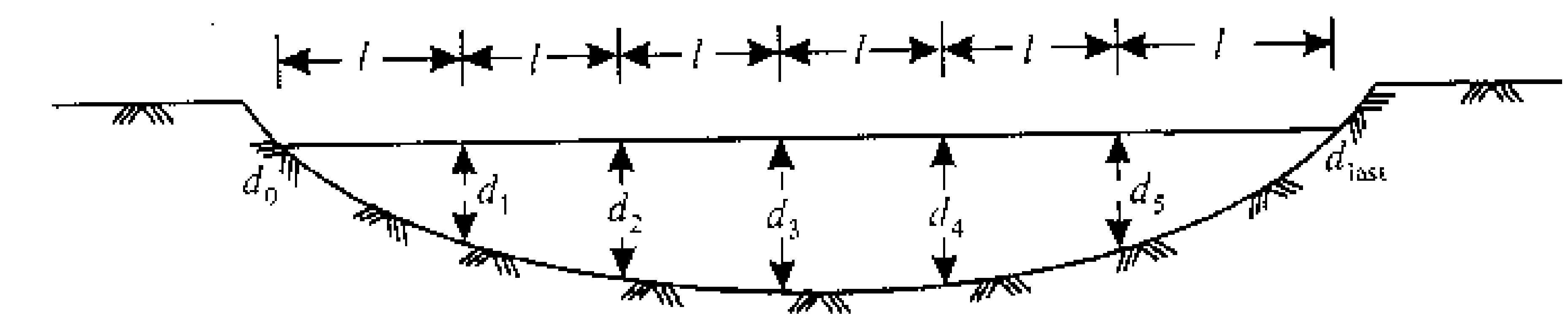


Fig. 16.37. Cross-section of river with equal segments (Simpson's) rule.

$$A = \frac{l}{3}(d_0 + d_{last}) + 2(d_1 + d_3 + d_5) + 4(d_2 + d_4 + d_6)$$

where, l = Length of each segment, and

d_0, d_1, d_2 ..... = Depths taken at the end of segments.

16.11.2. Mean Velocity of Flow

The mean velocity of flow may be *measured* by the following methods :

1. Pitot tube 2. Floats 3. Current meter.

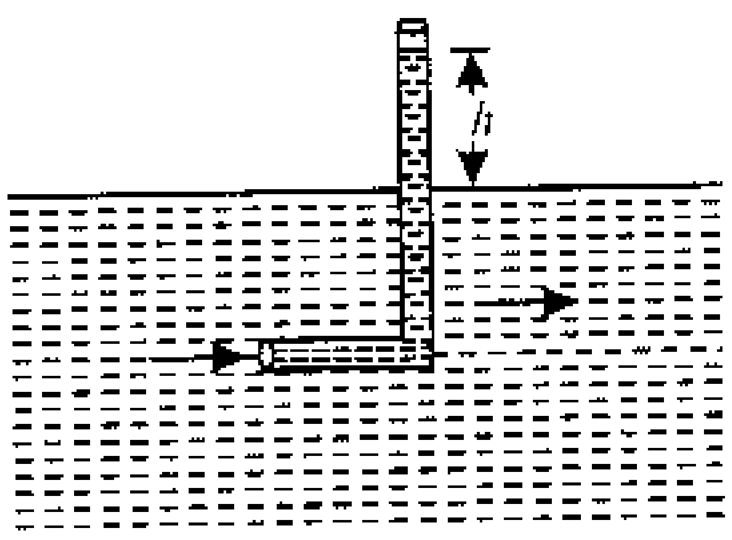


Fig. 16.38. Pitot tube.

1. **Pitot tube.** A pitot tube is a simple device used for measuring the velocity of flow at the required point in the flowing stream. In its simplest form it consists of a glass tube (large enough for capillary effects to be negligible) bent at *right angle*. The tube is dipped vertically in the flowing stream with its lower open end facing the direction of flow and upper open end projecting above the water surface in the stream as shown in the Fig. 16·38. The water rises up in the tube, due to pressure exerted by the flowing

water. By measuring the rise of water in the tube, the velocity of water (V) is calculated by using the following relation:

$$V = \sqrt{2gh}$$

where, h = Height of water in the tube above the water surface, and

g = Acceleration due to gravity.

2. **Floats.** The velocity of flow can be measured in a simple way by means of *floats*. A float is a small object made of wood or other suitable material which is lighter than water and thus capable of floating on the surface. The *surface velocity at any section may be obtained by using* a ***single float.*** The time taken by the float to traverse a known distance is measured; the velocity is then calculated by dividing the distance travelled (by the float) by the time taken to travel that distance. Since the mean velocity of flow is equal to 0.8 to 0.95 times the surface velocity, the approximate value of the mean velocity of flow may then be determined from the known value of the surface velocity.

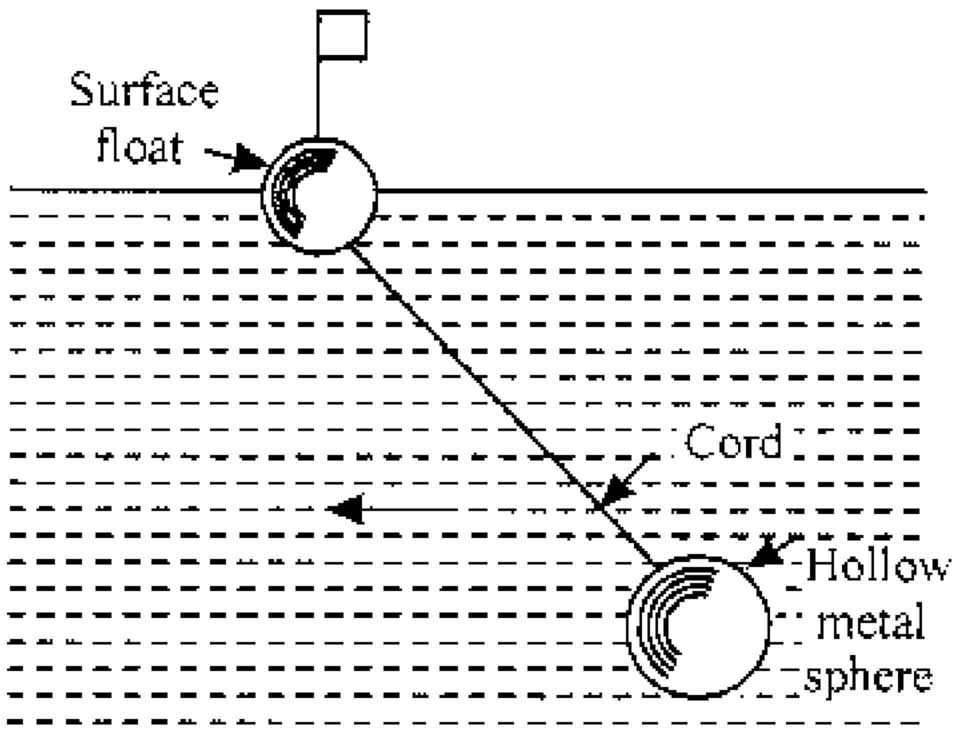


Fig. 16.39. Double float.

A better method is to use ***double floats.*** A double float consists of a surface float on to which is attached a hollow metal sphere, heavier than water, and suspended from it by a cord of known length (Fig. 16·39). The depth of the lower float may be regulated by the length of the cord. The velocity is then obtained by noting the time taken by the float to traverse a known distance (as explained in the previous case of single float)

The double float method *directly* gives the value of mean velocity of flow.

The *best type of float* is the ***rod float*** (Fig. 16·40). It consists of a vertical wooden rod which is weighted at the bottom to keep it vertical. The length of the rod is so adjusted that it reaches the bottom of stream (without touching the weeds, sand or mud at the bottom of the river) and its top should be above the water surface. Some types of rod are made telescopic, so that length may be adjusted to suit any depth. *The rod will travel with a velocity equal to the mean velocity of the section.*

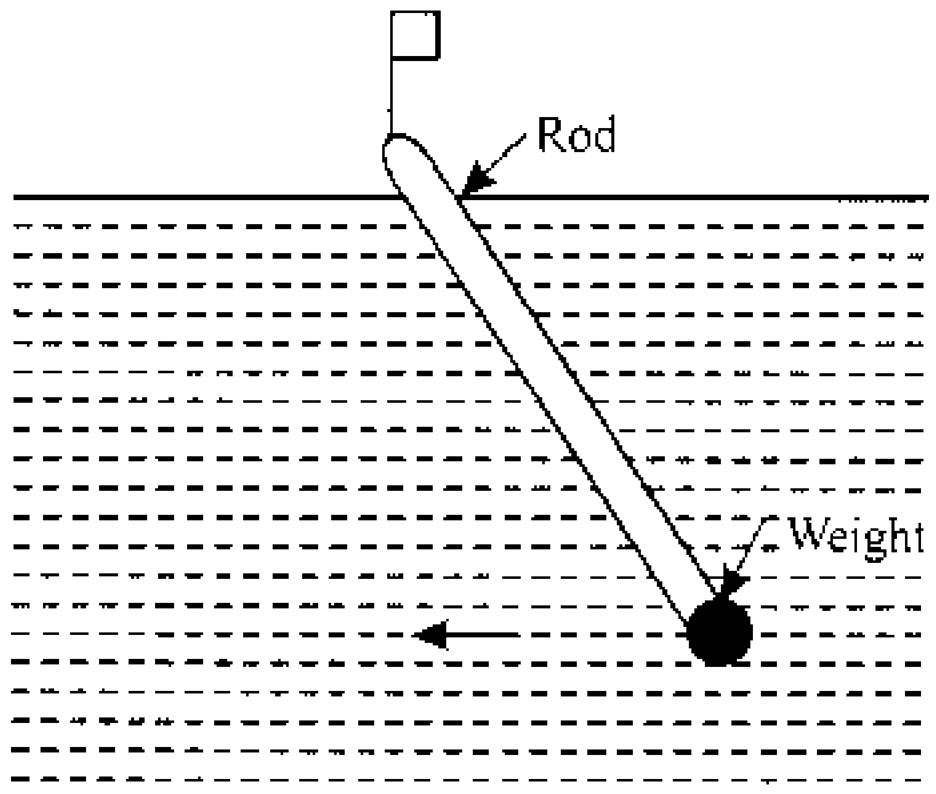


Fig. 16.40. Rod float.

3. **Current meter.** A current meter is an instrument used to measure the velocity of flow at a required point in the flowing stream. In general it consists of a *wheel or revolving element containing blades or cups, and a tail on which flat vanes or fins are fixed.* The current meters, according to the shape of the revolving element, may be classified as follows:

(*i*) Cup type (*ii*) Screw type or propeller type.

In a *Cup type current meter* (Fig. 16·41) the wheel or revolving element has the form of a series of conical cups, mounted on a spindle. The spindle is held vertical at right angle to the direction of flow.

In a *screw or propeller type current meter* (Fig. 16·42) the revolving element consists [illegible] with its axis parallel to the direction of flow, which carries a number of curved vanes [illegible] blades) mounted on the periphery of the shaft. This type of meter is more sensitive than cup type because it gives higher r.p.m. for the same velocity of flow.

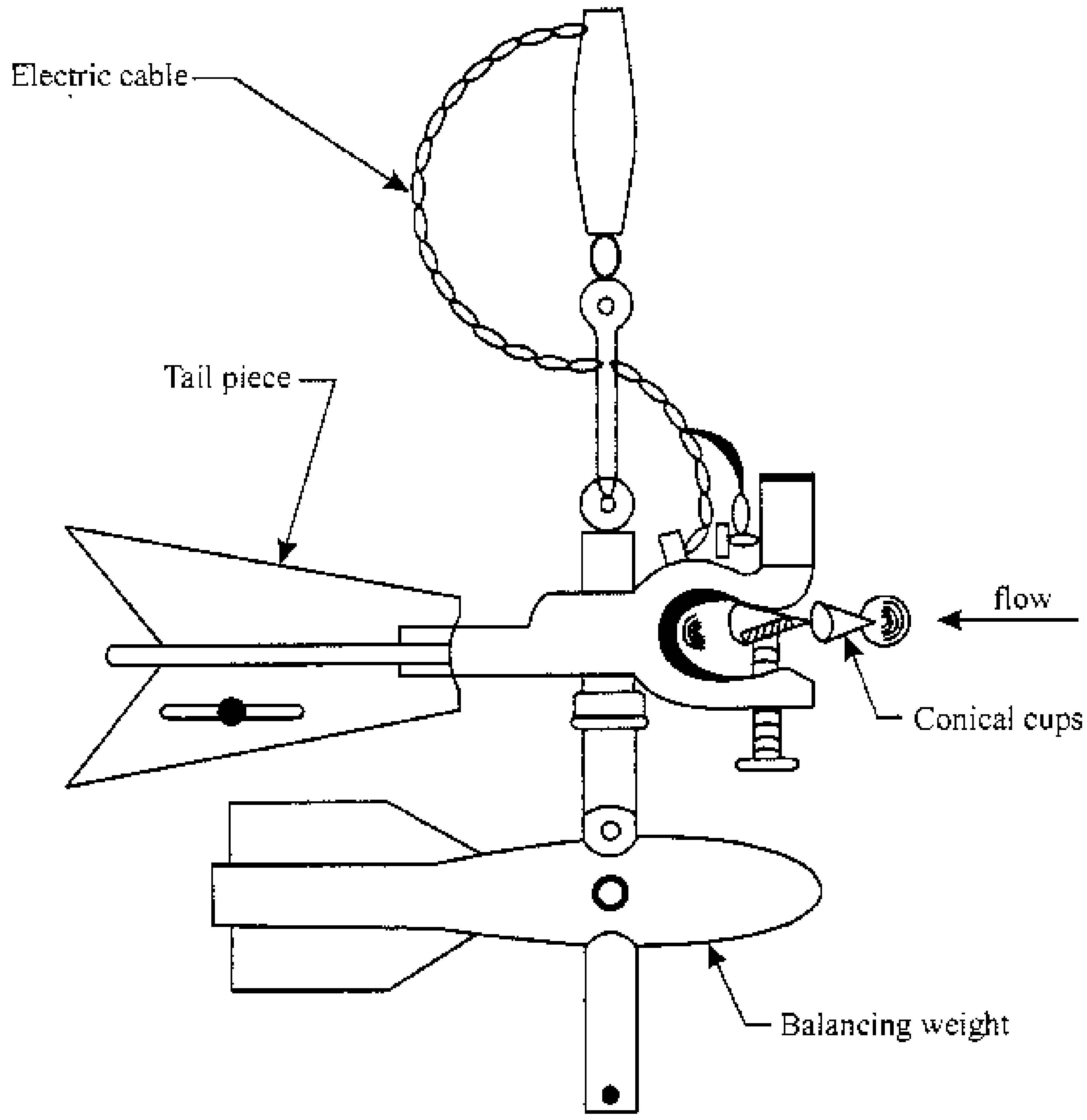


Fig. 16.41. Cup type current meter.

In order to measure the velocity of flow, meter is submerged under water and motion of water in the stream activates it, driving the wheel (or rotatory elements) at a *speed proportional to the velocity of flow*. An electric current is passed from the battery to the wheel by means of wire. The rotation of wheel makes and breaks the electric circuit, which causes an electric bell to ring. Thus by counting the ringing of bell, the rotations of the wheel and hence the velocity of flowing water is obtained.

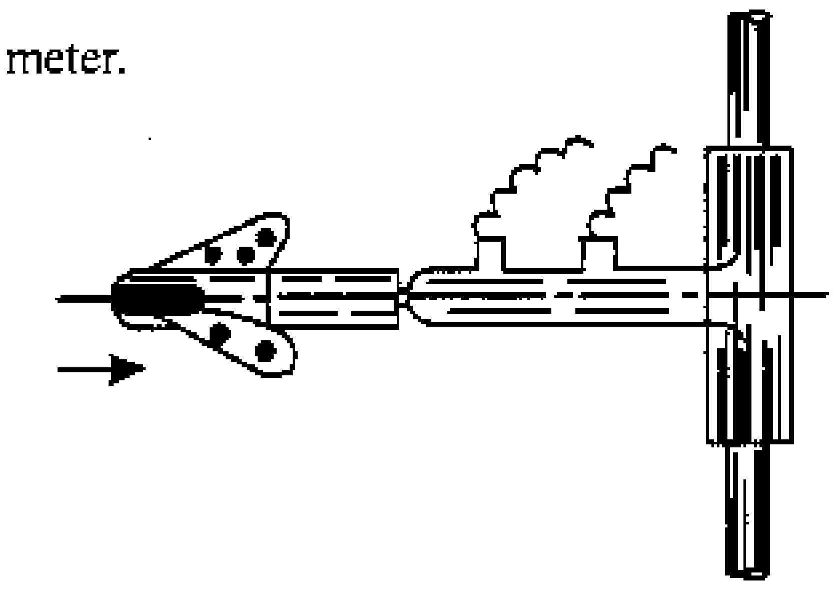


Fig. 16.42. Screw of propeller type current meter.

HIGHLIGHTS

1. An *open channel* may be defined as a passage in which liquid flows with its upper surface exposed to atmoshpere.
2. Flow in a channel is said to be *uniform*, if the depth, slope, cross-section and velocity remain constant over a given length of the channel. Flow in a channel is said to be *non-uniform* (or varied) when the channel depth *varies* continuously from one section to another.
3. The flow in the open channel may be characterised as laminar or turbulent depending upon the value of Reynolds number:

When $Re < 500$ ...flow is laminar;

When $Re > 2000$ ...flow is turbulent.

When $500 < Re < 2000$ ...flow is transitional.

4. If Froude number (Fr) is less than 1·0, the flow is subcritical or streaming. If Fr is equal to 1·0, the flow is critical. If Fr is greater than 1·0, the flow is supercritical or shooting.

5. Velocity by Chezy's formula is given by

$$V = C\sqrt{RS}$$

where, C = Chezy's constant,

R = Hydraulic radius (or hydraulic mean depth) $= \dfrac{A \text{ (area)}}{P \text{ (wetted perimeter)}}$, and

S = Slope of the bed.

6. Empirical relations for the Chezy's constant, C

(*i*) $$C = \frac{157 \cdot 6}{1 \cdot 81 + \dfrac{K}{\sqrt{R}}}$$ ...*Bazin's formula*

where, K = Bazin's constant,

R = Hydraulic radius (or hydraulic mean depth)

(*ii*) $$C = \frac{23 + \dfrac{0 \cdot 00155}{S} + \dfrac{1}{N}}{1\left(23 + \dfrac{0 \cdot 00155}{S}\right)\dfrac{N}{\sqrt{R}}}$$ ...*Kutter's formula*

where, N = Kutter's constant, and S = bed slope.

(*iii*) $$C = \frac{1}{N} R^{1/6}$$ ...*Manning's formula*

where, N = Manning's constant = Kutter's constant.

7. The *most economical section* (also called the best section or most efficient section) is one which gives the maximum discharge for a given amount of excavation.

8. Conditions for maximum discharge through different channel sections:

(*a*) *Rectangular section:*

(*i*) $b = 2y$; (*ii*) $R = \dfrac{y}{2}$

(*b*) *Trapezoidal section:*

(*i*) Half top width = Sloping side

or, $$\frac{b + 2ny}{2} = y\sqrt{n^2 + 1}$$

(*ii*) $$R = \frac{y}{2}$$

(*iii*) A semicircle drawn from the mid-point of the top width with radius equal to depth of flow will touch the three sides of the channel. *Best side slope* for most economical trapezoidal section is

$$\theta = 60° \quad \text{or} \quad n = \frac{1}{\sqrt{3}} = \frac{1}{\tan\theta}$$

(c) *Triangular section:*

(i) Each sloping side makes an angle of 45° with the vertical.

(ii) Hydraulic radius, $R = \frac{y}{2\sqrt{2}}$.

(d) *Circular section:*

(i) Condition for *maximum discharge:*

Depth of flow, $y = 0{\cdot}95$ diameter of circular channel;

Hydraulic radius, $R = 0{\cdot}29$ times channel diameter.

(ii) Condition for *maximum velocity:*

Depth of flow, $y = 0{\cdot}81$ diameter of circular channel;

Hydraulic radius, $R = 0{\cdot}305$ diameter.

9. For a *circular channel*:

Area of flow, $A = r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

Wetted perimeter, $P = 2r\theta$

where, r = Radius of circular channel, and

θ = Half the angle subtended by the water surface at the centre.

10. Channel sections of constant velocity are designed particularly in the case of large sewers in which the discharge ranges from a certain minimum value that flows daily to a very large value during rainy season.

11. The total energy of flow per unit weight of liquid is given by

Total energy $= z + y + \frac{V^2}{2g}$

12. Specific energy of a flowing liquid per unit weight,

$$E = y + \frac{V^2}{2g}$$

13. The depth of flow at which specific energy is minimum is called *criticial depth*, which is given by $y_c = \left(\frac{q^2}{g}\right)^{1/3}$, where g = discharge per unit width.

14. The velocity of flow at critical depth is known as *critical velocity*, which is given by

$$V_c = \sqrt{g \times y_c}$$

15. Minimum specific energy is given by

$E_{min} = \frac{3}{2} y_c$, where y_c = critical depth.

16. (i) A flow corresponding to critical depth (or when Froude number, $Fr = 1$) is known as critical flow.

(ii) When the depth of flow in a channel is greater than critical depth (when $Fr < 1$) the flow is said to be sub-critical or streaming flow.

(iii) The flow is supercritical (or shooting or torrential) when the depth of flow in a channel is less than the critical depth (when $Fr > 1$).

17. The condition for maximum discharge for given value of specific energy is that the depth of flow should be *critical*.

18. **Hydraulic jump.** In an open channel when rapidly flowing stream abruptly changes to slowly flowing stream, a distinct rise or jump in the elevation of liquid surface takes place, this phenomenon is known as *hydraulic jump*. The hydraulic jump is also known as '*standing wave*'.

The depth of flow after the jump is given by

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{2gy_1}} \qquad \text{...(in terms of } q)$$

$$= -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2V_1^2\, y_1}{g}} \qquad \text{...(in terms of } V_1)$$

$$= \frac{y_1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right) \qquad \text{...(in terms of } Fr_1)$$

(where, y_1 = depth of flow of water before the jump)

Height of hydraulic jump, $H_j = y_2 - y_1$

Length of hydraulic jump, $L_j = 5$ to $7\, H_j$

Loss of energy due to hydraulic jump, $E_L = \dfrac{(y_2 - y_1)^3}{4y_1y_2}$

19. **Gradually varied flow** (G.V.F.) is one in which the depth gradually over a long distance. Equation of gradually varied flow is given by

$$\frac{dy}{dx} = \frac{S_b - S_e}{\left(1 - \dfrac{V^2}{gy}\right)} \qquad \text{...(in terms of } V)$$

$$= \frac{S_b - S_e}{(1 - Fr^2)} \qquad \text{...(in terms of } Fr)$$

where, $\dfrac{dy}{dx}$ = Slope of free water surface,

S_b = Slope of the channel bed,

S_e = Slope of the energy line, and

V = Velocity of flow.

20. **Afflux** is the increase in water level due to some obstruction across the flowing liquid; the curved surface of the liquid with its concavity upwards, is known as **back water curve**.

Length of back water curve, $l = \dfrac{E_2 - E_1}{S_b - S_e}$

where, $E_1\left(= y_1 + \dfrac{V_1^2}{2g}\right)$ and $E_2\left(= y_2 + \dfrac{V_2^2}{2g}\right)$ represent the specific energies at the beginning and end of back water curve.