THIN-WALLED PRESSURE VESSELS

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- Cylindrical or spherical pressure vessels are commonly used in industry to serve as boilers or storage tanks.

- The stresses acting in the wall of these vessels can be analyzed in a simple manner provided it has a *thin wall*, that is, the inner-radius-to-wall-thickness ratio is 10 or more ($r/t \geq 10$)

- Specifically, when $r/t \geq 10$ the results of a thin-wall analysis will predict a stress that is approximately 4% *less* than the actual maximum stress in the vessel.

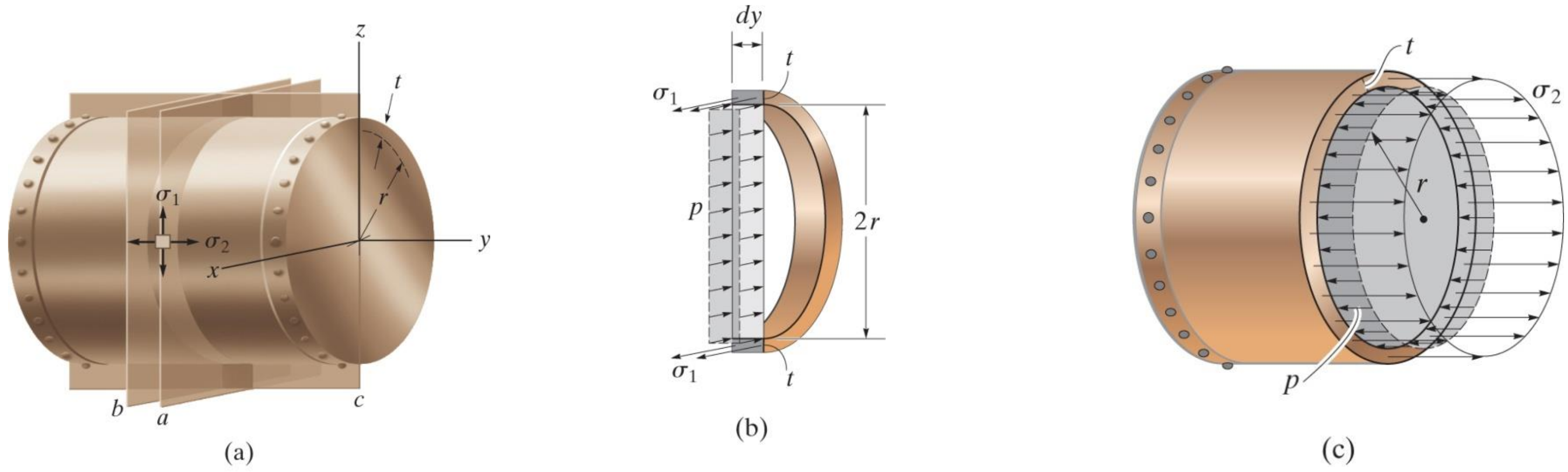
- For larger r/t ratios this error will be even smaller

THIN-WALLED PRESSURE VESSELS

Assumptions:

1. Inner-radius-to-wall-thickness ratio ≥ 10
2. Stress distribution in thin wall is uniform or constant

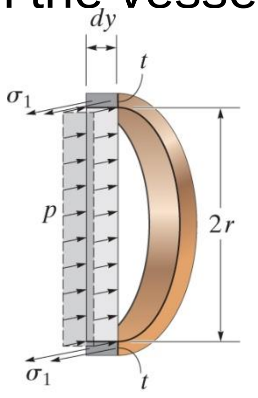
- Cylindrical vessels:



- The cylindrical vessel in Fig. *a* has a wall thickness *t*, inner radius *r*, and is subjected to an internal gas pressure *p*.
- Two types of stresses ***circumferential*** or ***hoop stress***, & ***longitudinal stress***

THIN-WALLED PRESSURE VESSELS

- To find the ***circumferential*** or ***hoop stress***, we can section the vessel by planes *a*, *b*, and *c*.



(b)

- Considering only loadings in the *x*-direction:

$$\sum F_x = 0; \qquad 2[\sigma_1(t\,dy)] - p(2r\,dy) = 0$$

$$\text{Hoop direction: } \sigma_1 = \frac{pr}{t}$$

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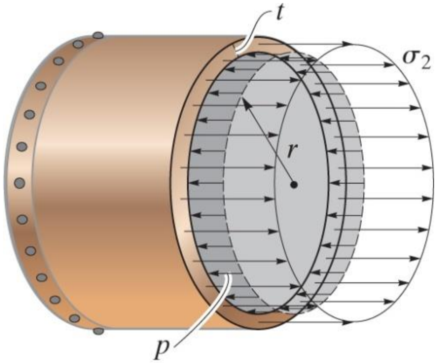
LONGITUDINAL STRESS

- In a FBD below, σ_2 a uniformly throughout the wall, and p acts on the section of the contained gas.

- Since the mean radius is approximately equal to the vessel's inner radius, equilibrium in the y direction requires:

$$\sum F_y = 0; \qquad \sigma_2(2\pi r t) - p(\pi r^2) = 0$$

$$\sigma_2 = \frac{pr}{2t}$$



THIN-WALLED PRESSURE VESSELS (cont)

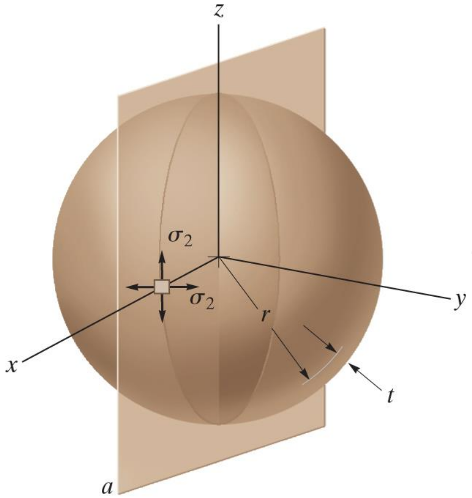
LONGITUDINAL STRESS

- Comparing the two stresses, it can be seen that the hoop or circumferential stress is *twice as large* as the longitudinal or axial stress.

- This implies that:
  - when fabricating cylindrical pressure vessels from rolled-formed plates, it is important that the longitudinal joints be designed to carry twice as much stress as the circumferential joints.

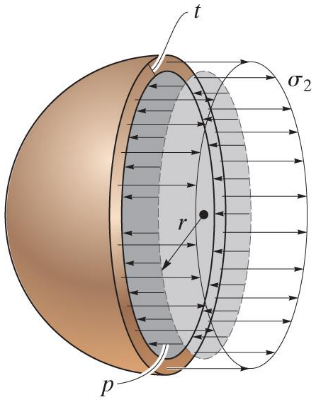
THIN-WALLED PRESSURE VESSELS (cont)

- Spherical vessels:



$$\sum F_y = 0; \qquad \sigma_2(2\pi r t) - p(\pi r^2) = 0$$

$$\sigma_2 = \frac{pr}{2t}$$



This is the same result as that obtained for the longitudinal stress in the cylindrical pressure vessel, although this stress will be the same regardless of the orientation of the hemispheric free-body diagram

THIN-WALLED PRESSURE VESSELS (cont)

- LIMITATIONS:

- The above analysis indicates that an element of material taken from either a cylindrical or a spherical pressure vessel is subjected to ***biaxial stress***, i.e., normal stress existing in only two directions.

- Actually, however, the pressure also subjects the material to a ***radial stress***, σ_3 , which acts along a radial line. This stress has a maximum value equal to the pressure p at the interior wall and it decreases through the wall to zero at the exterior surface of the vessel, since the pressure there is zero.

- For thin-walled vessels, however, we will *ignore* this stress component, since our limiting assumption of $r/t = 10$ results in σ_2 and σ_1 being, respectively, 5 and 10 times *higher* than the maximum radial stress, $(\sigma_3)_{max} = p$.

- Finally, note that if the vessel is subjected to an *external pressure*, the resulting compressive stresses within the wall may cause the wall to suddenly collapse inward or buckle rather than causing the material to fracture.

EXAMPLE 1

A cylindrical pressure vessel has an inner diameter of 1.2 m and a thickness of 12 mm.

- Determine the maximum internal pressure it can sustain so that neither its circumferential nor its longitudinal stress component exceeds 140 MPa.
- Under the same conditions, what is the maximum internal pressure that a similar-size spherical vessel can sustain?

EXAMPLE 1 (cont)

Solutions

- The maximum stress occurs in the circumferential direction.

$$\sigma_1 = \frac{pr}{t}$$

$$140{,}000{,}000 = \frac{p(0.6)}{0.012}$$

$$p = \ 28\ \text{MPa} \quad (\text{Ans})$$

•The stress in the longitudinal direction will be $\sigma_2 = \frac{1}{2}(140) = 70\ \text{MPa}$

•The *maximum stress* in the *radial direction* occurs on the material at the inner wall of the vessel and is

$$\sigma_{3(\max)} = p = 28\ \text{MPa}$$

EXAMPLE 1 (cont)

Solutions

- The maximum stress occurs in any two perpendicular directions on an element of the vessel is

$$\sigma_2 = \frac{pr}{2t}$$

$$140 = \frac{p(600)}{2(12)}$$

$$p = 5.6\ \text{N/mm}^2 = 5.6\ \text{MPa} \quad (\text{Ans})$$