STATIC INDETERMINACY

Deformation of statically indeterminate structures

Static Indeterminacy

- Structures for which internal forces and reactions cannot be determined from statics alone are said to be *statically indeterminate*.

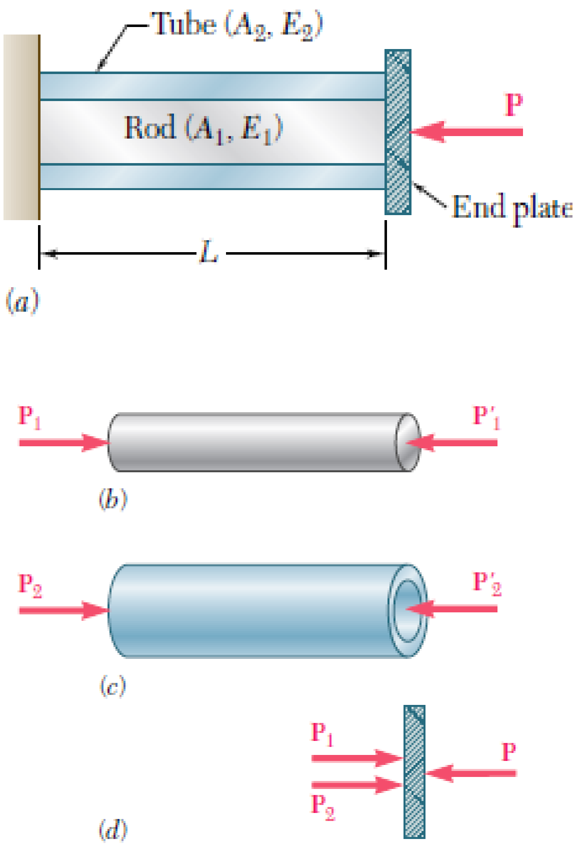
- A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.

- Redundant reactions are replaced with unknown loads which along with the other loads must produce compatible deformations.

- Deformations due to actual loads and redundant reactions are determined separately and then added or *superposed*.

$$\delta = \delta_L + \delta_R = 0$$

Illustration of statically indeterminacy



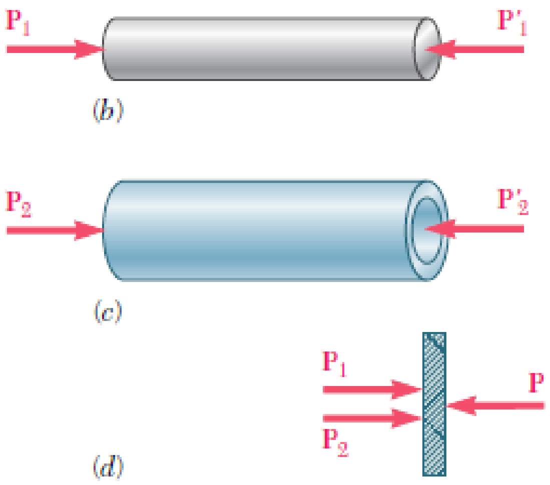
A rod of length *L*, cross-sectional area A_1, and modulus of elasticity E_1, has been placed inside a tube of the same length *L*, but of cross-sectional area A_2 and modulus of elasticity E_2 . What is the deformation of the rod and tube when a force **P** is exerted on a rigid end plate as shown?

Denoting by P_1 and P_2, respectively, the axial forces in the rod and in the tube, we draw free-body diagrams of all three elements (Fig.*b*,*c*, *d*). Only the last of the diagrams yields any significant information, namely:

$P_1 + P_2 = P$

Clearly, one equation is not sufficient to determine the two unknown internal forces P_1 and P_2. The problem is statically indeterminate. However, the geometry of the problem shows that the deformations δ_1 and δ_2 of the rod and tube must be equal.

Illustration of statically indeterminacy



$$\delta_1 = \frac{P_1 L_1}{A_1 E_1} \qquad \delta_2 = \frac{P_2 L_2}{A_2 E_2}$$

Equating both equations

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

Substituting above Eqn into $\boldsymbol{P_1 + P_2 = P}$

$$P1 = \frac{A1E1P}{A1E1 + A2E2} \;;\quad P2 = \frac{A2E2P}{A1E1 + A2E2}$$

STATIC INDETERMINACY – SUPERPOSITION

Superposition Method

- We observe that a structure is statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.
- This results in more unknown reactions than available equilibrium equations.
- It is often found convenient to designate one of the reactions as *redundant* and to eliminate the corresponding support.

STATIC INDETERMINACY – SUPERPOSITION

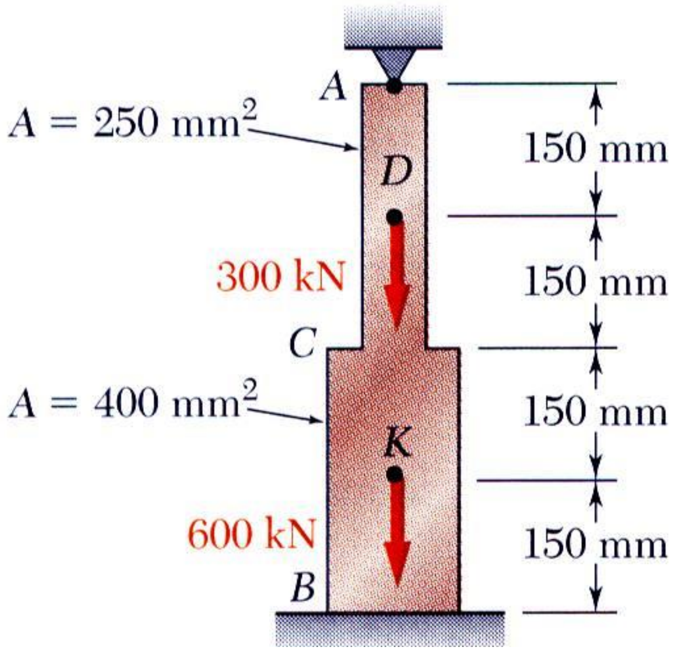
- Since the stated conditions of the problem cannot be arbitrarily changed, the redundant reaction must be maintained in the solution

- But it will be treated as an *unknown load* that, together with the other loads, must produce deformations that are compatible with the original constraints.

- The actual solution of the problem is carried out by considering separately the deformations caused by the given loads and by the redundant reaction, and by adding—or *superposing*—the results obtained

Example 1

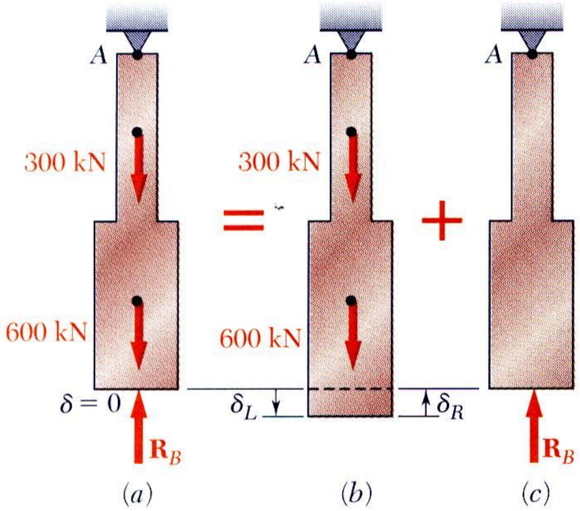
Determine the reactions at *A* and *B* for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.



Example 1

SOLUTION:

- Consider the reaction at *B* as redundant, release the bar from that support, and solve for the displacement at *B* due to the applied loads.

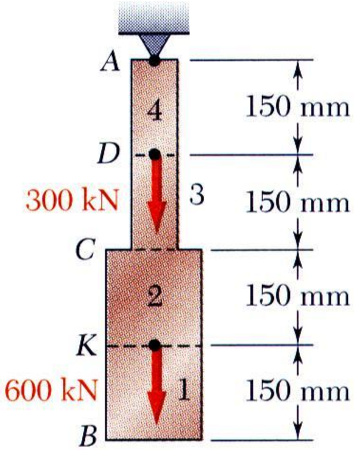


- Solve for the displacement at *B* due to the redundant reaction at *B.*

- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.

- Solve for the reaction at *A* due to applied loads and the reaction found at *B*.

Example 1 Continued



SOLUTION:

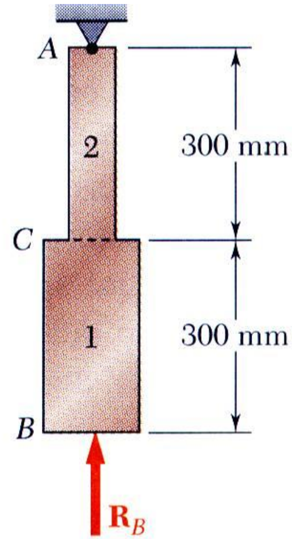
- Solve for the displacement at *B* due to the applied loads with the redundant constraint released,

$$P_1 = 0$$
$$P_2 = P_3 = 600 \times 10^3 \text{N}$$
$$P_4 = 900 \times 10^3 \text{N}$$

$$A_1 = A_2 = 400 \times 10^{-6} \text{m}^2$$
$$A_3 = A_4 = 250 \times 10^{-6} \text{m}^2$$
$$L_1 = L_2 = L_3 = L_4 = 0.150 \text{ m}$$

$$\delta_{\text{L}} = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125 \times 10^9}{E}$$

Example 1 Continued



- Solve for the displacement at *B* due to the redundant constraint, we divide the bar into two portions,

$$\delta_R = \frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E}$$

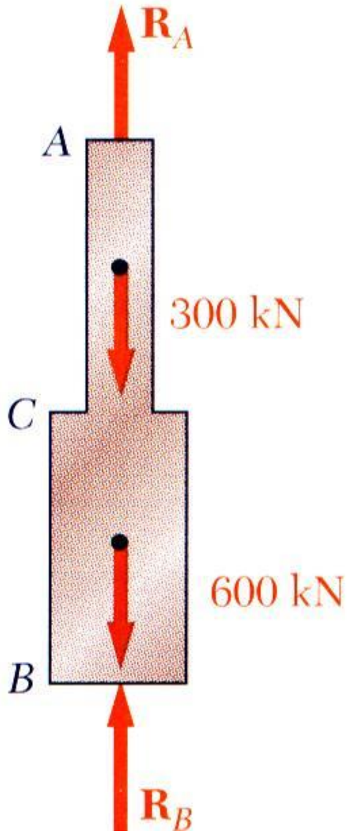
$$P_1 = P_2 = -R_B$$

$$A_1 = 400 \times 10^{-6}\,\mathrm{m}^2 \quad A_2 = 250 \times 10^{-6}\,\mathrm{m}^2$$

$$L_1 = L_2 = 0.300\ \mathrm{m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{\left(1.95 \times 10^3\right) R_B}{E}$$

Example 1 Continued



- Expressing that the displacements due to the loads and due to the redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0$$

$$\delta = \frac{1.125 \times 10^9}{E} - \frac{\left(1.95 \times 10^3\right) R_B}{E} = 0$$

$$R_B = 577 \times 10^3 \text{ N} = 577 \text{ kN}$$

- Find the reaction at *A* due to the loads and the reaction at *B*

$$\sum F_y = 0 = R_A - 300 \text{ kN} - 600 \text{ kN} + 577 \text{ kN}$$

$$R_A = 323 \text{ kN}$$

$$\boxed{\begin{aligned} R_A &= 323 \text{ kN} \\ R_B &= 577 \text{ kN} \end{aligned}}$$