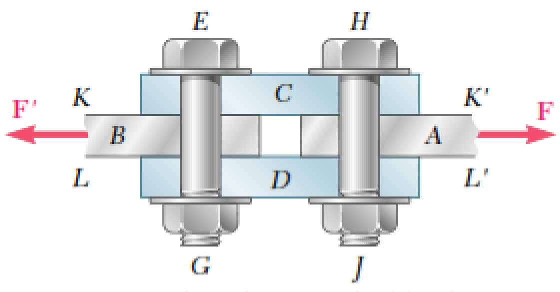
Structural connections

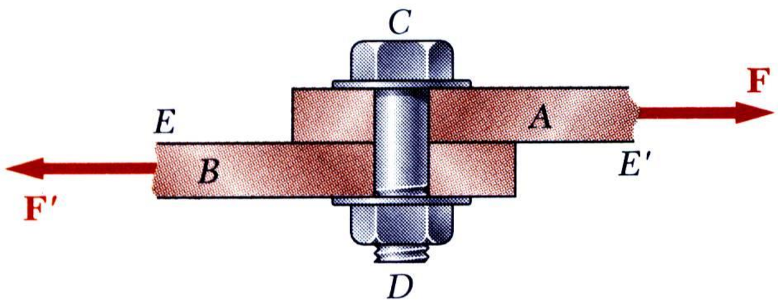
Eccentrically bolted and welded connections

Shearing Stress in connections

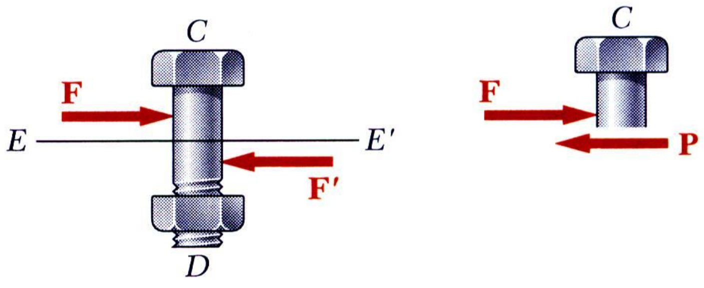
Double Shear

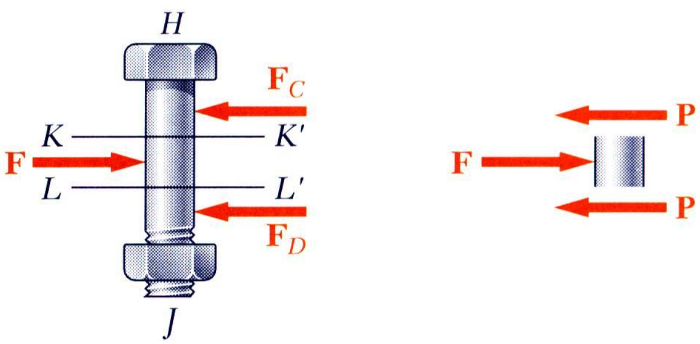


Single Shear



Observe that the shear P in each of the sections is $P = F/2$, we conclude that the average shearing stress is



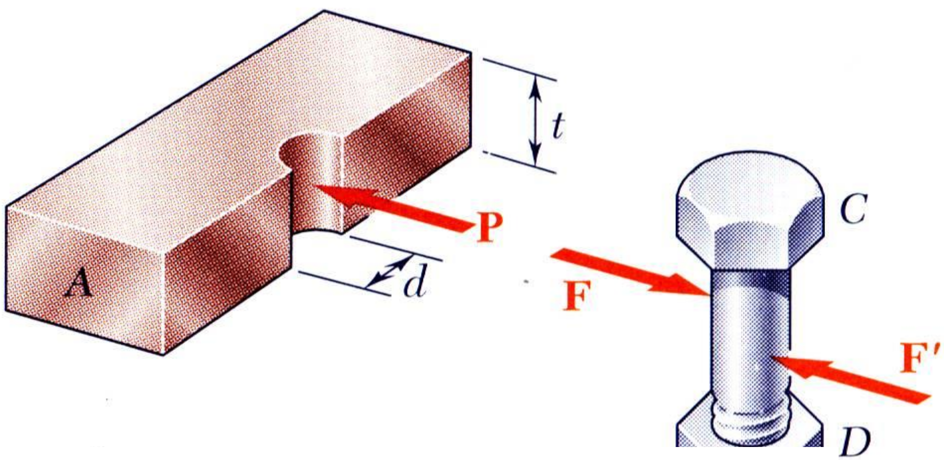


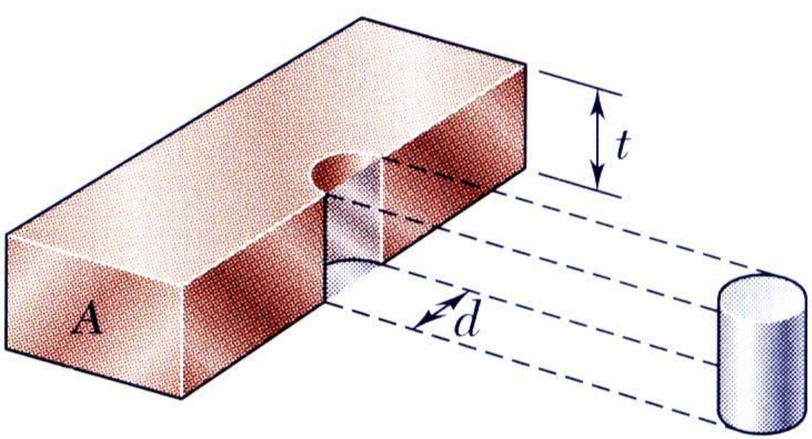
$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

Bearing Stress in Connections

Bearing stress (crushing stress) is the contact pressure between the separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces.





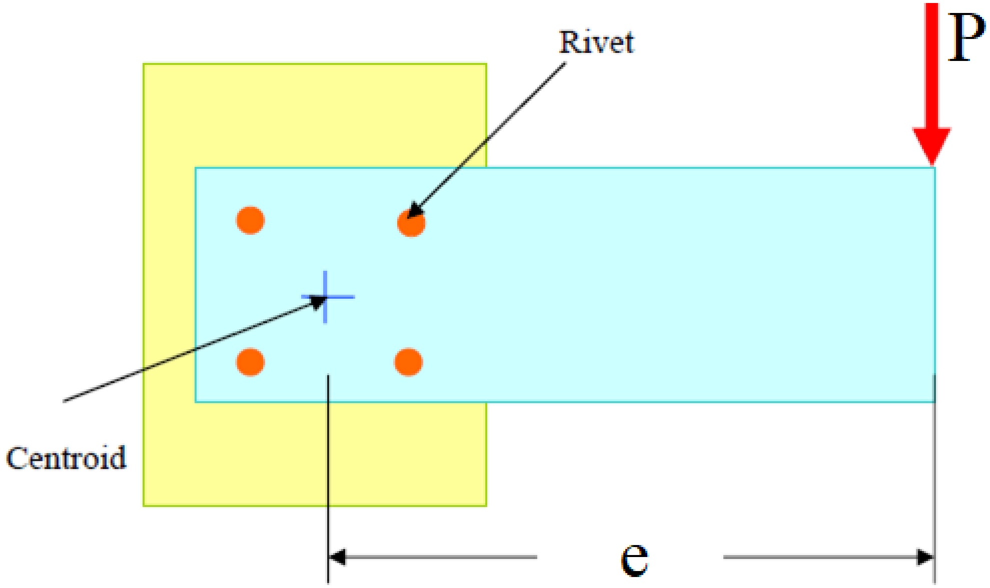
- Bolts, rivets, and pins create stresses on the points of contact or *bearing surfaces* of the members they connect.
- The bolt exerts on plate *A* a force **P** equal and opposite to the force **F** exerted by the plate on the bolt
- The resultant of the force distribution on the surface is equal and opposite to the force exerted on the pin.
- Corresponding average force intensity is called the bearing stress,

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

Eccentrically loaded connections

Eccentrically loaded: When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the joint is said to be an ***eccentric loaded riveted joint, as*** shown below.

The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or *primary shear*.

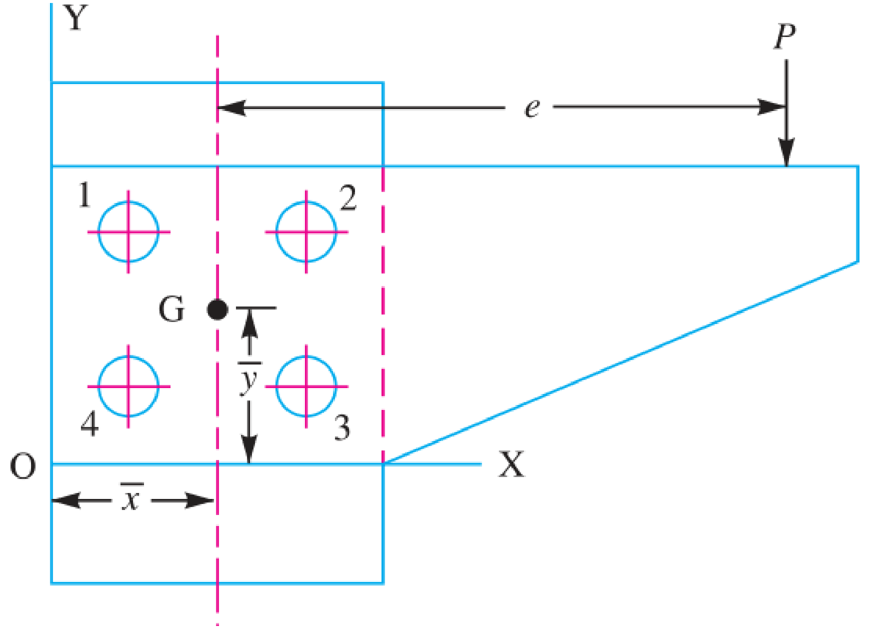


Eccentrically loaded connections

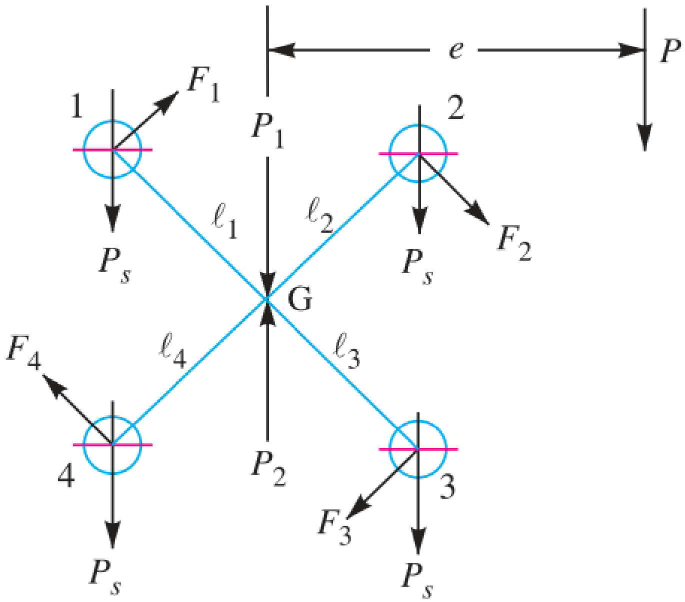
Where:

P = Eccentric load

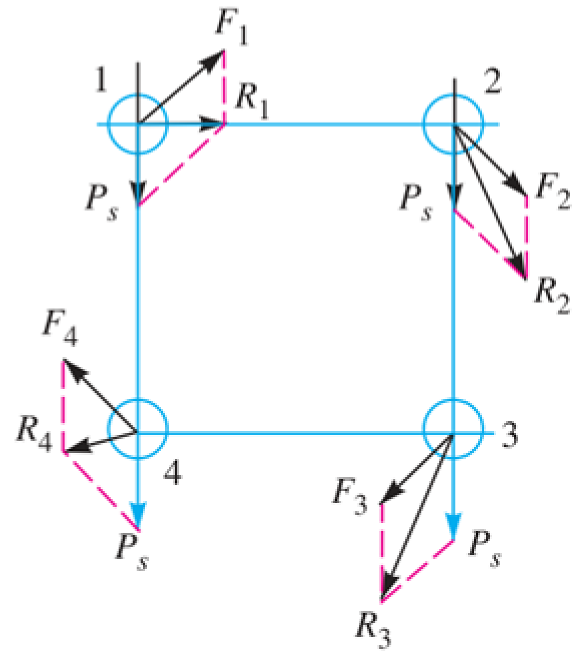
e = Eccentricity of the load (the distance between the line of action of the load and the centroid *G)*



Eccentrically loaded connections



(b)



(c)

Eccentrically loaded connections

1. First of all, find the centre of gravity G of the rivet system.

Let A = Cross-sectional area of each rivet,

x_1, x_2, x_3 etc. = Distances of rivets from OY, and

y_1, y_2, y_3 etc. = Distances of rivets from OX.

We know that

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \ldots}{A_1 + A_2 + A_3 + \ldots} = \frac{A x_1 + A x_2 + A x_3 + \ldots}{n . A}$$

$$= \frac{x_1 + x_2 + x_3 + \ldots}{n}$$ ...(where n = Number of rivets)

Similarly,

$$\bar{y} = \frac{y_1 + y_2 + y_3 + \ldots}{n}$$

Eccentrically loaded connections

2. Introduce two forces P_1 and P_2 at the centre of gravity. These forces are equal & opposite to P as shown in Fig. (b).

3. Assuming that all the rivets are of the same size, the effect of $P_1 = P$ is to produce direct shear load on each rivet of equal magnitude. Therefore, direct shear load P_s, on each rivet,

$$P_s = \frac{P}{n}$$

Eccentrically loaded connections

4. The effect of $P_2 = P$ is to produce a turning moment of magnitude $P \times e$ which tends to rotate the joint about the centre of gravity '*G*' of the rivet system in a clockwise direction. Due to the turning moment, *secondary shear* load on each rivet is produced.

In order to find the secondary shear load, the following two assumptions are made :

(*a*) The secondary shear load is proportional to the radial distance of the rivet under consideration from the centre of gravity of the rivet system.

(*b*) The direction of secondary shear load is perpendicular to the line joining the centre of the rivet to the centre of gravity of the rivet system.

Eccentrically loaded connections

Let

$F_1, F_2, F_3 \ldots$ = Secondary shear loads on the rivets 1, 2, 3...etc.

$l_1, l_2, l_3 \ldots$ = Radial distance of the rivets 1, 2, 3 ...etc. from the centre of gravity 'G' of the rivet system.

∴ From assumption (a),

$$F_1 \propto l_1 \,;\, F_2 \propto l_2 \text{ and so on}$$

or

$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} = \ldots$$

∴

$$F_2 = F_1 \times \frac{l_2}{l_1}, \text{ and } F_3 = F_1 \times \frac{l_3}{l_1}$$

Eccentrically loaded connections

We know that the sum of the external turning moment due to the eccentric load and of internal resisting moment of the rivets must be equal to zero.

$$P.e = F_1.l_1 + F_2.l_2 + F_3.l_3 + \dots$$

$$= F_1.l_1 + F_1 \times \frac{l_2}{l_1} \times l_2 + F_1 \times \frac{l_3}{l_1} \times l_3 + \dots$$

$$= \frac{F_1}{l_1} [(l_1)^2 + (l_2)^2 + (l_3)^2 + \dots]$$

From the above expression, the value of F_1 may be calculated and hence F_2 and F_3

The direction of these forces are at right angles to the lines joining the centre of rivet to the centre of gravity of the rivet system, as shown in Fig. (*b*), and

Eccentrically loaded connections

5. The primary (or direct) and secondary shear load may be added vectorially to determine the resultant shear load (R) on each rivet as shown in Fig. (c).

It may also be obtained by using the relation:

$$R = \sqrt{(P_s)^2 + F^2 + 2\,P_s \times F \times \cos\theta}$$

Where: θ = Angle between the primary or direct shear load (P_s) and secondary shear load (F).

Eccentrically loaded connections

When the secondary shear load on each rivet is equal, then the heavily loaded rivet will be one in which the included angle between the direct shear load and secondary shear load is minimum.

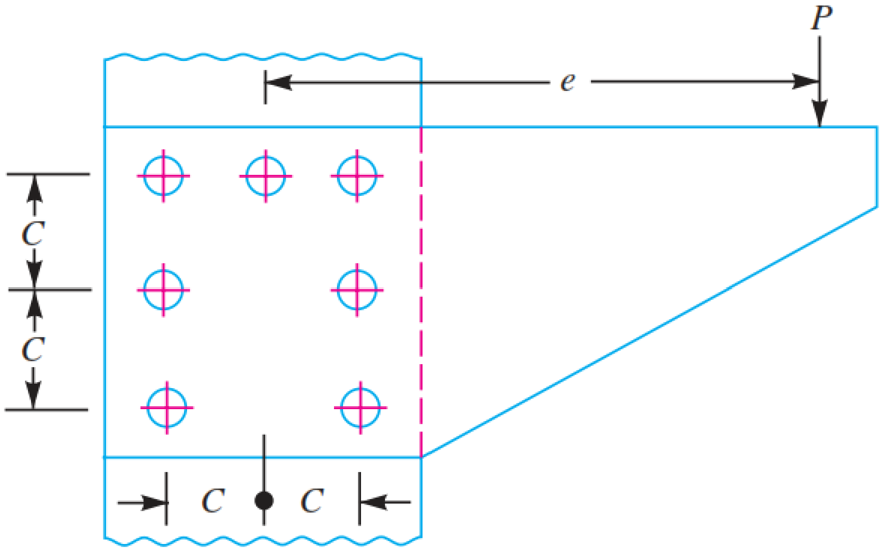
The maximum loaded rivet becomes the critical one for determining the strength of the riveted joint.

Knowing the permissible shear stress (τ), the diameter of the rivet hole may be obtained by using the relation:

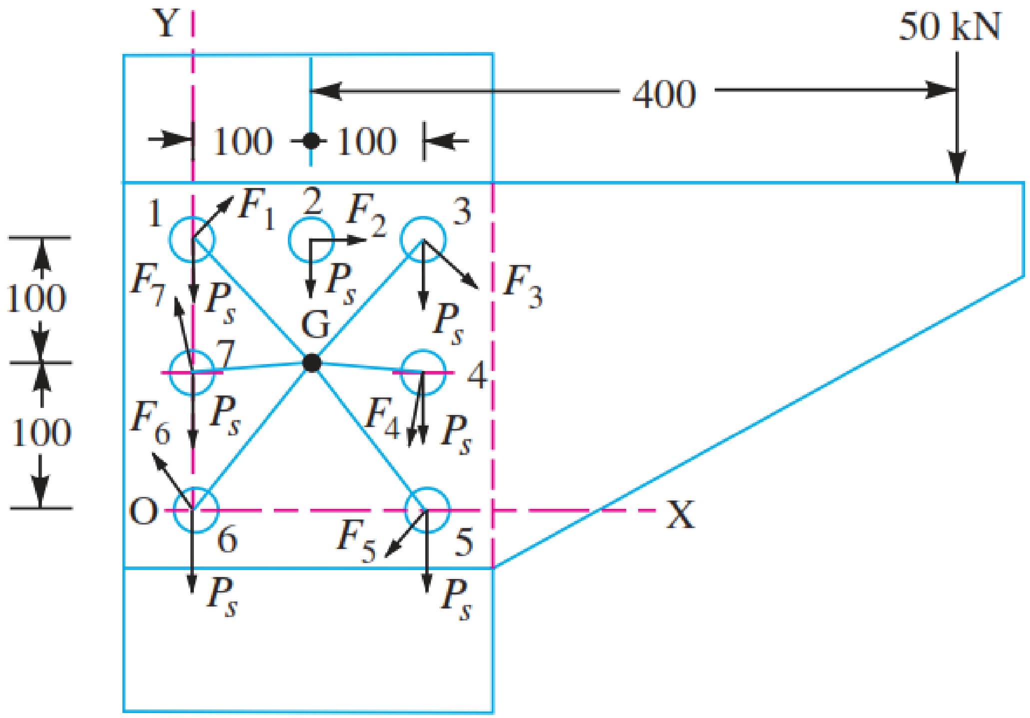
$$\text{Maximum resultant shear (R)} = \frac{\pi}{4} \times d^2 \times \tau$$

PROBLEM 1

An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown below: The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 kN ; rivet spacing, C = 100 mm; load arm, *e* = 400 mm. Permissible shear stress is 65 MPa and crushing (bearing) stress is 120 MPa. Determine the size of the rivets to be used for the joint.



PROBLEM 1



PROBLEM 1

First of all, let us find the centre of gravity (G) of the rivet system.

Let $\bar{x}$ = Distance of centre of gravity from OY,

$\bar{y}$ = Distance of centre of gravity from OX,

$x_1, x_2, x_3 \ldots$ = Distances of centre of gravity of each rivet from OY, and

$y_1, y_2, y_3 \ldots$ = Distances of centre of gravity of each rivet from OX.

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n}$$

$$= \frac{100 + 200 + 200 + 200}{7} = 100 \text{ mm}$$

$$\bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7}{n}$$

$$= \frac{200 + 200 + 200 + 100 + 100}{7} = 114.3 \text{ mm}$$

PROBLEM 1

We know that direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{50 \times 10^3}{7} = 7143 \text{ N}$$

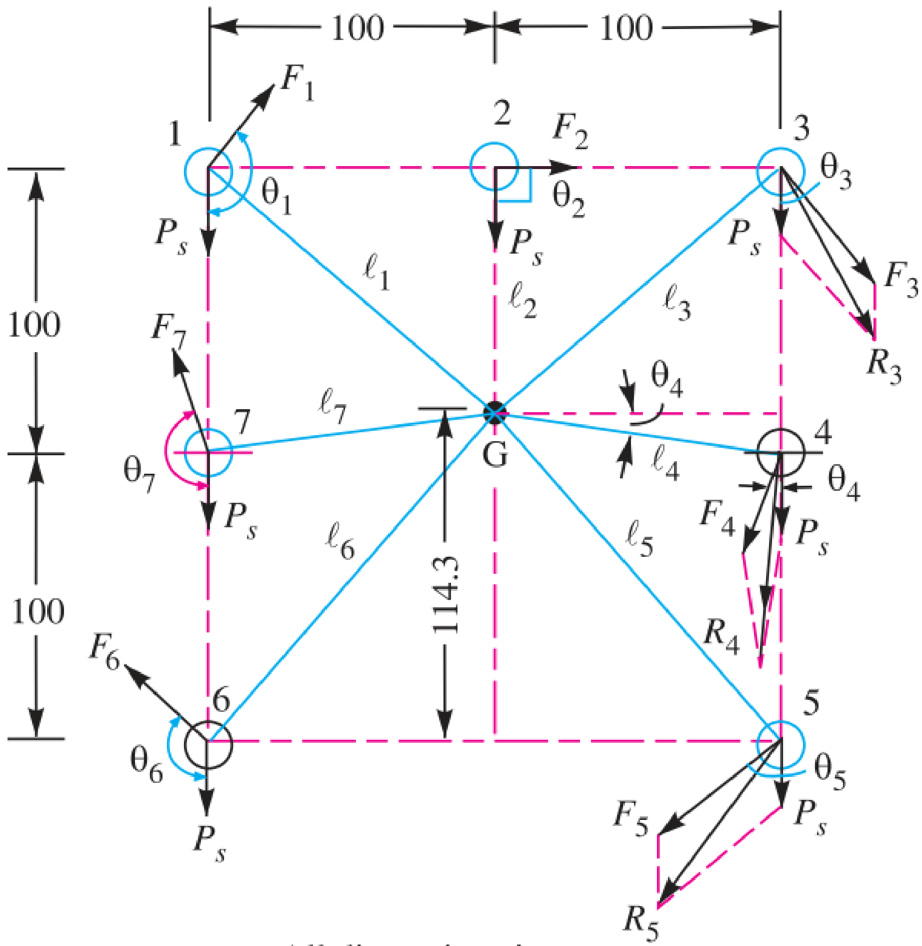
Turning moment produced by the load P due to eccentricity (e)

$$= P \times e = 50 \times 10^3 \times 400 = 20 \times 10^6 \text{ N-mm}$$

This turning moment is resisted by seven rivets as shown overleaf:

Let F_1, F_2, F_3, F_4, F_5, F_6 and F_7 be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at distances l_1, l_2, l_3, l_4, l_5, l_6 and l_7 respectively from the centre of gravity of the rivet system as shown in figure next page

PROBLEM 1



All dimensions in mm.

PROBLEM 1

$$l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm}$$

$$l_2 = 200 - 114.3 = 85.7 \text{ mm}$$

$$l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm}$$

$$l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = 152 \text{ mm}$$

Equating the turning moment due to eccentricity of the load to the resisting moment of the rivets:

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 + (l_7)^2\right]$$

$$= \frac{F_1}{l_1}\left[2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2\right]$$

PROBLEM 1

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2\right]$$

F_1= 24,244 kN

Secondary shear loads are proportional to their radial distances from the centre of gravity,

$$F_2 = F_1 \times \frac{l_2}{l_1} = 24\ 244 \times \frac{85.7}{131.7} = 15\ 776 \text{ N}$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 24\ 244 \text{ N}$$

$$F_4 = F_1 \times \frac{l_4}{l_1} = 24\ 244 \times \frac{101}{131.7} = 18\ 593 \text{ N}$$

PROBLEM 1

$$F_5 = F_1 \times \frac{l_5}{l_1} = 24\,244 \times \frac{152}{131.7} = 27\,981 \text{ N}$$

$$F_6 = F_1 \times \frac{l_6}{l_1} = F_5 = 27\,981 \text{ N}$$

$$F_7 = F_1 \times \frac{l_7}{l_1} = F_4 = 18\,593 \text{ N}$$

Rivets 3, 4 and 5 are heavily loaded.
Their angles between the direct and secondary shear load are:

$$\cos\theta_3 = \frac{100}{l_3} = \frac{100}{131.7} = 0.76$$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

$$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = 0.658$$

PROBLEM 1

Resultant shear loads:

$$R_3 = \sqrt{(P_s)^2 + (F_3)^2 + 2\,P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(7143)^2 + (24\,244)^2 + 2 \times 7143 \times 24\,244 \times 0.76} = 30\,033 \text{ N}$$

$$R_4 = \sqrt{(P_s)^2 + (F_4)^2 + 2\,P_s \times F_4 \times \cos\theta_4}$$

$$= \sqrt{(7143)^2 + (18\,593)^2 + 2 \times 7143 \times 18\,593 \times 0.99} = 25\,684 \text{ N}$$

$$R_5 = \sqrt{(P_s)^2 + (F_5)^2 + 2\,P_s \times F_5 \times \cos\theta_5}$$

$$= \sqrt{(7143)^2 + (27\,981)^2 + 2 \times 7143 \times 27\,981 \times 0.658} = 33\,121 \text{ N}$$

PROBLEM 1

From R_3, R_4 and R_5; the maximum resultant shear load is on rivet 5.

If d is the diameter of rivet hole, then maximum resultant shear load (R_5),

$$R_5 = \frac{\pi}{4} \times d^2 \times \tau$$

$$33{,}121 = \frac{\pi}{4} \times d^2 \times 65$$

$$d = 25.5 \text{ mm}$$

PROBLEM 1

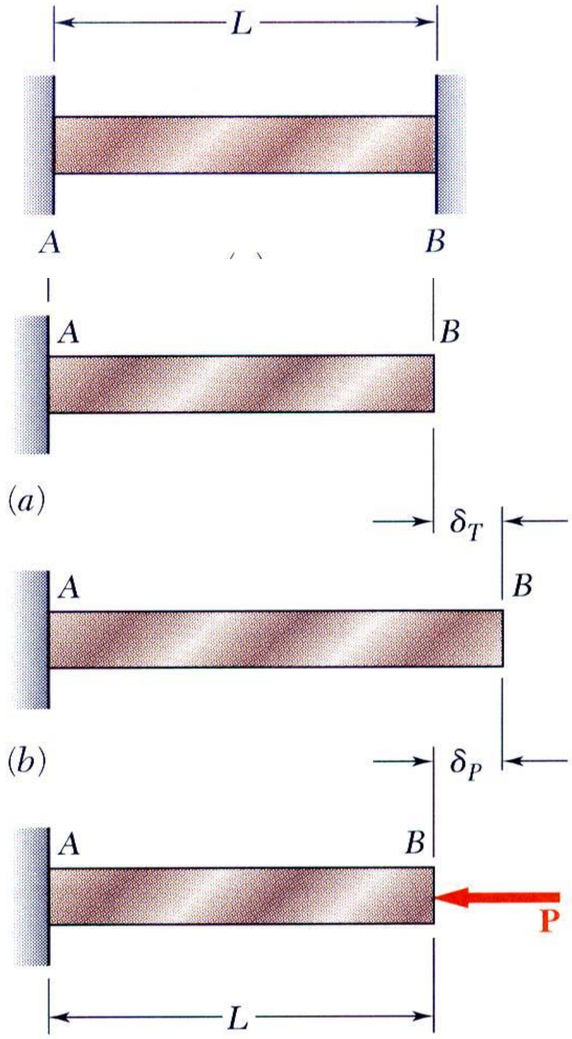
CRUSHING STRESS

$$\text{Crushing stress} = \frac{\text{Max. load}}{\text{Crushing area}} = \frac{R_5}{d \times t} = \frac{33\,121}{25.5 \times 25}$$

$$= 52.95 \text{ MPa}$$

The computed stress is well below the given crushing stress of 120 MPa, thus, the design is satisfactory.

Thermal Stresses



- A temperature change results in a change in length or *thermal strain*. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.
- Treat the additional support as redundant and apply the principle of superposition.

$$\delta_T = \alpha(\Delta T)L \qquad \delta_P = \frac{PL}{AE}$$

$$\alpha = \text{thermal expansion coef.}$$

- The thermal deformation and the deformation from the redundant support must be compatible.

$$\delta = \delta_T + \delta_P = 0$$

$$\alpha(\Delta T)L + \frac{PL}{AE} = 0$$

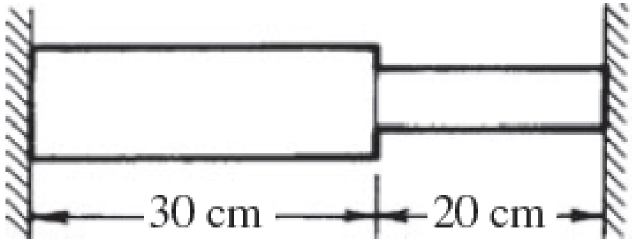
$$\delta = \delta_T + \delta_P = 0$$

$$P = -AE\alpha(\Delta T)$$

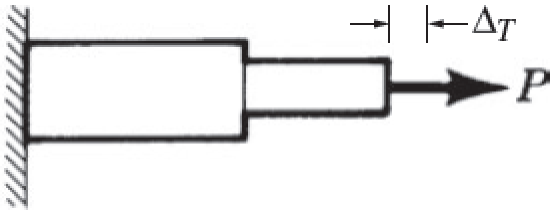
$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

EXAMPLE 1

The composite bar shown in Fig. is rigidly attached to the two supports. The left portion of the bar is copper, of uniform x-sectional area 80 cm^2 and length 30cm. The right portion is aluminum, of uniform x-sectional area 20 cm^2 and length 20cm. At a temperature of 26°C the entire assembly is stress free. The temperature of the structure drops and during this process the right support yields 0.025 mm in the direction of the contracting metal. Determine the minimum temperature to which the assembly may be subjected in order that the stress in the aluminum does not exceed 160 MPa. For copper E = 100 GPa, $a = 17 \times 10^{-6}$/°C, and for aluminum E = 80 GPa, $a = 23 \times 10^{-6}$/°C.



Example 1 – Continued



SOLUTION: It is perhaps simplest to consider that the bar is cut just to the left of the supporting wall at the right and is then free to contract due to the temperature drop ΔT. The total shortening of the composite bar is given by

$$\Delta_T = (\alpha L \Delta T)_{cu} + (\alpha L \Delta T)_{al}$$

$$= (17X10^{-6})(0.30)\Delta T + (23X10^{-6})(0.20)\Delta T$$

$$= 9.7X10^{-6}\Delta T$$

However, this is not the complete analysis because the reaction of the wall at the right has been neglected by cutting the bar there. Consequently, we must represent the action of the wall by an axial force *P* applied to the bar, as shown in above. For equilibrium, the resultant force acting over any cross section of either the copper or the aluminum must be equal to *P*. The application of the force *P* stretches the composite bar by an amount

Example - continued

$$\Delta = \frac{P(0.30)}{(80X10^{-4})(100X10^{9})} + \frac{P(0.20)}{(20X10^{-4})(80X10^{9})} = 16.25X10^{-10}P$$

If the right support were unyielding, we would equate the last expression to the expression giving the total shortening due to the temperature drop. Actually, the right support yields 0.025 cm and consequently we may write

$$(16.25X10^{-10})P = (9.7X10^{-6})\Delta T - 0.025X10^{-3}$$

The stress in the aluminum is not to exceed **160 MPa**, and since it is given by the formula

$$\sigma = \frac{P}{A} \qquad P = \sigma A \quad = (160X10^{6})X\,20X10^{-4} = 320000N$$

Substituting this value of P ; **ΔT = 56.2°C** Therefore the temperature may drop 56.2°C from the original **26°C**.

The final temperature would be –30.2°C.