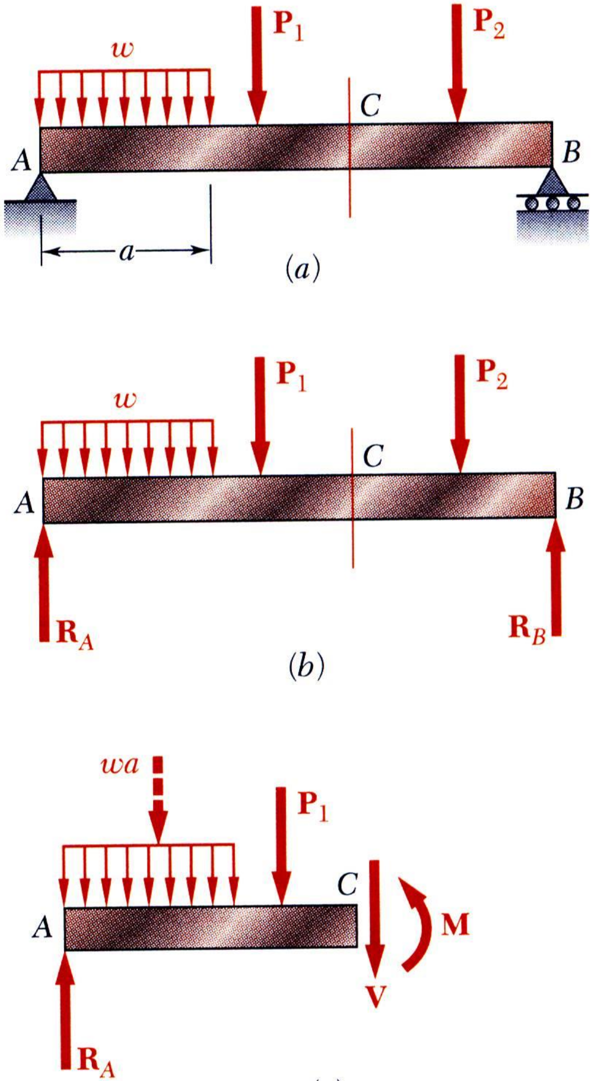
SHEARING FORCE AND BENDING MOMENT IN STRAIGHT BEAMS

Introduction



(a) (b)

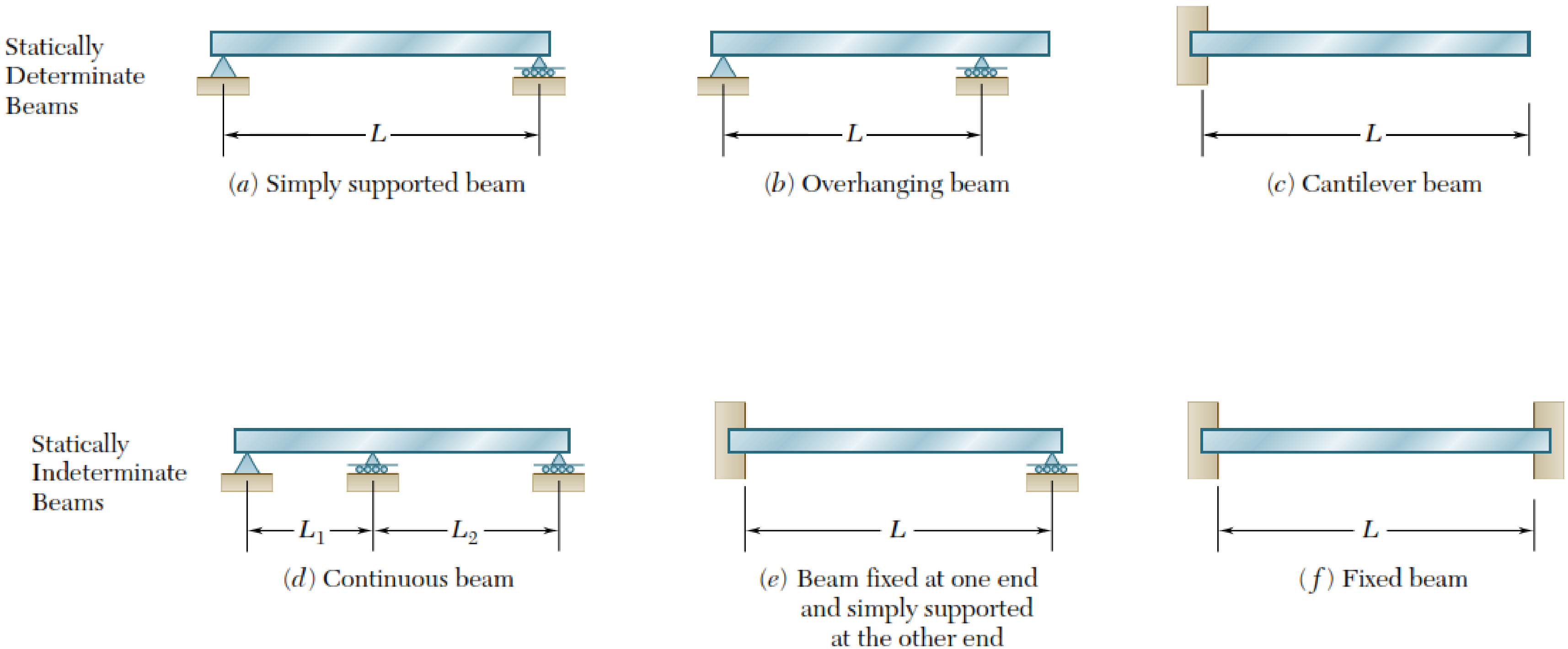
- Objective - Analysis and design of beams
- *Beams* - Members that are slender and support loadings that are applied perpendicular to their longitudinal axis
- Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)
- Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

Requires determination of the location and magnitude of largest bending moment

Introduction

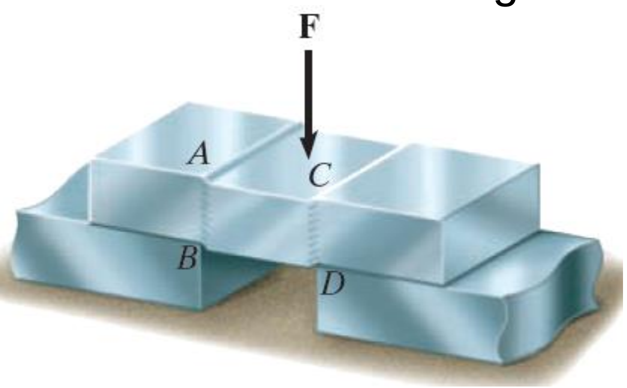
Classification of Beam Supports

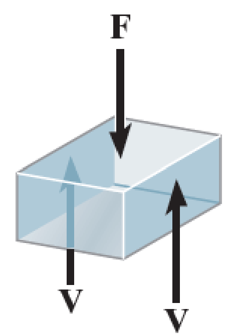


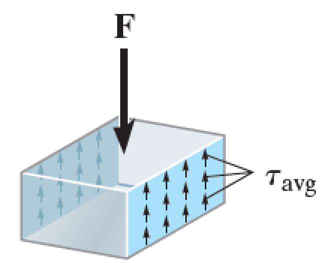
Shear Stress

Shear stress, τ (tau) is defined as the shear force per unit area.

If **F** is large enough, it can cause the material of the bar to deform and fail along the planes identified by *AB* and *CD*. A free-body diagram of the unsupported centre segment of the bar, indicates that the shear force $V = F/2$ must be applied at each section to hold the segment in equilibrium.







The shear stress is rarely uniform, but the average value can be found in terms of the shear force, V, and the area, A, on which it acts.

$$\tau_{avg} = \frac{V}{A}$$

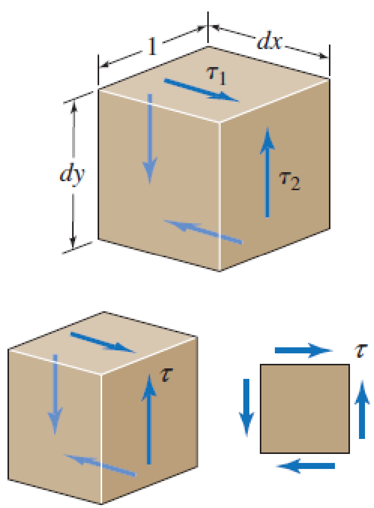
Shear Stress Equilibrium

The shear stress is shown as τ_1 acting on the top and bottom faces of a small element. There are likewise equal and opposite shear stresses, τ_2, on the vertical faces.

For this element to be in equilibrium, the moment about the center must be zero. If the element is dx by dy by 1 thick, then

$$\sum M = -\tau_1(dx)(1)(dy) + \tau_2(dx)(1)(dy) = 0$$

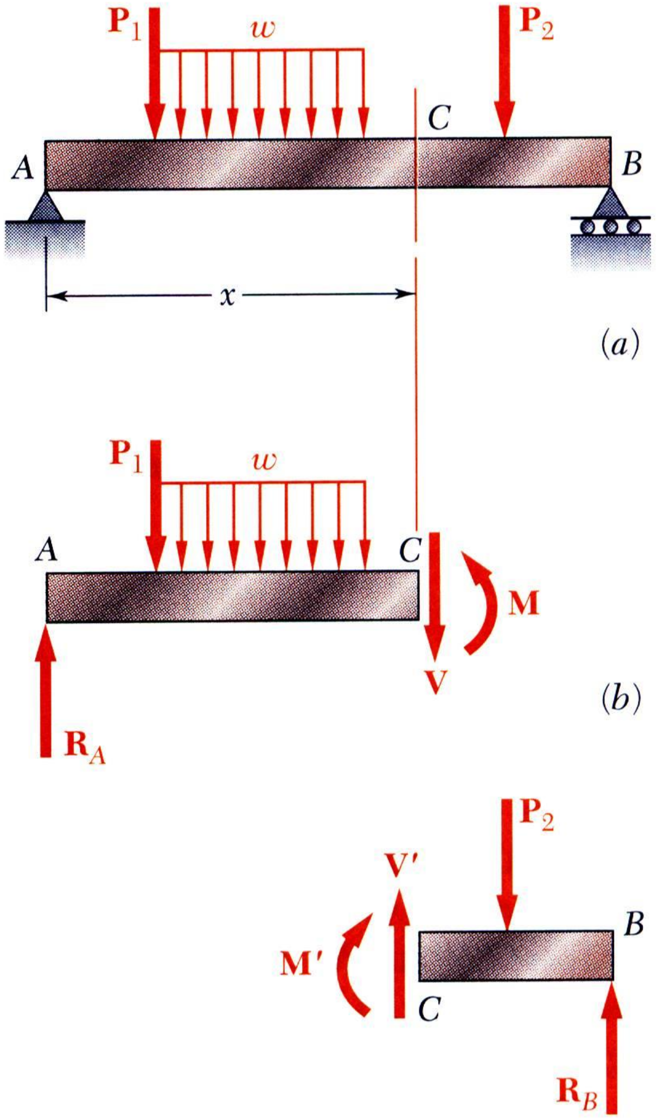
So, the shear stresses must be equal: $\tau_1 = \tau_2$. A single shear stress, τ , describes the shear force per area on horizontal and vertical faces.



All four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element,

This is referred to as the ***complementary property of shear***, and the element in this case is subjected to *pure shear*.

Shear and Bending Moment Diagrams

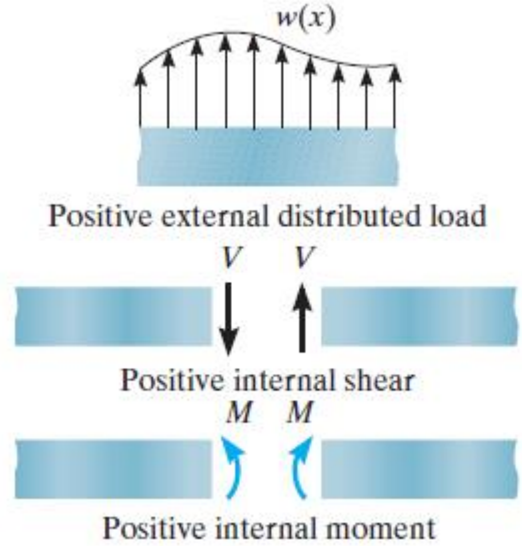


In order to properly design a beam it's necessary to determine the ***maximum*** shear and moment in the beam. One way to do this is to express ***V*** and ***M*** as functions of their arbitrary position ***x*** along the beam's axis. These ***shear*** *and* ***moment functions*** can then be plotted and represented by graphs called ***shear and moment diagrams***.

Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.

Beam Sign Convention

Before plotting shear and moment diagrams, it is first necessary to establish a ***sign convention*** so as to define "positive" and "negative" values for V and M.



Beam sign convention

The ***positive directions*** are as follows:

- The ***distributed load*** acts ***upward*** on the beam;
- The internal ***shear force*** causes a ***clockwise*** rotation of the beam segment on which it acts;
- The internal ***moment*** causes ***compression*** in the *top fibers* of the segment such that it bends the segment so that it holds water. Loadings that are opposite to these are considered negative.

SF & BM Diagrams – Procedure for Analysis

The shear and moment diagrams for a beam can be constructed using the following procedure.

Support Reactions.

• Determine all the reactive forces and couple moments acting on the beam, and resolve all the forces into components acting perpendicular and parallel to the beam's axis.

Shear and Moment Functions.

• Specify separate coordinates *x* having an origin at the beam's *left end* and extending to regions of the beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading.

• Section the beam at each distance *x*, and draw the FBD of one of the segments. Be sure **V** and **M** are shown acting in their positive sense

SF & BM Diagrams -Procedure for Analysis

- The shear is obtained by summing forces perpendicular to the beam's axis.

- To eliminate *V*, the moment is obtained directly by summing moments about the sectioned end of the segment.

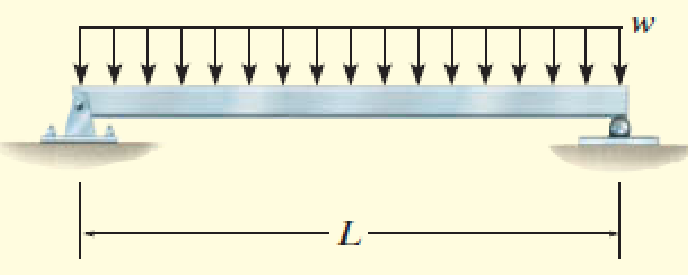
Shear and Moment Diagrams.

- Plot the shear diagram (*V* versus *x*) and the moment diagram (*M* versus *x*). If numerical values of the functions describing *V* and *M* are *positive*, the values are plotted above the *x* axis, whereas negative values are plotted below the axis.

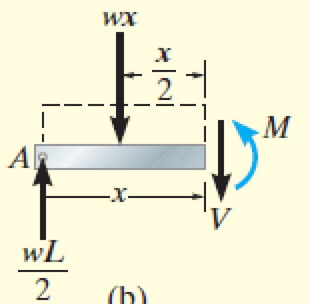
- Generally it is convenient to show the shear and moment diagrams below the free-body diagram of the beam.

GENERIC EXAMPLE

Draw the shear and moment diagrams for the beam shown



(a)



(b)

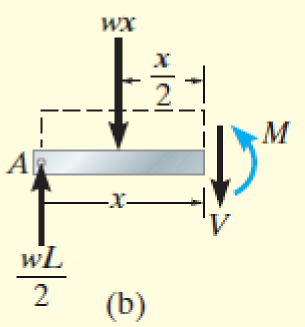
***Support Reactions*.**

The support reactions are: $\frac{\omega L}{2}$

***Shear and Moment Functions*.**

A FBD of the left segment of the beam is shown in *b*. The distributed loading on this segment, ωx *is* represented by its resultant force only *after* the segment is isolated as a FBD. This force acts through the centroid of the area comprising the distributed loading, a distance $\frac{x}{2}$ of from the right end. Applying the two equations of equilibrium yields

Generic Example



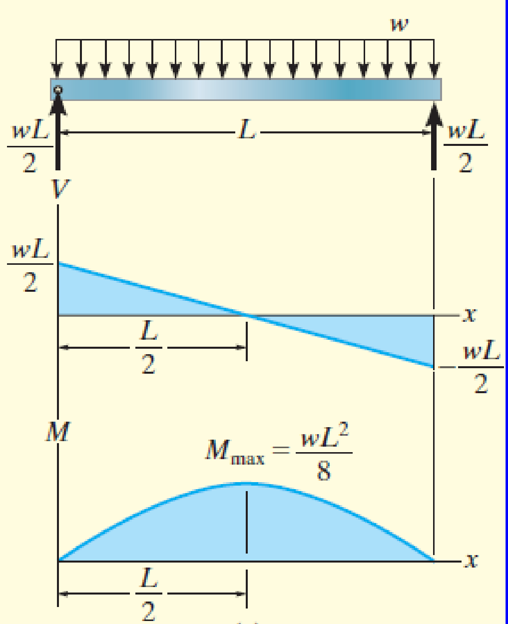
(b)

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \qquad (2)$$



(c)

Shear and Moment Diagrams.
The shear and moment diagrams shown in Fig.(c) are obtained by plotting *Eqs*. 1 and 2. The point of ***zero shear*** can be found from Eq. 1:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

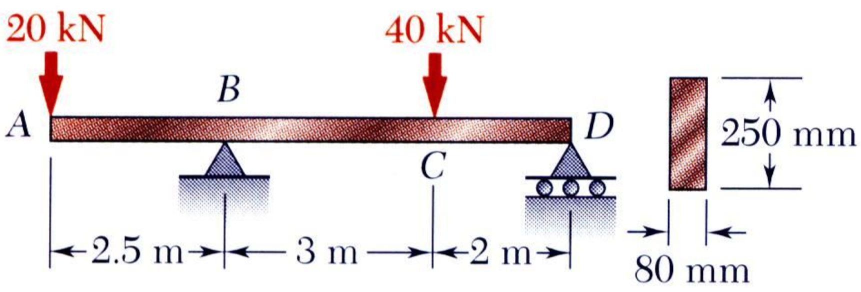
$$x = \frac{L}{2}$$

x represents the point on the beam where the *maximum moment*

$$M_{max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{wL^2}{8}$$

Sample Problem 1.0

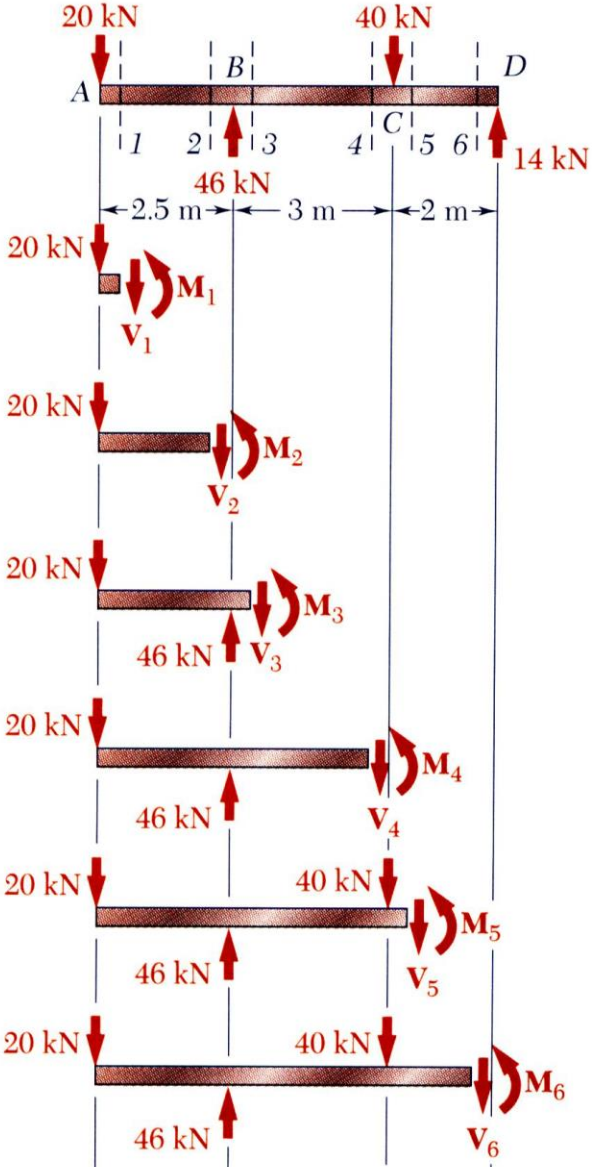


For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

Sample Problem 1



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces

  from $\sum F_y = 0 = \sum M_B : \quad R_B = 40\,\text{kN} \quad R_D = 14\,\text{kN}$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$+\uparrow \mathrm{F_y} \sum F_y = 0 \quad -20\text{ kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$

$$(+M)\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\text{ kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$

$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

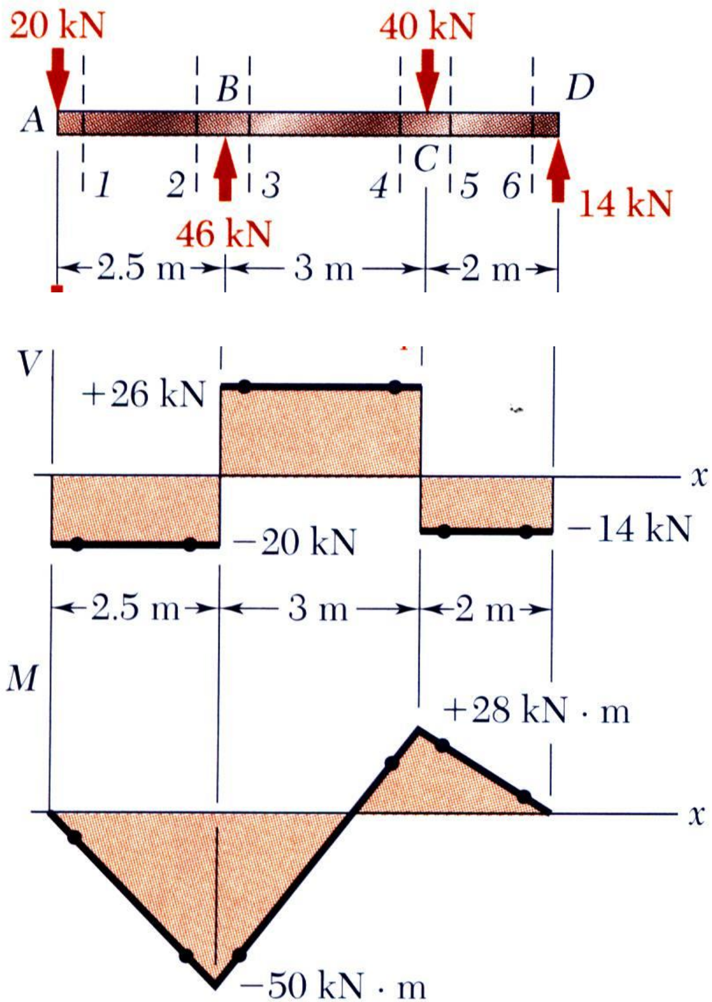
$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$

$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$

$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$

$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

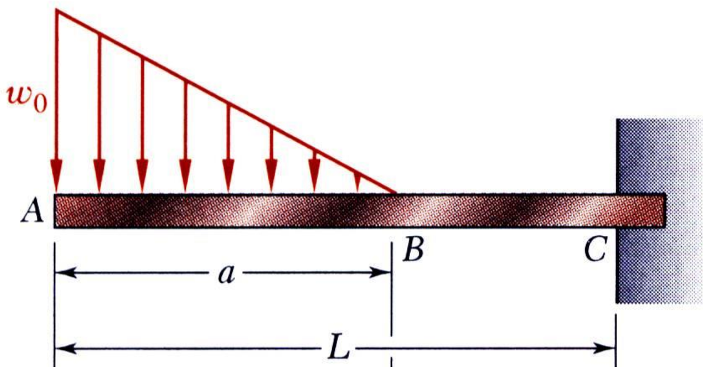
Sample Problem 1



- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\text{kN} \quad M_m = |M_B| = 50\,\text{kN}\cdot\text{m}$$

Sample Problem 2.

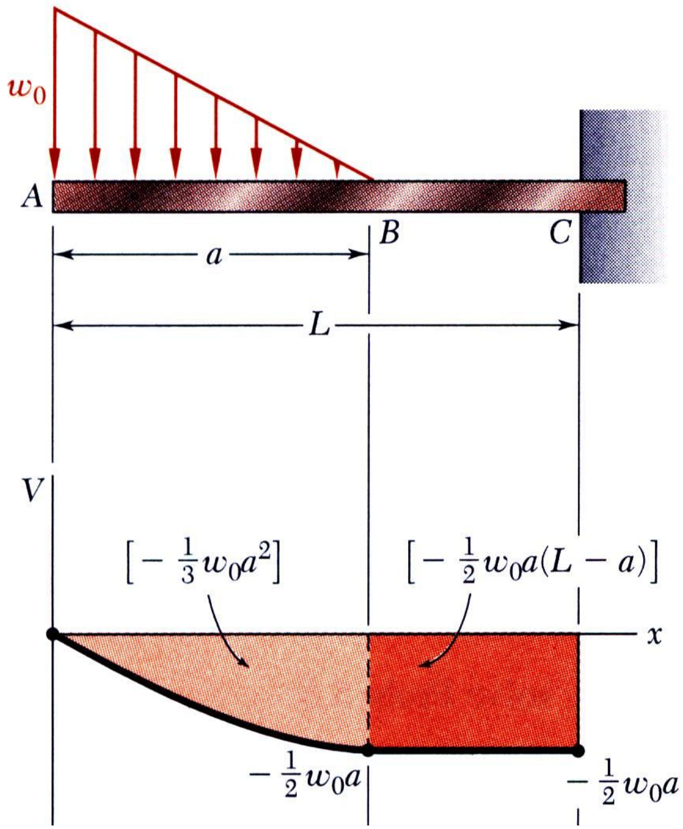


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 2



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.

$$\sum F_y = 0 = -\tfrac{1}{2}w_0 a + R_C \qquad R_C = \tfrac{1}{2}w_0 a$$

$$\sum M_C = 0 = \tfrac{1}{2}w_0 a\left(L - \frac{a}{3}\right) + M_C \qquad M_C = -\tfrac{1}{2}w_0 a\left(L - \frac{a}{3}\right)$$

Results from integration of the load and shear distributions should be equivalent.

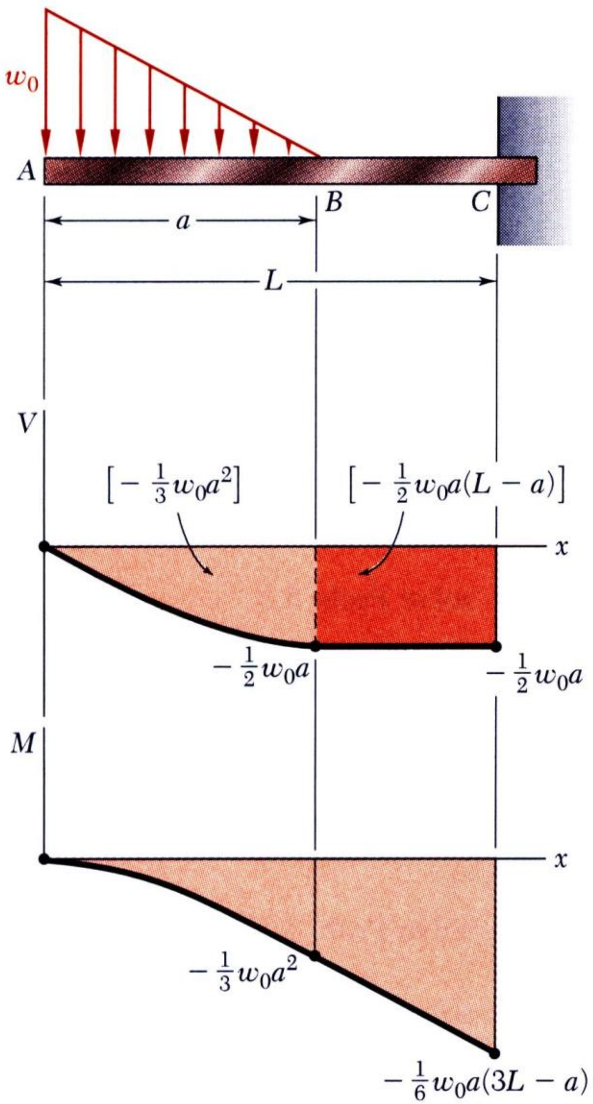
- Apply the relationship between shear and load to develop the shear diagram.

$$V_B - V_A = -\int_0^a w_0\left(1 - \frac{x}{a}\right)dx = -\left[w_0\left(x - \frac{x^2}{2a}\right)\right]_0^a$$

$$V_B = -\tfrac{1}{2}w_0 a = -(\,area\ under\ load\ curve\,)$$

- No change in shear between *B* and *C*.
- Compatible with free body analysis

Sample Problem 2.



- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$M_B - M_A = \int_0^a \left(-w_0\left(x - \frac{x^2}{2a}\right)\right)dx = \left[-w_0\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\right]_0^a$$

$$M_B = -\tfrac{1}{3}w_0a^2$$

$$M_B - M_C = \int_a^L \left(-\tfrac{1}{2}w_0a\right)dx = -\tfrac{1}{2}w_0a(L-a)$$

$$M_C = -\tfrac{1}{6}w_0a(3L-a) = \frac{a\,w_0}{2}\left(L - \frac{a}{3}\right)$$

Results at *C* are compatible with free-body analysis