Civil Engineer's Reference Book

Fourth Edition

Edited by

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Preface to the fourth edition

The aim of this edition, as of those that preceded it, is to give civil engineers a concise presentation of theory and practice in the many branches of their profession. The book is primarily a first point of reference which, through its selective lists of references and bibliographies, will enable the user to study a subject in greater depth. However, it is also an important collection of state-of-the-art reports on design and construction practices in the UK and overseas.

First published in 1951, the book was last revised in 1975. Although civil engineering is not normally regarded as involving fast-moving technologies, so many advances have occurred in the theory and practice of most branches of civil engineering during the past decade or so that the preparation of a fourth edition became essential. Some of these advances have taken the form of improvements in earlier practices, for example in surveying, geotechnics, water management, project management, underwater working, and the control and use of materials. Other radical changes have resulted from the evolving needs of clients for almost all forms of construction, maintenance and repair. Another major change has been the introduction of new national and Euro-codes based on limit state design covering most aspects of structural engineering.

The fourth edition incorporates these advances and, at the same time, gives greater prominence to the special problems relating to work overseas, with differing client requirements and climatic conditions.

As before, careful attention has been given to the needs of the different categories of readers. Students and graduates at the start of their careers need guidance on the practice of design and construction in many of the fields of civil engineering covered in Chapters 11 to 44. The engineer in mid-career will also find these chapters valuable as presentations of the state of the art by acknowledged experts in each field, in addition to the references and bibliographies they contain for deeper study of specific problems. Chapters 1 to 10 provide engineers, at all levels of development, with up-to-date 'lecture notes' on the basic theories of civil engineering.

Although the book was primarily prepared for civil engineers in the UK and elsewhere in the world, members of other professions involved in construction—architects, lawyers, mechanical engineers, insurers and clients—will also benefit by referring to it.

I am most grateful to the authors who have contributed chapters. They are all engineers of considerable standing—consultants, contractors, research workers or academics—who have devoted a substantial amount of time to presenting their expert knowledge and experience for the benefit of the profession.

L.S. Blake
Bournemouth
November 1988

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MATHEMATICS

1.1 Algebra

1.1.1 Powers and roots

The following are true for all values of indices, whether positive, negative or fractional:

$$a^p \times a^q = a^{p+q}$$
$$(a^p)^q = a^{pq}$$
$$(a/b)^p = a^p/b^p$$
$$(ab)^p = a^p b^p$$
$$a^p/a^q = a^{p-q}$$
$$a^{-p} = (1/a)^p = 1/a^p$$
$$p\sqrt{a} = a^{1/p}$$
$$a^0 = 1$$
$$0^p = 0$$

1.1.2 Solutions of equations in one unknown

1.1.2.1 Linear equations

Generally $ax+b=0$
of which there is one solution or root $x=-b/a$

1.1.2.2 Quadratic equations

Generally $ax^2+bx+c=0$
of which there are two solutions or roots

$$x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a} \tag{1.1}$$

where, if $b^2>4ac$, the roots are real and unequal, $b^2=4ac$, the roots are real and equal, and $b^2<4ac$, the roots are conjugate complex.

It is worth attempting to rearrange equations as, often, they can be put into a more familiar form simply by rearrangement, e.g.:

$$ax^{2m}+bx^m+c=0$$

is a quadratic equation in x^m

while $a/x^2+b/x+c=0$

is the quadratic $cx^2+bx+a=0$

1.1.2.3 Cubic equations

Generally $x^3+bx^2+cx+d=0$
If the substitution: $x=y-b/3$ is made the equation

becomes $y^3+ey+f=0$

where $e=(3c-b^2)/3$

and $f=(2b^3-9bc+27d)/27$

now define

$$A=\left[-\frac{f}{2}+\left(\frac{f^2}{4}+\frac{e^3}{27}\right)\right]^{1/3}$$

$$B=\left[-\frac{f}{2}-\left(\frac{f^2}{4}+\frac{e^3}{27}\right)\right]^{1/3}$$

and the three roots, in terms of y are:

$$y_1=[A+B]$$

$$y_{2,3}=[-(A+B)/2\pm\sqrt{-3}(A-B)/2]$$

and in terms of x the three roots are:

$$x_{1,2,3}=y_{1,2,3}-\frac{b}{3}$$

1.1.2.4 Equations of higher degree

Equations of degree higher than the second (quadratic equations) are not solvable directly as the method of solving the cubic equation above shows. Generally recourse must be had to either graphical or numerical techniques.

If the equation be of the form:

$$F(x)=0$$

e.g. $a_n x^n + a_{n-1}x^{n-1} \ldots + a0 = 0$

then plot the graph of $y=F(x)$ the values of x at which $y=0$ are the roots or solutions to the equation. Frequently this graphical approach may be used fairly roughly (and therefore quickly) to obtain an estimate of a root. This estimate can then be improved by numerical means. For instance, values of $F(x)$ may be calculated for values of x close to that given as a root by the graphical method. The difficulty (which is not serious for hand calculations) is guessing by how much to adjust x to get $F(x)$ nearer to 0.

1.1.3 Newton's method

This is a method of step-by-step iteration in which an estimate of a root is refined.

Suppose that a_1 is an approximation to a root of an equation then, for small q:

$$F(a_1+q)\simeq F(a_1)+qF(a_1)$$

So that if we assume (a_1+q) to be the better solution we are seeking, i.e.:

$$F(a_1+q)=0 \tag{1.2}$$

then:

$$q=\frac{-F(a_1)}{F'(a_1)} \tag{1.3}$$

and $a_2=a_1+q$ is a second and better approximation.

This is well illustrated by drawing a curve cutting the x-axis, assuming a value a_1 of x near to the intersection to have been found, drawing the ordinate to the curve $x=a_1$ and then constructing the tangent to the curve $y=F(x)$ at the point $x=a_1$.

The point $x=a_2$ where this tangent cuts the axis is plainly a better estimate of the intersection than is a_1.

This technique can be used successfully in automatic calculation on a computer. The problem then becomes that of determining when to stop the iteration process:

$$a_1, a_2, a_3 \ldots$$

which may be best done by stopping when the change between successive approximations, a_n and a_{n+1} becomes less than some small preset amount.

Graphical and numerical methods will generally be required to deal with transcendental equations although in some cases it may be more convenient to find the intersections of two graphs rather than try to compute where a more complicated graph cuts an axis

e.g. $x - \sin x = 0$

is best solved by plotting:

$y = x$

and $y = \sin x$

to find the intersection which will give an estimate which can be refined numerically.

1.1.4 Progressions

(1) Arithmetic progressions in which the difference between consecutive terms is a constant amount. Thus, the terms may be:

$a, a+d, a+2d, a+3d \ldots$

The nth term is $a+(n-1)d$ and the sum to n terms,

$$S_n = \frac{n}{2}\{2a+(n-1)\,d\} \tag{1.4}$$

(2) Geometrical progressions in which the ratio between consecutive terms is a constant. Generally terms are:

$a, ar, ar^2, ar^3 \ldots$

The nth term is ar^{n-1} and the sum of n terms is:

$$S_n = \frac{a(1-r^n)}{1-r} \tag{1.5}$$

If r is strictly smaller than 1 so $-1 < r < 1$, then r^n tends to zero as n becomes larger so that for such geometric progressions we can find the 'sum to infinity' of the series:

$$S_\infty = \frac{a}{1-r} \tag{1.6}$$

The geometric mean of a set of n numbers is the nth root of their product.

If we limit consideration to non-negative numbers then the arithmetic mean of a set of numbers will be greater than or equal to their geometric mean.

1.1.5 Logarithms

Logarithms, which, short of calculating machinery of some form, are probably the greatest aid to computation are based on the properties of indices.

Thus, if we consider logarithms to base a we have the following results:

$a^x = P$ is equivalent to $\log_a P = x$

$a^1 = a$ is equivalent to $\log_a a = 1$

$a^0 = 1$ is equivalent to $\log_a 1 = 0$

So that using rules for powers given on page **1**/3:

If: $a^x = P$ and $a^y = Q$

then: $PQ = a^{x+y}H$

so: $\log_a PQ = x + y = \log_a P + \log_a Q$

Similarly: $\log_a (P/Q) = \log_a P - \log_a Q$

Also: $P^n = a^{nx}$

so: $\log aP^n = nx = n \log_a P$

In computation, it is generally convenient to use as base the number 10, i.e. in the expressions given above $a = 10$. However, in fundamental work or integration natural logarithms (also known as Napierian or hyperbolic logarithms) are generally used. These are logarithms to base e a transcendental number given approximately by:

$$e = 2.71828 \tag{1.7}$$

and whose definition can be taken as: 'The value of the solution of the differential equation $dy/dx = y$ for $x = 1$.'

(Note the solution of $dy/dx = y$ is $y = e^x$.)

1.1.6 Permutations and combinations

If, in a sequence of N events, the first can occur in n_1 ways, the second in n_2, etc. then the number of ways in which the whole sequence can occur is:

$n_1 n_2 n_3 \ldots n_N$

1.1.6.1 Permutations

The number of permutations of n different things taken r at a time means the number of ways in which r of these n things can be arranged *in order*. This is denoted by:

$${}^nPr = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!} \tag{1.8}$$

where $n! = n(n-1)(n-2), \ldots 3.2.1$ is called factorial n.

It is clear that:

${}^nPn = n!$

and that:

${}^nP_1 = n$

If, of n things taken r at a time p things, are to occupy fixed positions then the number of permutations is given by:

$$^{n-p}Pr-p \tag{1.9}$$

If in the set of n things, there are g groups each group containing $n_1, n_2 \ldots n_g$ things which are identical then the number of permutations of all n things is:

$$\frac{n!}{n_1!n_2!\ldots n_g!}$$

1.1.6.2 Combinations

The number of combinations of n different things, into groups of r things at a time is given by:

$$^nCr=\frac{n!}{r!(n-r)!}=\frac{^nPr}{r!} \tag{1.10}$$

It is important to note that, whereas in permutations the order of the things does matter, in combinations the order does not matter. From the general expression above, it is clear that:

$$^nCn=1$$

$$^nC_1=n \tag{1.11}$$

If, of n different things taken r at a time p are always to be taken then the number of combinations is:

$$^{n-p}Cr-p \tag{1.12}$$

If, of n different things taken r at a time p are never to occur the number of combinations is:

$$^{n-p}Cr \tag{1.13}$$

Note that combinations from an increasing number of available things are related by:

$$^{n+1}Cr={}^nCr+{}^nCr-1 \tag{1.14}$$

also $\quad {}^nCr={}^nCn-r \qquad (1.15)$

1.1.7 The binomial theorem

The general form of expansion of $(x+a)^n$ is given by:

$$(x+a)^n={}^nC_0x^n+{}^nC_1\times{}^{n-1}a^r+{}^nC_2x^{n-2}a^2\ldots \tag{1.16}$$

Alternatively this may be written as:

$$(x+a)^n=x^n+nx^{n-1}a+\frac{n(n-1)}{1.2}x^{n-2}a^2+\frac{n(n-1)(n-2)}{1.2.3}x^{n-3}a^3 \tag{1.17}$$

It should be noted that the coefficients of terms equidistant from the end are equal (since $^nCr={}^nCn-r$).

1.2 Trigonometry

The *trigonometric functions* of the angle a (see Figure 1.1) are defined as follows:

$\sin a=y/r$	$\operatorname{cosec} a=r/y$
$\cos a=x/r$	$\sec a=r/x$
$\tan a=y/x$	$\cot a=x/y$

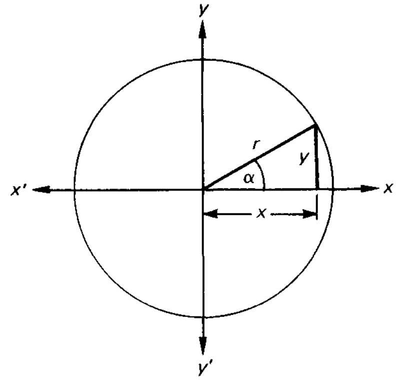

Figure 1.1 Trigonometric functions

These functions satisfy the following identities:

$$\sin^2a+\cos^2a=1$$
$$1+\tan^2a=\sec^2a$$
$$1+\cot^2a=\operatorname{cosec}^2a$$

1.2.1 Positive and negative lines

In trigonometry, lines are considered positive or negative according to their location relative to the coordinate axes xOx', yOy', (see Figure 1.2).

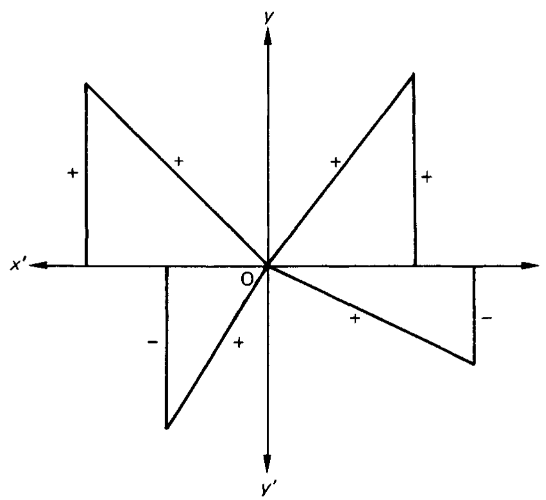

Figure 1.2 Positive and negative lines

1.2.1.2 Positive lines

Radial: any direction.
Horizontal: to right of yOy'.
Vertical: above xOx'.

1.2.1.3 Negative lines

Horizontal: to left of yOy'.
Vertical: below xOx'.

1.2.2 Positive and negative angles

Figure 1.3 shows the convention for signs in measuring angles. Angles are positive if the line OP revolves anti-clockwise from

Ox as in Figure 1.3a and are negative when OP revolves clockwise from Ox.

Signs of trigonometrical ratios are shown in Figure 1.4 and in Table 1.1.

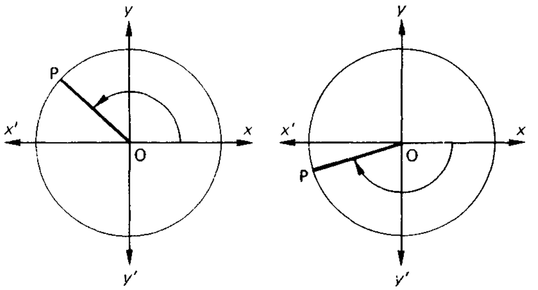

Figure 1.3 (a) Positive (b) negative angle

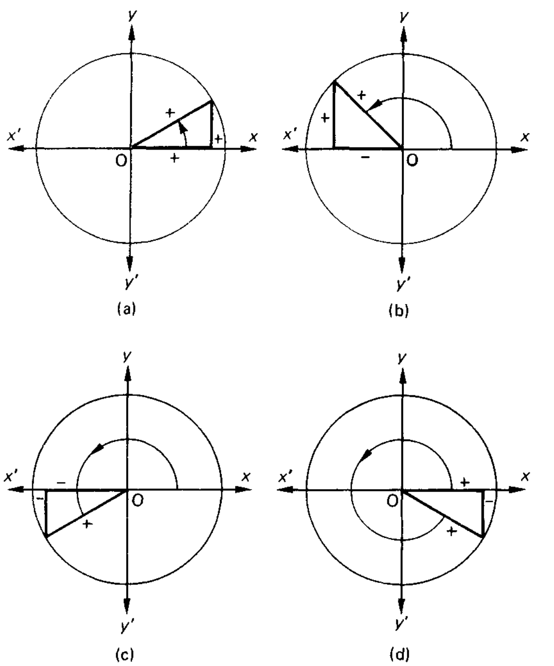

Figure 1.4 (a) Angle in first quadrant; (b) angle in second quadrant; (c) angle in third quadrant; (d) angle in fourth quadrant

1.2.3 Trigonometrical ratios of positive and negative angles

Table 1.1

Quadrant	*Sign of ratio*	
	positive	*negative*
First	sin cos tan cosec sec cot	
Second	sin cosec	cos sec tan cot
Third	tan cot	sin cosec cos sec
Fourth	cos sec	sin cosec tan cot

1.2.4 Measurement of angles

1.2.4.1 English or sexagesimal method

1 right angle $= 90°$ (degrees)
$1°$ (degree) $= 60'$ (minutes)
$1'$ (minute) $= 60''$ (seconds)

This convention is universal.

1.2.4.2 French or centesimal method

This splits angles, degrees and minutes into 100th divisions but is not used in practice.

1.2.4.3 The radian

This is a constant angular measurement equal to the angle subtended at the centre of any circle by an arc equal in length to the radius of the circle as shown in Figure 1.5.

$$\pi \text{ radians} = 180°$$

$$1 \text{ radian} = \frac{180}{\pi} = \frac{180}{3.141\,6} = 57°\ 17'\ 44'' \text{ approximately}$$

Table 1.2

$\sin(-\alpha) = -\sin\alpha$	$\tan(-\alpha) = -\tan\alpha$	$\sec(-\alpha) = \sec\alpha$
$\cos(-\alpha) = \cos\alpha$	$\cot(-\alpha) = -\cot\alpha$	$\operatorname{cosec}(-\alpha) = -\operatorname{cosec}\alpha$
$\sin(90°-\alpha) = \cos\alpha$	$\tan(90°-\alpha) = \cot\alpha$	$\sec(90°-\alpha) = \operatorname{cosec}\alpha$
$\cos(90°-\alpha) = \sin\alpha$	$\cot(90°-\alpha) = \tan\alpha$	$\operatorname{cosec}(90°-\alpha) = \sec\alpha$
$\sin(90°+\alpha) = \cos\alpha$	$\tan(90°+\alpha) = -\cot\alpha$	$\sec(90°+\alpha) = -\operatorname{cosec}\alpha$
$\cos(90°+\alpha) = -\sin\alpha$	$\cot(90°+\alpha) = -\tan\alpha$	$\operatorname{cosec}(90°+\alpha) = \sec\alpha$
$\sin(180°-\alpha) = \sin\alpha$	$\tan(180°-\alpha) = -\tan\alpha$	$\sec(180°-\alpha) = -\sec\alpha$
$\cos(180°-\alpha) = -\cos\alpha$	$\cot(180°-\alpha) = -\cot\alpha$	$\operatorname{cosec}(180°-\alpha) = \operatorname{cosec}\alpha$
$\sin(180°+\alpha) = -\sin\alpha$	$\tan(180°+\alpha) = \tan\alpha$	$\sec(180°+\alpha) = -\sec\alpha$
$\cos(180°+\alpha) = -\cos\alpha$	$\cot(180°+\alpha) = \cot\alpha$	$\operatorname{cosec}(180°+\alpha) = -\operatorname{cosec}\alpha$

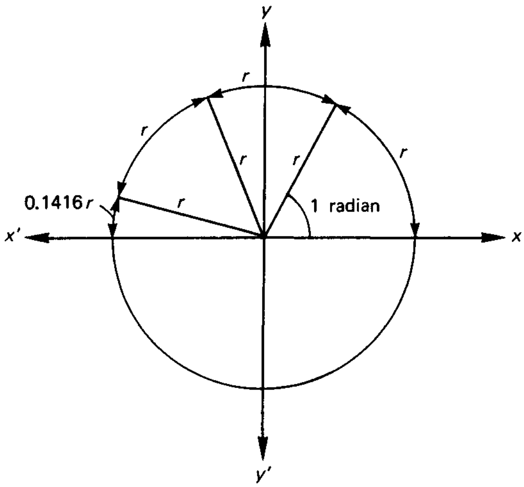

Figure 1.5 The radian

1.2.4.4 Trigonometrical ratios expressed as surds

Table 1.3

Angle in radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Angle in degrees	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\surd 2}$	$\frac{\surd 3}{2}$	1
cos	1	$\frac{\surd 3}{2}$	$\frac{1}{\surd 2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\surd 3}$	1	$\surd 3$	∞

Table 1.3 gives these ratios for certain angles.

1.2.5 Complementary and supplementary angles

Two angles are complementary when their sum is a right angle; then either is the complement of the other, e.g. the sine of an angle equals the cosine of its complement. Two angles are supplementary when their sum is two right angles.

1.2.6 Graphical interpretation of the trigonometric functions

Figures 1.6 to 1.9 show the variation with α of $\sin\alpha$, $\cos\alpha$, $\tan\alpha$ and $\operatorname{cosec}\alpha$ respectively. All the trigonometric functions are periodic with period 2π radians (or 360°).

1.2.7 Functions of the sum and difference of two angles

$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$
$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\pm\tan A\tan B}$$

1.2.8 Sums and differences of functions

$\sin A+\sin B=2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$
$\sin A-\sin B=2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$
$\cos A+\cos B=2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$
$\cos A-\cos B=-2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$
$\sin^2 A-\sin^2 B=\sin(A+B)\sin(A-B)$
$\cos^2 A-\cos^2 B=-\sin(A+B)\sin(A-B)$
$\cos^2 A-\sin^2 B=\cos(A+B)\cos(A-B)$

1.2.9 Functions of multiples of angles

$\sin 2A=2\sin A\cos A$
$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$
$\tan 2A=2\tan A/(1-\tan^2 A)$
$\sin 3A=3\sin A-4\sin^3 A$
$\cos 3A=4\cos^3 A-3\cos A$
$\tan 3A=(3\tan A-\tan^3 A)/(1-3\tan^2 A)$
$\sin pA=2\sin(p-1)A\cos A-\sin(p-2)A$
$\cos pA=2\cos(p-1)A\cos A-\cos(p-2)A$

1.2.10 Functions of half angles

$$\sin A/2=\sqrt{\left(\frac{1-\cos A}{2}\right)}=\frac{\surd(1+\sin A)}{2}-\frac{\surd(1-\sin A)}{2}$$

$$\cos A/2=\sqrt{\left(\frac{1+\cos A}{2}\right)}=\frac{\surd(1+\sin A)}{2}+\frac{\surd(1-\sin A)}{2}$$

$$\tan A/2=\frac{1-\cos A}{\sin A}=\frac{\sin A}{1+\cos A}=\sqrt{\left(\frac{1-\cos A}{1+\cos A}\right)}$$

1.2.11 Relations between sides and angles of a triangle (Figures 1.10 and 1.11)

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a=b\cos C+c\cos B$$

$$c^2=a^2+b^2-2ab\cos C \qquad (1.18)$$

$$\sin A=\frac{c}{bc}\surd\{s(s-a)(s-b)(s-c)\} \qquad (1.19)$$

where $2s=a+b+c$

Area of triangle $\triangle=\tfrac{1}{2}ab\sin C=\surd\{s(s-a)(s-b)(s-c)\}$

$$\tan\frac{A}{2}=\sqrt{\left\{\frac{(s-b)(s-c)}{s(s-a)}\right\}}$$

$$\cos\frac{A}{2}=\sqrt{\left\{\frac{s(s-a)}{bc}\right\}}$$

$$\sin\frac{A}{2}=\sqrt{\left\{\frac{(s-b)(s-c)}{bc}\right\}}$$

$$\tan\frac{B-C}{2}=\frac{(b-c)}{(b+c)}\cot\frac{A}{2}$$

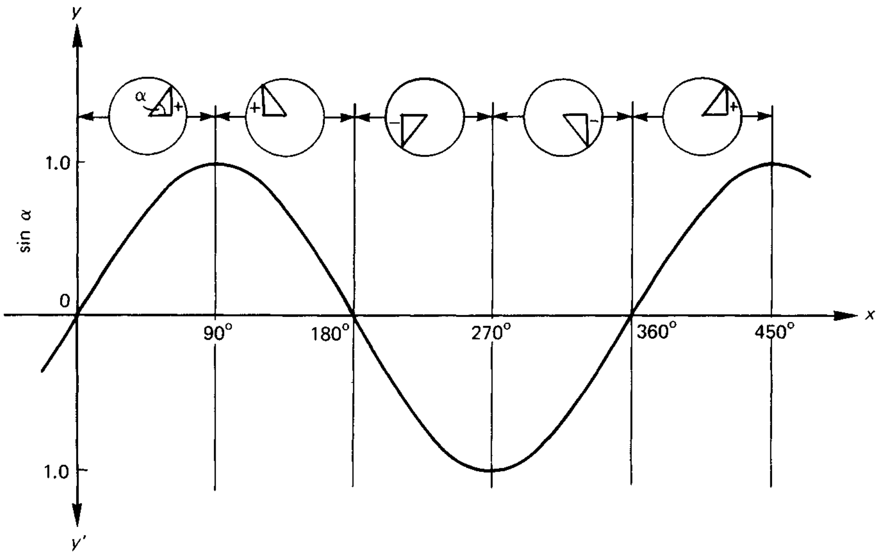

Figure 1.6 Sin α

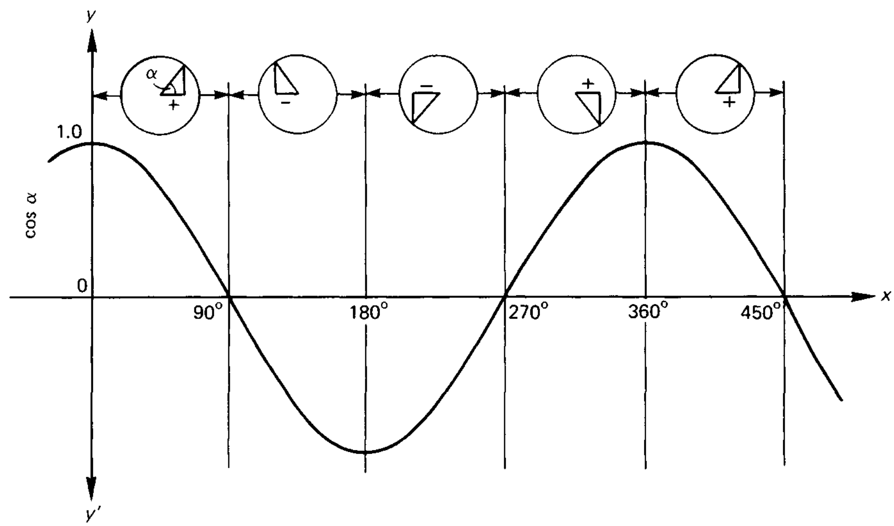

Figure 1.7 Cos α

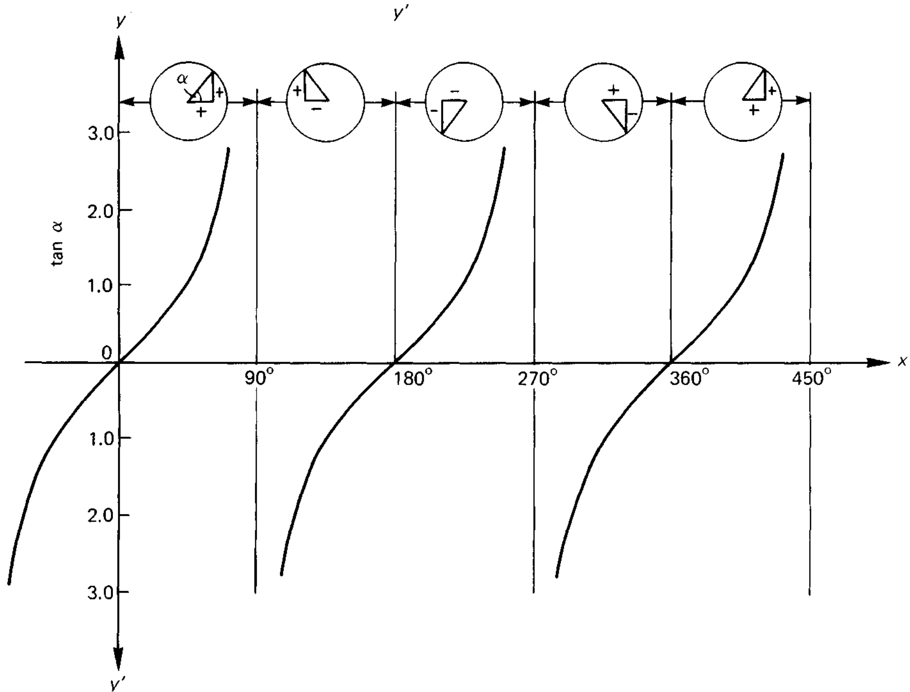

Figure 1.8 Tan α

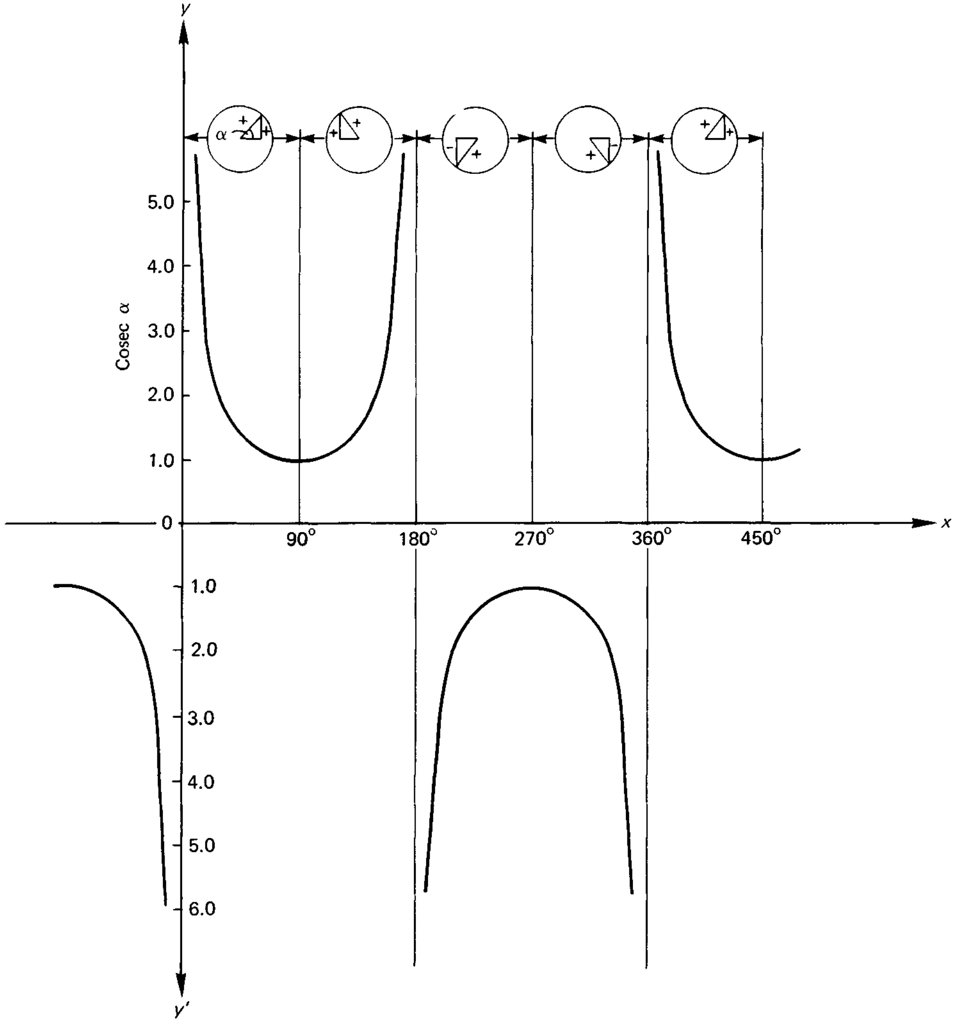

Figure 1.9 Cosec α

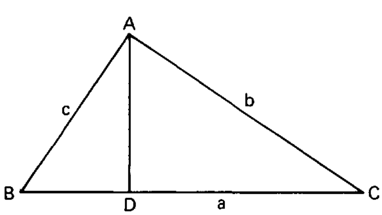

Figure 1.10

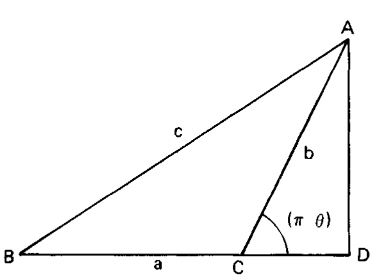

Figure 1.11

1.2.11.1 Any right angled triangle (Figure 1.12)

$a^2 + b^2 = c^2$; $A + B = 90°$; $\sin A = \cos B$; $\cot A = \tan B$ etc.

Area of $\triangle\ ABC = \frac{1}{2}ab = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$

1.2.11.2 Any equilateral triangle (Figure 1.13)

$a = b = c$; $A = B = C = 60° = \pi/3$

$h = \dfrac{b\sqrt{3}}{2}$; area $= \dfrac{b^2\sqrt{3}}{4}$

Circumscribed circle radius $R = \dfrac{b\sqrt{3}}{3}$

Inscribed circle, radius $r = \dfrac{b\sqrt{3}}{6}$

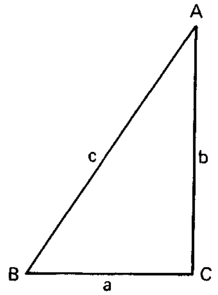

Figure 1.12

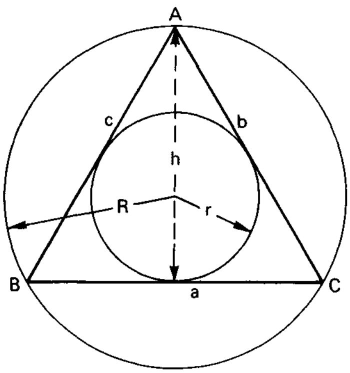

Figure 1.13

1.2.12 Solution of trigonometric equations

The method best suited to the solution of trigonometric equations is that described in the section on algebra which deals with the method of solving transcendental equations by means of graphs. The expression to be solved is arranged as two identities and two graphs drawn as shown in Figure 1.14. The points of intersection of the curves projected on to the coordinate axes give the values which will satisfy the trigonometric equation.

Example 1.1 Solve $\sin(x+30)=\frac{1}{3}\cos x$ for x between 0 and 2π.

Assigning values to x in Table 1.4 and calculating the corresponding values for $y=\sin(x+30)$ and $y=\frac{1}{3}\cos x$ gives the readings for plotting the curves in Figure 1.14.

Plotting the curves between $x=169°$ and $170°$ shows that the intersection is at $x=169.11°$ to the second approximation. Greater accuracy can be obtained by continuing the small range large scale plots of the type in Figure 1.15.

There is one further value of x between $x=300°$ and $360°$ which will satisfy the equation as can be seen on Figure 1.14.

1.2.13 General solutions of trigonometric equations

Due to the periodic nature of the trigonometric functions there is an infinite number of solutions to trigonometric equations. Having obtained the smallest positive solution, α, the general solution for θ is then given by:

if	$a=\sin^{-1}x$	then	$\theta=n\pi+(-1)^n a$
	$a=\cos^{-1}x$		$\theta=2n\pi\pm a$
	$a=\tan^{-1}x$		$\theta=n\pi+a$

where θ and a are measured in radians and n is any integer.

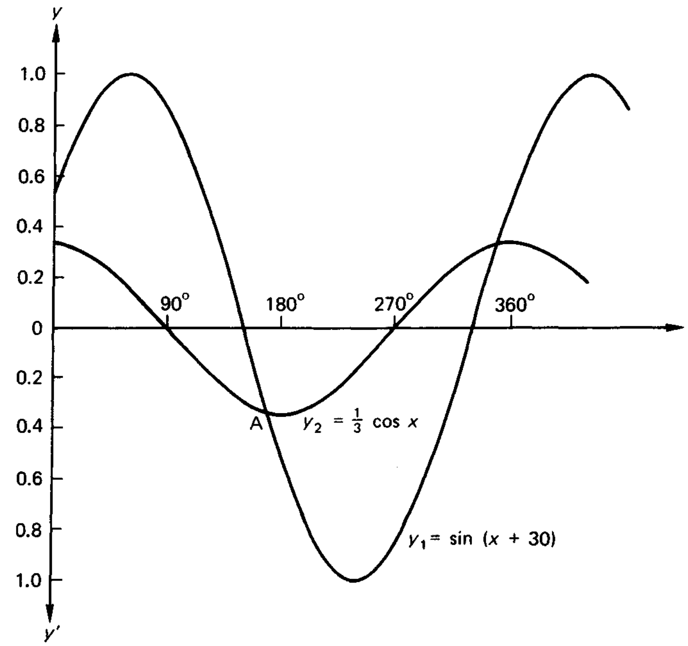

Figure 1.14 Solution of trigonometrical equations showing the intersection between $x=10$ and $x=\pi$ as $x=169°$ approximately

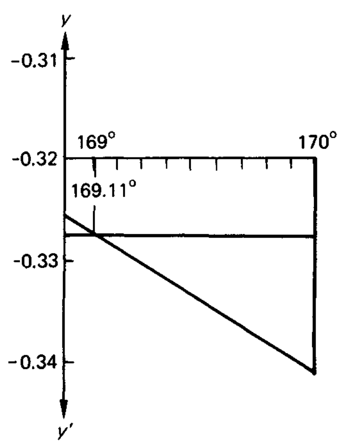

Figure 1.15 Enlargement at A of Figure 1.14

1.2.14 Inverse trigonometric functions

Inverse functions of trigonometric variables may be simply defined by the example: $y=\sin^{-1}\frac{1}{2}$ which is merely a symbolic way of stating that y is an angle whose sine is $\frac{1}{2}$, i.e. y is actually 30° or $\pi/6$ in radian measure but need not be quoted if written as $\sin^{-1}\frac{1}{2}$.

1.3 Spherical trigonometry

1.3.1 Definitions

Referring to Figure 1.16, representing a sphere of radius r:

Small circle The section of a sphere cut by a plane at a section not on the diameter of the sphere, e.g. EFGH.

Table 1.4

x	0	30	60	90	120	150	180
$y_1 = \sin(x+30)$	0.5	0.866	1.0	0.866	0.5	0	−0.5
$y_2 = \frac{1}{3}\cos x$	0.333	0.289	0.166 7	0	−0.166 7	−0.289	−0.333

x	210	240	270	300	330	360
$y_1 = \sin(x+30)$	−0.866	−1.0	−0.866	−0.5	0	+0.5
$y_2 = \frac{1}{3}\cos x$	−0.289	−0.166 7	0	0.166 7	0.289	+0.333

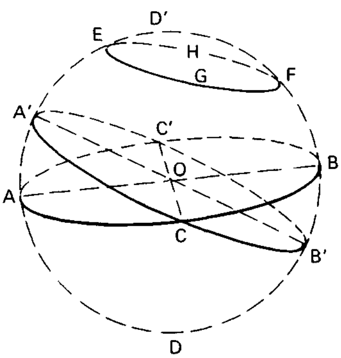

Figure 1.16 Sphere illustrating spherical trigonometry definitions

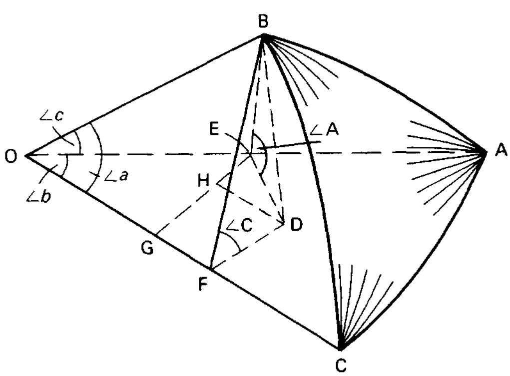

Figure 1.17 Spherical triangles

Great circle The section of a sphere cut by a plane through any diameter, e.g. ACBC′.

Poles Poles of any circular section of a sphere are the ends of a diameter at right angles to the section, e.g. D and D′ are the poles of the great circle ACBC′.

Lunes The surface areas of that part of the sphere between two great circles; there are two pairs of congruent areas, e.g. ACA′C′A; CBC′B′C and ACB′C′A; A′CBC′A′.

Area of lune If the angle between the planes of two great circles forming the lune is θ (radians), its surface area is equal to $2\theta r^2$.

Spherical triangle A curved surface included by the arcs of three great circles, e.g. CB′B is a spherical triangle formed by one edge BB′ on part of the great circle DB′BA the second edge B′C on great circle B′CA′C′ and edge CB on great circle ACBD′. The angles of a spherical triangle are equal to the angles between the planes of the great circles or, alternatively, the angles between the tangents to the great circles at their points of intersection. They are denoted by the letters C, B', B for the triangle CB′.

Area of spherical triangle $\text{CB'B} = (B' + B + C - \pi)r^2$.

Spherical excess Comparing a plane triangle with a spherical triangle the sum of the angles of the former is π and the spherical excess E of a spherical triangle is given by $E = B' + B + C - \pi$; hence, area of a spherical triangle can be expressed as $(E/4\pi) \times$ surface of sphere.

Spherical polygon A spherical polygon of n sides can be divided into $(n-2)$ spherical triangles by joining opposite angular points by the arcs of great circles.

$$\text{Area of spherical polygon} = [\text{sum of angles} - (n-2)\pi]r^2$$

$$= \frac{E}{4\pi} \times \text{surface of sphere.}$$

Note that $(n-2)\pi$ is the sum of the angles of a plane polygon of n sides.

1.3.2 Properties of spherical triangles

Let ABC, in Figure 1.17, be a spherical triangle; BD is a perpendicular from B on plane OAC and $\hat{\text{OED}}$, $\hat{\text{OFD}}$, $\hat{\text{OEB}}$, $\hat{\text{OFB}}$, $\hat{\text{OGE}}$, $\hat{\text{DHG}}$ are right angles; then $\hat{\text{BED}} = A$ and $\hat{\text{BFD}} = C$ are the angles between the planes OBA, OAC and OBC, OAC respectively. $\hat{\text{DEH}} = \hat{\text{COA}} = b$ also $\hat{\text{COB}} = a$, $\hat{\text{AOB}} = c$, and since $\text{OB} = \text{OA} = \text{OC} =$ radius r of sphere, $\text{OF} = r\cos a$, $\text{OE} = r\cos c$; then

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Also the sine formulae are:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

and the cotangent formulae are:

$$\sin a \cot c = \cos a \cos B + \sin B \cot C$$
$$\sin b \cot c = \cos b \cos A + \sin A \cot C$$
$$\sin b \cot a = \cos b \cos C + \sin C \cot A$$
$$\sin c \cot a = \cos c \cos B + \sin B \cot A$$
$$\sin c \cot b = \cos c \cos A + \sin A \cot B$$
$$\sin a \cot b = \cos a \cos C + \sin C \cot B$$

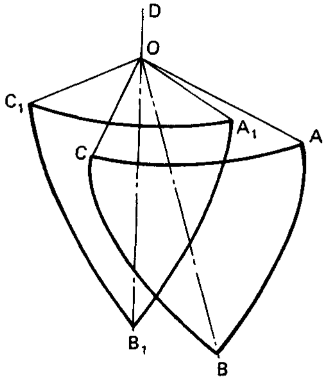

Figure 1.18 Polar triangles

In Figure 1.18, ABC, $A_1B_1C_1$ are two spherical triangles in which A_1, B_1, C_1 are the poles of the great circles BC, CA, AB respectively; then $A_1B_1C_1$ is termed the polar triangle of ABC and vice versa. Now OA_1, OD are perpendicular to the planes BOC and AOC respectively; hence $A_1\hat{O}D$ = angle between planes BOC and AOC $=C$. Let sides of triangle $A_1B_1C_1$ be denoted by $a_1b_1c_1$ then $c_1=A_1\hat{O}B_1=\pi-C$ also $a_1=\pi-A$ and $b_1=\pi-B$; $c=\pi-C_1$; $a=\pi-A_1$; $b=\pi-B_1$ and from these we get

$$\cos b=\frac{\cos B+\cos A\cos C}{\sin A\sin C} \tag{1.20}$$

$$\cos a=\frac{\cos A+\cos B\cos C}{\sin B\sin C} \tag{1.21}$$

$$\cos c=\frac{\cos C+\cos A\cos B}{\sin A\sin B} \tag{1.22}$$

1.3.2.1 Right-angled triangles

If one angle A of a spherical triangle ABC is 90° then $\cos a=\cos b\cos c=\cot B\cot C$

$$\cos B=\frac{\tan c}{\tan a};\quad \cos C=\frac{\tan b}{\tan a};\quad \sin B=\frac{\sin b}{\sin c};$$
$$\sin C=\frac{\sin c}{\sin a};\quad \tan B=\frac{\tan b}{\sin c};\quad \tan C=\frac{\tan c}{\sin b};$$
$$\cos B=\cos b\sin C;\quad \cos C=\cos c\sin B.$$

1.4 Hyperbolic trigonometry

The hyperbolic functions are related to a rectangular hyperbola in a manner similar to the relationship between the ordinary trigonometric functions and the circle. They are defined by the following exponential equivalents:

$$\sinh\theta=\frac{e^{\theta}-e^{-\theta}}{2} \qquad \operatorname{cosech}\theta=\frac{1}{\sinh\theta}$$

$$\cosh\theta=\frac{e^{\theta}+e^{-\theta}}{2} \qquad \operatorname{sech}\theta=\frac{1}{\cosh\theta}$$

$$\tanh\theta=\frac{\sinh\theta}{\cosh\theta} \qquad \coth\theta=\frac{1}{\tanh\theta}$$

1.4.1 Relation of hyperbolic to circular functions

$$\begin{aligned}
\sin\theta&=-i\sinh i\theta\\
\cos\theta&=\cosh i\theta\\
\tan\theta&=i\tanh i\theta\\
\operatorname{cosec}\theta&=i\operatorname{cosech} i\theta\\
\sec\theta&=\operatorname{sech} i\theta\\
\cot\theta&=i\coth i\theta\\
\sinh\theta&=-i\sin i\theta\\
\cosh\theta&=\cos i\theta\\
\tanh\theta&=-i\tan i\theta\\
\operatorname{cosech}\theta&=i\operatorname{cosec} i\theta\\
\operatorname{sech}\theta&=i\sec i\theta\\
\coth\theta&=i\cot i\theta
\end{aligned}$$

1.4.2 Properties of hyperbolic functions

$$\cosh^2\theta-\sinh^2\theta=1$$
$$\operatorname{sech}^2\theta=1-\tanh^2\theta$$
$$\sinh 2\theta=2\sinh\theta\cosh\theta$$
$$\cosh 2\theta=\cosh^2\theta+\sinh^2\theta$$

$$\operatorname{cosech}^2\theta=\coth^2\theta-1$$

$$\tanh 2\theta=\frac{2\tanh\theta}{1+\tanh^2\theta}$$

$$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$$
$$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$$

$$\tanh(x\pm y)=\frac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$$

$$\sinh x+\sinh y=2\sinh\tfrac{1}{2}(x+y)\cosh\tfrac{1}{2}(x-y)$$
$$\sinh x-\sinh y=2\cosh\tfrac{1}{2}(x+y)\sinh\tfrac{1}{2}(x-y)$$
$$\cosh x+\cosh y=2\cosh\tfrac{1}{2}(x+y)\cosh\tfrac{1}{2}(x-y)$$
$$\cosh x-\cosh y=2\sinh\tfrac{1}{2}(x+y)\sinh\tfrac{1}{2}(x-y)$$

1.4.3 Inverse hyperbolic functions

As with trigonometric functions, we define the inverse hyperbolic functions by $y=\sinh^{-1}x$ where $x=\sinh y$:

Therefore: $x=(e^{y}-e^{-y})/2$

Rearranging and adding x^2 to each side:

$$e^{2y}-2x.e^{y}+x^2=x^2+1$$

or: $e^{y}-x=\sqrt{(x^2+1)}$

and therefore: $y=\sinh^{-1}x=\log_e[x+\sqrt{(x^2+1)}]$ (1.23)

The other inverse functions may be treated similarly. We find:

$$\sinh^{-1}x=\log[x+\sqrt{(x^2+1)}];$$

$$\cosh^{-1}x=\log[x+\sqrt{(x^2-1)}];$$

$$\tanh^{-1}x=\tfrac{1}{2}\log\frac{1+x}{1-x};$$

$$\operatorname{cosech}^{-1} x = \log \frac{1+\surd(1+x^2)}{x}$$

$$\operatorname{sech}^{-1} x = \log \frac{1+\surd(1-x^2)}{x}$$

$$\coth^{-1} x = \tfrac{1}{2} \log \frac{x+1}{x-1}$$

The relationships with the corresponding inverse trigonometric functions are as follows:

$$\sinh^{-1} x = -i \sin^{-1} ix$$
$$\cosh^{-1} x = i \cos^{-1} x$$
$$\tanh^{-1} x = -i \tan^{-1} ix$$
$$\sin^{-1} x = -i \sinh^{-1} ix$$
$$\cos^{-1} x = -i \cosh^{-1} x$$
$$\tan^{-1} x = i \tanh^{-1} ix$$

1.5 Coordinate geometry

1.5.1 Straight-line equations

The equation of a straight line may be expressed as:

(1) $ax+by+c=0$ or $y=-\dfrac{a}{b}x-\dfrac{c}{b}=mx+n$ (1.24)

where a, b and c are constants and m is the slope of the line as shown in Figure 1.19.

(2) $\dfrac{x}{k}+\dfrac{y}{l}=1$ (1.25)

where k is the intercept on the x axis and l is the intercept on the y axis.

(3) $x \cos \alpha + y \sin \alpha = p$ (1.26)

where p = length of the perpendicular from the origin to the line and α the inclination of this perpendicular to Ox in Figure 1.20.

The length d of a perpendicular (see Figure 1.21) from any point $(x'y')$ to a straight line is given by $(ax'+by'+c)/\surd(a^2+b^2)$ if the straight line equation is as given in (1), or $(x' \cos \alpha + y' \sin \alpha - p)$ if the straight line equation is as given in (3).

The equation of a straight line through one given point $(x'y')$ is $y-y'=m(x-x')$.

The equation of a straight line through two given points (Figure 1.22) $(x_1y_1)(x_2y_2)$ is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \qquad (1.27)$$

The angle ψ between two straight lines (Figure 1.23) $y=m_1x+n_1$ and $y=m_2x+n_2$ is given by:

$$\tan \psi = \frac{m_1-m_2}{1+m_1m_2} \qquad (1.28)$$

For lines which are parallel $m_1=m_2$.
For lines at right angles $1+m_1m_2=0$.

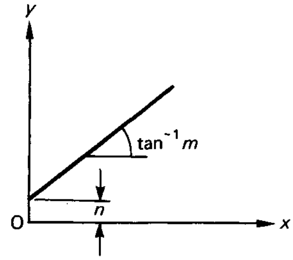

Figure 1.19 Straight-line equation $y=mx+n$

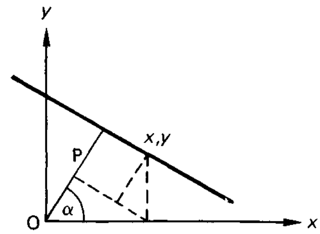

Figure 1.20 Straight-line equation $x \cos \alpha + y \sin \alpha = p$

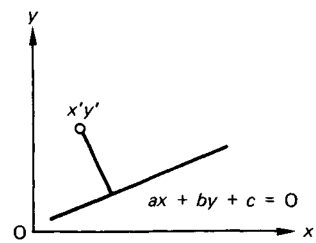

Figure 1.21 Perpendicular to straight line

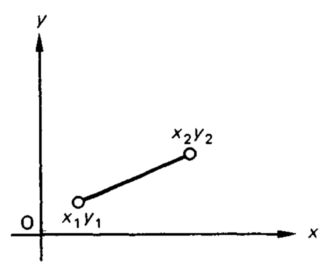

Figure 1.22 Straight line through two points

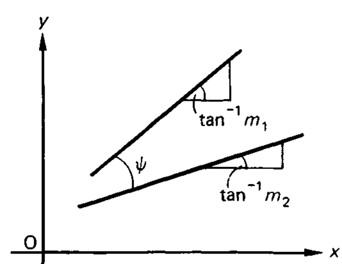

Figure 1.23 Angle ψ between two straight lines

1.5.2 Change of axes

Let the equation of the curve be $y=f(x)$ referred to coordinate axes Ox, Oy; then its equation relative to axes O′x', O′y' parallel to Ox, Oy with origin O′ at point (r, s) is given by $y+s=f(x+r)$ in which x and y refer to the new axes.

If the equation of a curve is given by $y=f(x)$ referred to coordinate axes Ox, Oy, then if these axes are each rotated an angle ψ anti-clockwise about O, the equation of the curve referred to the rotated axes is given by $x\sin\psi+y\cos\psi=f(x\cos\psi-y\sin\psi)$.

1.5.2.1 Tangent and normal to any curve $y=f(x)$

The tangent PT and the normal PN at any point x_1y_1 on the curve $y=f(x)$ in Figure 1.24 are given by the following equations:

Tangent: $y-y_1=\dfrac{dy}{dx}(x-x_1)$ where $\dfrac{dy}{dx}=m=$ the slope of the curve at P

Normal: $(y-y_1)\dfrac{dy}{dx}+(x-x_1)=0$

If ϕ be the angle which the tangent at P makes with the axis of x, then:

$$\tan\phi=\frac{dy}{dx};\ \cos\phi=\frac{dx}{ds};\ \sin\phi=\frac{dy}{ds}$$

where s is the distance measured along the curve.

1.5.2.2 Tangent and normal to any curve $f(xy)=0$

The function is implicit in this case so that partial differential coefficients are employed in the equations for the tangent and for the normal at $x_1\ y_1$.

Tangent: $(y-y_1)\dfrac{\partial f}{\partial y}+(x-x_1)\dfrac{\partial f}{\partial x}=0$

Normal: $\dfrac{(y-y_1)}{(\partial f/\partial y)}=\dfrac{(x-x_1)}{(\partial f/\partial x)}$

where $\dfrac{dy}{dx}=-\dfrac{\partial f}{\partial x}\Big/\dfrac{\partial f}{\partial y}$

1.5.2.3 Subtangent and subnormal to any curve $y=f(x)$

The subtangent is TQ and the subnormal is QN at any point $P(x_1y_1)$ on the curve $y=f(x)$ in Figure 1.24. Their lengths are given by:

Subtangent, $\mathrm{TQ}=y_1\Big/\left(\dfrac{dy}{dx}\right)_1$

and subnormal, $\mathrm{QN}=y_1\left(\dfrac{dy}{dx}\right)_1$

Example 1.2 Find the equation of the tangent and of the normal where $x=p$ on the curve $y=\cos\pi x/(2p)$

$\dfrac{dy}{dx}=-\dfrac{\pi}{2p}\sin\dfrac{\pi x}{2p}$ and when $x=p$, $\sin\dfrac{\pi x}{2p}=1$,

i.e. $\dfrac{dy}{dx}=-\dfrac{\pi}{2p}$ and $y=0$

Therefore:

the required equation of the tangent is $y=-\dfrac{\pi}{2p}(x-p)$

and the equation of the normal is $y=\dfrac{2p}{\pi}(x-p)$

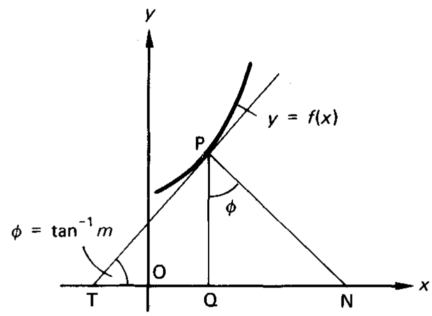

Figure 1.24 Tangent, normal, subtangent and subnormal to curve

1.5.3 Polar coordinates

The polar coordinates of any point P in a plane are given by r, θ where r is the length of the line joining P to the origin O and θ is the inclination of OP, the radius vector relative to the axis Ox (see Figure 1.25).

The relations between the rectangular coordinates x and y and the polar coordinates r and θ are:

$x=r\cos\theta$, $y=r\sin\theta$;

$r=\sqrt{(x^2+y^2)}$, $\theta=\tan^{-1}y/x$

If PT is a tangent to the curve at point P then:

$\tan\phi=rd\theta/dr$; $\cot\phi=(1/r)(dr/d\theta)$;

$\sin\phi=rd\theta/ds$ and $\cos\phi=dr/ds$

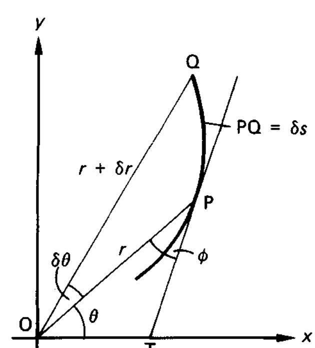

Figure 1.25 Polar coordinates

1.5.3.1 Polar subtangent and subnormal

In Figure 1.26 the polar subtangent is OR and the polar subnormal is OQ where QR is perpendicular to OP and their lengths are given by: polar subtangent $=r^2d\theta/dr$; polar subnormal $=dr/d\theta$.

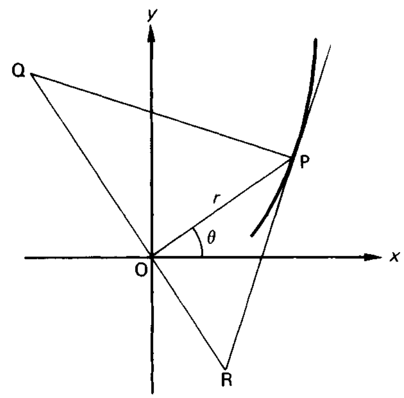

Figure 1.26 Polar subtangent and subnormal

1.5.3.2 Curvature

Let PQ in Figure 1.27 represent an elemental length δs of a given curve and PS, QT the tangents at the points P, Q then:

Curvature at P $= d\beta/ds$. For a circle centre at C, radius ρ, $ds = \rho\, d\beta$, i.e. curvature $= 1/\rho$.

Therefore: $$\rho = \frac{ds}{d\beta} = \text{radius of curvature} \quad (1.29)$$

Putting $\beta = \tan^{-1}\left(\dfrac{dy}{dx}\right)$ and differentiating:

$$\text{Curvature} = \frac{d\beta}{ds} = \frac{1}{\rho} = \frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\text{Radius of curvature } \rho = \frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$$

Where dy/dx is small (as in the bending of beams), the radius of curvature is given by:

$$\rho = \frac{1}{d^2y/dx^2} \quad (1.30)$$

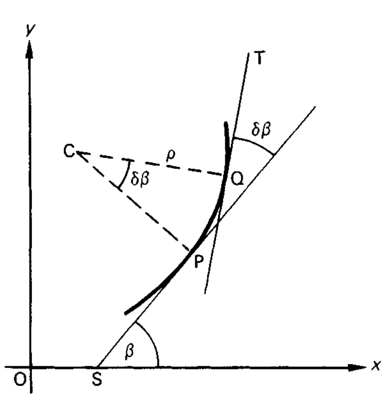

Figure 1.27 Curvature

Example 1.3 Find the radius of curvature at any point at the curve $y = a\cos x/a$:

$$\frac{dy}{dx} = \sinh\frac{x}{a}$$

Therefore:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \left[1+\sinh^2\frac{x}{a}\right]^{3/2}$$

$$= \left(\cosh^2\frac{x}{a}\right)^{3/2} = \cosh^3\frac{x}{a}$$

$$\frac{d^2y}{dx^2} = \frac{1}{a}\cosh\frac{x}{a}$$

Therefore:

$$\rho = \frac{a\cosh^3\dfrac{x}{a}}{\cosh\dfrac{x}{a}} = a\cosh^2\frac{x}{a} = \frac{y^2}{a}$$

1.5.4 Lengths of curves

1.5.4.1 General theory

From Figure 1.28:

$$ds^2 = dx^2 + dy^2$$

Hence:

$$ds = \sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}\,dx = \sqrt{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}}\,dy$$

Therefore: $$s = \int_a^b \sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}\,dx$$

or: $$s = \int_c^d \sqrt{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}}\,dy$$

For the evaluation of s for any given continuous function, use the first formula if x is single-valued, i.e. if one value of x corresponds to one point only in the function, e.g. Figure 1.29. If more than one point on the curve corresponds to one value of x, the second formula for a curve of the form shown in Figure 1.30, should be used.

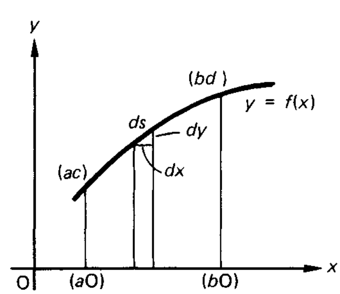

Figure 1.28

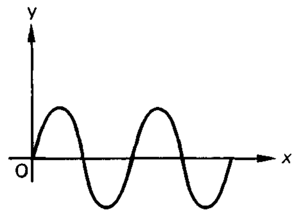

Figure 1.29

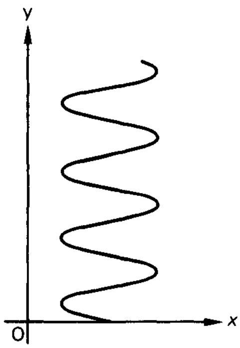

Figure 1.30

For polar coordinates, from Figure 1.31

$$ds=\sqrt{\{(\rho d\theta)^2+(d\rho)^2\}}=\sqrt{\left\{\rho^2+\left(\frac{d\rho}{d\theta}\right)^2\right\}}\,d\theta$$

$$s=\int_{\theta_1}^{\theta_2}\sqrt{\left\{\rho^2+\left(\frac{d\rho}{d\theta}\right)^2\right\}}\,d\theta \qquad (1.32)$$

or:

$$s=\int_{\rho_1}^{\rho_2}\sqrt{\left\{1+\left(\rho\frac{d\theta}{d\theta\rho}\right)^2\right\}}\,d\rho \qquad (1.33)$$

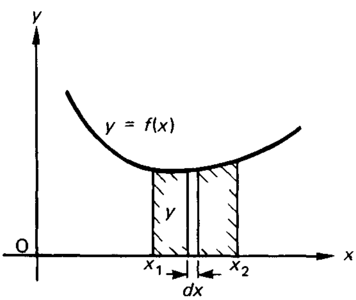

Figure 1.31

1.5.5 Plane areas by integration

See Figures 1.32 and 1.33.

1.5.5.1 General theory

From Figure 1.32, $A=\int_{x_1}^{x_2} y\,dx=\int_{x_1}^{x_2} f(x)\,dx$

1.5.5.2 Polar coordinates

From Figure 1.33, $dA=\frac{1}{2}\rho^2\,d\theta$

Therefore: $$A=\tfrac{1}{2}\int\rho^2 d\theta=\tfrac{1}{2}\int\{f(\theta)\}^2 d\theta \qquad (1.34)$$

(*Note*. For curve cutting x axis, equate $f(x)$ to zero, find values of x for $y=0$ and integrate between these values for the area cut off by the x axis.)

When the area lies above and below the x axis integrate the positive and negative areas separately and add algebraically.

Where the area does not extend to the x axis in the case of cartesian coordinates, or to the origin in the case of polar coordinates, then double integration must be used.

Thus: $$A=\iint dx\,.\,dy\,.\iint\rho\,.\,d\rho\,.\,d\theta \qquad (1.35)$$

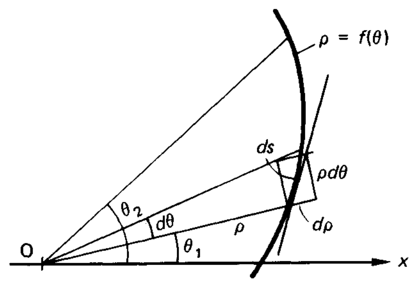

Figure 1.32

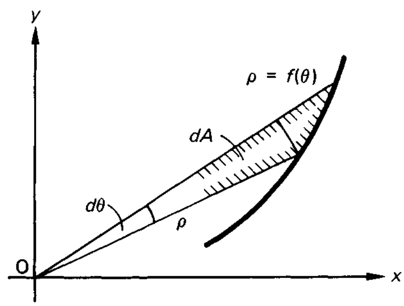

Figure 1.33

1.5.6 Plane area by approximate methods

See Figure 1.34.

1 *Trapezoidal rule*:

$$A=\frac{h}{2}\{y_0+2(y_1+y_2+\;\ldots\;+y_{n-1})+y_n\} \qquad (1.36)$$

(2) *Durand's rule*:

$$A=h(0.4y_0+1.1y_1+y_3+\;\ldots\;+y_{n-2}+1.1y_{n-1}+0.4y_n) \qquad (1.37)$$

(3) *Simpson's rule (*n *made even)*

$$A=\frac{h}{3}(y_0+4y_1+2y_2+4y_3+2y_4+\;\ldots\;+\;2y_{n-2}+4y_{n-1}+y_n) \qquad (1.38)$$

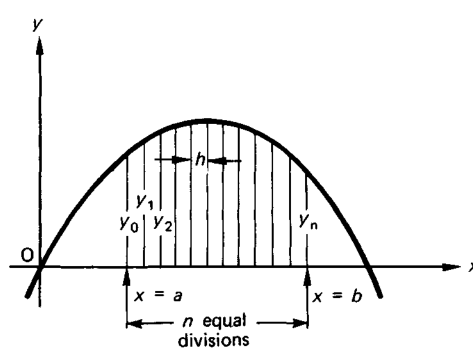

Figure 1.34

Of these, Simpson's is the most accurate. The accuracy is increased in all cases by increasing the number of divisions. Areas can often be determined more rapidly by plotting on squared paper and 'counting the squares' or by the use of a planimeter.

1.5.7 Conic sections

Conic sections refer to the various profiles of sections cut from a pair of cones vertex to vertex when intersected by a plane. Figure 1.35 shows a pair of cones generated by two intersecting straight lines AB, CD about the bisector EF of the angle between the lines.

Two straight lines. A section through the axis EF.

Circle. A section b–b parallel to the base of a cone.

Ellipse. A section c–c not parallel to the base of a cone and intersecting one cone only.

Parabola. A section d–d parallel to the side of a cone.

Hyperbola. A section e–e inclined to the side of a cone and intersecting both cones.

1.5.8 Properties of conic sections

A conic section is defined as the locus of a point P which moves so that its distance from a fixed point, the focus, bears a constant ratio, the eccentricity, to its perpendicular distance from a fixed straight line, the directrix.

Referring to Figure 1.36: the vertex of the curve is at V, the focus of the curve is at F, the directrix of the curve is the line DD parallel to yy'; the latus rectum is the line LR through the focus parallel to DD, $FL = FR = l$; the eccentricity of the curve is the ratio $FP/PQ = FV/VS = e$.

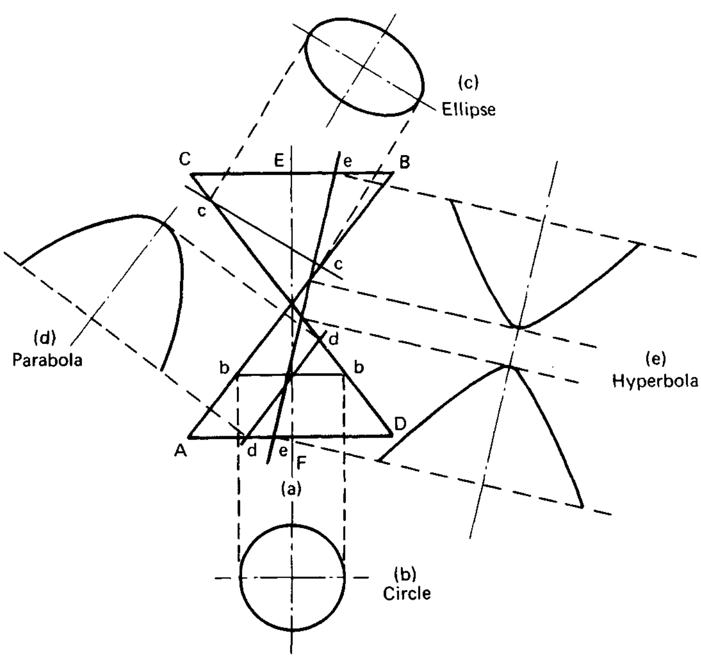

Figure 1.35 Circular cones generated by two intersecting straight lines

Then the curve is a parabola if $e = 1$, an ellipse if $e < 1$; and a hyperbola if $e > 1$. A circle is a particular case of an ellipse in which $e = 0$.

The polar equation of a conic is given by $l = \boldsymbol{\rho}(1 - e\cos\theta)$ where $\boldsymbol{\rho}$ is the radius vector of any point P on the curve, θ the angle the vector makes with VX and l the semi latus rectum.

Parabola ($e = 1$) (see Figure 1.36).

1.5.8.1 Equations

With origin at V and putting $a = VS = VF$ then for P at (x, y): $(x - a)^2 + y^2 = (x + a)^2$, i.e. $y^2 = 4ax$.

1.5.8.2 Tangents

Let PT be a tangent at any point P $(x_1\ y_1)$ then the equation of PT is given by:

$$y - y_1 = m(x - x_1) = (2a/y_1)(x - x_1)$$

or $yy_1 = 2a(x + x_1)$

since $d/dx(y^2) = 2y\ dy/dx = 4a$,

i.e. $m = dy/dx = 2a/v_1$ at P $(x_1\ y_1)$.

Alternatively, if any straight line $y = mx + c$ meets the parabola $y^2 = 4ax$ then $(mx + c)^2 = 4ax$ at the points of intersection and this expression will satisfy the condition for tangency if the roots of $m^2x^2 + 2(mc - 2a)x + c^2 = 0$ are equal, i.e. if $4(mc - 2a)^2 = 4m^2c^2$ or $c = a/m$ so that the equation for the tangent may be expressed as $y = mx + a/m$ for all values of m where $m = dy/dx$, and tangency occurs at the point $(a/m^2, 2a/m)$.

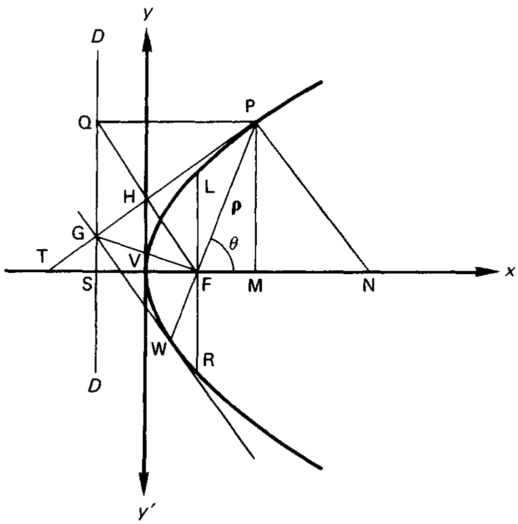

Figure 1.36 Properties of a conic section

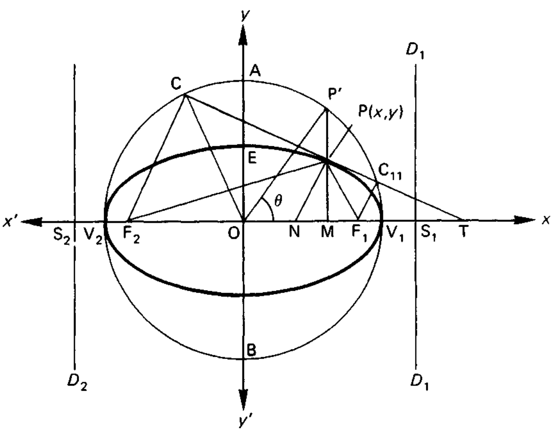

Figure 1.37 Ellipse in cartesian coordinates

1.5.8.3 Normal

Let PN be the normal at any point P $(x_1\ y_1)$; then the equation of PN is given by:

$$y-y_1=-(y_1/2a)(x-x_1) \tag{1.39}$$

1.5.8.4 General properties

Tangents:

(1) The tangent PT bisects $F\hat{P}Q$.
(2) The tangents PG, GW where PW is a focal chord intersect at G on DD.
(3) The tangent PT intersects the axis of the parabola at a point T where TV = VM; TF = SM = PF.
(4) The angles GFP, PHQ and PGW are right angles.

Normals: any normal PN intersects VX at N where FT = FN.

Subnormals: the subnormal MN is a constant length, i.e. MN = FS = 2*a*.

1.5.8.5 Ellipse $(e<1)$

Referring to Figure 1.37, F_1, F_2 and the foci; D_1D_1, D_2D_2 the directrices.

$$e=\frac{F_1V_1}{S_1V_1}=\frac{F_1V_2}{S_1V_2}=\frac{F_2V_1}{S_2V_1}=\frac{F_1P}{MS_1}=\frac{F_2P}{MS_2}=\frac{OF_1}{OV_1}=\frac{OF_2}{OV_2}=\frac{F_1F_2}{V_1V_2}$$

Let OV_1 the semi-major axis $=a$ and OE the semi-minor axis $=b$,

then $OF_1=OF_2=ae$ and $OS_1=OS_2=\frac{a}{e}$

also $F_1P=a-ex$; $F_2P=a+ex$ $\therefore F_1P+F_2P=2a$

$F_1E=eOS_1=a$; $(OE)^2=b^2=(F_1E)^2-(OF_1)^2=a^2(1-e^2)$,

or $e^2=1-\frac{b^2}{a^2}$

Hence, as OM $=x$ and PM $=y$ we have the following.

1.5.8.5 Equation of ellipses

$y^2=a^2(1-e^2)-x^2(1-e^2)$

or $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in cartesian coordinates.

Substituting $\rho\cos a$ for x and $\rho\sin a$ for y (see Figure 1.38) in the above equation for an ellipse we have for the polar equation for an ellipse:

$$\frac{1}{\rho^2}=\frac{\cos^2 a}{a^2}+\frac{\sin^2 a}{b^2} \tag{1.40}$$

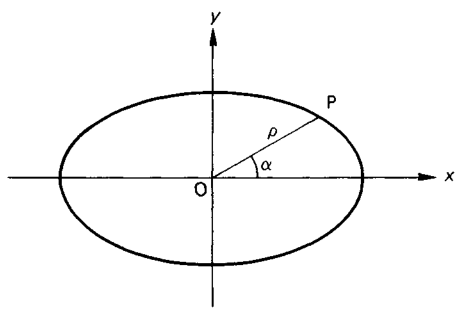

Figure 1.38 Ellipse in polar coordinates

1.5.8.6 Tangent

At any point $P\ (x_1\ y_1)$ on the ellipse
Let $f(xy)=0$ represent the curve, then:

$$\frac{dy}{dx}=-\frac{\partial f}{\partial x}\ \frac{\partial f}{\partial y} \tag{1.41}$$

Therefore $\partial f/\partial x=2x/a^2$ and $\partial f/\partial y=2y/b^2$ so that dy/dx at point $(x_1\ y_1)$ is given by $-b^2x_1/a^2y_1=m$. Substituting this value of m in the equation of a straight line $(y-y_1)=m(x-x_1)$ we have the equation of tangent PT: $xx_1/a^2+yy_1/b^2=1$.

Alternatively, the straight line $y=mx+c$ is a tangent to the ellipse $x^2/a^2+y^2/b^2=1$ when the roots of $x^2/a^2+(mx+c)^2/b^2-1=0$ are equal, i.e. when $c^2=a^2m^2+b^2$. Substituting we have for the equation of a tangent at any point P: $y=mx+\sqrt{(a^2m^2+b^2)}$.

The equation to the tangent may also be written in the form:

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1. \tag{1.42}$$

The coordinates of the point of contact are $(a\cos\theta, b\sin\theta)$, θ being known as the eccentric angle (see Figure 1.37).

1.5.8.7 Normal

Substituting the value of m above in the general equation for the normal PN to a curve at point P $(x_1\ y_1)$ given by: $(y-y_1)m+(x-x_1)=0$ we have as the equation for the normal $(y-y_1)b^2/y_1=(x-x_1)a^2/x_1$.

1.5.8.8 General properties

(1) The circle AV_2BV_1 is termed the auxiliary circle (Figure 1.37).
(2) $OM\times OT=a^2$.
(3) $F_2N=eF_2P$.
(4) $F_1N=eF_1P$.
(5) PN bisects $\angle F_1PF_2$.
(6) The perpendiculars from F_1, F_2 to any tangent meet the tangent on the auxiliary circle.

1.5.8.9 Circle (c=0)

The circle may be regarded as a particular case of the ellipse (see above). The equation of a circle of radius a with centre at the origin is $x^2+y^2=a^2$ or, in polar coordinates, $\rho=a$.

The equation of the tangent at the point $(x_1\ y_1)$ is $xx_1+yy_1=a^2$, or, $y=mx+a\surd(1+m^2)$. The equation of the normal is $xy_1-yx_1=0$.

1.5.8.10 Hyperbola (e>1)

This is shown in Figure 1.39 where F_1 F_2 are the foci, D_1D_1 and D_2D_2 the directrices and:

$$e=\frac{F_1V_1}{S_1V_1}=\frac{F_1P}{MS_1}=\frac{F_2P}{MS_2}=\frac{F_1V_2}{S_1V_2}=\frac{F_2V_1}{S_2V_1}=\frac{V_1V_2}{S_1S_2}=\frac{OV_1}{OS_1}=\frac{OV_2}{OS_2}$$

where O is the origin of the axes x and y.

Putting $OV_1=OV_2=a$ then $OF_1=OF_2=ea$ and $OS_1=OS_2=a/e$; also $F_1P=ex-a$ and $F_2F=ex+a$. Now $(F_1P)^2=(PM)^2+(F_1M)^2$, so $(ex-a)^2=y^2+(x-ae)^2$ which becomes $y^2=(e^2-1)x^2-(e^2-1)a^2$. Putting $(e^2-1)a^2=b^2$ then $y^2=(b^2/a^2)x^2-b^2$; therefore the equation of the hyperbola is given by $x^2/a^2-y^2/b^2=1$ in cartesian coordinates, or:

$$\frac{1}{\rho^2}=\frac{\cos^2\theta}{a^2}-\frac{\sin^2\theta}{b^2} \tag{1.43}$$

in polar coordinates.

Rearranging we have $y=b\surd(x^2/a^2-1)$, i.e. y is imaginary when $x^2<a^2$ and $y=0$ for $x=\pm a$. y is real when $x>a$ and there are two values for y of opposite sign.

1.5.8.11 Conjugate axis

The conjugate axis lies on yy' and is given by CC' where $OC=OC'=\pm b$.

1.5.8.12 Tangents

Let the straight line $y=mx+c$ meet the hyperbola $x^2/a^2-y^2/b^2=1$; then $x^2/a^2-(mx+c)^2/b^2-1=0$ will give the points of intersection. The condition for tangency is that the roots of this equation are equal, i.e. $c=\surd(a^2m^2-b^2)$ and the equation of the tangent is given by $y=mx+\surd(a^2m^2-b^2)$ at any point. Alternatively, the tangent to the hyperbola at $(x_1\ y_1)$ is given by $xx_1/a^2-yy_1/b^2=1$.

1.5.8.13 Normal

The equation for the normal at any point $(x_1\ y_1)$ on the curve is given by:

$$(y-y_1)b^2/y_1+(x-x_1)a^2/x_1=0. \tag{1.44}$$

1.5.8.14 Asymptotes

The tangent to the hyperbola becomes an asymptote when the roots of the equation $x^2/a^2-(mx+c)^2/b^2-1=0$ are both infinite, i.e. when $b^2-a^2m^2=0$ and $a^2mc=0$. Therefore: $m=\pm b/a$ and $c=0$. Substituting for m in $y=mx+c$ we have as the equation for an asymptote $y=\pm(b/a)x$. The combined equation for both asymptotes is given by:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0 \tag{1.45}$$

The equation of the hyperbola referred to its asymptotes as oblique axes is:

$$X.Y=\frac{a^2+b^2}{4} \tag{1.46}$$

1.5.8.15 General properties

(1) $F_2P-F_1P=2a$.
(2) The product of the perpendiculars from any point on a hyperbola to its asymptotes is constant and equal to $a^2b^2/(a^2+b^2)$.

1.5.8.16 Rectangular hyperbola

When the transverse axis V_1V (Figure 1:39) is equal to the conjugate axis CC' the hyperbola is a rectangular hyperbola, i.e. $a=b$ and the equation for the curve is given by $x^2-y^2=a^2$.

The equation for the asymptotes then becomes $y=\pm x$ which represents two straight lines at right angles to each other. The equation of the rectangular hyperbola referred to its asymptotes as axes of coordinates is given by $xy=$constant.

1.5.8.17 General equation of a conic section:
The general equation of a conic section has the form:

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \tag{1.47}$$

Let $D=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ and $d=\begin{vmatrix} a & b \\ h & b \end{vmatrix}$

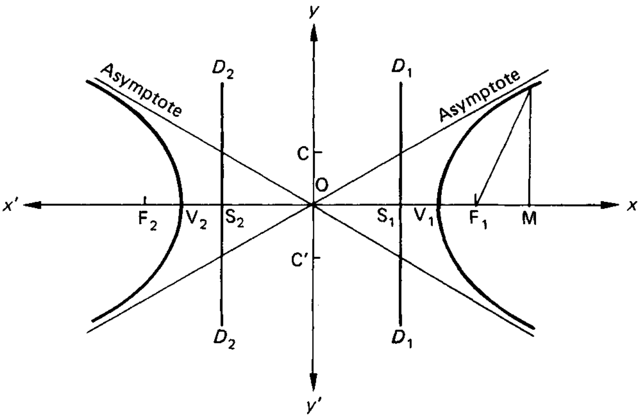

Figure 1.39 Hyperbola

Then, the general equation represents:

(1) An ellipse if $d>0$;
(2) A parabola if $d=0$;
(3) A hyperbola if $d<0$;
(4) A circle if $a=b$ and $h=0$;
(5) A rectangular hyperbola if $a+b=0$;
(6) Two straight lines (real or imaginary) if $D=0$;
(7) Two parallel straight lines if $d=0$ and $D=0$.

The centre of the conic $(x_0 y_0)$ is determined by the equations: $ax_0 - hy_0 + g = 0$, $hx_0 + by_0 + f = 0$.

1.6 Three-dimensional analytical geometry

1.6.1 Sign convention

1.6.1.1 Cartesian coordinates

This is shown in Figure 1.40, there being eight compartments formed by the right-angled intersection of three planes. The signs of x, y, z follow the convention that these are positive when measured in the directions Ox, Oy, Oz of the coordinate axes and negative when measured in the directions Ox', Oy', Oz' respectively.

1.6.1.2 Polar coordinates

The location of any point P in space (see Figure 1.41) is fully located by the radius vector $\boldsymbol{\rho}$ and the two angles θ and ϕ thus $(\boldsymbol{\rho}\theta\phi)$. From Figure 1.41:

$$\mathrm{OP} = \boldsymbol{\rho} = \sqrt{[(\mathrm{OD})^2 + (\mathrm{OB})^2 + (\mathrm{OC})^2]} = \sqrt{(x^2 + y^2 + z^2)}$$

and $\quad x = \boldsymbol{\rho} \sin\theta \cos\phi;\ y = \boldsymbol{\rho} \sin\theta \sin\phi;\ z = \boldsymbol{\rho} \cos\phi.$

1.6.1.3 Cylindrical coordinates

In this system the point P (Figure 1.41) is located by its perpendicular distance, z, from the x–y plane and the polar coordinates of the foot, A, of that perpendicular in the x–y plane. P is the point r, ϕ, z where OA $= r$.

1.6.1.4 Direction-cosines of a straight line

If the direction of the line OP in Figure 1.42 is determined by α,

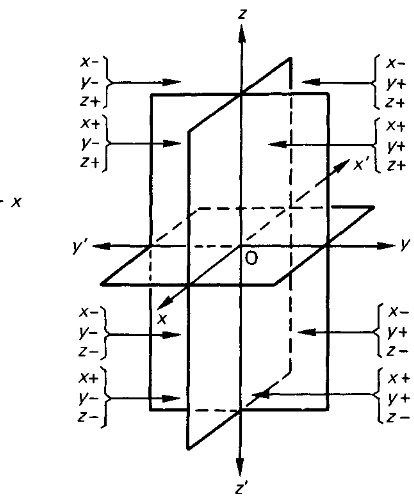

Figure 1.40 Sign conventions in analytical solid geometry

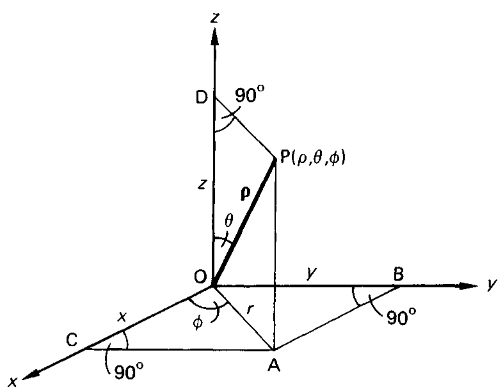

Figure 1.41 Polar coordinates in three dimensions

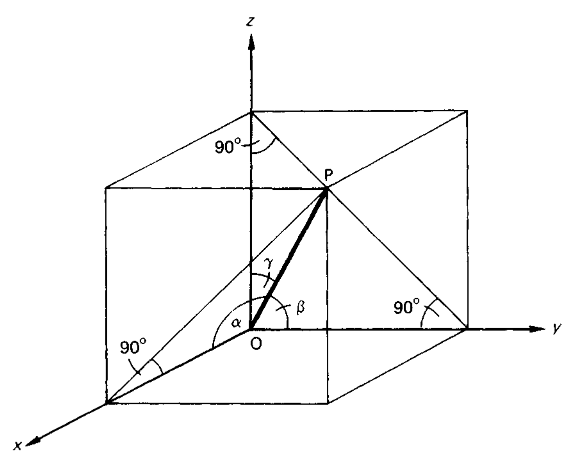

Figure 1.42 Direction-cosines

β, γ then the projections of a unit length of OP on to the axes Ox, Oy, Oz are given by $\cos\alpha$, $\cos\beta$, $\cos\gamma$ respectively, termed direction-cosines. Let $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ and CP $= \rho$; then $\rho^2(l^2 + m^2 + n^2) = x^2 + y^2 + z^2 = \rho^2$, i.e. $l^2 + m^2 + n^2 = 1$.

Also:

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = (1 - l^2) + (1 - m^2) + (1 - n^2) = 2$$

Again if:

$$l:m:n = s:t:u \text{ then } \frac{l^2}{s^2} = \frac{m^2}{t^2} = \frac{n^2}{u^2} = \frac{l^2+m^2+n^2}{s^2+t^2+u^2} = \frac{1}{s^2+t^2+u^2}$$

i.e.:

$$l = \frac{s}{\sqrt{(s^2+t^2+u^2)}};$$

$$m = \frac{t}{\sqrt{(s^2+t^2+u^2)}};$$

$$n = \frac{u}{\sqrt{(s^2+t^2+u^2)}}$$

1.6.1.5 General equations

The expression $F(xyz) = 0$ represents a surface of some kind and if we put $x = 0$ the resulting equation is for a curve in the y–z plane; similarly, for $y = 0$ the curve is in the x–z plane, etc. In general, any two simultaneous equations, $F(xyz) = 0$, $F'(xyz) = 0$ represent a line (either straight or curved) being the intersection of two surfaces. Any three such simultaneous equations represent a point (or several points).

1.6.2 Equation of a plane

The general equation of a plane is given by the expression $ax + by + cz + d = 0$ ($abcd$ being constants). By putting $y = 0$, $z = 0$ then $x = -d/a = a'$ which is the intercept of the plane on the x axis at a distance a' from the origin. Similarly, the intercepts on the y and z axes are b' and c'. Hence $a = -d/a'$; $b = -d/b'$; $c = -d/c'$ and substituting these values in the general equation for the plane we have the intercept equation for a plane as $x/a' + y/b' + z/c' = 1$.

In Figure 1.43 let P be any point on the plane ABC and let OQ of length p be at 90° to the plane ABC; then if l, m, n are the direction cosines of OQ we have $p = lx + my + nz$, which is the perpendicular form of the equation to a plane. The various forms of the equation to a plane are interchangeable since:

$$p = -\frac{d}{\sqrt{(a^2+b^2+c^2)}} = la' = mb' = nc'$$

and

$$\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = \frac{1}{p^2} \qquad (1.48)$$

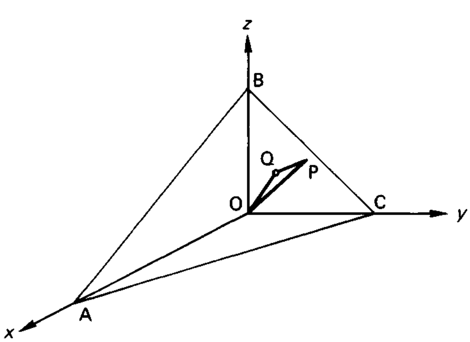

Figure 1.43 Equation of a plane

1.6.3 Distance between two points in space

Let the two points be P($x_1y_1z_1$); Q($x_2y_2z_2$). Assume origin shifted to P and axes kept parallel to original axes, then coordinates of Q relative to P are $(x_2 - x_1)$, $(y_2 - y_1)$, $(z_2 - z_1)$ and the length PQ = r, i.e. $r = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$ and the locus of Q is a sphere if r is constant.

1.6.4 Equations of a straight line

Using direction cosines for PQ, $l = (x_2 - x_1)/r$; $m = (y_2 - y_1)/r$; $n = (z_2 - z_1)/r$. If Q is taken as any point then the symmetrical equation of a straight line is given by $r = (x - x_l)/l = (y - y_1)/m = (z - z_1)/n$ and the coordinates of any point on the line are given by $x = x_1 + rl$; $y = y_1 + rm$; $z = z_1 + rn$. For a line through the origin this becomes:

$$r = \frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

The equation of the straight line through the points $(x_1y_1z_1)$ and $(x_2y_2z_2)$ is:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

1.6.4.1 Angle between two lines of known direction cosines

Let PA, QB be any two lines in space (Figure 1.44) and let P′O, Q′O be parallel to PA, QB respectively and having direction cosines $l_1m_1n_1$; $l_2m_2n_2$ respectively then $\cos a = l_1l_2 + m_1m_2 + n_1n_2$ where $a = \mathrm{P'\hat{O}Q'}$

1.6.4.2 The angle between two planes

Let the equations of the planes be:

$$a_1x + b_1y + c_1z + d_1 = 0$$

and: $$a_2x + b_2y + c_2z + d_2 = 0$$

then the direction-cosines of the normals to these planes are:

$$\frac{a_1}{\sqrt{(a_1^2+b_1^2+c_1^2)}}; \frac{b_1}{\sqrt{(a_1^2+b_1^2+c_1^2)}}; \frac{c_1}{\sqrt{(a_1^2+b_1^2+c_1^2)}}$$

and: $$\frac{a_2}{\sqrt{(a_2^2+b_2^2+c_2^2)}}; \frac{b_2}{\sqrt{(a_2^2+b_2^2+c_2^2)}}; \frac{c_2}{\sqrt{(a_2^2+b_2^2+c_2^2)}}$$

If θ is the angle between the planes, this is equal to the angle between the normals to these planes, i.e.:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{[(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)]}} \qquad (1.49)$$

The planes are perpendicular to each other if $a_1a_2 + b_1b_2 + c_1c_2 = 0$. They are parallel if $a_1/a_2 = b_1/b_2 = c_1/c_2$.

1.6.4.3 The angle between a plane and a straight line

The angle θ between the plane $l_1x + m_1y + n_1z = p$ and the line $(x - x_1)/l_2 = (y - y_1)/m_2 = (z - z_1)/n_2$ is given by $\sin\theta = (l_1l_2 + m_1m_2 + n_1n_2)$.

1.6.4.4 Length of the perpendicular from a point $x_1y_1z_1$ to a plane

(1) Where the equation of the plane is the perpendicular form $lx+my+nz=p$ then the equation of a plane containing the point $(x_1y_1z_1)$ and parallel to the given plane is given by $lx+my+nz=p'$ where p and p' are the lengths of perpendiculars from the origin. Therefore required length of perpendicular is $p'-p=lx_1+my_1+nz_1-p$, since the point $(x_1y_1z_1)$ lies on the second plane.
(2) Where the equation of the plane takes the general form $ax+by+cz+d=0$ the length of perpendicular from point $x_1y_1z_1$ is given by:

$$\frac{ax_1+by_1+cz_1+d}{\sqrt{(a^2+b^2+c^2)}}$$

In the above the equation of the perpendicular is given by:

$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

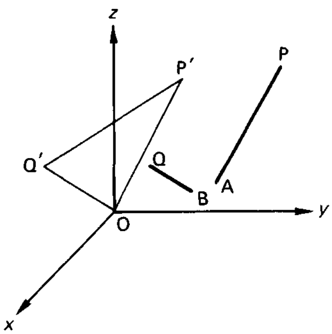

Figure 1.44 Angle between two lines of known direction-cosines

1.7 Calculus

The calculus deals with quantities which vary and with the rate at which this variation takes place.

Variables may be denoted by u, v, w, x, y, z and increments of these variables are denoted by $du, dv \ldots dz$. A simple example concerns the slope of a curve. Suppose that the curve is defined by some function:

$$y=f(x) \tag{1.50}$$

The slope at the point $x=x_1$ may be approximated to as follows. Let x_2 be close in value to x_1; then, provided the curve $f(x)$ is well behaved in the region of x_1, the line joining $f(x_1)$ to $f(x_2)$ is an approximation to the tangent to the curve at $x=x_1$. As x_2 is moved closer to x_1 the approximation becomes better and better, until, in the limit, when x_2 reaches x_1 the tangent (instead of the secant) is obtained and thereby the slope of the curve $y=f(x)$ is found at the point $x=x_1$. This process is known as 'differentiation'.

1.7.1 Differentiation

This process is used to find the value of dy/dx.
For combinations of functions u, and v of x:

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx} \tag{1.51}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v(du/dx)-u(dv/dx)}{v^2} \tag{1.52}$$

For polynomial functions, $y=ax^n$:

$$\frac{dy}{dx}=nax^{n-1} \tag{1.53}$$

The differentiation process may be carried out more than once. Thus:

$$\frac{d}{dx}\frac{(dy)}{dx}=\frac{d^2y}{dx^2} \text{ etc.} \tag{1.54}$$

As an example, if $y=f(x)=ax^4+bx^3+cx^2+dx+c$

then: $\frac{dy}{dx}=f'(x)=4ax^3+3bx^2+2cx+d$

$$\frac{d^2y}{dx^2}=f''(x)=12ax^2+6bx+2c$$

$$\frac{d^3y}{dx^3}=f'''(x)=24ax+6b$$

$$\frac{d^4y}{dx^4}=f^{iv}(x)=24a$$

$$\frac{d^5y}{dx^5}=f^{v}(x)=0$$

It is often convenient, when dealing with long, complicated expressions, to substitute a symbol for a part of a compound expression. Suppose we have:

$$y=f(x) \tag{1.55}$$

a complicated expression and we choose to make a substitution u then the differential, $f'(x)$ can be found from the rule:

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

e.g.: $y=(x^4+a^2)^6$

The substitution

$u=x^4+a^2$ is appropriate

so: $y=u^6$

thus: $\frac{dy}{dx}=6u^5$ and $\frac{du}{dx}=4x^3$

so: $\frac{dy}{du}=\frac{dy}{du}\frac{du}{dx}=6(x^4+a^2)^5 4x^3$

$$=24x^3(x^4+a^2) \tag{1.56}$$

In the case of trigonometric functions the differentiation process can be obtained via the expressions for multiple angles (see section 1.2).

For, suppose:

$$y = \sin\theta$$

We let:

$$y + \delta y = \sin(\theta + \delta\theta)$$

$$= \sin\theta\cos\delta\theta + \cos\theta\sin\delta\theta$$

$$= \sin\theta + \cos\theta.\delta\theta$$

So:

$$\delta y = \cos\theta.\delta\theta$$

$$\frac{\delta y}{\delta\theta} = \cos\theta \text{ or } \frac{\delta y}{\delta\theta} = \cos\theta \tag{1.57}$$

In cases where inverse trigonometric functions are involved, the principle of substitution is employed, for suppose:

$$y = \sin^{-1} u$$

then:

$$u = \sin y$$

so: $\frac{du}{dy} = \cos y = (1 - \sin^2 y) = (1 - u^2)$

so: $\frac{dy}{du} = 1 \quad \frac{du}{dy} = \frac{1}{(1-u^2)}$ (1.58)

In cases where exponentiation is involved, the principle of substitution may again be employed:

For, suppose $y = e^{3x^2/4}$

we write $y = e^u$

where $u = 3x^2/4$

and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u 6x/4$

$$= \frac{3x}{2} e^{3x^2/4} \tag{1.59}$$

1.7.2 Partial differentiation

The dependent variable u may be a function of more than one independent variable, x and y, and we wish to find the rates of changes of u with respect to u and v separately. These rates of change, the partial differentials with respect to x and y are denoted by:

$$\frac{du}{dx} \text{ and } \frac{du}{dy}$$

In these processes the normal rules of differentiation are followed except that in finding du/dx, y is treated as a constant and in finding du/dy, x is treated as a constant.

The total differential of a function:

$$u = f(x, y)$$

where both x and y are functions of t is given by:

$$\frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt} + \frac{du}{dy}\frac{dy}{dt} \tag{1.60}$$

1.7.3 Maxima and minima

Maxima and minima of functions occur when the function has zero slope or first differential. Thus, in order to determine a maximum or minimum of a function $y = f(x)$:

we set: $\frac{dy}{dx} = f'(x) = 0$ (1.61)

and solve this equation, say $x = x_1$.

To distinguish between maxima and minima it is necessary to evaluate:

$\frac{d^2y}{dx^2}$ at the point x_1

For a maximum: $\frac{d^2y}{dx^2} < 0$

For a minimum: $\frac{d^2y}{dx^2} > 0$

1.7.4 Integration

Integration is generally the reverse of the process of differentiation. It may also be regarded as equivalent to a process of summing a number of finite quantities but, in the limit the number of quantities becomes infinite and their size becomes infinitesimal.

By the reverse of the differentiation process the integral

$$\int ax^n.dx = \frac{ax^{n+1}}{n+1} + c \tag{1.62}$$

the c being an arbitrary constant which, for shortness, is frequently not written. This is called an indefinite integral as no range over which the integration is to be performed has been specified. If such a range is specified then we obtain the case of a definite integral, e.g.:

if $F(x)\,dx = f(x)$

then $\int_b^a F(x)\,dx = f(b) - f(a)$ (1.63)

In geometrical terms, this integral represents the area bounded by the curve $y = F(x)$, the x axis, and the two lines $x = a$, $x = b$.

1.7.5 Successive integration

This is the reverse process from that of successive differentiation, each cycle of operations consisting of the integration of the function resulting from the immediately previous integration. In general terms instructions to carry out successive integration are expressed thus: $y = \iiiint f(x)\ dx,\ dx,\ dx,\ dx$, which means integration is to be successively carried out 4 times with respect to x.

Another form of successive integration is $v=\iiint f(x, y, z)\, dx, dy, dz$; referred to as a volume integral. A surface integral would take the form $s=\iint f(x, y)\, dx, dy$.

Example 1.4 Find a general expression for the deflection of a simple span girder of span l loaded uniformly by a load w per unit length of span given that $w=EI\, d^4y/dx^4$ and taking E and I as constant and x as measured from one end.
Load:

$$EI\frac{d^4y}{dx^4}=w$$

Shear:

$$EI\int\frac{d^4y}{dx^4}dx=EI\frac{d^3y}{dx^3}=wx+c_1=wx-\frac{wl}{2}$$

$$\left(\text{Shear}=-\frac{wl}{2}\text{ for }x=0\right)$$

Bending moment:

$$EI\int\frac{d^3y}{dx^3}dx=EI\frac{d^2y}{dx^2}=\frac{wx^2}{2}-\frac{wlx}{2}+c_2=\frac{wx^2}{2}-\frac{wlx}{2}$$

(B.M. = 0 for $x=0$)

Slope:

$$EI\int\frac{d^2y}{dx^2}dx=EI\frac{dy}{dx}=\frac{wx^3}{6}-\frac{wlx^2}{4}+c_3=\frac{wx^3}{6}-\frac{wlx^2}{4}+\frac{wl^3}{24}$$

$$\left(\text{Slope}=0\text{ for }x=\frac{l}{2}\right)$$

Deflection:

$$EI\int\frac{dy}{dx}dx=EIy=\frac{wx^4}{24}-\frac{wlx^3}{12}+\frac{wl^3x}{24}$$

(Deflection = 0 for $x=0$)

i.e. at any distance x from one end the deflection

$$y=\frac{1}{24}\frac{w}{EI}(x^4-2lx^3+l^3x)$$

$$\iiiint\frac{d^4y}{dx^4}dx\,dx\,dx\,dx=\frac{w}{24EI}(x^4-2lx^3+l^3x)$$

which is in the general form.

For the mid-span deflection the range of integration is from $x=0$ to $x=\frac{1}{2}l$.

Hence:

$$\iiiint_0^{l/2}\frac{d^4y}{dx^4}dx.dx.dx.dx=\frac{wl^4}{24EI}\left(\tfrac{1}{16}-\tfrac{1}{4}+\tfrac{1}{2}\right)=5\frac{wl^4}{384EI}$$

1.7.6 Integration by substitution

The integration of functions can often be simplified by substituting a new variable for a part or the whole of the original function, thereby reducing it to one of the standard forms.

Example 1.5 Find the value of $\int\sqrt{(3+x)}\,dx$

Let $3+x=u$

Therefore $dx=du$

so that

$$\int\sqrt{(3+x)}\,dx=\int u^{\frac{1}{2}}\,du=\tfrac{2}{3}u^{3/1}=\tfrac{2}{3}(3+x)^{3/2}\text{ or }\tfrac{2}{3}\sqrt{(3+x)^3}$$

Example 1.6 Find the value of

$$2\int\frac{dx}{e^{3x}+c^{-3x}}$$

Let $e^{3x}=v$; then $3e^{3x}.dx=dv$, or $dx=dv/3v$.

Substituting

$$2\int\frac{dx}{e^{3x}+e^{-3x}}=\tfrac{2}{3}\int\frac{dv}{v(v+1/v)}=\tfrac{2}{3}\int\frac{dv}{v^2+1}=\tfrac{2}{3}\tan^{-1}v=\tfrac{2}{3}\tan^{-1}e^{3x}.$$

Example 1.7 Find the value of $\int\sqrt{(1-x^2)}\,dx$.
Put $x=\sin\theta$; then $\sqrt{(1-x^2)}=\cos\theta$

Therefore

$$\int\sqrt{(1-x^2)}\,.dx=\int\cos\theta.d\sin\theta=\int\cos^2\theta.d\theta$$

$$=\int\frac{1+\cos 2\theta}{2}\cdot d\theta$$

$$=\frac{\theta}{2}+\frac{\sin 2\theta}{4}=\frac{1}{2}\{\sin^{-1}x+x\sqrt{(1-x^2)}\}$$

1.7.7 Integration by transformation

The integration of trigonometric functions can often be simplified by transformation into a standard form of integral.

TYPE

$$\int\sin^m\theta\cos^n\theta d\theta$$

Case 1: m = positive odd integer, n = any positive integer.

TRANSFORMATIONS

$$\int\sin^{m-1}\theta\sin\theta\cos^n\theta d\theta$$

$$=\int(1-\cos^2\theta)^{(m-1)/2}\sin\theta\cos^n\theta d\theta$$

$$=-\int(1-\cos^2\theta)^{(m-1)/2}\cos^n\theta d(\cos\theta)$$

Example 1.8 Solve $\int\sin^3\theta\cos^2\theta d\theta$

$$\int\sin^3\theta\cos^2\theta d\theta$$

$$=\int(1-\cos^2\theta)\sin\theta\cos^2\theta d\theta$$

$$=\int\cos^2\theta\sin\theta d\theta-\int\cos^4\theta\sin\theta d\theta$$

$$= -\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5}$$

Case 2: m = any positive integer, n = positive odd integer.

TRANSFORMATION

$\int \sin^m\theta \cos^{n-1}\theta d\theta = \int (1-\sin^2\theta)^{(n-1)/2} \sin^m \theta d(\sin\theta)$.

Example 1.9 Solve $\int \sin^2\theta \cos^3\theta d\theta$

$\int \sin^2\theta \cos^3\theta d\theta = \int (1-\sin^2\theta) \sin^2\theta \cos\theta d\theta$

$= \int \sin^2\theta \cos\theta d\theta - \int \sin^4\theta \cos\theta d\theta$

$$= \frac{\sin^3\theta}{3} - \frac{\sin^5\theta}{5}$$

TYPE

$\int \tan\theta d\theta$ where n is an integer >1.

TRANSFORMATION

$\int \tan^{n-2}\theta \tan^2\theta d\theta = \int \tan^{n-2}\theta (\sec^2\theta - 1) d\theta$

$= \int \tan^{n-2}\theta . \tan\theta - \int \tan^{n-2}\theta . d\theta$

TYPE

$\int \cot^n\theta d\theta$ where n is an integer >1.

TRANSFORMATION

$\int \cot^{n-2}\theta \cot^2\theta d\theta = \int \cot^{n-2}\theta(\mathrm{cosec}^2\theta - 1) d\theta$

$= -\int \cot^{n-2}\theta . d\cot\theta - \int \cot^{n-2}\theta . d\theta$.

TYPE

$\int \sec^n\theta d\theta$ where n is positive and even.

TRANSFORMATION

$\int \sec^{n-2}\theta \sec^2\theta d\theta = \int (\tan^2\theta + 1)^{(n-2)/2} d\tan\theta$.

TYPE

$\int \mathrm{cosec}^n\theta d\theta$ where n is positive and even.

TRANSFORMATION

$\int \mathrm{cosec}^{n-2}\theta \,\mathrm{cosec}^2\theta d\theta = \int -(\cot^2\theta + 1)^{(n-2)/2} d\cot\theta$

TYPE

$\int \tan^m\theta \sec^n\theta d\theta$ where n is positive and even.

TRANSFORMATION

$\int \tan^m\theta \sec^{n-2}\theta \sec^2\theta d\theta = \int \tan^m\theta (\tan^2\theta + 1)^{(n-2)/2} d\tan\theta$.

TYPE

$\int \cot^m\theta \,\mathrm{cosec}^n\theta d\theta$ where n is positive and even.

TRANSFORMATION

$\int \cot^m\theta \,\mathrm{cosec}^{n-2}\theta \,\mathrm{cosec}^2\theta d\theta = \int -\cot^m\theta (\cot^2\theta + 1)^{(n-2)/2} d\cot\theta$.

TYPE

$\int \tan^m\theta \sec^n\theta d\theta$ where m and n are odd.

TRANSFORMATION

$\int \tan^{m-1}\theta \tan\theta \sec^{n-1}\theta \sec\theta d\theta$
$= \int (\sec^2\theta - 1)^{(m-1)/2} . \sec^{n-1}\theta . d\sec\theta$.

1.7.8 Integration by parts

The integration of functions can often be simplified by breaking up the function into two parts u and dv where u and v are the substituted variables in $\int u.dv = u.v - \int v.\mathrm{du}$, the fundamental formula for integration by parts, $\int u.dv$ representing the function to be integrated. In applying this method of integration $\int v.\mathrm{du}$ should not be more complex than $\int u.dv$. The integration of logarithmic, exponential, inverse trigonometric and products of algebraic expressions may be simplified by this procedure.

Example 1.10 $\int w \sin w\, dw$
Let $u = w$ and $dv = \sin w\, dw$ then $du = dw$ and $v = -\cos w$

Therefore:

$$\begin{aligned}\int w \sin w . dw = \int u . dv &= -\mathrm{w} \cos w + \int \cos w\, dw \\ &= -w \cos w + \sin w\end{aligned}$$

Example 1.11 $\int xe^x dx$
Let $u = x$ and $dv = c^x dx$ then $du = dx$ and $v = e^x$

Therefore:

$$\int xe^x dx = \int u.dv = xe^x - \int e^x dx = xe^x - e^x = e^x(x-1)$$

Example 1.12 $\int \cos^2\theta d\theta$
Let $u = \cos\theta$ and $dv = \cos\theta d\theta$ then $du = -\sin\theta d\theta$ and $v = \sin\theta$

Therefore:

$$\begin{aligned}\int \cos^2\theta d\theta = \int u.dv &= \cos\theta \sin\theta + \int \sin^2\theta d\theta \\ &= \frac{\sin 2\theta}{2} + \int (1-\cos^2\theta) d\theta\end{aligned}$$

i.e.:

$$2\int \cos^2\theta d\theta = \frac{\sin 2\theta}{2} + \theta \text{ hence } \int \cos^2\theta d\theta = \frac{\sin 2\theta}{4} + \frac{\theta}{2}$$

Example 1.13 $\int \sec^3\theta d\theta$
Let $u = \sec\theta$ and $dv = \sec^2\theta d\theta$ then $du = \sec\theta \tan\theta d\theta$ and $v = \tan\theta$

Therefore:

$$\int \sec^3\theta d\theta = \sec\theta \tan\theta - \int \tan^2\theta \sec\theta d\theta$$

$$= \sec\theta \tan\theta - \int \sec^3\theta d\theta + \int \sec\theta d\theta$$

i.e.:

$$2\int \sec^3\theta d\theta = \sec\theta\tan\theta + \log_e\left\{\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right\}$$

Therefore:

$$\int \sec^3\theta d\theta = \tfrac{1}{2}\left[\sec\theta\tan\theta + \log_e\left\{\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right\}\right]$$

1.7.9 Integration of fractions

The integration of functions consisting of rational algebraic fractions is best carried out by first splitting the function into partial fractions. It is assumed that the numerator is of lower degree than the denominator; if not, this should first be achieved by dividing out. It may be shown that the prime real factors of any polynomial are either quadratic or linear in form. This leads to four distinct types of partial fraction solutions which are now described.

1.7.9.1 Fractions type 1

The denominator can be factored into real linear factors all different. The partial fractions are then of the form $a/(bx+c)$.

Example 1.14 $\int \frac{2x+3}{x^2-4}dx$

Now

$$\frac{2x+3}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

i.e.: $2x+3 = A(x+2)+B(x-2)$

i.e. $A = \frac{7}{4}$ and $B = \frac{1}{4}$

Therefore:

$$\int \frac{2x+3}{x^2-4}dx = \frac{7}{4}\int\frac{dx}{x-2} + \frac{1}{4}\int\frac{dx}{x+2} = \frac{7}{4}\log_e(x-2) + \frac{1}{4}\log_e(x+2)$$

1.7.9.2 Fractions type 2

The prime factors of the denominator include quadratic functions and all factors are different. The partial fractions then include expressions of the form $(ax+b)/(cx^2+dx+e)$.

Example 1.15 $\int \frac{7x^2-3}{2x^3-3x^2+4x-6}\cdot dx$

Put $$\frac{7x^2-3}{2x^3-3x^2+4x-6} = \frac{Ax+B}{x^2+2} + \frac{C}{2x-3}$$

i.e.: $7x^2-3 = (Ax+B)(2x-3)+C(x^2+2)$

$$= (2A+C)x^2 + (2B-3A)x - (3B-2C)$$

and therefore $A=2$, $B=3$, $C=3$

Therefore:

$$\int \frac{7x^2-3}{2x^3-3x^2+4x-6}\cdot dx = \int\frac{2x+3}{x^2+2}\cdot dx + \int\frac{3}{2x-3}\cdot dx$$

$$= \int\frac{2x}{x^2+2}\cdot dx + \int\frac{3}{x^2+2}\cdot dx + \int\frac{3}{2x-3}\cdot dx$$

$$= \log_e(x^2+2) + \frac{3}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{3}{2}\log_e(2x-3)$$

1.7.9.3 Fractions type 3

The denominator can be factored into real linear factors, some of which are repeated. The partial fractions then include expressions of the form $a/(bx+c)^n$.

Example 1.16 $\int \frac{3x^2+8x+16}{x^3+3x^2-4}dx = \int\frac{f(x)}{F(x)}dx$

$$F(x) = (x-1)(x+2)^2 \text{ and } \frac{f(x)}{F(x)} = \frac{A}{x-1} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

Hence:

$$3x^2+8x+16 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

putting $x=1$ then $A=3$; $x=-2$ then $C=-4$; substitution gives $B=0$

Therefore:

$$\int\frac{f(x)}{F(x)}dx = 3\int\frac{dx}{x-1} - 4\int\frac{dx}{(x+2)^2} = 3\log_e(x-1) + \frac{4}{(x+2)}$$

1.7.9.4 Fractions type 4

The prime factors of the denominator include quadratic functions some of which are repeated. The partial fractions then include expressions of the form $(ax+b)/(cx^2+dx+e)^n$.

Example 1.17 $\int \frac{12x-1}{(x^2+1)^2(x+2)}dx = \int\frac{f(x)}{F(x)}dx$

$$\int\frac{f(x)}{F(x)}dx = \frac{Ax+B}{(x^2+1)^2} + \frac{Cx+D}{(x^2+1)} + \frac{E}{(x+2)}$$

i.e.
$12x-1 = (Ax+B)(x+2) + (Cx+D)(x^2+1)(x+2) + E(x^2+1)^2$.

Put $x=-2$; then $E=-1$.

Therefore:

$$x^4+2x^2+12x = Cx^4 + (D+2C)x^3 + (A+2D+C)x^2 + (2A+B+D+2C)x + 2(B+D)$$

Equating coefficients, we find $C=1$, $D=-2$, $B=2$ and $A=5$

Therefore:

$$\int\frac{f(x)}{F(x)}dx = \int\frac{5x+2}{(x^2+1)^2}dx + \int\frac{x-2}{x^2+1}dx - \int\frac{dx}{x+2}$$

$$= \frac{5}{2}\int\frac{d(x^2+1)}{(x^2+1)^2} + 2\int\frac{dx}{(x^2+1)^2} + \frac{1}{2}\int\frac{d(x^2+1)}{x^2+1}$$
$$-2\int\frac{dx}{x^2+1} - \int\frac{dx}{x+2}$$

$$= -\frac{5}{2}\int\frac{1}{x^2+1} + \frac{x}{x^2+1} + \int\frac{dx}{x^2+1} + \frac{1}{2}\log_e(x^2+1)$$
$$-2\tan^{-1}x - \log_e(x+2)$$

$$= \frac{2x-5}{2(x^2+1)} + \log_e\frac{\sqrt{(x^2+1)}}{x+2}\tan^{-1}x$$

1.8 Matrix algebra

A matrix is an array of mn numbers in m rows and n columns

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

The element in the ith row and jth column a_{ij} is called the (i, j)th element and the matrix is often denoted by $[a_{ij}]$ or A. When $m=n$ the matrix is square. An $m \times 1$ matrix is called a column vector or column matrix.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \tag{1.64}$$

A $1 \times n$ matrix is called a row vector

$$Y = [y_1, y_2 \dots y_n] \tag{1.65}$$

1.8.1 Addition of matrices

Two matrices may be added if and only if they are of the same order $m \times n$.

Then: $A + B = [a_{ij}] + [b_{ij}] = [(a_{ij} + b_{ij})]$

i.e. the sum is formed by adding corresponding elements.

1.8.2 Multiplication of matrices

(1) By a scalar.
Any matrix may be multiplied by a scalar.

Then $\lambda A = \lambda[a_{ij}] = [(\lambda a_{ij})] = A\lambda$

i.e. all the elements of A are multiplied by λ.

(2) Multiplication of two matrices.
Two matrices may be multiplied (A times B in that order) only if the number of columns of A is equal to the number of rows of B. If A is $[a_{ij}]$ of order $m \times n$ and B is $[b_{ij}]$ of order $n \times p$ then

$$AB = [a_{ij}][b_{ij}] = \left[\left(\sum_{k=1}^{n} a_{ik}b_{kj}\right)\right]$$

is of order $m \times p$.
It should be noted that, in general $AB \neq BA$.

1.8.3 The unit matrix

The unit matrix is a square matrix I for which:

$a_{ij} = 0$ for $i \neq j$
$a_{ij} = 1$ for $i = j$

1.8.4 The reciprocal of a matrix

The reciprocal matrix A^{-1} of A exists only if the determinant of A is nonzero and is given by:

$$AA^{-1} = I = A^{-1}A$$

1.8.5 Determinants

The determinant of a square matrix is defined as:

$$|A| = \|a_{ij}\| = \sum(\pm a_{1\alpha}a_{2\beta} \dots a_{n\nu})$$

the summation of $n!$ terms being over all the arrangements $(\alpha, \beta, \dots \nu)$ of the column suffixes and the sign $\pm$ being chosen according to whether the arrangement is even or odd.
In the simplest case,

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

and from this can be developed the expressions for the expansion of determinants of higher order than the second. The minor of a_{ij} in A is the determinant of the matrix obtained by deleting the ith row and jth column of A. The cofactor A_{ij} of a_{ij} in A is $(-1)^{i+j} \times$ minor of a_{ij}.
Now the expression for a determinant is given by:

$$|A| = a_{11}|A_{i1}| + a_{i2}|A_{i2}| \dots + a_{in}|A_{in}|$$

The value of a determinant is unaltered by interchanging rows with columns. Interchanging either two rows or two columns changes the sign of a determinant. Thus, if either two rows or two columns are identical the determinant is zero.

1.8.6 Simultaneous linear equations

Simultaneous linear equations can be arranged in matrix form and their solution obtained via determinants

$$\begin{matrix} a_{11}x_1 + a_{12} & x_2 + & \dots & + a_{in}x_n = b_1 \\ \vdots & & & \vdots \quad \vdots \\ a_{m1}x_1 + & \dots & & + a_{mn}x_n = b_m \end{matrix}$$

may be written $AX = B$

and now $X = A^{-1}B$

alternatively $x_j = \dfrac{|D_j|}{|D|}$

where D denotes the determinant $|a_{ij}|$ and D_j denotes the determinant D with the elements $a_{ij}a_{2j} \dots a_{mj}$ replaced by $b_1b_2 \dots bm$.

$$= \frac{5}{2}\int\frac{d(x^2+1)}{(x^2+1)^2} + 2\int\frac{dx}{(x^2+1)^2} + \frac{1}{2}\int\frac{d(x^2+1)}{x^2+1}$$
$$-2\int\frac{dx}{x^2+1} - \int\frac{dx}{x+2}$$

$$= -\frac{5}{2}\int\frac{1}{x^2+1} + \frac{x}{x^2+1} + \int\frac{dx}{x^2+1} + \frac{1}{2}\log_e(x^2+1)$$
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may be written $AX=B$

and now $X=A^{-1}B$

alternatively $x_j=\dfrac{|D_j|}{|D|}$

where D denotes the determinant $|a_{ij}|$ and D_j denotes the determinant D with the elements $a_{ij}a_{2j}\dots a_{mj}$ replaced by $b_1b_2\dots bm$.

STATISTICS

1.9 Introduction

Statistical techniques are used in engineering mainly in connection with the quality control of manufacturing of produced material and with the checking for compliance of such products, with whatever specifications or clauses are contained in the contracts covering their purchase and sale. In order to exercise quality control or to check for compliance it is necessary to make measurements of one sort or another. Now it is well established that the result of repeating a measurement (or of repeating an experiment) does not generally repeat the observation or original result. Further, repeat measurements will lead to further results and so appears the problem of variability.

Generally speaking, the variation in results arises both because the subjects of the measurement are themselves different and also because of errors introduced by the experiment or the measuring technique. Such variation is common experience in the measurement of, for example, the strengths of materials. It will often be desirable (if only from an economic viewpoint) to reduce the variation to as small an amount as can conveniently be arranged. However, it is not generally possible to reduce such variation to an unimportantly small value and so it becomes necessary to deal with the problem posed by the obtaining of different results from apparently identical experiments. It is to deal with the evaluation of such scattered experimental results that statistical techniques have been developed.

It is supposed that, were it possible to continue the experiments indefinitely, the results so obtained would cluster around some fixed value which would be the required value. (It is an implicit assumption that the indefinite series of experiments be conducted under identical conditions.) Since it is not possible to conduct indefinitely long experiments the problem becomes that of trying to determine, from a finite series of experiments, that fixed value (which is presumably the true value) about which the indefinite series of results would cluster. This attempt to determine is known as 'estimating', and while the use of that particular word does not imply that there has been any guesswork in obtaining it, there is an implication of uncertainty about the result. In statistical methods this uncertainty is calculated and specified in terms of confidence limits. A result obtained after statistical calculations should generally be given in terms of an estimate surrounded by confidence limits. Of course the more nearly certain we wish to be that the confidence limits contain the true value the wider those limits must be. In cases where the experiment or test is not aimed at the estimating of some particular quantity, the form of the estimate and confidence limit changes to one that such and such a result would not have arisen 'by chance' more than on so many per cent of occasions in an indefinitely long series of trials.

It is important that statistical results should be properly presented in the form of estimate and confidence limits: having decided upon such a form it is then sensible to use an appropriate precision for reporting the values. For example, when estimating the strength of concrete where an estimate might be of the form: 42 ± 5 N/mm^2 (at 95% confidence) there is clearly no point in reporting the result to several decimal places.

When an estimate of some quantity has been obtained, the interval between the confidence limits may be wider than it is desired they should be, in which case the interval may be narrowed by accepting a lower confidence. If this is not desirable it will be necessary to: (1) take more observations; or (2) improve the experimental techniques used to reduce the variability of the results.

It is important that the question of what is required by way of precision should be considered prior to an experiment so that the number of observations necessary to obtain the required precision may be assessed. In making that assessment it will be necessary to have information about the variability of parts of the experiment. This information may be available from previous experience, but if not it must be obtained by a pilot experiment.

It will be clear from the foregoing that any result which is obtained, being subject to error, may cause a wrong decision to be taken. Thus when dealing with, for instance, material to be checked for strength the contract for the supply of the material should indicate a test scheme to determine whether the strength of the material is correct or not.

Such a test scheme will involve experiments, and the possibilities for a wrong decision are:

(1) That the test will wrongly show as unsatisfactory, material with the correct strength. (This is known as the manufacturer's or supplier's risk.)
(2) That the test will wrongly show as satisfactory, material with an incorrect strength. (This is known as the consumer's risk.)

The performance of a test scheme is defined by its power and is represented by a graph showing, on one axis, the true value of the parameter in question (e.g. the strength of the material) plotted against the probability that material will pass the test and so be accepted. The calculation of such graphs is not a simple matter and requires full information about all aspects of the test scheme under consideration. The power curves of two test schemes represent, however, the only way in which the performance of the two schemes may be compared.

In the following sections are presented definitions of some of the terms used in statistical work, descriptions of statistical techniques and tests which may be used as a part of the experimenter's armoury of techniques and a description of central charts as a method of quality control. In the final section the references have, in the first cases, been selected for their readability as well as for their coverage of any particular point. Thus the works by Moroney[1] and Neville and Kennedy[2] are especially recommended as initial reading for anyone interested in statistical problems and techniques.

1.10 Definitions of elementary statistical concepts

1.10.1 Statistical unit or item

One of a number of similar articles or parts each of which may possess several different quality characteristics.

Example 1.18 A piece of glass tubing taken from a large number produced in quantity for which the diameter and other characteristics may be measured; a concrete cube for which the strength may be measured.

1.10.2 Observation – observed value

The value of a quality characteristic measured or observed on a unit.

Example 1.19 The diameter in millimetres of a piece of tubing; the strength of a concrete cube.

1.10.2.1 Sample

A portion of material or a group of units taken from a larger number which is used to obtain estimates of the properties of the larger quantity.

Example 1.20 Forty-eight pieces of tubing sampled from all the pieces produced during a day; the concrete cube made from a batch of concrete.

1.10.2.2 Random sample

A sample selected in such a manner that every item has an equal chance of inclusion.

1.10.2.3 Representative sample

A sample whose selection requires planned action to ensure that proportions of it are taken from different subportions of the whole.

Example 1.21 The forty-eight pieces of tubing selected two from every hour's production in one day; concrete cubes made, one from every batch, of a lot consisting of several batches.

1.10.2.4 Population

A large collection of individual units from one source. In particular circumstances this may be, for example, an output or batch: the bulk of material (concrete) or total collection of units (pieces of tube) produced by a set of machines or a factory in a specified time.

Example 1.22 Pieces of tubing made in a particular factory during a month; the concrete produced by a single plant during 1 day.

1.10.2.5 Statistic

A statistic is a quantity computed from the observations of a sample.

1.10.2.6 Parameter

A parameter is a quantity computed from the observations made on a sample. Thus, the value of a parameter for a population is estimated by the appropriate statistic for the sample.

1.11 Location

1.11.1 Measures

1.11.1.1 Arithmetic mean

The arithmetic mean, often called the 'mean' or the 'average', is the sum of all the observations divided by the number of observations:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{1.66}$$

Example 1.23 0.20 (2.540 + 2.538 + 2.547 + 2.544 + 2.541) = 2.542.

1.11.1.2 Median

The value which is greater than one-half of the values and less than one-half of the values.

Example 1.24 The value 2.541 is the median of the above five numbers. (Had there been an even rather than an odd number of numbers the median is the average of the two numbers either side of the median position.)

1.11.1.3 Midpoint or midrange

The value which lies half way between the extreme values.

Example 1.25 Using the numbers above the mid point is

0.5 (2.538 + 2.547) = 2.542 5

1.12 Dispersion

1.12.1 Measures

1.12.1.1 Range

The difference between the largest and the smallest values.

Example 1.26 2.547 − 2.538 = 0.009.

1.12.1.2 Deviation

The difference between a value and the mean of all the values.

1.12.1.3 Variance

The variance of a set of values is the mean squared deviation of the individual values and is normally represented by σ^2.

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2 \tag{1.67}$$

where μ is the mean value.

A frequently occurring problem is that of estimating the main properties (the mean to describe the location and the variance to describe the dispersion) of a population by measurements (x_i) taken on a sample. From the measurements on the sample we can calculate the sample mean, $\bar{x}$ which is an estimate of the population mean μ. The sum of the squared deviations is smallest about the arithmetic mean; thus, for the population an estimate of variance using the sample mean and sample variance will be an underestimate. In cases where we wish to estimate population parameters from sample observations, a correction is made by using $(n-1)$ as divisor instead of n. Thus, the estimate of the population variance from observations x_i on a sample is:

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{1.68}$$

1.12.1.4 Standard deviation

The standard deviation is the square root of the variance.

$$s = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right]^{1/2} \tag{1.69}$$

As in the case of variance, the divisor n is replaced by $(n-1)$ when working with sample observations to estimate a population standard deviation. The standard deviation has the same units as the original observations and their mean $\bar{x}$.

When carrying-out hand calculations, the identity:

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2 \tag{1.70}$$

frequently saves effort. However, this method is not recommended for use on computers because of the danger of loss of accuracy when n is large and x_i has several significant figures.

1.12.1.5 Coefficient of variation

The coefficient of variation is the standard deviation expressed as a percentage of the mean. This is useful for dealing with properties whose standard deviation rises in proportion to the mean, for instance the strengths of concrete as measured by compressive tests on cubes.

1.12.1.6 Standard error

The standard error is the standard deviation of the mean (or of any other statistic). If in repeated samples of size n from a population the sample means are calculated, the standard deviation calculated from these means is expected to have a value:

$$Sm=\sigma/\sqrt{n} \tag{1.71}$$

where σ is the standard deviation of the population.

An important result is that whatever the distribution of the parent population (normal or not) the distribution of the sample mean tends rapidly to normal form as the sample size increases.

1.13 Samples and population

1.13.1 Representations

1.13.1.1 Frequency

The number of observations having values between two specified limits. It is often convenient to group observations by dividing the range over which they extend into a number of small, equal, intervals. The number of observations falling in each interval is then the frequency for that interval. This allows a convenient representation of the information by means of a histogram.

1.13.1.2 Histogram or bar chart

A diagram in which the observations are represented by rectangles or bars with one side equal to the interval over which the observations occurred and the other equal to the frequency of occurrence of observations within that range (Figure 1.45).

1.13.1.3 Distribution curve

The result of refining a histogram by reducing the size of the intervals and correspondingly increasing the total number of observations. In the limit, when the intervals become infinitesimally small and the number of observations infinitely large, the tops of the rectangles of a histogram become a distribution curve (Figure 1.46).

1.13.1.4 Normal distribution (or Gaussian distribution)

A particular type of distribution curve given by:

$$y(x)=\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}\exp\left\{\frac{-\frac{1}{2}(x-\mu)^2}{\sigma^2}\right\} \tag{1.72}$$

where x is the observational scale value, μ the population mean and σ the population standard deviation.

These parameters of the distribution are estimated by the sample mean $\bar{x}$ and standard deviation s.

It has been found that a great many frequency distributions met with in practice fit quite closely to the normal distribution. However, one should beware of thinking that there is any law which says that this shall be so; it is simply a matter of experience. In circumstances where the observed frequency distribution does not appear to be normal it is often possible to transform the original data (e.g. by taking logarithms, square roots or squares) so that the transformed data is nearly normal. These two facts explain why so much of the effort in statistical theory has been devoted to treatment of normal-distribution problems.

For normal distributions the percentage of observations (in large samples) lying within certain limits of the observational scale are given in Table 1.5 and Figure 1.47.

1.14 The use of statistics in industrial experimentation

As has been stated, in experimental work units in a sample drawn from a parent population and the observations made on them are subject to error, and our task for which we use statistics is to make useful statements about the properties of the parent population. To achieve this, the most important statistics are the mean and the standard deviation. This section, therefore, considers the obtaining of sample means and standard deviations and confidence limits for them in situations where the parent population is normally distributed. Tests of significance for comparisons of means and variances are also described. Inevitably, only brief summaries are given and a study of standard works is advised before using the techniques on any important matters. As an alternative, the help of the statistical expert should be sought. If such assistance is to be obtained, it cannot be emphasized too strongly that it should be acquired right at the outset of the problem. It is rarely of much help to anyone (even though it happens only too frequently) for the statistician to be asked: 'Please tell me what these numbers show: they must mean something, I've collected so many, and they cost a great deal to obtain.'

1.14.1 Confidence limits for a mean value

If the form of the distribution were known together with the true mean μ and the standard deviation σ, then it is easy to make statements about the mean of a number of observations. If the population is normal then the mean $\bar{x}$ of a sample size n drawn randomly will, on average, satisfy:

$$\mu-\frac{3\sigma}{\sqrt{n}}<\bar{x}<\mu+\frac{3\sigma}{\sqrt{n}}$$

997 times out of 1000 (see Table 1.5).

Thus, if μ is actually unknown (and we are trying to estimate it) we may assert:

$$\bar{x}-\frac{3\sigma}{\sqrt{n}}<\mu<\bar{x}+\frac{3\sigma}{\sqrt{n}}$$

with 99.7% confidence. By this, we mean that if we go on making such assertions indefinitely we shall be wrong only 3 times in every 1000. We can make the containing interval narrower by reducing confidence so that we assert with 95% confidence that the limits for μ are $\bar{x}\pm 1.96\sigma/\sqrt{n}$ (Figure 1.48).

Very often we may be concerned with a limit on only one side, for instance we may require assurance that μ is greater than a certain value. Now, the probability of $\bar{x}$ falling above $\mu+2\sigma/\sqrt{n}$

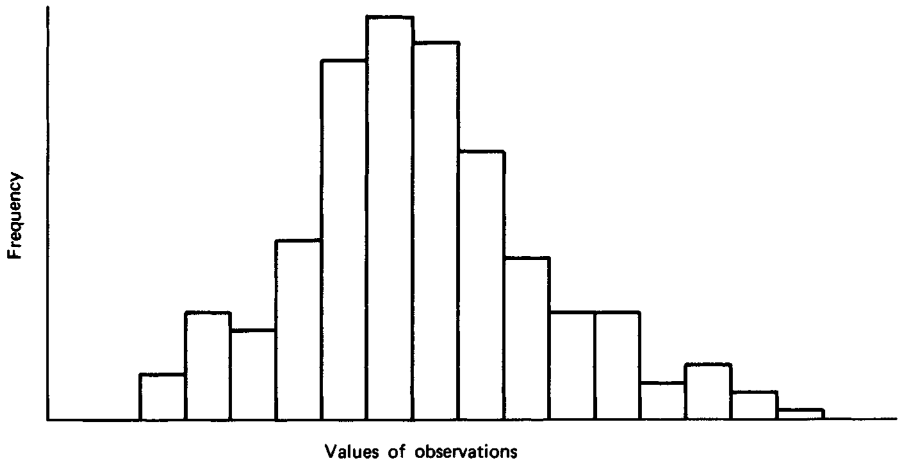

Figure 1.45 A histogram of observations from a sample

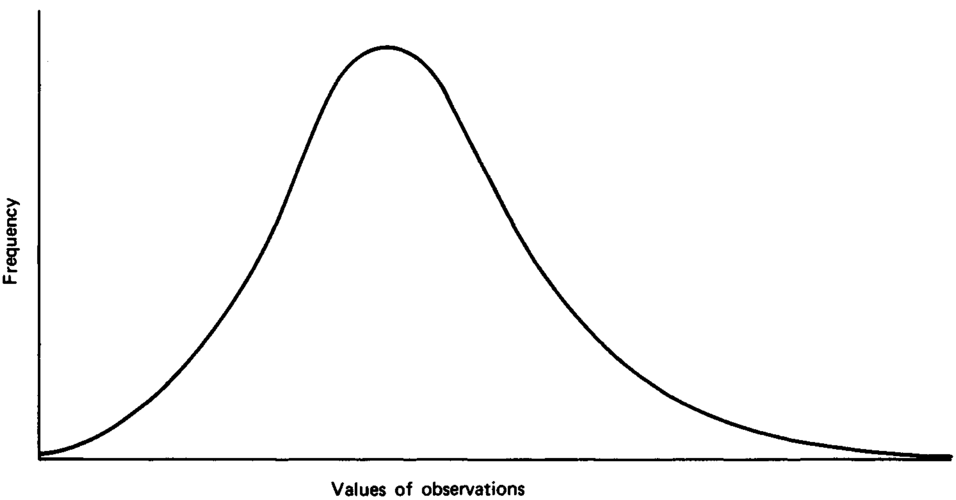

Figure 1.46 A continuous distribution curve

Table 1.5 The normal distribution

Range	*Observations within range* (%)
$\mu \pm \sigma$	68.27
$\mu \pm 2\sigma$	95.45
$\mu \pm 3\sigma$	99.73
$\mu \pm 1.96\sigma$	95
$\mu \pm 3.09\sigma$	99.8

is 2.27%. Thus, we may assert with 97.73% confidence that μ does not lie below $\bar{x} - 2\sigma/\sqrt{n}$.

Generally, the proportion of sample means $\bar{x}$ which exceed $\mu + u_a\sigma/\sqrt{n}$ is equal to a where u_a is the value given in a table of the normal distribution for a specified probability, say P. Because the distribution is symmetrical, α is also the proportion of sample means which are exceeded by $\mu - u_a\sigma/\sqrt{n}$. Thus, the whole range of values which μ may take is divided into three parts and three assertions can be made, to correspond one with each part:

(1) $\mu \geqslant \bar{x} - u_a\sigma/\sqrt{n}$ with confidence $100(1-a)\%$.
(2) $\mu \leqslant \bar{x} + u_a\sigma/\sqrt{n}$ with confidence $100(1-a)\%$.
(3) $\bar{x} - u_a\sigma/\sqrt{n} \leqslant \bar{x} + u_a\sigma/\sqrt{n}$ with confidence $100(1-2\alpha)\%$.

This shows two sorts of statement, the single-sided (cases 1 and 2) and the double-sided (case 3). When using statistical tables it is important to check whether the tabulation is for single-tailed testing or two-tailed testing. (This description arises because cases 1 and 2 are, in the practical cases where a useful level of confidence is being used, representable as the two tails of a curve shaped like the normal distribution curve.)

In the discussion of confidence limits for the mean value μ of a population estimated by the mean of the sample $\bar{x}$ above it was assumed that the population standard deviation was known. Generally this will not be the case and μ will have to be estimated as s, a sample standard deviation and used in place of μ in the calculations above. The confidence limits for μ are now

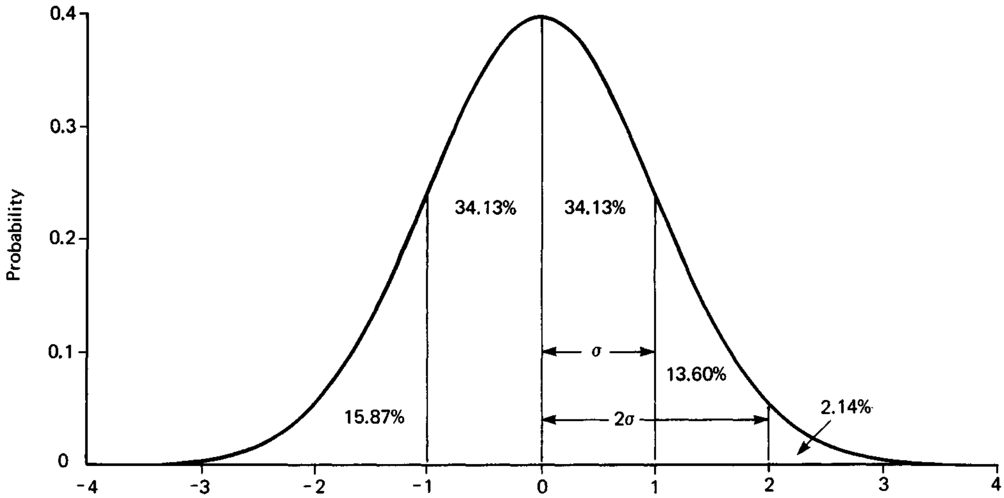

Figure 1.47 The normal distribution with limits

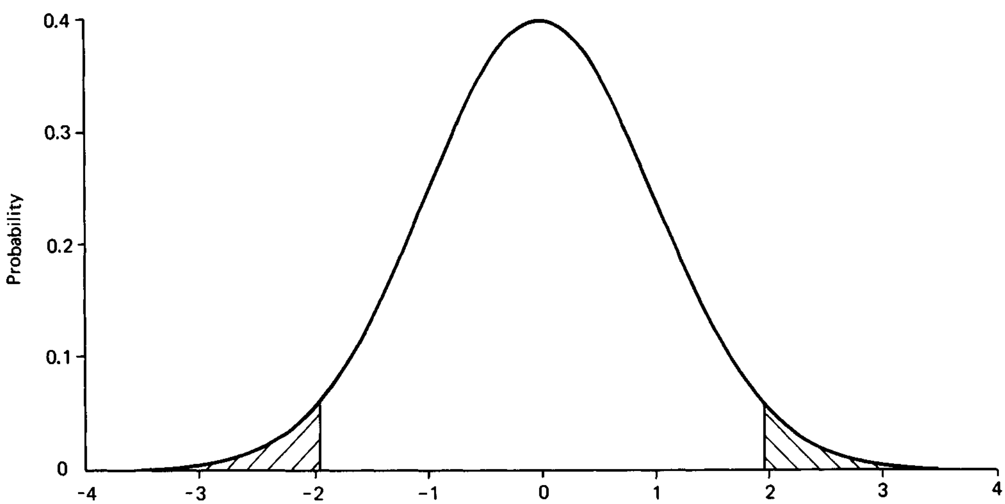

Figure 1.48 Diagrammatic representation of confidence limits (with 95% limits shown)

wider because of the uncertainty about s and, instead of using u from the normal distribution curve it becomes necessary to use tables of student's t. The particular value of t to be used depends on how good the estimate s of σ is, which in turn depends upon the number of degrees of freedom in making the estimate. In the case of a standard deviation of n observations, the number of degrees of freedom is $(n-1)$. The number of degrees of freedom is generally denoted by ϕ. Some values of t are given in Table 1.6. In using this table the $100(1-2a)\%$ confidence limits are:

(1) Lower limit $\bar{x}-t_a s/\sqrt{n}$.
(2) Upper limit $\bar{x}+t_a s/\sqrt{n}$.

using the value of t_a for the appropriate number of degrees of freedom.

Table 1.6 Significance points of the t-distribution (single-sided)

ϕ	*Probability: P*				
	0.1	0.05	0.025	0.01	0.005
1	3.08	6.31	12.70	31.80	63.70
2	1.89	2.92	4.30	6.96	9.92
5	1.48	2.01	2.57	3.36	4.03
10	1.37	1.81	2.23	2.76	3.17
20	1.32	1.72	2.09	2.53	2.85
40	1.30	1.68	2.02	2.42	2.70
∞	1.28	1.64	1.96	2.33	2.58

1.14.2 The difference between two mean values

A problem which arises frequently is that of determining if the difference between two means has occurred by chance because of natural variation or whether there is a real difference. A real difference can only be asserted in the form of a statistical statement that the difference is significant at a certain level, i.e. there is a probability that there is a real difference. This is done by calculating a t statistic from information about the samples and comparing the result with the tabulated t values. The means, standard deviations and number of observations of the two tests are denoted by $\bar{x}_1$, $\bar{x}_2$, s_1, s_2, n_1 and s_2.

Calculate $t=(\bar{x}_1-\bar{x}_2)/s_p$

where $s_p=\sqrt{\left[\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\left(\frac{\nu_1 s_1^2+\nu_2 s_2^2}{\nu_1+\nu_2}\right)\right]}$

and $\nu_1=n_1-1$, $\nu_2=n_2-1$

(*Note*: s_p is the pooled standard deviation for the samples 1 and 2.).

If this calculated value exceeds a tabulated value of t (for $\phi=\nu_1+\nu_2$) then the difference is significant at the level determined by the probability heading the column of the t table.

As an example, consider the comparison of two testing machines for crushing concrete cubes. The machines are to be compared by making a single batch of twelve concrete cubes and testing six cubes on each machine. The results obtained are:

Machine 1	39.2	38.4	44.7	41.0	41.0	44.1
Machine 2	41.1	33.8	42.4	36.8	32.0	40.1

From these observations we can calculate:

Sample sizes	$n_1=6$, $n_2=6$
Sample means	$\bar{x}_1=41.4$, $\bar{x}_2=37.7$
Sample standard deviations	$s_1=2.54$, $s_2=4.19$

$$s_p=\sqrt{\left\{\left(\frac{1}{6}+\frac{1}{6}\right)\left(\frac{5\times 2.54^2+5\times 4.19^2}{5+5}\right)\right\}}=2.0$$

so: $t=(41.4-37.7)/2.0=1.85$

The number of degrees of freedom $\phi=\nu_1+\nu_2=10$.

From Table 1.6 it is seen that for $\phi=10$ the single-sided 2.5% (or 0.025) point of the t distribution is 2.23 and so the calculated t value is not significant at the $2\times 2.5\%$ level.

(*Note*: It is necessary to double the probability value from the table because the question: 'Are the testing machines different?' requires a two-sided test to be carried out. By contrast the question: 'Is mean 1 greater than mean 2?' would require a single-sided test.)

1.14.3 The ratio between two standard deviations

In a similar way in which it may be desired to compare two means, it may be desired to compare two standard deviations. Whereas means are compared by calculating their difference, standard deviations are compared by calculating the ratio of the variances (the square of the standard deviation) and comparing the ratio with tabulated values in an F test. In all such calculations the value obtained for the ratio must be greater than unity so that the larger standard deviation (say s_1) must be placed over the smaller (s_2) where s_1 and s_2 are sample standard deviations and so estimates of the population standard deviations. Since we have of necessity $s_1 \geqslant s_2$ when we calculate

$$F=\frac{s_2^1}{s_2^2}$$

the F test is a one-sided test.

Values of F for comparison with the calculated value from the observed standard deviations are given in most statistical books.[2] Such tables are presented generally with one table for each specified probability and within such a single table the column headings are the values of ν_1 (the number of degrees of freedom, n_1-1, of the smaller standard deviation estimate s_2).

By way of illustration, consider the example used above for the comparison of means. In that example the observations lead to:

$n_1=n_2=6$

so: $\nu_1=\nu_2=5$

$s_1=2.54 \quad s_2=4.19$

In this example $s_2>s_1$ so the calculation of F is:

$$F=\left(\frac{4.19}{2.54}\right)^2=2.72$$

In the tables (e.g. in Neville and Kennedy[2]) the tabulated 1% confidence point of F is 10.97 and the 5% point is 5.05 both found for $\nu_1=\nu_2=5$. Since the calculated F ratio is not greater than the tabulated values the conclusion to be drawn is that there is not strong evidence that population standard deviations are different.

1.14.4 Analysis of variance

If a manufacturing process or a testing scheme involves a number of independent factors, each of which contributes to the variability of the results, then the variance of the whole system is equal to the sum of the component variances. (Note that the variance must be added, not the standard deviation.) This additive property permits the technique of analysis of variance, which can take many forms depending on the structure of the process which is being analysed. One of the major difficulties of analysis of variance lies in deciding what form of structure is appropriate to the process being modelled by the analysis of variance. In the majority of cases which are not both simple and short it will be sensible for the arithmetic to be performed by computer. However, in simple and short situations the calculations may reasonably be undertaken by hand.

Probably the most commonly occurring simple situation is that of analysis to determine variance between and within batches. The methods are best described by an example. The example will be one in which concrete cubes are made batches (each of three cubes) and strength tested at (say) 28 days. The first step is to define the statistical model which is being used:

$$Y_{ij}=Y+A_i+E_{ij} \tag{1.73}$$

where there are i batches each of j cubes, Y_{ij} is the observed strength of the jth cube in the ith batch, Y is the average strength (averaged over all tests), A_i is the difference between Y and the average strength of batch i, and E_{ij} is the difference between the jth cube of batch i and the average strength $Y+A_i$ of that batch.

If the data for four batches of cubes is:

19.8	21.1	19.8	(batch 1)
21.8	22.0	21.0	(batch 2)
21.2	21.5	21.2	(batch 3)
21.4	21.4	21.0	(batch 4)

it is found that $Y = 21.1$.

From this can now be found the sums of squares of the A_i and E_{ij}. Associated with each sum of squares is a number of degrees of freedom (as usual one less than the number of occurrences) so that dividing the sums of squares by the appropriate number of degrees of freedom gives the mean square. Thus is constructed an analysis of variance table as shown in Table 1.7.

The method of test is by F ratio so that the larger variance (the average of the sums of squares of errors) is divided by the smaller. Here, $1.07/0.23 = 4.61$ with 3 degrees of freedom for the column heading and 8 for the row heading when comparing with the tabulated F values. For $v_1 = 3$, $v_2 = 8$ the tabulated F value at 1% confidence level is 7.59. The observed value does not exceed this and so there is no assertion that can be made at the 1% level. However, the tabulated F value at the 5% confidence level is 4.07. The observed value exceeds this and so a result significant at the 5% level has been obtained. Thus, although there is not strong evidence there is some evidence of a real difference between batches.

In an example so small as this one the necessary arithmetic (especially if properly organized) may reasonably be tackled by hand. However, as can be deduced from examination of F tables, it is not always easy to get significant results with small experiments. Thus, the use of the technique will in many cases imply the use of a computer for handling the arithmetic. In such circumstances the engineer is likely to be using an existing computer program and need only concern himself with correctly presenting the data for the program to analyse and then with the interpretation of results and comparisons with tabulated F values. He has no need therefore to develop great skills in short-cut arithmetic methods.

Table 1.7

Model term	*Sum of squares*	*Degrees of freedom*	*Mean square*
A_i	3.2	3	1.07
E_{ij}	1.9	8	0.23

1.14.5 Straight-line fitting and regression

Experiments may be designed to examine whether two parameters are related. The circumstances may involve the effect on a property of a product of some parameter in the production process. In the experiment the parameter will be controlled or constrained to take a number n of prescribed values x_i over some range and the consequential observations y_i will be paired with them. The question now arises as to the 'best straight line' through the points x_i, y_i. It is assumed that the x_i values are error-free but that the observations y_i are subject to error. The method of obtaining the 'best' straight line in such circumstances is to choose the two parameters m and c of the straight line:

$$y = mx + c \tag{1.74}$$

in such a way that the sum of the squares of the errors in the y direction is a minimum. This is achieved by making:

$$m = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \tag{1.75}$$

and:

$$c = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2} \tag{1.76}$$

This line is called the line of regression of y on x and one of its properties is that it passes through the centroid $\bar{\bar{x}}$, $\bar{y}$ of the observed points. The usual statistical question now arises concerning the confidence limits which should be applied to the calculated line which is an estimage of a relationship. To examine this problem the errors or deviations must be calculated. At every observation point x_i y_i which does not actually lie on the calculated line there is an e_i. The variance of y estimated by the regression line is then:

$$s_y^2 = \frac{\Sigma e_i^2}{v} \tag{1.77}$$

where v is the number of degrees of freedom.

Since calculation of m and c impose two restraints the value of v is given by:

$$v = n - 2 \tag{1.78}$$

The variance of the mean value $\bar{y}$ is given by:

$$s_{\bar{y}}^2 = \frac{s_y^2}{n} \tag{1.79}$$

so that the confidence limits for $\bar{y}$ *are:*

$$\bar{y} \pm ts_{\bar{y}} \tag{1.80}$$

where, just as for a sample mean, the value of t is found from tables using the appropriate number of degrees of freedom.

The variance of the slope m is given by:

$$s_m^2 = \frac{s_y^2}{\Sigma(x - \bar{x})^2} \tag{1.81}$$

and the confidence band for slope is given by:

$$m \pm ts_m \tag{1.82}$$

It may be necessary to compare one regression line with another, theoretical one, to see if there is any significant difference between the theoretical slope, m_0, and the observed slope m. This test is performed by calculating a t statistic:

$$t = \frac{m - m_0}{s_b} \tag{1.83}$$

and comparing with the tabulated values. Just as in the case of comparison by means of samples we can compare the slopes of two observed lines by replacing $m - m_0$ by $m_1 - m_2$ in Equation (1.83) and using a pooled standard deviation from the variances of the slopes of both lines in place of s_b. The number of degrees of freedom used in the t table will be $n_1 + n_2 - 4$.

1.15 Tolerance and quality control

Material is often manufactured for supply according to a specification which will include compliance clauses for the performance of the product. As an example, CP 110[3] lays down (in Section 6.8) certain strength requirements and also suggests a testing plan. The *Handbook* to that code[4] discusses the problems of compliance and shows how different forms of testing plan after the operating characteristic of a test plan and so charge the risks run by the producer and by the customer. The customer has, in theory, the opportunity of reducing his risk by adopting a more vigorous testing plan. This, however, is likely to cost more and a customer may well deem this not worth while. The producer, on the other hand, must expect to have to meet the compliance clauses and needs to arrange his production methods so as to make a profit taking account of whatever limits or penalties may be imposed on him by the compliance clauses under which he has to operate. Thus, the manufacturer or producer is faced with a problem of how to control his product.

One example of a technique for exercising this control is shown by a system advocated for controlling the strength of ready-mixed concrete[5] by means of the cumulative sum chart which is an improved form of control chart especially developed and adapted to the problems of concrete manufacture.

In the process of manufacture and measurement of some property of the product natural variation will cause the results obtained to be distributed in some way. The problems facing the manufacturer are:

(1) To maintain adequate control over the process so that the variation in results does not become so large that an uneconomic number fall outside the specified tolerances.
(2) To detect any trend for the observations obtained to be moving out of the specified limits, sufficiently early to take useful corrective action.

As usual, samples are taken to estimate the properties of the parent population. To do this comparatively, many samples (25 or more) of comparatively small (but not less than about four and all the same) size are tested and the mean of the means $\bar{\bar{x}}$ used to estimate the population mean. The population standard deviation is estimated from the variance within samples, the average sample standard deviation from the average sample range.

Thus: $$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 \ldots + \bar{x}_k}{k} \quad (1.84)$$

$$\bar{x} = s/\sqrt{n}$$

for k samples of size n.

Now a chart is drawn with time or sample number in the horizontal axis and observation values on the vertical axis. A line drawn at $\bar{\bar{x}}$ represents the target performance of the process and two surrounding lines at $\bar{\bar{x}} \pm 1.96/(\sqrt{n})s$ represent warning levels for the process while surrounding lines at $\bar{\bar{x}} \pm 3.09/(\sqrt{n})s$ can be regarded as action levels.

The choice of the figures 1.96 and 3.09 has been made on the assumption that the process is functioning in such a way that the specified tolerance limits are reasonable, i.e. they are not so stringent that the chance of the product meeting the requirements is not high while on the other hand the process is not so 'good' (in which case it may be unnecessarily expensive) that all the results obtained lie well within limits.

The design and use of control charts is a valuable use of statistical methods. Generally they are robust in the sense that their usefulness is little affected by factors such as non-normality of the basic data. However, for their efficient use in some area experience of the particular technology is desirable and for a better understanding of the possibilities of the techniques the reader is recommended to works by the British Ready Mixed Concrete Association[5] and Davies and Goldsmith.[6]

COMPUTERS

Computers and computing have made a substantial impact on most walks of life, civil engineering not excepted. The pace of development in computing is substantially greater than for any other area of activity in the engineering world. Although other subject areas are subject to bursts of activity from time to time, when research or some specific project provides the necessary spur, computers are developing rapidly all the time, whether the engineering world wishes it or not. In consequence a great many organizations find it difficult to keep abreast of what is available or of what might actually be of benefit to them in their work. This difficulty is not eased by the wide discrepancy between the useful life of most civil engineering work and the life of computers.

Although, in principle, computers are simple machines which can perform simple arithmetic and make simple decisions (according to a set of coded instructions—the program) that fact is ever more frequently masked by the use of sophisticated techniques which appear to make computers behave more and more like human beings, and able to undertake tasks previously the province of human effort.

1.16 Hardware and software

One of the most important distinctions which must be understood when considering computers is the difference between *hardware* and *software*. A simple criterion is to imagine that the hardware consists of the material pieces which one can see—boxes, wires, screens, discs, chips etc.—while the software comprises the instructions which the hardware obeys. It is in the nature of the general developments in society that the cost of making the hardware is, in real terms, falling all the time. This fall in cost comes about through better design of components, automated manufacturing techniques and so on. All this is similar to the developments which have been taking place in other fields of manufacturing.

The software, on the other hand, consumes human effort and imagination very intensively. It is not easy to improve the techniques of manufacture here! In consequence, the total cost of a computer installation—if it is regarded, for simplicity, as being composed of the two elements of hardware and of software—has changed considerably. In the early days, when computers were harnessed in working offices, the cost of the hardware was the major consideration and the software, if considered at all, tended to be something of an afterthought. Now we are recognizing that the software is, or ought to be, the major consideration. Once the major details of the software suitable for the envisaged tasks have been settled it is logical to search for the 'best' hardware solution which will accommodate the chosen software.

It is, of course, unlikely that any office, let alone organization, will wish to 'computerize' just one activity. It is normal for a great multitude of tasks to benefit from being done by machine rather than by man. In this event the choice of hardware will be constrained by a, perhaps wide, variety of software. This emphasizes the fact that computers should be thought of (in the hardware sense) as general-purpose machines.

1.17 Computers

The changes which have taken place in recent years encompass the change from remote 'batch' computing to 'personal' computing where every person who needs one in order to do his job appears to have access to one on his desk. In truth, this revolution has come about via an intermediate stage, i.e. the change from the large mainframe machines which, while they could perform several tasks apparently concurrently, had to be run by dedicated operators remote from the users, to the mini machines operated via remote terminals. With this scheme, the many users all feel (most of the time) that the computer is dedicated to them alone while, actually, the centre of the machine is servicing up to some tens of users and it is only the terminal which is dedicated to the individual user.

The development of the 'personal' computer has had a bigger impact on this situation than is at first sight obvious. Personal computers came about because the huge improvements in computer technology allowed the production of a machine which can sit, complete, on a desk, but which has power greater than the mainframe machines of a decade ago. (Those mainframes had required a large dedicated room, and air-conditioning, as well as operating staff.)

The presence of the computer on the desk, with an impressive array of available software, encouraged a situation in which the users were often repeating tasks being performed by colleagues (especially the inputting of data). One description of the development as it affected the functioning of an organization was that it was leading to near anarchy with little managerial control of what was happening to the benefit of the organization.

This independence of the personal computers has therefore been both a benefit and a source of difficulty. Trying to get the best of all possible worlds has led to much emphasis on communications. Here is meant the communication between different computers, generally communicating with other computers within the same organization, but sometimes further afield. Increasingly, for organizations of a certain size, the plan followed is one with a major computer at the heart of operations with a network of personal computers around it. These personal computers can be connected to the 'heart machine', or can be operated in stand-alone mode, at the will of the user. In these circumstances it becomes possible for the heart machine to be the repository of the valuable corporate data (which should then be held once only) to which the individual users can have access as and when their work demands it. The individual users can then use their own 'personal' data and run the programs of interest to them on their personal computer without affecting anyone else. Should an individual user have available something (be it data or be it a program) to which other users require access, this is arranged via the heart machine.

The organization and control of such an arrangement is not simple and produces interesting problems of a managerial and human nature. But it is now possible to make arrangements which seem to be getting near to providing a situation in which men, machines, and the organization can all work reasonably efficiently.

The providers of computing solutions (hardware and software) are, of course, in business. They will therefore advise potential customers of the benefits of the particular solutions they purvey. It is not easy for the (computing) lay person to judge the advice received from such quarters. It seems likely, therefore, that many organizations will be well advised to adopt the strategy of ensuring that they have in-house expertise to judge such matters. This notwithstanding the fact that, as time goes by, the purveyors of computing solutions are making greater emphasis of the idea that their solution needs no computer expertise. As in other walks of life, the lack of expertise in an activity in which one is engaged is likely to be costly.

Since the advent of the personal computer (able to double as a terminal) sitting on the desk, has come the mobile or portable computer. This is depicted as sitting on the knees of the user and working off batteries, thus freeing the user from the need for access to mains electricity. While an obvious early use was, for example, for salesmen to enter their transactions, other applications are being found. The collection of technical data on site is an obvious parallel to those activities in other fields, so such machines are proving useful to the engineer. It can reasonably be said that the use of the fruits of computer invention are limited only by human imagination. Though trite, this statement has considerable importance. It can be very difficult indeed to think of a really new way of achieving some objective: the straitjacket of 'we've always done it this way' can be extremely strong.

1.17.1 The use of computers by civil engineers

Although engineers have appeared, at times, to lag behind in the use of computers, they actually began using them at a very early stage. The first 'obvious' application lay in the solution of the many simultaneous linear equations to which many problems of structural analysis can be reduced. This mathematical problem had received much attention in an effort to speed, refine, and make more reliable, hand methods. The ability of the computer to perform repetitive tasks reliably shifted the search to making the preparation, and input, of the data describing the problem more robust. This search was hampered for some time by limitations of the hardware. However, the availability of substantially increased computing power eventually allowed the problem of the data to be encompassed as well as the problem of solving the equations and presenting the results.

This theme of the availability of increasing computing power allowing new tasks to be tackled has recurred frequently in the history of computing.

The use of computers for structural analysis represented the limit of activity in engineering for some time. However, developments of other machines for drawing or plotting prompted an attack on another phase of engineering design activity. Conceptually, the operation of a design project can be split into four stages: (1) the concept and choice of solution; (2) the analysis of the whole structure; (3) the design of individual members; and (4) the preparation of detailed drawings.

The contribution computers can provide to (1) above has only comparatively recently become apparent in terms of rearranging scheme drawings and the holding of base data. Stage (2) was covered by the early computing endeavours and (4), the detail drawings, became possible when the plotters, developed for aero work, for example, became cheap enough for use in civil engineering. The development of a package for the production of drawings of reinforced concrete details was a major breakthrough. In use the detailer has available the information arising from the design of members. Using a desktop computer, for example, he supplies data of the basic dimensions of the member and then, via a question-and-answer dialogue, supplies information to define the reinforcement detail. At all stages the information supplied by the user is checked for logical consistency, geometric compatibility and compatibility with appropriate code or standard documents. If an attempt is made to do something impossible or contrary to regulation, then the user is not permitted to proceed until the error has been rectified. By contrast, an attempt to do something which, according to standards incorporated with the program, is unusual will result in a message which the user can heed, or ignore, at will. Such interactive programs were impossible with the earlier batch machines.

Having been developed in modern environments, such a program is now expected to be very 'user friendly'. To take a cynical view, a user-friendly program is one for which the user has no need to consult the (written) user manual!

The effect of using such an aid is that a small team of detailers can become very much more productive, producing many more drawings per week than by manual means. Further, the fact that the data defining the drawings is stored means that, in the event of changes becoming necessary, the revised drawings can be produced very much more quickly (and reliably) than if done by hand.

An interesting sideline to the development of such a tool is the attitude taken to drawings. While some drawings are required to make an impression, and so are treated as works of art, the drawing of a reinforcement detail is just a technical necessity and it is used only by technical people and therefore may be less impressive. In consequence, detail drawings, produced on comparatively cheap dot matrix printers, have become quite acceptable. Only a few years ago even these technical drawings were also treated as works of art.

The third stage of the design office exercise, that of designing the individual members has also been solved. More than one approach has been adopted but this has the benefit of providing potential purchasers and users with competitive choice.

If we consider the three technical stages of the design office activity and the solutions listed above, we find the appearance of some more common occurrences such as the requirements for compatibility and the transfer of information between different stages of the work. In this example, the information from the global structural analysis is required by both the design and the detailing activities. Similarly, the information from the member design is required for the detailing phase. There are different views about the extent to which this information transfer can, or indeed should, be made automatic. At the time of writing the general feeling is to limit the amount of automatic transfer, it being held that the contribution of man is too difficult to codify and too valuable to lose. Such views have held sway before and have, eventually, been overturned. It seems probable that the developments in expert systems and other advanced computer technology may have the same overturning effect here in due course.

The production of the software for such systems represents a very substantial expenditure and it is important that the solutions developed should not be excessively dependent on particular hardware. In fact the drawing part of the solution has used plotters and more recently dot matrix printers. (A likely change is that laser printing will be a practical tool for such work.) The actual computers used have covered a wide range although the operating system used by the computer has been important. This is another area where the general developments in computing towards standardized operating and filing systems will make the transport of software solutions from machine-to-(often successive)-machine a comparatively painless task.

This example of the solution of an engineering set of problems—analysis, design, detailing—has been described at some length because it typifies the problems which will require consideration in some form almost whenever a computer solution to a problem is being sought. It is foolish to underestimate the benefits which the computer can bring, but it is important to be aware of, and consider properly, the problems and side issues which can arise. Proper treatment of such matters can sometimes bring unexpected benefits.

1.17.2 Nontechnical computing

Although, when first invented, computers were largely used by technical people to perform technical tasks, it has long been the case that the bulk of computer sales and use have been in commercial fields. For some time this affected the design of computers but the picture now is one of much more general application as computers are becoming the user's workhorse. It is not practicable to have many different computers to perform the many different tasks which an individual may tackle.

1.17.2.1 Spreadsheet

One example of the change of use to which software can be put is the spreadsheet. A spreadsheet is essentially a rectangular array of boxes identified by cartesian-type coordinates. A box may contain either a value or a formula. Such formulae may relate to the values held in other boxes. After entry of data and formulae the user will request that calculation of values be performed. For whatever reason an item of data, or a formula, may need to be altered. After the change a recalculation can be requested and will usually seem to be performed almost instantaneously. It does not matter, of course, to the computer, whether the change was a correction of an error or a change of mind on the part of the user. The traditional use of spreadsheets has been in financial areas where the slogan about answering 'What if?' questions was meant to appeal to those with responsibility for profit margins, etc. However, a spreadsheet is nothing more than a general-purpose organization of calculation: there is no reason why a spreadsheet should not be used to calculate a set of sine or of logarithm tables. Increasingly, engineers are finding that, if they think about a problem from a different angle, a different solution tool may come to their aid. The spreadsheet is one example. The use of an improved solution tool will not come about unless knowledge of the tool and its capabilities exists together with a knowledge of the tasks tackled by the organization. As has been mentioned above, the best results will come only when there is a proper awareness of requirements and capabilities.

1.17.2.2 Word processing

Probably the most widespread computer application now is that of word processing. Although, traditionally, an author has passed his original (e.g. manuscript or dictated tape) to another person who has sole responsibility for production of the typed form, this may well change. With the increasing use of computers many workers who at some stage take on the role of author, are becoming more or less keyboard-competent. Now, while it would be too much to claim that such authors can key as well as a professional typist, there is an increasing number of these 'sometime' authors who can type quickly enough to keep up with their own creative thought processes. Also, by using a few of the more basic capabilities of the word processing system the author can produce a good result that is well ordered and cogent (even if the spelling and layout may leave something to be desired) more quickly than with the older techniques. Even the problems of spelling and of layout can, in part, be tackled by the computer. It seems hardly likely that the secretary is under serious threat from such developments of author capability but the notion of the copy-typing task may well be one that will disappear. Provided reasonable control can be exercised over aspects of detail and over the proper use of an individual's time, there may well soon be a substantial increase in the number of engineers producing their own reports.

1.17.2.3 Networks

It is this chameleon-like behaviour of the user at his desk, wanting to be structural analyst, financial analyst, typist, etc. which makes the proper arrangement of personal computers that are able to double as terminals so important. While, in some circumstances, it may be suitable to have these personal computer networks connected to one another without the existence of any 'heart' machine, as described above, it seems likely that the more frequent situation will be one in which the central computer is needed to provide not only backup facilities but also the corporate data (details of cost rates for example) which many users may require.

1.17.3 Specific vs. general-purpose software

When seeking software there is sometimes a choice between a program which has been designed for a strictly limited and well-defined purpose on the one hand and a program designed to be general-purpose on the other hand. To illustrate this, consider the problem of producing drawings of reinforcement details as discussed above. The end product drawing is nothing more than a set of lines drawn on a piece of paper. Some of these lines are straight, some curved and some are in the form of characters. Thus, a general-purpose drawing package, capable of putting lines on the paper according to data instructions defining, for example, cartesian coordinates of end points of line segments, could produce the required drawing if the data are prepared. On the other hand, the special-purpose detailing program will require far less data in order to produce the same end result. It will, partly in consequence, be very much easier for humans to understand the data of the special-purpose version at a glance. They are thereby more able to spot mistakes and correct them.

On the other hand, the detailing program will be no use for the production of general-arrangement drawings or for a host of other tasks. It will generally be the case that the general-purpose program will feel, to the user, much more cumbersome, than a program built to specific purpose. When this occurs, most operators begin to feel that they are not properly in control and thereby become a little careless, allowing mistakes to creep in.

It is not practicable to produce special-purpose programs for all problems; there are too many problems. Indeed, even a special-purpose program will be, to some extent, general-purpose. (A detail drawing program will, for instance, be capable of drawing a wide variety of beam types, though it may not be capable of drawing a column.)

There is no universal answer to this choice problem. It is a question which can be resolved only by harnessing a proper awareness.

1.17.4 Computers and information

The last half decade has seen an explosion in the amount of data stored in computers. (Technically this has become possible as the cost of unit storage has reduced.) However, there is no point in storing data in a computer if it cannot be accessed with both speed and ease, and then manipulated to meet the need.

Computers traditionally have been regarded as 'unintelligent' so that information would, of necessity, be stored only in carefully prearranged patterns in order to permit subsequent location and retrieval. The argument has been that, if there is no pattern, retrieval will be impossible, so do not store. (There have inevitably been 'squirrels' who have adopted the policy of storing everything, in case it may be useful. This philosophy has not been regarded as generally cost-effective.)

Information, which can be expensive to collect and to keep, has typically been stored in large databases to which accredited users can gain access. These databases have generally been in very well-defined structural forms. In consequence access has, in general, been rapid. However, the design of the database envisages 'all' possible accesses which might be used in future. Now, however, increased machine speeds and the production of 'intelligent' software is cutting across these restrictions. The way ahead is not clear, nor is it likely to be quick because of the sheer volume of information which is available to man. However, there will be movement towards making information, generally, more easily available.

1.17.5 Computers and management

The size of projects in which mankind engages has increased enormously; so has the complexity. The management of projects (and the training of managers) has become a major problem. Early tools to come to the aid of management have included bar-chart techniques, etc. The problem with most of these techniques is the volume of work necessary to cope with the inevitable alterations to the original plan. These alterations are liable to occur throughout the life of the project. Ideally, the manager would like to examine the effect of the change forced on him and then consider possible effects of changing his own plan for proceeding. Of course, the calculation power of the computer is the facility which makes such possibilities realistic.

However, this is only dealing with the techniques. There is also the problem of training managers, preferably without the trainees making mistakes (with very large cost consequences) on a real job. Developments in universities and research organizations have played a major part here.

1.17.5.1 Training games

These developments take the form of the simulation of a construction project, e.g. the construction of a manhole. This simulation is incorporated in a 'training game'. The game is set up by the tutor and included in it are details of the project and rates, e.g. for crane hire. Some of this information is made available to the player, who is invited to manage the construction. For each day's work his management will take the form of ordering types of labour and/or materials. The player has options, e.g. a cheap but not too reliable crane hire company, as opposed to a more reliable, more expensive one. The simulation makes available weather forecasts for the following day at the stage when the player is ordering. As in real life the weather is *generally* similar to the forecasts but differences do occur. With the labour and materials he has ordered, a certain amount of construction will get done in the day, and for this the player earns credit or payment. On the other side the labour and materials will be expensive. The actual progress of the work is subject to statistical interruptions whose level of occurrence is set by the tutor in advance. The objective for the players is to make a profit that is as large as possible.

The use of this training tool seems to be most effective when the player is actually a small team of about four students. The element of competition provided by three other teams working at the same time (but independently so they suffer different statistical 'accidents') increases the learning by sharing complementary experiences. There is clear evidence that this training is effective: it is certainly cheaper than making mistakes on a real job.

1.17.5.2 Project planning models

A further illustration of the way computers assist with tackling the unknown is to be found in a tool to be used in advance planning. For this application the project is modelled as a fairly conventional bar chart (possibly at more than one level). However, the model is not deterministic, i.e. it is recognized that when the chart is constructed, a bar is only a best advance guess and that it is subject, in the event, to variation. For many items (e.g. weather effects, rate of bricklaying, etc.) data is available about the variations which occur in practice. These variations are incorporated with the basic data. The best-guess bars represent just one way in which the project might be built. Changing one bar (within its allowed variation) produces another way the project might be built. What the computer does is to 'build' the project many hundreds of times allowing all the bars to vary stochastically. The result is an envelope of possible construction routes. Some will be quicker, others will be slower; some will be cheaper, others will be more expensive. Overall, however, the envelope will highlight potential holdups caused by delays, indicate cashflow requirements, etc. Clearly, the system can be run not only prior to construction but also during construction (when work already complete is, of course, no longer subject to variation). The tool ideally should be used collaboratively between client and contractor in a noncompetitive manner. At present, the world is far from ideal but there may be sufficient benefit for this route to appeal, especially to those involved in the very large projects for which it is best used. It is interesting that this potentially valuable exercise demands nothing more expensive than a fairly run-of-the-mill desktop personal computer in order to produce useful results.

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The following selection of works is not intended to be exhaustive. The literature of mathematics is vast and that relating to engineers is especially large. Thus, the reader should, in the first instance, turn to works with which he is already familiar as being the quickest way to find the answer to a problem. The list below includes books either of a wide range and general nature and intended for engineers or covering subject matter of recent development, and also some old books on specialist subjects.

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2

Strength of Materials

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Contents

2.1 Introduction

The subject 'Strength of Materials' originates from the earliest attempts to account for the behaviour of structures under load. Thus the problems of particular interest to the first investigators, Galileo and Hooke in the seventeenth century, and Euler and Coulomb in the eighteenth,[1] were the very practical problems associated with the behaviour of beams and columns; at a somewhat later stage, general mathematical investigations of the behaviour of elastic bodies were made by Navier (1821) and Cauchy (1822). The theory of structures has subsequently developed so that it now includes many different and sophisticated fields of interest. Nevertheless, the topic 'Strength of Materials' traditionally covers those aspects of the theory that were the subject of the original research: the theory of bars and the general theory of elasticity. This chapter, therefore, is essentially a review of the main features of these two somewhat disparate theories, and contains some of the results that are of immediate importance to civil engineers.

2.2 Theory of elasticity

2.2.1 Internal stress

Internal stress is the name given to the intensity of the internal forces set up within a body subject to loading. Consider such a body shown in Figure 2.1(a) and an imaginary plane surface within the body passing through a point P. The internal forces exerted between atoms across this surface are represented in the expanded view of Figure 2.1(b). They are described by stress vectors (having the dimensions of force per unit area), and the particular vectors at P give a measure of the intensity of the internal forces at this point. They are denoted by σ and called *internal stress vectors*. If they are directed away from the material as in Figure 2.1(c) they are called *tensile*, and if towards the material *compressive*.

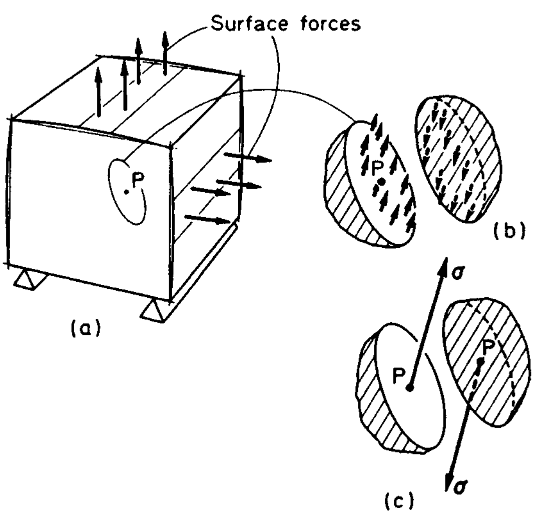


Figure 2.1

2.2.1.1 Components of stress

The complete state of stress at P is defined in terms of the internal stress vectors acting on three particular surfaces at P called the *positive coordinate surfaces*. (The positive x coordinate surface is the surface parallel to the y–z plane of an x, y, z coordinate system, with the material situated so that a vector directed outwards from the material and normal to the surface is in the positive direction of the x coordinate line as in Figure 2.2.)

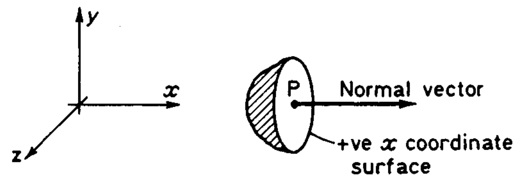


Figure 2.2

These internal stress vectors are distinguished by appropriate subscripts. Thus $\boldsymbol{\sigma}_x$ acts on the positive x coordinate surface, while $\boldsymbol{\sigma}_y$ and $\boldsymbol{\sigma}_z$ respectively act on the y and z surfaces. Their scalar components† are then denoted by two subscripts. Thus the components of $\boldsymbol{\sigma}_x$ are σ_{xx}, σ_{xy} and σ_{xz} and are shown in Figure 2.3(a). Similarly the components of $\boldsymbol{\sigma}_y$ are σ_{yx}, σ_{yy}, σ_{yz} and of $\boldsymbol{\sigma}_z$ are σ_{zx}, σ_{zy}, σ_{zz} as shown in Figure 2.3(b) and (c). σ_{xx}, σ_{yy} and σ_{zz} are called the *direct stress components* at P in the x, y and z directions respectively, while σ_{xy}, σ_{xz}, σ_{yx}, σ_{yz}, σ_{zx}, and σ_{zy} are called the *shear stress components*.

While the above notation is strictly logical and clarifies the basic concepts of stress, conventional engineering notation is somewhat different and emphasizes the physical differences between the components. Thus the direct stress components are written σ_x, σ_y and σ_z, while the shear stress components are written τ_{xy}, τ_{xz}, τ_{yx}, τ_{yz}, τ_{zx}, τ_{zy}. Except in section 2.2.1.3 (below), this latter notation is employed in the remainder of this chapter.

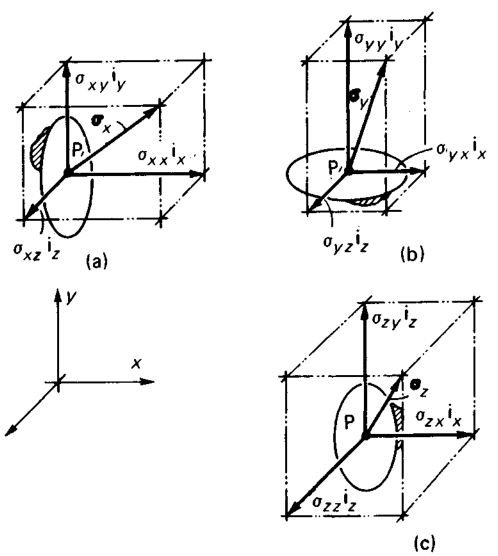


Figure 2.3

2.2.1.2 Stress on an arbitrary surface

Suppose a plane surface through P is defined in terms of the components n_x, n_y and n_z of the outward unit normal vector $\mathbf{n}$, as in Figure 2.4. The stress vector $\boldsymbol{\sigma}_n$ acting on this surface is

† A vector $\mathbf{F}$ at P is equal to $F_x\mathbf{i}_x + F_y\mathbf{i}_y + F_z\mathbf{i}_z$, where F_x, F_y and F_z are the scalar components of $\mathbf{F}$, and $\mathbf{i}_x$, $\mathbf{i}_y$ and $\mathbf{i}_z$ are unit base vectors parallel respectively to the x, y and z coordinate lines at P.

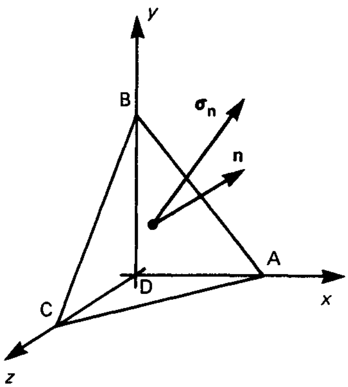


Figure 2.4

obtained in terms of the basic stress components defined in the previous section by considering the linear equilibrium of the differentially small trapezoidal element ABCD shown in the figure. Thus:

$$\sigma_{nx} = \sigma_x n_x + \tau_{yx} n_y + \tau_{zx} n_z \tag{2.1}$$

$$\sigma_{ny} = \tau_{xy} n_x + \sigma_y n_y + \tau_{zy} n_z \tag{2.2}$$

$$\sigma_{nz} = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z \tag{2.3}$$

where σ_{nx}, σ_{ny} and σ_{nz} are the components of σ_n.

2.2.1.3 *Transformation of stress*

Considering a new coordinate system x', y', z' rotated relative to the x, y and z system as in Figure 2.5, then the components of stress in the new system are defined as in section 2.2.1.1, so that $\tau_{x'y'}$ ($=\sigma_{x'y'}$), for example, is the component in the y' direction of the stress vector acting on the positive x' coordinate surface.

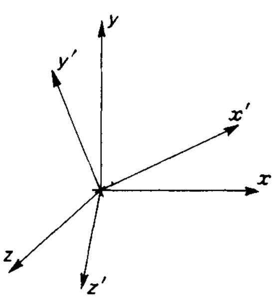


Figure 2.5

The components of stress in the two systems are related by equations of the following type (where for conciseness we employ the original notation of section 2.2.1.1):

$$\begin{aligned}\sigma_{x'y'} = {} & \frac{\partial x}{\partial x'}\frac{\partial x}{\partial y'}\sigma_{xx} + \frac{\partial x}{\partial x'}\frac{\partial y}{\partial y'}\sigma_{xy} + \frac{\partial x}{\partial x'}\frac{\partial z}{\partial y'}\sigma_{xz} \\ & + \frac{\partial y}{\partial x'}\frac{\partial x}{\partial y'}\sigma_{yx} + \frac{\partial y}{\partial x'}\frac{\partial y}{\partial y'}\sigma_{yy} + \frac{\partial y}{\partial x'}\frac{\partial z}{\partial y'}\sigma_{yz} \\ & + \frac{\partial z}{\partial x'}\frac{\partial x}{\partial y'}\sigma_{zx} + \frac{\partial z}{\partial x'}\frac{\partial y}{\partial y'}\sigma_{zy} + \frac{\partial z}{\partial x'}\frac{\partial z}{\partial y'}\sigma_{zz}\end{aligned} \tag{2.4}$$

Equation (2.4) and eight similar equations formed by permuting x', y' and z' are called the *transformation equations of stress*. The partial derivatives in Equation (2.4) are called direction cosines, since $\partial y/\partial x'$, for example, is equal to the cosine of the angle between the y and x' coordinate lines.

2.2.1.4 *Principal stresses*

For a particular orientation of x', y' and z' it is found that all the shear stress components vanish, i.e. that the stress vectors $\sigma_{x'}$, $\sigma_{y'}$ and $\sigma_{z'}$ are directed at right angles to their respective coordinate surfaces. Calling this coordinate system X, Y and Z, the matrix of stress components takes the form:

$$\begin{matrix} \sigma_X & 0 & 0 \\ 0 & \sigma_Y & 0 \\ 0 & 0 & \sigma_Z \end{matrix}$$

The direct stresses σ_X, σ_Y and σ_Z are called the *principal stresses* at P, while the X, Y and Z coordinate lines are called the *principal directions of stress*.

The values of the principal stresses in terms of the stress components in the x, y and z system are equal to the three roots of the equation:

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0 \tag{2.5}$$

where

$$I_1 = \sigma_x + \sigma_y + \sigma_z \tag{2.6}$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 \tag{2.7}$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 \tag{2.8}$$

The direction cosines of the Y coordinate line say, relative to the x, y and z coordinate lines (λ_{Yx}, λ_{Yy}, λ_{Yz}), are found by solving the equations

$$\begin{bmatrix} (\sigma_x - \sigma_Y) & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & (\sigma_y - \sigma_Y) & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & (\sigma_z - \sigma_Y) \end{bmatrix} \begin{bmatrix} \lambda_{Yx} \\ \lambda_{Yy} \\ \lambda_{Yz} \end{bmatrix} = 0 \tag{2.9}$$

$$(\lambda_{Yx})^2 + (\lambda_{Yy})^2 + (\lambda_{Yz})^2 = 1 \tag{2.10}$$

(Note that the three equations represented by Equation (2.9) are not independent.)

2.2.1.5 *Internal equilibrium equations*

Consideration of the equilibrium of a differentially small parallelepiped element of material surrounding an internal point P, leads to three equations of linear equilibrium:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + F_x = 0 \tag{2.11}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \frac{\partial \tau_{xy}}{\partial x} + F_y = 0 \tag{2.12}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + F_z = 0 \tag{2.13}$$

and three equations of rotational equilibrium:

$$\tau_{xy} = \tau_{yx} \qquad (2.14)$$

$$\tau_{yz} = \tau_{zy} \qquad (2.15)$$

$$\tau_{zx} = \tau_{xz} \qquad (2.16)$$

In Equations (2.11 to 2.13), F_x, F_y and F_z are the components of any body force vector **F** (units: force per unit volume) acting at P. Note, for example, that a body force vector of magnitude (ρg)/unit volume is exerted by the Earth at all points within a body situated in its gravitational field, ρ being the local density of the body and g being the acceleration due to gravity.

The shear stress components τ_{xy} and τ_{yx} being equal, are called *complementary shear stresses*. It is apparent from Equations (2.14 to 2.16) that if a body is in equilibrium then only six of the nine stress components can take different values at any point.

2.2.1.6 *Plane stress*

For structures made of elements whose dimensions in the z direction are much smaller than the dimensions in the x and y directions, such as thin plate girders, slabs, shear walls, etc., the following assumptions can be made: (1) the stress components σ_z, τ_{yz}, τ_{xz} can be ignored; and (2) the stress components are uniform across the thickness of the element. That is, they are independent of z.

Such a state of stress is called *plane stress.*

For plane stress, the transformation Equations (2.4) take a simple and important form. Suppose the x', y', z' system is formed by a rotation of $\alpha°$ about the z axis anticlockwise from the reader's viewpoint, as in Figure 2.6. The transformation equations between σ_x, σ_y, τ_{xy} and $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ are then as follows:

$$\sigma_{x'} = \tfrac{1}{2}(\sigma_x+\sigma_y)+\tfrac{1}{2}(\sigma_x-\sigma_y)\cos(2\alpha)+\tau_{xy}\sin(2\alpha) \qquad (2.17)$$

$$\sigma_{y'} = \tfrac{1}{2}(\sigma_x+\sigma_y)-\tfrac{1}{2}(\sigma_x-\sigma_y)\cos(2\alpha)-\tau_{xy}\sin(2\alpha) \qquad (2.18)$$

$$\tau_{x'y'} = -\tfrac{1}{2}(\sigma_x-\sigma_y)\sin(2\alpha)+\tau_{xy}\cos(2\alpha) \qquad (2.19)$$

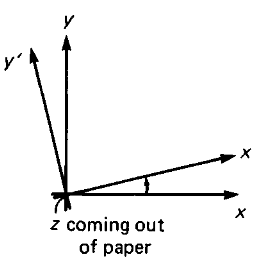


Figure 2.6

These equations can then be represented by the following graphical construction. Two axes are drawn, the vertical representing shear stress and the horizontal, direct stress, and a circle is constructed whose centre is at $(\sigma_x+\sigma_y)/2$ on the direct stress axis, and which passes through the point (σ_x, τ_{xy}) as in Figure 2.7. The line through the centre of the circle at an angle $2\alpha°$ *clockwise* to the line joining the centre and (σ_x, τ_{xy}) then intersects the circle at $(\sigma_{x'}, \tau_{x'y'})$. Produced backwards, it intersects the circle at a point whose abscissa is $\sigma_{y'}$. This construction was devised by Otto Mohr in 1882 and the circle is called *Mohr's circle of stress.*

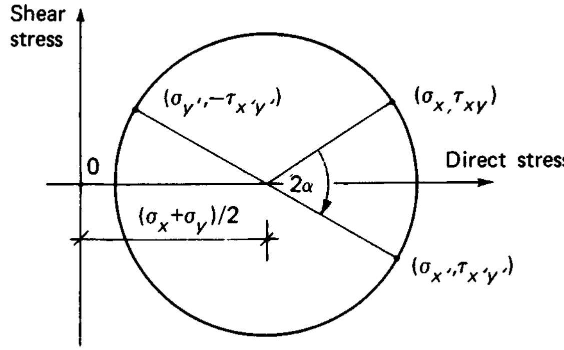


Figure 2.7 Mohr's circle of stress

2.2.2 Strain

Strain is the general name given to the deformation of a body subject to loading.

2.2.2.1 *Displacements*

A particular point P in a body before loading, occupies its initial position P_i say, and after loading its final position P_f. The line joining P_i to P_f is a vector which is denoted by **u** and called the *displacement vector* at P. In general, this vector varies continuously from point to point in the body, and its three components u_x, u_y and u_z are continuous functions of the coordinates of P.†

Consider two neighbouring points P(x, y, z) and P*(x+dx, y+dy, z+dz) in the body. Then

$$du_x = \frac{\partial u_x}{\partial x}dx + \frac{\partial u_x}{\partial y}dy + \frac{\partial u_x}{\partial z}dz \qquad (2.20)$$

$$du_y = \frac{\partial u_y}{\partial x}dx + \frac{\partial u_y}{\partial y}dy + \frac{\partial u_y}{\partial z}dz \qquad (2.21)$$

$$du_z = \frac{\partial u_z}{\partial x}dx + \frac{\partial u_z}{\partial y}dy + \frac{\partial u_z}{\partial z}dz \qquad (2.22)$$

where the differentials du_x, du_y and du_z are the differences between the components of **u** at the two points. As such, these differentials can be regarded as the components of the vector giving the displacement of P* *relative* to P.

2.2.2.2 *Components of strain*

In order to obtain a concise description of the deformation of the material at P it is convenient to define nine dimensionless components ε_{xx}, ε_{yy}, ε_{zz}, ε_{xy}, ε_{yz}, ε_{zx}, ω_{xy}, ω_{yz}, ω_{zx} by the following equations, called the *strain–displacement relations*:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z} \qquad (2.23, 2.24, 2.25)$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right), \quad \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right),$$

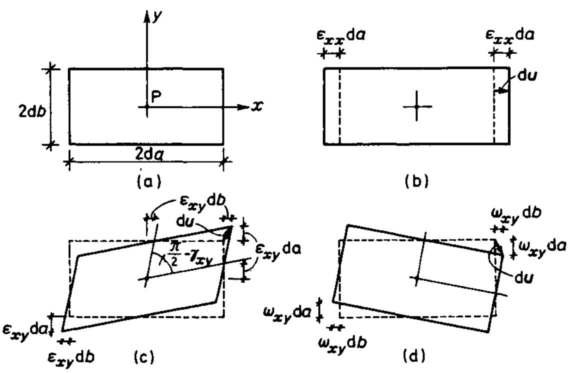
$$\varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right) \qquad (2.26, 2.27, 2.28)$$

† In most cases **u** is so small that the coordinates of P do not change appreciably during the loading.

$$\omega_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}-\frac{\partial u_y}{\partial x}\right),\quad \omega_{yz}=\frac{1}{2}\left(\frac{\partial u_y}{\partial z}-\frac{\partial u_z}{\partial y}\right),$$
$$\omega_{zx}=\frac{1}{2}\left(\frac{\partial u_z}{\partial x}-\frac{\partial u_x}{\partial z}\right) \qquad (2.29, 2.30, 2.31)$$

The physical meaning of these components is clarified by considering the deformation of the rectangular element of material containing P shown in Figure 2.8(a) for each component in turn.

Thus if $\varepsilon_{xx}\neq 0$, $\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon_{xy}=\varepsilon_{yz}=\varepsilon_{zx}=\omega_{xy}=\omega_{yz}=\omega_{zx}=0$ then, by using Equations (2.20 to 2.22), it can be shown that the element deforms as in Figure 2.8(b). ε_{xx}, corresponding to this type of longitudinal deformation is called the *direct strain component* in the x direction at P. If it is positive it is called *tensile* and the element lengthens and if negative, it is called *compressive* and the element shortens. Similarly, the components ε_{yy} and ε_{zz} corresponding respectively to longitudinal deformation in the y and z directions are called the direct strain components in these directions.



Note: deformation shown to an exaggerated scale

Figure 2.8

If $\varepsilon_{xy}\neq 0$, $\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon_{yz}=\varepsilon_{zx}=\omega_{xy}=\omega_{yz}=\omega_{zx}=0$ then $\partial u_x/\partial y=\partial u_y/\partial x=\varepsilon_{xy}$ and again by using Equations (2.20 to 2.22) it can be shown that the element deforms into a lozenge shape as in Figure 2.8(c). Deformation of this type is called shear strain, and ε_{xy} is called the *mathematical shear strain component* at P. The adjective 'mathematical' is used to distinguish between this and the engineering shear strain at P, which is denoted by γ_{xy} and is equal to the closure in radians of the angle between the x and y coordinate lines. From the geometry of Figure 2.8(c) we have

$$\gamma_{xy}=2\varepsilon_{xy} \qquad (2.32)$$

Similarly, the components ε_{yz} and ε_{zx} correspond to shear strain in the y–z and z–x planes respectively.

Finally, if $\omega_{xy}\neq 0$, $\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon_{xy}=\varepsilon_{yz}=\varepsilon_{zx}=\omega_{yz}=\omega_{zx}=0$ then $\partial u_x/\partial y=-\partial u_y/\partial x=\omega_{xy}$ and it can be shown that the element rotates without deformation about the z coordinate line as in Figure 2.8(d). ω_{xy} is called the *rotation* at P. Similarly ω_{yz} and ω_{zx} correspond respectively to local rotations about the x and y coordinate lines through P. These rotations are necessary in the theoretical discussion in order to define the displacement derivatives in Equations (2.20 to 2.22). However, since they do not define *deformation* directly, they are not considered further in elastic analysis.

As in the case of stresses, the conventional engineering notation for the strain components is somewhat different from the above and the direct strain components are written ε_x ε_y and ε_z. Except in section 2.2.2.4 (below), this latter notation is employed in the remainder of the chapter, and the shear strains are described in terms of γ_{xy}, γ_{yz} and γ_{zx}.

In the majority of civil engineering structures, the strain components are very small, of the order of magnitude 10^{-3}. Thus, the deformation of the elements in Figure 2.8 is exaggerated. The strain–displacement relations in Equations (2.23 to 2.28) assume that the *displacements* are small. If this is not the case, nonlinear terms involving the products of the derivatives are included.[2] These nonlinear terms are significant in defining the buckling characteristics of thin elements in compression.[3,4]

2.2.2.3 *Uniform strain*

If the displacement components u_x, u_y and u_z are linear functions of the coordinates of P then the corresponding strains given by Equations (2.23 to 2.28) are uniform. The overall changes in the geometry of a body are then simply related to the strain components. Thus consider, for example, a line AB in or on the surface of the body which originally coincides with an x coordinate line. If the original length of AB is l and its increase in length is Δl, then:

$$\varepsilon_x=\Delta l/l \qquad (2.33)$$

2.2.2.4 *Transformation of strain*

Considering again a new coordinate system x', y', z' rotated relative to the x, y and z system as in Figure 2.5, then the components of strain in this new system are defined by strain–displacement relations similar to Equations (2.23 to 2.28). Thus $\gamma_{x'y'}(=2\varepsilon_{x'y'})$, for example, is given by:

$$\gamma_{x'y'}=\left(\frac{\partial u_{x'}}{\partial y'}+\frac{\partial u_{y'}}{\partial x'}\right) \qquad (2.34)$$

where $u_{x'}$, $u_{y'}$ and $u_{z'}$ are the components of the displacement vector $\mathbf{u}$ relative to x', y' and z'. The components of strain in the two systems are related by equations of the same type as Equation (2.4) (where again for conciseness we employ the original notation of section 2.2.2.2). Thus:

$$\varepsilon_{x'y'}=\frac{\partial x}{\partial x'}\frac{\partial x}{\partial y'}\varepsilon_{xx}+\frac{\partial x}{\partial x'}\frac{\partial y}{\partial y'}\varepsilon_{xy}+\frac{\partial x}{\partial x'}\frac{\partial z}{\partial y'}\varepsilon_{xz}$$
$$+\frac{\partial y}{\partial x'}\frac{\partial x}{\partial y'}\varepsilon_{yx}+\frac{\partial y}{\partial x'}\frac{\partial y}{\partial y'}\varepsilon_{yy}+\frac{\partial y}{\partial x'}\frac{\partial z}{\partial y'}\varepsilon_{yz}$$
$$+\frac{\partial z}{\partial x'}\frac{\partial x}{\partial y'}\varepsilon_{zx}+\frac{\partial z}{\partial x'}\frac{\partial y}{\partial y'}\varepsilon_{zy}+\frac{\partial z}{\partial x'}\frac{\partial z}{\partial y'}\varepsilon_{zz} \qquad (2.35)$$

The nine equations formed by permuting x', y' and z' in Equation (2.35) are called the *transformation equations of strain.*

2.2.2.5 *Principal strains*

For a particular orientation of x', y' and z', all the shear strain components vanish, and in most materials this orientation is the same as that of the principal directions of stress discussed in section 2.2.1.4. Calling the coordinate system X, Y and Z as before, the direct strains ε_X, ε_Y and ε_Z are called the *principal strains* at P.

The values of the principal strains are equal to the three roots of the equation:

$$\varepsilon^3 - E_1\varepsilon^2 + E_2\varepsilon - E_3 = 0 \tag{2.36}$$

where:

$$E_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z \tag{2.37}$$

$$E_2 = \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x - \tfrac{1}{4}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2) \tag{2.38}$$

$$E_3 = \varepsilon_x\varepsilon_y\varepsilon_z + \tfrac{1}{4}(\gamma_{xy}\gamma_{yz}\gamma_{zx} - \varepsilon_x\gamma_{yz}^2 - \varepsilon_y\gamma_{zx}^2 - \varepsilon_z\gamma_{xy}^2) \tag{2.39}$$

2.2.2.6 Compatibility equations

The three displacement components u_x, u_y and u_z can be eliminated from the six strain–displacement relations in Equations (2.23 to 2.28) to produce three equations called the *compatibility equations*, which must be satisfied by the strain components. This elimination can be done in different ways to produce different sets of equations. Two such are:

$$\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} - \frac{\partial^2\gamma_{xy}}{\partial x\partial y} = 0 \tag{2.40}$$

$$\frac{\partial^2\varepsilon_y}{\partial z^2} + \frac{\partial^2\varepsilon_z}{\partial y^2} - \frac{\partial^2\gamma_{yz}}{\partial y\partial z} = 0 \tag{2.41}$$

$$\frac{\partial^2\varepsilon_z}{\partial x^2} + \frac{\partial^2\varepsilon_x}{\partial z^2} - \frac{\partial^2\gamma_{zx}}{\partial z\partial x} = 0 \tag{2.42}$$

$$\frac{2\partial^2\varepsilon_x}{\partial y\partial z} - \frac{\partial}{\partial x}\left(\frac{\partial\gamma_{xy}}{\partial z} - \frac{\partial\gamma_{yz}}{\partial x} + \frac{\partial\gamma_{zx}}{\partial y}\right) = 0 \tag{2.43}$$

$$\frac{2\partial^2\varepsilon_y}{\partial z\partial x} - \frac{\partial}{\partial y}\left(\frac{\partial\gamma_{yz}}{\partial x} - \frac{\partial\gamma_{zx}}{\partial y} + \frac{\partial\gamma_{xy}}{\partial z}\right) = 0 \tag{2.44}$$

$$\frac{2\partial^2\varepsilon_z}{\partial x\partial y} - \frac{\partial}{\partial z}\left(\frac{\partial\gamma_{zx}}{\partial y} - \frac{\partial\gamma_{xy}}{\partial z} + \frac{\partial\gamma_{yz}}{\partial x}\right) = 0 \tag{2.45}$$

2.2.2.7 Plane strain

Plane strain is said to exist when the strain components ε_z, ε_{yz} and ε_{zx} are equal to zero. It occurs when $u_z = 0$ at every point within a region of a body. From symmetry this is the case in the central region of a body which: (1) is very long in the z direction; (2) is of uniform cross-section; and (3) is subjected to loading in the z plane that is uniformly distributed along its length (Figure 2.9). It can therefore occur in structures such as gravity dams, tunnel linings or retaining walls.

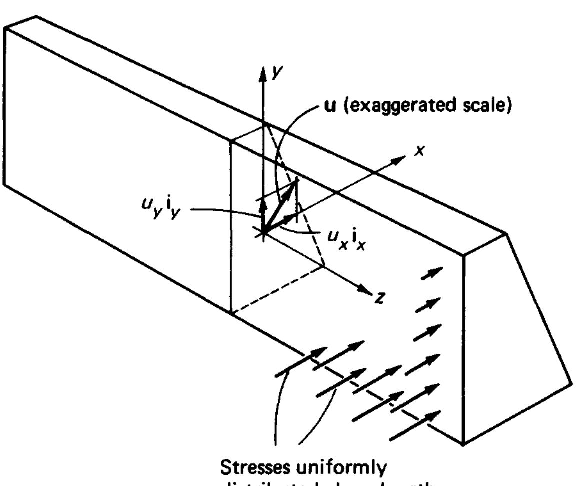


Figure 2.9

Considering again the new coordinate system x', y', z' formed by a rotation of $\alpha°$ anticlockwise about the z axis as in Figure 2.6, the transformation equations between ε_x, ε_y, ε_{xy} and $\varepsilon_{x'}$, $\varepsilon_{y'}$, and $\varepsilon_{x'y'}$ take the same form as Equations (2.17 to 2.19). These transformation equations are represented by a graphical construction called *Mohr's circle of strain*, whose function is the same as that of Mohr's circle of stress.

2.2.3 Elastic stress–strain relations

The relationship between the stress and strain components at a point in a body is a property of the particular material making up the body. For an isotropic elastic material the stress–strain relations are linear and are independent of the orientation of the x, y, z coordinate system. They take the following form:

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha\Delta T \tag{2.46}$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_z + \sigma_x)] + \alpha\Delta T \tag{2.47}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] + \alpha\Delta T \tag{2.48}$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy},\ \gamma_{yz} = \frac{1}{G}\tau_{yz},\ \gamma_{zx} = \frac{1}{G}\tau_{zx} \tag{2.49, 2.50, 2.51}$$

where ΔT is the temperature change from some initial state. E and G are constants having the dimensions of force per unit area and are called *Young's modulus* and the *shear modulus* respectively, ν is a dimensionless constant called *Poisson's ratio* and α is a constant having the dimensions °C^{-1} and is called the *temperature coefficient of expansion*. G in fact is related to E and ν by the following equation:

$$G = E/2(1+\nu) \tag{2.52}$$

Values of E, ν and α for a variety of practical materials are given in Table 2.1.

The corresponding inverse stress–strain relations are found by solving Equations (2.46 to 2.51) for the stresses and are as follows:

$$\sigma_x = 2\mu\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) - (3\lambda + 2\mu)\alpha\Delta T \tag{2.53}$$

$$\sigma_y = 2\mu\varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) - (3\lambda + 2\mu)\alpha\Delta T \tag{2.54}$$

$$\sigma_z = 2\mu\varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) - (3\lambda + 2\mu)\alpha\Delta T \tag{2.55}$$

$$\tau_{xy} = \mu\gamma_{xy},\ \tau_{yz} = \mu\gamma_{yz},\ \tau_{zx} = \mu\gamma_{zx} \tag{2.56, 2.57, 2.58}$$

where for conciseness we employ the *Lamé constants* λ and μ defined in terms of E and ν by the equations:

$$\lambda = \nu E/(1+\nu)(1-2\nu) \tag{2.59}$$

$$\mu = E/2(1+\nu) \tag{2.60}$$

Table 2.1 Properties of materials (representative)

Material	*Density* (kg/m³)	*E* (GN/m²)	μ	α (°C^{-1})	*Limit of proportionality* (MN/m²)	*Ultimate stress* (MN/m²)	*Uniform elongation*
Mild steel	7840	200	0.31	1.25×10^{-5}	280	370	0.30
High-strength steel	7840	200	0.31	1.25×10^{-5}	770	1550	0.10
Medium-strength aluminium alloy	2800	70	0.30	2.3×10^{-5}	230	430	0.10
Titanium alloy	4500	120	0.30	0.9×10^{-5}	385	690	0.15
Magnesium alloy	1800	45	0.30	2.7×10^{-5}	155	280	0.08
Concrete	2410	25	0.20	1.2×10^{-5}	—	3 (tension) 30 (compression)	—
Timber (Douglas fir)	576	7 (with grain)		0.6×10^{-5}	43 (compression with grain)	52 (compression with grain)	—
Glass	2580	60	0.26	0.7×10^{-5}	—	1750	—
Nylon	1140	2	—	10×10^{-5}	77	90	1.00
Polystyrene (not expanded)	1050	4	—	10×10^{-5}	46	60	0.03
High-strength glass-fibre composite	2000	60	—	—	—	1600	—
Carbon fibre composite	1600	170	—	—	—	1400	—

The stress–strain relations hold for a wide range of stresses in most practical materials. They become invalid when the inter-atomic bonds in the materials break down, this process being called *yielding* or *fracture*. Yielding in steel can be demonstrated by the tensile test, where a known stress system $\sigma_x \neq 0$, $\sigma_y=\sigma_z=\tau_{xy}=\tau_{yz}=\tau_{zx}=0$, called *uniaxial stress*, is induced in a specimen and the corresponding strain ε_x is measured. A typical plot of σ_x versus ε_x for a mild steel tensile specimen then takes the form shown in Figure 2.10(a). The initial straight section of the curve of slope equal to E corresponds to Equation (2.46), but at a certain stress of the order of 250 MN/m², the strain increases dramatically with little or no increase of load. This stress is called the *uniaxial yield stress* of mild steel. Subsequently, the stress–strain curve indicates that the specimen supports larger stresses up to a maximum value of the order of 400 MN/m² which is called the *ultimate tensile stress*. The uniaxial stress–strain curve for an aluminium alloy specimen shown in Figure 2.10(b) does not display a marked yield stress and the material is linear elastic up to a stress called the *limit of proportionality* which again is of the order of 250 MN/m². Two other properties frequently quoted in engineering literature, the 0.2% *proof stress* and the *uniform elongation*, are shown in the figure. Values for the limit of proportionality, ultimate stress and uniform elongation are included in Table 2.1.

For accounts of yield criteria and plastic stress–strain relations corresponding to more general stress systems see, for example, Bisplinghoff *et al*,[5] and Prager and Hodge.[6]

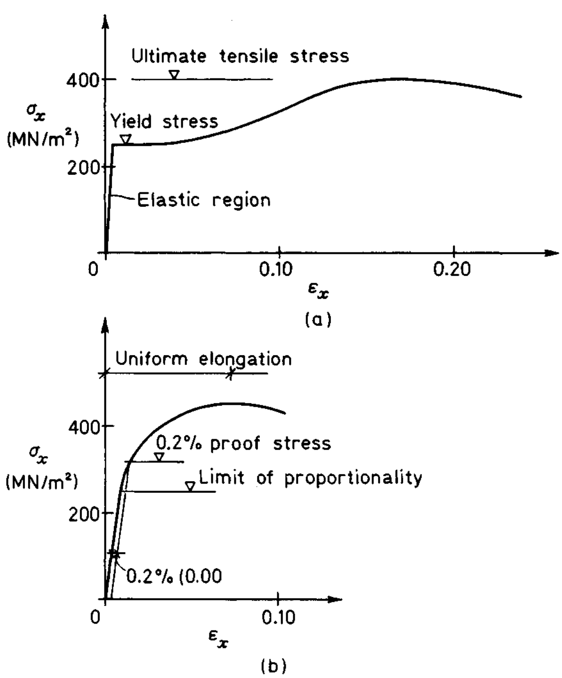


Figure 2.10 Definitions of material properties

2.2.4 Analysis of elastic bodies

The internal equilibrium Equations (2.11 to 2.16), strain–displacement relations Equations (2.23 to 2.28) and the stress–strain relations Equations (2.46 to 2.51) are eighteen differential equations in the unknowns of the analysis problem, namely the nine stress components, the six strain components and the three displacement components. These equations must be satisfied subject to boundary conditions.

2.2.4.1 Boundary conditions

The boundary conditions at a point P on the surface of a body are expressed in terms of the components S_x, S_y and S_z of the surface stress vector **S** acting at P, and the components u_x, u_y and u_z of the displacement vector **u** of P. They are of three types, as follows.

Static boundary conditions. The three stress vector components at P are specified. Thus at an unloaded point on the boundary $S_x=S_y=S_z=0$, while at a loaded point $S_x=k_1$, $S_y=k_2$, $S_z=k_3$, where k_1, k_2 and k_3 are known values at P.

Kinematic boundary conditions. The three displacement components at P are specified. Thus at a rigid support $u_x=u_y=u_z=0$, while at a point whose displacements are constrained by, say, a screw jack $u_x=j_1$, $u_y=j_2$, $u_z=j_3$, where j_1, j_2 and j_3 are known values at P.

Mixed boundary conditions. Certain displacement and certain stress-vector components at P are specified simultaneously. For example, at the point P on the roller support shown in Figure 2.11, $S_x=0$ and $u_y=u_z=0$.

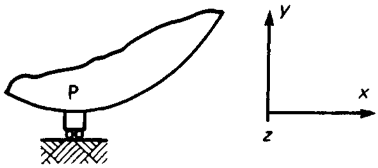


Figure 2.11

2.2.4.2 Solution in terms of displacements

A straightforward solution method involves treating the displacement components as the basic unknowns. The three linear equilibrium Equations (2.11 to 2.13) are expressed in terms of the displacements by using the stress–strain relations followed by the strain–displacement relations. The resulting differential equations in u_x, u_y and u_z are called the *Navier equations*. They are as follows:

$$\mu\nabla^2 u_x+(\lambda+\mu)\frac{\partial\Phi}{\partial x}+F_x=0 \tag{2.61}$$

$$\mu\nabla^2 u_y+(\lambda+\mu)\frac{\partial\Phi}{\partial y}+F_y=0 \tag{2.62}$$

$$\mu\nabla^2 u_z+(\lambda+\mu)\frac{\partial\Phi}{\partial z}+F_z=0 \tag{2.63}$$

where

$$\nabla^2 u_x=\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2} \tag{2.64}$$

and:

$$\Phi=\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z} \tag{2.65}$$

In order to solve these equations, the boundary conditions must all be expressed in terms of the displacements of the surface points. In the case of the static boundary conditions, equations for the internal stress components are obtained using Equations (2.1 to 2.3) with the components of σ_n replaced by the components of S. These are then converted to differential boundary conditions in displacements by again using the stress–strain and the strain–displacement relations. Thus at each internal point and each boundary point there are three simultaneous differential equations in the unknowns u_x, u_y and u_z. In most cases, a direct solution is obtainable only by a numerical procedure such as the finite-difference method.[7]

2.2.4.3 Solution in terms of stresses

A second solution method involves treating the nine internal stress components as the basic unknowns. Six equations in these unknowns are immediately available from the internal equilibrium Equations (2.11 to 2.16). A further three equations are obtained from the compatibility Equations (2.40 to 2.42) or (2.43 to 2.45) by using the stress–strain relations to express them in terms of the stress components. The resulting equations are called the *Beltrami–Michell* equations and are as follows:

$$\nabla^2\sigma_x+\frac{1}{(1+\nu)}\frac{\partial^2\Theta}{\partial x^2}=\frac{-\nu}{(1-\nu)}\Psi-\frac{2\partial F_x}{\partial x} \tag{2.66}$$

$$\nabla^2\sigma_y+\frac{1}{(1+\nu)}\frac{\partial^2\Theta}{\partial y^2}=\frac{-\nu}{(1-\nu)}\Psi-\frac{2\partial F_y}{\partial y} \tag{2.67}$$

$$\nabla^2\sigma_z+\frac{1}{(1+\nu)}\frac{\partial^2\Theta}{\partial z^2}=\frac{-\nu}{(1-\nu)}\Psi-\frac{2\partial F_z}{\partial z} \tag{2.68}$$

or:

$$\nabla^2\tau_{xy}+\frac{1}{(1+\nu)}\frac{\partial^2\Theta}{\partial x\partial y}=-\left(\frac{\partial F_x}{\partial y}+\frac{\partial F_y}{\partial x}\right) \tag{2.69}$$

$$\nabla^2\tau_{yz}+\frac{1}{(1+\nu)}\frac{\partial^2\Theta}{\partial y\partial z}=-\left(\frac{\partial F_y}{\partial z}+\frac{\partial F_z}{\partial y}\right) \tag{2.70}$$

$$\nabla^2\tau_{zx}+\frac{1}{(1+\nu)}\frac{\partial^2\Theta}{\partial z\partial x}=-\left(\frac{\partial F_z}{\partial x}+\frac{\partial F_x}{\partial z}\right) \tag{2.71}$$

where

$$\Theta=\sigma_x+\sigma_y+\sigma_z \tag{2.72}$$

$$\Psi=\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z} \tag{2.73}$$

The only problems than can be solved directly in terms of stresses conveniently are those in which all the boundary conditions are static boundary conditions. In such problems, three equations in the internal stress components are obtained using Equations (2.1 to 2.3) and these, together with the three equations of rotational equilibrium and the three compatibility equations, provide the required nine equations at the boundary. In problems where displacements are specified at various boundary points, the corresponding boundary stresses cannot usually be obtained in advance of the solution except for those special cases where the body is externally statically determinate.

Direct solutions in terms of stresses can in principle be obtained using numerical procedures. However, many solutions, especially to two-dimensional problems,[2,8] have been obtained using stress functions which automatically satisfy the equilibrium equations.

2.2.5 Energy methods

2.2.5.1 Virtual work

Consider a body which is in equilibrium under surface stresses S over part of its surface and body forces F. Suppose the corresponding internal stress system is given by σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx}. This is called an *equilibrium force system*.

Next consider an entirely independent system of displacements $\mathbf{u}^*$ which vary continuously from point to point in the body and satisfy the kinematic boundary conditions. Suppose the corresponding strain system is given by ε_x^*, ε_y^*, ε_z^*, γ_{xy}^*, γ_{yz}^*, γ_{zx}^*. This is called a *compatible displacement system*.

The virtual work W_e^* done by the external forces S and F, supposing they were to move through $\mathbf{u}^*$, is as follows:

$$W_e^*=\int_A(S_xu_x^*+S_yu_y^*+S_zu_z^*)\,dA+\int_V(F_xu_x^*+F_yu_y^*+F_zu_z^*)\,dV \tag{2.74}$$

where $\int_A(\ )\,dA$ represents an integral taken over the loaded surface of the body, and $\int_V(\ )\,dV$ represents an integral taken over its volume. By a purely mathematical operation[5] it can be shown that

$$W_e^* = W_i^* \tag{2.75}$$

where W_i^* is a quantity called the *internal virtual work* and is given by

$$W_i^* = \int_V (\sigma_x \varepsilon_x^* + \sigma_y \varepsilon_y^* + \sigma_z \varepsilon_z^* + \tau_{xy}\gamma_{xy}^* + \tau_{yz}\gamma_{yz}^* + \tau_{zx}\gamma_{zx}^*)\,dV \tag{2.76}$$

Equation (2.75) is called the *equation of virtual work*. Note that its derivation is independent of the nature of the stress–strain relations of the material making up the body.

2.2.5.2 Strain energy

Consider the body in equilibrium under **S** and **F** and suppose differential changes in the loading d**S** and d**F** occur causing corresponding differential changes in the *real* displacements d**u**. d**u** and the strains $d\varepsilon_x$, $d\varepsilon_y$, $d\varepsilon_z$, $d\gamma_{xy}$, $d\gamma_{yz}$, $d\gamma_{zx}$ can be regarded as a compatible system of displacements in Equation (2.75). The work terms on either side of Equation (2.75) are then differential quantities of real work caused by the loading change. In particular the internal work is given by

$$dW_i = \int_V (\sigma_x\,d\varepsilon_x + \sigma_y\,d\varepsilon_y + \sigma_z\,d\varepsilon_z + \tau_{xy}\,d\gamma_{xy} + \tau_{yz}\,d\gamma_{yz} + \tau_{zx}\,d\gamma_{zx})\,dV \tag{2.77}$$

Using the elastic stress–strain relations it is possible to integrate Equation (2.77) to obtain the total internal work done on an elastic body from the initial state with zero stress to the final state with stresses corresponding to **S** and **F**. This internal work is found to be independent of the loading path to the final state and is called the *elastic strain energy U*. It can be expressed in three forms:

$$\begin{aligned} U &= \int_V \frac{1}{2E}\Big[(\sigma_x^2+\sigma_y^2+\sigma_z^2) - 2\nu(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x) \\ &\quad + 2(1+\nu)(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\Big]\,dV \\ &= \int_V \tfrac{1}{2}(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\sigma_z\varepsilon_z+\tau_{xy}\gamma_{xy}+\tau_{yz}\gamma_{yz}+\tau_{zx}\gamma_{zx})\,dV \\ &= \int_V \Big[\mu(\varepsilon_x^2+\varepsilon_y^2+\varepsilon_z^2) \\ &\quad + \frac{\lambda}{2}(\varepsilon_x+\varepsilon_y+\varepsilon_z)^2 + \frac{\mu}{2}(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)\Big]\,dV \end{aligned} \tag{2.78}$$

2.2.5.3 Principle of stationary total potential energy

The external work done by the loading in the previous subsection is given by:

$$\begin{aligned} dW_e &= \int_A (S_x\,du_x + S_y\,du_y + S_z\,du_z)\,dA \\ &\quad + \int_V (F_x\,du_x + F_y\,du_y + F_z\,du_z)\,dV \end{aligned} \tag{2.79}$$

If the loading is *conservative*, so that all the loads on the body are independent of the displacements, it is possible to define a function V as follows:

$$V = U - \int_A (S_x u_x + S_y u_y + S_z u_z)\,dA - \int_V (F_x u_x + F_y u_y + F_z u_z)\,dV \tag{2.80}$$

so that the equation of virtual work for the differential change of the body in equilibrium takes the form:

$$d\Phi = 0 \tag{2.81}$$

Φ is called the *total potential energy* of the system of the body plus loads.

Equation (2.81) is the mathematical statement of the Principle of Stationary Total Potential Energy. Thus, *for a body in equilibrium, the total potential energy is stationary with respect to small changes in the actual displacements of the body*. This is the most important energy principle, and its method of application for the solution of structures involves expressing all the displacements of the structure in terms of a (usually limited) number of degrees of freedom. (This can be done exactly for frameworks, but only approximately for structures such as slabs.) The stationary position of the total potential energy is found by equating to zero the derivatives of Φ with respect to the degrees of freedom. The resulting equations are analogous to the stiffness equations in the stiffness method of structural analysis. They are solved for the degrees of freedom to yield the exact or approximate displacements of the structure corresponding to equilibrium.

If the structural displacements are assumed to be small so that the linear strain–displacement relations in Equations (2.23 to 2.28) are applicable, then it can be shown that the potential energy is a *minimum* for a structure in equilibrium.[9] The equilibrium is then said to be *stable*. If the displacements are not small, and the non-linear strain–displacement relations are used to obtain Φ, the equilibrium potential energy can either be a *minimum* or a *maximum*. In the latter case the equilibrium is said to be *unstable*. For certain values of load called the critical loads or *eigenvalues*, the equilibrium is *neutral*. This is indicated mathematically when the determinant of the coefficient matrix in the stiffness equations is zero. Extensive treatments of the eigenvalue problem have been given in many texts, e.g. by Croll and Walker[10] and by Thompson and Hunt.[11]

2.2.6 Measurement of stress and strain

2.2.6.1 Surface strain

The measurement of strain is usually limited to obtaining direct strains tangential to the surfaces of structures by means of mechanical or electrical strain gauges. If the complete state of tangential strain at a surface point is to be determined, then separate measurements of direct strain have to be obtained in three distinct directions at the point. In interpreting these measurements, we then use the fact that two of the principal directions of stress and strain are tangential to the surface whilst the third is normal to it. Thus using, for example, a 45° strain-gauge rosette, producing strain measurements ε_1, ε_2 and ε_3 as shown in Figure 2.12, it can be shown that the principal direction X is at $\theta°$ anticlockwise to the x coordinate line where:

$$\tan(2\theta) = \frac{(2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)}{(\varepsilon_1 - \varepsilon_3)} \tag{2.82}$$

The two principal surface strains ε_X and ε_Y are then given by:

$$\varepsilon_X = \frac{(\varepsilon_1+\varepsilon_3)}{2} + r \quad \varepsilon_Y = \frac{(\varepsilon_1+\varepsilon_3)}{2} - r \tag{2.83, 2.84}$$

where

$$r = \tfrac{1}{2}[(\varepsilon_1-\varepsilon_3)^2 + (2\varepsilon_2-\varepsilon_1-\varepsilon_3)^2]^{1/2} \tag{2.85}$$

Example 2.1. The strains measured by the three gauges of the 45° rosette shown in Figure 2.12 are respectively:

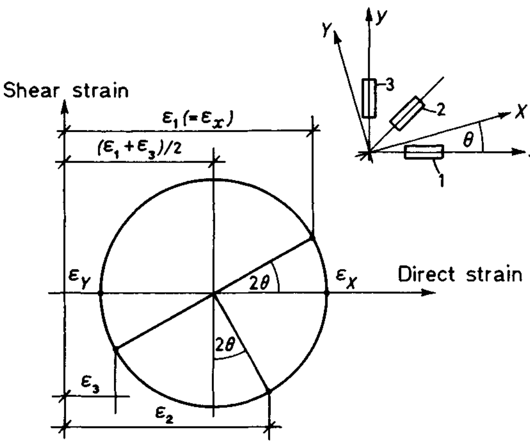


Figure 2.12

$\varepsilon_1 = -5.0\times10^{-4}$ $\varepsilon_2 = +3.0\times10^{-4}$ $\varepsilon_3 = +1.0\times10^{-4}$

What are the principal strains at the point and the orientation of the principal direction X, to the x coordinate line?
From Equation (2.85):

$$r=\tfrac{1}{2}[(-5.0-1.0)^2+(2\times3.0+5.0-1.0)^2]^{1/2}\times10^{-4}$$

$$=5.8\times10^{-4}$$

Thus:

$$\varepsilon_X=3.8\times10^{-4}\quad \varepsilon_Y=-7.8\times10^{-4}$$

From Equation (2.82):

$$\tan(2\theta)=-1.667$$

Thus:

$$2\theta=-59.0^\circ \quad\text{or}\quad 121.0^\circ$$

The ambiguity in the expression for θ is resolved by examining the position of the strains on the Mohr's circle of strain for the surface plane (Figure 2.13). Thus, it is clear that in this example, 2θ must be greater than 90°. The X coordinate line is therefore directed at 60.5° anticlockwise to the x coordinate line.

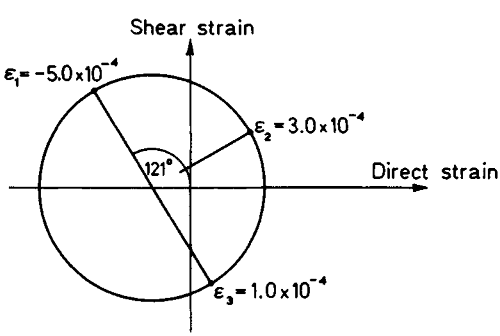


Figure 2.13

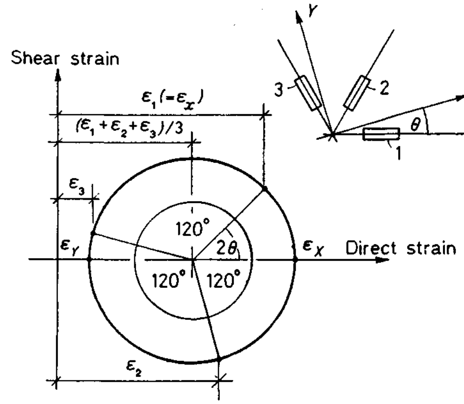


Figure 2.14

Another common layout for strain gauges is the 60° rosette shown in Figure 2.14. The principal direction X is then at θ° anticlockwise to the x coordinate line where:

$$\tan(2\theta)=\sqrt{3}(\varepsilon_2-\varepsilon_3)/(2\varepsilon_1-\varepsilon_2-\varepsilon_3) \tag{2.86}$$

while the principal surface strains ε_X and ε_Y are given by

$$\varepsilon_X=\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}+r \quad \varepsilon_Y=\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}-r \tag{2.87, 2.88}$$

where

$$r=\tfrac{2}{3}(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2-\varepsilon_1\varepsilon_2-\varepsilon_2\varepsilon_3-\varepsilon_3\varepsilon_1)^{1/2} \tag{2.89}$$

The complete state of surface stress corresponding to the strains measured above can be found from the stress–strain relations, noting that in the absence of surface loading the state of stress is one of plane stress.

2.2.6.2 The photoelastic method[12,13]

A good indication of the internal stresses in model structures can be obtained by making use of the property of certain materials such as glass and plastics, that they become double-refracting when subject to stress.

The apparatus for photoelastic stress analysis consists essentially of a light source L (Figure 2.15), a *polarizer* P, and an *analyser* A and the model M of photoelastic material, which is held in a reaction frame and subjected to loads. The lenses L_1 and L_2 are arranged so that a parallel beam of light passes through the model. An image containing bands of different colours then appears on the ground glass screen, these colours representing regions of equal principal stress difference $(\sigma_X-\sigma_Y)$ in the model. For further experimental and theoretical details see, for example, Hendry.[2]

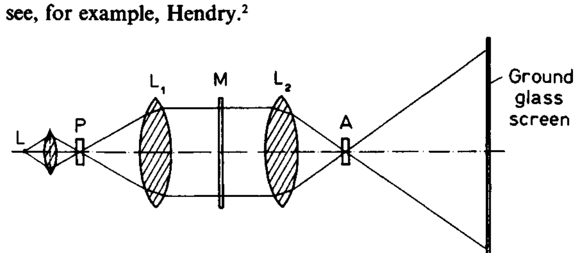


Figure 2.15

2.3 Theory of bars (beams and columns)

2.3.1 Introduction

A great many engineering structures contain components whose dimensions in two coordinate directions are small compared with their dimensions in the third. These components can be called *bars* as a means of general classification, although they are often given other names to denote the particular way they are loaded in structures. Thus if they are subjected to tensile forces they are called *ties*, to compressive forces they are called *struts* or *columns*, to lateral forces they are called *beams*, while if they are subjected to both compressive and lateral forces they are called *beam-columns*.

Structures completely composed of bars are called *frames*, and are either two-dimensional *plane frames*, or three-dimensional *space frames*.

This section reviews the engineering theory of straight bars of uniform cross-section.

2.3.2 Cross-section geometry

2.3.2.1 First moment of area

Consider a bar of some particular cross-sectional shape shown in Figure 2.16, and the two orthogonal axes y and z. (The choice of axes with y horizontal and z downwards, is quite arbitrary but has two advantages when applied to beams: (1) the displacements of a beam are usually vertically downwards, and therefore in the positive direction of z; and (2) as shown in section 2.3.5, a positive bending moment about the y axis causes tension on the bottom of the beam; and therefore positive stresses occur at points in the beam defined by positive values of z.) The first moment of area of the cross-section about the y axis G_y, is defined as the sum of the products obtained by multiplying each element of cross-sectional area dA by its distance z from the y axis. Thus:

$$G_y=\int_A z\,\mathrm{d}A \tag{2.90}$$

Similarly:

$$G_z=\int_A y\,\mathrm{d}A \tag{2.91}$$

The position of the *centroid* of the cross-section is such that the first moment of area about any axis passing through it is zero. Thus if C is the centroid in Figure 2.16, then

$$G_y=G_z=0$$

From this it is clear that C must lie on any axis of symmetry of the section. The centroid can be located in general by selecting any two orthogonal axes y' and z'. The coordinates of the centroid relative to this system, y'_c and z'_c, are then given by:

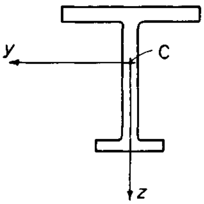


Figure 2.16

$$y'_c=G_{z'}/A \qquad z'_c=G_{y'}/A \tag{2.92, 2.93}$$

where A is the area of the cross-section. The positions of the centroids of various cross-sectional shapes are shown in Table 2.2.

The *longitudinal axis* of the bar is defined as the line passing through the centroids of its cross-sections.

2.3.2.2 Moments of inertia

The *moment of inertia*† of the cross-section about the y axis I_y, is defined as the sum of the products obtained by multiplying each element of cross-sectional area dA by the square of its distance z from the y axis. Thus:

$$I_y=\int_A z^2\,\mathrm{d}A \tag{2.94}$$

Similarly:

$$I_z=\int_A y^2\,\mathrm{d}A \tag{2.95}$$

The *product of inertia*, I_{yz} is defined as:

$$I_{yz}=\int_A yz\,\mathrm{d}A \tag{2.96}$$

where y and z are the respective distances of each element of area dA from the z and y axes.

The *polar moment of inertia* of the cross-section about the x axis, I_p, is defined as:

$$I_p=\int_A r^2\,\mathrm{d}A \tag{2.97}$$

where r is the distance of each element of cross-sectional area dA from the x axis. Note that since $r^2=(y^2+z^2)$

$$I_p=\int_A (y^2+z^2)\,\mathrm{d}A=I_z+I_y \tag{2.98}$$

If y' is an axis parallel to the centroidal axis y and distance c from it, then:

$$I_{y'}=I_y+Ac^2 \tag{2.99}$$

The relationship in Equation (2.99) is known as the *parallel axis theorem*. This theorem facilitates the calculation of the moments of inertia of a complicated cross-section, for the section can be divided into separate simpler elements of area A_e say, whose moments of inertia I_{ye} about their *own* centroidal axes are known. If then c_e is the distance of an element centroid from the y axis, we have:

$$I_y=\sum_{\text{elements}} (I_{ye}+A_e c_e^2) \tag{2.100}$$

The moments of inertia about their centroidal axes, of various sectional shapes are given in Table 2.2.

2.3.2.3 Transformation of moments of inertia

Consider a new system of centroidal axes, y' and z', formed by a rotation of α° anticlockwise about the x axis as shown in Figure

† The term 'moment of inertia' is commonly used in engineering texts because the quantity I_y defined by Equation (2.94) is directly proportional to the mechanical moment of inertia about the y axis, of a thin lamina of the same shape as the cross-section. A more precise term for I_y is the 'second moment of area'.

Table 2.2 Geometrical properties of plane sections

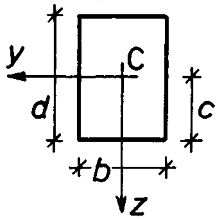
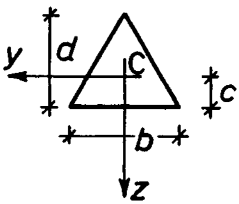
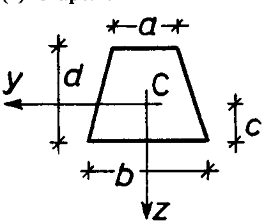
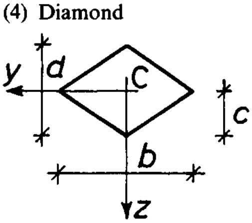
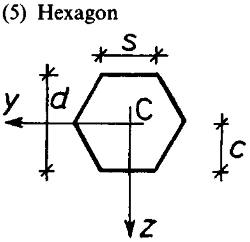
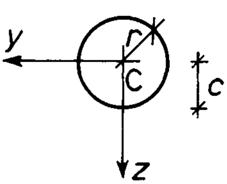
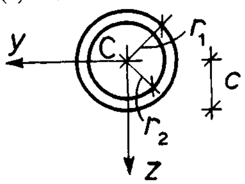
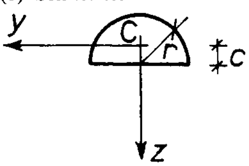
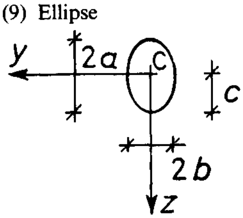
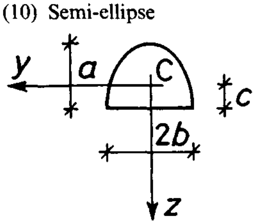
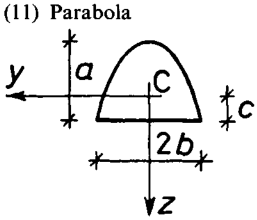
Section	*Area A*	*Position of centroid C*	*Moments of inertia*
(1) Rectangle 	$A=bd$	$c=d/2$	$I_y=bd^3/12$ $I_z=db^3/12$
(2) Triangle 	$A=bd/2$	$c=d/3$	$I_y=bd^3/36$ $I_z=db^3/48$
(3) Trapezium 	$A=d(a+b)/2$	$c=d(2a+b)/3(a+b)$	$I_y=d^3(a^2+4ab+b^2)/36(a+b)$ $I_z=d(a^3+a^2b+ab^2+b^3)/48$
(4) Diamond 	$A=bd/2$	$c=d/2$	$I_y=bd^3/48$ $I_z=db^3/48$
(5) Hexagon 	$A=0.866d^2$	$c=d/2$	$I_y=I_z=0.0601d^4$
(6) Circle 	$A=\pi r^2$ $=3.1416r^2$	$c=r$	$I_y=I_z=\pi r^4/4$ $=0.7854r^4$

Table 2.2 Geometrical properties of plane sections

Section	*Area A*	*Position of centroid C*	*Moments of inertia*
(7) Hollow circle 	$A=\pi(r_1^2-r_2^2)$ $=3.1416(r_1^2-r_2^2)$	$c=r_1$	$I_y=I_z=(\pi/4)(r_1^4-r_2^4)$ $=0.7854(r_1^4-r_2^4)$
(8) Semicircle 	$A=\pi r^2/2$ $=1.5708r^2$	$c=0.424r$	$I_y=[(\pi/8)-(8/9\pi)]r^4$ $=0.1098r^4$ $I_z=\pi r^4/8$ $=0.3927r^4$
(9) Ellipse 	$A=\pi ab$	$c=a$	$I_y=(\pi/4)ba^3=0.7854ba^3$ $I_z=(\pi/4)ab^3=0.7854ab^3$
(10) Semi-ellipse 	$A=\pi ab/2$	$c=0.424a$	$I_y=0.1098ba^3$ $I_z=0.3927ab^3$
(11) Parabola 	$A=4ab/3$	$c=2a/5$	$I_y=0.0914ba^3$ $I_z=0.2666ab^3$

2.17. Then the inertias $I_{y'}$, $I_{z'}$ and $I_{y'z'}$, being defined in the same way as I_y, I_z and I_{yz} in Equations (2.94 to 2.96), are related to I_y, I_z and I_{yz} by the equations:

$$I_{y'}=\tfrac{1}{2}(I_y+I_z)+\tfrac{1}{2}(I_y-I_z)\cos(2\alpha)-I_{yz}\sin(2\alpha) \qquad (2.101)$$

$$I_{z'}=\tfrac{1}{2}(I_y+I_z)-\tfrac{1}{2}(I_y-I_z)\cos(2\alpha)+I_{yz}\sin(2\alpha) \qquad (2.102)$$

$$I_{y'z'}=\tfrac{1}{2}(I_y-I_z)\sin(2\alpha)+I_{yz}\cos(2\alpha) \qquad (2.103)$$

Note that these transformation equations are similar in form to the transformation equations of plane stress in Equations (2.17 to 2.19), the difference being in the sign of α.

For a certain orientation of y' and z', the product of inertia $I_{y'z'}$ vanishes. Denoting these coordinates by Y and Z, then I_Y and I_Z are called the *principal moments of inertia* of the cross-section, and Y and Z are called the *principal axes*. Concerning their orientation, it can be shown in particular that one of the principal axes always coincides with an axis of symmetry in the section. Values of I_Y and I_Z for standard rolled sections are given in BS 4.[14]

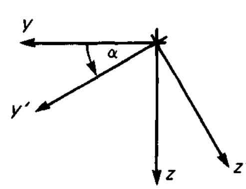


Figure 2.17

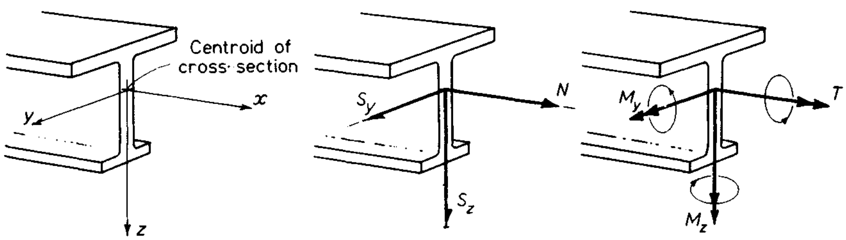


Figure 2.18

2.3.3 Stress resultants

The stresses acting across a particular cross-section of a bar under loads, are conveniently represented by their resultant forces and couples relative to the three coordinate axes x, y and z. Thus the resultants acting on the material of the bar on the negative† side of the cross-section are considered positive when acting in the directions shown in Figure 2.18 and are defined as follows:

Resultant	*Defining equation*	
Axial force N	$N=\int_A \sigma_x \,dA$	(2.104)
Bending moment about the y axis M_y	$M_y=\int_A \sigma_x z\,dA$	(2.105)
Bending moment about the z axis M_z	$M_z=-\int_A \sigma_x y\,dA$	(2.106)
Shear force in the y direction S_y	$S_y=\int_A \tau_{xy}\,dA$	(2.107)
Shear force in the z direction S_z	$S_z=\int_A \tau_{xz}\,dA$	(2.108)
Torque T	$T=\int_A(-\tau_{xy}z+\tau_{xz}y)\,dA$	(2.109)

These resultants are in equilibrium with the loads acting on that part of the bar which is on the negative side of the cross-section. Thus, if the bar is statically determinate, the resultants can be obtained directly by resolving and taking moments.

A *stress resultant diagram* represents the variation of the stress resultant with x for a specified bar loading. The diagram is drawn positive in the direction of the y and z coordinates. Thus given the beam subject to the vertical forces shown in Figure 2.19(a), the shear force (S_z) diagram and the bending moment (M_y) diagram take the form shown in Figures 2.19(b) and (c) respectively. Note that a positive bending moment M_y, causes tension on the bottom of the beam and therefore that the bending moment diagram is located on the tension side of the member. This orientation of the bending-moment diagram is very useful in reinforced concrete design leading to an immediate visual impression of where in the beam the tension reinforcement needs to be placed.

It is sometimes of interest in the case of beams to consider the value of a stress resultant (or any other parameter), at a particular point P in the beam, for various positions of a load moving across the beam. If, for example, we consider the bending moment about the y axis at P ($[M_y]_P$), caused by a unit vertical force at point x on the beam, then $[M_y]_P$ is a function of

† 'Negative' means the side in the negative direction of the x axis.

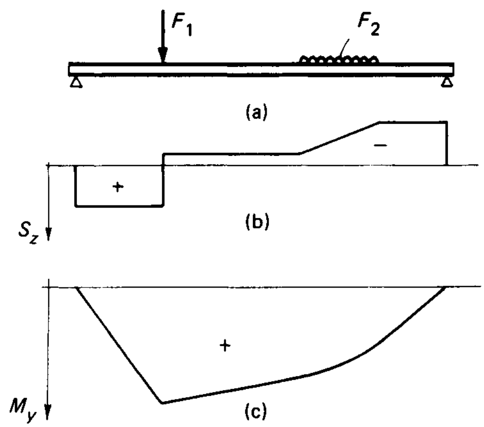


Figure 2.19

the coordinate x. The plot of $[M_y]_P$ versus x is called the *influence line* of M_y at P. Thus for the beam AB in Figure 2.20 the influence lines for $[S_z]_P$ and $[M_y]_P$ are as shown.

The stress resultants are not all independent of each other. Thus considering the rotational equilibrium about the y axis of a small element of a bar subject to a vertical distributed load q per unit length, as in Figure 2.21:

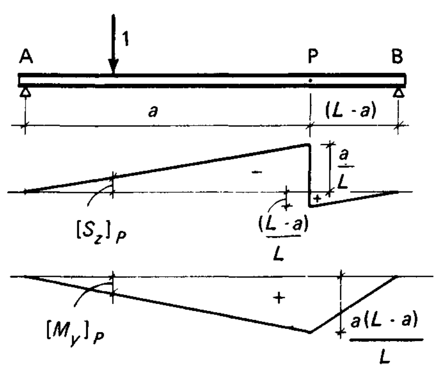


Figure 2.20

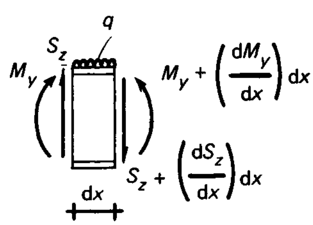


Figure 2.21

$$dM_y/dx = S_z \quad (2.110)$$

Further, considering vertical equilibrium:

$$dS_z/dx = -q \quad (2.111)$$

Whence, combining Equations (2.110) and (2.111) gives:

$$d^2M_y/dx^2 = -q \quad (2.112)$$

A similar set of equations relates M_z, S_y and the horizontal loading on the bar.

2.3.4 Bars subject to tensile forces (ties)

Consider a bar subject to axial forces N, produced by the loading shown in Figure 2.22.

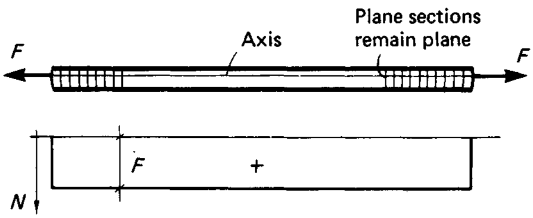


Figure 2.22

From the symmetry of the system at some distance from the loading points it can be *deduced* that plane sections originally normal to the longitudinal axis remain plane and normal to the axis after the deformation, while from the geometry of the bar, it can be assumed that the only nonzero component of stress is σ_x.[8]

The stress–strain relations corresponding to the uniaxial state of stress take the form:

$$\varepsilon_x = (\sigma_x/E) + \alpha\Delta T \quad (2.113)$$

$$\varepsilon_y = \varepsilon_z = -(\nu\sigma_x/E) + \alpha\Delta T \quad (2.114)$$

and it follows that at some distance from the loading points:

$$\sigma_x = N/A \quad (2.115)$$

$$\varepsilon_x = (N/EA) + \alpha\Delta T \quad (2.116)$$

2.3.5 Beams subject to pure bending

2.3.5.1 Beams symmetric about the vertical plane and subject to vertical loading

Consider a beam subject to a uniform bending moment M_y over part of its length, produced, for example, by the loading shown in Figure 2.23. (Note that Equation (2.110) implies that a uniform bending moment can only occur in the absence of shear forces.) From the symmetry of the system it can be *deduced* that: (1) the beam deforms in the vertical plane, and straight-line generators parallel to the longitudinal axis deform into segments of circles with a common centre; and (2) planes originally normal to the axis remain plane and normal to the axis after deformation.

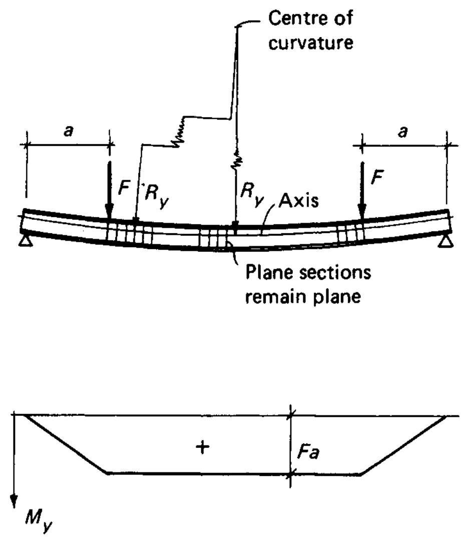


Figure 2.23

It can again be assumed that: (3) the only nonzero component of stress is σ_x.

The above three conditions are the fundamental assumptions made in the *engineering theory of the bending of beams.*

The surface containing those points in the beam at which $\varepsilon_x = 0$ is called the *neutral surface*. The intersection of the neutral surface with a cross-section produces a line called the *neutral axis*.

From the geometry of the deformation, the uniaxial stress–strain relations in Equations (2.113, 2.114), and the requirement of axial equilibrium ($N = 0$), it follows that:

(1) The neutral axis is given by the equation:

$$z = 0 \quad (2.117)$$

i.e. it is a horizontal straight line, coincident with the y coordinate line, and passing through the centroid of the section.

(2) $$\sigma_x = \frac{M_y z}{I_y} \quad (2.118)$$

and

$$\frac{1}{R_y} = \frac{M_y}{EI_y} \quad (2.119)$$

where R_y is the vertical radius of curvature of the beam axis.

2.3.5.2 Composite beams

Suppose the beam in the previous subsection is made of two materials of Young's modulus E_1 and E_2 respectively comprising areas A_1 and A_2 of the total cross-section, as in Figure 2.24. The three conditions of the engineering theory of the bending of beams discussed in the previous subsection still apply. It therefore follows that:

(1) The neutral axis is a horizontal straight line passing through a point C′ called the *equivalent centroid of the cross-section.*

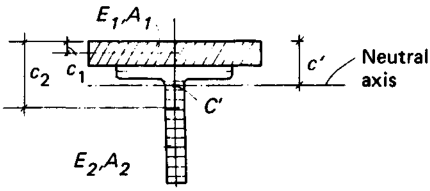


Figure 2.24

This is defined as being such that the first moment of *Young's modulus times area* about any axis passing through it is zero. Thus if c' is the distance of C' from the upper boundary of the beam and c_1 and c_2 are the distances of the respective centroids of the areas A_1 and A_2 from the upper boundary, then:

$$c' = \frac{E_1 A_1 c_1 + E_2 A_2 c_2}{E_1 A_1 + E_2 A_2} \tag{2.120}$$

(2) $[\sigma_x]_{A_1} = \dfrac{M_y z}{I'_y}$ $\quad [\sigma_x]_{A_2} = \dfrac{E_2}{E_1}\dfrac{M_y z}{I'_y}$ (2.121, 2.122)

and

$$\frac{1}{R_y} = \frac{M_y}{E_1 I'_y} \tag{2.123}$$

where $[\sigma_x]_{A_1}$ represents the axial stress in the area A_1, etc. I'_y is the *equivalent moment of inertia* of the cross-section defined as:

$$I'_y = \int_{A_1} (z^2)\, dA_1 + \frac{E_2}{E_1} \int_{A_2} (z^2)\, dA_2 \tag{2.124}$$

where $\int_{A_1} (\ )\, dA_1$ represents an integral taken over the area A_1, etc. In the above equations, the coordinates are relative to axes y and z passing through the equivalent centroid of the section.

2.3.5.3 Reinforced concrete beams

A reinforced concrete beam behaves as a composite beam, except that where the concrete is in tension (i.e. below the neutral axis for positive bending about the y axis) its stress-bearing capacity is taken to be zero (Figure 2.25). Otherwise the conditions of the engineering theory of the bending of beams still apply.

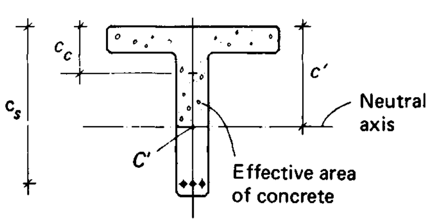


Figure 2.25

Let the subscripts c and s denote parameters associated respectively with the concrete and the steel. It then follows that:

(1) The neutral axis is a horizontal straight line passing through the equivalent centroid whose distance c' from the upper boundary of the beam is given by:

$$c' = \frac{E_c A_c c_c + E_s A_s c_s}{E_c A_c + E_s A_s} \tag{2.125}$$

(Note that since A_c and c_c are themselves functions of c', Equation (2.125) is an implicit equation.)

(2) $[\sigma_x]_c = \dfrac{M_y z}{I'_y}$ $\quad [\sigma_x]_s = \dfrac{E_s}{E_c}\dfrac{M_y z}{I'_y}$ (2.126, 2.127)

and

$$\frac{1}{R_y} = \frac{M_y}{E_c I'_y} \tag{2.128}$$

where

$$I'_y = \int_{A_c} (z^2)\, dA_c + \frac{E_s}{E_c} \int_{A_s} (z^2)\, dA_s \tag{2.129}$$

Example 2.2. A rectangular reinforced concrete beam with a single layer of reinforcement is shown in Figure 2.26. For this section:

$$c' = \frac{E_s A_s}{E_b\, b}\left[\left(1 + \frac{2E_c}{E_s}\frac{b(d-e)}{A_s}\right)^{1/2} - 1\right] \tag{2.130}$$

$$I'_y = \frac{bc'^3}{3} + \frac{E_s}{E_c} A_s [d - (c' + e)]^2 \tag{2.131}$$

Note that the ratio $E_s : E_c$ is generally taken to be 15.

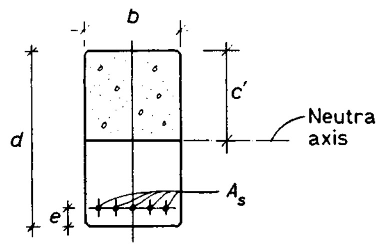


Figure 2.26

2.3.5.4 Beams of asymmetric section subject to both vertical and horizontal loading

Consider again a beam of homogeneous material. The general case of pure bending occurs when the beam is of asymmetric section and is subject to uniform bending moments M_y and M_z (Figure 2.27) over part of its length.

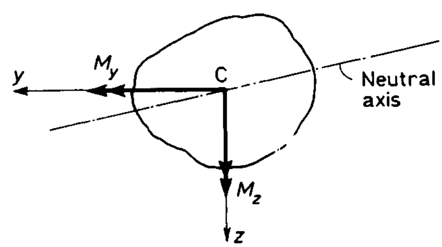


Figure 2.27

From the symmetry of the system it can be *deduced* that straight-line generators parallel to the axis of the beam deform into curves of constant horizontal and vertical curvature. The other conditions discussed in section 2.3.5.1 still apply.

From the geometry of the deformation, the uniaxial stress–strain relations and the requirement that $N = 0$, it follows that:

(1) The neutral axis is given by the equation:

$$(M_y I_z + M_z I_{yz}) z - (M_z I_y + M_y I_{yz}) y = 0 \tag{2.132}$$

i.e. it is a straight line passing through the centroid of the section, as shown in Figure 2.27.

(2) $$\sigma_x = \frac{(M_y I_z + M_z I_{yz}) z - (M_z I_y + M_y I_{yz}) y}{(I_y I_z - I_{yz}^2)} \tag{2.133}$$

$$\frac{1}{R_y} = \frac{(M_y I_z + M_z I_{yz})}{E(I_y I_z - I_{yz}^2)} \quad \frac{1}{R_z} = \frac{-(M_z I_y + M_y I_{yz})}{E(I_y I_z - I_{yz}^2)} \tag{2.134, 2.135}$$

where R_z is the horizontal radius of curvature of the beam axis.

Note:

(1) If the loading is vertical so that $M_z = 0$, Equation (2.135) indicates that the deformed beam is curved horizontally, i.e. $R_z \neq 0$.
(2) If y and z are principal axes, so that $I_{yz} = 0$, Equations (2.134, 2.135) indicate that the curvature about each axis is proportional only to the bending moment about that axis.

In some cases, where a standard commercial section is mounted obliquely, as in Figure 2.28(a) for example, $I_{y'}$, $I_{z'}$ and $I_{y'z'}$ will be known relative to the axes y', z', while the bending moments will be known about the axes y and z. In order to use the results in Equations (2.132 to 2.135) it is preferable to work in terms of the y' and z' axes and resolve the bending moments into equivalent moments about these axes, as in Figure 2.28(b).

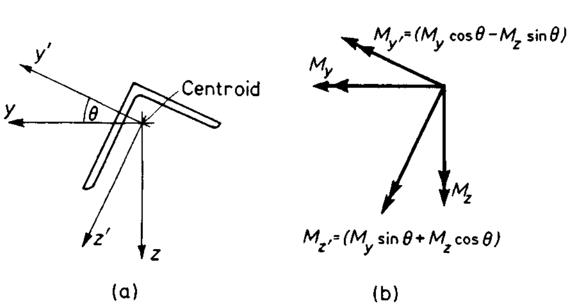


Figure 2.28

2.3.6 Beams subject to combined bending and shear

Practical loading arrangements on beams generally produce a combination of bending and shear stress resultants as, for example, in Figure 2.19.

2.3.6.1 Beams symmetric about the vertical plane and subject to vertical loading

Consider a point in a beam at which both M_y and S_z are nonzero. The presence S_z then implies the existence of the shear stresses τ_{xz} on the cross-section and corresponding shear strains γ_{xz}, and much of the symmetry of the deformation of a beam under a uniform bending moment is lost. In particular, plane sections no longer remain plane.

The following approximate analysis of the problem is due to St Venant.[15] It is assumed that the direct stresses σ_x and curvature $(1/R_y)$ are the same as they would be if M_y were acting alone. They are therefore given by Equations (2.118, 2.119). The shear stresses in the beam are then obtained by considering the longitudinal equilibrium of the element of length dx shown shaded in the cross-sectional view of Figure 2.29. Thus employing Equations (2.110) and (2.118), namely $dM_y/dx = S_z$ and $\sigma_x = M_y z / I_y$, it can be shown that the *mean* longitudinal shear stress τ on the surface ABCD is given by:

$$\tau = \frac{S_z A_e c_e}{b_e I_y} \tag{2.136}$$

where A_e is the cross-sectional area of the element, c_e is the distance of its centroid from the neutral axis, and b_e is the length of the curve joining AB (Figure 2.29).

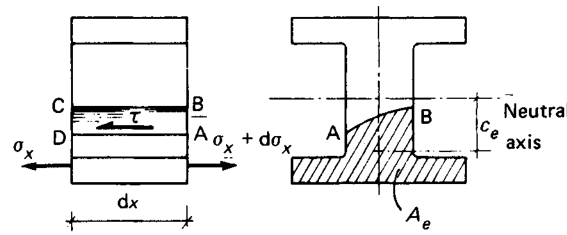


Figure 2.29

τ can then be related to the shear stresses τ_{xy} and τ_{xz} on the cross-section as follows. If the cut surface ABCD is a horizontal plane (i.e. it is a z-coordinate surface) then τ is the mean value of the shear stress component τ_{zx} on that surface. Whence, since $\tau_{zx} = \tau_{xz}$, it follows that τ is also the mean value of τ_{xz} on the line AB. For thin sections, we assume that τ_{xz} is uniformly distributed across the width so that:

$$\tau_{xz} = \tau \tag{2.137}$$

Thus for the rectangular section shown in Figure 2.30, Equation (2.136) gives the following parabolic distribution of shear stress on the cross-section:

$$\tau_{xz} = \frac{3S_z}{2bd^3}(d^2 - 4z^2) \tag{2.138}$$

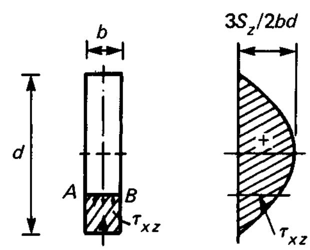


Figure 2.30

If the cut surface ABCD is a vertical plane (a y-coordinate surface) then τ is the mean value of the shear stress component τ_{yx} on that surface, or the mean value of τ_{xy} on the line AB. Thus for an I-section, the shear stresses in the flanges are as shown in Figure 2.31.

2.3.6.2 Composite beams

The existence of the longitudinal shear stress τ (Figure 2.29) is of

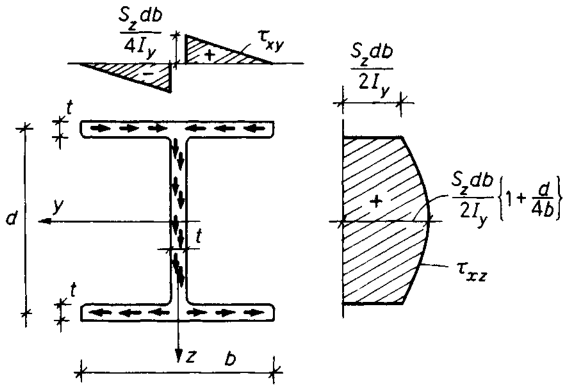


Figure 2.31

special significance in built-up composite beams, because this stress has to be transmitted between the separate components of the beams by means of suitable bonds such as welds, rivets or shear connectors.

Thus consider a beam composed of two materials of Young's modulus E_1 and E_2 respectively comprising areas A_1 and A_2 of the total cross-section (Figure 2.32). The position of the neutral axis and the equivalent moment of inertia of the cross-section are again given by Equations (2.120) and (2.124), whence, employing the assumptions of St Venant's theory, it can be deduced that the mean longitudinal shear stress at the interface AB is given by:

$$\tau=\frac{S_z A_1(c'-c_1)}{bI'_y} \tag{2.139}$$

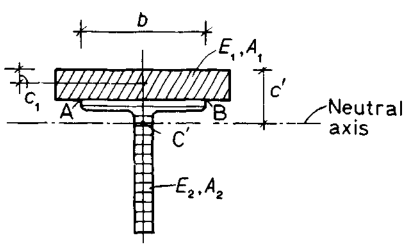


Figure 2.32

while the corresponding longitudinal shear force/unit length of beam F is given by:

$$F=b\tau \tag{2.140}$$

If the beam were composed, say, of a concrete slab connected to a steel T-section joist, then F would be transmitted by stud shear connectors of the type shown in Figure 2.33 which would be welded on to the top face of the T-section. Supposing that the factored shear strength of each connector were known experimentally to be F_s, then the connectors would need to be distributed at a concentration of F/F_s per unit length of beam.

2.3.6.3 The shear centre (beams asymmetric about the vertical plane)

In a beam of asymmetric cross-section the shear stresses given by St Venant's theory contribute to a torque T. Consider, for

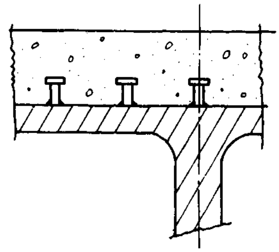


Figure 2.33

example, the shear stresses produced in the channel section shown in Figure 2.34. They are statically equivalent to the stress resultants S_f acting in the two flanges, and S_w in the web, where:

$$S_w=S_z \tag{2.141}$$

$$S_f=\frac{S_z b^2 dt}{4I_y} \tag{2.142}$$

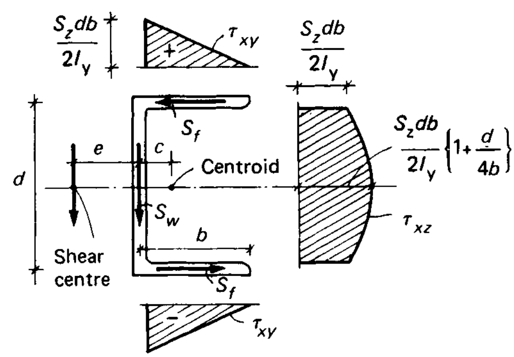


Figure 2.34

and because of the asymmetry of the section, they produce a torque T acting about the longitudinal axis of the channel given by

$$T=S_z c+\frac{S_z b^2 d^2 t}{4I_y} \tag{2.143}$$

An important assumption of St Venant's theory is that the beam deflects vertically without twist. Thus, it can be deduced that if the loading on the beam is such that it produces the torque T, then twisting does not, in fact, occur. (If the loading did not produce T then some twisting of the beam would be necessary in order to modify the torque obtained in Equation (2.143).) T can be applied by positioning the vertical loading so that its resultant at any cross-section lies at a suitable distance from the centroid. Thus the torque in the channel can be produced by the loading shown in Figure 2.35. The point at

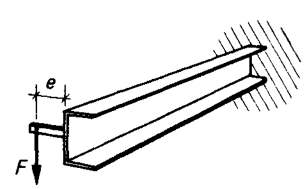
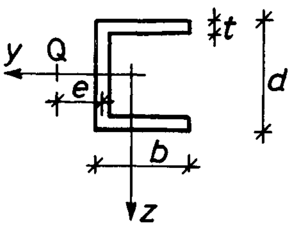


Figure 2.35

Table 2.3 Shear centres of the walled sections

Section	*Position of shear centre Q*

(1) Channel



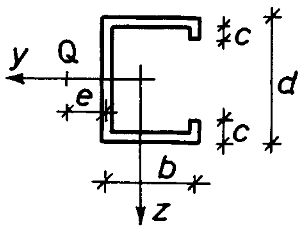
$$e=d\left(\frac{H_{yz}}{I_y}\right)$$

where H_{yz} is the product of inertia of the half section (above the y axis).

If t is uniform:

$$e=\frac{b^2d^2t}{4I_y}$$

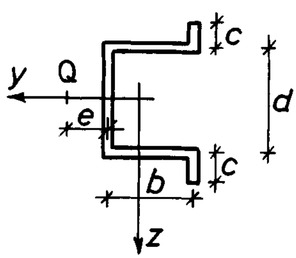
(2) Lipped channel



Values of (e/d)

c/d \ b/d	1.0	0.8	0.6	0.4	0.2
0.0	0.430	0.330	0.236	0.141	0.055
0.1	0.477	0.380	0.280	0.183	0.087
0.2	0.530	0.425	0.325	0.222	0.115
0.3	0.575	0.470	0.365	0.258	0.138
0.4	0.610	0.503	0.394	0.280	0.155
0.5	0.621	0.517	0.405	0.290	0.161

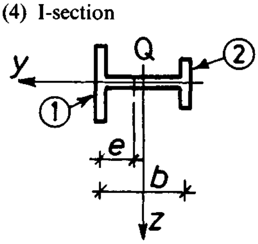
(3) Hat-section



Values of (e/d)

c/d \ b/d	1.0	0.8	0.6	0.4	0.2
0.0	0.430	0.330	0.236	0.141	0.055
0.1	0.464	0.367	0.270	0.173	0.080
0.2	0.474	0.377	0.280	0.182	0.090
0.3	0.453	0.358	0.265	0.172	0.085
0.4	0.410	0.320	0.235	0.150	0.072
0.5	0.355	0.275	0.196	0.123	0.056
0.6	0.300	0.225	0.155	0.095	0.040

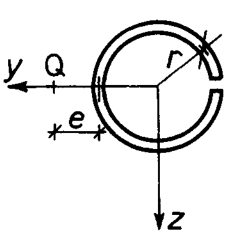
(4) I-section



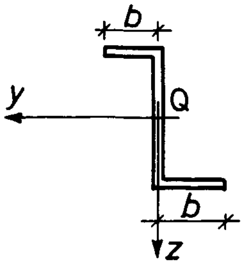
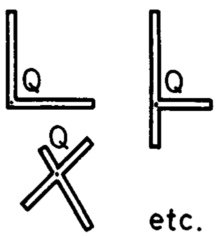
$$e=\frac{bI_2}{I_1+I_2}$$

where I_1 and I_2 respectively denote the moments of inertia about the y axis of flange 1 and flange 2

(5) Split circle



$$e=r$$

Section	*Position of shear centre Q*
(6) Z-section 	Shear centre coincides with centroid
(7) Sections with elements intersecting at a single point 	Shear centre lies at point of intersection

which the vertical resultant crosses the neutral axis is then called the *shear centre*, and for the channel section it is located at a distance e from the web (Figure 2.34) where

$$e = \frac{b^2 d^2 t}{4I_y} \tag{2.144}$$

The positions of the shear centres of various cross-sectional shapes are shown in Table 2.3.

The *shear axis* of the beam is defined as the line passing through the shear centres of its cross-sections, and by definition, the resultants of all lateral forces acting on the beam must pass through this axis if the beam is to deflect without twist.

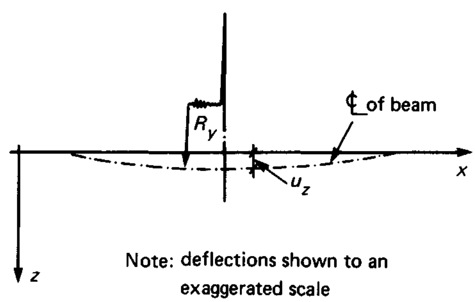


Figure 2.36

2.3.7 Deflection of beams

According to St Venant's theory, the curvature of a beam subject to combined bending and shear is given by Equation (2.119) thus: $1/R_y = M_y/EI_y$. Suppose u_z is the corresponding vertical deflection of the longitudinal axis of the beam, then from the geometry of the deformation (Figure 2.36), it can be shown that:

$$\frac{1}{R_y} = -\frac{d^2u_z}{dx^2} \Bigg/ \left(1 + \left(\frac{du_z}{dx}\right)^2\right)^{3/2} \tag{2.145}$$

In practice, the slopes of beams are extremely small and the denominator of the right-hand side of Equation (2.145) can be taken to be equal to unity, whence, combining Equations (2.119) and (2.145) gives the following differential equation:

$$\frac{d^2u_z}{dx^2} + \frac{M_y}{EI_y} = 0 \tag{2.146}$$

called the *differential equation of beams*. For statically determinate beams, where M_y can be found as a function of x, this second-order equation can be solved subject to boundary conditions by double integration. The solution $u_z(x)$ is then the deflected shape of a beam produced by the applied loading. Examples of the boundary conditions for particular cases are shown in Figure 2.37. Special techniques, such as the step function method[16a] and the moment-area method[15] have been devised to simplify the analysis.

The differential equation of beams can be expressed in two further forms using the results of Equations (2.110) and (2.112). Thus from Equation (2.110) we have:

$$\frac{d^3u_z}{dx^3} + \frac{S_z}{EI_y} = 0 \tag{2.147}$$

while from Equation (2.112) we have:

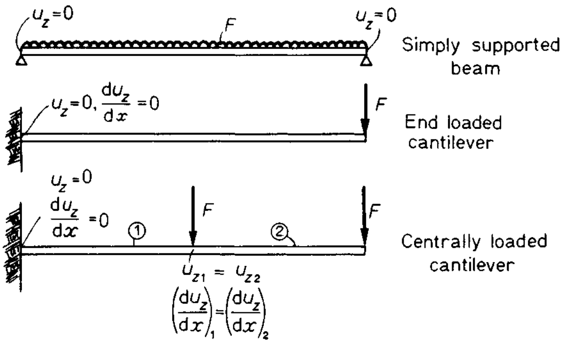


Figure 2.37

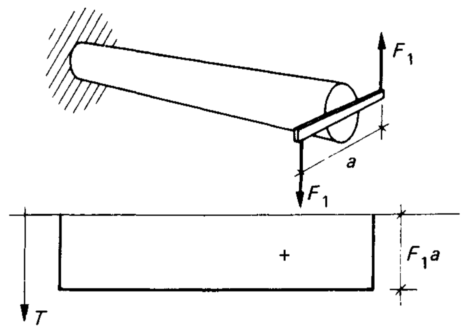


Figure 2.39

$$\frac{d^4u_z}{dx^4}-\frac{q}{EI_y}=0 \tag{2.148}$$

Equation (2.148), expressing the deflections of beams in terms of the lateral loading, is directly equivalent to the three-dimensional Navier Equations (2.61 to 2.63), and can be solved if the boundary conditions are expressed in terms of the displacements. The solution of this equation as opposed to Equation (2.146), is necessary when a beam is statically indeterminate, i.e. when M_y cannot be found in advance. Examples of the required displacement boundary conditions for particular cases are shown in Figure 2.38.

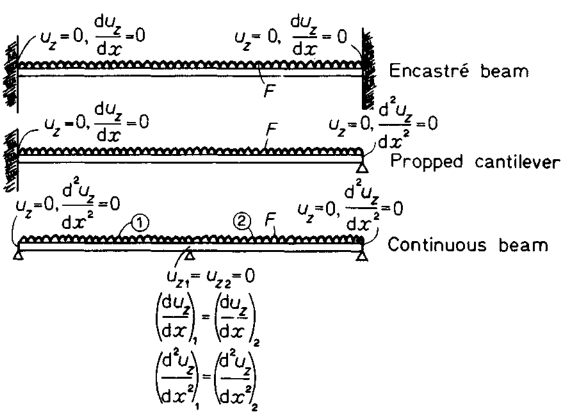


Figure 2.38

An interesting modification of Equation (2.148) occurs when a beam rests on an elastic foundation. Suppose the stiffness of the foundation is k per unit length of beam. Then in addition to the vertical applied loading q, the foundation resists the deflection of the beam with distributed forces equal to ku_z per unit length. Equation (2.148) then takes the form:

$$\frac{d^4u_z}{dx^4}+ku_z-\frac{q}{EI_y}=0 \tag{2.149}$$

Examples of the solution of this equation are given by Hetényi.[16b]

2.3.8 Bars subject to a uniform torque

2.3.8.1 Bars of circular cross-section

Consider a bar subject to a uniform torque T produced, for example, by the loading shown in Figure 2.39.

From the symmetry of the system it can be deduced that: (1) the bar twists about its longitudinal axis; (2) planes originally normal to the axis remain plane and normal to the axis and rotate like rigid laminae, and (3) the rotation θ of any plane is proportional to its distance along the beam.

From the geometry of the deformation and the shear stress–strain relations in Equations (2.49 to 2.51), it follows that:

$$\tau_{xt}=\frac{Tr}{J} \tag{2.150}$$

$$\frac{d\theta}{dx}=\frac{T}{GJ} \tag{2.151}$$

where τ_{xt} is the shear stress on the cross-section at a distance r from the axis, and tangential to the circle of radius r (Figure 2.40). J is a sectional constant, equal in this case to the polar moment of inertia I_p about the longitudinal axis.

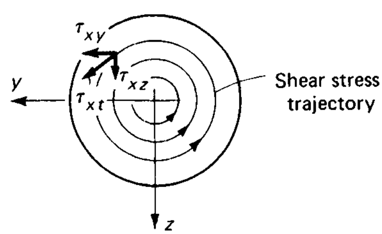


Figure 2.40

The quantity $d\theta/dx$ being the rate of change of rotation with x is called the *twist* of the bar, and is clearly uniform when the bar is subject to uniform torque.

2.3.8.2 Bars of arbitrary cross-section

The three assumptions of section 2.3.8.1 can be shown to lead to impossible values of τ_{xt} at the boundaries of an arbitrary section, since in order to satisfy longitudinal equilibrium conditions, τ_{xt} must be tangential to those boundaries (Figure 2.41).

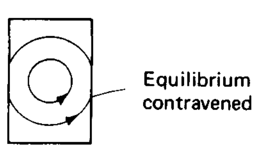


Figure 2.41

The theory for the analysis of bars of arbitrary section is again due to St Venant.[8] Thus the assumption in the previous subsection that plane sections remain plane is relaxed, and a point P is assumed to have an axial displacement u_x given by:

$$u_x = \frac{d\theta}{dx}\psi(y, z) \tag{2.152}$$

u_x is called the *warping* of the section, and is directly proportional to the twist, but is independent of x. The shear stresses τ_{xy} and τ_{xz} are then expressed in terms of a stress function $\phi(y, z)$ by the equations:

$$\tau_{xy} = \partial\phi/\partial z \quad \tau_{xz} = -\partial\phi/\partial y \tag{2.153, 2.154}$$

so that the internal equilibrium Equations (2.11) to (2.13) are identically satisfied. Satisfaction of the compatibility Equations (2.40) and (2.42) then leads to the following equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = -2G\left(\frac{d\theta}{dx}\right) \tag{2.155}$$

Equilibrium conditions require that ϕ is constant along the boundaries of the section, and if the section is solid ϕ can be conveniently taken as zero along the boundaries, whence it can be shown that:

$$T = 2\int_A \phi \, dA \tag{2.156}$$

Equations (2.155) and (2.156) are solved simultaneously, either numerically, or experimentally,[8] and the shear stresses corresponding to T are obtained from Equations (2.153) and (2.154). The results can be expressed in the following form:

$$[\tau_{xb}]_{max} = T/k \tag{2.157}$$

$$d\theta/dx = T/GJ \tag{2.158}$$

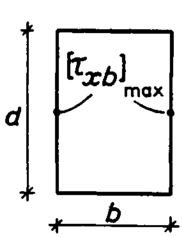
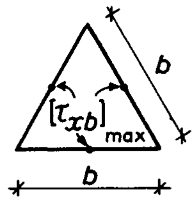
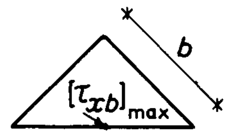
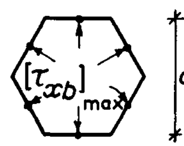
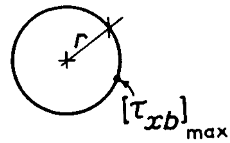
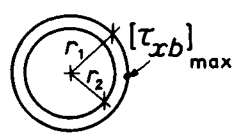
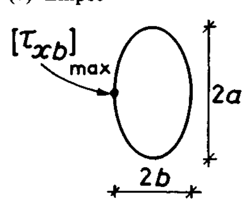
where $[\tau_{xb}]_{max}$ is the maximum shear stress on the boundary of the section and is tangential to the boundary. k and J are constants, and their values for various cross-sectional shapes are shown in Table 2.4.

For the narrow rectangular section shown in Figure 2.42:

$$k = t^2d/3 \quad J = t^3d/3 \tag{2.159, 2.160}$$

and the maximum shear stress occurs along the boundaries of greatest length. These results can be used to determine the

Table 2.4 Torsional constants

Section		*k*	*J*
(1) Rectangle	d/b		
	1.0	$0.208\,(b^2d)$	$0.1406\,(b^3d)$
	1.2	$0.219\,(b^2d)$	$0.166\,(b^3d)$
	1.5	$0.231\,(b^2d)$	$0.196\,(b^3d)$
	2.0	$0.246\,(b^2d)$	$0.229\,(b^3d)$
	2.5	$0.258\,(b^2d)$	$0.249\,(b^3d)$
	3.0	$0.267\,(b^2d)$	$0.263\,(b^3d)$
	4.0	$0.282\,(b^2d)$	$0.281\,(b^3d)$
	5.0	$0.291\,(b^2d)$	$0.291\,(b^3d)$
	10.0	$0.312\,(b^2d)$	$0.312\,(b^3d)$
	∞	$1/3\ (b^2d)$	$1/3\ (b^3d)$
(2) Equilateral triangle 		$b^3/20$	$\sqrt{3}b^4/80$
(3) Right isosceles triangle 		$0.0554\,b^3$	$0.0261\,b^4$
(4) Hexagon 		$0.217\,Ad$	$0.133\,Ad^2$
(5) Circle 		$\pi r^3/2$ $1.5708\,r^3$	$\pi r^4/2$ $1.5708\,r^4$
(6) Hollow circle 		$\frac{\pi}{2}\left(\frac{r_1^4 - r_2^4}{r_1}\right)$	$\frac{\pi}{2}(r_1^4 - r_2^4)$
(7) Ellipse 		$\pi ab^2/2$	$\pi a^3b^3/(a^2+b^2)$

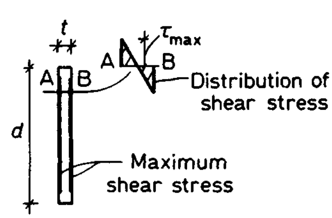


Figure 2.42

torsional properties of a thin-walled open-section bar, supposing that the cross-section can be divided into narrow rectangular elements of thickness t_e and d_e, for it can be shown that to a first approximation:

$$J=\sum_{\text{elements}}\frac{t_e^3 d_e}{3} \tag{2.161}$$

Thus for the I-section shown in Figure 2.43:

$$J=\frac{2d_f t_f^3+d_w t_w^3}{3} \tag{2.162}$$

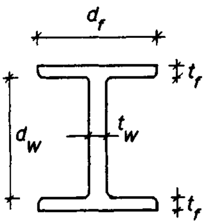


Figure 2.43

The maximum shear stress $[\tau_{xb}]_e$ along the boundaries of a particular element are given by:

$$[\tau_{xb}]_e=T/k_e \tag{2.163}$$

where

$$k_e=J/t_e \tag{2.164}$$

$[\tau_{xb}]_e$ however, is not the maximum shear stress on the cross-section, for this now occurs at the re-entrant corners. Thus, in a constant-thickness channel section (Figure 2.44(a)) $[\tau_{xb}]_{max}$ occurs at point P, and is related to $[\tau_{xb}]_e$ and the radius of the corner as shown in Figure 2.44(b).[8]

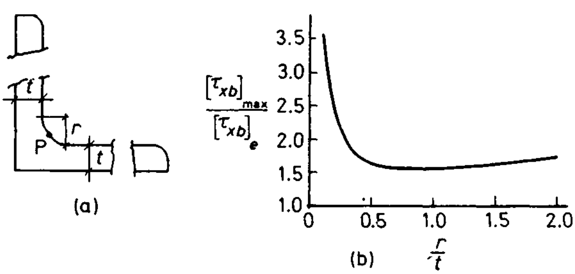


Figure 2.44

In a thin-walled closed-section bar, such as the tube of varying wall thickness t shown in Figure 2.45, the shear stress τ_{xb} is uniform across the thickness at any point and is tangential to the surface of the tube. It is given by:

$$\tau_{xb}=T/2At \tag{2.165}$$

where A is the gross cross-sectional area.
J in Equation (2.158) is given by:

$$J=4A^2\Big/\left(\oint\frac{ds}{t}\right) \tag{2.166}$$

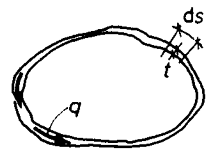


Figure 2.45

where ds is an element of length round the tube (Figure 2.45). A further quantity q called the *shear flow* is defined at a point in the tube wall by the equation

$$q=\tau_{xb}t \tag{2.167}$$

It is then apparent from Equation (2.165) that the shear flow is independent of t.

In multicell thin-walled bars, as shown in Figure 2.46, the concept of circulatory shear flows q_1, q_2, q_3 is introduced, a concept which automatically satisfies the conditions of longitudinal equilibrium at junctions such as A. The shear flow at point B, for example, is then given by (q_1-q_2). The shear flows and the twist of the bar corresponding to a certain applied torque are calculated from the four simultaneous equations:

$$T=2q_1A_1+2q_2A_2+2q_3A_3 \tag{2.168}$$

$$\frac{d\theta}{dx}=\frac{1}{2A_1G}\oint_1\frac{q}{t}ds_1=\frac{1}{2A_2G}\oint_2\frac{q}{t}ds_2=\frac{1}{2A_3G}\oint_3\frac{q}{t}ds_3 \tag{2.169, 2.170, 2.171}$$

where $\oint_1$ represents the contour integral taken round cell 1, etc.

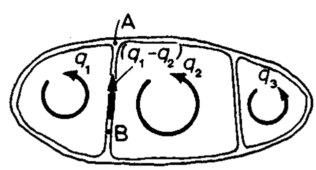


Figure 2.46

2.3.9 Nonuniform torsion

Nonuniform torsion in a bar is defined to occur when the twist $d\theta/dx$ varies along its length. This situation arises when the warping assumed in St Venant's theory is restrained at a rigid support, or when the torque exerted by the applied loading is nonuniform.

The nature of the modification necessary to St Venant's theory can be appreciated by considering the nonuniform torsion of the I-section cantilever shown in Figure 2.47. Since $d\theta/dx$ is not constant, the flanges are curved in the z plane. Considering the flanges as subsidiary beams, they contain shear forces $[S_z]_f$ which are related to this curvature. The torque T therefore includes an extra component $[S_z]_f d$. $[S_z]_f$ is given by Equation (2.147) as:

$$[S_z]_f=-E[I_y]_f\frac{d^3u_z}{dx^3} \tag{2.172}$$

where $[I_y]_f$ is the moment of inertia of each flange about the y axis, and u_z is its displacement. Whence noting that:

$$u_z=\pm\frac{d}{2}\theta \tag{2.173}$$

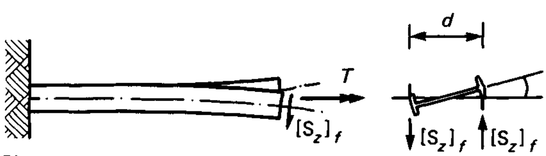
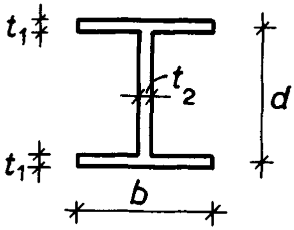
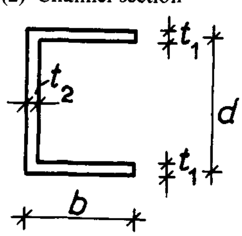
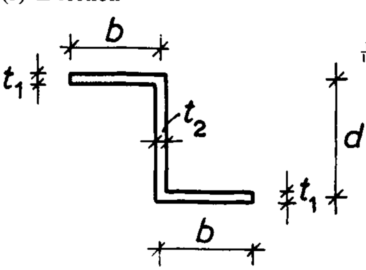
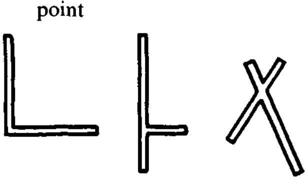


Figure 2.47

Table 2.5 Warping factors

Section	Γ
(1) I-section 	$\frac{1}{24}b^3d^2t_1$
(2) Channel section 	$\frac{1}{12}b^3d^2t_1\left(\frac{3bt_1+2dt_2}{6bt_1+dt_2}\right)$
(3) Z-section 	$\frac{1}{12}b^3d^2t_1\left(\frac{bt_1+2dt_2}{2bt_1+dt_2}\right)$
(4) Thin-walled sections with elements intersecting at a single point 	0

the additional torque component becomes:

$$-EI_y\frac{d^2}{4}\frac{\mathrm{d}^3\theta}{\mathrm{d}x^3}$$

where I_y is the *total* moment of inertia of the cross-section about the y axis. Combining this with the torque required for the uniform torsion of the bar, we obtain:

$$T=GJ\frac{\mathrm{d}\theta}{\mathrm{d}x}-EI_y\frac{d^2}{4}\frac{\mathrm{d}^3\theta}{\mathrm{d}x^3} \tag{2.174}$$

Equation (2.174) can be expressed in the more general form:

$$T=GJ\frac{\mathrm{d}\theta}{\mathrm{d}x}-E\Gamma\frac{\mathrm{d}^3\theta}{\mathrm{d}x^3} \tag{2.175}$$

where Γ is a constant called the *warping factor*. Its values for various cross-sectional shapes are given in Table 2.5. The differential Equation (2.175) can be solved for various values of T applied to the bar, subject to boundary conditions in θ. Examples of these boundary conditions are shown in Figure 2.48.

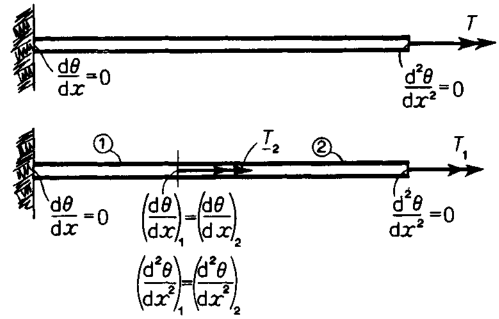


Figure 2.48

2.3.10 Bars subject to compressive forces (columns)

2.3.10.1 Short columns

If the geometry of a bar is such that its length is less than about 5 times its lateral dimensions, then it is usually stable under compressive forces. If therefore it is subjected to an axial compressive force F, then $N=-F$ and the corresponding stress σ_x is given by Equation (2.115) as: $\sigma_x=N/A$. If further, the bar is subjected to bending moments M_y and M_z acting about the principal axes y and z, then by superposition:

$$\sigma_x=\frac{N}{A}+\frac{M_yz}{I_y}-\frac{M_zy}{I_z} \tag{2.176}$$

and the neutral axis is given by the equation:

$$\frac{M_yz}{I_y}-\frac{M_zy}{I_z}+\frac{N}{A}=0 \tag{2.177}$$

Combined compressive forces and bending moments occur in the bar if the compressive force F is eccentrically positioned as shown in Figure 2.49. Thus if the resultant due to F passes at a distance n and m from the y and z axes respectively, then:

$$N=-F \quad M_y=-Fn \quad M_z=+Fm \tag{2.178}$$

and:

$$\sigma_x=-F\left(\frac{1}{A}+\frac{nz}{I_y}+\frac{my}{I_z}\right) \tag{2.179}$$

The neutral axis is then given by the equation:

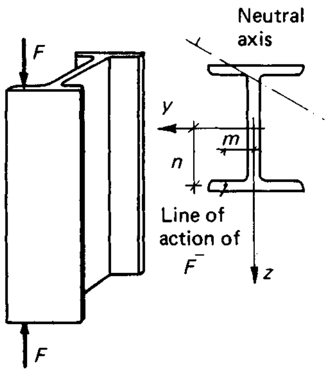


Figure 2.49

$$\frac{nz}{r_y^2}+\frac{my}{r_z^2}+1=0 \tag{2.180}$$

where r_y and r_z are the *radii of gyration* of the cross-section defined respectively by the equations:

$$r_y^2=\frac{I_y}{A} \quad r_z^2=\frac{I_z}{A} \tag{2.181}$$

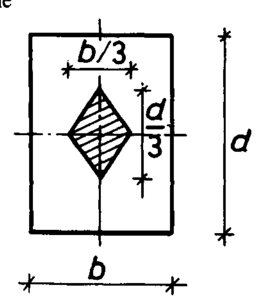
Note that if the location of the neutral axis is known, then the maximum and minimum stresses on the section are located at those points which are at the greatest perpendicular distance from this axis. Their positions can easily be found graphically.

It is apparent from Equation (2.180) that the location of the neutral axis depends only on the coordinates n and m defining the eccentricity of F. If this eccentricity is such that the neutral axis falls outside the section, then the stress σ_x is negative (or compressive) at all points in the section. This situation arises if the stress resultant lies within an area called the *core* of the section. The dimensions of the cores of regular sections can be found analytically and some examples are given in Table 2.6.

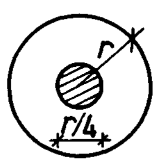
Table 2.6 Cores of sections

Section

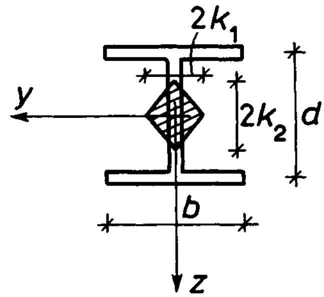
(1) Rectangle



(2) Circle



(3) I-section



$k_1=2r_z^2/b$

$k_2=2r_y^2/d$

2.3.10.2 Long columns

If the length of a bar is greater than about 5 times its lateral dimensions, it can become unstable under compressive forces. Consider, for example, the pin-ended bar subject to an axial compressive force F shown in Figure 2.50. If u_z is the lateral displacement in the z direction of a particular cross-section, then the moment M_y exerted by F at the section is Fu_z. Thus from Equation (2.146) we have the differential equation

$$\frac{d^2u_z}{dx^2}+\frac{Fu_z}{EI_y}=0 \tag{2.182}$$

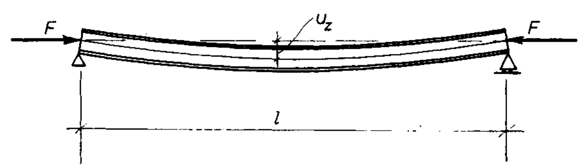


Figure 2.50

One solution of Equation (2.182) is $u_z=0$, i.e. the bar remains straight. However, further nonzero solutions for u_z occur for particular values of F called the *eigenvalues*. The lowest eigenvalue is the critical load F_{cr} of the bar, and can be regarded as the maximum load that can be carried before failure by lateral instability. It can be shown that F_{cr} is given by:

$$F_{cr}=\frac{\pi^2EI_y}{l^2} \tag{2.183}$$

while the corresponding deflected shape of the bar is sinusoidal and of arbitrary amplitude, taking the following form:

$$u_z=A\sin\left(\frac{\pi x}{l}\right) \tag{2.184}$$

The value for the critical load was first obtained by Euler, and a pin-ended bar subject to axial compression is often called an *Euler strut.*

Dividing Equation (2.183) by the area of the bar leads to the following expression for the *critical buckling stress* σ_{cr}:

$$\sigma_{cr}=\pi^2E/\lambda^2 \tag{2.185}$$

where λ ($=l/r_y$) is called the *slenderness ratio.*

When, as in most cases, $I_y \neq I_z$, the strut buckles first about the minor principal axis, about that axis for which the moment of inertia of the section is a minimum.

The critical buckling loads of struts with other than pin-ended boundary conditions are given in Table 2.7. The corresponding *effective lengths* l_e are then defined so that the critical stresses can be given by an equation analogous to Equation (2.185) namely

$$\sigma_{cr} = \pi^2 E/\lambda_e^2 \tag{2.186}$$

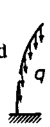
where $\lambda_e = l_e/r_y$. Values for l_e are included in the table.

Table 2.7 Critical buckling loads of struts

All struts are of length l; $I_z > I_y$

Lower end boundary condition	*Upper end boundary condition*	*Mode*	P_{cr}	l_e
(1) Hinge along y axis	Hinge along y axis		$\pi^2 EI_y/l^2$	l
(2) Clamped	Clamped		$4\pi^2 EI_y/l^2$	$0.5\,l$
(3) Clamped	Hinge along y axis		$20.19\,EI_y/l^2$	$0.7\,l$
(4) Clamped	Free		$\pi^2 EI_y/4l^2$	$2.0\,l$
(5) Hinge along z axis	Hinge along z axis		Smaller of $4\pi^2 EI_y/l^2$ or $\pi^2 EI_z/l^2$	$0.5\,l$
(6) Hinge along z axis	Hinge along y axis		$20.19\,EI_y/l^2$	$0.7\,l$

Special loading cases

(7) Pin-ended strut under end load P_1 and central load P_2		$(F_1+F_2)_{cr} = \pi^2 EI/(kl)^2$ where $k \simeq 1/(2-c^2)$ $c = F_1/(F_1+F_2)$
(8) Cantilever strut under uniformly distributed load q/unit length		$(q_{cr})l = \pi^2 EI/(1.122\,l)^2$

The above type of buckling is called *flexural buckling*, and occurs when the cross-section of the strut has two axes of symmetry. For unsymmetrical sections, buckling may be accompanied by torsion as well as flexure, producing a correspondingly reduced critical load. Results for such cases are given by Bleich.[3]

2.3.10.3 Formulae for the strength of columns

The plot of σ_{cr} versus λ for various column lengths is the hyperbola shown in Figure 2.51. Clearly, when λ is very small, the critical stress becomes much greater than the yield stress σ_Y of the material, and the failure of the column is brought about by the yielding of the material rather than by flexural buckling. If the columns were perfectly straight and the axial load had no eccentricity then the ultimate stresses σ_u would be given by the upper curve in Figure 2.51, i.e. the elastic buckling hyperbola intersected by the horizontal 'squash' line. However, tests show that the strengths of real columns are considerably reduced by initial imperfections when $\sigma_{cr} \simeq \sigma_Y$ as indicated by the lower curve in the figure. The following semi-empirical formulae have been devised to account for this, giving the ultimate stresses of columns in terms of their geometrical and material properties.

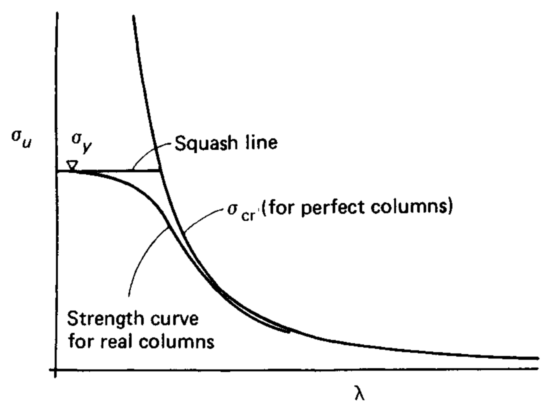


Figure 2.51

The Rankine formula.[17] A simple interaction formula relating σ_u, σ_Y and σ_{cr} is as follows:

$$\frac{1}{\sigma_u} = \frac{1}{\sigma_{cr}} + \frac{1}{\sigma_Y} \tag{2.187}$$

gives:

$$\sigma_u = \frac{\sigma_Y}{1 + \dfrac{\sigma_Y \lambda^2}{\pi^2 E}} \tag{2.188}$$

The interaction curve is tangential to the squash line at $\lambda = 0$, and to the buckling hyperbola at $\lambda = \infty$.

The Johnson parabola.[17] The formula:

$$\sigma_u = \sigma_Y \left(1 - \frac{\sigma_Y \lambda^2}{4\pi^2 E}\right) \tag{2.189}$$

gives a parabolic interaction curve in the nonelastic range which is tangential to the squash line at $\lambda=0$, and to the buckling hyperbola at the point $\sigma_{cr}=\frac{1}{2}\sigma_Y$.

The secant formula. The secant formula is derived assuming that the axial forces on the column have an initial *eccentricity e* (Figure 2.52(a)). In this case it can be shown that:

$$\sigma_u=\frac{\sigma_Y}{1+\eta\sec[(\pi/2)\surd(\sigma_u/\sigma_{cr})]} \tag{2.190}$$

where η is given by:

$$\eta=ec/r_y^2 \tag{2.191}$$

c is the distance from the neutral axis to the extreme fibre of the section.

The Perry–Robertson formula. Assuming that the column has an initial *curvature* and that its maximum misalignment is e (Figure 2.52(b)), Ayrton and Perry derived the following formula:

$$\sigma_u=\tfrac{1}{2}[\sigma_Y+(1+\eta)\sigma_{cr}]-\left[\left(\frac{\sigma_Y+(1+\eta)\sigma_{cr}}{2}\right)^2-\sigma_Y\sigma_{cr}\right]^{1/2} \tag{2.192}$$

where η is again given by Equation (2.191).

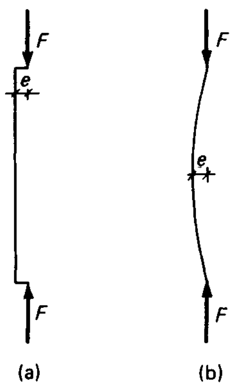


Figure 2.52

Robertson showed by experiment that a good but conservative prediction of the real strengths of columns can be obtained by making η proportional to λ, as follows:

$$\eta=0.003\,\lambda \tag{2.193}$$

Later experiments by Dutheil[17] led to the modified expression

$$\eta=0.3\,(\lambda/100v)^2 \tag{2.194}$$

2.3.10.4 Codes of practice for the design of columns

Section 2.3.10.3 summarizes the bases of simple empirical formulae for the strengths of columns. Current and projected codes of practice are somewhat more complicated, attempting to allow for the effects of variations in cross-sectional geometry and of residual stresses due to rolling and welding.

The *British* codes of practice are based on the Perry–Robertson formula. In the current standard for the design of steel bridges,[18] compression members are designed for η in Equation (2.191) which is linearly related to λ and a parameter α as follows:

$$\eta=0 \qquad (\lambda<\lambda_0) \tag{2.195}$$

$$\eta=0.001\alpha(\lambda-\lambda_0) \quad (\lambda>\lambda_0) \tag{2.196}$$

where λ_0 is the slenderness ratio below which the members are assumed to reach their full squash load. This is given as $0.2\,\lambda_1$ where λ_1 $(=\pi\sqrt{E/\sigma_Y})$ is the slenderness ratio for which the critical stress is equal to the yield stress. Four curves for σ_u are presented, curves A, B, C and D corresponding to $\alpha=2.5$, 4.5, 6.2 and 8.3 respectively. These are shown in a nondimensional plot in Figure 2.53. The curves appropriate for various cross-sections and fabrication methods are then selected according to

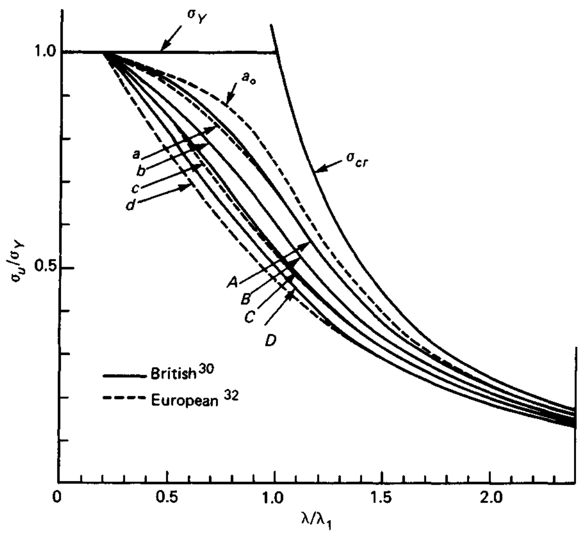


Figure 2.53 British and European column strength curves

Table 2.8.† The revised standard for steelwork in buildings,[19] adopts a similar approach, with slight differences in α for the different cases.

The *European Recommendations for Steel Construction*,[20] published by the European Convention for Structural Steelwork (ECSS) employ three basic column strength curves *a*, *b* and *c* again describing the strengths of groups of rolled and welded columns with various cross-sections. These curves are included as broken lines in Figure 2.53. The additional curves a_0 and *d* respectively deal with heat-treated sections in high-strength steel, and with sections with particularly thick plates (>40 mm). For welded sections the effective value of the yield stress is reduced by 6%. An extended account of the reasoning behind the Recommendations is given in Chapters 2 and 3 of the Second International Colloquium report.[21]

The current *American* codes of practice are based on the Johnson parabola. Thus the American Institute of Steel Construction[22] recommend that the *allowable* stresses are obtained by dividing the interaction curve given by Equation (2.189) by a safety factor ϕ which depends on the slenderness ratio. Thus defining λ_2 to be the slenderness ratio for which $\sigma_{cr}=\frac{1}{2}\sigma_Y$, then

† Extracts from BS 5400: Part 3: 1982 are reproduced by permission of the British Standards Institution, 2 Park Street, London, W1A 2BS from whom complete copies of the standard can be obtained.

Table 2.8 Selection of British column strength curves. British Standards Institution (1982) *Steel, concrete and composite bridges,* BS 5400: Part 3. BSI, Milton Keynes)

	Members fabricated by welding (excluding local welding of battens, lacing, etc.)	*All other members (including stress relieved welded members)*
$r_y/c \geqslant 0.7$	curve *B*	curve *A*
$r_y/c = 0.60$	curve *C*	curve *B*
$r_y/c = 0.50$	curve *C*	curve *B*
$r_y/c \leqslant 0.45$	curve *C*	curve *C*
All-rolled sections with flange thickness > 40 mm	curve *D*	
Hot-finished hollow sections	curve *A*	

Notes: (a) For intermediate values of r_y/c, linear interpolation may be used between the curves given.
(b) c is defined as for Equation (2.191).

$$\phi = \tfrac{5}{3} + \tfrac{3}{8}\left(\frac{\lambda}{\lambda_2}\right)^{\frac{1}{2}} - \tfrac{1}{8}\left(\frac{\lambda}{\lambda_2}\right)^{3/2} \quad (\lambda < \lambda_2) \tag{2.197}$$

$$\phi = \tfrac{23}{12} \qquad (\lambda > \lambda_2) \tag{2.198}$$

For slender bracing and secondary members for which $\lambda > 120$, the allowable stresses may be divided by $(1.6 - \lambda/200)$, giving stresses similar to those of the Rankine formula. The Structural Stability Research Council (SSRC)[23] describe three column-strength curves (1), (2) and (3) each one representing the computed strength of a group of rolled or welded sections with realistic residual stresses and an initial bow of $l/1000$. These are shown in Figure 2.54.

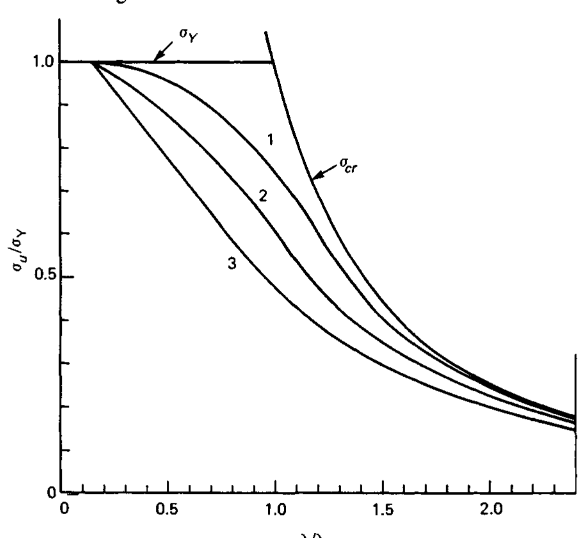


Figure 2.54 American column strength curves

2.3.11 Virtual work and strain energy of frameworks

The state of stress and strain at all points in a framework can be expressed in terms of the stress resultants at those points, using the appropriate equations of the previous sections and the stress–strain relations. The internal virtual work done in a framework corresponding to the general expression in Equation (2.76) is then given by:

$$W_i^* = \sum_{\text{bars}} \int_0^l \left(\frac{NN^*}{EA} + \frac{M_yM_y^*}{EI_y} + \frac{M_zM_z^*}{EI_z} + \frac{k_yS_yS_y^*}{GA} + \frac{k_zS_zS_z^*}{GA} + \frac{TT^*}{G}\right) \mathrm{d}x \tag{2.199}$$

where k_y and k_z are dimensionless form factors depending on the shape of the bar cross-section at each point in the framework. Values of the form factors for some common cross-sections are given in Table 2.9.

Table 2.9 Form factors

Section	k_y	k_z
1 Rectangle	1.20	
2 Circle	1.11	
3 Hollow circle	2.00	
4 I-section or hollow rectangle (approx.)	A/A_{flanges}	A/A_{webs}

Similarly the internal strain energy of a framework corresponding to the expression in Equation (2.78) is given by:

$$U = \sum_{\text{bars}} \int_0^l \left(\frac{N^2}{2EA} + \frac{M_y^2}{2EI_y} + \frac{M_z^2}{2EI_z} + \frac{k_yS_y^2}{2GA} + \frac{k_zS_z^2}{2GA} + \frac{T^2}{2GJ}\right) \mathrm{d}x \tag{2.200}$$

2.3.12 Note on the limitations of the engineering theory of the bending of beams (ETBB)

As noted in section 2.3.6, the basic assumptions of the ETBB, while quite correct when the beam is subject to pure bending, become invalid when the beam is also subject to shear. In particular, we can no longer assume that plane sections remain plane.

Some indication of the error involved in using the ETBB is obtained by analysing a thin-walled deep cantilever beam. Treating this as a plane stress problem, a complete solution is possible subject only to certain assumptions regarding the fixity at the encastre end.[8] Thus it can be shown that if the cantilever is loaded by a single vertical load F at its end so that the shear stress resultant is uniform along the length, the direct and shear stresses given by the ETBB are *exact*. However, the deflections u_x and u_z are given by:

$$u_x = \frac{F}{2EI_y}(-2lx + x^2)z + \frac{\nu F z^3}{6EI_y} - \frac{Fz^3}{6GI_y} \tag{2.201}$$

$$u_z = \frac{F}{6EI_y}(3lx^2 - x^3) + \frac{\nu F}{2EI_y}(l - x)z^2 + \frac{Fd^2x}{8GI_y} \tag{2.202}$$

and the corresponding deflected shape of the beam is composed

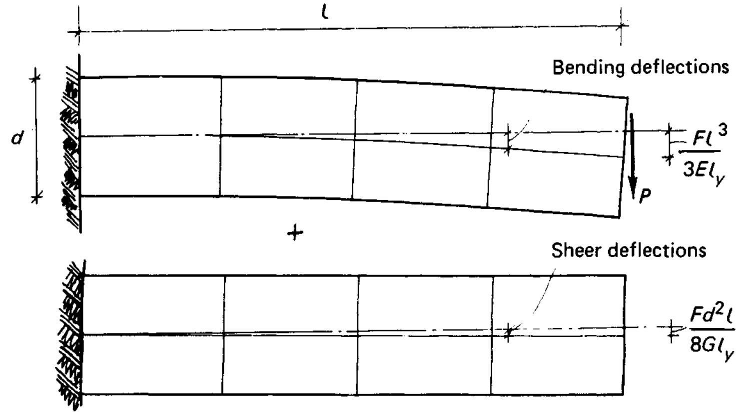


Figure 2.55

of two components as shown in Figure 2.55. One is the curved shape predicted by the ETBB, while the other is a linear vertical displacement due to the shear with the original plane cross-section taking up an S-shape in side view.

If the cantilever is loaded by a uniformly distributed load F/unit length so that the shear-stress resultant varies with x then the *stresses* given by the ETBB are also slightly inaccurate. However, it can be shown that the error is small, provided the span of the beam is large compared with its depth. Further the curvature of the beam is modified from Equation (2.119) to:

$$\frac{1}{R_y} = \left[\frac{M_y}{EI_y} + \frac{F}{EI_y}\frac{d^2}{4}\left(\frac{4}{5}+\frac{\nu}{2}\right)\right] \tag{2.203}$$

where the second term on the right-hand side represents the effect of the shear forces.

The preceding discussion concerns the behaviour of the webs of beams. However, in the flanges as well, it can be shown that plane sections no longer remain plane when beams are subject to shear. This phenomenon is called *shear lag*. It can be conveniently illustrated by the T-section cantilever shown in Figure 2.56(a). According to St Venant's theory (section 2.3.6), the forces in the flange are transmitted by longitudinal shear across the section A–B, so that the flange can be considered to behave like the cantilever plate shown in Figure 2.56(b) subjected to the uniformly distributed axial load along its centreline. It is then clear that the corresponding displacements u_x and the axial stress σ_x are nonuniform across the width of the flange. The analysis of shear lag for practical cases is complex, and the topic is dealt with at some length by Williams.[24]

Further departures from the ETBB occur when beams become geometrically unstable. This instability can take the form of local compressive buckling of the flanges,[3] local shear buckling of the webs[3] and overall torsional buckling.[3]

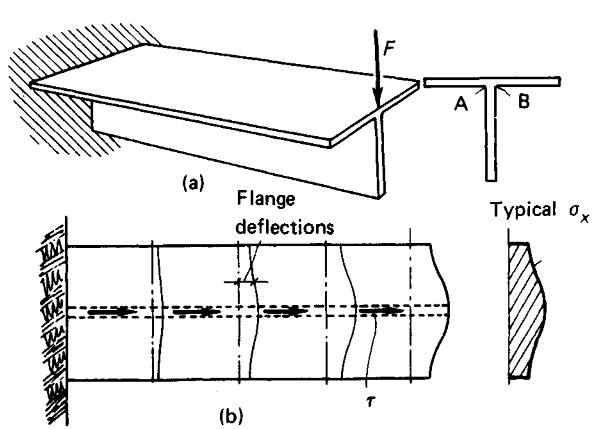


Figure 2.56

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3 Theory of Structures

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Contents

3.1 Introduction

3.1.1 Basic concepts

The 'Theory of Structures' is concerned with establishing an understanding of the behaviour of structures such as beams, columns, frames, plates and shells, when subjected to applied loads or other actions which have the effect of changing the state of stress and deformation of the structure. The process of 'structural analysis' applies the principles established by the Theory of Structures, to analyse a given structure under specified loading and possibly other disturbances such as temperature variation or movement of supports. The drawing of a bending moment diagram for a beam is an act of structural analysis which requires a knowledge of structural theory in order to relate the applied loads, reactive forces and dimensions to actual values of bending moment in the beam. Hence 'theory' and 'analysis' are closely related and in general the term 'theory' is intended to include 'analysis'.

Two aspects of structural behaviour are of paramount importance. If the internal stress distribution in a structural member is examined it is possible, by integration, to describe the situation in terms of 'stress resultants'. In the general three-dimensional situation, these are six in number: two bending moments, two shear forces, a twisting moment and a thrust. Conversely, it is, of course, possible to work the other way and convert stress-resultant actions (forces) into stress distributions. The second aspect is that of deformation. It is not usually necessary to describe structural deformation in continuous terms throughout the structure and it is usually sufficient to consider values of displacement at selected discrete points, usually the joints, of the structure.

At certain points in a structure, the continuity of a member, or between members, may be interrupted by a 'release'. This is a device which imposes a zero value on one of the stress resultants. A hinge is a familiar example of a release. Releases may exist as mechanical devices in the real structure or may be introduced, in imagination, in a structure under analysis.

In carrying out a structural analysis it is generally convenient to describe the state of stress or deformation in terms of forces and displacements at selected points, termed 'nodes'. These are usually the ends of members, or the joints and this approach introduces the idea of a structural element such as a beam or column. A knowledge of the forces or displacements at the nodes of a structural element is sufficient to define the complete state of stress or deformation within the element providing the relationships between forces and displacements are established. The establishment of such relationships lies within the province of the theory of structures.

Corresponding to the basic concepts of force and displacement, there are two important physical principles which must be satisfied in a structural analysis. The structure as a whole, and every part of it, must be in equilibrium under the actions of the force system. If, for example, we imagine an element, perhaps a beam, to be removed from a structure by cutting through the ends, the internal stress resultants may now be thought of as external forces and the element must be in equilibrium under the combined action of these forces and any applied loads. In general, six independent conditions of equilibrium exist; zero sums of forces in three perpendicular directions, and zero sums of moments about three perpendicular axes. The second principle is termed 'compatibility'. This states that the component parts of a structure must deform in a compatible way, i.e. the parts must fit together without discontinuity at all stages of the loading. Since a release will allow a discontinuity to develop, its introduction will reduce the total number of compatibility conditions by one.

3.1.2 Force–displacement relationships

A simple beam element AB is shown in Figure 3.1. The application of end moments M_A and M_B produces a shear force Q throughout the beam, and end rotations θ_A and θ_B. By the stiffness method (see page 3/11), it may be shown that the end moments and rotations are related as follows:

$$\left.\begin{aligned} M_A &= \frac{4EI\theta_A}{l}+\frac{2EI\theta_B}{l} \\ M_B &= \frac{4EI\theta_B}{l}+\frac{2EI\theta_A}{l} \end{aligned}\right\} \tag{3.1}$$

Or, in matrix notation,

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{2EI}{l}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix}$$

which may be abbreviated to,

$$\mathbf{S}=\mathbf{k}\boldsymbol{\theta} \tag{3.2}$$

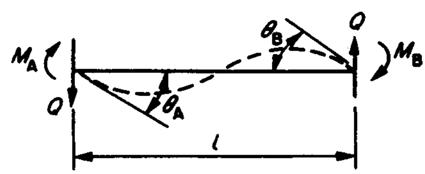


Figure 3.1

Equation (3.2) expresses the force–displacement relationships for the beam element of Figure 3.1. The matrices **S** and $\boldsymbol{\theta}$ contain the end 'forces' and displacements respectively. The matrix **k** is the *stiffness matrix* of the element since it contains end forces corresponding to *unit* values of the end rotations.

The relationships of Equation (3.2) may be expressed in the inverse form:

$$\begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} M_A \\ M_B \end{bmatrix}$$

or

$$\boldsymbol{\theta}=\mathbf{f}\mathbf{S} \tag{3.3}$$

Here the matrix **f** is the *flexibility matrix* of the element since it expresses the end displacements corresponding to *unit* values of the end forces.

It should be noted that an inverse relationship exists between **k** and **f**

i.e.

$$\mathbf{kf}=I$$

or,

$$\mathbf{k}=\mathbf{f}^{-1} \tag{3.4}$$

or,

$$\mathbf{f}=\mathbf{k}^{-1}$$

The establishment of force–displacement relationships for structural elements in the form of Equations (3.2) or (3.3) is an important part of the process of structural analysis since the element properties may then be incorporated in the formulation of a mathematical model of the structure.

3.1.3 Static and kinematic determinacy

If the compatibility conditions for a structure are progressively reduced in number by the introduction of releases, there is reached a state at which the introduction of *one further* release would convert the structure into a mechanism. In this state the structure is *statically determinate* and the nodal forces may be calculated directly from the equilibrium conditions. If the releases are now removed, restoring the structure to its correct condition, nodal forces will be introduced which cannot be determined solely from equilibrium considerations. The structure is *statically indeterminate* and compatibility conditions are necessary to effect a solution.

The structure shown in Figure 3.2(a) is hinged to rigid foundations at A, C and D. The continuity through the foundations is indicated by the (imaginary) members, AD and CD. If the releases at A, C and D are removed, the structure is as shown in Figure 3.2(b) which is seen to consist of two closed rings. Cutting through the rings as shown in Figure 3.2(c) produces a series of simple cantilevers which are statically determinate. The number of stress resultants released by each cut would be three in the case of a planar structure, six in the case of a space structure. Thus, the degree of statical indeterminacy is 3 or 6 times the number of rings. It follows that the structure shown in Figure 3.2(b) is 6 times statically indeterminate whereas the structure of Figure 3.2(a), since releases are introduced at A, C and D, is 3 times statically indeterminate. A general relationship between the number of members m, number of nodes n, and degree of static indeterminacy n_s, may be obtained as follows:

$$n_s = \frac{6}{3}(m - n + 1) - r \tag{3.5}$$

where r is the number of releases in the actual structure

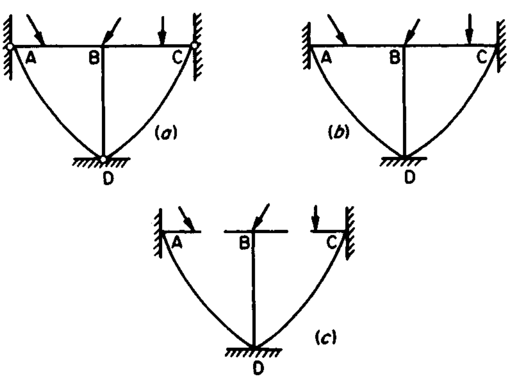


Figure 3.2

Turning now to the question of *kinematical determinacy*; a structure is defined as kinematically determinate if it is possible to obtain the nodal displacements from compatibility conditions without reference to equilibrium conditions. Thus a fixed-end beam is kinematically determinate since the end rotations are known from the compatibility conditions of the supports.

Again, consider the structure shown in Figure 3.2(b). The structure is kinematically determinate except for the displacements of joint B. If the members are considered to have infinitely large extensional rigidities, then the rotation at B is the only unknown nodal displacement. The degree of kinematical indeterminacy is therefore 1. The displacements at B are *constrained* by the assumption of zero vertical and horizontal displacements. A *constraint* is defined as a device which constrains a displacement at a certain node to be the same as the corresponding displacement, usually zero, at another node. Reverting to the structure of Figure 3.2(a), it is seen that three constraints, have been removed by the introduction of hinges (releases) at A, C and D. Thus rotational displacements can develop at these nodes and the degree of kinematical indeterminacy is increased from 1 to 4.

A general relationship between the numbers of nodes n, constraints c, releases r, and the degree of kinematical indeterminacy n_k is as follows,

$$n_k = \frac{6}{3}(n - 1) - c + r \tag{3.6}$$

The coefficient 6 is taken in three-dimensional cases and the coefficient 3 in two-dimensional cases. It should now be apparent that the modern approach to structural theory has developed in a highly organised way. This has been dictated by the development of computer-orientated methods which have required a re-assessment of basic principles and their application in the process of analysis. These ideas will be further developed in some of the following sections.

3.2 Statically determinate truss analysis

3.2.1 Introduction

A structural frame is a system of bars connected by joints. The joints may be, ideally, *pinned* or *rigid*, although in practice the performance of a real joint may lie somewhere between these two extremes. A *truss* is generally considered to be a frame with pinned joints, and if such a frame is loaded only at the joints, then the members carry axial tensions or compressions. *Plane* trusses will resist deformation due to loads acting in the plane of the truss only, whereas *space* trusses can resist loads acting in any direction.

Under load, the members of a truss will change length slightly and the geometry of the frame is thus altered. The effect of such alteration in geometry is generally negligible in the analysis.

The question of statical determinacy has been mentioned in the previous section where a relationship, Equation (3.5) was stated from which the degree of statical indeterminacy could be determined. Although this relationship is of general application, in the case of plane and space trusses, a simpler relationship may be established.

The simplest plane frame is a triangle of three members and three joints. The addition of a fourth joint, in the plane of the triangle, will require two additional members. Thus in a frame having j joints, the number of members is:

$$n = 2(j - 3) + 3 = 2j - 3 \tag{3.7}$$

A truss with this number of members is statically determinate, providing the truss is supported in a statically determinate way. Statically determinate trusses have two important properties. They cannot be altered in shape without altering the length of one or more members, and, secondly, any member may be altered in length without inducing stresses in the truss, i.e. the truss cannot be *self stressed* due to imperfect lengths of members or differential temperature change.

The simplest space truss is in the shape of a tetrahedron with four joints and six members. Each additional joint will require three more members for connection with the tetrahedron, and thus:

$$n = 3(j-4)+6 = 3j-6 \tag{3.8}$$

A space truss with this number of members is statically determinate, again providing the support system is itself statically determinate. It should be noted that in the assessment of the statical determinacy of a truss, member forces and reactive forces should all be considered when counting the number of unknowns. Since equilibrium conditions will provide two relationships at each joint in a plane truss (there is a space truss), the simplest approach is to find the total number of unknowns, member forces and reactive components, and compare this with 2 or 3 times the number of joints.

3.2.2 Methods of analysis

Only brief mention will be made here of the methods of statically determinate analysis of trusses. For a more detailed treatment the reader is referred to Jenkins[1] and Coates, Coutie and Kong.[2]

The *force diagram* method is a graphical solution in which a vector polygon of forces is drawn to scale proceeding from joint to joint. It is necessary to have not more than two unknown forces at any joint, but this requirement can be met with a judicious choice of order. The two conditions of overall equilibrium of the plane structure imply that the force vector polygon will form a closed figure. The method is particularly suitable for trusses with a difficult geometry where it is convenient to work to a scale drawing of the outline of the truss.

The method of *resolution at joints* is suitable for a complete analysis of a truss. The reactions are determined and then, proceeding from joint to joint, the vertical and horizontal equilibrium conditions are set down in terms of the member forces. Since two equations will result at each joint in a plane truss, it is possible to determine not more than two forces for each pair of equations. As an illustration of the method, consider the plane truss shown in Figure 3.3. The truss is symmetrically loaded and the reactions are clearly 15 kN each.

Consider the equilibrium of joint A,

vertically, $P_{AE}\cos 45° = R_A$; hence $P_{AE} = 15\sqrt{2}$ kN (compression)

horizontally, $P_{AC} = P_{AE}\cos 45°$; hence $P_{AC} = 15$ kN (tension)

It should be noted that the arrows drawn on the members in Figure 3.3 indicate the directions of forces acting *on the joints*. It is also seen that the directions of the arrows at joint A, for example, are consistent with equilibrium of the joint. Proceeding to joint C it is clear that $P_{CE} = 10$ kN (tension), and that $P_{CD} = P_{AC} = 15$ kN (tension). The remainder of the solution may be obtained by resolving forces at joint E, from which $P_{ED} = 5\sqrt{2}$ kN (tension) and $P_{EF} = 20$ kN (compression).

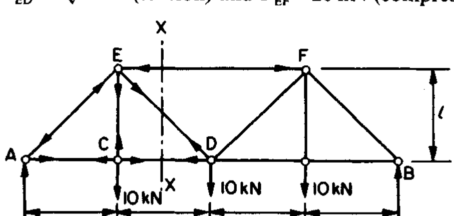


Figure 3.3

The *method of sections* is useful when it is required to determine forces in a limited number of the members of a truss. Consider, for example, the member ED of the truss in Figure 3.3. Imagine a cut to be made along the line XX and consider the vertical equilibrium of the part to the left of XX. The vertical forces acting are R_A, the 10 kN load at C and the vertical component of the force in ED. The equation of vertical equilibrium is:

$$15-10 = P_{ED}\cos 45° \quad \text{hence } P_{ED} = 5\sqrt{2}\text{ kN}$$

Since a downwards arrow on the left-hand part of ED is required for equilibrium, it follows that the member is in tension. The method of tension coefficients is particularly suitable for the analysis of space frames and will be outlined in the following section.

3.2.3 Method of tension coefficients

The method is based on the idea of systematic resolution of forces at joints. In Figure 3.4, let AB be any member in a plane truss, T_{AB} = force in member (tension positive), and L_{AB} = length of member.

We define:

$$T_{AB} = L_{AB}t_{AB} \tag{3.9}$$

where t_{AB} = tension coefficient.

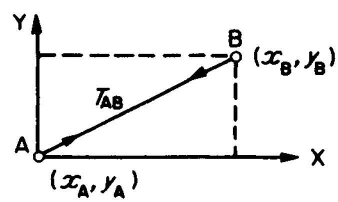


Figure 3.4

That is, the tension coefficient is the actual force in the member divided by the length of the member. Now, at A, the component of T_{AB} in the X-direction:

$$= T_{AB}\cos \text{BAX}$$

$$= T_{AB}\frac{(x_B - x_A)}{L_{AB}} = t_{AB}(x_B - x_A)$$

Similarly the component of T_{AB} in the Y-direction:

$$= t_{AB}(y_B - y_A)$$

At the other end of the member the components are:

$$t_{AB}(x_A - x_B),\ t_{AB}(y_A - y_B)$$

If at A the external forces have components X_A and Y_A, and if there are members AB, AC, AD etc. then the equilibrium conditions for directions X and Y are:

$$\left.\begin{aligned} t_{AB}(x_B - x_A) + t_{AC}(x_C - x_A) + t_{AD}(x_D - x_A) + \ldots + X_A = 0 \\ t_{AB}(y_B - y_A) + t_{AC}(y_C - y_A) + t_{AD}(y_D - y_A) + \ldots + Y_A = 0 \end{aligned}\right\} \tag{3.10}$$

Similar equations can be formed at each joint in the truss. Having solved the equations, for the tension coefficients, usually a very simple process, the forces in the members are determined from Equation (3.9).

The extension of the theory to space trusses is straightforward. At each joint we now have three equations of equilibrium, similar to Equation (3.10) with the addition of an equation representing equilibrium in the Z direction:

$$t_{AB}(z_B - z_A) + t_{AC}(z_C - z_A) + \ \ldots \ + Z_A = 0 \tag{3.11}$$

The method will now be illustrated with an example. The notation is simplified by writing AB in place of t_{AB} etc. A fabular presentation of the work is recommended.

Example 3.1. A pin-jointed space truss is shown in Figure 3.5. It is required to determine the forces in the members using the method of tension coefficients. We first check that the frame is statically determinate as follows:

Number of members = 6
Number of reactions = 9

Total number of unknowns = 15

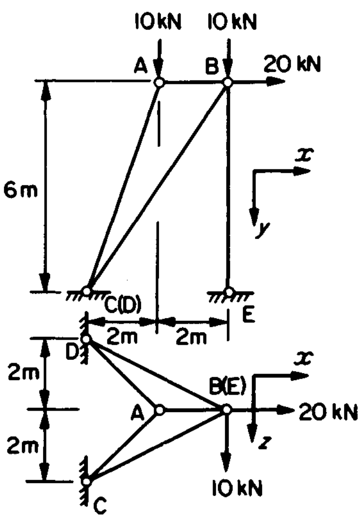


Figure 3.5

The number of equations available is 3 times the number of joints, i.e. $3 \times 5 = 15$. Hence, the truss is statically determinate. In counting the number of reactive components, it should be observed that all components should be included even if the particular geometry of the truss dictates (as in this case at E) that one or more components should be zero.

The solution is set out in Tables 3.1 and 3.2 where it should be noted that, in deriving the equations, the origin of coordinates is taken at the joint being considered. Thus, each tension coefficient is multiplied by the projection of the member on the particular axis.

The methods of truss analysis just outlined are suitable for 'hand' analysis, as distinct from computer analysis, and are useful in acquiring familiarity and understanding of structural behaviour. Much analysis of this kind is now carried out on computers (mainframe, mini- and microcomputers) where the stiffness method provides a highly organized and suitable basis. This topic will be further considered under the heading of the stiffness method.

Table 3.1

Joint	*Direction*	*Equations*	*Solutions*
A	x	$-2AC-2AD+2AB=0$	$AC=AD=-\frac{10}{12}$
	y	$6AC+6AD+10=0$	$AB=-\frac{10}{6}$
	z	$2AC-2AD=0$	$-4BC-4BD+\frac{10}{3}+20=0$; $2BC-2BD+10=0$
C	x	$-4BC-4BD-2AB+20=0$	$BC=\frac{10}{24}$
	y	$6BC+6BD+6BE+10=0$	$BD=\frac{130}{24}$
	z	$-2BD+2BC+10=0$	Hence $BE=-\frac{15}{2}$

Table 3.2

Member	*Length* (m)	*Tension coefficient*	*Force* (kN) (*tension* +)
AB	2	$-\frac{10}{6}$	−3.33
AC	6.62	$-\frac{10}{12}$	−5.52
AD	6.62	$-\frac{10}{12}$	−5.52
BC	7.48	$\frac{10}{24}$	+3.12
BD	7.48	$\frac{130}{24}$	+40.5
BE	6	$-\frac{15}{2}$	−45.0

3.3 The flexibility method

3.3.1 Introduction

The idea of statical determinacy was introduced previously (see page 3/4) and a relationship between the degree of statical indeterminacy and the numbers of members, nodes and releases was stated in Equation (3.5). A statically determinate structure is one for which it is possible to determine the values of forces at all points by the use of equilibrium conditions alone. A statically indeterminate structure, by virtue of the number of members or method of connecting the members together, or the method of support of the structure, has a larger number of forces than can be determined by the application of equilibrium principles alone. In such structures the force analysis requires the use of compatibility conditions. The flexibility method provides a means of analysing statically indeterminate structures.

Consider the propped cantilever shown in Figure 3.6(a). Applying Equation (3.5) the degree of statical indeterminacy is seen to be:

$$n_s = 3(2-2+1)-2 = 1$$

(Note that two releases are required at B, one to permit angular rotation and one to permit horizontal sliding, and also that an additional foundation member is inserted connecting A and B.) The structure can be made statically determinate by removing the propping force R_B or alternatively by removing the fixing moment at A. We shall proceed by removing the reaction R_B. The structure thus becomes the simple cantilever shown in Figure 3.6(b). The application of the load w produces the deflected shape, shown dotted, and in particular a deflection u at the free end B. Note also that it is now possible to determine the bending moment at A $= wl^2/2$, by simple statical principles. The

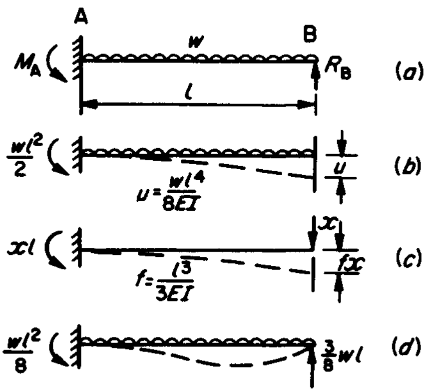


Figure 3.6 Basis of the flexibility method

deflection u may be obtained from elementary beam theory as $wl^4/8EI$. We now remove the applied load w and apply the, unknown, redundant force x at B. It is unnecessary to know the sense of the force x; in this case we have assumed a downwards direction for positive x. The application of the force x produces a displacement at B which we shall call fx; i.e. a unit value of x would produce a displacement f. The compatibility condition associated with the redundant force x is that the final displacement at B should be zero, i.e.:

$$u+fx=0 \tag{3.12}$$

and substituting values of u and f

$$x=-\tfrac{3}{8}wl$$

The process may be regarded as the superposition of the diagrams Figures 3.6(b) and (c) such that the final displacement at B is zero. The addition of the two systems of forces will also give values of bending moment throughout the beam, e.g. at A:

$$M_A=\frac{wl^2}{2}+xl$$

$$=\frac{wl^2}{2}-\tfrac{3}{8}wl^2 \qquad =\frac{wl^2}{8}$$

The actual values of reactions are as shown in Figure 3.6(d).

The displacement f is called a 'flexibility influence coefficient'. In general f_{rs} is the displacement in direction r in a structure due to unit force in direction s. The subscripts were omitted in the above analysis since the force and displacement considered were at the same position and in the same direction.

3.3.2 Evaluation of flexibility influence coefficients

As seen in the above example, flexibility coefficients are displacements calculated at specified positions, and directions, in a structure due to a prescribed loading condition. The loading condition is that of a single unit load replacing a redundant force in the structure. It should be remembered that at this stage the structure is, or has been made, statically determinate.

For simplicity we restrict our attention to structures in which flexural deformations predominate. The extension to other types of deformation is straightforward.[3] In the case of pure flexural deformation we may evaluate displacements by an application of Castigliano's theorem or use the principle of virtual work.[3] In either case a convenient form is:

$$\Delta_i=\int M\partial M/\partial F_i \,\frac{ds}{EI} \tag{3.13}$$

in which Δ_i is the displacement required, M is a function representing the bending moment distribution and F_i is a force, real or virtual, applied at the position and in the direction designated by i. It follows that $\partial M/\partial F_i$ can be regarded as the bending moment distribution due to unit value of F_i.

Consider the cantilever beam shown in Figure 3.7(a). Forces x_1 and x_2 act on the beam and it is required to determine influence coefficients corresponding to the positions and directions defined by x_1 and x_2. From now on we work with *unit* values of x_1 and x_2 and draw bending moment diagrams, as in Figure 3.7(b) and (c), due to unit values of x_1 and x_2 separately.

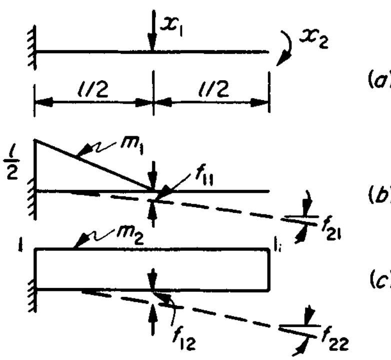


Figure 3.7 Evaluation of flexibility coefficients

These are labelled m_1 and m_2. Consider the application of unit force at x_1 ($x_2=0$). Displacements will occur in the directions of x_1 and x_2. Applying Equation (3.13) the displacement in the direction of x_1 will be:

$$\left.\begin{aligned} &f_{11}=\int m_1m_1\frac{ds}{EI} \\ &\text{and in the direction of } x_2\text{:} \\ &f_{21}=\int m_2m_1\frac{ds}{EI} \end{aligned}\right\} \tag{3.14}$$

Similarly, when we apply $x_2=1$, $x_1=0$, we obtain:

$$\left.\begin{aligned} &f_{22}=\int m_2m_2\frac{ds}{EI} \\ &\text{and:} \\ &f_{12}=\int m_1m_2\frac{ds}{EI} \end{aligned}\right\} \tag{3.15}$$

The general form is:

$$f_{rs}=\int m_rm_s\frac{ds}{EI} \tag{3.16}$$

The evaluation of Equation (3.16) requires the integration of the product of two bending moment distributions over the complete structure. Such distributions can generally be represented by simple geometrical figures such as rectangles, triangles and parabolas and standard results can be established in advance. Table 3.3 gives values of product integrals for a range of combinations of diagrams. It should be noted that in applying Equation (3.16) in this way, the flexural rigidity EI is assumed constant over the length of the diagram.

We may now use Table 3.3 to obtain values of the flexibility coefficients for the cantilever beam under consideration. Using Equations (3.14) and (3.15) with Figures 3.7(b) and (c) we obtain:

$$f_{11}=\frac{1}{3}\cdot\frac{l}{2}\cdot\frac{l}{2}\cdot\frac{l}{2}\cdot\frac{1}{EI}=\frac{l^3}{24EI}$$

$$f_{21}=\frac{1}{2}\cdot\frac{l}{2}\cdot 1\cdot\frac{l}{2}\cdot\frac{1}{EI}=\frac{l^2}{8EI}$$

$$f_{22}=l\cdot 1\cdot 1\cdot\frac{1}{EI}=\frac{l}{EI}$$

$$f_{12}=\frac{1}{2}\cdot\frac{l}{2}\cdot\frac{l}{2}\cdot 1\cdot\frac{1}{EI}=\frac{l^2}{8EI}$$

It is seen that f_{21} and f_{12} are numerically equal, a result which could be established using the Reciprocal Theorem. This is a useful property since in general $f_{rs}=f_{sr}$ and the effect is to reduce the number of separate calculations required. It should be further noted that whilst $f_{21}=f_{12}$, f_{21} is an angular displacement and f_{12} a linear displacement.

The evaluation of the flexibility coefficients f_{rs} provides the displacements at selected points in the structure due to unit values of the associated, redundant, forces. Before the compatibility conditions can be written down, it remains to calculate displacements (u) at corresponding positions due to the actual applied load. The basic equation (Equation 3.13) is applied once more. Now the bending moment distribution M is that due to the applied loads and we will re-designate this m_0. As before, $\partial M/\partial F_i=m_i$, and thus:

$$u_i=\int m_0 m_i\frac{ds}{EI} \qquad (3.17)$$

The table of product integrals, Table 3.3, can be used for evaluating the u_i in the same way as the f_{rs}.

Table 3.3

Product integrals $\int_0^l m_r\, m_s\, ds$ (EI uniform)

m_s \ m_r	a, l	a, l	a, b, l
c, l	lac	$\frac{l}{2}ac$	$\frac{l}{2}(a+b)c$
c, l	$\frac{l}{2}ac$	$\frac{l}{3}ac$	$\frac{l}{6}(2a+b)c$
c, l	$\frac{l}{2}ac$	$\frac{l}{6}ac$	$\frac{l}{6}(a+2b)c$
c, d, l	$\frac{l}{2}a(c+d)$	$\frac{l}{6}a(2c+d)$	$\frac{l}{6}\{a(2c+d)+b(2d+c)\}$
c, l	$\frac{2}{3}lac$	$\frac{l}{3}ac$	$\frac{l}{3}(a+b)c$

In cases where the bending moment diagrams do not fit the standard values given in Table 3.3 or where a member has a stepped variation in EI, the member may be divided into segments such that the standard results can be applied and the total displacement obtained by addition. In cases where the standard results cannot be applied, e.g. a continuous variation in EI, the integration can be carried out conveniently by the use of Simpson's rule:

$$\int m_r m_s\frac{ds}{EI}\simeq\frac{a}{3}(h_1+4h_2+2h_3+4h_4+\ \ldots\ +h_n)$$

where a = width of strip

$$h_i=\frac{m_r m_s}{EI}\text{ at section i.}$$

In using Simpson's rule it should be remembered that the number of strips must be even, i.e. n must be odd.

3.3.2.1 Sign convention

A flexibility coefficient will be positive if the displacement it represents is in the same sense as the applied, unit, force. The bending moment expressions must carry signs based on the type of curvature developing in the structure. Since the integrand in Equation (3.16) is always the product of two bending moment expressions, it is only the relative sign which is of importance. A useful convention is to draw the diagrams on the tension (convex) sides of the members and then the relative signs of m_r and m_s can readily be seen. In Figure 3.7(b) and (c), both the m_1 and m_2 diagrams are drawn on the top side of the member. Their product is therefore positive. Naturally, the product of one diagram and itself will always be positive. This follows from simple physical reasoning since the displacement at a point due to an applied force at the same point will always be in the same sense as the applied force.

3.3.3 Application to beam and rigid frame analysis

The application of the theory will now be illustrated with two examples.

Example 3.2. Consider the three-span continuous beam shown in Figure 3.8(a). The beam is statically indeterminate to the second degree and we shall choose as redundants the internal bending moments at the interior supports B and C. The beam is made statically determinate by the introduction of moment releases at B and C as in Figure 3.8(b). We note that the application of the load W now produces displacements in span BC only, and in particular rotations u_1 and u_2 at B and C. The bending moment diagram (m_0) is shown in Figure 3.8(c).

We now apply unit value of x_1 and x_2 in turn. The deflected shapes and the flexibility coefficients, in the form of angular rotations, are shown at (d) and (e). The bending moment diagrams m_1 and m_2 are shown at (f) and (g).

Using the table of product integrals (Table 3.3), we find:

$$EIf_{11}=\frac{2}{3}l$$

$$EIf_{22}=\frac{2}{3}l$$

$$EIf_{12}=EIf_{21}=\frac{l}{6}$$

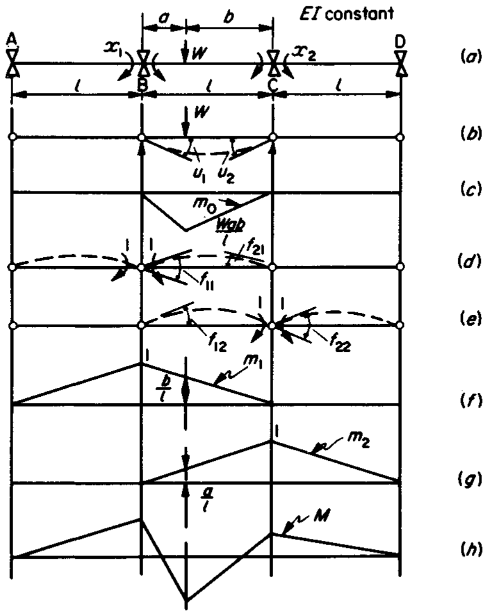


Figure 3.8 Flexibility analysis of continuous beam

$$EI\,u_1 = -\frac{a}{6}\left(1+\frac{2b}{l}\right)\frac{Wab}{l} - \frac{b}{3}\cdot\frac{b}{l}\cdot\frac{Wab}{l}$$

$$= -\frac{Wab}{6l}(a+2b)$$

and

$$EI\,u_2 = -\frac{Wab}{6l}(b+2a)$$

The required compatibility conditions are, for continuity of the beam:

at B, $f_{11}x_1 + f_{12}x_2 + u_1 = 0$
at C, $f_{21}x_1 + f_{22}x_2 + u_2 = 0$

or, in matrix form:

$$\mathbf{FX} + \mathbf{U} = \mathbf{0} \tag{3.18}$$

i.e.:

$$\frac{l}{6EI}\begin{bmatrix}4 & 1\\ 1 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \frac{Wab}{16EIl}\begin{bmatrix}(a+2b)\\ (b+2a)\end{bmatrix}$$

and the solutions are:

$$\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \frac{Wab}{15l^2}\begin{bmatrix}(2a+7b)\\ (2b+7a)\end{bmatrix}$$

The actual bending moment distribution may now be determined by the addition of the three systems, i.e. the applied load and the two redundants. The general expression is:

$$M = m_0 + m_1x_1 + m_2x_2 \tag{3.19}$$

In particular:

$$M_B = x_1 = \frac{Wab}{15l^2}(2a+7b)$$

$$M_C = x_2 = \frac{Wab}{15l^2}(2b+7a)$$

and the bending moment under the load W is:

$$M_W = -\frac{Wab}{l} + \frac{b}{l}x_1 + \frac{a}{l}x_2$$

$$= -\frac{2Wab}{15l^3}(4l^2 + 5ab)$$

The final bending moment diagram is shown in Figure 3.8(h).

Example 3.3. A portal frame ABCD is shown in Figure 3.9(a). The frame has rigid joints at B and C, a fixed support at A and a hinged support at D. The flexural rigidity of the beam is twice that of the columns.

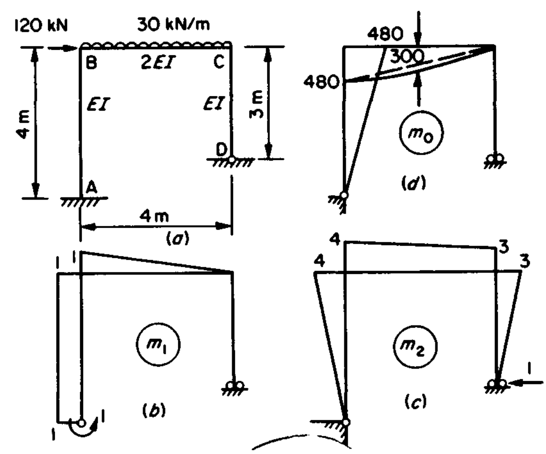


Figure 3.9

The frame has two redundancies and these are taken to be the fixing moment at A and the horizontal reaction at D. The bending moment diagrams corresponding to the unit redundancies, m_1 and m_2 and the applied load, m_0, are shown at (b), (c) and (d) in Figure 3.9.

Using the table of product integrals, Table 3.3, we obtain:

$$f_{11} = \int m_1^2 \frac{ds}{EI} = \frac{14}{3EI}$$

$$f_{22} = \int m_2^2 \frac{ds}{EI} = \frac{55}{EI}$$

$$f_{12} = f_{21} = \int m_1 m_2 \frac{ds}{EI} = \frac{35}{3EI}$$

$$u_1 = \int m_0 m_1 \frac{ds}{EI} = -\frac{1320}{EI}$$

$$u_2 = \int m_0 m_2 \frac{ds}{EI} = -\frac{4600}{EI}$$

Thus the compatibility equations are:

$$\frac{1}{3}\begin{bmatrix}14 & 35\\ 35 & 165\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = +\begin{bmatrix}1320\\ 4600\end{bmatrix}$$

from which

$$x_1 = +157 \text{ kNm}$$

and

$$x_2 = +\ 50 \text{ kN}$$

The bending moment at any point in the frame may now be determined from the expression:

$$M = m_0 + m_1 x_1 + m_2 x_2$$

e.g.:

$$M_{BA} = 480 - 1(+157) - 4(+50) = 123 \text{ kNm}$$

and

$$M_{CD} = 3x_2 = 150 \text{ kNm}$$

3.3.4 Application to truss analysis

The analysis of statically indeterminate trusses follows closely on that established for rigid frames; however, the problem is simplified due to the fact that for each system of loading investigated, the axial forces are constant within the lengths of the members and thus the integration is considerably simplified. We are now concerned with deformations in the members due to axial forces only and the flexibility coefficients are:

$$f_{rs} = \sum p_r\, p_s \frac{l}{AE} \tag{3.20}$$

and

$$u_i = \sum p_0\, p_i \frac{l}{AE} \tag{3.21}$$

in which the p_r system of forces is due to unit tension in the rth redundant member and similarly for p_s and p_i. The p_0 system of forces is that due to the applied load system acting on the statically determinate structure (i.e. with the redundant members omitted). Equations (3.20) and (3.21) should be compared with Equations (3.16) and (3.17) in the flexural case.

Example 3.4. The plane truss shown in Figure 3.10 has two redundancies which we will choose as the forces in members AE and EC. AE is constant for all the members and equal to 1×10^6 kN. The member EC is $l/10\,000$ short in manufacture and has to be forced into position. The member force systems p_0, p_1 and p_2 are found from a simple statical analysis and are listed in Table 3.4.

The flexibility coefficients may now be obtained as follows:

$$f_{11} = \sum p_1 p_1 \frac{l}{AE} = \frac{2l}{AE}(1+\surd 2)$$

$$f_{22} = f_{11}$$

$$f_{12} = f_{21} = \sum p_1 p_2 \frac{l}{AE} = \frac{l}{2AE}$$

$$u_1 = \sum p_1 p_0 \frac{l}{AE} = \frac{Wl}{AE}(1 + 1/\surd 2)$$

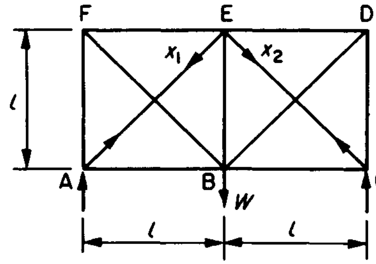


Figure 3.10

Table 3.4

Member	*Length*	p_0/w	p_1	p_2
AB	l	0	$-1/\surd 2$	0
BC	l	0	0	$-1/\surd 2$
CD	l	$-1/2$	0	$-1/\surd 2$
DE	l	$-1/2$	0	$-1/\surd 2$
EF	l	$-1/2$	$-1/\surd 2$	0
AF	l	$-1/2$	$-1/\surd 2$	0
FB	$\surd(2)l$	$1/\surd 2$	1	0
BE	l	0	$-1/\surd 2$	$-1/\surd 2$
BD	$\surd(2)l$	$1/\surd 2$	0	1
AE	$\surd(2)l$	0	1	0
EC	$\surd(2)l$	0	0	1

Ignoring, for the moment, the effect of the shortness in length of member EC, the compatibility equations are:

$$f_{11}x_1 + f_{12}x_2 + u_1 = 0$$

$$f_{21}x_1 + f_{22}x_2 + u_2 = 0$$

Clearly the symmetry will produce $x_1 = x_2$ and thus:

$$x_1 = x_2 = -W\frac{(2+\surd 2)}{(5+4\surd 2)}$$

The effect of the prestrain caused by the forced fit of member EC may be obtained by putting:

$$U = -\begin{bmatrix} 0 \\ 10^{-4}l \end{bmatrix} \tag{3.22}$$

and then solving $\mathbf{FX} + \mathbf{U} = \mathbf{0}$ obtaining:

$$x_1 = \frac{-200}{(47+32\surd 2)} \text{kN}$$

$$x_2 = \frac{800(1+\surd 2)}{(47+32\surd 2)} \text{kN}$$

The forces in the other members may now be obtained from $p = p_0 + p_1 x_1 + p_2 x_2$.

The sign of the lack of fit in Equation (3.22) should be studied carefully and it should be noted that the convention for the signs of forces is tension-positive throughout.

3.3.5 Comments on the flexibility method

For a more detailed treatment of the flexibility method the reader may consult any of the standard texts, e.g. Jenkins[1] and

Coates, Coutie and Kong.[2] The method has declined in popularity in recent years due to the widespread adoption of computerized methods based on *stiffness* concepts. In the context of automatic computation, the stiffness method, which will be considered in the next section, offers considerable advantages over the flexibility method. Methods based on flexibility offer some advantage for hand computation in structures with low (1 or 2) degrees of statical indeterminacy or with lack of fit, temperature change or flexible supports. The concept of flexibility influence coefficients is also useful in determining stiffness coefficients, e.g. in nonprismatic members.

3.4 The stiffness method

3.4.1 Introduction

This method has been very extensively developed in recent years and now forms the basis of most structural analysis carried out on digital computers. The method of 'slope-deflection' is an example of the application of the general stiffness method.

Consider the structure shown in Figure 3.11(a) which is fixed at A and C and has a rigid joint at B. The degree of kinematical indeterminacy, from Equation (3.6), is:

$$n_k = 3(n-1) - c + r$$

$$= 3(3-1) - 5 + 0$$

$$= 1$$

The five constraints are the zero displacements, three at C and two at B, related to the fixed point A. The single unknown

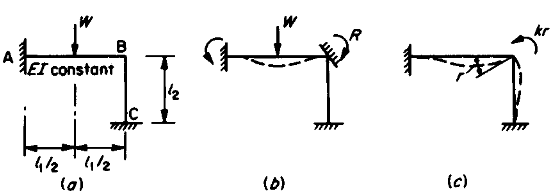


Figure 3.11 Basis of the stiffness method

displacement, nodal degree of freedom is, of course, the rotation of the joint B.

The procedure is to clamp the joint B so constraining the nodal degree of freedom r. On applying the load W, a constraining force, R, will be required at B to prevent the rotation of the joint. The constraining force R is now applied to the, otherwise unloaded, structure with its sign reversed and the nodal degree of freedom released. The result is a rotation of joint B through angle r. The external moment required to effect this rotation is kr where k is the stiffness of the structure for this particular displacement. Thus, for equilibrium:

$$kr = R \tag{3.23}$$

From the table of fixed-end moments, Table 3.5:

$$R = \frac{Wl_1}{8}$$

and from the force–displacement relationships of Equation (3.1)

$$k = \frac{4EI}{l_1} + \frac{4EI}{l_2}$$

Thus:

$$4EI\left(\frac{1}{l_1} + \frac{1}{l_2}\right) r = \frac{Wl_1}{8}$$

Hence:

$$r = \frac{Wl_1^2 l_2}{32EI(l_1 + l_2)}$$

The member forces are now obtained by adding the two systems (b) and (c) in Figure 3.11, e.g.:

$$M_{BA} = \frac{Wl_1}{8} - \frac{4EI(r)}{l_1} = \frac{Wl_1}{8}\left(1 - \frac{l_2}{l_1 + l_2}\right)$$

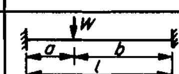
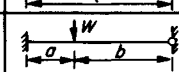
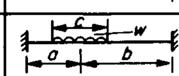
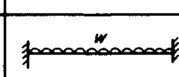
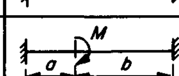
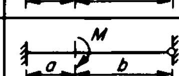
$$= \frac{Wl_1^2}{8(l_1 + l_2)}$$

and

$$M_{BC} = -\frac{4EI(r)}{l_2} = -\frac{Wl_1^2}{8(l_1 + l_2)}$$

Note that in the above, clockwise moments are considered positive.

Table 3.5 Fix-end moments for uniform beams (clockwise moments positive)

M_{FL}	Loading	M_{FR}
$-\frac{Wab}{l}\left(\frac{b}{l}\right)$		$\frac{Wab}{l}\left(\frac{a}{l}\right)$
$-\frac{Wab}{2l^2}(a+2b)$		0
$-\frac{wc}{12l^2}[12ab^2+c^2(a-2b)]$		$\frac{wc}{12l^2}[12a^2b+c^2(b-2a)]$
$-\frac{wl^2}{12}$		$\frac{wl^2}{12}$
$-\frac{wl^2}{30}$		$\frac{wl^2}{20}$
$-\frac{5}{96}wl^2$		$\frac{5}{96}wl^2$
$\frac{Mb}{l^2}(2a-b)$		$\frac{Ma}{l^2}(2b-a)$
$\frac{M}{2l^2}(l^2-3b^2)$		0

3.4.2 Member stiffness matrix

In the stiffness method, a structure is considered to be an assemblage of discrete elements, beams, columns, plates, etc. and the method requires a knowledge of the stiffness characteristics of the elements. In the 'finite element' method (see page 3/14) an artificial discretization of the structure is adopted. As an

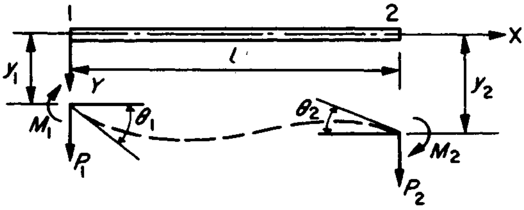


Figure 3.12 Structural beam element

example of the determination of stiffness influencing coefficients we shall consider the simple beam element shown in Figure 3.12. We neglect any axial deformation.

The expression for the bending moment in the beam with origin at end 1 and deflections y positive downwards is:

$$EI\,d^2y/dx^2 = P_1x - M_1$$

Integrating

$$EI\,dy/dx = \frac{P_1x^2}{2} - M_1x + C_1$$

$$= EI\theta_1 \text{ for } x=0$$

Hence:

$$C_1 = EI\theta_1$$

$$= EI\theta_2 \text{ for } x=l$$

Hence:

$$EI(\theta_2-\theta_1) = \frac{P_1l^2}{2} - M_1l \qquad (3.24)$$

Integrating again:

$$EIy = \frac{P_1x^3}{6} - M_1\frac{x^2}{2} + EI\theta_1x + C_2$$

$$= EIy_1 \text{ for } x=0$$

Therefore:

$$C_2 = EIy_1$$

$$= EIy_2 \text{ for } x=l$$

Hence:

$$EI(y_2-y_1) - EI\theta_1l = P_1\frac{l^3}{6} - M_1\frac{l^2}{2} \qquad (3.25)$$

Solving equations (3.24) and (3.25) for M_1 and P_1:

$$M_1 = \frac{4EI\theta_1}{l} + \frac{6EIy_1}{l^2} + \frac{2EI\theta_2}{l} - \frac{6EIy_2}{l^2} \qquad (3.26)$$

and

$$P_1 = \frac{6EI\theta_1}{l^2} + \frac{12EIy_1}{l^3} + \frac{6EI\theta_2}{l^2} - \frac{12EIy_2}{l^3} \qquad (3.27)$$

Two further relationships between the forces and displacements are obtained from statical equilibrium as follows:

For vertical equilibrium, $P_1+P_2=0$

Hence:

$$P_2 = -P_1 \qquad (3.28)$$

Taking moments about end 1:

$$M_2 = -M_1 - P_2l$$

$$= \frac{2EI\theta_1}{l} + \frac{6EIy_1}{l^2} + \frac{4EI\theta_2}{l} - \frac{6EIy_2}{l^2} \qquad (3.29)$$

Equations (3.26)–(3.29) may be combined in the matrix form:

$$\begin{bmatrix} M_1 \\ P_1 \\ M_2 \\ P_2 \end{bmatrix} = \frac{EI}{l^3}\begin{bmatrix} 4l^2 & 6l & 2l^2 & -6l \\ 6l & 12 & 6l & -12 \\ 2l^2 & 6l & 4l^2 & -6l \\ -6l & -12 & -6l & 12 \end{bmatrix}\begin{bmatrix} \theta_1 \\ y_1 \\ \theta_2 \\ y_2 \end{bmatrix}$$

$$\text{or } \mathbf{S} = \mathbf{k}\boldsymbol{\Delta} \qquad (3.30)$$

The matrix **k** is the stiffness matrix of the beam, and **S** and **Δ** are the matrices of member forces and nodal displacements respectively. Equation (3.30) expresses the force–displacement relationships for the beam in the *stiffness* form as distinct from the *flexibility* form. The symmetry of the matrix should be noted as consistent with the symmetry exhibited by flexibility coefficients (see page 3/9).

3.4.3 Assembly of structure stiffness matrix

The stiffness method involves the solution of a set of linear simultaneous equations, representing equilibrium conditions, which may be expressed in the form:

$$\mathbf{Kr} = \mathbf{R} \qquad (3.31)$$

Equation (3.31) is similar in form to Equation (3.23) with the important difference that now we are concerned with a multiple degree of freedom system as distinct from a single unknown displacement. **K** is the structure stiffness matrix, **r** is a matrix of nodal displacements and **R** a matrix of applied nodal forces.

The process of assembling the matrix **K** is one of transferring individual element stiffnesses into appropriate positions in the matrix **K**. Naturally, this has been the subject of considerable organization for digital computer analysis and the subject is well documented.[3] Some aspects of a computerized approach will be considered later but the basic process will be illustrated here using a simple example. Consider the structure shown in Figure 3.13(a). The two beams are rigidly connected together at B where there is a spring support with stiffness k_s. End A is hinged and end C fixed. The structure has three degrees of freedom, rotations r_1 and r_3 at A and B and a vertical displacement r_2 at B. The stiffness matrix for each beam has the form of Equation (3.30) from which **k** may be written in the general form:

$$\mathbf{k} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \qquad (3.32)$$

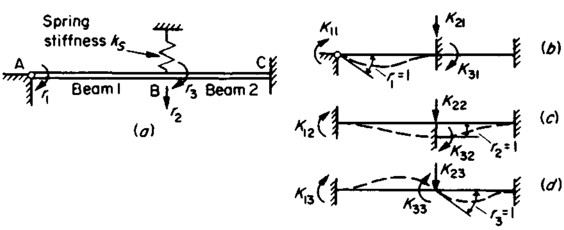


Figure 3.13

where $k_{11}=4EI/l$; $k_{12}=6EI/l^2$, etc.

We apply unit value of each degree of freedom in turn as shown in Figure 3.13(b), (c) and (d). It should be noted that when $r_1=1$ is applied, r_2 and r_3 are constrained at zero value and similarly with $r_2=1$ and $r_3=1$. The force systems necessary to achieve the unit values of the degrees of freedom are also shown at (b), (c) and (d). The equilibrium conditions are clearly:

$$K_{11}r_1+K_{12}r_2+K_{13}r_3=R_1$$

$$K_{21}r_1+K_{22}r_2+K_{23}r_3=R_2$$

$$K_{31}r_1+K_{32}r_2+K_{33}r_3=R_3$$

i.e. $\mathbf{Kr}=\mathbf{R}$

where $\mathbf{R}$ is the matrix of applied loads. Clearly, the forces shown in Figure 3.13(b), (c) and (d) constitute the elements of the stiffness matrix $\mathbf{K}$ and this may now be assembled by inspection. Using the individual beam elements from Equation (3.30) with the notation of Equation (3.32):

$$\mathbf{K}=\begin{array}{|c|c|c|}\hline (k_{11})_1 & -(k_{12})_1 & (k_{13})_1 \\ \hline -(k_{12})_1 & (k_{44})_1+(k_{22})_2+k_s & (k_{23})_2-(k_{14})_1 \\ \hline (k_{13})_1 & (k_{23})_2-(k_{14})_1 & (k_{33})_1+(k_{11})_2 \\ \hline \end{array} \tag{3.33}$$

and more specifically:

$$\mathbf{K}=\begin{array}{|c|c|c|}\hline 4\left(\frac{EI}{l}\right)_1 & -6\left(\frac{EI}{l^2}\right)_1 & 2\left(\frac{EI}{l}\right)_1 \\ \hline -6\left(\frac{EI}{l^2}\right)_1 & 12\left(\frac{EI}{l^3}\right)_1+12\left(\frac{EI}{l^3}\right)_2+k_s & 6\left(\frac{EI}{l^2}\right)_2-6\left(\frac{EI}{l^2}\right)_1 \\ \hline 2\left(\frac{EI}{l}\right)_1 & 6\left(\frac{EI}{l^2}\right)_2-6\left(\frac{EI}{l^2}\right)_1 & 4\left(\frac{EI}{l}\right)_1+4\left(\frac{EI}{l}\right)_2 \\ \hline \end{array} \tag{3.34}$$

3.4.4 Stiffness transformations

The member stiffness matrix $\mathbf{k}$ in Equation (3.30) is based on a coordinate system which is convenient for the member, i.e. origin at one end and X-axis directed along the axis of the beam. Such a coordinate system is termed 'local' as distinct from the 'global' coordinate system which is used for the complete structure. This subject is considered in detail in a number of texts[2,3] and we shall give only a brief indication of the type of computation required.

Consider a three-dimensional coordinate system $\bar{X}\bar{Y}\bar{Z}$ (global) which is obtained by rotation of the (local) coordinate system XYZ. In the local system the force–displacement relationships for a beam element may be expressed in the partitioned matrix form:

$$\begin{bmatrix}\mathbf{S}_1\\ \mathbf{S}_2\end{bmatrix}=\begin{bmatrix}\mathbf{k}_{11} & \mathbf{k}_{12}\\ \mathbf{k}_{21} & \mathbf{k}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2\end{bmatrix} \tag{3.35}$$

in which the subscripts refer to ends 1 and 2.

The stiffness expressed in the coordinate system $\bar{X}\bar{Y}\bar{Z}$ may be obtained as follows:

$$\begin{bmatrix}\bar{\mathbf{S}}_1\\ \bar{\mathbf{S}}_2\end{bmatrix}=\begin{bmatrix}\lambda\mathbf{k}_{11}\lambda^T & \lambda\mathbf{k}_{12}\lambda^T\\ \lambda\mathbf{k}_{21}\lambda^T & \lambda\mathbf{k}_{22}\lambda^T\end{bmatrix}\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2\end{bmatrix} \tag{3.36}$$

in which λ is a matrix of direction cosines as follows:

$$\boldsymbol{\lambda}=\begin{bmatrix}\lambda_{\bar{x}x} & \lambda_{\bar{x}y} & \lambda_{\bar{x}z} & 0 & 0 & 0\\ \lambda_{\bar{y}x} & \lambda_{\bar{y}y} & \lambda_{\bar{y}z} & 0 & 0 & 0\\ \lambda_{\bar{z}x} & \lambda_{\bar{z}y} & \lambda_{\bar{z}z} & 0 & 0 & 0\\ 0 & 0 & 0 & \lambda_{\bar{x}x} & \lambda_{\bar{x}y} & \lambda_{\bar{x}z}\\ 0 & 0 & 0 & \lambda_{\bar{y}x} & \lambda_{\bar{y}y} & \lambda_{\bar{y}z}\\ 0 & 0 & 0 & \lambda_{\bar{z}x} & \lambda_{\bar{z}y} & \lambda_{\bar{z}z}\end{bmatrix} \tag{3.37}$$

where $\lambda_{\bar{x}x}=\cos\bar{X}OX$, etc.

3.4.5 Some aspects of computerization of the stiffness method

The remarkable increase in popularity of the stiffness method is due to the widespread availability of relatively cheap computing power. The method is of limited practical use *except* on computers. The stiffness method is eminently suitable for computers because the setting up of the data describing the structure and loading system to be analysed is a comparatively simple operation. Although there is then generally considerable numerical computation to do, this is done by the computer. Thus the human effort required is minimized and the likelihood of errors being made also reduced. With the phenomenal development of cheap and powerful microcomputers, which are quite suitable for analysing most 'run-of-the-mill' structures, it is quite likely that in the very near future almost all structural analysis will be carried out on computers.

It will be useful to look briefly at the more important aspects of adapting the stiffness method for use on computers. The method may be viewed as a succession of six stages:

(1) Define the nodal degrees of freedom of the structure (n) (Equation (3.6)), the nodal 'coordinates'. The total number determines the size of the structure stiffness matrix $\mathbf{K}$. The ordering is a matter of convenience but in some programs a judicial ordering of coordinates is necessary to reduce the 'band width' of $\mathbf{K}$. An array $\mathbf{K}$ ($n\times n$) is now generated in the computer and all elements are zeroed. This is necessary since component stiffnesses are going to be added-in to this array thus 'accumulating' the stiffnesses element by element.
(2) The individual structural elements are now defined and their force–displacement relationships expressed in stiffness matrices, $\mathbf{k}$ (Equation (3.30)); $\mathbf{S}=\mathbf{k}\boldsymbol{\Delta}$. The dimensions of these matrices will depend on the type of element used but for most of the common elements (beam, column, pin-jointed truss member, etc.) the standard matrices are pub-

lished in the textbooks. The element stiffnesses are now transformed from local to global coordinates using matrix transformations as in Equation (3.36).

(3) The transformed stiffnesses are now transferred into appropriate locations of the structure stiffness matrix **K**. Suppose we are to transfer the stiffnesses of a particular element and suppose this element has two coordinates numbered 1 and 2. If the coordinates in the actual structure which correspond to 1 and 2 of the element are, say, *i* and *j* then the transfer of stiffnesses is carried out as follows:

$$\mathbf{k}_{11} \rightarrow \mathbf{k}_{ii}$$
$$\mathbf{k}_{12} \rightarrow \mathbf{k}_{ij}$$
$$\mathbf{k}_{21} \rightarrow \mathbf{k}_{ji}$$
$$\mathbf{k}_{22} \rightarrow \mathbf{k}_{jj}$$

There is considerable economy in organization and programming if the above procedure is applied to 'groups' of coordinates, e.g. *all* the displacements at one node. This can be achieved by *partitioning* the element stiffness matrices.

(4) Once **K** has been set up, the applied load matrix **R** is generated. This is simply a column matrix containing the applied (nodal) loads arranged in the same order as the nodal coordinates. If the structure is carrying loads other than at the defined nodes, then such loads must be converted to statically equivalent nodal loads. In rigid frames, for example, this is easily done using the standard values of 'fixed-end' effects. If a concentrated load does not coincide with the defined nodal coordinates then it is a simple matter, as an alternative, to introduce a node at the load point. This procedure, although it increases the size of the system to be solved, does have the advantage of yielding the displacements developing at the load point.

(5) The computer now solves the linear simultaneous equations (Equation (3.31)) $\mathbf{Kr} = \mathbf{R}$ to produce the nodal displacements $\mathbf{r}$.

(6) Lastly, the element forces are obtained from Equation (3.30) $\mathbf{S} = \mathbf{k}\Delta$. In this last operation, some logical organization is clearly needed to extract the element nodal displacements Δ from the structure displacement **Sr**.

The foregoing is a description of the fundamental basis of the stiffness method applied on computers. Of course, it is possible to incorporate many refinements and devices to simplify the input and output, to check the results and to make changes in data without having to re-input all data.

In its most general form the stiffness method is used to analyse complex structures in which not only simple elements such as beams and columns are used but 'continua' such as plates and shells. This is the 'finite element' method which will now be examined briefly.

3.4.6 Finite element analysis

This extremely powerful method of analysis has been developed in recent years and is now an established method with wide applications in structural analysis and in other fields. Space permits only the most brief introduction here but the method is extensively documented elsewhere.[4-6] We have discussed the application of the stiffness method to framed structures in which the structural elements, beams and columns, have been connected at the nodes and the method observes the correct conditions of displacement compatibility and equilibrium at the nodes. The finite element method was developed, originally, in order to extend the stiffness method to the analysis of elastic continua such as plates and shells and indeed to three-dimensional continua. The first step in the process is to divide the structure into a finite number of discrete parts called 'elements'. The elements may be of any convenient shape, e.g. a thin plate may be represented by triangular or rectangular elements, and the discretization may be *coarse*, with a small number of elements, or *fine*, with a large number of elements. The connection between elements now occurs not only at the nodal points but along boundary lines and over boundary faces.

The procedure ensures, as for framed structures, that equilibrium and compatibility conditions are satisfied at the nodes but the regions of connection between nodes are constrained to adopt a chosen form of displacement function. Thus, compatibility conditions along the interfaces between elements may not be completely satisfied and a degree of approximation is generally introduced. Once the geometry of the elements has been determined and the displacement function defined, the stiffness matrix of each element, relating nodal forces to nodal displacements, can be obtained. The remainder of the structural analysis follows the established procedures similar to those for framed structures. Naturally the best choice of element and discretization pattern, the precise conditions occurring at the interfaces and the accuracy of the solution, are matters which have received a great deal of attention in the literature.

A central stage in the process is the adoption of a suitable displacement function for the element chosen, and the subsequent evaluation of the element stiffnesses. This will be illustrated with one of the simplest possible elements, a triangular plate element for use in a plane stress situation.

3.4.6.1 Triangular element for plant stress

A triangular element ijk is shown in Figure 3.14. Under load, the displacement of any point within the element is defined by the displacement components *u*, *v*. In particular the nodal displacements are:

$$\Delta = \{u_i u_j u_k v_i v_j v_k\} \tag{3.38}$$

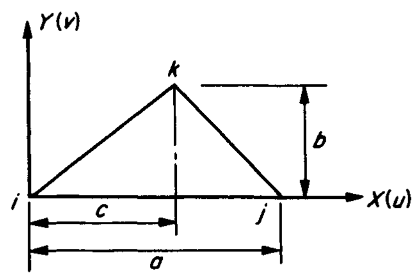


Figure 3.14

It is now assumed that the displacements *u*, *v* are linear functions of *x*, *y* as follows:

$$\begin{aligned} u &= \alpha_1 + \alpha_2 x + \alpha_3 y \\ v &= \alpha_4 + \alpha_5 x + \alpha_6 y \end{aligned} \tag{3.39}$$

The nodal displacements Δ are now expressed in terms of the displacement parameters α, from Equations (3.39) and Figure 3.14:

$$\begin{bmatrix} u_i \\ u_j \\ u_k \\ v_i \\ v_j \\ v_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & a & 0 & 0 & 0 & 0 \\ 1 & c & b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & a & 0 \\ 0 & 0 & 0 & 1 & c & b \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} \tag{3.40}$$

or, $\Delta = \mathbf{A\alpha}$

The strains in the element are functions of the derivatives of u and v as follows:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{bmatrix} \tag{3.41}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} \tag{3.42}$$

i.e.:

$$\boldsymbol{\varepsilon} = \mathbf{B}\boldsymbol{\alpha} = \mathbf{BA}^{-1}\boldsymbol{\Delta} \tag{3.43}$$

from Equation (3.40).

It should be noted that the matrix **B** in Equation (3.42) contains only constant terms and it follows that the strains are constant within the element.

The stress–strain relationships for plane stress in an isotropic material with Poisson's ratio ν and Young's modulus E are:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

i.e.:

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} = \mathbf{DBA}^{-1}\boldsymbol{\Delta} \tag{3.44}$$

Matrix **D** is the 'elasticity' matrix relating stress and strain. To obtain the element stiffness we employ the principle of virtual work and apply arbitrary nodal displacements $\bar{\boldsymbol{\Delta}}$ producing virtual strains in the element:

$$\bar{\boldsymbol{\varepsilon}} = \mathbf{BA}^{-1}\bar{\boldsymbol{\Delta}} \tag{3.45}$$

The virtual strain energy in the element, from Equation (2.78) of Chapter 2, is:

$$\int_{vol} \bar{\boldsymbol{\varepsilon}}^T \boldsymbol{\sigma} \, \mathrm{d}V$$

where V = volume of triangular element $= tab/2$, t = thickness

Substituting for $\bar{\boldsymbol{\varepsilon}}^T$ and $\boldsymbol{\sigma}$ from Equations (3.45) and (3.44) respectively, the virtual strain energy is:

$$\int_{vol} [\mathbf{BA}^{-1}\bar{\boldsymbol{\Delta}}]^T \mathbf{DBA}^{-1}\boldsymbol{\Delta}\, \mathrm{d}V$$

Now since all the matrices contain constant terms only and are thus independent of x and y, the expression for the virtual strain energy may be written:

$$\bar{\boldsymbol{\Delta}}^T\{[\mathbf{A}^{-1}]^T \mathbf{B}^T \mathbf{DBA}^{-1} V\}\boldsymbol{\Delta}$$

The external work is the product of the virtual displacements $\bar{\boldsymbol{\Delta}}$ and the nodal forces **S**, hence equating external virtual work and internal virtual strain energy:

$$\bar{\boldsymbol{\Delta}}^T \mathbf{S} = \bar{\boldsymbol{\Delta}}^T\{[\mathbf{A}^{-1}]^T \mathbf{B}^T \mathbf{DBA}^{-1} V\}\boldsymbol{\Delta}$$

The virtual displacements are quite arbitrary and in particular may be taken to be represented by a unit matrix, thus:

$$\mathbf{S} = \{[\mathbf{A}^{-1}]^T \mathbf{B}^T \mathbf{DBA}^{-1} V\}\boldsymbol{\Delta}$$
$$= \mathbf{k}\boldsymbol{\Delta}, \text{ from Equation (3.30)}$$

Thus:

$$\mathbf{k} = [\mathbf{A}^{-1}]^T \mathbf{B}^T \mathbf{DBA}^{-1} V \tag{3.46}$$

Before the matrix multiplications required in Equation (3.46) can be performed we need to find $\mathbf{A}^{-1}$. This is easily determined as:

$$\mathbf{A}^{-1} = \frac{1}{ab} \begin{bmatrix} ab & 0 & 0 & 0 & 0 & 0 \\ -b & b & 0 & 0 & 0 & 0 \\ (c-a) & -c & a & 0 & 0 & 0 \\ 0 & 0 & 0 & ab & 0 & 0 \\ 0 & 0 & 0 & -b & b & 0 \\ 0 & 0 & 0 & (c-a) & -c & a \end{bmatrix}$$

Hence finally, with $\lambda_1 = \frac{1}{2}(1-\nu)$ and $\lambda_2 = \frac{1}{2}(1+\nu)$ we obtain the stiffness matrix for the plane stress triangular element as shown in equation (3.47) below.

It is neither necessary nor economical to carry out these operations by hand; the computation of the element stiffness and, indeed, the entire computational process is easily programmed for the digital computer.

Computer 'packages' for finite element analysis of structures are highly developed, very powerful and readily available. Because of the comparatively heavy demands on computer storage, the use of the packages is generally confined to mainframe computers. A good example of a finite element system which is used very extensively is PAFEC.[6] The more important topics which should be studied in pursuing finite element analysis include: (1) shape (displacement) functions; (2) conforming and nonconforming elements; (3) isoparametric elements; and (4) automatic mesh generation.

$$\mathbf{k} = \frac{Et}{2(1-\nu^2)ab} \begin{bmatrix} b^2+\lambda_1(c-a)^2 & & & & & \\ -b^2-\lambda_1 c(c-a) & b^2+\lambda_1 c^2 & & & \text{Symmetric} & \\ \lambda_1 a(c-a) & -\lambda_1 ac & \lambda_1 a^2 & & & \\ -\lambda_2 b(c-a) & \lambda_1 cb+\nu b(c-a) & -\lambda_1 ab & \lambda_1 b^2+(c-a)^2 & & \\ \lambda_1 b(c-a)+\nu cb & -\lambda_2 bc & \lambda_1 ab & -\lambda_1 b^2-c(c-a) & \lambda_1 b^2+c^2 & \\ -\nu ab & \nu ab & 0 & a(c-a) & -ac & a^2 \end{bmatrix} \tag{3.47}$$

3.5 Moment distribution

3.5.1 Introduction

Although the stiffness method, described in the previous section has the merit of simplicity, the solution of the equilibrium equations (3.31) is generally a matter for the digital computer since only for the simplest structures can a hand solution be contemplated. An alternative procedure which is eminently suitable for hand computation is the method of moment distribution which is essentially an iterative solution of the equations of equilibrium.

As in the general stiffness method, we first imagine all the degrees of freedom, joint rotations and joint translations, to be constrained. We ignore axial effects in members and consider flexure only. The constraints are imagined to be clamps applied to the joints to prevent rotation and translation. The forces required to effect the constraints are applied artificially and in the moment distribution processes these clamping forces are systematically released so as to allow the structure to achieve an equilibrium state. It is important to note that in the method as generally applied, the rotational joint restraints are relaxed by one process and the translational restraints by another. Finally the principle of superposition is used to combine the separate results.

It is necessary to assemble certain standard results before we can consider the actual process.

3.5.2 Distribution factors, carry-over factors and fixed-end moments

For the time being we confine our attention to prismatic members. The treatment of nonuniform section members will be touched on later.

Standard member stiffnesses are required and these are illustrated in Figure 3.15. The member end forces are those required to produce the deflected forms shown. Diagrams (a) and (b) relate to rotation without translation (sway), and diagrams (c) and (d) relate to sway without rotation. The results in diagrams (a) and (c) may be deduced from the stiffness matrix in Equation (3.30). The other results may be obtained easily from elementary beam theory, e.g. in Figure 3.15(b), taking the origin of x at the left-hand end and y positive downwards:

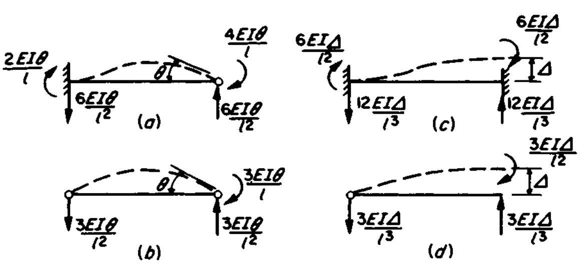


Figure 3.15

$EId^2y/dx^2 = \frac{Mx}{l}$, where M is the moment, to be determined, at the right-hand end,

$$EIdy/dx = \frac{M}{l}\frac{x^2}{2} + C_1$$

$$= EI\theta \text{ for } x = l; \text{ hence } C_1 = EI\theta - M\frac{l}{2}$$

$$EIy = \frac{M}{l}\frac{x^3}{6} + \left(EI\theta - M\frac{l}{2}\right)x + C_2$$

$$= 0 \text{ for } x = 0; \text{ hence } C_2 = 0$$

$$= 0 \text{ for } x = l; \text{ hence, } M = \frac{3EI\theta}{l}$$

When loads are applied to members which are constrained at the joints, fixed-end moments are required to prevent the end rotations. This is another standard type of result which is required in the moment distribution method. Table 3.5 lists fixed-end moments for a selection of loading cases on uniform section beams. Again, these results may be obtained from elementary beam theory. It should be noted that the sign convention is that a moment is *positive* if tending to produce clockwise rotation of the end of the member at which it acts. This convention is different to, and should not be confused with, the sign convention for constructing bending moment diagrams which must be based on the curvature produced in the member.

As an illustration of the basic process, consider the structure ABC shown in Figure 3.11. This structure was analysed by the stiffness method previously. Joint B is considered to be clamped and thus a system of fixed-end moments is set up in member AB. The end moments in the members are shown in line 1 of Table 3.6. The constraining moment at joint B is seen to be $Wl_1/8$ clockwise and we imagine this moment to be removed by the application of a moment $-Wl_1/8$. The subsequent rotation of joint B, anticlockwise through angle θ, will develop moments in both members. Referring to Figure 3.15 the moments induced will be:

$$M_{BA} = -\frac{4EI\theta}{l_1};\ M_{AB} = -\frac{2EI\theta}{l_1}$$

$$M_{BC} = -\frac{4EI\theta}{l_2};\ M_{CB} = -\frac{2EI\theta}{l_2}$$

For equilibrium of joint B, the applied moment $-Wl_1/8$ must equal the sum of the moments absorbed by the two members meeting at the joint:

$$-\frac{Wl_1}{8} = -\frac{4EI\theta}{l_1} - \frac{4EI\theta}{l_2} = -4EI\theta\left(\frac{I}{l_1} + \frac{I}{l_2}\right)$$

and it is seen that the moment is 'distributed' to the members in proportion to their I/l values.

Thus:

$$M_{BA} = \frac{-Wl_1}{8}\frac{I/l_1}{(I/l_1 + I/l_2)} = \frac{-Wl_1}{8}\left(\frac{l_2}{l_1 + l_2}\right)$$

and:

$$M_{BC} = \frac{-Wl_1}{8}\frac{I/l_2}{(I/l_1 + I/l_2)} = \frac{-Wl_1}{8}\left(\frac{l_1}{l_1 + l_2}\right)$$

The moments induced at A and C are from Figure 3.15, one-half of those induced at B and the factor of one-half is termed the *carry over* factor. This set of moments is shown in line 2 of Table 3.6.

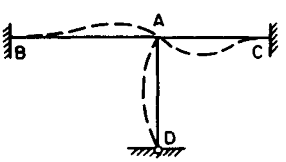
Joint B is now 'in balance' and since it was the only joint which was clamped we have reached an equilibrium state and no further distribution of moments is required. The final set of

Table 3.6

Stage	*Operation*	M_{AB}	M_{BA}	M_{BC}	M_{CB}
1	Fixed-end moments	$-Wl_1/8$	$+Wl_1/8$	0	0
2	Distribution at B	$-\frac{Wl_1}{16}\left(\frac{l_2}{l_1+l_2}\right)$	$-\frac{Wl_1}{8}\left(\frac{l_2}{l_1+l_2}\right)$	$-\frac{Wl_1}{8}\left(\frac{l_1}{l_1+l_2}\right)$	$-\frac{Wl_1}{16}\left(\frac{l_1}{l_1+l_2}\right)$
3	Total moments	$-\frac{Wl_1}{16}\left(\frac{2l_1+3l_2}{l_1+l_2}\right)$	$\frac{Wl_1^2}{8(l_1+l_2)}$	$-\frac{Wl_1^2}{8(l_1+l_2)}$	$-\frac{Wl_1^2}{16(l_1+l_2)}$

moments is obtained in line 3 of Table 3.6, by the addition of lines 1 and 2. This result is the same as that obtained from pure stiffness considerations. It should be noted that the zero sum of moments M_{BA} and M_{BC} indicates that joint B is in rotational equilibrium.

Two further points should be noted before we consider the moment distribution process in more detail. Referring to Figure 3.16, of the three members connected at joint A, member AD is hinged at the end remote from A whereas the other two members are fixed. Since D is hinged no moment can exist there and hence there is no carry-over to D. Furthermore, the moment–rotation relationship is different for a member pinned at the remote end, as may be seen by comparing Figures 3.15(a) and (b). In calculating distribution factors this is taken account of by taking $\frac{3}{4}(I/l)$ for such members as compared with I/l for members fixed at the remote end.



$$\Sigma k = \left(\frac{I}{l}\right)_{AB} + \left(\frac{I}{l}\right)_{AC} + \frac{3}{4}\left(\frac{I}{l}\right)_{AD}$$

AB : AC : AD =

$$\frac{(I/l)_{AB}}{\Sigma k} : \frac{(I/l)_{AC}}{\Sigma k} : \frac{\frac{3}{4}(I/l)_{AD}}{\Sigma k}$$

Figure 3.16 Distribution factors at typical joint

3.5.3 Moment distribution without sway

As an example of a structure with two degrees of freedom of joint rotation and no sway, consider the frame shown in Figure 3.17, EI (beams) = 3 × EI (columns).

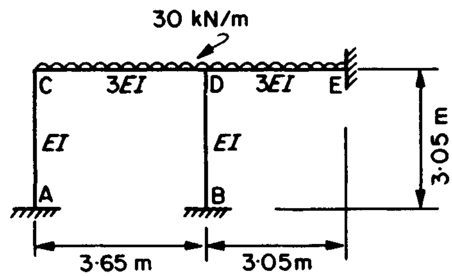


Figure 3.17

Table 3.7 Moment distribution for frame shown in Figure 3.17

	Joint	*A*	*C*		*D*			*B*	*E*
	Distribution factors		0.285	0.715	0.386	0.154	0.460		
	end moments	AC	CA	CD	DC	DB	DE	BD	ED
(1)	Fixed-end moments			−33.3	+33.3		−23.3		+23.3
(2)	Distribution at C		+9.5	+23.8					
(3)	Carry-over to A and D	+4.75			+11.9				
(4)	Distribution at D				−8.45	−3.38	−10.07		
(5)	Carry-over to C, B and E			−4.23				−1.69	−5.04
(6)	Distribution at C		+1.20	+3.03					
(7)	Carry-over to A and D	+0.60			+1.52				
(8)	Distribution at D				−0.59	−0.23	−0.70		
(9)	Carry-over to C, B and E			−0.30				−0.12	−0.35
(10)	Distribution at C		+0.09	+0.21					
(11)	Carry-over to A and D	+0.05			+0.11				
(12)	Distribution at D				−0.04	−0.02	−0.05		
(13)	Carry-over to C, B and E				May be neglected				
(14)	Total moments (kNm)	+5.40	+10.79	−10.79	+37.75	−3.63	−34.12	−1.81	+17.91

The fixed-end moments are, $(wl^2/12)$,

$$M_{FCD} = -30 \times \frac{3.65^2}{12};\ M_{FDC} = +30 \times \frac{3.65^2}{12} = 33.3\ \text{kNm}$$

$$F_{FDE} = -30 \times \frac{3.05^2}{12};\ M_{FED} = +30 \times \frac{3.05^2}{12} = 23.3\ \text{kNm}$$

and the distribution factors are:

$$\text{at C, CD:CA} = \frac{3/3.65}{(1/3.05)+(3/3.65)} : \frac{1/3.05}{(1/3.05)+(3/3.65)}$$
$$= 0.715:0.285$$

at D, DC:DB:DE =

$$\frac{3/3.65}{(3/3.65)+(1/3.05)+(3/3.05)} : \frac{1/3.05}{(3/3.65)+(1/3.05)+(3/3.05)} : \frac{3/3.05}{(3/3.65)+(1/3.05)+(3/3.05)}$$

$$= 0.386:0.154:0.460$$

The moment distribution is carried out in Table 3.7. It should be noted that after each distribution at a joint the distributed moments are underlined to indicate that the joint is balanced at that stage. At step 4, the out-of-balance moment to be distributed at D is $+33.3+11.9-23.3 = +21.9$; hence the distributed moments should total -21.9.

3.5.4 Moment distribution with sway

This process will be illustrated with reference to Example 3.3 (page 3/9), for which the structure is shown in Figure 3.9. We first ignore any horizontal movement (sway) of the joints B and C and carry out a moment distribution.

The fixed-end moments are $wl^2/12 = \pm 40$ kNm; and the distribution factors are:

$$\text{BA:BC} = \tfrac{1}{3}:\tfrac{2}{3}$$

$$\text{CB:CD} = \tfrac{2}{3}:\tfrac{1}{3} \text{ (noting } \tfrac{3}{4}I/l \text{ for CD)}$$

The result of this (no sway) moment distribution is given in line 3 of Table 3.8. We now consider the horizontal equilibrium of the beam BC, Figure 3.18(a), and find that a force F_1 is required to maintain equilibrium. F_1 may be calculated by evaluating the horizontal shear forces at the tops of the columns as follows:

$$F_1 = 120 + \frac{(20+10)}{4} - \frac{20}{3} = 120.8\ \text{kN}$$

This force cannot exist in practice and what happens is that the beam BC deflects to the right and a new set of bending moments is set up with the effect that the out-of-balance horizontal force F_1 is removed. We consider the effect of this sway separately. Referring to Figure 3.18(b), a movement to the right of Δ, without joint rotation, requires column moments as shown. From Figure 3.15(c) and (d), these column moments are,

$$M_{FBA} = M_{FAB} = -6\left(\frac{EI}{l^2}\right)\Delta_{AB}$$

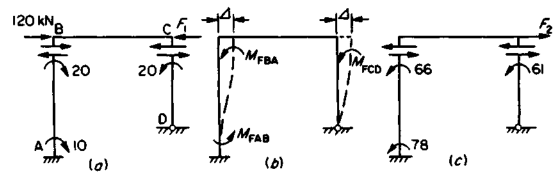


Figure 3.18

$$M_{FCD} = -3\left(\frac{EI}{l^2}\right)\Delta_{CD} \quad (\text{note } M_{FDC} = 0)$$

We cannot evaluate these moments unless Δ is known but we could proceed with an arbitrary value of Δ, and carry out a distribution to produce rotational equilibrium of the joints B and C. In fact, it is seen that any arbitrary values of moments can be used providing these are in the correct proportions between the two columns. The ratio in this example is:

$$\text{AB:CD} = \left(\frac{I}{l^2}\right)_{AB} : \frac{1}{2}\left(\frac{I}{l^2}\right)_{CD}$$

If we adopt

$$M_{FBA} = M_{FAB} = -90$$

and

$$M_{FCD} = -80$$

the moments are in the correct proportion. A second moment distribution is now carried out, using these values of fixed-end moments, and the result is shown in line 1 of Table 3.8. This set of moments is consistent with an applied horizontal force F_2, Figure 3.18(c), and:

$$F_2 = \frac{66+78}{4} + \frac{61}{3} = 56.3\ \text{kN}$$

Table 3.8

	Joint	A	B		C	
	End moments	AB	BA	BC	CB	CD
(1)	Arbitrary sway	−78	−66	+66	+61	−61
(2)	Corrected [(1) × λ]	−167	−141	+141	+131	−131
(3)	No sway moments	+10	+20	−20	+20	−20
(4)	Final moments [(2)+(3)]	−157	−121	+121	+151	−151

Now F_2 has to be scaled to equal F_1 and the scaling factor is $F_1/F_2 = \lambda = 120.8/56.3 = 2.14$.

The corrected moments are given in line 2 of Table 3.8 and the final moments are in line 4 obtained by adding lines 2 and 3.

3.5.5 Additional topics in moment distribution

Space has permitted only a brief introduction to the method of moment distribution. Additional topics which should be studied by reference to the standard texts,[3,4] are as follows:

(1) *Frames with multiple degrees of freedom for sway.* These are handled by carrying out an arbitrary sway distribution

for each sway in turn. Equilibrium conditions are then used to relate the out-of-balance forces and obtain the correction factors for each sway mode.

(2) *Treatment of symmetry.* In cases of symmetry the moment distribution process can be considerably shortened. Two cases arise and should be studied, systems in which it is known that the final set of moments is symmetrical and systems in which the final moments form an anti-symmetrical system.

(3) *Nonprismatic members.* If the flexural rigidity (EI) of a member varies within its length, then the effect is to change the values of end stiffnesses, carry-over factor and fixed end moments. A suitable general method for handling this situation is to evaluate end flexibilities by the use of Simpson's rule and then convert the flexibilities into stiffnesses.

3.6 Influence lines

3.6.1 Introduction and definitions

It is frequently necessary to consider loads which may occupy variable positions on a structure. For example, in bridge design it is important to determine the maximum effects due to the passage of a specified train or system of loads. In other cases the total load on a structure may be comprised of different loads which may be applied in various combinations and this again is a problem of variability of load or load position. The effect of varying a load position may be studied with the help of *influence lines*.

An influence line shows the variation of some resultant action or effect such as bending moment, shear force, deflection, etc. at a particular point as a unit load traverses the structure. It is important to observe that the effect considered is at a fixed position, e.g. *bending moment at* C, and that the independent variable in the influence line diagram is *the load position*. The following is a summary of influence line theory. For a more detailed treatment the reader should consult Jenkins.[1]

3.6.2 Influence lines for beams

Consider the simply-supported beam AB, Figure 3.19, carrying a single unit load occupying a variable position distant y from A. We require to obtain influence lines for bending moment and shear force at a fixed point X distant a from A and b from B.

If the unit load lies between X and B:

$$M_x = R_A \cdot a = 1\frac{(l-y)}{l}a \qquad (3.48)$$

If the unit load acts between A and X:

$$M_x = R_B \cdot b = 1 \cdot y/l \cdot b \qquad (3.49)$$

Equations (3.48) and (3.49) are linear in y and when plotted in the regions to which they relate, form a triangle as shown in Figure 3.19(b). We note that, in both cases, substitution of $y=a$ gives $M_x = 1 \cdot ab/l$. Thus the influence line for M_x is a triangle with a peak value ab/l at the section X.

Turning now to the influence line for shearing force at X. For unit load between X and B:

$$S_x = R_A = \frac{l-y}{l} \qquad (3.50)$$

(and now we have implied a sign convention for shear force

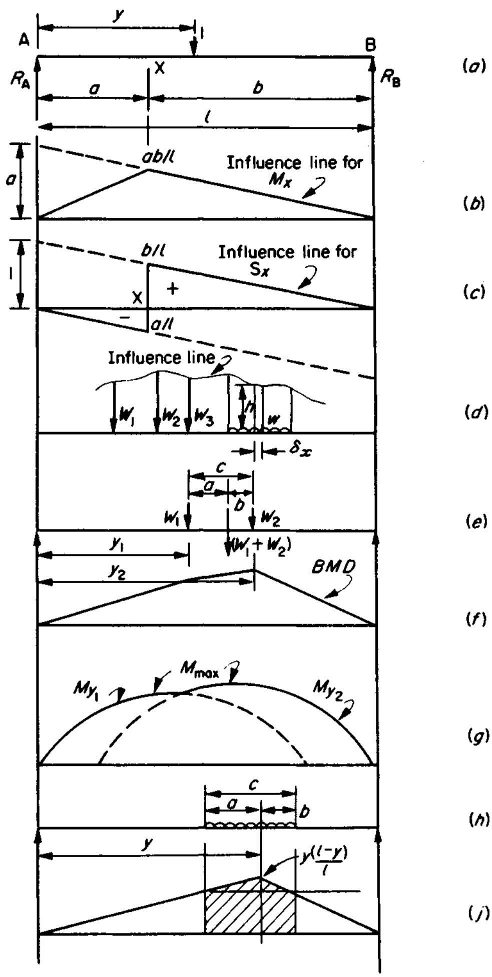


Figure 3.19 Influence lines and related diagrams for simply supported beams

namely that S_x is positive if the resultant force to the left of the section is upwards).

Where $y=a$, $S_x=b/l$

For unit load between A and X:

$$S_x = -R_B = -y/l \qquad (3.51)$$

when $y=a$, $S_x = -a/l$

We note that Equations (3.50) and (3.51) give different values of S_x for $y=a$ and moreover the signs are opposite. This means that the shear force influence line contains a discontinuity at X as shown in Figure 3.19(c).

In using influence lines with a given system of loads and having determined the locations of the loads on the span, the total effect is evaluated as:

$$\sum(W \times \text{ordinate}), \text{ for concentrated loads} \qquad (3.52)$$

and:

$$\int whdx = w \text{ (area under influence line)} \tag{3.53}$$

for distributed loads (Figure 3.19(d).

The maximum effect produced at a given position is of interest in the design process. In the case of concentrated loads, from Equation (3.52), this is obtained when:

$\sum(W \times \text{ordinate})$ is a maximum

The process of locating the loads to produce the maximum value is best done by trial and error. It follows from the straight-line nature of a bending moment diagram due to concentrated loads, that the maximum bending moment at a section will be obtained when one of the loads acts at the section. This may be illustrated by reference to the two-load system shown at (e) in Figure 3.19. The shape of the bending moment diagram is as shown at (f) and at (g) is drawn a diagram which shows the maximum value of bending moment at any section in the beam. This is the *maximum bending moment envelope* M_{max} which is seen to consist of two intersecting parabolic curves M_{y1} and M_{y2}.

The curve M_{y1} represents the maximum bending moment at all sections in the beam when this is obtained with load W_1 placed at the section. The curve M_{y2} represents the maximum bending moment at all sections in the beam when this is obtained with load W_2 at the section. It is seen that W_1 should be placed at the section towards the left-hand end of the beam, and W_2 at the section towards the right-hand end of the beam.

The expressions for M_{y1} and M_{y2} are as follows:

$$\left.\begin{aligned} M_{y1} &= (W_1 + W_2)\frac{y_1}{l}(l - y_1 - a) \\ M_{y2} &= (W_1 + W_2)\frac{(l - y_2)}{l}(y_2 - b) \end{aligned}\right\} \tag{3.54}$$

In the case of a distributed load which has a length greater than the span, then for an influence line of type (b) in Figure 3.19, the whole span would be loaded, whereas for an influence line of type (c) one would place the left-hand end of the load at X thus avoiding the introduction of a negative effect on the maximum positive value. For a short distributed load, as at (h), for maximum effect at y, the load must be placed so that the shaded area in (j) is a maximum.

The rule for this is:

$$y/l = a/c \tag{3.55}$$

3.6.3 Influence lines for plane trusses

In the analysis of plane trusses, the influence line is useful in representing the variations in forces in members of the truss.

Figure 3.20(a) shows a Warren girder AB of span 20 m. For the unit load acting at any of the lower chord joints, the force in member 1 is:

$$P_1 = \frac{4R_A}{2\sqrt{3}}$$

The peak value occurs when the unit load is at C, and thus:

$$P_{1\,max} = \frac{2}{\sqrt{3}} \times \frac{4}{5} \times 1 = \frac{8}{5\sqrt{3}}$$

The influence line for P_1 is shown at (b).

For member 2, if the unit load lies between A and E, we take:

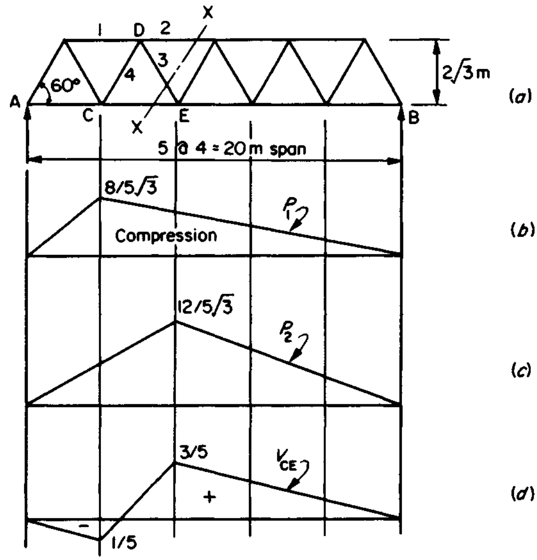


Figure 3.20 Influence lines for plane truss

$$P_2 = \frac{12R_B}{2\sqrt{3}}$$

or, if the unit load lies between E and B we take:

$$P_2 = \frac{8R_A}{2\sqrt{3}}$$

The result is a triangle with peak value $12/5\sqrt{3}$ at E, as shown in diagram (c).

It should be noted that both the P_1 and P_2 influence lines indicate compression for all positions of the unit load.

For members 3 and 4 it is useful to note that these members carry the vertical shear force in the panel CE, and we proceed by drawing the influence line for V_{CE} as at (d).

Considering now the force in member 3 and the section XX in diagram (a), it is clear that the relationship is:

$$P_3 = \frac{V_{CE}}{\sin 60°}$$

and that P_3 is tensile when V_{CE} is positive and compressive when V_{CE} is negative.

3.6.4 Influence lines for statically indeterminate structures

The use of influence lines in representing the effects of variable-position loads in statically determinate beams and trusses has been outlined. The concept is, of course, of general application. When dealing with statically indeterminate structures it is convenient to introduce some additional theorems to assist the analysis. It is possible to relate influence line shapes to deflected shapes of structures under particular forms of applied force. This involves an application of Mueller-Breslau's principle, which we shall look at in this section. The application of this principle can take the form of a model analysis, to which a simple form or model of the structure is made and particular distortions of the model produce scaled versions of influence lines.

With the enormous increase in computing power now available there is little need to use models in this way and it is generally more economical to produce influence lines by computer. It should be noted that it is always possible to construct influence lines by repeated analysis of the structure under a unit applied load, changing the load position for each analysis and thus producing a succession of ordinates to the influence line sought. This latter approach will be illustrated in section 3.6.8.

We now look at two important theorems concerned with influence lines.

3.6.5 Maxwell's reciprocal theorem

Consider the propped cantilever shown in Figure 3.21 to be subjected to a load W at A, producing displacements f_{11} and f_{21} as shown at (a), and then separately to be subjected to a moment M at B producing displacements f_{12} and f_{22} as at (b). Assuming a linear load–displacement relationship we may use the principle of superposition and obtain the combined effects of W and M by adding (a) and (b). Clearly it will be immaterial in which order the forces are applied. Applying W first and then M, the work done by the loads will be:

$$(\tfrac{1}{2}Wf_{11})+(\tfrac{1}{2}Mf_{22}+Wf_{12}) \tag{3.56}$$

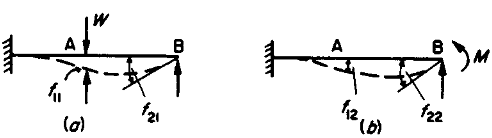


Figure 3.21

The first bracket in Equation (3.56) contains the work done during the application of W and the second bracket the work done (by both M and W) during the application of M.

In a similar way, if the order is reversed, the work done is:

$$(\tfrac{1}{2}Mf_{22})+(\tfrac{1}{2}Wf_{11}+Mf_{21}) \tag{3.57}$$

From Equations (3.56) and (3.57) it is evident that:

$$Wf_{12}=Mf_{21} \tag{3.58}$$

If the applied actions are taken to have unit values, then Equation (3.58) simplifies to:

$$f_{12}=f_{21} \tag{3.59}$$

Equation (3.59) is a statement of Maxwell's reciprocal theorem. A more general theorem, of which Maxwell's is a special case, is due to Betti. This latter theorem states that if a system of forces P_i produces displacements p_i at corresponding positions and another set of forces Q_i, at similar positions to P_i, produces displacements q_i, then:

$$P_1q_1+P_2q_2+\ldots+P_nq_n=Q_1p_1+Q_2p_2+\ldots+Q_np_n \tag{3.60}$$

3.6.6 Mueller-Breslau's principle

This principle is the basis of the indirect method of model analysis. It is developed from Maxwell's theorem as follows. Consider the two-span continuous beam shown in Figure 3.22(a). On removal of the support at C and the application of a unit load at C, a deflected shape, shown dotted in Figure 3.22(b), is obtained. If a unit load now occupies any arbitrary position D, as at (c), then from Maxwell's theorem the deflection at C will be δ_D. In other words, the deflected form (b) is the influence line for deflection of C.

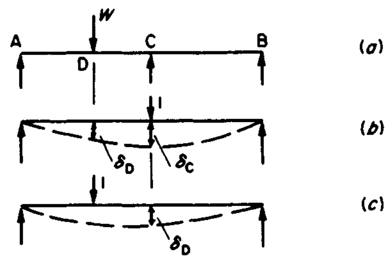


Figure 3.22

Now the force at C to move C through $\delta_C=1$

Hence, the force at C to move C through $\delta_D=1\times\delta_D/\delta_C$.

If a unit load acts at D, producing a deflection δ_D at C, then the upwards force needed to restore C to the level of AB is $1\times\delta_D/\delta_C$. Hence, the reaction at C for unit load at D is $1\times\delta_D/\delta_C$. Since D is an arbitrary point in the beam then it is seen that the deflected shape due to unit load at C, Figure 3.22(b), is to some scale, the influence line for R_C. The scale of the influence line is determined from the knowledge that the actual ordinate at C should equal unity. Hence, the ordinates should all be divided by δ_C.

This result leads to Mueller-Breslau's principle which may be stated as follows:

'The ordinates of the influence line for a redundant force are equal to those of the deflection curve when a unit load replaces the redundancy, the scale being chosen so that the deflection at the point of application of the redundancy represents unity.'

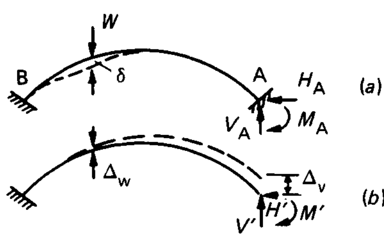


Figure 3.23

3.6.7 Application to model analysis

Consider the fixed arch shown in Figure 3.23(a). The arch has three redundancies which may be taken conveniently as H_A, V_A and M_A. We make a simple model of the arch to a chosen linear scale and pin this to a drawing board. End B is fixed in position and direction and the undistorted centreline is transferred to the drawing paper. We then impose a *purely* vertical displacement Δ_v at A and transfer the distorted centreline to the drawing paper. The distortion produced will require force actions at A, V', H' and M'. Let the displacement of a typical load point be Δ_w. Applying Equation (3.60) to the two systems of forces:

$$V_A(\Delta_v)+H_A(0)+M_A(0)+W(\Delta_w)=V'(0)+H'(0)+M'(0)+0(\delta)$$

Hence:

$$V_A\Delta_v+W\Delta_w=0$$

and if $W=1$:

$$V_A = \frac{-\Delta_w}{\Delta_v} \tag{3.61}$$

Similarly, we impose a *purely* horizontal displacement Δ_H and obtain:

$$H_A = \frac{-\Delta'_w}{\Delta_H} \tag{3.62}$$

then a pure rotation θ and obtain:

$$M_A = -\frac{\Delta''_w}{\theta} \tag{3.63}$$

In Equations (3.62) and (3.63) the displacements Δ'_w and Δ''_w represent the arch displacements due to the imposed horizontal and rotational displacements respectively. In each case the deflected shape, suitably scaled, gives the influence line for the corresponding redundancy.

3.6.7.1 Sign convention

The negative sign in Equations (3.61) to (3.63) leads to the following convention for signs. On the assumption that a reaction is positive if in the direction of the imposed displacement, then a load W will give a positive value of the reaction if the influence line ordinate at the point of application of the load is opposite to the direction of the load. This is evident in Figure 3.23(b) where the upward deflection Δ_w, being opposed to the direction of the load W, is consistent with a positive (upwards) direction for V_A.

3.6.7.2 Scale of the model

It should be noted that when using relationships (3.61) and (3.62) the ratios Δ_w/Δ_v and Δ'_w/Δ_H are dimensionless and thus the linear scale of the model does not affect the influence line ordinates. On the other hand, when using Equation (3.63) in obtaining an influence line for bending moment, Δ_w/θ has the dimensions of length and thus the model displacements must be multiplied by the linear scale factor.

In performing the model analysis, quite large displacements can be used providing the linear relation between load and displacement is maintained. Hence, the indirect method is sometimes called the 'large displacement' method.

3.6.8 Use of the computer in obtaining influence lines

With adequate computing facilities it is generally more economical to proceed directly to the computation of influence line ordinates by the analysis of the structure under a unit load, the unit load occupying a succession of positions. The actual method of analysis is immaterial but for bridge-type structures often the flexibility method offers some advantage especially if the structural members are 'nonprismatic'. An example of this type of computation is shown in Figure 3.24 where influence lines for bending moments at the interior supports of a five-span continuous beam are given. The beam is taken to be uniform in section over its length and, due to the symmetry of the spans, unit load positions need only be taken over one-half of the structure as shown.

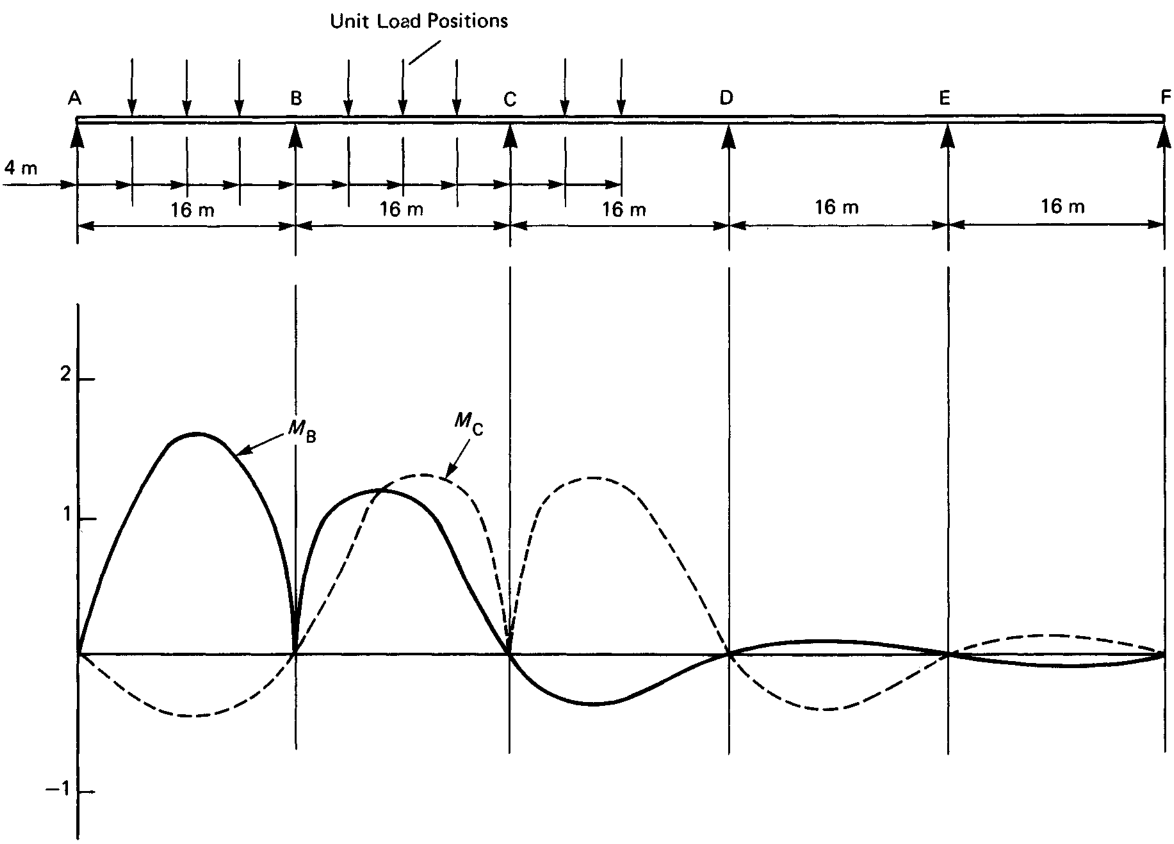


Figure 3.24 Influence lines for bending moments in a continuous beam obtained by computer analysis

3.7 Structural dynamics

3.7.1 Introduction and definitions

Structural vibrations result from the application of *dynamic* loads, i.e. loads which vary with time. Loads applied to structures are often time-dependent although in most cases the rate of change of load is slow enough to be neglected and the loads may be regarded as *static*. Certain types of structure may be susceptible to dynamic effects; these include structures designed to carry moving loads, e.g. bridges and crane girders, and structures required to support machinery. One of the most severe and destructive sources of dynamic disturbance of structures is, of course, the earthquake.

The dynamic behaviour of structures is generally described in terms of the displacement–time characteristics of the structure, such characteristics being the subject of vibration analysis. Before considering methods of analysis it is helpful to define certain terms used in dynamics.

(1) *Amplitude* is the maximum displacement from the mean position.
(2) *Period* is the time for one complete cycle of vibration.
(3) *Frequency* is the number of vibrations in unit time.
(4) *Forced vibration* is the vibration caused by a time-dependent disturbing force.
(5) *Free vibrations* are vibrations after the force causing the motion has been removed.
(6) *Damping.* In structural vibrations, damping is due to: (a) internal molecular friction; (b) loss of energy associated with friction due to slip in joints; and (c) resistance to motion provided by air or other fluid (drag). The type of damping usually assumed to predominate in structural vibrations is termed *viscous* damping in which the force resisting motion is proportional to the velocity. Viscous damping adequately represents the resistance to motion of the air surrounding a body moving at low speed and also the internal molecular friction.
(7) *Degrees of freedom.* This is the number of independent displacements or *coordinates* necessary to completely define the deformed state of the structure at any instant in time. When a single coordinate is sufficient to define the position of any section of the structure, the structure has a *single degree of freedom*. A continuous structure with a distributed mass, such as a beam, has an infinite number of degrees of freedom. In structural dynamics it is generally satisfactory to transform a structure with an infinite number of degrees of freedom into one with a finite number of freedoms. This is done by adopting a *lumped mass* representation of the structure, as in Figure 3.25. The total mass of the structure is considered to be *lumped* at specified points in the structure and the motion is described in terms of the displacements of the lumped masses. The accuracy of the analysis can be improved by increasing the number of lumped masses. In most cases sufficiently accurate results can be obtained with a comparatively small number of masses.

Figure 3.25

3.7.2 Single degree of freedom vibrations

The portal frame shown in Figure 3.26 is an example of a structure with a single degree of freedom providing certain assumptions are made. If it is assumed that the entire mass of the structure (M) is located in the girder and that the girder has an infinitely large flexural rigidity and further, that the columns have infinitely large extensional rigidities, then the displacement of the mass M resulting from the application of an exciting force $P(t)$, is defined by the transverse displacement y. The girder moves in a purely horizontal direction restrained only by the flexure of the columns.

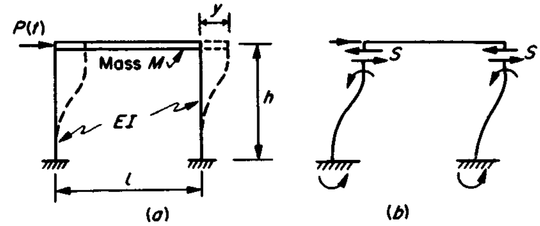


Figure 3.26

From Newton's second law of motion:

Force = mass × acceleration

i.e.:

$$\sum P = M\ddot{y} \tag{3.64}$$

Now from Figure 3.26(b), the force resisting motion is:

$$2S = 2\left(\frac{12EIy}{h^3}\right)$$

$$= 24\frac{EIy}{h^3} \tag{3.65}$$

Thus Equation (3.64) becomes:

$$P(t) - 24\frac{EIy}{h^3} = M\ddot{y}$$

or:

$$M\ddot{y} + 24\frac{EIy}{h^3} = P(t) \tag{3.66}$$

If the effect of damping is included then the equation of motion, Equation (3.66) is modified by the inclusion of a term $c\dot{y}$ where c is a constant. It should be noted that since the effect of damping is to resist the motion, then the term $c\dot{y}$ is added to the left-hand side of Equation (3.66). Thus:

$$M\ddot{y} + c\dot{y} + 24\frac{EIy}{h^3} = P(t) \tag{3.67}$$

Equation (3.67) may be generalized for any single degree of freedom structure by observing that the stiffness of the structure, i.e. force required for unit displacement horizontally, is given by:

$$k = 24\frac{EI}{h^3} \tag{3.68}$$

Combining Equations (3.67) and (3.68) we obtain the general single degree of freedom equation of motion:

$$M\ddot{y} + c\dot{y} + ky = P(t) \tag{3.69}$$

If in Equation (3.69) $P(t)=0$, we have a state of *free vibration* of the structure. The governing equation becomes:

$$M\ddot{y}+c\dot{y}+ky=0 \tag{3.70}$$

The situation envisaged by Equation (3.70) would arise if the beam were given a horizontal displacement and then released. The resulting vibrations would depend on the amount of damping present, measured by the coefficient c.

The solution of Equation (3.70) is:

$$y=A_1e^{\lambda_1 t}+A_2e^{\lambda_2 t} \tag{3.71}$$

where A_1 and A_2 are the constants of integration, to be evaluated from initial conditions, and λ_1 and λ_2 are the roots of the auxiliary equation:

$$M\lambda^2+c\lambda+k=0 \tag{3.72}$$

or, substituting:

$$\left.\begin{aligned} &p^2=k/M \\ &\text{and} \\ &2n=c/M \end{aligned}\right\} \tag{3.73}$$

Equation (3.72) becomes:

$$\lambda^2+2n\lambda+p^2=0 \tag{3.74}$$

Hence:

$$\lambda=-n\pm\sqrt{(n^2-p^2)} \tag{3.75}$$

Four cases arise:

Case 3.1 $p^2<n^2$

Here (n^2-p^2) is always positive and $<n^2$ and thus λ_1 and λ_2 are real and negative.

Equation (3.71) takes the form:

$$y=e^{-nt}(A_1e^{\sqrt{(n^2-p^2)}t}+A_2e^{-\sqrt{(n^2-p^2)}t}) \tag{3.76}$$

The relationship between y and t of Equation (3.76) is shown in Figure 3.27(a) and it is seen that the displacement y gradually returns to zero, no vibrations taking place.

Now, since $n^2>p^2$, then:

$$\frac{c^2}{4M^2}>\frac{k}{M}$$

or

$$c>2\sqrt{(Mk)} \tag{3.77}$$

A structure exhibiting these characteristics is said to be *over-damped*.

Case 3.2 $p^2=n^2$

From Equation (3.75), $\lambda-n$ (twice)
and hence,

$$y=e^{-nt}(A_1+A_2t) \tag{3.78}$$

Again, no vibrations result and Equation (3.78) has the form shown in Figure 3.27(a).

From Equation (3.73) the value of c for this condition is given by:

$$c_c=2\sqrt{(Mk)} \tag{3.79}$$

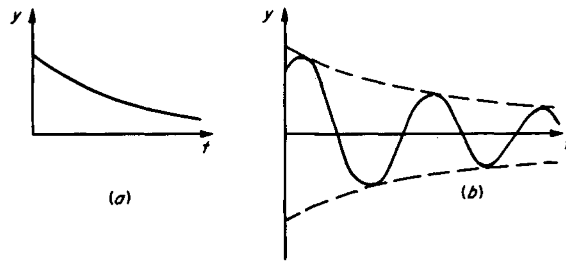


Figure 3.27

This is termed *critical damping* and the critical damping coefficient c_c is the value of the damping coefficient at the boundary between vibratory and nonvibratory motion. The critical damping coefficient is a useful measure of the damping capacity of a structure. The damping coefficient of a structure is usually expressed as a percentage of the critical damping coefficient.

Case 3.3 $p^2>n^2$

Here $c<c_c$ and the structure is *underdamped.*

From Equation (3.75), $\lambda=-n\pm i\sqrt{(p^2-n^2)}$

Hence:

$$y=e^{-nt}(A_1e^{i\sqrt{(p^2-n^2)}t}+A_2e^{-i\sqrt{(p^2-n^2)}t})$$

or, putting:

$$(p^2-n^2)=q^2$$

$$y=e^{-nt}(A_1e^{iqt}+A_2e^{-iqt})$$

or

$$y=e^{-nt}(A\cos qt+B\sin qt) \tag{3.80}$$

A typical displacement–time relationship for this condition is shown in Figure 3.27(b).

An alternative form for Equation (3.80) is:

$$y=Ce^{-nt}\sin(qt+\beta) \tag{3.81}$$

where C and β are new arbitrary constants

The period $T=\frac{2\pi}{q}=\frac{2\pi}{p\sqrt{\{1-(n/p)^2\}}}$

The period is constant but the amplitude decreases with time. The decay of amplitude is such that the ratio of amplitudes at intervals equal to the period is constant, i.e.:

$$\frac{y_{(t)}}{y_{(t+T)}}=e^{nT}$$

and $\log e^{nT}=nT=\delta$

δ is called the *logarithmic decrement*, and is a useful measure of damping capacity.

The percentage critical damping

$$= 100\frac{c}{c_c}$$

$$= 100\frac{\delta}{pT}$$

This is of the order of 4% for steel frames and 7% for concrete frames.

Case 3.4 $c=0$

In the absence of damping, Equation (3.70) becomes:

$$M\ddot{y}+ky=0 \tag{3.82}$$

The solution of which is:

$$y=A_1e^{\lambda_1 t}+A_2e^{\lambda_2 t}$$

where, from Equation (3.72):

$$\lambda_1=ip$$

$$\lambda_2=-ip$$

Thus:

$$y=A\sin pt+B\cos pt \tag{3.83}$$

The period is, $T=\dfrac{2\pi}{p}$

where p is the *natural circular frequency*

The *natural frequency* is $f=\dfrac{1}{T}=\dfrac{p}{2\pi}$

3.7.3 Multi-degree of freedom vibrations

Vibration analysis of systems with many degrees of freedom is a complex subject and only a brief indication of one useful method will be given here. For a more comprehensive and detailed treatment, the reader should consult one of the standard texts.[7]

For a system represented by lumped masses, the governing equations emerge as a set of simultaneous ordinary differential equations equal in number to the number of degrees of freedom. Mathematically the problem is of the *eigenvalue* or *characteristic value* type and the solutions are the *eigenvalues* (frequencies) and the *eigenvectors* (modal shapes). We shall consider the evaluation of mode shapes and fundamental, undamped, frequencies by the process of matrix iteration using the flexibility approach (see page 3/6). The method to be described, leads automatically to the lowest frequency, the fundamental, this being the one of most interest from a practical point of view. The alternative method using a stiffness matrix approach leads to the highest frequency.

Consider the simply-supported, uniform cross-section beam shown in Figure 3.28(a). The mass/unit length is w and we will regard the total mass of the beam to be lumped at the quarter-span points as shown in Figure 3.28(b). We may ignore the end masses $wl/8$ since they are not involved in the motion, and consider the three masses

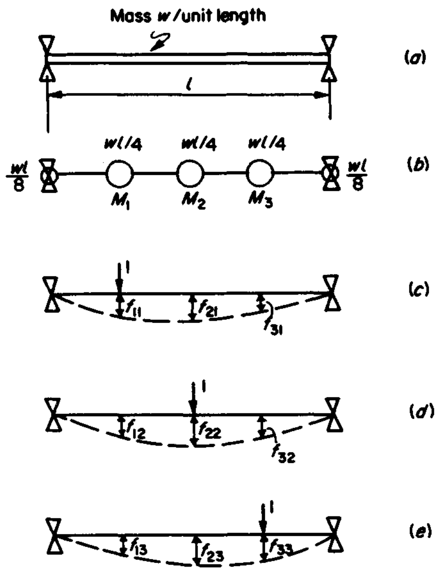


Figure 3.28

$$M_1=M_2=M_3=wl/4.$$

The appropriate flexibilities, f_{ij}, are shown at (c), (d) and (e). Using the flexibility method previously described, we may obtain a flexibility matrix as follows:

$$\mathbf{F}=\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = \frac{l^3}{256EI}\begin{bmatrix} 3.00 & 3.67 & 2.33 \\ 3.67 & 5.33 & 3.67 \\ 2.33 & 3.67 & 3.00 \end{bmatrix} \tag{3.84}$$

It should be noted that f_{ij} is the displacement of mass M_i due to unit force acting at mass M_j. Thus, if the forces acting at the positions of the lumped masses are $F_{1,2,3}$ and the corresponding displacements are $y_{1,2,3}$, then:

$$\left.\begin{aligned} y_1&=f_{11}F_1+f_{12}F_2+f_{13}F_3 \\ y_2&=f_{21}F_1+f_{22}F_2+f_{23}F_3 \\ y_3&=f_{31}F_1+f_{32}F_2+f_{33}F_3 \end{aligned}\right\} \tag{3.85}$$

For free, undamped vibrations, F_i is an inertia force $=-M_i\ddot{y}_i$.

Thus:

$$\left.\begin{aligned} y_1+f_{11}M_1\ddot{y}_1+f_{12}M_2\ddot{y}_2+f_{13}M_3\ddot{y}_3&=0 \\ y_2+f_{21}M_1\ddot{y}_1+f_{22}M_2\ddot{y}_2+f_{23}M_3\ddot{y}_3&=0 \\ y_3+f_{31}M_1\ddot{y}_1+f_{32}M_2\ddot{y}_2+f_{33}M_3\ddot{y}_3&=0 \end{aligned}\right\} \tag{3.86}$$

The solutions take the form:

$$y_i=\delta_i\cos(pt+a) \tag{3.87}$$

Hence:

$$\ddot{y}_i=-p^2y_i \tag{3.88}$$

Thus, Equations (3.86) become:

$$\left.\begin{aligned}\delta_1-f_{11}M_1p^2\delta_1-f_{12}M_2p^2\delta_2-f_{13}M_3p^2\delta_3=0\\ \delta_2-f_{21}M_1p^2\delta_1-f_{22}M_2p^2\delta_2-f_{23}M_3p^2\delta_3=0\\ \delta_3-f_{31}M_1p^2\delta_1-f_{32}M_2p^2\delta_2-f_{33}M_3p^2\delta_3=0)\end{aligned}\right\} \quad (3.89)$$

or:

$$\Delta=p^2\mathbf{FM}\Delta \quad (3.90)$$

where:

$$\Delta=\begin{bmatrix}\delta_1\\ \delta_2\\ \delta_3\end{bmatrix}; \quad \mathbf{M}=\begin{bmatrix}M_1 & 0 & 0\\ 0 & M_2 & 0\\ 0 & 0 & M_3\end{bmatrix}$$

The unknowns in Equation (3.90) are the displacement amplitudes δ_i and the frequency p; p has as many values as there are equations in the system, and for every value of p (eigenvalue) there corresponds a set of y (eigenvector).

We adopt an iterative procedure for the solution of Equation (3.90) and first of all rewrite the equations in the form:

$$\mathbf{FM}\Delta=\frac{1}{p^2}\Delta \quad (3.91)$$

We start with an assumed vector Δ_0, thus:

$$\mathbf{FM}\Delta_0=\frac{1}{p^2}\Delta_0$$

Putting $\mathbf{FM}\Delta_0=\Delta_1$

$$\Delta_1\simeq\frac{1}{p^2}\Delta_0 \text{ giving } p^2\simeq\frac{\Delta_0}{\Delta_1}$$

We cannot form Δ_0/Δ_1 since each Δ is a column matrix, so we take the ratio of corresponding elements in Δ_0 and Δ_1 and form the ratio δ_0/δ_1. It is best to use the numerically greatest δ for this purpose.

Continuing the process:

$$\mathbf{FM}\Delta_1\simeq\frac{1}{p^2}\Delta_1 \text{ giving } p^2=\delta_1/\delta_2$$

$$=\Delta_2$$

and again:

$$\mathbf{FM}\Delta_2\simeq\frac{1}{p^2}\Delta_2$$

$$=\Delta_3 \text{ giving } p^2=\delta_2/\delta_3$$

It can be shown that this iterative process converges to the largest value of $1/p^2$ and hence yields the lowest (fundamental mode) frequency.

Applying the iterative scheme to the beam of Figure 3.28, and assuming:

$$\Delta_0=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$$

then, $\Delta_1=\mathbf{FM}\Delta_0$

$$\text{where } \mathbf{FM}=\frac{l^3}{256EI}\begin{bmatrix}3.00 & 3.67 & 2.33\\ 3.67 & 5.33 & 3.67\\ 2.33 & 3.67 & 3.00\end{bmatrix}\begin{bmatrix}wl/4 & 0 & 0\\ 0 & wl/4 & 0\\ 0 & 0 & wl/4\end{bmatrix}$$

$$=\frac{wl^4}{1024EI}\begin{bmatrix}3.00 & 3.67 & 2.33\\ 3.67 & 5.33 & 3.67\\ 2.33 & 3.67 & 3.00\end{bmatrix}$$

$$\text{Thus: } \Delta_1=\frac{wl^4}{1024EI}\begin{bmatrix}12.67\\ 18.00\\ 12.67\end{bmatrix}=\frac{12.67wl^4}{1024EI}\begin{bmatrix}1.00\\ 1.42\\ 1.00\end{bmatrix}$$

$$\text{Hence: } p_1^2=\frac{\delta_0}{\delta_1}=\frac{2\times 1024EI}{12.67\times 1.42wl^4}$$

$$=114\frac{EI}{wl^4}$$

A second iteration gives:

$$\Delta_2=\mathbf{FM}\Delta_1=\frac{wl^4}{1024EI}\begin{bmatrix}3.00 & 3.67 & 2.33\\ 3.67 & 5.33 & 3.67\\ 2.33 & 3.67 & 3.00\end{bmatrix}\frac{12.67wl^4}{1024EI}\begin{bmatrix}1.00\\ 1.42\\ 1.00\end{bmatrix}$$

$$=12.67\left(\frac{wl^4}{1024EI}\right)^2\begin{bmatrix}10.54\\ 14.91\\ 10.54\end{bmatrix}$$

Hence:

$$p_2^2=\frac{\delta_1}{\delta_2}=\frac{12.67\times 1.42wl^4}{1024EI}\times\frac{1}{12.67(wl^4/1024EI)^2\times 14.91}$$

$$=97.5\frac{EI}{wl^4}$$

This result is very close to that produced by an exact method, i.e. $97.41EI/wl^4$.

3.8 Plastic analysis

3.8.1 Introduction

The plastic design of structures is based on the concept of a *load factor* (N), where

$$N=\frac{\text{Collapse load}}{\text{Working load}}=\frac{W_c}{W_w}' \quad (3.92)$$

A structure is considered to be on the point of collapse when finite deformation of at least part of the structure can occur without change in the loads. The simple plastic theory is based on an idealized stress–strain relationship for structural steel as shown in Figure 3.29. A linear, elastic, relationship holds up to a stress σ_y, the *yield stress*, and at this value of stress the material is considered to be in a state of perfect plasticity, capable of infinite strain, represented by the horizontal line AB continued indefinitely to the right. For comparison the dotted line shows the true relationship.

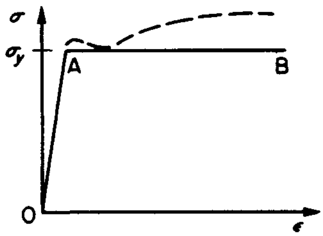


Figure 3.29

The term '*plastic analysis*' is generally related to steel structures for which the relationship indicated in Figure 3.29 is a good approximation. The equivalent approach when dealing with concrete structures is generally termed 'ultimate load analysis' and requires considerable modification to the method described here.

The stress–strain relationship of Figure 3.29 will now be applied to a simple, rectangular section, beam subjected to an applied bending moment M (Figure 3.30).

Under purely elastic conditions, line OA of Figure 3.29, the stress distribution over the cross-section of the beam will be as shown in Figure 3.30(b) and the limiting condition for elastic behaviour will be reached when the maximum stress reaches the value σ_y. As the applied bending moment is further increased, material within the depth of the section will be subjected to the yield stress σ_y and a condition represented by Figure 3.30(c) will exist in which part of the cross-section is plastic and part plastic. On further increase of the applied bending moment ultimately condition (d) will be reached in which the entire cross-section is plastic. It will not be possible to increase the applied bending moment further and any attempt to do so will result in increased curvature, the beam behaving as if hinged at the plastic section. Hence, the use of the term *plastic hinge* for a beam section which has become fully plastic.

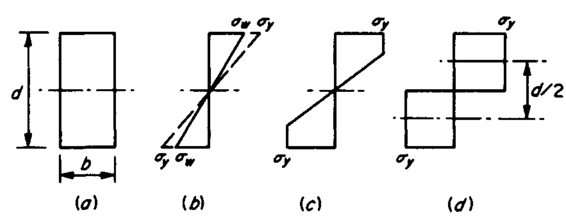


Figure 3.30

The moment of resistance of the fully plastic section is, from Figure 3.30(d):

$$M_p = b\frac{d}{2}\sigma_y\frac{d}{2} = \frac{bd^2\sigma_y}{4}$$

$$= Z_e\sigma_w \tag{3.93}$$

where Z_p = plastic section modulus

In contrast, the moment of resistance at working stress σ_w is, from Figure 3.30(b):

$$M_w = b\frac{d}{2}\frac{\sigma_w}{2}\frac{2}{3}d = \frac{bd^2}{6}\sigma_w \tag{3.94}$$

$$= Z_e\sigma_w$$

where Z_e = elastic section modulus

The ratio Z_p/Z_e is the shape factor of the cross-section. Thus the shape factor for a rectangular cross-section is 1.5.

The shape factor for an I-section, depth d and flange width b, is given approximately by:

$$\left(\frac{1+x/2}{1+x/3}\right)$$

where $x = \frac{t_w d}{2t_f b}$ and t_w and t_f are the web and flange thicknesses respectively

Values of plastic section moduli for rolled universal sections are given in steel section tables.

3.8.2 Theorems and principles

The definition of collapse, which follows from the assumed basic stress–strain relationship of Figure 3.29, has already been given. If the structural analysis is considered to be the problem of obtaining a correct bending moment distribution at collapse, then such a bending moment distribution must satisfy the following three conditions:

(1) *Equilibrium condition:* the reactions and applied loads must be in equilibrium.
(2) *Mechanism condition:* the structure, or part of it, must develop sufficient plastic hinges to transform it into a mechanism.
(3) *Yield condition:* at no point in the structure can the bending moment exceed the full plastic moment of resistance.

In elastic analysis of structures where several loads are acting, e.g. dead load, superimposed load and wind load, it is permissible to use the principle of superposition and obtain a solution based on the addition of separate analyses for the different loads. In plastic theory the principle of superposition is not applicable and it must be assumed that all the loads bear a constant ratio to one another. This type of loading is called 'proportional loading'. In cases where this assumption cannot be made, a separate plastic analysis must be carried out for each load system considered.

For cases of proportional loading, the uniqueness theorem states that the collapse load factor N_c is uniquely determined if a bending moment distribution can be found which satisfies the three collapse conditions stated.

The collapse load factor N_c may be approached indirectly by adopting a procedure which satisfies two of the conditions but not necessarily the third. There are two approaches of this type:

(a) We may obtain a bending moment distribution which satisfies the equilibrium and mechanism conditions, (1) and (2); in these circumstances it can be shown that the load factor obtained is either greater than or equal to the collapse load factor N_c. This is the 'minimum principle' and a load factor obtained by this approach constitutes an 'upper bound' on the true value.
(b) We may obtain a bending moment distribution which satisfies the equilibrium and yield conditions, (1) and (3), and in these circumstances it can be shown that the load factor obtained is either less than or equal to the collapse load factor N_c. This is the 'maximum principle' and its application produces a 'lower bound' on the true value.

It should be observed that whilst method (a) is simpler to use in practice, it produces an apparent load factor which is either correct or too high and thus an incorrect solution is on the unsafe side. A most useful approach is to employ both principles in turn and obtain upper and lower bounds which are sufficiently close to form an acceptable practical solution.

3.8.3 Examples of plastic analysis

This section contains some examples of plastic analysis based on the minimum principle. The method employed is termed the 'reactant bending moment diagram method'.

Example 3.5. The structure is a propped cantilever beam of uniform cross-section, carrying a central load W, as shown in Figure 3.31(a). The bending moment distribution under elastic conditions is shown in Figure 3.31(b) and it should be noted that the maximum bending moment occurs at the fixed end A.

As the load W is increased, plasticity will develop first at end A. As the load is further increased, end A will eventually become fully plastic with a stress distribution of the type shown in Figure 3.30(d) and the bending moment at A, M_A, will equal M_p the fully plastic moment of the beam. Further increase of load will have no effect on the value of M_A but will increase M_B until it also reaches the value M_p. The resulting bending moment distribution will now be as shown in Figure 3.31(c).

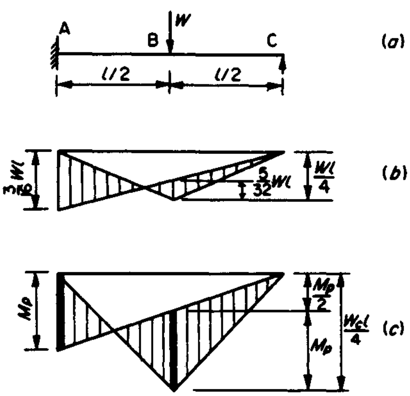


Figure 3.31

The geometry of the diagram produces a relationship between the load at collapse, W_c, and the plastic moment of resistance of the beam M_p, as follows:

$$\frac{W_c l}{4} = M_p + M_p/2$$

or:

$$W_c = 6\frac{M_p}{l} \tag{3.95}$$

If the working load is W_w then the load factor is given by:

$$N = \frac{W_c}{W_w} \tag{3.96}$$

Example 3.6. This is again a propped cantilever but here the load is uniformly distributed (Figure 3.32(a)). At collapse the bending moment diagram will be as shown in Figure 3.32(b) with plastic hinges at A and C. It should be noted that C is not at the centre of the beam. The location of the plastic hinge at C

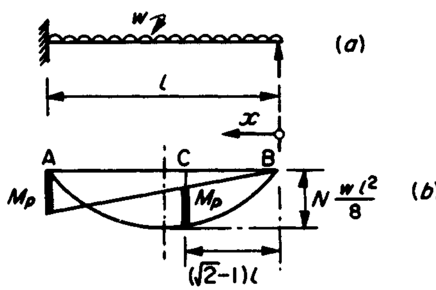


Figure 3.32

and the relationship between the load and the value of M_p may be obtained by differentiation as follows.

At C:

$$M_p = \left(N\frac{wlx}{2} - N\frac{wx^2}{2}\right) - M_p\frac{x}{l}$$

i.e.:

$$M_p = N\frac{wlx(l-x)}{2\ (l+x)} \tag{3.97}$$

$$\frac{dM_p}{dx} = N\frac{wl}{2}\frac{\{(l+x)(l-2x) - x(l-x)\}}{(l+x)^2}$$

$$= 0 \text{ for } M_{p\,max}$$

Hence: $x^2 + 2xl - l^2 = 0$

i.e.:

$$x = l(\sqrt{2} - 1) = 0.414l$$

which locates the point C.

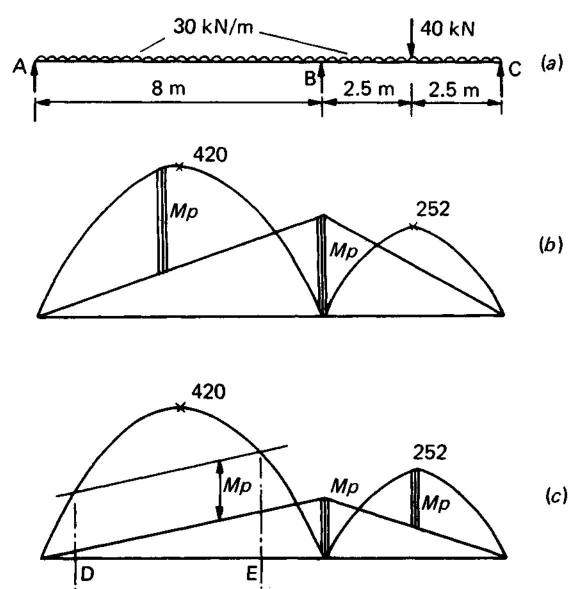


Figure 3.33

Also, substituting in Equation (3.97) for x:

$$M_p = \frac{Nwl^2(\sqrt{2}-1)}{2\sqrt{2}}(2-\sqrt{2})$$

$$= \left(\frac{Nwl^2}{8}\right) 4(3-2\sqrt{2})$$

$$= 0.686\left(\frac{Nwl^2}{8}\right)$$

Example 3.7. A two-span continuous beam is shown in Figure 3.33. The loads shown are maximum working loads and it is required to determine a suitable universal beam (UB) section such that $N = 1.75$ with a yield stress $\sigma_y = 250\ \text{N/mm}^2$. Effects of lateral instability are ignored for the purposes of this example.

With factored loads, the free bending moments are:

$$1.75 \times 30 \times \frac{8^2}{8} = 420\ \text{kNm}$$

$$1.75 \times 30 \times \frac{5^2}{8} + 1.75 \times 40 \times \frac{5}{4} = 252\ \text{kNm}$$

For collapse to occur in span AB, Figure 3.33(b)

$$420 \times 0.686 = M_p = 288\ \text{kNm}$$

For collapse in BC, assuming the span hinge in BC to occur at the centre (Figure 3.33(c)):

$$252 = \frac{3}{2}M_p; \quad M_p = 168 < 288$$

Hence the beam must be designed for $M_p = 288\ \text{kNm}$

$$= Z_p\sigma_y$$

Hence:

$$Z_p = \frac{288 \times 10^6}{250 \times 10^3}\ \text{cm}^3 = 1152\ \text{cm}^3$$

From section tables, select 406 × 178 UB 60 ($Z_p = 1194\ \text{cm}^3$).

This design may be compared with elastic theory from which we obtain $M_{max} = 198\ \text{kNm}$, $Z_e = 1200\ \text{cm}^3$ (using $\sigma_w = 165\ \text{N/mm}^2$). A suitable section would be 457 × 152 UB 67 ($Z_e = 1250\ \text{cm}^3$) or, 406 × 178 UB 74 ($Z_e = 1324\ \text{cm}^3$).

The plastic design may be improved by choosing different sections for spans AB and BC:

For BC, $M_{PBC} = 168$ giving $Z_p = \frac{168}{250} \times \frac{10^6}{10^3} = 672\ \text{cm}^3$

Select 356 × 171 UB 45 ($Z_p = 773.7\ \text{cm}^3$)

For AB, $M_{PAB} \simeq 420 - \frac{1}{2}M_{PBC}$

$$= 420 - \tfrac{1}{2} \times \frac{773.7 \times 10^3 \times 250}{10^6}$$

$$= 420 - 96.7 = 323\ \text{kNm}$$

$$\therefore Z_p = \frac{323}{250} \times \frac{10^6}{10^3} = 1293\ \text{cm}^3$$

Select 406 × 178 UB 67.

The weights of steel used in the different designs may be compared.

First plastic design	780 kg
Elastic design	871 kg
Second plastic design	761 kg

As an alternative to the second plastic design the lower value of M_p could be used, based on collapse in BC (356 × 171 UB 45, $Z_p = 773.7$, $M_p = 193\ \text{kNm}$), and flange plates welded on to the beam in the region DE, Figure 3.33(c).

The additional M_p required at the plated section

$$= 420 - \tfrac{3}{2} \times 193$$
$$= 130\ \text{kNm}$$

Using plates 150 mm wide top and bottom, the plastic moment of resistance of the plates is approximately:

$$2\left(150 \times t \times 250 \times \frac{356}{2}\right) \times 10^{-6}$$

$$= 13.4\,t$$

where t = plate thickness in millimetres

Hence:

$$t = \frac{130}{13.4} \simeq 10\ \text{mm})$$

Example 3.8. Here we consider the plastic analysis of a portal frame type structure as in Figure 3.34(a) and (b). At (a) the frame has pinned supports and at (b) fixed supports. A simple form of loading is used for illustration of the principles.

The frame is made statically determinate by the removal of H_A in both cases, and by the removal of M_A and M_E in case (b). The 'free' bending moment diagram is then as in diagram (c) and the reactant bending moment diagrams are as at (d) for H_A and at (e) for M_A and M_E combined. We now seek combinations of the diagrams which will satisfy the conditions of equilibrium, mechanism and yield (see page 3/27). We consider first the case of the two-hinged frame.

Diagram (f)

This is consistent with a pure sideway mode of collapse. From the geometry of the diagram:

$$M_p = \frac{Hh}{2} \tag{3.98}$$

The yield condition will be satisfied providing:

$$\frac{wl}{4} \leqslant \frac{Hh}{2} \tag{3.99}$$

Diagram (g)

This is a combined mechanism involving collapse of the beam and sidesway. From the geometry of the diagram:

At D:

$$M_p = Hh \mp H_A h$$

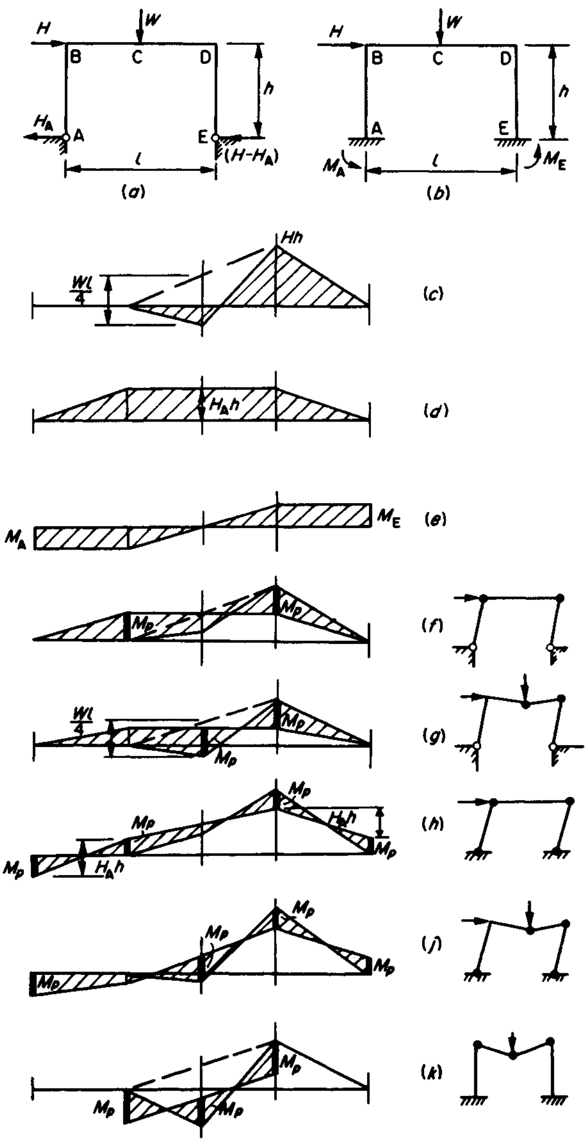


Figure 3.34

At C:

$$M_p = \frac{Wl}{4} - \frac{Hh}{2} \pm H_A h$$

Adding:

$$2M_p = \frac{Wl}{4} + \frac{Hh}{2}$$

or:

$$M_p = \frac{Wl}{8} + \frac{Hh}{4} \tag{3.100}$$

In the case of the frame with fixed feet, there are three possible modes of collapse. The corresponding bending moment diagrams are constructed at (h), (j) and (k) and the results are as follows:

Diagram (h):

$$M_p = \frac{H_A h}{2}$$

$$M_p = Hh - H_A h - M_p$$

Hence:

$$M_p = \frac{Hh}{4} \tag{3.101}$$

Diagram (j):

$$M_p = \frac{Wl}{4} - \frac{Hh}{2} \pm H_A h$$

$$M_p = Hh \mp H_A h - M_p$$

Adding:

$$3M_p = \frac{Wl}{4} + \frac{Hh}{2}$$

or:

$$M_p = \frac{Wl}{12} + \frac{Hh}{6} \tag{3.102}$$

Diagram (k)

This mode is the same as the collapse of a fixed end beam; the columns are not involved in the collapse apart from providing the resisting moment M_p at B and D. From the geometry of the diagram:

$$M_p = \frac{Wl}{8} \tag{3.103}$$

Example 3.9. Here we consider a pitched roof frame, a structure which is eminently suitable for design by plastic methods. The frame is shown in Figure 3.35(a). The given loads are already factored and we are to find the required section modulus on the basis of a yield–stress $\sigma_y = 280\ \text{N/mm}^2$, neglecting instability tendencies and the reduction in plastic moment of resistance due to axial forces.

The bending moment diagram at collapse is shown in Figure 3.35(b). The free bending moment diagram, EFGB, is drawn to scale after evaluating values of moment at intervals along the rafter members. The reactant line (H_A diagram) is then drawn by trial and error so that the maximum moment in the region BC is equal to the moment at D. This moment is the required M_p for the frame and is found to be:

$$M_p = 52\ \text{kNm} = \sigma_y Z_p$$

from which:

$$Z_p = \frac{52 \times 10^3 \times 10^3}{280 \times 10^3} = 186\ \text{cm}^3$$

Horne[8] and Baker and Heyman[9] should be consulted for a more

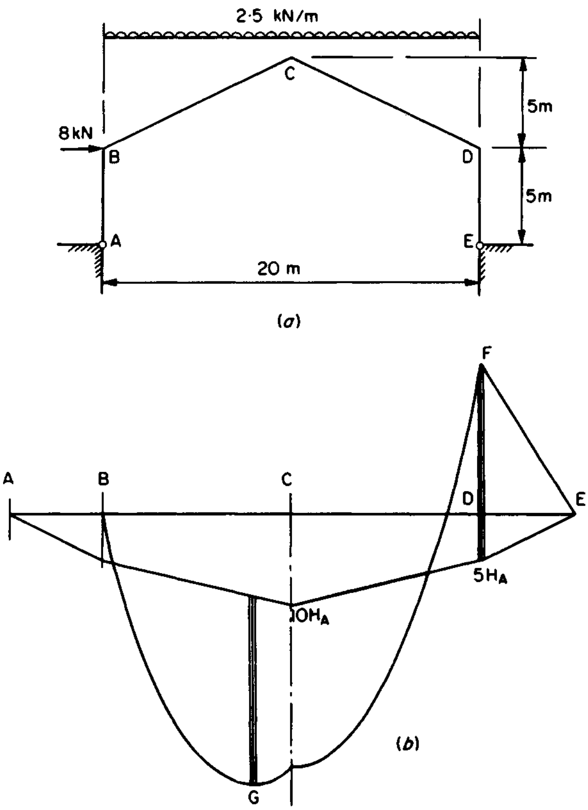


Figure 3.35

detailed study of plastic analysis. Among the topics deserving of further study are:

(1) Use of the principle of virtual work in obtaining relationships between applied loads and plastic moments of resistance.
(2) Effects of strain hardening.
(3) Evaluation of shape factors for various cross-sections.
(4) Application of the maximum principle in obtaining lower bounds.
(5) Numbers of independent mechanisms.
(6) Shakedown.
(7) Effects of axial forces.
(8) Moment carrying capacity of columns.
(9) Behaviour of welded connections.

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4

Materials

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Contents

4.1 Introduction

A good working knowledge of the materials used in civil engineering is very important to the engineer and in this book the characteristics and properties of many materials are described appropriately in other chapters as indicated below.

Material	*Chapter*
Soils	9
Rocks	8 and 10
Reinforcement	12
Steel	13
Aluminium	14
Bricks and masonry	15
Timber	16
Bituminous materials	23 (also 17 and 24)

This chapter is concerned with materials which are not covered elsewhere in the book and considers in detail only: concrete (pages **4**/3 to **4**/18), plastics and rubbers (pages **4**/18 to **4**/24) and paint (pages **4**/24 to **4**/26).

The authors gratefully acknowledge permission by Peter Pullar Strecker to include or update parts of his text from the 3rd Edition of the reference book (1974).

In the field of materials especially, the solution of problems often requires a full understanding of technologies outside the engineer's normal experience. Fortunately specialist help is usually readily available in the UK, although the enquirer does not always know where to look for it. Many sources are listed by the Construction Industry Research and Information Association (CIRIA)[1] *Guide to sources of construction information*. A selection of useful organizations and their addresses is as follows.

Aluminium Federation Ltd, Broadway House, Calthorpe Road, Five Ways, Birmingham B15 1TN.
Asbestos Information Centre, 40 Piccadilly, London W1V 9PA.
Association of Bronze and Brass Founders, 136 Hagley Road, Birmingham B16 9PN.
Brick Development Association, Woodside House, Winkfield, Windsor, Berks SL4 2DX.
British Aggregate Construction Materials Industries, 156 Buckingham Palace Road, London SW1W 9TR.
British Cast Iron Research Association, Alvechurch, Birmingham B48 7QB.
British Cement Association, Wexham Springs, Slough, Berks SL3 6PL.
British Ceramic Research Ltd, Queens Road, Penkhull, Stoke-on-Trent, Staffs ST4 7LQ.
British Constructional Steelwork Association Ltd, 35 Old Queen Street, London SW1H 9HZ.
British Glass Industry Research Association, Northumberland Road, Sheffield S10 2UA.
British Non-ferrous Metals Federation, 10 Greenfield Crescent, Edgbaston, Birmingham B15 3AU.
British Rubber Manufacturers' Association Ltd, 90–91 Tottenham Court Road, London, W1P 0BR.
British Standards Institution, 2 Park Street, London W1A 2BS.
British Steel Corporation, Corporate Research Laboratories, Swinden House, Moorgage, Rotherham S60 3AR.
British Wood Preserving Association, 150 Southampton Row, London WC1B 5AL.
Building Centres: London, Manchester, Bristol, Peterborough, Durham, Glasgow.
Building Research Establishment, Garston, Watford, Herts WD2 7JR.
Cement and Concrete Association, *see* British Cement Association.
Clay Pipe Development Association, Drayton House, 30 Gordon Street, London WC1H 0AN.
Concrete Pipe Association, 60 Charles Street, Leicester LE1 1FB.
Construction Industry Research and Information Association (CIRIA), 6 Storey's Gate, London SW1P 3AU.
Copper Development Association, Orchard House, Mutton Lane, Potters Bar, Herts EN6 3AP.
Flat Glass Council, 44–48 Borough High Street, London SE1 1XB.
Institution of Mining and Metallurgy, 44 Portland Place, London W1N 4BR.
Lead Development Association, 34 Berkeley Square, London W1X 6AJ.
National Physical Laboratory, Teddington, Middlesex TW11 0LW.
Paint Research Association, Waldegrave Road, Teddington, Middlesex TW11 8LD.
RAPRA Technology Ltd, Shawbury, Shrewsbury, Shropshire SY4 4NR.
Steel Construction Institute, Silwood Park, Ascot, Berks SL5 7QN.
Stone Federation, 82 New Cavendish Street, London W1M 8AD.
Timber Research and Development Association, Stocking Lane, Hughenden Valley, High Wycombe, Buckinghamshire HP14 4ND.
Zinc Development Association, 34 Berkeley Square, London W1X 6AJ.

4.1.1 Standards and codes of practice

British and some other standards and codes referred to in this chapter are listed separately in the bibliography.

4.2 Concrete

4.2.1 Cement

Hydraulic cement, i.e. a cement which hardens because of chemical reactions between the cement and water is the main, and often the only, binder used in concrete for civil engineering purposes. Portland cement or one of its variants is usually used, but high-alumina cement has advantages for some applications. The following list of cements is likely to be encountered in civil engineering. The relevant British Standards governing properties are given in the headings.

4.2.1.1 Ordinary Portland cement (OPC): BS 12

This is the most commonly used form of cement. It is made by heating together raw materials containing alumina and calcium. Clay and chalk or limestone are common sources. During the heating process the materials fuse to form clinker which is subsequently ground to a fine powder, gypsum usually being added at this stage to control the setting characteristics of the cement. Portland cements normally comprise four main phases or chemical compounds: tricalcium silicate, dicalcium silicate, tricalcium aluminate and calcium ferroaluminate. For convenience, these phases are usually given a shorthand notation of C_3S, C_2S, C_3A and C_4AF. This powder resulting from the grinding of clinker is the cement in its final form. The fineness of grinding, the raw materials and the conditions of the fusing process influence the nature and the reactivity of the cement, fine cement hardening more quickly than coarse cement of the same composition. The quality of British cement, although varying according to its source, usually exceeds the BS requirements by a considerable margin.

4.2.1.2 Rapid-hardening Portland cement (RHPC): BS 12

This is similar to OPC in composition but it is more finely ground. It gains strength more quickly than OPC, though the final strength is only slightly increased. Heat is generated more quickly during the hydration of the cement. This may have advantages in cold weather, or in precasting operations. The difference in strength development between OPC and RHPC has now become less marked.

4.2.1.3 Low-heat Portland cement: BS 1370

This cement is less reactive than OPC because it differs in composition, but it is nevertheless more finely ground than OPC. Heat is generated more slowly on hydration and lower concrete temperatures are reached. Early and eventual strengths are less than with OPC and the initial setting time is greater. This cement is made only to order in the UK.

4.2.1.4 Sulphate-resisting Portland cement: BS 4027

This cement is similar to OPC but the proportions of the cement phases are different and it is less prone to attack by sulphates principally by having a controlled low C_3A content. Heat may be generated more slowly than with OPC, but a little more quickly than with low-heat Portland cement.

4.2.1.5 Portland blast-furnace cement: BS 146

This cement is made by grinding together OPC clinker with granulated blast-furnace slag (see later). The granulated blast-furnace slag content must be less than 65% of the total weight. This cement is less reactive than OPC and gains strength a little more slowly. It has advantages in generating heat less quickly than OPC and in being more resistant than OPC to attack from sulphates. Portland blast-furnace cement is not widely available in the UK. (Low-heat Portland blast-furnace cement contains more slag but is manufactured only to order in the UK; BS 4246 governs its composition and properties.) Combination at the concrete mixer of Portland cement with ground granulated blast-furnace slag is more commonly used to achieve similar performance. By this method a wider range of OPC:slag ratios is readily achievable. These combinations are likely to be available in most parts of the UK.

4.2.1.6 Portland PFA cement: BS 6588

This cement is manufactured by intergrinding or combining at the cement plant pulverized fuel ash (PFA), complying with BS 3892, Part 1 (see later) with ordinary Portland cement. The PFA content should be between 15 and 35% by weight. The rate of strength development is slower than that of the respective Portland cement source. The cement may generate heat less quickly and be more chemically resistant in some circumstances.

Combination of PFA with ordinary Portland cement at the concrete mixer can produce concrete with a similar performance to that using this cement.

4.2.1.7 Pozzolanic cement with PFA as pozzolana: BS 6610

As for BS 6588 but the PFA content is between 35 and 50%. This cement is not referred to in BS 8110 or BS 5328 and is therefore unlikely to be used in reinforced concrete or other slender structural elements. The lower heat of hydration is useful property in massive structures.

4.2.1.8 White Portland cement

This cement is similar to OPC but with selected raw materials and processing to remove the normal OPC grey coloration; it would also comply with BS 12 for setting time and early and eventual strength.

4.2.1.9 Supersulphated cement: BS 4248

This cement is made from granulated blast-furnace slag, gypsum and not more than 5% of OPC clinker. It is more resistant to sulphate attack than sulphate-resisting cement, and it is not attacked by weak acids. This cement is much finer though less reactive than OPC, but eventual strengths are at least as high. It is not currently available in the UK. Good control of concrete mix is essential and its use has largely been superseded by other cement–slag combinations.

4.2.1.10 Water-repellent cement

This is made from OPC and stearates. It is used to reduce water permeability especially in screeds and rendering.

4.2.1.11 Masonry cement: BS 5224

This cement is made by mixing OPC with plasticizers and a fine powder (often whiting). It is used to give plasticity to bricklaying and rendering mortars, especially where the local sand is harsh.

4.2.1.12 High-alumina cement: BS 915

This cement is chemically different from OPC and its varieties. Concrete made with it has different properties from OPC concrete. High-alumina cement is very reactive and produces very high early strengths (the eventual strength may be reached in less than 1 day) but the initial setting is slower than with all varieties of Portland cement.

High-alumina cement is very resistant to attack from sulphates and is more resistant to acid attack than any variety of Portland cement but is attacked by alkalis. At temperatures above 700°C, high-alumina cement forms a ceramic bond with suitable aggregates and it can therefore be used for refactory concrete. Under moist conditions at temperatures of 40° to 100°C conversion takes place and high-alumina cement loses strength. Cement in this condition is less resistant to chemical attack.

It is widely believed that high-alumina cement should not be used in contact with hardened Portland cement. The scientific basis for this is, however, less well founded. Mixtures of unhardened Portland and high-alumina cements lead to very rapid 'flash' setting. This phenomenon has some practical applications where almost instantaneous setting is wanted, but the quality of the resulting concrete will be in most respects inferior to either Portland cement concrete or high-alumina cement concrete.

High-alumina cement concrete is not permitted for use in structural concrete in BS 8110. Applications such as floor toppings, hardstandings are still permissible.

4.2.1.13 Other cementing materials

Ground granulated blast-furnace slag. This is a by-product of the manufacture of iron from iron ore. The molten slag is removed from the furnace and quenched rapidly (granulation). Subsequent grinding can be either after combination with Portland cement clinker or more commonly of the granulated slag alone. The slag is composed mainly of calcium and magnesium silicates and alumino-silicates. Although some small strength gain or hardening would take place in water, the strengths developed are not likely to be sufficient for construction. Blending with a Portland cement produces a much faster and useful strength gain. Combinations of ground granulated blast-furnace slag and Portland cements have been used for many years both in the UK and overseas. An increase in the use and interest in these materials has taken place over recent years in the UK and BS 6699 gives composition and performance requirements. It is widely available in the UK.

Pozzolanas. Natural or artificial materials containing amorphous silica in a reactive form. The silica can react with lime to produce cementing compounds giving useful strength properties. This lime can be either hydrated lime or the calcium hydroxide produced during the hydration of Portland cements. The original pozzolana was volcanic ash from Pozzuoli, Italy. Using pozzolanas as a cementing component in Portland cement concretes can be useful to reduce heat of hydration or to improve resistance to some chemicals. Early age strength development may be affected unless the concrete is proportioned to allow for it.

Pulverized fuel ash (PFA). This is the most common pozzolana used in Portland cement concrete. It is electrostatically precipitated from the exhaust fumes of coal-fired power stations burning pulverized coal. It is widely available in the UK, and performance and compositional requirements are given in BS 3892, Part 1 (for use in structural concrete) and BS 3892, Part 2 (for miscellaneous uses in concrete).

Condensed silica fume. A high-purity silica pozzolana which has a very fine particle size much smaller than that of cement or PFA (mean particle size approximately 1 μm). Condensed silica fume is so fine it can be used to fill the interstices between cement particles and it reacts rapidly with the cement hydration products. Condensed silica flume is a by-product of the production of silicon and ferro-silicon being collected by cooling and filtering of furnace gases. Condensed silica flume can be used to produce very high strengths and good chemical resistance.

4.2.1.14 Non-UK standards

Many other national standards exist for Portland cements and combinations of Portland cements with blast-furnace slag or PFA. These standards cover similar ranges of materials to those in the British Standards given in the preceding pages although the overlap will not be complete for each country. Methods or terminology of classification vary for each country but common principles exist, e.g. sulphate-resisting cements are always low in C_3A content but the actual limiting value will be different.

Standards issued by the American Society for Testing Materials (ASTM) are widely used outside the US. Their standard C-150 has five main categories of Portland cement and a summary of these types is given in Table 4.1.

Other national standards for Portland cements which are likely to be encountered more widely are issued by Deutsches Institut für Normung (DIN) and in Japan as Japanese Industrial Standards (JIS). A wide range of cement specifications are incorporated within these standards and, hence, are not reproduced here.

4.2.2 Aggregates

Aggregates form more than three-quarters of the volume of concrete and the selection and proportioning of coarse and fine aggregates greatly influence the properties of both fresh and hardened concrete. The choice of grading, maximum aggregate size and aggregate:cement ratio are subjects for concrete mix design and are dealt with below. In this section the selection of aggregate type will be covered. Broadly, aggregates can be classified according to density as normal (particle density 2000 to 3000 kg/m^3), lightweight (less than 2000 kg/m^3) and heavy aggregates (greater than 3000 kg/m^3). Typical properties of concretes made with a range of aggregates are given in Table 4.2.

Table 4.1 Cement type classification in ASTM C-150

Type	*Use*	*Special requirements*
I	Where other special types not needed	
II	General use, moderate sulphate resistance or moderate heat of hydration	Max.C_3A (8%)
III	For high early strength	
IV	For low heat of hydration	Max. C_3S (35%) Min. C_2S (40%) Max. C_3A (7%)
V	For high sulphate resistance	Max C_3A (5%) Max. $C_4AF + 2\ C_3A$ (20%)

4.2.2.1 Normal aggregates

These usually consist of natural materials, hard crushed rock or crushed or natural gravel and their corresponding sands, but artificial materials like crushed brick and blast-furnace slag can also be used. The specific gravity of these materials usually lies between 2.6 and 2.7. Because satisfactory concrete for most purposes can be made with a very wide range of aggregates, local sources of supply usually determine which aggregate will be used. Where very high strength, resistance to skidding, good appearance or other special properties are required, appropriate aggregates will have to be selected, preferably on the basis of previous experience.

For example, the low-speed skidding resistance of concrete roads is affected by the hardness of the sand but only slightly by the polished-stone value of the coarse aggregate.Thus, a hard sand should be chosen for concrete which is to form the surface of a concrete pavement.

Some aggregates have undesirable influences on important concrete properties or are themselves unsound. They should be used with caution, if at all. Examples are aggregates with high drying shrinkages, which may lead to poor durability in exposed concrete, aggregates which react with alkalis in the cement paste, aggregates which are readily oxidized, aggregates which can cause surface staining, and aggregates made from weathered, partially decomposed, rocks.

Other aggregates, although making reasonably satisfactory hardened concrete, for most purposes, may give the fresh concrete poor handling characteristics. Aggregates with flat, flakey, very angular or hollow particles tend to have this effect. In general, aggregates with well-rounded particles in the case of gravels, or near-cubical particles in the case of crushed rock, produce concrete with better workability and fewer voids than aggregates with angular particles.

Natural sands have advantages over crushed rock sands because their particles tend to be more rounded and they contain less very fine material (of 150 μm or less), but crushed rock sands may be preferable, e.g. where the grading of locally occurring natural sands is poor, where the colour of natural sands would be unsatisfactory in weathered concrete (many sands weather to a yellowish colour) or where resistance to slipping is important. General requirements for aggregates to be used in concrete are given in BS 882.

Table 4.2 Properties of concrete using different aggregates

Aggregate	Typical range of dry density: Aggregate (kg/m^3)	Concrete (kg/m^3)	Compressive strength (N/mm^2)	Drying shrinkage (%)	Thermal conductivity at 5% moisture content (W/m°C)
Flint gravel or crushed rock	1350–1600	2200–2500	20–80	0.03–0.08	1.6–2.2
Crushed limestone	1350–1600	2200–2400	20–80	0.03–0.04	1.6–2.0
Crushed brick	1100–1350	1700–2150	15–30	—	0.85–1.50
Expanded clay, shale or slate and sintered pulverized fuel ash	300–1050	1350–1800	15–60	0.02–0.12	0.55–0.95
Foamed slag	500–950	1700–2100	15–60	0.04–0.10	0.85–1.40
Expanded clay, shale or slate and sintered pulverized fuel ash	300–1050	700–1300	2–7	0.03–0.07	0.24–0.50
Foamed slag	500–950	950–1500	2–7	0.03–0.07	0.30–0.65
Pumice	500–900	650–1450	2–15	0.04–0.08	0.21–0.63
Exfoliated vermiculite and expanded perlite	60–250	400–1100	0.5–7	0.20–0.35	0.15–0.39
Clinker	700–1050	1050–1500	2–7	0.04–0.08	0.35–0.65

4.2.2.2 Lightweight aggregates

These consist of various artificial and natural materials with specific gravities of between 0.1 and 1.2. They are used to make lightweight concrete for structural and insulating applications. In general, concrete made with lightweight aggregates has better fire resistance than dense concrete, but greater shrinkage and moisture movement.

Examples of lightweight aggregates are given below.

(1) *Sintered PFA* is made by heating pellets of PFA until they fuse to form hard spherical lumps.
(2) *Expanded clay, shale, slate and perlite* are made by heating suitable grades of these materials to their fusion temperature (about 1000°C) when they simultaneously fuse and are blown by gases generated within the material.
(3) *Pumice* is a natural lightweight aggregate consisting of a frothy volcanic glass.
(4) *Clinker* consists of fused lumps of fuel residues. To be suitable for use as a concreting aggregate it must be low in sulphates and residual fuel. Limits are given in BS 1156.
(5) *Foamed blast-furnace slag* is made by treating molten blast-furnace slag with water so that the steam which is generated blows the slag. Standards for this material are given in BS 877.
(6) *Exfoliated vermiculite* is made by heating vermiculite (a micalike mineral found in Africa and America) to a temperature of about 700°C when it expands to form a very light material.

Of these aggregates the sintered PFA, and the expanded clay, shale and slate and perlite are the most likely to be encountered.

4.2.2.3 Heavy aggregates

These consist either of natural or artificial materials and are used to make high-density concrete for radiation shielding or ballasting.

Examples of heavy aggregates are barytes, which is a naturally occurring rock consisting of 95% barium sulphate (specific gravity about 4.1; density of concrete up to 3700 kg/m^3); iron ores such as magnetite, goethite, limonite and ilmenite (specific gravity about 3.4 to 5.3, density of concrete up to 4200 kg/m^3) iron or steel shot (specific gravity 7.7; concrete density up to 5500 kg/m^3); lead shot (specific gravity 11.4; concrete density up to 7000 kg/m^3) and scrap-iron stampings and punchings. Provided the materials are sound and free from oil, satisfactory concrete of good structural strength can be made, especially if prepared by a method such as prepacking to avoid segregation. Consideration of the higher-density effect on mixing and batching facilities is important.

4.2.2.4 Contaminants, unsound aggregates and reactive aggregates

Aggregates may contain impurities which upset the hydration of the cement or coatings which interfere with bond, or the aggregates themselves may be unstable. To some extent, impurities and surface coating can be removed by suitable treatments, but aggregates which are unsound or reactive must be avoided.[2] Unsound or reactive particles may occur naturally with the aggregate source and may be detected by careful examination of the supply. It is also possible for a small percentage of contamination to occur during transportation or storage of aggregate.

Organic impurities. These may or may not delay or prevent the hydration of the cement and it is best to compare the strength of the concrete made with the contaminated aggregate with the strength of concrete made from similar but clean aggregate. Sugar, sugar-like substances and humic acid are among common contaminants which are known to retard or prevent cement hydration. Products of wood degradation such as 'cellibiose' have a similar effect.

Clay and fine material. These can contaminate aggregates either as a coating on the coarse aggregate or as a constituent of the fine aggregate. As coatings, these materials interfere with bond and therefore reduce concrete strength. As constituents of the mix they are less troublesome unless the quantity is great enough to require the addition of extra water to make the concrete workable. Clay, silt and fine material should not form more than 1% by weight of coarse aggregate, 3% by weight of gravel sand or 15% by weight of crushed rock sand (BS 882).

Salt is usually present in marine deposited or extracted aggregates and in small quantities it is harmless. Efficient washing of the aggregates before use in concrete is capable of reducing the salt to an acceptable level. The salt content should, however, be limited to the levels in Table 4.3 taken from BS 882:1983.

In addition to the limits given in Appendix C of BS 882, there is an overall limit given for the chloride ion from all sources calculated as a percentage by weight of cement given in Table 6.4 of BS 8110.

Table 4.3 Maximum chloride content of aggregates

Type or use of concrete	*Maximum total chloride content expressed as percentage of chloride ion by mass of combined aggregate*
Pre-stressed concrete Steam-cured structural concrete	0.02
Concrete made with cement complying with BS 4027 or BS 4248	0.04
Concrete containing embedded metal and made with cement complying with BS 12	0.06 for 95% of test results, with no result greater than 0.08

Note:
Marine aggregate and some inland aggregate contain chlorides. Both should be selected carefully and may need efficient washing to achieve the limit required for use in pre-stressed concrete.

Nondurable particles. These are sometimes found in aggregates which are otherwise satisfactory. Examples of such particles are lumps of clay, shale, wood or coal. Being soft, they are easily eroded and will lead to pitting or spalling of the concrete surface. If more than about 5% of such particles are present in the aggregate they will also cause strength to be reduced. Although no limits are given for these in BS 882, generally such particles should not form more than 1% of the aggregate by weight. The actual significance of the particles in the structure will be affected by the nature of the structure, e.g. a concrete paving will be more affected by soft particles floating to the surface than will a wall.

Reactive particles. Reactive particles found in some aggregates may be soluble in, or react with, water or the hydrating cement paste. Mica and sulphates, e.g. gypsum, react with cement paste, and iron sulphides, e.g. pyrites and marcasite, react with air and water to form products which then react with the cement paste and cause staining or pop-outs.

Unsound material. This may form the whole of the aggregate or unsound particles may merely contaminate it. Unsoundness is the property of some aggregates to expand or contract excessively as a result of freezing and thawing, wetting and drying, or temperature changes. Such movements can be large enough to cause the aggregate itself to break down or they may disrupt concrete made with it. Examples of unsound aggregates are rocks with very high water absorption, porous cherts, limestones and other sedimentary rocks if they contain laminae of clay, and some shales. Foreknowledge of how such aggregates behave in concrete is the only reliable guide, but freezing and thawing tests may give some indication of an aggregate's unsoundness.

Reactive aggregates. Reactive aggregates are those which react chemically with the cement paste, the most common reaction being between reactive silica and alkalis (in the form of sodium and potassium ions). Reactive silicas occur in opaline and chalcedonic cherts, siliceous limestone, rhyolites, andesite and phyllites. The actual susceptibility of particular aggregate sources needs to be assessed by tests or previous experience. The silica forms a gel with the alkali and this gel expands continuously as it absorbs water, exerting enough force to disrupt the surrounding cement paste in some cases.[1] This phenomenon of alkali silica reactions is well known and recorded. It was first identified some 46 years ago by Stanton in the US. Since then, workers in other countries around the world notably Denmark, Iceland, Germany and South Africa have identified similar reactions. It was believed until recently that the combination of high alkali cements together with reactive aggregates did not occur in the UK. However, a number of cases of alkali silica reaction have now been reported in UK structures built over many years. It is not clear at this time what the extent of these occurrences are or what significance they will have in structural performance. Guidance is available on minimizing the risks of the reaction.[3]

4.2.3 Admixtures

Relatively small quantities of other materials called admixtures can be added to concrete to modify its properties in either fresh or hardened state. There are several classes of admixtures which are listed below.

The British Standard for admixtures BS 5075 is in separate parts for each class of admixture.

4.2.3.1 Water-reducing admixtures and workability aids (BS 5075, Part 1)

These materials are also commonly called plasticizers and have the effect of making concrete more workable for a given water content. They can also reduce the water:cement ratio for a constant workability and can therefore be used to improve strength development.

These materials can also entrain a little air in the concrete or, if used in too high a dosage, can cause retardation of the cement setting. If used as a result of trial mixes or in accordance with the manufacturer's recommendations these side-effects should not be significant under normal site conditions.

Plasticizers for mortars are used to give plasticity or cohesion. They function by entraining large amounts of air which, as a side-effect, reduces strength. This modification to mortars should be carried out using only admixtures specifically formulated for the particular use.

4.2.3.2 Superplasticizers and high-range water-reducing admixtures (BS 5075, Part 3)

These more specialized admixtures perform similar functions to normal plasticizers but with increased effectiveness. Very high workability or flowing concrete is a common application. Because of their very effective action on the fluid properties of the concrete, much closer control of the initial mix design and subsequent batching is needed to prevent excessive bleeding or segregation of the mix. Many of the general superplasticizing admixtures have a relatively limited activity and the concrete workability may fall back to normal levels after approximately 30 to 45 min.

4.2.3.3 Air-entraining agents (BS 5075, Part 2)

These are widely used admixtures, especially for paving concrete. Their importance is related to the capacity of concrete containing a small amount of air in the form of well-distributed small bubbles to have greater resistance to the destructive action of freezing and thawing when the concrete is saturated than similar concrete made without air-entraining agents. The freezing and thawing action is made more severe when de-icing salts are used, or can be brought on to the surfaces by vehicles. In such circumstances the use of air entrainment is strongly recommended in codes of practice. This increased durability is gained at the expense of some strength and it is therefore important to control the amount of entrained air between close limits.

The amount of air that will be entrained with a given addition of an air-entraining agent is influenced by the grading of the sand, the workability of the concrete, the type of mixer and the duration of mixing. Trial mixes are essential to establish how much of each agent is to be added. Frequent regular measurements must be made throughout the work to ensure that the correct air content is being maintained (see page 4/17). Some difficulty may be experienced when using fine sands, sands with an organic or carbon content or when PFA and ground granulated blast-furnace slag materials are incorporated in the mix constituents.

As well as being more resistant to damage from de-icing salts, air-entrained concrete is somewhat more cohesive than concrete made without an air-entraining agent and tends to have slightly higher workability, a factor which can be used partly to offset the strength reduction.

4.2.3.4 Accelerators and 'antifreezes' (BS 5075, Part 1)

These are used to hasten the hardening of concrete, particularly in cold weather. The term 'antifreeze' is misleading because these admixtures merely lessen the period when frost damage is likely; they do not prevent concrete from freezing. Since the prohibition of the use of chloride-based accelerators as a result of corrosion of embedded steel, other proprietary products, often based on calcium formate, have been developed. Such admixtures are much less efficient at accelerating the strength development and therefore are less attractive to use. There may also remain some uncertainty about the risks of inducing corrosion. Alternative procedures for protecting concrete or mortars from frost, such as heated materials and adequate protection for the formed work, may be preferable.

4.2.3.5 Retarders (BS 5075, Part 1)

These have the effect of delaying the onset of hardening and usually also of reducing the rate of the reaction when it starts. Ultimate strengths are unaffected by retardation for several hours but may be reduced if the addition of retarder is excessive. Accidental overdosage may cause retardation of a few days or it may prevent hardening altogether. The fear that this may happen is probably one of the reasons why retarders are seldom used in the UK. Nevertheless, retarders can be beneficial where large volumes of concrete have to be poured in one operation or where high ambient temperature conditions prevail which lead to rapid setting. Care must be taken in this situation that the rapid set is not the result of rapid moisture loss by evaporation. Trial mixes are essential to determine the dosage at which the retarder is to be used.

4.2.3.6 Mixed admixtures

Mixed admixtures containing a variety of materials are available. Examples are combinations of an air-entrainment admixture with water-reducing admixture, or water-reducing and retarding admixtures.

4.2.3.7 Other admixtures

These include waterproofers, viscosity modifiers, resin bonding agents, fungicides, etc. They may be useful for specific applications, but the claims made for them should be supported by impartial test results. This applies particularly to the permanence of the effects claimed.

Pigments may be incorporated in concrete mixes. If bright or pastel shades are wanted, white cement and light-coloured sand must be used for the basic concrete, but low-key colours and dark shades can be obtained with ordinary concrete. The pigments must be stable in cement, fast to light and resistant to being washed out by weathering. Requirements are given in BS 1014.

Although a number of organic pigments can be used in concrete, the most commonly used are iron oxides for red, brown, yellow and black, and chromium oxide for green. Synthetic iron oxides have better staining power than natural ones and are available in a greater colour range. Although more expensive than natural oxides, they may be cheaper in use. Carbon black gives a more intense black than iron oxide, but because it is often greasy it is difficult to disperse and has the reputation of being easily washed out. Pigment additions vary typically from about 2 to 10% or more by cement weight. Some strength reduction should be expected with the larger rates of addition.

4.2.4 Concrete mix design

4.2.4.1 General

The purpose of concrete mix design is to choose and proportion the ingredients used in a concrete mix to produce economical concrete which will have the desired properties both when fresh and when hardened. The variables which can be controlled are: (1) water:cement ratio; (2) maximum aggregate size; (3) aggregate grading; (4) aggregate:cement ratio; and (5) use of admixtures.

Interactions between the effects of the variables complicate mix design and successive adjustments following trial mixes are usually necessary. Experience built up by ready-mix concrete producers should enable them to produce suitable mix designs more quickly than this. Many different methods of mix design have been developed, one relatively simple method is given by Teychenné, Franklin and Erntroy.[5]

4.2.4.2 Water: cement ratio

Many of the most important properties of fully compacted hardened concrete and strength in particular are for normal concrete virtually decided by the water:cement ratio of the mix. The importance of this parameter is due to the fact that any excess of water over that needed to hydrate the cement (about 25% by weight) forms voids in the concrete, thus reducing its density. The reduced density leads to reduced compressive, tensile and bond strengths, lower durability, lower resistance to abrasion and greater permeability to water. Excess water cannot be eliminated altogether because it is needed to lubricate the mix and make it workable, but it should be kept to a minimum.

Figure 4.1 shows how strength is influenced by water:cement ratio and the first step in concrete mix design is to fix the water:cement ratio from a knowledge of the strength required. The shape of the curves will be similar for all types of Portland cements but the actual relationship between strength and water:cement ratio will be different for each cement source.

4.2.4.3 Workability

When the concrete is fresh it must be workable or fluid enough to be compacted easily under the conditions in which it will be

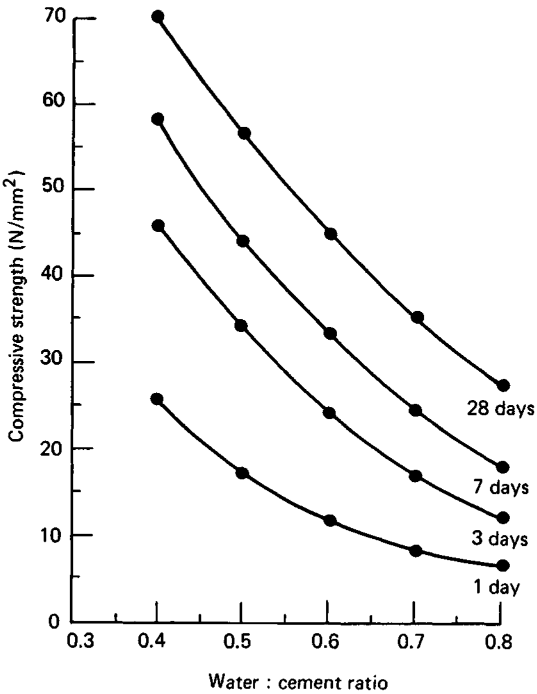

Figure 4.1 Influence of water: cement ratio on strength for typical UK OPC sources

placed. This is vitally important since loss of density has a very large effect in reducing strength. Table 4.4 gives suggested levels for workability suitable for different circumstances. Other factors may influence the selection of workability, e.g. some large foundations have been poured with very high workability to increase rates of placing. The cement paste is the lubricant which provides workability, but on grounds of economy as well as for technical reasons such as the limitation of shrinkage and thermal contraction, the amount of cement paste should be as small as possible. The next stages in the mix design process are intended to ensure that the mix will not be richer than is necessary.

4.2.4.4 Maximum aggregate size

Using a larger aggregate requires less fines to make up a volume of concrete; using a lower fines content will enable less cement and less water to be used for the same workability. The largest size of aggregate which can be used is governed by the dimensions of the section being cast and the spacing of the reinforcement. It is unusual to use aggregate with a maximum size of more than 20 mm for reinforced concrete, or with a maximum size greater than 25% of the section thickness for any work. Where large aggregates are to be used, consideration needs to be given to the use of appropriate-sized concrete testing equipment.

4.2.4.5 Overall grading

Sand and coarse aggregates frequently occur together, e.g. in gravels, but they seldom occur naturally in the best proportions for making concrete. Although all-in aggregates can be used, it is usually more satisfactory and more economical to separate sand from coarse aggregate and then recombine them in the required proportions. Further adjustments could be made by separating the aggregates into smaller-size groups and recombining them as required, but it is doubtful if this would repay its cost for most concreting requirements. Table 4.5 shows the British Standard requirements for aggregate gradings. The criterion for determining what proportions of sand and coarse aggregate should be used is that the concrete shall be cohesive enough to resist segregation but not so oversanded as to require higher cement and water contents.

Table 4.4 Suggested workabilities of concrete mixes

Workability	*Suitable use*	*BS 1881 recommendation for method of measuring workability*
Very low	Vibrated concrete in large sections	Vebe time
Low	Mass concrete foundations without vibration. Simple reinforced sections with vibration	Vebe time, compacting factor
Medium	Normal reinforced work without vibration, and heavily reinforced sections with vibration	Compacting factor, slump
High	Sections with congested reinforcements. Not normally suitable for vibration	Compacting factor, slump, flow
Very high	As for high workability plus large volume pours	Flow

Fine sands provide more cohesion than coarse ones so less sand will be needed if it is fine. Very fine sand is not recommended for structural concrete unless tests have shown that concrete made with it is satisfactory. Very coarse sand may cause difficulties in surface finishing if floors or pavements are being constructed. To resist segregation, high workability mixes need more sand than low workability ones, and the proportion of sand must also be increased as the maximum aggregate size is reduced. Crushed rock coarse aggregates need more sand than rounded gravel aggregates.

Table 4.6 taken from BS 5328 gives an indication of sand: coarse-aggregate ratios considered suitable for a range of 'prescribed' mixes.

4.2.4.6 Cement content

When the water:cement ratio has been fixed, the related variable of cement content can be chosen to ensure that there will be enough cement paste to produce a workable mix. The cement content that will be needed to do this depends on the grading and shape of the aggregates, more cement being needed for mixes with a finer overall grading or a more angular coarse aggregate. Trial mixes will usually be needed before the choice of cement content is finally made. Cement contents considered suitable for a range of prescribed mixes are also included in Table 4.6 and may also form the starting point for trial mixes if the work by Teychenné *et al.*[5] is not available.

4.2.5 Properties of hardened concrete

4.2.5.1 Compressive strength

This depends on water:cement ratio, degree of compaction, the type of cement and aggregates used, the curing and the age of the concrete. To the extent that compressive strength reflects water:cement ratio and density, it is a good indicator of general

Table 4.5 British Standard requirements for aggregates[3]

(i) Coarse aggregate

Sieve size (mm)	*Percentage by mass passing BS sieves for nominal sizes* *Graded aggregate* (mm) *40–5*	*20–5*	*14–5*	*Single-sized aggregate* (mm) *40*	*20*	*14*	*10*	*5**
50.00	100	—	—	100	—	—	—	—
37.50	90–100	100	—	85–100	100	—	—	—
20.00	35–70	90–100	100	0–25	85–100	100	—	—
14.00		—	90–100	—	—	85–100	100	—
10.00	10–40	30–60	50–85	0–5	0–25	0–50	85–100	100
5.00	0–5	0–10	0–10	—	0–5	0–10	0–25	45–100
2.36		—	—	—	—	—	0–5	0–30

*Used mainly in precast concrete products.

(ii) Fine aggregate

Sieve size	*Percentage by mass passing BS sieve* *Overall limits*	*Additional limits for grading* *C*	*M*	*F*
10.00 mm	100	—	—	—
5.00 mm	89–100	—	—	—
2.36 mm	60–100	60–100	65–100	80–100
1.18 mm	30–100	30–90	45–100	70–100
600 μm	15–100	15–54	25–80	55–100
300 μm	5–70	5–40	5–48	5–70
150 μm	0–15*	—	—	—

*Increased to 20% for crushed rock fines, except when they are used for heavy duty floors.

Note:
Fine aggregate not complying with this table may also be used provided that the supplier can satisfy the purchaser that such materials can produce concrete of the required quality.

(iii) All-in aggregate

Sieve size	*Percentage by mass passing BS sieves for nominal sizes* *40 mm*	*20 mm*	*10 mm*	*5 mm**
50.0 mm	100	—	—	—
37.5 mm	95–100	100	—	—
20.0 mm	45–80	95–100	—	—
14.0 mm	—	—	100	—
10.0 mm	—	—	95–100	100
5.00 mm	25–50	35–55	30–65	70–100
2.36 mm	—	—	20–50	25–100
1.18 mm	—	—	15–40	15–45
600 μm	8–30	10–35	10–30	5–25
300 μm	—	—	5–15	3–20
150 μm	0–8†	0–8†	0–8†	0–15

*Used mainly in precast concrete products.
†Increased to 10% for crushed rock fines.

(iv) Clay, silt and dust

Aggregate type	*Quantity of clay, silt and dust (max. % by mass)*
Uncrushed, partially crushed or crushed gravel	1
Crushed rock	3
Uncrushed or partially crushed sand or crushed gravel fines	3
Crushed rock fines	15 (8 for use in heavy duty floor finishes)
Gravel all-in aggregate	2
Crushed rock all-in aggregate	10

Note:
The nature of the material passing the 75 μm BS 410 test sieve used in the decantation method differs between crushed rock and gravel or sand.

concrete quality and it is an easy property to measure with reasonable consistency. The cube crushing strength is consequently an important test of both the structural and general quality of the concrete (see page **4**/16).

The development of compressive strength with age is greatly influenced by the temperature of the concrete, especially early in its life. Since the hydration of the cement itself generates heat, the temperature of the concrete is influenced not only by its initial temperature and the temperature of the surroundings, but also by the volume and shape of the section. Figure 4.2 indicates how the development of strength is related to the temperature of the concrete itself, and Table 12.4 in Chapter 12 relates the strength of various grades of concrete to the age at test. It should be remembered that because of these factors the properties and performance of the concrete in the structure will be different from the same concrete mix made into cubes which are stored and tested under controlled conditions.

The strength to which a concrete mix is designed depends on structural considerations and the fact that the concrete must be durable. Since there will be some variation in the quality of the concrete made on site and in the results of cube-crushing tests, the strength to which the mix is designed must exceed the strength actually needed by a safety factor which will depend on the degree of control which can be exercised over the concrete production process. These matters are discussed in Chapter 12; the question of the strength is also influenced by the overriding consideration that the concrete must be durable, and this factor often fixes the least cement content which can be used. This

Table 4.6 Cement contents and sand:coarse aggregate ratios considered suitable for a range of prescribed concrete mixes

Mass of dry aggregate to be used with 100 kg of cement

Grade of concrete (see Note 1)	*Nominal maximum size of aggregate* (mm)	*40*		*20*		*14*		*10*	
	Workability	*Medium*	*High*	*Medium*	*High*	*Medium*	*High*	*Medium*	*High*
	Range for standard sample (mm)	*50–100*	*80–170*	*25–75*	*65–135*	*5–55*	*50–100*	*0–45*	*15–65*
	Range for sample taken in accordance with 9.2 of BS 5328 (mm)	*40–110*	*70–180*	*15–85*	*55–145*	*0–65*	*40–110*	*0–55*	*5–75*
		(kg)	(kg)	(kg)	(kg)	(kg)	(kg)	(kg)	(kg)
C7.5P		1080	920	900	780	N/A	N/A	N/A	N/A
C10P		900	800	770	690	N/A	N/A	N/A	N/A
C15P	Total	790	690	680	580	N/A	N/A	N/A	N/A
C20P	aggregate	660	600	600	530	560	470	510	420
C25P		560	510	510	460	490	410	450	370
C30P		510	460	460	400	410	360	380	320

N/A not applicable
Source: BS 5328

Percentage by mass of fine aggregate to total aggregate

Grade of concrete	*Nominal maximum size of aggregate* (mm)	*40*		*20*		*14*		*10*	
	Workability	*Medium*	*High*	*Medium*	*High*	*Medium*	*High*	*Medium*	*High*
C7.5P									
C10P		30–45		35–50		N/A		N/A	
C15P									
	Grading zone 1	35	40	40	45	45	50	50	55
C20P	2	30	35	35	40	40	45	45	50
C25P	3	30	30	30	35	35	40	40	45
C30P	4	25	25	25	30	30	35	35	40

N/A not applicable
Notes on the use of tables:
(1) The proportions given in the tables will normally provide concrete of the strength in newtons per square millimetre indicated by the grade except where poor control is allied with the use of poor materials.
(2) For grades C7.5P, C10P and C15P a range of fine-aggregate percentages is given; the lower percentage is applicable to finer materials such as zone F sand and the higher percentage to coarser materials such as zone C sand.
(3) For all grades, small adjustments in the percentage of fine aggregate may be required depending on the properties of the particular aggregates being used.
(4) For grades C20P, C25P and C30P, and where high workability is required, it is advisable to check that the percentage of fine aggregate stated will produce satisfactory concrete if the grading of the fine aggregate approaches the coarser limits of zone C or the finer limits of zone F.

conflict between specifying strength for structural considerations and minimum cement contents to satisfy durability considerations has in the past led to confusion. For externally exposed concrete ensure that the more onerous requirement (usually durability) is properly achieved. Table 4.7 gives the requirements of BS 8110.

4.2.5.2 Tensile and flexural strength

The tensile strength of concrete is much smaller than the compressive strength and is in any case usually effectively eliminated by cracking, whether this cracking is visible or not. Consequently the tensile strength of concrete is not usually taken into account for design purposes, though it can be important inasmuch as it influences the spacing and control of cracks in structures[6] and contributes to the flexural strength of concrete paving.

Tensile strength is measured either directly by testing bobbins or cylinders to failure in tension, or indirectly by the cylinder splitting test or flexural tests on concrete beams. Results from the latter test are referred to as 'modulus of rupture'.

Table 12.5 in Chapter 12 gives figures for tensile and flexural strengths for several grades of concrete, and testing is discussed on page 4/16.

While tensile and flexural strength both increase with increasing compressive strength, there is no fixed relationship between

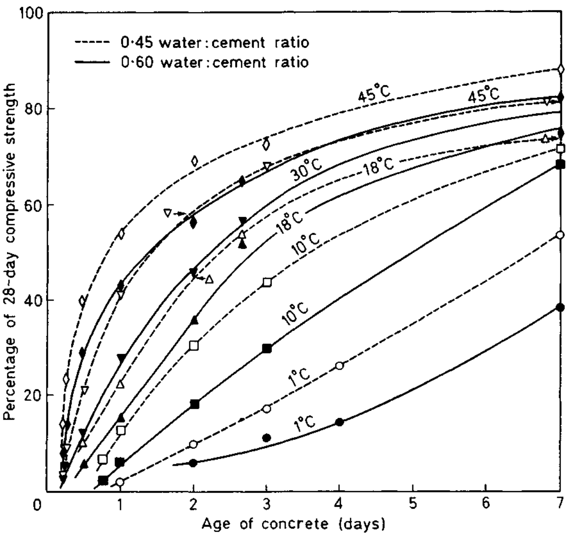

Figure 4.2 Influence of age and temperature on strength. (After Sadgrove (1970) *The early development of strength in concrete*. Construction Industry Research and Information Association)

them. Cylinder splitting tests have shown that the relationship is influenced by the nature of the aggregate, but that some surface characteristic of the aggregate, rather than whether the aggregate is crushed or rounded, is the cause of this influence.[8]

4.2.5.3 Elastic modulus

The elastic modulus of concrete is important in designing members to resist deflection, though concrete is not perfectly elastic and does exhibit significant creep behaviour. For design purposes, shrinkage, creep and elastic modulus are often allowed for together by designing on the basis of an 'effective modulus' which takes account of the three factors. This is discussed in Chapter 12.

The elastic modulus of concrete varies between about 7 and 50 kN/mm^2 depending on the strength of the concrete and the proportion and rigidity of the aggregate. The lowest figure would be applicable to low-strength concrete made with lightweight aggregate while normal structural concrete would have an elastic modulus of 25 to 30 kN/mm^2; some values are given in Table 12.5 in Chapter 12.

The elastic modulus of concrete can conveniently be measured by vibrating a suitable specimen; the value for the modulus found in this way is termed the 'dynamic' modulus and is considerably higher than the static modulus because no creep occurs under the test condition.

4.2.5.4 Creep

'Creep' is the term given to the tendency for concrete to continue to strain over a period of time when the stress is constant. For design purposes, creep is allowed for by using an 'effective' modulus which takes account of both short- and long-term stress–strain relationships. This is covered in Chapter 12.

Factors which tend to increase creep are low strength, low ambient relative humidity, low-modulus aggregates, and high stressing. Methods for calculating creep deflections usually assume that creep is increased by early loading, but other investigations suggest that this effect may not be significant.[8]

Table 4.7 Minimum cement content and other requirements in Portland cement concrete to ensure durability under specified conditions of exposure (from BS 8110)

(i) Conditions of exposure

Environment	*Exposure conditions*
Mild	Concrete surfaces protected against weather or aggressive conditions
Moderate	Concrete surfaces sheltered from severe rain or freezing whilst wet Concrete subject to condensation Concrete surfaces continuously under water Concrete in contact with nonaggressive soil (see class 1 of Table 4.9)
Severe	Concrete surfaces exposed to severe rain, alternate wetting and drying or occasional freezing or severe condensation
Very severe	Concrete surfaces exposed to sea-water spray, de-icing salts (directly or indirectly), corrosive fumes or severe freezing conditions whilst wet
Extreme	Concrete surfaces exposed to abrasive action, e.g. sea-water carrying solids or flowing water with pH ≤ 4.5 or machinery or vehicles

(ii) Minimum cement contents and other requirements for durability

Conditions of exposure	*Nominal cover to reinforcement*				
	(mm)	(mm)	(mm)	(mm)	(mm)
Mild	25	20	20*	20*	20*
Moderate	—	35	30	25	20
Severe	—	—	40	30	25
Very severe	—	—	50†	40†	30
Extreme	—	—	—	60†	50
Maximum free water:cement ratio	0.65	0.60	0.55	0.50	0.45
Minimum cement content (kg/m^3)	275	300	325	350	400
Lowest grade of concrete	C30	C35	C40	C45	C50

*These covers may be reduced to 15 mm provided that the nominal maximum size of aggregate does not exceed 15 mm.
†Where concrete is subject to freezing whilst wet, air-entrainment should be used.
Note: This table relates to normal-weight aggregate of 20 mm nominal maximum size.

4.2.5.5 Shrinkage and moisture movement

Concrete shrinks when it dries. Part of this shrinkage, usually about 30% but sometimes as much as 60% is reversible, and is known also as 'moisture movement'. Shrinkage leads to cracking or distortion in members which are restrained or reinforced, though in this respect it is now considered to be less important than thermal movement (see section 4.2.5.6 below).

Shrinkage is increased with increasing cement content or the water content of the mix. High workability mixes shrink more than low workability mixes of the same strength. Aggregates with high elastic moduli are more effective in restraining shrinkage than low-modulus aggregates, this influence being virtually confined to the coarse aggregate. The phenomenon can be viewed simply as a two-component system – cement paste which tends to shrink and aggregate which tends to resist. Changing the balance of these two components will affect the overall magnitude of the shrinkage.

Figure 4.3 shows the influence of ambient relative humidity on the rate and amount of shrinkage. From the latter it can be seen that shrinkage is a more serious problem in dry countries or inside dry buildings than outside in the UK where the relative humidity usually exceeds 75%; indeed, where concrete remains permanently moist, it increases somewhat in volume.

Lightweight aggregates usually have less effect in restraining shrinkage than normal-weight aggregates, and where aggregate is absent, e.g. in aerated concrete, products have to be autoclaved to keep the shrinkage and moisture movement within reasonable limits.

4.2.5.6 Thermal movement

The linear coefficient of thermal expansion of concrete varies from about 5 to 15 microstrain per degree centigrade, depending on the richness of the mix and the coefficient of expansion of the aggregate. Rich mixes have higher coefficients than lean ones, and siliceous aggregates have higher coefficients than limestone and granite. In the same way as for shrinkage, thermal movements can be seen as the summation of properties of the two primary components cement paste and aggregates.

Since concrete tends to become heated when the cement hydrates, thermal contraction on cooling and hardening can set up enough stress on restrained members to cause cracking. Even if a reduced coefficient is used for immature concrete (to take creep into account) cooling strains in walls of normal thickness can reach 200 microstrain or more within a few days of the concrete being cast. Table 4.8 gives general figures for coefficient of thermal expansions for various aggregate types.

4.2.5.7 Durability

This important property of concrete has already been referred to on page **4**/11 where the minimum cement content needed for durability was mentioned in relation to conditions of exposure. Durability considerations will need to be of both the concrete itself and any embedded steel reinforcement. There is a great deal of information on the durability of concrete. Codes of practice are now focusing much more closely on this aspect of concrete performance; careful consideration of the relevant codes of practice are therefore necessary before producing a specification for the concrete. Special care must be taken when concrete is exposed to sulphates, acids or salts used for de-icing, or other aggressive chemicals.

In general, concrete which has low permeability will be much more durable than concrete which has high permeability and the effect may be so marked that it outweighs the influence of specially resistant cements. Well-compacted dense concrete con-

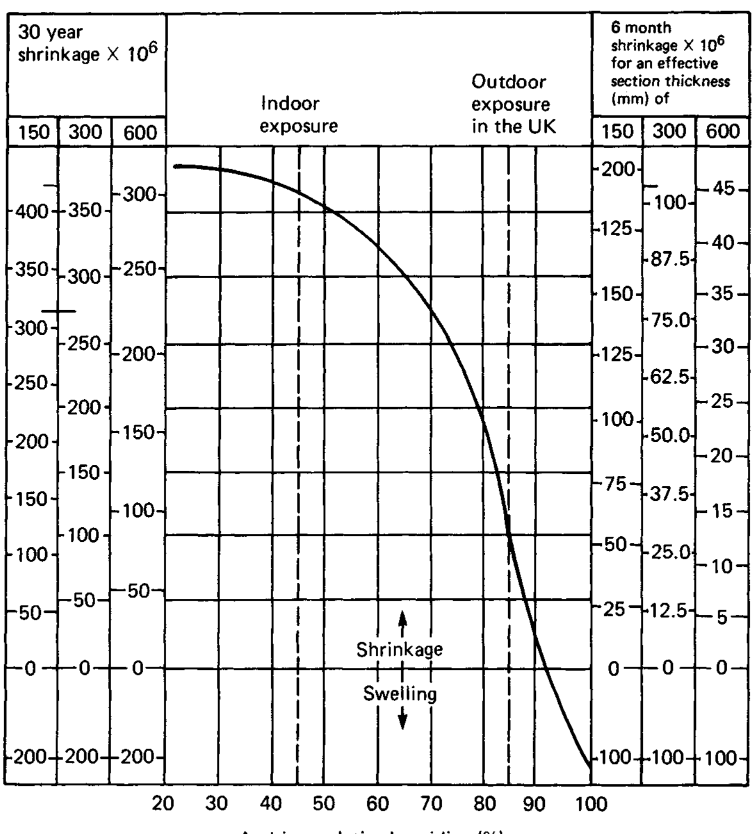

Figure 4.3 Drying shrinkage of normal-weight concrete (from BS 8110). The graph relates to concrete of normal workability with a water content of about 190 l/m³. Shrinkage may be regarded as proportional within the range of 150 to 230 l/m³

Table 4.8 Coefficients of thermal expansion

Coarse aggregate/rock group	*Thermal expansion coefficient (× 10⁻⁶/°C (microstrain/°C)*	
	Rock	*Saturated concrete*
Chert or flint	7.4–13.0	11.4–12.2
Quartzite	7.0–13.2	11.7–14.6
Sandstone	4.3–12.1	9.2–13.3
Marble	2.2–16.0	4.4–7.4
Siliceous limestone	3.6–9.7	8.1–11.0
Granite	1.8–11.9	8.1–10.3
Dolerite	4.5–8.5	Average 9.2
Basalt	4.0–9.7	7.9–10.4
Limestone	1.8–11.7	4.3–10.3
Glacial gravel	—	9.0–13.7
Lytag (coarse and fine)	—	5.6
Leca (10 mm)	—	6.7

taining sufficient cement and no unnecessary water should always be used where durability is important. Additional measures may also be needed where exposure conditions are severe.

Sulphates in solution can attack cement paste if the concentration is sufficiently high. Sources of sulphate are calcium and magnesium sulphate present in some groundwaters, sulphates contained in sea-water and sulphates formed from sulphur dioxide present in the air in urban and industrial areas. Sulphates from the last two sources would be too dilute to attack good-quality concrete unless circumstances had allowed them to become concentrated by evaporation. This situation can arise in coastal splash and tidal zones, and on the undersides of units from which contaminated water drips, e.g. copings on walls. Table 4.9 gives the recommendations of BS 8110.

Acids of all kinds attack concrete made with Portland cement. Sources of acids are flue gases (if condensation occurs), carbon dioxide dissolved in water (moorland water is frequently acid) and acid formed from sewer gas. Concrete can be protected to some extent by applying acid-resisting coatings, and limestone concrete (curiously) has been found to be more resistant than other concrete possibly because the large area which can be attacked neutralizes the acid before much local damage is done to the cement paste alone. It is not clear how often serious acid attack of concrete actually occurs in service; however, a report by Eglington for CIRIA[10] gives a review of available information.

Freezing and thawing cycles attack poor concrete, but very good-quality concrete is resistant unless de-icing salts are used. Even good-quality concrete may have a more porous top surface as the result of waterbleed and evaporation which may be more vulnerable to frost action. Air entrainment (see page 4/8) has been found to provide protection, though there are different explanations of the mechanism by which it works. Where no de-icing salt is to be used but the concrete is liable to become frozen when wet, air entrainment may not be specified since concrete with a very low water:cement ratio should be satisfactory. However, the concrete will need to have a cement content in excess of 400 kg/m^3 and a water:cement ratio less than 0.45. British Standard 8110 proposes a minimum concrete grade of C50 to ensure these requirements are met. It may be more practical and economic in these circumstances to use a lower-strength grade together with appropriate air-entrainment levels.

The corrosion of reinforcement in concrete is covered in Chapter 12.

4.2.6 Curing

If newly hardened concrete is to achieve its potential strength and durability, the hydration of the cement must be allowed to continue for as long as possible. The detrimental effects of inadequate curing on the durability of reinforced concrete may take many years to become apparent and therefore the relevance at the time of casting the concrete may be overlooked. For this purpose an excess of water must be present in the pores of the concrete and the act of ensuring that this is so is 'curing'. The excess water normally present in the concrete is enough to provide curing except in the case of very dry mixes, but near the surface of a member it will escape by evaporation unless this is prevented. Formwork is usually left in place long enough to provide initial curing, and where appearance and durability are not considered important, this may be enough in the UK climate. Where further curing is considered justified, spray-applied curing films or other means of preventing evaporation must be used. An alternative is to apply water to the surface for the curing period.[11] The curing of unformed surfaces should be commenced as soon as possible after placing the concrete to prevent rapid moisture loss and possible cracking in the plastic concrete.

4.2.7 Concreting in hot, arid climates

Reference should be made to Chapter 37 for the special requirements for mixes, production and curing in hot arid countries such as those in the Middle East.

4.2.8 Reinforcement and prestressing steel

These materials are covered in Chapter 12.

4.3 Concrete testing

Most of the tests which are described below have to be carried out on a sample of concrete which will inevitably be very small compared with the work which it is intended to represent. Sampling is therefore of the greatest importance and every care must be taken to ensure that the sample is as representative as possible if the test results are to have any real meaning. British Standard 1881:1983, Part 101 gives advice on methods of sampling.

As well as variations in the concrete there will also be variations in the test itself and it is necessary to carry out several tests on concrete which nominally represents the same part of the work. These variations have been called reproducibility R and repeatability r and can be quantified by careful repeat tests within and between test laboratories.

4.3.1 Workability tests

These are designed to measure the ease with which concrete can be compacted. Because none of the tests exactly reproduces the conditions under which concrete is compacted on site, each test has some limitations in applicability, though within limits any of the tests is suitable for monitoring uniformity of workability once site use has established what workability will be needed. Some measure of control of water content in the concrete is also possible by monitoring workability.

(1) Slump test (BS 1881:1983, Part 102):
   (a) *application*: quick approximate tests for medium and high workability concrete; suitable for site use; simple apparatus;

Table 4.8 Coefficients of thermal expansion

Coarse aggregate/rock group	*Thermal expansion coefficient (× 10⁻⁶/°C (microstrain/°C)*	
	Rock	*Saturated concrete*
Chert or flint	7.4–13.0	11.4–12.2
Quartzite	7.0–13.2	11.7–14.6
Sandstone	4.3–12.1	9.2–13.3
Marble	2.2–16.0	4.4–7.4
Siliceous limestone	3.6–9.7	8.1–11.0
Granite	1.8–11.9	8.1–10.3
Dolerite	4.5–8.5	Average 9.2
Basalt	4.0–9.7	7.9–10.4
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(1) Slump test (BS 1881:1983, Part 102):
   (a) *application*: quick approximate tests for medium and high workability concrete; suitable for site use; simple apparatus;

Table 4.9

Class	*Concentration of sulphates expressed as SO_3*			*Type of cement*	*Dense, fully compacted concrete made with 20 mm nominal maximum size aggregates complying with BS 882 or BS 1047*	
	In soil		*In groundwater* (g/l)		*Cement* content not less than* (kg/m³)	*Free water cement* ratio not more than*
	Total SO_3 (%)	*SO_3 in 2:1 water:soil extract* (g/l)				
1	< 0.2	< 1.0	< 0.3	All cements listed in BS 5328 BS 12 cements combined with PFA† BS 12 cements combined with ground granulated blast-furnace slag†	— Moderate exposure; see Table 4.7	—
2	0.2–0.5	1.0–1.9	0.3–1.2	All cements listed in BS 5328 BS 12 cements combined with PFA† BS 12 cements combined with ground granulated blast-furnace slag†	330	0.50
				BS 12 cements combined with minimum 25% or maximum 40% PFA‡ BS 12 cements combined with minimum 70% or maximum 90% ground granulated blast-furnace slag	310	0.55
				BS 4027 cements (SRPC) BS 4248 cements (SSC)	280	0.55
3	0.5–1.0	1.9–3.1	1.2–2.5	BS 12 cements combined with minimum 25% or maximum 40% PFA† BS 12 cements combined with minimum 70% or maximum 90% ground granulated blast-furnace slag	380	0.45
				BS 4027 cements (SRPC) BS 4248 cements (SSC)	330	0.50
4	1.0–2.0	3.1–5.6	2.5–5.0	BS 4027 cements (SRPC) BS 4248 cements (SSC)	370	0.45
5	Over 2	Over 5.6	Over 5.0	BS 4027 cements (SRPC) and BS 4248 cements (SSC) with adequate protective coating	370	0.45

*Inclusive of PFA and ground granulated blast-furnace slag content.
†For reinforced concrete see 3.3.5; for plain concrete see 6.2.4.2 of BS 8110.
‡Values expressed as percentages by mass of total content of cement, PFA and ground granulated blast-furnace slag.
Notes:
Within the limits given in this table, the use of PFA or ground granulated blast-furnace slag in combination with sulphate-resisting Portland cement (SRPC) will not give lower sulphate resistance than combinations with cements to BS 12.
If much of the sulphate is present as low solubility calcium sulphate, analysis on the basis of a 2:1 water extract may permit a lower site classification than that obtained from the extraction of total SO_3. Reference should be made to BRE Current Paper 2/79 for methods of analysis, and to BRE Digests 250 and 222 for interpretation in relation to natural soils and fills, respectively.

(b) *special apparatus*: mould in shape of inverted cone frustum, flat baseplate;
(c) *method*: Concrete is compacted into the mould in three approximately equal layers with a 16 mm diameter tamping rod giving 25 tamps per layer. Top surface is struck off and finished with trowel. The mould is then lifted off vertically and the concrete is allowed to slump;
(d) *result*: difference in height between moulded and slumped condition measured to nearest 5 mm, *or* if total collapse or shear occur, these facts are recorded.

(2) Compacting factor (BS 1881:1983, Part 103):
(a) *application*: concrete of all workabilities; suitable for simple site laboratory;
(b) *special apparatus*: compacting factor apparatus consisting of two hoppers and a measuring cylinder fixed in vertical alignment; balance to weigh 25 kg to 10 g accuracy;
(c) *method*: Concrete is filled loosely into top hopper and allowed to fall to next hopper. Concrete from this hopper is then allowed to fall into the measuring cylinder. The surplus is struck off;
(d) *result*: ratio of the weight of concrete in the cylinder filled as above to the weight of concrete fully compacted into the cylinder.

(3) 'V-B' consistometer (BS 1881:1983, Part 104):
(a) *application*: concrete of all workabilities; suitable for large site laboratory;
(b) *special apparatus*: consistometer consisting of conical mould, cylindrical container, transparent disc kept horizontal by a guide and a vibrating table of specified size, frequency and amplitude, and stopwatch;
(c) *method*: the mould is placed in the cylinder and filled with concrete as for the slump test. The mould is then removed and the disc is allowed to rest on top of the slumped concrete. Vibration is then applied and allowed to continue until the underside of the disc is just covered with grout;
(d) *result*: vibration time in seconds to nearest 0.5 s.

(4) Ball penetration test (ASTM C360-82)
(a) *application*: similar to slump test;
(b) *special apparatus*: kelly ball consisting of 30 lb, 6 in diameter hemisphere with support frame and graduated scale;
(c) *method*: the frame is placed on the surface of the concrete, e.g. in a wheelbarrow, with the bottom of the hemisphere just touching the concrete. When the weight is released, the penetration into the concrete is measured;
(d) *result*: penetration in inches.

(5) Flow test (BS 1881:1984, Part 105):
(a) *application*: high-workability concrete;
(b) *special apparatus*: mould in shape of inverted cone frustum: hinged flat baseplate;
(c) *method*: concrete is compacted into cone on hinged baseplate. Cone mould lifted off and top half of hinged plate lifted and dropped through predetermined height (40 mm) fifteen times. Diameter of concrete 'cowpat' measured in two directions;
(d) *result*: diameter of flow, in millimetres.

Other workability tests. Other test methods have been developed but have not been widely accepted. One test method which may become more widely used has been developed by Tattersall[12] from laboratory research. This method is a so-called two-point method and attempts to measure more scientifically the rheological properties of the plastic concrete by measuring the torque required to turn an impeller at various speeds when immersed in the concrete.

4.3.2 Strength tests

Strength tests are designed to measure the potential strength of concrete when cured and tested in a standard manner; the actual strength in a structural member depends on compaction, curing and uniformity as well as the potential strength. It cannot therefore be measured except by tests on the member (for core tests and nondestructive tests see sections 4.3.5 and 4.3.6). The primary reason for these strength tests is to maintain control over the batching and mixing of the concrete supply, thereby checking compliance with specified requirements.

The main requirements of BS 1881 for strength tests on concrete specimens are summarized in Table 4.10.

4.3.3 Accelerated strength tests

The curing and testing regimes shown in Table 4.10 are for 'standard' control tests and most specifications have a requirement for strengths to be tested at 28 days. For the majority of construction work this is acceptable. There are circumstances, e.g., where construction is likely to be on a fast timetable, in which waiting for 28 days before confirming compliance of the concrete is not preferred. In this situation it is possible to use an accelerated curing regime to give strength testing at, say, 24 h. The elevated temperatures used for this curing do not produce a constant effect on concreting materials and therefore it is normally required to do correlation testing in advance of the main concreting work. British Standard 1881:1983, Part 112 gives some guidance on accelerated curing.

4.3.4 Tests on cores

Tests on cores cut with diamond-tipped core cutters are described in BS 1881:1983, Part 120. The diameters of the cores should conform with the diameters of cylindrical specimens (see table in BS 1881), but the length cannot usually be chosen. The ends of the cores must be flat and perpendicular to the axis (this can be achieved by capping or grinding). The cores must be soaked in water for at least 48 h before being tested and are then tested while still wet. The failure stress is calculated from the load × the correction factor for the length/diameter ratio, divided by the average actual area of cross-section.

4.3.5 Nondestructive strength tests

A wide range of tests have been developed to give an indication of strength in concrete structures. The most commonly used in the UK are the rebound (Schmidt) hammer and ultrasonic pulse velocity tests. Others include Windsor probe, pull out, internal fracture and break-off tests. Most of these tests measure the properties of a relatively small volume of concrete near the surface and all of them need calibration against concrete of known strength. A summary of nondestructive test methods is given in BS 1881:1986: Part 201.

Surface hardness tests measure the rebound of impact hammers (e.g. the Schmidt hammer test) or the depth of indentation. A large number of tests is needed for a satisfactory assessment because the closeness of the aggregate to the test point affects the results. British Standard 1881:1986 Part 202 gives details of these tests.

Ultrasonic pulse velocity tests depend on the relationship between transmission time and the density and elastic properties of the concrete, both of which are related to concrete strength. Guidance on the method and interpretation is given in BS 1881:1986 Part 203.

4.3.6 Tests on aggregates

British Standard 812 gives details of numerous tests on aggregates. A number of these are frequently used in concrete mix

Table 4.10 Strength tests for concrete specimens

	Crushing strength	*Flexural strength*	*Indirect tensile strength*
Specimen size (mm)	150 × 150 × 150	150 × 150 × 750	150 dia × 150
Specimen size (mm)[a]	100 × 100 × 100	100 × 100 × 500	
Rammer size (for hand compaction)[b]	25 × 25 × 380	25 × 25 × 380	25 × 25 × 380
Blows per layer	⩾35	⩾150	⩾30
(smaller specimens)	(⩾25)	(⩾100)	
Rate of loading	0.2–0.4 N/mm²·s	0.06 ± 0.04 N/mm²·s	0.02–0.04 N/mm²·s
(smaller specimens)			
Test result	Failure stress (f_c)	Modulus of rupture (f_b)	Tensile strength (f_t)
Calculation	$f_c = \frac{\text{load}}{\text{nominal area}}$	$f_b = \frac{\text{load} \times \text{outer span}^c}{\text{breadth} \times \text{depth}^2}$	$f_t = \frac{2 \times \text{load}}{\pi \times \text{diameter} \times \text{length}}$
	(parts of broken beams may be used for crushing strength tests. The area of the platens is then the nominal area)	(for failure inside middle third)	

Notes:
[a]The smaller specimen size may be used where the maximum aggregate size is 25 mm or less.
[b]The weight of the rammer is 1.8 kg in each case.
[c]Outer span = 3 × inner span
Curing of specimens: until strong enough to be demoulded (usually after 24 h) the specimens are stored in their mould at a min RH of 90% and a temperature of 20°C ± 2° (for specimens to be tested at 7 days or less) or 20°C ± 5° (for specimens to be tested at 7 days or more). After being demoulded they are stored in water at 20°C ± 2°.

design or for quality control. These are briefly summarized in Table 4.11.

4.3.7 Measurement of entrained air

It is important to control the air content of air-entrained concrete for the reasons given on page 4/8. Entrained air is measured by compacting a sample of fresh concrete in three layers in a container of known volume (nominally 0.006 m³). Compaction must be sufficient to remove all entrapped air, but not so prolonged that entrained air is also removed. The container is then clamped to an airtight cover which incorporates a pressure gauge and a graduated sight tube. The space under the cover is filled with water and the vessel is pressurized with an air pump to compress the air contained in the concrete (the air pressure is usually about 1 atm). The change in volume is indicated by a drop in the water level in the sight tube which is calibrated directly in per cent of entrained air by volume. British Standard 1881:1983: Part 106 gives details of this test.

This test method is used as the basis for most entrained-air concrete specifications and is easily carried out on site. For development of admixtures and for demonstration of the effectiveness in resisting freezing and thawing the determination of the distribution of air bubbles in the hardened concrete may be preferred. This can be carried out using the methods described in ASTM C-457.

4.3.8 Analysis of fresh concrete

This is used to determine the proportions of the constituents and the grading of the aggregates before the concrete has hydrated sufficiently to bind the components together.

Several different approaches to this type of analysis have been developed. One method which was incorporated in BS 1881 comprised a set of sieves and used water wash to separate the concrete into material greater then 5 mm, between 5 mm and 150 μm, and less than 150 μm. These tests are not widely carried out on site and testing in a laboratory is difficult because of the need to retard the cement hydration.

A more convenient method of analysing fresh concrete quickly has been developed and is called the Rapid Analysis Machine (RAM). This machine separates-out the fine component of a concrete mix by elutriation in a water column, and by prior calibration an assessment of cement content can be made. As for accelerated strength testing this method can be very useful for construction work with a fast programme or where large volumes of concrete have to be placed.

Details of five methods of fresh concrete analysis are given in BSDD83:1983.

4.3.9 Analysis of hardened concrete

This is sometimes needed when a failure has occurred, or when, for some other reason, the constituents of the hardened concrete have to be determined. Full details of the methods used are given in BS 1881:Part 6:1971, although this method is shortly to be revised.

The chemistry of the analysis of hardened concrete is relatively straightforward; however, skill and experience are needed to ensure an accurate result is consistently achieved. This type of work should be carried out only in laboratories in which expertise is available and experience in interpreting results is possible.

Two main methods of analyses are: (1) determination of calcium oxide; or (2) of soluble silica. Both of these methods analyse a sample ground to a fine dust which is then dissolved in acid. Determination of either, or both, calcium oxide and soluble silica contents of the concrete can be related to the quantities in the original cement, assumed or from existing data, and hence give the cement content of the concrete. The analytical methods have inherent inaccuracies in them which can be defined and these should be investigated and accepted before an analysis is carried out.

Table 4.11 Tests on aggregates – summary of main tests in BS 812

Test	*Property measured*	*Principle/apparatus/method*
Sieve analysis	Aggregate grading, including clay and fine silt passing 75 µm	Dried sample of aggregate sieved over a number of test sieves conforming with BS 410; weight retained on each is measured
Sedimentation test	Proportion of clay, silt or dust	Fine material in suspension sampled with sedimentation pipette
Field settling test	Estimate of clay, silt or dust	Sample of aggregate shaken with salt solution; depth of material which has settled measured
Flakiness test	Percentage of flat particles	Sample of aggregate tested in specified slotted gauges
Specific gravity and water absorption	Specific gravity and percentage water absorption of coarse aggregates	Sample of saturated aggregate submerged – loss in weight indicates volume; sample dried to give dry weight and weight of absorbed water
Specific gravity and water absorption	Specific gravity and percentage water absorption of coarse aggregates	Sample of saturated aggregate submerged – displaced water indicates volume; remainder as above
Specific gravity and water absorption	As above, but for fine aggregates	Volume of sample measured by water displacement in a pycnometer: remainder of test as above
Bulk density	Bulk density and void volume of aggregate sample	Weight of aggregate required to fill container of known volume
Oven drying	Percentage moisture content	Weighed sample oven dried and reweighed
Siphon can	Percentage moisture content	Water volume determined by displacement in siphon can
Aggregate impact value	Resistance of aggregate to shock	Percentage of material passing 2.40 mm determined after specified impact test on aggregate sample
Aggregate crushing value	Resistance of aggregate to crushing	As above, but specified crushing test instead of impact
10% fines value	Resistance of aggregate to crushing	Determination of load required to produce 10% of material passing 2.40 mm in specified crushing test
Aggregate crushing strength	Compressive strength of rock	Crushing test on cylinder cut from rock sample
Aggregate abrasion value	Resistance of aggregate to surface wear	Determination of percentage loss in weight after specified lapping of aggregate sample

Aggregate type and grading are determined by breaking down a sample of concrete by heating it to 550°C for an hour or more. Cement is dissolved from the lumps of fine material with dilute hydrochloric acid, and a sieve analysis is carried out on the insoluble material which remains. Again, the method is difficult and rather approximate if the aggregates contain a substantial proportion of limestone.

The original water content is found by saturating a slice (sawn with a diamond saw) with carbon tetrachloride. This fills the pores left by the uncombined water and the volume of the pores is estimated from the weight gained. The combined water is found from the loss in weight on ignition of a sample of the concrete. The determination of original water content is also very approximate and, unless large variations in actual values are being sought, the test may not be beneficial.

4.4 Plastics and rubbers

4.4.1 Terminology

Standard definitions of terms relating to plastics (ASTM D883) includes the following.

Polymer A substance consisting of molecules characterized by the repetition (neglecting ends, branch junctions and other minor irregularities) of one or more types of monomeric units.

Plastic(s) A material that contains as an essential ingredient one or more organic polymeric substances of large molecular weight, is solid in its finished state and, at some stage in its manufacture or processing into finished articles, can be shaped by flow.

Rubber A material that is capable of recovering from large deformations quickly and forcibly, and can be, or already is, modified to a state in which it is essentially insoluble (but can swell) in boiling solvent, such as benzene, methylethylketone, and ethanol-toluene azeotrope.

A rubber in its modified state, free of diluents, retracts within 1 min to less than 1.5 times its original length after being stretched at room temperature (18 to 29°C) to twice its length and held for 1 min before release.

Elastomer A macromolecular material that at room temperature returns rapidly to approximately its initial dimensions and shape after substantial deformation by a weak stress and release of the stress.

Thermoplastic A plastic that repeatedly can be softened by heating and hardened by cooling through a temperature range characteristic of the plastic, and that in the softened state can be shaped by flow into articles by moulding or extrusion.

Thermoset A plastic that, after having been cured by heat or other means, is substantially infusible and insoluble.

4.4.2 Physical and chemical properties

4.4.2.1 Fusibility

Thermoplastics melt or soften when heated and return to their original state on cooling provided that they have not been degraded by overheating. Some thermoplastics, e.g. polystyrene, become very fluid when heated and can be used to make castings, others become soft and doughy but do not melt. These compounds, e.g. PVC, have to be shaped or formed under pressure.

4.4.2.2 Combustibility

All polymer materials should be considered combustible. However, the range of their behaviour in fire is wide. Some plastics based on, for example, chlorides, fluorides or formaldehyde, will be very difficult to ignite, and then will be self-extinguishing. Some plastics to which flame-retardant additives have been added may also behave in this way. On the other hand, some plastics which would normally be considered to be difficult to ignite or self-extinguishing may be rendered otherwise by the addition of combustible plasticizer. Plasticized PVC is an example of this.

4.4.2.3 Resistance to daylight and weathering

Ultraviolet light, present in daylight outside but effectively filtered-out by ordinary window glass, attacks many plastics and rubbers though some (acrylics for example), are largely unaffected. Those materials which are attacked can be made much more resistant by the incorporation of suitable pigments or ultraviolet absorbers in the formulation. The degree of attack naturally depends on the amount of ultraviolet light present, and performance data must relate to the appropriate conditions of exposure. Rain may leach out constituents of some formulations, and a few plastics and rubbers are not resistant to moisture.

4.4.2.4 Resistance to extremes of temperature

The flexibility of plastics compounds increases as the temperature rises and oxidation may degrade plastics and rubbers which are kept at high temperatures for long periods. Thermoplastics particularly are affected by temperature changes, and with some (bitumen is a familiar example), the ambient temperature range is enough to change them from the brittle to the fluid state. With others, e.g. polypropylene and thermosetting plastics, ambient temperature changes have little effect. These materials are stable at 100°C or more, and do not become brittle at normal low temperatures.

4.4.2.5 Thermal expansion

The coefficient of thermal expansion of polymers tends to be very high compared with conventional construction materials. Formulating compounds with high filler contents reduces this effect, but in design it must always be allowed for, e.g. by incorporating suitable movement joints and consideration when choosing fixing points. Rigid PVC formulations, such as those used for pipes and claddings, have coefficients of thermal expansion several times those of metals commonly used in construction.

4.4.2.6 Resistance to acids and alkalis

Polymers tend to be resistant to attack from acids and alkalis and are generally better than more common construction materials in this respect. The good chemical resistance of many polymers is made use of in formulating protective coatings and linings, but incorporating nonresistant fillers in compounds (chalk is a common filler for plastics) reduces or eliminates their resistance.

4.4.2.7 Resistance to oil and solvents

Polymers vary greatly in their resistance to oil and solvents. Many thermoplastics are attacked by a variety of solvents; thermosetting plastics and elastomers tend to be more resistant but may swell. Nylon and PTFE are notable among common thermoplastics for their solvent resistance. The solvent resistance of many polymers is highly specific, and polymers which are unaffected by one solvent may be readily attacked by another.

Applications of this behaviour of plastics that are worthy of note are: (1) joining by solvent welding; (2) the formulation of adhesives and paints; and (3) the possibility of incorporating 'plasticizers'. These are materials (usually liquids) which are compounded with certain plastics to make them more flexible – PVC is an example.

Problems associated with this behaviour are:

(1) Firstly, the migration of solvents and oils into or out of plasticized compounds. This occurs if the solvent or oil is miscible with the plasticizer in such compounds, even if the basic polymer would be immune from attack. Solvent welding and plasticizer migration are dealt with on pages 4/20 and 4/21.
(2) Environmental stress cracking and crazing of some polymers when stressed in an environment in which solvents are present. Polystyrene is an example of a material susceptible to this.

4.4.2.8 Resistance to oxidation and ozone

Some polymers are oxidized to a significant extent at high temperatures (the temperatures at which they fuse for example) and some (rubbers especially) are attacked at ambient temperatures by ozone. Formulation with suitable inhibitors can be used to make rubber and plastics compounds which are resistant to these effects.

4.4.2.9 Resistance to biological attack

Most polymers are immune from biological attack, though attack on ingredients used in the formulations of plastics compounds is not unknown. Casein (a protein) can be attacked; though not when it is crosslinked with formaldehyde. Borers, such as woodworm, have been known to make their way into plasticized PVC, but this is unusual and occurs only when the compound is in contact with some more palatable material. Rats sometimes bite through plastics water pipes (as they do through lead) but it is an uncommon hazard.

4.4.3 Mechanical properties

The mechanical properties of rubber and plastics compounds are greatly influenced by both the basic polymer and by the other ingredients used in formulating the compound. The compounding and manufacturing process itself also influences the mechanical properties, especially where molecular orientation occurs.

Data on mechanical properties are thus very difficult to tabulate concisely, also because values vary so much with test conditions such as temperature, duration of loading and method of loading. For such reasons the data given in Table 4.12 are incomplete in some cases, and may appear to be very imprecise in many others.

4.4.3.1 Strength

Tensile and compressive strengths of plastics compounds vary over a wide range. High tensile strength is a property of polymers such as nylon and some forms (films and fibres) of polyester and polypropylene. The relatively low elastic modulus of many polymers makes the compressive strength more difficult to assess in practical terms. Thermosetting resins tend to have high compressive and tensile strengths, the latter being capable of being greatly increased by the incorporation of reinforcing fibres. Orientation in films and fibres is also a means of increasing strength.

4.4.3.2 Elastic modulus

Rubbers and thermosetting polymers behave elastically over a large part of their strain range, but thermoplastic polymers tend to strain irreversibly after a relatively small proportion of their ultimate strain. There are a number of exceptions to this general rule. Unmodified polystyrene is noted among thermoplastics for its exceptionally low strain at failure and it shatters easily. Thermosetting polymers tend to be less flexible than rigid thermoplastics and when broken they often show a brittle fracture.

4.4.3.3 Hardness and abrasion resistance

Rubber and plastics compounds are soft compared with most construction materials, though they are not necessarily easily abraded. Rubber and flexible thermoplastics are softer than rigid thermoplastics, thermosetting plastics being generally harder still. Abrasion resistance depends on several factors including hardness, elasticity, surface friction and the ability for abrasive particles to become embedded. Factors increasing abrasion resistance for some of these reasons tend to reduce it for others and it is a property which is difficult to predict without tests.

4.4.3.4 Creep

Strength and elastic modulus measured at high rates of loading are much higher than those which are obtained at very low rates of loading for most plastics compounds, though rubbers and thermosetting polymers are less prone to creep than thermoplastic polymers. When creep deflection is considered important, care must be taken to choose suitable compounds and to limit stresses to those which will not lead to unacceptable creep. Where loads are to be applied intermittently, creep is unlikely to be a problem as recovery can take place over a relatively long period. Creep in plastics increases greatly with higher temperatures.

4.4.4 Compounding, processing and fabrication

4.4.4.1 Compounding

Some of the ingredients which are used in formulating plastics compounds have been mentioned on page **4**/19. Many polymers are used in commercial applications without addition, but the art or science of formulating PVC compounds suitable for particular applications is the converter's most important contribution in the manufacture of plastics articles and compounds based on this polymer. Guidance on formulation cannot be given here, but it is necessary that the engineer should understand that formulation is important.

4.4.4.2 Processing methods

There are many ways of making plastics compounds into useful articles or materials; some of the most usual methods are:

(1) Extrusion, where the compound is continuously forced through a die.
(2) Calendering, where the compound is forced between a series of rollers to form a sheet.
(3) Injection moulding, where the compound is forced into a die or mould.
(4) Spreading, where the compound (usually PVC) is spread on to a support (often temporary) to form a sheet.
(5) Casting, where the compound is allowed to flow into a mould under gravity or by centrifugal force.
(6) Dough moulding, where the compound is shaped under pressure by a die.
(7) Vacuum forming, where previously made sheet is shaped by being heated and forced on to an evacuated former under air pressure.

4.4.4.3 Influence of processing methods on properties

All processing methods except some used for thermosetting polymers need the polymer or compound to be heated and many thermoplastics compounds are degraded by prolonged heating. Thus, processing methods, like extrusion, which need the compound to be heated for only a short time have inherent technical advantages over methods like calendering where the compound may have to be kept hot over a long period.

Processing methods for compounds which do not become truly fluid on being heated shape the compound under pressure into a form which it will largely retain on cooling. However, some tendency to return to the unformed shape may remain and 'relaxation' of newly formed shapes (calendered sheet especially) in thermoplastics should be allowed for.

Where thermoplastics compounds are to be used at temperatures which even begin to approach the processing temperature, relaxation can be a severe problem. An example is vacuum-formed shapes which have been formed from sheet heated only enough to soften it slightly. Such shapes may relax enough to be considerably distorted even by the temperatures caused by sunshine on a summer's day.

4.4.4.4 Fabrication methods for materials and components

Materials made from thermoplastic polymers or compounds can be fabricated by heat or friction welding and, in the case of those which are soluble, by solvent welding. Mechanical methods of fabrication can also be used.

It is usually possible to find a solvent which can be used for solvent welding thermoplastics, though not all the solvents which attack a material are suitable for welding it. Important among thermoplastics which cannot be solvent welded are polyethylene, polypropylene, PTFE and nylon. The welding solvent may be modified by the addition of a separate polymer to make it tacky. This is useful in keeping joined parts in position while the solvent is doing its work. Properly made heat- or solvent-welded joints are often as strong as the parent material.

Materials made from thermosetting polymers and cross-linked rubbers cannot be welded, though many can be glued satisfactorily using thermosetting resin, or with some solvent-based adhesives made from other polymers. Best results are usually achieved if gluing is carried out as soon as possible after fabrication before cross-linking of the material is complete.

Glueing polymer materials together is likely generally to be less strong than welding. For this reason such applications as reservoir liners and waterproof membranes are far more reliable when welded. Often this is best carried out at works rather than on site. However, thermosetting adhesives can produce strong bonds. This is useful when other materials are involved, e.g. concrete, steel.

Fabrication can usually be limited to the joining of finished units because plastics materials are relatively simple to make in almost any shape, and even these can often have mechanical joints formed into them during manufacture.

4.4.4.5 Fabrication methods – direct fabrication from polymers and compounds: contact moulding

Manufacture of the material and fabrication into the required unit can often be combined into one operation. Glass-reinforced thermosetting polymers are often used in this way, and if the polymer can be cured under ambient site conditions fabrication on site is possible. When contemplating on-site fabrication of plastics components or the direct application of compounds such as, for instance, chemical-resistant epoxy surface coatings, it should be noted that full curing under ambient conditions (which might need to be modified by installing heating) will be needed. It sometimes happens that a compound which will harden under ambient conditions, and look as if it has cured fully, does not cross-link sufficiently to develop fully its desired properties of strength, durability and solvent-resistance.

On-site fabrication, or surface coating with thermosetting compounds, usually needs a curing agent to be added to the polymer shortly before fabrication. This is necessary because most compounds which will cure under ambient conditions would also cure during storage and could not be kept ready-mixed for more than a few hours. Exceptions include thermosetting compounds which cure through the absorption of atmospheric moisture and compounds whose storage life can be extended usefully by storing them under refrigeration.

Where heat can be applied to promote curing, ready-mixed thermosetting compounds which can be stored at ambient temperature are often convenient to use, since curing starts when heat is applied, and this time is under the fabricator's control. As well as thermosetting compounds which already contain the curing agent, pre-impregnated glass cloth can be fabricated in this way. This cloth is usually made with glass strand mats and compounds which have a high enough viscosity at ambient temperature to give a conveniently handled material. Before it is cured, the compound is in a thermoplastic condition, and the material can be shaped easily if it is slightly heated. Prolonged heating, or heating to a higher temperature, is then used to cure the compound after shaping and fabrication.

4.4.4.6 Application of plastics materials and components

The properties of plastics compounds described in the above sections should give the designer some guide on the virtues and limitations of the materials themselves, but in their application the interaction between plastics and other materials must also be taken into account. Two important limitations are the high coefficient of thermal expansion of plastics materials and the phenomenon of plasticizer migration.

In the case of flexible plastics, the high coefficient of thermal expansion causes few problems because the material's tendency to strain with temperature changes does not produce high stresses in the plastics materials or at the interface between plastics materials and the materials to which they are applied.

With rigid plastics materials, however, the stresses produced by restrained thermal expansion can be high enough to produce distortion, failure at the interfaces or even failure of the materials themselves. When designing fittings for rigid plastics components, provision must be made for thermal movement. Fixing through slotted holes or with clips is satisfactory provided that they are not fastened too tightly to allow free movement. Where weatherproofing has to be provided by plastics components, the design of joints which will remain weathertight while allowing movement is essential.

Plasticizer migration can be a serious problem when flexible thermoplastics containing plasticizers are bonded with adhesives which contain similar materials. In such cases the plasticizer and constituents of the adhesive diffuse into each other with the result that the plastics material may shrink and become brittle if there is a net loss of plasticizer, or soften excessively if there is a net gain and the adhesive may suffer similarly. The problem is best avoided by the choice of suitable adhesives, but where plasticized materials have to be applied over substrates into which plasticizer can migrate, e.g. over bituminous materials, a coating or an intermediate layer can be used to provide a barrier to the migration of the plasticizer.

4.4.5 Identification of polymers and plastics compounds

The suitability of any polymer or compound for any particular application will depend greatly on which compound is chosen, and it is therefore helpful to know how different compounds and polymers can be recognized. Although precise identification is often impossible without modern analytical equipment, a useful idea of the nature of the material can be obtained quite easily in many cases. The following is intended as a general guide.

(1) *Flexibility*. Rubbers can be bent without breaking or cracking and they snap back when released.
   Flexible thermoplastics can also be bent without breaking or cracking, though usually not as much as rubbers, and they do not snap back. Rigid thermoplastics can usually be bent a little, but continued attempts to bend them result in breaking or cracking. Polystyrene, however, is rigid and brittle unless modified. It cannot be bent. Thermosetting plastics are usually very rigid and break cleanly if an attempt is made to bend them.
(2) *Feel* is a difficult sensation to describe accurately, but polyethylene and PTFE have a waxy feel which other plastics do not have.
(3) *Bounce*. Most rubbers (but not butyl rubber) bounce.
(4) *Density*. A simple division can be made between polymers which float in water (a minority) and those which do not. (Table 4.12 lists specific gravities.)
(5) *Burning*. Many polymers support combustion and, of those that do, some burn with a smoky flame and others with a clear flame. Table 4.13 indicates behaviour on ignition.
(6) *Chemical tests*. Details of chemical tests are too long to be included here, but engineers who wish to carry out further tests for the identification of polymers and compounds will find that many of the simple tests can be carried out with rudimentary chemical knowledge and apparatus.

4.4.6 Foamed and expanded plastics

Thermal insulation is a very important application of plastics when they are in a foamed or expanded form. Very low bulk

Table 4.12 Properties and applications of some commonly used plastics and rubbers

Compound or polymer	*Combustibility*	*Specific gravity*	*Fusing temperature (°C)*	*Maximum working temperature (°C)*	*Ultimate tensile strength (N/mm²)*	*Minimum working temperature (°C)*	*Tensile elastic modulus (N/mm²)*	*Tensile strain at failure (%)*	*Coefficient of thermal expansion (°C × 10⁻⁵)*	*Resistance to weathering*	*Resistance to acids and alkalis: Concentrated inorganic acids*	*Diluted inorganic acids*	*Organic acids*	*Alkalis*	*Resistance to solvents: Petrol*	*Paraffin, diesel and fuel oil*	*Aromatic and chlorinated solvents*	*Ethers, ketones and esters*	*Alcohols*	*Typical applications*	*Relevant British Standards or codes of practice*
Acetal copolymers		1.41	160	80 to 120	35 to 80		1000 to 2000		8		−	−	−	+	+	+	① 0	+	+	Plumbing components, e.g. taps, door and window furniture	
Acrylic resins	② F C	1.18	100 to 120	80	40 to 70		3000	5	6	+	−	+	−	+	+	+	−	−	+	Moulded and shaped lights, e.g. rooflights and domelights, lighting fittings, sanitary ware	
Acrylonitrile-butadiene-styrene (ABS)	③ F S	1.10		80	40		400	1.5												Waste and drainpipes and fittings, pressure pipes and fittings	
Butyl rubber	F S	0.92	—	125	5 to 17	−50		500 to 800		+	+	+	+	+	−	−	−			Roof coverings, tank linings, BS 3227 damp-proof membranes, adhesives and mastics, sealants, bridge bearings	
Chlorosulphonated polyethylene (CSM)	B	1.10	—		14			300 to 500		+					+ to 0	+ to 0				Roof coverings, tank linings	
Epoxide resins	F S	1.25 to 1.30	—		55 to 70		5000	1.6 to 1.8		+	+	+	+	+	+	+	+		+	Adhesives, bedding and jointing mortars and grouts, concrete patching mortars; surface coatings	BS 4994 BS 6374
Melamine formaldehyde (laminates)	B	1.45	—	120	95		8000	0.7	3	0							①			Decorative laminates, cladding, surface coatings	BS 3794
Nylon	B S	1.10 to 1.40	220 to 265	80 to 120	50 to 100	−40	2000	75	8 to 10	+	−	−	−	+	+	+	0	+	+	Door and window furniture, cold water fittings, surface coatings, fairleads, ropes and straps	
Phenol formaldehyde (figures for laminates)	B	1.40 (1.40)	—	120 (120)	50 (80)		7000	0.5	5 (3)	+	+ ⑦	+		−	+	+	+	0 to −	−	Laminates for roofing and walling panels, adhesives for timber, surface coatings. (See Table 4.13 for applications of foamed material)	BS 1203, BS 1204, BS 2572, BS 6374

Polychloroprene	B S	1.23	—	120	5 to 20	−20		600 to 900		+					+ to 0	+	−			Seals, waterstops and gaskets, adhesives, surface coatings, bridge bearings	BS 2752
Polyester (figures for laminates)	③ F S	1.1 (1.6)	—	90 (90)	40 to 70 (≥300)		2000 (10 000)	2 (0.5)	5 to 10 (2)	④ ⑤ +	+	+	+	−	+	+	+	−	+	Surface coatings, as laminated material for: pipes, roofing and cladding, storage tanks, 'architectural' features	BS 3532, BS 4154, BS 4549, BS 4994, BS 6374
Polyethylene (low-density)	F ⑥	0.91 to 0.94	110 to 125	80	5 to 15	−60	100	100 to 400	14 to 24	④ +	⑦ +	+	0	⑦ +	−	0	−	−	0	Damp-proof membranes, protective sheeting (temporary), cold-water supply pipes, cold-water storage tanks, drains	BS 1972, BS 1973, BS 3012, BS 4646, BS 6515
Polyethylene (high-density)	F ⑥	0.94 to 0.97		105	20 to 35		1000	50 to 200	14 to 24	④ +	⑦ +	+	0	⑦ +	−	0	−	−	0		BS 4646
Natural rubber	F S	0.93	—	70	20	−55	up to 10	500 to 800		— −	⑦ +	+	+	⑦ +	−	−	−			Bridge bearings, adhesives, floor coverings	BS 1711, BS 6716
Polypropylene	F ⑥	0.90 to 0.91	165 to 175	120	18 to 30		1000 to 1500	5	9.0 to 13.5		⑦ +	+	+	⑦ +	0	+	−	0	+	Drain and waste pipes and fittings, containers, pressure pipes, ropes, geotextiles	BS 3867, BS 3943, BS 4159
Polysulphide	F ⑧	1.34	—	95	4 to 10	−50		200 to 350		+	−	−	−	−	+	+	①			Flexible sealants	BS 4254, BS 5215
Polytetrafluoro-ethylene (PTFE)	R	2.15 to 2.24	325	250	12		400	150	10	+	+	+	+	+	+	+	+	+	+	Bridge bearings, chemically resistant coatings, gaskets	BS 6564
Polyvinyl chloride (PVC) rigid	B S ⑨	1.39	75 to 85	65	35 to 55	−40	5500	10	6	④ +	−	+	−	+	+	+	−	−	+	Cold-water supply pipes, drains and wastes, rain-water goods, roofing and cladding, lining panels, ducting	BS 3505, BS 3506, BS 4203, BS 4346, BS 4514, BS 4576, BS 4660, BS 5481
Polyvinyl chloride (PVC) plasticized	B/F S	1.30	60 to 85	40 to 65	10 to 25		10	200		④ + to −	−	+ to −	−	+ to −	+ to −	+ to −	−	−	+ to −	Roof coverings, waterstops, preformed seals, surface coatings, floor coverings	BS 2571, BS 3869

Notes:
(The data given in this table must be considered in relation to the descriptions and guidance given in the text.)
① Resistant to aromatic solvents.
② Burns with sweet 'gassy' smell.
③ Very sooty flame, penetrating 'styrene' smell.
④ If suitably pigmented or ultraviolet stabilized.
⑤ Flare-retardant grades are less resistant to weathering.
⑥ Burns, drips and smells like candle wax.
⑦ Oxidizing agents may attack.
⑧ Burns with sulphurous fumes.
⑨ Burns with sweetish acrid smell.

Key:
Combustibility:
F Flammable; burning continues after ignition
B Combustible but self-extinguishing
R Difficult to ignite
C Burns with clear flame.
S Burns with smoky flame.
Chemical resistance:
\+ Resistant.
0 Some attack; use with caution if at all.
− Little or no resistance; unsuitable for use.

densities combined with sufficient strength for satisfactory handling and fixing can be obtained with these materials, and some of them have the additional advantage for low temperature insulation of low water vapour diffusance. Commonly used insulating materials made from plastics are listed in Table 4.13 together with their most important physical properties, typical applications and relevant British Standards and codes of practice.

4.4.6.1 Resin mortars

Resin mortars are usually based on epoxy or polyester. They can produce high strengths and are useful for setting-in and repair applications. When considering their use, a number of points should be borne in mind. In curing, these materials give out heat. The manufacturer's guidance should be sought as to the size of application at which this exotherm becomes too great. These materials are susceptible to creep under sustained load. In a fire, they will deteriorate and cease to perform their function. They do require care and proper conditions for their use on site, in particular: (1) correct proportioning of resin and hardener; (2) correct application and curing temperature; and (3) correct surface preparation.

4.4.6.2 Crack injection

There are a number of specialist contractors offering resin injection repair of cracked concrete. Such repairs are primarily used for restoring durability and should not be considered for structural use. Resins used are polyesters or epoxy of suitably low viscosity for penetration of hairline cracks. Different application techniques are offered by different contractors but most involve injection under pressure or application of a vacuum prior to injection. Again, care is needed to ensure correct proportioning and temperatures.

4.4.6.3 Geotextiles and liners

The key question is of course: If buried in the ground or underwater, how long will the material last? Plastics commonly used for these are: polypropylene, polyethylene polyester, PVC, polyamide, butyl, CPE and CSM. They will often be subject to oil, water, chemicals, biodeteriorating environments, and may be used in applications where there is a likelihood of mechanical damage or puncture. Rodents may be a problem. In general the materials will not usually be exposed to ultraviolet light, except perhaps where they emerge from the ground or water.

Guidance should be sought from manufacturers, but most of the information relating to the durability of the above materials can be found in Table 4.12.

4.5 Paint for steel

Painting of steelwork is covered by BS 5493:1977. Using this document, corrosion protection schemes can be chosen for most situations. This section discusses the key aspects of corrosion protection of steel and the coating types that will be met most frequently.

4.5.1 Zinc coatings

There are a number of ways in which a layer of zinc can be deposited on to the surface of steel: hot-dip galvanizing, metal spraying, sherardizing, and electro deposition. These will produce results of different thicknesses and composition through their thickness. It has been suggested that the life of the coat to first maintenance is proportional to the thickness. As such, the method of zinc deposition greatly influences the performance of the coating.

The mechanism by which zinc coating protects steel is that the difference in potential between the zinc coating and any of the steel surface which becomes exposed (scratches, etc.) and is subject to moisture which would otherwise cause it to corrode, causes an electric current to flow through the cell such that the more anodic metal, the zinc, will corrode preferentially. The zinc is lost sacrificially at the anode of the cell and the steel is cathodically protected.

Elsewhere, where the zinc coating remains intact over the steel and is not required to give cathodic protection yet, normal relatively slow corrosion rates of zinc will apply.

Some special cases should be mentioned. Firstly, zinc coating will not protect steel in hot water (in excess of about 60°C). Secondly, where zinc-coated steel is to be concreted it should be chromated beforehand. Consideration should also be given to the risks of hydrogen embrittlement and distortion when choosing the method of deposition.

4.5.2 Surface preparation

Surface preparation probably defines the quality of the overall scheme to a greater extent than any subsequent coats. It has a number of functions:

(1) Removal of millscale.
(2) Removal of existing paint.
(3) Removal of salts.
(4) Removal of contaminants, e.g. grease, dirt and weld fume.
(5) Roughening of the surface to improve adhesion.
(6) Preparation of galvanized or other zinc-coated surfaces to receive paint.

Removal of millscale. Millscale is a layer of dense oxide formed during the rolling processes at the steel mill. It is blue-grey in colour, reasonably shiny and initially is tightly adherent to the underlying steel. As such, it can easily be missed. It has the tendency to loosen and flake off with time even if coated. It is essential that it is removed before painting. Hand preparation alone (wire brushing, grinding, needleguns, etc.) is ineffective. If preceded by flame cleaning, however, it may be satisfactory. Blast cleaning is more efficient and reliable. There are a number of alternative forms. Possibilities exist for its use at works or on site.

Sand blasting is little used these days because of silicosis. Grit blasting is probably the most effective method but grit particles can get trapped in the surface of the steel. Shot blasting is common in automatic blasting plants there being problems recycling grit. It is not so efficient as grit for removing millscale because it tends to impact it into the surface.

Removal of existing paint. If paint is well adhered there is generally little point in removing it before overcoating unless an adhesion or interaction problem with the overlying coats is likely. Where existing paint is to be removed, possible methods are blast cleaning, water jetting, chemical paint strippers and hot-air strippers with mechanical cleaning. Removal of lead-based paints can be particularly problematic from a health and safety viewpoint.

Removal of salts. When rusting has occurred to the extent that even shallow pits have formed, hygroscopic iron salts will be present within the pits. These will cause the premature breakdown of paint films applied over them by underfilm corrosion. For their removal, hand preparation is useless. Normal blast cleaning is of limited effectiveness. Wet blasting or high-pressure water jetting after normal blasting are preferable.

Removal of contaminants. For oil or water-soluble contami-

Table 4.13 Properties of some foamed and expanded plastics

Expanded or foamed polymer	*Bulk density (kg/m³)*	*Thermal conductivity (W/m °C)*	*Maximum working temperature (°C)*	*Water absorption at 7 days (% by vol.)*	*Vapour resistivity (MNs/g)*	*Combustibility*	*Typical applications*	*British Standards or codes of practice*
Bead board polystyrene	16–24	0.033–0.035	80	2.5–3.0	100–600	*F	Lining walls and ceilings: insulating cold-water services Integral wall, floor and roof insulation	BS 3837:1986 BS 6203:1982
Extruded polystyrene	32–40	0.033–0.035	80	1.0–1.5	1200–1800	*F	Similar to above	BS 3290:1960 BS 6203:1982
Expanded PVC	24–125	0.035–0.055	65	3.0–4.0	1000–1800	B	Lining walls and roofs: integral insulation in sandwich panels	BS 3869:1965
Foamed phenol formaldehyde	32–64	0.036	130	Depends on cell structure	50–250	R	Roof insulation under hot-applied finishes	BS 3927:1986
Foamed urea formaldehyde	8	0.038	100	High	20	R	*In situ* cavity wall insulation	BS 5617:1985 BS 5618:1985
Foamed polyurethane	32	0.020–0.025	100	2.5	400–600	*F	Lining walls and ceilings, integral insulation in sandwich panels, insulating pipes, sprayed on *in situ* insulation	BS 4841:1975

Key:
* Flame retardant grades are available
F Flammable and continues to burn after ignition
B Combustible but self-extinguishing
R Resistant to ignition

nants, washing with a detergent solution followed by thorough rinsing is preferable to solvent washing, which tends to spread the contaminant rather than remove it. Weld slag and fume may be removed by blast cleaning or mechanical abrasion.

Roughening of the surface. Surfaces can be either so smooth that many paints will not stick well to them or too rough so that peaks penetrate the first primer coat, possibly leading to spot rusting. In general, a peak-to-trough height between 50 and 100 μm is sensible.

4.5.3 Preparation of galvanized or other zinc-coated surfaces

Galvanized and other zinc-coated surfaces are inherently difficult to paint. If zinc has weathered sufficiently that significant surface roughening has occurred, then adhesion will be improved. In new construction this is rarely the case. Calcium plumbate primers were a traditional way of solving the problem; however, being slow drying, poisonous and not compatible with many modern paints, their use is much decreased.

The two most common solutions to the problem are: (1) the use at works of an etch primer, e.g. two-pack PVB etch primer (a polyvinyl butyral, zinc tetroxychromate, phosphoric acid etch primer); (2) the use on site of British Rail 'T-Wash' – a nonproprietary product available from almost any paint supplier. This etches the surface and, in so doing, discolours it.

A number of paint products have recently been introduced to the market, claiming to be directly applicable over such surfaces. Some of these appear promising.

4.5.4 Coating types

Primers. Commonly encountered types of primer are as follows.

(1) *Lead-based primers.* The use of red lead, metallic lead and calcium plumbate primers has reduced due to their toxicity.
(2) *Zinc chromate primers.* Zinc chromate primers are based on mixtures of zinc chromate and other pigments. A variety of media are possible. They perform comparatively poorly in industrial atmospheres and marine environments.
(3) *Zinc phosphate primers.* These are good general-purpose pigments for primers. They are available in a variety of media and give good performance in most environments.
(4) *Zinc-rich primers.* Zinc-rich primers are made from not less than 85% metallic zinc dust in styrene, chlorinated rubber or epoxide media. The dry film should contain at least 90% of zinc which is a high enough concentration to give electrical contact between the zinc and the steel. Cathodic protection therefore results, though it is believed that this mechanism of protection is soon replaced by the protective action of a dense layer of zinc and its corrosion products which form through the action of contamination by the atmosphere. Very good surface preparation is essential for these primers to ensure good electrical contact and continued adhesion. The solubility of zinc corrosion products formed on the surface of the primer makes it essential to wash primed surfaces very thoroughly before applying subsequent coats of the painting system.
(5) *Other primers.* A number of paint manufacturers recently have introduced to their range products especially formulated, it is claimed, to cope with poorly prepared steel surfaces, i.e. wire brushing only. Some of these appear promising.

Barrier and finishing paints. Barrier coatings, as their name suggests, should keep the harmful elements of the environment (moisture, ultraviolet, etc.) from the primer and from the steel. In some cases they also give the steelwork an attractive appearance. Where this is not the case, a separate finishing paint may often be used over the barrier coat, although this is not always possible; bitumen paints, for instance, cannot be overcoated successfully in such a way.

Lamellar pigments. Plate-like (lamellar) particles are much used for barrier coats. Examples are micaceous iron oxide, flake glass and flake aluminium. The overlapping of the pigment particles in the paint film provides a tortuous path for moisture to penetrate through its thickness. They can be used in a variety of media. Oleoresinous, two-pack epoxy or chlorinated rubber are common.

Paint media. Pigments, whether inhibitive, lamellar, decorative, reinforcing or with other purposes, are dispersed in media. Characteristics of types commonly met are given below.

(1) *Alkyd paints.* These are cheap and easy to use but, comparatively, not very long-lasting. They can be modified with silicone to produce more durable decorative properties.
(2) *Oleoresinous paints.* Linseed oil used on its own as a paint media is slow-drying but does have advantageous qualities. Alkyd and/or other resins are often incorporated with it in paint formulations to improve this and confer other properties. Phenolic resin is often used.
(3) *Chlorinated rubber paints.* These are solutions of chlorinated rubber and plasticizer in suitable solvents. They produce generally chemically resistant, highly impermeable coatings but they do have the disadvantage of being comparatively soft and remain susceptible to solvents. This can lead to some wrinkling during overcoating. Generally, however, they are easy to maintain.
(4) *Epoxy paints (two-pack).* These are very hard, chemically resistant and confer a very high degree of protection. However, they do require a high quality of surface preparation and can be difficult to overcoat at age, being smooth and hard. At low temperatures curing may be very slow.
(5) *Polyurethane paints (two-pack).* These are tough and chemically resistant and give a high degree of protection. Often their flexural and decorative properties are superior to those of an epoxy but the difficulties of overcoating them are greater. Also, their spray application has health hazards associated with it.
(6) *Acrylics, vinyls, acrylated rubber paints.* These are one-pack products that are similar to chlorinated rubber in performance although, in general, somewhat less soft.
(7) *Coal-tar pitch and bitumen paints.* These are based on solutions of these materials in solvents. Pigments and thickening agents have to be added to these solutions if the paints are to give adequate film thicknesses and be resistant to ultraviolet light. High-build paints can be formulated to give dry film thicknesses of up to 250 μm in one coat. These paints are normally black, but a limited range of dark colours can be produced. They are slow-drying.
(8) *Pitch/epoxide paints.* Pitch/epoxide paints are similar to coal-tar pitch paints, but have much better chemical and weathering resistance because of the addition of 30% or more of epoxide resin. These paints can be formulated to give dry film thicknesses of 250 μm or more in one coat and they give good protection in very severe exposure conditions. These paints are black or very dark in colour.

Standards and codes of practice referred to in Chapter 4

BS 12 1978: *Specification for ordinary and rapid-hardening Portland cement.*
BS 146:1973:Part 2: *Specification for Portland blast-furnace cement.*
BS 812:1984:Part 101: *Guide to sampling and testing aggregates.*
BS 877:1973 (1977):Part 2: *Specification for foamed or expanded blast-furnace slag lightweight aggregate for concrete.*
BS 882:1983: *Specification for aggregates from natural sources for concrete.*
BS 915:1972 (1983):Part 2: *Specification for high alumina cement.*
BS 1014:1975 (1986): *Specification for pigments for Portland cement and Portland cement products.*
BS 1165:1985: *Specification for clinker and furnace bottom ash aggregates for concrete.*
BS 1203:1979: *Specification for synthetic resin adhesives (phenolic and aminoplastic) for plywood.*
BS 1204:1979: *Synthetic resin adhesives (phenolic and aminoplastic) for wood.*
BS 1370:1979: *Specification for low-heat Portland cement.*
BS 1711:1975: *Specification for solid rubber flooring.*
BS 1881: *Methods of testing concrete*:
Part 6:1971: 'Analysis of hardened concrete'.
Part 101:1983: 'Method of sampling fresh concrete on site'.
Part 102:1983: 'Method for determination of slump'.
Part 103:1983: 'Method for determination of compacting factor'.
Part 104:1983: 'Method for determination of vebe time'.
Part 105:1984: 'Method for determination of flow'.
Part 106:1983: 'Methods for determination of air content of fresh concrete'.
Part 112:1983: 'Methods of accelerated curing of test cubes'.
Part 116:1983: 'Method for determination of compressive strength of concrete cubes'.
Part 117:1983: 'Method for determination of tensile splitting strength'.
Part 118:1983: 'Method for determination of flexural strength'.
Part 201:1986: 'Guide to the use of nondestructive methods of test for hardened concrete'.
Part 202:1986: 'Recommendations for surface hardness testing by rebound hammer'.
Part 203:1986: 'Recommendations for measurements of velocity of ultrasonic pulses in concrete'.
BS 1972:1967: *Specification for polythene pipe (type 32) for above ground use for cold-water services.*
BS 1973:1970 (1982): *Specifications for polythene pipe (type 32) for general purposes including chemical and food industry uses.*
BS 2571:1963: *Specification for flexible PVC compounds.*
BS 2572:1976: *Specification for phenolic laminated sheet and epoxide cotton fabric laminated sheet.*
BS 2752:1982 (1987): 'Specification for chloroprene rubber compounds'
BS 3012:1970 (1982): *Specification for low- and intermediate-density polythene sheet for general purposes.*
BS 3227:1980: *Specification for butyl rubber compounds (including halobutyl compounds).*
BS 3290:1960: *Specification for toughened polystyrene extruded sheet.*
BS 3505:1986: *Specification for unplasticized polyvinyl chloride (PVC-U) pressure pipes for cold potable water.*
BS 3506:1969: *Specification for unplasticized PVC pipe for industrial uses.*
BS 3532:1962: *Specification for unsaturated polyester resin systems for low-pressure fibre-reinforced plastics.*
BS 3794:1986: *Decorative laminated sheets based on thermosetting resins.*
BS 3837:1986: *Expanded polystyrene boards.*
BS 3869:1965: *Specification for rigid expanded polyvinyl chloride for thermal insulation purposes and building applications.*
BS 3892:1982:Part 1: *Specification for pulverized fuel ash for use as a cementitious component in structural concrete.*
BS 3892:1984:Part 2: *Specification for pulverized fuel ash for use in grouts and for miscellaneous uses in concrete.*
BS 3927:1986: *Specification for rigid phenolic foam (PF) for thermal insulation in the form of slabs and profiled sections.*
BS 4027:1980: Specification for sulphate-resisting Portland cement.
BS 4154:1985: *Corrugated plastics translucent sheets made from thermosetting polyester resin (glass fibre reinforced).*
BS 4203:1980 (1987): *Extruded rigid PVC corrugated sheeting.*
BS 4248:1974: *Specification of supersulphated cement.*
BS 4254:1983: *Specification for two-part polysulphide-based sealants.*
BS 4346:1969/70/82: *Joints and fittings for use with unplasticized PVC pressure pipes.*
BS 4514:1983: *Specification for unplasticized PVC soil and ventilating pipes, fittings and accessories.*
BS 4549:1970: *Guide to quality-control requirements for reinforced plastics mouldings.*
BS 4576:1970 (1982): *Specifications for unplasticized PVC rainwater goods.*
BS 4646:1970 (1982): *Specification for high-density polythene sheet for general purposes.*
BS 4660:1973: *Specification for unplasticized PVC underground drain pipe and fittings.*
BS 4841:1975: (1987 parts 1, 2, 3): *Rigid polyurethane (PUR) and polyisocyanurate (PIR) foam for building applications.*
BS 4994:1987: *Specification for design and construction of vessels and tanks in reinforced plastics.*
BS 5075: *Concrete admixtures*:
Part 1:1982: 'Specification for accelerating admixtures retarding admixtures and water-reducing admixtures'.
Part 2:1982: 'Specification for air-entraining admixtures'.
Part 3:1985: 'Specification for superplasticizing admixtures'.
BS 5139:1974: *Classification for polypropylene plastics materials for moulding and extrusion.*
BS 5215:1986: *Specification for one-part gun grade polysulphide-based sealants.*
BS 5328:1981: *Methods for specifying concrete, including ready-mixed concrete.*
BS 5481:1977: *Specification for unplasticized PVC pipe and fittings for gravity sewers.*
BS 5493:1977: *Code of practice for protective coating of iron and steel structures against corrosion.*
BS 5617:1985: *Specification for urea formaldehyde (UF) foam systems suitable for thermal insulation of cavity walls with masonry or concrete inner and outer leaves.*
BS 5618:1985: *Code of practice for thermal insulation of cavity walls (with masonry or concrete inner and outer leaves) by filling with urea formaldehyde (UF) foam systems.*
BS 6203:1982: *Guide to fire characteristics and fire performance of expanded polystyrene (EPS) used in building applications.*
BS 6374:1984:Part 3: *Specification for lining with stoved thermosetting resins. Lining equipment with polymeric materials for the process industries.*
BS 6515:1984: *Specification for polyethylene damp-proof courses for masonry.*
BS 6564:1985: *Polytetrafluoroethylene (PTFE) materials and products.*

BS 6588:1985: *Specification for Portland pulverized fuel ash cement.*
BS 6610:1985: *Specification for pozzalanic cement with pulverized fuel ash as pozzolana.*
BS 6699:1986: *Specification for ground granulated blast-furnace slag for use with Portland cement.*
BS 6716:1986: *Guide to properties and types of rubber.*
BS 8110:1985: *Structural use of concrete.*
BS DD 83:1983: *Assessment of the composition of fresh concrete.*

Other standards

ASTM C-150: *Standard specification for Portland cement.*
ASTM C-457: *Standard recommended practice for microscopical determination of air void content and parameters of the air void system in hardened concrete.*
ASTM D-883: *Standard definitions of terms relating to plastics.*

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5 Hydraulics

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Contents

5.1 Physical properties of water

5.1.1 Density

For most purposes in hydraulic engineering, the density of fresh water may be taken to be 1000 kg/m^3. Correspondingly, the weight of 1 l is approximately 1 kg. In more precise work, usually of a laboratory or experimental nature, the variation of density with temperature may have to be taken into account in accordance with Table 5.1.

Table 5.1 Density of fresh water at atmospheric pressure

Temperature (°C)	*Density* (kg/m^3)
0	999.9
4	1 000.0
10	999.7
20	998.2
30	995.7
40	992.2
50	988.1
60	983.3
70	977.8
80	971.9
90	965.3
100	958.4

The density of sea water depends on the locality but for general calculations the open sea may be assumed to weigh 1025 kg/m^3. In a tidal river the density varies appreciably from place to place and time to time; it is influenced by the state of the tide and by the amount of fresh water flowing into the estuary from the higher reaches or from drains and other sources. At any one spot it may also vary through the depth of the water owing to imperfect mixing of the fresh and saline constituents.

5.1.2 Viscosity

Let us visualize a layer of a fluid as represented in Figure 5.1. The thickness of the layer is δy and particles in the plane AB are supposed to have a velocity v while those in CD have a different

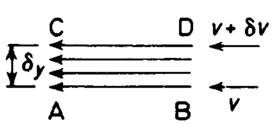


Figure 5.1 Layer of fluid illustrating laminar flow

velocity, say $v+\delta v$. The plan area of each plane, AB or CD, is a, say. Now the fluid bounded by AB and CD experiences a resistance to relative motion along AB analogous to shear resistance in solid mechanics. This force of resistance, divided by the area a, will give a resistance per unit area, or a stress f. Then:

$$f=\eta(\mathrm{d}v/\mathrm{d}y) \tag{5.1}$$

as δy tends to zero, or as the layer assumes an infinitesimal thickness, so that $\mathrm{d}v/\mathrm{d}y$ becomes the velocity gradient, i.e. the rate at which the velocity changes as we proceed outwards in a direction normal to the plane AB. η in Equation (5.1) is known as the coefficient of viscosity. If force is defined by force = mass × acceleration then η will have the units of $f/(\mathrm{d}v/\mathrm{d}y)$, or:

$$([M]\times[L]\times[T^{-2}]\times[L^{-2}])/([L]\times[T^{-1}]\times[L^{-1}])$$

i.e. $[ML^{-1}T^{-1}]$, where $[M]$ represents mass, $[L]$ length, and $[T]$ time.

Thus if newtons (i.e. kilogram metres per squared second) are adopted for the force of resistance, metres for length, metres squared for area, and metres per second for velocity, then the coefficient of viscosity takes the units kilograms per metre second. For example, the numerical value of η in the case of water at 10° C is 0.001 31 kg/m s. This is the same as 0.0131 poise, i.e. 0.0131 g/cm s.

5.1.2.1 Kinematic viscosity

Kinematic viscosity ν is defined as the ratio of the viscosity η to the density ρ of a fluid, or:

$$\nu=\eta/\rho \tag{5.2}$$

It follows from this definition that if η is in kilograms per metre second and ρ in kilograms per cubic metre then ν will be in square metres per second. Again, considering water at 10° C, ν is 1.31×10^{-6} m^2/s or 1.31×10^{-2} cm^2/s.

Typical values of η and ν, for both water and air, are given in Table 5.2, from which it will be seen that temperature has quite different effects on these two fluids.

Table 5.2 Viscosities of water and dry air at atmospheric pressure

Temperature	*Water*		*Air*	
°C	$10^3\eta$ (kg/m s)	$10^6\nu$ (m^2/s)	$10^5\eta$ (kg/m s)	$10^5\nu$ (m^2/s)
0	1.79*	1.79*	1.71*	1.32*
5	1.52	1.52	1.73	1.36
10	1.31	1.31	1.76	1.41
15	1.14	1.14	1.78	1.45
20	1.01	1.01	1.81	1.50
25	0.894	0.897	1.83	1.55
30	0.801	0.804	1.86	1.59
35	0.723	0.727	1.88	1.64
40	0.656	0.661	1.90	1.69
50	0.549	0.556	1.95	1.79
60	0.469	0.477	2.00	1.88
80	0.357	0.367	2.09	2.09
100	0.284	0.296	2.18	2.30

*To avoid any misinterpretation of the column headings note that, at 0° C, η for water is 1.79×10^{-3} kg/m s; ν is 1.79×10^{-6} m^2/s; η for air is 1.71×10^{-5} kg/m s; ν is 1.32×10^{-5} m^2/s.

5.1.3 'Non-Newtonian' fluids

Table 5.2 implies that for water and air, the coefficient of viscosity (sometimes called 'dynamic viscosity' or 'absolute viscosity') varies with temperature but otherwise is a constant coefficient in Equation (5.1), i.e. f is in simple proportion to $\mathrm{d}v/\mathrm{d}y$. This is true of very many other fluids, but there exist also the so-called non-Newtonian fluids. With some of these, η decreases as $\mathrm{d}v/\mathrm{d}y$ increases; with others the reverse is true. Among the examples of substances which, in phases of their fluid state, behave in a 'non-Newtonian' way, are certain lubricants, e.g. grease used in bearings, plastics, and suspensions of particles.

5.1.4 Compressibility

In the vast majority of engineering calculations, water may be treated as an incompressible fluid. Exceptions arise when large and sudden changes of velocity occur, as in certain problems associated with the rapid opening or closing of a valve. If we imagine a mass of water to have its volume changed from V to $V-\delta V$ by an increase of pressure δp applied uniformly round its surface, then the bulk modulus of compressibility K is defined as the stress or pressure intensity δp divided by the volumetric strain produced by δp. This volumetric strain is $-\delta V/V$. Hence:

$$K=-\delta p\Big/(\delta V/V) \qquad (5.3)$$

δV itself being treated as negative.

The value of K depends somewhat on the temperature and absolute pressure of the water, but in round numbers it is usually sufficiently accurate to take it as 2×10^9 N/m^2.

5.1.5 Surface tension

Surface tension is the property which enables water and other liquids to assume the form of drops, when it appears that the water is bounded by an elastic skin or membrane under tension. Another important manifestation is related to small waves or ripples where the form and motion are restricted or influenced by the tension in the surface. Surface tension depends on the liquid and gas in contact with one another. Suppose a portion of the liquid to have a bounding surface with radii of curvature in two mutually perpendicular directions R_1 and R_2 as in Figure 5.2. Then the excess of pressure intensity inside the boundary, over that outside it, is $\gamma(1/R_1+1/R_2)$, where γ is the surface tension, a force per unit length of the line to which it is normal.

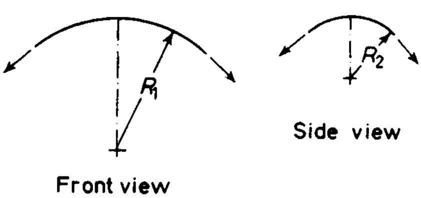


Figure 5.2 Bounding surface of a liquid

The surface tension of mercury in contact with air at 20° C is approximately 0.51 N/m.

Table 5.3 Surface tension of water in contact with air

(° C)	0	20	40	60	80	100
γ (N/m)	0.0756	0.0728	0.0700	0.0671	0.0643	0.0615

5.1.6 Capillarity

If a vertical tube is placed in a vessel containing water, the water will be drawn up the tube by capillary attraction. The angle α in Figure 5.3 is known as the angle of contact, and approaches the value zero for clean water in contact with a clean glass tube. In that case

$$h=(4\times 10^6\,\gamma/\rho dg)\text{ mm, approximately, or say }(4\times 10^5\,\gamma)/\rho d \qquad (5.4)$$

where γ is the surface tension in newtons per metre, ρ the density in kilograms per cubic metre, d the bore of the tube in millimetres, and g is in metres per second squared.

At 20° C, therefore, the elevation of the water in the glass tube amounts to about $30/d$ mm. It should be emphasized, however, that capillary attraction depends to a marked degree upon the state of cleanliness of the liquid and the tube.

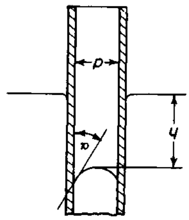


Figure 5.3 Capillary rise of water in a tube

If mercury is considered instead of water, there is a depression of the liquid in the tube as shown in Figure 5.4. The angle β is approximately 53° and h, at room temperature, is about $9/d$ mm.

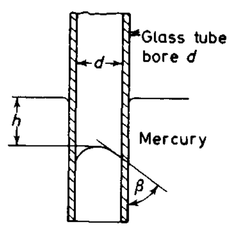


Figure 5.4 Capillary depression of mercury in a tube

Capillary attraction may be important in connection with the technique of measurement. For example, if the level of water in a tank is read, for convenience, on an external gauge as depicted in Figure 5.5, the bottom of the meniscus, or curved surface of the water in the gauge-tube, will stand higher than in the tank. Whether the effect is serious depends, of course, upon the standard of accuracy demanded, but it is generally advisable in such a case, or when using a differential gauge having two limbs, to use a tube not smaller than 9.5 mm bore, and as uniform as practicable throughout its length.

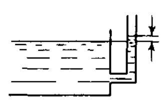


Figure 5.5 Effect of capillary attraction on measurement of height of water in a tank

5.1.7 Solubility of gases in water

At atmospheric pressure, water is capable of dissolving approximately 3, 2 and 1% of its own volume of air at temperatures of 0, 20 and 100° C respectively. Certain other gases, such as carbon dioxide, are dissolved in much greater volumes, but the presence of air alone, together with the phenomenon of vapour pressure of water, may lead to complications in certain pipelines or machines. To take an example, at the highest point of a siphon the pressure is below atmospheric and it is possible for air to come out of solution there which was originally dissolved in the water at atmospheric pressure. This accumulation of air may ultimately reduce the flow along the siphon very appreciably, or even break it entirely, unless precautions are taken to draw off the air and vapour as it collects. The suction-lift of pumps is also restricted in practice by the difficulty of release of air and generation of vapour so that, in practice, it is usual to limit the suction-lift to about 8.5 m instead of the full height of the water barometer, say 10.4 m.

5.1.8 Vapour pressure

If a liquid is contained within a closed vessel the space above it becomes saturated with its vapour and the space is subjected to an increase of pressure, which is the vapour pressure of the liquid at the temperature then obtaining.

Table 5.4 Vapour pressure p_v (N/m^2) of water at various temperatures

(°C)	0	5	10	15	20	30	40
$10^{-4}p_v$	0.0610	0.0875	0.123	0.170	0.235	0.423	0.736
(°C)	50	60	70	80	90	100	
$10^{-4}p_v$	1.23	2.00	3.09	4.76	7.00	10.1	

To obtain p_v in metres of water at a given temperature, divide pv, (N/m^2) by $g\rho$ where ρ (kg/m^3) is given in Table 5.1. For example, at 100°C, p_v is $(10.1 \times 10^4)/(9.81 \times 9.58 \times 10^2)$, i.e. 10.8 m of water.

5.2 Hydrostatics

(1) A fluid at rest exerts a pressure which is everywhere normal to any surface immersed in it.
(2) The pressure intensity at a point P in a liquid is equal to that at the free surface of the liquid together with ρgh, where h is the depth of P below the free surface and ρ is the density of the liquid.

5.2.1 Force on any area

In many engineering problems all pressures are treated relative to atmospheric pressure as a datum. Adopting that system, consider the force exerted on an elementary, or infinitesimal, portion δA of an area A immersed in a liquid (see Figure 5.6).

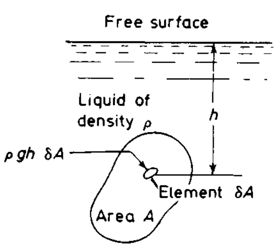


Figure 5.6 Elementary area δA immersed in liquid

The pressure intensity on δA is $p=\rho gh$. Hence, the force on δA is $\rho gh \cdot \delta A$, where h is the vertical depth of δA.

The total force on the whole area A of which δA is an element is the arithmetical sum of the forces on all its constituent elements, $\Sigma \rho gh \cdot A = \rho g \Sigma h \cdot \delta A$, assuming ρ to be constant throughout the liquid. Hence:

$$\text{total force} = \rho gH \cdot A \tag{5.5}$$

where H is the vertical depth of the centroid of the whole area.

This total force is only equal to the resultant force if the area under consideration is a plane one. If the area is curved, then the forces acting on its elementary portions are not all parallel to one another so their simple arithmetic sum is not the same as their resultant.

5.2.2 Force on plane areas (Figure 5.7)

Force on element $= (\rho gz \cos\theta)\delta A$

$$\begin{aligned}\text{Resultant force} &= \text{total force in this case}\\ &= \Sigma(\rho gz\cos\theta)\delta A\\ &= \rho g\bar{z}A\cos\theta \end{aligned} \tag{5.6}$$

where $\bar{z}$ is the inclined depth of centroid of A.

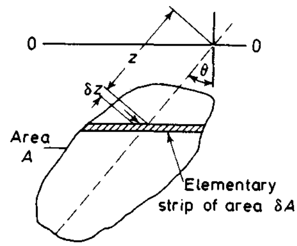


Figure 5.7 Plane area immersed in liquid

The resultant force will act through a point in the immersed area A known as its centre of pressure and such that its inclined depth Z is given by:

$$Z = \left(\int z^2\,dA\right)\Big/\bar{z}A = (\text{Second moment of area } A \text{ about } 00)/\bar{z}A$$
$$= I_{00}/Az \tag{5.7}$$

or:

$$Z = k_{00}^2/\bar{z} \tag{5.8}$$

where k_{00} is the radius of gyration about 00 and $k_{00}^2 = k^2 + \bar{z}^2$, where k is the radius of gyration about an axis through the centroid parallel to 00.

Examples 5.1 to 5.6. In the following examples C is the centroid, P the centre of pressure.

Example 5.1: Parallelogram (Figure 5.8(a)):

$$I_{00} = (bd^3/12) + bd(\bar{z})^2$$
$$Z = [bd^3/12 + bd(\bar{z}^2)]/bd\bar{z}$$
$$= (d^2/12 + \bar{z}^2)/\bar{z}$$
$$= \tfrac{2}{3}d \quad \text{if upper edge of parallelogram is in surface.}$$

Example 5.2: Circular area, diameter d (Figure 5.8(b)):

$$I_{00} = (\pi d^4/64) + (\pi d^2/4)\bar{z}^2$$

$$Z = \frac{(\pi d^4/64) + (\pi d^2/4)\bar{z}^2}{(\pi d^2/4)\bar{z}}$$
$$= (d^2/16 + \bar{z}^2)/\bar{z}$$
$$= 5d/8 \quad \text{if circle touches 00.}$$

Example 5.3: Triangular area, apex downwards (Figure 5.8(c)):

$$I_{00} = (bh^3/36) + (bh/2)\bar{z}^2$$
$$Z = \frac{(bh^3/36) + (bh/2)\bar{z}^2}{(bh/2)\bar{z}} = \frac{h^2/18 + \bar{z}^2}{\bar{z}}$$

where $\bar{z} = a + (h/3)$

$Z = h/2$ if a = 0.

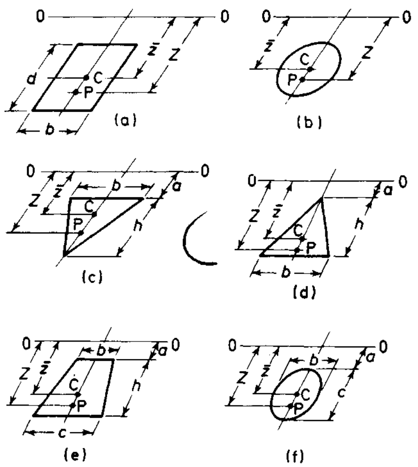


Figure 5.8 (a) Parallelogram; (b) circular area; (c) triangular area, apex downwards; (d) triangular area, apex upwards; (e) trapezium; (f) ellipse

Example 5.4: Triangular area, apex upwards (Figure 5.8(d)):

$$I_{00} = (bh^3/36) + (bh/2)\,\bar{z}^2$$

$$Z = \frac{h^2/18 + \bar{z}^2}{\bar{z}}$$

where $\bar{z} = a + (2/3)h$

$$Z = (3/4)h$$

if a = 0.

Example 5.5: Trapezium (Figure 5.8(e)):

$$Z = (k^2 + \bar{z}^2)/\bar{z}$$

where $k^2 = (h^2/18)[1 + 2bc/(b+c)^2]$

$$\bar{z} = \frac{h(2c+b)}{3(b+c)} + a$$

Example 5.6: Ellipse (Figure 5.8(f)):

$$Z = (k^2 + \bar{z}^2)/\bar{z}$$

where $k^2 = c^2/16$; $\bar{z} = a + (c/2)$

In the examples so far considered, the immersed areas have had a vertical plane of symmetry in which it is evident that the resultant force will act. All that has been necessary, therefore, was to determine the position of the resultant force in that plane of symmetry.

5.2.2.1 Force on an unsymmetrical plane area

Choose any convenient axes OX, OY. OX may be the line of intersection of the plane of the immersed area with the surface of the liquid. The elementary area δA has coordinates x and y relative to the chosen axes (Figure 5.9).

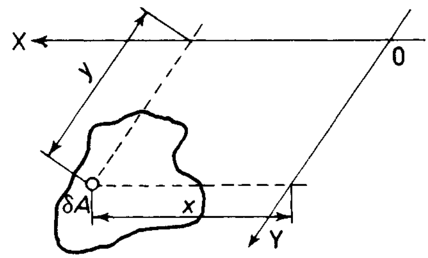


Figure 5.9 Unsymmetrical plane area

Let $\bar{x}$ and $\bar{y}$ be the coordinates of the centre of pressure of the whole area relative to these same axes. Then, by using the principle that the moment of the resultant force is equal to the sum of the moments of the individual elementary forces:

$$\bar{y} = (\Sigma y^2\,\delta A)/(\Sigma y\,\delta A) \qquad \bar{x} = (\Sigma xy\,\delta A)/(\Sigma y\,\delta A)$$

or:

$$\bar{y} = \frac{\iint y^2\,dx\,dy}{\iint y\,dx\,dy} \qquad \bar{x} = \frac{\iint xy\,dx\,dy}{\iint y\,dx\,dy} \qquad (5.9)$$

But:

$$\Sigma y^2\,\delta A \quad \text{or} \quad \iint y^2\,dx\,dy = Ak^2$$
$$\Sigma y\,\delta A \quad \text{or} \quad \iint y\,dx\,dy = Ay_0 \qquad (5.10)$$

where A is the total area, k its radius of gyration about OX and y_0 the ordinate of the centroid of the area.

5.2.3 Force on curved areas

The following examples will serve to illustrate some useful principles.

Hemispherical bowl, radius r, just full of water (Figure 5.10).

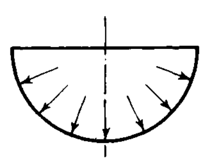


Figure 5.10 Hemispherical bowl just full of water

Total force = arithmetical sum of forces acting on the surface
= area × pressure intensity at centroid
= $2\pi r^2$ × density of water × depth of centroid × g
= $2\pi r^2 \times \rho \times (r/2)g$
= $\pi r^3 \rho g$

But the horizontal components of the corresponding forces on opposite sides of the vertical axis counterbalance one another, and:

Resultant force = weight of water contained
= volume of hemisphere × density of water × g
= $\frac{2}{3}\pi r^3 \rho g$

Cylindrical vessel with hemispherical end, just full of water.

(1) (Figure 5.11(a)). Force on lid due to water = 0

Resultant force (vertical) on hemispherical base $= (\pi r^2 h + \frac{2}{3}\pi r^3)\rho g$

(2) (Figure 5.11(b)). Resultant force (vertical) on flat base $= \pi r^2(h + r)\rho g$

Resultant force (vertically upwards) on dome $= \pi r^2(h + r)\rho g - (\pi r^2 h + \frac{2}{3}\pi r^3)\rho g = \frac{1}{3}\pi r^3 \rho g$

(3) (Figure 5.11(c)). Horizontal force on either end $= \pi r^2(r\rho)g = \pi r^3 \rho g$

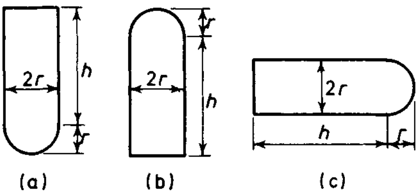


Figure 5.11 (a) Cylindrical vessel with hemispherical end just full of water; (b) the same, inverted; (c) the same, lying with axis horizontal

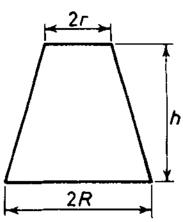


Figure 5.12 Truncated cone just full of water

Truncated cone, just full of water (Figure 5.12).

Resultant force (vertically downwards) on base $= \pi R^2 h\rho g$

Volume of water contained $= \frac{1}{3}\pi h(R^2 + Rr + r^2)$

Resultant force (vertically upwards) on curved side

$= [\pi R^2 h\rho - \frac{1}{3}\pi h(R^2 + Rr + r^2)\rho]g$

$= (\frac{2}{3}\pi R^2 h - \frac{1}{3}\pi Rrh - \frac{1}{3}\pi r^2 h)\rho g$

5.2.4 Buoyancy

A liquid of density ρ exerts a vertical upwards force $V\rho g$ on an immersed body of volume V (Figure 5.13). If the weight of the body is greater than $V\rho g$ it will sink. If the weight of the body is less than $V\rho g$ the body will float in such a way that the portion immersed has a volume V' which satisfies the following equation:

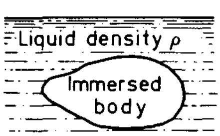


Figure 5.13 Body immersed in liquid

$$V'\rho g = \text{total weight of body} \tag{5.11}$$

Centre of buoyancy and metacentre. The centre of gravity of the displaced fluid is called the centre of buoyancy. When a body is floating freely, the weight of the fluid displaced equals the weight of the body itself, and its centre of buoyancy, for equilibrium, must be in the same vertical as the centre of gravity of the body. The degree of stability for angular displacements involves the conception of the metacentre. In Figure 5.14, XX represents the vertical axis of symmetry of a floating body with a centre of gravity, owing to the distribution of its weight, we suppose to be at G. B is the centre of buoyancy.

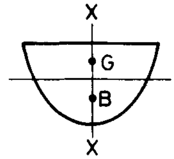


Figure 5.14
Centre of buoyancy

Let the body be displaced so that B_1 becomes the new centre of buoyancy, i.e. the centre of gravity of the liquid as displaced in the new position (Figure 5.15).

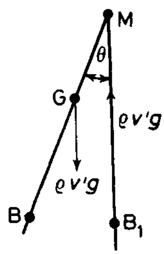


Figure 5.15 Metacentre

The new force of buoyancy acts vertically upwards through B_1, to intersect the deflected line BG in M, the metacentre. Strictly speaking, the metacentre is the position assumed by M as the angle of displacement θ tends to zero.

If M is above G, there will be a 'righting moment' $\rho V' \cdot GM \sin\theta \cdot g$.

The condition for initial stability, or stability during small displacements, is then that M shall be above G.

The metacentric height may be calculated from the equation:

$$\mathrm{GM} = (I/V') \pm \mathrm{GB} \tag{5.12}$$

where $I = Ak^2$: the plus sign is used if G is below B and the minus sign if G is above B.

A is the area of the water-line section and k is its radius of gyration about the axis Oy. V' is the volume of the immersed portion of body (shown in Figure 5.16).

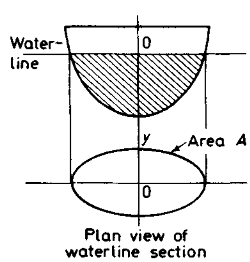


Figure 5.16 Metacentric height

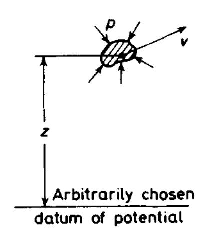


Figure 5.17 Mass of liquid subjected to p N/m^2 and moving with velocity v m/s

5.3 Hydrodynamics

5.3.1 Energy

A liquid possesses energy by virtue of the pressure under which it exists, its velocity and its height above some datum level of potential energy. These three forms of energy—pressure, kinetic and potential—may be expressed as quantities per unit weight of the liquid concerned. The result is the pressure, kinetic or potential head. Thus, referring to Figure 5.17, in which a mass of liquid is represented as subjected to a pressure p N/m^2, moving with a velocity v m/s and having its centre of mass at a height z above a datum of potential energy:

$$\text{total head} = (p/\rho g) + (v^2/2g) + z \qquad (5.13)$$

in metres, where ρ is the density in kilograms per cubic metre.

If a gas is considered, then account should be taken of its elasticity and the work done in compressing a given mass of it as it passes from a region of low to higher pressure.

5.3.2 Bernoulli's theorem

This states that in the streamline motion of an incompressible and inviscid fluid the total head remains constant from section to section along the stream tube, i.e.:

$$(p/\rho g) + (v^2/2g) + z = \text{constant} \qquad (5.14)$$

The idealized circumstances envisaged in this statement would seem, at first sight, to render the theorem useless for the solution of problems dealing with natural fluids, which are viscous, and especially when such fluids are not moving in streamlines, i.e. when the motion is turbulent, as it usually is in hydraulics. In fact, however, the theorem forms the basis of the majority of practical calculations if and when appropriate terms are added or coefficients introduced to allow for losses of head arising from various causes. The numerical values of these terms and coefficients are almost always the result of experiment or experience.

5.3.3 Streamline and turbulent motion

If we concentrate attention upon one point P in the cross-section of a pipe or channel along which a fluid is moving at a constant rate, the motion at P may be called 'streamline' if the velocity there is constant in magnitude and direction. On the other hand, the motion at P will be turbulent if the velocity there varies from time to time in magnitude and/or direction, despite the fact that the general rate of flow along the channel is constant. In this turbulent motion, the instantaneous velocity at P depends upon how the eddies are passing it at the moment under consideration. The eddies which characterize turbulent motion require energy for their creation and maintenance, and the law of resistance in streamline (sometimes called laminar) flow is quite different from that in turbulent flow.

5.3.3.1 Flow in pipes

Loss of head in smooth pipes The general equations for the loss of head in a pipe of uniform diameter d are as follows. Let h be the loss of head in metres, l the length of pipe considered in metres, v the mean velocity in metres per second $= Q/(\pi d^2/4)$, Q the rate of discharge (in cubic metres per second) and ν the kinematic viscosity of fluid (in square metres per second). Then:

$$h = K(l v^{2-n} \nu^n)/g d^{3-n} \qquad (5.15)$$

where K is a coefficient.

Both K and n depend upon R_e the Reynolds number, vd/ν. If R_e is less than 2100, $K = 32$ and $n = 1$.

These values give the equation for streamline flow, which may be deduced mathematically:

$$h = 32\nu l v/g d^2 \qquad (5.16)$$

Alternatively we may use the equation† commonly adopted by hydraulic engineers, i.e.:

$$h = f l v^2/2gm \qquad (5.17)$$

in which m represents the hydraulic mean depth or the ratio of the area of section to the wetted perimeter. In a cylindrical pipe running full, $m = d/4$.

For values of R_e up to 2100:

$$f = 16(vd/\nu)^{-1} \qquad (5.18)$$

For values of R_e between 3000 and 150 000 (Davis and White[1]):

$$f = 0.08(vd/\nu)^{-0.25} \qquad (5.19)$$

Alternatively, if R_e exceeds 4000, the Prandtl equation may be used:

$$1/\sqrt{(4f)} = 2.0 \lg[R_e\sqrt{(4f)}] - 0.8 \qquad (5.20)$$

These relationships apply to smooth pipes of, say, glass, drawn brass, copper or large pipes with a smooth cement finish.

In calculating Reynolds numbers, the units in which v, d and ν are measured should be consistent with one another; e.g. v in metres per second, d in metres and ν in square metres per second. Values of f for smooth pipes in the equation $h = flv^2/2gm$ are plotted against $\lg vd/\nu$ in Figure 5.18.[2]

At a temperature of 15°C, ν for water is 1.14×10^{-6} m^2/s.

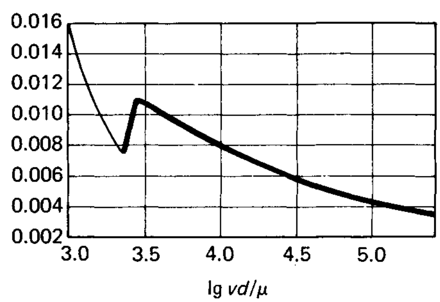


Figure 5.18 Values of f for smooth pipes. (After Stanton and Pannell (1914) Phil. Trans A, **214**; National Phys. Lab. Coll. Res., **11**)

5.3.4 Pipes of noncircular section

There is experimental evidence showing that if the flow is *turbulent*, the value of f for various shapes of section is approximately the same as for a cylindrical pipe at the same value of vm/ν, where m again represents the hydraulic mean depth.

The critical value of vm/ν, below which the motion is normally laminar, does depend to some extent, however, on the shape of the section. For a circular section it is 525 (i.e. $vd/\nu = 2100$). For rectangular sections the critical vm/ν varies with the ratio of the lengths of the sides and has approximate values of 525 for a square section, 590 for a section having one

† Some writers prefer to use $h = \lambda l v^2/2gd$, rather than $4fv^2/2qd$, for cylindrical pipes. Their friction factor λ is then $4f$.

side 3 times the other, and 730 for a section in which the length of one side is large compared with the other.

During truly *laminar* or *viscous* motion, the loss of head h for various shapes of section is as follows (v being the mean velocity through the section):

Circular section (diameter d): $h=32vlv/gd^2$

Rectangular section (one side $2a$, other side $2b$):

$$h=3vlv\Big/gb^2\left[1-\frac{192}{\pi^5}\frac{b}{a}\left(\tanh\frac{\pi a}{2b}+\frac{1}{3^5}\tanh\frac{3\pi a}{2b}+\ \ldots\right)\right]$$

Square section (each side $2a$):

$$h=7.12vlv/ga^2$$

Rectangular section having a large compared with b:

$$h\rightarrow 3vlv/gb^2$$

Circular annulus (mean velocity v through space of area $\pi(d_1^2-d_0^2)/4$):

$$h=32vlv\Big/gd_1^2\left[1+(d_0/d_1)^2+\frac{1-(d_0/d_1)^2}{\ln(d_0/d_1)}\right]$$

where d_1 is the outside diameter and d_0 the inside diameter.

5.3.5 Loss of head in rough pipes

Here the value of f also depends upon the ratio of the hydraulic mean depth to the height of the roughening projections from the wall, as well as upon the distribution and shape of these roughnesses. Figure 5.19 summarizes experimental results obtained by Nikuradse[3] with sand-roughened pipes, k being the mean size of the grain projecting from the wall.

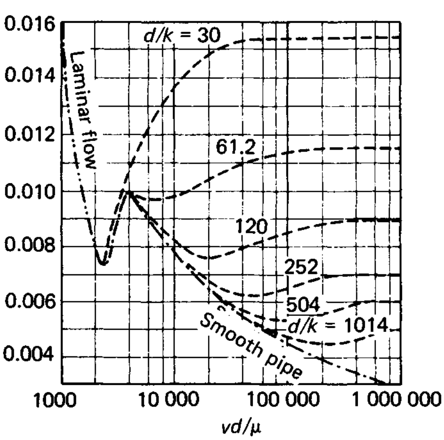


Figure 5.19 Values of f for rough pipes. (After Nikuradse (1933) *Forschungsh. Ver. dtsch. Ing.*, No. 361)

The general tendency is for f to become constant, for a given rough pipe, at sufficiently high Reynolds numbers. A commonly accepted explanation of this is that first of all the surface grains lie inside a very thin viscous layer at the wall of the pipe, even when the main motion is turbulent; at higher values of R_e, however, they begin to project from this layer and to shed eddies for the maintenance of which additional energy is required.

Prandtl and von Kármán[4] have shown than Nikuradse's results may be made to lie within one band by plotting the quantity

$$1/\sqrt{(4f)}-2\lg(d/2k)$$

against a new Reynolds number V_*k/v, in which V_* represents $v\sqrt{(f/2)}$.

Again, if V_*k/v exceeds 60, f becomes constant for a given pipe, the flow then being 'fully turbulent' and the resistance proportional to v^2.

Under those conditions ($V_*k/v>60$):

$$1/f=16\left(\lg\frac{3.7d}{k}\right)^2 \tag{5.21}$$

A pipe may be regarded as 'hydraulically smooth' if $V_*k/v<3$.

To take a specific example, namely, $d/k=252$: for values of the original Reynolds number vd/v less than about 11 500, f is the same as for a smooth pipe (see Figure 5.19), while if vd/v exceeds 250 000, f assumes a constant value of 0.007. Between the two there is a curve which represents a transition and which covers a wide range of Reynolds numbers (11 500 to 250 000 in the example $d/k=252$).

A large proportion of the cases which occur in engineering will be found to lie within the zone of transition, for which Colebrook and White[5,6] have evolved the equation:

$$1/\sqrt{(4f)}=-2\lg\left(\frac{k}{3.7d}+\frac{2.51}{R_e\sqrt{(4f)}}\right) \tag{5.22}$$

5.3.6 Formulae for calculating pipe friction (turbulent flow)

With the velocities commonly encountered in water pipes, the motion is turbulent. These velocities in fact are frequently within the range from 1.5 to 3.5 m/s, whereas in general the motion can only be expected to be streamline, considering water at ordinary temperatures, for velocities lower than $2.4/d$ m/s, where d is the pipe diameter (in millimetres).

Manning's formula. Among the many formulae which have been suggested from time to time, that of Manning[7] is much favoured:

$$v=(m^{2/3}i^{1/2}/n)\qquad \text{(m/s)} \tag{5.23}$$

In this form it applies to pipes and open channels.

m is the hydraulic mean depth (in metres), i the virtual slope of the pipe (i.e. h/l) or the actual slope of the open channel under conditions of uniform flow. n depends upon the material of which the conduit is made.

Alternatively:

$$v=(m^{2/3}i^{1/2}/100n)\qquad \text{(m/s)} \tag{5.24}$$

if m is expressed in millimetres.

For cylindrical pipes, the Manning formula in Equation (5.23) gives:

$$h=n^2lv^2/m^{4/3} \tag{5.25}$$

But:

$$m = d/4$$

Hence:

$$h = n^2(4)^{4/3}(lv^2/d^{4/3}) = 6.35n^2(lv^2/d^{4/3}) \qquad (5.26)$$

If we write this in the form:

$$h = A(lv^2/d^{4/3})$$

the following are appropriate values of A for new pipes (see Table 5.5).

Table 5.5

Material	n	$A = 6.35\,n^2$
Clean uncoated cast iron	0.013	0.001 1
Clean coated cast iron	0.012	0.000 92
Riveted steel	0.015	0.001 4
Galvanized iron	0.014	0.001 2
Brass, copper or glass	0.010	0.000 64
Wood-stave	0.012	0.000 92
Smooth concrete	0.012	0.000 92
Cement mortar finish	0.013	0.001 1
Vitrified sewer pipe	0.011	0.000 77

Comparing the formulae

$$h = 4flv^2/2gd \text{ and } h = Alv^2/d^{4/3}$$

it appears that

$$f = (4.91/d^{1/3})\,A \qquad (5.27)$$

or

$$f = 31.2n^2/d^{1/3} \qquad (5.28)$$

where d is in metres. If d is in millimetres:

$$f = (49.1/d^{1/3})\,A \quad = 312n^2/d^{1/3} \qquad (5.29)$$

Hydraulic Research Papers, Nos 1 and 2 (Ackers[8]), first published by HMSO in 1958, contain not only a fascinating review of the resistance of fluids in channels and pipes but also tables and graphs for the use of designers. The results are derived from the formula of Colebrook and White in Equation (5.22) and values of the effective roughness dimension k are quoted for a great variety of commercial pipes, while in addition, a supplementary note provides information concerning the *actual* diameters of different classes of pipe in relation to their nominal bores.

5.3.7 Deterioration of pipes

Pipes deteriorate with age and to allow for this reduction in their carrying capacity Barnes[9] has suggested the following (see Table 5.6).

None of these values can be at all precise, since the reduction of carrying capacity must depend not only upon the material but also upon the nature and velocity of the water and upon the diameter of the pipe; an increased roughness due to tuberculation will be more troublesome, proportionately, in small- than in large-diameter pipes.[10]

Table 5.6

Type of pipe	*Discharge for which to design, in terms of required discharge* Q
Uncoated cast iron	1.55 Q
Asphalted cast iron	1.45 Q
Asphalted riveted wrought iron or steel	1.33 Q
Wood-stave	1.08 Q
Neat cement or concrete	1.06 Q

5.3.8 Use of additives to reduce resistance

The literature dealing with this important subject has expanded greatly in the last 10 or 20 years and now includes a large number of papers giving information not only of laboratory studies but also of evidence adduced by full-scale applications.

Fortunately, the International Association for Hydraulic Research (IAHR) invited a panel or 'Task Force' consisting of R. H. T. Sellin, J. W. Hoyt, J. Pollert and O. Scrivener to compile a 'state-of-the-art review' and this has now been published in two parts in the Association's *Journal*.[11,12]

These reports deal largely with the addition of polymers and their observed effect in decreasing the resistance to the motion of the fluid. Attempts to explain the phenomenon are discussed and a wide range of applications are described: they include, *inter alia*, full-scale examples of sewers, oil pipelines, open channels, hydro-transport of solids, hydraulic machinery, ships and submerged bodies, in which significant reductions of resistance or increases of the carrying-capacity of pipes and channels have been recorded.

5.3.9 Losses of head in pipes due to causes other than friction

Sudden enlargement. With sufficient accuracy for practical purposes, loss due to sudden enlargement (Figure 5.20) = $(v_1 - v_2)^2/2g$

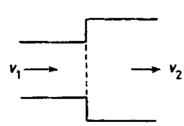


Figure 5.20 Loss of head due to sudden enlargement

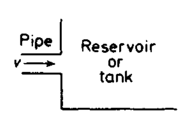


Figure 5.21 Pipe joined to tank or reservoir

If the enlarged section is very large, as when a pipe is joined to a tank or reservoir (Figure 5.21):

$$\text{loss} = v^2/2g \qquad (5.31)$$

Gradual enlargement (Figure 5.22). Loss of head (including friction) may be taken as:

$$k(v_1 - v_2)^2/2g \qquad (5.32)$$

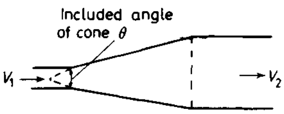


Figure 5.22 Gradual enlargement of pipe

where k depends upon the angle of divergence θ in the following manner (Gibson[13]).

Table 5.7

θ (degrees)	2	5	10	15	20	40	60	90	120	180
k (circular pipe)	0.20	0.13	0.18	0.27	0.43	0.91	1.12	1.07	1.05	1.00

Sudden contraction. The loss due to sudden contraction is almost entirely due to the subsequent re-enlargement of the contracted stream (Figure 5.23). For practical purposes:

$$\text{loss} = (1/2)(v^2/2g) \tag{5.33}$$

and this may be taken as the immediate loss of head experienced as water flows from a reservoir into a pipe in which it attains a velocity v.

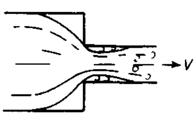


Figure 5.23 Loss of head due to sudden contraction

With a reasonably rounded entrance, the loss may be reduced to about:

$$(1/20)(v^2/2g)$$

If the pipe has a sharp entrance but projects into the reservoir and forms a re-entrant mouthpiece, the loss of head at the entrance is approximately $0.80\,v^2/2g$ (assuming that the pipe runs full).

5.3.10 Losses at pipe bends

Owing to the many variables involved (e.g. size of pipe, radius of bend, velocity of flow), it is impracticable at present to generalize with any degree of certainty, but the following data may be helpful in ordinary calculations (Figure 5.24).

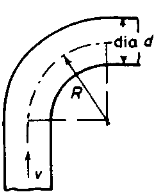


Figure 5.24 Pipe bend

Defining $Kv^2/2g$ as the loss in excess of that which would arise from friction in the same length of straight pipe, then approximate values of K are:

$R/d = 1$; $K = 0.50$ for either 90° or 180° bends;
$R/d = 2$ to 8; $K = 0.30$ for 90° and $K = 0.35$ for 180° bends.

For 90° elbows, $K = 0.75$ and for square, or sharp, elbows, $K = 1.25$.

The motion round the bend tends to take on the characteristics of a free vortex, having a greater velocity at the inside than at the outside. Correspondingly, the pressure at the inside is less than at the outside. Consider a section half-way round the bend and let $d = 2r$. If the discharge (volume per second) is Q, the velocities at the inside and outside of the bend are approximately:

$$v_i = \frac{Q}{2\pi(n-1)r^2[n-(n^2-1)^{1/2}]}$$

$$v_0 = \frac{Q}{2\pi(n+1)r^2[n-(n^2-1)^{1/2}]}$$

where $n = R/r$.

Correspondingly, the effect of the free vortex itself is to make the pressure head at the outside of the bend exceed that at the inside by an amount $(v_i^2 - v_0^2)/2g$.

5.3.11 Losses at valves

These depend, of course, upon the relative size and design, but the order of magnitude involved may be judged from Gibson's experiments[13] for:

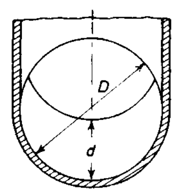


Figure 5.25 Circular sluice gate valve

(1) Circular sluice gate valves (Figure 5.25):

$$\text{Loss} = F(v^2/2g) \text{ where } v = \text{velocity in pipe} \tag{5.34}$$

Table 5.8

	F for d/D						
	0.2	0.3	0.4	0.5	0.6	0.8	1.0
$D=$ 50 mm	30	11	4.2	2.1	0.9	0.22	0
$D=$ 600 mm	36	11	3.0	1.6	1.0	—	—

(2) Butterfly valve (Figure 5.26):

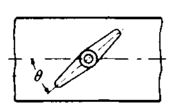


Figure 5.26 Butterfly valve

Table 5.9

θ	5°	10°	20°	30°	40°	50°	60°	70°
F	0.24	0.52	1.54	3.9	10.8	32.6	118	751

5.3.12 Graphical representation of pipe-flow problems

This may be illustrated by the example of a pipeline joining two reservoirs (Figure 5.27), and for the sake of the example we will suppose that the pipe is enlarged somewhere along its length. Consider a particle of water to find its way, in effect, from the surface in the upper reservoir to that in the lower. The velocity of fall of the upper surface, or of rise of the lower one, may be treated as negligible in comparison with the velocity through the pipe.

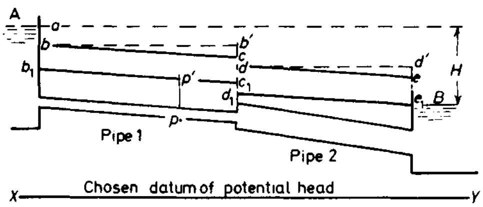


Figure 5.27 Graphical representation of pipe flow

Accordingly, the particle starting in surface A has originally no kinetic head. Taking atmospheric pressure as the datum of reference for pressure energy, it has also no pressure head. Its head is, in fact, entirely potential and may be represented diagrammatically by the height of A above any arbitrarily chosen datum of potential, XY. As the water enters the pipe there is a loss ab due to the sudden contraction: this is followed by a friction loss $b'c$ in pipe 1. Then comes the loss cd caused by the enlargement and this is followed by the friction loss $d'e$ in the second length of pipe.

The height of the line $abcde$ above XY therefore represents the total head.

Now drop bc by a distance bb_1, representing the kinetic head $v_1^2/2g$ in pipe 1, and de by a distance dd_1, representing the kinetic head $v_2^2/2g$ in pipe 2.

The height of $ab_1c_1d_1e_1$ above XY must then represent total head minus kinetic head, i.e. pressure plus potential head. $b_1c_1d_1e_1$ is then the line of virtual slope or hydraulic gradient of the pipe, and its height above the centreline of the pipe gives the pressure head in the pipe. thus, Pp' is the pressure head at P. If p' were below P, it would follow that the pressure in the pipe at P was below atmospheric and one of the advantages of this graphical method is to reveal clearly such points of suction.

Further, considering the bottom end of the pipeline, if the pipe 2 has a sharp, or flanged, connection to the reservoir, the whole of the kinetic head $v_2^2/2g$ will be lost in the creation of eddies at the enlargement. The point e_1 will then define the surface level in the lower reservoir, the total head under which flow is taking place being H. With a gradual enlargement, joining in a curved bell mouth to the lower reservoir, a proportion of $v_2^2/2g$, up to possibly $\frac{3}{4}(v_2^2/2g)$ might be regained and the level in B would be correspondingly higher than e_1 by this, say, $\frac{3}{4}(v_2^2/2g)$ amount of reconverted kinetic head.

5.3.13 Pipes in parallel

Consider two reservoirs connected by three pipes as shown in Figure 5.28. Pipe 1 is of diameter d_1, length l_1. Pipe 2 has diameter d_2, length l_2. Pipe 3 is of diameter d_3, length l_3. H is the loss of head in any one of the pipes. Neglecting end-effects:

$$H=\frac{4f_1l_1v_1^2}{2gd_1}=\frac{4f_2l_2v_2^2}{2gd_2}=\frac{4f_3l_3v_3^2}{2gd_3} \qquad (5.35)$$

or, in the Manning form:

$$H=\frac{A_1l_1v_1^2}{d_1^{4/3}}=\frac{A_2l_2v_2^2}{d_2^{4/3}}=\frac{A_3l_3v_3^2}{d_3^{4/3}} \qquad (5.36)$$

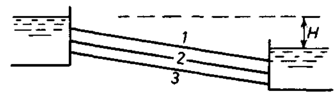


Figure 5.28 Pipes in parallel

The total rate of flow along the pipes:

$$=v_1(\pi d_1^2/4)+v_2(\pi d_2^2/4)+v_3(\pi d_3^2/4)+\ \ldots \qquad (5.37)$$

5.3.14 Siphons

Consider two reservoirs connected by a siphon pipeline as shown in Figure 5.29a, Figure 5.29b being the diagrammatic equivalent.

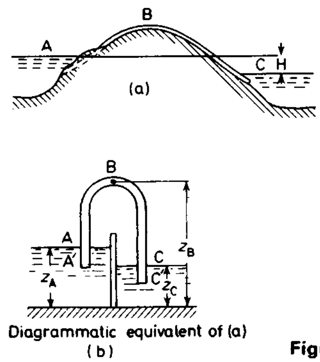


Figure 5.29 Siphon

If v is the velocity through the siphon, then H= loss at entry + friction loss + loss at exit, i.e.:

$$H=0.80(v^2/2g)+\text{friction loss}+(v^2/2g)$$

for submerged sharp entrance and exit ends of pipe. The end-effects are negligible in a reasonably long pipe.

Let F_1 be the friction loss of head in portion A'B of length l_1. Then pressure head at crown B is:

$$p_B/\rho g=z_A-z_B-\left(F_1+0.80\frac{v^2}{2g}+\frac{v^2}{2g}\right) \qquad (5.38)$$

assuming a loss of $0.80(v^2/2g)$ at A'.

Hence p_B is negative, i.e. less than atmospheric.

In practice, to avoid undue difficulty arising from accumulation of air and vapour at B, the numerical value of $p_B/\rho g$ should not exceed 8.5 m of water, an absolute pressure of about 2 m of water being then left at the crown.

The analysis giving Equation (5.38) assumes that the pressure over the pipe-section at B is uniform. If the curvature of the pipe is pronounced, however (as it frequently is in the case of siphon-spillways), the free-vortex phenomenon mentioned earlier (section 5.3.10) becomes important. The difference of pressure between the inside and the outside of the bend has already been given for a *circular section*. In the case of a *rectangular section* having a breadth b, an inside radius r_i and an outer radius r_o, the pressure head at the outside of the bend exceeds that at the inside by an amount, due to the free vortex alone:

$$K^2/2g[(1/r_i^2)-(1/r_o^2)]$$

where $K=Q/b \ln r_o/r_i$ and Q is the volume flowing per second.

Difficulties may arise, therefore, in bends of severe curvature, through high suction at the inside of the bend.

5.3.15 Nozzle at the end of a pipeline (Figure 5.30)

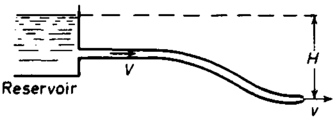


Figure 5.30 Nozzle at end of pipeline

Let H be the total head (in metres), V the velocity in the pipe (in metres per second), v the velocity in the jet (in metres per second), D the diameter of the pipe (in metres) and d the diameter of the nozzle end (in metres):

Discharge $= v(\pi d^2/4)$ m³/s. Then $v=V(D/d)^2$

Effective head behind nozzle $= H -$ loss at entrance to pipe $-$ friction in pipe. Hence $v^2/2g = H -$ friction head in pipe, neglecting entrance effect, or, more precisely:

$$v=C_N[2g(H-F)]^{1/2} \tag{5.39}$$

where F is the friction loss of head in pipe and C_N the coefficient of the nozzle.

Although C_N depends upon the Reynolds number vd/ν, for practical purposes it is usually sufficiently accurate to give it the value 0.98, either for elaborately streamlined nozzles or for a straight taper form ending in a cylindrical portion of length $d/2$ and diameter d.

Writing F in the form $F=4flV^2/2gD$:

$$v^2=\frac{2gC_N^2H}{1+[4flC_N^2/D(D/d)^4]} \tag{5.40}$$

The energy delivered in the jet $=\rho av^3/2$ N m/s, where ρ is the density of water ($=1000$ kg/m³) and $a=\pi d^2/4$.

This energy is a maximum, for a given H, if:

$$d^2=\frac{D^2}{C_N}\sqrt{(D/8fl)} \tag{5.41}$$

The velocities are then such that the friction head F is very nearly equal to $H/3$.

5.3.16 Multiple pipes

Consider the example shown in Figure 5.31, in which it is desired to calculate the flow along the three pipes.

Height of A, B, C, J above some chosen datum of potential

$= z_A, z_B, z_C, z_J$ respectively.

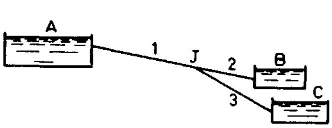


Figure 5.31 Multiple pipes

Neglecting losses other than those due to pipe friction F:

$$z_A=(p_J/\rho g)+(v_1^2/2g)+z_J+F_1 \tag{5.42}$$

$$z_B=(p_J/\rho g)+(v_2^2/2g)+z_J-F_2 \tag{5.43}$$

$$z_C=(p_J/\rho g)+(v_3^2/2g)+z_J-F_3 \tag{5.44}$$

Also:

$$v_1(\pi d_1^2/4)=v_2(\pi d_2^2/4)+v_3(\pi d_3^2/4) \tag{5.45}$$

Calling:

$$F_1=(4f_1l_1v_1^2/2gd_1);\quad F_2=(4f_2l_2v_2^2/2gd_2);\quad F_3=(4f_3l_3v_3^2/2gd_3)$$

and assigning values to f_1, f_2, f_3, Equations (5.42) to (5.45) are sufficient for the determination of the four unknowns p_J, v_1, v_2, v_3, and if necessary the solutions may be modified by further calculation should the resulting v suggest values of f_1, f_2, f_3 different from those originally assumed.

The solution of (in this example) four simultaneous equations is, however, cumbersome and full of possibilities of arithmetical slips. A simpler and quicker method[14] is as follows.

If *assumed* total head difference between two ends of a pipe $= h$, then:

$$h=\frac{4flv^2}{2gd}=\frac{4fl}{2gd}\left(\frac{Q}{a}\right)^2=KQ^2 \tag{5.46}$$

where Q is the rate of flow $= va$, a is area of section $=\pi d^2/4$, and $K=2fl/gda^2$.

If the correct values of h and Q are $h+\delta h$ and $Q+\delta Q$, then $\delta h=2h\cdot\delta Q/Q$ to a first approximation.

Now the error δQ for any one pipe is unknown, but the sum of the errors, $\Sigma(\delta Q)$, is known, being equal to the unbalanced flow, and:

$$\delta h=\Sigma(\delta Q)/\Sigma(Q/2h) \tag{5.47}$$

Example 5.7. $z_A=30.50$; $z_J=12.20$; $z_B=6.10$; $z_C=3.05$, all in metres.

(These might be levels with reference to ordnance datum, say.)

Pipe 1 3.0 km long, 600 mm diameter, $f=0.007$
Pipe 2 1.5 km long, 300 mm diameter, $f=0.007$
Pipe 3 1.5 km long, 300 mm diameter, $f=0.007$

Let:

$$H=(p/\rho g)+(v^2/2g)+z$$

Steps in solution

(1) Assign some value to the total head at J. Evidently it must be less than that at A, which is 30.5 m. Hence we might first try $H_J=20$ m, say.

(2) It then follows that:

$$F_1=(4\times 0.007\times 3000v_1^2)/(2\times 9.81\times 0.600)=30.5-20.0=10.5$$

Hence:

$v_1=\sqrt{(1.47)}=1.21$ m/s
$Q_1=0.342$ m³/s

(3) Similarly, $F_2 = 20.0 - 6.1 = 13.9$
therefore:

$$\frac{4 \times 0.007 \times 1500 v_2^2}{2 \times 9.81 \times 0.300} = 13.9$$

$$v_2 = 1.40 \text{ m/s}$$

$$Q_2 = 0.099\,0 \text{ m}^3/\text{s}$$

$$F_3 = 20.0 - 3.05 = 16.95$$

therefore:

$$\frac{4 \times 0.007 \times 1500 v_3^2}{2 \times 9.81 \times 0.300} = 16.95$$

$$v_3 = 1.54 \text{ m/s}$$

$$Q_3 = 0.109 \text{ m}^3/\text{s}$$

Hence, our original assumption (that total head at J = 20 m) has led to an out-of-balance flow of $0.342 - (0.099\,0 + 0.109)$, or $0.134 \text{ m}^3/\text{s}$. Hence:

$$\Sigma\delta Q = 0.134$$

(4) But $Q/2h$ for pipe $1 = 0.342/(2 \times 10.5) = 0.016\,3$

therefore $\Sigma(Q/2h) = 0.023\,1$

But $Q/2h$ for pipe $2 = 0.099\,0/27.8 = 0.003\,57$

But $Q/2h$ for pipe $3 = 0.109/33.9 = 0.003\,22$

so that $\Sigma\delta Q/[\Sigma(Q/2h)] = 0.134/0.023\,1 = 5.80$

(5) Now it is evident that we must aim at decreasing our original estimate of Q_1 while increasing those of Q_2 and Q_3.
We now try total head at J: $20.0 + 5.8 = 25.8$ m:

Our new estimate of

$$Q_1 = 0.342 \times \sqrt{\left(\frac{30.5 - 25.8}{30.5 - 20.0}\right)} = 0.229 \text{ m}^3/\text{s}$$

Our new estimate of $Q_2 = 0.099\,0 \times \sqrt{\left(\frac{19.7}{13.9}\right)} = 0.118 \text{ m}^3/\text{s}$

Our new estimate of $Q_3 = 0.109 \times \sqrt{\left(\frac{22.75}{16.95}\right)} = 0.126 \text{ m}^3/\text{s}$

and:

$$\Sigma\delta Q = 0.118 + 0.126 - 0.229 = 0.015$$

$Q/2h = 0.229/9.40 = 0.0244$ for pipe 1

$Q/2h = 0.118/39.4 = 0.0030$ for pipe 2

$Q/2h = 0.126/45.5 = 0.0028$ for pipe 3

Therefore:

$$\Sigma(Q/2h) = 0.030\,2$$

$$\frac{\Sigma\delta Q}{\Sigma(Q/2h)} = 0.015/0.030\,2 = 0.497$$

(6) Now make total head at J: $25.8 - 0.5 = 25.3$:

$$Q_1 \text{ then} = 0.342 \times \sqrt{(5.20/10.5)} = 0.241$$

$$\left.\begin{array}{l} Q_2 \text{ then} = 0.099\,0 \times \sqrt{(19.2/13.9)} = 0.116 \\ Q_3 \text{ then} = 0.109 \times \sqrt{(22.25/16.95)} = 0.125 \end{array}\right\} 0.241$$

The flows are now in balance; the number of steps required in the process of successive approximation depends on the accuracy of the original guess at the total head of J.

Having obtained a sensibly accurate balance, we may if we choose carry out refined calculations based upon more acceptable values of f and including losses due to other causes such as the junction J, any bends, and so forth.

The example considered above is, however, comparatively simple. The more complicated cases frequently encountered in practice are nowadays analysed with the aid of analogue or digital computers.[15–18]

5.3.17 Flow measurement in pipes

5.3.17.1 The Venturi meter

The proportions shown in Figure 5.32 are not essential, but are fairly representative; the gently tapering divergent portion following the convergent tube (which is the real meter) is intended to minimize the overall loss of head due to eddies and friction between 1 and 3.

$$Q = \text{rate of flow} = va = C(\pi d^2/4)\left[\frac{2g(h_1 - h_2)}{(a_1/a_2)^2 - 1}\right]^{1/2} \tag{5.48}$$

where h_1 $(= p_1/\rho g)$ and h_2 $(= p_2/\rho g)$ are the pressure heads at sections 1 and 2 respectively.

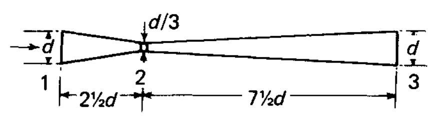


Figure 5.32 Venturi meter

If the throat diameter is too small, the pressure p_2 may be so low as to encourage release of air and vapour which will cause the flow to fluctuate and will introduce an uncertainty. To avoid this, it is advisable to use proportions which will not cause p_2 to be less than 21×10^3 N/m²); i.e. h_2 not less than 2.1 m of water (absolute).

Values of C. The value of C, the coefficient of the meter, is influenced by the Reynolds number and for the sort of designs of meter commonly used in practice, the following are approximately correct values[19] (Table 5.10).

5.3.17.2 Pipe orifice as meter

The pipe orifice as a measuring device is conveniently installed at a flange joint but has the disadvantage of creating an appreciable obstruction and consequent loss of head. In Figure 5.33, *a* and *b* are pressure tappings:

$$Q = CA\sqrt{2gh/[(D/d)^4 - 1]} \tag{5.49}$$

Table 5.10

Reynolds number vd/v as measured at throat	*C*
2 000	0.91
6 000	0.94
10 000	0.95
100 000	0.98
1 000 000	0.99

where $A = \pi D^2/4$, and h is the pressure head difference between points a and b. C is of the order of 0.61 for values of $(d/D)^2$ between 0.3 and 0.6, but it should be noted that the accuracy of the machining is of great importance, since the quantity $(D/d)^4$ occurs in the formula. Similarly, it is important to have a high degree of accuracy in the measurement of the diameter D of the pipe in which the plate is installed.

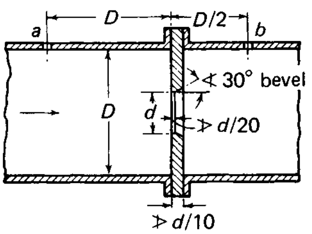


Figure 5.33 Pipe orifice as a meter

If a well-shaped convergent nozzle is used instead of a sharp-edged orifice plate, C has a value of 0.98 to 0.99 if $(d/D)^2$ does not exceed 0.2.

5.3.17.3 General notes on meters

The pressure holes used for meters or other purposes should be finished flush with the inside of the pipe. A reasonable length of straight pipe should precede the meter, and, though less important, should follow the meter.

For laboratory purposes it is always most satisfying to calibrate any meter *in situ* by comparison with the collection of a known weight of water in a measured time, but considerable accuracy may be expected from observance of the recommendations in the British Standard Code,[20] BS 1042:1943, which covers Venturi tubes, orifice plates and nozzles, and pitot tubes: it deals with gases as well as liquids. The US Standard[21] is *ASME Fluid Meters Report.*

5.3.18 Water hammer in pipes

If a valve is closed suddenly, successive masses of the water in the pipe are brought to rest; their kinetic energy is converted to strain energy and the effect is transmitted along the pipe with the velocity of sound waves in water. Some energy is, in fact, expended in stretching the pipe walls, thus reducing the water hammer pressure, but if this effect is neglected the rise of pressure p at the valve is given by:

$$p = v\sqrt{(K\rho)} \quad (\mathrm{N/m^2}) \tag{5.50}$$

where v is the velocity of flow before the valve is closed (in metres per second), K the bulk modulus of compressibility of the water, equal to about 2×10^9 N/m² and ρ is the density of water, 1000 kg/m³.

This formula leads to the result:

$$\text{Water hammer pressure} = 1.4 \times 10^6 v \quad (\mathrm{N/m^2}) \tag{5.51}$$

where v is the original velocity of flow (in metres per second).

Pressures of this order of magnitude will result if a valve is closed in a time not exceeding $2l/V_p$ s, where $V_p = \sqrt{(K/\rho)}$ is the velocity of sound waves in the water and where l is the length of pipe (in metres). In round numbers, V_p may be taken as 1400 m/s.

If the time of closure exceeds $4l/V_p$ the stoppage becomes gradual. Supposing the valve to be then closed in such a manner as to cause a constant retardation α m/s² of the water column, the resulting rise of pressure will be $\rho l\alpha$ N/m², or $l\alpha/g$ m head of water.

5.3.19 Flow in open channels

5.3.19.1 Formulae for open channels

Consider the portion of an open channel shown in Figure 5.34. AB represents the surface of a stream: section A is distance l along the channel, section B a distance $l + \delta l$ along. h is the depth at A, $h + \delta h$ the depth at B. v is the mean velocity at A, r the depth of surface at A below some arbitrary datum and $(r + \delta r)$ the depth of surface at B below the same datum. m is the hydraulic mean depth, equal to the ratio of area to wetted perimeter, and f is the friction coefficient. Then:

$$\mathrm{d}r/\mathrm{d}l = (v/g)(\mathrm{d}v/\mathrm{d}l) + fv^2/2gm \tag{5.52}$$

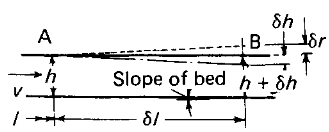


Figure 5.34 Portion of open channel

If the flow is uniform and the mean velocity constant from section to section along a channel of constant cross-section, this assumes the familiar form:

$$i = \text{slope of bed} = \text{slope of water surface}$$

$$= \text{fall per unit length}$$

$$= fv^2/2gm \tag{5.53}$$

alternatively, as in the Chézy equation, Equation (5.53) can be written:

$$v = c\sqrt{(mi)} \tag{5.54}$$

where c is known as the Chézy coefficient and is related to the friction coefficient f in the formula:

$$\text{friction head} = flv^2/2gm \quad \text{by} \quad c^2 = 2g/f \tag{5.55}$$

The numerical value of c depends on the units adopted; it has the units of $[L^{\frac{1}{2}}]/[T]$. Consequently, in the metre second system it is measured in metres$^{\frac{1}{2}}$ per second. On the other hand, f is dimensionless; it has the same numerical value in either system.

Chézy's c depends upon the nature of the channel and also upon the hydraulic mean depth of a channel of given material.

To some extent it also depends upon the mean velocity of the stream, although in most practical examples of open channel flow with which the engineer is concerned, this effect is of minor importance.

Although old fashioned in the sense that it dates back to the last century, a formula due to Bazin[22] is very reliable:

$$c = 158/(1.81 + N/\sqrt{m}) \quad (m^{\frac{1}{2}}/s) \tag{5.56}$$

the hydraulic mean depth m being measured in metres and N having the values given in Table 5.11.

Table 5.11

Class	*N*	*Application*
I	0.109	Smoothed cement or planed wood
II	0.290	Planks, bricks or cut stone
III	0.833	Rubble masonry
IV	1.54	Earth channels of very regular surface, or revetted with stone
V	2.35	Ordinary earth channels
VI	3.17	Exceptionally rough earth channels (bed covered with boulders) or weed-grown sides

As one of the many proposed alternatives to the Chézy–Bazin treatment ($v = c\sqrt{(mi)}$ where $c = [158/(1.81 + N/\sqrt{m})]$, the formula due to Manning[7] is much favoured and regarded by many as more convenient, though giving much the same result.

For the classes of channel already described in Table 5.11 in connection with Bazin's N, Manning's n may be taken as given in Table 5.12 and in Equation (5.23): $v = (m^{2/3}i^{1/2})/n$ (m/s) where m is in metres.

With rather more precision, the values of n given by Parker[23] are quoted in Table 5.13.

Table 5.12

Class	*n*
I	0.009 3
II	0.012 9
III	0.018 2
IV	0.022 5
V	0.025 8
VI	0.028 4

Table 5.13

Nature of channel	*n*
Timber, well planed and perfectly continuous	0.009
Planed timber, not perfectly true	0.010
Pure cement plaster	0.010
Timber, unplaned and continuous; new brickwork	0.012
Rubble masonry in cement, in good order	0.017
Earthen channels in faultless condition	0.017
Earthen channels in very good order or heavily silted in the past	0.018
Large earthen channels maintained with care	0.0225
Small earthen channels maintained with care	0.025
Channels in order, below the average	0.0275
Channels in bad order	0.030

Note that by comparing $v = c\sqrt{(mi)}$ with $v = (m^{2/3}i^{1/2})/n$ we may obtain the result:

$$c = m^{1/6}/n$$

or:

$$f = 2g/c^2 = 19.6n^2/m^{1/3} \tag{5.57}$$

Incidentally the formula $c = 20.7 + 17.7 \lg m/k$ $m^{\frac{1}{2}}/s$ covers a remarkably wide range of both rough pipes and rough open channels.[24] Here k represents the size of roughening excrescences.

5.3.19.2 Form of channel for maximum v and Q

Q = rate of discharge (in cubic metres per second) = vA, where A is now the area of section (in square metres) and v is the mean velocity (in metres per second).

Adopting the Manning formula:

$$Q = vA = (Am^{2/3}i^{1/2})/n \tag{5.58}$$

for a given material of channel, and a given slope i, v is a maximum when m, or when A/P, is a maximum, P being the wetted perimeter.

For Q to be a maximum, however, $Am^{2/3}$ must be a maximum. Hence:

$$\left.\begin{array}{r} \text{for max } v,\ P\,dA - A\,dP = 0 \\ \text{for max } Q,\ 5P\,dA - 2A\,dP = 0 \end{array}\right\} \tag{5.59}$$

Examples 5.8 to 5.10.

Example 5.8: Rectangular channel (Figure 5.35).

$A = bd$; $P = b + 2d$. If A, n and i are fixed, maximum v and maximum Q will occur when $b = 2d$.

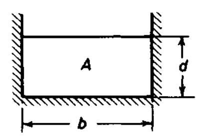


Figure 5.35 Rectangular channel

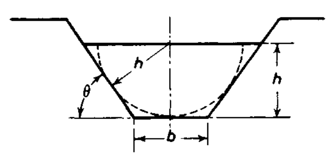


Figure 5.36 Trapezoidal channel

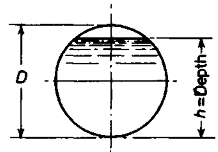


Figure 5.37 Circular channel

Example 5.9: Trapezoidal channel (Figure 5.36):
For maximum v and maximum Q with given A:

$$\sqrt{(1 + s^2)} = (b + 2sh)/2h$$

where $\tan\theta = 1/s$

This is satisfied if a semicircle can be drawn, centred in the water surface and touching both sides and bottom.

Example 5.10: Circular channel (Figure 5.37).
For maximum v, $h=0.813D$; for maximum Q, $h=0.938D$.

5.3.19.3 Resistance of natural river channels

This resistance is complicated by the losses of energy at bends and at relatively sudden changes of cross-sectional area. As these depend on the precise dimensions and shapes, it is quite impossible to generalize, but they are nevertheless important. For example, it has been shown that in a 13-km tortuous stretch of the River Mersey[25] the textural roughness of the bed and sides accounts for only 25 to 50% of the total loss of head depending on the rate of flow, the rest of the resistance being due principally to the bends.

Somewhat similar conclusions have been reached in a study of the River Irwell.[26]

5.3.19.4 Velocity distribution in open channels

Side and bottom friction cause the stream to be retarded. The highest velocity in any vertical at a particular section is usually found some distance below the surface; the mean velocity in a vertical line occurs at about 60% of the depth, whether the wind is blowing up- or downstream. This is the basis of one method of stream-gauging, in which the section is considered divided into strips of equal width and the velocity in these strips, or panels, is measured by current meter at 60% of the depth of each individual panel (Figure 5.38). The area of a strip, as found by sounding the bed, multiplied by the velocity so measured is assumed to give the flow through the strip and the addition for the total number of strips gives the flow through the whole section.

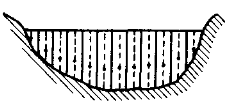


Figure 5.38 Measurement of velocity in open channel

5.3.19.5 Energy of a stream in an open channel, or 'specific energy'

If D is the depth and v the mean velocity, the energy head H_e, taking atmospheric pressure as the datum of pressure and the bottom of the channel as the datum of potential, is $D+v^2/2g$, or $D+Q^2/2gA^2$.

5.3.19.6 Critical depth

Critical depth is the depth at which maximum discharge occurs for a certain energy head, or, alternatively, the depth at which a given discharge takes place with the minimum energy head. It represents an unstable condition, often accompanied by water surface undulations. Under these conditions $D=v^2/g$ in a rectangular channel and Q then equals $(0.544b\sqrt{g})H_c^{3/2}$.

5.3.19.7 Nonuniform flow in open channels

If h is the depth at a distance l along the channel with slope i

$$\mathrm{d}h/\mathrm{d}l=\frac{i-(fv^2/2gm)}{1-(v^2/gh)} \tag{5.60}$$

This condition of varying depth, even in a stream of constant width and constant rate of discharge, as here assumed, may be brought about by obstructions or irregularities.

h is now the depth actually found at a particular section, whereas H may be called the depth which would apply under conditions of uniform flow corresponding to the simple Equation (5.54). In other words, h becomes equal to H if $\mathrm{d}h/\mathrm{d}l=0$.

Suppose now, in order to examine general trends, that the width of the stream is large compared with its depth. In such a case the hydraulic mean depth m is approximately the same as h, at any rate in channels having approximately uniform depth across their width. Then:

$$\mathrm{d}h/\mathrm{d}l=\frac{i-(fv^2/2gh)}{1-(v^2/gh)}$$

and $Q=vbh$. Q is also equal to VbH, where V and H are the velocity and depth which would be obtained with uniform flow.

It then follows that:

$$\mathrm{d}h/\mathrm{d}l=\frac{i[1-(H/h)^3]}{1-(2i/f)(H/h)^3} \tag{5.61}$$

5.3.19.8 Special cases of nonuniform flow

(1) *Sluice gate in channel with small slope and/or rough bed*
$2i/f<1$, $h^3<(2i/f)H^3$ (Figure 5.39). $\mathrm{d}h/\mathrm{d}l$ becomes infinite when

$$h=(2i/f)^{1/3}H \tag{5.62}$$

A 'hydraulic jump' then results.

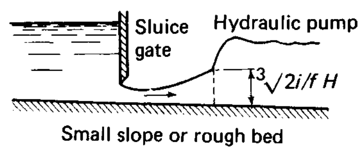


Figure 5.39 Sluice gate in channel, small slope and/or rough bed

(2) *Sluice gate in channel with steep slope and/or smooth bed*
$2i/f>1$, $h<H$. $\mathrm{d}h/\mathrm{d}l$ is again positive, but tends to zero, i.e. the depth increases gradually until it reaches that appropriate to uniform flow (Figure 5.40).

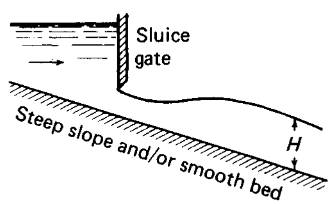


Figure 5.40 Sluice gate in channel, steep slope and/or smooth bed

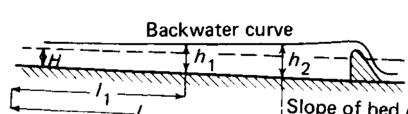


Figure 5.41 Weir or dam in channel, small slope and/or rough bed

(3) *Weir or dam in channel with small slope and/or rough bed*
$2i/f<1$, $h>H$ (Figure 5.41):

$$h_1-h_2=i(l_1-l_2)+H[1-(2i/f)][\Phi(y_1)-\Phi(y_2)] \tag{5.63}$$

where $\Phi(y)=$ backwater function $=-\int \mathrm{d}y/(y^3-1)$, and $y=h/H$.

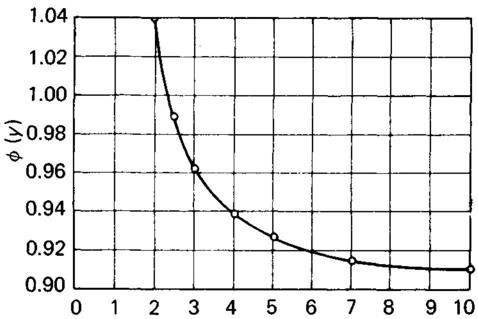


Figure 5.42 Backwater function

For values of the backwater function, see Figure 5.42.

Specific values are tabulated below:

y	1.000	1.005	1.010	1.015	1.020	1.050	1.100
$\Phi(y)$	∞	2.555	2.326	2.192	2.098	1.803	1.587
y	1.200	1.500	1.800	2.000	2.500	5.000	10.000
$\Phi(y)$	1.387	1.162	1.073	1.039	0.989	0.927	0.911

Example 5.11. The following example illustrates the application of the backwater function. A dam is built across a stream which was previously flowing with a depth of 1 m. The effect of the dam is to raise the level just behind it by 4 m. The slope of the bed is 1:2000 and $f=0.01$. What is the effect of the dam on the levels upstream? (Figure 5.43.)

$i=1/2000$; $2i/f=1/10$; $H=1$ m; $H(1-2i/f)=0.9$
$h_2=$ depth behind dam $=5$ m
$y_2=h_2/H=5$; $\Phi(y_2)=0.927$

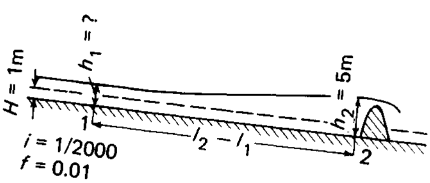


Figure 5.43 Application of backwater function

Therefore from Equation (5.63):

$$l=2000[4.166-h_1+0.9\Phi(y_1)] \tag{5.64}$$

where $l=l_2-l_1$; $h_1=$ depth at distance l from dam; $y_1=h_1/H=h_1/1$.

Now assign values to h_1 and calculate l from Equation (5.64). This process gives:

h_1 (m)	5	4	3	2	1.5	1.2
l (m)	0	2020	4070	6200	7420	8430

Inspection of the data given for this example will show that the water surface is virtually horizontal over the length extending 2000 m above the dam.

Although this theory of backwater is based upon the assumption of a rectangular channel of great width compared with its depth, and of f independent of depth, it nevertheless gives reasonably accurate results in practical cases provided that the value of H for the actual channel is known. Thus Gibson[27] has applied it to a circular conduit, in which the central depth was increased by a weir from its normal value H of 1.04 m to 1.67 m in the vicinity of the weir.

5.3.19.9 Flow over a horizontal bed

In this case, the slope i of the bed is zero and the simple Chézy Equation (5.54) ceases to be applicable. Instead of being uniform, the depth h decreases in the direction of flow and the fall of the water surface provides for the head required to overcome friction together with that needed to increase the kinetic head.

Considering such a rectangular channel of constant breadth b and writing $i=0$ in Equation (5.60):

$$\mathrm{d}h/\mathrm{d}l=-\left[\frac{(fv^2/2gm)}{1-(v^2/gh)}\right]$$

Let $Q=$ (constant) discharge (volume/s) supplied to the channel.

Then $v=Q/bh$

and:

$$\mathrm{d}h/\mathrm{d}l=f/[2gm(b^2h^2/Q^2-1/gh)]$$

If $Q_1=$ discharge per unit width:

$$\mathrm{d}h/\mathrm{d}l=-f/[2gm(h^2/Q^2-1/gh)]$$

Substituting $m=bh/(b+2h)$ and integrating between distances l_1 and l_2 along the channel in the direction of flow:

$$4Q_1^2fl/bg=4Q_1^2/g+b^3/2)\log_e\alpha-4/3(h_2^3-h_1^3)$$
$$+b(h_2^2-h_1^2)-b^2(h_2-h_1) \tag{5.65}$$

where $\alpha=(2h_2+b)/(2h_1+b)$ and $l=l_2-l_1$.

Even this complicated result assumes that f is constant all along the channel, whereas strictly f is influenced by the depth (which changes). More refined calculations may be made by considering small lengths with an adjustment for f from one to the next.

In the case of a comparatively broad channel, $m\rightarrow h$,

and:

$$\mathrm{d}h/\mathrm{d}l=-f/2g(h^3/Q_1^2-1/g)$$

from which:

$$l_2-l_1=\frac{1}{f}\left[\frac{g(h_1^4-h_2^4)}{2Q_1^2}-2(h_1-h_2)\right] \tag{5.66}$$

In practical cases, h_2 is often known from the conditions imposed at the end of the channel by, say, a weir. Another possibility is that the bed of the channel drops sharply at its end and the stream then issues as a waterfall. In such an example, the depth h_2 at or near the downstream end is $(Q_1/\sqrt{g})^{2/3}$, and by assuming a value for f, the depth h_1 at a distance l $(=l_2-l_1)$ upstream can be estimated. Alternatively, the distances l

upstream of h_2 may be found for a succession of chosen values of h_1.

5.3.19.10 The hydraulic jump (sometimes called 'standing wave')

An hydraulic jump is illustrated in Figure 5.44.

$$h_1 - h_2[(h_1 + h_2)/2 - (h_1 v_1^2/h_2 g)] + (h_2 - d/2)d = 0$$

or, in a practically horizontal channel ($d = 0$):

$$h_2 = -h_1/2 + [(h_1^2/4) + (2h_1 v_1^2/g)^{1/2}]$$

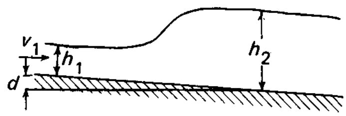


Figure 5.44 Hydraulic jump

and:

$$(h_2 - h_1) = [(h_1^2/4) + (2h_1 v_1^2/g)]^{\frac{1}{2}} - 3h_1/2 \tag{5.67}$$

For information concerning the length of channel covered in forming the jump, see Allen and Hamid.[28]

5.3.19.11 The Venturi flume

The flume is a device for measuring rates of flow in an open channel and is not so liable to damage as a weir and does not offer the same obstruction to the flow. In order to preserve its surface and its hydraulic characteristics, it is sometimes lined with stainless steel.

It may be formed by inserting 'streamlined' humps on the sides (Figure 5.45a) and/or the bed (Figure 5.45b) of the channel.

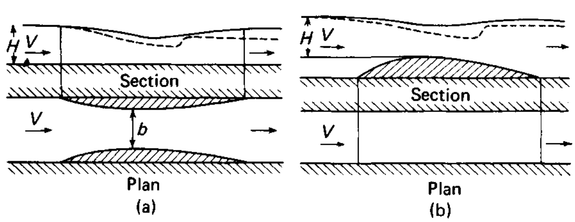


Figure 5.45 Venturi flume

If the discharge is 'free' as represented by the broken lines in Figure 5.45 and accompanied by the formation of a standing wave, the rate of discharge depends only upon the depth upstream of the constriction. With a throat of rectangular section and width b, Q is approximately $0.54g^{\frac{1}{2}}(H + V^2/2g)^{3/2}$, V itself of course depending upon H.

The general equation is:

$$Q = C_D\{b_2 d_2/[1 - (b_2 d_2/b_1 d_1)^2]^{\frac{1}{2}}\}[2g(d_1 - d_2)]^{1/2} \tag{5.68}$$

where b_2 is the breadth at throat, d_2 the depth at throat, d_1 the depth upstream of the constriction, b_1 the breadth upstream of the constriction, and C_D is the coefficient of discharge.

For particular designs, C_D is best found by scale-model experiments.

Details as to proportions, shapes and types of flow may be found in papers by Engel.[29] See also Elsden.[30]

5.3.20 Orifices

5.3.20.1 Sharp-edged orifice (Figure 5.46)

Velocity at *vena contracta* $= C_v\surd(2gh)$, where C_v is the coefficient of velocity, about 0.985.

Neglecting air resistance:

$$x^2 = 4yhC_v^2 = 3.88yh \tag{5.69}$$

Discharge $Q = C_v a_c \surd(2gh)$

$$= C_v C_c a \surd(2gh) \tag{5.70}$$

where C_c is the coefficient of contraction, or:

$$Q = C'a\surd(2gh) \tag{5.71}$$

where a is the area of the orifice and C' is the coefficient of discharge.

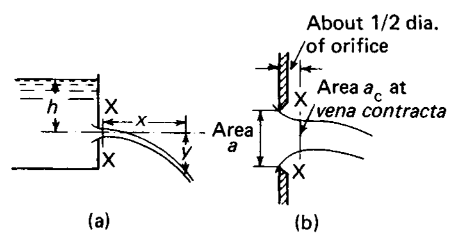


Figure 5.46 Sharp-edged orifice

Consideration of various published data[31] indicates that for orifices of 6.35 cm diameter or over, under heads of at least 0.43 m, $C' = 0.60$, provided $h \nless 3d$, where d = diameter of orifice. Other typical results are given in Table 5.14.

It is doubtful whether a third significant figure is of any value, as a 1% error in measuring the mean diameter of the orifice at once makes 2% difference to the computed discharge. The absolute sharpness of the edge must also have some bearing upon the results.

Table 5.14 Values of C' for sharp-edged orifice

Head, h	d *of circular orifice, or side of square orifice*					
(m)	0.64 (cm)	1.27 (cm)	2.54 (cm)	6.35 (cm)	15.2 (cm)	30.5 (cm)
0.12	—	0.64	0.63	0.61	—	—
0.18	0.66	0.64	0.62	0.61	0.60	—
0.31	0.65	0.63	0.62	0.61	0.60	0.60
0.61	0.63	0.62	0.62	0.61	0.60	0.60
1.22	0.62	0.61	0.61	0.61	0.60	0.60
3.05	0.61	0.61	0.61	0.60	0.60	0.60
15.24	0.60	0.60	0.60	0.60	0.60	0.60
30.48	0.60	0.60	0.60	0.60	0.60	0.60

The value of C' for a rectangular orifice appears to be somewhat higher than for a circular or square one of the same area. The difference amounts to about 2% for rectangles having a 4:1 ratio of sides and 4% for a ratio of 10:1.

5.3.20.2 Rounded or bell-mouthed orifice

For a design such as that shown in Figure 5.47, $C_c = 1$ and $C' = C_v$:

$$Q = 0.95(\pi d^2/4)\sqrt{(2gh)} \quad \text{to} \quad 0.99(\pi d^2/4)\sqrt{(2gh)} \qquad (5.72)$$

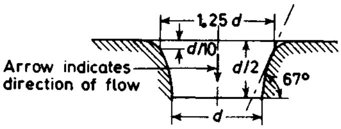


Figure 5.47 Rounded orifice

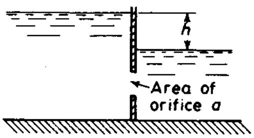


Figure 5.48 Submerged orifice

5.3.20.3 Submerged orifices

For a submerged orifice as shown in Figure 5.48:

$$Q = C'a\sqrt{(2gh)} \qquad (5.73)$$

where C' is substantially the same as for free discharge into the atmosphere.

5.3.20.4 Time of discharge through an orifice

The equation for time of discharge through an orifice from a tank (Figure 5.49), without any simultaneous inflow is:

$$dh/dt = C'(a/A)\sqrt{(2gh)}$$

or

$$t_2 - t_1 = -\frac{1}{C'a\sqrt{(2g)}}\int_{H_1}^{H_2} Ah^{-1/2}\,dh \qquad (5.74)$$

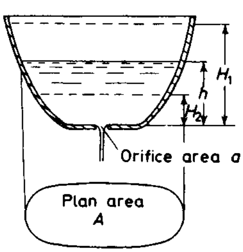


Figure 5.49 Discharge through an orifice

This can be solved if A can be expressed in terms of h, the instantaneous head at time t.

If A is constant, or independent of h:

$$[C'a\sqrt{(2g)}/A](t_2 - t_1) = 2(\sqrt{H_1} - \sqrt{H_2}) \qquad (5.75)$$

treating C' as independent of h.

The time of discharge through a submerged orifice (Figure 5.50) is given by the equation:

$$(t_2 - t_1) = \frac{2}{C'(1/A_1 + 1/A_2)a\sqrt{(2g)}}(\sqrt{H_1} - \sqrt{H_2}) \qquad (5.76)$$

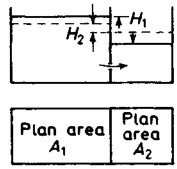


Figure 5.50 Discharge through submerged orifice

5.3.21 Weirs and notches

The term 'notch' is used for the smaller weirs common in laboratories, as distinct from outdoors.

5.3.21.1 Rectangular sharp-crested weir

For a rectangular weir as shown in Figure 5.51 in which $a \nless 4H$ and $c \nless 3H$, and H is measured at a distance 6 to 10 times H behind the weir, then:

$$Q = [0.410(2g)^{1/2}](b - H/10)H^{3/2} \qquad (5.77)$$

This formula is probably accurate within 2% for all values of H from 0.08 to 0.61 m provided $b/H > 2$ and provided b is $\nless 0.305$ m.

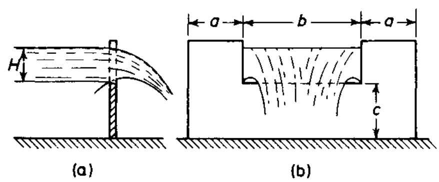


Figure 5.51 Rectangular sharp-crested weir

The effect of the velocity of the approaching stream is automatically allowed for in this formula, as in all others to be presented.

5.3.21.2 Suppressed rectangular weir

If a rectangular weir crest occupies the full width of the channel, the end contractions are suppressed. Under these conditions the 'nappe' or stream discharging over the crest, the sides of the channel and the front of the weir plate form the boundaries of a pocket of air, some of which may be dissolved in the turbulent mass of water at the foot of the weir on its downstream side and carried away (Figure 5.52). The effect of this would be to reduce the pressure below the nappe and, hence, to increase the

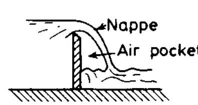


Figure 5.52 Suppressed rectangular weir

discharge for a given head. In itself this is no detriment but it introduces an element of uncertainty and of variation. To overcome this it is generally supposed that air vents should be provided through the sides of the channel in communication with the air-pocket with the object of maintaining atmospheric pressure and preserving a standardized condition.

Under such conditions (ventilated suppressed rectangular weir of height P m above the bed of the channel), the Rehbock formula[32] is perhaps accurate to within 1 or 2%. It reads:

$$Q = \tfrac{2}{3}\sqrt{(2g)}b\left(0.605 + \frac{1}{1050H - 3} + \frac{0.08H}{P}\right)H^{3/2} \quad (\text{m}^3/\text{s}) \qquad (5.78)$$

Writing this as:

$$Q = Cb\sqrt{(2g)}H^{3/2} \tag{5.79}$$

values of C are as given in Table 5.15.*

Table 5.15 Values of C

P (m)	H 0.06 (m)	0.15 (m)	0.31 (m)	0.46 (m)	0.61 (m)	0.91 (m)	1.22 (m)	1.52 (m)
0.15	0.436	0.461	0.512	0.565	0.617	0.724	0.831	0.936
0.31	0.425	0.434	0.459	0.486	0.511	0.564	0.617	0.672
0.61	0.421	0.422	0.433	0.446	0.458	0.484	0.510	0.537
0.91	0.418	0.417	0.423	0.431	0.440	0.458	0.475	0.492
1.52	0.416	0.413	0.416	0.421	0.426	0.436	0.446	0.457
3.05	0.415	0.411	0.411	0.412	0.416	0.421	0.425	0.431

5.3.21.3 The 90-degree vee-notch (sharp-edged)

Measuring the head with reference to the point v (Figure 5.53):

$$Q = 1.34H^{2.48} \quad (\mathrm{m^3/s}) \tag{5.80}$$

over a wide range, H being in metres.

This notch is more convenient than the rectangular form for the measurement of small quantities but it should be remembered that an error of 1% in the measurement of H means 2.5% in the resulting estimate of Q, whereas with a rectangular notch the corresponding error is 1.5%.

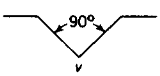


Figure 5.53 Sharp-edged 90° vee-notch

5.3.21.4 The Cippoletti weir (sharp-crested)

The discharge over this type of weir (Figure 5.54) is:

$$Q = 0.420\sqrt{(2g)}b[(H+h)^{\frac{3}{2}} - h^{3/2}] \quad \text{if } \tan\theta = \tfrac{1}{4} \tag{5.81}$$

where $h = v^2/2g$, v being the mean velocity in the approach channel.

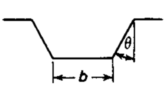


Figure 5.54 Sharp-crested Cippoletti weir

v cannot be allowed for until Q is known. Hence, as a first approximation, find Q from $Q = 0.420(2g)^{\frac{1}{2}}bH^{3/2}$. Then calculate $v = Q$/area of section of approach channel, and re-evaluate Q from Equation (5.81).

5.3.21.5 Weirs without sharp crests

Some typical examples are shown in Figure 5.55(a), (b), (c) and (d). The discharge $Q = C(2g)^{\frac{1}{2}}bH^{3/2}$

5.3.21.6 Submerged weirs

If the downstream level rises above the crest of the weir, a 'drowned weir' results. The effect of this upon the discharge for a given upstream head is surprisingly small: in general, the reduction in discharge will not amount to more than 2 or 3% for 'downstream heads' or submergences up to 20% of the upstream head.

* $Q = 4.43CbH^{1/2}\ \mathrm{m^3/s}$ if b and H are in metres.

5.3.21.7 Time of discharge over a weir (Figure 5.56)

The time of discharge over a weir or spillway of length b may be calculated as follows.

(1) *With no inflow*:

$$A\,\delta H = -C(2g)^{\frac{1}{2}}bH^{\frac{3}{2}}\,\delta t$$

therefore:

$$\int_{t_1}^{t_2} \mathrm{d}t = -\int_{H_1}^{H_2} \frac{A}{C(2g)^{\frac{1}{2}}b} H^{-\frac{3}{2}}\,\mathrm{d}H$$

Or, time for head to fall from H_1 to H_2 is:

$$(t_2 - t_1) = t = -\frac{1}{b}\int_{H_1}^{H_2} \frac{A}{C(2g)^{\frac{1}{2}}} H^{-\frac{3}{2}}\,\mathrm{d}H$$

This may be solved by splitting the change between H_1 and H_2 into stages over which mean values of A and C are applied.

If A and C are treated as constant:

$$t = \frac{2A}{bC(2g)^{\frac{1}{2}}}\left(\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right) \tag{5.82}$$

(2) *Reservoir with inflow as well as outflow* (Figure 5.56)

Let A be the surface area of reservoir, Q the inflow and h the instantaneous head over spillway of length b.

Then excess of inflow over outflow $= Q - C(2g)^{\frac{1}{2}}bh^{3/2}$. Hence:

$$\mathrm{d}h/\mathrm{d}t = [Q - C(2g)^{\frac{1}{2}}bh^{3/2}]/A$$

Let H be the head over spillway which would make the rate of outflow equal to the rate of inflow. Then:

$$Q = C(2g)^{\frac{1}{2}}bH^{3/2}$$

Let:

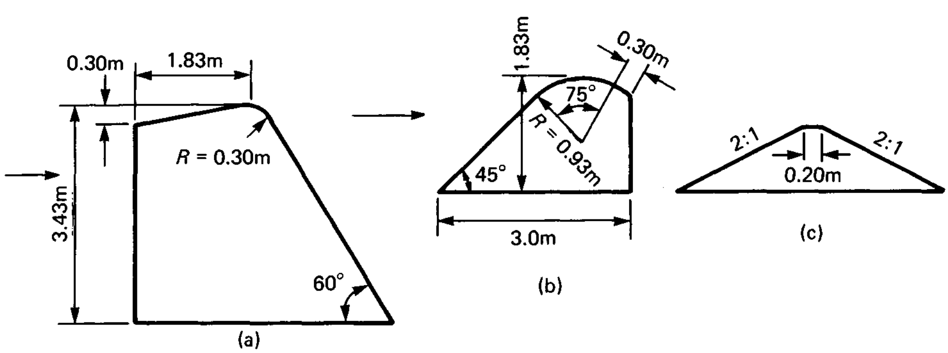
$$r = h/H \qquad \text{and} \qquad K_1 = C(2g)^{\frac{1}{2}}b.$$

The time taken for the head to change from h_1 to h_2 is given by:

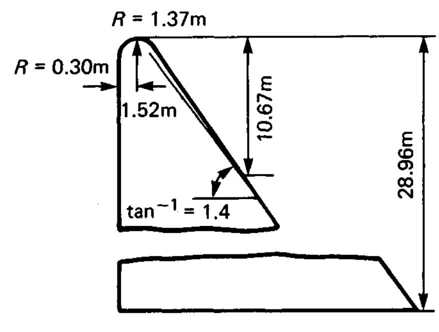
$$t_2 - t_1 = (A/K_1\sqrt{H})[\Phi(r_2) - \Phi(r_1)] \tag{5.83}$$

In this equation, $r_1 = h_1/H$, $r_2 = h_2/H$ and Φ represents Gould's function[33] of h/H, i.e. of r.

Detailed values of $\Phi(r)$ for use when the time-interval is expressed in the form given by Equation (5.83) (where b as well as $C\sqrt{(2g)}$ is absorbed in K_1) have been calculated by Mathieson.[34]



(a)		(b)		(c)		(d)	
H (m)	*C*	*H* (m)	*C*	*H* (m)	*C*	*H* (m)	*C*
0.15	0.400	0.15	0.402	0.15	0.392	0.61	0.368
0.31	0.426	0.31	0.407	0.31	0.426	0.76	0.379
0.46	0.441	0.46	0.424	0.46	0.439	0.91	0.384
0.61	0.442	0.61	0.431	0.61	0.450	1.07	0.387
0.91	0.411	0.91	0.445	0.91	0.456	1.22	0.386
1.22	0.392	1.22	0.455	1.22	0.456	1.37	0.384
						1.42	0.384



(d)

Figure 5.55 Weirs without sharp crests, with corresponding *H*, *C* values
See also *Hydraulics Research*, HMSO Annual Reports, e.g. 1964 p. 15 and 1965 p. 7 (the 'Crump' weir)

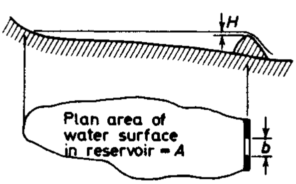


Figure 5.56 Discharge over a weir

When $r < 1$:

$$\Phi(r) = \frac{2}{3}\left\{ \ln\left[\frac{(r+\sqrt{r}+1)^{\frac{1}{2}}}{1-\sqrt{r}}\right] - \sqrt{3}\left[\tan^{-1}\left(\frac{2\sqrt{r}+1}{\sqrt{3}}\right) - \frac{\pi}{6}\right]\right\}$$

When $r > 1$:

$$\Phi(r) = \frac{2}{3}\left\{ \ln\left[\frac{(r+\sqrt{r}+1)^{\frac{1}{2}}}{\sqrt{r}-1}\right] - \sqrt{3}\left[\tan^{-1}\left(\frac{2\sqrt{r}+1}{\sqrt{3}}\right) - \frac{\pi}{2}\right]\right\}$$

Some of the values given by Mathieson are quoted below.

r	0	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000
$\Phi(r)$	0	0.1013	0.2076	0.3220	0.4482	0.5920	0.7615
r	0.7000	0.8000	0.9000	0.9900	1.0100	1.0200	1.0400
$\Phi(r)$	0.9729	1.2619	1.7414	3.2925	4.4948	4.0426	3.5795
r	1.0600	1.1000	1.5000	2.0000	5.0000	10.0000	∞
$\Phi(r)$	3.3202	2.9838	1.9708	1.5730	0.9155	0.6376	0

Example 5.12. A reservoir has a surface area of 2.5 km². It is provided with an overflow weir of length 25 m, $C = 0.400$. Initially there is a steady head of 0.25 m over the weir, but superimposed upon the discharge corresponding with this condition, additional flood water enters the reservoir as detailed below.

Time (h)	0	1	2	3	4	5	6	7	8
Additional inflow (m³/s)	0	10	35	50	40	20	10	0	−2.75

Investigate the variation of water level.

$$K_1 = Cb\sqrt{(2g)} = 0.4 \times 25.0 \times 4.43 = 44.3$$

$$A/K_1 = 2.5 \times 10^6/44.3 = 5.64 \times 10^4;\ 3600K_1/A = 0.0637$$

Initial inflow = initial outflow = $44.3(0.25)^{3/2} = 5.53$ m³/s.

First hour

Mean $Q = 5.53 + 5 = 10.53$ m³/s
therefore $H^{3/2} = 10.53/44.3 = 0.238$
and $H = 0.384$ m; $\sqrt{H} = 0.62$
$r_1 = h_1/H = 0.25/0.384 = 0.651$; $\Phi(r_1) = 0.863$
$3600(s) = (5.64 \times 10^4/0.62)[\Phi(r_2) - 0.863]$
$\Phi(r_2) = 0.903$; $r_2 = 0.67$
$h_2 = 0.67 \times 0.384 = 0.258$ m
= head over spillway at end of 1 h

Second hour

$$\text{Mean } Q = 5.53 + 22.5 = 28.03 \text{ m}^3/\text{s}$$
$$H^{3/2} = 28.03/44.3 = 0.634$$
$$H = 0.738 \text{ m}; \sqrt{H} = 0.859$$
$$r_1 = 0.258/0.738 = 0.35; \Phi(r_1) = 0.384$$
$$\text{therefore } 0.0637 = (1/0.859)[\Phi(r_2) - 0.384]$$
$$\Phi(r_2) = 0.438; r_2 = 0.392$$
$$\text{and } h_2 = 0.392 \times 0.738 = 0.289 \text{ m}$$
$$= \text{head at end of 2 h.}$$

Proceeding in this way, we obtain:

Hour	0	1	2	3	4	5	6	7	8
Head (m)	0.250	0.258	0.289	0.348	0.407	0.442	0.453	0.449	0.438

So the maximum head is 0.453 m at 6.10 h. This head would give a maximum *out*flow of $44.3(0.453)^{3/2}$, or $13.5 \text{ m}^3/\text{s}$, as compared with the maximum *in*flow of $55.5 \text{ m}^3/\text{s}$ at 3.10 h.

5.3.22 Impact of jets on smooth surfaces

5.3.22.1 *Single moving vane*

Let v be the absolute velocity of jet, u the absolute velocity of vane, assumed parallel to v, and a the area of section of jet (Figure 5.57).

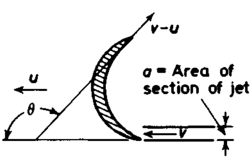


Figure 5.57 Impact of a jet on a single moving vane or series of moving vanes

Velocity of jet relative to vane $= v - u$.

Therefore mass striking vane $= \rho a(v-u)$.

This is unaltered in flow over the vane. (If roughness is taken into account, the final relative velocity $= k(v-u)$ where $k < 1$.)

Initial momentum per second of jet $= \rho a(v-u)v \leftarrow$

Final momentum per second of jet $= \rho a(v-u)[u + (v-u)\cos\theta] \leftarrow$

Therefore force exerted on vane $= \rho a(v-u)^2(1-\cos\theta) \leftarrow = x$, say

Work done on vane $= \rho a(v-u)^2(1-\cos\theta)u$

Initial kinetic energy of jet $= \rho a v^3/2$

$$\text{Therefore efficiency} = [2(v-u)^2(1-\cos\theta)u]/v^3 \qquad (5.84)$$

Initial momentum per second of jet, in $\uparrow$ direction $= 0$

Final momentum per second of jet, in $\uparrow$ direction $= \rho a(v-u)^2 \sin\theta$

Therefore force exerted on vane in direction $\downarrow = \rho a(v-u)^2 \sin\theta = y$, say

$$\text{Resultant force on vane} = \sqrt{(x^2 + y^2)} \qquad (5.85)$$

5.3.22.2 *Series of moving vanes*

Figure 5.57 also applies. Since successive vanes intercept the jet, the mass of water striking them per second now is $\rho a v$. ρ is again the density of the water.

Force exerted in direction $\leftarrow = \rho a v(v-u)(1-\cos\theta) = x$, say

Work done on vanes $= \rho a v u(v-u)(1-\cos\theta)$

$$\text{Efficiency} = 2u(v-u)(1-\cos\theta)/v^2 \qquad (5.86)$$

$$\text{Force exerted in direction } \downarrow = \rho a v(v-u)\sin\theta = y, \text{ say} \qquad (5.87)$$

Resultant force $= \sqrt{(x^2 + y^2)}$

5.3.22.3 *Cubical block resting on the bed of a stream (Figure 5.58)*

Let ρ' be the density of the material of the block (in kilograms per cubic metre), μ the coefficient of friction between the block and bed of stream and v the velocity of stream at height y above bed (in metres per second).

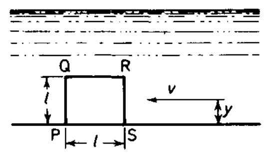


Figure 5.58 Impact of jet on cubical block resting on the bed of a stream

Resistance to overturning about P $= \{[(\rho' - \rho)l^4]/2\}g$

Resistance to sliding $= \mu(\rho' - \rho)l^3 g$

where μ is the coefficient of sliding friction.

Force of impact of stream against face RS $= K'\rho l \int_0^l v^2 \, dy$

where K' is a coefficient, ~ 0.70, to allow for the fact that not all the forward momentum of the stream is 'destroyed'.

Therefore, block will overturn if:

$$K' \int_0^l v^2 y \, dy > \frac{g(\rho' - \rho)l^3}{2\rho} \qquad (5.88)$$

or will slide if:

$$K' \int_0^l v^2 \, dy > \frac{\mu g(\rho' - \rho)l^2}{\rho} \qquad (5.89)$$

These results neglect other effects such as reduction of pressure on top and lee faces. They serve to show, however, that the stability of the block is essentially dependent upon the way in which the velocity is distributed through the depth of the stream.

Experiments in laboratory flumes indicate that:

$\bar{v}$ = mean velocity of stream for which block overturns (in metres per second)
= rate of discharge divided by (area of section of channel − area of section of block)

$$= \frac{L}{l}\left(5.52 - \frac{2930}{(32 + h/l)^2}\right)\left[\left(\frac{\rho' - \rho}{\rho}\right)l\right]^{1/2} \qquad (5.90)$$

where L is the length of block measured parallel to direction of current (in metres), l the depth of block (in metres), h the depth of water above top of block (in metres), ρ' the density of material of block (in kilograms per cubic metre) and ρ the density of water (in kilograms per cubic metre).

Example 5.13. A concrete cube 1 × 1 × 1 m of density 2400 kg/m³.

Total depth D (m)	*Depth over cube h* (m)	$\bar{v}$ (m/s)
2	1	3.34
3	2	3.53
4	3	3.70
6	5	3.99
11	10	4.56

For stability of more than one block, arranged in rows and tiers, or heaped at random, see Allen.[35]

5.3.22.4 Stability of flat beds and mounds of broken stone and sand

The paper just cited[35] suggests that, over a wide range including materials having 'equivalent cube lengths' of 0.098 to 26.2 mm and specific gravities of 2.016 to 3.89:

$$\bar{v}=k[(L+B)/2l]^{0.44}(\sigma'-\sigma)^{0.22}l^{0.22}M^{-0.22}\,h^{0.06}\,m^{0.22} \qquad (5.91)$$

where v is the mean velocity (in metres per second) of stream flowing over a flat bed, L is the average value of maximum length of individual grains, B the average value of maximum breadth of individual grains, l the length of side of a cube of the same weight as the average piece, σ' the specific gravity of material, σ the specific gravity of water (say unity), M the uniformity modulus of material, h the depth of stream, m the hydraulic mean depth and k is a constant.

k is equal to 0.067 for movement of the first few particles if L, B, l, h and m are all in millimetres.

The uniformity modulus M is defined as indicated in Figure 5.29.

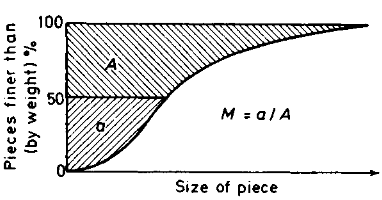


Figure 5.59 Definition of a uniformity modulus

Similarly, for mounds:

$$\bar{v}=0.079(\sigma'-\sigma)^{0.27}l^{0.27}\,M^{-0.27}\,h^{0.21}\,m^{0.02} \qquad (5.92)$$

for initial disturbance.

$$\bar{v}=0.083[(L+B)/2l]^{0.54}(\sigma'-\sigma)^{0.27}l^{0.27}\,M^{-0.27}\,h^{0.21}\,m^{0.02} \qquad (5.93)$$

for 'flattening the crest of the mound'.

Here $\bar{v}$ is the velocity (mean) of the stream flowing over the top of the mound and h the depth over the top (in millimetres).

Equations (5.92) and (5.93) apply to materials of irregular shape (sand or broken stone), but the case of mounds formed of *cubes* laid in random fashion has also been investigated by Allen.[35]

Example 5.14: 1 m cubes weighing 2400 kg so that $\sigma'=2.4$.
Example 5.15: Broken stone, equivalent cube length $l=1$ m, $\sigma'=2.64$, $M=0.9$

Depth over crest of mound laid in random fashion (m)	*v for initial movement* (m/s): 1 m *cubes*	*Broken stone* $l=1$ m, $M=0.9$
1	1.6	2.9
2	1.9	3.4
5	3.1	4.2
10	4.0	5.0

Note that σ', the true specific gravity, of silica or quartz sands is usually 2.64, or very nearly so.

5.3.23 Sediment transport

Considerable effort continues to be applied to the theoretical and experimental study of the transport of sand and suspended silt in both laboratory channels and natural waterways. One approach used for at least 100 years has been to try to relate the bed movement to the shear stress or tractive force per unit area exerted by the water on the bed of the channel, especially for the critical condition necessary to initiate motion of the grains. This stress is $\rho f v^2/2$, where f depends on the material which forms the bed and sides of the channel and on the hydraulic mean depth (see Equation (5.57)). The stress so defined has formed the basis of many theories and the means of interpreting observations of bed movement. The results cover a wide range of complexity, one of the most elementary being to write τ_c, the critical shear stress required to start movement of the sand, as proportional to the median diameter d_s of the grains. For example, $\tau_c=1.2d_s$, where τ_c is in newtons per square metre if d_s is in millimetres. While such a simple relationship makes no allowance for the shape of the grains, or the variation in their individual sizes, or any cohesion such as is present with fine particles, it nevertheless gives the right order of magnitude of τ_c if d_s is not less than about 0.25 mm. Thus, for $d_s=0.25$ mm, τ_c becomes 0.3 N/m². Equating this to $\rho f v^2/2$ and taking ρ for water as 1000 kg/m³, and supposing f to be of the order of 0.003 if the depth is, say, 1 m, it then follows that the mean velocity v in such a channel would be about 0.5 m/s. At this velocity, grains would begin to move.

Other researchers, such as G. M. White[36] and R. A. Bagnold,[37] have introduced a force of upward lift on a grain in addition to the longitudinal drag, while some methods of assessing the *quantity* of bed material transported by a given stream have been based on the amount by which the shear stress at the bed exceeds the critical shear stress τ_c. Yet another concept is to relate the sediment transport to the 'streampower' τ_v. More detailed accounts of these matters appear in many books and journals, e.g. a paper by P. A. Mantz.[38]

5.3.24 Vortices

5.3.24.1 Forced vortices Forced vortices are of the type caused by stirring a liquid in a dish or by rotating the dish (Figure 5.60).

$$h=(\omega^2r^2)/2g \qquad (5.94)$$

where ω is the angular velocity of rotation (rad/s).

5.3.24.2 Free vortices

Free vortices are of the type which results when a liquid flows

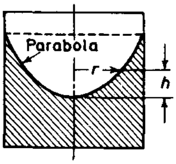


Figure 5.60 Forced vortex

through a hole in the bottom of a vessel. The water moves in stream lines spirally towards the centre, where an air-core tries to form. The coefficient of discharge through the orifice is greatly reduced as compared with its value in the absence of a vortex, i.e. with larger heads.

If the hole is now closed, the vortex motion persists for a time (until damped out by viscous resistance), the lower part having the characteristics of a forced vortex and the upper of a free cylindrical vortex (Figure 5.61):

$$c_1 - z = \text{constant}/2gr^2 \tag{5.95}$$

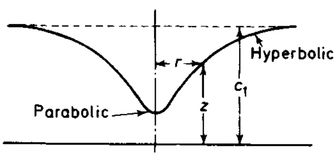


Figure 5.61 Free vortex

5.3.25 Waves

5.3.25.1 Waves of transmission or translation

Waves of transmission or translation are of the type formed when a sluice gate is suddenly opened to admit water to a channel, or when a tidal bore advances along a river.

Let h be the depth of the stream, v the velocity of the stream moving in the direction opposite to the wave, k the height of the wave crest above the surface of the stream and V the velocity of propagation of the wave. Then:

$$V = \left[\frac{2g(h+k)}{1+h/(h+k)}\right]^{1/2} - v \tag{5.96}$$

If k is small compared with h

$$V = [g(h+k)]^{1/2} - v$$

If k is very small compared with h

$$V = (gh)^{1/2} - v$$

5.3.25.2 Waves of oscillation

Waves of oscillation occur in comparatively deep water (Figure 5.62).

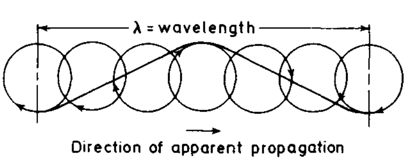


Figure 5.62 Waves of oscillation

Particles in the surface describe circular orbits, giving the appearance of a wave crest advancing with velocity $\sqrt{(g\lambda/2\pi)}$. The period of oscillation is $\sqrt{(2\pi\lambda/g)}$. In shallower water the orbits become distorted into approximate ellipses – a condition intermediate between a wave of oscillation and one of translation – and finally the wave breaks on the shore.

5.3.25.3 Dynamic effects on hydraulic structures

These are not confined to the direct impact of streams or waves. They may also result from eddies giving rise to periodic forces and may become serious, e.g. with long, slender piles in deep tidal waters. An important occurrence of this phenomenon was investigated at Immingham Oil Terminal.[39–41]

5.3.26 Dimensional analysis

Dimensional analysis is a valuable tool in reducing the apparent chaos of experimental results involving many variables and also in the systematic design of experimental procedure or technique.

Consider the resistance R of a certain shape of submerged body. Let l be any representative dimension, such as the length, v the velocity relative to the stream, ρ the density of fluid, μ the viscosity of fluid and g the acceleration due to gravity.

$$\text{Suppose } R \text{ depends upon } l, v, \rho, \mu \text{ and } g \tag{5.97}$$

Six quantities are involved here, and together they depend upon the three fundamental units or quantities of mass, length and time.

We therefore expect to find three dimensionless groups of the original quantities R, l, v, ρ, μ and g, related to one another.

Let N be a dimensionless group.

Choose any three quantities from Equation (5.97) such that together they include mass M, length L, and time T.

Suppose we choose R, l, ρ.

R (a force) has the dimensions of mass × acceleration, or $[MLT^{-2}]$

l has the dimensions of length $[L]$

ρ has the dimensions of $[ML^{-3}]$

Now write $N_1 = R^{a_1} l^{b_1} \rho^{c_1}$, the last quantity v being chosen at random from the remaining symbols of Equation (5.97).

Now N_1 is to have no dimensions in mass, length or time. Hence, dimensionally, $0 = [M^{a_1}][L^{a_1}][T^{-2a_1}][L^{b_1}][M^{c_1}][L^{-3c_1}][LT^{-1}]$.

Equating indices of $[M]$, $[L]$, $[T]$ in turn

for $[M]$, $0 = a_1 + c_1$
for $[L]$, $0 = a_1 + b_1 - 3c_1 + 1$
for $[T]$, $0 = -2a_1 - 1$

These simultaneous equations give $a_1 = -1/2$, $c_1 = 1/2$, $b_1 = 1$. Hence:

$$N_1 = \rho^{\frac{1}{2}} l v / R^{1/2}$$

Similarly, writing $N_2 = R^{a_2} l^{b_2} \rho^{c_2} \mu$ and remembering that the units of μ are $[M][L^{-1}][T^{-1}]$, we should find

$$N_2 = \mu / R^{\frac{1}{2}} \rho^{1/2}$$

Again, writing

$$N_3 = R^{a_3} l^{b_3} \rho^{c_3} g$$

we should get:

$$N_3 = R^{-1} l^3 \rho g$$

Now write:

N^1 = a function of N_2, $N_3 = \Phi(N_2, N_3)$

i.e.

$$\rho^{\frac{1}{2}} l v / R^{1/2} = \Phi(\mu / R^{\frac{1}{2}} \rho^{\frac{1}{2}}, l^3 \rho g / R)$$

which result is not invalidated if we multiply any one of the dimensionless 'groups', N_1, N_2, N_3, by a function of itself or of one of the others, so long as we do not reduce the total number of such distinctive groups.

Therefore:

$$R/\rho l^2 v^2 = \Phi(vl\rho/\mu,\ v^2/lg) \tag{5.98}$$

In this, $vl\rho/\mu$ is the so-called Reynolds number, and v^2/lg is the so-called Froude number, although some writers define it as $v/\sqrt{(lg)}$.

For alternative methods of dimensional analysis, see Duncan[42] and Whittington.[43]

5.3.26.1 *Application to scale models*

The principle of dynamical similarity indicates that if a model is operated at a speed truly corresponding with the full size project, then:

$$R_1/R_2 = \rho_1 l_1^2 v_1^2 / \rho_2 l_2^2 v_2^2$$

where the suffix 1 refers to the full size and the suffix 2 to its model.

As far as the resistance of submerged bodies is concerned therefore, it follows from Equation (5.98) that the Reynolds number in the model should be the same as in the actual, and (at the same time) the Froude numbers should be the same in model and full-size project.

The one condition then requires:

$$v_2 = v_1 (l_1/l_2)(\rho_1/\rho_2)(\mu_2/\mu_1) \tag{5.99}$$

and the other requires:

$$v_2 = v_1 \sqrt{(l_2/l_1)} \tag{5.100}$$

These two conditions cannot, in general, be satisfied at one and the same time and one of the features of scale model technique is to choose one or other as the more important and then to discover, as a result of experiments on models of different scales, or by comparison of model results with prototype values, what is the inaccuracy or 'scale effect' caused by neglect of the other requirement. In more complicated examples, more than two requirements may ideally be necessary.

5.3.26.2 *Ship models*

In testing the resistance of a ship's hull, the model is towed at a speed given by $v_2 = v_1 \times (1/S)^{1/2}$, where $1/S$ represents the geometrical scale. Its total resistance R_T is then measured and from this is subtracted the skin-frictional resistance R_S as estimated from experiments on long 'boards' of similar surface roughness. The residue $(R_T - R_S)$ represents eddy and wave-making resistance R_E:

$$R_E \text{ multiplied by } \rho_1 l_1^2 v_1^2 / \rho_2 l_2^2 v_2^2, \quad \text{or} \quad (\rho_1/\rho_2) S^3$$

then gives the eddy and wave-making resistance of the full-size ship at the corresponding speed $v_1 = v_2\sqrt{S}$, and to this is added the estimated skin friction of the ship at that speed.

5.3.26.3 *Falling particles – terminal velocity*

A particle descending through a fluid will accelerate until its effective weight is balanced by the resistance to its motion – it will then have attained its terminal velocity V_T.

Let ρ_1 = density of particle, ρ = density of fluid, η = viscosity of fluid, and R_T = force of resistance when velocity of particle reaches its terminal value V_T.

Then, once the terminal velocity is attained,

$$g(\rho_1 - \rho) \times \text{volume of particle} = R_T.$$

A general treatment of the resistance to relative motion of the solid through the fluid would involve considering inertia (arising from the property of mass possessed by the fluid) and viscosity (brought about by the molecular structure of the fluid).

In his classical study of a sphere moving through a fluid, Sir George Gabriel Stokes assumed that at sufficiently low velocities the inertia terms could be neglected and so established the result that under such conditions the resistance would be $3\pi\eta Vd$, where V is the velocity of the sphere of diameter d relative to the fluid.

Applying this to the terminal velocity V_T:

$$g(\rho_1 - \rho)[(1/6)\pi d^3] = 3\pi\eta V_T d,$$

from which $V_T = (gd^2/18)\left(\dfrac{\rho_1 - \rho}{\eta}\right)$.

Experiments confirm this provided that $V_T d\rho/\eta$ does not exceed approximately 0.5. Subject to that proviso, the formula for V_T may be used to determine: (1) the diameter of a small sphere from its observed terminal velocity in a fluid of known viscosity; and (2) the viscosity of a fluid by measuring the terminal velocity of a small sphere of known diameter.

In more general terms, the terminal velocity, even when $V_T d\rho/\eta$ exceeds 0.5, may be used as a measure of: (1) the size of particles in comparison with the well-established experimental results for spheres; and (2) the capacity of particles to be lifted into suspension by an upwards current.

(Incidentally, it will be seen that Stokes's $R = 3\pi\eta Vd$ is equivalent to $R/\rho l^2 V^2 = R/\rho d^2 V^2 = 3\pi/(\rho V d/\eta)$, or, if A is the projected area of the sphere, $\pi d^2/4$, $R/\rho A V^2 = 12/(Vd/\nu)$.)

5.3.26.4 *Models of sluice-gates, weirs, etc*

In models of sluice-gates, weirs and spillways, the Froude number is to be adopted, provided the scale is well chosen. For example, a model of an overflow spillway for a dam may usually be relied upon if the lowest head on which deductions are to be based is not less than 6.4 mm in the model. Under heads smaller than this, surface tension and viscosity are generally responsible for appreciable scale effects. With this reservation, however, if we call h the head observed in the model and q the discharge, then the discharge of the full-size spillway under a head of Sh will be $qS^{5/2}$, the model scale being $1:S$.

5.3.26.5 *River models*

River models are frequently constructed to different scales horizontally and vertically. Many considerations influence the scales actually to be chosen. If the horizontal scale is $1:x$ and the vertical scale $1:y$, there will be a vertical exaggeration, or distortion, of scale, equal to x/y. This exaggeration is desirable in order to improve the prospects of: (1) the flow being turbulent in the model as it is in nature; (2) the water-surface slopes being reproduced; (3) bed material being shifted by the currents available in the model. The smaller the value of x, the smaller must be the exaggeration x/y. The nominal velocity scale for sensibly horizontal stream velocities in such a model is then $1:\sqrt{y}$ and the scale of time $1:x/\sqrt{y}$. If q is the rate at which water is fed to the model in order to simulate a flow Q in nature, then q should be equal to $Q/xy^{3/2}$. Some river models have, however, been operated with such flows and velocities, discovered by trial, as to give the proper gradients and bed movements irrespective of these ideal conditions which should, if practicable, be observed.

Sand or other material used on the bed of river models to give qualitative or approximately quantitative indications of scour is not necessarily reduced to scale. Frequently such materials are of the same order of size as in nature, the feasibility of this depending on the fact that the scouring property of a shallow stream is greater than that of a deep one of the same mean velocity.

5.3.26.6 Harbour models

Models of harbours specifically concerned with surface waves produced by storms have been successful with scales (undistorted) in the region of 1:50, 1:100 or 1:180, and with model waves in the exposed area outside the harbour works about 20 mm or more in height. The velocity scale of such a model having a geometrical scale of 1:100 would be $1:\sqrt{100}$; or 1:10, and its time scale would also be 1:10.

5.3.26.7 'Mathematical models'

Mathematical solutions to complex problems of fluid motion have developed on a wide front in recent years, in conjunction with the increasing availability of computers. They cover a range of problems embracing the analysis of data, the study of surges, the hydraulic behaviour of rivers and estuaries, sediment transport, etc. The representation or modelling by mathematical formulae and equations frequently contains terms and/or coefficients which have been derived from observations in the field, the laboratory, or on physical models. The choice of which kind of model to use – the mathematical or the physical – depends on the nature of the engineering problem concerned; in some cases, it is advantageous to combine the two methods, so treating one as complementary to the other. Recent literature provides a guide, and in the particular case of estuaries, the potential and the limitations of both kinds of model are discussed by McDowell and O'Connor.[44]

Notwithstanding the undoubted powers of the mathematical approach, engineers experienced in these matters are likely to endorse the view of McDowell and O'Connor that 'the greatest single merit of physical models is their capacity to reproduce, on an easily observed scale, the intricate three-dimensional flow in a large estuary. They are, in consequence, an aid to thought and planning without equal. . . .' This applies not only to physical models of rivers and estuaries but equally to those of hydraulic structures, e.g. spillways, channels and other devices for the 'dissipation' of energy.

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6

Engineering Surveying

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Contents

6.1 Introduction

The work of the land surveyor can be classified into three main areas of responsibility. Firstly, he is concerned with the recording of measurements which allow the size and shape of the Earth to be determined. Secondly, and primarily, he is involved in the collection, processing and presentation of the information necessary to produce maps and plans. Thirdly, he may be required to locate on the surface of the Earth the exact positions to be taken up by new roads, dams or other civil engineering works.

As a consequence of the diverse nature of the land surveyor's duties, several distinct branches of the subject have evolved.

6.1.1 Branches of surveying

Geodetic surveys are carried out on a national or international basis in order to locate points large distances apart. This type of survey acts as a framework for 'lower order' surveys. In order to ensure high accuracy, the effect of factors such as the curvature of the Earth on observations must be considered and the necessary corrections applied.

Topographic surveys are concerned with the small-scale representation of the physical features of the Earth's surface. Frequently, the data necessary for such an operation will be provided by the use of aerial photography. The science of taking measurements from photography in order to produce maps is known as photogrammetry. Topographic surveys are often the responsibility of a national organization such as, for example, the Ordnance Survey in the UK.

Hydrographic surveys, in contrast, involve the representation of the surface of the seabed. The end-product is normally a navigational chart. In recent years this branch has become increasingly important with the development of the offshore oil industry. In this case, in addition to the production of charts, the surveyor may be required to position large structures such as oil production platforms. This type of operation would normally necessitate the use of ground and satellite electronic position-fixing equipment.

Cadastral surveys relate to the location and fixing of land boundaries. In many countries in the world, e.g. Australia, the information supplied by the cadastral surveyor may be an integral part of a land registration system.

Finally, engineering surveys are required for the preparation of design drawings relating to civil engineering works such as roads, dams or airports. The surveys are normally at a large scale, with scales of 1:500 and 1:1000 being most common.

Many of these branches require highly specialized knowledge, beyond the scope of this chapter. In view of this, the aim in this chapter will be to discuss: (1) those aspects of the subject which are required in order to carry out simple surveys for engineering projects; (2) the processes involved in carrying out precise surveys for deformation monitoring projects; and (3) the use of computers in surveying for digital mapping and ground modelling.

6.1.2 Principles of surveying

In spite of the diverse nature of land surveying, it is possible to define certain basic principles which are common to all branches of the subject. These principles have proved over the years to be vital if accurate surveys are to be conducted.

The first and most important principle is the provision of an initial framework before observing and fixing the detail of a survey. This process is often known as providing control. It is essential to ensure that the positions of the control points are known to a higher order of accuracy than those of the subsidiary points. By satisfying this principle it is possible to ensure that errors, which inevitably occur, do not accumulate but are contained within the control framework.

A second and perhaps more obvious principle is that of planning. All too often it is tempting to rush into a survey without consideration for an overall plan. Of particular importance is the need to define a job specification. This is indispensable since the relationship between cost and accuracy is not linear and an increase in accuracy may have a disproportionate effect on cost. For example, if a distance of 500 m is to be determined to an accuracy of either 5 or 0.5 mm, the cost ratio of the respective accuracies may be of the order of 1:300. It is important, therefore, to choose techniques and instruments appropriate to the survey specification. Of equal importance is the need to plan the reconnaissance stage. Before starting a task it is essential to examine the area carefully, considering all the possible ways of doing the survey and then selecting the most suitable method. Remember, 'time spent on reconnaissance and planning is never wasted'.

A third principle is the need to ensure that sufficient independent checks are incorporated into the survey to eliminate or minimize errors. It is important that the checking system is included at all stages of the survey from fieldwork and computations to the final plotting. In addition, the checking system should be independent and not solely a repeat of the initial measurement. Examples of independent checks are:

Fieldwork:
- measure both diagonals of a quadrilateral
- measure distances in both directions
- measure angles using different parts of the theodolite circle

Computations:
- use the summation check on angle observations of geometric figures, e.g. sum of interior angles $(2n-4)\times 90°$
- levelling booking cross-checks

Plotting:
- plot positions of important points by using angles and distance and also using coordinates.

The final principle is that of safeguarding. Safeguarding is equally important at all stages of the survey, and refers to the process of ensuring that the survey results can be replicated if accidental, or other, damage occurs to the survey markers or field observations. Thus, it is important when constructing permanent survey markers to take 'witness or reference measurements' to points of prominent detail in the vicinity of the point. Linear measurements of this type enable the point to be relocated if it is damaged, or alternatively if it is difficult to find. The latter situation can often occur with road projects. In many instances there may be a gap of many years between initial survey and final setting-out. During this time the permanent survey markers may become overgrown with vegetation and hence difficult to locate. The use of witness marks and measurements can often be of crucial importance if the permanent marks are to be relocated.

Safeguarding of field observations is also of paramount importance. Thus, it is considered good practice to produce abstract sheets from the surveyor's fieldbook at the end of each day. These abstract sheets should summarize the major results from the fieldbook (e.g. rounds of angles, mean distance etc.) and should be carefully filed in the survey office. By such a process the possibility of several days' work being lost if the fieldbook is damaged or misplaced can be eliminated.

6.1.3 Errors in surveying

It is an unfortunate and often misunderstood fact that all measurements are affected by errors. So often, when confronted with the question: 'How accurate do you want the survey to be?', or 'How accurately do you want this point located?', the glib answer 'Exactly!', or 'Spot on!', is given as the reply by a prospective client. If, in addition, the question of errors is raised, it is quickly dispensed with the comment: 'Errors don't occur if you do it properly.' The answer is correct in one respect, i.e. in relation to mistakes, or, more correctly, gross errors. These should not occur if a survey is carried out according to the basic principles of surveying. However, other types of errors do occur which can be much more difficult to handle.

Systematic errors, as the name suggests, are errors which follow a pattern or system. Errors of this type are normally related to the variations in physical conditions which can occur when a measurement is made. For example, a steel tape is normally known to be a certain length at some standard temperature. If the temperature under which a measurement is made varies from this standard, a systematic error will occur. By knowing the coefficient of linear expansion a value for the expansion of the tape can be determined and a correction applied. Systematic errors whose effect can be modelled mathematically are, hence, eliminated.

Random errors, in contrast, do not follow a standard pattern and are entirely based on the laws of probability. These errors, or rather variations in measurement, will occur after gross and systematic errors have been eliminated. The measurement of a distance by taping can again be taken as an example. It is often not appreciated that the same distance measurement made under the same physical conditions with the same tape will produce different answers. Since it is assumed that the measurements will follow a normal distribution they can be examined using standard statistical techniques.

The following formula can therefore be applied to the analysis of random errors:

$$\text{arithmetic mean} = \bar{x} = \frac{\Sigma x_i}{n} \tag{6.1}$$

where $i = 1, 2, \ldots, n$ are the observed values and n denotes the number of observed values.

The arithmetic mean is significant because it is often taken to be the closest approximation to the 'true' value and as such is known as the most probable value (m.p.v.). The difference between the m.p.v. and the observed value is known as a residual (v).

A term often used in order to estimate the precision of a series of measurements is the standard error, where standard error of a single observation is:

$$\sigma_s = \pm \left(\frac{\Sigma v^2}{n-1}\right)^{1/2} \tag{6.2}$$

and standard error of the arithmetic mean is:

$$\sigma_M = \pm \left(\frac{\sigma_s}{n^{1/2}}\right) \tag{6.3}$$

For surveying purposes, the terms 'standard error', 'standard deviation' and 'root mean square (r.m.s.) error' are synonymous. All such terms are used to give an indication of the precision of the result, i.e. the degree of agreement between successive measurements. High precision may not be indicative of high accuracy, since accuracy is related to the proximity of the measurement to the true value. If, however, all the effects of the bias caused by systematic errors have been eliminated, these indices of precision may also be used as indices of accuracy.

For example, suppose an angle has been measured 9 times and the subsequent error analysis indicates that $\sigma_s = 3''$ and $\sigma_m = 0.81''$. What does this information tell us? Firstly, it indicates that the angle has been measured to a high precision. Secondly, it indicates that, statistically, there is a 68% chance or probability of the standard error of a single measurement being less than 3″. Furthermore, if one extends the confidence limit to a value 3 times the standard error, or 9″, then statistically the probability that the error will be less than 9″ is now 99.7% with only a 0.3% chance of the error being greater than 9″. This confidence limit is often applied as a rejection criterion to a group of observations. Any observation with a residual greater than 3 times the standard error may then be rejected, on the basis that it is highly unlikely that the variation is solely a consequence of random effects. Similar reasoning would apply to the standard error of the arithmetic mean. Further information on errors and their treatment can be found in Cooper[1] and Mikhail and Gracie.[2]

6.2 Surveying instrumentation

Surveying is essentially concerned with the direct measurement of three fundamental quantities: (1) the angle subtended at a point; (2) the distance between two points; and (3) the height of a point above some datum, normally mean sea-level. From the measurement of these three quantities, it is then possible to compute the three-dimensional positions of points.

With the exception of electronic methods of determining distance, the instruments used by the surveyor have not radically changed in principle for 40 to 50 years. The advances in technology may have reduced the size and increased the efficiency of the instruments, but the fundamental principles remain unchanged.

6.2.1 Angular measurement using the theodolite

The theodolite is used for the measurement of horizontal and vertical angles. In simple terms, a theodolite consists of a

Table 6.1 Characteristics of some modern theodolites

Type of theodolite:	1″ Precise	20″ Engineers	10′ Builders	Compass	Electronic
Typical example:	Kern DKM 2A-E	Sokkisha TM20ES	Zeiss (Ober.) TH51	Wild TO	Kern E-2
Country of manufacture:	Switzerland	Japan	W. Germany	Switzerland	Switzerland
Direct reading to	1″	20″	10′	1′	1″
By estimation to	0.1″	5″	1′	30″	—
Telescope magnification	32 ×	28 ×	20 ×	20 ×	32 ×
Telescope aperture (mm)	40	45	30	28	45
Sensitivity of plate					
Level per 2 mm run	20″	30″	45″	8′	—
Weight of instrument (kg)	6.2	4.2	2.2	2.9	8.7

telescope mounted on a platform which may be levelled to form a horizontal plane by means of a simple spirit bubble. Angles are measured by pointing the telescope at targets and establishing the difference between readings on a circular protractor mounted on the level platform.

There is a bewildering choice of theodolites available. Table 6.1 lists the characteristics of a selection of commonly available modern theodolites. The broad distinction can be made between those instruments which measure angles and those such as compass and gyro theodolites which measure bearings, relative to magnetic north and to true north respectively.

6.2.1.1 General construction of the theodolite

There are certain fundamental relationships and components which are common to all theodolites. Before examining the detailed construction of a modern glass arc theodolite, it is important to appreciate the geometrical arrangement of the axes of a theodolite, as illustrated in Figure 6.1.

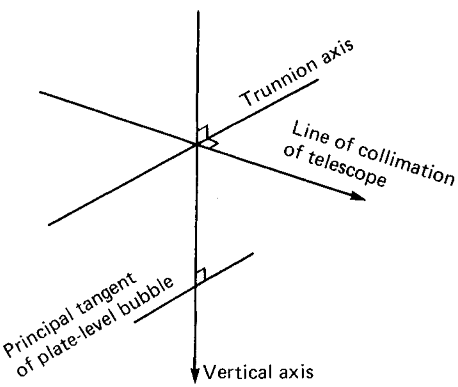


Figure 6.1 Theodolite axes

In this ideal arrangement the vertical axis is vertical, the trunnion axis is perpendicular to it and hence horizontal, and the line of collimation is perpendicular to the trunnion axis. Unfortunately, it is not possible during the manufacturing process to ensure that these orthogonal relationships occur exactly. Similarly, during use over a period of years, wear may occur which may also alter these conditions. The extent to which a theodolite fails to satisfy them can be measured by a series of instrument tests which may be carried out in the field. If, subsequently, the instrument is found to be out of adjustment, the instrument should be returned to the manufacturer or a specialist instrument technician for adjustment. Details of the field tests and methods of adjustment may be found in Cooper.[3] If, however, a modern theodolite is treated with care, and a suitable observational technique is employed, regular servicing should be all that is required in order to obtain good results.

The detailed construction of a modern 1 s precise theodolite is shown in Figures 6.2 and 6.3. Examination of these figures illustrates that the theodolite consists essentially of three distinct parts:

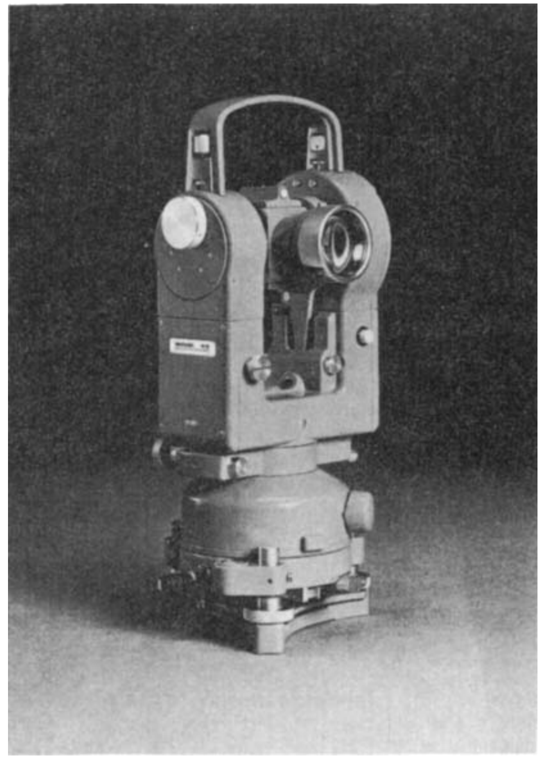


Figure 6.2 Wild T-2 one second theodolite (Wild Heerbrugg)

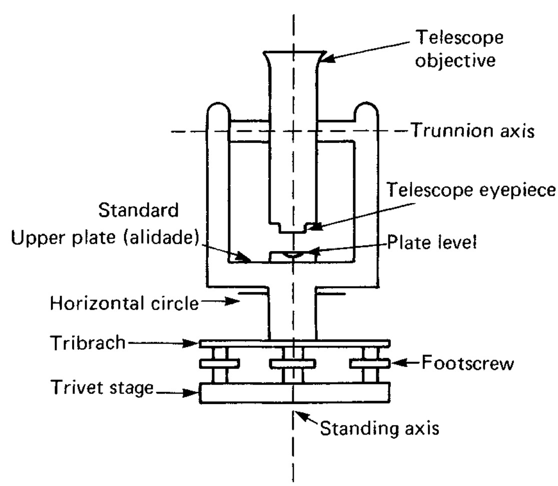


Figure 6.3 Construction of a theodolite

(1) *Base.* This consists of two main components: the tribrach and the horizontal circle. The tribrach can be screwed securely to the tripod and, by means of three footscrews, the instrument may be levelled. The circle is made of glass with photographically etched graduations. It is normally graduated in a clockwise manner. A circle-setting screw is also usually provided and the base will generally house an optical plummet. This consists of a small eyepiece with a line of sight which is deviated by 90° in order to point vertically down. By this process it is possible to centre the instrument precisely over a ground point. In some cases the optical plummet may be housed in the alidade.

(2) *Alidade.* This rotatable upper part of the theodolite may also be known as the upper plate. The alidade rotates about the vertical axis. Mounted on the alidade is the plate-level bubble which indicates whether the instrument is level. By

means of clamps and slow-motion screws it is possible to rotate and clamp the alidade relative to the base.

(3) *Telescope.* Attached to the trunnion axis of the theodolite is the telescope. The telescope magnifies the object and, by the use of cross-hairs, allows the exact bisection of the target. Focusing of the object and the cross-hairs is carried out using separate focusing screws. A further clamp and slow-motion screw allow precise pointing of the telescope in a vertical plane.

Angles of elevation or depression are measured using a vertical circle also attached to the trunnion axis. Prior to measuring a vertical angle it may be necessary to set the altitude bubble. However, most modern theodolites employ an automatic compensating mechanism. In these cases vertical angles may be recorded after the plate level has been set, without recourse to an additional bubble setting.

When the vertical circle is to the left of the telescope, the theodolite is in what is conventionally called the face left (FL) position. Conversely, when the vertical circle is to the right of the telescope as it views an object, the theodolite is in the face right (FR) position.

6.2.1.2 Circle reading

By projecting daylight through the standards of the theodolite, it is possible to illuminate the glass scale of both the horizontal and vertical circles.

In order to resolve a direction to a higher precision than that to which the circle has been graduated, an optical micrometer is employed. Optical micrometers are the modern equivalent of verniers. The principle of operation involves the use of a plane parallel-sided block of glass as shown in Figure 6.4. When the glass is in the normal position, as shown by position (a), light passing through will be uninterrupted. Rotation of the block of glass, however, produces a lateral shift of the incident beam as shown by position (b). This rotation is controlled by the micrometer screw of the theodolite. Movement of this screw enables the observer to read, on an auxiliary scale, the lateral shift required in order to bring the image of the main-scale degree graduations into coincidence with the index marks which are built into the optical path. Using this technique it is possible to resolve directly to 20″ of arc if the micrometer is reading from one side of the circle. Resolution direct to 1″ is possible if a mean-reading optical micrometer is used. In this case, readings from two points diametrically opposite are meaned in order to eliminate the effects of any circle eccentricity.

Figures 6.5 and 6.6 illustrate two typical examples of the circle reading systems for both the single- and mean-reading optical micrometers.

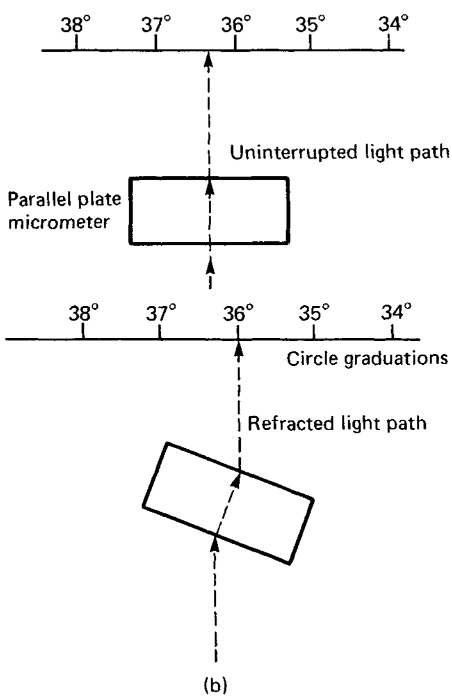


Figure 6.4 Parallel plate micrometer

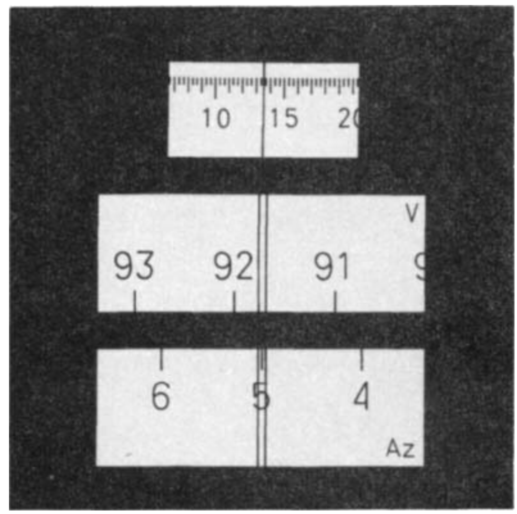


Figure 6.5 Single reading optical micrometer: circle reading Wild T-1A 05° 13′ 30″ (Wild Heerbrugg)

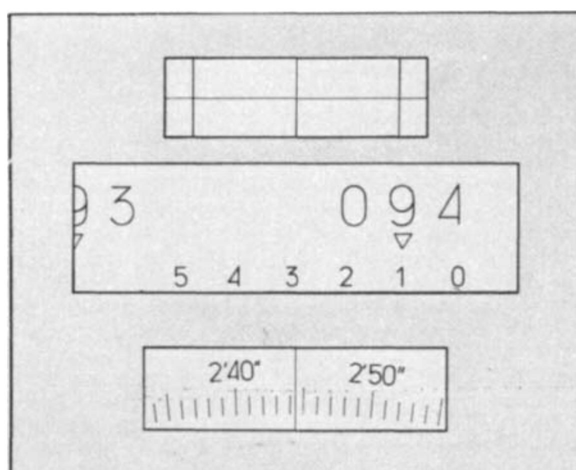


Figure 6.6 Mean reading optical micrometer: circle reading Wild T-2 94° 12′ 44.3″ (Wild Heerbrugg)

6.2.1.3 Field procedure

Potentially the theodolite is a very precise instrument. It is, however, necessary to follow a strict procedure both in setting-up the instrument and in observing if this potential is to be realized. Incorrect use of a theodolite will undoubtedly result in poor results, regardless of how precise the instrument may be.

Setting-up. Setting-up a theodolite prior to observations being taken consists of three separate operations: centring, levelling and focusing.

Centring involves positioning the instrument exactly over a ground point. This may be achieved by means of either a plumb bob suspended from the instrument or a centring rod or an optical plummet. The process of centring and levelling should be considered as iterative in nature, becoming increasingly more precise after each operation.

Levelling the theodolite carefully is a necessary prerequisite for precise measurements. The following sequence of operations must be carried out in order to level a theodolite:

(1) Approximately level the instrument using the small circular bubble.
(2) Set the plate-level bubble parallel to any two footscrews, such as A and B in Figure 6.7(a). Rotate both footscrews together or apart until the bubble is in a central position.

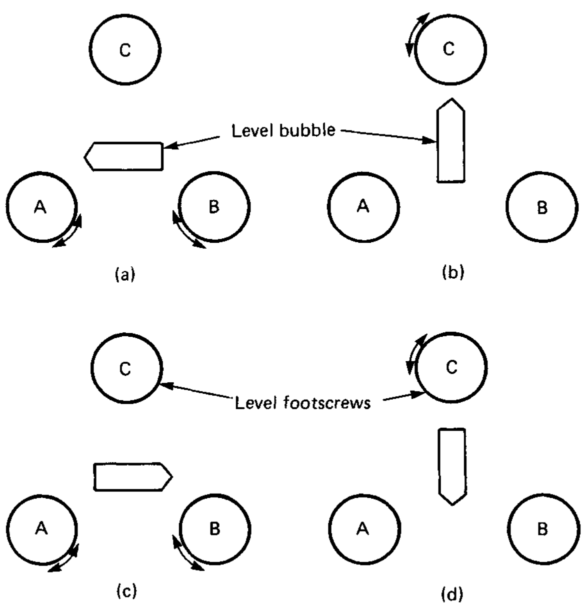


Figure 6.7 Levelling the theodolite

(3) Rotate the alidade until the bubble is now approximately perpendicular to the initial position, as shown in Figure 6.7(b). Using footscrew C only, centralize the bubble.
(4) Return to the initial position and again centralize the bubble using footscrews A and B. Repeat (2) and (3) until the bubble is central in both positions.
(5) Rotate the alidade through 180° until the position shown by Figure 6.7(c) is achieved. If the bubble does not remain in a central position, move the bubble until it is in a position midway between a central position and its initial position.
(6) Rotate the alidade until the position illustrated by Figure 6.7(d) is achieved. Using footscrew C, move the bubble into the same position as in Figure 6.7(c). The bubble should then remain in the same off-centre position for any alignment of the alidade.

The final step before observations begin is to focus both the cross-hairs and the object to which observations will be made. It is important to ensure that both images appear clear and sharp. In addition, it is critical that parallax does not exist. Parallax refers to the apparent movement of the cross-hairs and objects relative to each other when the observer moves his head. It is caused by the image of the object not lying in the same vertical plane as the cross-hairs. If this occurs, the focusing operation must be repeated until it is eliminated.

Observational procedure. A strict observational procedure is essential if both human and instrumental errors are to be reduced to a minimum. Consider the problem of measuring the angle shown in Figure 6.8.

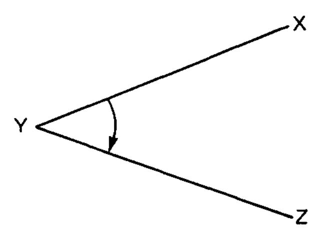


Figure 6.8 Angle measurement

The observational procedure which should be adopted is as follows. A booking procedure is illustrated by Table 6.2.

(1) Point with telescope in the FL position to the target at X and record the horizontal circle reading, e.g. 90° 20′ 30″.
(2) Point on FL to target Z and record, e.g. 130° 25′ 40″.

Table 6.2 Booking and reduction of theodolite readings

Station Y   Observer ....Date ....
Height of Inst: ....Booker ....Weather ....

	TO	FL			FR			MEAN			ANGLE			COMMENTS
		°	′	″	°	′	″	°	′	″	°	′	″	
Round 1	X	90	20	30	270	20	40	90	20	35	40	05	10	
	Z	130	25	40	310	25	50	130	25	45				
Round 2	X	135	30	15	315	30	25	135	30	20	40	05	15	
	Z	175	35	30	355	35	40	175	35	35				
	Mean angle: 40° 05′ 13″													

(3) Change face to FR and point to target Z and record circle reading 310° 25′ 50″.
(4) Point on FR to target X and record direction 270° 20′ 40″.

In order to reduce the effect of instrument maladjustments to a minimum, the mean of the FL and FR minutes and seconds readings to the same point is averaged and the value entered in the mean column. The difference between the mean circle readings is then derived and entered in the angle column.

This constitutes one round of angles. The base setting screw should then be adjusted and the process repeated in order to increase the precision of the angle measurement. A minimum of two rounds is necessary for the least precise measurements; up to sixteen rounds may be required for very precise operations.

6.2.2 Distance measurement

The second fundamental quantity which it is necessary to measure is distance.

A wide variety of techniques can be used for the determination of distance. The general distinction can, however, be made between direct, optical and electronic methods. All of the techniques discussed are capable of varying levels of precision depending on the degree of sophistication of the instrumentation and the observational techniques adopted.

6.2.2.1 Direct distance measurement (DDM)

The simplest method of measuring distance is that of physically measuring the distance with a tape. In the past invar tapes were used for the precise measurement of baselines for triangulation networks. Nowadays, DDM is generally confined to either the precise measurement of short distances for setting-out or control purposes or the less precise measurement of the detailed dimensions of a building or land parcel.

There are basically two types of tape in common use. Fibreglass measuring tapes are manufactured from multiple strands of fibreglass coated with PVC. They are waterproof and normally either 30 or 50 m in length. Fibreglass tapes are generally used for detail measurements and have largely superseded the linen tapes which were available previously. For more precise measurements it is necessary to use steel bands. These are typically either 30, 50 or 100 m in length.

In order to obtain high precision with either type of tape, it is essential that it is periodically checked against a standard reference tape, the length of which is known to a higher order of accuracy than that of the tape being checked. If a significant variation exists, a standardization correction should be applied. In addition, it is vital that suitable attention is paid to the effect of variations in slope, temperature and tension which may necessitate appropriate corrections being applied to the measured distance. The corrections (C_1, C_2 and C_3 respectively) are:

Slope:

$$C_1 = -L(1-\cos\theta) \quad (6.4)$$

where θ = slope angle, and L = measured slope distance

or:

$$C_1 = -\Delta h^2/2L$$

where Δh = height difference between end-points

Temperature:

$$C_2 = \pm\alpha L(t_m - t_s) \quad (6.5)$$

where t_m = measured temperature in the field, t_s = temperature at which the tape was standardized, usually 20°C, and α = coefficient of linear expansion (0.000 011 2 for steel bands)

Tension:

$$C_3 = \pm L(T_m - T_s)AE$$

where T_m = measured tension, T_s = standard tension, A = cross-sectional area of tape, and E = Young's modulus for the tape, typically 200 kN/mm^2 = 200 000 N/mm^2

Miller[4] details the typical accuracy levels which can be achieved with steel tapes.

6.2.2.2 Optical distance measurement (ODM)

As an alternative to the direct method of measuring distance, it is also possible to measure distance indirectly by optical methods.

The development of ODM began over two centuries ago. James Watt is recorded as having used this approach in his survey of the West of Scotland in 1774. Although many instruments and improvements have been introduced since then, they all essentially involve the solution of the same geometrical problem.

All methods of ODM are based on the solution of an isosceles triangle, as shown in Figure 6.9. The triangle consists of three important components: the parallactic angle α, the base length B (which may be either in a horizontal or vertical plane), and D, the horizontal bisector of the base of the triangle. By knowing the relationship between the three components, the horizontal distance D between two points can be determined.

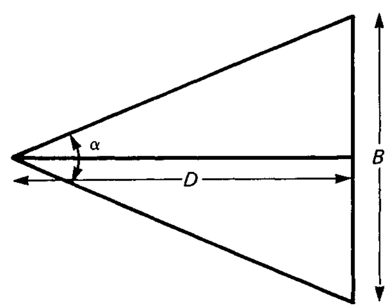


Figure 6.9 Optical distance measurement (ODM)

Two methods of solution are possible: either an instrument with a fixed parallactic angle is used and the variable base B is measured (Figure 6.10) or a base of fixed length is set up and the variable parallactic angle is measured (Fig. 6.11). In both cases, the variable quantity is proportional to the horizontal distance. By defining the mathematical relationship between the fixed and variable quantities it is therefore possible to determine the horizontal distance.

Tacheometry. The first approach described above (fixed angle, variable base) is commonly known as tacheometry or more correctly as vertical staff stadia tacheometry. It is normally used for the measurement of distance where a proportional error of between 1/500 and 1/1000 is acceptable, e.g. in picking-up survey detail points.

All modern theodolites have a diaphragm consisting of a main horizontal cross-hair and two horizontal stadia lines

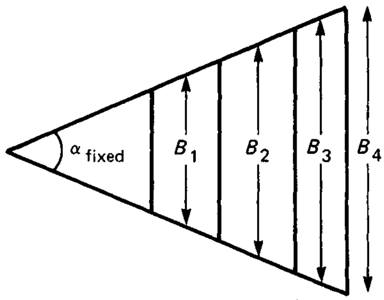


Figure 6.10 ODM: fixed angle, variable base

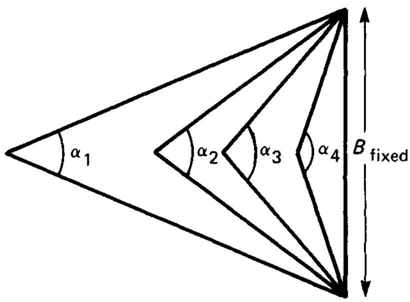


Figure 6.11 ODM: fixed base, variable angle

spaced either side of it. These stadia lines define the fixed parallactic angle. If the theodolite telescope is sighted on to a levelling staff and the readings of the outer lines noted, the difference in the readings, the staff intercept (s), will be directly proportional to the horizontal distance between the instrument and the staff. Generally, the distance between the stadia lines is designed in such a manner that the horizontal distance D_H between the instrument and staff is given by:

$$D_H = 100s \tag{6.7}$$

For inclined sights the geometry is as shown in Figure 6.12. Hence:

$$D_s = 100(s \cos \theta) \tag{6.8}$$

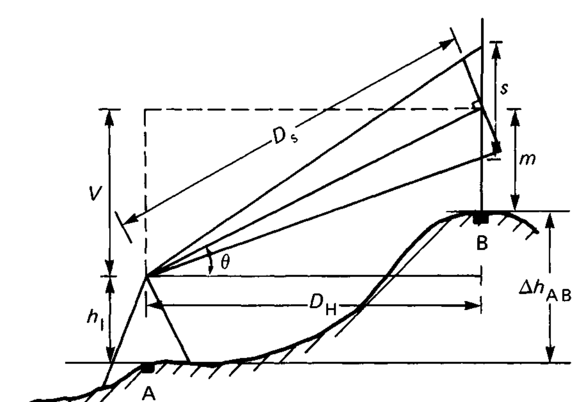


Figure 6.12 Stadia tacheometry: inclined sights

where D_s = slope distance, and θ = vertical angle measured by the theodolite. Therefore:

$$D_H = 100s \cos^2 \theta \tag{6.9}$$

$$\Delta h_{AB} = h_I + V - m \tag{6.10}$$

where Δh_{AB} = difference in height between A and B, h_I = height of instrument (trunnion axis to ground), m = middle hair reading, and V = difference in height between middle-hair reading and trunnion axis = $50s \sin 2\theta$ (6.11)

Several self-reducing tacheometers have also been designed. The main advantage of these instruments is their ability to compensate for the effect of the inclination of the theodolite telescope and, hence, allow the direct determination of horizontal distance without additional computation.

Two notable examples of this type of instrument are the Wild RDS vertical staff self-reducing tacheometer and the Kern DK-RT horizontal bar double-image self-reducing tacheometer. Details of the construction and use of these instruments may be found in Hodges and Greenwood,[5] and Smith.[6] In recent years, the manufacture of these precise optical devices has ceased, their place being taken by low-cost electronic measuring devices.

Subtense bar. The second approach (fixed base, variable angle) is commonly known as the subtense or horizontal subtense bar method. The method is normally confined to the measurement of distance for control purposes. Using this approach, distances may be determined with a proportional error of up to 1/10 000.

The instrumentation required consists of a subtense bar, normally 2 m long, and a one-second theodolite, such as the Wild T2. The bar has targets mounted at each end of an invar strip. The strip is protected by a surrounding aluminium strip in order to ensure that, for all practical purposes, the length of the bar remains constant at 2 m. The bar is set up and oriented at right angles to the line of sight of the theodolite, as shown in Figure 6.13.

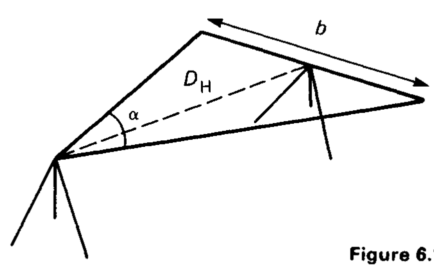


Figure 6.13 Subtense bar

The horizontal parallactic angle α is measured with the theodolite. Irrespective of the vertical angle to the bar, the horizontal distance is given by:

$$D_H = \tfrac{1}{2} b \cot(\alpha/2) \tag{6.12}$$

with $b = 2m$

$$D_H = \cot(\alpha/2) \tag{6.13}$$

For distances greater than 100 m it is advisable to subdivide the distance to be measured or, alternatively, to use the auxiliary base method (Hodges and Greenwood,[5] and Smith.[6]).

Subtense methods are also tending to be superseded by low-cost electronic methods. Nevertheless, many organizations still possess this type of equipment and for many projects it is a very suitable technique to adopt.

6.2.2.3 Electronic distance measurement (EDM)

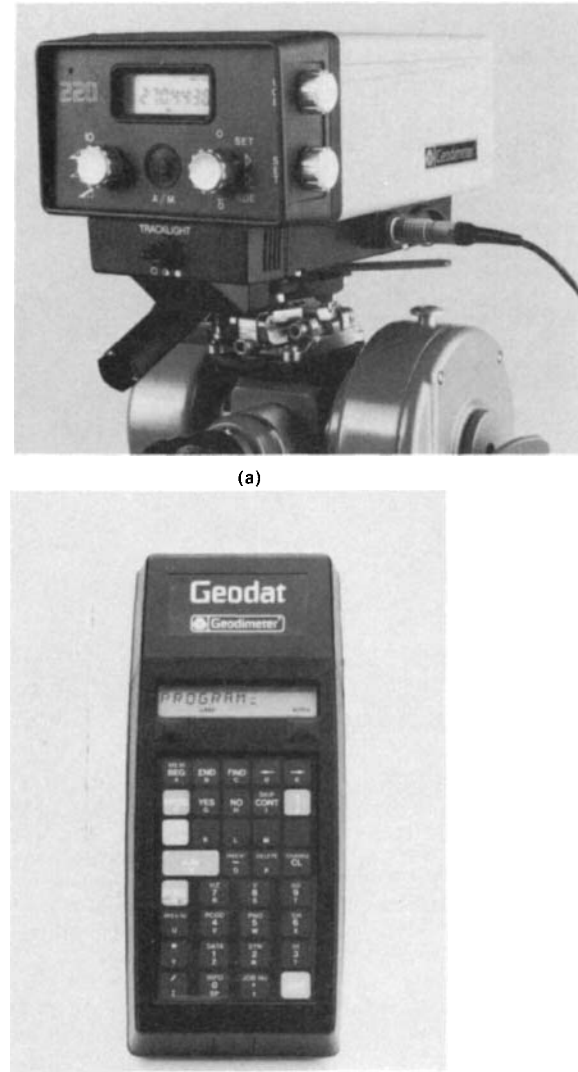
Development. The first generation of EDM instruments was developed in the early 1950s. Typical of the early meters were the Swedish Geodimeter (GEOdetic DIstance METER) and the South African Tellurometer instrument. The former, an electro-optical instrument, used visible light measurement, whilst the latter used high-frequency microwaves. Both instruments were primarily developed for military geodetic survey purposes and had the ability to measure long distances, up to 80 km in the case of the Tellurometer, to a precision of a few centimetres. They were also, however, bulky, heavy and expensive in comparison to their modern-day equivalents.

During the late 1960s, developments in microelectronics and low-power light-emitting diodes led to the emergence of a second generation of EDM instruments. These electro-optical instruments utilized infra-red radiation as the measuring signal and were developed for the short range (< 5 km) market. In addition, they were considerably smaller, lighter and less expensive than their predecessors. Probably the best-known example is the Wild DI-10 Distomat.

The introduction of microprocessors into the survey world in the early 1970s led to the introduction of a third series of EDM instruments. With this group it became possible, not only to determine slope distance, but also to carry out simple computational tasks in the field. For example, the facility became available to compute automatically the corrected horizontal distance and difference in height between two points by manual input of the vertical angle read from the theodolite. Electronic distance measurement instruments of this type had also been reduced in size to the extent that the EDM unit could be theodolite-mounted. The Wild DI-3 is a typical example of this type of instrument.

The most recent short-range EDM instruments are similar to the previous group, but have several additional features worthy of mention. Firstly, the technology now exists to sense automatically the inclination of the EDM unit and therefore to be able to compute automatically the horizontal distance between two points. The Geodimeter 220 (Figure 6.14) has this facility. This instrument also has the ability to measure to a moving target, or track, a useful feature for setting-out purposes. By using an additional unit it is also possible to have one-way speech communication between the instrument and target positions, again valuable when setting-out. This instrument can also be connected to a Geodat 126 hand-held data collector (Figure 6.14), which is able to store automatically distance information from the EDM unit. Other relevant information (numeric or alphanumeric) can be input manually via the keyboard. The Geodimeter 220 has a range of 1.6 km with one prism and 2.4 km with three prisms determined to a standard error of ± 5 mm ± 5 parts per million (p.p.m.) of the distance.

The last development in the field of EDM instrumentation is the electronic tacheometer or 'total station'. The former term is more appropriate in view of the different interpretations, by the instrument manufacturers, of the term total station. In essence, an electronic tacheometer is an instrument which combines an EDM unit with an electronic theodolite. Hence, such instruments are capable of measuring, automatically, horizontal and vertical angles and also slope and/or horizontal distance. The majority also have the facility to derive other quantities such as heights or coordinates and store this data in a data collector. Two designs of instrument have evolved during the last 5 years.



(a)

(b)

Figure 6.14 (a) Geodimeter 220; (b) Geodat 126 data collector (Geotronics)

The first, the integrated design, consists of one unit which, generally, houses the electronic circle-reading mechanism and the EDM unit. The Wild TCI Tachymat and the Geodimeter 140 (Figure 6.15) are representative of this range of instrument. The second design approach is the modular concept. In this case, the EDM instrument and the electronic theodolite are separate units which can be operated independently. This approach tends to be more flexible and enables units to be exchanged and upgraded as developments occur; it may also be a more cost effective solution for many organizations. The Kern E-2 and Wild T-2000 Systems (Figure 6.16) are representative of this design of electronic tacheometer.

Finally, mention should be made of high-precision EDM instruments. These instruments have been designed for projects such as dam deformation or foundation monitoring where extremely high precision is necessary. Instruments which are

Figure 6.15 Electronic tacheometer, integrated design: Geodimeter 140

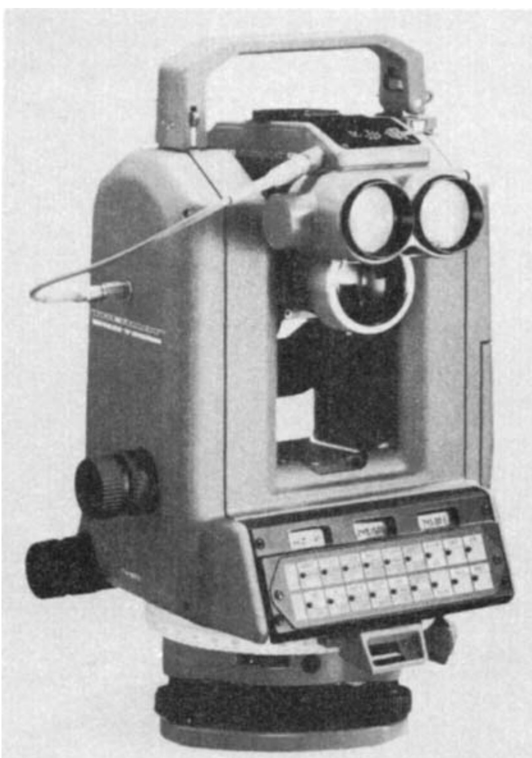


Figure 6.16 Electronic tacheometer, modular design: Wild T-2000, with DI4

representative of this design include the Tellurometer MA-100 Jaakola,[7] the Kern ME-3000 Mekometer (see Froome,[8] Meir-Hirmer,[9] and Murname[10]) and the Comrad Geomensor 204 DME (Figure 6.17). The latter instrument has a range of up to

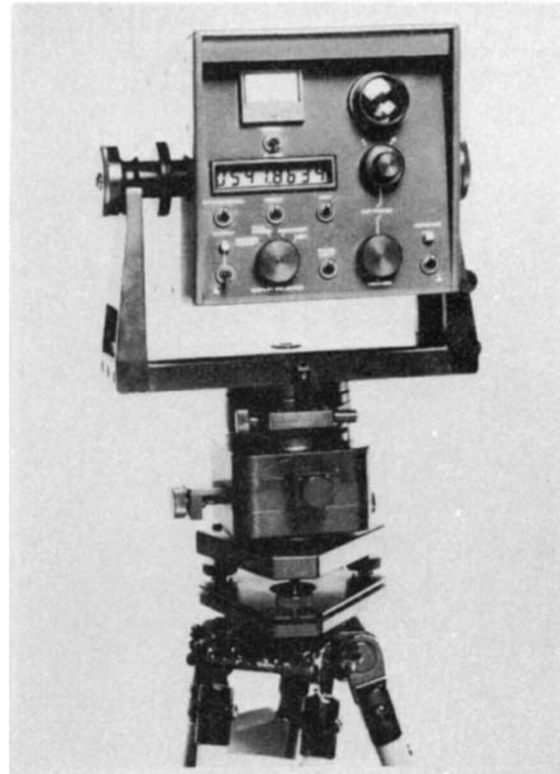


Figure 6.17 High-precision EDM: Comrad Geomensor 204DME

10 km with a standard error of ± 0.1 mm ± 0.5 p.p.m. Further up-to-date technical information on many modern EDM instruments can be found in Burnside.[11]

Principle of measurement. Although there is a wide variety of EDM instruments on the market, they all measure distance using the same basic principle. This can be most clearly illustrated by means of the flow diagram (Figure 6.18), which relates specifically to electro-optical instruments.

An electromagnetic (EM) signal of wavelength equal to either 560 nm (visible light), 680 nm (HeNe laser) or 910 nm (infra-red) is generated. This signal is subsequently amplitude-modulated before being transmitted through the optical system of the instrument towards a retro-reflector mounted at the end of the line to be measured. The signal is then retro-reflected, or redirected through 180°, by a precisely ground glass corner cube. Cheaper acrylic corner cubes may also be used.[12,13] This reflected signal is consequently directed towards the receiving optical system. On entering the optical system of the instrument, the signal is converted by means of a photomultiplier into an electrical signal.

The next stage involves the measurement of the phase difference between the transmitted and received signals and the conversion of this information into distance. Figure 6.19 shows the path taken by an EM signal radiated by an EDM instrument together with the instantaneous phase of the signal. It is apparent that the distance X–Y–X travelled by the EM signal is equivalent to twice the distance to be measured. Also, this distance can be seen to be related to the modulation wavelength (λ) and the fraction of the wavelength ($\Delta\lambda$) by the following relationship:

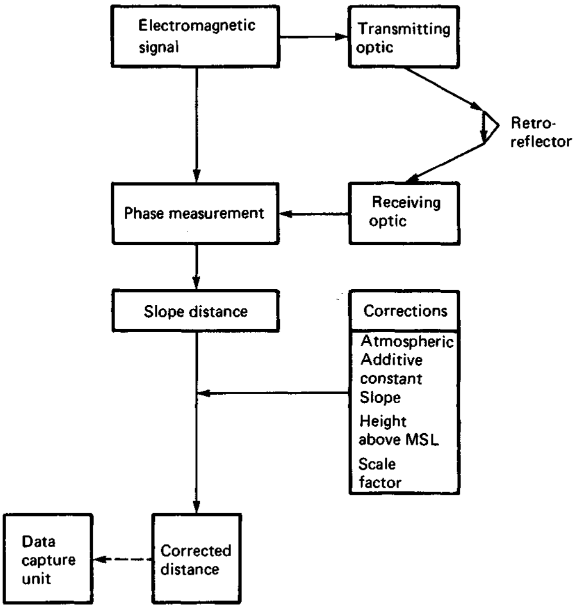


Figure 6.18 Principle of operation: electro-optical distance measurement

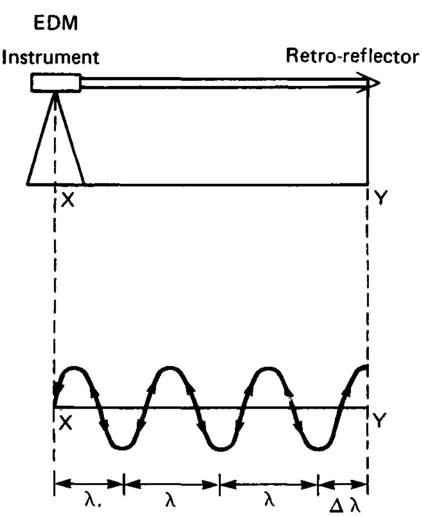


Figure 6.19 Double path measurement using EDM

$$2D = n\lambda + \Delta\lambda \qquad (6.14)$$

where n is an unknown integer number of wavelengths. The determination of the distance therefore involves resolving both $\Delta\lambda$ and n. Phase detectors are used to determine $\Delta\lambda$ which effectively measures the phase difference between the transmitted and received signal and, hence, allows the fractional part of the distance less than one full wavelength to be determined. The value of n can be determined by using two or more EM signals of slightly varying wavelengths. For example, assume $2D = 25.5$, $\lambda_1 = 2.5$ m and $\lambda_2 = 2.4$ m then:

$$2D = 2.5n_1 + 0.5 \qquad (6.15)$$

and

$$2D = 2.4n_2 + 1.5 \qquad (6.16)$$

Assuming $n_1 = n_2$ for short distances, solving for n leads to

$$n = 10 \qquad (6.17)$$

Substituting in Equations (6.15) or (6.16)

$$D = 12.75\text{ m} \qquad (6.18)$$

This entire process is fully automatic in modern instruments, taking approximately 10 to 20 s to complete.

As with any other method of distance measurement, it is necessary to apply several corrections to the measured slope distance in order to determine the corrected horizontal distance. The first correction to be applied is the atmospheric or refractive index correction. Just as a steel tape varies in length with variations in temperature and pressure so, too, does the modulation wavelength of an EDM instrument. It is therefore necessary to measure the temperature, pressure and, in some cases, relative humidity during measurements. A correction is then applied to compensate for the variation in modulation wavelength caused by variations in atmospheric conditions.

Many instruments have the facility to compute automatically and apply this correction to observations directly in the field. Temperature, pressure and relative humidity readings are taken and the appropriate reading to be set on the refractive index correction dial is read from a nomogram.

A second important correction is the additive zero or prism constant. This correction represents the difference between the electro-optically determined distance and the correct length of line. It is a combination of the errors due to prism offset and the variation in the physical and electrical centres of the EDM instrument. Many manufacturers design their corner cube reflectors in order to eliminate this correction totally. However, if several different types of corner cube are being used, it is essential that a full field calibration be undertaken in order to determine the correction. (See Schwendener,[14] Ashkenazi and Dodson,[15] and Sprent and Zwart[16] for further details of the procedure for instrument calibration.) The slope correction is the same as for DDM. For distances measured above or below mean sea-level (MSL) a correction is necessary in order to reduce the distance to its equivalent at MSL. The correction (C_4) is given by:

$$C_4 = (-LH_m)/R \qquad (6.19)$$

where H_m is the mean height of the instrument and reflector above MSL and R is the radius of the Earth (6370 km). Finally, if the distance is to be used for computation of coordinates on the national grid, the horizontal distance at MSL must be multiplied by the local scale factor. For the Transverse Mercator projection, the local scale factor (F) may be approximately calculated from:

$$F = 0.999\,601\,27 + [1.228 \times 10^{-14} \times (E - 400\,000)^2] \qquad (6.20)$$

where E is the mean local national grid easting in metres of the line to be measured.

6.2.3 Height measurement using the level

The third and final quantity which is measured is height or, more correctly, height difference. This is achieved by means of a level.

The fundamental principle of the level is illustrated by Figures 6.20 and 6.21. Figure 6.20 represents the situation which normally exists when the level is initially set up. In this case, the standing axis of the level and the vertical do not coincide. Hence, the line of collimation of the level will not be horizontal. Figure 6.21 represents the geometrical arrangement of the axis when the instrument has been levelled using the procedure outlined in 'Setting up' in section 6.2.1.3. It can be seen that completion of this procedure ensures that, firstly, the standing and vertical axes are made coincident and, secondly, if the instrument is in perfect adjustment, the line of collimation of the level is coincident with a horizontal line of sight.

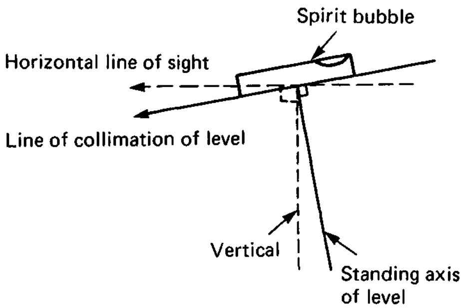


Figure 6.20 Geometry of the level axes: before levelling

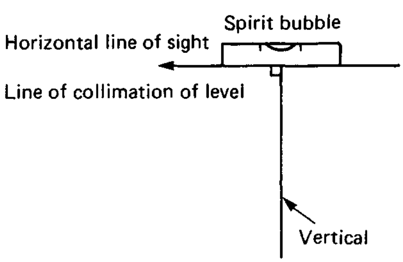


Figure 6.21 Geometry of the level axes: after levelling

Three distinct types of level are available for engineering survey purposes: (1) the dumpy level; (2) the tilting level; and (3) the automatic level.

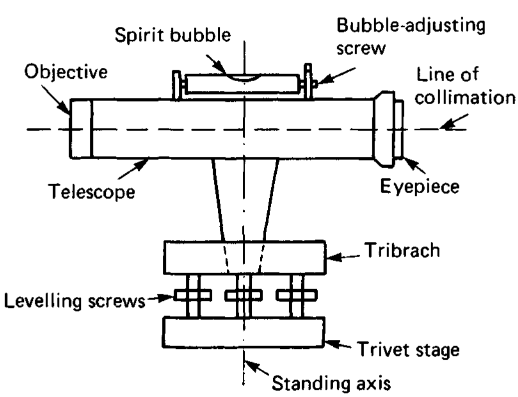


Figure 6.22 Level construction: dumpy level

6.2.3.1 Dumpy level

The dumpy level was so named because of the rather short telescopes which were used with early versions of this instrument.

The construction of a typical dumpy level is shown in Figure 6.22. The most distinctive feature of this type of level is that the axis of the telescope is fixed rigidly to the standing axis of the instrument. In order to satisfy the condition that both the vertical and standing axes are coincident, the standard levelling procedure outlined in section 6.2.1.3 is carried out. Rotation of the telescope will now define a horizontal plane.

In the past, this type of level was very popular for general engineering work. It has, however, been replaced in recent years by the automatic level.

6.2.3.2 Tilting level

The tilting level is a more precise instrument than the dumpy level. Figure 6.23 illustrates the main features. In contrast to the dumpy level, the telescope is not rigidly attached to the standing axis but is able to be tilted in a vertical plane about a pivot point X, by means of a tilting screw.

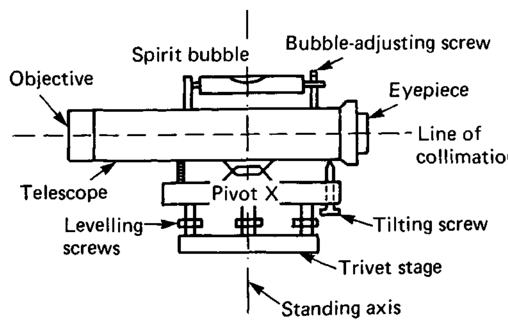


Figure 6.23 Level construction: tilting level

Prior to recording an observation, the instrument is approximately levelled. This is normally achieved by means of a 'ball-and-socket' arrangement and a small circular bubble. In order to set the standing axis exactly vertical, the tilting screw is turned and the main bubble altered until a coincident position (Figure 6.24), as viewed through a small auxiliary eyepiece, is achieved.

If the telescope is now rotated horizontally to sight a second or subsequent point, it is important to relevel the main bubble by means of the tilting screw.

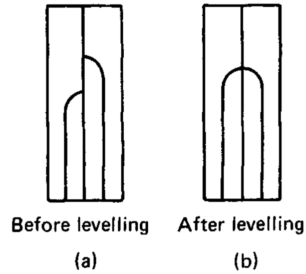


Figure 6.24 Coincidence bubble-reading system

6.2.3.3 Automatic level

This type of level is not, as the name suggests, totally automatic. Human intervention is still necessary. However, one major source of human error, that of setting the bubble, is replaced by an automatic compensating system. In common with the tilting level, approximate levelling is still necessary. The tedious and error-prone bubble-setting process, however, is eliminated. As with the dumpy level, the instrument defines a horizontal plane when rotated. The automatic level therefore combines the speed of operation of the dumpy level with the precision of the tilting level.

Figure 6.25 illustrates the main components of this type of level. The essential feature of the instrument is the incorporation of an automatic optical–mechanical compensating mechanism. The use of such a system ensures that the line of collimation as defined by the centre cross-hair will trace out a horizontal plane irrespective of the fact that the optical axis of the instrument may not be exactly horizontal. It is, however, necessary to level the instrument approximately in order to ensure that the line of sight is within range of the compensating mechanism.

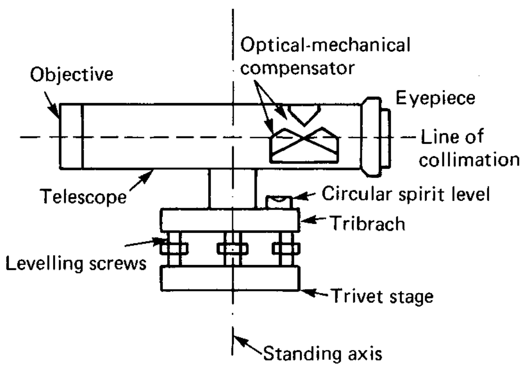


Figure 6.25 Level construction: automatic level

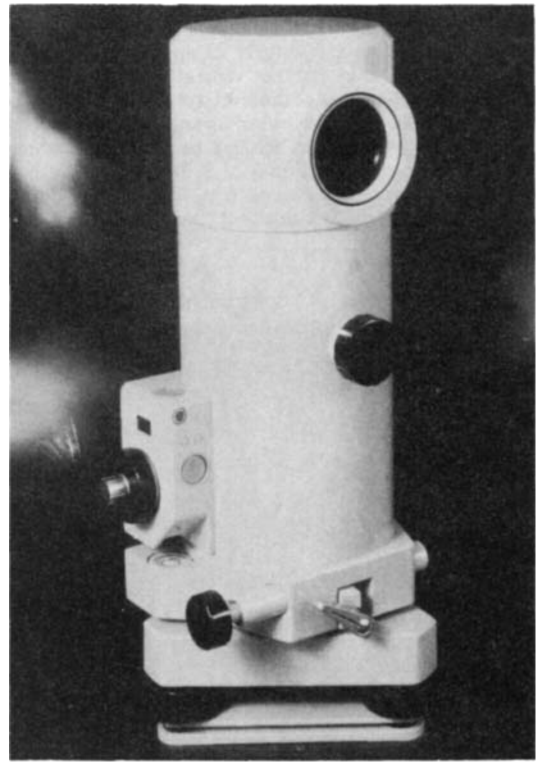


Figure 6.26 Zeiss (Jena) Ni 007 automatic precise level

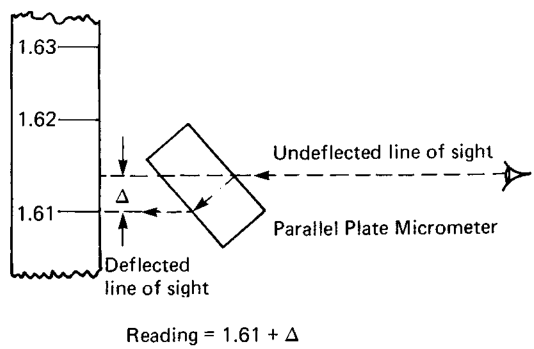


Figure 6.27 Operation of a parallel plate micrometer for precise levelling

For high-precision levelling, e.g. in order to detect the settlement of a building, a parallel plate micrometer (PPM), attached to the front of the objective of the telescope normally forms part of the construction of the level. Almost all precise levels in use nowadays are of the automatic design and, ideally, should be designed so that the PPM forms an integral part of the instrument, rather than being an 'add-on' attachment. One such instrument is the Zeiss (Jena) Ni 007 (Figure 6.26). This particular instrument also has an unusual compensating mechanism which results in the 'periscope'-type appearance of the instrument. The PPM operates by deflecting the line of sight to the nearest whole staff graduation, the amount of displacement which is required being measured by a micrometer. This value is then added to the staff reading to give the final staff reading. This is illustrated in Figure 6.27. Clearly in an operation such as precise levelling it is important to minimize the effects of systematic errors. This is partially overcome by a suitable field procedure,[17] and partially by ensuring that the staff is maintained at a constant length. In order to achieve this, an invar staff with stabilizing arms and a level bubble attachment is normally used.

6.2.3.4 Laser level

Lasers are monochromatic, coherent and highly collimated light sources, initially developed in the 1940s. Until relatively recently, their use has tended to be restricted to the field of pure scientific research. Nowadays, however, the laser is a widely used tool in land surveying for distance measurement, alignment,[18] and levelling purposes.

There are essentially two types of laser in use in civil engineering: (1) the fixed-beam; and (2) the rotating beam laser. The fixed-beam laser projects a single highly collimated light beam to a single point. This design is particularly suited to alignment problems. The rotating-beam laser, in contrast, takes the fixed-beam source and rotates it at high speed, so forming a plane (either in the horizontal or vertical sense), of laser light. This design is more appropriate for levelling or grading purposes.

The Spectra-Physics EL-1 shown in Figure 6.28 is a typical example of the laser levels currently in use. The laser beam in

Figure 6.28 Spectra-Physics EL-1 (Spectra-Physics)

this case forms a 360° horizontal plane which is detected by a portable sensing device also shown in Figure 6.28. The laser unit automatically corrects for any error in level of the instrument, providing it has been roughly levelled to within 8° of the vertical. An accuracy of ± 5 to 6 mm per 100 m up to a maximum range of 300 m can be achieved with this type of instrument.

6.2.3.5 Collimation error

So far the assumption has been made that once the standing axis of the level has been set truly vertical, then the line of collimation will be horizontal. This may not always be the case.

If this condition does not occur, then a collimation error is said to exist. This is illustrated in Figure 6.29. If accurate levelling is to be achieved, it is essential that a regular testing procedure is established in order to check the magnitude of any collimation error that may exist.

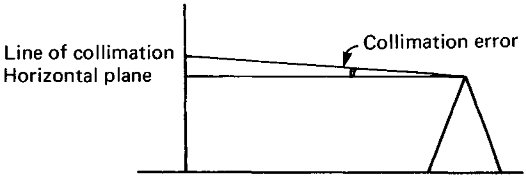


Figure 6.29 Collimation error

A common field procedure which can be used to test a level is known as the 'two-peg test'. The procedure is as follows:

(1) Set out two points A and B approximately 50 m apart, as shown in Figure 6.30(a). The level is set up at the mid-point of AB and levelled as in section 6.2.1.3.
(2) A reading is taken on to a staff held at points A and B and the difference between the two readings calculated. This value represents the true difference in height between A and B. Any collimation error which exists will have an equal effect on both readings and, hence, will not affect the difference between the readings. In this case the difference in height is $1.415 - 0.932 = 0.483$ m.
(3) The instrument is now moved to a point C close to the staff at B (about 3 to 5 m away), as in Figure 6.30(b). The reading on staff B is recorded (1.301). If no collimation error exists, the reading on staff A should be equal to the reading on staff B ± the true difference in height as established in (2), i.e. $1.301 + 0.483 = 1.784$ m.
(4) The actual observed reading on staff A is now recorded (1.794). Any discrepancy between this value and that derived previously in (3) indicates the magnitude and direction of any collimation error. For example, in this case, the error would be $1.794 - 1.784 = 10$ mm per 50 m.

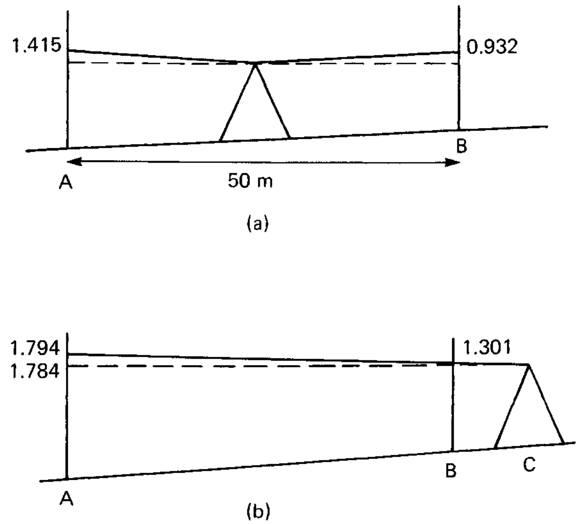


Figure 6.30 Two-peg test

An error of up to 2 to 3 mm over this distance would be acceptable. If, however, the error is greater than this, the instrument should be adjusted. Unlike theodolite adjustments, this type of adjustment can normally be performed without any great difficulty by the engineer and the procedure is as follows.

For the dumpy and automatic level: alter the position of the cross-hairs until the centre cross-hair is reading the value which should have been observed from step (3) above. This is achieved by loosening the small screws around the eyepiece which control the position of the cross-hairs.

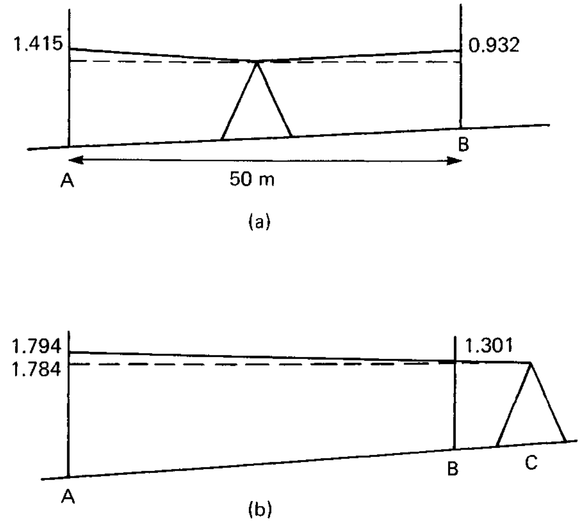
For the tilting level: again alter the position of the centre cross-hair until it is reading the value previously determined in (3), in this case by tilting the telescope using the tilting-screw. Unfortunately, this will displace the bubble. The bubble must, therefore, be centralized by means of the bubble-adjusting screw.

6.3 Surveying methods

6.3.1 Horizontal control surveys

Any engineering survey or setting-out project, regardless of its size, requires a control framework of known co-ordinated points. Several different control methods are available as des-

Figure 6.28 Spectra-Physics EL-1 (Spectra-Physics)



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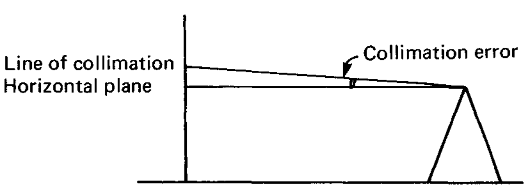

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6.3 Surveying methods

6.3.1 Horizontal control surveys

Any engineering survey or setting-out project, regardless of its size, requires a control framework of known co-ordinated points. Several different control methods are available as described below. The choice of which method to use depends on many factors, e.g. the purpose for which the control is required, the accuracy required, the density of control points which is required, the type of equipment and computing facilities which are available and, lastly, the physical nature of the ground.

6.3.1.1 Triangulation

Triangulation is the oldest, and in the past was the most common, method of control for large civil engineering projects. The principles are well known and essentially involve the establishment of a measured baseline from which a network of triangles is formed, all of the angles of the triangle being measured. The development of EDM has, however, led to the establishment of several alternative control methods, such as trilateration and traversing. The introduction of EDM has therefore tended to make 'classical triangulation' obsolete as a method of control.

6.3.1.2 Trilateration

Trilateration is a method of establishing control which involves the direct measurement, normally using EDM, of all the sides of a network of triangles, in contrast to triangulation which involves the measurement of angles. Although the method has been used in this 'classical form', it does not offer any significant advantages over the method of triangulation. The method has therefore not become particularly common.

6.3.1.3 Traversing

A traverse is a method of establishing control by measuring the distance between successive points and also the horizontal angle between adjacent stations, as shown in Figure 6.31.

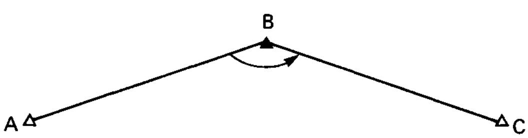


Figure 6.31 Traversing

The method is very popular for several reasons. Firstly, it is a more flexible method than triangulation. In the case of triangulation, the positions of the control stations must be chosen so that not only are they intervisible, but also that the triangles formed are well conditioned. For this reason the reconnaissance stage in triangulation projects is extremely important and often very time-consuming. In contrast, with traversing, much less attention has to be paid to the reconnaissance stage, since it is only required that adjacent stations be intervisible. This allows the surveyor much greater flexibility in the choice of control station positions. The stations can then be positioned in areas close to the detail to be picked up, or close to the project for which they are required. This can be of enormous benefit in areas which are either very flat or, alternatively, heavily forested. A second reason for the popularity of traversing is the computational simplicity, both of determining provisional coordinates and also of adjusting any misclosure which may exist. It should, however, be mentioned that in recent years more rigorous techniques of adjustment, based on the principle of least squares, have become more common for the adjustment of traverse.

Traversing does, however, suffer from one serious drawback: the lack of redundant observational data. As a consequence, the effect of small errors of measurement is not only difficult to detect but is also cumulative in nature. To counteract this problem, additional angle and distance observations are often taken in order to strengthen the control framework. In the past, this additional information tended to be used solely for the detection of gross errors. Nowadays, by using a suitable adjustment technique, these additional observations can be used to improve the precision of the coordinates.

The normal procedure adopted for traverse adjustment is firstly to determine and adjust the angular misclosure and, secondly, to determine and adjust the misclosure in easting and northing. The angular misclosure is determined, in the case of a closed polygon, by summing the internal angles. These should total $(2n-4)$ right angles, where n is the number of traverse sides. A misclosure of $>(20''\sqrt{n})$, for example, would not be acceptable for site traverses.

The question of which method to use for the adjustment of any misclosure in eastings and northings is a matter which has been examined by Schofield.[19] Traditionally, the Bowditch and Transit methods have been used.

Bowditch: $$\Delta E_c = M_E \frac{d}{\Sigma d} \text{ and } \Delta N_c = M_N \frac{d}{\Sigma d} \tag{6.21}$$

Transit: $$\Delta E_c = M_E \frac{\Delta E}{\Sigma|\Delta E|} \text{ and } \Delta N_c = M_N \frac{\Delta N}{\Sigma|\Delta N|} \tag{6.22}$$

where M_E, M_N, is the misclosure in easting, northing, d is the length of a traverse leg, $\Sigma|\Delta E|$, $\Sigma|\Delta N|$ is the absolute sum of the provisional ΔE, ΔN, and ΔE_c, ΔN_c is the correction to be applied to the provisional ΔE, ΔN.

For most small site traverses observed using a theodolite and steel tape both techniques will give acceptable results. Schofield,[19] however, also discussed the problems which arise when semi-rigorous methods of adjustment such as Bowditch and Transit are used to adjust modern EDM traverses. The main conclusion reached is that both methods are based on assumptions which are not applicable to EDM. For example, it is assumed that the expected error in an EDM measurement is proportional to the distance measured, which is clearly not true. It is suggested that all EDM-based traverses should be adjusted by a rigorous method such as variation of coordinates.

6.3.1.4 Survey networks

The combination of angle, distance and orientation measurements to form a control framework is now commonly referred to as a survey network. The advantages of such an approach are considerable. Firstly, the scale and orientation errors associated with classical triangulation can be reduced by the inclusion of additional distance and azimuth measurements. Secondly, the control framework does not suffer from the serious propagation of errors which can occur in traversing. Thirdly, the optimum number of observations (angular and distance) can be determined before the fieldwork commences, using computer simulation techniques. Finally, by using the method of variation of coordinates a least squares procedure can be used to determine the most probable values of the coordinates.

The principle of least squares states that the most probable value of a sample is that for which the sum of the squares of the residuals is a minimum. *Variation of coordinates* is a computational method, based on this principle, which is used for the determination of coordinates in a survey network.

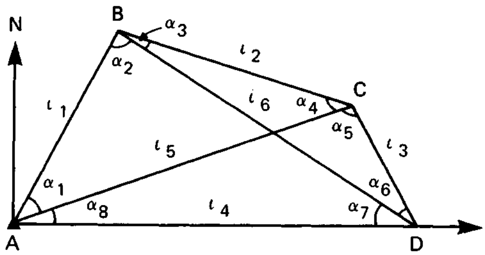


Figure 6.32 Adjustment of a braced quadrilateral by variation of coordinates

Consider Figure 6.32, which illustrates a small network in which all the eight angles $\alpha^{o}1$ to 8 and all distances $l^{o}1$ to 6 have been observed. Point A is fixed, the bearing to point D is fixed in order to orient the network and the a priori standard errors of the observations are estimated. There are, therefore, fifteen observations and six unknowns to be determined, i.e. the eastings and northings of points B, C and D. The method of solution is as follows: (1) assume provisional coordinates for all points. These may either be scaled graphically from a plan or computed using a selection of the measurements; (2) using the provisional coordinates, compute values for the angles $\alpha^{c}1$ to 8 and the distance $l^{c}1$ to 6; and (3) set up an observation equation for each individual observation.

For a distance l_{ij}:

$$-\cos\beta_{ij}\mathrm{d}N_i - \sin\beta_{ij}\mathrm{d}E_i + \cos\beta_{ij}\mathrm{d}N_j + \sin\beta_{ij}\mathrm{d}E_j = (l^{o}_{ij} - l^{c}_{ij}) + v \qquad (6.23)$$

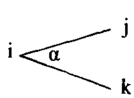
or

$$\mathrm{Pd}N_i - \mathrm{Qd}E_i + \mathrm{Rd}N_j + \mathrm{Sd}E_j = (l^{o}_{ij} - l^{c}_{ij}) + v \qquad (6.24)$$

For bearing φ_{ij}

$$\frac{\sin\beta_{ij}}{l_{ij}}\mathrm{d}N_i - \frac{\cos\beta_{ij}}{l_{ij}}\mathrm{d}E_i - \frac{\sin\beta_{ij}}{l_{ij}}\mathrm{d}N_j + \frac{\cos\beta_{ij}}{l_{ij}}\mathrm{d}E_j = (\varphi^{o}_{ij} - \varphi^{c}_{ij}) + v \qquad (6.25)$$

For angle α_{ijk}:



$$\left(\frac{\sin\beta_{ik}}{l_{ik}} - \frac{\sin\beta_{ij}}{l_{ij}}\right)\mathrm{d}N_i + \left(\frac{-\cos\beta_{ik}}{l_{ik}} + \frac{\cos\beta_{ij}}{l_{ij}}\right)\mathrm{d}E_i$$

$$+\frac{\sin\beta_{ij}}{l_{ij}}\mathrm{d}N_j - \frac{\cos\beta_{ij}}{l_{ij}}\mathrm{d}E_j - \frac{\sin\beta_{ik}}{l_{ik}}\mathrm{d}N_k + \frac{\cos\beta_{ik}}{l_{ik}}\mathrm{d}E_k$$

$$= (\alpha^{o}_{jik} - \alpha^{c}_{jik}) + v \qquad (6.26)$$

For a position

$$\mathrm{d}N_i = (N^{o}_i - N^{c}_i) + v \qquad (6.27)$$

$$\mathrm{d}E_i = (E^{o}_i - E^{c}_i) + v \qquad (6.28)$$

where β_{ij} = direction of line ij, $\mathrm{d}N_i$, $\mathrm{d}E_i$ = the unknowns, the corrections to the provisional coordinates, and v = residual.

These observation equations can be expressed in matrix form:

$$\underset{15}{\mathbf{A}} \quad \underset{6}{\mathbf{X}} = \underset{15}{\mathbf{b}} + \underset{15}{\mathbf{V}} \qquad (6.29)$$

$$\begin{bmatrix}\text{Matrix of}\\ \text{coefficients}\end{bmatrix}^{6}_{15}\begin{bmatrix}\text{Vector of}\\ \text{unknowns}\end{bmatrix}^{1}_{6} = \begin{bmatrix}\text{Vector of}\\ \text{o–c terms}\end{bmatrix}^{1}_{15} + \begin{bmatrix}\text{Vector of}\\ \text{residuals}\end{bmatrix}^{1}_{15}$$

The solution is found by forming the normal equations.

$$A^{\mathrm{T}}WA\mathbf{X} = A^{\mathrm{T}}Wb \qquad (6.30)$$

where W is the weight matrix, the diagonal elements of which are equal to $1/\sigma^2$ where σ refers to the a priori standard error of the observation.

The normal equations may then be solved using a Choleski's triangular decomposition method or by matrix inversion.

$$\mathbf{X} = (A^{\mathrm{T}}WA)^{-1}A^{\mathrm{T}}Wb \qquad (6.31)$$

The column vector $\mathbf{X}$ therefore contains the corrections ($\mathrm{d}N_i$, $\mathrm{d}E_i$) to the provisional coordinates of the unknown points. Using these new values for the coordinates, the entire computational procedure is repeated until there is no further change in the coordinates.

Usually when this method is used a complete error analysis of the results is carried out. By this process information about the precision and reliability of the coordinates can be obtained. For further reading on this aspect of the method see Ashkenazi,[20,21] Ashkenazi *et al.*,[22,23] and section 3.4.2.

6.3.2 Detail surveys

After the main control survey has been observed and computed, a detail survey is carried out in order to locate the positions of features which are to be presented on the map.

6.3.2.1 Tacheometry

The most common method of carrying out a detail survey by ground methods at scales smaller than 1:500 is that of stadia tacheometry. The basic principles have been outlined in section 6.2.2.2.

In practice, the observational and booking procedure should be as follows. The theodolite should be set up and levelled over the point from which observations are to be taken. This point may coincide with one of the main control survey stations, or more commonly be one of a series of subsidiary control stations which have been established by traversing from the main control. By this process not only are any errors contained within the control network, but also the positions of the subsidiary control will be closer to the detail which is to be surveyed.

The circle of the theodolite should then be oriented until a horizontal circle reading of 0° coincides with the direction to an adjacent control point. This is known as the reference object (RO). This procedure simplifies the subsequent plotting of the field observations. The height of instrument should be recorded.

The staffman is then directed to the various points which are to be mapped. The staff is held vertical over the point and the stadia readings, vertical angle and horizontal circle reading to the point are observed from the theodolite. Several techniques for speeding-up the field recording of the stadia hair readings

Table 6.3 Booking and reduction of tacheometric readings

Observer: TJMK
Booker: DRG
Date: 22/9/85

Station B
Height of Instrument: 1.690 m
Reduced Level: 58.35 m

Point	Horizontal circle	Staff readings U M L	Staff intercept (*S*)	Vertical circle	Horizontal distance (D_H) (100 $\cos^2 \theta$)	*V* (50 *s* sin 2θ)	Reduced level
A(RO)	0° 00′	—	—	—	—	—	—
1	10° 30′	1.430 1.215 1.000	0.430	91° 00′	42.99	0.75	59.58

have been advocated.[24] Assuming that a pocket calculator, preferably programmable, will be used to reduce the observations, the most efficient field method is to set the bottom hair to the 1 m or 2 m point of the staff. Using this method, the mental calculation of the stadia intercept becomes much quicker.

The booking procedure for stadia observations is illustrated by Table 6.3. It is also essential to draw, whilst in the field, a good sketch-map of the area being surveyed. This is invaluable when the results are being plotted.

6.3.2.2 Chaining

This is the simplest form of detail surveying. The method involves measuring the lengths of the sides of a series of triangles or braced quadrilaterals. Points of detail are then picked-up by measuring offsets from these lines. The procedure is illustrated in Figure 6.33.

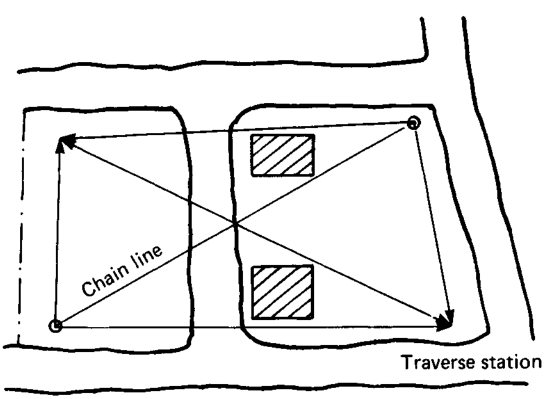


Figure 6.33 Chaining

In this case, two traverse stations from an existing control survey are within the boundary of the site which is to be surveyed.

The first stage in the survey involves breaking-down this existing control into either well-conditioned triangles or braced quadrilaterals. In this case, a braced quadrilateral is chosen. The inclusion of diagonal measurements provides an independent field check on the measurements. These 'chain lines' are normally measured with a steel band.

The second stage in the survey process involves the measurement of offset or ties from these chain lines in order to pick-up the survey detail. Figure 6.34 illustrates the distinction between offsets and ties.

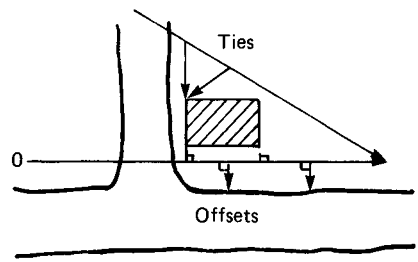


Figure 6.34 Offsets and ties

Ties, which involve two measurements to one point, are normally employed when the offset distance is long and, hence, the accuracy with which the perpendicular to the chain line can be set out is low. Offset and tie measurements are normally made with a fibreglass tape.

A standard method of booking is conventionally adopted for chain surveying. Details of this and other points relating to chain surveying can be found in any of the standard surveying textbooks listed in the bibliography.

6.3.3 Vertical control surveys

In general, all civil engineering projects require not only planimetric control, but also vertical or height control points. The two most common methods of obtaining this height control are by levelling and by trigonometrical heighting.

6.3.3.1 Levelling

Levelling is the name given to the process of determining, by means of a surveyor's level, the height of a point above some datum, normally mean sea-level.

As is the case with horizontal control, it is important to work from 'the whole to the part'. It is therefore normal practice to design a levelling control framework in a hierarchical manner, in order to contain small errors. Typical maximum allowable misclosures for level loops in one such hierarchical design are shown in Table 6.4.

The basic principle of levelling involves taking horizontal backsight (BS), foresight (FS) and intermediate sight (IS) readings, as defined by the line of collimation of the level, on to vertical staves as shown in Figure 6.35. The difference between successive readings indicates the difference in height between points. By this process it is therefore possible to determine the reduced level (RL) of a series of points.

Table 6.4 Accuracy of levelling (K is the distance in kilometres)

Order	*Maximum allowable misclosure* (m)
1st	$0.004\sqrt{K}$
2nd	$0.008\sqrt{K}$
3rd	$0.012\sqrt{K}$
4th	$0.024\sqrt{K}$

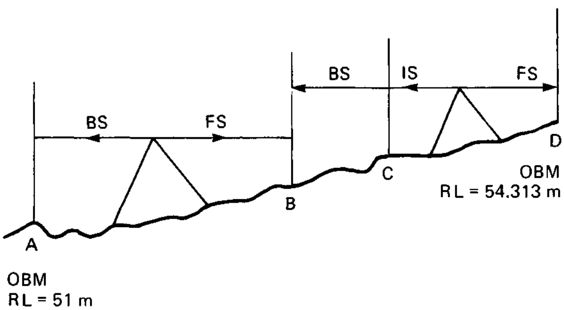


Figure 6.35 Levelling

The reduced level of a point is defined as its height above some datum. In the UK, the fundamental datum established by the Ordnance Survey is mean sea-level at Newlyn, Cornwall, known as Ordnance Datum. A further series of points of known height have been established throughout the UK and these are known as Ordnance bench-marks (OBM).

The results of a levelling operation are, by convention, booked in a standard manner. Two methods of booking are used. The rise and fall method (Table 6.5) is usually employed when running lines of levels between bench-marks in order to establish additional supplementary height points. The height of collimation method (Table 6.6), on the other hand, is more suitable for tasks such as recording cross-sectional information, in which many intermediate sights have been taken.

Contouring. A contour is an imaginary line joining points of equal elevation. The level can be used in a variety of ways for contouring. One of the simplest methods is by means of grid levelling.

With this technique, the area to be contoured is covered by an imaginary grid of lines forming squares of 10, 20 or 30 m. The level is set-up in a central position and levels are then taken to a site temporary bench-mark (TBM) and at the intersections of the grid lines, as shown in Figure 6.36.

Contours may then be interpolated either graphically or mathematically from the grid of levels.

Quantity determination. The method of grid levelling provides a convenient means of determining earthwork quantities of, for example, borrow pits.

The depth of cut (h) at each intersection point is established. This will be equal to the difference between the ground-level and the proposed formation level of the borrow pit. Figure 6.37 illustrates a simple example for a 20-m grid of levels.

The volume of excavation consists of a series of rectangular prisms each having a base area (A), in this case equal to 400 m^2. The total volume (V) for the general case is therefore:

$$V=\frac{A}{4}(\Sigma h_1+2\Sigma h_2+3\Sigma h_3+4\Sigma h_4) \tag{6.32}$$

Table 6.5 Rise and fall method of booking

	Backsight	Intermediate sight	Foresight	Rise	Fall	Reduced level	Remarks
	2.345					51.000	OBM
	1.935		0.632	1.713		52.713	B
		1.213		0.722		53.435	C
			0.335	0.878		54.313	OBM
Σ	4.280		0.967	3.313	0.000		

Checks: ΣBS $-$ ΣFS = 3.313 m ΣRISE $-$ ΣFALL = 3.313 m.
First RL $-$ last RL = 3.313 m

Table 6.6 Height of collimation method of booking

	Backsight	Intermediate sight	Foresight	*Height of Collimation (HC)		Reduced level (RL)	Remarks
	2.345			53.345		51.000	OBM
	1.935		0.632	54.648		52.713	B
		1.213				53.435	C
			0.335			54.313	OBM
Σ	4.280		0.967				

Checks: ΣBS $-$ ΣFS = 3.313 m First RL $-$ last RL = 3.313 m
*Height of collimation = reduced level + backsight
RL = HC $-$ FS or IS

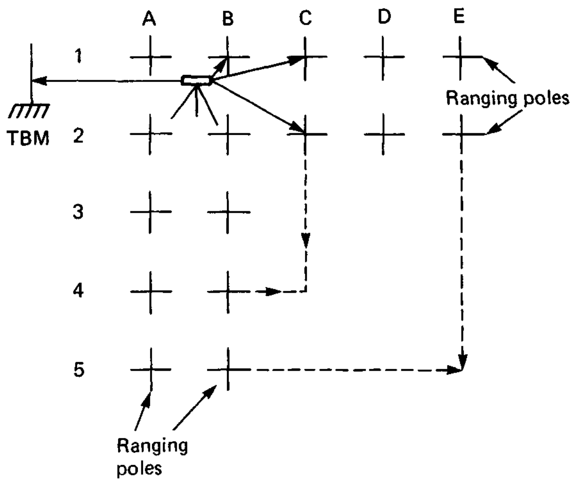


Figure 6.36 Grid levelling

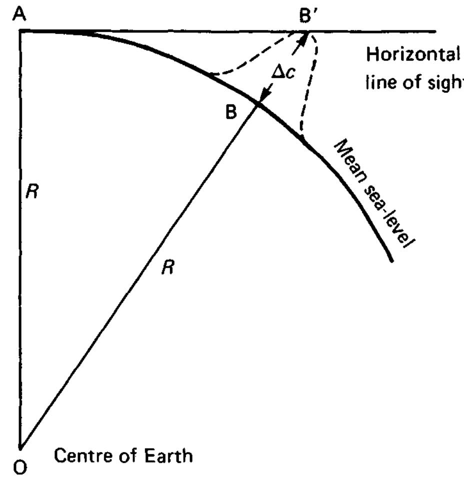


Figure 6.38 Trigonometrical heighting: Earth curvature

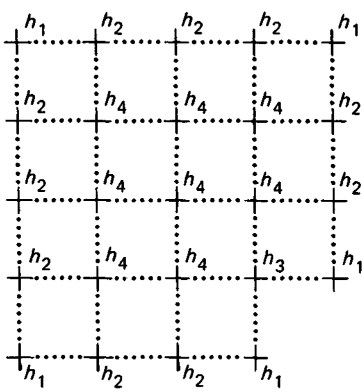


Figure 6.37 Quantity determination using a grid of levels

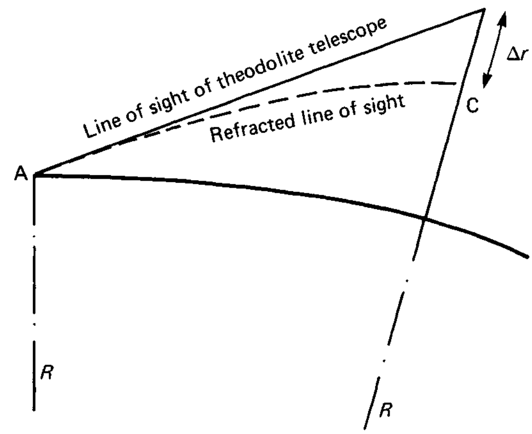


Figure 6.39 Trigonometrical heighting: atmospheric refraction

where h_1 = depth of cut used once, h_2 = depth of cut used twice, etc.

The volume determination assumes that the ground slope between grid intersections is constant. Clearly, therefore, by reducing the size of the grid, the accuracy of the quantity determination will be increased.

6.3.3.2 Trigonometrical levelling

For projects where the acceptable accuracy requirements of the height control are lower than would be obtained by levelling, the method of trigonometrical heighting is normally used.

The method is based on the measurement of the vertical angle between two points by means of a theodolite, together with either the slope or horizontal distance. By simple geometry, the difference in height can therefore be calculated. As the distance between the points increases, however, the effects of two phenomena, Earth curvature and refraction, become more significant. Figures 6.38 and 6.39 illustrate the highly exaggerated effect of these phenomena.

In Figure 6.38, a distant point B′ appears too low by an amount Δc. A positive Earth curvature is therefore necessary. It can also be seen in Figure 6.39 that the effect of refraction is to refract the line of sight of the telescope so that a distant point C appears too high by an amount Δr. A negative refraction correction is therefore required.

It can be shown that the difference in height between A and C can be determined by the equation:

$$\Delta h_{AC} = d \tan \theta + (\Delta c - \Delta r) \tag{6.33}$$

where $$\Delta c - \Delta r = \frac{d^2(1-2K)}{2R} \tag{6.34}$$

and K = coefficient of refraction $\simeq 0.07$, R = radius of Earth = 6378 km, d = horizontal distance, and θ = vertical angle.

If $k = 0.07$ and d is in kilometres, then $\Delta c - \Delta r = 0.0675d^2$ m.

The results obtained using this simple approach can be increased significantly by observing vertical angles from both ends of a line. Reciprocal observations of this type, particularly if they are observed simultaneously, can eliminate completely the necessity for Earth curvature and refraction corrections. In this case, the difference in height between A and C can be computed by the general expression

$$\Delta h_{AC} = \frac{d \tan \theta_A + \theta_c}{2} + \frac{(h_{S_A} - h_{T_C}) - (h_{S_C} - h_{T_A})}{2} \qquad (6.35)$$

where θ_A = vertical angle from A to C, θ_c = vertical angle from C to A, h_{S_A} = height of signal at A, h_{S_C} = height of signal at C, h_{T_A} = height of theodolite at A, h_{T_C} = height of theodolite at C

6.3.4 Deformation monitoring surveys

Deformation monitoring surveys using conventional land-surveying methods, photogrammetric survey methods or specialist geotechnical methods are used to quantify the amount by which an engineering structure has moved both vertically and horizontally over specific periods of time.

The information which surveys of this type provide can be of critical importance and may be used to indicate that either: (1) the structure is stable and consequently safe; (2) the structure is experiencing small random movements which are not imposing significant forces on the structure; (3) the structure is experiencing small localized systematic deformations, e.g. as caused by seasonal effects, which may or may not be significant; (4) lastly and most significantly, the structure is experiencing deformation which is increasing as a function of time. This may indicate that remedial measures have to be taken, in some cases immediately, in order to avoid catastrophic consequences.

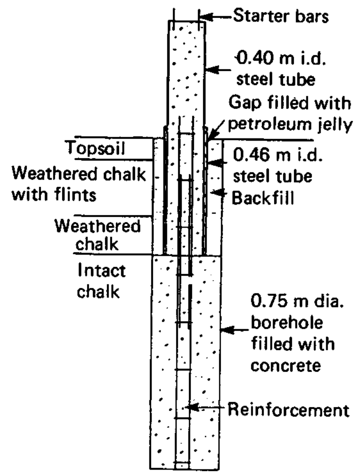
For a variety of reasons, surveys of this type have been increasingly more important in recent years. Several interrelated factors have been responsible for this increase in interest. Firstly, for many types of structures it is now a mandatory element of the civil engineering process that a monitoring survey be commissioned. Notable in this respect in this country are, for example, reservoirs, which now have to be monitored following the implementation of the 1975 Reservoirs Act. A further example, from Switzerland, is the requirement by the Swiss Federal Government for all dams with a height greater than 15 m and a cubic capacity greater than 50 000 m^3 to be monitored.[25] A second factor which has led to this increase in interest has been the speed of development both in the manufacture of precise survey instrumentation and also in high-speed, low-cost computing facilities. This, in conjunction with improvements in very elegant and highly sophisticated software for the design and analysis of surveying observations, now provides the surveyor and engineer with a highly accurate measurement system. Thirdly, the tolerances to which civil engineers are now designing and constructing many modern structures necessitates a much higher order of accuracy in the initial dimensional control and also in the subsequent deformation monitoring. Indeed, this factor may have acted as a 'springboard' for many of the developments in software and instrument design. It is also hoped that to some extent civil engineers are now more aware of the possibilities offered by modern survey techniques and are therefore requesting this type of survey more frequently than was the case in the past.

Most of the instruments which are used for deformation monitoring projects have been discussed in several of the previous sections of this chapter. It will therefore be the aim in this section to concentrate on two other important aspects of deformation monitoring surveys. The first is the design of suitable reference and monitoring points. Clearly, when very small displacements are being measured it is crucial that the points from which measurements are being recorded are not subject to movement. The second aspect which will be discussed is the computational processes associated with the horizontal control networks which are often used to quantify the extent of any structural deformation. Finally, it should be noted that deformation monitoring can often be carried out, in some cases more efficiently, using close-range photogrammetric techniques. Reference should be made to Chapter 7 for further details.

6.3.4.1 Design of reference and monitoring points

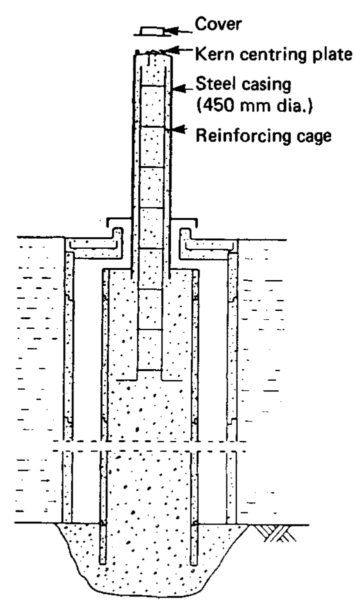
The full accuracy potential currently offered by modern surveying equipment for measuring small structural displacements can only be fully realized if care and attention is paid to the design of the reference points from which observations will be taken. Equally important is the need to design appropriate monitoring points to be placed on the structure under investigation.

Reference points. Two distinct types of survey reference point can be identified. The first, typically a survey pillar, forms the reference framework for the horizontal and vertical control measurements, whilst the second, a steel or concrete pile, forms the datum for levelling observations.



Reference monument

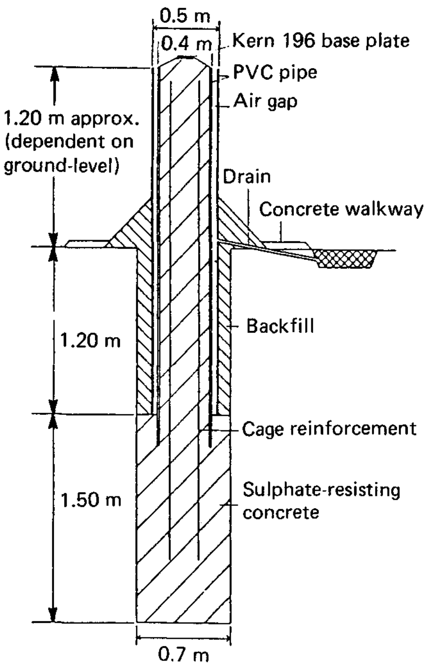
(a) Survey pillar (Penman and Charles[26])



Scammonden reference monument

(b) Survey pillar (Penman and Charles[26])

Figure 6.40 Survey pillar designs



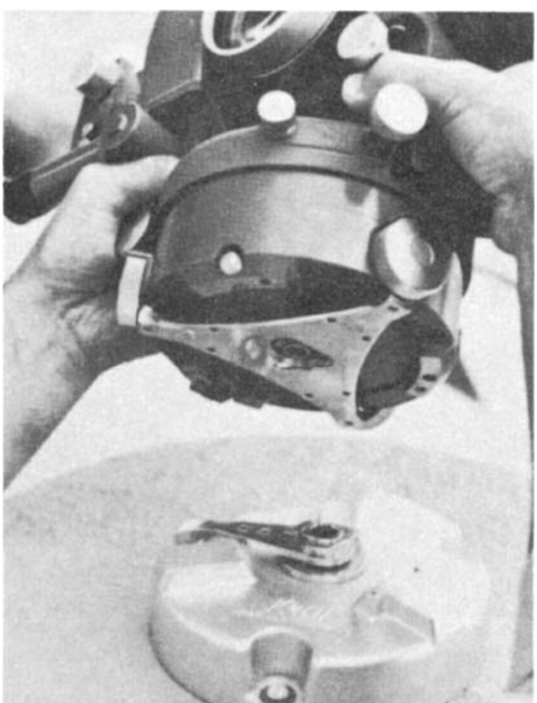
(c) Cross section of Base Line Pillar (Deeth *et al*[27])

Figure 6.40 (Continued) Survey pillar designs

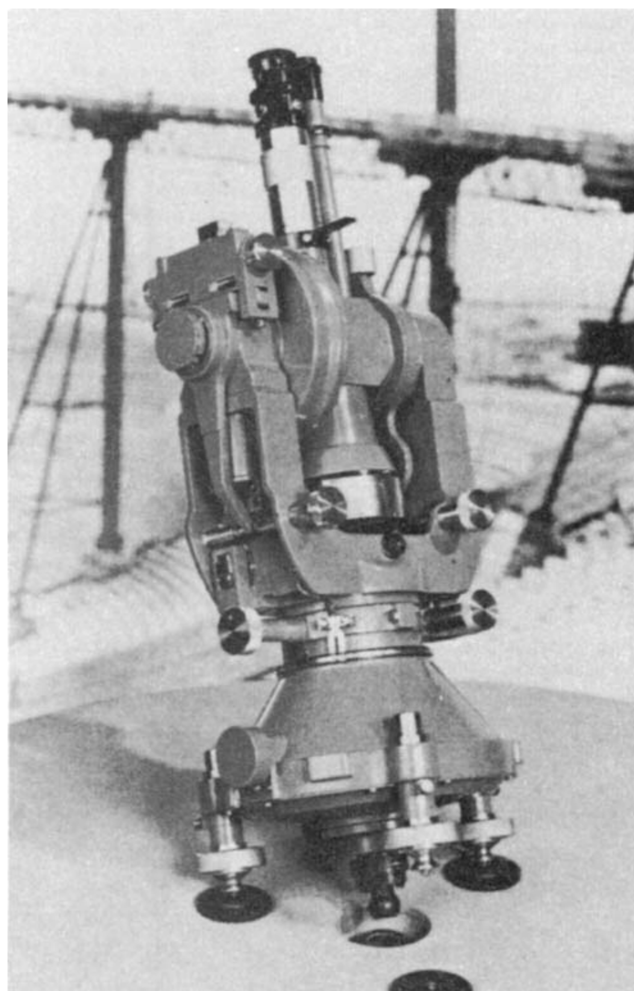
Three distinct designs of survey pillar are illustrated in Figure 6.40. The main requirement in siting pillars is the need to ensure that they are founded on stable ground outside the zone of influence of the structure under investigation. It is, however, often difficult to assess whether this is the case before observations are recorded. It is therefore important to incorporate into the design an insulating gap which ensures that the central pillar is not in contact with the surrounding ground. This should ensure that the effects of diurnal or seasonal earth movements are minimized.

A further common design feature is the incorporation of some system of forced centring. The two most common forced centring systems for deformation monitoring are the Wild ball centring system, and the Kern system. Both designs are illustrated in Figure 6.41. Use of either system should ensure that errors from this source do not exceed ± 0.1 mm.

This concept of an insulating sleeve around the reference point is also evident in the levelling datum designs illustrated in Figure 6.42. Figure 6.42(a) can be seen to consist of a steel foot driven into the ground at the bottom of a 5 m deep borehole. This steel foot is connected to a central rod which extends



(a)



(b)

Figure 6.41 Pillar centering systems (a) Kern system; (b) Wild system

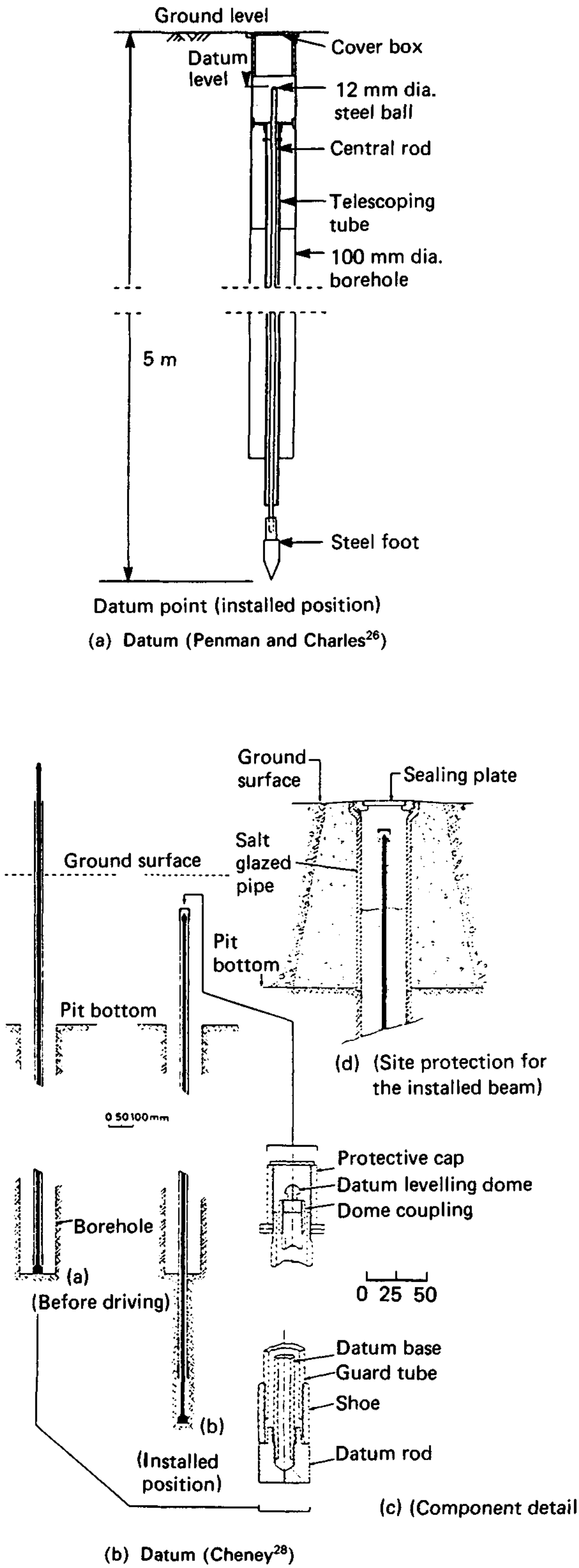


Figure 6.42 Levelling datum designs (from Penman and Charles,[26] and Cheney[28])

almost to ground-level. In order to minimize the potential effect of ground movement influencing the datum, the central rod is surrounded by a telescopic sleeve. An alternative design is illustrated in Figure 6.42(b). In this case, the inner datum rod consists of a 10 mm bore galvanized tube surrounded by a guard tube of 25 mm bore. The guard tube is driven into a 150 mm diameter hole which has been bored to a depth of 5.5 m. The datum point consists of a dome-shaped steel ball about 0.1 m below ground-level. The ball is covered by a protective cap and by a sealing plate (possibly a manhole cover), at ground-level. Further details relating to the installation procedure may be found in Cheney.[28] It is also important to install a sufficient number of datums to enable any settlement or uplift of the datums to be detected. Ideally, three should be installed.

Monitoring points. The main requirement of a survey monitoring point is that it can be either permanently affixed to, or precisely relocated, on the structure being investigated. Again, two distinct types of monitoring point can be identified, those to which angle/distance measurements will be taken and those which will be used as precise levelling settlement points.

The measurement of angles and distances to monitoring points on the structure will normally require that the target points be designed so that they are capable of accepting both conventional survey targets and also corner cube reflectors. The simplest approach, and that commonly used for dam deformation work, is to build a series of pillars on the structure. The pillars are fitted with a suitable forced centring system which enables both survey targets and corner cube reflectors to be interchanged very accurately. An example of such a system is illustrated in Figure 6.43.[29] An alternative approach may be used in situations where the distances to be measured are short. In these cases it may be possible to permanently affix targets and reflectors to the structure. Details of one particular arrangement which involves the use of reflex acrylic reflectors is reported by Kennie.[13] Cheney[30] also discusses the use of permanently mounted reflectors, in this instance for use with the Kern Mekometer ME 3000.

For settlement measurements, one of the most common designs of monitoring point is that which has been designed by the Building Research Establishment (BRE). The components of the settlement system are illustrated in Figure 6.44. It can be seen that the system consists of four components: (1) a stainless steel socket 65 mm long by 22 mm diameter which is grouted into the structure; (2) a detachable settlement bolt; (3) a protective Perspex cap; and (4) an alloy wrench to allow removal of the cap. The main advantages of such a system are, firstly, a high degree of accuracy in relocating the settlement point; Cheney[28] states that the settlement bolts may be repositioned in the socket to within 0.03 mm. The second advantage is the level of protection offered to the settlement point from accidental or other damage.

6.3.4.2 *Computational processes*

Horizontal control surveys for deformation monitoring purposes are essentially no different from control surveys for other purposes (e.g. national control surveys). Whilst they may differ in size (generally much smaller), and accuracy requirements (generally much higher), the basic principles in terms of observational and computational techniques are common. It is not surprising to note, therefore, that in recent years the type of control survey generally adopted for deformation purposes consists of a mixed set of angle and distance observations or a 'survey network'. The advantages of such an approach in reducing scale and orientation errors have been discussed previously. However, the primary advantage of this approach occurs when the data are subjected to a rigorous least squares

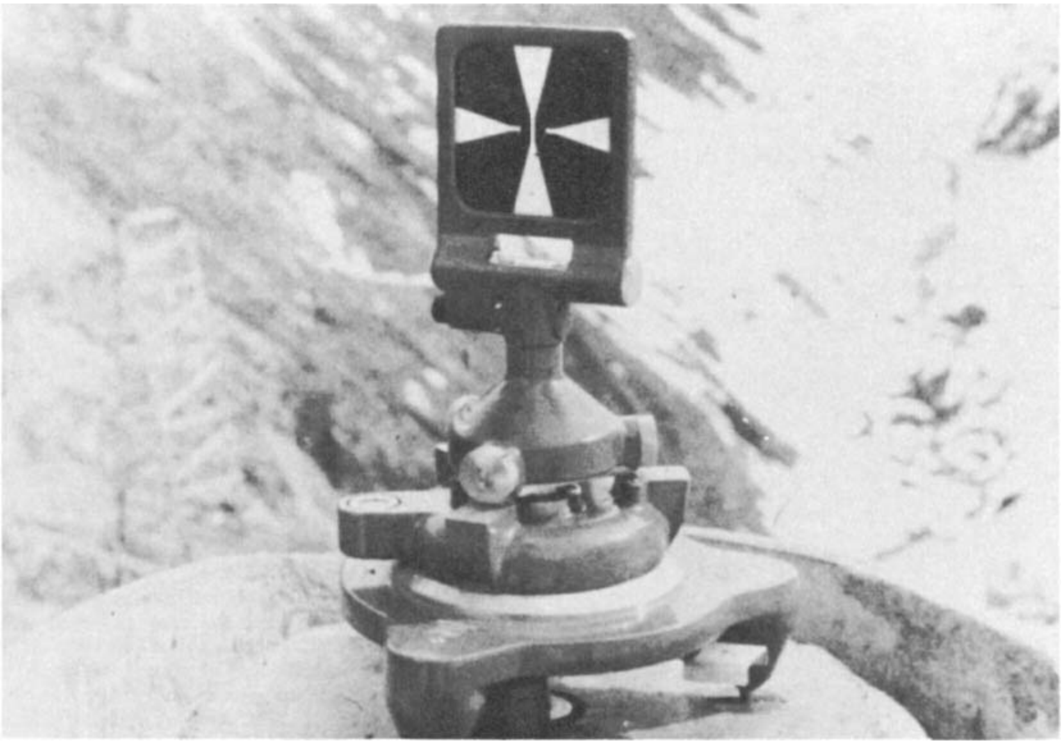


Figure 6.43 Pillar target point (from Egger[29])

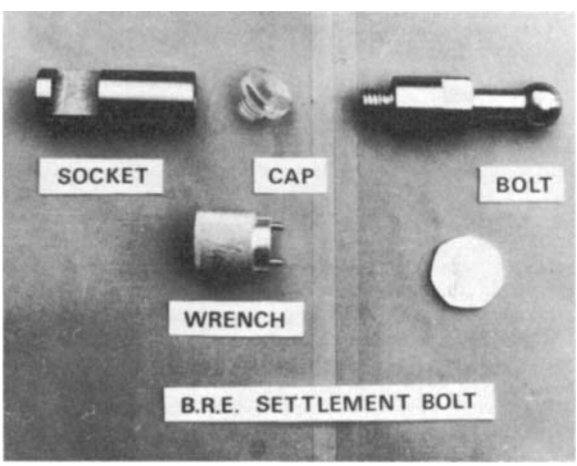


Figure 6.44 Building Research Establishment settlement bolt system

adjustment using the method of variation of coordinates (see section 6.3.1.4). This not only enables the most probable values of the positions to be arrived at, but also statistical data about the precision and reliability of the network to be determined. Furthermore, by interpreting these statistical indices and then the results of a network adjustment from two different epochs it is possible to evaluate the statistical significance of any changes which are observed. The various stages in the computational process are shown in Figure 6.45.

The various statistical indices which can be derived from a variation of coordinates adjustment is well documented in the literature, e.g. Cooper.[1] It is therefore intended to make only brief mention of the primary features of each statistical index.

Three distinct types of statistical indices can be identified: (1) those which are concerned with network precision; (2) network reliability; and (3) for deformation analysis.

(i) Network precision.

Residuals

The residuals ($\mathbf{v}$) of the observation equations are obtained from:

$$\mathbf{v} = \mathbf{A}\mathbf{X} - \mathbf{b} \qquad (6.36)$$

where A, $\mathbf{X}$ and $\mathbf{b}$ are as defined in Equation (6.29). In cases where a large number of observations exist, the ratio $\mathbf{v}/\sigma$, where σ refers to the standard error of the observation, may be used in order to reject suspect observations. For example, if $\mathbf{v}/\sigma$ is greater than 2.5, this indicates that there is only a 1.2% probability that this variation is caused by random observational effects and is more likely to be indicative of a poor observation. On this basis, therefore, the observation should be rejected.

Unit variance

The quantity σ_0^2 is computed from the expression:

$$\sigma_0^2 = \frac{\mathbf{v}^{\mathrm{T}} W \mathbf{v}}{m-n} \qquad (6.37)$$

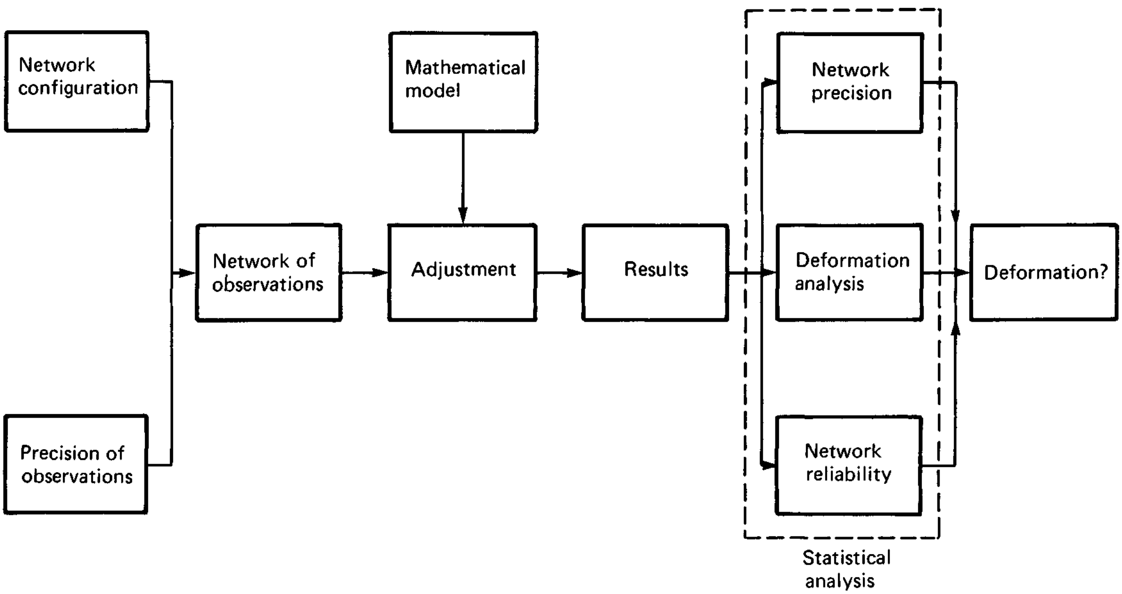


Figure 6.45 Stages of computation: deformation monitoring surveys

where m is the number of observations, n the number of unknowns and W is the weight matrix. It can be shown that, theoretically, σ_0 (the standard error of unit weight) should equal 1. A large departure from unity is usually indicative of some gross blunder, whilst a small departure may give an indication whether the initial a priori standard errors of the observations have been correctly estimated. In certain circumstances it is possible to correct the initial standard errors by a 'trial and error' process in order to bring the value of σ_0 closer to unity.

Variance – covariance matrix

The variance – covariance matrix ($\boldsymbol{\sigma}_{xx}$), is an extremely useful by-product of the adjustment process. It is computed from the following expression:

$$\boldsymbol{\sigma}_{xx} = \boldsymbol{\sigma}_0^2(\mathbf{A}^{\mathrm{T}}\mathbf{W}\mathbf{A})^{-1} \qquad (6.38)$$

The diagonal elements of the matrix ($\sigma^2_{E_i}$, $\sigma^2_{N_i}$, ...), refer to the variances or squares of the standard errors associated with the unknowns E_i, N_i, etc. It is therefore an extremely useful means of measuring the precision of the network. The off-diagonal elements, or covariances ($\sigma_{E_i N_i}$, ...) may be used in order to construct error ellipses.

Error ellipses

The 'absolute error ellipse' of a calculated point is an ellipse defined by σ_{min} and σ_{max} with the semimajor axis $a = \sigma_{max}$ and the semiminor axis $b = \sigma_{min}$. When drawn around a point the ellipse is generally interpreted as depicting the region within which there is 39% confidence that it contains the position of the point. The values of σ_{max} and σ_{min} are determined by the following expressions:

$$\sigma_{max} = \tfrac{1}{2}(\sigma_E^2 + \sigma_N^2) + [\tfrac{1}{4}(\sigma_E^2 - \sigma_N^2) + \sigma_{EN}]^{1/2} \qquad (6.39)$$

$$\sigma_{min} = \tfrac{1}{2}(\sigma_E^2 + \sigma_N^2) - [\tfrac{1}{4}(\sigma_E^2 - \sigma_N^2) + \sigma_{EN}]^{1/2} \qquad (6.40)$$

Two drawbacks exist with error ellipses of this type. Firstly the size of the error ellipse is not 'invariant' and depends on the position of the point which is chosen as the origin. Secondly, no information is given about any inter-station correlation which may exist. An alternative type of error ellipse which overcomes the first drawback is the 'relative error ellipse'. In this case the ellipse is computed on the basis of the variances and covariances of the differences in coordinates between points. It is therefore a measure of the relative positional accuracy between the points.

A posteriori standard errors

The a posteriori or post adjustment standard errors of the adjusted quantities are a further set of 'invariant' statistics which can be computed. For example, the a posteriori standard error associated with the adjusted distance between any two points in the network is given by:

$$\sigma_i^2 = [\mathrm{P\,Q\,R\,S}]\begin{bmatrix} 4\times 4 \text{ Sub-matrix} \\ \text{of } \sigma_{xx} \end{bmatrix}\begin{bmatrix} \mathrm{P} \\ \mathrm{Q} \\ \mathrm{R} \\ \mathrm{S} \end{bmatrix} \qquad (6.41)$$

where P, Q, R and S are the trigonometric functions defined in Equation (6.24).

(ii) Network reliability. Network reliability in this context is defined as the ability of the network to detect gross errors in the observed quantities. Ashkenazi[21] has suggested that the following statistic should be used in order to assess the reliability of the network:

$$\text{reliability} = \frac{\text{a posteriori standard error of the observation}}{\text{a priori standard error of the observation}} \qquad (6.42)$$

Theoretically the greater the number of observations indirectly affecting a particular observation, the lower the ratio should be. Thus low ratios are indicative of high reliability, whilst a ratio of one would indicate complete unreliability. Ashkenazi suggests that the maximum acceptable value of this ratio should be 0.9

(iii) Deformation analysis. The statistical indices discussed so far relate to a single set of network observations. A much more common problem in deformation monitoring work, however, is the situation where two or more sets of observations of the same network, taken at different times, exist. In these circumstances, further analysis is required in order to determine whether any differences in coordinates which exist are statistically significant.

Currently this particular problem is the subject of considerable research interest. Following the second FIG symposium on Deformation Measurements in Bonn, 1978, a committee was established in order to investigate different approaches into the analysis of deformation measurements using the same measurement data. The provisional results of this committee's work were reported by Chrzanowski.[31] The primary problems which this group, and others, have been examining are, firstly, what is the most appropriate means of eliminating systematic effects from the data sets, and secondly, what are the most appropriate types of statistical test to apply to the recorded deformations in order to test their significance.

The first problem, that of eliminating systematic effects, such as a scale bias in EDM measurements, has been discussed by Ashkenazi *et al.*[23] and Ashkenazi.[21] In cases where a systematic bias is considered to be present in one set of data, the systematic bias parameters (scale, orientation and translation) may be determined by means of a least squares four-parameter Helmert transformation. By the use of this procedure it is therefore possible to eliminate from one data set a series of systematic effects which, if not eliminated, may be mistaken as systematic deformations.

The main dangers associated with the application of this technique to networks is that it may lead to the removal of a genuine systematic deformation under the mistaken believe that it is caused by observational error. It has therefore been suggested by Dodson,[32] that it is preferable for any systematic effects to be eliminated by careful instrument calibration, or by designing the network so that 'the influence of such effects does not influence the acceptance or rejection of any hypothesis of deformation'.

The second problem, that of devising the most appropriate statistical tests, has also been examined by Ashkenazi and Dodson.[33] The approach which they have adopted involves comparing the detected coordinate differences with the corresponding coordinate standard errors. The basis of this approach can be illustrated by considering two points P and Q in a fictitious network. If the position of P in the network is stronger (i.e. more accurately determined because of a good geometrical configuration and higher precision observations) than point Q, then quite clearly a smaller difference between two successive values of the coordinates of P, may be more significant than a larger difference corresponding to point Q. Thus periodic coordinate differences may only be considered to be significant if the standard error associated with the deformation at that point is also examined. Therefore, if:

$$\Delta^2 = (\Delta E^2 + \Delta N^2)^2 \qquad (6.43)$$

where Δ is the computed deformation based on planimetric coordinate differences ΔE, and ΔN, and the standard error in Δ is given by:

$$\sigma_\Delta^2 = (\sigma_{\Delta E}^2 + \sigma_{\Delta N}^2)/2 \qquad (6.44)$$

where $\sigma_{\Delta E} \approx \sqrt{2}\sigma_E$ and $\sigma_{\Delta N} \approx \sqrt{2}\sigma_N \qquad (6.45)$

then, the ratio Δ/σ may then be used in order to test the significance of any deformation. For example, if the ratio Δ/σ is 2 (95% probability level) then there is only a slight chance (19:1 against) that the movement has been caused by random observational errors and is much more likely to have been caused by movement of the point. In contrast, a ratio of 0.7 would indicate that there was an even chance (50% probability level) that the difference was caused by movement, it being equally likely that the variation was simply a manifestation of a random observational error. Ashkenazi[22,23] discusses the application of this technique to sets of data observed at Cattleshaw reservoir.

6.4 Computers in surveying

Computers have, throughout their development, been extremely important in the fields of surveying and mapping. Initially, their use was almost exclusively restricted to the 'number crunching' requirements of large organizations carrying out geodetic computations or the adjustment of major control frameworks. Operations of this type were carried out on large mainframe computers in batch mode. Whilst slow and cumbersome to operate by modern computing standards, these early computers offered enormous benefits to the surveyor: their computational speed, and their ability to deal with the application of rigorous computational techniques, such as the method of least squares, on a much greater scale than was possible previously. In recent years, the trend, in computational terms, has been towards the development of more 'user friendly' software which can be operated on smaller computers, particularly microcomputers (see Milne[34] and Walker and Whiting[35]).

Whilst the use of computers for solving mathematical problems in surveying continues to develop, the main thrust area of interest in recent years has been in the development of interactive graphics systems for processing and displaying surveying and mapping data. Thus, the emphasis is becoming concentrated more on the capability of the computer to store, search, retrieve and display digital map data, than on computational speed. Such systems offer considerable benefits both in terms of speed of access and flexibility of use, e.g. the ability to be able to display selectively map data at a user-defined scale. Indeed, there is little doubt that over the next decade the storage of map data in digital form will continue to expand at an ever-increasing speed as the costs of computer memory drop and the availability of data in digital form increases. The engineer will therefore come into increasing contact with these types of data, primarily at the design stage of a project, using large-scale digital maps and ground-modelling systems, but also at the project planning stage using data from land/geographic information systems.

6.4.1 Digital mapping and ground-modelling systems

In the UK, responsibility for national mapping lies with the Ordnance Survey. The Ordnance Survey are also involved in the production of digital maps (Thompson[36] and Logan[37]), and actively considering the possibility of producing a national digital topographic database (Thompson[38] and Rhind[39]). The task of compiling large-scale digital maps to cover the entire country is an enormous task and one which requires substantial resources if it is to be produced within a reasonable time-scale. To date, the success of the Ordnance Survey in providing large-scale digital mapping coverage has been very limited (10 to 15% coverage). It is hoped that as technology develops and, perhaps more significantly, as funding becomes more available, that this situation will improve. Nevertheless, in spite of the lack of

Ordnance Survey-derived data, many organizations are involved in the acquisition, processing and sale of digital survey data in the commercial sector. Several sophisticated suites of software have also evolved for the manipulation of this digital survey data, particularly in conjunction with civil engineering design information. Two examples which are representative of this range of software are MOSS and the Eclipse Interactive Ground Modelling System.

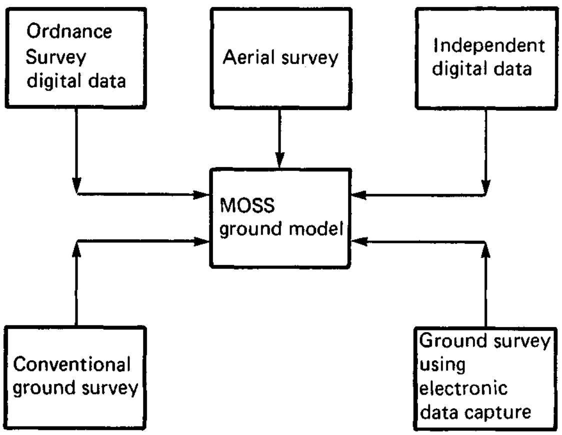


Figure 6.46 Alternative methods of creating a MOSS ground model

6.4.1.1 MOSS system

MOSS is a combined surveying and engineering design system which records information in the form of three-dimensional 'strings'. These 'strings' consist of a series of linked coordinated points representing features such as kerbs, roads, railway lines, or ground level detail, such as contours. Each string is given a label and by covering an area with a sufficient number of strings and/or point information it is possible to represent the terrain in digital form in the form of a MOSS 'model'. A wide variety of survey methods can be used in order to create a string ground-model, and Figure 6.46 illustrates a few of the options which are available. The design features within the system also enable a model of any proposed works to be generated. By comparison of both models it is therefore possible to generate other data such as earthwork quantities.

The MOSS system was initially launched in the mid 1970s by a consortium of several county councils within the UK. Although it was not initially developed with the production of large-scale plans as its primary aim, it has been upgraded and enhanced since its inception and is now widely used for large-scale survey purposes. Figure 6.47 is an example of the use of MOSS in this mode. MOSS is, however, much more widely used as an integrated survey and design system, rather than as a stand-alone survey system, and it has been used in this mode for a wide variety of projects including: highway design (Hougham[40] and Fawcett[41]), railway design (Bedingfield and Craine[42]) and the design of land reclamation schemes (Wilson[43]). A new interactive version of MOSS has recently been launched by MOSS Systems Ltd, a company which was formed by several members of the original consortium in 1983. This version will offer considerable benefits to its users, particularly in terms of speed and ease of use.

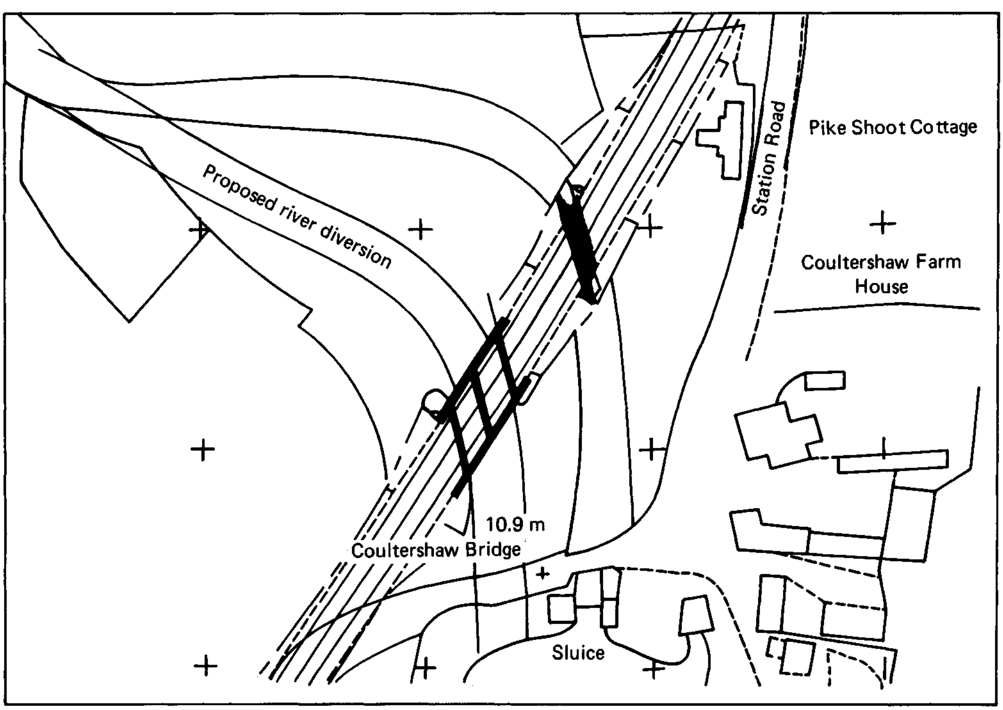


Figure 6.47 MOSS model taken from a graphics VDU showing an Ordnance Survey digital map, updated with a small road improvement, bridge detail, proposed river diversion, and culvert. (MOSS Systems Ltd, model courtesy of West Sussex County Council.)

6.4.1.2 Eclipse system

The Eclipse interactive ground modelling system is a further example of an integrated system for survey processing, digital mapping and engineering design. In common with the MOSS system the software is designed so that it can be installed on a variety of different computer systems. However, unlike MOSS which is designed for use with a variety of mainframe (IBM, ICL, etc.), minicomputers (DEC/VAX, Prime, etc.) and engineering workstations (Sun, Apollo, etc.), the Eclipse system has been designed primarily for Wang minicomputers and, recently, for the Wang PC and the IBM PC/AT.

The program modules within the system include software for processing and editing – interactively – survey control and detail observations. Data input can be by manual keyboard entry, or automatically by transfer from one of the electronic data collectors currently on the market. The design software includes facilities for road design, drainage design, building layout design and land reclamation design. Throughout, facilities exist for deriving additional information such as areas and earthwork quantities. For high-quality output a series of options within the draughting software enable the user to enhance cartographically the screen image in order to produce final contract drawings if necessary.

Other systems currently available which perform operations similar to those outlined include the Wild System-9, Intergraph, HASP, AXIS and ProSurveyor.

6.4.2 Land information systems

Land information systems (LIS), are currently one of the major growth areas in computing as applied to surveying and mapping. They are of particular relevance to engineers involved with feasibility and planning studies. The concept of a LIS can best be described by considering the definition offered by Andersson:[44]

> A land information system is a tool for legal, administrative and economic decision making, and as an aid for planning and development which consists, on the one hand, of a database containing spatially referenced land related data and, on the other hand, of procedures and techniques for the systematic collection, updating, processing and distribution of the data. The base of a LIS is a uniform spatial referencing system for the data in the system which also facilitates the linking of data within the system with the other land related data.

Two other terms, Geographic Information System (GIS) (Hallam[45]) and Urban Information System (UIS) (Parker and Bray[46]), are also used to describe systems which operate in a similar manner but within a regional or local urban area respectively.

All of these systems are very much in their infancy at present. However, many organizations concerned with, for example, public utilities (gas, electricity, waste water disposal) and land taxation/valuation/registration are already investing significant sums of money in the development of such systems. A review of the history and future possibilities for LIS in the UK is provided by Dale.[47] The LIS will have enormous impact on the amount of information available to the engineer and planner. The management and efficient use of this data is the challenge which has to be faced in the future.

6.5 Acknowledgements

The author would like to thank the following persons and organizations who helped in the preparation of this chapter: Geotronics (UK) Limited, Moss Systems Limited, Spectra Physics (UK) Limited, and Wild Heerbrugg Limited for permission to reproduce photographs of their products. Also, to Mrs Veronica Brown for her valuable comments on the initial draft.

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7 Photogrammetry and Remote Sensing

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Contents

7.1 Introduction

Photogrammetry and remote sensing are two indirect methods of obtaining both quantitative and qualitative data about the Earth's surface or other features of interest. Since some debate exists regarding the demarcation between the two subjects, the following broad distinction will be used throughout this chapter. Photogrammetry will be considered to be concerned with the scientific methods of obtaining reliable measurements from ground or airborne imagery (primarily photographic) in order to produce a precise representation (graphical or digital) of the feature of interest. Remote sensing, in contrast, will be defined as encompassing those methods of detecting variations in radiant energy from the Earth's surface using airborne or satellite sensors. Interpretation (either visually or using computer techniques) of these recorded patterns is used to create thematic maps. The aim of this chapter will therefore be to: (1) provide an introduction to the fundamental principles of photogrammetry and remote sensing; (2) review the current state of development of instrumentation in both fields; and (3) examine the application of both techniques to problems in civil engineering.

7.2 Principles of photogrammetry

Since measurements may be taken from both air and ground images (normally photographs) two separate branches of the discipline are generally recognized: aerial and close range (or terrestrial) photogrammetry.

Aerial photogrammetry is a well-established technique in civil engineering for the production of topographic maps. Aerial photographs produced for such purposes can be obtained either with the optical axis of the camera pointing, nominally, vertically downwards so producing vertical aerial photographs or, with the axis intentionally tilted, to produce oblique aerial photographs. The former are almost exclusively used nowadays for photogrammetric purposes, although the latter have received a revival in recent years with the advent of analytical techniques. Vertical aerial photographs are normally acquired to achieve a 60% forward and a 20 to 25% lateral overlap in coverage. This enables a stereoscopic view of the terrain to be obtained and also a stereoscopic 'model' to be produced. The latter forms the basis of almost all of the photogrammetric instruments and techniques discussed in section 7.3.

Close-range, terrestrial or, to use a less attractive but commonly used alternative, nontopographic photogrammetry, is concerned with photographs taken on, or near the ground, rather than from an aerial platform. This branch of photogrammetry has developed considerably in the past 20 yr or so and has been applied to many problems in civil engineering. A selected number of these applications are presented in section 7.4.

A further distinction which is becoming used increasingly as a means of classification, both in aerial and close-range photogrammetry (CRP), is that between the analogue and analytical approaches to the subject. Apart from a few exceptions, until relatively recently photogrammetric instruments and techniques consisted of optical and mechanical analogue systems which were used for the production of a graphical end product (map, plan, elevation, section, etc.). However, the emphasis in photogrammetry is now moving towards analytical systems which use rigorous mathematical models to simulate the problem under investigation. By using these mathematical models in conjunction with modern computers, new photogrammetric instruments have evolved which offer significant advantages over their analogue counterparts in terms of speed of operation, accuracy and flexibility. Furthermore, the output products of these computer-based systems consist not only of graphical products, but also of other information sources such as digital terrain models.

7.2.1 Geometry of a single photograph

A photograph differs fundamentally in geometric terms from a map (unless the terrain is flat and the photograph vertical). Image displacement causes scale variations over the format due to the alteration in the position of points, compared with their corresponding map positions, as a result of the effects of ground relief and tilt of the photograph at the moment of exposure. The geometrical influence of both factors is illustrated by Figure 7.1 which shows that a tall tower is not imaged as a single point, as it would be on a map, but rather as a displaced line t–t and t'–t' on the vertical and tilted photographs respectively. In this case, the displacement is greatest on the vertical photograph. A similar displacement will exist for all points above the datum level e.g. point X. It is also evident that the tilted and vertical photographs are coincident at point i, the isocentre, and that tilt displacement radiates from this point. This radial nature of the displacements can be used to devise a simple method of plotting from a pair of photographs.

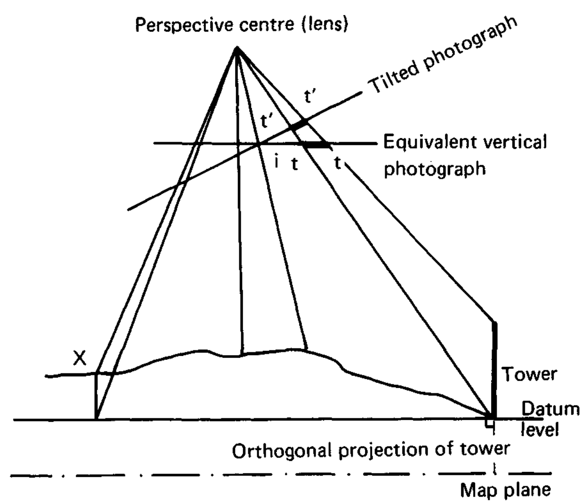


Figure 7.1 Relief and tilt image displacements on a vertical and near-vertical aerial photograph

The effect of image displacement on scale can be seen in Figures 7.2 and 7.3 which illustrate the separate results of tilt and variation in ground relief. It can be seen that, unlike a map which has a constant scale, the combined effects of relief and tilt produce a photograph which will be of constantly changing

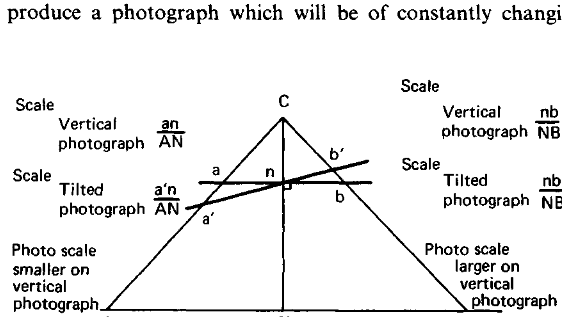


Figure 7.2 Scale changes on a vertical aerial photograph caused by variations in the elevation of the ground

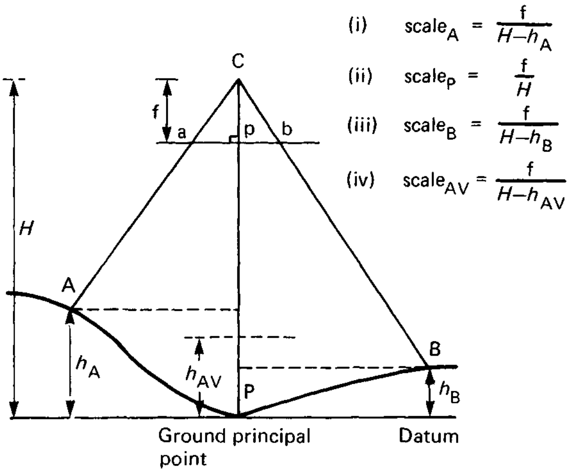


Figure 7.3 Scale changes on a tilted and vertical aerial photograph

scale, although an average nominal value for the scale can be calculated (often referred to as the contact scale).

7.2.2 Stereoscopy and parallax

Stereoscopic viewing and the measurement of the perceived stereomodel is fundamental to photogrammetry and thus of great importance. If two photographs taken from different viewpoints of the same area are viewed simultaneously, the difference in position of a common image point on the two photographs results in a discrepancy in image coordinates and this leads to the concept of x parallax and y parallax.

All points appearing on successive overlapping photographs exhibit x parallax in the direction of flight and this is the principal reason why a stereomodel exists. It can be seen from Figure 7.4 that the combined movement of point A across the focal plane of both photographs relative to the central axis is greater than for point B, thus point A is defined as exhibiting a greater absolute x parallax than B. It is also evident from the diagram that the absolute x parallax of a point is related to the height of that point and that it increases with increasing height. This simple concept forms the basis of the methods used to derive heights from aerial photographs. Section 7.3.3 outlines one such technique.

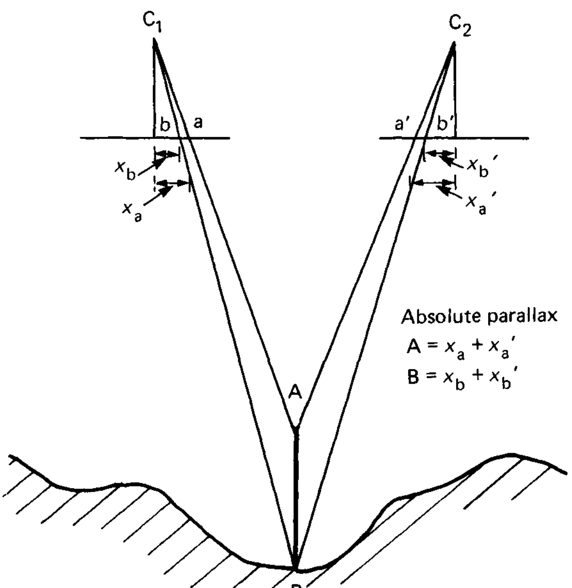


Figure 7.4 Variations in x-parallax on an overlapping pair of aerial photographs

In contrast, y parallax is not related to height variations. It is produced if there is a difference in orientation of the two photographs at the time of exposure. The effect of y parallax on a stereomodel can be eliminated by reproducing the original camera orientations to ensure that corresponding rays from the two photographs will intersect. Such a process is termed the 'relative orientation' phase in setting up a stereoplotter.

7.2.3 Analytical photogrammetry

Many photogrammetric operations which were previously carried out by graphical or analogue methods are now being performed digitally using analytical instruments and computational techniques. Since most analytical methods involve some form of coordinate transformation, it is first necessary to examine the coordinate reference systems which are used. On the plane of the photograph, the reference system known as the 'image space' system is defined with respect to the principal point and the fiducial marks, as shown in Figure 7.5. It may also be important to take into account any variation in position which exists between the fiducial centre (the intersection of the fiducial marks) and the principal point (where the perpendicular line from the lens intersects the focal plane). This is defined by a camera calibration procedure. On the ground, positions are usually referred to a cartesian coordinate system, e.g. the National Grid. This is illustrated by Figure 7.6 in which X_c, Y_c and Z_c represent the 'object space' coordinates of the camera exposure station.

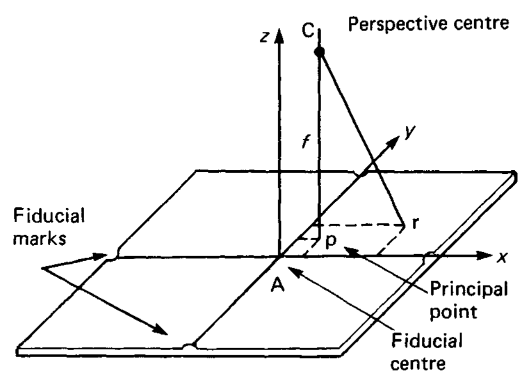


Figure 7.5 Image space coordinate reference system

An appreciation of the two basic assumptions which are used in order to define positions is also essential. Firstly, the line in space joining a point in the object space – the perspective centre of the camera lens and the position of the point on the photograph – is assumed to be a straight line, i.e. line p–C in Figure 7.6. In analogue stereoplotters, this assumption is imposed by optical projection or mechanical 'space rods'.

Analytically, it is expressed by means of the collinearity equations for a *single* photograph:

$$x_r = -f\frac{M_{11}(X_r-X_c)+M_{12}(Y_r-Y_c)+M_{13}(Z_r-Z_c)}{M_{31}(X_r-X_c)+M_{32}(Y_r-Y_c)+M_{33}(Z_r-Z_c)} \quad (7.1)$$

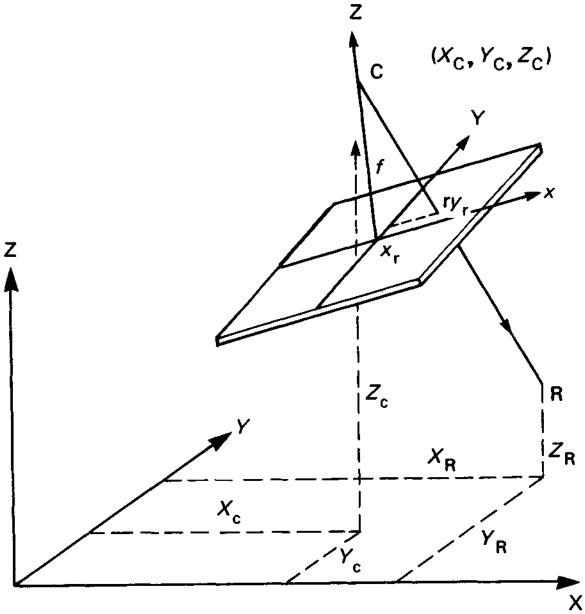


Figure 7.6 Object space coordinate reference system and collinearity condition

$$y_r = -f\frac{M_{21}(X_r - X_c) + M_{22}(Y_r - Y_c) + M_{23}(Z_r - Z_c)}{M_{31}(X_r - X_c) + M_{32}(Y_r - Y_c) + M_{33}(Z_r - Z_c)} \quad (7.2)$$

where $M_{11} \ldots M_{33}$ represent terms of an orthogonal rotation matrix and the other terms are as defined in Figure 7.6. The collinearity equations are used extensively in analytical photogrammetry to relate image coordinates, ground coordinates and the coordinates of the perspective centre. The technique may then be used to derive the coordinates of a series of unknown positions.

A further analytical technique which is used by photogrammetrists is that defined by the coplanarity equations. Coplanarity equations can be established for a pair of photographs and they may be used to reconstruct the orientation of the cameras at the moment of exposure, i.e. an analytical method of relative orientation. In order to satisfy the condition, the two exposure stations C_1 and C_2, the object point R and its corresponding image points r_1 and r_2 are assumed to all lie in a common plane, known as the epipolar plane (Figure 7.7).

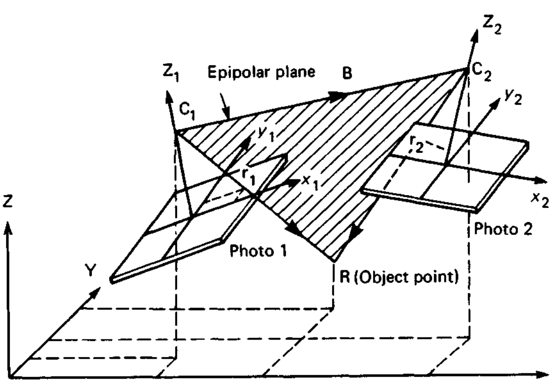


Figure 7.7 Coplanarity condition in analytical photogrammetry

7.3 Photogrammetric instrumentation

7.3.1 Cameras

Most photogrammetric measurements are recorded from photography produced by a high-quality camera specifically designed for photogrammetric purposes. The main feature which distinguishes this type of camera from others is that the internal geometrical characteristics are known precisely. Thus, data relating to the lens distortion, focal length and principal point location are established, and monitored periodically by a camera calibration procedure.

Both aerial and close-range cameras are used for data acquisition. Aerial mapping cameras are normally classified on the basis of the angular field of view of the lens, a parameter which relates directly to both the focal length (f) and the format size of the camera. The focal length of an aerial camera is defined as the distance from the optical centre of the lens to the image plane. For the general case of a 230 × 230 mm format, the classification shown in Table 7.1 can be produced. Normal angle (NA) photography is only occasionally used and it can be seen from Figure 7.8 that the principal advantage of the super wide

Table 7.1 Classification of aerial mapping cameras

Camera type	*Approximate angular field of view* (°)	*Approximate focal length* (mm)	*Examples*
Super wide angle (SWA)	120	90	Zeiss (Ober.) RMK A 8.5/23 Wild RC-10 (Super Aviogon lens) Zeiss (Jena) MRB 9/2323
Wide angle (WA)	90	150	Zeiss (Ober.) RMK A 15/23 Wild RC-8 Wild RC-10 (Universal Aviogon lens) Zeiss (Jena) MRB 15/2323
Normal angle (NA)	60	300	Zeiss (Ober.) RMK A 30/23 Wild RC-10 (Normal Aviogon lens)

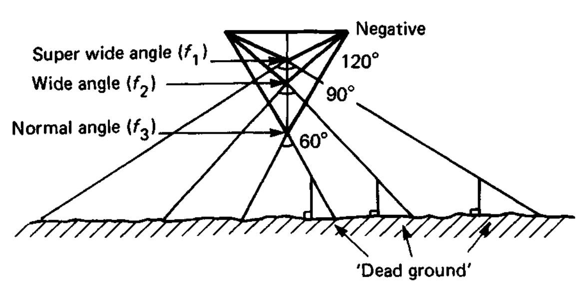


Figure 7.8 Variation in ground coverage of normal, wide and superwide angle cameras

angle (SWA) lens is its greater ground coverage for a given flying height. This is particularly important since it not only reduces the number of photographs required to cover an area, but also reduces the number of control points which are required for controlling the mapping. However, because SWA photography is more susceptible to the production of 'dead ground' (loss of detail because of the screening effect of features), a compromise solution is normally adopted by using a camera which enables wide angle (WA) photography to be produced.

Three distinct types of close range or terrestrial photogrammetric camera are currently available: (1) metric; (2) stereometric; and (3) nonmetric. The metric camera models include those in which a camera and a theodolite form one integrated unit, the classical phototheodolite, and those where the camera is a separate unit from the theodolite. The latter case tends to be more common and the UMK10/1318 is a typical example (Figure 7.9). Details of the characteristics of several cameras of this type are listed in Table 7.2.

The second design, the stereometric system, consists of two matched camera units mounted on each end of a base bar, typically 0.4 to 3 m long. The camera axes are set at right angles to the bar and since the relative position and orientation of the cameras at the instant of exposure are known, the subsequent analysis phase is considerably simplified. The Zeiss (Oberkochen) range of cameras illustrated by Figure 7.10 are representative of this design. Further details of other cameras of this type are listed in Table 7.3.

A nonmetric camera is one which was not designed specifically for photogrammetry. Although such cameras offer advantages in terms of size and cost, their use is limited, unless analytical techniques are used to compensate for the unstable interior geometry of the camera. A varied selection of nonmetric cameras has been used for photogrammetric purposes including

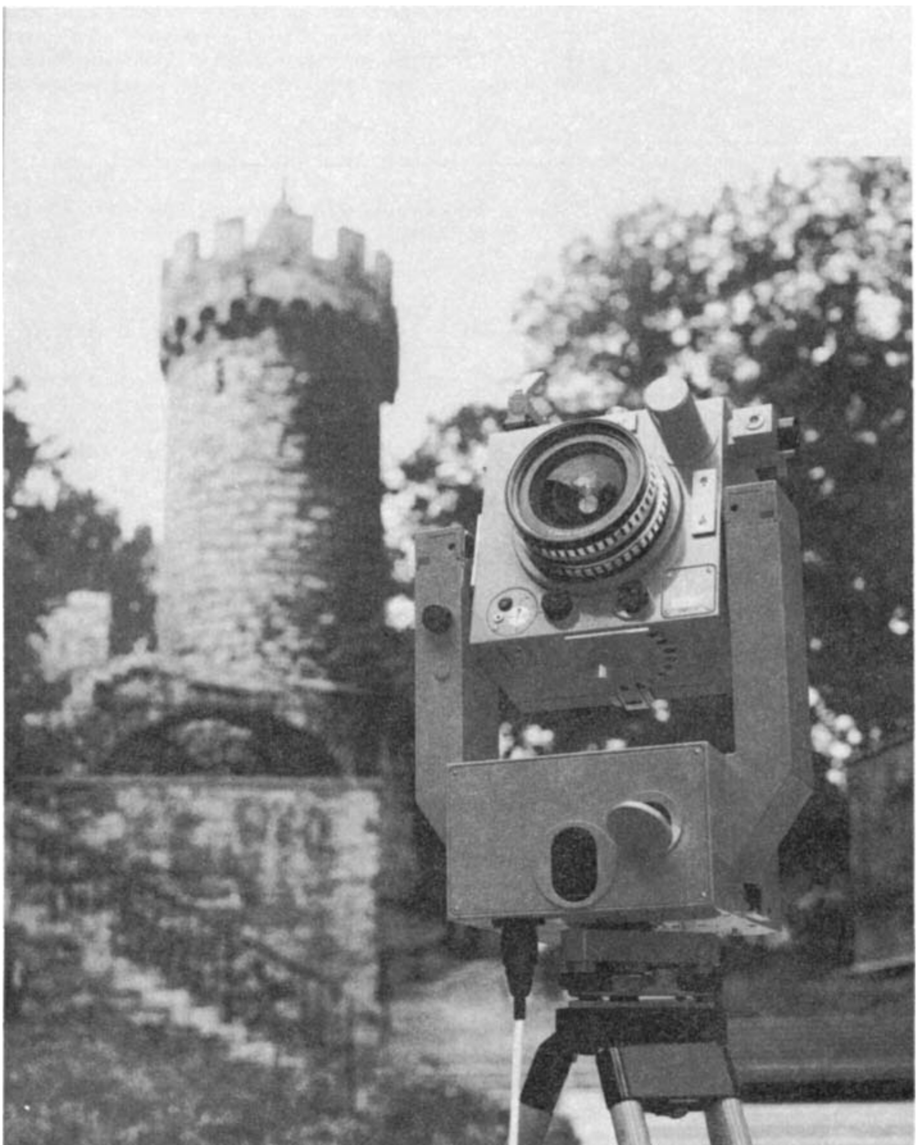


Figure 7.9 Zeiss (Jena) UMK 10/1318 close-range camera

Table 7.2 Summary of main features of a selection of close-range photogrammetric cameras

Manufacturer	*Model*	*Nominal focal length* (mm)	*Principal distance* (F = fixed, V = variable)	*Format* (mm)	*Minimum range* (m)
Wild	P30	165	F	100 × 150	20
Wild	P31	45, 100, 200	F, V, V	100 × 150	7, 1.4, 8
Wild	P32	64	F	65 × 90	3.3
Officine Galileo	Verostat	100	V	90 × 120	2
Zeiss (Jena)	UMK 10/1318	100	V	130 × 180	1.4
Zeiss (Ober.)	TMK 6	60	V	90 × 120	5

(Adapted from Atkinson (1978) 'Photography in non-topographic photogrammetry'. In: Newman (ed.) *Photographic techniques in scientific research*, vol. III. Academic Press, London)

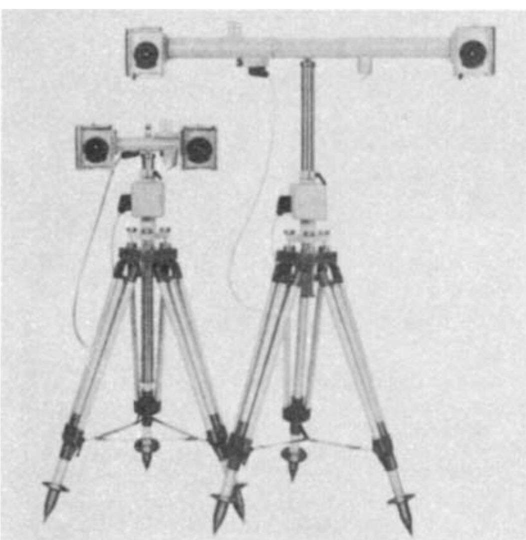


Figure 7.10 Zeiss (Oberkochen) SMK 120 and SMK 40 stereometric camera

medium-priced 'amateur' cameras, such as the Olympus OM-2, Canon TX, Minolta XG-7 and Rolleiflex SL66, as well as the more expensive 'professional' cameras such as the Linhof Technica and the Hasselblad 500EL.

7.3.2 Single-photograph-based instruments

In certain circumstances it may be appropriate to attempt to derive metric plan information from a single photograph. Reasonable results can be obtained provided that relatively flat ground has been imaged on an almost tilt-free photograph.

The camera lucida principle, whereby images of an existing map and the corresponding photograph are superimposed, has been adopted to produce a simple mapping instrument such as the Zeiss (Jena) Sketchmaster and the Bausch and Laumb Zoom Transfer Scope. Adjustments can be made either mechanically or optically in order to remove the effect of minor tilt distortions and to make a correction for small variations in scale between map and photograph. This type of instrument is of particular value for map revision. When detail shown on both photograph and map has been aligned, new details imaged on the photograph can be traced on to the map.

It is also possible to apply a digital approach to measurement on a single photograph using a monocomparator. This instrument essentially consists of a travelling microscope moving

Table 7.3 Summary of main features of stereometric close-range photogrammetric cameras

Manufacturer	*Model*	*Nominal principal distance* (mm)	*Principal distance* (F = fixed, V = variable)	*Format* (mm)	*Focusing range*
Wild	C40	64	F	65 × 90	1.5–7.0
Wild	C120	64	F	65 × 90	2.7–∞
Officine Galileo	Veroplast	100	V	90 × 120	2–∞
Zeiss (Jena)	SMK5.5/0808	56	F	80 × 80	5–∞
Zeiss (Ober.)	SMK120	60	F	90 × 120	5–∞

(Adapted from Atkinson (1978) 'Photography in non-topographic photogrammetry'. In: Newman (ed.) *Photographic techniques in scientific research*, vol. III. Academic Press, London) along orthogonal axes in order to determine image coordinates. Various refinements to the basic principle have been made; the Surveying and Scientific Instruments PI-1a, for example, employs a miniature CCTV observation system and provides digital readout with the MDR 1S/3 display, whilst the Zeiss PSK-1 instrument uses optical scales based on Moiré fringes. With the aid of a microcomputer and suitable software, a plot can be produced from such equipment.

7.3.3 Approximate solution stereoscopic instruments

When reconnaissance mapping or revision of existing maps from new photography is being carried out, it is often more cost effective to use a simple, approximate stereoscopic instrument rather than a sophisticated and expensive rigorous stereoplotter.

Several manufacturers produce stereoscopic versions of the monoscopic instruments which are based on the camera lucida principle, e.g. the Cartographic Engineering Stereosketch and the Bausch and Laumb Stereo Zoom Transfer Scope. The image of the stereoscopic view of an overlap is superimposed on the map image and photographic plan detail can be traced; no facilities are provided for height measurement. The Cartographic Engineering Radial Line Plotter (Figure 7.11) is a simple, lightweight instrument which can be used for small-scale planimetric mapping. The instrument design is based on the radial line assumption which states that angles measured at the principal point are true if the photographic tilt and variation in ground relief are below some limiting value (normally 3° and 10% of the flying height). Other approximate instruments include the Zeiss (Oberkochen) G2 Stereocord and the Officine Galileo Stereomicrometer.

A parallax bar (Figure 7.12) is a portable device for obtaining measurements of differences in x parallax which can be used to calculate differences in height. Spot heights at changes of slope can hence be obtained and contours interpolated. The eye observes the measuring mark on the glass plates through a stereoscope. When adjusted to the correct x separation the two marks appear to fuse into a single point which may appear to 'float' above the level of the terrain. In order to make some correction for possible photographic tilt and variation in flying height for the stereopair, a minimum of five control points of known ground height are required for the overlap; ideally eight to ten points will be used. Each of the control points is observed in turn and the micrometer screw is turned to remove x parallax and, hence, bring the floating mark to ground level.

Any y parallax can be eliminated by adjusting the right-hand stage plate in the y direction. The micrometer readings are recorded for all the control points. In order to calculate spot heights, the mean flying height must be determined from scaled measurements on the photographs and the absolute parallax of one point must be determined using a travelling microscope. A computer program is then normally used to calculate crude

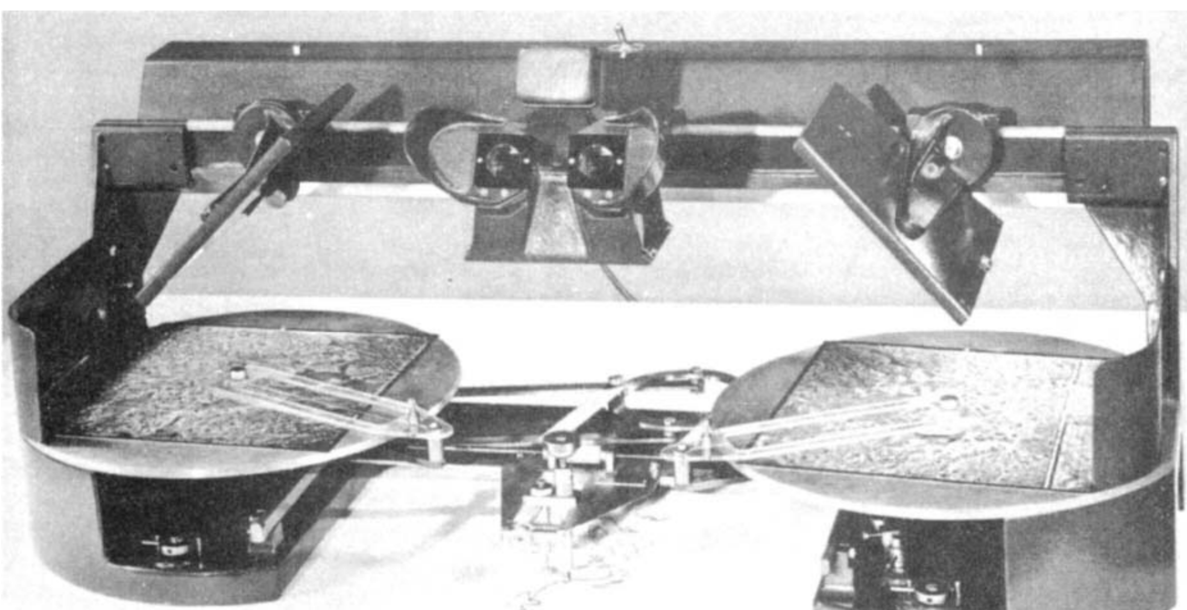


Figure 7.11 Radial line plotter. (*Courtesy*: Cartographic Engineering)

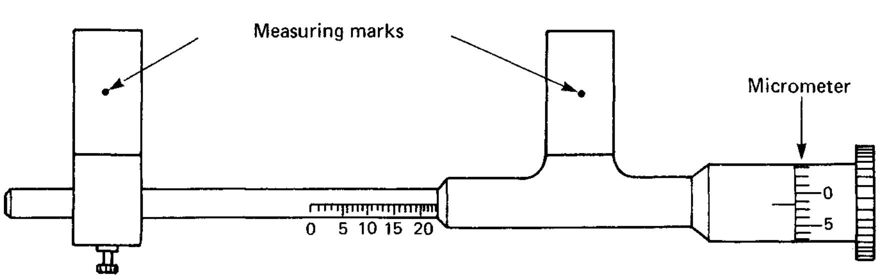


Figure 7.12 Parallax bar

heights from Equation (7.3) and to obtain corrected spot heights from Equation (7.4). If redundant control has been provided, residuals will be available and the heighting accuracy can be investigated.

$$h_B = h_A - \frac{(H - h_A)\Delta_{p_{AB}}}{p_A - \Delta_{p_{AB}}} \tag{7.3}$$

where h_A = height of A (a reference point of known height), h_B = crude height of point B (point of required height), $\Delta_{p_{AB}}$ = difference in parallax bar readings between A and B (care must be taken with sign) and p_A = parallax of A.

$$h_{B'} = h_B + a_0 + a_1x + a_2y + a_3xy + a_4x^2 \tag{7.4}$$

where $h_{B'}$ = corrected height of point B and x, y are image coordinates of B, and $a_0 \ldots a_4$ are coefficients, constant for a given overlap. This method of height correction was first proposed by Thompson[1] and was subsequently modified by Methley.[2] Contours may be interpolated between spot heights, continuous reference being made to the stereomodel as they are drawn.

7.3.4 Rigorous solution stereoscopic instruments

7.3.4.1 Analogue stereoplotters

Analogue stereoplotters create and measure an exact three-dimensional model of the ground. Direct optical projection in the form of Multiplex equipment was the earliest method of realizing this objective. The Zeiss (Ober.) DP 2 stereoplotter (Figure 7.13) is a modern version of Multiplex equipment. Plotters such as these, which employ direct optical projection, have the disadvantage of relatively poor viewing conditions and an inflexible plotting scale, usually approximately double the photograph scale. However, their uncomplicated operation and minimum of maintenance make them very suitable for medium-scale mapping and map revision, although in recent years they have tended to be superseded by mechanical and analytical instruments.

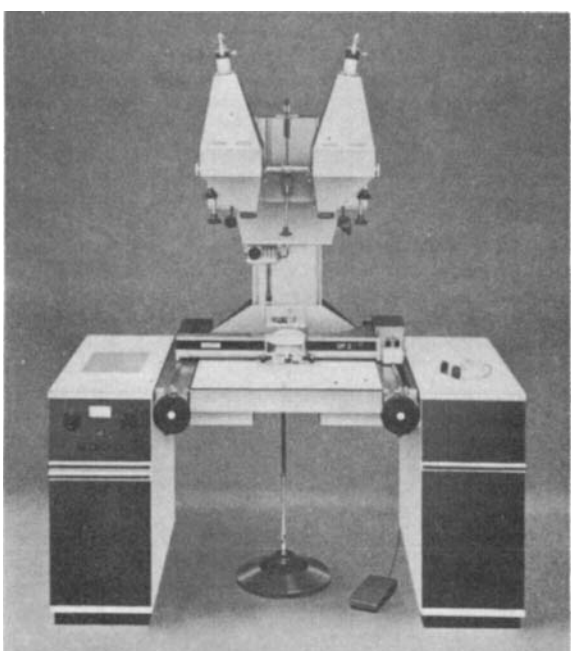


Figure 7.13 Zeiss (Oberkochen) DP-2 stereoplotter. (*Courtesy*: Zeiss (Oberkochen))

Mechanical rods replace light rays when a rigorous model is formed in instruments employing mechanical projection (Figure 7.14). Full-sized diapositives are placed in projector heads which can be tilted; a stereoview is obtained through a binocular eyepiece linked to viewing microscopes fitted to each projector. The movement of each microscope is communicated via a tie rod to a sleeve at the top of a space rod which pivots about a gimbal joint representing the projection centre. The two space rods take up the attitude of the space rays from the ground point to the camera stations; they normally intersect in a model point which is connected to a plotting pencil by gears. The principal distance of the taking camera is represented by the separation of the sleeve on the space rod which represents the image point and the gimbal joint; this distance can easily be varied so that mechanical projection plotters can accommodate photography taken with a wide variety of cameras. Considerable flexibility of plotting scales (up to 6 times the photographic scale) is provided by altering the gears linking the model point to the plotting point (Table 7.4).

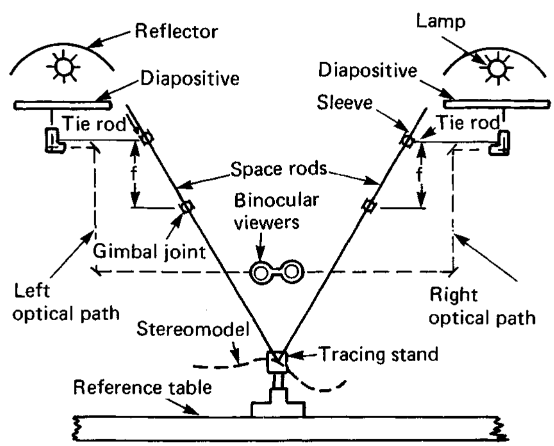


Figure 7.14 Mechanical projection stereoplotter

Table 7.4

Photographic scale	*Flying height (m)*	*Enlargement factor*	*Mapping scale*
1:3000	450	6	1:500
1:5000	750	5	1:1000
1:10 000	1500	4	1:2500
1:25 000	3750	2.5	1:10 000

Digital readout of model coordinates is now standard practice and recent developments include computer-assisted prompts, to guide the orientation procedures, and computer-aided drawing. The Wild A10 stereoplotter (Figure 7.15) is a typical example of a mechanical instrument.

7.3.4.2 Analytical stereoplotters

With the advent of digital computers, especially mini- and microcomputers together with electronic encoders for digitizing the output from stereoplotting equipment, a new generation of

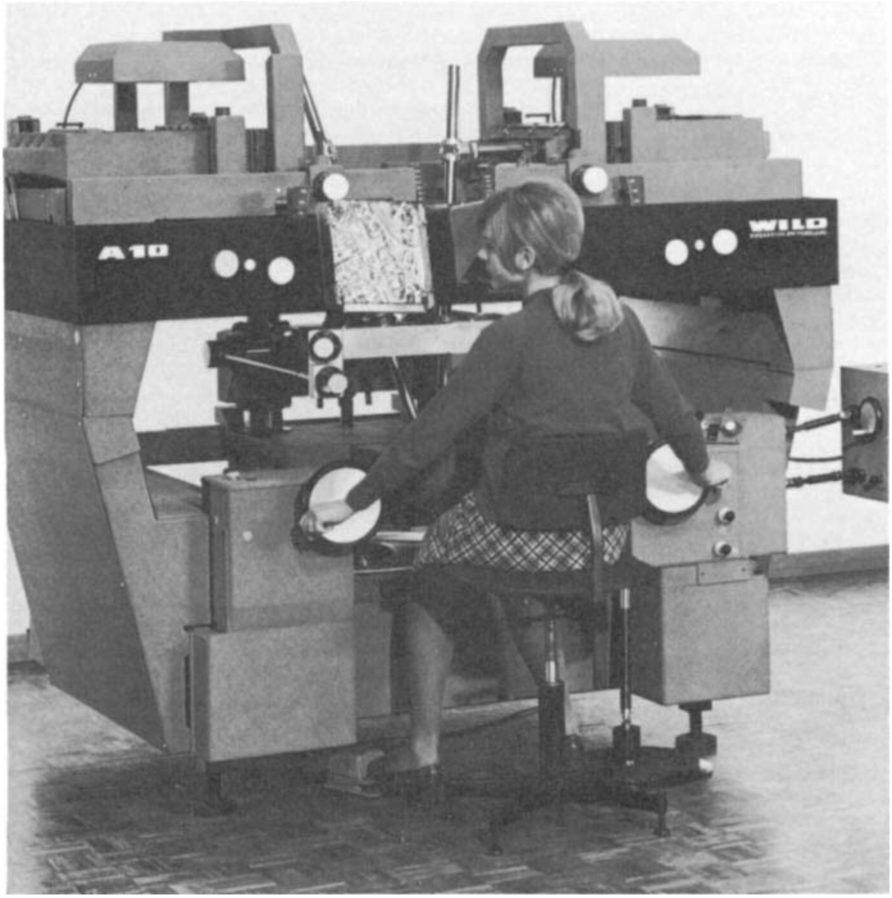


Figure 7.15 Wild A-10 stereoplotter. (*Courtesy*: Wild Heerbrug)

photogrammetric stereoplotter has emerged: the analytical stereoplotter. With this type of instrument a mathematical model replaces the space rods and other mechanical linkages which are found in more conventional analogue stereoplotters. An instrument which is representative of this design is the Officine Galileo Digicart. This instrument consists of a stereoviewing system, linear encoders to digitally record the position of the carriages on which the photographs are mounted, a minicomputer to process the data, and a plotting table. Unlike a conventional analogue instrument where manual elimination of y-parallax is performed initially over the entire model, with this design any parallax existing within the model is continuously eliminated by a feedback loop which ensures that corrections are transmitted to the optics in real time (50 times a second) to provide a model free from parallax. Figure 7.16 illustrates the principle of operation.

The advent of analytical techniques has had a considerable influence on the practical application of photogrammetry to engineering problems. In particular, analytical techniques offer the following advantages over conventional analogue techniques:

(1) Higher accuracy of measurement can be achieved than was previously possible (up to ± 3 μm at photoscale with suitable care).
(2) Photography taken with differing format sizes and focal lengths can be used. In certain cases, provided suitable

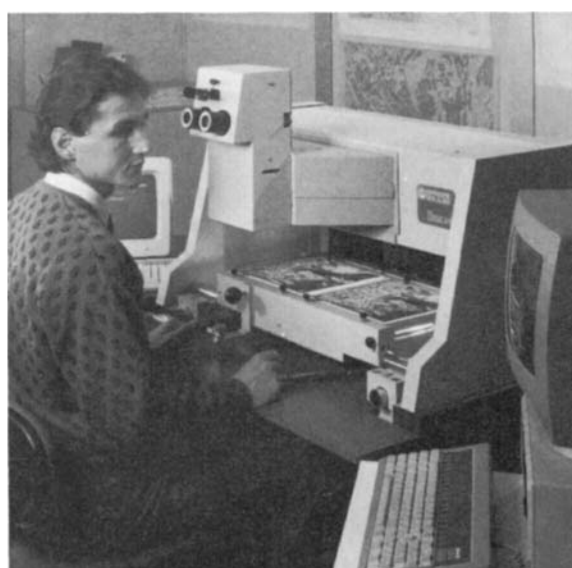


Figure 7.16 Officine Galileo Digicart analytical stereoplotter. (*Courtesy*: Officine Galileo)

precautions are taken to eliminate any systematic errors such as lens distortion, nonmetric cameras can be used.

(3) The setting up of any stereomodel presents little difficulty; most models can be set up in 10 to 15 min by an experienced operator, thus leaving more time for the actual measuring phase.

(4) Photography can be obtained in the field without too great a concern about the orientation of the camera. This can save a great deal of time (as much as 50% over the conventional approach). Where access is difficult, and vertical or normal case geometry is impeded, oblique photography may be taken. Such an approach would have been precluded using traditional methods.

(5) Data are captured in ground coordinates and stored on magnetic disks. This allows for plotting off-line on graphical plotters at any scale. Also, with data being collected digitally rather than graphically, much more flexibility exists in data handling, e.g. the data can be manipulated on a Computer Aided Drafting and Design (CADD) system.

7.3.4.3 Orientation of stereoplotters

Whatever type of equipment is used, before drawing can commence three stages of orientation must be carried out in order to set up the model correctly.

Inner orientation consists of the careful setting of the diapositives on glass stage plates. For analogue instruments, either the principal point or the fiducial marks on both diapositive and register glass must be aligned. Unless the principal distance is fixed (as is the case for direct optical projection instruments), this must be set to the value obtained from the camera calibration certificate. If an analytical plotter is used, the reference marker is taken to each fiducial mark in turn and the coordinates are recorded; the camera principal distance is entered into the computer as are any known lens distortion characteristics.

Relative orientation is next carried out in order to produce a true undistorted model. It does not require any reference to ground control and is achieved in analogue instruments by removing y parallax for clear points of detail selected at five standard positions. The entire model should then be clear of y parallax. For analytical models, y parallax is measured at five or more positions to enable the computer to deduce the movement of the servomotors subsequently required to view a parallax-free model.

Absolute orientation is required in order to scale the model and level it to ground datum. Scaling is basically accomplished by comparing a model distance with a known ground distance and adjusting the machine base in analogue instruments to achieve the required model scale. For heighting, the model and ground heights of a minimum of three points are compared and the model is rotated about the machine X and Y axes until differences in height agree. The machine datum will then be parallel to ground datum and its exact level can be determined. True ground heights can then be read directly from the instrument height scale. Analytically, photo coordinates and corresponding ground coordinates of three or more control points are compared in order to enable the required transformation coefficients to be computed.

The machine is now set up and plotting can commence. For plan detail, the operator moves the floating mark to follow image detail, keeping it in contact with the model surface by constantly adjusting its height. In correspondence, the plotting pencil traces out the detail on the map sheet. In order to plot a contour, the floating mark is kept at the required constant height and is moved in contact with the model surface so that the contour line is drawn on the plot. A similar procedure is followed for the analytical plotter as far as the movement of the floating mark by the operator is concerned, but computer-assisted plotting offers additional options such as defining the end points of a straight line and instructing the pencil to join them.

The machine plot will only contain information that can be obtained directly from the stereopair and field completion is required to add ground detail which might be obscured on the photographs, e.g. within woodland or below the eaves of houses, and also to assist annotation of, for example, place names and road classifications.

7.3.4.4 Orthophoto systems

An orthoprojector is used to generate a photograph that is the geometrical equivalent of a map by a process of differential rectification. An exact model is set up in a stereoplotter, the effects of tilt being removed by the standard procedures of relative and absolute orientation. The operator scans the model systematically with the floating mark, adjusting the effective height of the mark continuously so that it remains on the surface of the ground. The orthoprojector operates in the darkroom conditions and holds a duplicate of one of the photographs of the stereopair. The tilt of the corresponding plotter projector, and the linear motion and height movement of the floating mark are transmitted either mechanically (Zeiss (Oberkochen) DP Ortho-3), optically (Wild PPO-8) or analytically (Zeiss (Oberkochen) Orthocomp Z-2) to the orthoprojector which allows exposure through a small slit on to light-sensitive paper. Thus a corrected print of the overlap area, known as an orthophotograph, is gradually built up as the slit moves in sympathy with the floating mark, the effects of height difference being removed by the continuous adjustment of the orthoprojector. Cartographic enhancement may be added to the print (e.g. contour lines, grid lines and place names), resulting in an orthophotomap.

7.4 The use of photogrammetry in civil engineering

7.4.1 Aerial photogrammetry

7.4.1.1 Topographic mapping

The use of aerial survey is generally considered to be the standard method of producing a topographic map or plan at scales smaller than 1:500. For scales greater than 1:500, ground survey would almost invariably be used.

The basic sequence of operations required for the production of a topographic map is shown by the flow diagram in Figure 7.17. The major air survey companies will have the equipment and manpower to carry out all the stages indicated but smaller concerns might, for example, subcontract the photography. It is very important that the exact requirements for the survey output should be known at the outset and that detailed discussion should take place between the civil engineer and the air survey company in order to identify any specific problems which might arise in connection with a particular project.

Flight planning. Before a survey flight can commence, a flight plan must be prepared detailing all the information required by the pilot, navigator and camera operator.

Aerial photographs which are to be used for mapping purposes must be taken in a regular sequence along parallel lines of flight. Navigation is usually visual by identifying landmarks shown on the map, mosaic or satellite image on which the

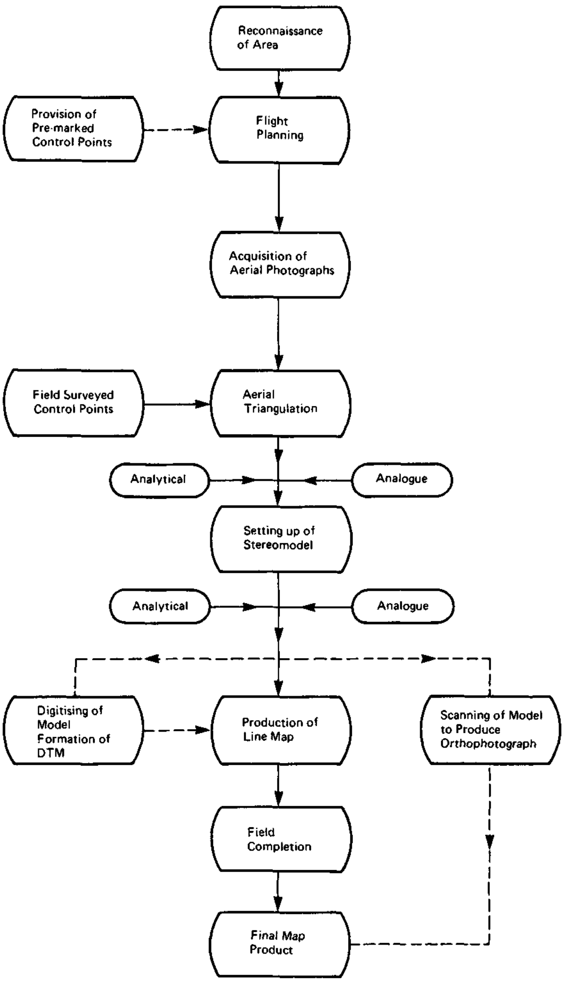


Figure 7.17 Flow diagram illustrating the sequence of photogrammetric operations involved in the production of a topographic map

required lines of flight have been marked. Increasingly, reliance may be placed on some form of radio navigation system. Flight lines will normally be drawn parallel to the longes edge of the area to be covered. However, if there is any marked grain of relief, this direction may be taken for the flight lines in order to attempt to maintain a constant scale along a given strip. Each line of flight produces a *strip* of photography, successive overlapping strips forming a *block* of photography.

Since a stereoscopic view of the entire area is needed, each ground point must be photographed from two camera stations giving a minimum overlap stipulation of 50% in the direction of the line of flight (the fore and aft direction). However, in order to give additional overlap to satisfy aerial triangulation requirements and to provide a safety margin, a value of 60% is usually specified.

A standard *Specification for vertical aerial photography* has been prepared by the Royal Institution of Chartered Surveyors[3] in collaboration with the British Air Survey Association. This specification is used by air survey companies throughout the world as a contract document for providing black-and-white aerial photography. The specification includes the air camera and its calibration, photographic coverage and operation, flying conditions, the type of aerial film, processing and drying conditions, the resultant products to be supplied, and documentation and annotation.

Aerial triangulation. The object of aerial triangulation is to establish a network of control points with known ground coordinates which are required before a map can be plotted from aerial photographs. In order to perform absolute orientation when setting up a model in a stereoplotter, the minimum theoretical control required for each model is 2 points with known plan coordinates and 3 points with known heights (in practice, 3 plan points and 4 height points are usually considered to be the minimum). Thus, a large project would require the provision of an extensive amount of control; it would be very expensive and time-consuming if this had to be carried out exclusively by field survey methods. Aerial triangulation also has the advantages that it is easy to provide more than the bare minimum of control and it will usually be possible to obtain points in very convenient positions for their subsequent use in a stereoplotter.

Analogue methods of aerial triangulation have evolved steadily with the development of analogue stereoplotters. The method of aerial triangulation by independent models (AIM) is now used almost universally with modern analogue instruments. Since the technique requires model coordinates to be measured in an analogue plotter, a substantial amount of computation is required to obtain ground coordinates and it could therefore be described as a semi-analytical method.[4]

Whereas model coordinates measured in an analogue plotter form the basic data for the calculation of aerial triangulation by independent models, x, y coordinates measured either in a mono- or stereocomparator provide the raw data for the fully analytical methods. Much less observation time is required than with the equivalent analogue methods, but much greater emphasis is, however, placed on the data processing phase, both for the location of gross and systematic errors and for the computation of the coordinates. As outlined in section 7.2.3, it is possible to establish the mathematical equations of the bundle of rays from the optical centre of the camera lens to all image points observed on a given photograph by using the collinearity method. The combination of all the bundles of rays for the entire block can be used to carry out relative and absolute orientation simultaneously.[5] The calculated ground coordinates obtained by this method are adjusted to a best mean fit by least squares. It is clearly advantageous if both the necessary hardware and software are available to consider the block as an entity rather than to adjust a strip and then connect adjacent strips in a separate operation.

Having carried out aerial triangulation, each stereomodel is subsequently set up in turn using the orientation procedures mentioned in section 7.3.4.3. Plotting can then commence, so creating a topographic map or the operator may measure spot heights to form a digital terrain model.

7.4.1.2 Production of digital terrain models

The term digital terrain model (DTM) was originally coined in 1958 by Miller and La Flamme. Since then, the subject has developed considerably and is currently an area of widespread activity in surveying, geology and geophysics, civil and mining engineering and other disciplines in the earth sciences.

Although a variety of different techniques exist for the creation of DTMs in general, the distinction can be made between those which make use of height data which have been collected in a regular grid and those based on a network of randomly located height points. The former, simpler, approach is generally adopted when photogrammetric equipment is being used for the generation of the DTM. While the square grid is the most common form, rectangular-, hexagonal- and triangular-based grids are also used.

Although the regular grid is the simplest technique to adopt it has one serious limitation: the distribution of data points is not related to the characteristics of the terrain. If the data point sampling is conducted on the basis of a regular grid, then the density must be high enough to portray accurately the smallest terrain features present in the area being modelled. If this is done, then the density of data collected will be too high in most areas of the model, in which case there will be an embarrassing and unnecessary data redundancy in these areas.

One solution to this problem is to use progressive sampling and its development, composite sampling.[6] Instead of all points in a dense grid being measured, the density of the sampling is varied in different parts of the grid, being matched to the local roughness of the terrain surface. This approach has been widely implemented on a variety of grid-based terrrain modelling packages such as HIFI,[7] and SCOP.[8]

Terrain models derived by photogrammetric techniques are also used by modelling packages such as MOSS, ECLIPSE, and Intergraph DTMN. A comprehensive review of the use of such terrain modelling packages in surveying and civil engineering can be found in Petrie and Kennie.[9]

7.4.1.3 Monitoring

Aerial photogrammetry has been used extensively in the past as a means of monitoring changing conditions such as the depletion of coal stocks or the degree of coastal erosion occurring over a period of years. In more recent years, its use has also developed for monitoring the deformation of natural or man-made features.

Fraser[10] and Fraser and Gruendig[11] describe the planning and subsequent execution of a photogrammetric survey of the Frank landslide/rockslide region of Turtle Mountain in Alberta, Canada. A rockslide occurred in this area over 80 yr ago when more than 30 million m^3 of rock moved down the east face of the mountain covering an area 3 km square to a depth of 14 m. Since 1933, crack monitoring surveys of the area have been undertaken to determine the stability of the rock wedge forming the southern peak of the mountain. These surveys have been supplemented in recent years by an *in situ* monitoring program using crack motion detectors. Whilst these devices are capable of very high accuracy (± 0.1 mm/yr), they can only, realistically, be deployed over a very small area. Also, if hazardous movements are detected it may be considered unsafe to continue recording measurements. Aerial photogrammetry, by covering large areas in a noncontact manner, can overcome these limitations. Therefore, it was considered appropriate to evaluate the potential of photogrammetry for this project. Fraser[10] established that the optimum flight plan consisted of obtaining large-scale (1:2000) multiple photographs (10 to 15) of the area containing the thirty monitoring target points. Photography was obtained using a Wild RC8 camera (focal length 152 mm) on two occasions approximately 1 yr apart. A subsequent analysis of the results indicated that statistically significant deformations had occurred at eight of the thirty points, the maximum movement being 63 mm.

7.4.2 Close-range photogrammetry (CRP)

Whereas it is possible to identify a standard procedure for mapping from aerial photographs, each project which requires the use of close-range photographs will tend to be regarded as unique due to the wide variation in circumstances which can arise for both taking the photographs and the subsequent analysis. For example, the positioning of the cameras for photography needs to be adapted to the particular demands of an individual task and may vary from a simulated aerial case with parallel, near vertical axes, through inclined and convergent axes to parallel horizontal axes. At the measurement stage, use of either analogue or analytical equipment may be specified according to the type of camera and whether a graphical or a numerical output is demanded.

Although some air survey companies also have a department which specializes in CRP, as do a few land survey organizations and universities, much of this type of work could be carried out by engineering firms themselves, perhaps within a research and development department as, for example, at Rolls Royce.[12] It is even more vital for CRP than for aerial photogrammetry that the engineer should be very closely associated with all stages of the work so that he can ensure that his exact requirements are met.

7.4.2.1 Monitoring

Deformation of structures. Close-range photogrammetry has been used widely in the field of structural engineering. The ability of the technique to record both the shape of a structure at one moment and the deformations which occur between two epochs have been exploited. Notable in the first case is the extensive work which has been carried out in architectural photogrammetry. In the latter case, comprehensive applications reviews have been written by Atkinson[13] and Cheffins and Chisholm.[14] A review of the various methods which can be adopted has been prepared by Cooper.[15] Cooper identifies four main methods including those which make use of a single camera (time parallax), controlled stereomodels, resection/intersection, and bundle adjustment. The latter is considered to be the most general case and most suitable for high-accuracy applications.

The range of large engineering structures which have been investigated by photogrammetric methods includes cooling towers, box girder bridges, elevation of St Paul's Cathedral, tower cranes, offshore platforms, and ships. In addition, photogrammetry has been used for the measurement and dimensional control of smaller structures such as microwave antennae, robots, and large compressors (see Welsh[16] for further details). Increasingly, the emphasis is being placed on the development of a 'real-time' photogrammetric system using video technology and solid-state cameras. Wong and Wei-Hsin[17] outline the development of such a system.

Earth and rockfill dam monitoring. The use of CRP for monitoring earth and rockfill dams has been discussed by Moore[18] and Brandenberger,[19] among others. In the case of Moore,[18] the author describes the use of a Wild P30 phototheodolite for monitoring the three-dimensional displacements of the Llyn Brianne rockfill dam, in mid Wales, during several stages of construction. The predicted displacements were in the range of ± 0.5 to 1.0 m, and whilst high absolute positional accuracy was deemed to be important, it was considered that the definition of the direction of movement was of equal importance. By using a Wild A7 stereoplotter, measurements were taken at over 80 targets, at differing levels of fill. The results indicated a range of displacements, from 0.1 to 0.6 m (with a standard error of ± 0.05 m).

Retaining wall monitoring. The use of a modified KA-2 aerial camera (focal length 610 mm, format 23 × 23 cm) to monitor the deflection of a gabion wall is described by Veress, Jackson and Hatsopoulos.[20] The wall, situated near Seattle, Washington State, was over 400 m long and varied in height from 2 to 18 m. The gabions were constructed of 1-m steel mesh rockfilled cubes. Photographs were obtained from fixed control points up to 1000 m from the wall. Over 100 target points were observed with an analytical plotter and ground coordinates computed. The wall was measured on eleven occasions. The authors suggest that CRP used in conjunction with an internal monitoring system such as an inclinometer would be an ideal system for monitoring new structures.

7.4.2.2 Slope/rock stability studies

One of the earliest reports of rockface mapping is given in Cheffins and Rushton.[21] This paper describes the procedures used to produce 1:50-scale contoured elevations of the north face of Edinburgh Castle Rock. The elevations were required by a team of engineering geologists who were carrying out rock bolting and grouting as stabilizing measures to reduce the chance of rock falls from the face. The survey was carried out by taking four pairs of overlapping photographs from a baseline approximately parallel to the face. The photographs were obtained using a Wild RC5A aerial camera mounted on a hydraulic platform. The elevations with 0.25-m contours were subsequently used, in conjunction with ultrasonic data about the joint structure, to plan the positions of the necessary boreholes for rock bolting.

Moore[22] discusses the use of a phototheodolite for mapping vertical faces, in this case in clay pits. The problem under investigation involved the determination of information about the continuity and spacing of major joints in the clay face. Photogrammetric techniques enabled a three-dimensional model of the spatial disposition of the joints to be constructed. Other references which mention the use of photogrammetry for slope and rockface stability monitoring include Torlegard and Dauphin[23] and Robertson *et al.*[24]

Landslides are a commonly occurring hazard in road design and construction. Heath, Parlsey and Dowling[25] describe the use of a Wild P32 camera to obtain photographs of areas susceptible to landslipping in Columbia and Nepal. The photographs were observed using a Zeiss Topocart and Wild A40 Terrestrial Stereoplotter, and 1:200- to 1:10 000-scale contoured plots were produced. The plans were subsequently used to design remedial and preventative measures associated with the landslip areas. They also enabled a classification of landslide characteristics to be produced.

Kennie and McKay[26] discuss the use of CRP to monitor the erosion of chalk cliffs around a road tunnel in East Sussex. Control stations were monumented at both the top and base of the cliffs, ensuring that a minimum of six control points were visible within each stereomodel. The stations were surveyed by EDM and theodolite and targets positioned over the monuments for precise viewing and control pointing in the stereo model. Photography was taken with a Zeiss UMK 10/1318 camera using glass negatives for greater stability.

Due to the angle of slope of the cliff face from the camera a wide variation in photoscale resulted in this case, from 1:400 to 1:900. For each of the four faces under investigation, the control network was rotated about two points on top of the cliff thus forming a datum line parallel to the face itself. The shape of the cliff was then defined by contours at 0.25-m intervals of depth – i.e. as differences in the horizontal distance from this datum plane – rather than as more conventionally vertical elevations above a height datum. The accuracy of points shown on both the map and sections was within ± 5 cm of true ground position.

7.4.2.3 Tunnel profiling

Recent developments associated with the use of photogrammetry for producing tunnel profiles are described by Anderson and Stevens.[27] Anderson and Stevens outline the development and use of a 'mono-photogrammetric' tunnel profile measuring system. The two main elements of the system are a high-intensity light plane generator, which illuminates the section to be profiled, and a camera which records the line of light which is generated. By digitizing the line, applying corrections for lens distortion, and scaling the photographs, an accuracy of ± 10 mm is claimed. Furthermore, if the quoted progress rate of 100 sections per hour can be achieved in practice, the system would appear to be a highly cost-effective solution.

7.4.2.4 Laboratory-based applications

Close-range photogrammetry has also been applied to measurement problems in the laboratory; for example, Andrawes and Butterfield,[28] Wong and Vonderhoe,[29] and Davidson[30] have used the motion or 'false parallax' technique to measure the planar displacement fields associated with soil models. In the first two cases cited, the technique involved taking repeat photographs, from a single camera position, of a glass-sided tank which contained the soil under investigation. Photographs were taken using a nonmetric 35 mm camera before and after the sand within the tank was subjected to movement by a moving wall or wedge situated in the tank. By examination of enlargements of the photographs in an analogue stereoplotter, the relative positions of features could be measured and displacement contour diagrams produced. Analysis of these diagrams enabled the displacement component along the sand bed to be determined to within 5 μm. The latter two authors, in contrast, used analytical techniques to investigate the movement of soil particles around a model tunnel and soil penetration probe respectively.

7.5 Principles of remote sensing

The interpretation of aerial photography has for many years proved to be a valuable source of data for civil engineers. Until the early 1960s civilian 'remote sensing' was concerned primarily with the interpretation of such imagery. Since then, however, developments in orbiting satellites, sensor technology and computing have led to the creation of a discipline which now impinges on many areas in science and engineering. Although still in an embryonic state, particularly in the field of image processing, it has already proved to be a cost-effective method of investigating engineering phenomena of both large aerial extent, e.g. surface drainage, and those which are more localized, e.g. landslides, unstable land, etc.

7.5.1 The electromagnetic spectrum

Electromagnetic (EM) energy is all energy which travels in a periodic harmonic manner at the velocity of light. Electromagnetic energy is normally considered to consist of a continuum of wavelengths referred to as the EM spectrum (Figure 7.18).

It can be seen from Figure 7.18 that several regions of the EM spectrum are of particular importance in remote sensing. For example, the visible and reflected infra-red regions of the spectrum are important since they enable *reflected* solar radiation to be measured. In contrast, at longer wavelengths in the infra-red (8 to 14 μm waveband) the sensing of *emitted* thermal energy is of more significance. Measurements within the visible and infra-red (reflected and emitted) regions are considered to be *passive* in nature since the radiation being recorded occurs naturally. As the wavelength increases to the order of several millimetres it becomes more convenient *actively* to generate EM radiation of this wavelength and record the reflected radiation from the terrain. Thus, a typical *active* system would be side-

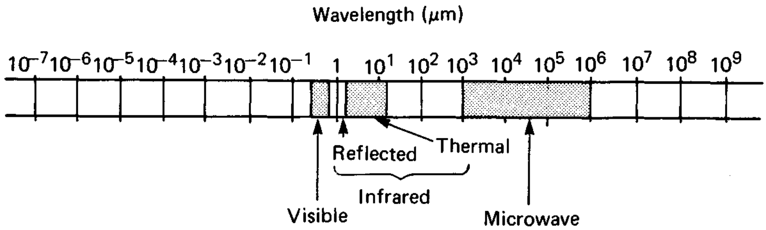


Figure 7.18 The electromagnetic spectrum

looking airborne radar (SLAR). It should be noted that instruments also exist for the measurement of *passive* microwave emission. However, since the emitted EM energy at this wavelength is very small, microwave radiometers are much less common than SLAR instruments.

Depending on the nature of the radiation being measured, it is possible to record the reflected or emitted energy by using either a lens and photographic emulsion or by using a linescanner and crystal detector. The geometrical distinction between the two approaches is illustrated in Figure 7.19. The primary advantage of using a linescanner approach is that it is possible to record radiation of wavelengths greater than about 0.9 μm. It is also possible to measure the variations in radiation within narrow spectral regions and to record directly these variations in digital form.

Mention should also be made here of the distinction between the terms 'photograph' and 'image'. A 'photograph' is an image which has been detected by photographic techniques and recorded on photographic film. In contrast, an 'image' is a more general term used to describe any pictorial representation of detected radiation data. Therefore, although scanner data may be used to create a photographic product, this result is normally referred to as an 'image' since the original detection mechanism involved the use of crystal detectors creating electrical signals, rather than a lens focused on to photographic film.

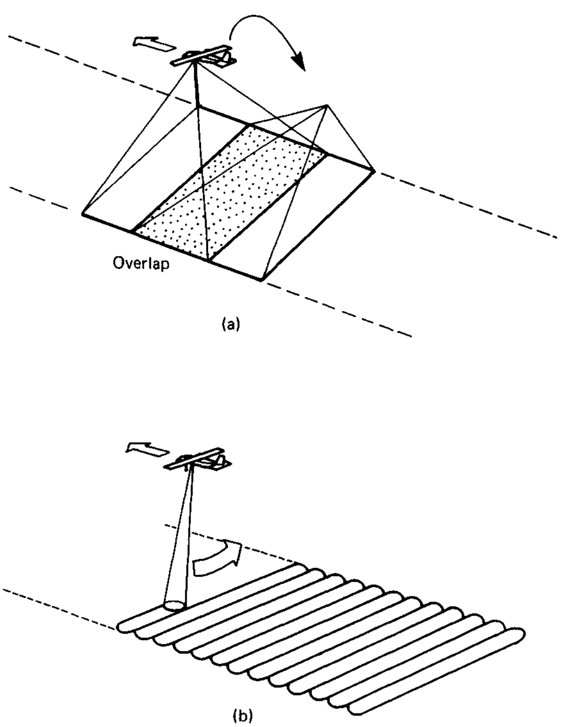


Figure 7.19 Geometry of data acquisition. (a) Camera; (b) linescanner

7.5.2 Classification of remote sensing systems

Remote sensing systems can be classified using various criteria, such as sensor sensitivity range, mode of recording (photographic or scanning), mode of operation (active or passive) or type of sensor platform (aircraft or satellite). Table 7.5 provides a framework for the classification of data acquisition systems in remote sensing by using the latter two criteria; it also provides some common examples of each category.

Table 7.5 Classification of data acquisition systems

	Aircraft	*Satellite*
Passive systems	Wild RC-10 camera Daedalus 1268 MSS Daedalus 1230 Thermal Scanner Barr and Stroud IR18 TVFS	Spacelab metric camera NASA large-format camera Landsat MSS, RBV, TM SPOT MOMS
Active systems	Goodyear GEMS Westinghouse SLAR SAR 580	Seasat SIR A/B ERS-1 Radarsat

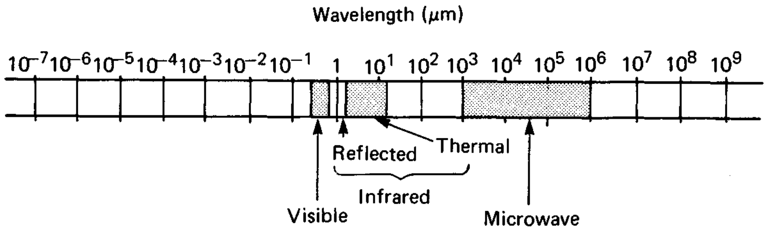
7.6 Data acquisition

Data can be acquired for remote sensing from a variety of aerial platforms, although fixed-wing aircraft and unmanned orbiting satellites are the most common. Both have specific and complementary advantages. Aircraft, for example, enable small localized phenomena to be investigated at high levels of resolution, whereas satellites enable wide synoptic views of the terrain to be obtained, often on a repeatable basis, but at much lower resolution.

7.6.1 Airborne systems

7.6.1.1 Photography

Vertical panchromatic aerial photography taken with a high-



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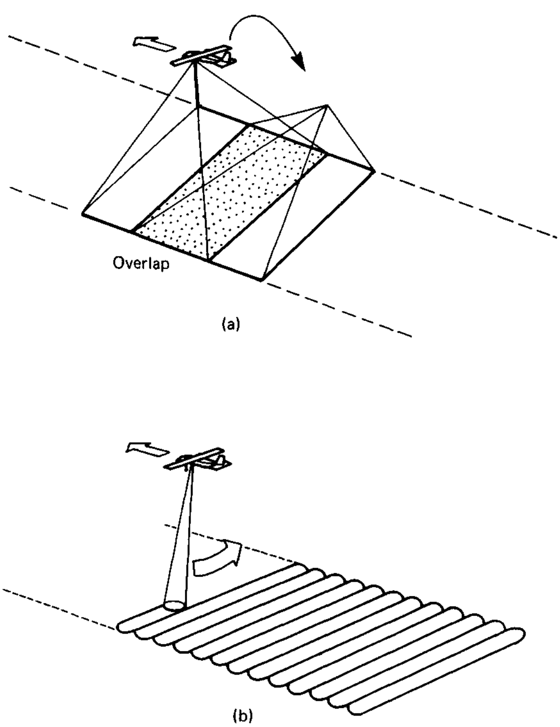

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7.6.1 Airborne systems

7.6.1.1 Photography

Vertical panchromatic aerial photography taken with a high-precision mapping camera is the most commonly available source of airborne remote sensing data (Figure 7.20(a) and Table 7.1). In addition to conventional aerial photography obtained using the system described above, it is also possible to utilize oblique aerial photography[31] (Figure 7.20(b)), low-cost or small-format photography,[32] or multiple-camera multispectral designs where each camera is filtered in order to record the reflected radiance in a particular waveband.[33] Alternatively, lower cost platforms such as microlight aircraft[34] or remotely piloted aircraft[35] may be used to obtain photography over a small site. A more recent development has been to produce photography from airborne video systems.[36]

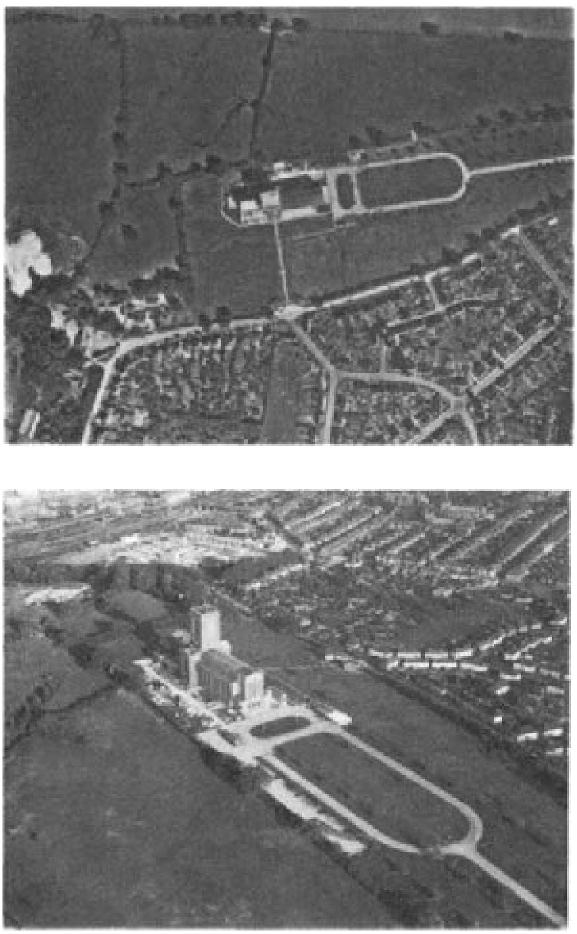

Figure 7.20 Aerial photographs of Guildford Cathedral. (a) Vertical; (b) oblique

7.6.1.2 *Multispectral scanner*

The principle behind the use of the multispectral scanner (MSS) concerns the detection of radiation from the Earth's surface. This radiance can be quantified by measuring the proportion of energy reflected or emitted by an element of the Earth's surface at various specific wavelengths. Each element is referred to as a pixel. An image can be created using an airborne linescanner by detecting the radiance in a linear pass across the ground perpendicular to the line of flight. Successive passes can be conducted at a rate commensurate with the aircraft flying speed, which can then be used to create an image. The image is therefore built up pixel by pixel in a sequential form, line by line. Crystal detectors in the MSS transform the incident radiation into electrical signals which are recorded in digital form on magnetic tape.

The technique used in the MSS is to detect both radiation *emitted*, and solar radiation *reflected*, from the Earth's surface. These forms of radiation, within several different bands of wavelength, can be detected simultaneously using this method. The advantage of using airborne systems rather than satellite systems (such as Landsat MSS) is the much higher spatial resolution of the former (less than a metre) and the greater number of available spectral wavebands.

One MSS which has been used in the UK for a variety of engineering projects is the Daedalus AADS 1268 Airborne Thematic Mapper (ATM). This instrument has an instantaneous field of view (IFOV) of 2.5 mrad and the spectral specifications of the instrument are presented in Table 7.6. Data are recorded in flight on high-density digital tape and subsequently reproduced in a standard form on to computer compatible tape (CCT) for analysis on a digital image processing system. Figure 7.21 illustrates a typical example of imagery obtained with this type of scanner.

Table 7.6 Spectral bands for the Daedalus 1268 AADS ATM

Spectral band	*Wavelength* (μm)	*Spectral region*
1	0.42–0.45	Blue
2	0.45–0.52	
3	0.52–0.60	Green
4	0.605–0.625	Red
5	0.63–0.69	
6	0.695–0.75	Near IR
7	0.76–0.90	
8	0.91–1.05	
9	1.55–1.75	
10	2.08–2.35	Mid IR
11	8.50–13.00	Far (Thermal) IR

7.6.1.3 *Thermal infra-red (IR) scanning*

Thermal IR scanning techniques are similar to those utilized by MSSs but concentrate on a single wavelength within the far IR region of the spectrum. Two regions can be sensed by such instruments, corresponding to atmospheric 'windows' in the 3 to 5 μm and 8 to 14 μm regions. The former region is used primarily for examining very hot objects, e.g. forest fires, and is rarely used for engineering applications. The latter region, corresponding to the peak value of emitted radiation from the Earth, is of much greater importance.

The most common thermal IR sensor is the linescanner which is a widely used instrument based on a scan geometry identical to that for MSS devices. A cadmium mercury telluride (CMT) detector is used to record the temperature variations over the scene and a calibrated internal blackbody reference enables precise quantitative temperature variations to be measured. Several thermal IR linescanners are currently in operation in the UK, including a Daedalus 1230 dual-channel scanner and as mentioned previously a Daedalus 1268. Figure 7.22 shows a typical example of a thermal image obtained with this scanner which could be used to assess heat loss.

An alternative technique for sensing in the thermal IR region of the spectrum which has recently been evaluated for remote sensing applications in environmental engineering is the thermal video frame scanner (TVFS).[37] Several benefits accrue from the

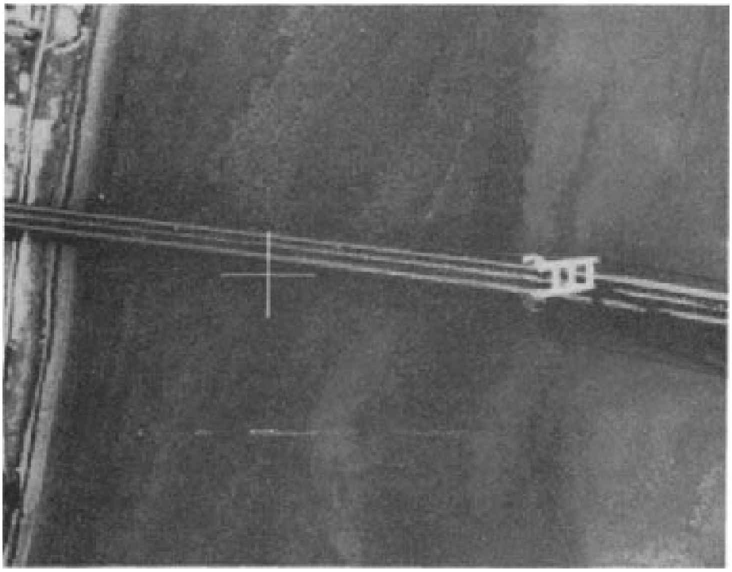

Figure 7.21 Airborne multispectral scanner image of the Humber Bridge illustrating sediment transport patterns. (*Courtesy*: Huntings Geology and Geophysics)

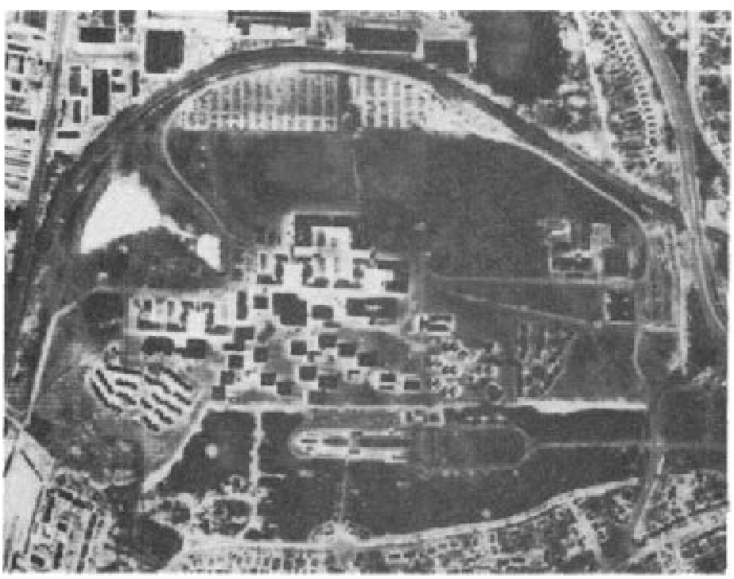

Figure 7.22 Thermal infra-red linescan image of the University of Surrey. (*Courtesy*: Clyde Surveys Ltd)

use of video-based instruments. These include system portability, almost real time verification of data capture, rapid turnaround (since there is no processing stage), low cost of operation (since the sensor can be operated from a light aircraft) and ease of duplication of data. Unlike linescanning systems, however, these instruments normally do not include a calibrated internal blackbody reference for quantitative temperature measurements. For some applications this could prove to be a severe limitation. However, for many engineering applications an assessment of the relative temperatures within a scene can be just as important as a detailed knowledge of the absolute ground temperatures. It should be noted that the quantitative determination of temperatures can, nevertheless, be carried out by correlating grey scale levels and ground control points of known temperature.[38] One example of this type of instrument which is currently being used for remote sensing applications is the Barr and Stroud IR18 TVFS system.

7.6.1.4 Side-looking airborne radar (SLAR)

Side-looking airborne radar was first developed for military purposes in the early 1950s. It was not, however, until the early 1970s that it became available commercially. The main impetus for the development of SLAR arose out of its two significant advantages over optical photographic or scanning systems: (1) its ability to produce imagery both day and night; and (2) its ability to 'sense' through haze, smoke, cloud and even rain. These radar systems are consequently in great demand for the production of imagery in equatorial regions of the Earth's surface.

The main reason for these twin advantages stems from SLAR's use of microwaves as the imaging source. The image which is produced by SLAR is very different from that produced by conventional optical systems. The view which is obtained is a record of the Earth's reflective properties at microwave wavelengths. Consequently, the nature and intensity of the reflections by SLAR will be influenced by factors such as ground conductivity and surface roughness, which are much less significant with optical systems. In addition, the geometrical properties of the image differ significantly from a conventional aerial photograph.

The principle of operation of a typical SLAR system is illustrated in Figure 7.23.[39] It involves the measurement of the time interval between the transmission and reflection of a microwave pulse. This indirectly provides the range from the aircraft to the ground feature. Successive pulses can then be sensed sequentially so providing a radar picture of the terrain. Figure 7.24 is a large-scale image obtained using an airborne radar system.

Two distinct designs of SLAR exist. The earliest designs were termed real aperture radar (RAR) systems. Although able to sense through cloud their ground resolution was limited by the size of the antennae which could be mounted on the side of the aircraft. In order to overcome this limitation, an alternative design, synthetic aperture radar (SAR) has been developed. Using suitable computer processing such a system is able to provide much higher resolution imagery than that which can be provided by a RAR system. Table 7.7 outlines some of the characteristics of a selection of SLAR systems.

7.6.2 Satellite systems

Satellite remote sensing systems range from low-resolution systems such as the meteorological satellites to high-resolution photographic missions such as the recently flown metric camera on board the space shuttle. Although both have some application to civil engineering the low ground resolution of the former system and the limited coverage of the latter restrict their application considerably. Of much greater importance to civil engineers are the Earth resources satellites operating the visible, infra-red and microwave regions of the spectrum.

Figure 7.24 Side-looking airborne radar image

7.6.2.1 Earth resources satellites – passive

A selection of the characteristics of the most common Earth resources satellites is given in Table 7.8.

Landsat. The Landsat satellite system, previously known as the Earth Resources Technology Satellite (ERTS), was initiated in 1967 by NASA in conjunction with the US Department of the

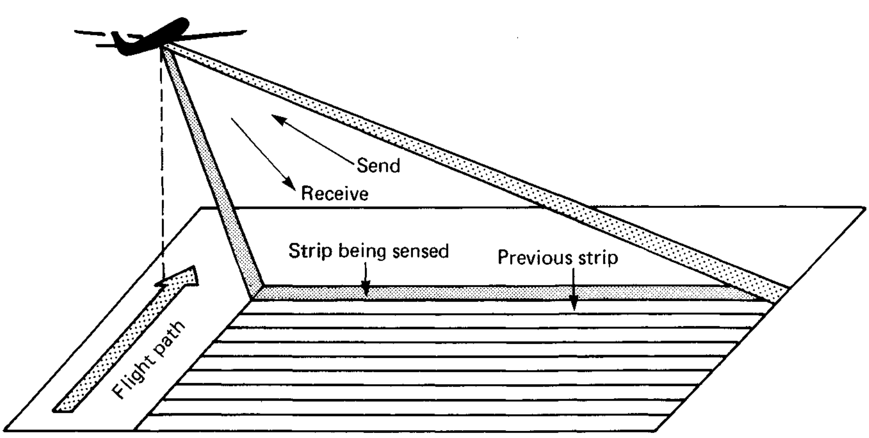

Figure 7.23 Side-looking airborne radar. (After Rudd (1974) *Remote sensing–a better view.* Duxbury Press, Massachusetts)

Table 7.7 Side-looking airborne radar systems

	Motorola AN/APS	*Westinghouse*	*Goodyear GEMS*	*SAR-580*
Type	RAR	RAR	SAR	SAR
Wavelength band	× (3 cm)	K(0.8 cm)	× (3 cm)	× (3 cm)
Resolution (m)	15	15	12	3

Table 7.8 Characteristics of passive Earth resources satellites

	Landsat's 1-5 multispectral scanner (MSS)	*Landsat's 1-3 return beam vidicon (RBV)*	*Landsat's 4-5 thematic mapper (TM)*	*Modular optoelectronic multispectral scanner (MOMS)*	*Le Système Probatoire d'Observation de la Terre (SPOT)*
Operated by (country)	EOSAT (USA)	EOSAT (USA)	EOSAT (USA)	DFVLR (W. Germany)	SPOT Image (France)
Date of launch	1972	1972–81	1982	1983	1986
Orbital altitude (km)	900	900	705	300	830
Ground resolution (m)	80	80, 30	30	20	10, 20
Spectral range (μm)	0.5–12.6	0.505–0.75	0.45–12.5	0.575–0.975	0.5–0.89
No. of wavebands	5	1	7	2	3
Further reading	NOAA[40]	NOAA[40]	NOAA[40]	—	Chevrel, Courtois and Wells[41]

Interior. The system was initially designed as an experiment in order to assess the feasibility of collecting Earth resource data from unmanned satellites. However, following the commercialization of remote sensing activities in 1985, the current operational activities of Landsat have been transferred to EOSAT, a joint venture formed by Hughes and RCA.

Three distinct sensors have been carried by Landsat: (1) a MSS; (2) a return beam vidicon (RBV); and (3) a thematic mapper (TM). The MSS is a linescanning device which uses an oscillating mirror to scan at right angles to the satellite flight direction. The IFOV of the sensors operating in the visible and near IR produce a resolution cell of approximately 56 × 79 m. The fifth channel has an IFOV of 0.258 mrad, or a ground resolution of about 235 m.

The MSS scans each line from west to east with the southward motion of the satellite providing the along-track progression of the scan lines (Figure 7.25).[40] Each Landsat MSS scene covers an area approximately 185 × 185 km. In view of the extremely high mirror oscillation rate which would be required using this approach if only one line was scanned, the system is designed to scan six lines simultaneously with each oscillation of the mirror. This results in an area 474 m × 185 km being recorded with each sweep.

A typical Landsat scene, 185 × 185 km, consists of 2340 scan lines with about 3240 pixels per line; each image therefore consists of over 7.5 million digital values. With four spectral observations per pixel, this means that over 30 million values have to be recorded for every Landsat scene.

The simplest Landsat MSS product is a photograph consisting of a black-and-white image for each spectral band (Figure 7.26). Although the resulting image is vaguely familiar, because

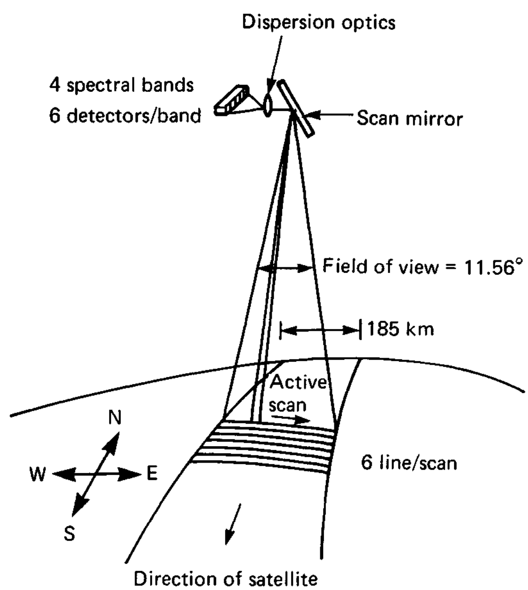

Figure 7.25 Landsat multispectral scanner[40]

it is similar to a conventional panchromatic aerial photograph, much of the information content of the image is lost. An alternative approach is to assign a different colour to each

Figure 7.26 A black-and-white image as presented by Landsat of Southeast England created by a MSS. (*Courtesy*: National Remote Sensing Centre)

spectral and superimpose these to produce a colour-composite image. Since the sensor also senses-in the reflected infra-red region of the spectrum, the consequent composite does not provide a true colour-coded image of the terrain but rather a 'false' colour image.

The second sensor, the RBV, was designed to provide high-resolution imagery suitable for mapping. In addition to having a high ground resolution (30 m on Landsat 3) it also had a réseau grid superimposed on the image. In recent years however, the advent of the TM has tended to reduce the importance of both MSS and RBV imagery, apart from forming a historical record.

The thematic mapper has been operational since 1984 and it provides data of higher spatial resolution (30 m) and finer spectral resolution (seven bands) than that available from the MSS or TM sensors. The comparative spatial resolution of the sensors is illustrated in Figure 7.27.

Modular optoelectronic multispectral scanner (MOMS). The modular optoelectronic multispectral scanner (MOMS) was designed by the West German MBB company for the German Aerospace Research establishment (DFVLR), primarily as a research system. The first satellite-borne imagery was obtained with the system during the seventh space shuttle flight in June 1983. The scanner has several unique design features of which the most significant are the dual lens system and four linear arrays of 1728 pixels which enable a continuous line of 6912 pixels to be swept out by the scanner (equivalent to 20 m on the ground).

A second-generation MOMS system is currently under development by MBB which will offer the possibility of obtaining stereoscopic imagery and which will also have an extended range of spectral bands.

Le Système Probatoire d'Observation de la Terre (SPOT). This is a commercial remote sensing system developed by the French Government and aerospace industry and operated worldwide by SPOT image. The sensors on board the satellite consist of two high-resolution visible (HRV) imaging instru-

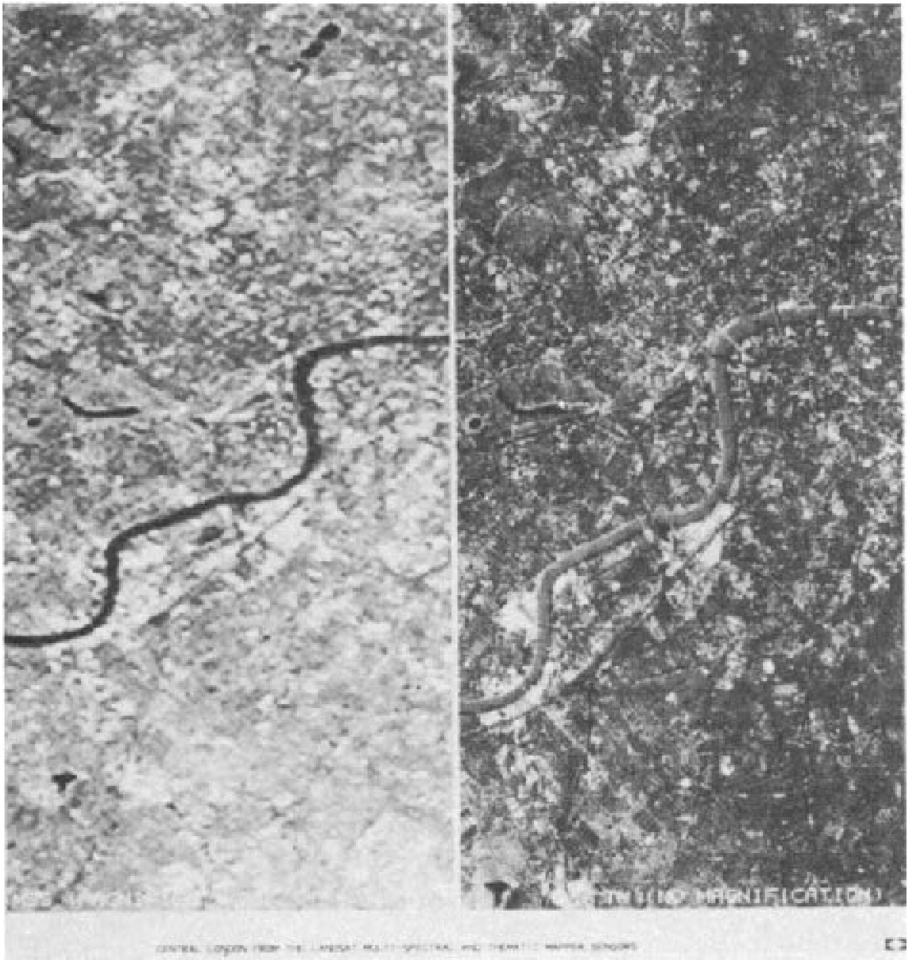

Figure 7.27 A comparative spatial resolution of the Landsat MSS (left) and thematic mapper sensors. (*Courtesy*: National Remote Sensing Centre)

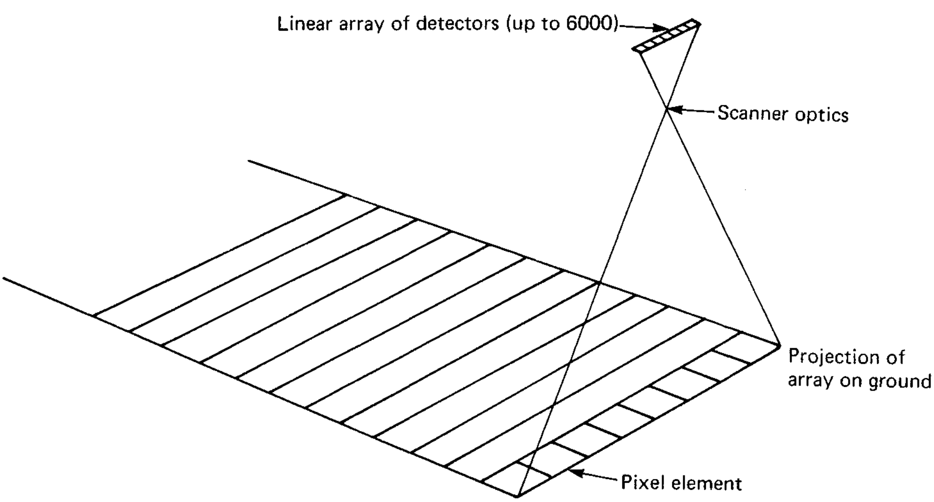

Figure 7.28 SPOT: pushbroom scanner design

ments employing the multilinear array or 'pushbroom' design of MSS. This design of MSS differs from the optical–mechanical design discussed previously. In this case each line of the image is formed by measuring the radiances which are imaged directly on to a one-dimensional linear array of small detectors located in the instrument's focal plane. Each line is subsequently scanned electronically and the radiance values recorded on to magnetic tape. As before, successive lines of the image are produced by the forward motion of the satellite along its orbital path. This pushbroom concept is illustrated diagrammatically in Figure 7.28.

A further important feature of the SPOT system from an engineering point of view is its capacity to provide stereoscopic coverage of the Earth's surface from the lateral overlap between successive scenes. A rotatable mirror in the SPOT sensor package also permits scenes to be acquired over areas up to 400 km left or right of the normal vertical vantage point of the satellite. This feature will permit much easier acquisition of high-priority scenes (Figure 7.29). Figure 7.30 shows a typical example of a SPOT panchromatic image (10 m pixel).

7.6.2.2 Earth resources satellites – active

Following the success of the Seasat satellite SAR mission in 1978 (Table 7.9 and Figure 7.31), a great deal of interest has been shown in the development of further side-looking radar satellites. For example, the forthcoming ERS-1 satellite will be used to provide continuous monitoring of ocean parameters such as wave and wind height and pattern, which may help to improve the engineering design of oil platforms. The SAR data will also be used to aid the planning of shipping routes particularly in Arctic regions. Similarly, the Canadian Radarsat will provide imagery to enable the positions of icebergs to be monitored more precisely. This data will be used by oil tankers transporting oil and natural gas through the Northwest passages from oil fields in and around the Arctic islands of northern Canada.

7.7 Digital image processing (DIP)

Digital image processing is a crucial stage in the effective use of

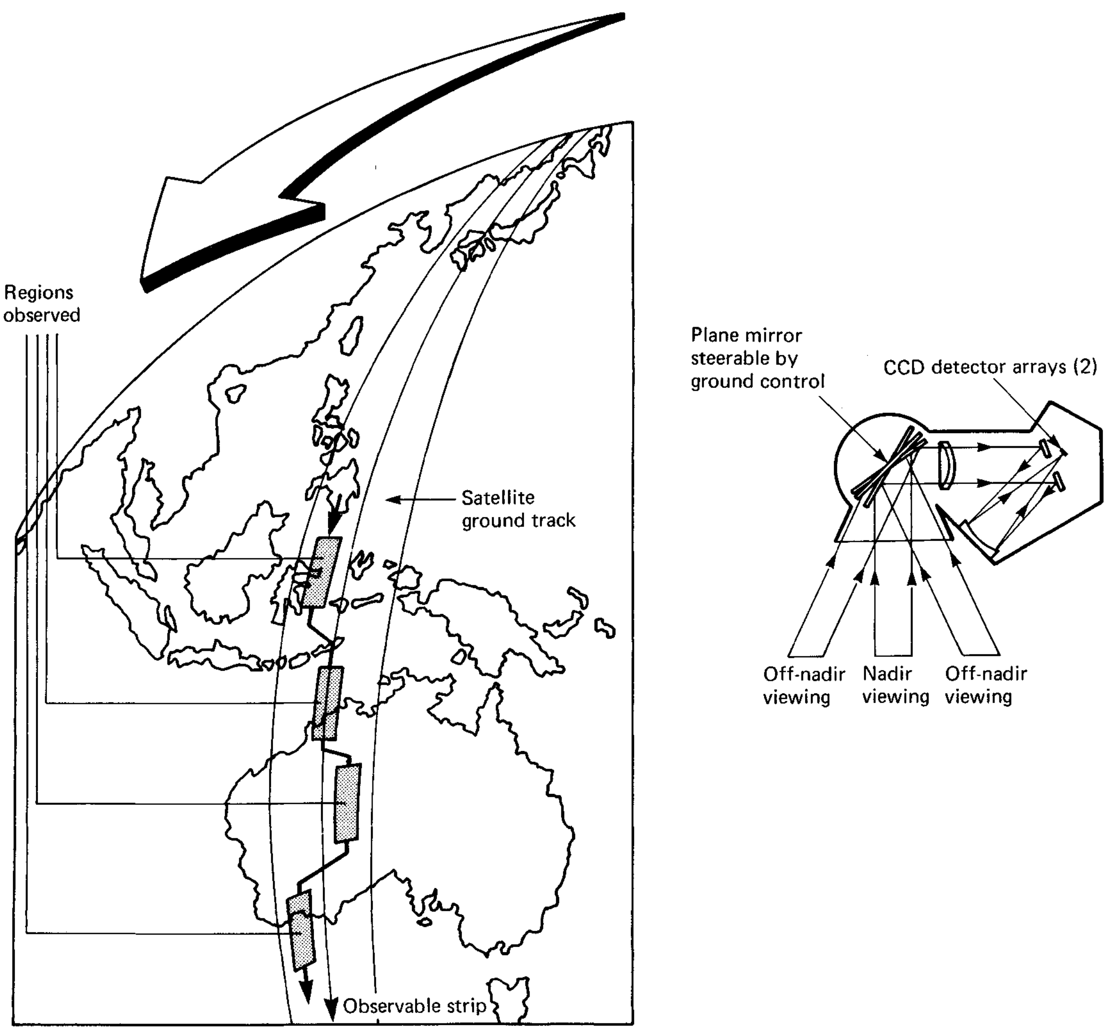

Figure 7.29 SPOT: nadir and off-nadir viewing. (*Courtesy*: SPOT Image)

Figure 7.30 SPOT: panchromatic image of part of Montreal (SPOT data copyright CNES, 1986, image provided by Nigel Press and Associates)

modern remote-sensing data. Although a great deal of information can be gleaned from the interpretation of a photographic image produced from, for example, Landsat, the full potential of the data can only be realized if the original data, edited and enhanced to remove systematic errors, is used with a suitably programmed image processing system.

7.7.1 Hardware

Until relatively recently, most DIP of remote sensing data was carried out on large, expensive, dedicated systems such as that illustrated in Figure 7.32. Such a system enables data to be read into the system from a magnetic tape reader, to be displayed on a colour TV monitor and, after suitable processing, to be output to a colour filmwriter for the production of high-quality imagery. The trend, however, is towards smaller and cheaper image processing based on the new generation of 32-bit super minicomputers and 16-bit IBM-compatible personal computers.[42] Table 7.10 lists a selection of the remote-sensing systems currently available.

7.7.2 Software

The range of algorithms which are used to restore and classify remote-sensing data is extremely wide and varied. Furthermore, an appreciation of the choice of the most appropriate technique,

Table 7.9 Characteristics of a selection of active Earth resources satellites

	Seasat	*Shuttle imaging radar (SIR)-A*	*SIR-B*	*European radar satellite (ERS-1)*	*Radarsat*
Date of launch	1978	1981	1984	1989	1991
Operated by	NASA	NASA	NASA	European Space Agency	Canada
Altitude (km)	800	245	255, 274, 352	675	1000
Wavelength (mm)	230	230	540	540	540
Look angle (°)	20	50	57	—	30–45
Resolution (m)	25	40	30	25	30
Swath width (km)	100	50	20–25	80	150

Figure 7.31 SEASAT SAR image of the Tay estuary. (*Courtesy*: National Remote Sensing Centre)

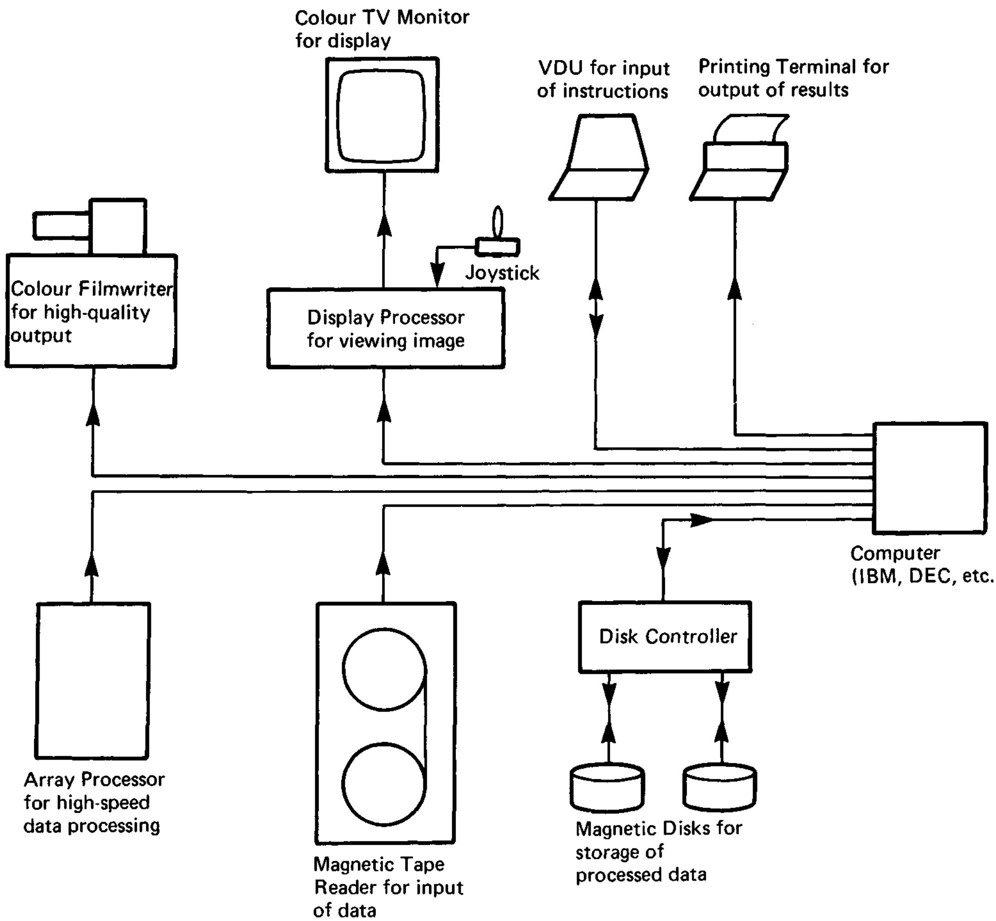

Figure 7.32 Digital image processing system

and the advantages and limitations of each technique, is a subject of some complexity beyond the scope of this chapter. Consequently, only a very brief description of the most commonly used techniques will be provided. For further details Bernstein,[43] Avery and Graydon[44] and Bagot[45] should be consulted.

7.7.2.1 Image restoration

Geometric correction. Several preliminary transformations are normally carried out on raw satellite data to reduce the effects of the Earth's rotation and curvature and variations in the attitude of the satellite. If a sufficient number of ground control points are available the imagery can be fitted to the control using a least squares technique. Such a procedure is also necessary if the remote sensing data is to be combined with existing map data.

Radiometric correction. Variations in the output from the detectors used in a MSS may cause a striping effect on the final image. Such an effect can be eliminated by a process termed 'destriping', which effectively interpolates the missing radiance values from the outputs of adjacent pixels. In some cases, complete scan lines may be missing from the data; preprocessing can also be carried out to overcome this type of problem.

Table 7.10 A selection of remote-sensing systems currently available

	Name of system	*Company*
Dedicated image-processing system	Gemstone 33	GEMS of Cambridge, UK
	I²S Model 75	Int. Imaging Systems, California, USA
	Vicom VDP Series	Vicom Systems, San Jose, USA
	Dipix Aries II	Dipix Systems, Ottawa, Canada
Personal computer-based systems	Diad Systems	Diad Systems, Edenbridge, Kent, UK
	LS10	CW Controls, Southport, Lancashire, UK
	Microimage	Terra Mar, California, USA
	ERDAS	Earth Resources Data Analysis System, Atlanta, USA
	ImaVision	RBA Associates, Ottawa

7.7.2.2 Image enhancement

Contrast stretching. Contrast stretching is a technique which is used to brighten an image and produce one which uses the full dynamic range of the data. It is often carried out automatically and is generally one of the first operations carried out by a user when viewing a new image.

Spatial filtering. Spatial filtering is carried out in order either to enhance boundaries between features (edge enhancement or high pass filtering) or to smooth and eliminate boundaries (smoothing or low pass filtering). The former approach is most suitable for enhancing geological features, whereas the latter may be used to reduce random noise from an image.

Image ratioing. Image ratioing is the process of dividing the radiance values of each pixel in one waveband by the equivalent values in another waveband. Ratioed images reduce radiance variations caused by local topographic effects and consequently may be used to enhance subtle spectral variations.

Principal components analysis. Principal components analysis is a statistical technique which is used to improve the discrimination between similar types of ground cover. It is based on the formation of a series of new axes which reduce the correlation existing between successive wavebands in a multispectral data set.

7.7.2.3 Image classification

Density slicing. Density slicing is the simplest form of automated classification. It enables a single band (often of thermal IR data) to be colour-coded according to the grey level of the individual pixels. Thus, for example, pixel values 0 to 19 may be coloured blue, 20 to 25 red and so on. This not only aids interpretation but if the density slices are related to temperature levels, the resulting image can be used quantitatively to assess the variations in temperature across the scene.

Supervised classification. In cases where ground data exists about the terrain under investigation, it is possible to use this data to 'train' the computer to perform a simplified type of automated interpretation. For example, a particular group of pixels may be known to represent an area of water. Examination of the maximum and minimum numerical values of the radiances in each of the spectral bands being used (seven for Landsat TM) would enable a 'spectral pattern' for water to be determined. If the computer then compared this multidimensional spectral pattern with the spectral pattern of every pixel in the scene it would be possible to display only those areas which satisfied the criteria set by the training sample. Consequently, all water areas over a large area (185 × 185 km for Landsat) could be determined.

The process outlined above refers to the simplest form of classifier, the 'box' classifier. Other more sophisticated techniques are also available.

Unsupervised classification. In unsupervised classification of multispectral data, no attempt is made to 'train' the computer to interpret common features. Instead, the computer analyses all the pixels in the scene and divides them into a series of spectrally distinct classes based on the natural clusters which occur when the radiances in *n*-bands are compared. The user may now analyse other evidence to determine the nature of each of the distinctive classes which have been identified by the computer. Unsupervised classification techniques are generally more expensive in terms of processing requirements than supervised techniques.

7.8 The use of remote sensing in civil engineering

Although aerial and satellite remote-sensing imagery (other than black-and-white aerial photography) have been used for topographic mapping, the primary role of this type of imagery has been for the production of thematic maps. Generally, for such applications users are satisfied with a relatively low level of positional accuracy and are also willing to accept a lower level of completeness than is the case with topographic maps. The type of remote-sensing data which is appropriate will depend largely on the stage of the civil engineering project and the degree of economic development of the region where the project is being carried out.

Five main stages can be identified for a civil engineering project: (1) reconnaissance; (2) preliminary planning/feasibility; (3) design; (4) construction; and (5) post construction/maintenance. The potential role of remote sensing at each stage will vary from project to project, but the following sections indicate some of the possible uses of the techniques at each stage.

7.8.1 Reconnaissance level investigations

The first stage of any major civil engineering project generally involves some form of preliminary reconnaissance study of the region or project area. The primary aim of this study is to collect together all available data concerning the physical characteristics of the terrain in order to assess its likely influence on the overall design of the engineering works.

Traditionally, the engineer has carried out this reconnaissance stage by examining existing maps of the region. Both topographic maps and specialist thematic maps (e.g. geological, geomorphological and pedological) are of greatest use at this initial stage of the project. In addition, examination of engineering reports produced for previous projects in the region may also provide valuable reference material. The main problems associated with this approach are that: (1) in many regions existing maps may be at a very small scale (< 1:500000), be significantly out of date and, in some instances, may not exist at any scale; and (2) that whilst engineering reports may give valuable data about a specific site they may not be appropriate or representative of regional terrain conditions. Consequently, many engineering projects have involved an examination of satellite and aerial imagery at this early stage.

The use of Landsat MSS data for projects of this type has been reported by Beaumont.[46] In this case, use of Landsat data for water resources planning in northeast Somalia is reported. Visual interpretation of Landsat data enabled a series of transparent overlays to be produced illustrating drainage, surface water, groundwater potential and land capacity over the region.

The use of remote sensing for terrain evaluation purposes is also of considerable importance. Terrain evaluation is a form of thematic mapping in which the terrain is classified in a hierarchical manner into units having common landscape patterns and engineering characteristics. This process can be carried out very cost-effectively using satellite imagery in conjunction with aerial photography.[47]

Interpretation of Landsat data may also be useful for the provision of additional supplementary information on hydrological phenomena, e.g. flooding, water quality, groundwater potential and water depth. A more general review of hydrological applications can be found in Blyth.[48]

Landsat has also been used for estimating urban populations over large areas,[49] and using multitemporal imagery for monitoring change, e.g. the expansion of urban areas, changes in soil erosion, varying river channels, the impact of desertification and so on.

7.8.2 Preliminary planning/feasibility

The aim of the preliminary planning/feasibility stage of a project is: (1) to select potential routes/sites; and (2) to select from these the best from the available options. Depending on the state of economic development of the region, it is likely that different factors will influence the choice of the optimum route/site. For example, in developing regions the location of low-cost construction materials such as calcrete may be important[50] or the identification of potential bridge crossing-points. Both activities can be carried out cost-effectively using satellite imagery in conjunction with aerial photography. A comprehensive review of the role of remote sensing for highway feasibility studies can be found in Lawrance and Beaven.[51]

7.8.3 Design

Remote sensing has a particularly valuable role to play at the design stage of a project. At this stage a detailed site investigation of the proposed site or route will be required, together with supplementary environmental information which may influence the design of the project being considered. A critical stage of a site investigation is the desk study. The desk study provides the engineer with an overview of the project area and is normally carried out using aerial photography. Not only can valuable information be obtained during this interpretation about possible engineering problems but it can also be very important for the planning of the subsequent ground investigations. A general review of the potential of aerial photography in this respect is provided by Dumbleton[52] and Matthews,[53] and a selected number of papers which consider the use of remote sensing for geotechnical investigations are presented in Table 7.11.

Of increasing importance in recent years has been the role of remote sensing in providing environmental engineering data which may provide useful supplementary data to the engineer to input into the design process. A comprehensive review of remote sensing in this context is provided by Mason and Amos.[54] Information which may be extracted from remote-sensing imagery (often thermal IR linescan) includes data on energy conservation (heat loss), power station cooling-water patterns, groundwater and spring detection, pipeline and field drainage patterns, hydraulic leakage from earth and concrete dams and the location of ice-prone regions on roads.

7.8.4 Construction

The role of remote sensing is much smaller during construction than at the preceding planning and design stages. Nevertheless, aerial photography can be used to provide a valuable historical record of construction activities and may also be used to plan construction activities. Singhroy[67] provides an interesting case-history concerning the use of large-scale colour IR aerial photography and aerial video during pipeline construction in Canada. Both types of data were used to plan construction activities, determine site conditions and assess environmental effects before, during and after construction activity.

7.8.5 Post-construction maintenance

As mentioned in the previous section, remote sensing can be used to assess the environmental impact of construction activities. Remote sensing may also be used to assess the use of a new road by conducting aerial traffic investigations.[68] Nevertheless, the use of remote sensing at this stage is again of much less importance than during the previous four stages which have been discussed.

In conclusion, Table 7.12 illustrates the recommended use of remote-sensing techniques at each stage of a civil engineering project.

7.9 Sources of remote-sensing data

7.9.1 Satellite data

The most general source of satellite data and information on the availability and cost of data are the national points of contact (NPOC) which exist in most countries throughout the world. In the UK, for example, the NPOC is the National Remote Sensing Centre, Royal Aircraft Establishment, Farnborough.

Several other organizations may have archives of satellite data or may act as distributors for Landsat or SPOT data. Table

Table 7.11 Remote sensing and site investigations – selected references

	Aerial photography	*MSS*	*Thermal IR linescanning*
Landslides/slope instability	Norman, Leibowitz and Fookes[55] Matthews and Clayton[31]	Liu[56]	Chandler[57]
Solution features	Norman and Watson[58]	Coker, Marshall and Thompson[59]	Kennie and Edmonds[60]
Derelict/contaminated land	Bullard[61]	Coulson and Bridges[62]	Ellyet and Fleming[63]
Surfacial/subsurface materials surveys	Caiger[64]	Lynn[65]	Singhroy and Barnett[66]

Table 7.12

		Satellite						Aircraft				
Project phase	*Photograph/ image scale*	*Photog.*	*Landsat MSS*	*Landsat TM*	*SPOT*	*MOMS*	*Side-looking radar*	*Photog.*	*MSS*	*Thermal*	*SLAR*	*Video*
Reconnaissance	1:1 000 000 to 1:50 000	***	****	***	***	***	***	**	—	—	—	—
Planning/feasibility	1:100 000 to 1:20 000	***	**	***	***	***	***	**	—	—	—	—
Design	1:20 000 to 1:2000	—	—	—	*	—	—	****	***	***	***	**
Construction	1:2000 to 1:500	—	—	—	—	—	—	****	*	*	*	***
Post construction/ maintenance	1:2000 to 1:500	—	—	—	—	—	—	****	*	*	*	**

****Very useful ***Very useful (but coverage limited) **Useful *Of limited use — inadequate

7.13 lists a selected sample of the organizations who provide remote-sensing services and may also hold copies of satellite data.

7.9.2 Aerial photography

As mentioned in Table 7.13, several of the commercial remote-sensing organizations offer facilities for obtaining airborne thermal and MSS data and should be consulted if such data are required. However, aerial photography remains the most popular source of data for remote-sensing applications in civil engineering. Table 7.14 lists some of the organizations which have archives of aerial photographs. In addition, several of the organizations listed in Table 7.13 may also have holdings of aerial photographs.

Table 7.13 Sources of airborne and satellite data

Name of Company/Organization	*Based in*	*Comments*
Clyde Surveys Ltd	Maidenhead, Berks, England	Operate Daedalus thermal scanner
DFVLR	Oberpfaffenhofen, W. Germany	Distributors of MOMS and metric camera data
EOSAT Corp.	Arlington, Virginia, USA	Responsible for operation of Landsat
GeoSurvey Ltd	East Molesey, Surrey, England	—
Huntings Geology and Geophysics Ltd	Borehamwood, Herts, England	Operate Daedalus airborne MSS
EROS Data Centre	Sioux Falls, S. Dakota, USA	Prime inter. distribution centre for Landsat
Nigel Press and Associates	Edenbridge, Kent, England	Distributors of SPOT data in UK
SPOT Image	Toulouse, Cedex, France	Main operator for SPOT
SPOT Image Corp.	Washington DC, Reston, Virginia, USA	Distributors of SPOT data in N. America

Table 7.14 Sources of aerial photography

Name of Organization	*Based in:*
Aerofilms Ltd	Borehamwood, Herts, England
BKS Surveys Ltd	Coleraine, Co. Londonderry, N. Ireland
Cartographical Services Ltd	Salisbury, Wilts, England
Central Register of Aerial Photography of N. Ireland (Ordnance Survey)	Belfast, N. Ireland
Central Register of Aerial Photography of Wales	Cardiff, Wales
ERSAC Ltd	Livingston, W. Lothian, Scotland
Scottish Development Department	Edinburgh, Scotland
J. A. Story and Partners	Mitcham, Surrey, England
Ordnance Survey and Overseas Survey Directorate	Southampton, Hants, England
Royal Air Force Film Library	Whitehall, London
University of Cambridge Committee for Aerial Photography	Cambridge, England

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8 Geology for Engineers

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Contents

This chapter introduces civil engineers to some basic geology and outlines the broad concepts of the subject.

Geology is concerned with the science of the Earth and the materials comprising the Earth. This includes *physical geology* or *geomorphology* (the surface form of the Earth), *palaeontology* (study of fossils), *stratigraphy* (the chronological sequence of rocks), *mineralogy* (study of minerals), *petrology* (study of the composition of rocks) and *structural geology* or *tectonics* (the broad structure of rocks). Together with newer and closely related branches such as geochemistry, geophysics or mathematical geology, and applied and biological aspects, the whole subject is rapidly developing and is now generally being called Earth Science.

Engineering geology is the branch of geology applied to civil engineering and, in Britain particularly, is applied to all aspects of foundation and excavation design, construction and performance. The extremes of the subject merge into the practices of soil mechanics, rock mechanics and some aspects of the extractive industries, as sand and gravel or opencast mining (Price[1]).

8.1 Basic geology

8.1.1 Introduction

Rock is strictly defined in geology as any natural solid portion of the Earth's crust which has recognizable appearance and composition. Some rocks are not necessarily hard, and in discussion a geologist may call peat or clay a rock as he would granite or limestone.

There are three major classes of rocks:

(1) *Sedimentary rocks* formed by the deposition of material at the Earth's crust, e.g. sandstone, clay.
(2) *Igneous rocks* formed from molten rock magma solidifying either at the Earth's surface or within the crust, e.g. basalt, granite (*s.l.*).
(3) *Metamorphic rocks* produced deep in the Earth by the transformation of existing rocks through the action of heat and pressure, e.g. marble, slate.

The interrelation and continual recycling of rock over long periods of geological time is illustrated in Figure 8.1.

8.1.2 Principles of stratigraphy

Sedimentary rocks cover some 75% of the Earth's land surface but form only a discontinuous and relatively thin cover to the underlying igneous and metamorphic rocks of the mantle.

The sedimentary layers (*strata*) normally lie one above another in order of decreasing age, but where there has been structural disturbance they are faulted and folded. Study of the strata in a particular area enables their sequence to be recorded, and this can then be compared with other local sequences. From such observations the general succession of sedimentary rocks over a wider area can be established: this has been done, for example, for nearly the whole of the British Isles. A list of strata for England and Wales was compiled by William Smith, 'the father of English geology'; in 1815 he produced the first simple coloured geological map of the country. As a result of his studies he stated two basic principles of stratigraphy, that 'the same strata are always found in the same order of superposition, and contain the same peculiar fossils'. These principles are still used to determine the relative ages of strata, i.e. in the order of superposition for an undisturbed series of sedimentary beds, the oldest (i.e. the first deposited) is at the bottom, and successively younger beds lie upon it. Sedimentary strata in different localities can usually be correlated by the diagnostic fossil remains they contain. Rapidly evolving fossils act as horizon markers so that a specimen of one of these enables the particular level of the rock outcrop in which it occurs to be identified in the geological column wherever in the world it is found.

The whole sequence of rocks comprising the geological column is broadly divided into the systems and groups shown in Table 8.1; this column applies particularly to British strata. The column shows the age of each group relative to the others, and was in use long before any of the recent radiometric methods of determining the absolute age in years was developed. Names of the geological systems, and of the larger groups are of worldwide application; they are also used to express the periods of time during which the rocks of the different systems were formed, e.g. the Jurassic system and the Jurassic period, or Mesozoic group and the Mesozoic era. The times of major mountain-building episodes (*orogenies*) and of phases of igneous activity in Britain are given in the third column of the table.

There are numerous further subdivisions down to 'zones' and even 'horizons', many of the smaller divisions being based on specific fossils.

In any given area the deposition of sediments was not continuous throughout the geological periods. There are breaks in the sequence of deposits, marked by *unconformities* which represent intervals of time during which there was no deposition

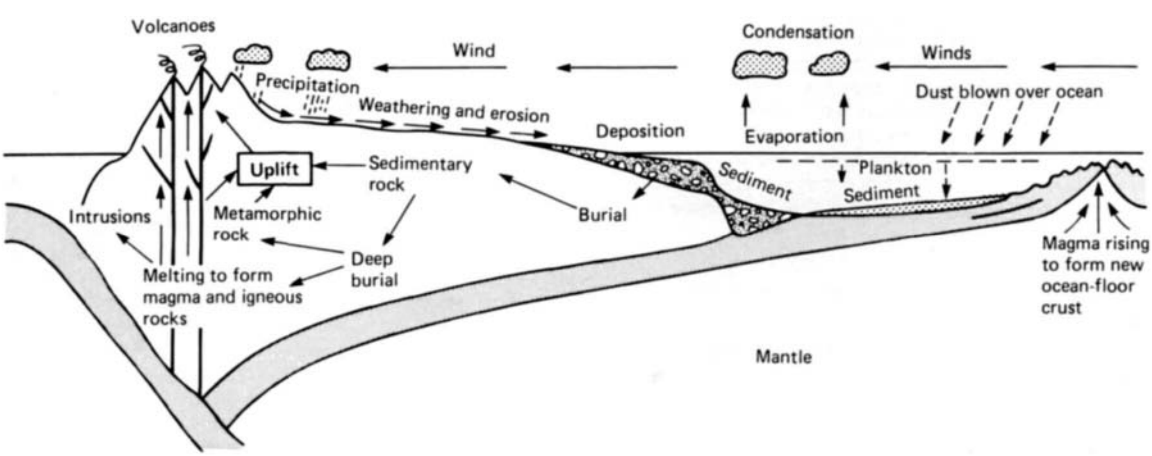


Figure 8.1 Diagrammatic representation of the long-term cycling of rocks. (After Bradshaw, Abbot and Gelsthorpe (1978) *The Earth's changing surface*. Hodder and Stoughton, London)

and erosion took place. The sea floors with their sediments were raised and became subject to erosion by wind and water. There were also periods of quiet sedimentation, when seas covered the land, and intervening episodes of disturbance when uplift and folding took place. This broad pattern of events – the transgression of the sea over the lands, then the regression of the sea, followed by orogenic upheaval – has been repeated many times throughout geological history (see Figure 8.2 which shows the typical simplified borehole sequence of such a chain of events).

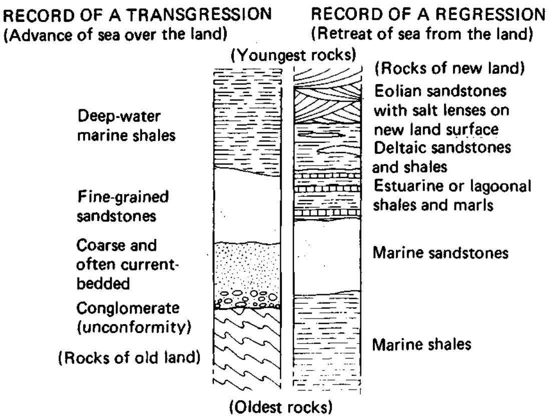


Figure 8.2 Marine transgression and regression as seen idealized in borehole core, tens of metres long. (After Read and Watson (1971) *Beginning geology*, 2nd edn. Macmillan/Allen and Unwin, London)

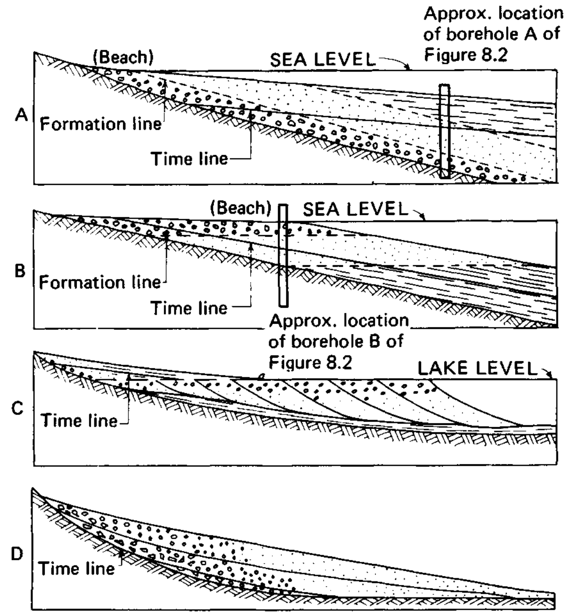


Figure 8.3 Examples of common marine and freshwater transgressions and regressions showing types and geometric distribution of sediment deposited. (After Lahee (1961) *Field geology*, 6th edn. McGraw-Hill, New York.) Lines parallel to lake and sea floors are time lines as they join sediment deposited contemporaneously. Lines essentially parallel to gravel, sand or clay deposits are formation lines. A, a marine transgression over the land; B, a marine regression from the land; C, a lake regression from the land; lake bottom muds are gradually covered by coarser sediments. Later transgression is shown left of a; D, an alluvial transgression by growth of a cone of river alluvium in mountainous area overlooking a desert plain

Unconformities are often marked by beds of pebble gravel, the beach deposits of a sea which gradually inundated the land during its submergence (see Figure 8.3). Examples of this are the pebbly quartzites at the base of the Cambrian, or the rounded flints at the base of the Eocene deposits of southeast England overlying the Chalk, both marking the oncoming of marine transgression. Boulder beds and hill or mountain screes formed on an old land surface during erosion, after uplift has taken place, may also be preserved as the lowest members of a newer series of rocks resting unconformably on older rocks; an example is the boulders and coarse sands at the base of the Torridonian in northwest Scotland which lie unconformably on an old land surface carved in the underlying Lewisian rocks.

An old land surface may be shown by the presence of a 'dirt bed' in which some of the old soil has been preserved, as at Purbeck, Dorset, or by other land-formed deposits. It indicates an interval of time during which there was locally no deposition of waterborne sediments. In marine deposits a minor unconformity (or nonsequence), representing a local cessation in deposition, can be marked by the absence of a metre or so of beds over a relatively small area. This can be found by comparison with other areas where the sequence is complete.

8.1.3 Plate tectonics and the evolution of the Earth

The close association of volcanic and earthquake activity has been known for some time but it is only during the last few years that it has been more or less understood. This association, together with the coincidence of young narrow fold mountain ranges on the continents, and trenches and ridges deep in the ocean basins also in narrow zones, has led to a new theory of Earth evolution known as plate tectonics. This idea was proposed in the late 1960s and has been received with widespread acceptance as more evidence has been found to fit the general model.

The concept suggests that the Earth's surface layers are divided into large segments or plates. Plates are approximately 100 km thick and therefore include the Earth's crust and the upper mantle, and measure several thousand kilometres across. One scheme considers there are six major plates and several smaller ones, covering the entire Earth. Plates slowly move over the face of the Earth with new plate rock formed from the solidification of slowly upwelling molten rock at the constructive margin as more new rock forms and travels towards the destructive margin where it is subducted, and rock material is moved downwards and returned to the lower mantle.

A plate may eventually accumulate a mass of lower density sedimentary rocks on its top to form a continent. Whilst the ocean-floor plate material is constantly being formed and destroyed, the continents are not consumed downwards at the destructive margin because their low density provides buoyancy. The continents are subjected to changes due to erosion and deposition by surface processes, but this has the overall effect of causing relatively light rocks to accumulate. The oldest known continental rocks are 3900 million yr old (Table 8.1) but nowhere are the ocean floor rocks known to be more than 200 million yr old.

Table 8.1 The geological column

Name of geological group or era		*Name of geological system or period* (ages in millions of years)	*General nature of deposits, major orogenies, and igneous activity in Britain*
CAINOZOIC	Quaternary	Recent Pleistocene (2)	Alluvium, blown sand, glacial drifts, etc. At least five ice ages separated by warmer periods. The Devensian (Weichselian or Newer Drift) is the last ice age
	Tertiary	Pliocene Miocene Oligocene Eocene (70)	Sands, clays, and shell beds *Alpine orogeny* *Igneous activity in Scotland and Ireland*
MESOZOIC (or Secondary)		Cretaceous Jurassic Triassic (225)	Sands, clays and chalk Clays, limestones, some sands Desert sands, sandstones and marls
PALAEOZOIC (or Primary)	Newer	Permian Carboniferous Devonian (and Old Red Sandstone) (*c.* 400)	Breccias, marls, dolomitic limestone *Hercynian orogeny* *Igneous activity* Limestones, shales, coals and sandstones Marine sediments (Lacustrine sands and marls) *Igneous activity*
			Caledonian orogeny
		Silurian Ordovician Cambrian (*c.* 600)	Thick shallow-water sediments, shales and sandstones. Older Volcanic activity in the Ordovician
PRE-CAMBRIAN		*Dalradian*	Schists
		Moinian (740+)	Schists and granulites
		Torridonian Uriconian Lewisian (3500+)	Sandstones and arkoses Lavas and tuffs (Shropshire) *Pre-Cambrian orogenies* Orthogneisses, etc.

8.2 Geological description and classification of rock

Engineering classification of rock is discussed in Chapter 10, and engineering classification of soils in Chapter 9.

8.2.1 Sedimentary rocks

Sediments originate mainly from the weathering of all rocks, especially igneous rocks. Certain resistant minerals in igneous rocks such as quartz survive unchanged and are eventually incorporated in the new sediments; often they tend to be concentrated in certain types of sediment (e.g. sands). Other igneous minerals, such as the feldspars and ferromagnesian minerals, break down during weathering to give rise to new minerals and to colloidal and dissolved substances. The new minerals, chiefly clay-minerals, are concentrated in a second group of sediments (e.g. clays) and the colloidal matter, usually iron hydroxides, in a third. The substances taken into solution include calcium and magnesium salts which are precipitated by chemical and organic processes as carbonate rocks, and sodium and potassium salts which may in certain circumstances crystallize out to give evaporites. Another group of sediments including coal and peat is produced by the piling up of decaying plant matter.

The products of weathering can be related, as is shown diagrammatically in Figure 8.4, into fairly distinct chemical and geological groups. This natural differentiation provides a simple classification of sediments into two broad groups:

(1) Detrital sediments made by the accumulation of fragmented particles of minerals or rocks, represented by (a) the pebbly rocks, and (b) the sands, made chiefly of inherited minerals or rocks, and (c) the clays made chiefly of new minerals.
(2) Chemical–organic sediments formed by the precipitation of material from solution or by organic processes, represented mainly by the limestones, the evaporites and the coals.

The sediments produced go on changing after deposition; e.g. they may be saturated by groundwater carrying salts in solution, or deformed by the weight of new sediments laid down on top of them. Changes produced by such means are called *diagenetic*

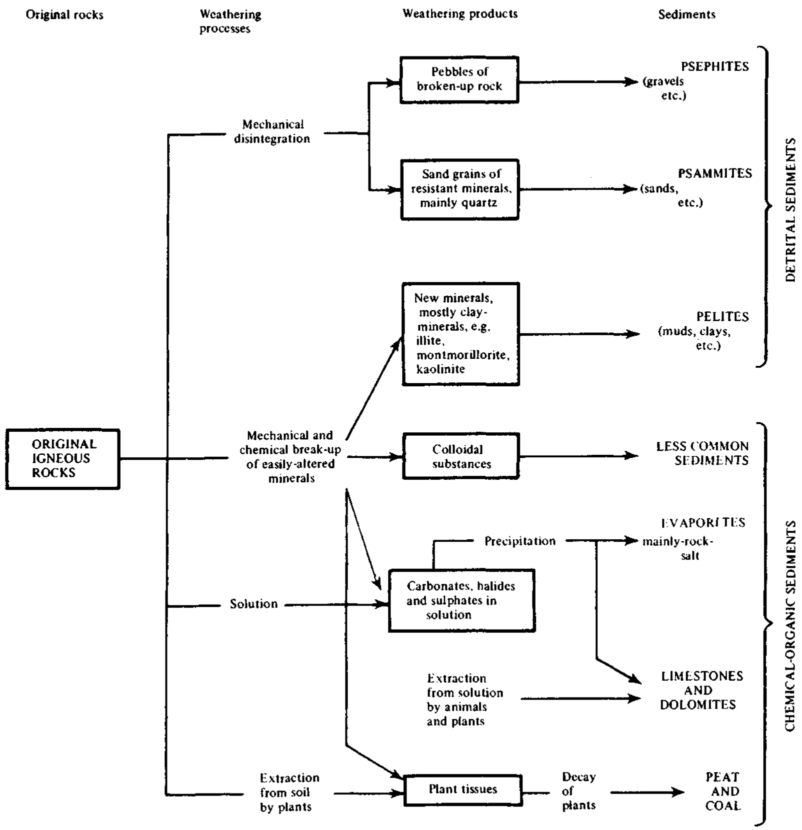


Figure 8.4 Sedimentary differentiation. (After Read and Watson (1971) *Beginning geology*, 2nd edn. Macmillan/Allen and Unwin, London)

changes and convert the *sediments* into consolidated or lithified (hardened) *sedimentary rocks*, e.g. a sand becomes a sandstone.

8.2.1.1 Deposition environments and textures of sedimentary rock

The characteristics and to a certain extent the engineering performance of recent sediments can be directly related to the environment occurring at their location of deposition, because the agents of deposition can still be seen in action. In the older sedimentary rocks, the environment of deposition can be reconstructed from the characters of the rocks themselves. The evidence for this reconstruction is provided by the *composition* and *texture* of the rock, the type of bedding, the fossil content and the relationship between any one bed and its neighbours. The sum of all these features decides its sedimentary *facies* and from this it is generally possible to deduce the conditions under which each rock was formed. This is summarized in Table 8.2.

The most obvious and characteristic feature of sedimentary rocks is *bedding*, i.e. the presence of recognizably different beds or strata in a sedimentary succession, and the presence within any one bed of depositional surfaces which are the bedding planes (see Figure 8.5).

Although many beds are homogeneous, some show considerable variation, especially *graded beds*, in which there is a passage from coarser to finer particles towards the top; lateral gradation may also be found. Thin laminae or layers, differing somewhat in colour or texture, may be present without causing a bed to lose its individuality. A bed is characterized by all of its lithological features. These indicate that it was laid down in a particular environment, either uniform, or varying systematically. Although it may be arbitrary, some very thin strata may best be regarded as beds rather than as laminae within a bed. For example, in glacial varves each annual deposit of summer silt and winter clay is an individual bed even though its thickness is measured in millimetres, whereas sandy laminae in a graded greywacke are parts of the whole graded unit (see Figure 8.6).

In describing bedding it is necessary to distinguish firstly between bedding planes which are individual structures where each planar surface may be distinguished, and also depositional textures, which result from the parallel orientation of particles throughout a bed. Both are primary depositional features, and may be either parallel or inclined to the separation planes, bounding individual beds (Figure 8.6). In addition, various textures, the parallel orientation of mica-flakes, for example, may be induced by post-depositional effects such as consolidation. These are post-depositional *fabrics* but in many instances they are very difficult to separate from true depositional fabrics.

8.2.2 Igneous rocks

The important characteristics of igneous rocks are the chemical composition, the mineral composition and the texture.

8.2.2.1 Chemical composition

The chemical composition depends on the magma from which the igneous rock was derived. Some 99% of the various igneous

Table 8.2 Environments of deposition of sedimentary rocks

Environment of deposition			*Common sedimentary rocks produced by the environment*
SEA			
Shallow seas (continental shelf)	Littoral (beaches, sandbanks, tidal flats)		Conglomerate, sandstone, shale
	Neritic	Shelf seas in stable areas	Orthoquartzite, current-bedded sandstone, shale, organic and chemical limestones
		Restricted deep basins	Black shale
Deep seas		Geosynclinal seas in mobile belts Deep seas in stable areas	As for shelf seas with in addition greywackes and other turbidite deposits
Abyssal seas			Calcareous ooze, siliceous ooze, Red Clay
LAND/SEA			
Deltas			Mainly sandstone, shale
Estuaries, lagoons			Shale
LAND			
Floodplain			Conglomerate, sandstone, shale
Lakes	with outlet to sea		Sandstone, shale, freshwater limestone
	in basins of interior drainage		Sandstone, shale, evaporates
Deserts			Sandstone, conglomerate, breccia
Piedmont (intermontane basins, alluvial fans)			Conglomerate, breccia, arkose, sandstone
Areas of glaciation			Tillite

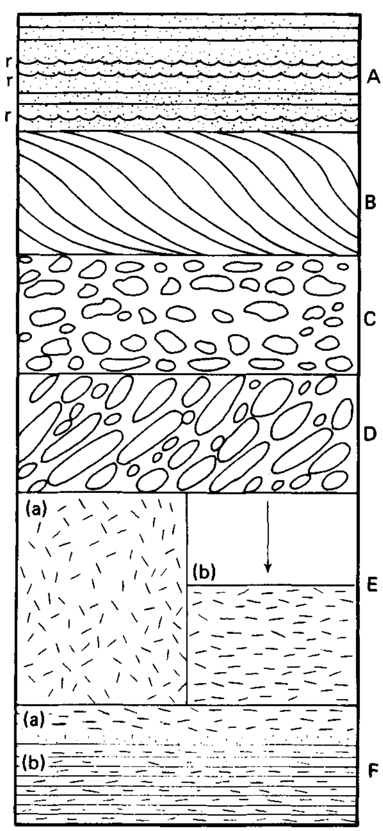


Figure 8.5 Idealized types of sedimentary bedding. (After Sherbon Hills (1972) *Elements of structural geology*, 2nd edn. Chapman and Hall, London) A, sandstone with discrete bedding planes parallel to separation planes. Some beds ripple-marked (r); B, sandstone with discrete bedding planes inclined to separation planes (false or cross-bedding; an inclined deposition texture); C, conglomerate with long axes of pebbles approximately parallel to separation planes (a parallel depositional texture); D, edgewise conglomerate with long axes of pebbles inclined to separation planes (an inclined depositional texture); E(a), unconsolidated mud with random orientation of mica flakes and clay particles (a random depositional texture); E(b), consolidated clay or lithified mudstone with flaky particles approximately parallel, and parallel with separation planes (a parallel consolidation texture); F(a), mudstone with mica flakes deposited parallel to separation planes, but lacking discrete bedding planes (a parallel depositional texture, cf. C above); F(b), mudstone with mica flakes deposited parallel to separation planes, and showing discrete bedding planes. A thin bed of sandstone lies between the two mudstones

rocks are made up by combinations of only eight elements. Of these, oxygen is dominant, next is silicon and then aluminium, iron, calcium, sodium, potassium and magnesium. In terms of oxides, silica (SiO_2) is by far the most abundant, ranging from 40 to 75% of the total. The silica percentage therefore forms the basis of a fourfold chemical classification of the igneous rocks, the limits being given on Figure 8.7.

8.2.2.2 Mineral composition

Mineral composition depends largely upon the chemical composition. The chief minerals present will normally be silicates of the six common metal cations noted, together with quartz, when silica is present in excess. The minerals which actually form will be controlled by the silica percentage and the relative abundance of the cations. For example, silica-poor silicates such as olivine

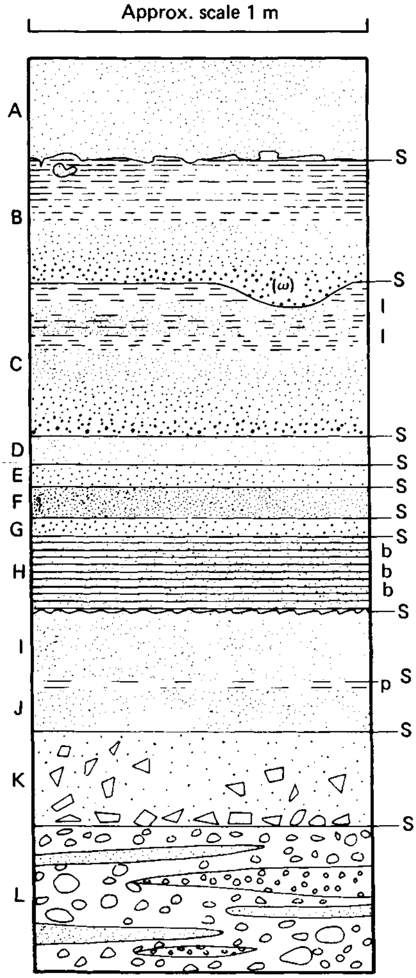


Figure 8.6 Idealized types of sedimentary beds. Beds are bounded by separation planes (S). A, uniform, massive sandstone with bottom structures at its base; B, simple graded bed, with uniform grading from coarse sandstone below to shale above and a washout (w); C, complex graded bed with thin sandstone laminae (l, l) in the shales; D, E, F, G, individual thin beds; H, single sandstone bed with discrete bedding planes (b, b, etc.); I, J, two sandstone beds separated by shale parting (p); K, heterogeneous bed of sandstone containing angular shale fragments; L, heterogeneous bed of conglomerate containing lenses of sand and gravel

will be most abundant in the ultrabasic and basic rocks and absent from the silica-rich acid rocks.

The chief minerals are quartz, orthoclase and plagioclase feldspars, micas, amphiboles, pyroxenes and olivines. Their distribution in the four chemical groups – ultrabasic, basic, intermediate and acid – established by silica percentage is shown diagrammatically in Figure 8.7. Many of the names given to igneous rocks are defined according to the presence of two or three particular minerals which are the essential minerals for that rock type. Other accessory minerals may also be present in small quantities, e.g. the essential minerals of granite are quartz, feldspar and mica; common accessories are zircon and iron oxide.

The predominant minerals of an igneous rock may determine its general appearance and it is usually possible to get some idea

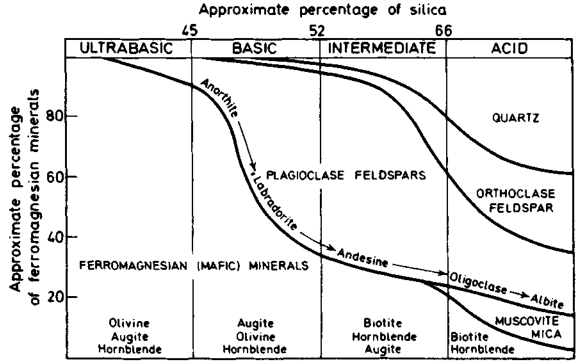


Figure 8.7 A classification of igneous rocks based on a silica percentage

of its composition from its colour and density. Quartz is commonly colourless and transparent, feldspars opaque but pale coloured. Rocks made mostly of these minerals (i.e. acid and intermediate rocks) are therefore usually pale in colour and relatively light in weight. The coloured ferromagnesian silicates, olivines, pyroxenes and amphiboles, are abundant in basic and ultrabasic rocks which are usually dark and relatively heavy. Two important exceptions are very fine-grained or glassy rocks which tend to look dark whatever their composition, and weathering or other alteration which changes the colours of minerals. It is, therefore, usually necessary to look at fresh-broken surfaces to diagnose the parent rock type.

8.2.2.3 Texture

The texture of an igneous rock is shown by the arrangement of the constituent minerals and the relation of each mineral to its neighbours. The main textural character is the grain size and in a general way this depends on the rate of cooling of the magma. Coarse-grained rocks are the result of slow cooling which allowed time for the growth of large crystals; fine-grained rocks are produced by rapid cooling. By extremely rapid cooling, no time at all is given for crystallization and glasses are formed. *Holo-crystalline* rocks are entirely crystalline, *hypo-crystalline* are partly crystals, partly glass. A common distinctive texture is the *porphyritic* texture in which crystals of two different sizes occur: large *phenocrysts* are scattered through a finer-grained or glassy groundmass. The texture is an important controlling factor in the engineering performance of the rock.

8.2.2.4 Classification

Classification of the common igneous rocks is usually made on the basis of grain size and silica percentage as given in Table 8.3. The characteristic minerals of rocks of different compositions are shown in Figure 8.7 which should be studied with Table 8.3.

8.2.2.5 Form

A body of magma which is under pressure in the sial may be forced upwards intruding into the upper rocks of the crust. During the process of intrusion it may incorporate some of the rocks with which it comes into contact, by assimilation. In some cases it may also give off mobile fluids which penetrate and change the rocks in its immediate neighbourhood and mineralization may occur. If the intrusive magma cools at some depth below the surface, the rocks which result are called *plutonic* rocks and are coarsely crystalline; a large mass of this kind constitutes a major intrusion, e.g. a granite batholith which may

Table 8.3 A classification of igneous rocks on silica percentage and grain

Basic	*Intermediate*	*Acid*
Coarse-grained (plutonic) rocks. Grain size larger than about 5 mm. Liable to be brittle owing to presence of large crystals		
Gabbro	Syenite	Granite
Norite	Diorite	Granodiorite
(Not very common in the British Isles)	(Comparatively rare in the British Isles)	(Widely distributed in the British Isles)
Medium-grained (hypabyssal) rocks. Grain size between about 1 and 5 mm. Very frequently possess intergrown texture: include some of the best roadstones		
Dolerite	Porphyry	Microgranite
Diabase	Porphyrite	Granophyre
(Widely distributed in the British Isles)	(Fairly common in the British Isles)	(Fairly common in the British Isles)
Fine-grained (volcanic) rocks. Grain size below about 1 mm, i.e. below the limit of visible recognition. Similar to medium-grained rocks, but sometimes liable to be brittle and splintery		
Basalt	Trachyte	Rhyolite
Spilite	Andesite	Felsite
(Widely distributed in the British Isles)	(Not very common in the British Isles)	(Not very common in the British Isles)
←	Continuous variation in properties	→
Dark colour ←	(Due to increase in ferromagnesian minerals)	→ Light colour
High specific gravity ← (2.9)	(Due to increase in ferromagnesian minerals)	→ Low specific gravity (2.6)

have an aureole of thermally altered rock. When magma rises and fills fractures or other lines of weakness in the crust, it forms minor intrusions. These include *dykes*, which are steep or vertical wall-like masses, with more or less parallel sides, and *sills*, which are sheets of igneous rock intruded between bedding planes of sedimentary rocks and lying more or less horizontal. Dyke and sill rocks commonly have a fine-grained texture. Veins are smaller and irregular bodies of igneous material, filling cracks which may run in any direction.

Magma which rises to the Earth's surface and flows out as a lava, is called extrusive, and under these conditions it loses most of its gas content. These are the *volcanic* rocks, and since they have cooled comparatively quickly in the atmosphere they are frequently glassy (i.e. noncrystalline), or very fine-grained with some larger crystals.

These forms are summarized in Figure 8.8.

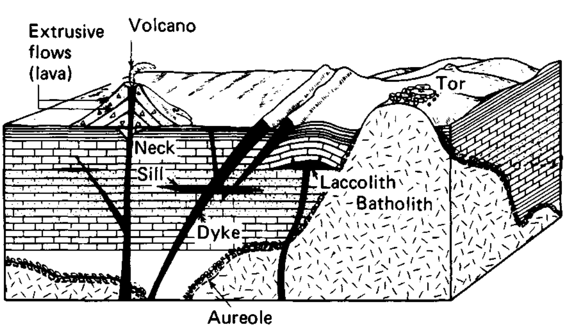


Figure 8.8 Idealized forms of intrusive plutonic rocks

8.2.2.6 Structure

The use of the term *structure* is reserved for more pronounced features of a rock than those described by the term 'texture'. In igneous rocks the structure may indicate a relative arrangement of different spatial features of the rock, both small (*microscopic*) and large (*macroscopic*). For example, gas bubble holes in an igneous rock may be characteristic of its structure. A *vesicular* structure is the presence of small holes, or vesicles, throughout the igneous rock, such as are found in pumices and some basalts. Holes larger than vesicles are *vugs* and are generally filled with minerals other than those forming the rock.

An important macroscopic structural feature is jointing of the rock. Joints are fractures and may be open or closed and run in various directions. They usually occur in more-or-less regular systems and may tend to break the rock into cubes or other regular blocks. This is an important engineering property and is discussed further later. Fractures or cracks are also macroscopic features and may run in any direction and may intersect each other at any angle. A fracture usually has an irregular surface in contrast to the planar or even surface of a joint.

8.2.2.7 Fabric

'Fabric' is a controversial term which sometimes is considered as a generalization of the term 'texture'. Here, igneous fabric denotes the spatial pattern of the rock particles which includes grain sizes and their ratios, grain shapes, grain orientation, microfracturing, packing and interlocking of particles, the character of the matrix, and so on, all of which help control the engineering performance of the rock.

8.2.3 Metamorphic rocks

Rocks formed by the complete or incomplete recrystallization, i.e. the change in crystal shape or in composition, of igneous or sedimentary rocks by high temperatures, high pressures, and/or high shearing stresses, are metamorphic rocks. A platy or foliated structure in such rocks indicates that high shearing stresses have been the principal agency in their formation.

Foliation is not always visible to the naked eye, but individual grains may exhibit strain lines when seen under the microscope.

Table 8.4 Metamorphic rock classification

Structure and texture	*Composition*	*Rock name*
FOLIATED OR PLATY	Various tabular and/or prismatic minerals (generally elongated)	Schist, some serpentines, slate, phyllite
MASSIVE:		
Banded, consisting of alternating lenses	Various tabular, prismatic, and granular minerals (frequently elongated)	Gneiss
Granular, consisting mostly of equidimensional grains	Calcite, dolomite, quartz, in small particles	Marble or quartzite

Metamorphic rocks formed without intense shear action have a massive structure. In Table 8.4 the most common metamorphic rocks are subdivided into two basic classes according to their structure. Foliated rocks usually have directional engineering properties.

8.2.4 A field identification of common rocks

Table 8.5 gives a simple field guide to the identification of the more common rocks. It is after the scheme by Krynine and Judd[2] for engineers with little training in geology and has been devised to present those features first seen when picking up a hand specimen. It is based primarily on texture and structure. They consider the scheme fits the most common occurrences of the rock but some variations will occur.

Textbooks such as the one by Lahee[3] give more specialized field identification techniques. Difficult or contentious identification should be carried out by a geologist who may require thin-section examination of a slice of the rock or even geochemical methods for complete identification.

8.2.5 Rock properties

Engineering characteristics of *rocks* are given more fully in Chapter 10 on rock mechanics and of *soils* in Chapter 9 on soil mechanics. Geological characteristics are given in the engineering geology and mineralogy and petrology textbooks listed in the bibliography.

Table 8.6 (from Shergold[4]) gives some general properties of common rocks and Table 8.7 (in part from Attewell and Farmer[5]) gives a range of mechanical properties of rocks identified by their British Standard (BS) 812:1951 trade group classification. This classification should be used with caution as the rocks listed in each group do not necessarily have close mechanical affinities. The results listed are probably on fairly fresh rock types, i.e. not weathered in the manner following.

8.3 Rock deformation in Nature – fractures and folds

When rocks of the Earth's upper mantle are subject to large stresses, they either break or bend with the production of fractures or folds. The kind of structure formed depends on the condition of the rocks and the rate at which deformation takes place. Most rocks are brittle at surface conditions and tend to fracture under stress though they may yield slowly by bending. At deeper levels where temperatures and pressures are high the majority of rocks become ductile and deform without breaking. Many special conditions at the Earth's surface cause minor fractures and folds, e.g. cooling of igneous lava, thermal stress by daily temperature changes, ground ice movement, and soil desiccation.

8.3.1 Joints

Joints are fractures without any displacement. They may appear to be somewhat random in direction, but a careful field examination will usually show that they have some relation to the host rock, e.g. with the bedding in sedimentary rock or with flow lines in igneous rock.*

In igneous rocks there are often three regular sets of joints (Figure 8.9). In an ideal situation one set lies approximately horizontal and parallel to the flow lines and is termed flat-lying. Another set, the cross-joints, is roughly perpendicular to the flow lines. The third set, the longitudinal joints, dips steeply and strikes parallel to the flow lines if the latter are projected to a plane surface such as a map.

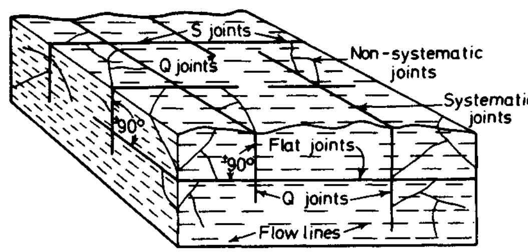


Figure 8.9 Block diagram of simple joint systems in an igneous rock. Systematic S joints are more commonly called longitudinal joints, and Q joints are more commonly called cross-joints perpendicular to the flow lines of the original molten rock

In sedimentary rocks, there are often two systems of mutually perpendicular joints, both perpendicular to the bedding plane.

Joints also may be grouped into strike joints and dip joints. Figure 8.10 illustrates the terms 'strike' and 'dip' where the rock bed is assumed to be an oblique plane. Strike is the direction of contour lines or lines of equal elevation on the surface of the rock mass, and the dip is the maximum slope of its surface. In Figure 8.10 the dip is the angle α made by the line AB with the horizontal. In measurements of dip, it is important to measure the 'true' dip, i.e. the angle located in a plane perpendicular to the strike; otherwise, a misleading *apparent* dip, β in Figure 8.10, is recorded. These terms also apply to beds, faults and other geometric features.

Joints and their orientation with respect to other structures have been widely studied in the field and it has been established that systematic joints usually show well-defined relationships to folds and faults which develop during the same tectonic episode.

The spacing of joints varies considerably and is of importance in engineering. Some rocks, such as sandstones and limestones in which the joints may be widely spaced, yield large blocks and

*Lines showing the flow of the originally liquid magma and indicated by the long axes of crystals

Table 8.5 Field identification of rocks (specimens should be unweathered and not altered in any way)

[LIGHT-COLOURED]

GRAINS OR CRYSTALS VISIBLE TO NAKED EYE								
Angular particles				*Rounded particles*			*Erratic large grains*	
Large	*Fine to medium*	*Very fine*	*Foliated or banded*	*Large*	*Fine to medium*	*Very fine*	*Rounded*	*Angular*
Pegmatite Granite (+Q, +F)* Granodiorite (+Q, +F)* Monzonite (−Q, +F)* Syenite (−Q, +F)* Marble (reacts with HCl) Arkose (usually bedded)	Tuff (contains glasslike fragments)	Felsite* (rhyolite +Q and trachyte −Q)	Schist (shiny) Gneiss (may have sub-angular particles)	Conglomerate (+10% of grains over 2 mm diameter) Sandstone (bedded) (if it reacts to HCl = calcareous sandstone; if it gets slick when wet = argillaceous sandstone)	Quartzite (not friable and very hard)	Siltstone	Depositional breccia	Volcanic breccia and agglomerate fault breccia (may have clay)

NO GRAINS OR SPARSE CRYSTALS VISIBLE TO NAKED EYE									
Glassy lustre	*Dull lustre*	*Shiny lustre*	*Earthy appearance*		*Laminated*				
			Spongy, light wt	*Porous, moderate wt*	*Slick when wet*	*Not slick*	*Slick when wet*	*Not slick*	
								Reaction to HCl	*No reaction to cold HCl*
Quartzite	Felsites* (rhyolite, trachyte)	Schist (foliated)	Pumice Volcanic ash	Chalk (HCl reaction)	Shale	Shale Slate (dull) Phyllite (shiny)	Claystone Mudstone Serpentine (usually greasy and may be banded)	Limestone Chalk (earthy)	Dolomite

[DARK COLOURED (DARK GREY OR GREEN TO BLACK)]

GRAINS OR CRYSTALS VISIBLE TO NAKED EYE		
Angular particles		*Rounded to subangular particles*
Fine to medium	*Very fine to glassy*	Graywacke (fine- to medium-grained) Dark sandstones
Peridotite (−Q, −B)* Gabbro (−Q, −B)* Diorite (−Q, +B)* Dolerite (−Q, +B)*	Andesite* Basalt (usually vesicular)*	

NO GRAINS OR SPARSE CRYSTALS VISIBLE TO NAKED EYE			
Glassy lustre	*Dull lustre – laminated*		*Dull lustre – not laminated*
	Slick when wet	*Not slick*	
Obsidian	Shale	Shale (flexible) Slate (brittle) (dull) Phyllite (shiny)	Basalt* Serpentine (usually greasy and may be banded)

*Rocks may contain occasional large crystals embedded in a very fine-grained matrix or occasional very large crystals in a medium-grained matrix – in either case the term 'porphyry' is appended to the rock name, e.g. syenite porphyry.
(+Q) = contains numerous white or colourless quartz crystals.
(−Q) = contains little or no quartz.
(+F) = contains numerous white to pink feldspar crystals.
(+B) = contains numerous flakes of black mica (biotite).
(−B) = contains little or no black mica.

Table 8.6 Summary of means and range of values for mechanical tests in each trade rock-group

Trade Group classification (BS 812: 1951)		*Aggregate* crushing value*	*Aggregate* impact value*	*Aggregate* abrasion value*	*Water* absorption (per cent)*	*Specific gravity*	*Polished-stone coefficient*
Artificial	Mean	28	27	8.3	0.7	2.71	0.50
	Range	(15–39)	(17–33)	(3–15)	(0.2–1.8)	(2.6–3.4)	(0.35–0.60)
	Number of samples	55	18	18	19	19	9
Basalt	Mean	14	15	6.1	1.1	2.80	0.56
	Range	(7–25)	(7–25)	(2–12)	(0.0–2.3)	(2.6–3.0)	(0.45–0.70)
	Number of samples	123	79	65	68	68	25
Flint	Mean	18	23	1.1	1.0	2.54	0.35
	Range	(7–25)	(19–27)	(1–2)	(0.3–2.4)	(2.4–2.6)	(0.30–0.40)
	Number of samples	63	32	45	24	24	4
Granite	Mean	20	19	4.8	0.4	2.69	0.56
	Range	(9–35)	(9–35)	(3–9)	(0.2–0.9)	(2.6–3.0)	(0.45–0.70)
	Number of samples	41	32	28	16	16	13
Gritstone	Mean	17	19	7.0	0.6	2.69	0.69
	Range	(7–29)	(9–35)	(2–6)	(0.1–1.6)	(2.6–2.9)	(0.60–0.80)
	Number of samples	81	45	31	33	33	18
Hornfels	Mean	13	12	2.2	0.4	2.82	0.45
	Range	(5–15)	(9–17)	(1–4)	(0.2–0.8)	(2.7–3.0)	(0.40–0.50)
	Number of samples	28	24	13	15	15	4
Limestone	Mean	24	23	13.7	1.0	2.66	0.43
	Range	(11–37)	(17–33)	(7–26)	(0.2–2.9)	(2.5–2.8)	(0.30–0.75)
	Number of samples	164	61	34	42	42	33
Porphyry	Mean	14	14	3.7	0.6	2.73	0.51
	Range	(9–29)	(9–23)	(2–9)	(0.4–1.1)	(2.6–2.9)	(0.45–0.60)
	Number of samples	62	29	23	30	30	13
Quartzite	Mean	16	21	3.0	0.7	2.62	0.57
	Range	(9–25)	(11–33)	(2–6)	(0.3–1.3)	(2.6–2.7)	(0.45–0.65)
	Number of samples	57	37	29	21	21	8
All groups†	Mean	19	19	5.7	0.7	2.68	0.53
	Range	(5–39)	(7–35)	(1–26)	(0.0–3.7)	(2.3–3.4)	(0.30–0.80)
	Number of samples	724	370	311	313	313	134

*In these tests a numerically lower result indicates a better performance in the test. †Including results from unclassified samples.

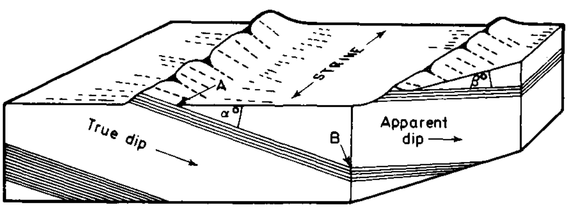


Figure 8.10 Idealized block diagram to show dip and strike relationships

may be suitable for masonry, for example, whereas other rocks may be so closely jointed as to break up into small pieces and may be suitable for aggregate or other purposes. Some joints in sedimentary rocks run only from one bedding plane to the next, but others may cross several bedding planes, and are called *master joints.*

The ease of quarrying, excavating or tunnelling in hard rocks largely depends on the regular or irregular nature of the joints and their surface characteristics, e.g. attitude, size, frequency, openness and spacing. Joints and other fractures control groundwater and air flow in otherwise intact rock and help to promote rock weathering.

8.3.2 Faults

Faults are fractures in the crust along which there has been displacement of the rocks on one side relative to those on the other.

The surface on which movement takes place during faulting is the fault plane. It may be vertical, steeply inclined or gently inclined as with thrust faults. The intersection of a fault with the ground surface is known as the fault line or fault trace. The upper side of an inclined fault, and the rock which lies above it, is referred to as the hanging wall. Rock below it is the foot wall; dip faults strike parallel to the local direction of dip of the beds, strike faults are parallel to the strike and oblique faults cut across both strike and dip directions.

Movements on a fault may be in any direction. The displace-

Table 8.7 Typical rock strengths, porosity and bulk densities of rock materials

Rock	*Strength* N/mm^{-2}			*Bulk density* (Mg/m^{-3})	*Porosity* (n%)
	Compressive	*Tensile*	*Shear*		
Granite	100–250	7–25	14–50	2.6–2.9	0.5–1.5
Diorite	150–300	15–30	—	2.7–3.05	0.1–1.0
Dolerite	100–350	15–35	25–60	2.7–3.05	0.1–0.5
Gabbro	150–300	15–30	—	2.8–3.1	0.1–0.2
Basalt	150–300	10–30	20–60	2.8–2.9	0.1–1.0
Sandstone	20–170	4–25	8–40	2.0–2.6	5–25
Shale	5–100	2–10	3–30	2.0–2.4	10–30
Limestone	30–250	5–25	10–50	2.2–2.6	5–20
Dolomite	30–250	15–25	—	2.5–2.6	1–5
Coal	5–50	2–5	—	—	—
Quartzite	150–300	10–30	20–60	2.6–2.7	0.1–0.5
Gneiss	50–200	5–20	—	2.8–3.0	0.5–1.5
Marble	100–250	7–20	—	2.6–2.7	0.5–2.0
Slate	100–200	7–20	15–30	2.6–2.7	0.1–0.5
Rhyolite	—	—	—	2.4–2.6	4–6
Andesite	50–200	—	—	2.2–2.3	10–15

ment or slip is the sum of all the previous effects of movement and is shown by the relative positions on either side of the fault of two originally contiguous features as a bedding plane. The vertical component of the slip, taken by itself, is called the throw of the fault (see Figure 8.11).

Faults can be classified, according to the direction of movement that has taken place on them, into normal faults, reverse faults and transcurrent or strike–slip faults.

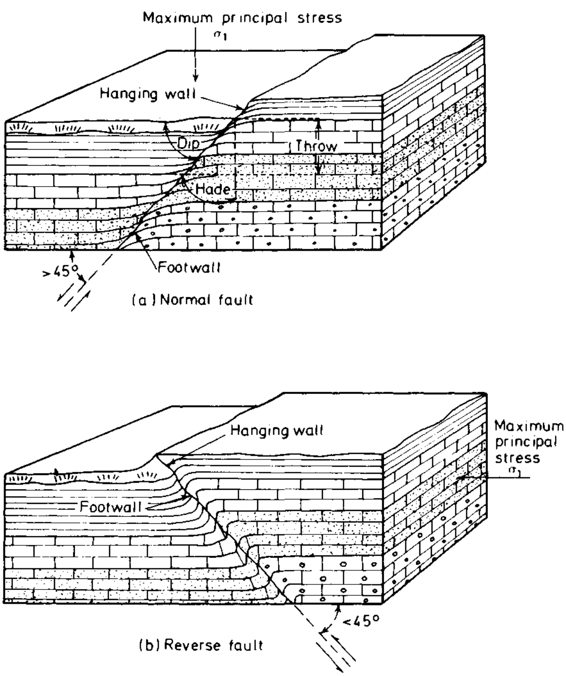


Figure 8.11

Normal faults. Normal faults (originally so-called because they are the normal type found in coalfields in the UK) are those in which the hanging-wall rocks have moved down the dip of the fault plane. Small normal faults are extremely common in almost all geological situations and may even occur in Quaternary sediments. Large normal faults, occurring in groups, produce a considerable effect of lengthening and are especially common in the more stable areas of the Earth's crust. Groups of faults are arranged so that alternate dislocations dip in opposite directions and produce the effect of block faulting illustrated in Figure 8.12; the crust is separated into high blocks or *horsts* between outward-dipping faults and low blocks, troughs or *graben* between inward-dipping faults.

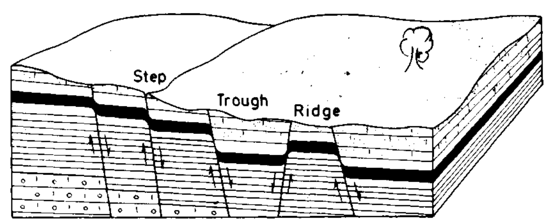


Figure 8.12 Idealized block diagram of some common fault groups. Note there is little effect on topography here as the surface bed is the same in all locations, but where difficult beds are exposed by the faulting, scarp topography may be found

Reverse faults. Reverse faults are those on which the rocks of the hanging wall move up the dip of the fault plane. They result in shortening across the fault and in duplication of strata; reverse faults with low dips are thrusts.

Transcurrent faults. These are wrench faults, tear faults or strike–slip faults on which horizontal movement takes place. The fault planes are almost vertical and the effect of faulting when seen on a map is to shift rocks laterally, even for many tens of kilometres. Examples of block diagrams to illustrate mapped outcrop patterns of faults are shown in Figure 8.13.

An example of the relationship between faulting and jointing in one complete episode is shown in Figure 8.14 from the textbook by Price.[6] Techniques and the use of stereographic projection in geology is given in Phillips.[7]

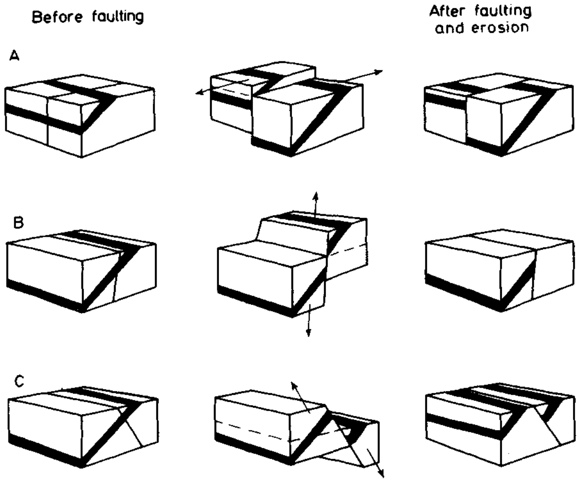


Figure 8.13 Idealized outcrop patterns of faulted beds. A, dip fault, i.e. movement in the dip direction; B, strike fault with downthrow in dip direction; C, strike fault with downthrow against the dip angle. (After Read and Watson (1971) *Beginning geology*, 2nd edn. Macmillan/Allen and Unwin, London)

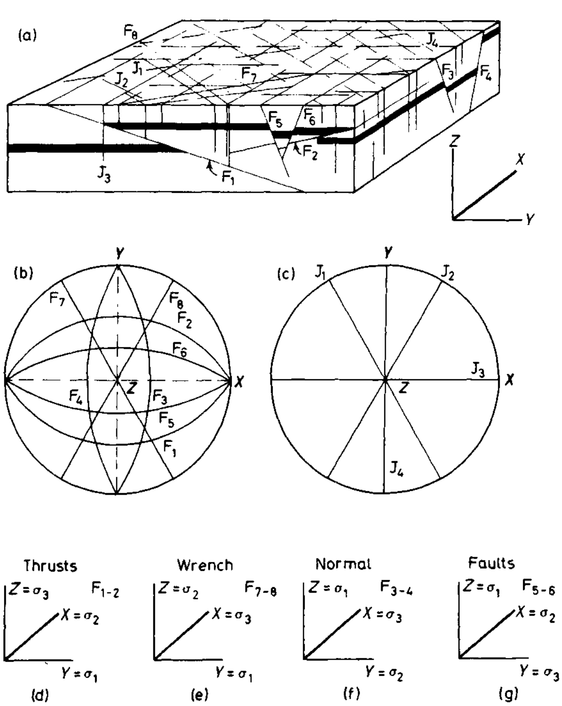


Figure 8.14 (a) Block diagram showing orientation of faults and joints in unfolded rocks which may result from various phases of compression and tension related to one complete tectonic episode; (b) stereogram of fault orientations shown in (a); (c) stereogram of joint orientation shown in (a); (d)–(g) orientation of stress fields when the various groups of faults were initiated. (Redrawn from Price (1966) *Fault and joint development in brittle and semi-brittle rock*. Pergamon Press, Oxford)

8.3.3 Folds

In geology weak rocks which deform under stress are termed *incompetent* whereas strong rocks that buckle and fracture are termed *competent*. These terms should not be confused, however, with similar terms describing the bearing capacity of foundation rocks.

A complete fold is composed of an arched portion, or *anticline*, and a depressed trough or *syncline* (Figure 8.15a). The highest point of an anticline is called the crest, and the inclined parts of the strata where anticline and syncline merge are the limbs of the fold. The youngest beds outcrop in the middle of a syncline and the oldest in the middle of an anticline.

The plane bisecting the vertical angle between equal slopes on either side of the crest line is the axial plane. Where this is vertical, as in Figure 8.15a, the fold is upright and symmetrical; where it is inclined the fold is asymmetrical (Figure 8.15b). Sometimes the middle limb has been brought into a vertical position by the compression which buckled the strata, and under still more severe conditions an *overturned fold*, or overfold, is produced (Figure 8.15c). Here the middle limb is inclined in the same attitude as the axial plane, and the beds of which it is composed have a reversed dip, i.e. upper beds are now brought to dip steeply beneath lower beds, an inversion of the true sequence.

If the compression is so extreme as to pack a series of folds together so that their limbs are all virtually parallel and steeply dipping, the structure is referred to as *isoclinal folding*, i.e. all limbs have the same slope (Figure 8.15c).

Where the axial plane is inclined at a low or zero angle, the fold is said to be *recumbent* (Figure 8.15d), a type which is often found in intensely folded mountain regions such as the Alps.

The term *monocline* is for the kind of flexure which has two parallel gently dipping limbs with a steeper middle limb between them: it is in effect a local steepening of the dip in gently dipping (or horizontal) beds.

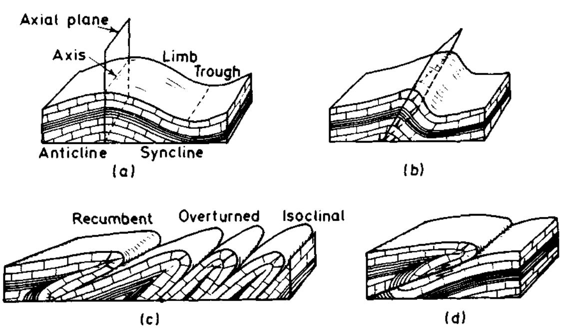


Figure 8.15 Idealized fold types. (a) Simple or gentle symmetrical; (b) simple or gentle asymmetrical; (c) tight assymetrical, recumbent, overturned and isoclinical; (d) recumbent passing into a thrust fault

The dimensions of anticlines and synclines vary between wide extremes, from small puckers millimetres across in sharply folded sediments, to broad archings of strata whose extent is measured in kilometres. The growth of such structures is, in general, a process which goes on slowly as stresses develop in any particular part of the Earth's crust; but superficial folds may develop in a comparatively short space of time, e.g. earthquake ripples forming quickly, in weak sediments or some types of hillcreep. Simple land topography largely controlled by folding is illustrated in Figure 8.16.

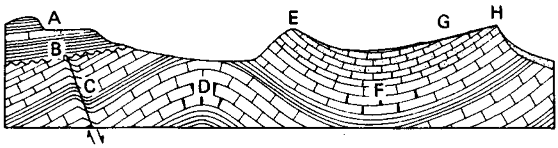


Figure 8.16 Simple fold forms and related topography. A, step topography; B, unconformity; C, normal fault; D, anticline; E, hog's back ridge; F, syncline; G, dip slope; H, scarp slope

8.3.4 Some engineering aspects of faults and folds

Any geological structure that influences one of the mass properties of the *in situ* rock, such as the strength, modulus of deformation or permeability, is highly significant. The most common structural features of significance are joints, bedding planes and foliation surfaces and 'shears' or faults. These are all planar or near-planar discontinuities, and have a strong anisotropic effect on the mass properties.

A search for discontinuities and other faults is not always

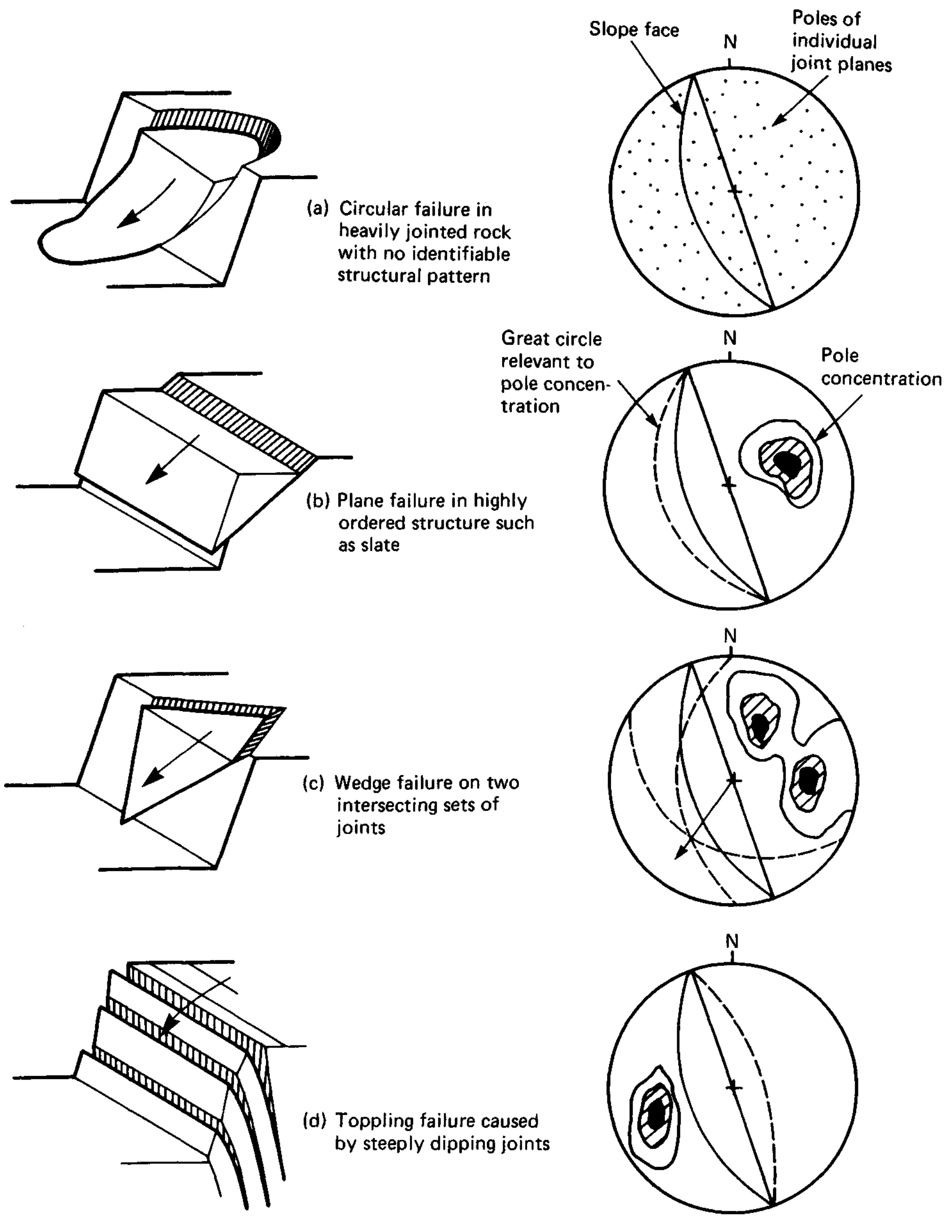


Figure 8.17 Representation of structural geology data concerning four possible slope failure modes, plotted on equatorial equal-area nets as poles and great circles. (After Hoek and Bray (1974) *Rock slope engineering*, 2nd edn. Institute of Mining and Metallurgy, London)

effective during site investigation, and significant faults, for example, are sometimes not discovered until construction or even afterwards. Stability of hillsides, cut slopes, quarry faces and so on may often be controlled by the geometric arrangement of joints and faults. (For examples see Figure 8.17.) Also the groundwater pattern may be controlled by the condition of the joints and faults whether they are open or closed or filled with debris or gouge and the persistence or continuity of such fractures may be important.

On large works the determination of whether a fault is *active*, *inactive* or *passive* may be important. Active faults are those in which movements have occurred during the recorded history and along which further movements can be expected any time (such as the San Andreas and some other faults in California). Inactive faults have no recorded history of movement and are assumed to be and probably will remain in a static condition. Unfortunately, it is not possible yet to state definitely if an apparently inactive fault will remain so. The fault may reopen, either because of a new stress accumulation in the locality or from the effect of earthquake vibrations.

From the alteration products of faulting, gouge is probably of the most concern in foundation problems. This is usually a relatively impervious clay-grade material and may hinder or stop the movement of groundwater from one side of the fault to the other and so create hydrostatic heads, e.g. if encountered in a tunnel. It may also reduce sliding friction along the fault plane. The presence of soft fault breccia or gouge may cause sudden squeezes in a tunnel that intersects the fault. Arch action of rocks in tunnels may be reduced by the presence of joints and faults. Rock falls on cuts and in tunnels, patterns of rock bolts, grout holes and so on are all controlled to a large extent by the joint and fault pattern.

In foundations, folds are generally not so critical as faults though they may give stability problems if their geometry is unfavourable. Occasionally, folds may influence the selection of a dam site; e.g. when the reservoir is located over a monocline containing pervious strata, there may be excessive seepage if the monocline dips downstream. If the monocline were to dip upstream, the reservoir might have little seepage providing the monocline contained some impervious layers such as shale which were not fractured in the folding. Serious water problems may arise in the construction and maintenance of tunnels intersecting synclines containing water-bearing strata. In deep cuts, analogous water problems arise that may create continuous maintenance problems.

Dipping beds, which must be part of a fold system, may cause stability problems if the dip is unfavourable into a cut face (Figure 8.17b).

8.4 Engineering geology environments

A geological environment is the sum total of the external conditions which may act upon the situation. For example, a 'shallow marine environment' is all the conditions acting offshore which control the formation of deposits on the sea bed: the water temperature, light, current action, biological agencies, source of sediment, sea bed chemistry and so on.

The concept of geological environment forms a suitable basis to study systematically the engineering geology of the deposits formed in or influenced by the various environments, as they condition the *in situ* engineering behaviour of the various deposits. A knowledge of the parameters of the environment enables predictions and explanations of the engineering behaviour to be attempted. Geomorphology is the study of the geology of the Earth's surface (see Fookes and Vaughan[8]).

8.4.1 Processes acting on the Earth's surface

A *landform* may be defined as an area of the Earth's surface differing by its form and other features from the neighbouring areas. Mountains, valleys, plains and even swamps are landforms.

The principal processes that are continually acting on the Earth's surface are gradation, diastrophism and vulcanism.

(1) *Gradation* is the building up or wearing down of existing landforms (including mountains), formation of soil and various deposits. Erosion is a particular case of gradation by the action of water, wind or ice.
(2) *Diastrophism* is the process where solid, and usually the relatively large, portions of the Earth move with respect to one another as in faulting or folding.
(3) *Vulcanism* is the action of magma, both on the Earth's surface and within the Earth.

With the exception of vulcanism and sometimes erosion, these processes may take hundreds and even millions of years to change the face of the Earth significantly. The sudden eruption of a volcano, for example, with the ensuing flow of lava or deposition of volcanic ash, can abruptly change land overnight.

Origin of soils. The majority of the soils are formed by the destruction of rocks. The destructive process may be physical, as the disintegration of rock by alternate freezing and thawing or day–night temperature changes. It may also be by chemical decomposition, resulting in changes in the mineral constituents of the parent rock and the formation of new ones.

Soils formed by disintegration and chemical decomposition may be subsequently transported by the water, wind or ice before deposition. In this case they are classified as alluvial, aeolian, or glacial soils and are generally called *transported* soils. However, in many parts of the world, the newly formed soils remain in place. These are called *residual* soils.

In addition to the two major categories of transported and residual soils, there exist a number of soils that are not derived from the destruction of rocks. For example, peat is formed by the decomposition of vegetation in swamps; some marly soils are the result of precipitation of dissolved calcium carbonate.

Soil-forming processes. There are very many and varied processes that take place in weathered rock and soils that affect the formation of soil profiles to varying degrees, but the major soil-forming processes are: (1) organic accumulation; (2) eluviation; (3) leaching; (4) illuviation; (5) precipitation; (6) cheluviation; and (7) organic sorting.

The soil-forming processes produce an assemblage of soil layers at horizons, called the *soil profile*. In its simplest it is categorized as three layers A, B and C but numerous varieties of this and many other soil classifications exist. Probably the most generally accepted one is that based on a geographical approach. This is the zonal scheme thought to reflect zones of climate, vegetation and other factors of the local environment.

8.4.2 Engineering significance of selected geomorphological environments

Much of what can be called 'classical' geotechnical engineering has developed in temperate climate regions of the Earth. As a result many of the concepts of soil and rock behaviour and their properties have been conditioned by the soil and rock found there. The climate and local geology play a major role in determining the local geotechnical characteristics of the soils and rocks. Figure 8.18 shows the generalized distribution of the four principal climatic engineering soil zones after Sanders and Fookes.[9]

effective during site investigation, and significant faults, for example, are sometimes not discovered until construction or even afterwards. Stability of hillsides, cut slopes, quarry faces and so on may often be controlled by the geometric arrangement of joints and faults. (For examples see Figure 8.17.) Also the groundwater pattern may be controlled by the condition of the joints and faults whether they are open or closed or filled with debris or gouge and the persistence or continuity of such fractures may be important.

On large works the determination of whether a fault is *active*, *inactive* or *passive* may be important. Active faults are those in which movements have occurred during the recorded history and along which further movements can be expected any time (such as the San Andreas and some other faults in California). Inactive faults have no recorded history of movement and are assumed to be and probably will remain in a static condition. Unfortunately, it is not possible yet to state definitely if an apparently inactive fault will remain so. The fault may reopen, either because of a new stress accumulation in the locality or from the effect of earthquake vibrations.

From the alteration products of faulting, gouge is probably of the most concern in foundation problems. This is usually a relatively impervious clay-grade material and may hinder or stop the movement of groundwater from one side of the fault to the other and so create hydrostatic heads, e.g. if encountered in a tunnel. It may also reduce sliding friction along the fault plane. The presence of soft fault breccia or gouge may cause sudden squeezes in a tunnel that intersects the fault. Arch action of rocks in tunnels may be reduced by the presence of joints and faults. Rock falls on cuts and in tunnels, patterns of rock bolts, grout holes and so on are all controlled to a large extent by the joint and fault pattern.

In foundations, folds are generally not so critical as faults though they may give stability problems if their geometry is unfavourable. Occasionally, folds may influence the selection of a dam site; e.g. when the reservoir is located over a monocline containing pervious strata, there may be excessive seepage if the monocline dips downstream. If the monocline were to dip upstream, the reservoir might have little seepage providing the monocline contained some impervious layers such as shale which were not fractured in the folding. Serious water problems may arise in the construction and maintenance of tunnels intersecting synclines containing water-bearing strata. In deep cuts, analogous water problems arise that may create continuous maintenance problems.

Dipping beds, which must be part of a fold system, may cause stability problems if the dip is unfavourable into a cut face (Figure 8.17b).

8.4 Engineering geology environments

A geological environment is the sum total of the external conditions which may act upon the situation. For example, a 'shallow marine environment' is all the conditions acting offshore which control the formation of deposits on the sea bed: the water temperature, light, current action, biological agencies, source of sediment, sea bed chemistry and so on.

The concept of geological environment forms a suitable basis to study systematically the engineering geology of the deposits formed in or influenced by the various environments, as they condition the *in situ* engineering behaviour of the various deposits. A knowledge of the parameters of the environment enables predictions and explanations of the engineering behaviour to be attempted. Geomorphology is the study of the geology of the Earth's surface (see Fookes and Vaughan[8]).

8.4.1 Processes acting on the Earth's surface

A *landform* may be defined as an area of the Earth's surface differing by its form and other features from the neighbouring areas. Mountains, valleys, plains and even swamps are landforms.

The principal processes that are continually acting on the Earth's surface are gradation, diastrophism and vulcanism.

(1) *Gradation* is the building up or wearing down of existing landforms (including mountains), formation of soil and various deposits. Erosion is a particular case of gradation by the action of water, wind or ice.
(2) *Diastrophism* is the process where solid, and usually the relatively large, portions of the Earth move with respect to one another as in faulting or folding.
(3) *Vulcanism* is the action of magma, both on the Earth's surface and within the Earth.

With the exception of vulcanism and sometimes erosion, these processes may take hundreds and even millions of years to change the face of the Earth significantly. The sudden eruption of a volcano, for example, with the ensuing flow of lava or deposition of volcanic ash, can abruptly change land overnight.

Origin of soils. The majority of the soils are formed by the destruction of rocks. The destructive process may be physical, as the disintegration of rock by alternate freezing and thawing or day–night temperature changes. It may also be by chemical decomposition, resulting in changes in the mineral constituents of the parent rock and the formation of new ones.

Soils formed by disintegration and chemical decomposition may be subsequently transported by the water, wind or ice before deposition. In this case they are classified as alluvial, aeolian, or glacial soils and are generally called *transported* soils. However, in many parts of the world, the newly formed soils remain in place. These are called *residual* soils.

In addition to the two major categories of transported and residual soils, there exist a number of soils that are not derived from the destruction of rocks. For example, peat is formed by the decomposition of vegetation in swamps; some marly soils are the result of precipitation of dissolved calcium carbonate.

Soil-forming processes. There are very many and varied processes that take place in weathered rock and soils that affect the formation of soil profiles to varying degrees, but the major soil-forming processes are: (1) organic accumulation; (2) eluviation; (3) leaching; (4) illuviation; (5) precipitation; (6) cheluviation; and (7) organic sorting.

The soil-forming processes produce an assemblage of soil layers at horizons, called the *soil profile*. In its simplest it is categorized as three layers A, B and C but numerous varieties of this and many other soil classifications exist. Probably the most generally accepted one is that based on a geographical approach. This is the zonal scheme thought to reflect zones of climate, vegetation and other factors of the local environment.

8.4.2 Engineering significance of selected geomorphological environments

Much of what can be called 'classical' geotechnical engineering has developed in temperate climate regions of the Earth. As a result many of the concepts of soil and rock behaviour and their properties have been conditioned by the soil and rock found there. The climate and local geology play a major role in determining the local geotechnical characteristics of the soils and rocks. Figure 8.18 shows the generalized distribution of the four principal climatic engineering soil zones after Sanders and Fookes.[9]

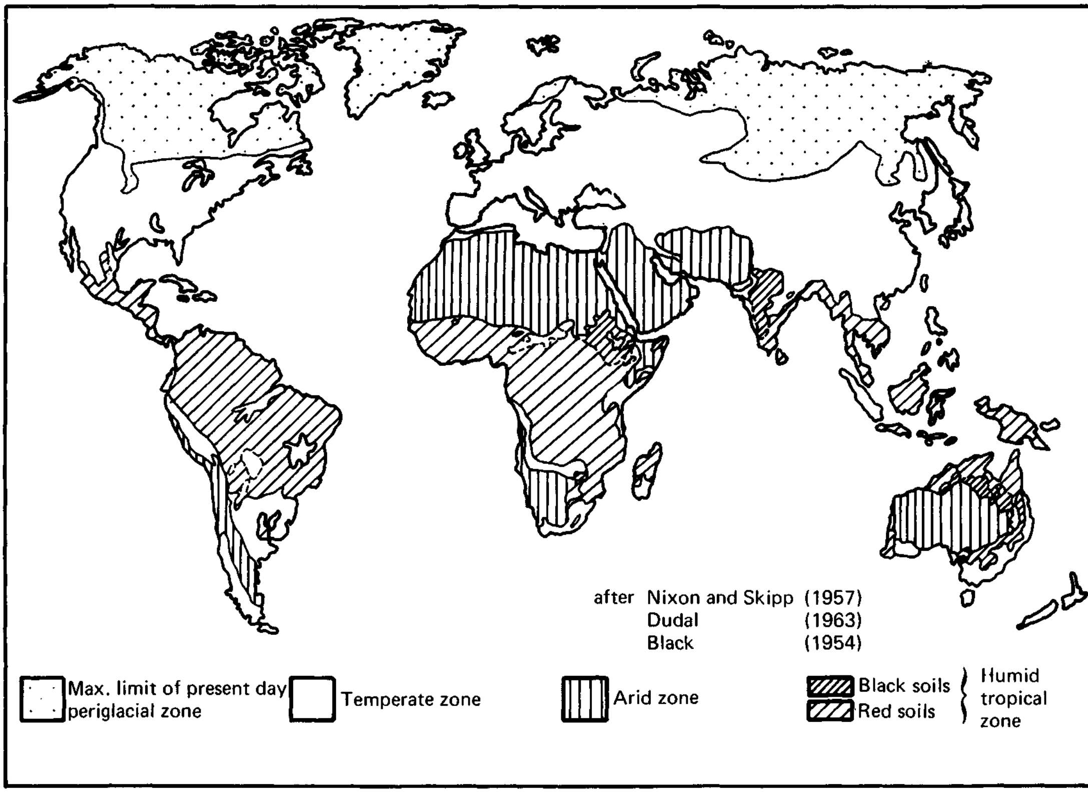


Figure 8.18 Generalized map showing present-day distributions of the four principal climatic engineering soil zones

The following section briefly describes the engineering geomorphology characteristics of the four principal climatic zones, together with other geological environments of particular interest.

8.4.2.1 Rock weathering

A rock can weather by physical breakdown without considerable change of its constituent minerals by disintegration. The residual or transported soil derived from this process consists of an accumulation of mineral and rock fragments virtually unchanged from the original rock. This type of weathering is found mainly in arid or cold climates. *Chemical decomposition* leads to the thorough alteration of a large number of minerals and only a few, among them quartz, may remain unaffected. The greater the percentage of weatherable minerals in the original rock, particularly the ferromagnesian minerals, the more conspicuous is the change from rock to soil. This type of weathering is generally found in warm and hot, wet climates and can lead to great thickness of weathered rock and soil. Biological weathering is generally of less importance than physical or chemical weathering and is a combination of biochemical and biophysical effects.

The weathering process produces a gradational and often quite irregular change in the rock from fresh some distance below ground surface to more or less completely weathered at the surface: Figure 8.19 shows somewhat schematically two examples of weathered rock profiles. Corestones need not always be present and in sedimentary rocks in particular are often missing.

There are several generalized weathering rock classifications for engineering purposes; the one reproduced here, Table 8.8, is from Fookes, Dearman and Franklin,[10] which discusses weathered rock mainly in the UK. For information on the engineering performance of weathered rock elsewhere, see also Little,[11] Deere and Patton[12] and Irfan and Dearman.[13]

8.4.2.2 Humid tropical residual soils

A residual soil is the end product (i.e. grades V and VI) of rock weathering. Different types of residual soils are produced in different environments (see Figures 8.20 and 8.21).

In tropical regions of high temperature and abundant surface water rock weathering it is at its most intensive. It is characterized by rapid breakdown of feldspars and ferromagnesian minerals, the removal of silica and bases and the concentration of iron and aluminium oxides. Kaolinite and related clay minerals form in well-drained areas. Because of the high iron concentration the resulting soils are usually red in colour. When dried, such materials may harden as a result of the cementing action by iron and aluminium oxides and they are commonly referred to as *laterites*. Large, fairly durable concretions may be formed this way to give lateritic gravels which are often important sources of aggregate for road construction and other uses, particularly in East and West Africa.

Tropical weathering of volcanic ash and rock leads to the formation of allophane and halloysite together with concentration of iron and aluminium oxides. Soils of this type, particularly common in the Far East, are known as *adisols*.

Nearer to the edges of the tropics to the north and south there

Table 8.8 Engineering grade classification of weathered rock

Grade	Degree of decomposition	FIELD RECOGNITION Soils (i.e. soft rocks)	Rocks (i.e. hard rocks)	Engineering properties of rocks
VI	Soil	The original soil is completely changed to one of new structure and composition in harmony with existing ground surface conditions.	The rock is discoloured and is completely changed to a soil in which the original fabric of the rock is completely destroyed. There is a large volume change.	Unsuitable for important foundations. Unsuitable on slopes when vegetation cover is destroyed, and may erode easily unless a hard cap present. Requires selection before use as fill.
V	Completely weathered	The soil is discoloured and altered with no trace of original structures.	The rock is discoloured and is changed to a soil, but the original fabric is mainly preserved. The properties of the soil depend in part on the nature of the parent rock.	Can be excavated by hand or ripping without use of explosives. Unsuitable for foundations of concrete dams or large structures. May be suitable for foundations of earth dams and for fill. Unstable in high cuttings at steep angles. New joint patterns may have formed. Requires erosion protection.
IV	Highly weathered*	The soil is mainly altered with occasional small lithorelicts of original soil. Little or no trace of original structures.	The rock is discoloured; discontinuities may be open and have discoloured surfaces and the original fabric of the rock near the discontinuities is altered; alteration penetrates deeply inwards, but corestones are still present.	Similar to grade V. Unlikely to be suitable for foundations of concrete dams. Erratic presence of boulders makes it an unreliable foundation for large structures.
III	Moderately weathered*	The soil is composed of large discoloured lithorelicts of original soil separated by altered material. Alteration penetrates inwards from the surfaces of discontinuities.	The rock is discoloured; discontinuities may be open and surfaces will have greater discoloration with the alteration penetrating inwards; the intact rock is noticeably weaker, as determined in the field, than the fresh rock.	Excavated with difficulty without use of explosives. Mostly crushes under bulldozer tracks. Suitable for foundations of small concrete structures and rockfill dams. May be suitable for semipervious fill. Stability in cuttings may depend on structural features, especially joint attitudes.
II	Slightly weathered	The material is composed of angular blocks of fresh soil, which may or may not be discoloured. Some altered material starting to penetrate inwards from discontinuities separating blocks.	The rock may be slightly discoloured, particularly adjacent to discontinuities which may be open and have slightly discoloured surfaces; the intact rock is not noticeably weaker than the fresh rock.	Requires explosives for excavation. Suitable for concrete dam foundations. Highly permeable through open joints. Often more permeable than the zones above or below. Questionable as concrete aggregate.
I	Fresh rock	The parent soil shows no discoloration, loss of strength or other effects due to weathering.	The parent rock shows no discoloration, loss of strength or any other effects due to weathering.	Staining indicates water percolation along joints; individual pieces may be loosened by blasting or stress relief and support may be required in tunnels and shafts.

*The ratio of original soil or rock to altered material should be estimated where possible.

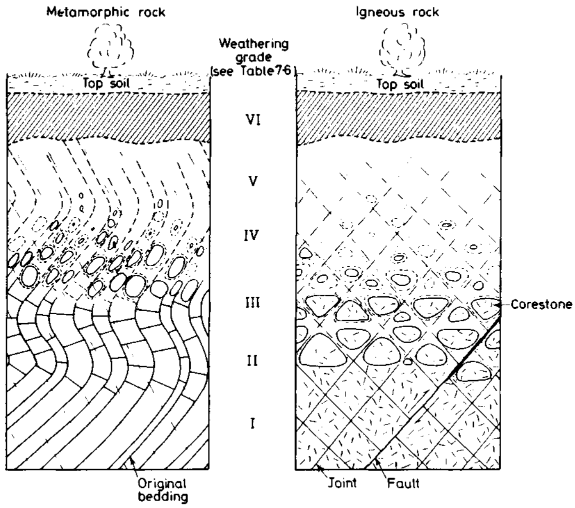


Figure 8.19 Diagrammatic weathering profile of an igneous and a metamorphic rock. (After Deere and Patton (1971) 'Stability of slopes in weathered rock.' *Proceedings, 4th Panamerican Conference*)

is decreased rainfall, alternating wet and dry seasons and often poor drainage. Under these conditions smectite (montmorillonite) clays are found and the highly active *black cotton soils* develop. (See Figures 8.18, 8.21 and 8.22.)

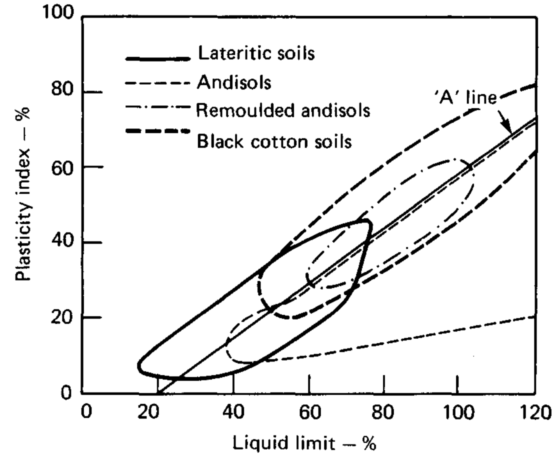
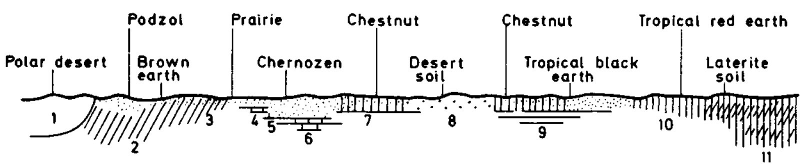


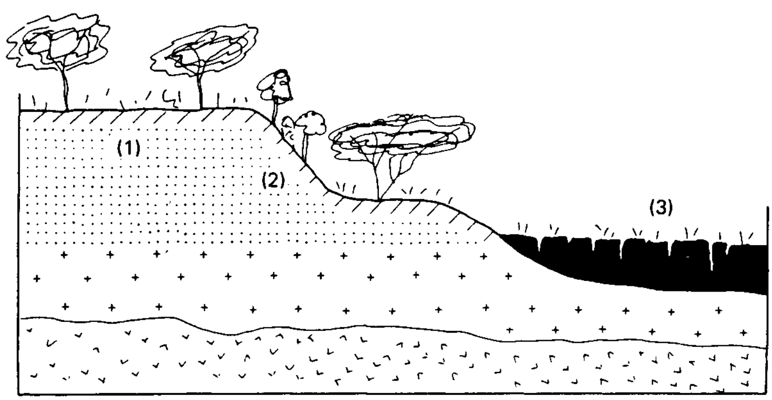
Figure 8.22 Approximate ranges of Atterberg limits for lateritic andisols and black cotton soils

In situ layers of tropical residual soils may be markedly nonhomogeneous. High geochemical activity in tropical environments can result in rapid changes in clay mineralogy and fabric both laterally and with depth. Cementation of particles into clusters and aggregates by sesquioxides and the hydrated state of some of the minerals are responsible for high void ratios (low densities), high strength, low compressibility and some-



1. Frozen ground
2. Intense leaching and illuviation
3. Less leaching more mixing
4. Carbonate appears
5. Organic matter increases
6. Carbonate prominent
7. Organic matter decreases
8. Gypsum and salt accumulate
9. Organic matter increases
10. Ferrallitisation becomes dominant
11. Ferricrete formation

Figure 8.20 Diagrammatic cross-section of zonal soils from pole to equator. (After Ollier (1975) *Weathering*, 2nd edn. Oliver and Boyd, Cambridge)



(1) Reddish-brown soil developed in deeply-weathered material beneath acacia grassland
(2) Reddish-brown soil with surface erosion developed in highly-weathered material
(3) Black soil of the depressions, fine-textured and salt-enriched in dry climates, deep cracking

Figure 8.21 Diagrammatic representation of topographic relationships of local soils in savanna lands of Africa. (After Thomas (1974) *Tropical geomorphology*. Macmillan, London)

Table 8.9 Physical properties of unremoulded, remoulded and sesquioxide-free lateritic soil (in Anon (1982) 'Engineering construction in tropical and residual soils'. *American Society of Civil Engineers, Geotechnical Engineering Division Special Conference*, Hawaii)

Property	*Unre-moulded*	*Remoulded*	*Sesquioxide-free*
Liquid limit (%)	57.8	69.0	51.3
Plastic limit (%)	39.5	40.1	32.1
Plasticity index (%)	18.3	28.0	19.2
Specific gravity	2.80	2.80	2.67
Proctor density (kN/m^3)	13.3	13.0	13.8
Optimum moisture content (%)	35.0	34.5	29.5

times high permeability in relation to high plasticity and small particle size. Collapsing soils can occur. (See Table 8.9.)

Many of the red lateritic soils and adisols are susceptible to breakdown on manipulation which makes index property determination difficult as well as earthwork construction, changing a predominantly granular soil which excavates easily to a plastic mess that cannot be compacted easily. Irreversible changes in property of many tropical soils result from drying. (See, for example, Table 8.10.)

Most of the black cotton soils have high plasticity and marked swelling and shrinkage characteristics which in areas with marked dry and wet seasons cause special problems for foundations of structures and roads. Reference should be made to the appropriate literature for the particular soil type, though this itself may be difficult since in many of the published engineering articles the soil type may not be described sufficiently to characterize it. In addition, a great variety of different residual soils seem to be dubbed with the title of 'laterite' often quite erroneously. See Sanders and Fookes[9] for a general study of foundation conditions related to four principal climate zones: (1) periglacial; (2) temperate; (3) arid; and (4) humid tropical. See Deere and Patton[12] for an extensive treatise on slope stability aspects; Little[11] for 'laterites' and the *Proceedings* of the Institution of Civil Engineers[14] for a symposium on road and airfield construction in tropical soils; Chen[15] and Gidigasu[16] on swelling soils; and Anon[17] on engineering and construction.

8.4.2.3 Hot desert soils

Desert soils are formed in dry environments where the evaporation exceeds the precipitation and are generally associated with the world's hot deserts. Rainfall is low (say less than 150 mm per annum) and often seasonal. Physical weathering is dominant and the disintegration of the rock mainly results from insolation, but often other factors such as abrasion by windborne particles and salt weathering may contribute.

The products of this type of weathering are mainly of coarser-grained materials near hills or mountains. Parent materials of a high-silica content produce detrital sands and gravels, which, when sand is transported away from high land by wind, give sand-dune deposits and possibly loess or, when transported by water, give alluvial sands and gravels. Calcareous parent materials result in calcareous sands and gravels and evaporite salts are often present throughout the soil profile especially in internally draining areas as playa or salina flats.

Cooke *et al.*[18] relate potentially suitable sources of aggregates to some desert landforms as shown in Figure 8.23 and Table 8.11.

Fookes and Knill[19] (Figure 8.24) divided inter-montane desert basins into four sediment deposition zones which may be correlated with the degree of disintegration of the parent material.

Engineering problems provided by desert conditions are principally those related to the grading of the material. Coarse, angular, ill-sorted material generally occurs in zone II (Figure 8.24) and intermittent stream flow and occasional flash floods indicate that carefully designed drainage and runoff measures are required for engineering works. Better sorted and finer material occurs in zone III and the danger of sheet flood here may be greater. In zone IV, mobile sand dunes may require stabilization and loess soils can suffer metastable collapse on loading. Evaporite salts in the soil may cause expansive problems under thin pavements and attack concrete.

Desert engineering problems in general are discussed in Fookes and Knill[19] and Cooke *et al.*[18], metastable loess soils in Holtz and Gibbs,[20] soluble salts in soils in Weinert and Clauss[21] and Fookes and French,[22] and construction materials by Fookes and Higginbottom[23] and Oweis and Bowman.[24]

8.4.2.4 Glacial soils ('drift' in the UK)

The five major glaciations of the Pleistocene period, begun about 2 million yr ago, constitute the last major episode in the shaping of much of the world's land surface. During each glaciation ice advanced over large areas of the northern and southern hemispheres. Post-glacial changes which have only occurred within the last 15 000 yr or so have been relatively limited. They are mostly confined to low ground, where alluvial deposits have tended to accumulate in response to the world-wide rise in sea-level caused by the latest recession of the ice sheets and glaciers. The deposits of one or more of the Pleistocene ice advances lie at the surface over, very approximately, 50% of the land area of the UK, whilst roughly another 10% is covered with post-glacial alluvium sometimes concealing glacial

Table 8.10 Effect of air-drying on index properties of a hydrated laterite clay from the Hawaiian Islands (In Gidigasu ((1975) *Laterite soil engineering*. Elsevier.)

Index properties	*Wet (at natural moisture content)*	*Moist (partial air drying)*	*Dry (complete air drying)*	*Remarks*
Sand content (%)	30	42	86	Dispersion prior to hydrometer test with sodium silicate
Silt content (%) (0.05–0.005 mm)	34	17	11	
Clay content (%) (<0.005 mm)	36	41	3	
Liquid limit (%)	245	217	NP	Soaking in water for 7 days did not cause regain of plasticity lost due to the air drying
Plastic limit (%)	135	146	NP	
Plasticity index (%)	110	71	NP	

Table 8.11 Major landforms as aggregate resources in hot deserts

Mountains
Including peaks, ridges, plateau surfaces, steep (excluding precipitous) slopes,* deep valleys and canyons, wadis, river terraces* and alluvial fans,* bounding scarp slopes.* Forms vary with rock type and the evolutionary history of the area
Pediments and alluvial fans
Rock pediment,† fan* and bajada,† with occasionally inselbergs* or salt domes° forming locally high ground
Plains
Occur downslope of pediments or alluvial fans without a distinct boundary and may include a whole variety of features including: alluvial† and colluvial plains,† wadi channels and flood plains, dune fields,° salt domes° inselbergs,* and extensive stone pavement surfaces°
Playa basins
Enclosed depressions receiving surface runoff from internal catchments or within escarpment zones.† They frequently contain lakes (either temporary or permanent), lake beaches, evaporite deposits° and may be strongly influenced by aeolian, fluvial and salt processes in their base zones
Coastal zones
These include beach ridges† (formed at periods of higher sea-level or during exceptional storms), sabkhas,° mud flats,° beach° and foreshore,° estuaries° and deltas°

*Normally a major source of aggregate, conditional on suitable mineralogy
†May be a reasonable source, depending on specific characteristics
°Normally should *not* be used for aggregates
(Symbols refer to Figure 8.23)

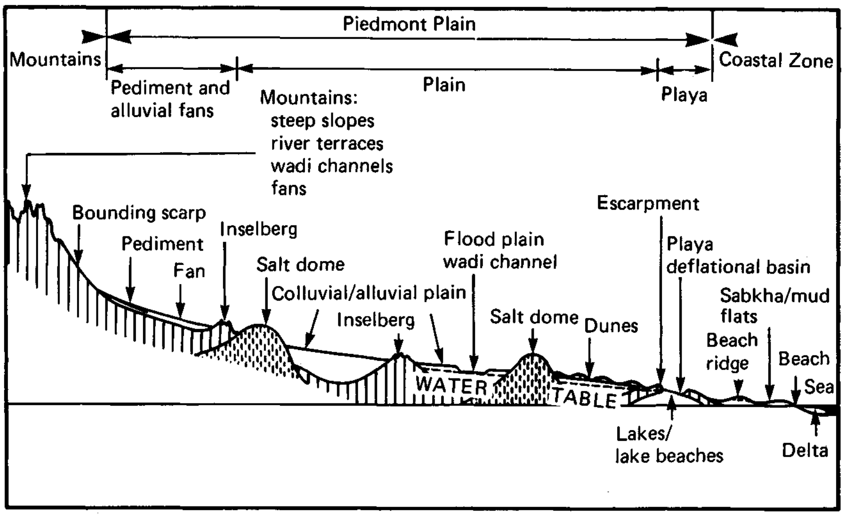


Figure 8.23 Some landforms of hot deserts and their potential suitability as sources of aggregates. (See Table 8.11 for explanation of terms)

materials at various depths. These deposits are generally known as 'drift' in the UK.

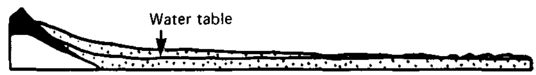
A large proportion of British site investigations therefore encounter glacial materials in one or more of their varying forms, and the property of rapid lateral and vertical change shown by some types of deposit has become notorious since construction has often revealed features undisclosed by the site investigation. It is commonly thought of as random and unpredictable but this is not always true. An understanding of the different facets of the glacial environment, each with its characteristic landforms, erosional processes and assemblages of deposits, can be of great value in predicting not only the range of variation but often also the actual location of anomalous geotechnical features.

Glacial till and outwash. Moving glaciers excavate soils and rocks in their paths which are carried along and released as the ice melts away. The material deposited directly by the glacier as it melts is called 'boulder clay' or much better *till.* If the ice front remains more-or-less stationary for a long period of time, a considerable amount of till moraine may accumulate along the ice front. Sub glacial and other tills are also laid down in the form of extensive plains revealed as the glaciers retreat during periods of melting and are characterized by lack of stratification and large range in particle size (Figure 8.25). Many tills deposited by continental glaciers contain substantial amounts of clay-size particles and these are sometimes overconsolidated and form fairly stiff clays. Even though a deposit of till may be extensive in size and uniform in texture, its strength may vary considerably from place to place.

Along the front of a glacier, water from the melting of the ice gathers to form large torrential streams which are capable of transporting great quantities of sediments. As the streams spread out over the plains most of the coarse sediment is deposited as *fluvioglacial* alluvium which has the characteristics of braided-stream deposits. This consists of granular soils with lenses of gravel, sand, or silt, which generally occur in front of a till moraine. In some localities extensive areas are covered by

Piedmont plain

Physiographic		Pediment →	Alluvial plain →	Base level
Zone	I	II	III	IV
Principal engineering soil types		Rock fans Boulder gravels	Silty stony desert and sandy stony desert, some evaporites. Could thinly cover a rock pediment	Sand dunes, sand sheets loess and evaporites.
Slope angle of desert surface		2–12°	½ – 2°	0 – ½°
Principal transporting agent of the environment		Gravity and as III Wadis	Intermittent stream flow and sheet floods Shallow anastomosing channels	Wind evaporation
Geotechnical features		Good for foundation and fill	Generally very good foundation and fill material. May be salty. May be pervious in foundation.	Erratic behaviour to load bearing, migrating dunes, metastable loess, salty absence of coarse material



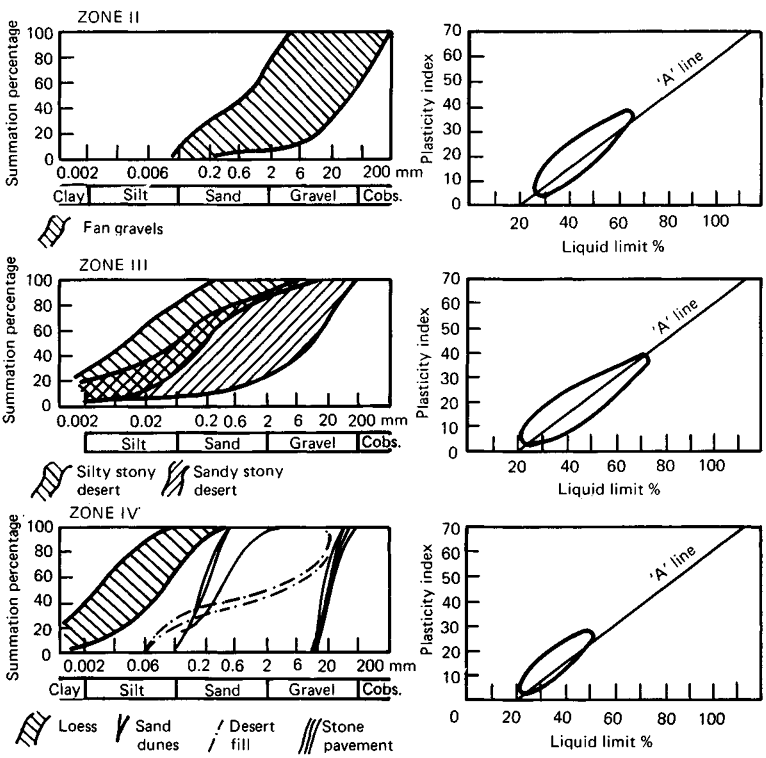


Figure 8.24 Idealized profile across mountain and plain desert terrain showing engineering zones I—IV and grading envelope, grading curves and Plasticity chart data from zones II, III and IV

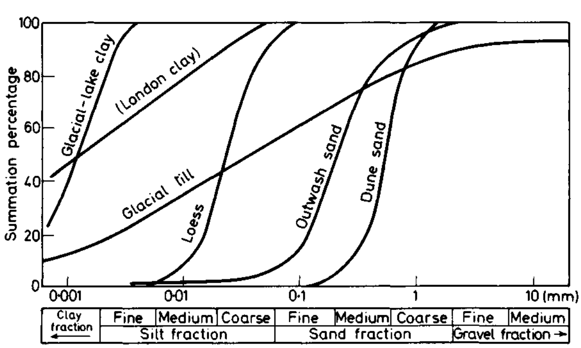


Figure 8.25 Examples of particle-size curves of some glacial soils and a London Clay for comparison

fluvioglacial deposits up to 30 m in thickness. In other areas they exist as thin lenses of limited lateral extent included between layers of till or peat.

Deposits of fluvioglacial soils may also occur in river valleys (valley trains) that once served as drainage outlets for glacial meltwater or locally in the form of ridges (eskers) and terraces (kames) as a result of various complications in the drainage system around glaciers. These fluvioglacial soils are composed primarily of silt, sand and gravel. Some of them are unstratified and others may exhibit irregular stratification. Glacial-lake deposits are often varved clays which exhibit characteristic thin stratification of silt and clay.

The principal engineering problem produced by the glacial environment is generally the difficulty of satisfactorily investigating the site. This is variously due to the rapidly changing soil type and engineering properties, large boulders in the till, lenses of clay, silt, sand or open gravel in other materials, concealed and weathered rockhead topography, structural disturbance of glacial deposits and complex groundwater conditions. Figure 8.26 shows some of the features associated with glaciers. Differential settlement may occur with heavy bearing structures and for water-retaining structures permeability may be a problem.

Linell and Shea,[25] symposium proceedings[26] and McGowan and Derbyshire[27] discuss geotechnical properties of glacial sediments.

8.4.2.5 Periglacial soils ('drift' and 'head' in the UK)

The term 'periglacial' is used to denote conditions under which frost action is the predominant weathering process. Mass transportation, wind action, or both, may occur, but only in association with very cold climatic conditions such as those near the margins of glacial ice. Perennially frozen ground (permafrost) is an important characteristic, but is not essential to the definition of the periglacial zone. The inner boundary of the zone is sharply defined by the current margin of the ice sheet, but the outer edge is gradational and the radial width of the periglacial zone is indefinite.

The distribution of permafrost may be strongly influenced by ground conditions and topographic features. The surface strata must be sufficiently porous or jointed to contain water, and their thickness must be greater than the potential thickness of the *active* layer. Permafrost may be thin or absent under surface features such as large bodies of water but it is still extensive near the northern and southern polar ice caps, in parts of Canada, Siberia and elsewhere.

Most periglacial effects from the Pleistocene glaciations on the topography and surface deposits are well preserved in southern England, between the limits of the last (Devensian) glaciation and the loess belts of North and Central Europe. However, they are not restricted to this area but can be found over the whole of the UK since the periglacial zone moved northward in the wake of the receding ice sheet. In the unglaciated areas, the relationship of frozen-ground features to older or younger glacial drifts often establishes them as independent of specifically glacial processes.

The phenomena associated with the periglacial environment almost defy classification since they are essentially overlapping aspects of a continuously evolving situation. Factors such as surface relief, lithology and geological structure have an important influence on the purely climatic effect. Some of the features which may occur concurrently are shown in an idealized manner in Figure 8.27.

From the figure it can be seen that the engineering problems can be classed under three principal headings:

(1) *Superficial structural disturbance*, e.g. frost shattering, glacial shear, hill creep, ice wedges and involutions of chemical weathering.
(2) *Mass movements*, e.g. cambering and valley bulging, landsliding or mudflows.

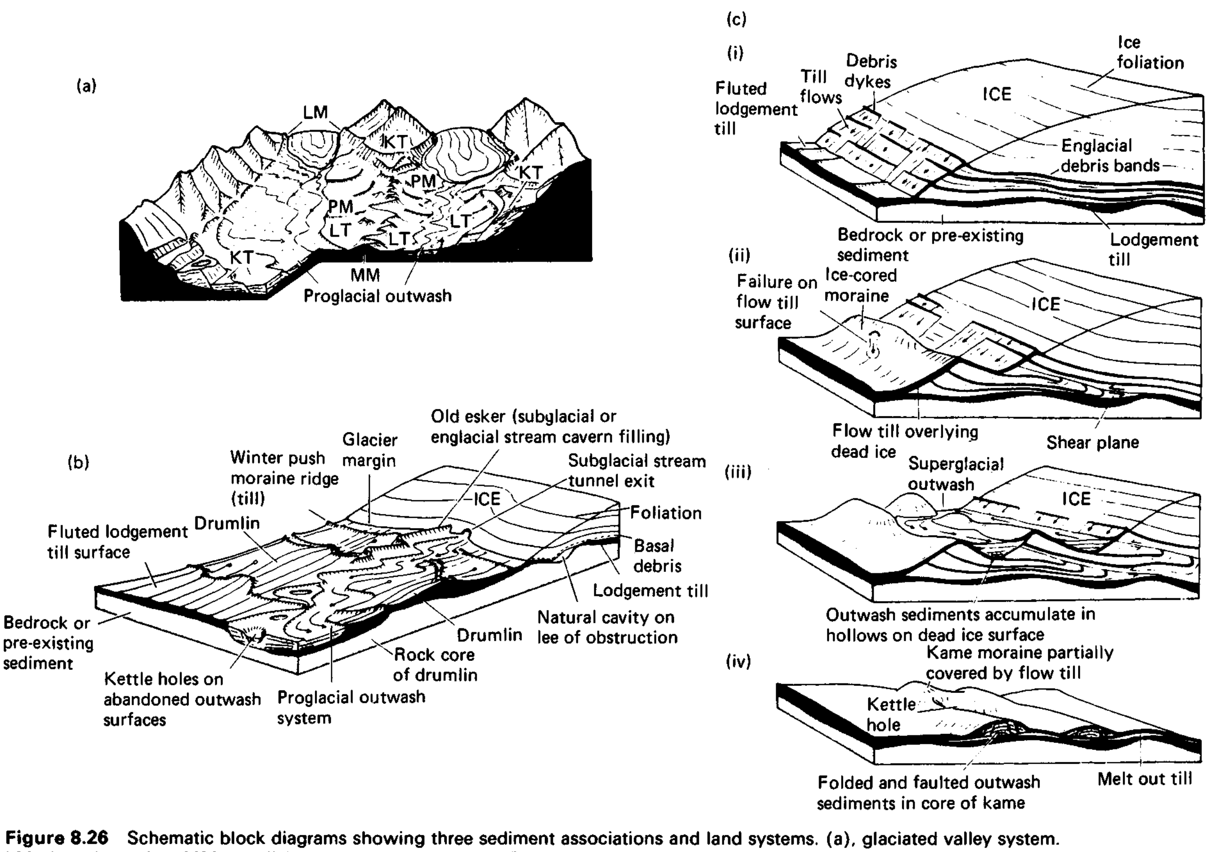


Figure 8.26 Schematic block diagrams showing three sediment associations and land systems. (a), glaciated valley system. LM – lateral moraine; MM – medial moraine (superglacial); LT – lodgement till; PM – push moraine; KT – kame terrace; (b), subglacial/proglacial system. Simple stratigraphy illustrated consists of outwash deposits on top of till resulting from single glacial advance followed by retreat; (c), superglacial system; progressive differential downwasting of ice margin produces ice-cored moraines between which meltwater streams flow and ultimately, an inversion of relief occurs as ice cores finally melt out. (After Derbyshire, Gregory and Hails (1979) *Geomorphological processes*. Butterworth, London, p. 312)
deflection indicates vertical strata; b, the outcrop which parallels

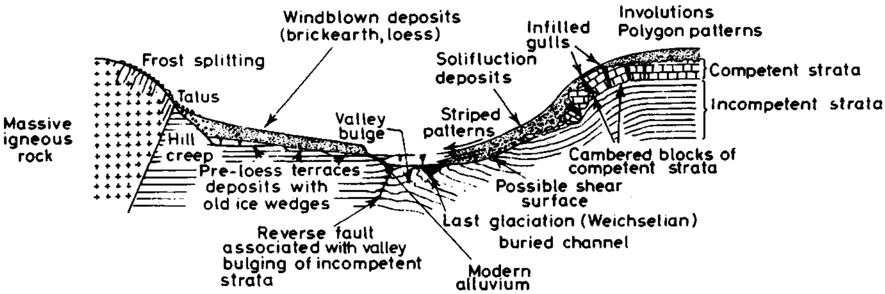


Figure 8.27 Idealized cross-section showing some periglacial features and deposits

(3) *Periglacial deposits*, e.g. loess, head or solifluxion soils.

A full discussion of these problems in the UK is given in Higginbottom and Fookes.[28] Weeks[29] discussed periglacial slope problems and Fookes and Best[30] discuss periglacial metastable soils.

8.4.2.6 Limestone landforms

Limestones usually develop more distinctive surface features of engineering relevance than other rock types, primarily as a result of its jointing, permeability and solubility in water containing carbon dioxide or humus acids. The lithology of the limestone is also significant, as strong, well-jointed limestones possess different features from those of weak limestones such as chalk.

Landforms on massive limestones. Rocks such as the Carboniferous Limestone of parts of England, the resistant Mesozoic Limestones of the Causses of Central France and similar rocks in the Karst region of Yugoslavia, are said to possess typical limestone landforms. The main process involved in producing the limestone features is the widening of fractures, joints and faults by solution, and it is helpful for this action if the water table is well below the surface to allow water to percolate continually downwards through the rock.

The surface of the limestone, due to the irregular solvent action of acid waters which pick out more readily attacked zones such as joints and faults, is often conspicuously furrowed and fretted. The vegetation, usually herbaceous plants, grows in the furrows where there is most likely to be a little soil, so that the real depth of the fretting may not be immediately apparent.

Joints are slowly enlarged by solution into holes, the shape of which will depend largely on the control exercised by the minor structural features of the rock. Two main types of solution holes are distinguished; funnel-shaped depressions with a hole at the centre (*doline*, *sink hole*, *swallow hole*, *swallet*) and shaft-like holes. With continued solution such holes may enlarge and in places several may coalesce to form larger, compound solution holes (*uvala*). Certain parts of the Carboniferous Limestone are dotted with grass-grown solution holes; the outcrop of the north of the South Wales coalfield near Penderyn, for example, has solution holes scattered over the hillside.

The largest depressions of Yugoslavia, the *poljes*, are probably not solution forms at all but tectonic depressions modified by solution of the limestone preserved in them.

If for some reason, such as the erosion of adjacent areas of impermeable rocks, the water table in a mass of limestone becomes lowered, the main underground channels will be displaced to successively lower levels. At the same time, general lowering of the limestone surface is thought to thin the rock above the underground caverns by solution loss so that eventually the roofs collapse and the drainage reappears at the surface in deep narrow gorges. Certain narrow valleys in Yugoslavia have been attributed to such cavern collapse and the same hypothesis has been applied to Cheddar Gorge in Britain. It must not be thought, however, that every narrow limestone valley is a collapsed cavern. The lowering of the water table in a limestone region is effected largely by the entrenchment of the valleys. During the process some rivers cut down their valleys more rapidly than others and, by the underground abstraction of drainage, the majority of the valleys become dry. The rivers which survive receive no surface tributaries but, instead, a supply of water from springs at river level. Streams having headwaters outside the limestone region will be assured of a supply of water, and consequently may survive as the main surface streams.

In the Yugoslavian Karst region, which is probably unique both on account of its area and because of the great thickness of its limestones, it is possible to formulate a Karst cycle of erosion. The cycle includes three important assumptions: (1) a thick and extensive mass of limestone; (2) an underlying impermeable stratum; and (3) a surface layer of impermeable rocks for the initiation of a stream pattern (see Figure 8.28).

Chalk landforms. Chalk and other weak limestones form relief which differs greatly from that developed on Carboniferous and similar massive limestones. Chalk does not often possess such a regular series of joints so there is usually little or no joint control of the relief comparable with that of Carboniferous Limestone districts, nor is the rock hard enough to be fretted at the surface by solution, nor usually is it strong enough to allow a development of large caves. A few caves and gaping fissures can occur, but they are not common, perhaps because the weight of fractured chalk above tends to close up any fissures widened by solution. In the UK, periglacial (freeze–thaw) disturbance of the upper part of the Chalk is very common (see section 8.4.2.5).

The general form of chalk landscapes, dominated by smooth convexo–concave curves is ascribed to the permeability of the rock and its residual soil.

See Dearman[31] for engineering data on limestone and Hobbs[32] for chalk.

8.4.3 Alluvial soils of rivers

8.4.3.1 Cycle of valley erosion

A river flowing in a valley erodes the material of its bed and local surface runoff contributes to the erosion of the walls of the valley. The eroded materials are transported in the form of sediment by the river and are eventually deposited.

Considered simply, rivers and the valleys along which they flow may be youthful, mature or old. At these three stages in the life of a river or valley its longitudinal profile, cross-section, and

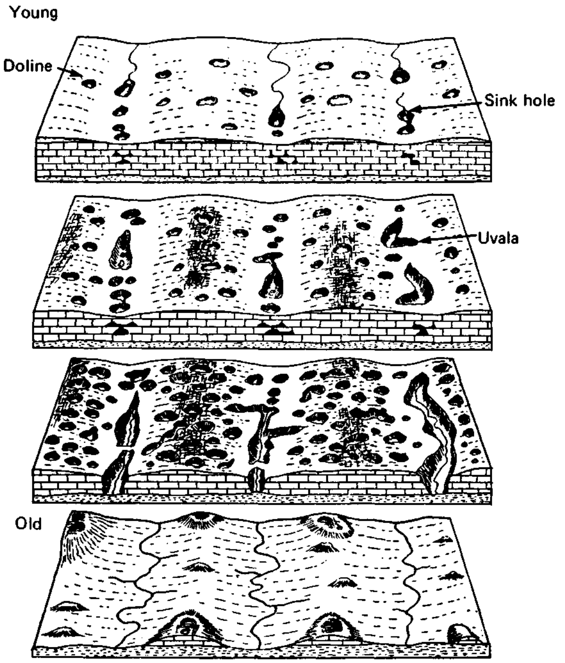


Figure 8.28 Simplified stages of the karst cycle of erosion (for exlanation, see text). The depth and size of the solution holes is greatly exaggerated in relation to the thickness of the limestone. (After Sparks (1972) *Geomorphology*, 2nd edn. Longman, London)

plan undergo gradual changes. At the *youthful* stage of a valley, its longitudinal profile is irregular and contains rapids, falls and even lakes because of local obstructions, such as hard rock strata, and its cross-section tends to be V-shaped. The plan of a youthful valley or river is somewhat angular or zigzag.

As erosion progresses, the river reaches *maturity*, irregularities gradually disappear and the plan acquires the shape of a smooth sinusoidal curve. The longitudinal profile also becomes reduced in gradient, decreasing gradually towards the mouth of the river. The valley at its mature stage is wide; its slopes are flatter than in its youth and often covered with talus (hillside rock debris).

Periodic floods contribute to the gradual widening of the valley until at its *old* age it becomes a wide floodplain. Between the floods, the old river meanders, changes its plan, but stays within a certain meander belt at the central part of the floodplain. In shifting from one location to another, a meandering river may leave behind oxbow lakes or abandoned oxbow-shaped depressions. Examples of a meandering river are the Thames, Rhine and Mississippi (see Figures 8.29 and 8.30).

During any period of geological time when climatic changes remain approximately constant, and in the absence of uplift (e.g. due to formation of folds) or change of base level (e.g. falling sea-level during a glacial period), downcutting of the river is slow enough for the lateral swinging of the river usually to make the valley wider than the channel itself. However, when the base level is lowered, the old floodplain is dissected and perhaps left in part at least, as a terrace, i.e. an abandoned floodplain. Rises in base level (e.g. the sea-level rising after a glacial period) causes the river to fill its channel (i.e. bury it) and to raise its bed level to keep pace with the rising base level. Figure 8.31 illustrates this by reference to a glacial cycle in the UK (e.g. the River Thames). More complex sequences can occur by the interacting of falling and rising base levels.

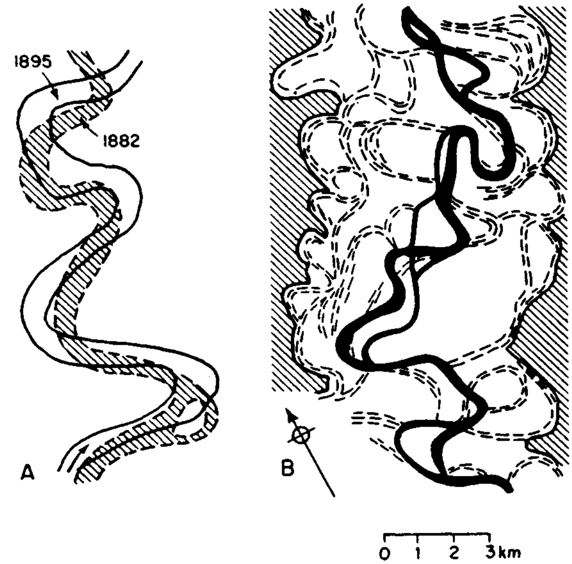


Figure 8.29 Meanders: A, downstream migration of meanders in the Mississippi; B, the river channel (black) and abandoned meanders of the Rhine

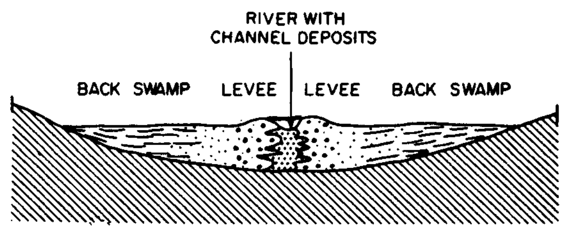


Figure 8.30 Deposits in the floodplain of a river

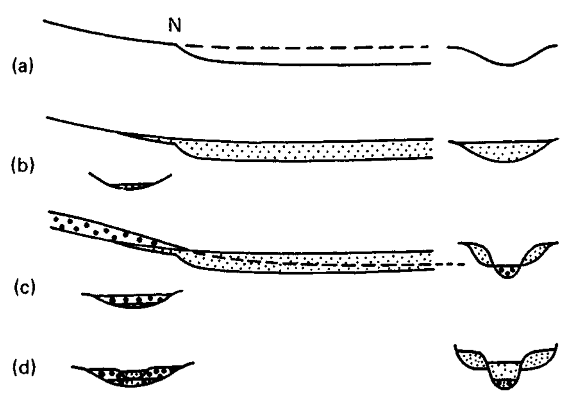


Figure 8.31 Long profiles of rivers with down-cutting and aggradation phases associated with changing sea-levels. Fine dots, alluvium of temperate stages; large dots, sands and gravels of cold stages: a, preglacial valley, river rejuvenated by low sea-level of glacial times. Nick-point marks the head of rejuvenation N; b, aggradation as a result of a rising interglacial sea-level; c, a further glacial low sea-level results in down-cutting in the lower parts of the valley and aggradation of outwash and weathering debris in the upper part; d, further aggradation during a second interglacial stage of higher sea-level

Table 8.12 Extended Casagrande classification of some alluvial soils

Material	*Major divisions*	*Subgroups*	*Casagrande group symbol*	*Drainage characteristics*	*Potential frost action*	*Shrinkage or swelling properties*	*Value as a road foundation when not subject to frost action*	*Bulk unit weight before excavation* (kg/m^3)		*Co-efficient of bulking* (%)
								Dry or moist	*Saturated*	
	Boulders and cobbles	Boulder gravels	—	Good	None to very slight	Almost none	Good to excellent	—	—	
		Well-graded gravel and gravel–sand mixtures, little or no fines	GW	Excellent	None to very slight	Almost none	Excellent	1920–2165	2080–2325	
		Well-graded gravel–sand mixtures with excellent clay binder	GC	Practically impervious	Medium	Very slight	Excellent	2000–2245	2160–2405	
		Uniform gravel with little or no fines	GU	Excellent	None	Almost none	Good	1520–1765	1840–2085	10–20
		Poorly-graded gravel and gravel–sand mixtures with little or no fines	GP	Excellent	None to very slight	Almost none	Good to excellent	1600–1845	1760–2005	
Coarse-grained soils	Gravels and gravelly soils	Gravel with fines, silty gravel, clayey gravel, poorly-graded gravel–sand–clay mixtures	GF	Fair to practically impervious	Slight to medium	Almost none to slight	Good to excellent	1760–1925	1920–2085	
		Well-graded sands and gravelly sands, little or no fines	SW	Excellent	None to very slight	Almost none	Excellent to good	1840–2005	2000–2165	
		Well-graded sand with excellent clay binder	SC	Practically impervious	Medium	Very slight	Excellent to good	1920–2085	1920–2325	
	Sands and sandy soils	Uniform sands with little or no fines	SU	Excellent	None to very slight	Almost none	Fair	1520–1845	1840–2165	5–15
		Poorly-graded sands with little or no fines	SP	Excellent	None to very slight	Almost none	Fair to good	1440–1685	1520–1765	
		Sands with fines, silty sands, clayey sands, poorly-graded sand–clay mixtures	SF	Fair to practically impervious	Slight to high	Almost none to medium	Fair to good	1520–1765	1760–2005	

Table 8.12 *Continued*

Material	*Major divisions*	*Subgroups*	*Casagrande group symbol*	*Drainage characteristics*	*Potential frost action*	*Shrinkage or swelling properties*	*Value as a road foundation when not subject to frost action*	*Bulk unit weight before excavation* (kg/m^3)		*Co-efficient of bulking* (%)
								Dry or moist	*Saturated*	
Fine-grained soils	Soils having low compressibility	Silts (inorganic) and very fine sands, rock flour, silty or clayey fine sands with slight plasticity	ML	Fair to poor	Medium to very high	Slight to medium	Fair to poor	1520–1765	1600–1765	20–40
		Clayey silts (inorganic)	CL	Practically impervious	Medium to high	Medium	Fair to poor	1600–1765	1760–1925	
		Organic silts of low plasticity	OL	Poor	Medium to high	Medium to high	Poor	1440–1685	1520–1765	
	Soils having medium compressibility	Silty and sandy clays (inorganic) of medium plasticity	MI	Fair to poor	Medium	Medium to high	Fair to poor	1520–1765	1600–1765	—
		Clays (inorganic) of medium plasticity	CI	Fair to practically impervious	Slight	High	Fair to poor	1600–1765	1765–1925	
		Organic clays of medium plasticity	OI	Fair to practically impervious	Slight	High	Poor	1440–1685	1520–1765	
	Soils having high compressibility	Micaceous or diatomaceous fine sandy and silty soils, elastic silts	MH	Poor	Medium to high	High	Poor	<1680	<1925	—
		Clays (inorganic) of high plasticity, fat clays	CH	Practically impervious	Very slight	High	Poor to very poor	<1925	<1925	
		Organic clays of high plasticity	OH	Practically impervious	Very slight	High	Very poor	<1765	<1925	
Fibrous organic soils with high compressibility		Peat and other highly organic swamp soils	Pt	Fair to poor	Slight	Very high	Extremely poor	<1765	<1765	—

8.4.3.2 Alluvial soils

Eroded soil transported by water and deposited is alluvial (water-laid) soil, or alluvium. Immediately adjacent to the steep portion of the valley, boulders and coarser gravel might be expected and there will be a minimum of fine sizes. At a distance of several kilometres from the place of original erosion, fines may predominate.

Alluvial deposits are in many respects similar to glacial but are generally more stratified and their properties might be determined from fewer boreholes than under equal conditions in glacial soils. The alluvial deposits are somewhat heterogeneous. It is not unusual, for example, to find a bed of alluvial clay several metres long, although it may be fairly narrow and only a few tens of millimetres thick. Rather uniform sand and gravel beds of varying dimensions may be found and, although there may be lens-like inclusions of sand in gravel beds and vice versa, these deposits as a whole are fairly continuous.

Besides forming terraces and benches in the valley itself, deposition of alluvium also may occur on river plains and form relatively flat deposits. Large plains are not necessarily continuous but may be interrupted by isolated hills and occasional valleys. The sediment carried by a flow moving across a plain during a flood may be spread if the gradient of the stream decreases gradually and in this case a floodplain is formed. However, if the gradient decreases abruptly, a larger part of the sediment carried by the stream drops in one place and forms an alluvial fan, a broad cone with the apex at the point where the gradient breaks.

Particular cases of recent alluvium are organic silt and mud. These are fine outwash from hills and mountain ridges, deposited in estuaries and in the rivers flowing into them, especially in the lower reaches of these rivers. The greater part of the organic silt consists of angular fragments of quartz and feldspar, abundant sericite (fine mica), and clayey matter; numerous microorganisms are also present. In the natural state, organic silt is dark and smells unpleasant; after drying it can become light grey and lose its characteristic odour. Table 8.12 gives the Casagrande classification of soils deposited from river systems.

For further reading see the bibliography on general and physical geology, and for engineering behaviour see Chapter 9.

8.5 Geological maps

8.5.1 General geological maps

Maps of the British Geological Survey in the UK are published in two principal forms. 'Solid' maps show the rock outcrops as they would appear with the overburden removed. 'Drift' maps show overburden, usually with dotted lines to indicate the probable extent of the underlying outcrops. All show outcrop patterns, not just the actual exposure of the rock at the surface. As it is only exposures which can be seen, geological maps are necessarily in part conjectural.

Mapping is based on various techniques and the completed map is not simply a survey, but the sum of all the information gathered from various sources. The small-scale map, e.g. 1:625 000 or larger, is useful to obtain a general appreciation of the country over a relatively wide area, whilst the large-scale map, 1:50 000 or less, is more for detailed information. Regional guides and detailed memoirs are also published by the British Geological Survey as well as water and mineral memoirs and so on, to supplement their maps.

One of the first things to be considered on looking at a geological map is to determine whether the rocks shown are igneous, metamorphic or sedimentary, by checking the main outcrops shown against the key provided on the map. Assuming the rocks are sediments, the strike is usually the long axis of the outcrop if it is a small-scale map. A more accurate picture is obtained by comparing the outcrop with the contours. By definition, in dipping beds the strike is the direction in which similar horizons of the strata are at the same elevation, so it follows that where the top (or bottom) boundary of a bed twice crosses the same contour line, the top of the bed will be at the same altitude at those two points and that a line drawn connecting them will show the strike direction.

The dip can be identified by noting the direction of dip arrows if these are shown, otherwise it can be deduced. The key will show which are the older of two successive beds; a boundary line on flat ground indicates that the older beds have emerged from below the newer beds, or in other words that the older beds are dipping towards the newer beds. If the boundaries follow contour lines, the beds are more-or-less horizontal, and if the boundaries cross contour lines at right angles, the beds are vertical. The relation of the outcrops to the contour lines in valleys is particularly helpful in diagnosing the general dip of the bed. (See examples in Figure 8.32.)

The degree of dip is obtainable by drawing parallel strike

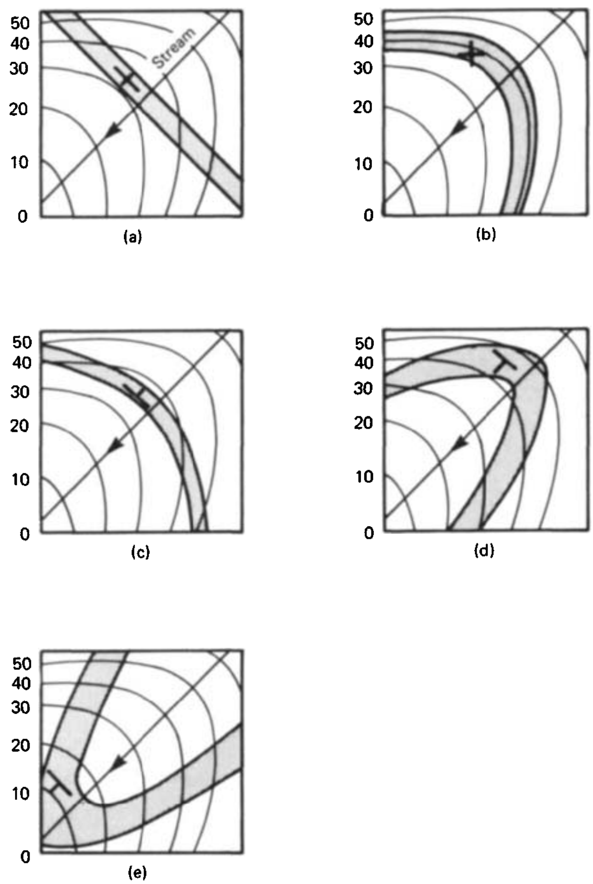


Figure 8.32 Outcrop patterns of a geological stratum related to topography: a, the outcrop crossing valley contours without defection indicates vertical strata; b, the outcrop which parallels topographic contours indicates horizontal strata; c, the outcrop which forms a blunt V pointing up a valley indicates a strata dipping upstream; d, the outcrop which forms a narrow V pointing up a valley indicates a strata dipping downstream but at an angle smaller than the valley gradient; e, the outcrop which forms a V pointing downstream across a valley indicates strata dipping downstream but at an angle greater than that of the valley gradient

lines, at different contour levels, using the same boundary between beds. Thus if a boundary between two beds crosses, say, the 100 m, 200 m and 300 m contours on each side of a valley, and the strike lines are drawn in at these levels, the distance between each strike line on the map is the distance over which the bed has dipped 100 m. The degree of dip may then be calculated or obtained graphically. Examples are given in Figures 8.33 and 8.34.

There are many other problems that can be solved by a study of geological maps, and textbooks dealing with this are given in the bibliography.

8.5.2 Special geological maps

8.5.2.1 Engineering geology maps

These are simplified maps in which the details and stratigraphy

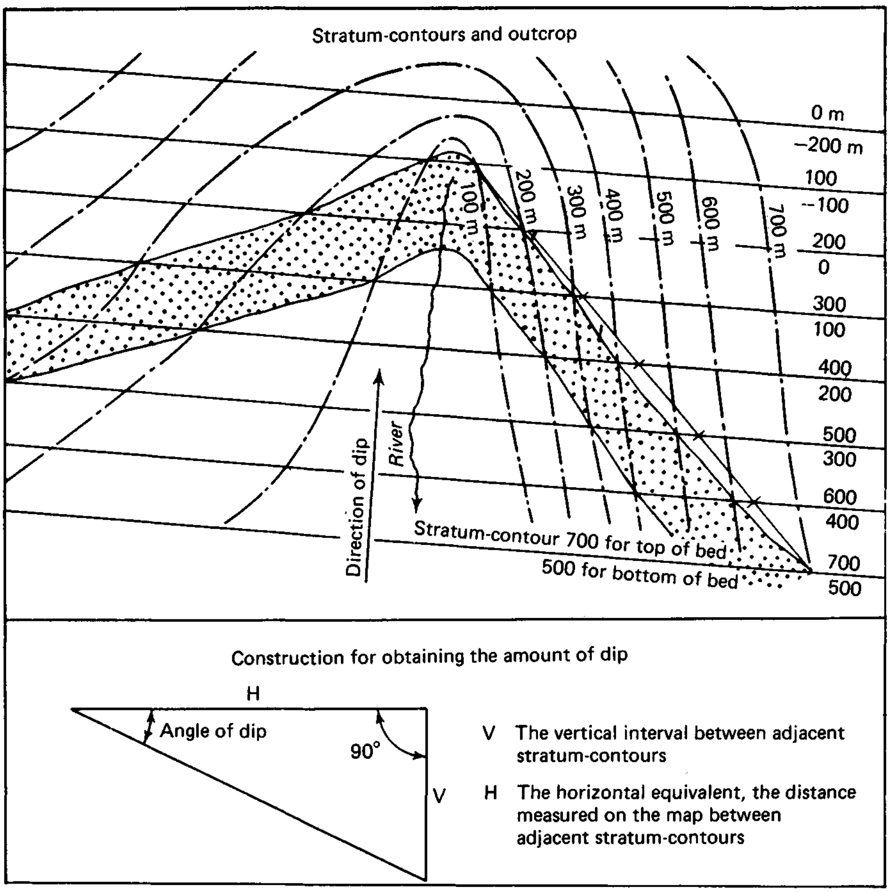


Figure 8.33 Stratum contours used to plot the outcrop of a bed and to calculate the dip. The construction is general, not specifically related to the example

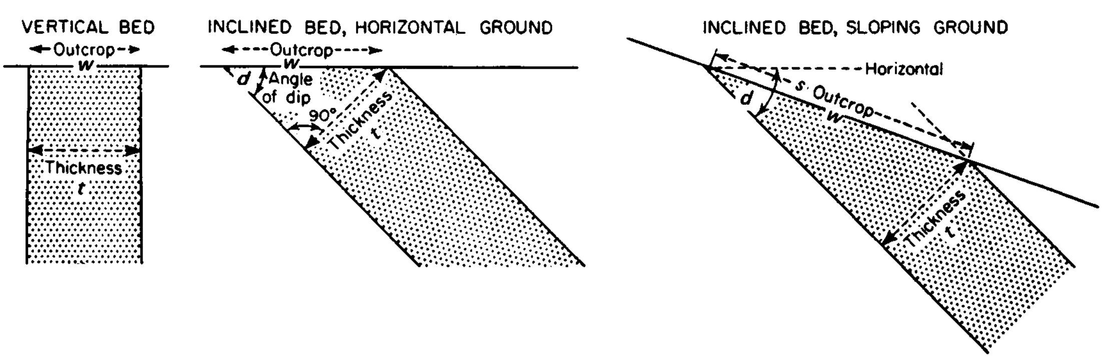


Figure 8.34 Width outcrop in relation to dip. Thickness t=outcrop width, w/cosec $d - s$ where s is the angle of slope

are omitted as far as possible, and physical characteristics of the rocks and soils, and their uses, are given. The legends of such maps are generally prepared so as to give facts and inferences of importance to the engineer. Small-scale maps may, however, be over-simplified. The particular preparation of maps and plans in terms of engineering geology is described in a Geological Society report.[33] Such maps can also be produced by an engineering geologist from the standard geological maps, and Dumbleton and West[34] list references which describe the procedure. British Standard Code of Practice 5930[35] gives current terminology.

8.5.2.2 Hydrogeological maps

The purpose of hydrogeological maps is to enable various areas to be distinguished according to their hydrological character in relation to the geology. At present, only limited areas in the UK or elsewhere have been mapped, either to show the structure and distribution of the various water-bearing strata, and/or to show the depth, and sometimes the seasonal variation, of the water table. New maps being produced in the UK indicate, on a regional basis, the extent of the principal groundwater bodies, the scarcity of groundwater, the known or possible occurrence of artesian basins, areas of saline groundwater and the potability of groundwater. They also show, according to scale, information of a local character, such as the locations of boreholes, wells and other works, contours of the groundwater table, the direction of flow and variations in quality of water.

In general, any information leading to a better understanding of occurrence, movement, quantity and quality of groundwater in the preparation of such maps should be shown with sufficient geology to lead to a proper understanding of the hydrogeological conditions.

8.5.2.3 Landform maps

Land use and landform are interrelated. Some maps are available in the UK and elsewhere which show present land utilization. Other maps indicate the relative value of land for agricultural use. Where such maps exist, they may serve as a preliminary indication of land values and engineering characteristics (see Dumbleton and West[34] for information concerning England and Wales). Landform is itself a reflection of the type of climate and the nature of the geology. Patterns of landforms can be mapped as land systems, subdivided into facets and elements, which broadly relate to the geology of the ground and more particularly to its agricultural and engineering properties. The advantage of such maps is that they can be prepared easily from air photographs; the disadvantage is their small scale (usually 1:500 000 to 1:50 000).

8.5.2.4 Soil maps

Soil maps exist in many countries including the UK but generally are concerned only with the top 1 to 1.5 m of material at the Earth's surface. They indicate what kinds of soils are present, where they are located and to some extent what use they can best serve. The classification adopted varies with intended use. Most soil maps are prepared for agricultural purposes, and although there is no agreed worldwide system which will provide for the precise classification of all varieties of physical and chemical composition existing in the soils of the world, discrimination between units is generally on the basis of geographical association, parent material, texture, subsoil characteristics and drainage class. Some but not all such criteria have engineering significance.

8.5.2.5 Resource maps

These maps indicate the occurrence and distribution of economic rocks (e.g. gravel, limestone, minerals, coal, oil, etc.) They are often on a small scale and generalized, but may record useful, if rapidly outdated, production information. Maps showing economic deposits for construction (e.g. sand and gravel, limestone, etc.) are published for parts of the UK.

8.5.2.6 Subsidiary map types

All the above categories of maps are potentially useful to the engineer and are generally published by a national agency. There are also more specialized maps sometimes obtainable through various government or commercial sources which may meet a particular need. Examples of some of these are:

(1) *Structural and tectonic maps*, which indicate the geological structure of an area but not necessarily the rock types.
(2) *Geophysical maps*, such as aeromagnetic and gravity anomaly maps.
(3) *Geochemical and mineral maps*, which indicate the distribution of specified substances.
(4) *Single feature maps*, to emphasize the distribution of a single rock type or rock property.

8.6 Geological information

Although geological information is available in the form of maps and in written texts and both published and unpublished data may have to be acquired, *Military engineering*, vol. XV[36] suggests the following.

8.6.1 Published data

Most countries now publish geological maps with supporting literature. This basic literature, which is usually readily available and understandable to a non-geologist, may take the form of a memoir, dealing with the geology of one map sheet or area (or with one aspect of the geology of several map sheets); a book broadly describing the geology of a country or significantly large region; or a brief summary of the geology of an area or information printed on the reverse side of a map.

Supplementary literature, more difficult to obtain or assess, but often incorporating maps, generally exists as papers in the bulletins of various institutions and in scientific journals. Where the basic literature is inadequate, this supplementary literature could be used. However, problems are often caused by the magnitude of published data (some 50 000 geological papers are published annually) and sifting and extracting relevant information may take time and the services of a geologist.

Some principal sources of geological information in England are listed in Table 8.13.

8.6.2 Unpublished data

Although a wealth of geological information is available in published form, much that is detailed and therefore potentially useful is never published in full, such as, for example, the following.

(1) Borehole logs.
(2) Specialist reports.
(3) Field notebooks, maps and detailed records from which publications have been summarized.
(4) University theses and dissertations.

Because of its detail such information may be of great practical

Table 8.13 Sources of geological information in the UK

Location	*Information available*	*Notes*
General Information British Geological Survey Exhibition Road South Kensington, London SW7 2DE Tel. 01-589 3444 and/or Nickerhill, Keyworth, Nottingham NG12 5GG (Regional offices in Edinburgh, Aberystwyth, Belfast, Exeter, Leeds and Newcastle)	Maps and literature worldwide coverage	Constituent body of Natural Environment Research Council. Specialist advisory service is also available through staff members. Library of the Overseas Division contains information on most developing countries. Visits should be made by appointment
National Reference Library of Geology, Geological Museum, Exhibition Road, South Kensington, London SW7 2DE	All scales of maps of Britain. Many publications and maps of countries worldwide. Comprehensive filing system	Available for copying. Also much unpublished data such as large-scale maps and borehole logs. Access free. (Part of British Geological Survey)
Library of Geological Society of London, Burlington House, Piccadilly, London W1V 0JU	Information on the UK and overseas countries, also contains an engineering geology section	Consultation fee must be paid by non-members
Libraries of British Museum, (Natural History) Cromwell Road, South Kensington, London SW7 5BD	All British and many foreign publications (including geological maps) relevant to the work of the museum	Access free
Geologists Association Library, University College, Department of Geology, Gower Street, London WC1E 6BT	Information on the UK and overseas countries	Access free
Specialized Information National Coal Board, Hobart House, Grosvenor Place, London SW1X 7AE (also area and regional offices)	Detailed information about coalmining areas	
Nature Conservancy (Geology and Physiography Section) Foxhold House, Thornford Road, Headley, Newbury, Berks	Geology of conserved areas, sites of special scientific interest, some coastlines, new roads	Constituent body of Natural Environment Research Council
Institute of Hydrology Maclean Building, Crowmarsh Gifford, Wallingford, Berks (also Regional Water Authorities)	Hydrogeological data	Constituent body of Natural Environment Research Council
Institute of Mining and Metallurgy 44 Portland Place, London W1N 4BR	Information on the UK and overseas countries	
Soil Survey of Great Britain, Rothamstead Experimental Station, Harpenden, Herts.	Pedological soil surveys	Constituent body of Agricultural Research Council
Macaulay Institute for Soil Research, Craigiebuckler, Aberdeen AB9 2QJ	Information on the UK and overseas countries	
Department of the Environment, Lambeth Bridge House London SE1 7SB	Data on some resources and sites, UK and overseas	Minerals Division deals with geological aspects of planning and environment, also sand/gravel supplies
Transport and Road Research Laboratory, Crowthorne, Berks G11 6AU	Geological information relevant to road construction	Department of the Environment

Table 8.13 *Continued*

Location	*Information available*	*Notes*
Water Resources Board, Reading Bridge House Reading RG1 8PS	Hydrogeological data	Department of the Environment
Building Research Station, Garston, Watford, Herts	Some geotechnical data relevant to building construction	Department of the Environment
Civil Engineering Laboratory, Cardington, Bedfordshire	Data relevant to foundations and to slope stability	Department of the Environment

use for engineering purposes, but problems may be encountered in locating it and securing permission for its release.

8.6.3 Books

The bibliographies of the Geological Society of America have been issued since 1933 and are available at most university or national reference libraries. They list all the geological information published about any country for a given year and in its present form have separate location and author indices. The value of the bibliographies lies in the detail of their references, but many of the publications listed may be difficult to obtain other than from the largest national lending libraries, e.g. the British Library Lending Division, Boston Spa, Wetherby, West Yorks LS23 7BQ, telephone (0937) 843434.

Many countries have an established Geological Society which publishes papers primarily related to the country. Copies of many of these papers are kept at the Geological Society of London. Major libraries throughout the world are listed in the *World of learning*, published annually by Europa Publications.

8.6.4 Institutions

Government institutes of geology and allied sciences provide the main reference source for geological information. They publish maps and literature, store unpublished data, and may provide a specialist advisory service through their staff members. Most countries now possess an equivalent to the British Geological Survey in the UK, although this may be called a Geological Survey Department, Bureau or Department of Mines and Mineral Resources. The work produced by such an institution will usually be in the language of that country and a technical translation of high quality may be essential.

Many universities have a department of geology or earth sciences and may possess information not available elsewhere, comprising research work in progress and unpublished theses, dissertations and reports. British universities have research interests overseas; details of staff and their research interest are published annually by the Department of Education and Science. The address of overseas universities and staff are given annually in the *World of learning*.

Local museums frequently serve as depositories for geological data, both published and unpublished. Certain national organizations in any country accumulate specialized geological information.

Similar organizations exist in some developing countries and for many of these areas particularly useful information may be contained in unpublished reports of oil companies, mining companies and civil engineering firms. The oil companies have produced geological maps for many otherwise unsurveyed areas and some of these have been published. A request for geological information, particularly about *near-surface* formations, may be received sympathetically depending on company policy and circumstances.

Besides the methods of obtaining geological information mentioned above, there are data retrieval organizations and geological abstracting services which will provide information for a fee.

Another source of information is satellite photographs. A catalogue and price list of satellite photographs can be obtained from Audio-visual Branch, National Aeronautics and Space Administration, Washington DC, US.

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British Mesozoic Fossils
British Cainozoic Fossils
$\Big\}$ Handbooks published by the British Museum (Natural History) London SW7
Geologists' Association Guides: a series of excursion guides to selected British localities (available from The Scientific Anglican, 30/30A St Benedict's St, Norwich)
Publications of the Geological Society of London, Burlington House, Piccadilly, London W1V 0JU: various monographs and authoritative works

9 Soil Mechanics

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Contents

9.1 The basics of soil behaviour

In engineering terms, soil is the generally softer, weaker and more weathered material overlying rock. All soils consist of solid particles assembled in a relatively loose packing. The voids between the particles may be filled completely with water (fully saturated soils) or may be partly filled with water and partly with air (partly saturated soils).

Soil and rock materials can, very simply, be divided into the groups shown in Table 9.1.

The primary engineering problems which we attempt to solve in soil mechanics are those of predicting the strength, compressibility and time-dependent compression of soil materials. For this it is necessary to understand the principle of effective stress.

9.1.1 Effective stress

Soil can be considered as a two-phase system consisting of a solid phase—the skeleton of soil particles—and a fluid phase—water plus air in a partly saturated soil, and water alone in a saturated soil. It follows that the normal stress across a plane within a soil mass will have two components: (1) an intergranular pressure, known as the effective pressure or effective stress; and (2) a fluid pressure known as the pore pressure or neutral pressure u. The sum of these will constitute the total normal stress. The volume change characteristics and the strength of a soil are controlled by the effective stress, the pore pressure being significant only in so far as it determines the magnitude of the effective stress for a given total stress.

The simplest illustration of pore pressure and effective stress is given by consideration of the vertical stresses acting on a horizontal plane at a depth h under equilibrium conditions with a horizontal water table. The total vertical stress σ is given by the weight per unit area of soil and water above the plane:

$$\sigma = \gamma h \tag{9.1}$$

where γ is the bulk density of the soil, i.e. its total weight/unit volume (see section 9.7 for a definition of terms).

The pore pressure will be the water pressure, and if the plane is at a depth h_w below the water table then $u = h_w \cdot \gamma_w$.

The effective vertical stress is the difference between these:

$$\sigma' = \sigma - u \tag{9.2}$$

In partly saturated soils, there is a pore air pressure u_a as well as a pore water pressure u_w and the effective stress Equation (9.2) is then modified as follows:

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \tag{9.2a}$$

The parameter χ is related to the degree of saturation S_r. For full saturation, $\chi = 1$ and Equation (9.2a) reduces to (9.2). Equation (9.2a) is rarely used in practice.

A change in total stresses arising from a change in external loading conditions will give rise to a change Δu in pore pressure. The excess pore pressure, positive or negative, will dissipate with time, the rate at which equilibrium pore pressure conditions are re-established being governed by the permeability of the soil. In coarse grained granular soils, such equilibrium conditions will be achieved immediately and changes in effective stress are equal to changes in total stress. At the other limit, with clays of low permeability, equilibrium conditions may take considerable time, up to tens of years, to be re-established. The relation between pore pressure change and the change in principal stresses can be expressed by the use of pore pressure parameters A and B. The basic relationship is in terms of the major and minor principal stresses, σ_1 and σ_3:

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \tag{9.3}$$

It is also useful to relate the pore pressure change to the change in deviator stress ($\Delta\sigma_1 - \Delta\sigma_3$) alone and also to the change in the major principal stress ($\Delta\sigma_1$). For these purposes, two further parameters $\bar{A}$ and $\bar{B}$ are used as follows:

$$\begin{aligned} &\Delta u = B \cdot \Delta\sigma_3 + \bar{A}(\Delta\sigma_1 - \Delta\sigma_3) \\ \text{or}\quad &\Delta u = \bar{B} \cdot \Delta\sigma_1 \end{aligned} \tag{9.4}$$

If the soil structure behaved in an elastic manner, the values of the pore pressures could be established theoretically, e.g. A would have a value of 1/3. However, soils behave nonelastically and A can have values ranging between +1.3 and −0.7 (values at failure in a triaxial compression test). Typical values of the pore pressure parameters are given by Bishop and Henkel.[2] For a full discussion of the parameters see Skempton.[1]

It is the effective stress, rather than the total stress, which controls the key properties of strength and compressibility and, to a certain extent, permeability.

9.1.2 Shear strength

Shear strength of a soil is commonly thought of as having two components: cohesion and frictional resistance. Clays are often described as cohesive soils in which the shear strength or cohesion is independent of applied stresses, and sands and gravels are described as noncohesive or frictional soils in which the shearing resistance along any plane is directly proportional to the normal stress across that plane:

$$s = p \cdot \tan\phi \tag{9.5}$$

The concepts of cohesion and friction were combined in Coulomb's equation for the shear strength of soil:

$$s = c + p \cdot \tan\phi \tag{9.6}$$

where c is the cohesion and ϕ is the 'angle of internal friction'.

Such simple concepts are, however, inadequate to deal with the complex problem of the shear strength of soils. The early history of the study of shear strength is somewhat confused.

Table 9.1 A simple grouping for soils and rocks

	Strength	*Compressibility*	*Speed of drainage*
Organic materials, e.g. peat	Very low	Very high	Generally rapid
Cohesive materials, e.g. clay	Low–medium	High	Slow
Granular soils, e.g. silts, sands and gravels	Medium–high	Low	Very rapid
Rocks	Very high	Very low	Often rapid

Attempts were made to represent the shear strength of a soil by the envelope to a Mohr circle diagram of stress,[3] the intercept on the vertical axis being taken as cohesion c, and the slope of the envelope being taken as the friction angle ϕ. It was found that, except in sands and gravels, the results for a given soil varied considerably depending on the test procedure used, particularly the rate of testing and the conditions of drainage of the specimens during test. However, following the realization that the strength of a soil is governed by the effective stress, it was possible to achieve a better understanding of the shear strength characteristics of soils. The shear strength can be expressed as:

$$\tau_f = c' + (\sigma - u) \tan \phi' \tag{9.7}$$

where c' and ϕ' are effective stress parameters, c' is the apparent cohesion, ϕ' the angle of shearing resistance and u the pore water pressure.

The Mohr circle diagram can be plotted in terms of effective stress, with c' as the cohesion intercept and ϕ' as the slope of the envelope (Figure 9.1).

In terms of effective principal stresses in the Mohr diagram the Coulomb failure criterion may be expressed as:

$$(\sigma_1' - \sigma_3') = \sin \phi'(\sigma_1' + \sigma_3') - 2c' \cos \phi' \tag{9.8}$$

and if c' is zero, then:

$$\sigma_3' = \sigma_1'(1 - \sin \phi')/(1 + \sin \phi') \tag{9.9}$$

which is the well-known Rankine failure condition.[4]

A special condition exists where the soil is loaded rapidly and no drainage is permitted. In this case, the shear strength is found to be independent of the total stress applied to the soil for a saturated unfissured clay. Changes in total stress lead to equal changes in pore water pressure u so that the effective stress in the soil remains unchanged. Because the soil is undrained its moisture content does not alter. Therefore, it is common practice to define the undrained shear strength of a clay at failure as:

$$\tau_f = c_u \quad (\phi_u = 0) \tag{9.10}$$

This parameter is used in so-called $\phi_u = 0$, 'short-term' or 'end of construction' analyses (see section 9.5).

9.1.3 Permeability

Permeability is that property of a soil which controls the rate of flow of water through the soil. In soil mechanics, permeability is defined by the equation derived from Darcy's law:

$$v = k \cdot i \tag{9.11}$$

where v is the superficial velocity of flow through the soil, i is the hydraulic gradient and k is the permeability.

k therefore has the dimensions of a velocity; it depends chiefly on particle size and grading, and to a lesser extent on the particle packing.

Typical values of permeability for soils range from 1×10^{-2} m/s for a coarse sand to 1×10^{-10} m/s for a clay. A very rough estimate of permeability for a relatively uniform sand can be obtained from Hazen's law:

$$k = D_{10}^2/100 \tag{9.12}$$

where D_{10} is the 10% size or effective size in millimetres and k is in metres per second.

The 10% size or effective size is the particle size at which the grading curve crosses the 10% line (Figure 9.2).

Research by Loudon[5] has shown that the permeability (in metres per second) of clean sand may be computed from simple soil tests, using Kozeny's formula, with an accuracy of ±20% (SI units):

$$kS^2 = 1.5 \times 10^{-4} \frac{n^3}{(1-n)^2} \tag{9.13}$$

He suggests an alternative formula which is easier to use and of equal accuracy:

$$\log_{10}(kS^2) = 1.365 + 5.15n \tag{9.14}$$

In both of these formulae, n is porosity and S denotes specific surface in square metres per cubic millimetre:

$$S = f(x_1S_1 + x_2S_2 + \ldots x_nS_n) \tag{9.15}$$

where f is an angularity factor, varying between 1.1 for a rounded sand and 1.4 for an angular sand, $x_1, x_2, \ldots$ is the

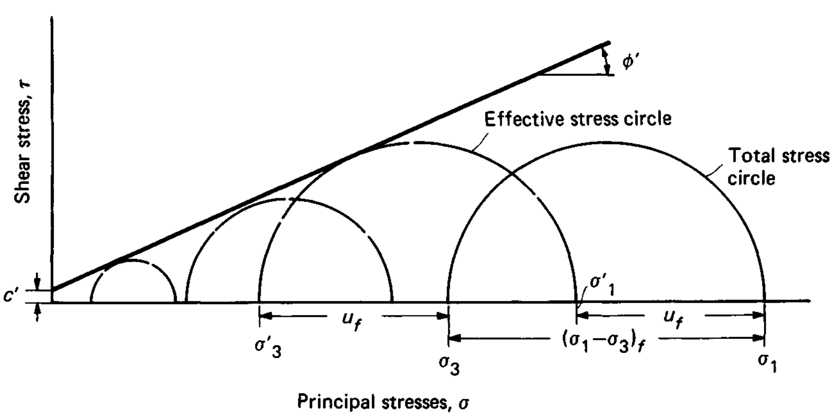


Figure 9.1 Mohr circle diagram

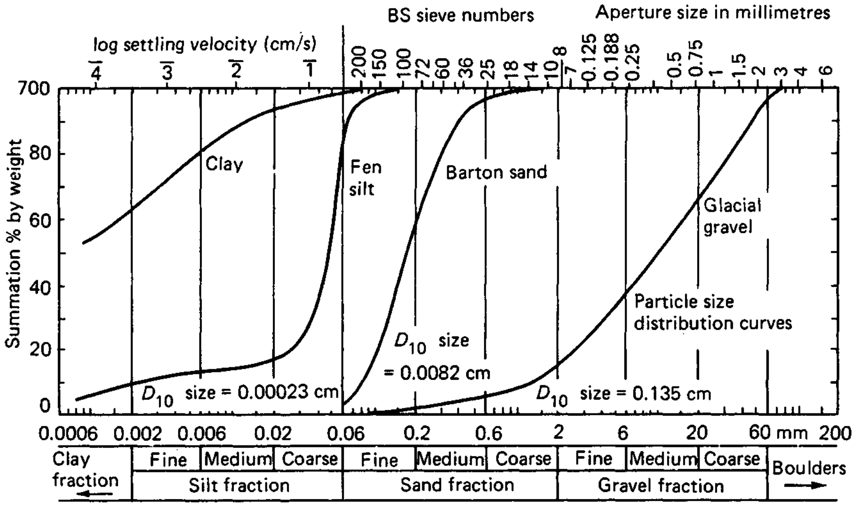


Figure 9.2 Particle size distribution curves (on standard sheet) showing D_{10} size. (The D_{10} size is now given in millimetres)

fraction in each sieve range and $S_1, S_2, \ldots$ is the specific surface for each sieve range, from Loudon's tables. (Note that Loudon's tables are given in square centimetres per cubic centimetre units for S. It is therefore necessary to convert Loudon's values for S using $1\ \mathrm{cm}^{-1} = 0.1\ \mathrm{mm}^{-1}$.)

9.1.4 Consolidation

The ultimate change in volume of a soil occurring under a change in applied stress depends on the compressibility of the skeleton of soil particles. However, the water in the voids of a saturated soil is relatively incompressible and, if no drainage takes place, change in applied stress results in a corresponding change in pore pressure, and the volume change is negligible. As drainage takes place by flow of water from zones of high excess pore pressure to zones of less or zero excess pore pressure, and the excess pore pressures dissipate, the applied stress is transferred to the soil skeleton and volume change takes place. It is this volume change of cohesive soils resulting from dissipation of excess pore pressures which is known as consolidation.

A study of consolidation requires knowledge of the compressibility of the soil skeleton and of the rate at which excess pore pressures dissipate, which is related to the permeability. In Terzaghi's consolidation theory the relation between these factors can be expressed for the one-dimensional case as:

$$c_v \partial^2 u/\partial z^2 = \partial u/\partial t \qquad (9.16)$$

where c_v is the coefficient of consolidation and is given by

$$c_v = k/m_v \cdot \gamma_w \qquad (9.17)$$

u is excess pore water pressure, z the thickness of the stratum, t is time, and m_v is the modulus of volume compressibility, defined as:

$$m_v = -(\mathrm{d}e/\mathrm{d}p)/(1+e_o) \qquad (9.18)$$

where e is the void ratio and p the effective pressure.

The solution of the consolidation equation has been given by Taylor[3] and values have been tabulated for the degree of consolidation U against the time factor T, where:

$$T = c_v t/H^2 \qquad (9.19)$$

and H is the length of the drainage path. Values of c_v and m_v are determined by laboratory tests known as oedometer, or consolidation, tests.

The relationship between the degree of consolidation, U, and the time factor T is dependent on the initial distribution of excess pore water pressure. Figure 9.3 gives a plot of U against t for various ratios of the initial excess pore pressure at the top and bottom of the compressible stratum u_1/u_2. The cases given are all for single drainage.

For double drainage (i.e. where drainage can take place at the top and bottom of the layer) values corresponding to $u_1/u_2 = 1$ can be used for all ratios of initial pore pressure, but it should be noted that in the double drainage case, H is taken as only half of the layer thickness.

One-dimensional drainage is seldom fully realized in practice and for important calculations, particularly where the loaded area is small in comparison with the thickness of the compressible stratum, two- or three-dimensional consolidation should be considered.[6,7]

Furthermore, in many deposits, lateral permeability can be up to two orders greater than vertical owing to the presence of a laminar structure of thin layers and partings of silt and fine sand. This will have a very marked effect on rate of consolidation.[8] Problems involving a number of layers having different consolidation characteristics have been solved numerically using the finite-difference method.[9]

Methods of test to obtain the soil parameters described in the previous sections are detailed in Appendix 9.1.

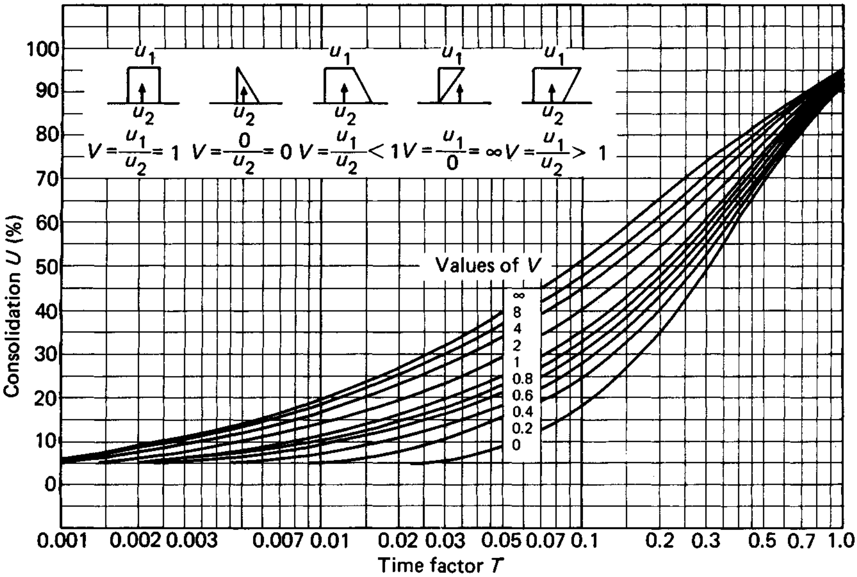


Figure 9.3 Values of time factor T

9.2 Design and limit states in soil mechanics and foundation engineering

Unlike virtually all other materials dealt with by civil engineers during the course of their work, soil is naturally occurring. It is inherently variable, not only from site to site but also at different levels and plan locations at any one site. The extent of its variability can be judged by examining the typical limits of some of its most important properties:

Undrained shear strength c_u	5–300 kN/m^2
Coefficient of permeability k	10^{-2}–10^{-10} m/s
Coefficient of compressibility m_v	0.01–3.00 m^2/MN

If the variability of soil is to be understood and allowed for, then a knowledge of geology and the processes leading to the formation and induration of the soil will be important. For any single soil type it is rarely reasonable to assume that the soil is uniformly variable, in the sense of conforming to a single Gaussian distribution for a given soil property, or that the full ranges of its properties are known.

The basic steps involved in soil mechanics design are:

(1) Arbitrary division of the soil into layers thought to have similar engineering behaviour. This process is carried out during site investigation (see Chapter 11, and Clayton, Simons and Matthews[10]) using sample description and classification testing. Approximate assessment of the principal soil mechanics parameters (e.g. undrained strength) for each soil group.
(2) Envisaging all the mechanisms by which the structure–soil combination may lead to a limit state for the structure (e.g. the structure may fail to perform as required because of foundation bearing capacity failure, excess differential settlement, chemical attack on foundations, etc.). Table 9.2 lists some of these factors as they affect low-rise construction.
(3) Testing of soil, both *in situ* and in the laboratory, to obtain detailed parameters suitable for sound geotechnical engineering calculations in order to assess the risk of 'failure' by all of the mechanisms envisaged in (2) above.
(4) Geotechnical calculation, associated with changes in structural design, to achieve maximum economy while avoiding unacceptable behaviour of the structure.

Table 9.2 Ground problems and low-rise building. (After Building Research Establishment (1987) *Site investigation for low-rise building: desk studies*. BRE Digest Number 318. HMSO, London)

Differential settlement or heave of foundations (or floorslabs)
Soft spots under spread footings on clays
Growth or removal of vegetation on shrinkable clays
Collapse settlements on pre-existing made ground
Mining subsidence
Self-settlement of poorly compacted fill
Floor slab heave on unsuitable fill material
Soil failure
Failure of foundations on very soft subsoil
Instability of temporary or permanent slopes
Chemical processes
Groundwater attack on foundation concrete
Reactions due to chemical waste or household refuse
Variations during construction
Removal of soft spots to increase depth of footings
Dewatering problems
Piling problems

Geotechnical engineers recognize an important division between soils which drain rapidly (e.g. silts, sands, gravels) and those which are slow-draining (e.g. clays and clayey soils). In the case of free-draining, noncohesive soils the important consequence of their rapid dissipation of excess pore water pressures is that they undergo increasing effective stress as load is applied; therefore, their strength increases as shear stresses brought about by loading also increase. The consequence is that when foundations are placed on granular soils, bearing capacity is rarely a problem.

Cohesive soils, on the other hand, do not drain rapidly; loading or unloading leads to excess pore water pressures which may take years to dissipate. For these materials the geotechnical engineer recognizes two types of loading which lead to critical conditions at two different times during the life of the structure.

Consider an element of soil beneath a foundation constructed on a clay foundation. As the loads are applied to the foundation during the relatively rapid process of constructing the superstructure of the building, the shear stress applied to the soil is increased. Because of the increase in load applied to the soil, excess pore pressures develop. Because clay is slow-draining, the excess pore pressures do not dissipate significantly, the effective strength does not change, and the shear strength of the soil remains constant. Thus, the factor of safety against bearing capacity failure decreases up to the end of the construction (Figure 9.4(a)), after which it increases. Normal practice, for almost all cases where a load increase is applied to the soil, is therefore to calculate a so-called 'short-term' factor of safety using the initial undrained shear strength of the soil combined with the structural loads and bearing pressures expected from the completed structure.

In those cases where unloading takes place (Figure 9.4(b)), the same logic leads to the conclusion that the critical time may be many years after the end of construction, in the 'long-term', once the pore pressures once again come to equilibrium. Examples of unloading situations include excavated slopes and retaining walls. In these cases, the shear strength used in calculations must take into account the changes in pore pressure and effective stress that have occurred. For this reason, long-term calculations are carried out using effective strength parameters.

9.3 Foundations

There are two ways in which a foundation can fail to perform satisfactorily: (1) by shear failure; and (2) by settlement. In the first case, a surface of rupture is formed in the soil, the foundation settles considerably and probably tilts to one side and heaving of the soil occurs on one or both sides of the foundation. In the second case, failure of the soil in shear does not occur, but the existing deformations are large enough to cause failure of the structure which the foundation is supporting.

Failure by settlement is therefore a function of the particular structure as well as the underlying soil. Skempton and Macdonald[11] have given a criterion for framed buildings based on angular distortion which is expressed by the ratio of differential settlement, δ, to the distance l, between two points, usually the column positions. From a detailed study of field data, a limiting value of $\delta/l = 1/300$ has been determined. More flexible structures, oil tanks for example, may undergo considerably greater settlements without sustaining damage. On the other hand, some sensitive machinery and stiff reinforced concrete slabs will tolerate very little settlement (see Chapter 17).

The ultimate bearing capacity of a foundation is the value of the net loading intensity at which the ground fails in shear. Before discussing bearing capacity, several definitions are necessary.

(1) The gross loading intensity, p, is the pressure due to the applied load and the total weight of foundation, including any backfill above the foundation.
(2) The net loading intensity, p_n, is the gross foundation pressure less the weight of material (soil and water) displaced by the foundation (and by the backfill above the foundation). Alternatively, the net pressure can be considered as equal to the gross pressure less the total overburden pressure, $p_n = p - p_o$.
(3) The safe bearing pressure is the ultimate bearing capacity divided by the factor of safety, $q_s = q_u/F$.
(4) The allowable bearing pressure, q_a, is less than, or equal to, the safe bearing capacity, depending on the settlements which are expected and which can be tolerated.

The term 'presumed bearing value' was introduced in CP 2004,[12] and was defined as the net loading intensity considered appropriate to the particular type of ground for preliminary design purposes. Table 9.3 gives the current presumed bearing values from BS 8004.[13]

9.3.1 Bearing capacity of shallow foundations

There are two groups of methods of determining ultimate bearing capacity: (1) analytical methods; and (2) graphical methods. The graphical methods are very flexible and will cover any conditions likely to be found in practice, but they are rather cumbersome in use. The analytical techniques, which are only strictly applicable in cases in which the soil is uniform, are quicker and easier to use, and therefore are the most often used.

The most general formula for the ultimate bearing capacity of a strip footing is that of Terzaghi,[14] which in terms of effective stress is:

$$q_u = p_o + c' \cdot N_c + p_o'(N_q - 1) + 0.5 \cdot \gamma \cdot B \cdot N_\gamma \tag{9.20}$$

where p_o is the total vertical stress (overburden pressure) at foundation level, p_o' is the effective vertical stress at foundation level, c' is the effective cohesion intercept of the soil, γ is the bulk density of the soil, and N_c, N_q and N_γ are bearing capacity factors which depend upon the geometry of the foundation and the effective angle of friction (ϕ') of the soil.

For a circular footing of diameter D:

$$q_u = p_o + 1.3 \cdot c' \cdot N_c + p_o'(N_q - 1) + 0.3 \cdot \gamma \cdot B \cdot N_\gamma \tag{9.21}$$

and for a square footing of width B:

$$q_u = p_o + 1.3 \cdot c' \cdot N_c + p_o'(N_q - 1) + 0.4 \cdot \gamma \cdot B \cdot N_\gamma \tag{9.22}$$

Unfortunately, there is some disagreement in the literature on appropriate values of the bearing capacity factors. According to Vesic,[15] there is reasonable agreement on the values of N_c and N_q, but N_γ values vary considerably. Conservative values, which assume no friction between the soil and the underside of the foundation, are given in Table 9.4. These factors are only valid for foundations under uniformly distributed loads. If the foundation load is neither vertical nor central, then various factors must be applied. For the strip footing, for example, the equation for ultimate bearing capacity becomes:

$$q_u = p_o + c' \cdot N_c \cdot f_{c_i} + p_o'(N_q - 1) \cdot f_{q_i} + 0.5 \cdot \gamma \cdot B' \cdot N_\gamma \cdot f_{\gamma_i} \tag{9.23}$$

B' is an effective (i.e. reduced) foundation width equal to:

$$B' = B - 2e$$

(Figure 9.5). R_h and R_v are the horizontal and vertical components of the resultant force on the foundation. Brinch Hanson[16] and Sokolovski[17] have proposed that:

$$f_{q_i} = \left[1 - \frac{R_h}{(R_v + B' \cdot c' \cdot \cot\phi')}\right]^2 \tag{9.24}$$

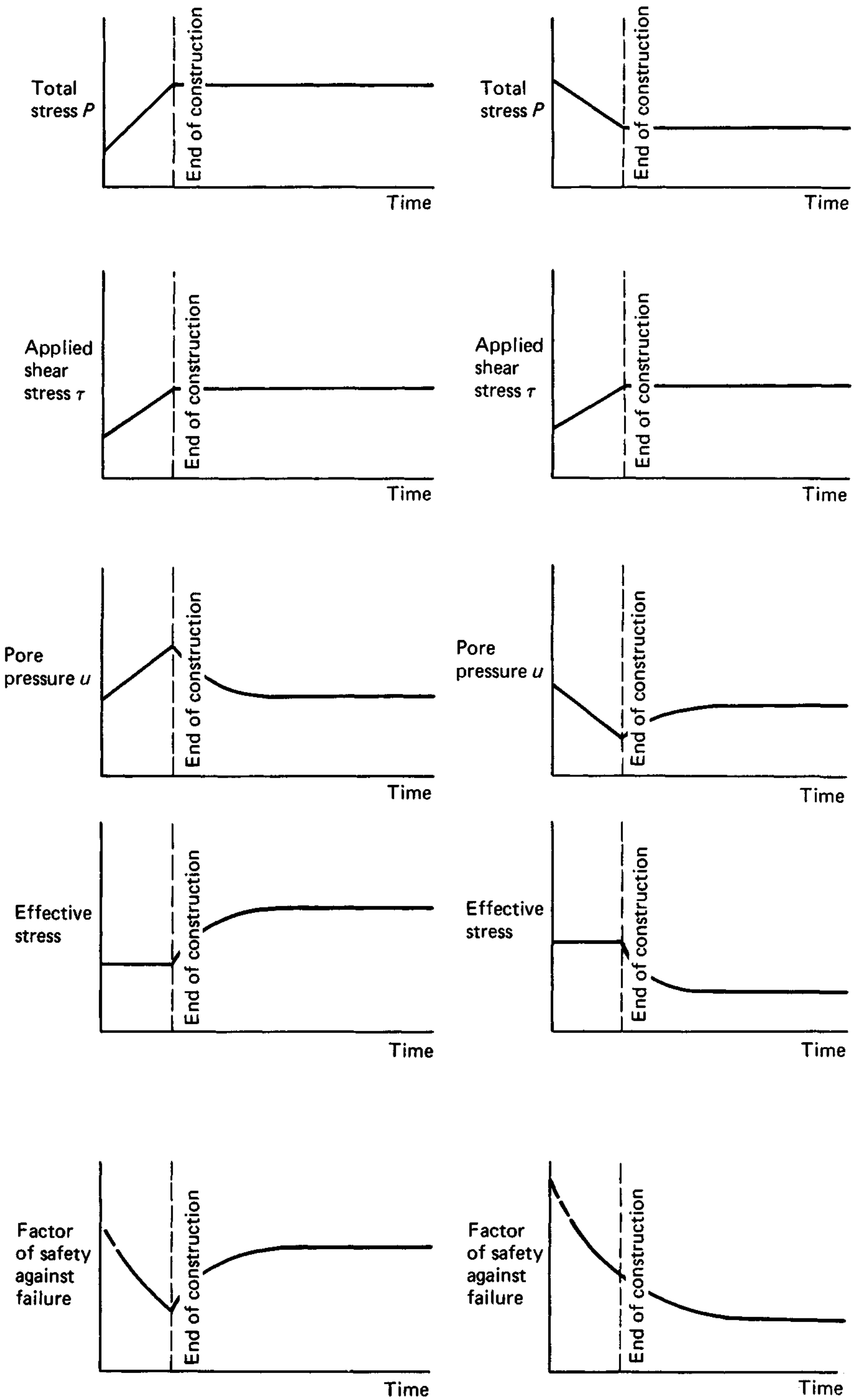


Figure 9.4 Load, pore pressure, shear strength and factor of safety. (a) For a clay beneath a foundation (loading case); (b) for a clay beneath a motorway excavation (unloading case). (After Clayton and Milititsky (1986) *Earth pressure and earth-retaining structures*. Surrey University Press, Glasgow)

Table 9.3 Presumed bearing values under vertical static loading. Refer to British Standards Institution (1986) *Foundations*, BS 8004 for further details)

Category	*Types of rocks and soils*	*Presumed allowable bearing value* (kN/m²*)	(kgf/cm²* tonf/ft²)	*Remarks*
Rocks	Strong igneous and gneissic rocks in sound condition	10 000	100	These values are based on the assumption that the foundations are taken down to unweathered rock.
	Strong limestones and strong sandstones	4 000	40	(For weak, weathered and broken rock, see BS 8004)
	Schists and slates	3 000	30	
	Strong shales, strong mudstones and strong siltstones	2 000	20	
Noncohesive soils	Dense gravel, or dense sand and gravel	>600	>6	Width of foundation not less than 1 m. Groundwater level assumed to be a depth not less than below the base of the foundation.
	Medium dense gravel, or medium dense sand and gravel	<200–600	<2–6	(For effect of relative density and groundwater level, see BS 8004)
	Loose gravel, or loose sand and gravel	<200	<2	
	Compact sand	>300	>3	
	Medium dense sand	100–300	1–3	
	Loose sand	<100	<1	
		Value depending on degree of looseness		
Cohesive soils	Very stiff boulder clays and hard clays	300–600	3–6	Cohesive soils will undergo long-term consolidation settlement
	Stiff clays	150–300	1.5–3	
	Firm clays	75–150	0.75–1.5	
	Soft clays and silts	<75	<0.75	
	Very soft clays and silts	Not applicable		
Peat and organic soils		Not applicable		
Made ground or fill		Not applicable		

Note. These values are for preliminary design purposes only, and may need alteration upwards or downwards. No addition has been made for the depth of embedment of the foundation.
* 107.25 kN/m² = 1.094 kgf/cm² = 1 tonf/ft².

$$f_{c_i} = f_{q_i} - \frac{(1 - f_{q_i})}{N_c \cdot \tan\phi'}$$

$$f_{\gamma_i} = (f_{q_i})^{3/2}$$

For a granular soil, or when $c' = 0$, with a foundation approximately at ground level ($p_o = 0$), these equations reduce to:

$$q_{ult} = \frac{1}{2} \cdot \gamma \cdot B \cdot N_\gamma \left(1 - \frac{R_h}{R_v}\right)^3 \tag{9.25}$$

and the vertical force per unit length of foundation at failure is then:

$$R_{v_{ult}} = q_{ult} \cdot B' \tag{9.26}$$

For clays, the short-term or end of construction case is critical. For short-term, '$\phi_u = 0$' analysis, the ultimate uniform bearing pressure that a foundation can apply is:

$$q_{ult} = p_o + N_c \cdot c_u \tag{9.27}$$

where p_o is the total vertical stress in the soil adjacent to the foundation, at footing level (as before), N_c is a bearing capacity factor, dependent on the geometry of the foundation, and c_u is the average undrained shear strength of the foundation equal to its width, from unconsolidated undrained triaxial compression tests on undisturbed samples.

Values of N_c for a strip footing are given in Table 9.5.

As before, this equation must be modified for nonvertical and eccentric levels. In this case:

$$q_u = c_u \cdot N_c \cdot f_{c_i} + p_o \tag{9.28}$$

where

$$f_{c_i} = 1 - \frac{2 \cdot R_h}{B' \cdot c_u \cdot N_c} \tag{9.29}$$

(See Figure 9.5.)

9.3.2 Bearing capacity of deep foundations

The Terzaghi bearing capacity equation has been generally accepted as a basis for the design of shallow foundations, but not for foundations where the depth greatly exceeds the width as in the case of piers and piles. Meyerhof[18] suggested that the shear surface beneath the base would return upwards and inwards to reach the shaft, and produced graphs of a general bearing capacity factor $N_{\gamma q}$ for various values of ϕ' and foundation depth ratio $D:B$. Large-scale experiments by Kerisel[19] have indicated that, in addition to ϕ', both depth and size influence the bearing capacity factor N_q. In a comprehensive study of deep foundations, Vesic[20] has observed that there is no evidence of failure surfaces reverting to the shaft.

Table 9.4 Bearing-capacity factors for shallow strip footage. (After Clayton and Milititsky (1986) *Earth pressure and earth retaining structures*. Surrey University Press, Glasgow)

Effective angle of friction of soil ζ' (deg.)	N_c^*	N_q^*	N_γ^*
0	5.14	1.00	0.00
15	10.98	3.94	2.65
20	14.83	6.40	5.39
25	20.72	10.66	10.88
30	30.14	18.40	22.40
35	46.12	33.30	48.03
40	75.31	64.20	109.41
45	133.88	134.88	271.76

* Bearing-capacity factors based on
$N_q = e^{\pi \tan\zeta'} . \tan^2(45 + \zeta'/2)$
$N_c = (N_q - 1)\cot\zeta'$
$N_\gamma = 2(N_q + 1)\tan\zeta'$

Table 9.5 Bearing-capacity factors for clay in the short term. (After Skempton (1951) *The bearing capacity of clays*. Building Research Congress Division 1, Part 3, p. 180)

Ratio of depth to width D/B	*Bearing-capacity factor* N_c *for*	
	strip footings	*circular and square footings*
0	5.14	6.2
0.25	5.6	6.7
0.50	5.9	7.1
0.75	6.2	7.4
1.00	6.4	7.7
2.00	7.0	8.4
4.00	7.5	9.0

The design of deep foundations in granular materials is a highly complex matter. Terzaghi's bearing capacity factor N_q is too conservative for higher values of ϕ' and is independent of the depth: width ratio D:B. Berezantsev's[21] curves relating N_q to ϕ' (Figure 9.6) are now recognized as giving the best representation of the ultimate bearing capacity of deep foundations in terms of N_q and D:B, thus:

$$Q_u = A \cdot p'_o \cdot D \cdot N_q \tag{9.30}$$

where A is the base area of the pier or pile. See also Tomlinson[22] for a discussion on deep foundations.

For clays, in the short term, experience has shown that Skempton's bearing capacity factors are reasonable. An N_c value of 9 is therefore generally adopted (Table 9.5).

9.3.3 Settlement

The problem of predicting settlement is a three-part one: (1) predicting the maximum settlement; (2) predicting differential settlement; and (3) predicting the rate at which settlement will occur. Differential settlements cause the majority of damage to structures because of the bending movements that they induce in the structure, although excessive total settlements can lead to problems where services enter buildings. Unfortunately, differential settlements are usually very difficult to predict; for routine problems with flexible buildings, maximum differential settlements are often as much as 75% of the maximum total settlement, so that the designer will normally seek to control total settlement in order to avoid problems due to differential settlements. For heavily loaded, relatively rigid structures, some form of fairly complex computer analysis will be necessary if differential settlements are to be estimated.

The rate at which consolidation settlements occur is of particular importance to two types of problems:

(1) Multistorey structures, where the rigidity of the structure increases as its height (and hence its load) goes up. If consolidation is relatively rapid, then a higher proportion of differential settlement will occur whilst the structure is flexible, thus inducing less bending moment.
(2) Stage-constructed embankments, where it is necessary to allow the subsoil (i.e. beneath the embankment) to gain strength before the next lift of fill can be placed.

Different methods of settlement prediction are used depending principally upon the type of soil to be analysed, but also to a certain extent upon the type of test data used to characterize the compressibility of the soil. The principal soil types for which settlement methods exist are: (1) normally consolidated clays; (2) lightly overconsolidated clays; (3) heavily overconsolidated clays; and (4) sands and gravels.

The main types of data used to characterize the compressibility of soil are derived from: (1) oedometer tests; (2) the triaxial test; (3) stress path tests; (4) the standard penetration test; (5) the static (Dutch) cone test; and (6) plate loading tests.

9.3.3.1 Settlement of clays

An idealized representation of settlement of foundations in clays and of the heave on excavation is given in Figure 9.7. On excavation, heave occurs and the majority of this is recovered when the original total overburden pressure p_o is replaced. This recovery of heave is often neglected in settlement calculations. With further application of load, i.e. with increase in the net applied pressure, an immediate settlement occurs without volume change of the clay, followed by consolidation settlement as the excess pore pressures set up by the applied load are dissipated. In practice, the consolidation settlement starts immediately the net pressure is greater than zero, but at a very slow rate, so that it is convenient to ignore the consolidation settlement occurring during construction or, alternatively, to consider it as starting at halfway through the construction period. See Taylor[3] for a fuller treatment of this problem.

The net final settlement is the sum of the immediate settlement and the consolidation settlement:

$$\rho_{final} = \rho_i + \rho_c \tag{9.31}$$

The net settlement at any time t is given by:

$$\rho_t = \rho_i + \bar{U} \cdot \rho_c \tag{9.32}$$

where $\bar{U}$ is the degree of consolidation after time t.

Immediate settlement Immediate settlement occurs without change in the volume of the soil and as a result of the shear stresses imposed by the foundation or embankment loads. Because it involves a change in shape without a change in volume (and therefore no drainage of pore water is involved) immediate settlement takes place as the load is applied. The immediate settlement below the corner of a uniformly loaded rectangular area can be calculated from elastic theory using the Steinbrenner[23] equation (see also Terzaghi[24]):

$$\rho_i = \frac{3}{4} q \frac{B}{E} I_p \tag{9.33}$$

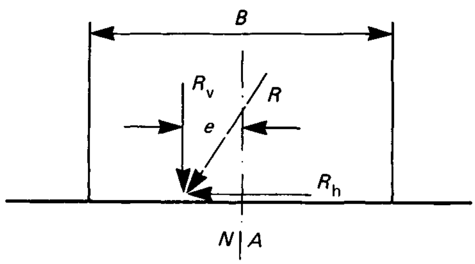


Figure 9.5 Bearing capacity for inclined eccentrically loaded strip foundations. Effective width $B'=B-2e$. (After de Beer (1949) *Grondmekanica*. NV Standard Boekhandel, Antwerp; Meyerhof (1955) 'The bearing capacity of foundations under eccentric and inclined loads', *Proceedings, 3rd international conference on soil mechanics and foundation engineering*, Vol. III)

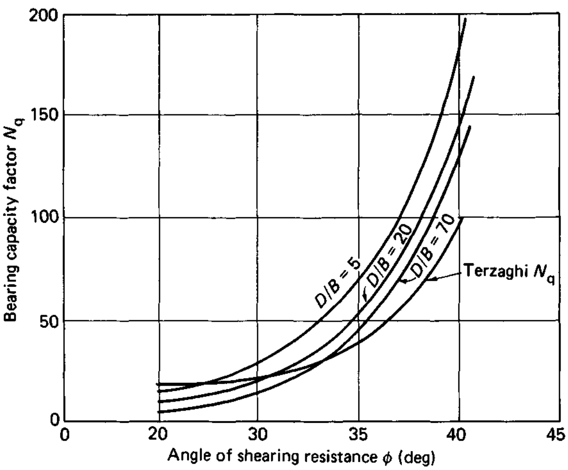


Figure 9.6 Berezantsev's bearing capacity factor N_q

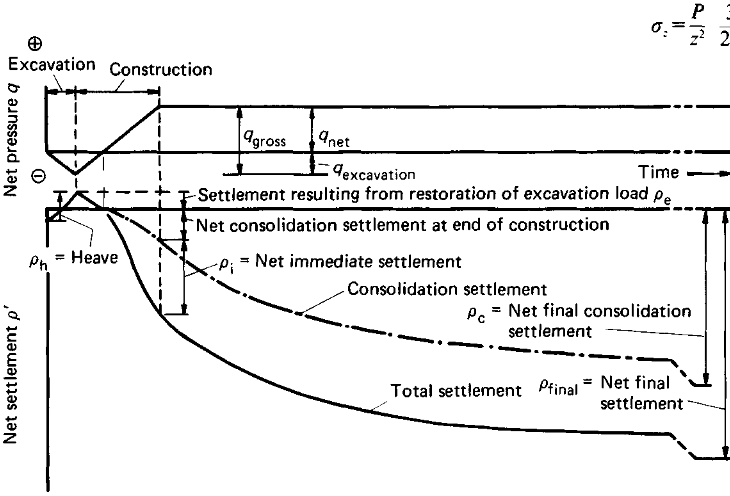


Figure 9.7 Idealized representation of settlement of foundations in clay

where E is Young's modulus for the clay, as measured by the appropriate tangent modulus of stress–strain curves from triaxial tests, and I_ρ is an influence factor which is a function of the length and width of the foundation and the thickness of the compressible layer below foundation level. Values of I_ρ for a Poisson's ratio value of 0.5 are given in Figure 9.8.

Settlements at other points below a rectangular area can be calculated by splitting the area into a number of rectangles and using the principle of superposition.

Consolidation settlement In practice, the contribution of consolidation to total settlement is calculated in different ways for clays with different stress histories.

Soft normally consolidated clays: oedometer data for normally consolidated clays should yield a straight line when plotted as a voids ratio (e) vs. logarithm of applied pressure ($\log_{10} p$) curve. Due to sample disturbance, they normally yield a curve at least in the early part of compression. To correct for this, the oedometer test should be continued until a void ratio of less than 0.42 times its initial value is reached. The 'virgin consolidation curve' can then be reconstructed by drawing a straight line through the point of the initial void ratio for the *in situ* effective vertical stress, p'_o, and the consolidation curve where $e=0.42\varepsilon_o$. The slope of this straight line is termed the compression index, C_c (Figure 9.9(a)).

Consolidation settlement is calculated by dividing the compressible strata into layers (at least five layers are normally required to achieve adequate accuracy) and calculating the contribution of each layer. Thus:

$$\rho_c = \sum_{x=1}^{x=n} \frac{C_c}{(1+e_o)} \cdot z_x \cdot \log_{10}\left(\frac{P'_o + \Delta\sigma_v}{P'_o}\right) \tag{9.34}$$

where z_x is the thickness of the xth layer, $\Delta\sigma_v$ is the average vertical stress increase in the xth layer due to foundation or embankment loading, C_c is the compression index, e_o is the initial voids ratio of the xth layer, and P'_o is the initial effective vertical stress at the centre of the xth layer.

The increase in vertical stress at any level, due to a flexible foundation or embankment loading, can be obtained from elastic theory. Boussinesq[25] gave the following equation for the vertical stress increase at depth z due to a point load P on the surface of a semi-infinite solid:

$$\sigma_z = \frac{P}{z^2}\,\frac{3}{2\pi}\left(\frac{1}{[1+(r/z)^2]^{5/2}}\right) \tag{9.35}$$

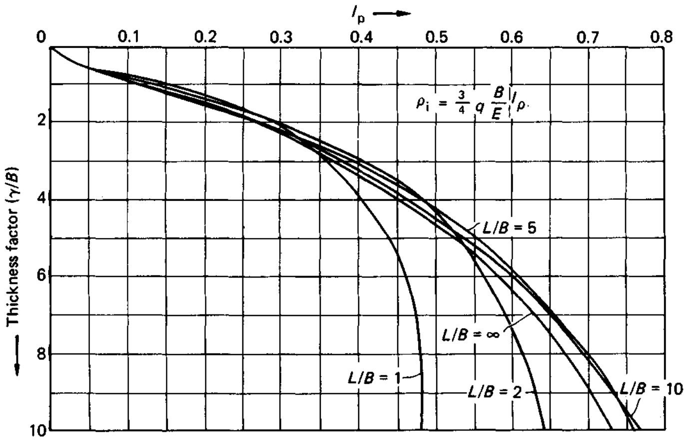


Figure 9.8 Steinbrenner's influence factors for loaded area $L \times B$ on compressible stratum of thickness Y

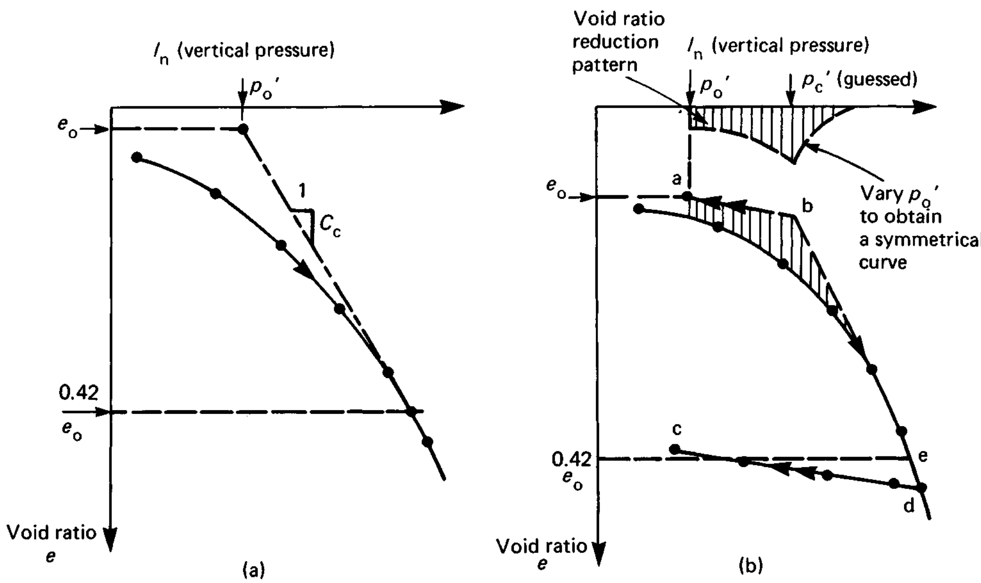


Figure 9.9 Schmertmann's methods of reconstructing the e vs $\log_{10} p'$ curve. (a) Normally consolidated soils; (b) lightly overconsolidated soils

where z and r are defined in Figure 9.10. This can be written:

$$\sigma_z = P \cdot K / z^2 \tag{9.36}$$

Values of K are given in Table 9.6.

Equation (9.36) has been integrated and tabulated by Newmark[26] to give the pressure below the corner of a rectangle uniformly loaded at the surface, and the values are given in Table 9.7. In order to obtain the pressure below any other point it is necessary to regard that point as the corner of four adjoining rectangles (not necessarily the same shape or size), calculate the pressure below the corner of each and add these pressures. For example, the pressure below the centre of a rectangle is 4 times the pressure beneath the corner of a rectangle whose sides are half the sides of the original rectangle. The principle can be extended to points outside the original rectangle by addition and subtraction of rectangles. It is implied in the above that the pressure is uniformly distributed at the surface of the ground.

A flexible load is one in which the contact reaction from the ground is identical to the applied pressure at each point. Uniformly flexible loads on relatively thick foundation soils produce a dish-shaped settlement profile. Under rigid foundations the settlement is uniform or planar but the contact pressure varies with soil type. Intermediate cases occur depending on the degree of relative rigidity of the structure and ground, but present analytical difficulties. In practice, foundations are generally considered as being either flexible (e.g. oil tanks) or rigid (e.g. high buildings on stiff rafts). The pressure distribution for the latter case has been worked out by Fox,[27] who gives a series of curves (Figure 9.11) for the mean vertical stress σ_z at a depth z beneath a rectangular area $a \times b$ uniformly loaded with a pressure q, the rectangle being on the surface.

For more important cases, it has been the recent practice to use the finite-element method to investigate soil–structure interaction, stress distribution within the structure and the ground, and the associated deformations. Alternatively, an excellent collection of elastic solutions has been made by Poulos and Davis.[28]

For lightly overconsolidated clays, where the maximum past vertical effective pressure, p'_c, is somewhat in excess of its current value, p'_o, sample disturbance and bedding effects are likely to mask the value of p'_c. The value of p'_c is required with some accuracy because the application of stress between p'_o and p'_c will lead to very little consolidation settlement, but once p'_c is exceeded then large consolidation settlements can be expected.

To overcome this problem, Schmertmann[29] has proposed the method of reconstruction of the e vs. $\log p'$ curve to find p'_c, as shown in Figure 9.9(b).

The method is as follows:

(1) Plot (e_o, p'_o), point a.
(2) Estimate the maximum preconsolidation pressure p'_c.
(3) Draw a straight line through a, parallel to the rebound curve, c–d, to intersect the vertical line through p'_c at b.
(4) Through b draw a straight line to point e on the consolidation curve, where point e has a void ratio equal to $0.42e_o$.
(5) Construct the 'void ratio reduction pattern', which is the difference between the reconstructed curve abed and the curve obtained from the laboratory test.

Schmertmann suggests that the best estimate of p'_c is obtained when the most symmetrical voids ratio reduction pattern is obtained.

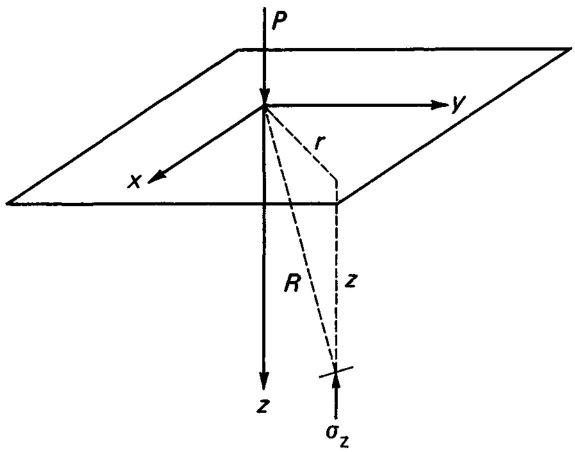


Figure 9.10 Diagram showing z and r in Boussinesq equation for concentrated load

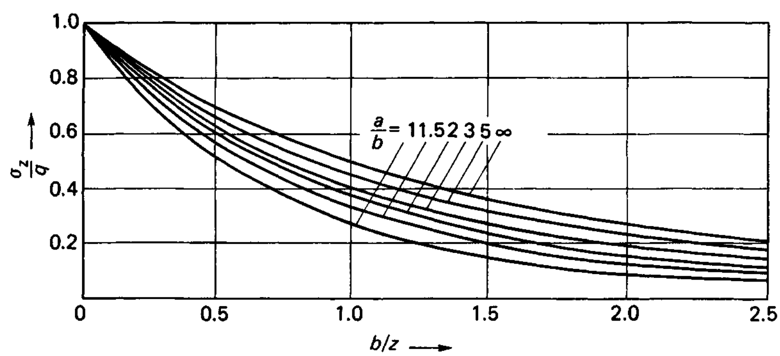


Figure 9.11 Mean pressure under a rigid foundation at ground level. (After Fox (1948) 'The mean elastic settlement of a uniformly loaded area at a depth below the ground surface', *Proceedings, 2nd international conference on soil mechanics and foundation engineering*, Vol. II)

Table 9.6 Values of coefficient K in Equation (9.36)

Ratio $\frac{r}{z}$	Coefficient K	Ratio $\frac{r}{z}$	Coefficient K	Ratio $\frac{r}{z}$	Coefficient K	Ratio $\frac{r}{z}$	Coefficient K	Ratio $\frac{r}{z}$	Coefficient K	Ratio $\frac{r}{z}$	Coefficient K
0.00	0.477 5	0.60	0.221 4	1.20	0.051 3	1.80	0.012 9	2.40	0.004 0	2.84	0.001 9
								45	0.003 7	2.91	0.001 7
0.10	0.465 7	0.70	0.176 2	1.30	0.040 2	1.90	0.010 5	2.50	0.003 4	2.99	0.001 5
0.15	0.451 6	0.75	0.156 5	1.35	0.035 7	1.95	0.009 5	2.55	0.003 1	3.08	0.001 3
0.20	0.432 9	0.80	0.138 6	1.40	0.031 7	2.00	0.008 5	2.60	0.002 9	3.19	0.001 1
0.25	0.410 3	0.85	0.122 6	1.45	0.028 2	2.05	0.007 8	2.65	0.002 6	3.31	0.000 9
0.30	0.384 9	0.90	0.108 3	1.50	0.025 1	2.10	0.007 0	2.70	0.002 4	3.50	0.000 7
0.35	0.357 7	0.95	0.095 6	1.55	0.022 4	2.15	0.006 4	2.72	0.002 3	3.75	0.000 5
0.40	0.329 4	1.00	0.084 4	1.60	0.020 0	2.20	0.005 8	2.74	0.002 3	4.13	0.000 3
0.45	0.301 1	1.05	0.074 4	1.65	0.017 9	2.25	0.005 3	2.76	0.002 2	4.91	0.000 1
0.50	0.273 3	1.10	0.065 8	1.70	0.016 0	2.30	0.004 8	2.78	0.002 1	6.15	0.000 1
0.55	0.246 6	1.15	0.058 1	1.75	0.014 4	2.35	0.004 4	2.80	0.0002 1		

Table 9.7 Vertical pressure σ_z under corner of rectangle $a \times b$ loaded uniformly with intensity q. Table gives σ_z/q for values of $\alpha = a/z$ and $\beta = b/z$

α/β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	∞
0.1	0.004 7	0.009 2	0.013 2	0.016 8	0.019 8	0.022 2	0.024 2	0.025 8	0.027 0	0.027 9	0.029 3	0.030 1	0.030 6	0.030 9	0.031 1	0.031 4	0.031 5	0.031 6	0.031 6	0.031 6	0.031 6	0.031 6	0.031 6
0.2	0.009 2	0.017 9	0.025 9	0.032 8	0.038 7	0.043 5	0.047 4	0.050 4	0.052 8	0.054 7	0.057 3	0.058 9	0.059 9	0.060 6	0.061 0	0.061 6	0.061 8	0.061 9	0.062 0	0.062 0	0.062 0	0.062 0	0.062 0
0.3	0.013 2	0.025 9	0.037 4	0.047 4	0.055 9	0.062 9	0.068 6	0.073 1	0.076 6	0.079 4	0.083 2	0.058 6	0.087 1	0.088 0	0.088 7	0.089 5	0.089 8	0.090 1	0.090 1	0.090 2	0.090 2	0.090 2	0.090 2
0.4	0.016 8	0.032 8	0.047 4	0.060 2	0.071 1	0.080 1	0.087 3	0.093 1	0.097 7	0.101 3	0.106 3	0.109 4	0.111 4	0.112 6	0.113 4	0.114 5	0.115 0	0.115 3	0.115 4	0.115 4	0.115 4	0.115 4	0.115 4
0.5	0.019 8	0.038 7	0.055 9	0.071 1	0.084 0	0.094 7	0.103 4	0.110 4	0.115 8	0.120 2	0.126 3	0.130 0	0.132 4	0.134 0	0.135 0	0.136 3	0.136 8	0.137 2	0.137 4	0.137 4	0.137 4	0.137 5	0.137 5
0.6	0.022 2	0.043 5	0.062 9	0.080 1	0.094 7	0.106 9	0.116 8	0.124 7	0.131 1	0.136 1	0.143 1	0.147 5	0.150 3	0.152 1	0.153 3	0.154 8	0.155 5	0.156 0	0.156 1	0.156 2	0.156 2	0.156 2	0.156 2
0.7	0.024 2	0.047 4	0.068 6	0.087 3	0.103 4	0.116 8	0.127 7	0.136 5	0.143 6	0.149 1	0.157 0	0.162 0	0.165 2	0.167 2	0.168 6	0.170 4	0.171 1	0.171 7	0.171 9	0.171 9	0.172 0	0.172 0	0.172 0
0.8	0.025 8	0.050 4	0.073 1	0.093 1	0.110 4	0.124 7	0.136 5	0.146 1	0.153 7	0.159 8	0.168 4	0.173 9	0.177 4	0.179 7	0.181 2	0.183 2	0.184 1	0.184 7	0.184 9	0.185 0	0.185 0	0.185 0	0.185 0
0.9	0.027 0	0.052 8	0.076 6	0.097 7	0.115 8	0.131 1	0.143 6	0.153 7	0.161 9	0.168 4	0.177 7	0.183 6	0.187 4	0.189 9	0.191 5	0.193 8	0.194 7	0.195 4	0.195 6	0.195 7	0.195 7	0.195 8	0.195 8
1.0	0.027 9	0.054 7	0.079 4	0.101 3	0.120 2	0.136 1	0.149 1	0.159 8	0.168 4	0.175 2	0.185 1	0.191 4	0.195 5	0.198 1	0.199 9	0.202 4	0.203 4	0.204 2	0.204 4	0.204 5	0.204 6	0.204 6	0.204 6
1.2	0.029 3	0.057 3	0.083 2	0.106 3	0.126 3	0.143 1	0.157 0	0.168 4	0.177 7	0.185 1	0.195 8	0.202 8	0.207 3	0.210 3	0.212 4	0.215 1	0.216 3	0.217 2	0.217 5	0.217 6	0.217 7	0.217 7	0.217 7
1.4	0.030 1	0.058 9	0.085 6	0.109 4	0.130 0	0.147 5	0.162 0	0.173 9	0.183 6	0.191 4	0.202 8	0.210 2	0.215 1	0.218 4	0.220 6	0.223 6	0.225 0	0.226 0	0.226 3	0.226 4	0.226 5	0.226 5	0.226 6
1.6	0.030 6	0.059 9	0.087 1	0.111 4	0.132 4	0.150 3	0.165 2	0.177 4	0.187 4	0.195 5	0.207 3	0.215 1	0.220 3	0.223 7	0.226 1	0.229 4	0.230 9	0.232 0	0.232 4	0.232 5	0.232 6	0.232 6	0.232 6
1.8	0.030 9	0.060 6	0.088 0	0.112 6	0.134 0	0.152 1	0.167 2	0.179 7	0.189 9	0.198 1	0.210 3	0.218 4	0.223 7	0.227 4	0.229 9	0.233 3	0.235 0	0.236 0	0.236 4	0.236 7	0.236 8	0.236 8	0.236 9
2	0.031 1	0.061 0	0.088 7	0.113 4	0.135 0	0.153 3	0.168 6	0.181 2	0.191 5	0.199 9	0.212 4	0.220 6	0.226 1	0.229 9	0.235 5	0.236 1	0.237 8	0.239 1	0.239 5	0.239 7	0.239 8	0.239 9	0.239 9
2.5	0.031 4	0.061 6	0.089 5	0.114 5	0.136 3	0.154 8	0.170 4	0.183 2	0.193 8	0.202 4	0.215 1	0.223 6	0.229 4	0.233 3	0.236 2	0.240 4	0.242 0	0.243 4	0.243 9	0.244 1	0.244 3	0.244 3	0.244 3
3	0.031 5	0.061 8	0.089 8	0.115 0	0.136 8	0.155 5	0.161 1	0.184 1	0.194 7	0.203 4	0.216 3	0.225 0	0.230 9	0.235 0	0.237 8	0.242 0	0.243 9	0.245 5	0.246 1	0.246 3	0.246 5	0.246 5	0.246 5
4	0.031 6	0.061 9	0.090 1	0.115 3	0.137 2	0.156 0	0.171 7	0.184 7	0.195 4	0.204 2	0.217 2	0.226 0	0.232 0	0.236 0	0.239 1	0.243 4	0.245 5	0.247 3	0.247 9	0.248 2	0.248 4	0.248 4	0.248 4
5	0.031 6	0.062 0	0.090 1	0.115 4	0.137 4	0.156 1	0.171 9	0.184 9	0.195 6	0.204 4	0.217 5	0.226 3	0.232 4	0.236 4	0.239 5	0.243 9	0.246 1	0.247 9	0.248 6	0.248 9	0.249 1	0.249 1	0.249 2
6	0.031 6	0.062 0	0.090 2	0.115 4	0.137 4	0.156 2	0.171 9	0.185 0	0.195 7	0.204 5	0.217 6	0.226 4	0.232 5	0.236 7	0.239 7	0.244 1	0.246 3	0.248 2	0.248 9	0.249 2	0.249 4	0.249 5	0.249 5
8	0.031 6	0.062 0	0.090 2	0.115 4	0.137 4	0.156 2	0.172 0	0.185 0	0.195 7	0.204 6	0.217 7	0.226 5	0.232 6	0.236 8	0.239 8	0.244 3	0.246 5	0.248 4	0.249 1	0.249 4	0.249 6	0.249 7	0.249 8
10	0.031 6	0.062 0	0.090 2	0.115 4	0.137 5	0.156 2	0.172 0	0.185 0	0.195 8	0.204 6	0.217 7	0.226 5	0.232 6	0.236 8	0.239 9	0.244 3	0.246 5	0.248 4	0.249 1	0.249 5	0.249 7	0.249 8	0.249 9
∞	0.031 6	0.062 0	0.090 2	0.115 4	0.137 5	0.156 2	0.172 0	0.185 0	0.195 8	0.204 6	0.217 7	0.226 6	0.232 6	0.236 9	0.239 9	0.244 3	0.246 5	0.248 5	0.249 2	0.249 5	0.249 8	0.249 9	0.250 0

For heavily overconsolidated clays, the shear stresses which bring about immediate settlement also cause changes in pore pressure. This generally has the effect of reducing the consolidation settlement undergone by the structure relative to those that straightforward use of oedometer data would suggest. In practice, consolidation settlement is calculated by dividing the stressed zone of the overconsolidated clay into layers and calculating the average vertical stress change, as before, and then applying the equation:

$$\rho_c = \mu \sum_{x=1}^{x=n} m_v \cdot \Delta\sigma_v \cdot z_n \tag{9.37}$$

where μ is the settlement coefficient proposed by Skempton and Bjerrum,[30] which is related to the pore pressure coefficient A by the equation:

$$\mu = A + a(1 - A) \tag{9.38}$$

where a is a coefficient depending on the geometry of the problem. The results for circular and strip footings, with various ratios of the thickness of the clay, z, to the width of the footing B, are given in Figure 9.12 in terms of the consolidation history.

9.3.4 Settlement of granular soils

Whilst, with clays, it is normally possible to obtain samples of sufficiently good quality to allow meaningful laboratory (oedometer) tests to be carried out, it is not feasible to obtain undisturbed samples of sand and gravels. In such cases, the engineer must rely upon *in situ* tests to obtain parameters from which to judge the compressibility of the ground. The most commonly used tests are penetration and plate loading tests.

At the outset, it is important to realize that neither the standard penetration test (SPT) nor the static cone test can provide an accurate measure of compressibility; both tests cause continuous soil failure as they penetrate the ground and are therefore more likely to give results related to the effective strength parameters and *in situ* stress levels of the soil. Nonetheless, the SPT is in widespread use. Recent evaluations of the accuracy of the numerous methods of estimating settlements from the SPT N value suggest that Schultze and Sherif's[31] method is one of the more accurate of those available.

Schultze and Sherif[31] used statistical methods to obtain a settlement equation from case-history data. The basic information for using the method is shown in Figure 9.13. Settlements are calculated from the equation:

$$\rho = \frac{s \cdot p}{N^{0.87}(1 + 0.4D/B)} \tag{9.39}$$

where the granular soil extends for more than $2B$ below the foundation, p is the applied stress at foundation level, N is the average SPT N value over a depth of $2B$ below the foundation level, or d_s if the depth of granular soil is less than $2B$.

Methods based upon the cone test would be expected, in general, to give slightly more accurate results than those based upon the SPT, because of better standardization of the test and the lack of borehole disturbance. N values can be obtained from cone test data using the relationship:

$$N = q_c/107K \tag{9.40}$$

where K can be obtained from Table 9.8.

While it is clear that the value of K increases with the particle size, comparison between tests on coarse materials becomes somewhat uncertain once the particle size becomes comparable with the diameter of the instruments, i.e. the penetrometer cone

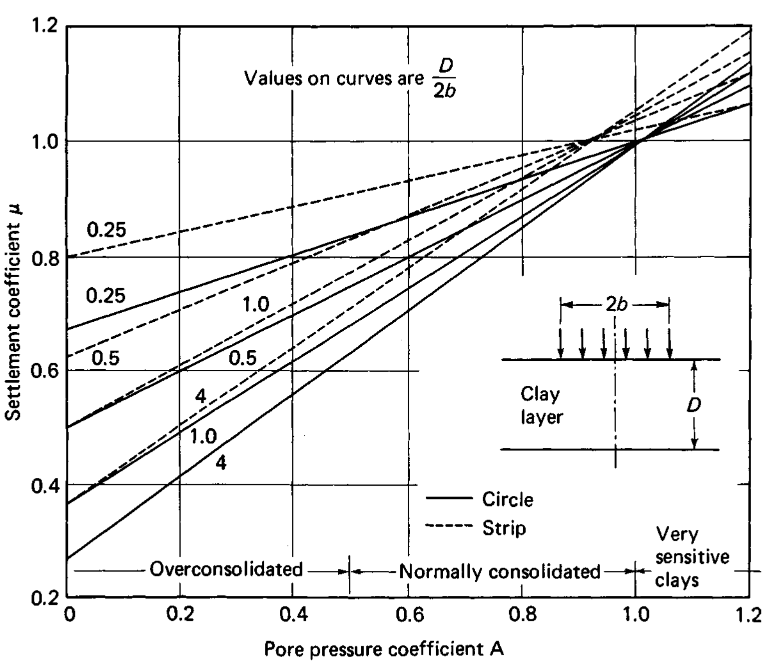
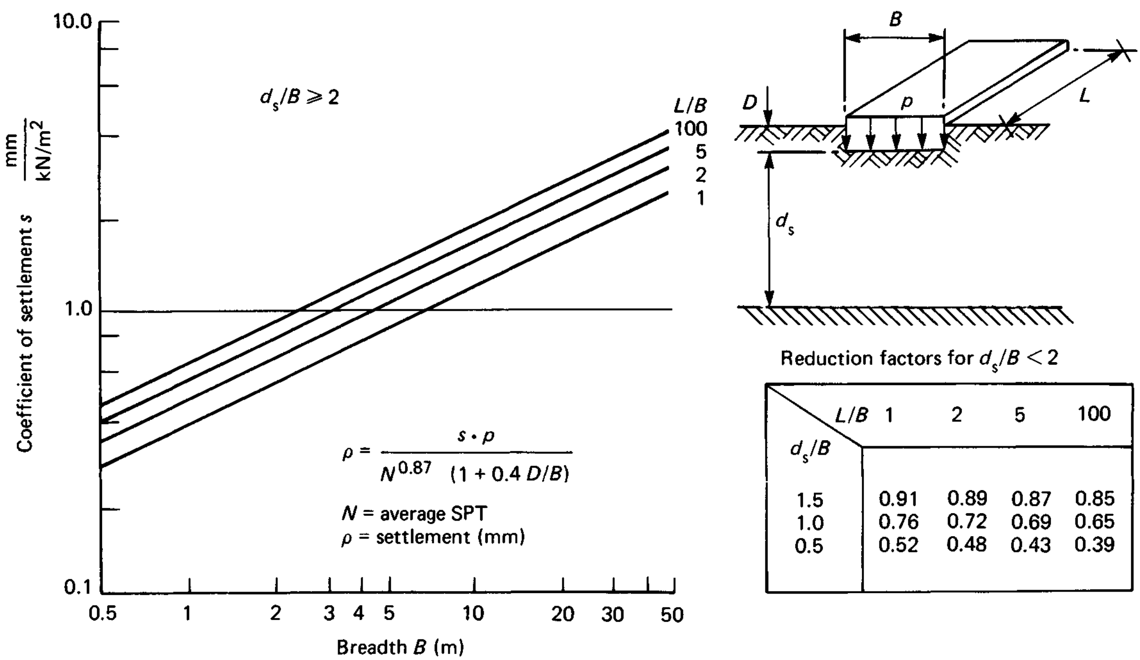


Figure 9.12 Settlement coefficient μ as a function of pore pressure coefficient A. (After Skempton and Bjerrum (1957) 'A contribution to the settlement analysis of foundations on clay', *Géotechnique*, **7**)



d_s/B \ L/B	1	2	5	100
1.5	0.91	0.89	0.87	0.85
1.0	0.76	0.72	0.69	0.65
0.5	0.52	0.48	0.43	0.39

Figure 9.13 Schultze and Sherif's method. (After Schultze and Sherif (1973) 'Prediction of settlement from evaluated settlement observations for sand', *Proceedings, 8th international conference on soil mechanics and foundation engineering*, Vol. I)

Table 9.8 Values of coefficient K in Equation (9.40). (After Simons and Menzies (1977) *A short course in foundation engineering*. Butterworths, London)

Soil description	K
Sandy silt	2.5
Fine sand	4.0
Fine to medium sand	5.0
Sand with sandy gravel	8.0
Medium to coarse sand	8.0
Gravelly sand	8–18
Sandy gravel	12–16

and the SPT spoon. Difficulties also arise on making comparisons at the other end of the scale owing to the tendency of finer sands to 'pipe'; this results in an underestimate of the N value.

A number of methods exist to predict the settlements of foundations directly from cone data. According to Clayton, Simons and Matthews,[10] there is little difference in the accuracy of these methods. In one of the earliest, de Beer[32] has suggested that the compressibility C_s of a granular deposit could be related to the cone resistance q_c and the effective overburden pressure p_o' by the relation:

$$C_s = \frac{1.5 q_c}{p_o'} \tag{9.41}$$

the compression of a layer of thickness H being given by the expression:

$$\rho = \frac{H}{C_s} \log_e \left(\frac{p_o' + \Delta\sigma_v}{p_o'} \right) \tag{9.42}$$

where $\Delta\sigma_v$ is the increase in stress due to the net foundation pressure at the centre of the soil layer.

Schmertmann[33] proposed a different approach to the use of SPTs in the calculation of the settlement of footings on sands. He noted that the distribution of vertical strain under the centre of a footing on a uniform sand is not qualitatively similar to the distribution of the increase in vertical stress, the greatest strain occurring at a depth of about $B/2$.

Schmertmann's equation for calculating settlement is:

$$\rho = C_1 \cdot C_2 \cdot \Delta p \sum_0^{2B} \left(\frac{I_z}{E} \right) \Delta z \tag{9.43}$$

where Δp is the increase in effective vertical pressure at foundation level, Δz is the thickness of layer under consideration, C_1 is the depth embedment factor, and I_z is the strain influence factor given in Figure 9.14.

$$C_1 = 1 - 0.5 \left[\frac{p_o'}{\Delta p} \right] \tag{9.44}$$

where p_o' is the initial effective overburden pressure at foundation level and C_2 is the empirical creep factor,

$$\text{i.e.:} \quad C_2 = 1 + 0.2 \log_{10} \left[\frac{t}{0.1} \right] \tag{9.45}$$

where t is the period in years for which the settlement is to be calculated and E is the deformation modulus:

$$E = 2 \cdot q_c \tag{9.46}$$

9.3.5 Depth corrections

Where foundations are below ground level, and where elastic theory based upon a load applied at ground surface has been used to obtain foundation stress increases for settlement analysis, a correction factor is generally made to allow for the effect of the soil above foundation level. Traditionally the values of Fox[27] have been used; these lead to a maximum possible reduction in settlement (for deep foundations) of 50%. Because the figures are based upon elastic analysis they tend to overestimate the reductions to be applied. The more conservative values of Burland[34] are therefore preferred (Figure 9.15).

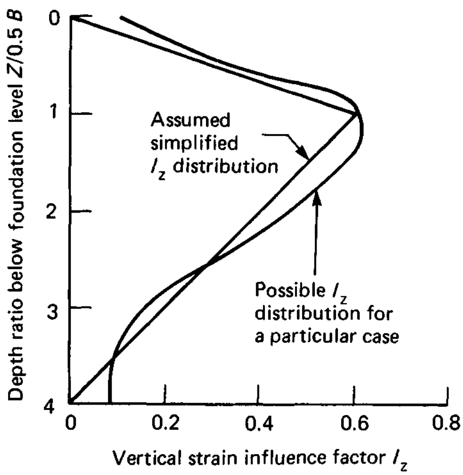


Figure 9.14 Variation of strain influence factor with depth. (After Schmertmann (1970) 'Static cone to compute elastic settlement over sand', *Proc. Am. Soc. Civ. Engrs,* **98**; Simons and Menzies (1977) *A short course in foundation engineering.* Butterworths, London)

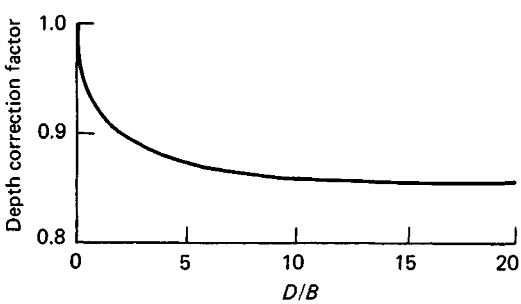


Figure 9.15 Depth correction factor. (After Burland (1970) *Proceedings, conference on* in situ *investigations in soils and rocks.* British Geotechnical Society, London)

9.4 Earth pressure

The pressure on a wall or structure in contact with soil is a complex matter, depending upon: (1) how the wall moves; (2) how the soil–wall system is placed; (3) the strength parameters of the soil; and (4) groundwater conditions.

Classical earth pressure analysis, which dates back to the eighteenth century, considers only the restricted cases of rigid walls which rotate about their base, either into or away from the soil. It is implicitly assumed that the soil is placed before the wall and that the wall can be placed without disruption to the soil. But in practice it is common to compact the soil behind a wall, and for the wall to slide or rotate in a way which is different to that implied by classical analyses. This leads to rather different pressure distributions. For a full discussion of earth pressure and the design of earth-retaining structures the reader is referred to Clayton and Milititsky.[35]

9.4.1 Active and passive conditions

The problem of earth pressure is the oldest soil mechanics problem. Active pressure is the lowest pressure which a retaining structure must be capable of resisting in order to prevent a soil mass from collapsing. The highest pressure which a structure can exert on a bank of earth without causing it to move in the direction of the pressure is the passive pressure or passive resistance. Examples of both are given in Figure 9.16.

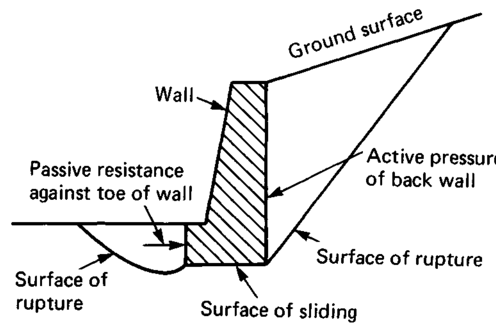


Figure 9.16 Active pressure and passive resistance

Below a level ground surface, the horizontal pressure is known as the 'pressure at rest'. This pressure lies between the active and passive pressures and is usually designated by the factor K_o which is the ratio of the horizontal and vertical effective pressures at any given depth. The factors K_a and K_p similarly relate the active and passive horizontal effective pressures to the vertical effective pressure.

The value of K_o depends on the depositional conditions and stress history of the ground. For loose sands the value of K_o is about 0.45, falling to about 0.35 in dense sands, following the relationship given by Jaky, $K_o = (1 - \sin\phi')$. A wider range is encountered in clays, typically 0.4 to 0.7, but in some heavily overconsolidated clays values considerably in excess of unity, and perhaps as large as 3, occur.

Tables 9.9 and 9.10 give typical values for K_a and K_p for cohesionless materials, vertical walls with horizontal ground where ϕ' is the angle of friction for the soil and δ' the angle of wall friction.

Experimental work by Rowe and Peaker[36] has emphasized the dominant role of the angle of wall friction, but has shown that the code values of K_p can be as much as 50% too high in loose and dense sands.

In order that the lateral pressure may change from the pressure at rest to either the active or passive value, movement must take place to mobilize shear forces. This generally occurs during the construction of the retaining structure.

Table 9.9 Values of K_a from CP2. (After Institution of Structural Engineers (1951) *Earth retaining structures.* ISE CP2, ISE, London)

Values of δ'	*Values of ϕ'*				
	25°	30°	35°	40°	45°
	Values of K_a				
0°	0.41	0.33	0.27	0.22	0.17
10°	0.37	0.31	0.25	0.20	0.16
20°	0.34	0.28	0.23	0.19	0.15
30°	—	0.26	0.21	0.17	0.14

Table 9.10 Values of K_p from CP2. (After Institution of Structural Engineers (1951) *Earth retaining structures.* ISE CP2, ISE, London)

Values of δ	Values of ϕ			
	25°	30°	35°	40°
	Values of K_p			
0°	2.5	3.0	3.7	4.6
10°	3.1	4.0	4.8	6.5
20°	3.7	4.9	6.0	8.8
30°	—	5.8	7.3	11.4

9.4.2 Active pressure

For an ideal material, the problem of determining the total active pressure is comparatively simple and is based on the wedge theory which was originally due to Coulomb[37] (1776) who solved it for a material having both friction and cohesion. Later workers omitted the cohesion, changed the coefficient of friction into the tangent of the angle of internal friction, which was taken as equal to the angle of repose, and extended the expression to include sloping walls, wall friction (anticipated in part by Coulomb), and inclined surcharges. Not all of these changes were improvements.

In the wedge theory it is assumed that the pressure on the wall is due to a wedge of earth which tends to slip down an inclined plane as shown in Figure 9.17. The forces acting on the wedge are also shown in the figure. The inclination of the plane BD is altered until the position which gives the greatest value for the force is found. This can either be done analytically or graphically.

The general formula for the total force over depth H exerted by a frictional material having no cohesion is:

$$P_a = \tfrac{1}{2}\gamma H^2 \left(\frac{K_a}{\sin a \cos \delta'}\right) \tag{9.47}$$

where the coefficient of active earth pressure K_a is:

$$K_a = \frac{\sin^2(a+\phi')\cos\delta'}{\sin a \cdot \sin(a-\delta)\left\{1+\left[\dfrac{\sin(\phi'+\delta')\sin(\phi'-\beta)}{\sin(a-\delta')\sin(a+\beta)}\right]^{1/2}\right\}^2} \tag{9.48}$$

where ϕ' is the effective angle of friction, δ' the angle of wall friction and a, β and H are as shown in Figure 9.17.

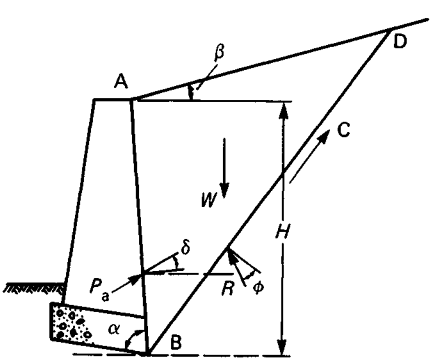


Figure 9.17 Coulomb or general wedge theory

For the case of a vertical wall and horizontal backfill, the values of K_a to be used in Equation (9.47) are as in Table 9.8. For the special case of no wall friction, vertical wall and horizontal backfill, this reduces to the Rankine formula:

$$P_a = \tfrac{1}{2}\gamma H^2(1-\sin\phi')/(1+\sin\phi') = \tfrac{1}{2}\gamma H^2 K_a \tag{9.49}$$

No analytical solution exists for the general case of a soil having both friction and cohesion, but for the special case of a vertical wall, horizontal backfill and no wall friction or adhesion, the total pressure is given by:

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a} \tag{9.50}$$

For a purely cohesive material, Equation (9.50) reduces to $P_a = \tfrac{1}{2}\gamma H^2 - 2cH$ for the case of no wall adhesion, or if wall adhesion is taken as equal to the cohesion the formula becomes:

$$P_a = \tfrac{1}{2}\gamma H^2 - 2(\sqrt{2})cH \tag{9.51}$$

The equation for the intensity of horizontal total stress at any level is:

$$p_a = K_a(\sigma_v - u) + u \tag{9.52}$$

Since earth retaining structures generally are involved with unloading of the soil, it is normal to use effective strength parameters c' and ϕ' to determine K_a. The use of total stress analysis, even in the case of temporary works design, should be avoided. It is better to use an effective stress analysis with a relatively low factor of safety.

When using the earth pressure coefficient approach, the procedure is as follows:

(1) Establish a design profile from existing borehole records or available information.
(2) Determine ground levels, and levels of each soil layer.
(3) Determine the assumed position of the groundwater table (this may be different on either side of the wall); with good drainage, or a low groundwater table, there may be no influence from groundwater.
(4) Calculate the vertical total stress at ground level (which may not be zero, e.g. when there is a uniform surcharge), at groundwater level, and at the boundary of each soil type:

$$\sigma_v = \sum_0^n \gamma_n \cdot z_n + p_s \tag{9.53}$$

where γ_n is the bulk weight of the soil in soil layer n, z_n is the thickness of soil layer n and p_s is the vertical uniform surcharge at ground level.
(5) Calculate the pore water pressure at each boundary. It is generally assumed that:

$$u = \gamma_w \cdot z \tag{9.54}$$

where γ_w is the bulk unit weight of water (which can be taken as 10 kN/m³) and z is the depth below groundwater level which implies no flow of water in the vicinity of the structure.
(6) Calculate the vertical effective stress at each level:

$$\sigma_v' = \sigma_v - u \tag{9.55}$$

where σ_v' is the vertical effective stress, σ_v is the vertical total stress (see (4) above) and u is the pore water pressure (see (5) above).
(7) Calculate the horizontal effective stress *for each soil type*,

and each level. Where two soil types meet, the earth pressure coefficient will change. Using the value of vertical effective stress on the boundary, calculate two values of horizontal effective stress, and for each soil type:

$$\sigma'_{h_n} = K_n \cdot \sigma'_v \tag{9.56}$$

$$\sigma'_{h_{n+1}} = K_{n+1} \cdot \sigma'_v \tag{9.57}$$

where σ'_{hn} is the horizontal stress in layer n, and $\sigma'_{h_{n+1}}$ is the horizontal stress level in layer n+1. K_n and K_{n+1} are the relevant earth pressure coefficients for layers n and n+1.

For passive pressures, the earth pressure coefficients are normally factored-down, typically by 2.

(8) Calculate the total horizontal stress at each level, for each soil type:

$$\sigma_h = \sigma'_h + u \tag{9.58}$$

where σ_h is the horizontal total stress to be supported by the wall, σ'_h is the horizontal effective stress (see (7) above) and u is the pore water pressure (see (5) above).

9.4.2.1 *Graphical method*

A simple graphical solution can be used for walls with irregular outlines, for irregular backfills, external loads on the backfill, and variation in the properties of the backfill. The principle is that of the wedge theory, the most dangerous surface of slip being found by trial.

The polygons of forces are drawn for a number of wedges ABD_1, ABD_2, etc. as shown in Figure 9.18. The forces acting on each wedge are its weight W, the reaction R across the plane BD, the cohesion C acting up the plane BD, and the force P which it is required to find. For limiting equilibrium, R is inclined at ϕ' to the normal to the plane BD, and $C = c \times \text{BD}$. If the direction of P is assumed, the polygon of forces can be completed, giving the value of P. It is convenient to plot all the polygons from a common origin of W as shown. The maximum value of P is then given by the point at which the envelope of the R lines cuts the line representing E. This P line can be drawn from the origin in any assumed direction, thus allowing for wall friction. Wall adhesion (cohesion) can be included by subtracting it from the weight of the wedge.

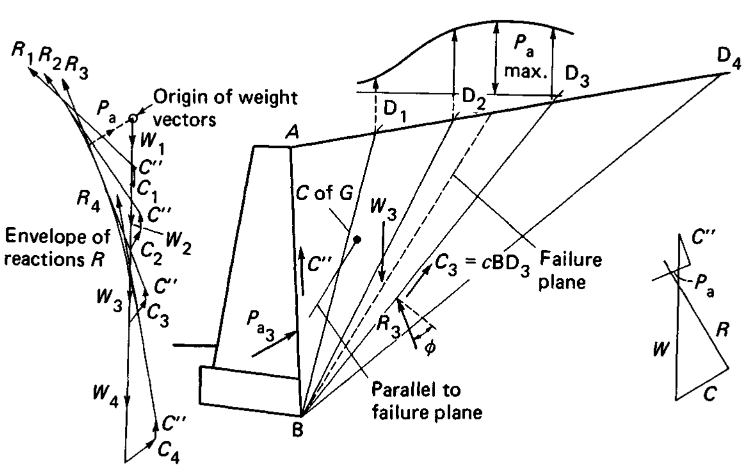


Figure 9.18 Engesser's method for determination of active pressure

Alternatively, the force polygons can be drawn separately for each wedge and the value of P_a can be plotted above the position of the corresponding slip surface.

9.4.2.2 *Surcharge loads*

The effect of a distributed surcharge of magnitude p_s is to increase the earth pressure over the whole height of the wall by an amount $K_a p_s$ in the case of granular backfills and by an amount p_s in the case of cohesive backfills. The effect of a line load W_l can be estimated with sufficient accuracy for most designs by the construction shown in Figure 9.19.

The effect of a point load W_i on the backfill is more difficult to estimate. An approximate method given in CP2[38] is to assume that the load is spread through the backing at an angle of dispersion of 45° on each side of the load. The lateral pressure at any point due to the surcharge is then taken as K_a times the vertical pressure at the point. This method, however, tends to give results on the unsafe side. The following tentative approximate method is suggested. The line of action of the resultant force is obtained by a construction similar to that for a line load (Figure 9.19), the 40° line being constructed from the centre of the loaded area. It is assumed that, if the length of the loaded area be L and the distance between the back of the wall and the near edge of the area be x, the resultant lateral thrust will be distributed along a length of wall equal to $L+x$. Then if W_i be the load on the area, the resultant thrust per unit length of wall will be $K_a \cdot W_i(L+x)$.

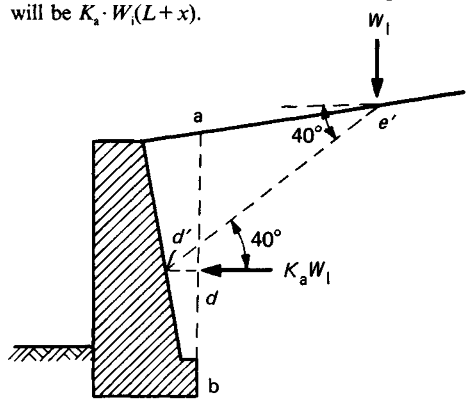


Figure 9.19 Method of estimating magnitude and line of action of pressure due to a line load

9.4.3 Passive resistance

In the case of active pressure the assumption of a plane surface of failure gives results which are within 3 or 4% of the true value, and this is close enough for all practical purposes. With passive pressure, however, this may lead to results which differ significantly from the true values if wall friction is taken into account. Only for the case of no wall friction does the wedge theory give the true value. The passive pressure in this case is given by the formula:

$$P_p = \tfrac{1}{2}\gamma H^2 K_p + 2c' H\sqrt{K_p} \qquad (9.59)$$

Wall friction adds greatly to the passive resistance, but not so much as the wedge theory indicates; it is seldom, therefore, that it can be neglected. The wedge theory does not give the correct answer because the surface of failure is not a plane but is curved, as shown in Figure 9.17.

The effect of curved failure surfaces can be analysed using the log-spiral method, a graphical procedure which is given in detail by Terzaghi and Peck,[4] and which forms one basis for the computation of bearing capacity.

For the simple case of horizontal ground and vertical wall, in cohesionless soil, Table 9.3 gives values of the passive earth pressure coefficient P_p for use in the equation:

$$P_p = \tfrac{1}{2}\gamma H^2 K_p \sec \delta' \qquad (9.60)$$

For earth pressure coefficients for more complex geometrics and soil conditions, the reader is referred to Clayton and Milititsky.[35]

9.4.4 Distribution of pressure

It is generally assumed that pressure increases uniformly with depth. This is only true in certain special cases, although the assumption is not unreasonable in some other cases for which it is not strictly true. Cases in which the assumption should not be made are dealt with below.

The wedge theory gives the total force; the distribution of pressure cannot be obtained from this theory as it depends on the lines of action of the forces involved and on the way in which the wall yields and will only be triangular if the wall yields by turning about its base; if the wall is not founded on rock and is very rigid in itself this is usually the way in which it will yield. It is for this reason that the assumption of triangular distribution can often be made in practice. The centre of pressure on the wall using the wedge method can be estimated as shown in Figure 9.18.

In the case of cohesive soils, calculations indicate a zone of tension at the top of the wall. In this region there will be no pressure on the wall, and in practice deep tension cracks are often observed behind walls supporting cohesive soils. The value of this tension must not be subtracted from the pressure diagram. If it is possible for the tension cracks to become filled with water, the value of the water pressure must be included in the pressure calculations.

9.4.5 Strutted excavations

For excavations below groundwater level, a sheeted excavation is often used in preference to an open excavation with battered sides, particularly where space is limited and where the piles can be driven into a relatively impervious stratum to provide a cutoff against groundwater in overlying pervious strata. Such sheeting is usually braced by frames consisting of walings and struts. Calculations of the earth pressures on the sheeting follow the same lines as for retaining walls. However, during progressive excavation and placing of frames, deflections of the sheeting occur which lead to frame loads which are not in agreement with those calculated from the earth pressure diagram, assuming hinge points at all frame levels below the top frame. The load in the struts does not increase in general with depth below about one-quarter of the depth of the excavation.

For sands, Terzaghi has proposed an empirical design rule, based on a number of field observations, as shown in Figure 9.20(b); and the same rule, with a modification for use in clay, has been adopted in CP2.[38] Peck[39] and Tomlinson[40] have discussed this question and that of adjacent associated movements in some detail. Typical pressure distributions are given in Figure 9.20(b) and (c).

It should be emphasized that the redistributed pressure diagrams described above are, in effect, design devices for obtaining frame loads and do not necessarily imply any actual redistribution of earth pressure. In fact, Skempton and Ward[41] have described the results of strut and waling load measurements in a cofferdam at Shellhaven and have interpreted results to show that the frame loads can be accounted for in terms of deflections of the sheet prior to placing the struts in position and without the need for assuming any redistribution of earth pressure.

The two useful rules in determining levels at which frames should be placed have been given by Ward[42] as follows:

(1) In a deep deposit of normally consolidated clay the uppermost frame of struts should be placed across a cofferdam before the depth of excavation H_1 reaches a value given by $H_1 = 2c_u/\gamma$.
(2) The second frame of struts should be placed before the depth of excavation reaches a depth H_2 given by $H_2 = 4c_u/\gamma$.

Recent practice, particularly for larger excavations, is to avoid internal strutting by the use of ground anchors, placed at suitable levels as excavation proceeds, and which with advantage can be prestressed to stipulated loads. Reference should be made to articles by Littlejohn[43] for information on anchor design and construction.

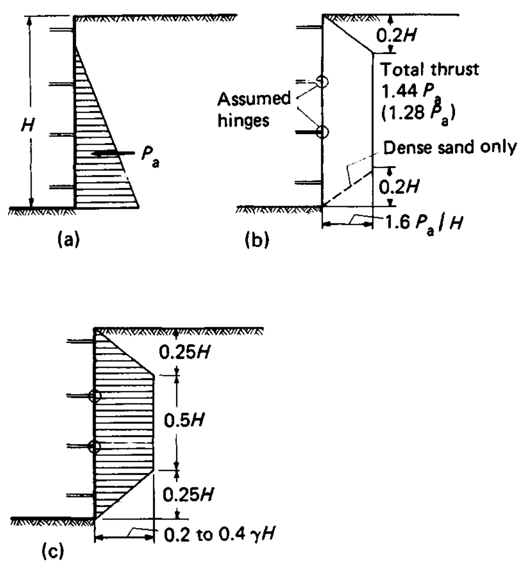


Figure 9.20 Earth pressure in strutted excavations. (a) Calculated earth pressure; (b) earth pressure distribution by CP2, based on Terzaghi's method; (c) stiff fissured clay. (After Terzaghi and Peck (1967) *Soil mechanics in engineering practice*. Wiley, New York)

9.4.6 Anchored bulkheads

An anchored bulkhead is usually in the form of a steel sheet-pile wall supported by ties at one level only and by passive pressure against the toe. However, anchored bulkheads may also be constructed with timber, precast reinforced concrete sheet piles, or continuous-bored piles. Calculations of active and passive earth pressures follow the same lines as for retaining walls but analysis of the stability of an anchored bulkhead requires the determination of bending moments in the piling and of the magnitude of the anchor pull.

The magnitude of maximum bending moment occurring in the bulkhead will be influenced by the relative rigidity of the soil and the section of the bulkhead.

The various dimensions and forces entering into a bulkhead calculation are indicated in Figure 9.21. A design procedure based on Rowe's method, and similar to that described by Terzaghi,[44] is given in sections 9.4.6.1 to 9.4.6.4.

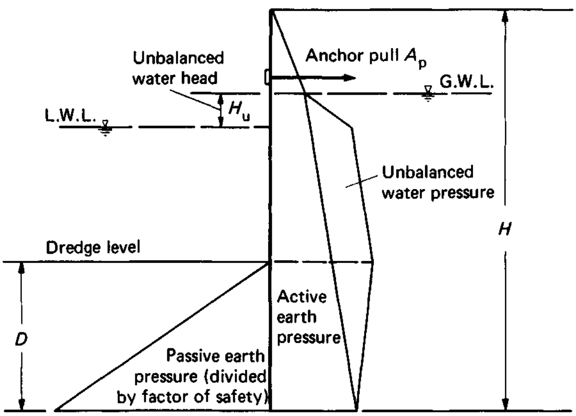


Figure 9.21 Dimensions and forces in anchored bulkhead calculation. Earth pressure diagrams illustrated are for homogeneous cohesionless soil. No pressures from surcharge loads have been shown

9.4.6.1 Forces acting on faces of bulkhead

The active and passive pressures are first calculated. The active pressure calculations must allow for the maximum possible unbalanced water pressure and for any surcharge in the form of distributed, line or point loads supported directly on the backfill.

Strictly speaking, the water pressures should be calculated from a flow net (Figure 9.22(a)) taking into account the effects of stratification in the soils present. However, if the soils do not vary widely in their permeabilities it is sufficient to use the simplified pressure distribution shown in Figure 9.22(b). Allowance must also be made, where the passive pressure is provided by a permeable stratum, for a reduction in passive pressure due to seepage gradients (Figure 9.22(c)).

9.4.6.2 Computation of depth of penetration (free-end method)

A diagram of active and passive pressures is drawn as shown in Figure 9.21 for a trial penetration of the piling. The effects of any surcharge loads should also be included, as described on page 9/19. Before passive pressures are plotted, a factor of safety F_p is applied. The choice of a value for this factor of safety for a given design depends on the accuracy of the data on which the earth pressure has been based, but in general it should not be less than 2.

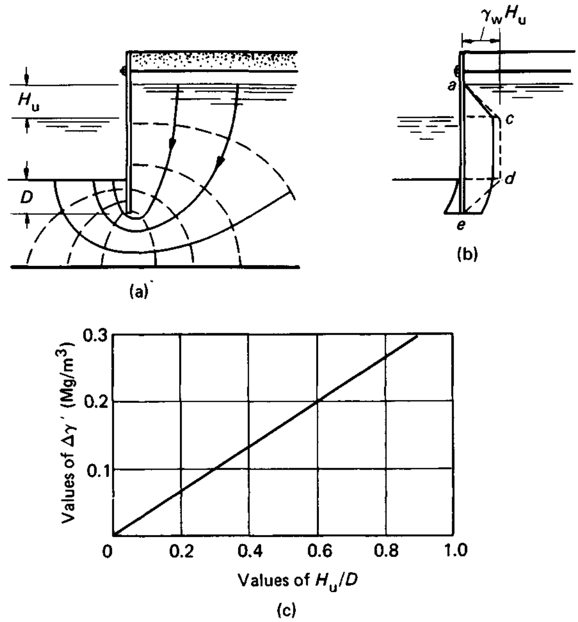


Figure 9.22 Unbalanced water pressure. (a) Flow net; (b) distribution of unbalanced water pressure; (c) average reduction of effective unit weight of passive wedge due to seepage pressure exerted by the upward flow of water. (After Terzaghi (1953) 'Anchored bulkheads', *Proc. Am. Soc. Civ. Engrs,* **79**)

The earth pressure diagram is then divided up into a number of convenient areas and the total load on each of these and its point of application is estimated. Moments of these loads are then taken about the line of the anchor pull. This is repeated for other trial depths of penetration until a depth giving zero total moment is obtained. Alternatively, the pressures below dredge level may be expressed in terms of the penetration D and the required depth found analytically. It is usual to increase the calculated depth of penetration by 20% to allow for the possibility of scour or overdredging.

9.4.6.3 Anchor pull

The anchor pull is determined by resolving horizontally all the forces acting on the bulkhead. There are, however, a number of factors which may lead to an anchor pull somewhat greater than that computed, and conservative design stresses should therefore be adopted.

Where the ties are taken back to blocks or beams the position of these should be such that no overlapping of active and passive zones occurs, as illustrated in Figure 9.23.

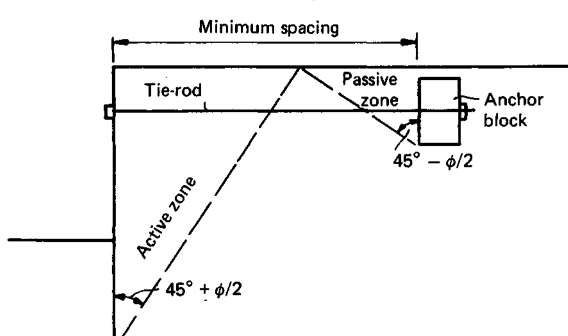


Figure 9.23 Minimum length of anchor ties

9.4.6.4 Computation of maximum bending moments

The bending moments are first calculated on the assumption of 'free earth support', and the maximum bending moment is determined. This is a straightforward calculation based on the resultant forces of the areas into which the pressure diagram has been divided, using either analytical or graphical methods.

The normal procedure is to construct a shear force diagram for the bulkhead, starting from the top and proceeding downwards only as far as is necessary to find the point of zero shear force. The maximum free earth support bending moment is then calculated at this point.

If the sheet piles are to be driven into fairly homogeneous stratum of clean sand with a known relative density, the calculated maximum bending moment for free earth support can be reduced on the basis of Rowe's investigations, as illustrated in Figure 9.24.

As a first step, the flexibility number is calculated for a trial section of bulkhead. For the calculated value of the flexibility number, a value of the moment reduction ratio M/M_{max} can be read off, depending on the relative density and, hence, the reduced moment can be obtained and compared with the moment of resistance of the piling. The trial is repeated until the most suitable section of the bulkhead is obtained, i.e. until the reduced moment is equal to, or just less than, the moment of resistance of the section. If required, the calculation can be extended to cover alternative construction materials.

If the sheet piles are to be driven into a homogeneous stratum of dense or medium-dense silty sand, the moment reduction curves for medium and loose sand should be used instead of those for dense and medium-dense sand. Sheet piles to be driven into loose silty sand should be calculated on the figure for free earth support, since compressibility of such sands may be high. Work by Rowe[45] has shown that in some cases moment reduction can be made where piles penetrate clay below dredge level, but this is rarely done in practice.

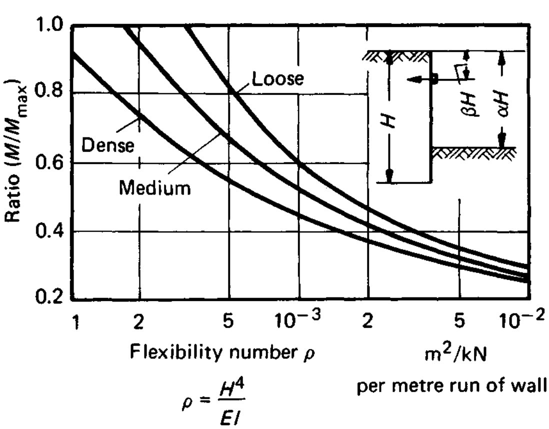


Figure 9.24 Relation between the flexibility number ρ of sheet piles and bending moment ratio M/M_{max} (logarithmic scale). (After Terzaghi (1953) 'Anchored bulkheads', *Proc. Am. Soc. Civ. Engrs,* **79**)

9.4.7 Overall stability

The design of a retaining wall, whether a mass wall or a sheet-pile wall, should always be considered from the point of view of overall stability, i.e. failure as a bank of earth. The forces are shown in Figure 9.25.

$$\text{Disturbing moment} = \Sigma\, TR + Pr \tag{9.61}$$

$$\text{Resisting moment} = (\Sigma\, N \tan \phi' + c'L)R \tag{9.62}$$

where L is the length of the arc over which c' acts and T and N are the tangential and normal components of W.

$$\text{Factor of safety} = \frac{(\Sigma\, N \tan \phi' + \Sigma\, c'L)R}{\Sigma\, TR + Pr} \tag{9.63}$$

The methods described in section 9.5 are directly applicable.

9.5 The stability of slopes

The analysis of slopes is important because of the dangers to both structures and life that can be caused by two types of problem:

(1) Where construction or excavation causes stress changes in the soil which lead to failure in previously stable ground (the so-called 'first-time slide').
(2) Where construction or excavation reactivates movement on a pre-existing shear surface in the soil, usually part of an ancient and pre-existing landslide.

As with other areas of soil mechanics, an important distinction is made between short- and long-term conditions. Short-term conditions are the most critical when load increases are applied to the soil, but they may also be relevant for temporary works when load decrease takes place on clays, because in this situation the depressed pore pressures which are induced by excavation, for example, may not have time to dissipate. For short-term conditions the $\phi_u = 0$ analysis is applied, using the undrained shear strength C_u. In fissured clays, it is observed from back-analysed failures that the mobilized undrained shear strength at failure is considerably less than that measured from small-scale tests. Typically, the mobilized undrained shear strength is only 45 to 60% of that measured on samples of the order of 40 to 50 mm diameter, because these samples do not contain representative fissures (which are planes of weakness).

There is little available evidence of the rate at which negative pore pressures due to unloading decay, and therefore for how long the $\phi_u = 0$ analysis can reasonably be applied. As they do so, however, the soil swells and its strength decreases. In the long term, when pore pressures have equalized, peak effective strength parameters c' and ϕ' are used, provided the soil has not previously failed. If, on the other hand, it is known that a pre-existing shear surface exists (due to previous slope instability on the site) the residual strength parameters (c'_r and ϕ'_r must be used. These are normally derived either from stress reversal shear box testing or from ring shear testing.

If analysis is to be carried out in terms of effective stress, then the equilibrium pore pressures in the slope must be estimated. This is a complex matter, depending on climatic conditions, local groundwater conditions before construction, subsoil geometry and properties, and the effective stress dependency of soil permeability. In practice, in the UK, a value of r_u is normally selected on the basis of experience with monitored and back-analysed slopes. In the brown London Clay, a long-term value of $r_u = 0.3$ is sometimes used.

Stability analyses assume that shear strength is mobilized evenly over the slip surface, but in reality it is thought that progressive failure occurs. It is also known that while stability analysis is capable of giving a reasonable guide in terms of the factor of safety it yields to the stability of a slope, instability can occur at failure on a different surface to that predicted by analysis. A very useful review of European experience of slope stability problems and parameter selection is given by Chandler.[46]

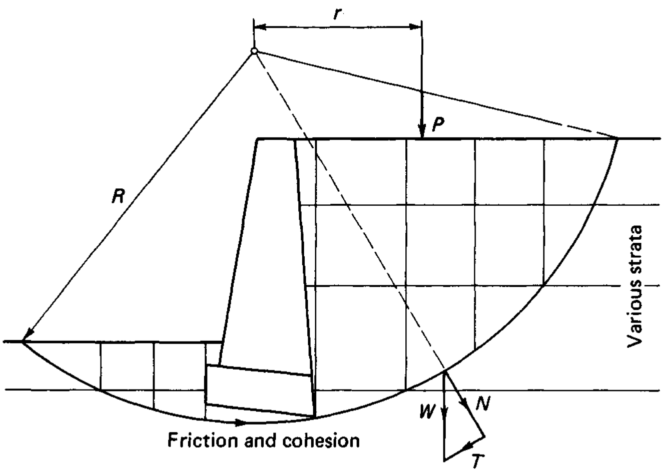


Figure 9.25 Overall stability of a retaining wall

9.5.1 Stability analysis

Slope stability problems in engineering works are usually analysed using limit equilibrium methods. Many such methods are available in practice and the most common ones call on the principle of slices. In this method, the failure mass is broken up into a series of vertical slices and the equilibrium of each of these slices is considered. This procedure allows both complex geometry and the variable soil and pore pressure conditions of a given problem to be considered.

The methods most commonly used are:

(1) The method of slices of Fellenius.[47]
(2) The modified, or simplified, method described by Bishop.[48]
(3) Janbu's generalized method of slices.[49]
(4) Morgenstern's and Price's method.[50]
(5) Spencer's method.[51]

The first two methods do not satisfy all the moment and force equilibrium equations and can only accommodate circular slip surfaces. The last three methods satisfy all equilibrium equations (although simplifying assumptions are required) and may be used to calculate the factor of safety along any shape of slip surfaces. Those methods which satisfy all the conditions of equilibrium, and Bishop's modified method, give accurate results which do not differ by more than 5% from the 'correct' answer, obtained by the log-spiral method.

The main conclusions of such studies can be found in La Rochelle and Marsal,[52] who prefer the simplified methods of Bishop and Janbu due to the simplicity of computer programming and the low cost of running such programs.

In practice, the design engineer must choose between circular and noncircular methods of analysis. Because of their relative simplicity, it is common to carry out routine slope-stability analyses using a circular failure surface. This assumption will be justified in relatively homogeneous soil conditions, since experience shows that the analysis can make good estimates of the factor of safety when failure is imminent. Noncircular analyses are used when: (1) a pre-existing surface has been found in the ground, and is known to be noncircular; and (2) circular failure is prevented, perhaps by the presence of a stronger layer of soil at a shallow depth. Under either of these conditions, the use of a circular shear surface will overestimate the factor of safety against failure.

9.5.1.1 Method of slices

In order to cater both for complex soil conditions and variable geometry it is common to divide the slipping mass of soil into slices (Figure 9.26). The factor of safety is normally defined in terms of the ratio between the average shear strength available on the shear surface and the average shear strength mobilized for stability, i.e.:

$$F=\frac{\bar{T}_{\text{available}}}{\bar{T}_{\text{mobilized}}} \tag{9.64}$$

where $F=1$ at failure.

In order to obtain a solution to the problem, slope stability analyses examine the equilibrium of the mass of soil which is being considered. A number of different solutions, therefore, can be derived, depending on the approach taken in the analysis. Methods used to derive the basic equations are:

(1) Force equilibrium of a single slice.
(2) Moment equilibrium of a single slice.
(3) Force equilibrium of the total mass of soil above the slip surface.
(4) Moment equilibrium of the total mass of soil above the slip surface.

For example, Bishop's method combines (1) and (4), Janbu uses (1) and (3), and Morgenstern and Price use a combination of (1), (2) and (3). A solution cannot be obtained for the complete stability of the slipped mass without making simplifying assumptions, and therefore it is common to make assumptions concerning the interslice forces E and X (Figure 9.26), for example, in deriving a factor of safety for a circular failure surface. From the definitions of factor of safety:

$$\text{Available shear strength}=\frac{\text{peak shear strength}}{\text{factor of safety}} \tag{9.65}$$

For moment equilibrium about the centre of the circle:

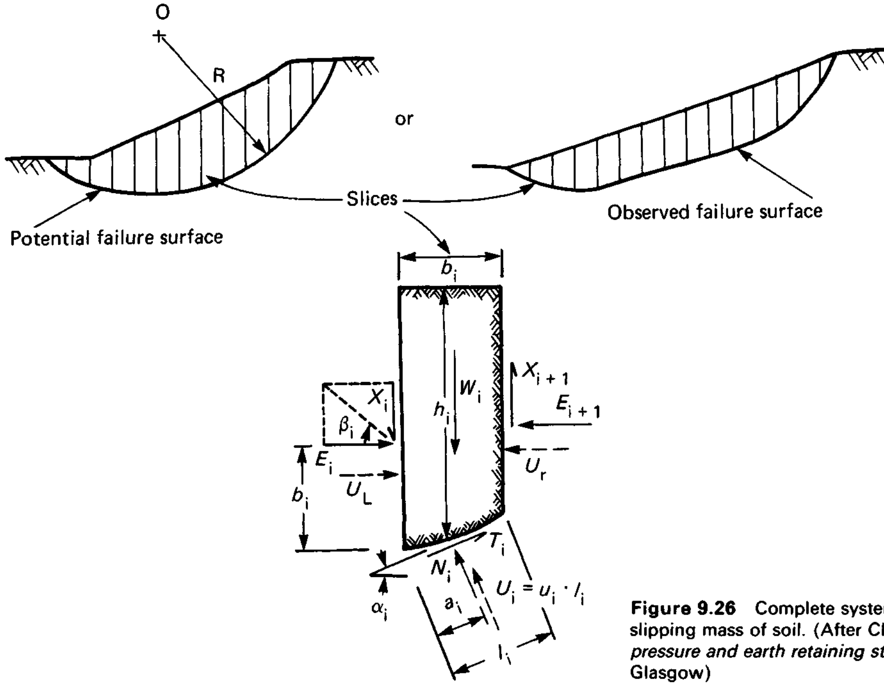


Figure 9.26 Complete system of forces on a single slice of a slipping mass of soil. (After Clayton and Milititsky (1986) *Earth pressure and earth retaining structures.* Surrey University Press, Glasgow)

(moment of available shear strength on shear surface) =

(moment of weight of soil mass)

i.e. disturbing moments must equal restoring moments, at failure.

A summary of the most significant features of each method recommended is presented below. Slope stability analyses for earth retaining structures will normally be carried out in terms of effective stress using estimates of long-term, stabilized pore water pressure, since this will give the lowest factor of safety for an unloading case, e.g. excavation. Where load increase takes place on clays, e.g. for fill placed behind a sheet-pile wall, short- or intermediate-term stability analysis may give the lowest factor of safety.

9.5.1.2 Short-term stability analysis

Short-term, total, stress analysis is most conveniently carried out by the method of slices, since this method can take into account irregular ground surface profiles, changes in soil type, and variations in undrained shear strength around the slip surface. Partial submergence of slopes can also be analysed.

Interslice forces E_i, E_{i+1}, X_i, X_{i+1}, are ignored, and the strength at the base of each slice T_i is independent of the effective normal force N_i and is equal to the undrained shear strength of the soil, divided by the factor of safety.

For equilibrium:

$$R\,\Sigma\, W_i \cdot \sin a_i = \frac{R}{F}\Sigma\, C_{u_i}\frac{b_i}{\cos a_i} \tag{9.66}$$

Therefore:

$$F = \frac{\Sigma\, C_{u_i} \cdot b_i/\cos a_i}{\Sigma\, W_i \cdot \sin a_i} \tag{9.67}$$

The solution is obtained by dividing the slipping mass of soil into slices. For hand calculation, the use of 10 to 15 slices is recommended. Calculations are most easily handled in table form as suggested in Figure 9.27.

A search must be made for the slip surface which will yield the lowest factor of safety.

9.5.1.3 Long-term stability analysis: circular slips

Fellenius's method[47] Like the short-term stability method described above, Fellenius's method for long-term analysis uses moment equilibrium of the slipped mass, and ignores interslice forces.

Considering the equilibrium of the *i*th slice, and resolving perpendicular to the slip surface,

$$W_i \cos a_i = N'_i + u_i \cdot l_i \tag{9.68}$$

Therefore:

$$N' = W_i \cos a_i - u_i \cdot l_i \tag{9.69}$$

By definition, $T_i = 1/F(c'l_i + N'_i \cdot \tan\phi')$ in terms of peak effective strength parameters and the weight of the slice.

$$W_i = \gamma \cdot h_i \cdot b_i \tag{9.70}$$

Summing the moments for all slices:

$$\Sigma_{\text{disturbing moments}} = \Sigma\, W_i \cdot R \cdot \sin a_i = R\,\Sigma\, W_i \sin a_i \tag{9.71}$$

$$\Sigma_{\text{restoring moments}} = \Sigma\, T_i R = \frac{R}{F}\Sigma\, c'l_i + N'_i \tan\phi')$$

$$= \frac{R}{F}\Sigma\,(c'l_i + (W_i \cos a_i - u_i l_i)\tan\phi') \tag{9.72}$$

Slice no.	b (m)	α (deg.)	h (m)	C_u (kN/m^2)	γ_b (kN/m^3)	W	$W \sin \alpha$	$C_u.b/\cos \alpha$
							Σ	Σ

Figure 9.27 Format for calculating slipping slices of soil

For equilibrium of the entire slipped mass:

$$R\Sigma W_i \sin a_i = \frac{R}{F}\Sigma (c'l_i + (W_i \cos a_i - u_i \cdot l_i) \tan \phi') \tag{9.73}$$

Therefore:

$$F = \frac{\Sigma (c'l_i + (W_i \cdot \cos a_i - u_i l_i) \tan \phi'}{\Sigma W_i \cdot \sin a_i} \tag{9.74}$$

As before, hand calculations are best carried out in tabular form.

A search must be made for the slip surface which gives the lowest factor of safety. Because of the simplifying assumption that the interslice forces are zero, or at least are in equilibrium for each slice, the Fellenius solution underestimates the factor of safety by between 5 and 20%. Errors are greatest when the variation of a is large, i.e. for deep circles. Results are conservative, but this may lead to uneconomical design. This method is not widely used in the UK, that by Bishop (see below) being preferred.

Bishop's method[48] As with Fellenius's method, Bishop's method uses force equilibrium of each slice and moment equilibrium of the entire slipped mass to derive an equation for the factor of safety. Unlike Fellenius's solution, however, the method does not entirely ignore interslice forces. The result is that the method is considerably more accurate than that of Fellenius.

As before, the factor of safety is defined in terms of the mobilized shear strength, i.e.:

$$T_i = \frac{1}{F}(c'l_i + N_i' \tan \phi_i') \tag{9.75}$$

For force equilibrium within each slice, resolving vertically, and for moment equilibrium of the entire slip mass about zero:

$$F = \frac{1}{\Sigma W_i \sin a_i} \Sigma \frac{1}{m_{a_i}} [c'l_i \cos a_i + (W_i - u_i l_i \cos a_i + (X_i - X_{i+1})) \tan \phi_i'] \tag{9.76}$$

where

$$m_a = \frac{1}{\sec a_i}\left[\frac{1 + \tan \phi' \tan a_i}{F}\right] \tag{9.77}$$

The Bishop simplified solution assumes that $(X_i + X_{i+1}) = 0$ and thus:

$$F = \frac{1}{\Sigma W_i \sin a_i} \Sigma \frac{1}{m_{a_i}} [c'_i l_i \cos a_i + (W_i - u_i l_i \cos a_i) \tan \phi_i'] \tag{9.78}$$

Improvements on the accuracy achieved by Fellenius's solution derive from the fact that no assumptions are made with regard to the normal interslice forces E_i and E_{i+1}. Unfortunately, since m_a is a function of the factor of safety F (see Equation (9.77)), Equation (9.78) must be solved by assuming a trial factor of safety, to input into the right-hand side of the equation, in order to evaluate the left-hand side of the equation. By iteration, the condition $F_{\text{trial}} = F_{\text{calculated}}$ is satisfied.

Convergence may be achieved rapidly by calculating the initial value of F_{trial} (to input into the right-hand side of the equation) using $m_a = 1$ for every slice. Adequate convergence occurs in hand calculations when F_{trial} and $F_{\text{calculated}}$ agree to two decimal places.

It is often convenient to express the pore water pressure at the base of each slice in a dimensionless form, where:

$$r_{u_i} = \frac{u_i}{\gamma \cdot h_i} \tag{9.79}$$

On this basis, Equation (9.8) becomes:

$$F = \frac{1}{\Sigma W_i \sin a_i} \Sigma \frac{1}{m_{a_i}} [c_i' l_i \cos a_i + W_i(1 - r_{u_i}) \tan \phi_i'] \tag{9.80}$$

Hand calculations are carried out in tabular form as, for example, in Figure 9.28.

After iteration to determine the value of factor of safety for a given slip surface, the procedure must be repeated to locate the slip surface which will give the lowest factor of safety. This is normally achieved by: (1) moving the centre of the circle; or (2) changing the depth of the circle.

For a given depth of circle, contours of factor of safety are obtained as shown in Figure 9.29. The critical shear surface will give the lowest factor of safety.

9.5.1.4 Long-term stability analysis: noncircular slips

Janbu's method[49] Janbu's method is the only hand-calculation method in widespread use for estimating the stability of noncircular slips. The method uses the force equilibrium of the individual slice, coupled with the horizontal force equilibrium of the entire slip mass, to derive a solution. Interslice forces are assumed to be zero.

As with the other methods, the factor of safety is defined in terms of the mobilized shear strength of the soil, i.e.:

$$T_i = \frac{1}{F}(c_i' \cdot l_i + N_i \cdot \tan \phi_i') = t_i \cdot l_i / F \tag{9.81}$$

for internal force equilibrium within each slice.

The factor of safety by this method is:

$$F = \frac{\Sigma (c_i' + (p_i - u_i) \tan \phi_i') b \cdot (sec^2 a_i / (1 + \tan a_i \cdot \tan \phi' / F))}{\Sigma W \tan a_i} \tag{9.82}$$

Once again, hand solution is carried out in tabular form.

Once the solution is achieved (by iteration, as with Bishop's method),[48] it is corrected by a factor introduced by Janbu to take account of the fact that interslice forces are ignored in the derivation (Figure 9.30):

$$F = f_0 \cdot F_{\text{calculated}} \tag{9.83}$$

Slice no.	Width b (m)	h (m)	γ (kN/m^2)	r_u	c' (kN/m^2)	ϕ' (deg.)	α	W	$W \sin \alpha$
								Σ	

$c'b$ W(1 − r_u) tan ϕ'	M_α for $F = F_x$	(1) + (2) ÷ (3)
	Σ	

Figure 9.28 Format for recording hand calculations

where f_0 depends on the strength parameters used in the analysis, and also upon how curved or flat the shear surface is; f_0 varies between 1.0 and about 1.1.

Janbu's method is normally used where site observations and measurement indicate a noncircular failure surface. It is common to use this method when the geometry of the critical slip surface is known. If the position of a critical shear surface is unknown then trial surfaces must be used to determine the minimum factor of safety.

Morgenstern and Price's method.[50] Morgenstern and Price's method takes into account normal and shear forces on vertical planes within a slip mass bounded by a noncircular failure surface. The method appears at first sight to use slices as with the other methods previously discussed; in fact, the 'slices' are used to discretize the slip mass, allowing linear variations of a number of functions to be assumed within each slice.

The method is not suitable for hand calculation, but can be solved on relatively small desktop computers.

The equations of the method derive partly from considerations of equilibrium of an infinitely thin vertical slice. Moments are taken about the centre of the base of this infinitely thin slice, and further equations are developed by resolving normal and parallel to the slip surface. These equations are then used to numerically integrate across the slip mass.

A satisfactory solution is obtained when this integration yields zero lateral force, and zero moment about the slip surface, at the end of the last slice. The method therefore gives both moment and force equilibrium.

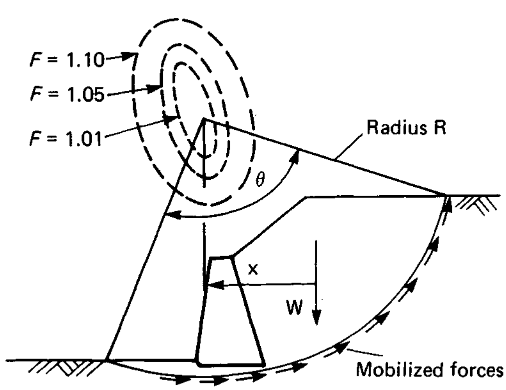


Figure 9.29 Contours of factor of safety. (After Clayton and Milititsky (1986) *Earth pressure and earth retaining structures*. Surrey University Press, Glasgow)

When using a computer program to make calculations by this method the engineer must not only define the geometry of the problem, soil parameters, and the desired division of the slip mass into 'slices', he must also determine the shape of the $f(x)$ distribution across the slip, where:

$$f(x) = \frac{1}{\lambda}\frac{X}{E} \tag{9.84}$$

on any vertical plane within the slip mass. The program defines the available shear strength in terms of the factor of safety F and in common with other methods assumes that F is constant across the slip. The condition that zero thrust and moment should be obtained from integration across the slip mass is obtained using the Newton–Raphson method to converge on the values of F and λ.

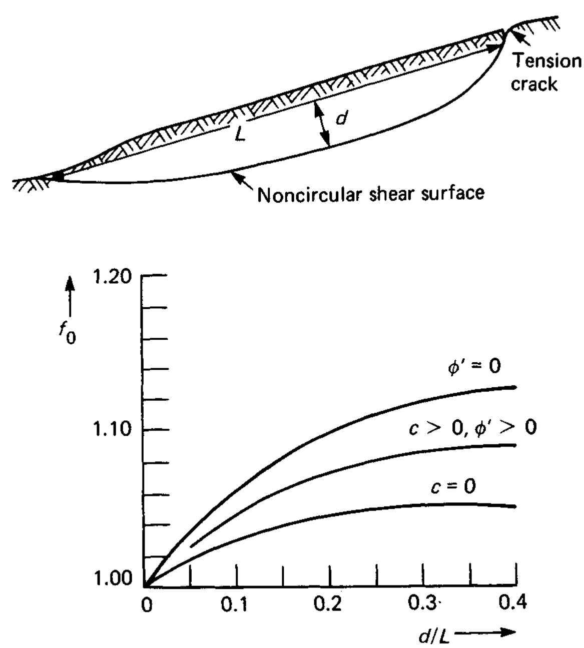


Figure 9.30 Correction factor f_0 for Janbu's slope stability method. (After Clayton and Milititsky (1986), *Earth pressure and earth retaining structures*. Surrey University Press, Glasgow)

It is common practice to start the analysis by assuming a uniform distribution of $f(x)$ (say $f(x) = 1.0$) across the slip. Some authors have indicated that variations in the shape of the $f(x)$ distribution have little effect on the factor of safety determined by the analysis, but other experience suggests that this depends upon the geometry of the problem and the parameters used. In some cases the $f(x)$ distribution can affect the result significantly, and therefore different distributions should be used. These may take the form of either: (1) a half-sine curve; or (2) a distribution where $f(x)$ is proportional to the curvature of the slip surface.

The output from such a computer analysis should not be accepted without a check that:

(1) The shear stress on any vertical surface within the slip mass does not exceed the available shear strength.
(2) No tension is implied on any vertical surface within the slip mass. For this to be true the resultant force on any section should pass within the bounds of the ground surface and the slip surface.

It is therefore necessary to obtain values for the vertical shear force and position of the resultant thrust across the section in order to make these checks.

9.5.1.5 Flow slides

Not all slides are deep-seated shear slides. (For a classification of landslides see Skempton and Hutchinson.[53]) A fairly common type which should be mentioned is the flow slide. Flow slides generally take place in saturated masses of loose, fairly impervious soils such as fine sands, silts or silty clays. They can occur on quite flat slopes and can travel long distances, the soil and water flowing as a liquid mass. They are caused by a sudden reduction of the shear strength to zero, or close to it, by the transfer of all the pressure to the water in the voids, a process known as liquefaction. The chief factor here is the relative density of the soil, loose soils having a relative density of less than a critical value below which volume reduction occurs on shearing contrary to dense soils which tend to dilate, thus throwing a tension on to the pore water. Casagrande considers in Green and Ferguson[54] that sands having a relative density greater than 50% would be safe against liquefaction. Casagrande and others have drawn attention to a more general phenomenon called 'fluidization' where the fluid phase is mainly air. Such phenomena are not necessarily confined to fine grained soils. They can occur under a variety of circumstances, the chief of which is a sudden disturbance of a loose solid by a heavy shock or vibration. Drainage and compaction are the two main remedial and preventative measures.

In some clays, the existing undisturbed strength is much greater than the strength after remoulding. The ratio of these two strengths is called the sensitivity of the clay. In general, the undisturbed strength of soft clays is measured *in situ* by means of a vane test. Much work on this problem has been done in Norway and Sweden and is described by Skempton and Northey.[55] In England, sensitivities are usually below 10 and in these cases an analysis can be made using the undrained shear strength and the $\phi_u = 0$ method.

With clays of high sensitivity, i.e. over 20 and up to 100 such as occur in southern Norway and Sweden, experience shows that an analysis based on the $\phi_u = 0$ case and the undisturbed strength of the clay measured by a vane test, gives results which do not necessarily agree with practice (Bjerrum[56]). When failure takes place in a clay of high sensitivity, the result is in many respects similar to a flow slide described in the paragraph above, the clay becoming practically fluid and flowing through quite small apertures down flat valleys or hillsides for a long distance. The formation of these very sensitive clays is believed to be due to the clay being laid down in salt water, then being raised above water level by isostatic readjustment, the salt later being leached out by the percolation of fresh water; thus, the liquid limit of the clay is reduced but the moisture content remains high. On disturbance, therefore, the moisture content greatly exceeds the liquid limit, and the clay acts as a heavy fluid. A description of the leaching process is given by Bjerrum.[57]

9.5.1.6 Protective and remedial measures

Certain protective and remedial measures can be taken to prevent a slip or to stabilize a slip which has occurred. Of these the most important is drainage. The majority of troubles are due to water; well-designed and adequately maintained drainage can prevent the ingress of water to a bank and so stop, or considerably delay, any softening which may take place and prevent the build-up of high pore pressures.

On sidelong ground, a drain should be installed at the top of the slope to catch surface water. Water from the slope itself should be caught in a toe drain and led away from the toe. On a long slope it may be necessary to catch surface water in herringbone drains or even to lead it to a longitudinal drain on a berm halfway up the slope. It is important to line the inverts of these drains.

A considerable degree of protection can be obtained by grassing the slope and the level area at the top where shrinkage cracks are likely to appear. Cracks once formed can become filled with water, the pressure of which exerts a considerable disturbing force on the bank. A good carpet of grass prevents not only drying and cracking but also surface erosion. Bushes and trees with strong root systems can also be of help, but trees which grow rapidly and absorb large amounts of water from the soil (e.g. poplars and elms) should be avoided.

Deeper drainage can be achieved by drilling horizontally into the slope—or in some cases adits can be used, either alone or in conjunction with vertical drainage holes. Counterforts are also used to improve drainage. These consist of trenches excavated back into the slope, backfilled with free-draining granular material.

In addition to drainage methods, the stability of deep-seated slides can be improved either by reducing the disturbing forces or by increasing the resisting forces by one or other of the following methods:

(1) Reducing the disturbing forces by loading the toe, or removing material from the top, or replacing material at the top by a lighter material, or altering the bank profile, e.g. introducing berms.
(2) Increasing the resisting forces by increasing the shear strength, by drying or adding frictional counterforts or keys through the slip surface.
(3) It may in some instances be possible to improve stability by introducing 'rigid' elements, such as sheet-piling driven through the toe, with or without ground anchors. However, great caution is required, since the rigid element will attract load and very high bending moments will develop.

9.6 Seepage and flow nets

A flow net is a graphical representation of the pattern of the seepage or flow of water through a permeable soil. It is possible, by means of a flow net, to calculate the hydrostatic uplift on a structure such as a dam or barrage, the amount of seepage

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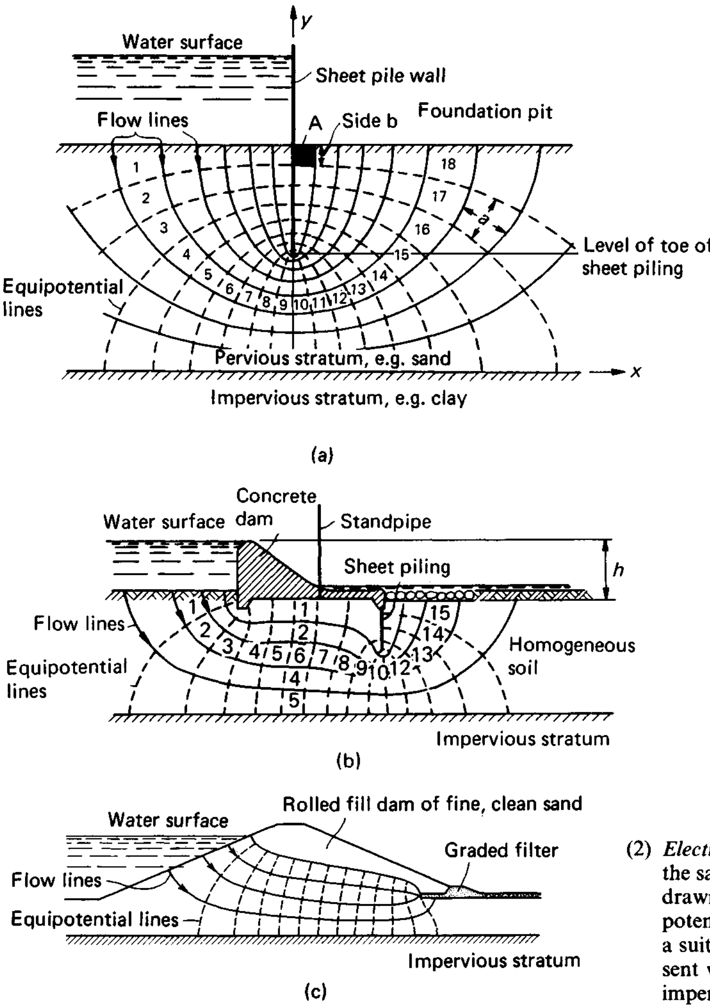

Figure 9.31 Examples of flow nets for simple cases. (a) Beneath sheet pile wall; (b) beneath concrete dam on sand with sheet pile cutoff wall; (c) through rolled fill dam with toe drain

through an earth dam or under a barrage, or estimate the probability of piping occurring in a cofferdam, for example.

A flow line is the path followed by a particle of water flowing through a soil mass. Flow lines are always smooth, even, curves as shown in Figure 9.31. An equipotential line is a line joining points at which the hydraulic head is equal; therefore, if standpipes are inserted into any two points on an equipotential line, the water will rise to the same level in each standpipe. Flow lines and equipotential lines are always at right angles to each other. For a discussion on the theory of flow nets, see Cedergren.[58]

9.6.1 Construction of flow nets

Four methods of constructing flow nets are in general use.

(1) *Mathematical.* For simple boundary conditions, the governing differential equation (Laplace) can be solved mathematically. Many computer-finite element packages now contain seepage programs which can be used to solve the more complex steady-state seepage problems, e.g. with layered soils and complex boundary conditions.

(2) *Electrical analogy.* The differential equation for flow nets is the same as that for flow of electricity, and flow nets can be drawn by using an electrical model and tracing lines of equal potential with a wandering probe. The soil is represented by a suitably shaped conducting paper, strips of copper represent water surfaces and the edges of the card represent an impervious surface. Once the equipotential lines are drawn, the flow lines can easily be drawn at right angles to them.

This method assumes that the permeability in both directions, horizontally and vertically, is the same. In the electrical resistance network method a scaled network of resistances of values proportional to the permeability is set up, the electrical potential at each node being a direct measure of the hydraulic potential at that point.

(3) *Hydraulic models.* An obvious approach is to construct a model of the problem in sand behind glass, to allow water to flow through it and to trace the flow lines by inserting a small drop of dye as it flows through the soil. This trace is then drawn on the glass with a wax pencil and the procedure repeated from a different point.

(4) *Graphical method.* After a little practice, it is quite possible to sketch a flow net for many problems, which is quite accurate enough for most practical purposes. The cross-section is drawn and the boundary conditions clearly marked. The flow net is then tentatively sketched in, bearing in mind that flow lines and equipotential lines are at right angles to each other, that flow lines always start at right angles to a free water surface and equipotential lines start or finish at right angles to an impervious surface. The number of flow and equipotential lines is chosen to divide the seepage area into shapes which are approximately square and which are bounded by two flow lines and two equipotential lines.

9.6.2 Examples of hydraulic problems by flow nets

9.6.2.1 Uplift pressure

In Figure 9.31(b), let the number of squares along a flow line be $n(=15)$ and the number along an equipotential line be $f(=5)$. Then if the total drop in head is h, the drop in head across each square is h/n, and at an imaginary standpipe through the concrete at the sixth equipotential line the loss in head will be:

$$6\times h/n=6h/15=0.4h \tag{9.85}$$

The uplift pressure at this point will be the remaining head times the density of water, i.e.:

$$(h-0.4h)\gamma_w=0.6h\gamma_w \tag{9.86}$$

Note that this result is independent of the number of squares in the flow net, since if $n=30$ instead of 15, the borehole in the position shown would be on the twelfth equipotential and loss in head would be:

$$12h/30=0.4h$$

9.6.2.2 Hydraulic gradient and D'Arcy's law

The hydraulic gradient is defined as $\Delta h/\Delta l$ (i.e. the ratio of head loss to distance) and is related to the velocity of flow v and the permeability k by D'Arcy's law:

$$v=ki \tag{9.87}$$

9.6.2.3 Amount of seepage

The quantity of water Q flowing under the dam in Figure 9.31(b) is given by $Q=Atki$, where A is the area of flow, t is time, k is the coefficient of permeability and i is the hydraulic gradient. The hydraulic gradient i across any square of the flow net of side b is given by $i=h/nb$. The flow in unit time through the square is $Q=bkh/nb=kh/n$.

If the number of flow channels is f, the total flow is $Q=fkh/n$, and is independent of the size of the squares.

9.6.2.4 Factor of safety against piping

The factor of safety against piping is the ratio of the critical hydraulic gradient to the existing hydraulic gradient at exit.

In Figure 9.31(a) piping will occur at A when the upward force of the water issuing at A is greater than the effective weight of the particles.

The seepage force on the base of the last square is $\gamma_w ib^2=$ effective weight of soil $=b^2\gamma_w(G_s-1)/(1+e)$. Therefore, piping occurs when $i=(G_s-1)/(1+e)$.

For sand, G_s is about 2.7, and in the loose state e is about 0.7, giving:

$$i=(2.7-1)/(1+0.7)=1 \tag{9.88}$$

i.e. the critical hydraulic gradient is about unity.

The exit hydraulic gradient at A in Figure 9.31(a) is $(h/n)/b$.

Therefore, the factor of safety against piping is:

$$F=1/(h/nb)=nb/h \tag{9.89}$$

Note that nb is independent of the number of squares.

It is generally considered that the value of the factor of safety against piping should be 4 or greater.

9.7 Definitions of terms used in soil mechanics

All soils consist of solid particles assembled in a relatively open packing. The voids may be filled completely with water (fully saturated soils) or partly with water and partly with air (partially saturated soils).

The relationships between void space and the volume occupied by the particles are fundamental and are characterized by the following definitions.

Porosity $n=$ volume of voids V_v/total volume of soil V_t.
Voids ratio $e=V_v$/volume of soil particles V_s.

$$\text{Hence}\quad e=n/(1-n)\quad\text{and}\quad n=e/(1+e) \tag{9.90}$$

Degree of saturation $S_r=$ volume of water$/V_v$.
Water content $w=$ weight of water/weight of soil particles.

Hence, if G_s is the specific gravity of soil particles:

$$w=S_re/G_s$$

For fully saturated soils, $S_r=1$ and $w=e/G_s$.
Bulk density $\gamma=$ total weight of soil and water: unit volume W_t/V_t.

$$\text{Hence}\quad \gamma=(G_s+S_re)\gamma_w/(1+e) \tag{9.91}$$

where γ_w is the density of water (1 Mg/m³).

$$\text{When}\quad S_r=1,\ \gamma=(G_s+e)\gamma_w/(1+e) \tag{9.92}$$

dry density $\gamma_d=W_s/V_t$.

$$\text{Hence}\quad \gamma_d=G_s\gamma_w/(1+e) \tag{9.93}$$

$$\text{and}\quad \gamma=\gamma_d(1+w) \tag{9.94}$$

$$\text{Also}\quad n=1-\gamma_d/G_s\gamma_w \tag{9.95}$$

$$\text{and}\quad S_r=w/(\gamma_w/\gamma_d-1/G_s) \tag{9.96}$$

The submerged density, of soils below water table, is given by:

$$\begin{aligned}\gamma_s&=(G_s-1)\gamma_w/(1+e)\\&=(G_s-1)\gamma/G_s(1+w)\end{aligned} \tag{9.97}$$

Percentage air voids $A=$ volume of air $\times 100/V_t$

$$A=1-\gamma_d(1+wG_s)/\gamma_wG_s \tag{9.98}$$

All the foregoing definitions and relationships are in constant use in soil mechanics problems.

Appendix 9.1 Laboratory testing of soils

This appendix gives a brief outline of some of the main laboratory tests required to classify individual soils and to indicate their compaction and strength characteristics.

In order to obtain reliable results, it is essential to follow the recommendations in Chapter 11 with regard to sampling and then to follow closely the practices recommended in the appropriate standards for sample preparation, testing and reporting.

The outline in this section is based on British Standard BS 1377: 1975[59] but other national standards may be relevant for work in some countries, such as, for example, in:

(1) *Australia*: AS 1289 – Methods of testing soils for engineering purposes.
(2) *France*: Norme X31 – Qualité des sols.
(3) *West Germany*: A series of DINs (Deutsche Normen), e.g. 18121–18127, 18130–18137 and 18196.
(4) *USA*: A series of ASTM Standards.

Other standards may be applicable elsewhere in the world, and the reader should refer to the appropriate national standards office for further information. The reader should also ensure that the standard to which he works is the latest version. For example, it is known that BS 1377 is being revised and that a new version will be published after 1989.

A9.1.1 Soil classification: physical

A9.1.1.1 Liquid limit (LL): cohesive soils

The liquid limit is the moisture content at which a soil passes from the plastic to the liquid state.

The preferred method of determining the LL is now by use of a standard cone penetrometer, under a load of 80 g, on to prepared samples 55 mm diameter and 40 mm deep. A series of at least four tests on the sample at increasing moisture contents allows a linear plot between penetration and moisture content, from which the LL is interpolated as the moisture content corresponding to 20 mm penetration.

The alternative traditional, but less reliable, method is to use the Casagrande apparatus, in which the prepared soil sample is placed in a cup, grooved with a standard tool and then lifted mechanically and dropped on to a standard rubber block at a rate of 2 blows/s, until the two parts of the soil come together for a distance of 13 mm along the groove. The results of a series of at least four tests on the sample, at increasing moisture content, are then plotted with moisture content on a linear scale and number of blows on a logarithmic scale. The LL is interpolated as the moisture content corresponding to 25 blows.

A9.1.1.2 Plastic limit (PL): cohesive soils

The plastic limit is the moisture content at which a soil becomes too dry to be in a plastic condition, as determined by the PL test. The 20-g sample of soil, with moisture content sufficient for it to be moulded into a ball between the palms of hands, is rolled between fingers and palm until slight cracks appear on its surface. The sample is divided equally and each subsample divided into four for tests that each comprise forming a thread 6 mm diameter by rolling between first finger and thumbs, and rolling the thread on a glass plate, using fingertips with a uniform pressure, until the thread reduces to 3 mm or the number of rolling passes exceeds 10. Each subsample should be remoulded by finger and thumb, to reduce moisture, and the thread rolled on glass until a reduction to 3 mm is achieved within ten passes and, simultaneously, the thread shears transversely and longitudinally. The average of the moisture contents at which crumbling (shearing) occurs is the PL of the soil.

A9.1.1.3 Plasticity Index (PI): cohesive soils

The Plasticity Index of a soil is the numerical difference between the liquid and plastic limits of the soil, i.e. PI = LL − PL. Where PI is zero (i.e. LL = PL) or when the soil is insufficiently cohesive for a PL to be measured, the soil is termed nonplastic (NP).

A9.1.1.4 Classification: cohesive soils

The Casagrande classification of cohesive soils is illustrated in Figure A9.1.1, in which PI is plotted against LL. Above the A line are CL, CI and CH (low, intermediate and highly plastic clays) and below the line are ML, MI and MH (low, intermediate and highly plastic silts) or OL, OI and OH (low, intermediate and highly plastic organic clays). The Casagrande classification provides the engineer with a good indication of the characteristics of a soil both for design and construction purposes, but an extended classification system is given in Chapter 11, section 11.5.

A9.1.1.5 Particle size distribution: granular soils

In the majority of cases, wet sieving is required first to remove and record the loss of silt and clay-size particles. The BS sieves used in the test range from 75 mm to 63 μm and the results are recorded as the dry weight expressed as the percentage, by weight, of the total sample passing each sieve. Grading curves in the form shown in Figure A9.1.2, give cumulative percentages passing 12 or so of the BS test sieves between 75 mm and 63 μm. The procedure for washing and sieving and the use of sodium hexametaphosphate solution to help remove and to break down fine particles should be followed very closely.

From the particle size distribution curve the soil will be classified as gravel (G), sand (S), or silt (M) or combinations of these, and the shape of the curve indicates whether it is well graded (e.g. GW), poorly graded (e.g. GP) or silty (GM). Further information on soil classification is given in Chapter 11, section 11.5.

A9.1.1.6 Particle size distribution: fine grained soils

The particle size distribution of fine grained soils, e.g. silts and clays or fractions of these in coarser grained soils, is determined by sedimentation tests in which the material is brought into suspension in a solution of sodium hexametaphosphate and allowed to settle. The mass of solids in a given volume of solution is measured at a specific point, either by sampling (using a pipette or by the use of an hydrometer) at specific time intervals corresponding roughly to the equivalent particle diameters (e.g. 0.02 mm, 0.006 mm and 0.002 mm) for which information is required.

The basis of this test is Stokes's law, which gives the relationship for a spherical particle falling through a column of liquid:

$$v = \frac{d}{t} = \frac{2(\gamma_s - \gamma_l)\,gr^2}{9\eta}$$

where v is the velocity of the falling particle, d is the distance through which it falls in time t, γ_s is the density of the particle, γ_l is the density of the liquid, r is the radius of the particle and η is the viscosity of the liquid. A monographic chart in BS 1377[59] facilitates the calculation of the equivalent particle diameter.

A9.1.1.7 The specific gravity of soil particles (G_s)

It is only rarely necessary to know the G_s of a soil for classification purposes, but values of G_s are required in the calculations and interpretation of some other test results and for design and construction purposes.

In essence, the determination of G_s is by measurement of the dry weight of a sample of soil, the weight of the same sample plus water required to fill a container and then the weight of water alone to fill the same container. For coarse grained soils, a gas jar of 1 litre is a suitable container but, for soil particles finer than about 2 mm, standard 50 ml density bottles (pycnometers)

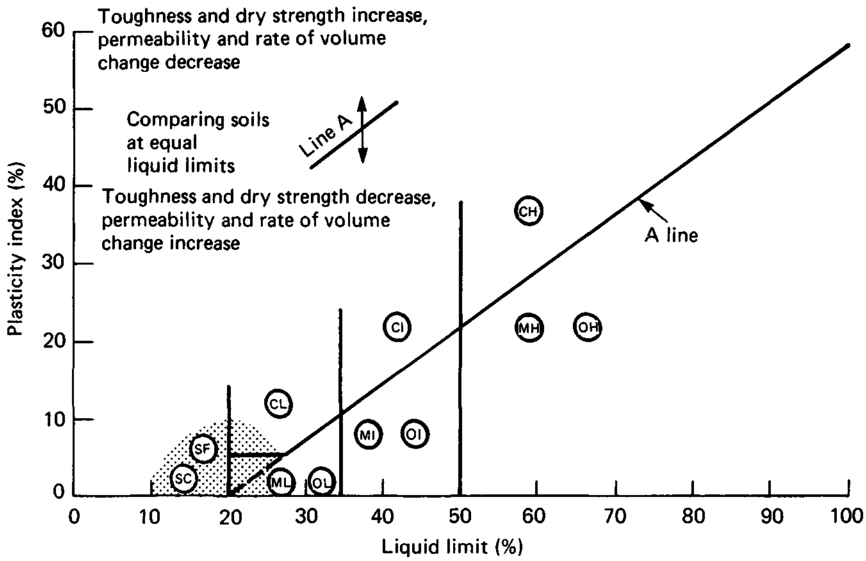

Figure A9.1.1 Plasticity chart used in the Casagrande soil classification. (After *Soil mechanics for road engineers* (1968). HMSO, London)

Figure A9.1.2 Particle size distribution chart. (After British Standards Institution (1975) *Methods of test for soils for civil engineering purposes*. BS 1377:1975. BSI, Milton Keynes) are often used. In either case, care is required to exclude air from the samples, involving the use of air-free distilled water in the tests on fine grained soils.

A9.1.2 Soil classification: chemical

A9.1.2.1 Organic matter content

Organic content is expressed as the percentage by mass of organic matter present in the soil.

The organic matter in a sample of soil dried in an oven between 105 and 110° C is oxidized in a 500 ml conical flask by the addition of 10 ml potassium dichromate solution and 20 ml concentrated sulphuric acid. The sample of soil should weigh between 0.2 g for peaty soil and up to 5 g for a soil with low organic content. Processes of titration determine the volume of potassium dichromate used to oxidize the organic matter and, hence, the organic content. Soils containing sulphides or chlorides require some special treatment.

A9.1.2.2 Total sulphate content of soil

This is determined as the percentage mass of sulphate (as SO_3) present in the soil sample, represented by the material passing, or which can be crushed to pass, the 2 mm sieve. The sample for test is then pulverized until it passes the 425 μm sieve. Stones, other than gypsum, may be assumed to contain no sulphate.

The test procedure involves treatment of the sample with hydrochloric acid and ammonia to form an acid extract, which is boiled and barium chloride solution added, to form a precipitate of barium sulphate. The mass of the precipitate formed is determined by filtration and ignition, and gives a measure of the SO_3 present in the sample.

A9.1.2.3 Sulphate content of groundwater

To determine the percentage of sulphate (as SO_3) present in the soil water or groundwater requires the extraction of water from a soil sample by means of a centrifuge or other means or the collection of a sample of groundwater. The extract or sample is passed through an ion exchange column comprising a strongly acidic cationic exchange resin; the sulphate content of the soil water extract or the groundwater can be separately determined by titration of a standardized sodium hydroxide solution against the resultant solution. In both cases, the SO_3 content is expressed in grams per litre.

A9.1.2.4 pH value

The pH value indicates the acidity or alkalinity of a soil, with values above 7 indicating alkalinity and values below 7 increasing acidity. The pH of groundwater can be determined in similar ways, using either an electrometric or a calorimetric method. The former method is most usual.

For the electrometric method, standard pH meter electrodes are placed in suspension of the soil sample in water and the readings obtained record the pH value. Buffer solutions are used to calibrate the pH meter.

A9.1.2.5 Additional chemical tests for contaminants

Although outside the scope of this chapter, the increasing proportion of construction on sites previously used for a range of industrial purposes has led to the need for additional and, often, more extensive soil testing. The problems involved are referred to in Chapter 11 and attention is drawn to the bibliography in that chapter relating to contaminated sites.

On contaminated sites, the presence of methane and of a variety of potentially dangerous or toxic chemicals needs to be considered from the point of view of safety of workmen during construction. Corrosive materials that could affect the durability of materials in a new construction should also be identified and special precautions taken in the design of the new construction to limit such damage. Common examples of aggressive materials are chlorides, sulphates and electrolytic, chemical or bacteriological agencies of various types.

Information and experience on the subject of contaminated sites is increasing rapidly and the reader with a special interest is advised to consult, for example, the Building Research Establishment for the most up-to-date information.

A9.1.3 Soil compaction

A9.1.3.1 Dry density: moisture content relationships

Laboratory compaction tests determine the mass of dry soil per cubic metre obtained when the soil is compacted in a specified manner at a specific moisture content. Repetition of the test over a range of moisture contents provides a compaction curve indicating the optimum moisture content and maximum dry density obtainable for the compactive effort applied.

The original 'Proctor' test simulated the compactive effort of construction plant in the 1930s. Later, the modified American Society of State Highway Officials (AASHO) test was developed to model heavier construction plant and, in the UK, a vibratory test was introduced in 1967 to simulate the effect of heavy compaction by vibrating rollers and plates. A comparison of the British and American tests is shown in Table A9.1.1.

In each test, soil of predetermined moisture content is placed in a specified number of layers into a cylindrical mould, and each layer is compacted by a specified number of blows with a standard rammer or, in the case of the vibration method, for a period of 60 s. A typical compaction curve obtained from a series of tests is shown in Figure A9.1.3.

While the modified AASHO test or its BS equivalent is suitable for fine grained as well as coarse grained granular soils up to 20 mm, the BS vibratory method is applicable to soils up to 37.5 mm and is preferred for soils such as clean gravels or rocks and for uniformly graded and coarse sands.

A9.1.3.2 Measurement of dry density

Reference is made in Chapter 11 of the sand replacement test method for the measurement of dry density of compacted soil *in situ*, and to nuclear and other tests available. For fine grained soils it is sometimes more convenient to cut cores, trim the soil core to the cutter size (usually 100 mm diameter by 130 mm long) and to determine the weight of dry soil within those dimensions.

Table A9.1.1. Comparison of standard American and British compaction tests

Test	*No. of layers*	*Blows per layer*	*Weight of hammer* (kg)	*Height of drop* (mm)	*Volume of mould* (cc)
Proctor	3	25	2.5	305	944
Modified AASHO	5	27	4.55	457	944
BS 1377					
test 12	3	25	2.5	300	1000
test 13	5	27	4.5	450	1000
test 14 (vibratory)	3	60 s with 300–400 N down force			*c*.2300

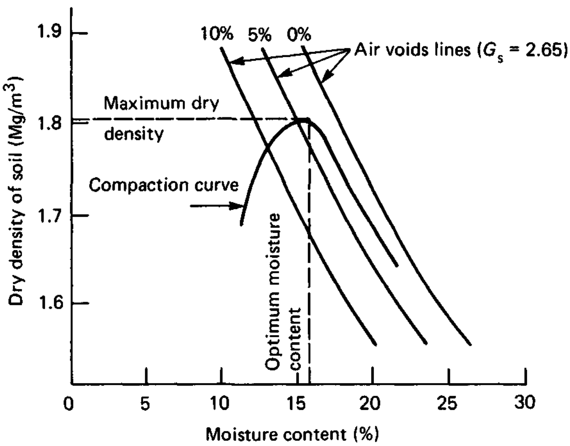

Figure A9.1.3 Typical compaction curve and the terms used. (After British Standards Institution (1975) *Methods of test for soils for civil engineering purposes.* BS 1377:1975. BSI, Milton Keynes)

An alternative laboratory method appropriate to lumps of soil, preferably approximately cubical or cylindrical, involves coating them with paraffin wax and subsequent determination of the soil volume by weighing the sample immersed in water or by a water displacement method.

A9.1.4 Strength tests

A9.1.4.1 Californian bearing ratio (CBR)

The Californian bearing ratio (CBR) test was developed in 1938 to evaluate Californian highway subgrade strengths and became the basis for the design of road and airfield pavements throughout the world. It is used both *in situ* and on prepared samples in the laboratory, but is limited to materials of particle sizes up to a maximum of 20 mm.

The test determines the relationship between force and penetration when a cylindrical plunger 1935 mm² in cross-section is pressed into soil at a given rate of 1 mm/min. For any given penetration, the ratio is expressed as a percentage of a standard force derived for crushed stone.

For CBR tests in the laboratory, the soil specimen is prepared at a predetermined moisture content and is compacted into a cylindrical mould 152 mm in diameter and 127 mm high either by continuous tamping, compression in three equal layers or dynamic compaction in layers, using either the rammers or the vibrating hammer used in the compaction tests. In all cases, the mass of soil poured into the mould is calculated as that required to provide the chosen dry density or air voids percentage on completion of compaction. These usually correspond to the optimum, determined from compaction tests, or are the values measured on the soil *in situ*.

Results of the test are plotted as shown in Figure A9.1.4. The CBR is calculated at penetrations of 2.5 and 5 mm, and the higher value taken. The standard forces corresponding to 100% CBR at these two penetrations are 13.24 and 19.96 kN respectively. The force–penetration curve is normally convex upwards, but curves beginning as concave upwards require correction by shifting the zero on the penetration axis.

A9.1.4.2 Other plate bearing tests

A variety of plate bearing tests has been used in site investigations and are mentioned in Chapter 11. However, for airfield pavements and use in Westergaard's analysis of strains and deflections in concrete slabs, a test developed in the US[60] is used to measure the 'modulus of subgrade reaction'. The plate is usually 750 mm in diameter and a linear plot of settlement against pressure applied gives a curve which is convex upwards. The modulus of subgrade reaction k is calculated as:

$$k = p/1.27 \text{ g/mm}^2\text{/mm}$$

where p is the pressure required to cause settlement of 1.27 mm.

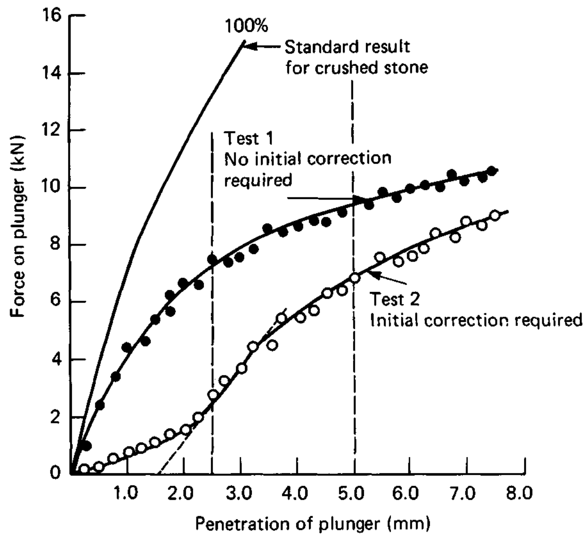

Figure A9.1.4 Typical California bearing ratio test results. (After British Standards Institution (1975) *Methods of test for soils for civil engineering purposes.* BS 1377:1975. BSI, Milton Keynes)

A9.1.4.3 Determination of shear strength

The vane test For *in situ* measurement of shear strength, a vane of cruciform section 50 or 75 mm in diameter and of length equal to twice the diameter is lowered into a borehole and pushed, without twisting, to a depth not less than 3 times the borehole diameter into the undisturbed soil. A torque head fitted to the top of the vane rods turns the vane at a rate between 10 and 20°/s until the soil is sheared.

For vanes with height twice the diameter, the vane shear strength S is given by:

$$S = \frac{M \times 10^6}{3.66D^3} \text{ kN/m}^2$$

where M is the torque to shear the soil in newton metres and D is the measured width of the vane in millimetres.

Direct shear test In this test, a soil sample is cut and trimmed carefully to fit closely into a metal box, either circular or square in plan, which is constructed to allow displacement along its horizontal midplane. The upper surface of the sample is confined by a normal load and a shear load is applied to the lower half of the box until the soil shears across the midplane.

The triaxial test The triaxial cell (Figure A9.1.5) provides the means of applying horizontal pressures to a cylindrical specimen by means of lateral hydraulic pressure in the cell chamber simultaneously with a vertical load applied by a ram. The

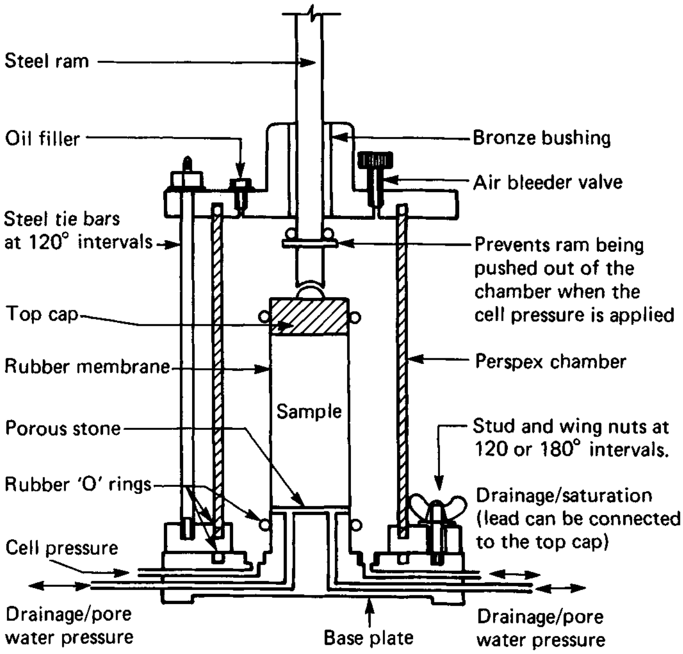

Figure A9.1.5 A triaxial cell. (After Clayton, Simons and Matthews (1982) *Site investigation*. Granada, London)

specimen in various types of test may be consolidated or unconsolidated, drained or undrained, and there is a facility to measure pore pressure.

The unconsolidated undrained compressive test is used to determine the undrained shear strength of undisturbed samples. At least three samples are tested over a range of cell pressures. At each pressure the ram force is increased and the vertical deformation recorded, until the maximum value of the stress had been passed or an axial strain of 20% reached. The principal stress difference (deviator stress $\sigma_1 - \sigma_3$) is calculated as the ram force divided by the cross-sectional area of the specimen. However, because the specimen diameter increases during the test, a corrected value of cross-sectional area is used in the calculation, such that:

$$A = A_0/(I - \varepsilon_a)$$

where A is the cross-sectional area of the specimen at a given time, ε_a is the measured axial strain at that time and A_0 is the initial cross-sectional area of the specimen.

The results of the test are plotted as curves of principal stress difference against strain. For conditions of maximum stress (i.e. failure) Mohr circles are plotted to give values for c_u, the apparent cohesion, and ϕ_u, the angle of shearing resistance. This is discussed further in section 9.1.2 and Mohr circles are shown in Figure 9.1.

A9.1.4.4 Penetration resistance N using the split barrel sampler

The penetration resistance N of a soil, relevant particularly to piling, is measured on site as the number of blows required to drive a standard sampler, fitted with a driving shoe of 35 mm internal, and 50 mm external, diameter, specific distances into a soil surface at the bottom of a borehole. The sampler is driven by a 65 kg hammer falling freely through a height of 760 mm. The number of blows to penetrate 300 mm is termed the penetration resistance N. For gravelly soils, the driving shoe can be replaced by a 60° solid cone (BS 1377: 1975).

A9.1.5 Consolidation tests

A9.1.5.1 The oedometer test

One-dimensional consolidation properties can be determined as the magnitude and rate of consolidation of a disc of saturated soil confined laterally and subject to vertical pressure. The equipment consists of a ring, usually 76 mm in diameter and 19 mm high, to restrain the carefully cut soil disc, and a loading cell capable of being filled with water into which the ring is placed between porous plates. The loading device must be capable of maintaining constant load, giving pressures from the range 10, 20, 50, 100, 200, 400, 800, 1600 and 3200 kN/m^2. The compression movement is measured as the relative movement between the base of the cell and the loading cap.

Immediately after application of the initial load, water is poured into the cell and, if this causes swelling, the load is increased to the next stage until there is no swelling. The load is maintained for a period of up to 24 h and the results plotted as the compression movement against $\sqrt{}$time, and also as the movement against log–time. Methods are given in BS 1377[59] for calculating the consolidation coefficient from these graphs as c_v in square metres per year. Further information on the calculation of consolidation settlement is given in section 9.3.3 (page 9/9).

A9.1.5.2 Triaxial dissipation test

An alternative to the use of an oedometer is a triaxial test, in which volume change is plotted against log–time and, in addition, pore pressure is measured at the base of the specimen. The compressibility measured as a result of triaxial dissipation is greater than that determined in the oedometer test.

Appendix 9.2 Pile capacities

Piles are used to transfer foundation loads to a deeper stratum when the surface soils are too weak or too compressible to carry the load without excessive settlement. Details of pile types and their design and use are given in Chapter 17. The reader's attention is drawn to the references in Chapter 17 for further information, particularly to BS 8004[13], Tomlinson[61,62] and to series of CIRIA/PSA piling guides.[63]

In this appendix, methods are given for estimating the carrying capacity of piles in various types of soil conditions.

A9.2.1 Groups of piles

A piled foundation generally consists of a group of several piles, the behaviour of which should be considered as an entity.

The piled group will consist of either point-bearing piles which transfer their load to a hard stratum of soil on which their points bear (e.g. piles to rock) or friction piles which transfer their load mainly by friction on the sides of the piles to a firm stratum into which they penetrate. Friction piles into a firm clay stratum will usually penetrate about 20 to 30 times the pile diameter into the clay. Piles driven through soft material into compact sand or gravel will usually penetrate about 5 times the diameter and will be partly point-bearing and partly frictional.

When friction piles are used in conditions in which the increase in strength of the soil with depth is only gradual, they must be of a length comparable to the size of the building to be of much advantage. This is shown in Figure A9.2.1, in which the stress distribution with and without piles is shown for two buildings of different widths but with piles of the same length. Unless the use of piles changes the stress pattern radically their use is probably not economic.

The design of a foundation on friction piles should always be checked from the point of view of overall stability, and assuming that the whole of the load is distributed uniformly over the area of the building and acts as a block foundation with its base at the foot of the piles. The friction round the circumference of the block of soil containing the piles should be subtracted from the foundation load.

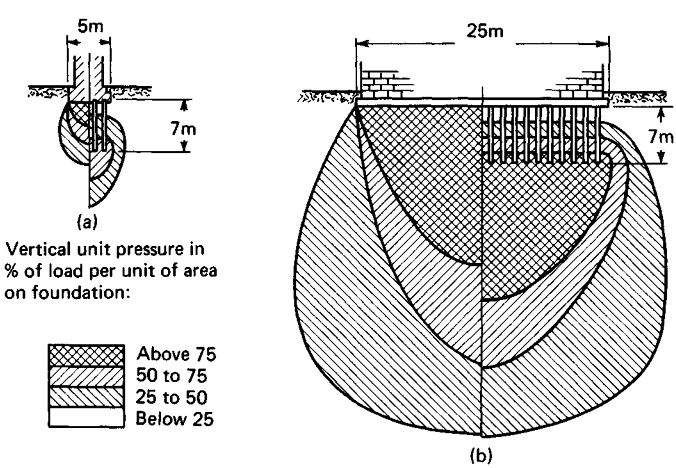

Figure A9.2.1 The increase of vertical pressure in soil beneath friction pile foundations having piles of equal lengths carrying equal loads. (a) Width of foundation small compared with pile length; (b) width of foundation large compared with pile length

If the foundation on friction piles is supported by a bed of clay, consolidation settlements will occur which can be estimated as described for deep foundations above.

A9.2.1.1 Single piles

Although the carrying capacity of a group of piles is not simply that of a single pile times the number of piles in the group, it is useful to know the load which a single pile will carry. Until the practice developed to treat pile groups as an equivalent deep raft, the capacities of groups were based on the sum of individual capacities with empirical efficiency ratios ranging from 1.0 for groups in sand down to 0.7 for widely spaced piles (i.e. 3 × diameter) in clay.

A9.2.1.2 Pile bearing on rock

Where the bedrock is massive and strong, bearing capacity is usually not a problem. However, if the rock surface is steeply sloping, it may be necessary to provide a driven pile with a special point to make sure that it is adequately toed-in. Bored piles into strong rock will only need a small penetration, say half to one pile diameter, in order to develop a working load equal to the maximum allowable working stress in the concrete – which is taken as 5000 kN/m^2. With small penetrations, particular care is needed to get a good contact between the pile toe and the rock.

With weaker and fractured rocks, it is still generally possible to develop the full allowable working stress in the concrete, but this will require penetration in order to carry some of the load in shear in the 'rock socket'. The required depth of penetration can be determined from a knowledge of the unconfined compression strength and fracture, or joint, spacing of the rock mass.[62]

A9.2.1.3 Piles bearing in deep deposits of sand and gravel

The ultimate point bearing capacity of a pile of end area A_B in sand may be estimated directly from the cone resistance C_r of the Dutch deep sounding test, thus:

$$Q_{uB} = A_B C_r \qquad \text{(A9.2.1)}$$

When using this method, however, it is necessary to take due account of the difference in scale between the cone (end area

10 cm^2) and the pile by ensuring that the pile penetrates the bearing layer some 4 to 6 or more diameters for piles of less than 450 mm diameter. For larger piles, greater penetration ratios are necessary (see Tomlinson,[62] for further information on this problem). The bearing capacity can also be determined from Berezantsev's[64] curves. However, results of theoretical calculations may be unreliable, and past experience is a better guide.

In deeply embedded piles it is also necessary to take account of the side friction, a factor which depends upon the horizontal earth pressure coefficient K_s, the overburden pressure $\gamma' D$ and the angle of shearing resistance δ between the ground and the pile; thus the ultimate shaft resistance is given by:

$$Q_{us} = A_s\bar{K}_s(\gamma'\bar{D} \tan \delta) \tag{A9.2.2}$$

where A_s is the shaft area and $\bar{K}_s$ and $\gamma'\bar{D}$ refer to average values of K_s and $\gamma' D$.

The horizontal coefficient K_s depends not only upon the relative density and stress history of the deposit, but also upon the method of forming the pile. In bored piles it can be as low as 0.7× at-rest earth pressure coefficient, rising to twice this coefficient for large displacement. The value of δ depends upon the value of ϕ and the pile material; δ/ϕ varies from 0.50 to 0.80 for steel and precast concrete respectively (see Tomlinson[62] for a full discussion on the question of shaft resistance).

The frictional resistance of the ground for driven piles can also be determined by use of the friction sleeve adaptation to the Dutch deep-sounding apparatus. This gives the term $K_s\gamma' D$, tan δ throughout the depth of the deposit directly (see Chapter 11).

The ultimate bearing capacity of the pile is given by the sum of Equations (A9.2.1) and (A9.2.2):

$$Q_u = Q_{uB} + Q_{us} \tag{A9.2.3}$$

Frequently, pile bearing capacities have to be based on standard penetration test data. Meyerhof[65] has proposed the following empirical relationships for the components Q_u and Q_{us}:

$$Q_u = A_B(107)4N + A_s \frac{\bar{N}}{50}(107) \tag{A9.2.4}$$

where Q_u is in kilonewtons and A_B and A_s are in square metres and $\bar{N}$ is the average N value along the pile shaft.

An upper limit of 60 kN/m^2 is suggested for the shaft resistance.

It is suggested that the point bearing capacity term in equation (A9.2.4) might be written as follows, to take account of the actual soil type into which the pile is driven:

$$Q_{uB} = A_B(107)KN \tag{A9.2.5}$$

When it is possible to carry out a loading test on a full-scale pile this should be done, the test being carried to failure, i.e. until settlement continues under constant load.

The use of dynamic pile-driving formulae to calculate the ultimate load is not to be recommended for two reasons, namely: (1) the information is obtained too late (i.e. at the construction stage instead of the design stage unless previous expensive tests are carried out) in which case loading tests should be included); and (2) a very wide range of answers can be obtained depending on the formula used and the choice of constants in the formula.

Dynamic pile-driving formulae have their uses, however, and records of set and energy should always be kept as they are guides to the variation in ultimate loads over a site on which one or two loading tests have already been carried out. Engineers of wide experience can also make estimates of the load-carrying capacity from the results of a driving test, provided always that their experience was obtained with conditions similar to those relating to the test. The relation between the true ultimate load and that given by the dynamic formula is empirical and should be recognized as such in spite of the mathematical basis of Newtonian impact mechanics on which such formulae appear to be founded.

A9.2.1.4 Piles bearing in clay

A pile bearing in clay receives support from the adhesion along the shaft and from the resistance at its base, the relative contribution of these two components depending upon the strength–depth profile of the clay, the ratio of the length to diameter of the pile and the manner of its installation, i.e. whether bored, driven preformed, or cast-in-place. Skempton[66] has given the following expression for the bearing capacity of a bored pile in London Clay:

$$Q = 9c_uA_b + a\bar{c}_uA_s \tag{A9.2.6}$$

where a is an empirical factor which depends on the type of pile, the clay and its condition, and varies between 0.3 and 0.6 for a bored pile in London Clay, with a mean value of 0.45 for a well-constructed pile in typical unweathered clay, and $\bar{c}_u$ is the average shear strength along the pile shaft, with an upper limit of 100 kN/m^2 for the average ultimate skin friction resistance.

The bearing capacity of a bored pile can be increased greatly by constructing an enlarged base (belling or under-reaming). The value of a for under-reamed piles should not exceed 0.3.

The shear strength of the clay in commercial practice is determined on 38 mm specimens cut from 100-mm driven samples, and experience over the last decade has shown that, owing to the fissured nature of London Clay, this practice results in considerably higher strengths being used in the assessment of end bearing capacity than obtain *in situ* (see Whitaker and Cooke[67] and also Burland, Butler and Dunican[68]). It is necessary therefore to allow for this factor by introducing an empirical coefficient ω related to the base diameter B of the pile, thus:

$$Q = 9\omega cA_B + a\bar{c}_uA_s \tag{A9.2.7}$$

where $\omega = 0.8$ for $B < 1$ m and 0.75 for $B > 1$ m, and ac_u has an approximate limit of 100 kN/m^2.

In the event of the strength being determined on larger specimens or from *in situ* piston tests, upward modification in the value of ω is necessary. Load factors used in the design range from 2 for straight-shafted piles to 2.5 for base diameter less than 2 m as these values will generally restrict the short-term settlement of the single pile to less than 10 mm. With diameters larger than 2 m the working load should be checked by calculating the settlement using the curves relating Q_a/Q to settlement shown in Figure A9.2.2.

The range of a values discussed above is based on experience of London Clay. In dealing with other overconsolidated clays (e.g. the Lias, Kimmeridge and Oxford Clays and in weathered marls) some caution is needed in assigning a values, and where previous experience is not available, loading tests designed to establish a values should be undertaken.

Piles driven into soft sensitive clays result in loss of strength in the clay in contact with the pile, and thus test loading should be delayed for as long as possible after driving, preferably at least a month, to allow thixotropic strength to be regained.

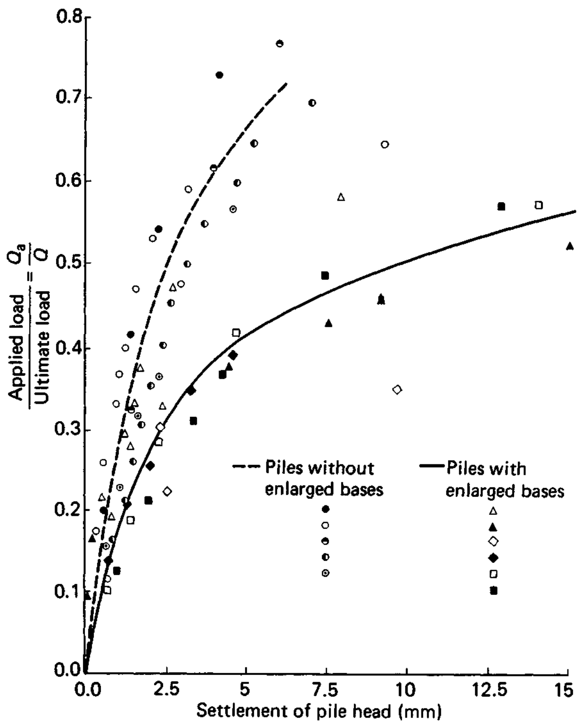

Figure A9.2.2 Q_a/Q against settlement in loading tests, showing the mean curves for piles with and without enlarged bases. (After Whitaker and Cooke (1966) 'An investigation of the shaft and base resistance of large bored piles in London Clay', *Proceedings, conference on large bored piles*. Institution of Civil Engineers, London)

Piles driven into stiff clay cause severe disturbance and fracturing to the clay and experience has shown that the effective cohesion can be extremely erratic, even on the same site (Tomlinson[62]). The lower limit of the adhesion factor, Tomlinson suggests, ranges from about 1.0 for soft clays to 0.2 for stiff and very stiff clays. When piles are driven to deep penetrations into stiff clays, the adhesion factor is influenced by the effective overburden pressure at any point in the pile shaft.[62]

Loading tests should be carried out whenever possible.

Piles driven into ground which is subject to consolidation from loading, or which is still settling under prior loading, are subject to downdrag forces (negative skin friction) which place an additional load on the toe. These forces can be severe and should be taken into account in the design and test loading. Negative skin friction can also occur, when driving piles through soft sensitive clays to bear on a harder stratum, owing to dissipation of the pore pressures induced by driving. Negative skin friction can be reduced by slicking the pile with the appropriate grade of bitumen. Johannessen and Bjerrum[69] recommend the evaluation of downdrag using effective stresses.

Appendix 9.3 Ground improvement

There are numerous cases in which the properties of naturally occurring soil or fill material can be improved or changed to help solve engineering problems arising either in temporary or permanent works. The methods of ground improvement cover a wide range of techniques – often referred to as geotechnical processes – and include compaction, moisture control, stabilization, grouting and reinforcement. Reference should also be made to the use of geotextiles for reinforcement, separation and filtration in the ground and of mild steel reinforcement strips to produce reinforced earth structures with increased shear properties in embankments and fills. Various other processes have been developed and the whole subject of ground improvement deserves special study starting, for example, with Chapters 29 to 38 of the *Ground engineer's reference book*.[70]

This appendix lists and gives a brief description of the main ground improvement methods.

A9.3.1 Drainage and water lowering

Drainage systems to control groundwater in engineering works may include seepage reduction measures, such as impervious barriers of sheet piles, membranes or grouts, together with drains incorporating a filtering material, sometimes enveloped within a plastic fabric filter. In sands and gravels, water-lowering systems can be used to allow an excavation to be carried out in the dry or to reduce the water pressure on the sides and base of the excavation.

A9.3.1.1 Site investigation

A thorough site investigation to establish the hydrogeological characteristics of a site is an essential preliminary to the design of any groundwater lowering project. This is discussed further in Chapter 11.

9.3.1.2 Permeability and filters

The permeability (transmissibility divided by the depth of the aquifer) can be determined by pumping from a large-diameter well while monitoring the drawdown in a number of adjacent observation wells. The shape of the drawdown–time curve is matched to type curves derived theoretically, and the transmissibility and storage coefficients so deduced. A wide range of available type curves enables varying aquifiers and hydrologic boundary conditions to be considered. Alternatively, for a fully penetrating well into an unconfined aquifier, permeability can be estimated from the equilibrium drawdown–distance curve, using Equation (A9.3.1). This latter method, which is the older of the two described, requires the establishment of equilibrium conditions, which may take many days; in contrast the time-variant method can be applied within a matter of hours after commencement of pumping.

Pumping tests give the most reliable value for k but an order of permeability can be obtained from grading curves (see Loudon[71]). Undisturbed samples of fine sands are essential in order to see if the material is laminated – a fact which naturally can have a marked effect on the horizontal permeability.

Grading curves are also used to choose suitable sand as a filter medium using Terzaghi's empirical rule which states that the grading curve for the filter material should be the same shape as that for the material to be filtered, and:

$$\frac{D_{15}\text{ (filter)}}{D_{85}\text{ (soil)}} \leqslant 4 \text{ to } 5 \leqslant \frac{D_{15}\text{ (filter)}}{D_{15}\text{ (soil)}}$$

where D_{15} and D_{85} are the grain sizes corresponding to those at which 15% and 85% pass on the grading curves.

A9.3.1.3 Pumping capacity

The quantity of water to be pumped from a fully penetrating well into an unconfined aquifier can be calculated from the equation:

$$Q=\frac{\pi k(H^2-h^2)}{\log_e R/A} \quad (\mathrm{m^3/s}) \tag{A9.3.1}$$

where k is the permeability in metres per second, H is the depth from normal water level to the impermeable stratum in metres, h is the depth from lowered water level to the impermeable stratum in metres, R is the radius of cone of depression in metres and A is the radius of circle of area equal to area surrounded by wells in metres.

R can be obtained from the empirical relation

$$R=30(H-h)\sqrt{k} \quad (\mathrm{m}) \tag{A9.3.2}$$

The number of wells required can be obtained from the empirical relationship

$$Q=3.63\times 10^{-6} r_0 h_0 n k^{\frac{1}{2}} \quad (\mathrm{m^3/s}) \tag{A9.3.3}$$

where r_0 is the radius of a well in metres. h_o is the water level outside a well in metres, k the permeability in metres per second and n is the number of wells required.

A9.3.1.4 Well points, suction wells and deep wells

Three systems of water lowering are in common use and each has certain advantages and disadvantages.

Well points When the lowering of the water level required is 4.5 m or less, a well point system can be used. If the system operates very efficiently, greater lowering can be obtained, but this should not be assumed at the planning stage.

A well point is a metal tube about 50 mm diameter carrying a gauze filter about 1 m long at its lower end. New types of well points are now on the market with slotted plastic outer covering and metal centre tubes. The well points are jetted into the ground at intervals of 1 or 2 m and then connected to a header main through which the water is extracted by a well-point pump for exhausting the main and well points, and a centrifugal pump to remove the water.

Well points are cheap to install and, for a long progressive excavation such as a pipe or sewer trench, they are usually the most economical system. If greater lowering than 4.5 m is required, a two-stage system can be used.

A special trenching machine is now available for laying a horizontal porous pipe of 100 mm diameter at depths down to 5 m below ground level. A pipe up to a maximum length of about 230 m can be installed, and for continuous trench work the pipes are overlapped by about 4.5 m.

Shallow wells These are, in principle, the same as well points but the wells are bored into the ground. The wells are usually about 600 mm diameter with a 300 mm filter tube and a 75 to 100 mm diameter suction pipe. The space outside the 300 mm tube is filled with a gravel filter as the boring tube is withdrawn. Because of their greater diameter, the wells usually can be spaced about 10 to 15 m apart, and since there are many fewer connections than in a well-point system, the efficiency is greater. The wells are connected to a ring main and a well-point pump. Alternatively ordinary suction-lift pumps can be installed individually in the wells for small excavations.

The cost of pumping is much the same for a given lowering of the water level from either a well-point or a shallow well system, but the cost of installation of the shallow wells is higher. The shallow well installation is often to be preferred for an excavation of rectangular shape (as opposed to a long trench) where pumping must continue for many months, and in fine sands where a graded filter is necessary or in laminated soils where a definite vertical connection between the aquifers is required.

As in the case of well-points, two-stage systems can be used for a lowering of more than 4.5 m but, in general, deep wells will prove cheaper.

Deep wells With the deep-well system lowering of the water level can be achieved in one stage. This is because the pumps are placed at the bottom of the wells and deliver the water against pressure; there is no suction lift. The pumps used are electrically driven submersible pumps.

The well cannot be less than 450 mm diameter, which allows a 75 mm thickness of filter gravel, and if a two-stage filter is desired the well will be 600 mm diameter. For this reason the capacity of the wells is much greater than in the case of shallow wells and they can therefore be spaced further apart, in general up to 30 m, but this will vary considerably on different installations.

The cost of a deep-well system is high but it generally gives safe dry excavation and can often reduce the time of construction considerably. For safety, two independent sources of electric power must be provided for the pumps, since once they have started pumping it might be disastrous if they failed.

A9.3.1.5 Vacuum drainage

In soils of low permeability, such as coarse silts, drainage can sometimes be effected by sealing the wells or well points and exhausting the air from them. The pressure of the atmosphere then acts as a surcharge on the soil causing it to consolidate, and water is squeezed out of the soil into the filters of the wells. The amount of water removed is very small but the increase in strength of the silt is marked, and excavation is greatly facilitated.

A9.3.1.6 Electro-osmosis

Electro-osmosis is a further drainage process which can be used in silts. It is based on the principle that if a direct electric current is passed through the soil a flow of water takes place from anode to cathode. The cathode is made into a well and the water which reaches it is pumped out. The amount of water removed is small. The success of the method depends, as with the vacuum method, on the fact that the flow of water is away from the excavation, the free water surface is lowered and the water which remains in the soil above this surface is in tension, and that the capillary tensions add greatly to the strength of silt. To some extent also the water content of the soil is reduced, thus resulting in an increased strength. Electro-osmosis is an expensive process and should only be considered if more normal methods of construction are inapplicable. In many silts the vacuum method of drainage is probably nearly as effective and much cheaper.

A9.3.1.7 Settlements caused by water lowering

When the water level is lowered the effective weight of the soil between the original and the lowered water levels is increased because the buoyancy effect has been removed. Where the soil concerned is sand and gravel any settlements due to this increase in weight are normally small, but where silt, clay or peat occurs in the zone referred to, settlement will occur with time owing to the consolidation of this material under its own increased weight. Advantage is taken of this in the methods given in section A9.3.2 to accelerate settlement.

Before installing a groundwater lowering system it is essential, therefore, to consider what effect such settlements may have on structures within the zone of influence. Important structures will probably be founded below the compressible material, either directly or on piles, and will be unaffected. For structures founded on the compressible strata, it is necessary to calculate the probable settlement and to estimate what damage, if any, to the structure would result.

In order to limit the radius of influence of a groundwater lowering system, some of the pumped water can be 'recharged' or fed back into the aquifer by means of infiltration wells sited close to the structure below which it is desired to limit the potential settlement.[72]

A9.3.2 Vertical drains to accelerate settlement

Vertical drains are used to accelerate the settlement of layers of soft clay or silt under applied loads. In many cases settlement can be tolerated provided it occurs quickly, preferably during the construction period, e.g. road embankments on soft clay (Figure A9.3.1).

The rate at which a uniform thin clay layer consolidates is inversely proportional to the square of the drainage path, which is either the thickness or half the thickness of the layer depending on the drainage conditions. The principle of this method is to provide vertical drains in the clay. The drainage path is then reduced to half the spacing of the drains. When the load (e.g. a fill) is applied, the settlements take place quickly in, say, a few months instead of many months or even years. It is important to note that vertical drains do not reduce the amount of settlement.

Vertical drains are particularly effective in deposits where the horizontal permeability is high compared with its vertical permeability. However, care must be taken to avoid local reduction of horizontal permeability by the process of installation of the drain. In deposits of exceptionally high lateral permeability, vertical drains may not be necessary. It is important, therefore, to investigate the horizontal drainage characteristics with great care.[73]

The consolidation of the clay under load increases its strength, a fact which can sometimes be made use of by construction, in stages, of a fill which would cause foundation failure if placed in one operation.

A9.3.2.1 Vertical sand drains

Vertical sand drains usually have been formed by driving a hollow mandrel and filling the space formed by the forcibly displaced soil with sand as the mandrel is withdrawn. Jetting and augering methods have also been used. The drains are generally between 0.15 and 0.5 m in diameter at between 2 and 5 m centres and up to 30 m long, depending on conditions. A horizontal drainage blanket is required at ground level to link the vertical drains together before the fill is placed, unless the fill itself is permeable.

The disturbance of the foundation soil during the installation of vertical sand drains is an undesirable feature, particularly if the method of installation could cause remoulding of sensitive clays. Various attempts have been made to develop thinner drains to reduce disturbance, such as the band drains described in section A9.3.2.3.

A9.3.2.2 Prefabricated drains (sandwicks)

The original Kjellmann wick drain of treated cardboard has been replaced by the use of fabric stocking filled pneumatically with sand or grooved plastic cores with nonwoven textile or geotextile filter coverings. Several different designs of plastic drains are available.

A9.3.2.3 Prefabricated band drains

An extension and, in many respects, an improvement on the prefabricated sandwicks are the band drains[74] which consist of a flat core with internal drainage grooves surrounded by a filter

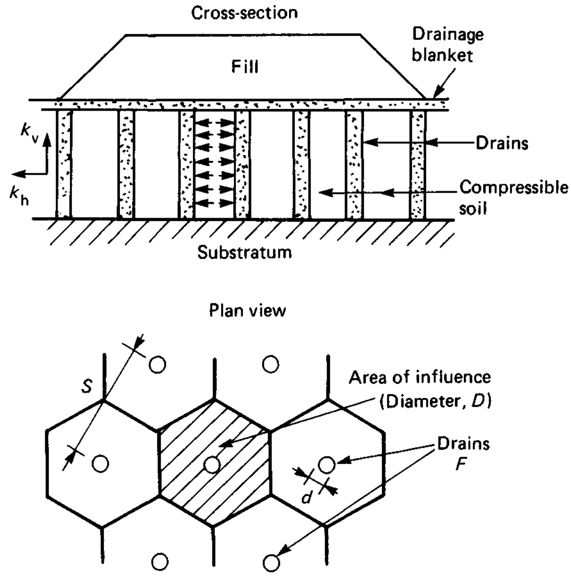

Figure A9.3.1 Typical vertical drain installation. (After Gambin (1987) 'Deep soil improvement', in: Bell (ed.) *Ground engineer's reference book* (Figure 36.6). Butterworth Scientific, Guildford)

cover, hence the name 'band'. The mandrel used to place these prefabricated bands may be circular, rectangular or of any other convenient section that reduces the disturbance of the surrounding soil. The equivalent diameters of band drains lie between 5 and 10 mm with a typical spacing of 1.2 to 1.5 m and a maximum length of 60 m (Figure A9.3.2).

The particular advantages of band drains are their simplicity, speed and cost of installation, together with minimum disturbance of the foundation soil.

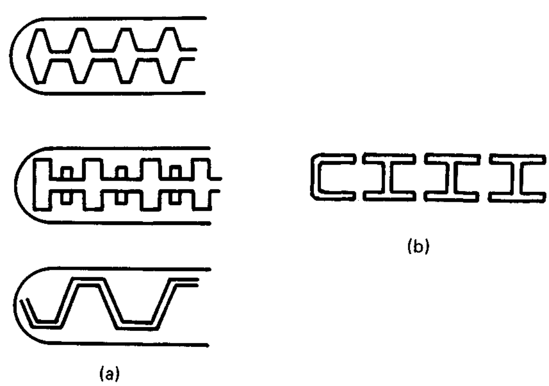

Figure A9.3.2 Examples of cross-sections of plastic band drains. (After Gambin (1987) 'Deep soil improvement', in: Bell (ed.) *Ground engineer's reference book* (Figure 36.10). Butterworth Scientific, Guildford)

A9.3.3 Exclusion of groundwater

Retaining systems excluding groundwater from a site, or from an area of a site, include sheet piled walls, *in situ* concrete diaphragm walls and contiguous bored pile diaphragm walls. The use of compressed air to exclude water from underground workings is another well-developed technique. Freezing and grouting are further possibilities which have the added potential advantage of strengthening the ground locally.

A9.3.3.1 Sheet piling

The use of standard interlocking steel sheet piles is described in Chapter 17 (section 17.3.2.5) in relation to the construction of basements. For all except shallow excavations, the interlocking piles require support either by struts, shores or by the use of ground anchors.

A9.3.3.2 In situ *concrete diaphragm walls*

Reinforced concrete diaphragm walls can be constructed to considerable depths by placing concrete by tremie tube in a narrow trench supported by bentonite mud. The trench is excavated in short panels, 2 to 7 m in length, using special cutters with circulating mud, or grabs and stationary mud. Junctions between panels are formed by means of steel tubes acting as end shutters. The mud displaced by the rising concrete is used in subsequent panels provided it is still in good condition.[75] It is possible to construct such walls in all types of ground, though difficulties have been known to occur when concreting in very soft alluvium owing to displacement of the clay under the high head of liquid concrete. In very permeable ground, significant mud losses may occur.

A9.3.3.3 Contiguous bored pile walls

Walls consisting of soldiers of bored piles can also be made with or without the use of mud depending upon the ability of the ground to support itself. The spacing can be varied and in the case of contiguous bored pile walls each is bored slightly into the completed adjacent pile while the concrete is still 'green'.

A9.3.3.4 Compressed air

The use of compressed air is well known as a construction expedient in underground work. It can be used in sands and gravels, silts and clays. The air pressure, acting on the surface of the soil in the excavation or, more correctly, on the water surfaces in the voids of the soil, prevents the flow of water through the soil and acts as a support. The air pressure theoretically must be equal to the water pressure. In practice, a pressure somewhat lower than the theoretical is often satisfactory. In gravels, the losses of air through the gravel may be serious and these can sometimes be cut down by injections of clay suspensions into the gravel before commencing excavation in order to reduce the permeability.

The cost of compressed-air working can be high in areas of silt, fine sand and some clays which require considerable support. However, it is often the most effective method for subaqueous tunnels in soft ground. Health hazards in compressed-air working, as in diving (Chapter 42), include the 'bends', and in the longer term, bone necrosis. The CIRIA medical code[76] should be applied, using the appropriate decompression procedure and equipment. See also section 17.3.4.4.

A9.3.3.5 Ground freezing

Another temporary method of preventing the access of groundwater to excavations and of strengthening the soil locally is by freezing the water. This is normally done by boring vertical holes into the ground, installing pipes in them and circulating brine or cryogenic liquids, cooled to below the freezing point of water, through the pipes. The freezing process is expensive and slow but once the water is frozen excavation can safely take place inside the frozen ring. The freezing must, of course, be continued until the permanent work is completed. One of the disadvantages of brine is that if a leak occurs in the pipes it will escape into the groundwater and it may then prove impossible to freeze it. To overcome this objection the Dehottay process was introduced, in which liquid carbon dioxide is circulated instead of brine. More recently, liquid nitrogen has been used. The freezing process in general is applied to narrow, deep excavations such as mineshafts, but cases are on record of its use in foundation work (Figure A9.3.3).

A9.3.3.6 Grouting

Injection processes using various types of grout are dealt with separately in section A9.3.4 because of their wide applications in reducing permeability, increasing strength and reducing compressibility in soils and rocks.

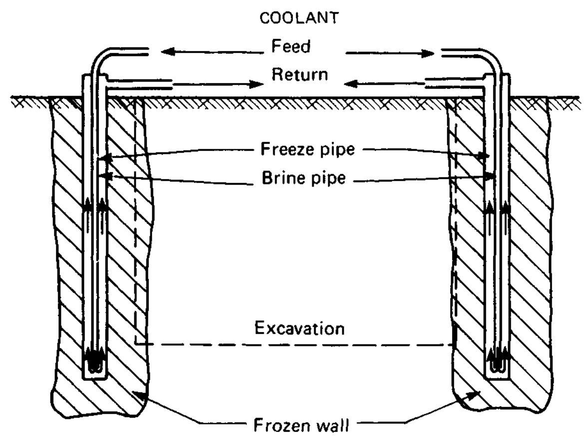

Figure A9.3.3 Scheme of ground freezing for support of an excavation. (After Jeffberger (1987) 'Artificial freezing of the ground for construction purposes', in: Bell (ed.) *Ground engineer's reference book*. Butterworth Scientific, Guildford)

A9.3.4 Injection processes: grouting

It is sometimes possible to change the properties of the ground encountered by injecting materials of various sorts into the voids of the soil. These changes include: (1) reduction in permeability; (2) increase in strength; and (3) decrease in compressibility, or a combination of these. A major use is for filling voids in mine workings and karstic limestone.

Cases in which the reduction in permeability is important include: (1) the formation of grouted cutoffs under dams; (2) grouting fissured rocks; (3) grouting sand and gravel to reduce air losses during construction work in compressed air; and (4) sealing gaps in sheet piling. The increase in strength is important in underpinning problems and in support of excavation in tunnelling. Injection processes can also be used to lift tanks and pavement slabs by hydraulic pressure, the grout later setting and supporting the structure in the raised position.

A9.3.4.1 Materials that can be grouted

Grouts can be injected into the fissures in a rock. This is probably one of the earliest applications of the process. Rockfill and rubble masonry can also be grouted. Cement grouts, containing sand or PF ash, are used in cases where the voids are fairly large.

Gravels and sands can be grouted successfully by a variety of different processes as described below, but clays and silts cannot because their voids are too small and their permeabilities too low. An exception to this is the use of the technique called claquage grouting, in which tongues of high-pressure grout penetrate planes and zones of weakness within the soil body.

A9.3.4.2 Grouting materials

Grouts generally consist of suspensions, solutions or aerated emulsions, the choice depending upon the nature of the work and the type of material to be grouted.

Suspensions The most commonly used grout is cement in water but, because of its high sedimentation rate, it can be relatively unstable, depending on the water/cement ratio. Pure cement grouts cannot be used for injecting sands or clays. Suspensions of cement with bentonite (e.g. in the proportions of 4 or 5:1) are more stable, easier to pump and, generally, produce a better result than pure cement grouts. If the voids to be filled are sufficiently large, sand is added to reduce shrinkage and cost. Another group of grouts comprises suspensions in solutions, e.g. in a solution of sodium silicate. An example would be a combined cement–bentonite–silicate grout.

Solutions There are several grouting processes in which solutions of chemicals based on sodium silicate are injected. The chemical processes can be used down to the fine sand range. They divide into the two-solution and the single-solution processes.

In the two-solution processes (Joosten and Guttman processes) the first chemical injected is sodium silicate, and this is followed immediately by calcium chloride or some such salt. The reaction is almost immediate and for this reason the solutions cannot penetrate far from the injection pipes which are therefore spaced at about 600 mm centres. The process gives considerable strength to the soil and also reduces the permeability to a very small fraction of its previous value.

In the single-solution processes, two chemicals are mixed before injection, possibly with a third chemical to delay the setting action for some time. The injection pipes can therefore be further apart. The processes reduce the permeability but do not give strengths comparable to the two-solution process.

In addition, a range of 'liquid' grouts is available, having acrylic, formaldehyde, lignin and epoxide bases. These grouts have low viscosities and therefore considerable penetration power, and achieve comparatively high strengths. They are, however, more expensive than the common grouts.

Aerated emulsions These are cement- or organic-based grouts into which a gas is emulsified. The properties of the resulting foam depend upon the distribution of the gas bubbles which, in turn, depends upon the materials and method of preparation. The foams are not particularly strong and this type of grout is used mainly as a filling.

Other types of grout Cement grouts with a low water/cement ratio (e.g. 0.4) are stable and pastelike in consistency. For grouting purposes, they can be made sufficiently fluid by the use of admixtures, such as plasticizers and swelling agents, or by vigorous stirring. 'Colgrout' is an example of the latter treatment and 'Prepakt' of the former. As the high potential strength of these low water/cement ratio grouts is often unnecessary, a large proportion of the cement can be replaced by pulverized fuel ash (PFA).

Bituminous emulsions can also be used as grouts, e.g. to reduce the permeability of soils down to the fine sand range.

A9.3.4.3 Methods of grout injection

In nearly all grouting work the injections are made by drilling or driving pipes into the ground and pumping the grout in under pressure through hoses attached to the pipes.

The spacing of the pipes varies widely with the process and the conditions, from 600 mm for the two-solution chemical process in sand, to about 3 m for clay injections in alluvium, and up to 6 m or more for cement grouts in fissured rocks.

When filling fissures in rock with cement grout it is usual to use piston pumps which will give a pressure up to 7500 kN/m^2. The same pumps can be used for clay injections with alluvial sands and gravels but the pressures must be controlled carefully in relation to the depth and nature of the overburden to avoid undue lifting of the ground surface. If the limitation of ground heaving is important, suitable instrumentation for the monitoring of heave may be necessary. In the Joosten and Guttman two-solution chemical processes the amount of grout required to fill the voids in the soil between injection pipes is pumped in with less regard to the pressure, subject to a maximum pressure of about 3000 kN/m^2. Piston pumps are used.

For very simple grouting jobs, a grout pan may be used. The cement grout is mixed in the pan by a paddle driven by hand or by a compressed air motor, an air pressure up to 750 kN/m^2 is then applied to the surface of the grout in the pan and this drives the grout through the hose into the injection pipe and so into the ground. This suffers the limitation that the injection pressures cannot easily be varied to suit the ground conditions.

For more complex jobs, sleeve grouting is frequently used using a 'tube-à-manchettes'. The system comprises a PVC tube of about 30 mm bore which is installed into a borehole of about 90 mm diameter and sealed into the ground with a relatively weak bentonite–cement sleeve grout. The tube is equipped at short intervals, normally 300 mm, with rubber sleeves covering perforations. An injection device is located against selected perforations in turn between upper and lower packers. The grout lifts the sleeve, fractures the sleeve grout and enters the ground. With this device it is possible to return to any position and regrout.

Grouting work in general is not simple and damage can be caused by the indiscriminate use of high pressures by inexperienced operators. The work should be planned and carried out by engineers and operators experienced in the use of grouting methods. The results need to be observed and monitored stage by stage.

A9.3.5 Reinforced soil

An example of reinforced soil as a constructional material consists of frictional soil backfill reinforced by linear flexible strips, usually placed horizontally. It was introduced by Vidal[77] in 1963 and has been since developed into a system comprising interlocking precast concrete or metallic wall-facing panels to which are fixed 5 mm thick galvanized ribbed mild steel strips, which provide the reinforcement. The facing plays no structural role, apart from helping to retain the backfill as it is compacted in layers and in locating the reinforcement strips in the backfill under compaction.

The performance of a reinforced soil structure depends on the friction developed between the soil and strip.

The choice of galvanized ribbed mild steel, instead of other metals or plastics, is based on durability, friction, creep and elastic property considerations.

Another form of reinforced soil is the use of woven plastic mesh placed on and wrapped around successive layers of compacted fill in embankment construction.

In Chapter 17, section 17.5.9, further information on both systems is given.

The range of uses for reinforced soil include retaining walls, sea walls, dams, bridge embankments and foundation slabs.

A9.3.6 Geotextiles

'Geotextile' is the generic name given to a wide variety of materials based on synthetic fibres, such as polyester, polyethylene, polypropylene, polyamine, etc. They may be woven, needle punched or formed into nets. Their potential applications in ground engineering are separation, filtration, drainage and reinforcement.

Most progress has been made in the use of these materials for filtration and drainage, but geotextiles are relatively new materials that are still developing. Their use needs to be considered in geotechnical applications.

A9.3.7 Ground anchors

Rock anchors and bolts are discussed in Chapter 10 but there are some similar requirements for anchorages and ties in ground engineering.

Diaphragm and pile walls are thin and generally require support which nowadays is provided by anchors, rather than strutting and bracing, as this facilitates excavation. However, for narrow excavations, strutting is often cheaper. With modern boring and injection techniques it is possible to install anchors at reasonably flat angles in all manner of soils.

The method comprises boring a hole using augers in clay and rotary percussion with water flushing and casing support in granular soils. Bar or strand is inserted into the hole and the predetermined anchor length grouted up with neat cement under pressure, the free end of the bar being sleeved off. The anchor can be stressed to loads in excess of the working load, if necessary, to test its capacity. When pulled to failure, special test anchors are installed.

The design procedure in clays is somewhat similar to that for bored piles. In sands, a semi-empirical approach is used based upon the density and grain size of the sand, the overburden pressure and the injection pressure.[78]

A9.3.8 Deep ground improvement

A variety of methods is available to improve the bearing capacity and decrease the compressibility of natural soils and manmade fills on site. They include preloading, vibro or dynamic compaction and the use of stone columns.

A9.3.8.1 Preloading

Improvement of soils by preloading is one of the techniques. The method is most applicable to loose sands, silts and waste materials. The types of work for which the method is most appropriate are those in which column loads will be relatively low, such as for embankments, low-rise buildings and light industrial developments (see Chapter 17, section 17.2.7).

The preload is applied by surcharging with imported fill, or water tanks, for the period of time necessary to achieve the required precompression. If it is required to accelerate the process, consideration should be given to the use of vertical drainage in conjunction with preloading. The surcharge load is restricted by the stability of the original ground but, if necessary, can be increased in stages as the ground improves with time.

A9.3.8.2 Vibrocompaction

Vibrocompaction (vibroflotation) is used for the deep compaction of cohesionless soils and fill materials to achieve improvements at depths to 20 to 30 m (see also Chapter 17, section 17.2.7). Increases achieved in density are greater for coarse grained than for fine grained material. The equipment consists of a probe (vibroflot) of about 400 mm diameter, fitted with a vibrator giving horizontal amplitudes of 2 to 12 mm at between 30 and 50 Hz. The probe and its extension tubes are lowered by crane at penetration rates generally between 1 and 2 m/min until the required depth is reached. The tip of the probe has jetting holes for water supplied under pressure to assist penetration. Granular backfill is sometimes placed over the area of treatment and used as supplementary fill for the hole as the probe is withdrawn.

Other forms of vibrocompactors apply vertical vibrations using a vibrator, similar to that for piledriving, to drive a steel pipe of I-beam section into the material. However, this method is less effective in compacting the top 2 or 3 m (Figure A9.3.4).

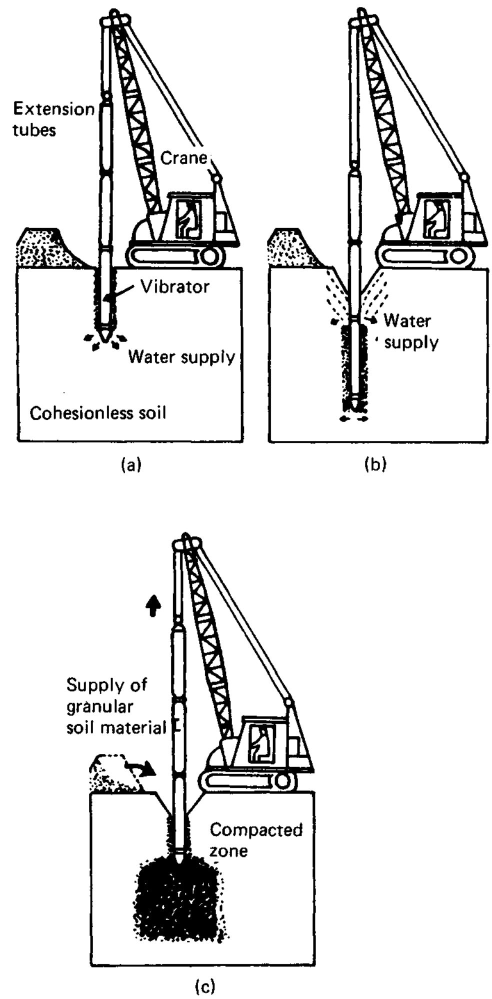

Figure A9.3.4 The vibrocompaction process. (After Gambin (1987) 'Deep soil improvement', in: Bell (ed.) *Ground engineer's reference book.* Butterworth Scientific, Guildford)

A9.3.8.3 Installation of stone columns

Cohesive materials and layered systems that are unsuitable for vibrocompaction can be reinforced by sand, gravel or stone columns formed by the vibrating probe or tubes driven by vibrators on top.

In partly saturated clays or fully saturated nonsensitive clays, the deep vibrating probe displaces the soil radially and the hole so formed is filled with well-graded 75 to 100 mm angular stone. No water is used in the probe, but compressed air is necessary for extraction of the probe. The fill is placed in layers, each layer being compacted by re-inserting the probe. The stone columns of 600 to 800 mm diameter allow partial mobilization of the passive resistance of the soil at small horizontal radial strains. In very soft cohesive soils (undrained shear strength 30 kN) the stone columns are formed by replacement of the soil under water pressure. The vibrating probe is operated in a similar way as for deep compaction, using the water jets, and the hole is washed out by repeated up and down movements of the probe. The stone column is formed and compacted as described earlier.

In very weak clays it is possible to couple three or four probes together to form a sufficiently stable hole to backfill with large self-supporting stone columns.

A9.3.8.4 Dynamic consolidation

A method of compaction introduced by Menard involves the dropping of a heavy weight, such as a concrete block or an assembly of plate steel of up to 20 t, from heights of about 20 m on to the ground. The compactive effect can reach depths in excess of 10 m. It is a method applicable particularly to the more freely draining soils and can strengthen cohesive soils although, with saturated cohesive soils, a combination of dynamic compaction and vertical sand drains may be necessary. It can also be used with manmade fills, including industrial wastes and some domestic waste tips (see Chapter 17, section 17.2.7).

Tamping weights can be selected to suit the depth and extent of improvement required. The site is first covered with a working blanket of free-draining material, about 1 m thick, and several impacts are applied at each centre in a predetermined grid. Several coverages of the area, including a perimeter strip, are required at intervals of up to several weeks, to allow the pore water pressure to dissipate.

A9.3.9 Shallow compaction

Shallow compaction refers to the compaction of material in layers, typically of 200 to 250 mm or less, in the construction of embankments, earth dams, pavement bases and sub-bases and in the process of fill, including the disposal of certain types of waste. Compaction increases the resistance to deformation, reduces permeability and increases the shear strength of a material. The principles of achieving a well-compacted material are illustrated by the laboratory soil compaction tests described in section A9.1.3. The maximum density achievable, measured either in terms of dry density or air voids content, depends upon the characteristics of the material, the moisture content at which it is compacted and the compactive effort applied, the latter depending on the number of passes as well as the weight or vibrating energy of the roller.

The most commonly used rollers were of the deadweight type (up to 20 t) including smooth-wheel, sheepsfoot or tamping rollers with variations such as grid rollers and pneumatic-type rollers which could apply higher local pressures. Vibrating rollers are now more common, ranging from self-propelled pedestrian-operated rollers to self-propelled and towed vibrating rollers of up to 20 t. Other important items of compaction equipment are vibrating plate compactors but, in addition, there are power rammers, dropping weight compactors and, more recently, an impact roller.

The choice of compaction equipment depends on the characteristics of the material to be compacted. Smooth wheel deadweight and vibrating rollers are suitable for most materials, but grid rollers and pneumatic tyred compactors are generally unsuitable on uniformly graded granular materials and silty clays.

Although the ideal is to bring the moisture content of material to be compacted to the optimum value determined by test or, preferably, by field compaction trial, adjustment of moisture content is difficult and sometimes impossible. In arid areas, compaction at moisture contents below optimum may have to be accepted. Where an adequate supply of water is available, the moisture content of the soil can be increased by mixing water into each layer using disc harrows or cultivators; however, quality of results may vary. In temperate and other countries with wet and dry seasons, earthmoving and compaction is usually confined to certain parts of the year.

A9.3.10 Soil stabilization

The term 'soil stabilization' is applied to a range of treatments which improve the properties of the existing ground materials, including changing their grading (mechanical stabilization) and chemical action (chemical stabilization). Ground freezing and grouting, described in section A9.3.3, are also forms of soil stabilization.

A9.3.10.1 Soil–cement

Soil stabilization includes treatments with cement, which can produce materials of considerable strength. For example, the flexural strength and elastic modulus of a fine grained soil–cement may be in the order of 0.5 to 1.5 MN/m^2 and 5 to 15 GN/m^2 respectively. For coarse grained soil–cement (e.g. cement-bound granular material) the strength and elastic properties approach those of lean concrete.

A9.3.10.2 Bitumen stabilization

Bitumen has also been used for stabilization in arid climates, but it has little application in moist soil conditions. Addition of bitumen to a granular soil or crushed rock improves its cohesion and resistance to water penetration. The most usual form of addition is bitumen emulsion but foam bitumen is a recent development.

A9.3.10.3 Lime stabilization

Lime stabilization is widely used, particularly in the warmer climates and developing countries. The reaction of lime with the clay content of a soil rapidly reduces the soil's plasticity; but the subsequent gain in strength of the soil–lime mix is slower and less than that obtained with cement. In some cases, lime is used to modify the properties of a soil rather than to produce a material with appreciable strength, i.e. soil modification rather than soil stabilization.

A9.3.10.4 Methods of construction

The most commonly used forms of stabilization – mechanical, cement, bitumen, lime – all require efficient mixing and compaction. Their main application is for road, airfield and other pavement areas, but soil–lime and soil–cement have been used to provide stable fill and embankments.

Mix-in-place methods, where the required thickness of the stabilized layer is less than about 200 to 250 mm, are generally the most appropriate, using single-pass or multipass machines with blades or tyres, similar to, but more sophisticated than, agricultural cultivators, or travelling mixers which pick up the soil from preformed windrows into cylindrical-drum mixers. Most of these specially developed machines have the capacity to disperse controlled amounts of water and to bring the mix to optimum moisture content; others each include a cement dispenser, otherwise the required amounts of cement and water are spread ahead of the mixer.

While it is possible to stabilize successive layers by mix-in-place methods, it requires considerable care and control. Mixing by central stationary mixing plant is often preferred, using either batch or continuous mixers fitted with paddles or blades to give a positive mixing action. Free-fall-type concrete mixers are not suitable. A range of spreading equipment is available to spread the mixed material *in situ* prior to compaction.

Compaction is usually by smooth wheel or vibrating rollers, but impact compactors are also available. Maximum density at optimum moisture content is the requirement for strength and durability (see section A9.3.9).

A9.3.10.5 Limitations to soil stabilization

Not all soils or other materials are suitable for stabilization and the following gives some indication of the main limitations.

Grading Uniformly or poorly granular soils which cannot be compacted adequately; stones should generally be less than 75 mm.

Plasticity Cohesive soils with LL > 45% cannot be mixed efficiently by most mix-in-place plant; soils containing more than about 10% plastic fines cement cannot be mixed effectively by most stationary mixers.

Organic content Peaty soils are unsuitable and soils with more than about 1% organic content give low strength with lime or cement.

Sulphate content Unsuitable for lime or cement stabilization without special precautions.

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10

Rock Mechanics and Rock Engineering

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Contents

10.1 Introduction

10.1 The scope of rock mechanics and rock engineering

Rock mechanics is a term for science and engineering applied to rock masses. As such, the term has relevance in numerous fields such as the recovery of hydrocarbons in rock reservoirs, development of geothermal energy resources, studies of the Earth's crust, seismicity studies, as well as mining and civil engineering. The area of activity restricted to construction works which require or essentially comprise excavation into the surface of, or within, rock masses might appropriately be referred to as rock engineering. Theoretical methods of analysing the behaviour of rock, based on the understanding of certain material or mass properties, have advanced rapidly in recent years. This has been driven partly by the need for effective nuclear waste management strategies. In turn, engineering practice as applied to construction in rock is also advancing, albeit at a slower pace because design processes often depend upon the use of empirical rules based on established precedents and on the knowhow gained from practical experience.

Civil engineering works which impose significant foundation loads on to rock, such as dams or massive nuclear containment buildings, or which involve an excavation such as a pit or cavern, demand that rock engineering principles be applied in order to achieve a stable structure. In many cases, such as tunnels, deep road cuttings, hydro-powerhouse caverns, storage caverns and underground repositories, deep bunkers, existing cliffs or natural cavities, the 'structure' is largely formed of the rock mass and its 'design' will have to take into account the inherent flaws, variability and weaknesses of the natural constituents.

For many engineering projects the main steps followed are:

(1) Investigation to determine geotechnical properties.
(2) Classification and characterizing the site.
(3) Initial design.
(4) Excavation and support.
(5) Performance monitoring and design re-evaluation.

For the investigation, the compiling of relevant geological and geotechnical information is facilitated by the introduction of standard schemes for terminology and methods (see Chapter 8). A number of rock classification schemes assist in the assimilation of data and characterizing of sites (see section 10.3).

Design methods which make use of advanced numerical calculations are now widely available as computers have become commonplace, although they have not necessarily replaced other methods. Rock conditions can be modelled with some degree of realism in order to predict rock behaviour when numerical methods are appropriately applied. However, uncertainties will inevitably exist concerning the geology, the appropriateness of the design model, the parameters used (the scatter may reflect real variability or random errors), the loads applied and unforeseen factors. Various statistical methods and risk and reliability analyses may be useful as aids to assess confidence limits, but they do not replace the need to monitor actual performance. Further benefits from computer technology have been derived from using automated data acquisition and results analysis for the performance monitoring of rock structures which provides feedback for design assessments,[1] as shown on Figure 10.1.

Improvements in excavation methods are evident with increasing production rates as rippers, excavators, tunnel boring machines and roadheader machines have become more powerful and resilient. Blasting practice has advanced through the continued development of hydraulic drilling machinery and a better understanding of explosives usage in obtaining the required fragmentation with the least damage to the surrounding rock forming the structure. Particular advances have been made in construction procedures to form large excavations with the least disturbance to the surrounding rock by applying an improved understanding of rock mechanics stress analysis to the sequencing of excavation and support works. Support is provided either actively, i.e. with preload applied, or passively by rock reinforcement using dowels or rockbolts, rock anchors, shotcrete, concrete or steel supports or cast *in situ* concrete liners. These expedients may be used separately or in combination, or in conjunction with grouting or drainage measures.

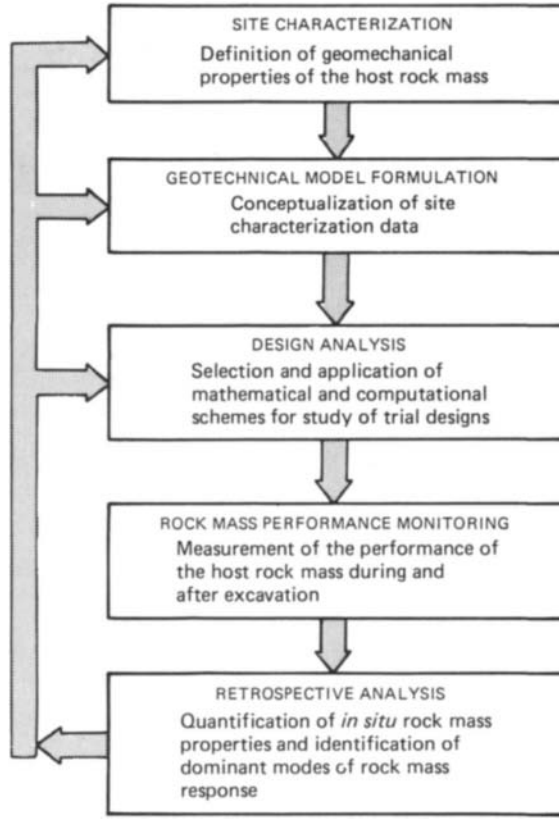


Figure 10.1 Components of a generalized rock mechanics programme. (After Brown (1985) 'From theory to practice in rock engineering'. *Proceedings, 4th international symposium on tunnelling 85.* Institute of Mining and Metals, London)

10.1.2 Rock engineering principles

The principles of rock engineering have developed from the disciplines of civil and mining engineering, structural geology, engineering geology and, where possible, the practical application of analytical rock mechanics. All of these subjects are developing continually, with papers describing research, theory and practice being published frequently. Some relevant journals and text books are listed in the bibliography to this chapter together with abstract bulletins that relate to rock mechanics topics. The International Society for Rock Mechanics (ISRM) has established commissions to attempt the standardization of terminology and test methods and to investigate various topics including rock mechanics teaching, research and rock classification.

In addition to engineering knowhow, an overall appreciation of relevant aspects of geology (see Chapter 8), is an essential prerequisite to enable rock engineering principles to be applied effectively. The relevant *properties* of the rock essentially must be determined so that its *behaviour* can be predicted adequately and subsequently monitored. Parameters describing the following are needed to describe or characterize the rock mass for an engineering project and it is important to appreciate how these are linked together and interact. They are:

(1) *Rock mass structure*: intact rock and discontinuities (mainly strength and deformability).
(2) *Hydrology and void space*: groundwater flow, permeability and pressure.
(3) In situ *rock stresses*: principal stress magnitudes and directions.
(4) *Construction*: excavation and support sequence and methods.

A rock mechanics or rock engineering interaction matrix has been devised[2] and is shown in Table 10.1. These paremeters are further described below and are dealt with later in this chapter.

10.1.3 Rock mass structure

Geological appraisals allow the presence of important features of engineering relevance to be anticipated and, if peculiar unexpected conditions are later encountered, will serve as a warning that something is wrong. Knowledge of the origin of rock at a site, whether sedimentary, metamorphic or volcanic, will immediately suggest certain properties such as respectively bedding features, local zones of alteration or characteristic fracture patterns. Geological processes or upheavals known to have affected a site will all serve as pointers to the likely prevailing rock conditions. Examples would include folding, faulting, heating and cooling, tectonic compressive and tensile stresses, extreme loading by ice age cover, weathering, surface denudation relieving stresses, solutioning and chemical alteration causing volume and, hence, stress changes. Subsequent site investigations can thus be scheduled accordingly. Awareness during investigations or construction works of the 'expected but unpredictable', such as solution features in limestones, is also extremely helpful.

The geological appraisal might also assist at an early stage in the selection of an appropriate analysis method for designing the proposed works, in deciding on essential parameters to be measured and on whether or not material should be modelled as a continuum. For example, it will be known whether intact material is likely to be uniform, or zoned into bands or graded units by metamorphism or sedimentation such that possible anisotropic material properties can be predicted. Regular discontinuity patterns may exist as joint sets, bedding and cleavage planes, or irregular fracture orientations may be present due to faults and slump features. The likely planarity or undularity and persistence of discontinuities can sometimes be anticipated from their geological origin.[3,4] The mechanical behaviour of rock is controlled very largely by the presence and characteristics of the discontinuities contained within it.

The mechanical behaviour of intensely fractured rock can sometimes be approximated to that of a soil. At the other extreme, e.g. in very deep underground works where the rock is massive and the fractures confined, the rock can sometimes be considered as a continuous medium. More often, rock must be regarded as a discontinuum in which the mechanical properties of the discontinuities are of considerable relevance. Discontinuities also play a major role in determining water and stress environments in the mass.

Roughness, *aperture*, *rock wall strength* and *filling* in particular control the shear strength and deformability of fractures.[5] Even a thin weathered layer in a joint will considerably reduce the strength which is otherwise afforded by tightly interlocking roughness asperities.[6] Discontinuities that *persist* smoothly and without interruption over extensive areas, e.g. certain types of fault and bedding plane, offer considerably less resistance to shearing than discontinuities of irregular and interrupted pattern.[7] The *orientation* of fractures relative to the exposed rock surface is also critical in determining rock mass stability. Techniques are available for rapid assessment of measured fracture orientations using stereogrammetric methods for presentation of fracture orientations in statistical and graphical forms and for processing this type of data in rock mechanics calculations.[8-10] Fracture *spacing* (typically 20 to 2000 mm) and the *number of sets* of joints are important, since they determine the *block size* of the rock mass. Fracture spacing is relevant to problems of excavation stability and the design of bolted support systems, also in determining the ease or difficulty in rock excavation and in establishing its suitability for use as construction material.

10.1.4 Hydrogeology and void space

Intact rock material contains grains and intergranular pores

Table 10.1 Rock engineering interaction matrix. The four main parameters are drawn on the leading diagonal of the matrix and the cause–effect interaction of these is demonstrated by a clockwise motion to each off-diagonal element of the matrix. (After Hudson (1986) 'Rock engineering interaction matrix', *Symposium on rock joints*. Geological Society, London)

ROCK MASS STRUCTURE	Discontinuities influence permeability	Strong rock sustains high stress	Rock conditions dictate support requirements
Water flow causes weathering and erosion	HYDROGEOLOGY AND VOID SPACE	Water pressure reduces total stress	Drainage or grouting works
Stress induces fractures	High stresses influence fracture apertures	IN SITU ROCK STRESS	Adjust excavation geometry
Blast damage and convergence	Modify water flows and pressures	Induced stresses	CONSTRUCTION

filled with air and water. The relative volumes and weights of these three constituents determine porosity, density, saturation and various related parameters as shown on Table 10.2. The presence of pores and pore water in the fabric of a rock material generally decreases its strength and increases its deformability. A small volume fraction of pores and the associated pore water pressure can produce an appreciable mechanical effect.[11-13]

The rock mass contains void space in the form of fissures as well as pores. The volume occupied by fissures is usually much less than that of pores, however, so that the porosity and bulk density of the rock mass and of intact material are usually similar. Fissures, however, have a much greater influence on mass permeability.[14] Fluid migration in rock masses is generally regulated by the intersecting network of conducting fissures which afford a less tortuous path for water flow than does a network of pores. It is possible usually to ignore the effect of intact material permeability in studies of water flow, except in some coarse-grained sedimentary rocks.

Construction works can cause the permeability of a rock mass to alter significantly depending on the deformations experienced by the joints. The hydraulic behaviour (i.e. conductivity) of rock joints is influenced by changes in effective normal stress because of the mechanical interaction between roughness, wall strength and aperture. The construction of dams, tunnels, caverns and slopes in jointed, water-bearing rock causes complex interactions between joint deformation and effective stress, so that deformation can take the form of normal closure, opening, shear and dilatation. The resulting changes in aperture can cause as much as three orders of magnitude changes in hydraulic conductivity of joints at moderate stress levels. This could influence both the design and scheduling (i.e. time of construction) of a grouted cutoff beneath a dam, for example.

The pressure of water in fissures imposes normal loads on the fissure walls and so forces are applied to the discrete rock blocks. These forces must be accounted for especially in stability analyses of rock slopes and in considering uplift forces in dam foundations. Groundwater pressure is measured with piezometers but, where fissure flow is expected, numerous measurements at specific locations are needed to establish the hydraulic gradients, i.e. spatial distribution of pressure differences. The use of specific drainage measures will reduce water pressures and thus generally improve the stability of rock structures.

10.1.5 Rock stress

The state of *in situ* stress refers to the stresses which exist naturally in a rock mass prior to any influence from engineering works. These stresses arise from gravitational and tectonic movements, crustal cooling and other major geomorphological influences. Complex stresses can develop in steep mountainous areas and near deeply incised river valleys. Given these possible origins, the state of *in situ* stress of a rock mass cannot readily be predicted and values can only be reliably obtained by direct measurement.[15] For major underground mining or civil engineering works, the *in situ* rock stress is a design parameter directly relevant to the layout, shape, size and orientation of excavations, to stability and support, and to sequence of excavation. Measured values of *in situ* rock stress are also useful for oilfield and geothermal development and studies related to evaluations of seismic hazard.

The state of stress existing at any point in a rock mass (or any material) is represented by a stress tensor which is a complete three-dimensional definition of stress magnitudes and directions. Local inhomogeneities or discontinuities within the rock mass can lead to variations in stresses such that measured values, besides showing some scatter, may be misleading in relation to the general regional stress field. In these circumstances it is necessary to consider the scale of the proposed excavations, the rock mass structure and the disposition of tests. It is also helpful to be aware of the possible constituents of the *in situ* stress field[16,17] which are shown in Figure 10.2 and Table 10.3.

Induced stresses (see Figure 10.3) are a function of the shape of the excavations and the previously existing *in situ* stresses. During early site investigations when direct access underground is not available, a practical method of measuring *in situ* stresses is hydrofracturing[18] which may be carried out in deep boreholes. When underground access via tunnels is available then stress

Table 10.2 Porosity and density definitions and interrelationship formulae

The components of a rock sample are defined as follows:

Grains: weight G_w, volume G_v	
Pore water: weight W_w and volume W_v	
Pore air: zero weight and volume A_v	
Pores: volume	$P_v = W_v + A_v$
Bulk sample: weight	$B_w = G_w + W_w$
Bulk sample: volume	$B_v = P_v + G_v$
Density of water:	ρ_w = mass of water per unit volume

The physical properties – density, porosity, etc. – may be defined in terms of the above components as follows:

Water content	$w = (W_w/G_w) \times 100\%$	
Degree of saturation	$S_r = (W_v/P_v) \times 100\%$	
Porosity	$n = (P_v/B_v) \times 100\%$	
Void ratio	$e = P_v/G_v$	
Dry density of rock	$\rho_d = G_w/B_v$	(dry specific gravity $d_d = \rho_d/\rho_w$)
Density of rock	$\rho = (G_w + W_w)/B_v$	(specific gravity $d = \rho/\rho_w$)
Saturated density of rock	$\rho_s = (G_w + P_v\rho_w)B_v$	(saturated specific gravity $d_s = \rho_s/\rho_w$)
Grain density	$\rho_g = G_w/G_v$	(grain specific gravity $d_g = \rho_g/\rho_w$)

Having defined the three properties, water content, porosity and dry density, the remaining properties may be calculated from the following interrelationships:

$S_r = 100(w\rho_d)/n\rho_w$

$e = n/(100 - n)$

$\rho = (1 + w/100)\rho_d$

$\rho_g = \rho_d(1 - n/100)$

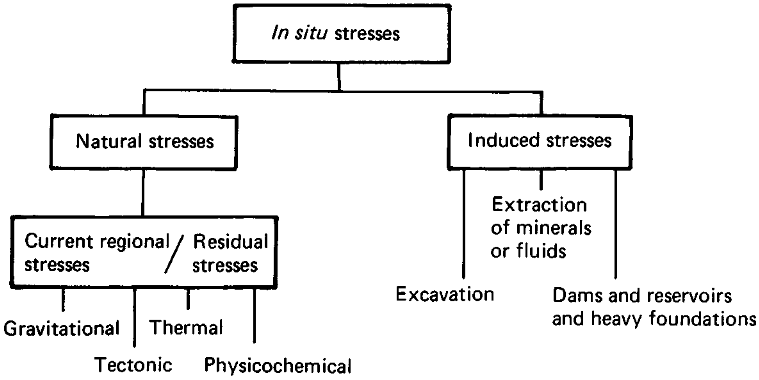


Figure 10.2 The constituents of *in situ* stress fields. (After Hyett, Dyke and Hudson (1986) 'A critical examination of basic concepts associated with the existence and measurements of *in situ* stress'. *Proceedings, international symposium on rock stress*, Sweden)

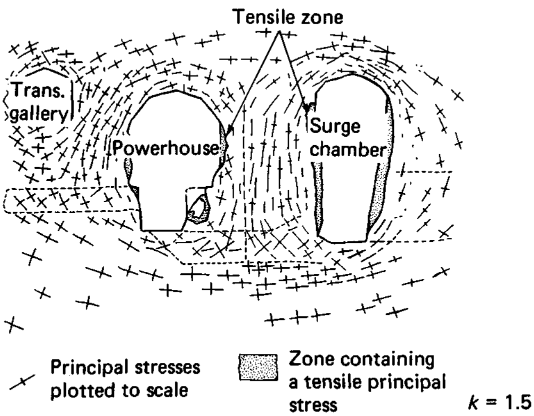


Figure 10.3 Induced stresses caused by excavation. (After Benson, Murphy and McCreath (1970) *Modulus testing of rock at the Churchill Falls underground powerhouse, Labrador.* American Society for Testing and Materials. Special Technical Publication Number 477, pp.89–116.

relief methods, such as overcoring[19,20] may be used to determine *in situ* stress. Provided sufficient care is exercised in making such measurements, the results are usually adequate for design purposes.

Other less direct methods may be used to determine natural stresses on a wider regional scale. These include the use of various geological indicators (e.g. foliations, cleavage, lineations, stylolites, slickensides, etc.) earthquake focal mechanisms[21] and borehole breakout studies.[22] Favourable comparisons have been obtained for direct measurements by overcoring and stress orientations from earthquake focal mechanism solutions in a seismically active area.[23]

10.2 Rock tests

Table 10.4 illustrates the various categories of testing, currently presented as *Suggested methods* by the ISRM.[24] Tests for classification and characterization of rock (index tests) are used for rock quality description and mapping and are generally quick and relatively cheap. Also, proposed construction methods can be evaluated from index test results, for example abrasiveness and hardness are relevant to tunnelling machine

Table 10.3 Glossary of *in situ* stress terms. (After Hyett, Duke and Hudson (1986) 'A critical examination of basic concepts associated with the existence and measurements of *in situ* stress'. *Proceedings, international symposium on rock stress.* CENTEK, Sweden)

Natural stress: The stress state which exists in the rock prior to any artificial disturbance. The stress state is the result of various events in the geological history of the rock mass. Therefore, the natural stresses present could be the product of many earlier states of stress. Synonyms include 'virgin', 'primitive', 'field' and 'active'.

Induced stress: The natural stress state as perturbed by engineering.

Residual stress: The stress state remaining in the rock mass, even after the originating mechanism(s) has/have ceased to operate. The stresses can be considered as within an isolated body that is free from external tractions; neither are they caused by the action of body forces or thermal gradients, etc. Sometimes the word 'remanent' is used as a synonym for 'residual'.

Tectonic stress: The stress state due to the relative displacement of lithospheric plates.

Gravitational stress: The stress state due to the weight of the superincumbent rock mass.

Thermal stress: The stress state set up as a result of temperature variation.

Physicochemical stress: The stress state set up as a result of chemical alteration and/or physical changes in the rock, e.g. recrystallization, absorption of water and fluctuation of groundwater levels.

Paleostress: A previously active *in situ* stress state no longer in existence. It can be considered as old and no longer present; whereas a residual stress is old and remains. Paleostress can be inferred from geological structures and from crystallography but cannot be measured.

Near field stress: The stress state perturbed by a heterogeneity (usually caused by engineering activities, e.g. a tunnel as a low-modulus inclusion).

Far field stress: A stress state which is not perturbed by a heterogeneity.

Regional stress: The stress state in a relatively large geological domain.

Local stress: The stress state in a small geological domain – usually of the dimensions of an engineering structure.

Active stress: A stress state with an associated strain energy state.

Table 10.4 Test categories published, or scheduled to be published, as *Suggested methods* by the International Society of Rock Mechanics. (After Brown (1981) *Rock characterization, testing and monitoring*. Pergamon, Oxford)

Field index tests for characterization	*Field design tests*
Discontinuity orientation	Deformability using a plate test
Discontinuity spacing	Deformability plate test down a borehole
Discontinuity persistence	Deformability radial jacking test
Discontinuity roughness	Deformability flexible borehole jack
Discontinuity wall strength	Deformability rigid borehole jack
Discontinuity aperture	Deformability flatjack
Discontinuity filling	Deformability *in situ* uniaxial triaxial test
Discontinuity seepage	Shear strength – direct shear
Discontinuity number of sets	Shear strength torsional shear
Discontinuity block size	Piezometric head (3 methods)
Discontinuity drill core recovery/RQD	Permeability transmissivity (5 methods)
Geophysical logging of boreholes	Flow velocity logs
Seismic refraction (2 methods)	Flow velocity – tracer dilution
Acoustic logging	Flow paths using tracers (4 methods)
Seismic measurements between boreholes	Stress determination – flatjack
Sonic log	Stress determination – surface coring
Caliper log	Stress determination – 'doorstopper'
Temperature log	Stress determination – strain gauge cell
SP log	Stress determination – USBM-type gauge
Resistivity logs (2 methods)	Stress determination – hydraulic fracturing
Focused current logs	
Induction log	*Field monitoring*
Gamma ray log	Movements: probe inclinometer
Neutron log	Movements: fixed-in-place inclinometer
Gamma-gamma log	Movements: tiltmeter
	Movements: borehole extensometers
Field 'quality control tests'	Movements: convergence meter
Rockbolt anchor strength	Movements: joints and faults
Rockbolt tension (torque wrench)	Movements: survey triangulation
Rockbolt tension (load cells)	Movements: survey levelling
Cable anchor tests	Movements: survey offset
Shotcrete: visual assessment	Movements: survey EDM
Shotcrete: pull tests	Vibration and blast monitoring
Shotcrete: box mould tests	Pressure – hydraulic cells
Shotcrete: core tests	Rock stress variations
Gas level measurements	Pendulum and inverted pendulum
	Strains in linings and steel ribs
Laboratory index tests for characterization	*Laboratory 'design tests'*
Water content	Triaxial strength
Porosity/density (4 methods)	Direct tensile strength
Void index (quick absorption)	Indirect (Brazil) tensile strength
Swelling pressure	Direct shear test (+field method)
Swelling strain (2 methods)	Permeability
Slake-durability	Time-dependent and plastic properties
Uniaxial compressive strength	
Uniaxial deformability (E, ν)	
Point load strength index	
Resistance to abrasion (Los Angeles test)	
Hardness (Schmidt rebound)	
Hardness (Shore scleroscope)	
Sound velocity	
Petrographic description	

selection. Engineering design tests provide data specifically required for design calculations and often may be complex and expensive. The quality control and monitoring categories of test relate to specific materials, rockbolts and shotcrete, and to methods of observation. They are of relevance to practical rock engineering, e.g. tunnelling or landslide studies, and assist in the development of practices based on empirical knowledge and provide feedback for comparisons with design calculations. Research tests devised for academic investigations into rock properties and behaviour fall beyond the scope of this section. Tests should be selected to suit a specific and well-defined requirement. For example, shear strength parameters or rock mass deformation moduli may be required for design using either limiting equilibrium methods or the finite element method respectively. Some of the more commonly used tests are summarized in Table 10.5[25] and some are shown in Figures 10.4 to 10.7. Test procedures mentioned below may generally be found in the ISRM *Suggested methods*.[24]

10.2.1 Laboratory index tests

10.2.1.1 Petrographic description

This is performed by microscopic examination of a thin section of the rock material to obtain information relevant to its mechanical behaviour such as mineral composition, grain size, texture, fabric (e.g. anisotropy), degree of alteration or weathering, and microfractures. The method does not provide a sufficiently accurate estimate of volumetric pore content (i.e. porosity), but can provide supplementary information on shape, size and interconnection of pores. The examination can be carried out only by a trained petrographer.

The information obtained can be useful in assessing abrasive properties. In particular, quartz content and intensity of cementation have been studied[26] and a cementation coefficient has been devised in an attempt to quantify the information (see Table 10.6).

10.2.1.2 Water content, porosity, density, absorption and related properties

These laboratory tests require measurement of the volumes and weights of rock constituents and of the bulk sample (Table 10.2). Bulk volume can be measured directly from caliper measurements on specimens of regular geometry, alternatively using displacement of a fluid such as mercury or water, or from buoyancy measurements using Archimedes' principle. Grain volume can be found by crushing the specimen to a powder. Pore volume is obtained using a water saturation technique or in Boyle's law gas-pressure cells. A simple quick absorption method is available that gives a void index for rocks that do not appreciably disintegrate when immersed in water. The test calls for a minimum of equipment and may be suitable for rock classification purposes as an index of porosity, or degree of weathering or alteration.

10.2.1.3 Swelling, slake durability index and durability tests

Clay-bearing rocks (shales, mudstone and some weathered igneous rocks) can swell or disintegrate when relieved of *in situ* confining stresses and when exposed to atmospheric wetting and drying. Swelling tests[24] similar to soil consolidation tests can be used to measure swelling pressure or strain during wetting of the specimen; alternatively, the swelling of an unconfined rock cube or cylinder can be measured as an index property. A quick slake durability index test can be used to measure the disintegration of rock subjected to wetting and atmospheric weathering. These tests are index tests, best used in classifying and comparing one rock with another. The swelling strain index, for example, should not be taken as the actual swelling strain that would develop *in situ*, even under similar conditions of loading and water content.

Sodium sulphate soundness and other salt crystallization tests[27] subject rock specimens to the disruptive action of salt crystals growing from solution in the pore space, and are appropriate in assessing the durability of aggregates and building stones exposed to saline conditions. Freezing and thawing tests[28] are available to measure susceptibility to frost damage.

10.2.1.4 Hardness, abrasion and attrition

The hardness and abrasiveness of rock are dependent on the type and quantity of the various mineral constituents of the rock and the bond strength that exists between the mineral grains. Quartz content in particular can be a useful guide. Tests for each property have been developed to simulate or to correlate with field experience. Many of the tests now used for rock have been adapted from highway materials, concrete and metals testing.

Abrasion tests measure the resistance of rocks to wear. These tests include wear when subject to an abrasive material, wear in contact with metal and wear produced by contact between the rocks. Abrasiveness tests can also measure the wear on metal components (e.g. tunnelling machine cutters) as a result of contact with the rock. These tests can be grouped in three categories: (1) abrasive wear impact tests; (2) abrasive wear with pressure tests; and (3) attrition tests.

The first category includes the Los Angeles test (in which the rock sample and an abrasive charge of steel spheres are rotated

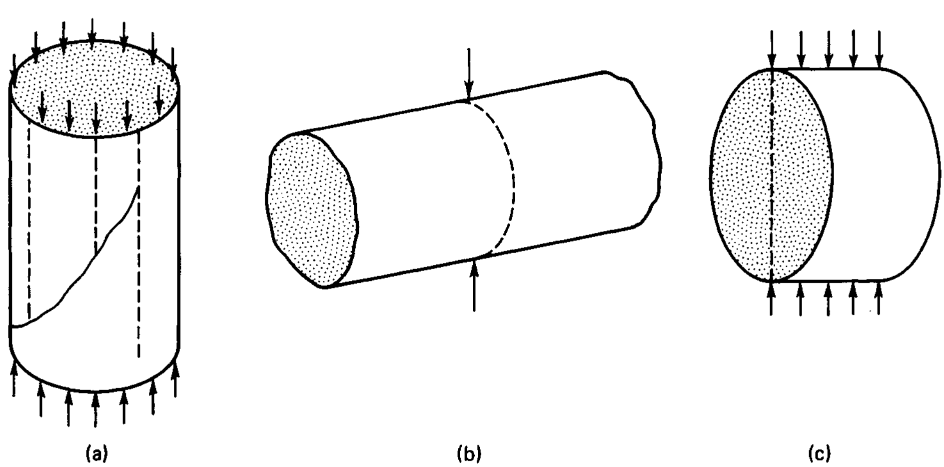


Figure 10.4 Strength index tests showing the loading configuration, specimen shape and failure pattern in: (a) The uniaxial compressive strength test; (b) the diametral point-load test; (c) the Brazilian test

Table 10.5 Summary of rock mechanics field and laboratory tests. (After Hoek (1977) 'Rock mechanics laboratory testing in the context of a consulting engineering organization', *Int. J. Rock Mech. Min. Sci.*, **14**)

Text	*Purpose*	*Specimen preparation*	*Comments*
(1) Point load test	Originally used to determine the tensile strength of rock (Brazilian test); now used as an index test or to estimate the uniaxial compressive strength of rock material	None	Simple, quick and inexpensive field test which gives reliable results provided that it is used on brittle, isotropic rock cores
(2) Slake durability test	Determination of the rate of breakdown of rock material under varying moisture-content conditions	None	Oven drying and weighing of sample adds complexity to simple test equipment
(3) Field shear test	Determination of shear strength of rock discontinuities in small field specimens	Specimens have to be mounted in plaster or cement with discontinuity surface horizontal	Simple test giving reasonable values for angle of friction within limitations of sample size
(4) *In situ* shear test	Shear strength determination on undisturbed samples	Area surrounding specimen has to be cleared and load frame mounted parallel to discontinuity surface	Specimen preparation relatively expensive and load limitation restricts use of apparatus to weak materials such as coals
(5) Large specimen shear tests	Determination of shear strength of discontinuities in samples of reasonable size under laboratory conditions	Trimming of specimens is usually required and specimens have to be mounted in plaster or cement with discontinuity horizontal	Tests are relatively expensive and are normally only justified on large jobs where improved accuracy of results can be used to design steeper slopes
(6) Triaxial testing of granular materials	Determination of shear strength of broken rock or weathered rock	None	Maximum lump size is limited by size of triaxial cell
(7) Uniaxial compression tests	Determination of elastic moduli and unconfined compressive strength of rock cores	Careful preparation of end of core specimens essential	Most commonly quoted rock strength value and one of the most difficult to obtain economically
(8) Triaxial compression tests	Determination of the triaxial unconfined compressive strength of rock cores	Careful preparation of ends of core specimens essential and mounting of specimen in cell critical	Only usually justified for special projects concerned with underground excavation design
(9) Anisotropic triaxial compression tests	Determination of triaxial compressive strength variations with direction in schistose rock materials	Core must be drilled at specified angles to schistosity directions. End preparation essential	Only usually justified for underground excavation design in highly anisotropic materials such as slate
(10) Triaxial shear test	Determination of shear strength of discontinuities in rock core with controlled pore-pressure conditions	As for (9) Discontinuity must be inclined in core specimen	Most effective means for carrying out shear tests with controlled pore pressure Platens must be free to move laterally to accommodate component of shear displacement and resulting mechanical complexity limits popularity of test
(11) Creep tests	Determination of deformation and failure characteristics of rocks which exhibit marked time-dependent behaviour	Preparation of core specimen ends critical and accurate control of temperature usually required	Normally only justified when designing underground excavations in evaporites such as salt or potash
(12) Stiff machine tests	Determination of the post-failure deformation behaviour of rock materials	Very accurate specimen end preparation essential	Information required for the design of yielding pillars in underground mines
(13) Scaled material model test	Simulation of behaviour of complete rock structure/applied load interaction	Choice and preparation of model materials and loading system critical if similitude requirements are to be satisfied	Normally only justified by research institutes for national scale problems

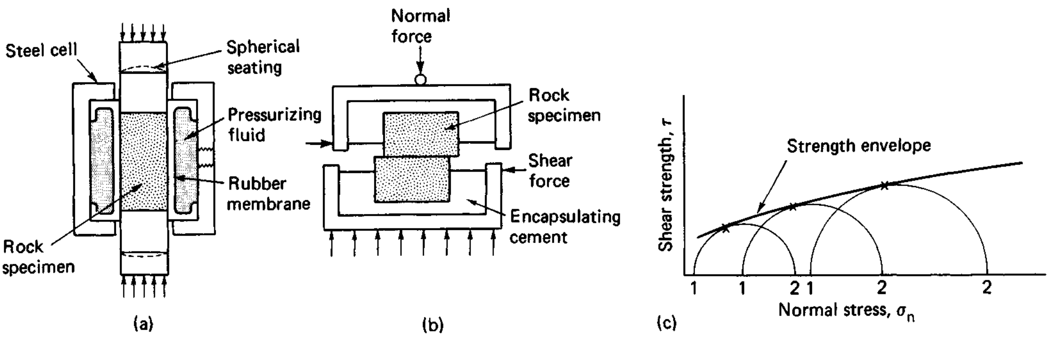


Figure 10.5 Triaxial and direct-shear tests in the laboratory. (a) The triaxial test is most often used for intact rock; (b) the shear test is used for determining the strength of joints or weakness planes; (c) the Mohr–Coulomb diagram is used for plotting the results from direct-shear tests or triaxial tests in order to obtain a strength envelope for the rock

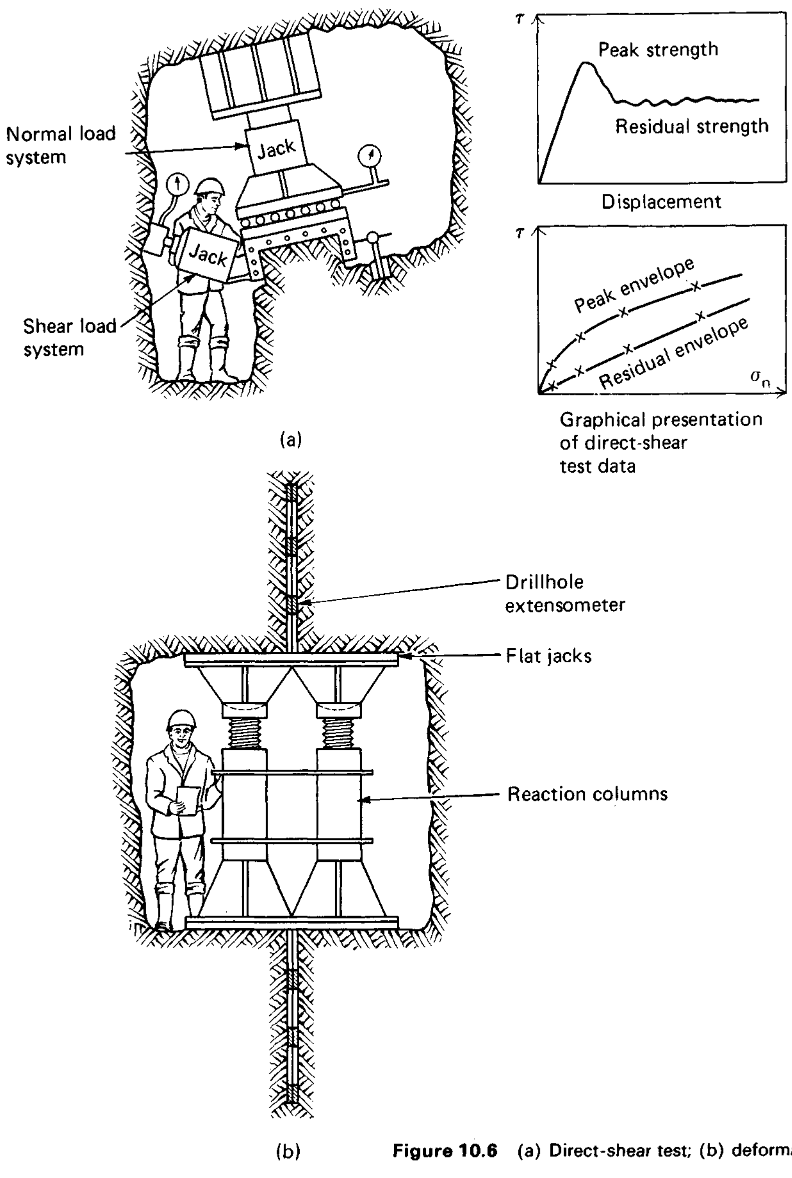


Figure 10.6 (a) Direct-shear test; (b) deformability test

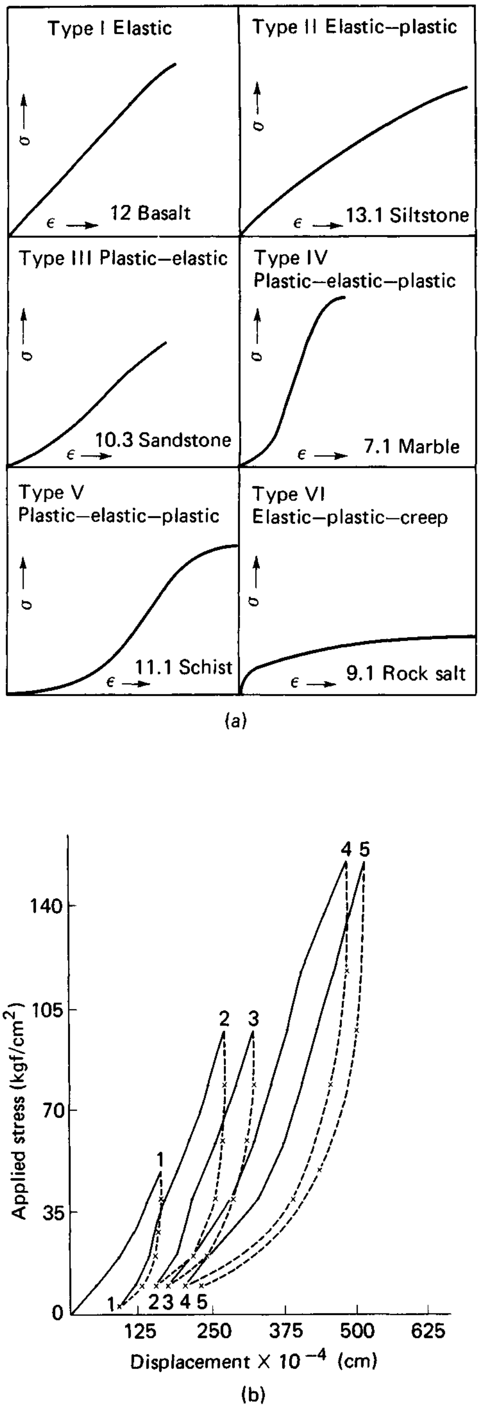


Figure 10.7 Stress–strain curves for laboratory and field deformability tests. (a) Laboratory stress–strain curves showing the influence of rock type on the shape of the curve. (After Hendron (1968) 'Mechanical properties in intact rock', in: Stagg and Zienkiewicz (eds) *Rock mechanics*. Wiley, Chichester, pp.21–53); (b) result of a field plate loading test on granulite rock at Monar dam, Scotland, showing the effect of repeated cycling of the applied load. A deformability modulus must be selected to suit the problem, and depends on the stress level, number of cycles, and whether load is increasing or decreasing

Table 10.6 Cementation coefficient. (After McFeat-Smith (1977) 'Rock property testing for the assessment of tunnelling machine performance'. *Tunnels & Tunnelling*, **9**, 3, 29–33)

(1) Noncemented rocks or those having greater than 20% voids
(2) Ferruginous cement
(3) Ferruginous and Clay cement
(4) Clay cement
(5) Clay and calcite cement
(6) Calcite (or Halite) cement
(7) Silt; Clay or Calcite with quartz overgrowths
(8) Silt with quartz overgrowths
(9) Quartz cement, quartz mozaic cements
(10) Quartz cement with less than 2% voids

in a test drum), a sand blast test[29] (where the surface of the test sample is abraded by an air blast containing abrasive powder) and the Burbank test (in which the sample abrades a metal testpiece). In the second category is the Dorry test (in which the test specimen is pressed against a rotating steel disc in the presence of an abrasive powder) and the Taber abraser test (in which an NX core sample is rotated against an abrasive wheel). Weight loss of both provides indices of abrasiveness. Various tests have been devised to determine the wear on rock drilling bits. Finally, attrition tests examine the wear produced merely by the rubbing of surfaces together. The Deval test is similar to the Los Angeles test but without the abrasive charge of steel spheres.

Hardness is not regarded as a fundamental material property because a quantitative measure of hardness depends on the type of test employed. Three types of test have been used: (1) indentation test; (2) dynamic or rebound test; and (3) scratch test.

A number of indentation tests normally used on metals have been variously applied to rocks and include Vickers hardness test and the Knoop test. A test used more extensively in the UK coal measures rocks is the National Coal Board's cone indenter test.[30] A scale of hardness and some typical values are illustrated in Table 10.7.[26]

The Shore scleroscope measures mineral hardness by recording the rebound height of a small diamond-tipped plunger after its impact with the rock surface. The Schmidt rebound hammer works on a similar principle but uses a larger plunger so that a measure of 'average rock hardness' is obtained. Care must be taken in standardizing the methods for holding the specimen and for preparing the surface at the point of impact. However, the instrument is portable and has frequently been used in the field. The use of the Schmidt hammer has been adopted particularly for the assessment of the strength of rock on joint surfaces (see section 10.3.3 below). Moh's scale of hardness commonly employed in petrographic studies is based on a scratch test

Table 10.7 Scale of hardness for the standard cone indenter test. (After McFeat-Smith (1977) 'Rock property testing for the assessment of tunnelling machine performance'. *Tunnels & Tunnelling*, **9**, 3, 29–33)

Standard hardness	*Description*
0–1.00	Soft
1.01–1.80	Moderately soft
1.81–2.50	Moderately hard
2.51–4.00	Hard
4.01–6.00	Very hard
6.00–	Extremely hard

applied to minerals. Micro-indentation tests are also available to measure mineral hardness under the microscope for purposes of mineral identification.

A number of derived indices have been devised based on the results of the individual tests mentioned above, and attempts have been made to correlate these with construction or excavation performance.

For the Shore scleroscope[26] the coefficient of plasticity:

$$K = \frac{H_2 - H_1}{H_1} \times 100\%$$

where H_1 is average Shore hardness and H_2 is Shore hardness after 20 tests on the same point.

In some respects this is a measure of 'work-hardening'.

Similarly, for the Schmidt hammer,[31] the deformation coefficient:

$$D = \frac{R_2 - R_1}{R_2} \times 100\%$$

where R_1 is the initial rebound number at a point and R_2 is the rebound number after 20 tests on the same point

Also for the Schmidt hammer when used on rock exposures of jointed weathered rock:[32]

$$\text{Reduction Index} = \frac{R_1 - R_F}{R_1} \times 100\%$$

where R_1 is the average rebound number from intact areas of the rock exposure free of discontinuities, and R_F is the mean value of the rebound number from randomly located tests on an exposure of fractured rock (such as a tunnel face).

To assess the tenacity of rock against the effort of rock cutters or picks such as on tunnelling machines, a number of parameters are obtained from *rock cutability tests* (or core grooving tests).[33] The test may be carried out on either core samples or on block samples. Four cuts are normally made in the rock sample at a constant depth of 5 mm with a tungsten carbide chisel-shaped tool of a standard geometry and composition, mounted on an instrumented shaping machine. The shaping machine is instrumented with a strain-gauged triaxial force dynamometer. For the standard instrumented cutting test, forces are analysed in the cutting and normal directions, since sideways forces are balanced due to the symmetrical design of the cutting tool. The strain gauge output from the dynamometer is recorded together with other information, such as weight of debris and length of cut. The analysis provides the following cutting parameters:

(1) Cutting and normal mean, mean peak and peak force components acting on the cutting tool.
(2) *Specific energy*. This is defined as the work done by the resistance forces within the rock against the cutter tip per unit volume of rock excavated.
(3) *Cutting wear*. This is the weight of tungsten carbide lost by the cutting tool during the four experimental cuts. Alternatively, cutting wear may be expressed in millimetres of wear-flat generated per metre of cut.

10.2.1.5 Strength tests on intact rock

The *uniaxial (unconfined) compression test* is mainly used for strength classification.[24,34] Rock cylinders are crushed by axial loading between steel platens to give compressive strength, defined as the ratio of failure force to specimen cross-sectional area (Figure 10.4). The length:diameter ratio for specimens should be between 2.0 and 2.5 to avoid the confining effect due to platen friction, and the end faces of the cylinder should be machined flat and parallel. Alternative crushing tests on cubes have been used for classification of rock aggregates but are not recommended in other applications.

The *point-load strength test* is an alternative strength classification.[35] Rock in the form of either core or irregular lumps is tested by compressing the specimen between conical platens, and a strength index is obtained as the ratio of failure load to the square of the distance separating the platen contact points. The test does not require machined specimens, employs portable testing equipment, and may be carried out in the field to produce a strength log for rock core. The *Brazilian test*[24,36] employs line loading of machined discs rather than point loading. The test provides an index of the tensile strength, which may also be determined by a direct tensile test.[34] *Impact tests* employing a pendulum or falling weight[37] are sometimes used for strength classification of rock aggregates.

10.2.1.6 Index tests on chalk

Chalk requires special consideration as an engineering material[38] and there are specific tests available.[39] The saturated moisture content is a measure of the porosity, and the higher the value the more water that can be released to generate an unworkable slurry when chalk lumps are broken down by compaction equipment. The chalk crushing value is a measure of the resistance to mechanical breakdown of pieces of chalk.

10.2.2 Field index tests

The quantitative description of discontinuities in rocks masses is in this category (Table 10.4), and this aspect is discussed below in section 10.3.3 and in Chapter 8. Further rock mass index properties can be assessed from the following methods.

10.2.2.1 Geophysical methods

Seismic, resistivity, magnetic and gravimetric techniques[40] were developed for mineral and oil prospecting, but have been successfully adapted to the more detailed surveys required in civil engineering. The techniques are used mainly to map rock structure but can also be used to give index values related to the mechanical character of rocks and soils.

Refraction seismic surveys measure the velocity of sound emitted usually by an explosive charge and received by one or more geophones after travelling through the ground. Velocity is usually highest in rocks of low porosity or when pores are filled with water. Therefore velocity measurements can be used to indicate the spacing and openness of fissures, particularly if the velocity of sound *in situ* is compared with that through unfissured laboratory specimens of the same rock. The method has proved useful in mapping the quality of rock in dam foundations and abutments and in assessing the effectiveness of grouting treatment.[41]

Hammer seismograph equipment, where the explosive source is replaced by a sledgehammer blow, is convenient for small-scale engineering surveys. Sound velocity can be measured between adits or, using a piezoelectric or sparker source, can be measured between drillholes or in a single drillhole, to give a more complete picture of rock quality variation.

Sound velocity values can be used in deriving dynamic elastic parameters (Young's modulus and Poisson's ratio) for the rock,[42] but the calculated values are usually quite different from the 'static' values measured, for example, by plate testing. The reason is that the rock strains produced by a travelling stress wave are of much smaller amplitude and higher stress rate and gradient than in static loading. Sound velocity can, however, be used as an index to rock deformability if a suitable correlation is established.

Field resistivity, magnetometry and *gravimetric surveys* are generally only used for structural mapping, and then in relatively few applications in comparison with seismic methods. However, these and other techniques are becoming increasingly used in *downhole geophysical logging*.[43,24] Instrumental probes are lowered to the base of a drillhole on a wireline and then drawn up the hole producing a record or 'log' of various rock properties. Common instruments measure sound velocity, spontaneous electric potentials in drilling fluid and natural or induced nuclear radiation. The drillhole logs can be used to evaluate rock porosity, density, saturation, clay content and other mechanically relevant information. These methods of geophysical logging of boreholes are summarized in Table 10.8. Caliper logging, which shows variations in hole diameter, has particular significance to the study of *in situ* stress-related 'breakouts', mentioned above in section 10.1.5.

10.2.3 Design tests

10.2.3.1 Permeability tests

The majority of flow through the rock mass usually occurs along fissures rather than pores, so that tests on *laboratory specimens* usually have limited significance. Such tests are, however, useful if the rock is very porous. Often they employ air or an inert gas rather than water as the test fluid, in order to speed up the testing procedure.

Permeability can be measured in the field using a *packer test* where a section of borehole is isolated by pneumatically inflated packers.[44,45] Tests in rock require packers that are long in relation to the test section and, preferably, also to the fracture spacing in the rock. Borehole permeability tests can be carried out by 'pumping in' under conditions of either constant or falling head, or by 'pumping out'. A graph of flow rate against test pressure is usually linear at lower pressures where laminar flow occurs, but may become nonlinear at higher pressures owing to turbulence or to the effect of water pressures in increasing the width of existing fissures. A summary of the main permeability test methods is given in Table 10.9. Analysis of test results and water flow problems in general should account for the influence of both total stress and pore pressure on rock permeability. Anisotropic permeability may require that tests be carried out with holes drilled at different inclinations.

10.2.3.2 Strength testing for design

Triaxial strength tests[46] are most often required for stress/strength design particularly of deep underground excavations. Cylindrical specimens are tested in the laboratory by applying a confining pressure to the curved surface of the specimen, using a pressurizing fluid confined in a triaxial cell and separated from the specimen by a flexible membrane to avoid generation of pore pressures. The confining pressure is radially symmetric and provides the (equal) intermediate and minor principal stresses acting on the specimen. The major principal stress is usually applied with a hydraulic ram acting through steel platens along the axis of the cylinder. This stress is increased, usually at constant confining pressure, until the specimen fails. The test is repeated for a range of confining pressure in order to define a 'strength criterion' for the material (Figure 10.5 and section 10.3.2).

Direct-shear tests are usually employed to provide data for limit equilibrium analyses, particularly for analysis of the stability of rock slopes, dam foundations and abutments.[47] They are best suited to testing a well-defined discontinuity such as a joint, bedding plane or the interface between concrete and rock rather than to testing of intact material. Rock samples containing joint planes can be tested in the field, or in a laboratory, using a portable shear box.[9,48] Typically an *in situ* test block with dimensions 700 × 700 × 350 mm is first isolated by sawing or line drilling. Stress is applied normal to the discontinuity to be tested, and a force is then applied to shear the discontinuity (Figure 10.6). To avoid generation of tension at the heel of the test block, the shearing force may be inclined so as to pass through its centre of area. A graph showing shear displacement as a function of shear force is used to find values of 'peak' and 'residual' strength. Further tests at different normal stress values allow strengths to be plotted as a function of normal stress. The shearing resistance of the discontinuity, thus evaluated, may be applied in limit equilibrium design calculations. The strength of discontinuities in rock is discussed in more detail in section 10.3.3.

10.2.3.3 Deformability testing for design

Deformability test data are required for designs involving analysis of stresses and displacements in rock, e.g. analysis of foundation settlements on softer rocks, analysis of stresses around underground openings or design of tunnel linings. The most commonly used design methods assume linear elastic behaviour for the rock and require values for the elastic parameters (Young's modulus and Poisson's ratio).

Laboratory deformability tests can be used in situations where the rock material is likely to be much more deformable than the discontinuities, e.g. in soft rocks and at depths where the discontinuities are tight. A cylindrical specimen is loaded along its axis, and strains occurring in the central third of the specimen are compared with the corresponding levels of stress. Strains are typically measured with electric resistance strain gauges or transducers bonded or clamped to the specimen surface. The stress–strain curves (Figure 10.7) are typically nonlinear, reflecting inelasticity due to the closing of pores at low stress levels and to the generation of failure cracks at higher levels of stress.[49] Elastic parameters can be obtained by approximating the slopes of these curves to straight lines over the restricted ranges of stress that are of relevance to the problem. Hysteresis effects (where the curve for unloading does not correspond to the loading curve) are observed in both laboratory and field tests and are probably due to friction acting on the surfaces of cracks and pores. The deformability of rock samples during and after 'failure' (i.e. beyond peak strength) can be examined using 'stiff' testing machines, and the relevance of such 'post-peak' behaviour is reviewed below in section 10.3.2.

Rock fissures and joints are typically more open near the surface owing to weathering and stress relief, and in such materials laboratory tests can indicate that rocks are less deformable than is really the case. Field tests are designed to affect as large a volume of rock as possible in order to fully reflect the contribution of fissures to deformability.

The most economic field deformability tests are carried out in drillholes using a jack or *dilatometer*.[50,51] This instrument applies a radial pressure through either a rigid split cylinder or a flexible membrane, and the resulting radial displacements may then be analysed in terms of elastic parameters. In softer ground these tests can also give an approximation to the strength of the rock.

Plate-loading tests[17] (Figure 10.6) can be used to obtain deformability values that reflect the behaviour of a larger volume of rock. Tests using smaller-diameter plates in drillholes have the advantage of simplicity when a large number of strata

Table 10.8 Geophysical (borehole logging) methods. (After International Atomic Energy Agency (1982) *Site investigations for repositories for solid radioactive wastes in deep continental geological formations.* IAEA, Geneva)

Mode	*Name of geophysical log*	*Basis of method*	*Application*
Electrical (requires uncased boreholes and presence of borehole fluid, except where indicated)	(1) Spontaneous potential (SP)	Measurement of variations in natural potentials of lithologies in borehole	Lithological variations – presence of conductive or oxidizing/reducing minerals, dykes, etc.; delineation of shale–sandstone sequences. Usually run in association with normal resistivity
	(2) Normal resistivity	Measurement of formation resistivity using electrode spacings from a few to about 6 m	Variations in lithology, porosity and groundwater salinity; formation penetration depends on the spacing of the electrodes; fracture analysis
	(3) Focused resistivity laterologs and (guard logs)	Measurement of formation resistivity by focusing the current into thin sheets	Porosity and groundwater salinity; indication of occurrence of fractures, and conductive or oxidizing/reducing minerals. Higher resolution than normal resistivity logs
	(4) Induction	Measurement of formation conductivity by the measurement of secondary magnetic fields within the formation generated by an alternating magnetic field created by a borehole sonde	Measurement of porosity in dry sections of borehole indication of occurrence of fractures (can be made in dry boreholes)
	(5) Microresistivity (micrologs and microlaterologs)	Measurement of small volumes of the formation immediately adjacent to the borehole wall. Done with miniature focused resistivity logs	Porosity and lithological variations; fracture analysis relative permeability (with mud-drilling fluids only)
	(6) Dipmeter	Specialized use of microresistivity measurements; data from 4 probes are fed to a computer for the computation of dips of bedding planes and fractures	Determination of the dip (inclination) and strike of planar structures (bedding planes, fractures, etc.) intersected in the borehole
	(7) Radar	Measurement of reflected pulses of induced electromagnetic energy	Gives 3-dimensional picture of inhomogeneities inside a rock mass within a radius of approximately 80 m of a borehole. In development stage
	(8) Fluid resistivity	Measurement of variations in resistivity (conductivity) of borehole fluids	Groundwater salinity; elevations of groundwater in-flow. Important for corrections to other logs
Radiometric (can be run in cased or uncased boreholes with or without a borehole fluid except where indicated)	(1) Natural gamma	Measurement of natural gamma radiation by a scintillation counter located in a sonde	Radioactive elements are normally concentrated in clays and shales so the logs give data on lithological variations
	(2) Gamma-gamma	Measurement of reflected radiation from induced radiation by gamma scource	Bulk density. Results can be disturbed by presence of high natural gamma background
	(3) Neutron-neutron	Measurement of neutrons emitted from hydrogen atoms in the rock or pore fluid. The hydrogen atoms capture neutrons from a neutron source and emit neutrons with a different energy	Porosity. Results can be interpreted in terms of water content, both free water and in minerals
	(4) Neutron-gamma	Measurement of gamma emitted from hydrogen atoms in the pore fluid. The hydrogen atoms capture neutrons from a neutron source and emit gamma radiation	Porosity. Results can be interpreted in terms of water content. Requires uncased borehole
	(5) Spectral gamma	Measurement of natural gamma radiation over a spectrum of energies	Determination of lithological and mineralogical variations, including detection of potassium, uranium and thorium. May provide interrelationship of different fissure systems
Acoustic (requires uncased borehole, preferably with a borehole fluid)	(1) Acoustic or sonic	Measurement of time required for sound to travel a certain distance through formations surrounding a borehole	Used to measure local sonic velocity. The response for a given formation depends on its lithology, porosity and fracturing. Provides a check on cement emplacement behind casing

Mode	Name of geophysical log	Basis of method	Application
	(2) 3-dimensional sonic	Measurement of time required for reflection of induced sonic energy	Determination of number and extent of fractures
	(3) Sonic waveform	Depiction of individual waveforms at discrete depths	Determination of number and extent of fractures
	(4) Downhole seismic	Measurement of surface signal recorded by geophone clamped to side of borehole	In combination with seismic data collected from surface geophones yields information on subsurface geology without the need to rely on closely spaced boreholes
	(5) Televiewer	Measurement of time for reflection of high-frequency acoustic pulses from borehole surfaces. Records a continuous image of a borehole wall	Determination of fractures along the borehole
Miscellaneous (uncased boreholes)	Television camera	Gives visual image of borehole wall, capable of recording image on videotape for playback	Examination and measurement of fractures and other discontinuities. Requires clear borehole fluid
	Caliper	Measurement of borehole diameter. Mechanical arms in contact with the borehole wall give a log of borehole diameter	Identifies fractures at the borehole; identifies cavitation or closure of certain formations intersected by the borehole; provides corrections to other logs for hole diameter variations; checks condition of casing
	Fluid temperature	Direct measurement of temperature variations of borehole fluid	Heat flow within a particular region. Under pumping and nonpumping conditions can be used to derive information on groundwater flow
	Flowmeter	Direct measurement of the flow of water, in a vertical direction, within the borehole	Determination of vertical hydraulic gradients, groundwater conditions, levels and amounts of inflow of groundwater; identifies casing leaks and condition of screens
	Gravimetric	Measurement of *in situ* rock density	Improves interpretation of regional data on rock density
	Magnetic	Measurement of magnetic intensity and/or orientation	Detection of magnetite or pyrrhotite-rich layers or bodies; improved interpretation of regional magnetic data

are to be tested. The trend, however, is to employ loaded areas of increasingly large diameter. Tests at diameters in excess of 1 m require *flat jack techniques*[52] in order to ensure uniformity of loading and to facilitate application of the high forces that are required. Flat jacks comprise two thin steel plates circumferentially welded and inflated by hydraulic pressure. Pressure is applied in increments and the corresponding rock displacements are recorded by extensometers either at the surface or at depths beneath the loaded plate or flat jack. Reaction can be provided by the opposite wall of an adit or test chamber, or methods are available where a flat jack is grouted into a slot machined into the rock face. Even larger volumes of rock can be tested by *radial jacking in a test chamber* using flat jacks distributed around the circumference of a ring beam, or by applying hydraulic pressure to a section of adit that has been lined and plugged with concrete.

In situations where rock behaviour cannot reasonably be assumed to be elastic, because of viscous effects, the creep or time-dependent behaviour of the rock must be measured.[53] These properties are of greatest relevance to geological processes involving crustal deformations over long periods of time and in engineering their relevance is restricted to certain rock types such as evaporates, salt deposits and clay-bearing rocks under conditions where flow and squeezing of rock is likely.

10.2.4 Rock as an engineering material

Rock materials used in engineering construction include building stone, riprap and rockfill, also concrete and road aggregates.

Rocks suitable for building stone are typically homogeneous and have well-defined, planar and persistent discontinuities. Materials with a fracture spacing of about 1000 mm are suitable for monumental stone, and smaller sizes may be useful for general-purpose building. Facing stone, flags and slates may be naturally flaggy, with fracture spacing in two orthogonal directions much greater than in the third direction, or may be sawn from blocks of more cubic shape. Appearance of the stone and ease of dressing are also important. Weathering deterioration of building stones is more often associated with incipient weakness planes (e.g. cleavage or poorly cemented bedding) than with the intact material, although porous materials in particular are subject to the action of frost and salt crystallization.[54]

Rockfill and riprap selection[55] is based on similar considerations although there is more flexibility in shapes, sizes and heterogeneity of materials, and visual appearance is seldom of great importance. Large-sized material is essential for marine works to inhibit removal by tide and wave forces. An optimum size grading is essential for adequate placement and compaction and to achieve size density and placed permeability. This is most

Table 10.9 Summary of permeability test methods. (After International Atomic Energy Agency (1982) *Site investigations for repositories for solid radioactive wastes in deep continental geological formations.* IAEA, Geneva)

Type of test	*Description*	*Duration*	*Results obtained*
1 FIELD TESTING			
1.1 Withdrawal	Removal of water from wells		
1.1.1 Single well	Pumping from single well; measurement of rate of withdrawal and drawdown	Minutes–hours	Transmissivity
1.1.2 Multiple wells	Pumping from single well; measurement of discharge and drawdown from pumped well; measurement of drawdown in one or more radially spaced observation wells	Hours–weeks–months	Transmissivity; storage coefficient (related to porosity (n)); identification of aquifer boundaries including aquitards and aquicludes
1.2 INJECTION	Injection of a measured volume of water into a well		
1.2.1 Slug test	Injection of measured volume of water into well and measurement of rate of recovery to pretest water level	Minutes–hours	Transmissivity
1.2.2 Single packer	Injection of measured volume of water at predetermined pressure into well below the level of an emplaced packer; measurement of water flow and pressure changes	Minutes–hours	Transmissivity; size of fracture apertures
1.2.3 Double packer	Injection of measured volume of water of predetermined pressure into interval of well between packers; measurement of water flow and pressure changes	Minutes–hours	Permeability; size of fracture apertures
1.2.4 Pulse test	Injection of small volume of water into well interval isolated by double packers; measurement of hydraulic pressure decay	Minutes–hours	Permeability; size of fracture apertures
1.2.5 Tracer tests	Injection of identifiable materials into single or multiple wells; may be employed in zones of a well isolated by packers; periodic sampling of adjacent wells to determine tracer concentrations	Hours–months	Groundwater velocity fracture interconnectivity; aquifer anisotropy; hydraulic gradient direction; sorption characteristics of aquifer

economically achieved if the natural fracture spacing is such as to give approximately the correct block shapes and size grading without appreciable secondary blasting and crushing. Tests to evaluate the suitability of building stone or rockfill might, for example, include point-load strength evaluation both parallel and perpendicular to weakness planes, porosity, density measurements, and evaluation of cementing materials and rock texture by an examination of hand specimens and thin sections under the microscope. Tests on small pieces of rock cannot usually be used to predict deterioration caused by extensive planes of weakness, and an examination of the weathering of rock that has been exposed for a number of years in a quarry or rockface can often provide the answer.

Concrete and road aggregate[56] can take the form of either natural gravels, artificial materials or crushed rock. For the latter, the crushability of the potential source of aggregate may be of even greater importance than its properties in use. The material should break readily into approximately cubic fragments without an excess of fines. Brittle, dense and crystalline materials are better from this point of view than porous or friable rocks. Surface roughness is advantageous for a satisfactory bond between the aggregate and cement or bitumen. A summary of relevant test methods is given in Table 10.10.

The mineralogy of the rock may also affect this bond, but probably to a lesser extent than roughness. Porosity is a major factor; some of the porous, yet strong, limestones give excellent bonding characteristics. Road surfacing materials also require to be resistant to polishing, the best polishing resistance being afforded by rocks with minerals of contrasting hardness or rocks with grains that are plucked rather than worn smooth by traffic. Rock constituents that are undesirable in that they react chemically with cement or bituminous substances can often be detected by a mineralogical examination, but tests on concrete made from the aggregates are usually also required to evaluate this hazard.[57]

Table 10.10 Summary of tests for aggregates. (After Collis and Fox (1985) *Aggregates: sand, gravel and crushed rock aggregates for construction purposes.* Geological Society, London)

Physical tests:	
(1) Aggregate grading	BS 882 and 1201:1973 BS 812:1975 ASTM Designations C33 and 136
(2) Aggregate shape, angularity, sphericity, roundness, surface texture	BS 812:1975:Part 1
(3) Relative density, bulk density, unit weight	BS 812:1975 ASTM Designations C29 and 127
(4) Water absorption	BS 812:1975 ASTM Designations C127 and 128
(5) Aggregate shrinkage	BRS Digest 35
Petrographic examination	ASTM Designation C295
Mechanical tests:	
Strength	
(1) Aggregate impact value	BS 812:1975
(2) Aggregate crushing value	BS 812:1975
(3) 10% fines value	BS 812:1975
(4) Franklin point load test	ISRM 1985
(5) Schmidt rebound number	Duncan 1969
Durability	
(1) Aggregate abrasion value	BS 812:1975
(2) Aggregate attrition value	BS 812:1943
(3) Los Angeles abrasion value	ASTM Designation C131
(4) Polished stone value	BS 812:1975
(5) Slake durability value	Franklin 1970
(6) Sulphate soundness	ASTM Designation C88
Chemical tests:	
(1) Chloride content	BS 812:Part 4:1976
(2) Sulphate content	BS 1377
(3) Organic content	BS 1377
(4) Adhesion tests	HMSO 1962

10.3 Characterizing rock mass properties

10.3.1 Rock mass classification

Rocks may be classified using geological names only, but this approach can mislead because the names are sometimes general and depend on properties that are of little engineering significance. For example, 'granite' can be a crumbly sand or a broken rubble rather than the monolithic material implied by the name. Shales, mudstone and limestone can also exhibit an extremely broad range of engineering properties. On the other hand, there are over 2000 igneous rock names in existence, reflecting minor mineralogical changes that are usually mechanically insignificant.

Various index tests can be introduced to supplement the classification, but the samples tested in the laboratory usually represent only a very small fraction of the total volume of the rock mass. Since often only those specimens which survive the collection and preparation process are tested, the results of these tests could represent a highly biased sample.

The classification can be made more realistic by including properties characteristic of *in situ* conditions as well as hand-specimen conditions, e.g. characteristics of the fractures and discontinuities. Fracture spacing has been used together with intact strength in a number of rock classification schemes[58,59] and others have been formulated on the basis of Young's modulus and uniaxial compressive strength tests[60] (Figure 10.8) and using porosity, density and crystallinity.[61] One objective of a rock classification scheme is to allow compilation of maps and cross-sections to show the 'geotechnical' or 'geomechanical' (rather than geological) variations with depth and extent to facilitate design and to simplify analytical models. For example, a finite element mesh constructed to represent variations in geomechanical properties is likely to be more appropriate for design analysis than one based solely on geological boundaries.

A number of rock mass classification systems have been developed in an attempt to provide guidance on the properties of rock masses upon which the selection of tunnel support systems can be based. Details of the more useful systems and of how to apply them have been compiled by Hoek and Brown.[62]

A simple yet useful rock tunnel classification system based on combinations of different parameters considered to be the main causes of ground instability was devised by Terzaghi[63] for cases where steel arch supports were to be used, but the system is purely descriptive. There is no objective assessment of rock quality, although later Deere[64] and Cording, Hendron and Deere[65] further developed Terzaghi's classification to include rock quality designation (RQD).

Other systems have been devised, such as the concept of rock structure rating (RSR)[66] which takes account of the local geological structure, the pattern and orientation of joints relative to the tunnel direction and groundwater and joint condition. The RSR is then related empirically to a support requirement in terms of rib ratio (RR) for steel arch supports.

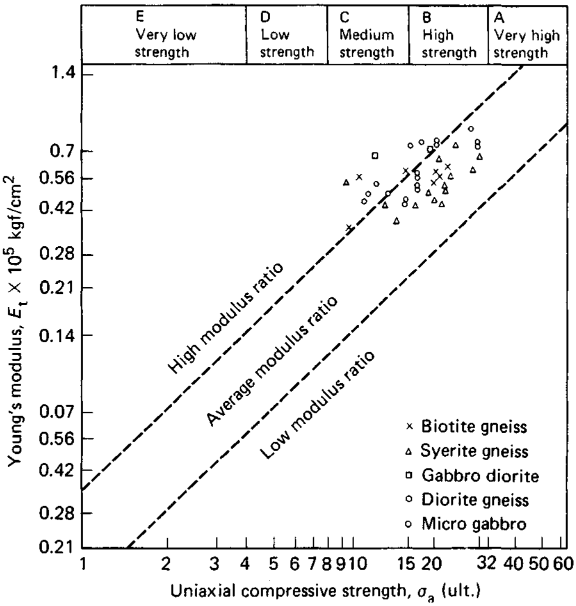
The most useful of these classifications take account of the interrelation of several key rock properties and relate them empirically to precedent case-histories. The two foremost classifications are those published by Bieniawski[67] and by Barton, Lien and Lunde.[68] These classifications include: (1) information on the strength of the intact rock materials; (2) the spacing, number and surface properties of the structural discontinuities as well as allowances for the influence of groundwater; and (3) *in situ* stresses and the orientation and inclination of dominant discontinuities.

Bieniawski developed the Council for Scientific and Industrial Research (CSIR) scheme in South Africa for classifying rock mass stability using input parameters including RQD, state of weathering, rock strength, joint spacing, separation and continuity, and groundwater flow. This scheme also considers the orientation of discontinuities with respect to the excavation, and it rates rock quality using a 'merit points' system. Graphical relationships between this classification, the span of the excavation and the time which may elapse before an unsupported length of tunnel would start to fail (the 'stand-up time') allow empirical correlations to be made between these different factors for any span up to 20 m.

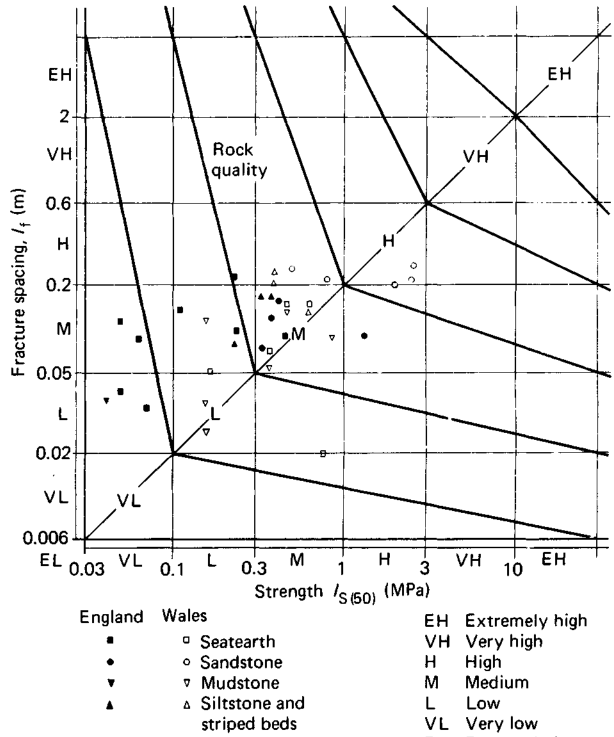
The Norwegian Geotechnical Institute (NGI) method of Barton, Lien and Lunde[68] is based on an evaluation of about 200 Scandinavian tunnel or cavern case-histories and proposes an index, Q, describing 'tunnelling quality'.

The numerical value of index Q is defined by an expression in terms of a derived 'joint structure number' J_n, 'joint roughness number' J_r, 'joint alteration number' J_a, 'joint water reduction factor' J_w, and 'stress reduction factor' (SRF) as follows:

$$Q=\frac{\text{RQD}}{J_n}\times\frac{J_r}{J_a}\times\frac{J_w}{\text{SRF}}$$



(a)



(b)

Figure 10.8 Rock classification methods. (a) Rock classification using laboratory measurements of Young's modulus and uniaxial compressive strength (σ_a). (After Deere (1968) 'Geological considerations', in: Stagg and Ziekiewicz (eds) *Rock mechanics.* Wiley, Chichester, pp.1–20). The plotted results relate to specimens from the Churchill Falls underground powerhouse (see: Benson, Murphy and McCreath (1970) *Modulus testing of rock at the Churchill Falls underground powerhouse.* American Society for Testing and Materials. Special Technical Publication Number 477); (b) rock classification using observations of fracture spacing and point-load strength. The plotted results relate to specimens of Coal Measures rock core. (After Franklin, Broch and Walton (1971) 'Logging the mechanical character of rock', *Trans Inst. Min. Metall.*, **80**, A1–A10; discussion, **81**, A43–A51 (1972))

The value of Q can be related to width of span and support requirements, using tables and graphs based on precedent practice for several categories for rock mass quality. The scheme includes a factor, excavation support ratio (ESR), which varies for temporary or permanent openings and is similar to the concept of factor of safety in engineering works. The index Q is essentially a function of three parameters which represent:

(1) Block size RQD/J_n.
(2) Inter-block shear strength J_r/J_a.
(3) Active stress J_w/SRF.

These rock mass classification systems have proved to be very useful practical engineering tools not only because they provide a starting point for the design of tunnel support but also because the properties of the rock mass must be examined in a very systematic manner. The familiarity and understanding gained from this systematic study are themselves of great value to enable sound engineering judgements to be made.[69]

Caution should be exercised, however, when considering the use of these classification schemes in rock types significantly different from those from whence the case-history information is taken.

10.3.2 Strength, deformation and failure criteria

In recent years, rock mechanics has advanced such that the demands on testing work have changed. From merely providing information to classify and to put into groups rocks on a quantitative as well as descriptive basis, the results of tests must now contribute input parameters for some very powerful analytical methods. These analyses require realistic constitutive relations for stress, strain and failure criteria in order to be useful. They draw upon quantitative data from laboratory tests, field tests, descriptive observations or the classification schemes mentioned in section 10.3.1.

Deformability tests can be used to determine the stress–strain relationship (i.e. modulus and Poisson's ratio) for rocks prior to the yield point or the peak strength being attained. In addition, the development of servo-controlled stiff testing machines[70] has permitted the determination of the complete *post-peak* stress–strain curve for rocks. This information is important in the design of underground excavations since the properties of the 'failed' rock surrounding the excavations have a significant influence upon the stability of the excavations. The term 'failure' is sometimes taken to mean the attainment of the peak strength. However, when the rock is confined, the *post-peak behaviour* is also relevant, and failure may alternatively mean the rock can no longer sustain the forces applied to it. This may involve considerations other then peak strength, e.g. excessive deformation may be a more appropriate criterion of 'failure'.[71]

Post-peak stress–strain curves are used, for example, in rock-support interaction analysis, described in section 10.4.7. Also various types of elastic–plastic, or elastic–strain softening and brittle behaviour can be applied to the analysis of 'yield zones' around tunnels.[72]

The stress–strain behaviour of jointed rock masses has been studied theoretically to determine equivalent overall elastic constants for use in design analyses taking into account the presence of numerous joint sets,[73] although the modulus of rock masses in relation to empirical classification schemes has also been studied. This information is summarized in Figure 10.9, which is a useful guide to selecting initially a modulus of deformation of a rock mass for preliminary calculations.

Rock mass strength concepts have been examined using several theoretical formulations based on expressions of three-dimensional stresses or equations of strain energy. Alternatively, a number of empirical approaches have been used which in general are more useful in solving practical engineering problems.[74] The simpler failure criteria use the maximum (major) and minimum (minor) principal stresses in their formulation, generally ignoring the effects of the intermediate principal stress. Some of the criteria provide an insight into theoretical behaviour, especially in the study of *fracture mechanics*, but they rarely agree with experimental results for a broad range of rocks or rock mass characteristics.[75]

Determination of the strength of a rock mass by laboratory testing is generally not practical (Table 10.11). Hence, this strength must be estimated from geological observations and from test results on individual intact rock pieces removed from the rock mass. This question has been discussed extensively by Hoek and Brown[62] who used the results of theoretical and model studies and the limited amount of available strength data, to develop an empirical failure criterion for jointed rock masses.

The empirical equation of Hoek and Brown's strength criterion which describes a Mohr failure envelope is:

$$\sigma_1' = \sigma_3' + (m\sigma_c\sigma_3' + s\sigma_c^2)^{1/2}$$

where σ_1' is the major principal effective stress at failure, σ_3' is the minor principal effective stress or the confining pressure in the case of a triaxial test, σ_c is the uniaxial compressive strength of the intact rock material, and m and s are empirical constants.

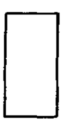
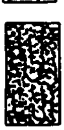
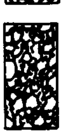
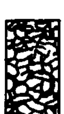
The equation may be normalized with respect to σ_c by division to give:

$$\sigma_{1n}' = \sigma_{3n}' + (m\sigma_{3n}' + s)^{1/2}$$

which is a very useful form when comparing the shape of these Mohr failure envelopes for different rocks (Figure 10.10).

The criterion thus contains three constants m, s and σ_c. Both m and s depend on the properties of the rock and the extent to which it is broken before the application of failure stresses. The value of s is zero for a broken rock with no tensile strength and no cohesion, ranging up to 1 for intact rock. It is found that m has values characteristic of certain rock types. The higher values (15 to 25) are associated with brittle igneous rocks with high friction angles such as granites, and lower values (3 to 7) with more ductile rocks such as limestomes. The uniaxial compressive strength σ_c was adopted for the criterion as it is such a widely quoted index of rock property and can be reasonably estimated at the initial stages when perhaps no firm data is available. The three constants can be determined statistically from experimental data, although values for m and s have been

Table 10.11 Summary of range of rock mass characteristics. (After Hoek (1983), 'Strength of jointed rock masses', *Géotechnique*, **33**, 3)

	Description	*Strength characteristics*	*Strength testing*	*Theoretical considerations*
	Hard intact rock	Brittle, elastic and generally isotropic	Triaxial testing of core specimens in laboratory relatively simple and inexpensive and results usually reliable	Theoretical behaviour of isotropic elastic brittle rock adequately understood for most practical applications
	Intact rock with single inclined discontinuity	Highly anisotropic, depending on shear strength and inclination of discontinuity	Triaxial testing of core with inclined joints difficult and expensive but results reliable. Direct shear testing of joints simple and inexpensive but results require careful interpretation	Theoretical behaviour of individual joints and of schistose rock adequately understood for most practical applications
	Massive rock with a few sets of discontinuities	Anisotropic, depending on number, shear strength and continuity of discontinuities	Laboratory testing very difficult because of sample disturbance and equipment size limitations	Behaviour of jointed rock poorly understood because of complex interaction of interlocking blocks
	Heavily jointed rock	Reasonably isotropic. Highly dilatant at low normal stress levels with particle breakage at high normal stress	Triaxial testing of undisturbed core samples extremely diffficult due to sample disturbance and preparation problems	Behaviour of heavily jointed rock very poorly understood because of interaction of interlocking angular pieces
	Compacted rockfill	Reasonably isotropic. Less dilatant and lower shear strength than *in situ* jointed rock but overall behaviour generally similar	Triaxial testing simple but expensive because of large equipment size required to accommodate representative samples	Behaviour of compacted rockfill reasonably well understood from soil mechanics studies on granular materials
	Loose waste rock	Poor compaction and grading allow particle rotation and movement resulting in mobility of waste rock dumps	Triaxial or direct shear testing relatively simple but expensive because of large equipment size required	Behaviour of waste rock adequately understood for most applications

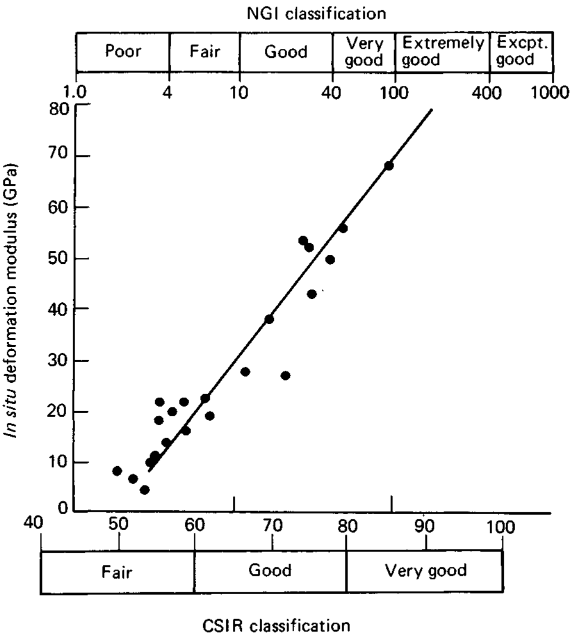


Figure 10.9 The relationship between *in situ* deformation modulus of rock masses and rock mass classification. (After Hoek and Brown (1980) *Underground excavations in rock*. Institution of Mining and Metallurgy, London)

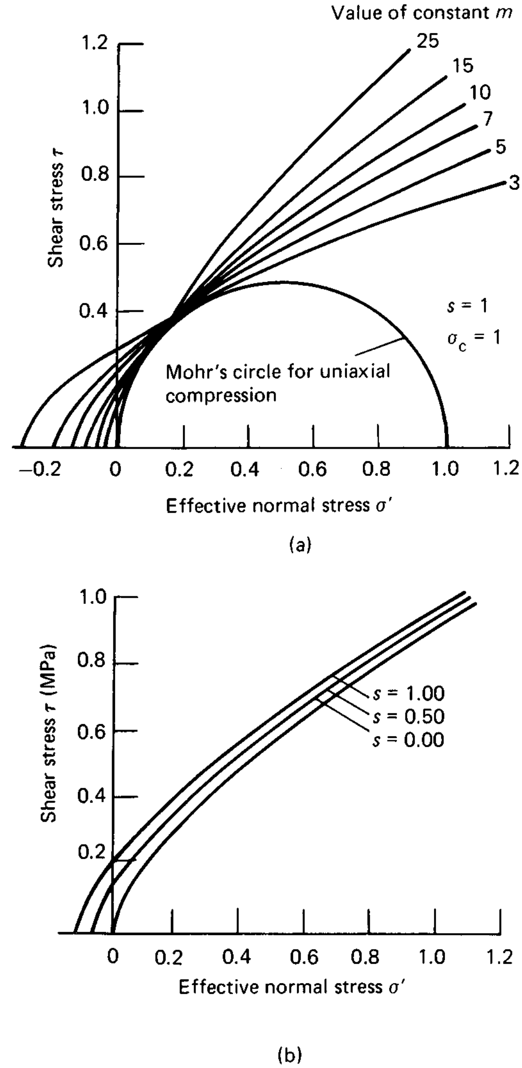


Figure 10.10 The influence of the values of *m* and *s* in Hoek and Brown's failure criterion. (a) Influence of the value of the constant *m* on the shape of the Mohr failure envelope at different effective normal stress levels; (b) influence of the value of the constant *s* on the shape of the Mohr envelope at different normal stress levels. (After Hoek (1983) 'Strength of jointed rock masses', *Géotechnique*, **33**, 3, 187–223)

proposed based on the rock mass classification systems and these are shown in Table 10.12. The effect of comparative size of the proposed excavation and the spacing of discontinuities in the rock mass (see Figure 10.11) is important to the design. Table 10.12 should be used only for rock masses containing several joint sets or for intact rock. There are other specified procedures necessary to apply this method to anisotropic or schistose conditions influenced by dominant discontinuities.

This criterion gives reasonably good estimates for disturbed hard rock masses and tends to be conservative in tightly interlocking hard rock masses. A similar empirical criterion by Johnston[75] appears to extend the principle to intact materials ranging down to soft rocks and firm clays.

10.3.3 Properties of joints and discontinuities

Standard methods of observing, recording and describing the characteristics of discontinuities mentioned above in sections 10.1.3 and 10.1.4 have been put forward by the ISRM[24] in which many practical aspects are described. A joint survey of a rock mass can be carried out either in a *subjective* (biased) manner where only those discontinuities which appear to be important are described, or in an *objective* (random) manner where all joints within a delineated area are described. A subjective survey is best applied where structural patterns can be identified clearly and will save time and effort, whereas an objective survey is time-consuming even though automatic data processing may be used to advantage to analyse all the data. The principal survey methods are:

(1) Outcrop or excavation (e.g. tunnel wall) exposure description.
(2) Drillcore or drillhole wall description.
(3) Terrestrial photogrammetry.

For method (1) 'scanlines' are often used to define a sample of the exposure and as such are similar to the linear sampling of the rock mass afforded by drillholes in method (2). The statistical significance of these samples in relation to the three-dimensional rock structure, the nature of probable errors and statistical methods of analysing the various data have been examined.[8,10]

There are three fundamental properties of discontinuities which significantly affect the rock mass and they are: (1) strength (or weakness); (2) deformability (or stiffness); and (3) conductivity (or permeability).[76] They are all interrelated, or coupled and they are stress-dependent. The parameters' strength and stiffness are strongly stress-dependent and may vanish under tensile stress. When under compression, however, they vary between fairly well-understood limits. The parameter which varies most of all under varying compression and shear is the joint aperture and, hence, the hydraulic conductivity.

Table 10.12 Approximate relationship between rock mass quality and material constants *m* and *s*. (After Hoek (1983) 'Strength of jointed rock masses', *Géotechnique*, **33**, 3)

Empirical failure criterion $\sigma_1' = \sigma_3' + (m\sigma_c\sigma_3' + s\sigma_c^2)^{1/2}$ σ_1' = major principal stress σ_3' = minor principal stress σ_c = uniaxial compressive strength of intact rock m, s = empirical constants		Carbonate rocks with well-developed crystal cleavage, e.g. dolomite, limestone and marble	Lithified argillaceous rocks, e.g. mudstone, siltstone, shale and slate (tested normal to cleavage)	Arenaceous rocks with strong crystals and poorly developed crystal cleavage, e.g. sandstone and quartzite	Fine-grained polymineralic igneous crystalline rocks, e.g. andesite, dolerite, diabase and rhyolite	Coarse-grained polymineralic igneous and metamorphic crystalline rocks, e.g. amphibolite, gabbro, gneiss, granite, norite and quartzdiorite
Intact rock samples Laboratory size samples free from pre-existing fractures		$m=7$ $s=1$	$m=10$ $s=1$	$m=15$ $s=1$	$m=17$ $s=1$	$m=25$ $s=1$
Bieniawski (1974b) (CSIRO)* rating	100					
Barton *et al.* (1974) (NGI)† rating	500					
Very-good-quality rock mass Tightly interlocking undisturbed rock with rough unweathered joints spaced at 1–3 m		$m=3.5$ $s=0.1$	$m=5$ $s=0.1$	$m=7.5$ $s=0.1$	$m=8.5$ $s=0.1$	$m=12.5$ $s=0.1$
Bieniawski (1974b) (CSIRO)* rating	85					
Barton *et al.* (1974) (NGI)† rating	100					
Good-quality rock mass Fresh to slightly weathered rock, slightly disturbed with joints spaced at 1–3 m		$m=0.7$ $s=0.004$	$m=1$ $s=0.004$	$m=1.5$ $s=0.004$	$m=1.7$ $s=0.004$	$m=2.5$ $s=0.004$
Bieniawski (1974b) (CSIRO)* rating	65					
Barton *et al.* (1974) (NGI)† rating	10					
Fair-quality rock mass Several sets of moderately weathered joints spread at 0.3–1 m, disturbed		$m=0.14$ $s=0.0001$	$m=0.20$ $s=0.0001$	$m=0.30$ $s=0.0001$	$m=0.34$ $s=0.0001$	$m=0.50$ $s=0.0001$
Bieniawski (1974b) (CSIRO)* rating	44					
Barton *et al.* (1974) (NGI)† rating	1					
Poor-quality rock mass Numerous weathered joints at 30–500 mm with some gouge. Clean, compacted rockfill		$m=0.04$ $s=0.00001$	$m=0.05$ $s=0.00001$	$m=0.08$ $s=0.00001$	$m=0.09$ $s=0.00001$	$m=0.13$ $s=0.00001$
Bieniawski (1974b) (CSIRO)* rating	23					
Barton *et al.* (1974) (NGI)† rating	0.1					
Very-poor-quality rock mass Numerous heavily weathered joints spaced at 50 mm with gouge. Waste rock		$m=0.007$ $s=0$	$m=0.010$ $s=0$	$m=0.015$ $s=0$	$m=0.017$ $s=0$	$m=0.025$ $s=0$
Bieniawski (1974b) (CSIRO)* rating	3					
Barton *et al.* (1974) (NGI)† rating	0.01					

*CSIRO Commonwealth Scientific and Industrial Research Organization.
†NGI Norwegian Geotechnical Institute.
(For references used in this table, consult source, above.)

Stresses applied normal to the plane of the discontinuity (normal stresses) will cause closure (i.e. reduction of the aperture) and the 'normal stress' to 'normal displacement' ratio is the 'normal stiffness'. Shear stresses cause shear displacements and this ratio is the shear stiffness; shear displacements, however, usually result in dilatation (i.e. increase in aperture) of the discontinuity. The above are all governed by the rock-to-rock contact of asperities across discontinuities which is described by the 'roughness' and 'waviness', 'wall strength' of the discontinuity surfaces and the friction angle.

An important factor affecting shear strength is the magnitude of the effective normal stress σ_n' acting across the joint. In many rock engineering problems the maximum effective normal stress will lie in the range 0.1 to 2.0 MPa for those joints considered critical for stability.

It has been customary to fit Coulomb's linear relation $\tau = c + \sigma_n \tan\phi$ to the results of shear strength investigations on rock joints (τ is peak shear strength, c is the cohesion intercept and ϕ is the friction angle). If this equation is applied to the results of shear tests on *rough joints*, under both high normal

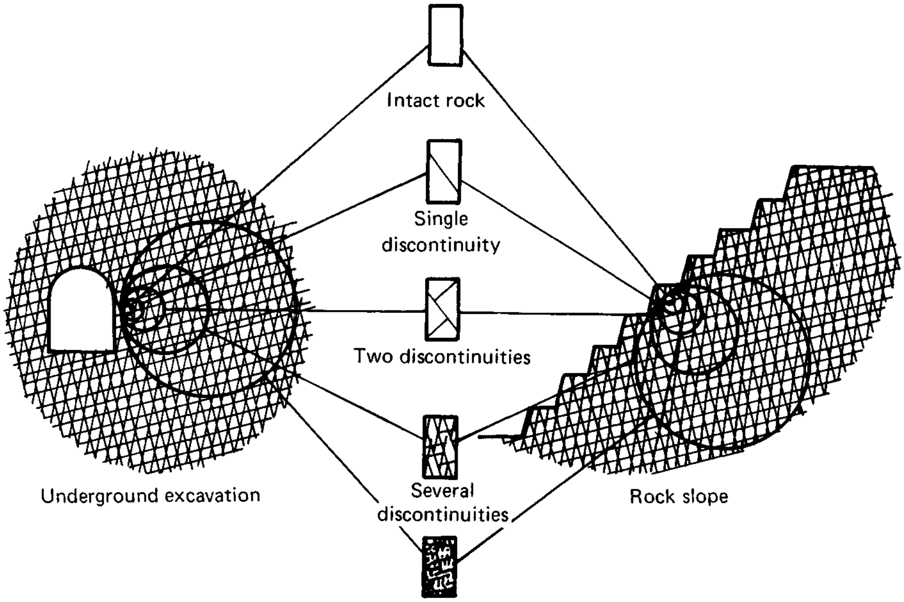


Figure 10.11 A simplified representation of scale on the type of rock mass behaviour model which should be used in designing underground excavations or rock slopes. (After Hoek (1983) 'Strength of jointed rock masses', *Géotechnique*, **33**, 3, 187–223)

stress and low normal stress, one finds the former recording a cohesion intercept of tens of megapascals and a friction angle of perhaps only 20°, while the latter has a friction angle of perhaps 70° and zero cohesion. The peak shear strength envelopes for *nonplanar* rock joints are strongly curved.

This behaviour can be explained[77] by assuming two mechanisms, illustrated for practical situations in Figure 10.12(a):

(1) Dilatation, in which the moving block rides up and over the surface irregularities – assumed to be applicable when the normal stress on the surface is small.
(2) Shear or fracture through the irregularities resulting in a planar surface and nondilatational slip – assumed to be applicable when the normal stress on the surface is large.

These ideas suggest a bilinear strength envelope such as illustrated in Figure 10.12(b). The angle ϕ represents the friction angle between clean planar rock surfaces and the angle i is the angle of dilatation.

For the first mechanism the apparent friction angle is $(\phi+i)$ and the joint behaviour is described by the relation:

$$\tau=\sigma_n \tan(\phi+i)$$

It should be noted that extrapolation back to low values of stress of the upper part of the bilinear strength envelope results in gross overestimation of strength represented by the cohesion intercept, c.

Empirical methods of quantifying roughness and wall strength and utilizing them in shear strength relations were developed by Barton and Choubey.[78] They proposed that the peak shear strengths of joints τ in rock could be represented by the empirical relation:

$$\tau=\sigma_n' \tan\left[\mathrm{JRC}\log_{10}\left(\frac{\mathrm{JCS}}{\sigma_n'}\right)+\phi_r'\right]$$

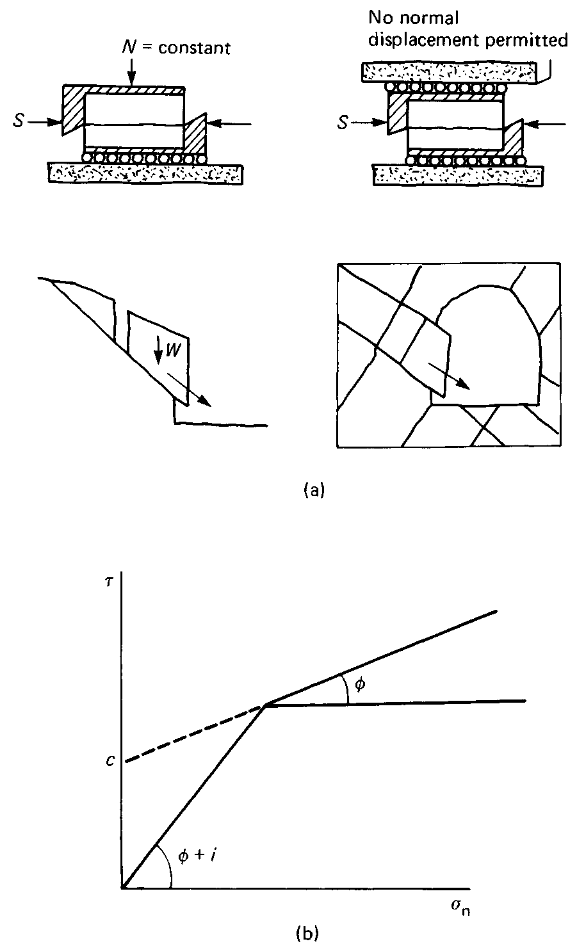


Figure 10.12 (a) Examples of two types of shear behaviour on joints; (b) bilinear shear strength envelope

where σ_n' is effective normal stress, JRC is joint roughness coefficient on a scale of 1 for the smoothest to 20 for the roughest surfaces, JCS is joint wall compressive strength and ϕ_r' is drained, residual friction angle.

This equation suggests that there are three components of shear strength – a frictional component given by ϕ_r', a geometrical component controlled by surface roughness (JRC) and an asperity failure component controlled by the ratio (JCS/σ_n'). As Figure 10.13 shows, the asperity failure and geometrical components combine to give the net roughness component *i*. The total frictional resistance is then given by $(\phi + i)$. The equation and Figure 10.13 show that the shear strength of a rough joint is both scale-dependent and stress-dependent.

The JRC and ϕ_r' can be obtained indirectly from extremely simple tilt tests using pieces of intact and jointed core, as shown in Figure 10.14. The ideal sample would be jointed axially, but routine testing can also include obliquely jointed samples, as typically recovered from a drilling program. The JRC, ϕ_r' are obtained from such tests as follows:

$$\mathrm{JRC} = \frac{a - \phi r'}{\log\,(\mathrm{JCS}/\sigma'_{n0})}$$

where α is the tilt angle when sliding occurs and σ'_{n0} is the corresponding value of effective normal stress when sliding occurs (weigh upper sample, correct for cos α, measure joint area).

The value of JRC typically varies from 0 to about 15, 0 corresponding to residual nondilatant joint surfaces for which:

$$\alpha = \phi_r'$$

The residual friction angle ϕ_r' may be lower than the *basic friction angle* ϕ_b obtained from the tilt tests on core cylinders (lowest sketch Figure 10.14) due to weathering or alteration effects. A simple empirical relation was developed to estimate ϕ_r' from ϕ_b using the Schmidt hammer. It is based on rebound tests on both the unweathered, dry rock (rebound *R*) and on the weathered and saturated joint wall (rebound *r*):

$$\phi_r' = (\phi_b - 20) + 20\frac{r}{R}$$

The *Schmidt hammer* test is also used to estimate the JCS of the fresh or altered joints in the saturated state, if this is appropriate to *in situ* conditions. The relationship derived by Miller[79] (Figure 10.15) is used to convert the rock density and the rebound *r* to estimate the compression strength JCS. Full details of the above characterization tests were given by Barton and Choubey.[78]

The three parameters (JRC, JCS, ϕ_r) are all that are needed to develop shear strength, displacement, dilatation and normal stress-closure curves for any given joint. Coupling conductivity with these processes requires additional information concerning the initial joint aperture but similar methods are available for such analyses.[7]

Discontinuities may contain infilling materials such as clay 'gouge' in faults, silt in bedding planes, weak low-friction minerals such as chlorite or stronger materials such as quartz or calcite in veins. Clay gouge infill will decrease both joint stiffness and shear strength, low friction mineral coatings will decrease friction angles (hence shear strength), while vein materials like quartz will increase the strength and stiffness. Occasionally the last category mentioned is described as a 'healed joint' but this term is not encouraged here. Clearly, major joints infilled with weak materials such as clay or silt are of particular concern as they may present the dominant failure mechanism. These weak materials are usually analysed as soils using an effective stress Coulomb relationship. The behaviour of filled discontinuities often has two characteristics, the first reflecting the deformability of the infill material without rock-to-rock contact, and the second reflecting the deformability and shear strength of rock asperities in contact. Swelling clay infill is dangerous as it can develop high swelling pressures and will lose strength on swelling.

10.4 Design methods

The purpose of designing a rock structure (e.g. a tunnel, cavern, open cut or foundation) is to decide the size and shape of the excavation, to determine whether measures to improve the rock conditions such as grouting, rock reinforcement, anchoring or drainage are needed, whether rock support such as shotcrete, mesh, rockbolts, steel arches, or concrete liners are required and to select and design the appropriate systems as necessary.

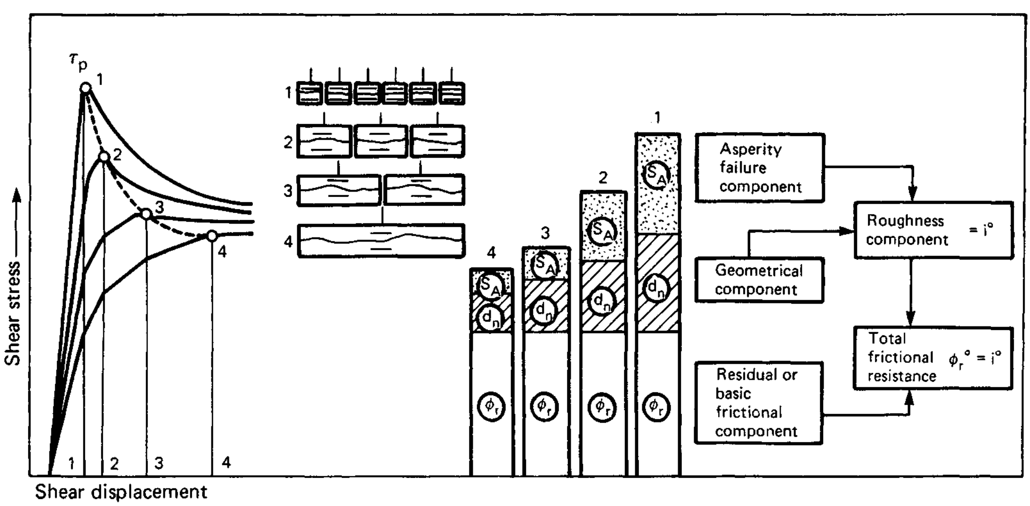


Figure 10.13 An illustration of the size-dependence of shear stress-deformation behaviour for nonplanar joints. (After Bandis, Lumsden and Barton (1981) 'Experimental studies of scale effects on the shear behaviour of rock joints'. *Int. J. Rock Mech. Min. Sci.*, **18**)

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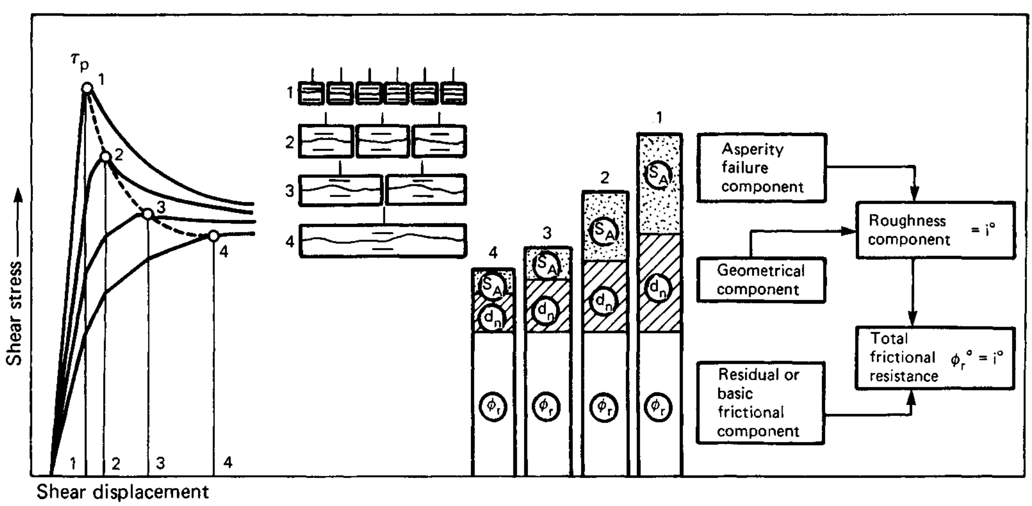

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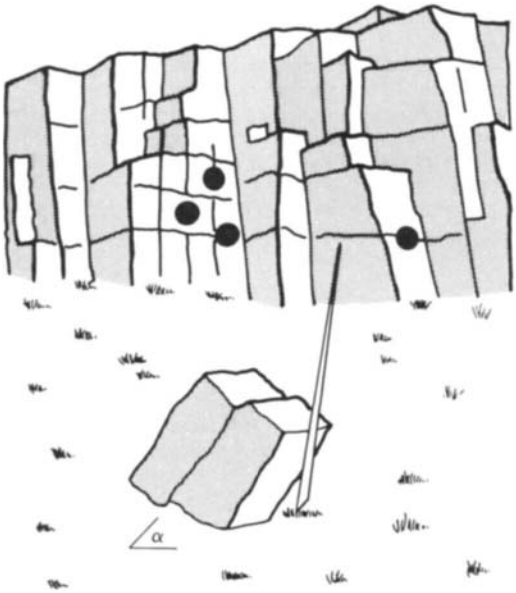
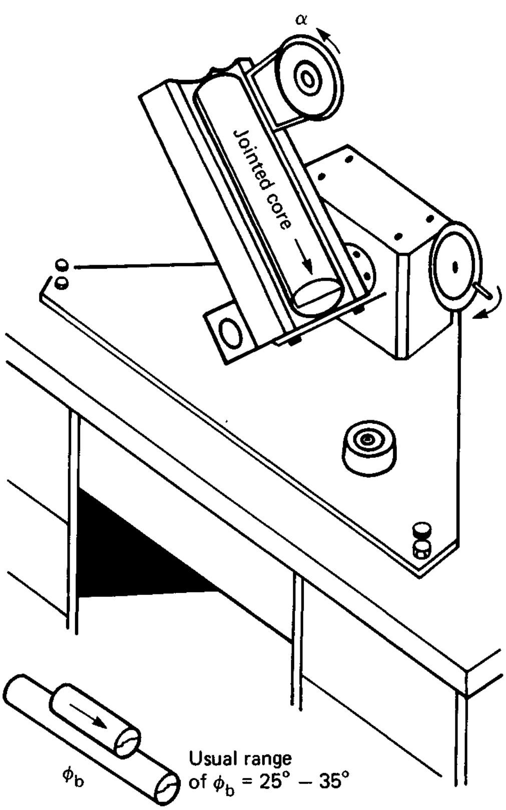


(a)



(b)

Figure 10.14 Tilt tests for obtaining joint roughness and basic friction parameters. (After Barton (1986) 'Deformation phenomena in jointed rock'. *Géotechnique*, **36**, 2)

The analysis of a rock structure should not start without first preparing a complete statement of the factors involved. These usually include the geometry and intended purpose of the structure together with the main elements of the rock engineering matrix described previously. That is, the structure of the rock, mechanical properties of intact rock materials and the discontinuities, the nature of water and stress conditions in the mass and the proposed construction/excavation method. It is not usually possible to take each factor into full account, so that analysis is often based on simplified mathematical or physical 'models'. The choice of simplifying assumptions and the errors that these assumptions are likely to introduce are matters for engineering judgement, and it is often advisable to check a design by carrying out more than one type of analysis wherever possible. 'Failure' of the structure needs to be defined (e.g. excessive displacements) and potential failure mechanisms need to be identified so that design calculations are addressed to these specifically. The adequacy of the initial design can be checked by instrumentation and monitoring and modified if necessary.

Design methods for the stability of the rock structure consider either the resulting *stresses* in the surrounding rock or the *displacements* which are induced or which become kinematically feasible, e.g. wedge failure on joint planes. In applying methods of stress analysis, judgement is required in deciding whether or not the rock should be regarded as a continuum, and then whether as an infinite space or half space. Various conditions of anisotropy and of inelastic behaviour can be simulated with some models. In discontinuous media, methods are available to analyse the displacements, provided parameters describing rock joint behaviour can be determined.

10.4.1 Empirical methods

A useful first approach is design by 'rule of thumb' using design principles that experience has shown to give satisfactory results. Where no precedent exists simple rules can be established by undertaking a programme of field observations to determine relationships between 'cause' and 'effect'. This is more likely to succeed if it is preceded by an attempt to establish, using theory and simple methods, those parameters that might prove fundamental to the design, and the trends to be expected.[9,80] Empirical design rules are usually only safe to apply in the context for which they were originally formulated, and extrapolation can be unreliable, particularly if the method has no theoretical basis.

An example of empirical methods based on precedent practice is the classification of rock in relation to support requirements for underground openings, namely the *Q* system, RSR and others (see section 10.3.1).

10.4.2 Physical models and analogue methods

Physical, as opposed to mathematical, models can be used in a laboratory analysis of stresses or displacements in a rock structure, and can sometimes also be applied to the study of fracture and failure.[81]

Elastic models may be used to analyse stresses mainly but are applicable where the rock may reasonably be assumed to behave elastically. The most common type, a *photoelastic* model, is machined from a stress-birefringent material such as glass or plastic. When loaded and viewed in polarized light the model exhibits coloured fringes (isochromatics) that follow contours of maximum shear stress in the model. Black fringes (isoclinics) visible in plane polarized light indicate the principal stress directions. Stress-freezing and other methods are available for analysis of problems in three dimensions. *Other types of elastic model* have also been used, e.g. metal sheet with resistance strain gauges or moire fringe grids mounted on the surface. Elastic modelling has been largely superseded in most applications by

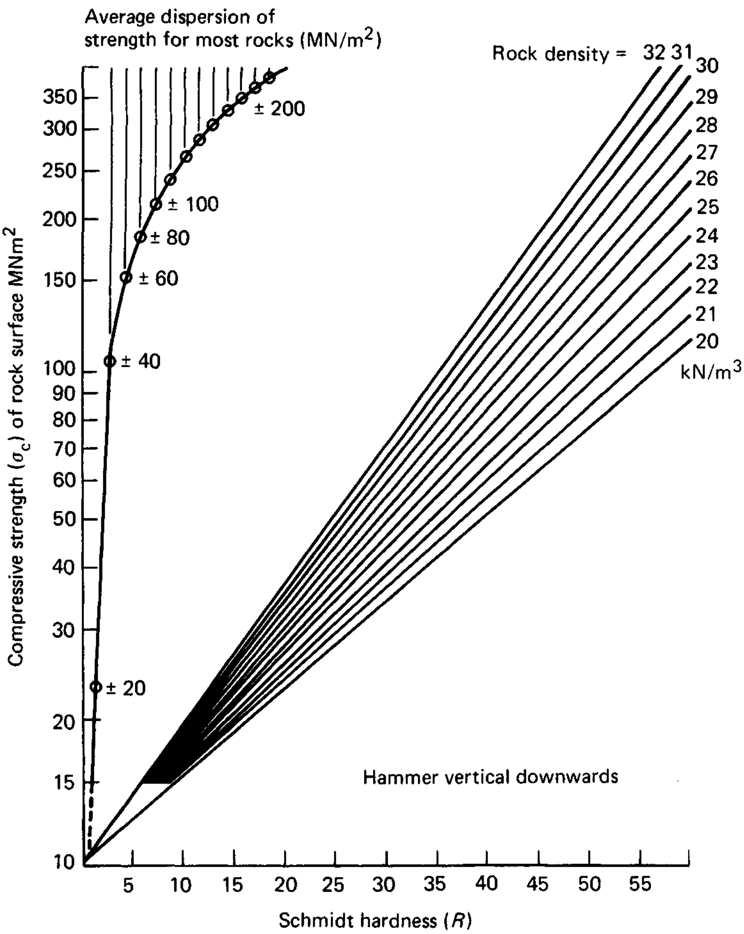


Figure 10.15 Correlation chart for the Schmidt hammer, relating rock density, compressive strength and rebound number. (After Miller (1965) 'Engineering classification and index properties for intact rock'. PhD thesis, University of Illinois)

computer analyses that afford greater flexibility, require fewer simplifying assumptions and less preparation time, and can also solve inelastic problems. Photoelastic models, however, can still be useful in presenting a visual and easily understood representation of stress distributions, and can cope easily with complex geometrical configurations.

Inelastic physical models and *block models* to study displacements are built from materials chosen, according to principles of dimensional similarity, to scale-down prototype properties such as density, strength and deformability.[82] Physical models have been used, for example, to investigate the behaviour of rock slopes, underground excavations at various stages of construction, and subsidence above mine workings.[83] Gravity can be simulated by building the model lying on a flat surface with a movable backing sheet which can be drawn in the downward direction: the friction forces on the blocks simulate gravity forces. Such models are known as 'base-friction models'.[84] Simple and approximate physical models can be valuable at the early stages of analysis in helping to visualize possible kinematic mechanisms and in formulating the problem, but care is needed to select appropriate scaling factors.

The equations governing electric potential differences and currents are analogous to the equations governing stress distributions or water flow in the rock mass. Thus, stress or water problems can be simulated and solved by electric analogue methods.[85,86] *The conducting paper method*, of limited accuracy and flexibility but simple, uses an impregnated paper with probes to monitor surface potential differences. *The resistance network method* uses a grid of interchangeable or variable resistors, can solve anisotropic and heterogeneous problems and is more accurate, but has the disadvantage of restricting measurements to a limited number of nodal points. Analogues other than electric ones are possible, e.g. the Hele–Shaw method uses the flow of viscous fluids between closely spaced parallel plates.[87]

10.4.3 Numerical methods of analysis

Analysis of stresses, strains and displacements based on principles of classical stress analysis and continuum mechanics[88] assume that the material is continuous throughout and that conditions of equilibrium and compatibility of displacements are satisfied for given boundary conditions. The constitutive equation, i.e. the relationship between stress, strain and time for the rock mass, must be known or assumed in order to formulate the problem. This relationship can in theory be established by testing, although in practice, tests serve only to measure the parameters in an idealized constitutive equation such as one of linear elasticity. A satisfactory constitutive equation should account for rock behaviour both before, during and after failure in intact material. In most rock structures, zones of fractured rock can develop owing to the induced stresses exceeding the rock strength (zones of 'overstress') which, because they are confined by unfractured material, do not lead to collapse. It is important to recognize when it is or is not reasonable to assume that a problem may be analysed as a continuum. The presence of fractures or discontinuities does not invalidate the premises of continuum mechanics provided that a constitutive equation can be formulated for an 'element' or test specimen that incorporates a large number of such discontinuities. Soil materials, for example, contain discrete grains bounded by discontinuities but can be tested in this way, thus allowing continuum mechanics to be applied to soil problems.

Having formulated an appropriate equation for the material the problem is solved taking into account the geometry and boundary conditions for the rock structure, and ensuring that conditions of equilibrium and compatibility of displacements are satisfied. Particular solutions which are most useful in considering tunnels or underground openings are those for circular holes in stressed elastic media. Equations for analysing thick-walled cylinders or circular holes in an infinite elastic solid (Kirsch equations) will determine the radial and tangential stresses around the surface and within the rock and the displacements surrounding the opening.[89] A wide range of *exact* or *'closed form' solutions* are available for solving two-dimensional problems of various geometries, particularly for linear elastic behaviour, but few solutions have been formulated for three-dimensional problems. Boussinesq and Cerruti give solutions for normal and shear point-loading on a three-dimensional elastic half-space that can be used to build up, by a process of simple superposition, the distribution of stresses and displacements for any system of applied loads.[90–91] Savin[92] gives closed-form solutions for stress concentrations around holes in an elastic plate; these and other solutions are reviewed by Obert and Duvall.[93] In practice, it is unnecessary to derive solutions to particular problems because published collections exist of most analytically tractable problems. Such a collection by Poulos and Davis[94] is most thorough. It is important, however, to ensure that an appropriate solution is being applied to each design problem.

10.4.4 Computational methods of analysis

For many design problems in rock mechanics it is necessary to seek a more detailed understanding of stress distribution than can be obtained by superposition of standard analytical solutions. Conditions of complex geometry, rock mass anisotropy, nonlinear constitutive behaviour and nonhomogeneity require more versatile methods of solution. To solve the many stress analysis problems for which no solution in closed form is available one must resort to *numerical approximation methods* for solving the continuum mechanics equations. These methods are now widely used since computers have become generally available. There are two categories of computational methods of analysis, namely differential methods and integral methods (see later). For the first of these the problem is divided up into a set of discrete elements and the solution based on numerical approximations of the governing equations, i.e. the differential equations of equilibrium and the stress–strain–displacement relations.

The *finite difference (relaxation) method*[90] has had a long-established use in civil engineering. Partial differential equations that define material behaviour and boundary conditions are replaced by finite-difference approximations at a number of discrete points throughout the rock mass. The resulting set of simultaneous equations is then solved. A finite difference computer program is available specifically designed for modelling soil and rock behaviour and is based on the Lagrangian method (FLAC).[95] It is capable of modelling elastic, anisotropic-elastic and elasto-plastic material properties and can simulate joint slip planes. Its main advantage is in solving for large displacements and collapse due to plastic flow, and is generally applicable to slope and foundations analyses as well as underground excavations.

The finite element method[96,97] is similar in many respects, except that the rock mass is subdivided into a number of structural components or interacting elements that may be of irregular and variable shape (Figure 10.16). A judicious selection of element is critical to the efficiency of computation. The elements are assumed to be interconnected at a discrete number of points on their boundaries, and a function is chosen to define uniquely the state of displacement within each element in terms of nodal displacements at element boundaries.

Strain may then be defined and, hence, stress using the constitutive equation for the material. Nonlinear and heterogeneous material properties may readily be accommodated, but the outer boundaries of the model (or 'problem domain') must be defined arbitrarily. The boundary conditions, in terms of relative fixity and degrees of freedom, may influence the area of interest to the analysis depending upon how distant these boundaries can be set in the model. There are practical limitations on the number of elements used in relation to computer storage, and the elements themselves need to be well-conditioned shapes (triangles or rectangles). Nodal forces are determined in such a way as to equilibriate boundary stresses, and the stiffness of the whole model may then be formulated as the sum of contributions from individual elements. The response of the structure to loading may then be computed by the solution of a set of simultaneous equations. Finite element computer programs have been written for a variety of rock mechanics problems, to tackle both two- and three-dimensional situations, elastic, plastic, and viscous materials, and to incorporate 'no-tension zones', joints, faults and anisotropic behaviour. The method is also used to solve water-flow problems, heat-flow problems, and an even wider scope of situations unrelated to rock engineering.

Analytical techniques, which are based on continuum idealization, are not always suitable for jointed rock problems. Numerical methods such as the finite element method or finite difference method in which rock masses are simply considered as elastic or elasto-plastic continua, are not suitable to model the geometric irregularity in natural jointing.

Several methods have been proposed to simulate such discontinuous media. They are divided into two groups: (1) 'equivalent' continuous analyses in which the jointed rock mass is represented by a homogeneous, anisotropic and continuous medium;[98,99] and (2) the methods which can deal with discontinuities directly and can express positively the behaviour of discontinuous rock masses.

Finite element techniques using joint elements[100] have been used in the analysis of certain problems, particularly in configurations involving a relatively small number of major faults or joints. However, for models with dense jointing, a large number of degrees of freedom are required.

The *distinct element method* or *dynamic relaxation method*[101] allows a problem to be formulated assuming rock blocks to be rigid, with deformation and movement occurring only at the joints and fissures so that for this type of analysis in its simplest form no information is needed on the deformability and strength of intact rock. The method is a discontinuum modelling approach which is suitable in cases where the behaviour of the rock mass is dominated by the properties of joints or other discontinuities. In such cases the discontinuity stiffness (i.e. force/displacement characteristics) is much lower than that of intact rock. Calculations are based on laws relating forces and

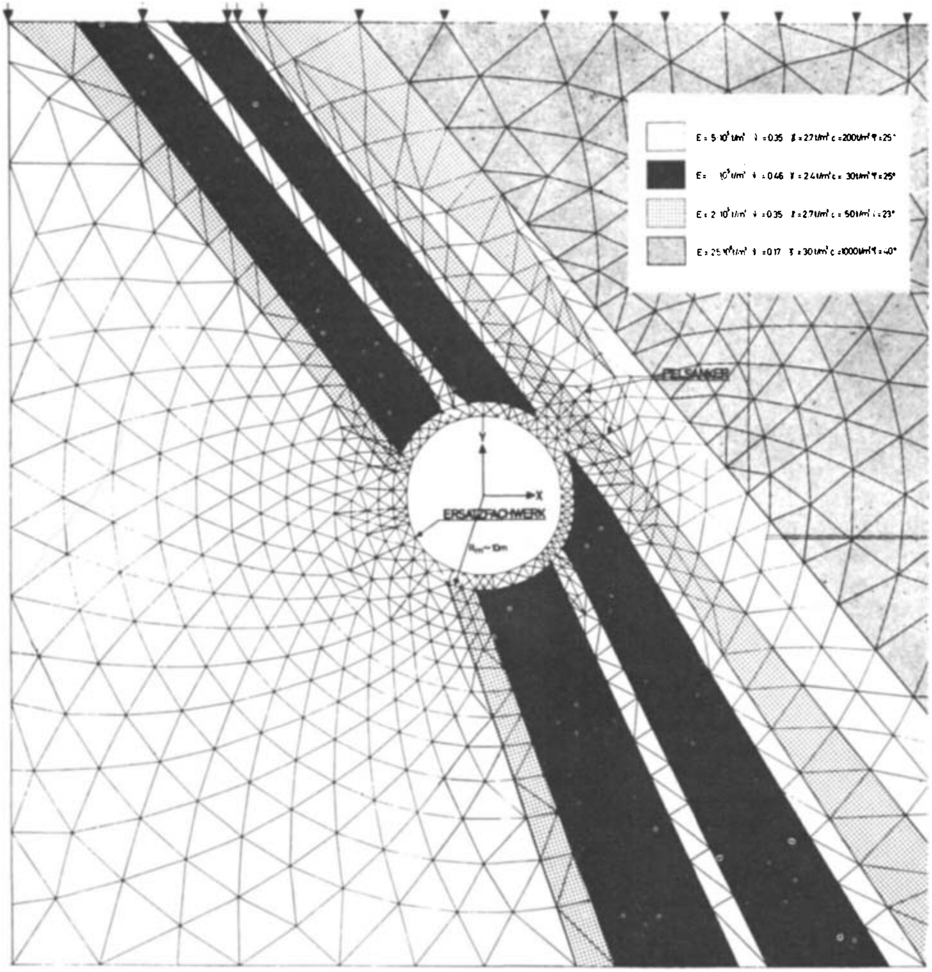


Figure 10.16 Finite-element method. An example showing finite-element mesh for the analysis of stresses acting on a pressure tunnel lining. Elements of varying stiffness have been used to simulate rock zones of varying competence. (Grob, *et al.* (1970) *Proceedings, 2nd international conference on rock mechanics,* Belgrade. Paper 4–69)

displacements between blocks (e.g. laws of elasticity or friction) and on the laws of motion (e.g. creep, viscosity or Newton's laws). Behaviour of the model is constricted to be compatible with boundary force or displacement conditions. Large movements can be modelled – not normally possible with any accuracy using a finite-element method. The method of computation is ideally suited for considering the development of rock movements incrementally with time (Figure 10.17). The distinct element method first described by Cundall[101] treats the rock as an assemblage of blocks interacting across deformable joints of definable stiffness. It is a development of the relaxation method and the dynamic relaxation method described by Otter, Cassell and Hobbs.[102] A force–displacement relationship governs interaction between the blocks and laws of motion determine block displacements caused by out-of-balance forces. Several forms of distinct element codes have been developed to cover a variety of *in situ* conditions. The Universal Distinct Element Code (UDEC), has been developed recently which provides, in one code, all the capabilities that existed separately in previous programs. Features exist for modelling variable rock deformability, nonlinear inelastic behaviour of joints, plastic behaviour and fracture of intact rock, and fluid flow and fluid pressure generation in joints and voids. An automatic joint generator produces joint patterns based upon statistically derived joint parameters. The program can simulate the influence of the far-field rock mass for both static and dynamic conditions as it is coupled to a boundary element program which represents the effects of a static, elastic far-afield response, and nonreflecting boundary conditions are available for dynamic simulations.[103] The technique has three distinguishing features which make it well suited for discontinuum modelling:

(1) The rock mass is simulated as an assemblage of blocks which interact through corner and edge contacts.
(2) Discontinuities are regarded as boundary interactions between blocks; joint behaviour is prescribed for these interactions.
(3) The method utilizes an explicit time-stepping algorithm

which allows large displacements and rotations and general nonlinear constitutive behaviour for both the rock matrix and the joints.

The *boundary element method*[104] and the *displacement discontinuity method*[105] are integral methods of stress analysis, in which the problem is specified and solved in terms of forces (or tractions) and displacements on the surface or boundary of the model. The boundary (such as a tunnel perimeter) is divided into discrete elements whilst the far-field boundary may be infinite (or semi-infinite). Thus 'discretization' errors are restricted to the problem boundary, and variations in stresses and displacements are fully continuous. The field equations at a point within the continuum are satisfied exactly, and errors are associated with the approximations occurring at the boundary only. The boundary forces or tractions determine the stresses in the surrounding medium which are evaluated using expressions for stress components at any point in an infinite medium.[106] Elastic displacements around the excavation are calculated by making use of standard solutions for displacements in an infinite medium due to point or line loads. The methods are not particularly well suited to heterogeneous, anisotropic or nonlinear material behaviour. A reasonably clear and concise description of the method is presented in the appendices of a practical manual on underground excavations.[62] This further provides a collection of stress distributions calculated using the boundary element method around single openings of various shapes within different stress fields. Reference to these is useful as a first assessment of stress concentration around proposed excavations.

More recently[107] a boundary element formulation has been presented for modelling structural discontinuities, joints, faults and heterogeneous rock. The model is divided into regions, each one homogeneous, separated by interfaces which can represent discontinuities. The solution, however, is an iterative approximation to account for nonlinear joint equations.

The displacement discontinuity method is particularly suited to the analysis of tabular openings (such as coalmines). It is able to work in three dimensions, data input is relatively easy, and will model multiple seam-mining layouts or folded or faulted single-seam deposits.

Various *coupled computational analyses* (or 'hybrid' methods) have been devised to make advantageous use of the boundary element integral method for modelling the far-field region of a problem, and to couple this with an appropriate differential method (relaxation, finite element or distinct element) to model the immediate surroundings of the excavation. A domain of complex behaviour is thus embedded in an infinite elastic continuum. Lorig and Brady[108,109] have described the coupling of the discrete element method with boundary elements, and Ushijima and Einstein[110] a three-dimensional finite element code with boundary elements.

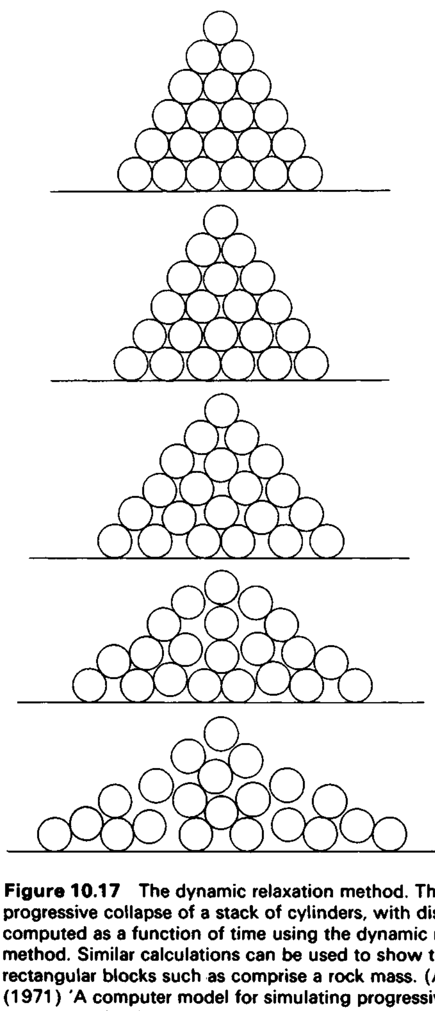


Figure 10.17 The dynamic relaxation method. The figure shows progressive collapse of a stack of cylinders, with displacements computed as a function of time using the dynamic relaxation method. Similar calculations can be used to show the collapse of rectangular blocks such as comprise a rock mass. (After Cundall (1971) 'A computer model for simulating progressive large-scale movements in blocky rock systems'. *Proceedings, international symposium on rock mechanics rock fracture,* Nancy. Paper 2–8)

10.4.5 Slope design

In *the limiting equilibrium method*[111] a rock mass is considered under conditions where the mass is on the point of becoming unstable. The method gives no information on magnitudes of displacement or on rock behaviour prior to failure, so that the design calculations cannot readily be checked by instrumentation and monitoring of rock movements.

Equilibrium is examined by relating the shear and normal forces on the sliding surface to the sliding resistance of that surface. Shear tests are necessary to evaluate sliding resistance, but otherwise a constitutive equation for the rock mass is not required. The geometry and position of the sliding surface must be predicted in advance, and for this reason the method has been most commonly applied to slope stability problems where the sliding surface is more readily predicted than in underground situations. The development of computers has made it possible to use conveniently some very powerful limit equilibrium methods.[69]

Slope design[9] usually employs limiting equilibrium analysis. A first step is to assess whether any kinematic mechanisms of potential slope failure are likely to be more closely approximated by a plane failure, a sliding wedge, rotational slip or a toppling failure model, and to identify the beds, joints or faults that could conceivably control such a failure, as illustrated in Figure 10.18. Throughgoing discontinuities such as faults, beds or older pre-existing failure surfaces are likely to be of considerably greater significance than impersistent or rough features. The presentation and analysis of geological structure data using the method of stereographic projections (stereonets)[10] is invaluable for such an assessment. Clearly, the collection in the field of the geological data relevant to the problem is of paramount importance, as described in Chapter 8.

Quick and approximate calculations at this stage help to assess whether there is indeed a problem, and whether a more detailed analysis is justified. These can employ hand calculations or design charts[9] using data for rock strength and water pressures estimated after examining the rock *in situ*. Worst and

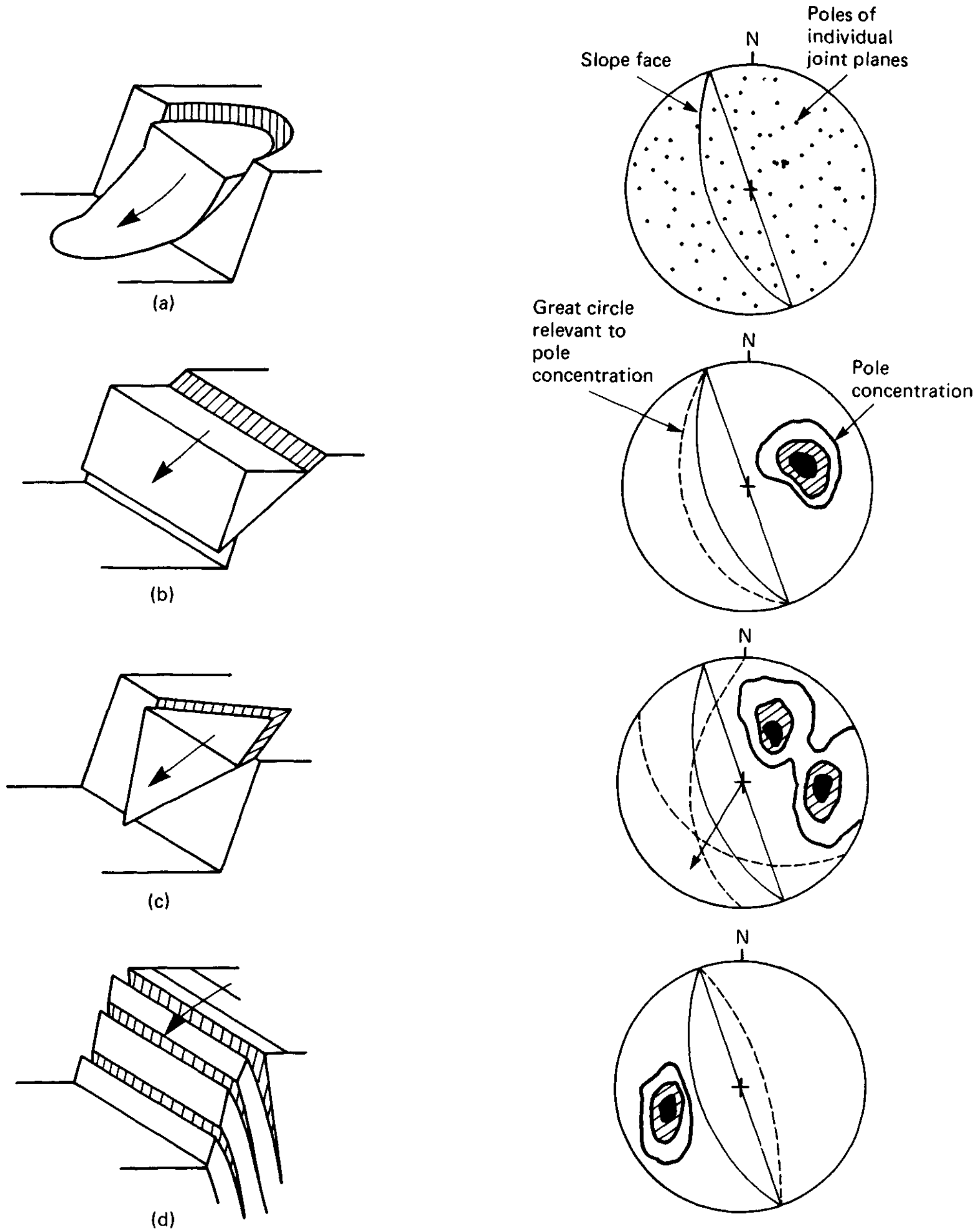


Figure 10.18 Representation of structural data concerning four possible slope failure modes, plotted on equatorial equal-area nets as poles and great circles. (a) Circular failure in heavily jointed rock; (b) plane failure in highly ordered structure such as slate; (c) wedge failure on two intersecting sets of joints; (d) toppling failure caused by steeply dipping joints. (After Brown (1981) *Rock characterization, testing and monitoring*. Committee Testing Methods International Society Rock Mechanics. Pergamon, Oxford)

best estimates can be used to give upper and lower bounds in the stability calculations. More rigorous calculations then require *in situ* measurement of shear strength or the back analysis of existing slides, water pressure monitoring and permeability testing. A flowchart showing the main steps in assessing the stability of a rock slope is presented in Figure 10.19. A most comprehensive practical manual[9] presents full details of the methods mentioned above, to which reference should be made.

The two-dimensional methods of analysis most often used in soil mechanics should not normally be applied to rock problems although Hoek[74] describes the use of the nonvertical slice method of Sarma.[112] This limit equilibrium method is ideally suited to many rock slope problems because it can account for specific structural features such as faults. Vector methods[9] are particularly suited to the limiting equilibrium analysis of three-dimensional wedges. The kinematics of stability are also of greater relevance in rock than in soil, and techniques are available for selecting probable from improbable slides on the basis of kinematic considerations.[113]

Natural or excavated rock slopes might be shown to be stable overall whilst the possibility remains of minor rockfalls occurring due to loosened blocks. These may be controlled by full stabilization methods, or by protection methods such as catch fences and ditches to arrest or retard the tumbling blocks. Experimental work, notably by Ritchie[114] and others,[115] has examined the trajectory of falling blocks and enabled guidelines to be developed for slope–toe rock traps. A design chart for ditch and fence rock traps is shown in Figure 10.20. Protection measures are generally not expensive but require continual maintenance, whereas stabilization measures such as rockbolting, buttressing, trimming, mesh and shotcrete which may need little maintenance can be very expensive to install, especially for high slopes.

Owing to the inherent variability of the orientation of discontinuities, even though they occur in 'sets', the design of rock slopes may in some circumstances lend itself to a probabilistic analysis.[116–118] For example, the resisting forces are due to the shear resistance of the joint planes and the disturbing forces are due to the weight of the rock block or wedge, both of which are functions of joint orientation. Whereas normal 'deterministic' analysis is based on a factor of safety for the ratio of resisting to disturbing forces, probability density functions or distributions can be derived to describe each of these. The probability of failure is then a function of these two distributions.

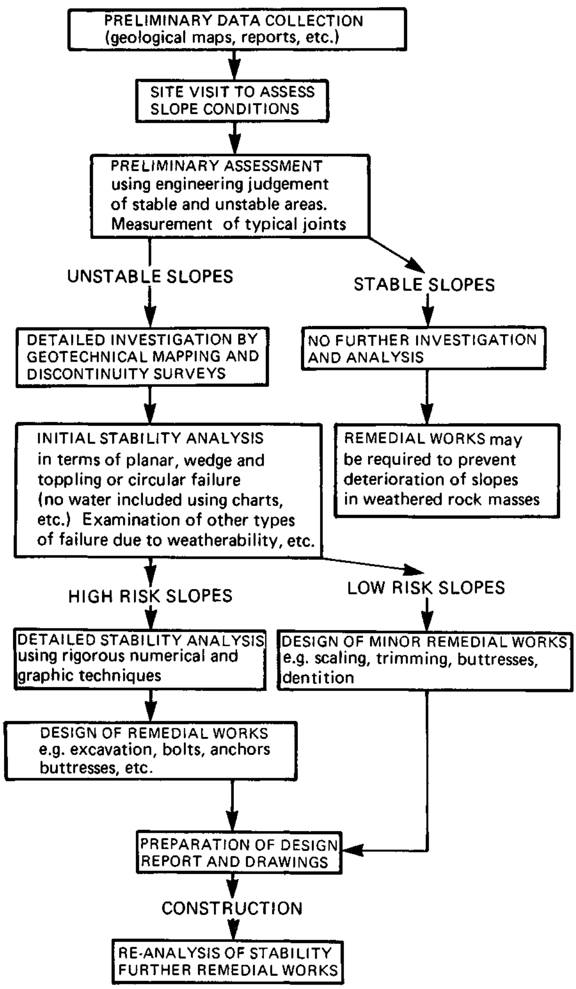


Figure 10.19 Flowchart for rock slope stability assessment and remedial works. (After Powell and Irfan (1986) *Slope remedial works in weathered rocks for differing risks. Rock engineering and excavation in an urban environment.* Institute Mining and Metallurgy, Hong Kong)

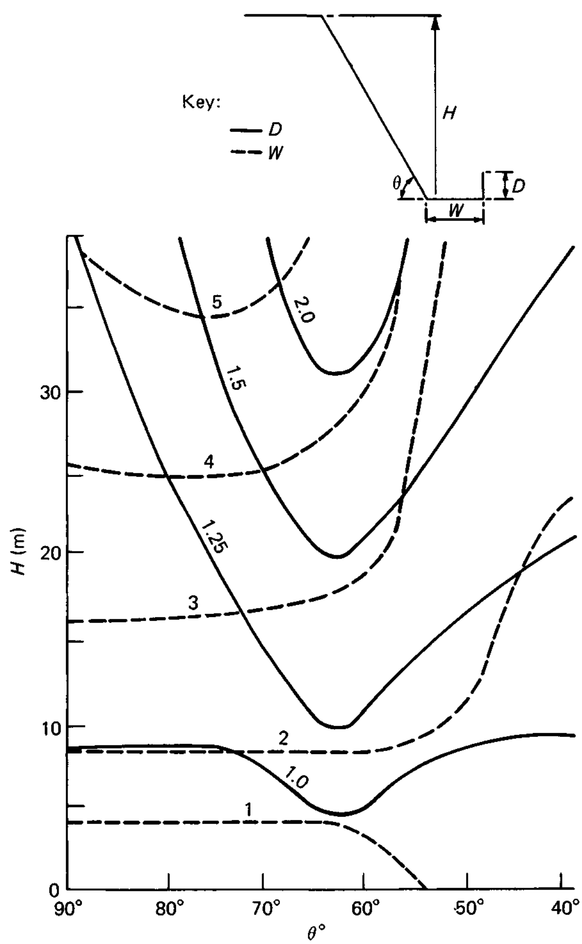


Figure 10.20 Rock trap guidelines. (After Whiteside (1986) Discussion, *Proceedings, symposium on rock engineering in an urban environment.* Institute of Mining and Metallurgy, Hong Kong)

10.4.6 Foundation design

Rocks generally have a high allowable bearing pressure which may be reduced by the presence of weak layers, discontinuities and weathering. The allowable bearing pressure depends on the compressibility and strength of the rock mass and the permissible settlement of the structure.

Detailed design calculations for foundation rock are generally required only if the rock is weak and/or broken or the loading is unusually high, and in these cases the problems are usually associated with settlement prediction rather than foundation bearing failure. The compressibility of the rock mass is related to the strength and modulus of intact rock, the lithology, and the frequency, nature and orientation of the discontinuities. Guidance on allowable bearing pressures and a method of calculation may be obtained from the British Standard Code of Practice.[119] These values are not necessarily suitable for very large heavy foundations nor for structures sensitive to settlement which should be considered having regard for the size of foundation, the variation in strength and nature of fracturing both with depth and laterally, and the variation in modulus with intensity of stress.[120]

Typical problems that require a more detailed study include the design of end-bearing piles or caissons carried to rock, particularly when the depth of overlying less competent materials is such as to require a minimum diameter of excavation with correspondingly high bearing pressures. The design of rock-socketed piers has been reviewed by Rowe and Armitage[121] and they have produced a number of design charts for their proposed method. Piling in chalk has been reviewed comprehensively by Hobbs and Healy.[122] Certain types of structure are particularly vulnerable to differential settlements, and others are particularly massive so as to impose high foundation loads (e.g. arch dams and the heavier types of nuclear reactor)[120,123] and in these instances rock foundations require careful design.

Site exploration is primarily aimed at locating suitable foundation levels, and the relative, rather than the absolute, competence of strata. Rock-quality maps can be useful in making this choice. The depth of rock weathering (Chapter 8) is often of particular significance. Approximate allowable bearing pressures can be estimated empirically[119] and often at this stage the foundation design can be modified or improved by grouting.

More detailed analyses of foundation behaviour may employ closed-form elastic or plastic solutions or the various computational methods of analysis described in section 10.4.4. These require information of rock deformability that is usually obtained by *in situ* plate-loading tests or from borehole dilatometer tests. Seismic refraction and other geophysical methods may be useful to assess the character of the rock mass on a large scale. Foundations on argillaceous rocks can be subject to plastic deformation under high contact stresses and may require a study of long-term (time-dependent) behaviour. Stability analyses – taking into account particularly any uplift forces due to water pressure – in addition to settlement calculations are necessary when designing dam foundations and abutments, and when a foundation is situated above a rock slope. Limiting equilibrium methods are appropriate for these analyses, although rock foundation–structure interaction analyses using computational methods of analysis are appropriate for any major structures.

10.4.7 Design of underground openings

Underground openings for civil engineering purposes require perhaps the most rigorous use of rock mechanics practice in their design and construction. They are so demanding because of the severe consequences of being unsuitable or unsafe for their intended purpose. Tunnels and caverns for, say, railway systems or hydro-electric power complexes are not only occupied by the public in some cases, but even minor instability or surface ravelling of small blocks is wholly intolerable to the function of the works. The responsibility on the designer given the natural variability of the materials is very great and it is also necessary to recognize the essential requirements of the various uses to which caverns may be put. In the above examples stability in every sense is essential but the effects on the groundwater regime may be of lesser importance. There are some caverns used for storage of water, oil or gas, and mining openings in which limited minor instability might be permissible, but to maintain, for example, an unlined gastight cavity, it is essential that drainage of the surrounding rock pore water and fissure water does not occur.

The most demanding of purposes for caverns for the designer is the storage of radioactive waste, and this requirement is one explanation for the recent most rapid advance in rock mechanics field experimentation and development of understanding. Stability is essential and must be considered both for the very long term and in relation to extreme thermal effects and radionuclide absorption on rock mass properties, as well as to the dynamic disturbing forces of possible future seismic events. The prediction of groundwater flow around and away from nuclear repositories must consider the stress and thermal effects on hydraulic conductivity of joints and fissures, and the hydrothermal migration or convection effects on groundwater. Such details can be examined theoretically using the computational methods described above, but the determination of realistic input parameters remains a major problem.

The shapes and sizes of underground excavations are often dictated by economic and functional considerations, but their precise location and orientation should be adjusted to suit ground conditions wherever possible. Optimum orientation requires a knowledge of the geological structure, rock mass structure and also of the directions and relative magnitudes of principal stresses in the ground prior to excavation, and so detailed layouts of proposed works should be held in abeyance until after the investigation stage if possible.

A guide to the most important steps in the stability design of underground openings is given in Chapter 1 of a comprehensive manual by Hoek and Brown[62] and full details of the various methods are given. In the design process it is necessary to characterize and zone the rock mass 'geo-mechanically', to determine constitutive relations and strength criteria for each zone, and then bearing these and the proposed geometry of the opening in mind to select the appropriate method of analysis which will render where potential zones of instability lie. The criteria for support design must then be decided upon, e.g. it is common to provide rockbolts of sufficient length and number to support the deadweight of tension zones or 'overstressed' rock determined by stress analysis by anchoring back into 'sound' rock as illustrated in Figure 10.21. The definition of relevant 'failure' criteria which may be influenced by water pressure or seepage considerations, dynamic seismic forces or limiting displacements is a significant design input. Each failure state must then be tested for various stages of the excavation progress because areas of stress concentration will vary. When the excavation sequence and support element dimensioning are decided, verification of performance monitoring is necessary.

A useful design method which is valuable in developing an understanding of the mechanics of rock support is known as *rock-support interaction analysis*.[62] Although the method makes numerous simplifying assumptions (e.g. a circular excavation in a uniform *in situ* stress field is assumed) the principles may potentially be extended to more general cases. The method analyses stress and displacement in the surrounding rock and in the support elements, taking into account rock mass properties, the *in situ* stress, the development of a 'zone of plastic failure'

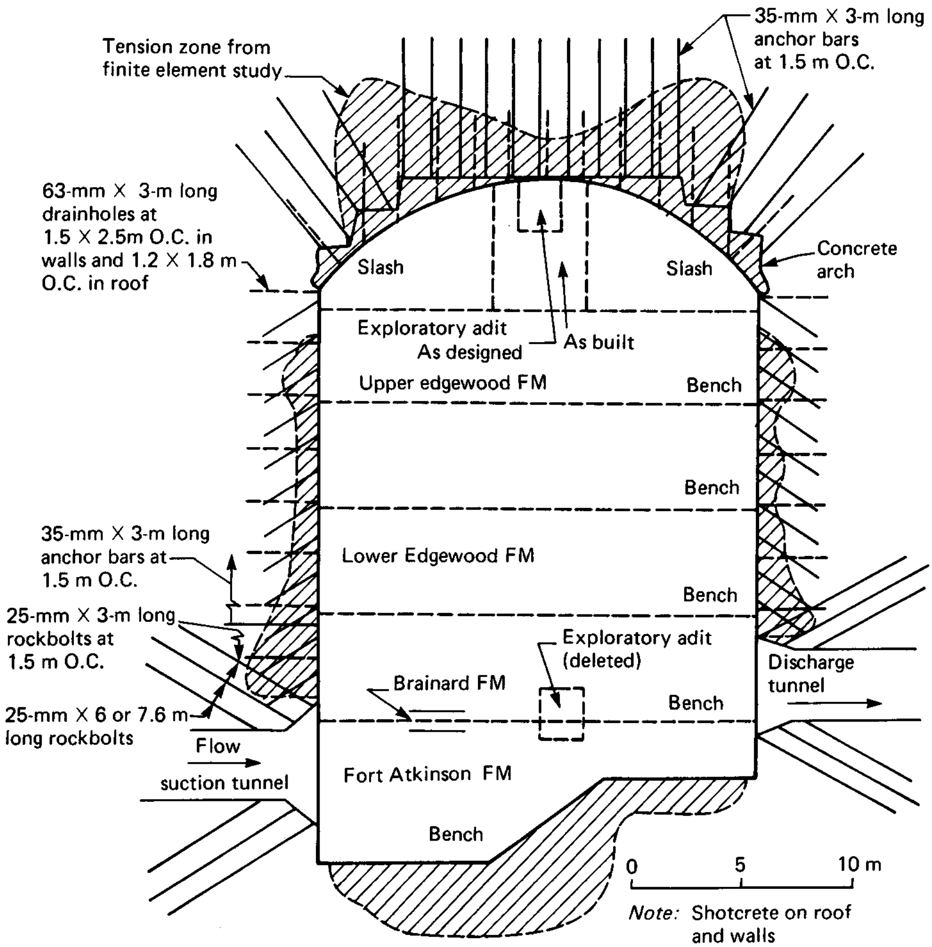


Figure 10.21 Example of rockbolt support for a major cavern. (After Cikanek and Goyal (1986) 'Experiences from large cavern excavation for TARP'. *Proceedings, symposium for large rock caverns,* Helsinki. Pergamon, Oxford)

around the opening, the stiffness of the support and the timing of its installation after excavation. Whenever an excavation is made there will be inward radial movements (convergence) of the surrounding rock which, in practice, are not instantaneous. In the meantime, supports such as rockbolts or arches are installed and, as convergence continues, so the supports will provide reaction forces. The characteristics of the rock are represented by a 'ground response curve' and the support pressure by a 'support reaction line'. Methods of response curve calculation which make use of nonlinear peak strength and residual strength criteria, and the method for pressure tunnel design have been described.[124,125]

Certain types of civil engineering excavation may give rise to subsidence problems, e.g. unsupported chambers for storing water and gas. Furthermore, the civil engineer is often affected in his surface construction operations by mining excavations beneath the site. Subsidence can be predicted to some extent although, since the phenomenon is essentially time-dependent, the analysis is complex and often based on empirical observations.[126]

10.5 Construction methods and monitoring

10.5.1 Excavation

Processes of rock fragmentation are known collectively as comminution processes. In spite of a considerable amount of research aimed at improving these techniques the gap between theory and practice is still great and an empirical approach is more often used. Much research has been directed towards understanding the mechanisms of fragmentation during drilling, in order to improve the design of conventional mechanical bits (diamond bits, percussion or rotary drag bits) and to develop new ways of drilling such as by water-jet cutting, flame cutting and pellet impact. Mechanical drilling techniques suffer from energy losses due to inefficient transfer of energy from the bit to the rock. The drilling process ideally should produce fragments small enough to be flushed from the drillhole but not so intensely crushed as to absorb a considerable proportion of the input energy. Other factors such as drilling rate and for core drilling, the quality of core recovery are of primary importance.

The theory and practice of blasting are reviewed in detail by Langefors and Kihlstrom[127] and details of techniques and explosives are given in Chapter 32.

Blast pressure in a drillhole can exceed 100 000 atmospheres. This pressure tends to shatter the area adjacent to the drillhole and a stress wave is generated that travels outward from the hole at a velocity of 3000 to 5000 m/s. The leading front of the stress wave is compressive, but is closely followed by tensile stresses that are responsible for the major part of rock fragmentation. When the stress wave is reflected from a nearby joint surface or exposed rock surface it again gives rise to tensile stresses which may cause scabbing of the superficial rock. The stress wave is the initial cause of fracturing, gas pressure serving to widen and extend the cracks previously generated.

Variables in designing a blasting pattern include the degree of fragmentation required (size of fragments), the explosive used, the diameter, inclination and method of loading the drillhole, the burden (distance from the drillhole to the free face) and the spacing between holes. Also the sequence of firing can be varied (e.g. with delayed charges) to minimize vibration levels and unwanted rock damage and to give a more efficient pattern of rock removal. To predict and control excessive blast vibrations and assess their likely effects involves the use of an attenuation law for vibrations through the ground. Various theoretical relationships exist between instantaneous charge weights, distance and vibration intensity but frequently site-specific experiments are necessary.[128] A well-designed blast gives maximum yield, controls the size and shape of fragments (critical to subsequent crushing processes for aggregate production and to utilization of material in rockfill constructions), controls the throw and scatter of fragments, and minimizes the amount of drilling and explosive required.

In road cuts, blasting is complicated by the continually varying height of bench; heights of more than 10 m are usually blasted in more than one lift. In foundation blasting it is particularly important to control throw and also vibration levels, usually by means of short-delay multiple-row blasting with small-diameter drillholes and reduced charges.

In tunnel blasting (Chapter 32) the first holes to be detonated should create an opening towards which the rest of the rock is successively blasted. The holes of this 'cut' are usually arranged in a wedge, fan or cone pattern. The remainder of the round is designed to leave the intended excavation contour undamaged. In long excavations of diameter larger than 8 m the upper section is often removed first, followed by removal of the remaining bench in one operation after installation of roof support. This can give a more economic result and also facilitates both mucking out and the installation of support. Several successive benches are excavated for caverns, as in Figure 10.21.

Smooth-wall blasting is a comparatively recent innovation that greatly improves rock stability and at the same time reduces the amount of concrete required for lining the excavation. The techniques are well proven[129] but not as widely used as they might be. The greater level of control needed may represent an additional cost, but experience of projects in which carefully controlled blasting has been used generally shows that the amount of support can be reduced significantly. The overall cost of excavation and support thus is lower than in the case of poorly blasted excavations. In *presplit blasting* cracks for the final contour are created prior to firing holes for the rest of the pattern. Spacing for the contour holes is typically 10 to 20 times the hole diameter, and holes are loaded with a reduced charge density. The contour holes should be ignited simultaneously.

Machine excavation causes very little disturbance to the surrounding rock. The development of larger and more efficient ripping and tunnelling machines together with the continuing increase in manpower costs has resulted in the increasing use of mechanical excavation as an alternative to blasting. Ripping can be highly competitive particularly in the larger-scale surface mining operations.

The mechanics of rock excavation with a ripper or with the cutting head of a tunnelling machine are in some respects similar to those of drilling, although on a larger scale the natural fractures and planes of weakness in the rock play an increasingly important role. Also a tunnelling machine, unlike a drill bit, cannot be changed at will to suit rock conditions. Machines where the spacing, size and type of cutting disc or pick as well as the thrust and speed of cut can be varied, may be desirable but the construction of machines of such wide versatility is not generally feasible. Hence, the considerable capital investment associated with the purchase of ripping and, particularly, tunnelling plant requires a careful study of rock conditions prior to selecting a machine. Rock quality classifications can help in making this choice when used in association with site investigation and geological studies.[130] Fracture spacing is often the most relevant property to note, since it determines the sizes of rock block to be excavated. Intact material strength, abrasiveness and specific energy are also of relevance. Sonic velocity mapping has sometimes been used to assist in assessing the state of fracturing of the ground and, hence, its rippability.

10.5.2 Rock support methods

Unstable rock conditions can very often be improved by rockbolting and support, grouting or drainage. The cost is frequently offset by the benefits, e.g. a rock slope can in some cases be steepened by 10° or more if an efficient drainage system is used; in deep rock cuts this appreciably reduces excavation costs.

10.5.2.1 Rockbolting and anchoring

A rockbolt assembly usually comprises a bar with an anchor at one end and a faceplate assembly (faceplate, nut and wedge or spherical washer) at the other. The anchor may be mechanical or grouted, and the bar may be substituted by a cable to achieve greater lengths and loads. For rockbolts to be used for permanent civil works a careful study of corrosion resistance is essential.

The *dowel*, or *fully bonded rockbolt*, comprises a bar installed in a drillhole and bonded to the rock over its full length. A cement-grouted bar can be installed by packing the drillhole with lean quick-set mortar into which the bar is driven. The 'Perfo' system uses a split perforated sleeve to contain the mortar, which is extruded through the perforations as the bar is driven home. Fully bonded resin anchors usually employ a polyester or epoxy resin and catalyst in the form of cartridges that are ruptured and mixed by a bar driven into the drillhole with a small rotary drill.

Point-anchored rockbolts comprise a bar anchored over a comparatively limited length. The slot and wedge system uses a bar with a longitudinally sawn slot. A wedge inserted into the slot expands the slotted section against the rock when the bar is driven home. Sliding wedge anchors employ a pair of wedges drawn over each other when the bolt is rotated. Expansion shell anchors employ two or more wedges or 'feathers' that are expanded by a threaded cone. Explosive anchors have also been used in softer rocks and comprise a split tube, sections of which are driven into the rock by an explosive detonated within the tube. The resin point anchor is identical to the fully bonded resin anchor described above, except that a limited quantity of resin is used to provide an anchor only at the base of the drillhole.

Point-anchored rockbolts must essentially be *tensioned* to work efficiently, since the action of opposing anchor and bearing plate forces effectively tightens the superficial zone of loose rock. This zone can then make a significant contribution to the support of rock at greater depth. Dowels cannot gain from tensioning during installation but tension is induced when the rock begins to move and dilate. Rockbolts should be installed as soon as feasible after excavation, before the rock begins to move with consequent loss of interlocking resistance.

Grouting of point-anchored rockbolts reinforces the anchor, protects against bolt corrosion, and is particularly necessary in softer rocks where point anchorages are seldom reliable in the long term. Grout should be injected at the lowest point in the drillhole, using a bleeder tube to remove air. Resin-bonded bolts can be arranged so that a fast-setting resin is introduced first in the drillhole, followed by slower setting resin cartridges. The bolt is tensioned when the point anchor provided by the fast-setting cartridge has gained sufficient strength, and the slow-setting cartridges then polymerize to grout the remainder of the bolt length. The use of fully resin-bonded wooden, plastic or glass-fibre rockbolts has the advantage that the bolts will not damage excavating machinery and so can be used if necessary for temporary support at, say, a tunnel portal or advancing face.

Friction anchored rockbolts develop anchorage loads along their full length within the drillhole. The *split-tube bolt* is hammered into a drillhole of slightly smaller diameter than the bolt itself. The bolt, which is manufactured from sprung steel and has a C-shaped cross-section, recoils against the drillhole wall. The *inflatable bolt* is manufactured from malleable steel as a reniform tube which is inserted into the drillhole. It is then inflated to take up the form of the drillhole by the use of water at high pressure.

Rockbolts should be field-tested in the rocks in which they are to be installed in order to establish their design performance. A bolting pattern may then be designed on the basis of test results, taking into account the rock structure and the size and shape of the slope or underground excavation.[62,131]

10.5.2.2 Sprayed concrete and other lining methods

A first essential in rock support is to prevent even small quantities of material from ravelling from the rock face, since this can lead to general loosening and progressive failure. The size of rockbolt faceplates is often adjusted to minimize ravelling, and *wire mesh* or *ribs* can be installed beneath the plates to give added protection. *Sprayed concrete* (gunite or shotcrete) can be used to supplement bolting and mesh, or may under some circumstances provide adequate support on its own,[132] it is a particularly appropriate technique for preventing the slaking deterioration of mudstones and shales. A thin sprayed concrete lining, unlike more rigid methods of support, will crack to reveal zones of instability before they develop fully, allowing the placing of additional local support. Sprayed linings as thin as 5 to 10 mm have been used effectively in some instances. Several proprietary systems for a more rigid tunnel lining using, for example, interlinked expanded metal sheeting and pumped or sprayed concrete, are in use and the methods are described in greater detail in Chapter 32.

10.5.2.3 Grouting

The injection of a grout into the rock mass so that air or water in fissures is replaced by a solid material or gel will inhibit percolation of water and may also provide added rigidity and possibly strength. Injection into rock normally requires a grout consisting of a mixture of Portland cement and water. Sand, clay, or other inert materials may be added to reduce cost provided that these filler grouts can flow, without undue segregation, through the sizes of fissure present in the rock. High grouting pressures can be necessary for adequate grout emplacement, depending on rock mass permeability and on the fluidity of the grout. Grouting at high pressure can result in hydraulic fracturing and lifting of beds. Although this assists emplacement it can be detrimental to the resulting strength of grouted fissures, can result in damage to nearby structures, and in the worst case can itself initiate rock collapse. The efficient grouting of narrow fissures can require grouts of greater than usual fluidity, and in these cases chemical grouting materials can be used. Permeability tests are usually needed to select appropriate grouting pressures and materials. Grouting is commonly used to reduce leakage beneath dams and into tunnels or excavations beneath the water table. It is also used in association with drainage measures to control uplift and pore pressures in dam foundations and abutments, and to consolidate loose rock in foundations or in the vicinity of an excavation. *Consolidation grouting* (distinct from the term 'consolidation' as applied in soil mechanics) serves to improve rock strength but, more importantly, it considerably reduces rock deformability. Cementitious materials are usually required. *Grout curtains* on the other hand are used to reduce permeability (e.g. beneath a dam) and may employ lower-strength gel-type materials. Temporary control of water flow, and temporary rock mass consolidation, can sometimes be provided by *freezing techniques*, used, for example, in the driving of shafts through highly fractured rock. The efficiency of rock grouting is usually assessed by monitoring of the grouting operation itself, by visual inspection, or using sonic velocity techniques.[41]

10.5.2.4 Drainage

A drainage system installed around a rock structure under construction can have the immediate advantage of improving working conditions (e.g. in a tunnel or cut excavation) but its principal objective is usually to reduce water pressures within the rock mass and hence improve its stability.[133,134] Groundwater will present a problem either by the seepage of copious quantities into excavations leading to inundation of the works, or by the destabilizing effects of water pressure acting in pores, joints and fissures. The former is more likely in permeable formations and the latter in formations of lower permeability. Hence, the control of seepage by grout injection usually results in a build-up of water pressures which must be relieved by suitable drainage methods to ensure stability. Sprayed concrete linings, for example, must usually be provided with drain holes if they are to remain intact under conditions of high water pressure.

The design of drainage systems requires a comparison of water pressures and flow paths in the drained and undrained structure (Figure 10.22). Darcy's law – that the flow velocity is proportional to the change in pressure per unit distance (hydraulic gradient) along the flow path – can be assumed to be valid in most cases. Flow through the rock may be solved relatively simply using graphical methods (flow-net sketching), analogue methods or numerical techniques as discussed in section 10.4. The graphical methods are particularly suitable for an initial examination of the problem, and can be relatively accurate given some practice in flow-net construction and reliable data on rock conditions.[135]

10.5.3 Monitoring

Instrumentation and monitoring gives a check on the design and its inherent assumptions and simplifications. Alternatively, it can be used in cases where a detailed analytical design is not justified by the nature of the problem, but when rock stability remains in question. The object of performance monitoring is to give sufficient warning of adverse or unpredicted behaviour to

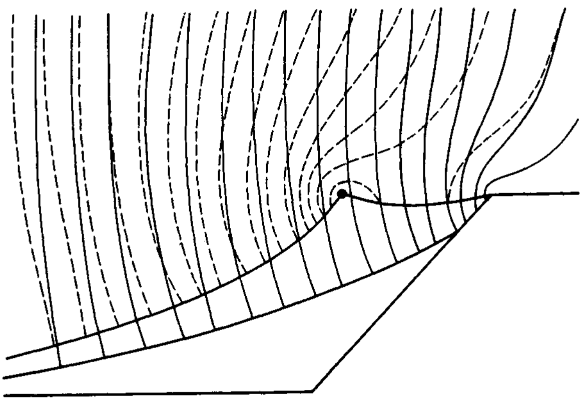


Figure 10.22 Comparison between drawdown curves and equipotential lines for a 45° slope in isotropic material with and without drainage *(after Sharp* et al. *Ref 133)*

allow timely remedial action, and to assess the effect of remedial works.

10.5.3.1 Movement monitoring

Conventional survey techniques (e.g. precise levelling and triangulation) provide the simplest and perhaps the most reliable control but their accuracy is not always sufficient for detection of movements smaller than a few millimetres.

Electronic distance measuring methods are capable of good accuracy, with the added advantages of speed and of a range greater than 1 km.

Photogrammetric techniques (ground photogrammetry in particular) are particularly useful for monitoring large rock slopes. Although their accuracy is usually not better than 100 or 200 mm, depending on the object distance, movements can be detected at every visible point even though they do not coincide with prelocated survey markers. *Lasers* can be used to detect changes in alignment, and *surface extensometers* to monitor the development of tension cracks, tunnel convergence, and the superficial movements of rock slopes.

Movement monitoring devices can also be installed in boreholes to supplement, or sometimes to replace, surface monitoring. *Borehole extensometers* measure changes in length of the borehole. Multiple position extensometers are available to measure differential movement at as many as ten or fifteen anchors at varying depth. Simple settlement devices often work on a hydraulic or 'U-tube' principle. Multiple-position instruments may employ rods or wires and either a mechanical or an electric transducer system for recording movements.

Borehole inclinometers record changes in borehole inclination. Moving-probe inclinometers employ a capsule or probe that travels along the borehole which for this purpose is cased with flexible plastic or aluminium grooved tubing. The probe may comprise, for example, a cantilever pendulum, a short length of pivoted rod, an inertial system or a system where the position of a bubble in a 'spirit level' is monitored electrically. Inclinometers of the fixed-position type usually employ a system of pivoted rods anchored at various positions along the borehole. Another device, the *shear strip*, comprises a set of electric resistors connected in parallel at regular intervals along the length of a printed circuit conductor and is used to detect the depth at which the strip, grouted into the borehole, is sheared by ground movements.

Blasting, earthquake and vibration studies require monitoring of *dynamic movements* with an array of geophones located in drillholes or at the rock surface. Seismic arrays can be used either to locate the sources of 'rock noise' (e.g. in monitoring landslide movements or rockburst phenomena) the development of hydraulic fractures, or to record the waveforms associated with blast or earthquake tremors, traffic or machinery vibrations.

10.5.3.2 Water pressure and flow

Piezometers (water-pressure measuring devices) are installed in drillholes to provide information for design analysis, or to record changes in water pressures so as to monitor one of the major causes of instability. They may take the form of *simple standpipes* where pressure is measured as a change in water level using a probe lowered into the standpipe tube. Artesian pressures and local pressure anomalies are common in rock, and the drillhole must usually be sealed with grout over its complete length except for the test sections at which pressures are to be measured. Several piezometers recording pressures at test sections of different elevation may be installed in a single drillhole.

Standpipe piezometers are suitable only for pressure monitoring in near-vertical drillholes and also have a comparatively slow response to water pressure fluctuations. This response can be improved by use of a *pressure transducer* method. Pressure transducers can be used in drillholes at any inclination and usually employ a flexible diaphragm exposed to water pressure on one side. They incorporate a device for measuring diaphragm deflection that typically uses bonded resistance strain gauges or a vibrating-wire system. Pressure sensors which determine back-pressures or balancing pressures across diaphragms and which use pneumatic or hydraulic readout equipment are also commonly used.

Water-flow monitoring is most often required in connection with reservoir leakage problems, pollution and hydro-geological studies, but a knowledge of flow velocities, directions and the identity of particular fissures carrying the majority of flow is often needed for other types of problem. Flow directions and velocities can be evaluated using radioactive or dye tracer techniques where a concentrated tracer is injected into one drillhole and the time taken for the tracer to appear in nearby observation holes recorded. Flow can also be measured in a single drillhole by observing the rate of dilution of a tracer or saline solution. Flow velocities and directions can also be logged in the drillhole by a probe incorporating a small turbine or electrically heated wires, the object being to find horizons of greatest permeability prior to installation of piezometers or to permeability testing.

10.5.3.3 Stress measurement

Rock stress measurements have the object of evaluating the natural stress field for purposes of design (see section 10.1) or of monitoring stress changes (i.e. induced stresses) for purposes of control and warning. Overcoring methods make the assumption that rock core is relieved of its *in situ* stress by core drilling and will return to its initial unstressed configuration. The expansion of the rock core is measured by first bonding a stressmeter device into a small-diameter drillhole, recording initial readings, and then drilling a concentric hole of larger diameter to remove an annulus of core with the stressmeter inside.[136] The stress prior to overcoring is then computed from initial and final strain measurements assuming elastic properties for the rock.

Borehole stressmeters have the advantage of measuring stresses sufficiently deep in the rock, away from disturbing influences due to the ground surface or the presence of excavated cavities. However, they need considerable expertise in their installation. They can be installed and left in place without overcoring if the object is to measure stress changes.

A further method for stress measurement is that of *hydrofracturing*, in which fluid is pumped at high pressure into a section of the drillhole isolated between two inflatable packers, and an accurate record kept of the relationship between pressure and flow volume. A sudden increase in volume at constant pressure indicates that the rock has been fractured (often this involves joint or bed separation rather than fracture of intact material) and gives an estimate of the pre-existing rock pressure normal to the fracture plane. The direction of the fracture plane must be known for meaningful interpretation of results.

Flat-jack techniques can also be used to measure rock stresses in close proximity to an excavation. A pattern of displacement measuring points is bonded to the rock surface, and after taking initial readings a slot is cut between the points. A flat jack is then grouted into the slot and inflated until the initial displacement values have been reinstated. The pressure at which this is achieved gives a measure of the rock stress that existed perpendicular to the plane of the slot prior to slot cutting. Slots at various positions and orientations around the excavations are required, and the evaluation of stresses must depend on assumptions as to the effect of the excavation on the natural stress field.

10.5.3.4 Pressure and load

Pressures on retaining walls and tunnel linings are usually monitored with hydraulic flat jacks or load cells. The flat jack or load cell may be connected to a pressure gauge as a sealed system so that changes in pressure are recorded directly, or there may be provision for inflation of the jack prior to taking readings, so that a null displacement condition is achieved.

Load cells (force transducers) can be used to record tension in rockbolts and anchors, compression in ribs, steel sets and prop supports, or can be incorporated into walls and tunnel linings as a means of measuring pressure. Essentially they use a 'proving ring' principle with a semi-rigid member to carry the load to be measured, and a means for monitoring the strain in this member. Electric-resistance load cells use one or more metal columns on to which the strain gauges are bonded. Hydraulic load cells employ a sealed hydraulic capsule with measurement of internal fluid pressure. Rockbolt and anchor load cells are available that work on an electric-resistance or vibrating wire principle, use a photo-elastic glass plug or employ a sandwich of rubber between metal plates whose separation is measured with a micrometer. Load cells often incorporate a spherical seating to ensure that the measured load is coaxial with the cell.

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11 Site Investigation

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Contents

11.1 Preliminary assessment

11.1.1 General

Site investigation in the overall sense is the process by which the various factors influencing the selection and use of the most appropriate location for a project are evaluated. Identification of the primary factors aids the initial selection of the site. Thus whereas topography and the geology determine the site of a dam, minimal environmental pollution requirements often define the location of an airport and preferential government aid that of a new industrial development. Each of the primary factors should be considered in sufficient depth to disclose any adverse item that may be critical before proceeding in detail to examine the technical feasibility of using the site. Guidelines for this preliminary assessment are given below.

11.1.2 Environmental considerations

This refers to the local conditions and resources both natural and those already existing including the infrastructure. A preliminary site reconnaissance should be carried out as early as possible utilizing available data, in order to consider the surroundings in relation to the project. Aerial photographs can be a valuable aid. The principal topics in this group are given below:

(1) *Topography* – Suitability of surface features at the site on land or over water.
(2) *Public and private services* – Availability of a suitable workforce, transportation facilities for access, water supply, power and telecommunications, sewerage and drainage, disposal of wastes.
(3) *Living amenities* – Facilities available or required for accommodation during construction and afterwards. The extent and standard of the community services either existing or planned.
(4) *Geology* – The local ground conditions at the site and in the surrounding area would normally be indicated sufficiently at this stage from the geological survey maps where available, otherwise by a site visit and/or an enquiry addressed to the local public authority. Check for adverse natural conditions such as unstable ground, underground caverns and subsidence potential.
(5) *Construction materials* – If *in situ* deposits are of interest refer to the geology or earlier uses of the site (e.g. old tips). For new roads in virgin country use aerial survey and remote sensing.
(6) *Hydrology* – Surface and groundwater conditions, river and tide levels, currents and stream flow, flood levels and drainage conditions. Periodic occurrence of springs.
(7) *Other uses of site area* – Past, current and proposed other uses at and around the site, such as mine workings (underground or open-cast), tunnels and underground bulk storage. Former industrial areas, refilled gravel pits, refuse tips, reclamation, waste and spoil dumps, buried pipelines, services, drains, pollution, radioactivity and other hazards. Ecological and conservational impacts. Consider both the site and its surroundings.
(8) *Meteorology* – Regional temperature, rainfall, humidity, prevailing winds and fog. Seasonal effects. Local microclimatical conditions before and after construction of the project.
(9) *Earthquakes and ground tremors* – See Chapter 8.

11.1.3 Administrative considerations

National or local plans for development or redevelopment, where they exist, should be inspected. Any restrictions such as those pertaining to access, noise, atmospheric pollution and site rehabilitation should be examined. The existence of mine workings, mineral rights, ancient monuments, burial grounds and rights of light, support and way, including any easements, should be established.

Proposal for development with outline plans should be submitted to the local planning authority and an application made for approval of use of the site before the preparation of a detailed scheme. It may be necessary to present evidence at a public enquiry.

11.1.4 Financial considerations

Although outside the scope of this chapter, another preliminary consideration of a proposed project should be a cost/benefit study covering both initial capital cost for construction along with subsequent running costs, wherein financial and economic factors, together with any social or amenity benefit are considered with respect to project feasibility and its alternatives.

In cases where the cost of the project may be significantly influenced by the ground conditions, such as for dams and major highways, including where comparisons are needed between alternatives, it would normally be advisable to extend the initial feasibility stage to include the preliminary appreciation of the site at least and its alternatives as described in section 11.2.

11.1.5 Environmental surveys

Engineering projects cannot properly be evaluated in isolation from their environment.

With minimal infrastructure there is the opportunity to select the most suitable site from a relatively simple environmental study of the natural features and facilities in the locality in order to consider the best compromise of aspects related, say, to topography, geology, biology and meteorology. As the intensity of local development and/or conflicting interests increases, site location becomes more restricted, and before it is examined in detail it may be necessary for impact studies to be made of the effects of the proposed development on the local human, natural and man-made environment or vice versa. Such studies take on a special significance where a public inquiry may ensue before final approval for a project can be given.

Some examples of major considerations that may be involved include: (1) preservation of natural vegetation, wildlife and land quality; (2) preservation of areas of archaeological, historical, or other special interest; (3) prevention of pollution of atmosphere in quality or by noise; (4) prevention of contamination of surface water, the ground or groundwater; (5) prevention of erosion on land, siltation and scour by water action; (6) preservation of social, public and private amenities; and (7) acceptable disposal or re-use of waste materials.

The principle to be adopted for undertaking an environmental survey should be to consider, comprehensively, all the pertinent factors whether or not at first sight they appear relevant having regard to the possible influence of the project. Full advantage should be made of available data and to ensure that sufficient territory has been taken into account, aerial photography generally provides the most convenient means of studying local topographical conditions that may be affected. Moreover, aerial photographs and multi-spectral techniques (satellite imagery) often reveal features not otherwise easily discernible, such as man-made buried workings and morphological changes. Trained interpreters, properly briefed, are able to extract much information.

A bibliography on this subject is given at the end of this chapter.

11.2 Site examination

11.2.1 General

Once the basic feasibility of a site for a new project has been established, the next step is to undertake a more detailed examination of the site itself to further the assessment of the relevant aspects required for the design and construction of the project.

11.2.2 Topographical surveys

The first stage in a detailed examination is to prepare an accurate survey from which plots on any required scale may be made. All levels should be referred to a reliable datum and the site preferably related to the national mapping and levelling system. Where derelict land, underground cavities or old workings have been identified, they should be included in this survey.

On large sites a local graticule of coordinates should be established, employing permanent beacons for more detailed surveys, extensions and setting out the works. Aerial photography can be advantageous, particularly on extended sites such as cross-country highways. Vertical photographs with stereoscopic overlap permit measurements to be made provided there is ground control. Photographs can be rectified to produce a true map and are then known as orthophotos. Such photographs can have contours plotted directly on to them. All aerial photographs can be produced as black and white with or without enhanced tones or infra-red-sensitive. Natural colour or false colour may also be used to identify special features. Existing air-photo cover sufficient for preliminary purposes may sometimes be available from the photo libraries of air survey companies and a few other organizations.

11.2.3 Hydrographic surveys

Hand-sounding may be sufficient for small areas of work. For larger works in tidal areas, deep water and high flows, more reliable methods will be required. Echo-sounding may be employed to provide a bed profile and under favourable conditions would be more convenient and accurate than hand-soundings. Surface profiling may be combined with sub-surface work by employing continuous seismic profiling or side-scan sonar systems, although the accuracy would be less. A comprehensive description of hydrographic surveying is given in the two standard works in the bibliography at the end of this chapter.

Photogrammetry is very useful for surveying coastal and inter-tidal zones. Offshore rocks, coral pinnacles, islands, sandbanks, shelves and buoys are easily located. Techniques are also available for reliably measuring nearshore depths. The disadvantages include unseen underwater hazards and drying-out areas which cannot usually be delineated.

Multi-spectral techniques may sometimes reveal detailed hydrological information not otherwise visible.

Estimates of peak flood levels should involve specialist advice and may require an extended survey far beyond the boundaries of the site.

11.2.4 Ground investigation

This refers to the collection and interpretation of data on the ground conditions at and surrounding the site for the design, construction, operation and maintenance of the project. Further details are given in the remainder of this chapter.

11.3 Principles of ground investigation

11.3.1 Primary objectives

Particular primary objectives of a ground investigation are to:

(1) Ascertain geological conditions at the site and groundwater hydrology to assess general suitability and for geotechnical study.
(2) Collect geotechnical data on relevant formations for quantitative design study of permanent and temporary works.
(3) Consider changes in ground stability and groundwater regime after construction due to the structure and/or future changes in ground such as mining or seismic activity.
(4) Evaluate effects of alternative excavation and construction methods, also temporary works.

In the case of existing structures, other factors that may be involved are:

(5) Need to ascertain reasons for structural defects, instability or failure.
(6) Consideration of remedial measures.

11.3.2 Contaminated site hazards

Additional investigation work, sometimes allied to that referred to above, is required when there has been some earlier use of a site that may have given rise to some significant disturbance or change in the conditions. A particularly difficult case is when the ground has become chemically contaminated with toxicants. This is a growing problem in developed countries.

The presence of chemical contaminants or ground liable to subsidence creates risks to personnel, for which reason special precautions are necessary during the investigation.

Objectives in this case, which may concern risks to construc-

Figure 11.1 Ground investigation using percussion boring equipment on waste tip containing hazardous materials. (*Courtesy*: Wimpey Laboratories Ltd)

tion workers, eventual users, animals, plants and building structures, are:

(1) To identify the types, extent and importance of the hazards, so that an assessment of their potential dangers to personnel, plants and/or the proposed end-use of the site can be made.
(2) To advise on suitable remedial measures to overcome identified hazards such as:

   (a) Settlement problems, e.g. subsidence due to decomposition, weathering and natural compaction, leaching and sudden collapse;
   (b) Obstructions, e.g., old foundations, piles, buried seawalls.
   (c) Other problems which include:

      (i) fire, smoke, noxious fumes, gases and explosions from combustible material, microbial reaction of organic matter, volcanic areas;
      (ii) deleterious attacks on personnel from toxic powders, asbestos, fibres, liquids, explosive and asphyxiant gases, radioactivity and biological contamination;
      (iii) deleterious effects on the growth of plants or to the safe consumption of edible plant material;
      (iv) deleterious attacks on construction materials from residual chemicals (see also corresponding problem with natural ground as described in section 11.3.7.5(5) aggressive ground and groundwater);
      (v) pollution of streams and aquifers and the control of leachates, which involve the determination of the type of contaminant, its source and drainage plume. Wind action on contaminated dusts.

Table 11.1 indicates the range of potentially hazardous areas that may or may not include toxic materials. Future changes that possibly might occur in the ground at the site need also to be considered. Examples of this include underground mining, tunnelling and underground storage.

Table 11.1

Major landfills	*Derelict works*	*Old workings and cavities*
Domestic waste	Gasworks	Underground workings
Colliery waste	Sewage works and sludge disposal	for coal, stone, lime and flints, etc.
Ash (PFA) and clinker	Ferrous and non-	Opencast workings
Slurry lagoons	ferrous works	Metalliferrous mines
Chemical wastes	(smelting, refining	Abandoned shafts and adits
Metallurgical slag	and processing)	Cellars and basements
Hospital wastes	Pickling tanks	Sewers and tunnels
Scrapyards	Plating works	Salt mines
Industrial fill	Chemical works	Underground storage
Radioactive waste	Tanneries	Wells and tanks
Backfilled quarries and pits	Oil refineries	

11.3.3 Governing factors and limitations

Sufficient knowledge and experience exist of the difficulties in predicting ground conditions locally without a proper study, and the inherent weaknesses in many soils and rocks, to provide a justification for ground investigations in order to ensure safe, practical and economic designs.

Neither surface inspection nor information from outside the site is usually sufficient to provide reliable data on the ground conditions below the site so that exploration penetrating into the ground is used, at points on the site and related to the project.

The intensity of the investigation depends upon the character and variability of the ground as well as the magnitude of the project. The investigation depends upon the collection of representative data at sufficient points of exploration to enable the relevant geotechnical properties to be inferred for any part of the project.

The wide variety in ground conditions coupled with the range of design and construction problems to be solved make the subject complex, so that precise rules on the manner and extent of any study are not possible. Both experience and judgement are necessary. Too little investigation may not reveal a potential hazard, or involve extra costs for safety, while too much would be uneconomical.

An investigation should be planned and executed sufficiently far in advance of the commencement of design and construction to allow for a full study and the most effective use of the conclusions.

Codes of practice for ground investigation are now available in a number of countries and where appropriate should always be studied as they are likely to embody important local experience. Some are referred to in the bibliography under 'Main Investigation' given at the end of this chapter.

11.3.4 Cost

The cost of the investigation cannot be measured solely according to the size of the site or the magnitude of the project. It also depends upon having a knowledge of project details together with as much information as is available on the ground conditions. Even so, adjustments may still arise as the investigation proceeds depending upon whether simpler or more complex geotechnical solutions to those originally contemplated are appropriate.

Particular conditions may exist at a site that will involve higher costs than normal even for the same kind of development. One reason for this would be the presence of naturally occurring 'problem' soils or rocks, e.g. exceptionally weak soils such as peats and unstable material such as loess. The site may have become contaminated. Another reason would be the location of the site in a high seismic risk area or a cavernous region.

Because of the wide variety of soil and rock conditions many different investigation techniques have been developed varying in range of application and accuracy depending on general and particular requirements.

As an approximate guide, ground investigation may cost about 0.1 to 0.5% of the capital cost of new works and about 0.1 to 2% of earthworks and foundation costs although exceptionally the cost may be several times these ranges.

Sometimes the cost may be related to an overall cost saving; more often, though, the value of the investigation lies in the assurance against costly over/under design, unforeseen ground conditions with consequential delays in construction and poor in-service performance.

11.3.5 Ground investigation stages

The investigation should be a systematic expansion in knowledge of the ground conditions, directed towards solving the geotechnical problems. It is convenient to distinguish three stages in the complete process: (1) preliminary appreciation; (2) main investigation; and (3) construction review. Each of these stages is described in the following sections and is embodied in Figure 11.2 which presents in outline the sequence of

Contaminated derelict sites		*Geotechnical operations*
	Project conception	
Consult recorded data, site visit	**Preliminary assessment**	Geological outline
Consult specialist	Environmental/impact surveys	Geomorphological study
	Detailed topographical/ hydrographical survey of site/alternatives	
Specialist decides safety precautions and makes site visit	**Preliminary appreciation** Desk study and site reconnaissance leading to PRELIMINARY REPORT on: ground conditions, possible engineering problems and main investigation programme with estimated costs	Surface inspection, possibly with engineering geologist. Very occasionally borings/ pits, or overwater geophysics
Plan sampling patterns and methods	**Main investigation** SELECT RESOURCES for field exploration (1) In-house contribution (2) Contract fieldwork only, or (3) Contract fieldwork, testing and analysis. CHECK: staff competence, equipment adequacy and fieldwork supervision responsibility. Ensure flexibility of programme and methods.	Plan timing access, availability of resources, etc.
Maintain safety precautions Fieldwork	FIELDWORK EXECUTION Regular liaison between person in charge of investigation, field supervisor, geologist and engineering design.	Borings/pits etc., geophysics, *in situ* testing, instrumentation
Laboratory tests	Minimal delays for submission of preliminary results. Samples to laboratory for testing leading to	Laboratory testing by stages with regular reviews
Specialists analysis and report	Record of results: FACTUAL REPORT ENGINEERING DESIGN REPORT	
Check sampling and testing	**Construction review** Compare predicted conditions with ground revealed in excavations, samples from bored piling, pile tests borrow pit conditions, trial embankments RECORD DATA AND ENTER ON DRAWINGS	Inspection, check results, full-scale trials instrumentation

Figure 11.2 Sequence of operations for ground investigation

events from commencement of the ground investigation to the completed development.

The particular investigation where a site is possibly contaminated is conveniently and economically carried out at the same time as the geotechnical study, as some of the processes are of mutual benefit, and knowledge of the results has an equal bearing on the predictions to be made.

11.3.6 Preliminary appreciation

Selected works on this subject are given in the bibliography, at the end of this chapter, particularly in the section headed 'Main Investigation'.

The preliminary appreciation of the available data on a site is an invaluable first stage in a ground investigation. By this means, full benefit is taken of experience to prevent wasteful exploration work and normally it provides a sound approach for the planning and commencement of the detailed study which should be considered as a separate exercise.

The time required to search and the amount of available data to be studied often cannot be predicted so that it is generally not a suitable subject for competitive bidding, especially on a lump-sum basis.

In order to proceed it is necessary to have some knowledge of the project besides knowing its location. Minimal information initially would be the approximate overall size, layout and purpose of the principal structural units. It needs to be sufficient to establish the main geotechnical problems in relation to the ground and site conditions revealed by the appreciation. However, at an early stage and before deciding upon the detailed plan for the main investigation, it will be advisable to have more particulars such as loadings, floor levels and settlement tolerances of buildings, to enable the scope of the testing to be determined. Clearly, the more detailed information that is available at this stage the more effective the planning of the investigation.

11.3.6.1 Objectives

The preliminary appreciation consisting of both a desk study and site reconnaissance, which should be properly recorded for future reference with a description of the project, should have the following objectives:

For natural ground:

(1) As clear a conception as is possible of the ground structure and groundwater regime underneath and neighbouring the site; the formations present and likely to be affected; their degree of complexity, the presence of problem soils and natural hazards, e.g. seismic activity, and subsidence; the possible alternatives in interpretation of the data and the probable degree of accuracy of each.
(2) The principal ground engineering problems anticipated and possible solutions in the design and for the construction, e.g. need for piling, excavation below water table, stability of natural slopes, suitable fill for embankments, slopes for cuttings (see list of primary objectives in section 11.3.1).
(3) Detailed proposals for the main investigation having regard to the factors outlined in the next section, including the methods to be used and the amount of work necessary in each. The budget cost and possible extent of contingencies that may be required having regard to the probable degree of accuracy of the preliminary information.

Additionally, for contaminated and derelict land:

(4) A carefully executed survey of available evidence on the earlier uses of the site and its surroundings to assess what is possibly present that is hazardous, where it is and how much (see list of potential hazardous areas earlier in section 11.3.2).
(5) Precautions to safeguard personnel and equipment during site visits and the investigation work.
(6) Main investigation programme, kinds of specialist services and personnel required.

Where the available data on a site are found to be inadequate to indicate the course of the main investigation or where important inconsistencies arise at this stage that raise doubts on the best exploratory method to be employed, it may be necessary to make a preliminary survey by carrying out a limited amount of field and laboratory work for the preliminary appreciation. Such an approach could apply to any type of investigation. Whilst the amount of such work should be kept to a minimum for economical reasons, it is important that this initial work is of a sufficiently high quality to enable the preliminary appreciation to be soundly based and that it is carried out sufficiently far in advance of the main investigation in order to provide adequate time to consider the results and to make the appropriate arrangements.

One particular form of a preliminary survey, well suited to investigation along a line, such as for a highway, railway or pipeline, is to utilize a system of land form mapping, sometimes called 'land surface evaluation' or 'terrain evaluation'. Predictions on conditions below ground-level, however, should always be checked, e.g. by geophysics.

The preliminary survey for an offshore structure is usually made by geophysics in the absence of drilling data and at the same time an inspection of the seabed topography is made to select a suitable site.

11.3.6.2 Desk study

The collection of available evidence on the site that may be relevant to the investigation and project is conveniently referred to as the 'desk study'. The time spent on this exercise will depend upon the amount of recorded data, the complexity of the site conditions and the magnitude of the project. Where the amount of information is significant, study of this prior to the initial site reconnaissance can be of advantage.

The sources of the information will vary according to the country in which the site is located. The following list represents the better-known potential sources, to act as an *aide-mémoire* in the search. For major projects, important evidence may be available in another country.

(1) Topographical maps. Past surveys can help identify filled land, subsidence and erosion.
(2) Geological maps and memoires. Mining records. The latest information is often only in manuscript form.
(3) Local administrative authority and museum records. Personal visits are likely to be the best approach. Enquire for national code of practice on ground investigation. Building by-laws. Regional code of practice for seismic areas. Records of unstable ground or flooding. Always ask about earlier uses. Archival search and specialist consultants can be helpful where mining was done.
(4) Aerial photographs and satellite imagery (the latter requires specialist interpretation). Commercial and government sources. Photography from model aircraft and balloons can be useful as accurate scaling is generally not needed.

Whilst aerial photography is excellent to gain an overall view and for interpreting surface features, e.g. old slips or swamps, caution should be exercised when predicting conditions below ground-level as these can only be inferred and changes may occur at very shallow depths. Experience aids interpretation.

(5) Previous investigations or construction work at or near the site. Valuable source in developed countries.
(6) Agricultural data. Often confined to surface data but can show local variations.
(7) Public services, water, electricity, sewerage, etc.

Advantage should also be taken of the growing body of recorded soil and rock mechanics' information on the more important ground formations throughout the world, e.g. laterites, decomposed granite and London Clay. These offer useful quantitative data for making tentative predictions on the engineering characteristics of the ground as well as suggesting the particular problems pertaining to the formations in question that have been noted from experience and which might not necessarily be revealed by an individual investigation at a site but nevertheless may deserve safeguards in the design. For example, where cavities (natural or man-made) may possibly be present as in chalk or limestone formations, whether or not any are revealed, foundations should be reinforced against local collapse and in certain cases permanent precautions taken during the subsequent use of the site against initiating subsidence by restricting the use of soakaways and garden hoses and by employing flexible joints on services.

Where the site is potentially contaminated, the desk study can be done in parallel with that for the natural ground and similar sources of information are used. However, specialists should interpret the hazards likely to be present and provide general guidance on the procedures to be used during the main investigation.

11.3.6.3 Site reconnaissance

Prior knowledge and interpretation of the available data on a site greatly aids the value to be gained from a preliminary inspection. It should best be done on foot to make a thorough visual examination on the topographical and geotechnical features; to note the layout of the project in relation with the ground and services, also to ascertain the overall suitability. The reconnaissance is very helpful for planning the main investigation, particularly with respect to the methods and means of access for rigs and the larger items of plant.

For all major projects an inspection should also be made by an experienced engineering geologist to interpret the geological features and to assess their engineering implications.

In the case of contaminated sites, particularly where chemical contamination is suspected, a specialist, preferably knowledgeable in the suspected contaminants, should be included in the team for the reconnaissance. Moreover, whatever the hazard it is vital that there is prior consultation between all members of the team to ensure that every reasonable precaution is taken not to endanger their health or safety during the site visit.

Guidelines for undertaking a site reconnaissance include:

Preparation – Take, if possible, project layout to scale, local and district map or chart, geological data, aerial photos and notebook and, if needed, simple surveying equipment, compass and geological hammer. For soil sampling, take hand auger, plastic bags and labels.

Ensure permission has been obtained to make the visit.

General – Position the project and assess effect on existing boundaries, topography and geology. Check access. Consider effect of earlier uses of site. Check services that may be available.
Ground – Study surface features in relation to available data. Note differences. Sample main soil types for simple identification. Study neighbouring geological features and existing structures. Inspect vegetation and make deductions on the soils present. Note presence of any problem soils such as peat, weak clays or loose sands. Look for chemical waste, significant odours, discoloured soil and blighted vegetation.
Main investigation – Inspect access to and around the site. Look for possible obstructions, such as power cables, buried pipelines and services, fences, etc. Seek water sources and power supply if required. Select location of offices, sample store and laboratory. Consider accommodation and communications.

11.3.7 Main investigation

11.3.7.1 General

The object is to develop, in sufficient detail, the initial concept of the ground and groundwater conditions formed from the preliminary appreciation to enable a final choice of site and layout to be made; a safe and economic design to be prepared of the works, with alternatives where appropriate; potential construction problems anticipated and hazards identified. This will almost invariably entail the use of specialized equipment in the field to establish the geological structure, soil and rock types and groundwater conditions, with *in situ* and laboratory tests, in conjunction with experience to assess the values of the engineering parameters.

The investigation should be a reiterative process whereby information gathered in the early stages is used in the checking of the preliminary appreciation and in the directing of the later stages. It is not unusual for a preliminary appreciation to be found inaccurate and it is important therefore that the investigation programme should be flexible enough and, at all stages, to permit, if needed, changes to be made in the amount, location and type of investigation methods and tests employed.

The scope of the main investigation involves taking into account a number of considerations as set out in sections 11.3.7.2 to 11.3.7.6, together with the selection of the appropriate methods of ground investigation.

The ground conditions usually determine the methods of field exploration and sampling but the numbers and types of tests needed are usually governed as much by the requirements of the project as by the ground conditions. All methods of investigation have their limitations and these must be continually borne in mind (see section 11.4).

The main investigation will usually consist overall of field and laboratory work, the results of which will be under continual scrutiny. Upon completion, all the results, their interpretation together with the conclusions should form the basis of a formal report.

11.3.7.2 Types of main investigation

Initial considerations affecting the scope of the main investigations are the influence of the basic engineering requirements as indicated below.

Whilst the majority of investigations concern new works, there are a number of other types each with particular requirements as referred to below.

New works. Attention always needs to be given to every aspect particularly where 'greenfield' sites are concerned, including the effects on adjacent properties. It may be necessary to consider alternative locations. The least favourable conditions should be taken into account for design purposes and to safeguard subsequent performance.

The type of new work can strongly influence the quantity and quality required for the main investigation, e.g. nuclear power stations, large industrial developments, petrochemical complexes and other potentially hazardous sites would require detailed investigation of the locations of the key plant sites to ensure that any hazard to the environment was limited to an acceptably low level. Major water-retaining structures could also come into the same category because of the potential hazard to the environment. It is usually necessary for the more sensitive projects to carry out a seismic risk analysis even if the site is located in a relatively low-risk area.

At the other end of the scale, single or small groups of houses, minor extensions to existing structures, small sewers and pipelines may only require relatively rudimentary investigation with shallow auger holes, trial pits and visual observations.

Extensions to existing works. Data from previous investigations should be sought and used together with design and construction experience of the existing works and their subsequent performance. It will be necessary to consider the effect of the new works on the old as this may influence radically the type of foundation to be employed. See also remarks in *Safety of existing works* above.

Damaged works. It is very important initially to establish the causes of the problems, so that with the collection of other relevant information required for the design of the remedial works, the outcome is that a more detailed investigation is often required than for a new project of similar size. Litigation is another reason for an extended investigation to provide ample justification for the redesign.

Measurements should be made to check for continuing movement.

Safety of existing works. This situation can occur where there has been a change of use entailing heavier loading conditions or ground conditions encountered which were not anticipated in the original design. Where safety is concerned, the key factor is to establish all the possible problems that could adversely affect it and where conditions indicate a marginal situation a careful and often detailed investigation is needed, including monitoring of the performance of the works.

Materials for constructional purposes. In soils, pits or, better still, trenches are generally more suitable than boreholes since these enable more detailed examination of local variations, can give some indication of excavation problems, and allow for larger samples for testing. Classification testing generally needs to be more detailed than for foundation investigations.

Rock classification should be on the basis of the size of material needed for its proposed use to take account of its actual jointing and planes of fracture. Drillings into bedrock should be planned to determine joint structure to predict excavation costs and mode of extraction. A blasting trial should be considered.

11.3.7.3 Lateral extent of exploration

Fieldwork, in conjunction with available geological information, is used to explore the ground conditions at points distributed over the plan area of the project and extending at least up to its boundaries. Where the project could affect or be affected by adjacent areas or structures outside the site boundary, the points of exploration should cover these areas also. For example, proposed basements may extend below adjacent existing foundations or there may be sloping ground just beyond the site boundary. This principle should be followed even if the effect is only temporary, e.g. during construction.

By carrying out exploration initially at widely spaced points the overall conditions become known at an early stage. Further exploration can then proceed within this framework by comparison of the results at two or more points. There are dangers in assuming that an investigation at a point is representative of some undefined area all around it and there is no indication of the dip of bedding. Investigation should normally be made at the extremities of a structure, with additional intermediate exploration if variable conditions exist, and at points of concentrated loading.

Spacing of the points of exploration depends on the interaction of such factors as the type of ground conditions, the significance of the project, the requirements for the investigation, the relative merits of methods of investigation and their availability. At the lower end of the scale, several machine-dug trial pits or hand-auger holes could be more appropriate for a small investigation on reasonable ground conditions than the cost equivalent in rig boreholes. However, in general terms boreholes (or their equivalent) are often as close as 10 to 30 m, with not less than three per 200 to 900 m^2 of project plan area. With increasing number of points of exploration the intensity of investigation tends to be reduced where the ground conditions are relatively uniform.

Large complex projects require the points of exploration to be concentrated in the areas of the more significant units, e.g. deep basements, high retaining walls, large, tall or heavily loaded structures, liquid retaining structures, structures sensitive to

Figure 11.3 A site laboratory usefully minimizes sample disturbance by reducing delays in testing and transport. This illustration shows the interior of one of four mobile air-conditioned laboratories, belonging to the China National Coal Development Corporation, that incorporates computer-controlled triaxial, oedometer and shear box equipment. (*Courtesy*: ELE International Ltd)

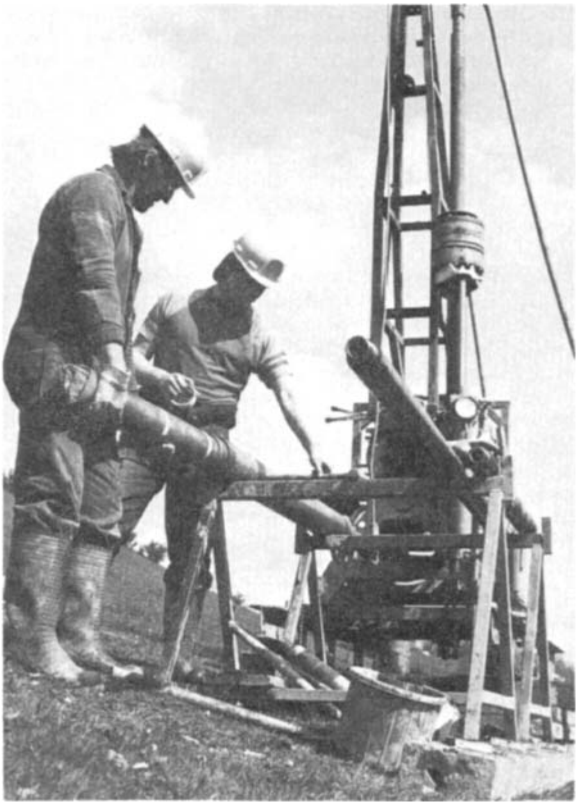


Figure 11.4 Rotary core drilling showing sealing of rock core in plastic tubing upon removal from double tube core barrel

movement, units with a potential hazard risk. For minor units located within a complex, fewer points of exploration are used provided they reveal ground conditions consistent with interpolations between the major areas of investigation.

Very large projects such as reservoirs, stockyards, spoil tips and reclamation schemes should be subjected initially to a broad study using a gridded pattern of points of exploration at, say, 300 m centres.

Linear structures such as pipelines, channels, roads, railways, airport runways and tunnels have points of exploration located along the centreline with some straddling it to detect lateral variations. Structures en route are investigated as separate units; otherwise, points of exploration are spaced about 50 to 200 m apart.

Construction on sidelong ground, that might entail retaining walls, or where there is an actual or potential slope stability problem, would usually include three to five or more points of exploration on line in the critical direction across, as well as beyond, the area.

11.3.7.4 Depth of exploration

In principle, the investigation should extend to such a depth that it identifies all the strata and groundwater regimes that will be affected by or will affect the project and provides sufficient data for design and construction. It is advisable at an early stage of the main investigation to establish or confirm the overall ground profile to the maximum depth required at least at one point under each major structure. Provided ground conditions are consistent and satisfactory it may not be necessary for the full profile to be established at all exploration points. Any excavation work (surface or underground) that is planned should always be adequately supported by investigation beyond its full depth.

Where bearing capacity and settlement are the controlling factors, as for single or multiple foundations, the investigation should extend to a depth at which the increase in vertical stress caused by the foundations will have negligible effect on the ground. This is usually taken as that depth at which the increase in vertical stress is less than 10% of the applied bearing pressure and less than 5% of the effective vertical stress in the ground. Where loadings are not known, the initial depth of exploration should be at least one-and-a-half times the width of the building, not less than 10 m unless very strong ground is encountered at shallow depth precluding any problem. Such a stratum should be investigated to a depth of at least 3 m. Where this stratum is rock, any very weathered zone should be fully penetrated to

Figure 11.5 Location of gravel-filled channel under St James's Park, London, using ground radar (electro-magnetic profiling) equipment. (*Courtesy*: Wimpey Laboratories Ltd)

ensure an improving profile with depth and that a boulder has not been mistaken for bedrock. Where loadbearing piles or other deep foundations, cantilever walls, ground anchors or other similar forms of temporary or permanent construction may be employed, the depth of exploration should be reckoned below the lowest possible founding level in order to assess overall stability and settlement. The depth of exploration should also extend below the depth of any proposed ground treatment such as freezing, chemical injection or dynamic compaction. Exploration depth below excavations and basements should be assessed by the change in vertical stress criteria as for foundations given above. If artesian or sub-artesian conditions are suspected the depth should extend to below the aquifer or to

Figure 11.6 Self-boring pressuremeter in operation complete with on-site data acquisition and monitoring system. (*Courtesy*: Soil Mechanics Ltd)

such a depth below which such conditions if they existed would not be significant.

Some relaxation of the above depth guides are permissible for high fills provided the ground conditions are shown to be satisfactory and settlement is not a problem. Side slopes should be investigated for stability by exploring to depths of half to one-and-a-half times the side slope width with the greater factor for the steeper slopes. Slope stability problems should be investigated to depths below any potential failure surface or to a hard stratum below the slope toe.

Where ground permeability is an important factor, as for dams and water-retaining structures, the depth of exploration should be sufficient to enable flow nets to be drawn, i.e. about 1 to 2 times the height of retention or half the base width, whichever is the greater. Where a vertical cut-off could be considered greater depths may be necessary.

Highway and airfield pavements require investigation to about 3 m depth below subgrade in cuts or ground-level under low fills unless the ground is very weak, such as in peaty areas when exploration needs to extend through the weak material.

Lightly loaded areas may involve an extended depth of exploration to penetrate all weak and compressible ground and particularly the zone of ground influenced by seasonal climatic changes and vegetation. In temperate conditions such as in Britain the zone affected by seasonal wetting, drying and frost may only extend to 1 or 2 m depth in open sites. However, the effect of tree roots can extend to about 5 m. In hotter, drier, climates the zone can extend to 20 to 30 m under trees. In arctic climates the temperature effect can extend to depths of 15 to 30 m.

11.3.7.5 Natural problem conditions

Particular types of natural ground conditions, representing problem conditions, are known from experience to require more careful exploration and testing because of the difficulties they can cause. Some examples are:

(1) Organic soils, peat, soft alluvial clays and silts, black cotton soil, quick clays leached by freshwater percolation, sensitive clays that weaken significantly on disturbance, swelling (expansive) and shrinking clays which are markedly affected by changes in moisture content.
(2) Weak granular soils such as dune (rounded grain) sand; or very loose sand which can settle significantly when subject to minor vibrations from, say, nearby pile driving. Earthquakes in some cases have caused spontaneous liquefaction in such soils.
(3) Metastable soils, such as loess, having been deposited in an exceptionally loose state and which can collapse on saturation leading to catastrophic settlement. In the dry state, such soils are very stable. This can be misleading during investigation and more so because collapse can take place in the boring process.
(4) Duricrusts. Hard crust sometimes occurs, normally near the top of a particular soil profile, beneath which much weaker soils exist. One example is the caprock at the top of a lateritic soil profile, others are lava flows and basaltic layers. The thickness and strength of the crust is often extremely variable.
(5) Aggressive ground and groundwater, that may contain constituents such as gypsum which attack Portland Cement concrete; or electrolytic, chemical or bacteriological agencies that attack metals, particularly cast iron. Saline ground, groundwater, as well as sea water and soft water may also require special consideration.
(6) Permafrost and frost-susceptible ground. Dealing with permanently frozen ground is a subject requiring special expertise, while frost-susceptible ground includes silts,

chalk and some shales which expand or disintegrate on freezing due to the development of internal ice lenses.

(7) Noxious and explosive gases are occasionally present in soils, rock and groundwater. These may cause a hazard during construction and in unlined excavations such as tunnels. Methane is soluble in groundwater, increasingly so above ambient pressure, and it has been known to cause explosions in boreholes and underground workings. A certain type of bacteria exists naturally in some ground, that can deplete the oxygen supply in poorly ventilated underground areas.

(8) Rocks subject to rapid weathering or swelling (see Chapters 9 and 10).

(9) Unstable profiles, geological or topographical anomalies, faults, ancient slip planes and extended discontinuities. Buried channels which may affect the project. Inclined drill holes may be of value to assist in the location of faults.

(10) Ground liable to subsidence such as in cavernous limestone and chalk areas, or collapse associated with valley cambering.

11.3.7.6 Contaminated site surveys

It is essential that a most thorough and careful preliminary appreciation has first been made to ascertain as much information as is available on the previous long-term history of the site and its surroundings to give the best indication of what potential hazards to expect from all previous uses. The next most important stage is to assess the immediate risks to personnel and plant in order to decide the safety precautions to be taken during the main investigation. Established procedures often exist for these precautions and specialist advice should be sought according to the risks. There may be national legal obligations to be complied with such as the Health and Safety at Work Act.

In the case of physical hazards, such as the location of derelict underground structural work or the size and extent of old mine workings, the problems are similar to natural ones and the main investigation represents simply an extension of the use of the established methods of ground exploration with the intensity of the points of investigation being programmed according to the available information. A grid pattern of boreholes is often used with the spacing being steadily reduced until sufficient confirmation of the hazard and its extent is obtained. Geophysics may also be employed with advantage.

Where contaminants are suspected, in liquid, gas, solid, bacteriological or radiological form, experience shows that investigation can become very complex to locate and identify what is the amount of risk, so that suitable specialists should be engaged to determine land quality and the precautions necessary for its safe development. The principal objective is to locate those parts of the site or its surroundings where concentrations of contaminants remain that are sufficiently high to impose a risk to the development envisaged.

To give some idea of what may be involved a systematic sampling strategy is usually employed in conjunction with a thorough visual inspection of ground and vegetation, including noting unusual smells. Machine-excavated pits (entry may be dangerous and backfilling should be prompt) are preferred to boreholes for ascertaining what variations there are in the concentration of the contaminants from one part of the site to another, laterally and in depth. Places with the highest levels are the important ones and probably would be sampled in a second stage in more detail. Initial sampling of soil and groundwater, might be on a 25 to 100 m grid and, say, at 1-m intervals in depth depending upon the kind of development. Sampling should be ample, it may include taking vegetation for test, and could involve special precautions to obviate contamination from the container. Surface water would need to be sampled. Where groundwater contamination is involved geological and hydrological surveys become important with a careful assessment of ground permeability.

The wide range of contaminants makes testing a specialist subject, the more so because the quantities may be small. Inorganic, organic, bacteriological and radiological testing may be involved so that a comprehensive study becomes multidisciplinary although chemical testing usually predominates. While some guidelines have been proposed on the level of concentration that involves precautions for the more common contaminants it will vary with the kind of development proposed and considerable judgement is involved. In fact, the presence of contamination does not necessarily mean a problem exists and many contaminated sites can be safely re-used.

11.3.7.7 Interpretation and the geotechnical report

Interpretation of the data should be a continuous process from the commencement of the investigation, leading firstly to reliable ground and groundwater profiles, then realistic values for the ground characteristics and ultimately solutions where possible to the ground engineering and site problems. As the fieldwork explores the stratigraphy, *in situ* tests are carried out and samples taken for examination and laboratory work. The types of sampling and testing are chosen compatible with the methods of exploration (see section 11.4) and the engineering problems. The amounts are based on previous experience of what is appropriate to give sufficient information. In view of the fundamental importance of the fieldwork, an experienced geotechnical engineer or engineering geologist should be employed full-time on site to supervise. Changes and adjustments in procedures can also be effected more competently and economically. Such a person, or a trained assistant, should personally inspect all samples and plot the logs. These should represent what the actual ground conditions are considered to be, at that point of exploration, weighing all the evidence from the boring-records, tests and sample descriptions, taking account of the inherent disturbance that sometimes occurs due to the boring and sampling operations.

The report will be the only lasting record of the investigation and therefore should contain a statement of the purpose of the investigation, a plan showing the site and its location, surface conditions, earlier uses, existing structures and topographical features, time of the fieldwork and for whom it was carried out. Along with the description of the proposed works would be given a summary of the local geology and a full record of the types and results of the field and laboratory work. Information from boreholes and trial pits should be recorded graphically and in cross-sections. Classification tests should be used to check the sample descriptions making due allowance for sample disturbance. Full correlations should be made with the geological information. Test results including water-level records should be tabulated and where appropriate plotted graphically. Up to this point a description of the interpretation of the ground conditions with a record of the results of the investigation is generally referred to somewhat irrationally as a factual or, more appropriately, descriptive report and may complete the work of a ground-investigation contractor. (Such reports should always be provided complete to all tenderers for the construction work.)

Access difficulties sometimes mean that it is not possible to investigate at the preferred locations. Such situations are undesirable and should be recorded in the report on the main investigation.

Where an engineering interpretation is required, the terms of reference should be recorded, with information on proposals supplied by the client. The derivation of values of ground

Table 11.2 Exploration methods on land

Method	*Geology*	*Technique*	*Applications and limitations*
Boreholes *Prime safety precautions:* ● *Overall collapse above cavities, buried mine shafts, etc.* ● *Gas and fires from peat beds and organic fills.* ● *Hand and head injuries.*	*Clays, silty clays* and *peats.*	*Hand or power auger boring* (single blade or continuous spiral). Usually without addition of water.	*Shallow reconnaissance.* Power operation fast. Limited to non-caving ground except for power-operated hollow continuous augers.
	As above, also *silts* and *sands*	*Wash boring*, with water or drilling mud.	*Preliminary exploration*, with disturbed sampling, frequently includes SPT. Unsatisfactory for precision work but inexpensive.
	As above with *gravel*, occasional *cobbles*, and *boulders* also *decomposed rocks.*	*Light cable percussion boring* with casing. (Shell, auger and clay cutter cable-operated boring tools)	*Standard for soil exploration.* Water added below water table to stabilize base of boring. Before core sampling cohesive soils, the borehole should be properly cleaned out.
	As above and up to *moderately weak rocks*	*Non-coring drilling*, with pneumatic chisel or rotary tricone bit.	*Limited to location of hard ground* (check for presence of boulders), *cavities, or testing, at pre-arranged levels*, in suitable soils.
	All *rocks* and occasionally *soils*	*Rotary core drilling*, usually with water flush. Drilling mud stabilizes wall and counters stress relief at base. Alternatives: air flush, foam and other liquid additives.	*Standard for rock exploration.* Reliability depends on correct selection of core barrels, bits and flush fluid. Water table observations difficult. For *proving rock* at base of cable percussion boring with casing use pendant drilling attachment. For *clays, sands and very weathered rocks*, use triple tube retractor barrels.
Pits and shafts *Prime safety precautions:* ● *Beware collapse of walls* ● *Asphyxiation without ventilation* ● *Quick condition in bottom* ● *Refer to BS 5573**	*Clays* and *peats*	Excavation by hand, power grab, or auger with support as required. Tractor-mounted hydraulic back hoe excavators particularly suitable.	*Detailed study of local soil variations.* Direct access gives best opportunity for inspection of ground *in situ*, presence of stratification and thin clay layers. Depth usually limited by problems of groundwater lowering.
	Silts, sands and *gravels*	Close timbering or piling, groundwater lowering essential below water table.	
	Weak to *moderately weak rocks*	Hand excavation, power grab, or auger.	*Detailed study of bedrock conditions*, weathering, fissures and joints. Depth usually limited as above.
Trenches *Safety: See above*	All *soils* above water	Excavation usually by machine such as hydraulic powered excavators. Support as required.	*Exploration of borrow areas.* Direct access with extended inspection of lateral variations.
Adits *Safety: As for tunnels*	All *soils* and *rocks*	Appropriate forms of hand excavation and timbering as for tunnelling.	*Established method for detailed exploration of dam abutments and underground structures.* Sub-surface exploration of steeply inclined rock strata.

Notes: (1) Locate pits and trenches outside proposed foundation areas to obviate soft spots under structures.
(2) Seal boreholes with impermeable backfill where it is necessary to prevent access for groundwater upwards or downwards into excavations.
(3*) BS 5573:1978 Code of Practice for Safety Precautions in the Construction of Large-diameter Boreholes for Piling and Other Purposes (formerly CP 2011).

parameters from the investigation data for design purposes should be explained with an assessment of reliability. The interpretation may take the form of recommendations or comments on a client's proposals, and may be qualified and subject to confirmation by further work. Various topics may be referred to depending on the project but could include comments upon: pad and raft foundations, working loads for piles, earth pressures for retaining walls, flotation of basements, the need for special construction techniques (e.g. anchors, groundwater control, chemical treatment), chemical attack on foundations, pavement design, temporary and permanent stability of slopes, subsidence, methods of excavation and filling, sources of construction materials. For further information see bibliography under 'Main Investigation'.

11.3.8 Construction review

The degree of confidence placed in the conclusions and recommendations in any investigation must recognize that they are based on a first-hand knowledge of only a minute proportion of the ground influencing or influenced by the project. Accordingly, during construction the results of the investigation should be verified. In simple cases, this will consist of comparing the conditions revealed in any excavations with the predicted soil profile. Significant differences that arise may require design amendments, possibly after further investigation. Such differences should be recorded properly for use later when modifications may be introduced or extensions added.

In the case of specialist geotechnical processes, e.g. piling, grouting, ground anchors and diaphragm walls, check tests may sometimes be required to compare local conditions with design criteria established from the main investigation. This third stage would also normally include full-scale trials made at the commencement of the contract.

The full extent of the bedrock structure can only be seen properly in an excavation and full allowance should be provided in the design for all reasonable eventualities. This applies to earthworks and especially dam construction where modifications in the design to suit the conditions revealed is frequently normal practice.

Groundwater observations may have to extend over a wet season or even several years and into the construction period. Records are also needed when groundwater lowering is being used to note its effect on the excavation work and outside the site where it may affect water supply and cause ground settlement.

Instrumentation measurements are often usefully continued throughout and after construction to observe the performance of the project, particularly where, because of complex ground conditions, predictions from tests are less reliable than usual. This is particularly desirable in the case of, say, dams where instrumentation can provide the only possibility of an early warning of the onset of unacceptable conditions.

In some cases, where the interaction of the project and ground conditions is a complex one, the construction review becomes more fundamental in the solution of the problem. This is considered further in section 11.4.2.5.

Table 11.3 Over-water ground exploration

Method	*Technique*	*Applications and limitations*
Cable percussion boring with casing. Conventional core rotary drill with rods to prove bedrock	As on land, from fixed platform or from floating pontoon, barge or ship fixed in position by anchors. (Wash boring with *in situ* sampling and testing may suffice for preliminary surveys.)	*River, lake and coastal structures in water depths up to about 50 m.* Sampling and borehole tests as on land. Proving bedrock facilitated by using pendant attachment mounted on boring casing to combat wave and tidal effects.
Rotary wireline drilling through guide tube from surface vessel or platform	Extension of above technique with cable extractable rotary core barrels. Vessels typically 50 m long for water depths up to 200 m. Greater depths require dynamic positioning gear.	*Widely used for offshore platforms* using percussion wash boring through sediments and rotary coring through rock. Heave compensation needed for rotary drilling from floating craft, or use pendant attachment.
Submerged rotary drills operated by divers	Hydraulic power from support vessel anchored above, using wireline coring technique and *in situ* testing.	*Projects in harbours and open water depths up to 40 m.* Total penetration typically 20–60 m. Maximum current 2–3 knots. At depth around 20 m may require 4–6 divers. At depths around 30/40 m may require minimum of 8 divers.
Submerged remote-controlled rotary corers and seabed samplers	Power supply and control via umbilical cable from support vessel to seabed unit with rotating head. Some incorporate magazine of drill pipes to increase penetration.	*Offshore structures.* Low penetration units limited to 5 m. Larger corers designed for penetrations around 50 m or more in water depths 200 m and more. Core sizes typically 50–100 mm dia.
Flexodrilling	Power supply, flushing media and remote control via special flexible non-rotating cable on the lower end of which is a motor-driven rotary drill that may incorporate coring and non-coring facilities.	*Coastal and offshore structures.*

Table 11.4 Sampling methods on land

Source	*Geology*	*Disturbed*		*Undisturbed*	
Boreholes	Clays, silty clays and peats	Hand auger	Normally representative of composition for classification, but unreliable for examination of structure.	Open tube samplers Area ratio (AR)	General purpose 100 mm dia × 450 mm long, heavy duty < 30% A Suitable for local stratigraphical identification and soil mechan testing on cohesive soils and weak rocks excluding pore-water pressure measurement on softer materials. Thin walled samples < 10% AR. 75 to 250 mm dia. better for soft and firm without stones.
		Clay cutter	As above, but liable to more mixing.		
	As above, also silts and sands	Shell	Standard for non-cohesive strata to examine composition (particle size and distribution). Best when whole contents of shell is emptied into tank and allowed to settle before taking representative sample from sediment.	Piston samplers	Less disturbance and better recovery than for open tube samplers. Fixed piston superior to free piston. Non-cohesive strata retain only within mud filled borehole. Improved quality helpful whei testing soft recent clays and for effective stress analysis. Reliabi aids studies of specific horizons. Sample diameters range from 250 mm and lengths up to 1 m.
		Powered auger	Liable to considerable disturbance and mixing except when conditions in depth are very uniform.		
		Water flush	Liable to serious disturbance and mixing (strata identification only).	Continuous samplers (usually commenced from ground surface)	(a) Delft 29 mm dia. (Nylon stocking) rapid method with individt samples up to 18 m long in recent alluvium (Dutch cone resista below 10 MN/m^2) for stratigraphical identification. (b) Delft 66 mm dia. (Nylon stocking) as for 29 mm sampler, also all standard soil mechanics testing. (c) Swedish 68 mm dia. (Steel foils) individual samples up to abou 29 m in soft recent alluvial clays and laboratory strength tests correspond to *in situ* vané results. Can also be used in silts and sands of medium and low density.
		Standard penetration test sampler (SPT)	Provides small specimens of both cohesive and non-cohesive soils for classification purposes but is not normally suitable for retaining structural features.		
		Flow-through sampler	Self-contained incremental sampling technique commenced from ground surface for strata identification. Size of sample similar to that of an SPT.	Compressed air sampler (60 mm dia.)	For recovery of silt and sand from above or below water table wi use of mud, to study laminar structure and composition, densit and permeability.
				Rotary core barrels (Total volume sampling)	Triple tube types (see below under rocks), including those with spring-loaded inner barrel (retractor type, Mazier) and face discharge tungsten carbide bits with removable inner liner usua plastic, e.g. Mylar. Synthetic polymer flushing fluid can be advantageous using low flow rates. Larger core sizes preferable typical 100 mm nom.
	Gravel, cobbles and boulders	Shell	Standard for gravel, but grading may be unreliable.		No common method in use, although injection of chemical grout been tried, also freezing where saturated.
		Power auger	Specimens up to gravel size may be recovered without reliance on source.	Rotary core barrels	Double and triple tube types to core boulders.
	Weak rocks (including *hard clays*)	Auger	Sample identification generally misleading due to remoulding which produces a weaker material.	Driven samplers	Shatter during driving causes serious structural disturbance which affect results of soil mechanics tests.
		Air flush (vacuum recovery)	Possible study of mineral composition above water table	Rotary core barrels	Double and triple tube types (see below and above under fine grai soils) and pitcher sampler.
		Flow through sampler	See note above.		
	All rocks	Water flush	Rock sludge samples provide opportunity for identification by microscope when conditions are uniform if no core is recovered.	Rotary core barrels	*Single tube.* Simplest type suitable only for massive uniformly strc rock. *Double tube* types support and protect core during drilling. Inner tube rigid: least likely to jam but liable to cause serious sa disturbance in variable and broken rock. Inner tube swivel: *internal discharge*: adversely affects core recove variable and broken rock which is minimised w discharge is below core lifter. *face discharge:* although expensive is considere best method to minimise losses in variable and broken rock. *Triple tube* types provide extra split inner tube which assists in rer of core from barrel with least disturbance. Other special barrels in *spring loaded inner barrel* which extends to protect core in weak la *Wire line barrels* provide facility to withdraw and return inner ba and core from bottom of hole independently of outer barrel and t *Water flush* is generally used to cool bit and remove cuttings. *Air* requires special equipment to maintain air speeds, can have advan when coring above the water table. *Mud flush* can be helpful to re erosion of core. *Foam flush* helps reduce bit wear in hard rocks an increases speed. Rock cutting is usually with diamond bits but tur carbide inserts are applicable for uniform soft rocks. *Chilled steel* is used only for large diameter cores (over 150 mm dia.) when son loss is acceptable. Fissures must be grouted to prevent loss of sho Suitable ancillary equipment as well as skilful operation are essen for good core recovery and the greater the complexity in ground conditions the higher the degree of skill required. The more broke ground the shorter each drill run should be to ensure good recove The core should be preserved in 'lay-flat' plastic tubing for labora testing or in polyurethane foam as described below.
Pits, trenches & adits	Clays and peats, silts, sands and gravels. Up to moderately strong rock.	Hand excavation	For identification purposes particularly useful to study local variations and anomalies. Ensure fresh *in situ* surface is exposed before sampling.	Open tube and piston samplers. Block samples	See notes for boreholes. Offers opportunity for horizontal and inc as well as vertical tube samplers, silts and sands as well as clays. Ensure fresh surface is exposed before sampling. Hand-cut *block samples* of self-supporting soil or weak rock, carefully cut and tri *in situ* to provide undisturbed sample with minimal disturbance. Samples, often 150 mm cube are coated in wax reinforced with m as each face is exposed or wrapped in foil and encapsulated in polyurethane foam.
Groundwater		Bail out borehole or pool, sample after the water has returned to its former level. Rinse the container thoroughly beforehand, preferably using water from test source. Ensure surface or rain water has not diluted water to be tested.			

11.4 Methods of ground investigation

11.4.1 General

Primarily, the very large number of methods in use can be divided into two groups; those that rely upon samples and those that provide *in situ* measurements. In any major investigation, both are normally required, the relative amounts depending upon the available information and the magnitude of the project. To aid basic selection, it is important to note that first the stratigraphy and the groundwater conditions must be interpreted adequately, before it is possible to decide upon the required engineering properties. In certain circumstances the general ground profile is reasonably well known in advance, in which case it is often possible to depend solely upon *in situ* measurements to interpret the soil conditions and determine the properties (see also section 11.4.3.1).

The merit of sampling is the opportunity it gives for direct inspection, although sample disturbance must always be taken into account. In the laboratory, more control is possible in the boundary conditions during a test, and adjustments can be made prior to the main test, e.g. in stress conditions or moisture content where these are of consequence in the design. Long-term tests are also more conveniently carried out.

11.4.2 Stratigraphical methods

11.4.2.1 Geological mapping

The structure in depth is inferred from mapping of the surface features. This gives a general indication of the ground conditions and, where there are numerous surface features to aid identification, may provide a very good indication of the structure. However, it may fail to reveal comparatively minor geological features, which have a decisive influence on the project. This method is more fully discussed in Chapter 9.

11.4.2.2 Exploration by boreholes, pits, trenches and adits

Geological mapping should always be supplemented by exploration using boreholes, pits, trenches and adits in which the ground in depth is exposed for direct examination and representative samples retained for identification and laboratory testing (section 11.4.3.2). The various techniques of exploration and their application to the ground conditions are given in Table 11.2, for use on land, from platforms or staging and, in Table 11.3, in offshore situations where floating craft are necessary (section 11.4.4).

Table 11.5 Deep-water soil sampling

Method	*Technique*	*Applications and limitations*
Dredges	Sheet metal or chain bag with open end, towed under way.	Disturbed samples of loose material from surface. *Reconnaissance only.*
Grabs	Variety of methods force jaws to close before withdrawal on winch rope.	*Bulk samples of bed material.* Unconsolidated clays, silts and sands. Limited use in compact soils and gravels. Difficult to use in rough conditions.
Drop or gravity corers	Similar to open-tube soil sampler with weight at the top end, and trip device for release at pre-selected distance above bed. Normally with plastic liner. Some types are driven in with an explosive charge. *Free-fall corers* for deep water obviate the need for winch and use release of buoyant chamber after sampling to return core to water surface.	*Open-drive tube* samples of soft cohesive soils up to few metres long. Fine non-cohesive soils recovered using tulip-type core catchers. Tube dia. typically 40–120 mm. Can be used at depths exceeding 400 m.
Piston corers	Drop corers in the form of a free-piston sampler. Overall weight 300–1500 kg	Offers opportunity to recover longer and better cores typically up to 10 m, than with drop corer. Operation simple but requires large boom and calm conditions to handle heavy weight on cable. Can be used at depths exceeding 400 m.
Vibrocorers	Core barrel, with cutter and catcher, is vibrated from the bed by hydraulic, pneumatic or electric motors, all mounted in a frame. May also rotate. Sometimes with piston and plastic liners.	*Widely used technique.* Lengths 2–10 m, dia. 100–300 mm. Cores subject to disturbance. Sampling possible in most unconsolidated sediments up to gravel size, as well as stiff clays and soft chalk. Time per core typically 2/3 h. Maximum current about 2 knots.
Air lift	A vertical pipe from the bed to above the water surface has injected into it, close to its base, compressed air which induces a strong upward flow that lifts the sediment to the surface.	*Large bulk samples* of unconsolidated sediments. Samples very disturbed and suffer same limitations as those from wash borings.

Besides the ground conditions, information must also be obtained, where possible, on the groundwater, e.g. the level at which it is struck, and presence of artesian conditions. However, such observations can be affected by the exploration method and it is advisable, therefore, to use observation wells or piezometers, which also take account of tidal and seasonal variations in level, in order that measurements may be made from time to time to establish the worst conditions. When drilling in rock, levels at which the circulating water fails to return should be noted as these denote the existence of open fissures.

Sufficient samples of the right size and type should be taken in order to fully represent the ground being investigated. In soils, this means that each stratum should be sampled at regular intervals, typically every metre, over its whole depth. Samples are either 'disturbed', i.e. taken from the spoil from the borehole, pit, etc. and not, therefore, representative of the soil structure, or 'undisturbed', i.e. showing the undisturbed soil structure. The latter, however, are still subject to some disturbance depending upon the method of sampling used. Further information on this and the methods used for both disturbed and undisturbed sampling is given in Table 11.4 for land and Table 11.5 for over water situations.

The size of the standard undisturbed sample is normally sufficient for the usual laboratory tests, although larger-diameter or block samples are sometimes required.

The size of the disturbed sample should be governed by the nature of the soil and the type and number of tests which are to be made upon it. Typical sizes are as shown in Table 11.6.

Table 11.6

Purpose of sample	*Type of soil*	*Minimum amount of sample required* (kg)
Soil identification natural moisture content and chemical tests	Cohesive soils and sands	1
	Gravelly soils	3
Compaction tests	Cohesive soils and sands	12
	Gravelly soils	25
Comprehensive examinations of construction materials including soil stabilization	Cohesive soils and sands	25–45
	Gravelly soils	45–90

11.4.2.3 Exploration by penetration tests

Advantage of these less expensive and quicker methods may be taken on occasion in preference to boring or pitting, to determine sufficient information of the ground formations as well as their engineering properties, albeit in an empirical form. The methods available for testing soils and their relative merits are set out in Table 11.7.

11.4.2.4 Geophysical methods

The techniques used are *in situ* methods of measuring contrasts in particular physical properties of strata and, hence, determining the stratigraphy and occasionally the water table. Where appropriate, the techniques represent a valuable and economic means of extending ground profile information outwards from a point of exploration. The methods are summarized in Tables 11.8 and 11.9. Further information is given in the selected bibliographies at the end of this chapter. Although there are no great difficulties in carrying out the site measurements, excluding adverse marine conditions, experience and a knowledge of the geology are essential for interpreting the data correctly. The results should always be checked with some form of direct exploration, such as rotary core-drilling.

11.4.2.5 The observational method

Sometimes, because of the complexity of the ground conditions or a need for some flexibility in the project plan, as often required with a dam, it is not economically feasible to assess completely the problems after the main investigation. However, by adjusting the construction programme, it is possible to monitor the construction so that the design can be checked and modified where necessary. A good example of this observational method is the construction of road embankments over soft ground, where the preliminary assessment indicates a very low factor of safety. Construction is monitored, the design checked and, if necessary, side slopes, rate of earthmoving, etc. adjusted.

The method is equally applicable to investigations other than those for new works and especially for investigations into failures. It should be noted that successful application of the method to any project depends upon obtaining reliable relevant field data, which, in turn, requires the correct field instrumentation. Basically, instrumentation is to enable measurements to be made of displacement, earth pressure and pore-water pressure. Two important points need always to be borne in mind: firstly, to select the simplest form of apparatus consistent with the required accuracy and, secondly, always to make provision for some breakdowns due to the difficulties arising during the installation and subsequently as a result of the severity of the operating environment.

11.4.3 Measurement of engineering properties

11.4.3.1 In situ testing and instrumentation

Normally a main advantage of *in situ* testing over laboratory work is that the ground under test is less disturbed, and occasionally this includes the retention of the natural *in situ* ground stress pattern. The amount of ground tested by each measurement may also be larger than would otherwise be economically possible to test in the laboratory. Against this, the boundary conditions are no longer precise compared with those in a laboratory. The method may also relate to a direction of testing different from that which will be subsequently imposed.

The various methods of *in situ* testing in soils and instrumentation with their main applications and limitations are set out in Tables 11.7 and 11.10. Exploration and *in situ* testing in rocks is described in Chapter 10.

11.4.3.2 Laboratory testing of representative samples

The samples obtained from the exploration are generally tested in the laboratory to assist in the identification of strata and to determine their relevant engineering properties. The various laboratory tests are summarized in Table 11.11 with their application to routine engineering problems. Compared to *in situ* testing, laboratory testing is under controlled boundary conditions and to defined testing procedures. Moreover, there is no doubt as to the soil type, state and structure under examination.

11.4.3.3 Geophysical methods

Although most geophysical work falls into the category of 'stratigraphical methods', referred to above, some techniques can be used for ascertaining certain engineering properties as given in Tables 11.7 and 11.10.

Table 11.7 *In situ* testing in soils for foundations

Location	*Method*	*Technique*	*Applications and limitations*
Normally on land or from platform	**Borehole tests**		
	Standard Penetration Test (SPT)	Standardized intermittent dynamic test. Provides small disturbed sample excepting gravel when solid cone is used. Maintain positive head of water or drilling mud.	*Most widely used preliminary field test.* Relative densities. *Bearing values of non-cohesive soils.* Unreliable in gravel. Correction applied in fine-grained soils. Indicates settlement of spread footings in granular soils. Aids estimation of liquefaction potential.
	Pressuremeter test	Lateral pressure and deformation tests from an expanding cell. Types include: – *Menard.* Used in pre-formed hole. Slotted steel casing for gravel. – *Stuttgart.* Split metal cylinder expanded hydraulically. – *Stressprobe.* Pressed below borehole and takes core. – *Camkometer.* Self-boring with pore pressure cell. – *Marchetti* dilatometer (DMT). Steel plate containing stress cell on one side, pushed edgewise into soil.	*Strength and deformation properties in most fine-grained soils.* Direct bearing values, but may not correspond to vertical loading. Less expensive than vertical loading tests and larger volume stressed than in laboratory test. Camkometer most sensitive, where boring is possible, able to measure k_0 and effective stress.
	Downhole bearing test	Plate loading test on base of borehole. Alternatively, simple screw plate augered through disturbed zone.	In-situ *bearing values for clays.* Not commonly used. Baseplate restricted to above the water table.
	Hydraulic fracturing	In hydraulic piezometers	Measurement of minor stress.
	Penetration tests (continuous record)		
	Simple probe	Driving a rod by drop hammer or pneumatically.	Location of hard ground beneath weak strata. Beware of boulders and influence of friction of the rods.
	Dynamic Probing (DP)	Standardized dynamic cone penetration testing procedures.	Bearing values where local specialized experience exists. Mainly used in non-cohesive soils. Caution needed at depths when rod friction may be high.
	Weight Sounding Test (WST)	Standardized procedure using dead weights on screw point, via rods followed by rotation to provide profile of half-turns/0.2 m.	*Well-established Scandinavian technique.* Inexpensive. Used in most soils except dense sediments and compact layers. Applied to footings and pile design, also compaction control.
	Cone Penetration Test (CPT)	Standardized procedure using shielded cone rod with slow constant rate of penetration. Local friction just behind cone also measured. Piezometer can be fitted in cone. Adaptable for small piston sampling at selected levels. Electric cones preferable to mechanical. Special equipment measures tilt, temperature and density. Piezocones measure pore-water pressure.	*Bearing value and length of piles in silts and sands.* More rapid and less costly than boreholes, suited to generally known conditions. Indicates soil types. Empirical formula for foundation design in sands and clays. Penetration affected by coarse-grained soils and cemented layers. Strength relationships also vary considerably in cohesive soils. Results in weak clays and silts can be suspect, particularly with the mechanical equipment. Accessory measurements include: (a) inclinometer to observe verticality, (b) temperature, e.g. beneath permafrost or cold stores, (c) acoustic for qualitative enhancement to differentiate soil types.
	Static-dynamic penetration tests	CPT procedures with dynamic sounding in dense layers or for extra penetration.	Investigating coarse soils and compact layers. Should be complemented by boreholes.

Location	Method	Technique	Applications and limitations
On land	**Independent tests**		
	Vane test	Direct penetration from surface and in boreholes or pit.	Undrained shear strength, for sensitive clays with cohesion up to 100 kN/m^2 Cross-check results, beware of silt or sand pockets and fibrous peat.
	Plate bearing tests	Incremental load/deformation test with plate encastré. Ensure plate unaffected by test load support.	*Bearing value of 'stoney' clays, weak and weathered rocks for foundation design.* Test by boring for softer deposits at depth.
	Load settlement test	Waste skip or metal tank incrementally loaded with settlement observations typically over 1–6 months period.	Immediate and short-term settlement data on fill or recent alluvium. Correlate with borehole data when extrapolating results.
On land or over water	Pile tests	Loading, pulling and lateral as required. (a) Maintained load method (ML) (b) Constant rate of penetration method (CRP) (c) Equilibrium load method (EL). Requires fairly even temperatures and leakproof ram. In all types of tests, end-load can be measured separately by load cell.	Pile design. Ratio of settlement in sands between individual test and group suggested by Skempton. ML method represents conventional technique. CRP method is very quick for load-carrying behaviour. EL method is compromise for determining load-carrying behaviour quickly. Load increment is applied and load system sealed so that as settlement occurs load decreases until equilibrium is reached.
Offshore marine	**Cable-operated equipment from vessel** (for wireline operation)		
	Penetrometers	Short-drive dynamic and static devices. Capacity 3–10 m approximately.	Intermittent empirical resistance diagrams. CPT (see above) very widely used. Results can be affected by the drilling operations.
	Pressuremeters	Operation may be below borehole casing, remote from seabed unit, or direct via seabed probe. Menard commonly used, but stressprobe has specially suited features.	*Used in most soil conditions,* excluding only the coarse types. For design of piles and spread foundations.
	Vane	Remote-controlled torque measuring device lowered to base of borehole, acting on short-drive vane rod.	Intermittent strength determinations in very weak cohesive soils, particularly where undisturbed sampling proves difficult.
	Logging	Seismic, nuclear, dip, calipers.	
	Seabed equipment		
	Remote-controlled static cone penetration test	Capacity typically about 30 m. Usually Dutch electric cone.	*Preferred method for determining mechanical properties of sands and consolidated clays.* Unit may sink into very weak seabeds. Very dense or coarse soils penetrated only a few metres.

Notes: (1) Test equipment should be regularly recalibrated and these results should be available on site.
(2) Seabed reaction frame may facilitate testing offshore with cable-operated equipment.

11.4.3.4 Model and prototype tests

It may be necessary to carry out full-scale trials in the field or model tests in the laboratory to check the parameters used in the preliminary analyses. For example, trial embankments may be constructed in the field on soft ground to check for stability or settlement, pile-loading tests carried out to measure shaft adhesion and/or end bearing, or compaction trials to test the suitability of fill.

Table 11.8 Geophysical methods on land

Method	*Technique*	*Applications and limitations*
Electrical resistivity	The form of flow of an induced electric current is affected by variations in ground resistivity, due mainly to the pore or crack water. Current is passed through an outer pair of electrodes whilst the potential drop is measured between the inner pair.	Simplest and least expensive form of geophysical survey. – Location of simple geological boundaries: *depth to bedrock beneath clay, and water bearing granular stratum over clay* (sub-surface saline bodies). – *'Expanding' electrode* method for changes in sequence with depth. Limited to 3 or 4 layers of similar thickness.
	Extension of direct measurements of porosity, saturation and permeability.	– *Constant separation* method for lateral delineation of boundaries, e.g. *location of faults, dykes, shafts and caverns.*
	Analysis is most often done by theoretical curve-fitting techniques.	Reliability affected by metal pipes, electrical conductors, complex and sloping strata, railway lines and power cables.
Seismic	The speed of propagation of an induced seismic impulse or wave is affected by the dynamic elastic properties and density of the ground. Impulse generated by falling weight and on open sites by explosive charges.	Most highly developed form of geophysical survey. Can be quite accurate under suitable conditions, particularly for horizontally layered structures. – Also for ground vibration problems.
	'Refraction' method with single shots concerns travel times of refracted waves which travel through sub-strata and are rebounded to the surface. Valid only when seismic velocities increase with depth. Short separate traverses used to check this.	*Determination of depth of bedrock beneath sands and gravel with low water table.* Variations laterally in rock, also buried channels and domes. – Direct evidence of seismic velocities in refracting strata. – For checking effectiveness of cement grouting of rock.
	'Reflection' method with single shots concerns the directly reflected impulses from horizons of abrupt increase in seismic velocity.	– Interpretation generally possible only for depths greater than is normally required for civil engineering on land.
Gravimetric	The Earth's natural gravitational field is affected by local variations in ground density. Measurements are made of differences between stations in the vertical component of the strength of gravity, which is then accurately corrected for latitude, height and topography to reflect only changes due to sub-surface geology. Precise topographical survey of exact station positions is necessary to obtain reliable results as differences are small.	– The interpretation of regional geology, without depth control, mainly where some geological information is already available. – For distinguishing local anomalies such as *buried rock faces in infilled quarries, large faults.* Also for positioning buried channels, large cavities and old shafts. – Fitting techniques based on simplified structures can be applied for studying anomalies.
Magnetic	Many rocks are weakly magnetic and the strength varies with the rock type depending upon the amount of ferromagnetic minerals present. This modifies the Earth's field. Surveys are similar to those for gravity measurements. Although the fieldwork is simpler, the interpretation is more difficult.	– For locating the hidden boundaries between different types of crystalline rock and positions of faults, ridges, dykes and *large ferrous ore bodies.* – *Field detectors for locating buried pipes and cables.*
Ground radar	An impulse radar system, representing the electromagnetic equivalent to echo-sounding. Variants including mapping and profiling techniques.	– For locating very shallow buried anomalies, such as disused hidden shafts and cavities, and simple shallow geological boundaries. – Unsuitable if clay topsoil or otherwise relatively high conducting surface layer present.
Borehole logging	The application of geophysical methods in boreholes.	Electrical and sonic methods to distinguish between strata especially where core recovery is difficult. See also Tables 11.7 and 11.10.

Notes: (1) All methods rely upon strong contrast in density, void space or resistivity across the boundaries to be identified. Transitional zones lead to uncertainty.
(2) Results should not be considered in isolation but correlated always with direct exposures, e.g. boreholes.
(3) Value of the results is very dependent upon use of the appropriate method or a combination of complementary methods, the specialist experience used in the analysis and the amount of geological knowledge that is available.
(4) Assessment of engineering parameters with geophysics is given in Table 11.10.

In the laboratory, dams, embankments and cuttings can be modelled and tested in a centrifuge and problems of permeability and seepage can be investigated in a flow tank. Model piles and footings can also be tested.

11.4.4 Ground investigations over water

Although in simple cases a land-type investigation is used with the additional facilities needed for access and the depth of water, as the difficulties with these two added complications increase so

Table 11.9 Over-water geophysics

Method	*Technique*	*Applications and limitations*
Echo-sounding	The times taken for a short pulse of high-frequency sound to travel from a source normally on the vessel's hull, vertically down and back to a detector via reflecting surfaces at and beneath the seabed.	– A *continuous water depth profile.* – Qualitative interpretation to limited depths of boundaries of higher density material beneath soft seabed deposits.
Side-scan sonar	Directional echo-sounding analogous to oblique aerial photography. Transducer source normally trailed at an elevation of 10–20% of maximum range.	– *Quantitative guide to position and shape of surface anomalies such as rock outcrops, wrecks and pipelines.* Qualitative guide to material on seabed.
Continuous seismic reflection profiling	The reflected wave trace of the seabed and underlying strata from a high rate of acoustic (sonar) impulses of short period for high resolution. *Pingers* are of frequencies typically 3–7 kHz penetrating a few tens of metres in soft silts, less in sands, few metres in stiff clay and none in compact gravel. *Boomers* 400 Hz–3 kHz penetrate over 100 m in soft sediments and only a few tens of metres in gravels and stiff clays. *Sparkers* 200–800 Hz of high resolution can penetrate more than 1 km in overburden or rock. *Air guns* are for much deeper penetrations. Optimum towing arrangements for sensors and detectors are adjusted to suit noise characteristics of survey vessel, its speed and the tides.	– *Valuable complementary aid to exploratory borings* for intermediate interpretation of stratigraphical horizons, location of *aggregate deposits, buried pipelines,* etc. within range of the impulses. Also applicable beneath rivers and lakes. Velocities of transmission obtained by seismic refraction for quantitative analysis. – *Type of acoustic source must be suited to local ground conditions.* Unable to distinguish between different formations with similar geophysical responses, e.g. coarse boulder clay and very weathered bedrock. – *Ineffective in water depths less than about 2 m.* Also noise of rough seas can cause signal losses.
Magnetic	See Table 11.8. Sensor trailed close to seabed and behind towing vessel, if iron about 2 ship lengths or more.	– Location of local buried structural changes with strong magnetic contrasting *dykes,* also *buried iron vessels* and pipelines.
Radioactive	Counter for detection is towed at or close to the seabed for location of geological anomalies. Alternatively *artificial isotopes can be introduced* for subsequent detection.	– Strong natural contrast between granite and basalt. Tracers indicate effluent dispersion and sediment mobility. Probes give estimate of seabed density.
Gravimetric	See Table 11.8. Seabed or ship-borne.	– Unlikely to be justified.

Note: Tidal corrections and good survey control essential, including, when appropriate, electronic navigation.

it becomes necessary to employ specially adapted techniques as given in Tables 11.3, 11.5, 11.7 and 11.9.

Selection of a suitable means for supporting the boring plant at the exploration positions is of fundamental importance in order that the work can be carried out safely and with minimal delays. It is generally not economical where craft are needed to provide a method that will permit working to continue during adverse weather and tidal conditions. Alongside jetties or river banks it is often possible to use a staging, but great care is essential to safeguard against overturning a cantilever platform, particularly during withdrawal of the casing. In protected waters, scaffold platforms or small pontoons are adequate. Larger pontoons or dumb barges usually suffice in more open conditions in the better weather or in estuaries near a safe haven, although the alternative of a jack-up pontoon platform may offer attractive advantages including overcoming difficulties due to tidal conditions. Offshore in the more exposed locations self-propelled craft, sometimes of special design and of sufficient size to remain stable are advisable. Adequate stability against the normal wave and tidal conditions with floating craft can be significantly improved by using ballast. At least four anchors suited to the seabed are essential to maintain position, preferably with an additional two in the direction of the tides or main current flow. In deep water over about 80 m special craft with computer-controlled thrust devices are used.

Separate facilities are also required in the form of an auxiliary vessel or helicopter for transporting personnel, materials and for visits to the boring-platform. Generally, an auxiliary vessel is also used for positioning dumb craft and to help with the time-consuming operation of handling and laying the anchors. Another use would be for sounding and geophysical surveys.

Position-fixing of the points or lines of exploration must be reliable and properly related to permanent stations. Sextants and theodolites are employed for simple cases very near land. Offshore electronic methods provide an accuracy of about 3 m with a range of up to 50 km. Seabed markers may also be employed.

Due consideration needs to be given to shipping, harbour and other regulations with respect to carrying out the investigation at the site and for permission to use the overwater facilities proposed.

Offshore investigations, say for oil production platforms,

Table 11.10 *In situ* testing and field instrumentation for earthworks, groundwater and other purposes

Nature of works	*Geology*	*Technique*		*Applications and limitations*
Earthworks, soil and rock slopes	Soils	*In situ* shear strength.	Normally undrained direct shear test.	Undrained *in situ* shear strength.
		Plate loading.	See static loading test below.	Refined cycled test for modulus or simple load test for bearing capacity.
		Sand replacement.	(Also water balloon device). Calibrate sand at natural humidity.	Bulk density during construction. Standard techniques unsuitable in coarse non-cohesive material: then use water replacement.
		Water replacement.	Water filled pit, lined with plastic.	
		Nuclear devices at surface.	Radioactive sources and counting unit.	Bulk density (preferably by attenuation method) and *in situ* moisture content.
		Nuclear density probe.	Usually back-scatter method with radioactive isotopes.	Bulk density measurements above and below water table, with casing if required.
		Proctor needle.	In earthwork construction.	Field control and consistency of fine-grained soils.
		In-situ CBR.	In earthwork construction and for roads.	Only appropriate in clay soils and subject to climatic changes.
		Piezometers.	High air value for partially saturated soils.	
		Total pressure cells.	Require very careful positioning.	Total earth pressure against sub-structures and within a soil mass.
		Settlement and heave instruments.	Types: Water, mercury, magnetic ring, buried plates, rods and notched tubes.	Total and relative settlement.
		Conventional survey methods.	Laser, photogrammetry.	Total and relative surface movement.
	Soils and rocks	Inclinometers and deflectometers. Extensometers.	Portable and installed.	Creep and slip detection. Expansion due to relief of stress and across tensile zones arising from differential settlement.
Groundwater permeability, etc.	Soils and rocks	Observation wells and piezometers.	Use effective filter, test regularly and seal from extraneous infiltration.	Level of water table, artesian and sub-artesian conditions.
		Rapid-response recorders.	Electric or pneumatic transducer type.	Tidal measurements, effect of surges, rainstorms and earthquakes.
	Clays and silts	Constant head seepage tests.	*In situ* measurement of permeability.	Coefficient of consolidation. (Certain advantages over laboratory tests.) Test is time-consuming. Groundwater must be at equilibrium at start of test.
	Sands and gravels	Pumping tests.	Pump to equilibrium conditions measuring transients during draw-down and recovery. Use at least two lines of observation wells.	Best form of test for natural permeability measurement. Transient measurements provide storage coefficient.
		Two-well pumping test.	Established technique.	Estimation of difference between horizontal and vertical permeability.
		Radioactive tracers.	Various	
		In situ permeability.	Careful shelling beforehand. Make both rising and falling head tests.	Local measurement of *in situ* permeability either through base of borehole or after placing coarse filter and withdrawing casing. Treat results with caution. A considerable number of tests are required to compensate for scatter.
		Infiltration above the water table.	Soakaway design.	
		Electrical resistivity.	Four electrodes. Wenner or Schlumberger configuration.	Extension of direct measurement of porosity, degree of saturation, and permeability.
	Rocks	Formation tests.	Expanding packers isolate zone under test.	Joint seepage and condition of joints by measuring flow under varying pressures, rising and falling.
Foundations for dynamic loads	Soils and rocks	Static loading test	Extra sensitive plate test cycled over expected stress range to give a modulus of reaction.	'Spring constant' for foundation design.
		Dynamic loading test.	Small vibrators mounted on soil to give resonance response.	Values of dynamic modulus. Poisson's ratio and damping.
		Seismic velocity measurements.	In various modes.	Dynamic and possibly static moduli for small strains.
Miscellaneous	Soils	Thermocouples and thermistors.		Ground temperature of coal tips on fire, beneath boilers and refrigeration plant.
		Electrical resistivity.	Four electrode system. Wenner configuration or two electrode probe.	'Apparent' resistivity for corrosion survey.
		Corrosion probe. (redox potential)	Short circuit current between reference cell and platinum electrode to earth.	Measure of oxygen in soil to assess microbial corrosivity.
		Stray current measurement.		For corrosive effect.
	Soils and rocks	Periscope calipers and borehole cameras, with video-tape recording.		Defining cavities, fractures, etc.
	Rocks	Noise detectors.	Considerable amplification required.	Incipient ground movement at faults, slopes, tunnels.

Note: Test equipment should be regularly recalibrated and these results should be available on site.

Table 11.11 Laboratory tests

Category	*Test*	*	*Remarks*
Identification and classification	Moisture content	FR	standard for all fine soils
	Atterberg limits	F	standard for all fine soils
	Particle size distribution	FC	standard for all coarse soils
	Particle density (specific gravity)	FR	used in conjunction with other tests
	Linear shrinkage and shrinkage limit	F	shrinkage/swell behaviour
	Saturation moisture content	R	
Compaction	*In-situ* density	FR	used with other tests particularly strength and deformation
	Compacted density-moisture content	FCR	standard, heavy and vibrating plate compaction tests for all fill materials
	Maximum and minimum density	C	used with *in-situ* density to indicate relative density
	Moisture condition value	FCR	suitability of fill for compaction. Adapted for chalk as the crushability test
Pavement design	California bearing	FCR	pavement thickness
	Frost heave test	FCR	susceptibility to frost heave
Permeability	Constant head	C	
	Variable head	FC	
	Triaxial consolidation	F	
	Rowe cell consolidation	F	
Erodability	Pinhole test	F	
Strength	Quick undrained triaxial compression	F	bearing capacity and short term stability. Variations include specimen sizes, single/multi-stage, quick/slow tests.
	Uniaxial compression	R	bearing capacity
	Shear vane	F	soft soils (not peats) bearing capacity
	Shear box	C	bearing capacity of recompacted soils.
		F	peak and residual effective strengths.
	Slow triaxial compression with/without pore-pressure measurements.	C	effective strength and pore pressure parameters for long term stability.
	Ring shear	F	residual strengths.
Deformation	Oedometer consolidation	F	drained modulus
	Triaxial consolidation	FR	drained modulus
	Rowe cell consolidation	F	drained modulus
	Cyclic undrained triaxial	FR	undrained modulus
Chemical corrosivity	Bacteriological content		Risk of attack on buried metals.
	Redox potential		Risk of attack on buried metals.
	Resistivity		Risk of attack on buried metals.
	Organic contents		Risk of attack on buried metals. / Risk of attack on buried concrete
	pH value		Risk of attack on buried metals. / Risk of attack on buried concrete
	Sulphate content		Risk of attack on buried concrete
	Carbonate content		Risk of attack on buried concrete
	Chloride content		Risk of attack on buried concrete
	Methane content		Health risk
	Full chemical analysis		Health risk

*Legend F = fine soils (clays and silts); C = coarse soils; R = soft rocks, mainly cohesive

Notes:

1. Every test specimen should have a complete soil rock description in order to assist interpretation of the test result.
2. Peat generally treated as fine soil

Table 11.12 Identification and description of soils

	Basic soil types	*Particle size* (mm)	*Visual identification*	*Particle nature and plasticity*[1]	*Composite soil types (mixtures of basic soil types)*
Very coarse soils	BOULDERS	200	Only seen complete in pits or exposures	Particle shape: Angular, Subangular, Subrounded, Rounded, Flat, Elongated	Scale of secondary constituents with coarse and very coarse soils. Term either before or after principal constituent (see below)
	COBBLES	60	Often difficult to recover from boreholes		
Coarse soils	GRAVELS	Coarse 20	Easily visible to naked eye; particle shape can be described; grading can be described		
		Medium 6			
		Fine 2			
	SANDS	Coarse 0.6	Visible to naked eye; very little or no cohesion when dry; grading can be described	Texture: Rough, Smooth, Polished	
		Medium 0.2			
		Fine 0.06			
Fine soils[2]	SILTS	Coarse 0.02	Only coarse silt barely visible to naked eye; exhibits little plasticity and marked dilatancy; slightly granular or silky to the touch. Disintegrates in water; lumps dry quickly; possesses cohesion but can be powdered easily between fingers.	Non-plastic or low plasticity	Scale of secondary constituents with fine soils. Term either before or after principal constituent. (see below)
		Medium 0.006			
		Fine 0.002			
	CLAYS		Dry lumps can be broken but not powdered between the fingers; they also disintegrate under water but more slowly than silt; smooth to the touch, exhibits plasticity but no dilatancy; sticks to the fingers and dries slowly; shrinks appreciably on drying, usually showing cracks. Intermediate and high plasticity clays show these properties to a moderate and high degree, respectively.	Intermediate plasticity (Lean clay); High plasticity (Fat clay)	
Organic	ORGANIC CLAY, SILT OR SAND	Varies	Contains substantial amounts of organic vegetable matter.	Examples of composite types (indicating preferred order for description)	
	PEATS	Varies	Predominantly plant remains usually dark brown, or black in colour, often with distinctive smell; low bulk density.	Loose, brown, subangular, very sandy, coarse GRAVEL with small pockets of soft grey clay. Firm, brown, thinly laminated SILT and CLAY. Dense, light brown, clayey, fine and medium SAND.	

Scale of secondary constituents with coarse and very coarse soils. Term either before or after principal constituent

Term before	Principal	Term after	Approximate % secondary
Slightly (sandy*)	SAND, GRAVELS, COBBLES OR BOULDERS	With a little (sand*) or occasional (cobbles†)	< 5
–(sandy*)		With some (sand*) or some (cobbles†)	5–20 +
Very (sandy*)		With much (sand*) or many (cobbles)	20–40 +
⌐		and (sand*) or and (cobbles†)	50 +

* Fine or coarse soil type as appropriate
† Very coarse soil type as appropriate
+ Or described as fine soil depending on mass behaviour

Scale of secondary constituents with fine soils. Term either before or after principal constituent.

Term before	Principal	Term after	Approximate % secondary
Slightly (sandy*)	CLAY OR SILT	With a little (sand*)	< 35
(Sandy*)		With some (sand*)	35–65
Very (sandy*)		With much (sand*)	> 65 +

* Coarse soil type as appropriate
Or described as coarse soil depending on mass behaviour

Notes: (1) Plasticity, compaction and strength can be defined in more detail, see text.
(2) Fine soils. Rarely exist as either silt or clay, but as mixtures, typified by intermediate behaviour. Such soils may be described as above, i.e. with an appropriate plasticity or in such terms as silty CLAY, very clayey, SILT, etc.

Compactness/strength		*Structure*				*Colour*
Term	*Field test*	*Term*	*Field identification*	*Interval scales*		
Loose	By inspection of voids and particle packing	Homo-geneous	Deposit consists essentially of one type	*Scale of bedding spacing*		Red Pink Yellow Brown Green Blue White Grey Black, etc. Supplemented as necessary with Light Dark Mottled, etc. and Pinkish Reddish Yellowish Brownish, etc.
Dense				*Term*	*Mean spacing* (mm)	
		Inter-stratified	Alternating layers of varying types or with bands or lenses of other materials. Interval scale for bedding; bedding spacing may be used	Very thickly bedded	Over 2000	
Loose	Can be excavated with a spade; 50 mm wooden peg can be easily driven.			Thickly bedded	2000–600	
		Hetero-geneous	A mixture of types	Medium bedded	600–200	
Dense	Requires pick for excavation, 50 mm wooden peg hard to drive			Thinly bedded	200–60	
		Weathered	Particles may be weakened and may show concentric layering	Very thinly bedded	60–20	
Slightly cemented	Visual examination; pick removes soil in lumps which can be abraded			Thickly laminated	20–6	
				Thinly laminated	Under 6	
Soft or loose*	Easily moulded or crushed in the fingers	Fissured	Break into polyhedral fragments along fissures. (Interval scale for spacing of discontuities may be used)	*Scale of spacing of other discontinuities*		
Firm or dense*	Can be moulded or crushed by strong pressure in the fingers			*Term*	*Mean spacing* (mm)	
Very soft*	Exudes between fingers when squeezed in hand	Intact	No fissures	Very widely spaced	over 2000	
Soft*	Moulded by light finger pressure	Homo-geneous	Deposit consists essentially of one type	Widely spaced	2000–600	
Firm*	Can be moulded by strong finger pressure			Medium spaced	600–200	
Stiff*	Cannot be moulded by fingers Can be indented by thumb.	Inter-stratified	Alternating layers of varying types. Interval scale for thickness of layers may be used	Closely spaced	200–60	
Very stiff*	Can be indented by thumb nail	Weathered	Usually has crumb or columnar structure	Very closely spaced	60–20	
* Mainly silt † Mainly clay				Extremely closely spaced	under 20	
Firm	Fibres already compressed together	Fibrous	Plant remains recognizable and retains some strength.			
Spongy	Very compressible and open structure					
Plastic	Can be moulded in hand, and smears fingers	Amor-phous	Recognizable plant remains absent			

have become a specialist activity for a limited number of organizations in view of the particular personnel and plant requirements. The relevant selected bibliography deals with the subject in some detail.

11.4.5 Personnel

Competent execution of an investigation not only requires selection of the appropriate methods and equipment but the use of properly trained and experienced persons to carry out the field and laboratory work. In charge of a major investigation there should be a suitably qualified senior engineer whose control should cover all aspects of the work. Throughout the investigation this person should meet the senior design engineer regularly.

Site work in the field should be fully supervised by a resident qualified geotechnical engineer or engineering geologist, who should describe all samples and prepare the logs. Operatives carrying out boring and testing should have been trained and should execute their work according to standardized procedures, to include full documentation of the results.

11.4.6 Contracts

Where specialist contractors are to be employed for the main investigation they should be selected on the basis of having adequate resources in the following respects: (1) specialist equipment for the field and laboratory work, with trained personnel for its operation; (2) experience in executing investigations of equivalent magnitude and to the required quality; and (3) professional staff for supervision and interpretation with knowledge and experience covering the range of techniques likely to be employed.

Although supervision may sometimes be supplied separately the intricacies of much ground investigation continues to demand an adequate level in the knowledge and experience of the contractor in order to execute the work competently and to produce reliable results.

There should be discussions before the contract is signed in order that the contractor may become acquainted with the results of the preliminary appreciation and have an opportunity to contribute from their experience on the formulation of an efficient and adequate programme for the field and laboratory work. They will visit the site except in the simplest of cases in order to plan their work.

Model conditions of contract and specifications exist (see bibliographies) that are specifically suited for ground investigations. Legal advice should always be sought if changes are contemplated in the wording of the clauses.

Flexibility should be an essential feature of the contract to deal with unexpected requirements and changes in the project. Moreover, a close working relationship should be formed between the client's engineering representative and the specialist geotechnical contractor for the provision of detailed and continuous communication to provide the best opportunity for a successful investigation. Ground investigation work awarded purely on the basis of the most competitive price for individual operations or the overall content has strict limitations in scope, may suffer in quality and could actually be misleading.

11.5 Description of soils and rocks

11.5.1 General

Soils and rocks need to be described in terms which readily convey their engineering properties. As these are largely physical properties they are first assessed from a visual inspection of samples or exposures using a few simple hand-tests. Samples or areas of exposures described as similar are grouped together as forming a geotechnical unit often with relatively uniform properties throughout, e.g. a stratum on a log. This grouping forms the basis for the selection of samples for laboratory testing and/or zones for *in situ* testing to obtain representative values of the engineering properties of each unit. A final assessment is then made taking into account the test results. For descriptions to fulfil their purposes they must be to a common system accepted by all concerned.

11.5.2 Systematic soil description

A universally accepted system does not exist at present so the following is presented as probably the most widely used. Non-organic soils are primarily described in terms of particle size, plasticity, compactness/strength, structure, and organic soils in terms of strength and structure (see Table 11.12). The particle size limits of the basic soil types are defined by various national standards which are all similar and broadly correspond to significant changes in the engineering properties. Soils are usually a wide-ranging mixture of particle sizes, with engineering properties largely controlled by the finer particles, particularly where they are clay-size and even where that constituent is a relatively minor percentage. The influence of the fines content varies widely with percentage, mineralogy, fabric and the particular engineering property under consideration.

For descriptions to be complete all the information indicated by the column headings in Table 11.12 should be recorded, either assessed visually, or as determined by the appropriate field and/or laboratory tests. Some typical descriptions are shown in the table embodying the results of particle-size distribution tests (examples of which are given in Chapter 9), standard penetration test (SPT) N-values, and shear strength tests (see Tables 11.13 and 11.14 respectively). These three tables are based on British practice.

The degree of plasticity of the amount passing 425 μm sieve fraction can be further described by liquid and plastic limits and

Table 11.13 State of compaction of coarse soils

State of compaction	*SPT* N *value* (blows per 300 mm penetration)*	*Approximate relative density (%)*
Very loose	0–4	0–15
Loose	4–10	15–35
Medium dense	10–30	35–65
Dense	30–50	65–85
Very dense	Over 50	85–100

* Standard penetration test N values may be corrected for overburden pressure, etc. in refined analyses.
SPT method using triggered hammer and free fall.

Table 11.14 Consistency of fine soils

Consistency	*Undrained shear strength range (kN/m^2)*	*Equivalent SPT* N *value (very approximate)*
Very soft	Under 20	Under 2
Soft	20–40	2–4
Firm	40–75	4–8
Stiff	75–150	8–15
Very stiff	Over 150	Over 15

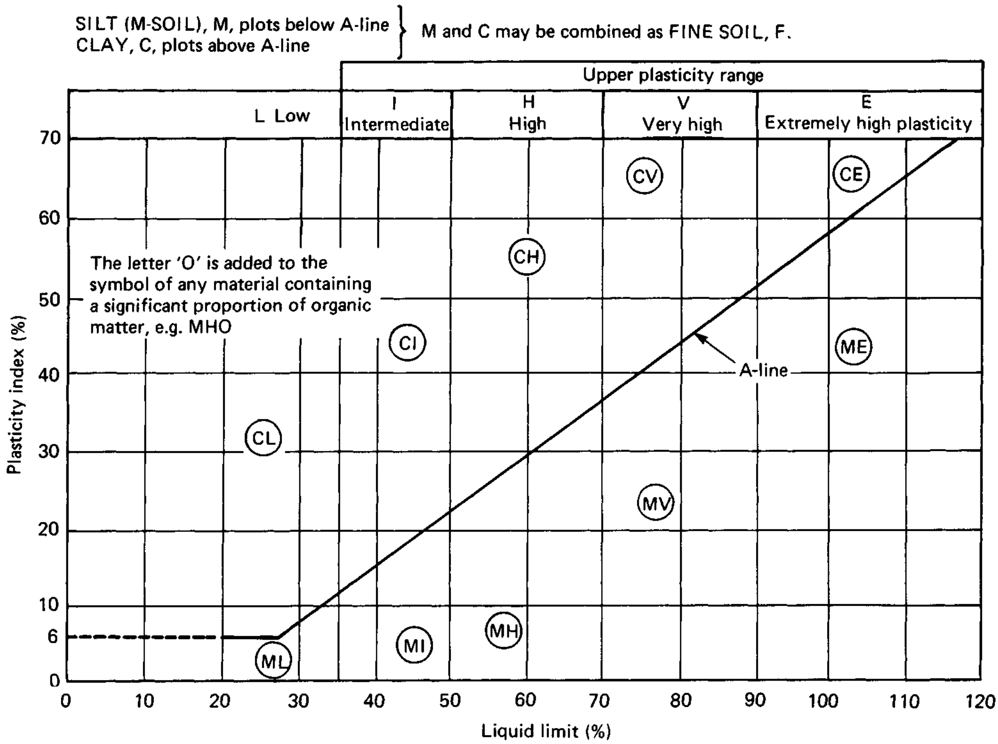


Figure 11.7 Plasticity chart for the classification of fine soils (measurements made on material passing a 425 μm BS sieve). (After BS 5930:1981.)

the plasticity chart (see Figure 11.7). Soils which plot below the A-line are predominantly silt and those above predominantly clay. High organic content moves the plots significantly to the right.

In the British Isles the principal non-organic soil types usually occur as siliceous coarse soils and silts and as alumino-siliceous clays, but worldwide many chemically and mineralogically different soils occur giving rise to particular mechanical and chemical characteristics (see Chapter 9). For such soils the standard description would need to be modified, e.g. for carbonate soils (and rocks) (see Table 11.15).

11.5.3 Classification of soils

Soils may be classified on any basis relevant to a particular project or problem, e.g. strength, aggressiveness to concrete, permeability, compaction. Probably the most widely used is the American Unified Soil Classification System (USCS) (see Figure 11.8). The British Soil Classification System for Engineering Purposes (BS 5930), of which Figure 11.7 is a part, is an expansion of the USCS system but as yet less widely used. Both classification systems assess soils objectively on the basis of specific tests, and therefore sometimes a classification may be at variance with a description which can be more subjective.

11.5.4 Systematic rock description

Rock description is generally more complex than soil description: not only is a description of the intact rock material required but a complete description of the rock mass *in situ* (i.e. structure and discontinuity pattern) is usually essential, together with the weathering state of the material and mass. The preferred scheme of description is set out in Table 11.16, together with definitions of terms used. Because weathering can be caused by mechanical and/or chemical agencies acting in varying degrees and ways on the intact rock and along discontinuities it has to date proved impossible to propose a single universally applicable scale of weathering, but this table includes a general scale which can be used. Rock colours should be in accordance with the scheme for soils.

Grain size can be assessed visually as for soils, but as grain size can imply a particular rock type, some guidance is given in Table 11.17. The texture refers to individual grains and their arrangement, the rock fabric. Structure is the inter-relationship of textural features. Sometimes texture and structure are implied in the rock name and therefore need not be described separately or structure may be better described as a minor lithological characteristic. Spacing of structural and lithological features should be described and oriented by direction and dip. Comments on degree of openness of discontinuities, irregularity of their surfaces and type of infilling should be included, as well as the size and shape of rock blocks.

Apart from the descriptive assessment, mechanical or fracture logging should be undertaken to complete a rock core or exposure log (see Table 11.16). For further comments on this topic see Chapter 9.

11.5.5 Boreholes and trial pit logs

Typical borehole and drillhole logs are shown in Figures 11.9 and 11.10 respectively, with standard legends in Figure 11.11 and a trial pit log in Figure 11.12. Apart from the soil and rock descriptions, each aims to present all factual data which may assist at design and construction stages. The log itself should take into account all disturbance caused by the ground exploration technique and sampling and thus be an interpretation of what that core or block of ground was and what the groundwater conditions were before exploration at that location.

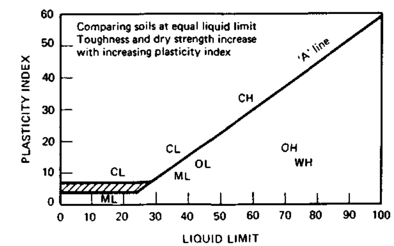
Table 11.15 Classification of carbonate soils and rocks (based on Middle East experience)

Degree of induration	Approximate unconfined compressive strength	Additional descriptive terms based on origin of constituent particles: Not discernible		BIOLASTIC (organic) OOLITE (inorganic)	SHELL (organic) CORAL (organic) ALGAL (organic) PISOLITES (inorganic)	
		← Increasing grain size of particulate deposits →				
		0.002 mm	0.060 mm	2 mm	60 mm	TOTAL CARBONATE CONTENT % (constituent particles plus matrix)
Non-indurated	Very soft to hard (<36 to >300 kN/m^2)	CARBONATE MUD	CARBONATE SILT	CARBONATE SAND	CARBONATE GRAVEL	90%
		Clayey CARBONATE MUD	Siliceous CARBONATE SILT[1]	Siliceous CARBONATE SAND[1]	Mixed carbonate and non-carbonate GRAVEL[2]	50%
		Calcareous CLAY	Calcareous SILT[1]	Calcareous silica SAND[1]		10%
		CLAY	SILT	silica SAND	GRAVEL	
Slightly indurated	Hard to moderately weak (0.3–12.5 MN/m^2)	CALCILUTITE (carb. clayst.)	CALCISILTITE (carb. siltst.)	CALCARENITE (carb. sandst.)	CALCIRUDITE (carb. conglom. or breccia)	90%
		Clayey CALCILUTITE	Siliceous CALCISILTITE	Siliceous CALCARENITE	Conglomeratic CALCIRUDITE[2]	50%
		Calcereous CLAYSTONE	Calcareous SILTSTONE	Calcareous SANDSTONE	Calcareous CONGLOMERATE	10%
		CLAYSTONE	SILTSTONE	SANDSTONE	CONGLOMERATE OR BRECCIA	
Moderately indurated	Moderately strong to strong (12.5–100 MN/m^2)	Fine-grained LIMESTONE		Detrital LIMESTONE	CONGLOMERATE LIMESTONE	90%
		Fine-grained Argillaceous LIMESTONE	Fine-grained Siliceous LIMESTONE	Siliceous detrital LIMESTONE	Conglomeratic LIMESTONE[2]	50%
		Calcareous CLAYSTONE	Calcareous SILTSTONE	Calcareous SANDSTONE	Calcareous CONGLOMERATE	10%
		CLAYSTONE	SILTSTONE	SANDSTONE	CONGLOMERATE OR BRECCIA	
Highly indurated	Strong to extremely strong (70 to >200 MN/m^2)	CRYSTALLINE LIMESTONE OR MARBLE (tends towards uniformity of grain size and loss of original texture)				50%
		Conventional metamorphic nomenclature applies in this section				

Notes: (1) Non-carbonate constituents are likely to be siliceous apart from local concentrations of minerals such as feldspar and mixed heavy minerals.
(2) In description the rough proportions of carbonate and non-carbonate constituents should be quoted and details of both the particle minerals and matrix minerals should be included.
(3) The preferred lithological nomenclature has been shown in block capitals; alternatives have been given in brackets and these may be substituted in description if the need arises.
(4) Calcareous is suggested as a general term to indicate the presence of unidentified carbonate. Where applicable, when mineral identification is possible calcareous referring to calcite or alternative adjectives such as dolomitic, aragonitic, sideritic etc. should be used.

UNIFIED SOIL CLASSIFICATION
INCLUDING IDENTIFICATION AND DESCRIPTION

Major divisions			FIELD IDENTIFICATION PROCEDURES (Excluding particles larger than 3 inches and basing fractions on estimated weights)			GROUP SYMBOLS	TYPICAL NAMES	INFORMATION REQUIRED FOR DESCRIBING SOILS	LABORATORY CLASSIFICATION CRITERIA	
COARSE GRAINED SOILS More than half of material is *larger* than No. 200 sieve size U (The No 200 sieve size is about the smallest particle visible to the naked eye)	GRAVELS More than half of coarse fraction is larger than No. 4 sieve size (For visual classifications the ¼ size may be used as equivalent to the No. 4 sieve size)	CLEAN GRAVELS (Little or no fines)	Wide range in grain size and substantial amounts of all intermediate particle sizes			GW	Well graded gravels, gravel-sand mixtures, little or no fines	Give typical name, indicate approximate percentages of sand and gravel, max size, angularity, surface condition, and hardness of the coarse grains; local or geological names and other pertinent descriptive information; and symbol in parentheses.	Use grain size curves in identifying the fractions as given under field identification. Determine percentages of gravel and sand from grain size curve. Depending on percentage of fines (fraction smaller than No. 200 sieve size) coarse grained soils are classified as follows:– Less than 5%: GW, GP, SW, SP; More than 12%: GM, GC, SM, SC; 5% to 12%: *Borderline* cases requiring use of dual symbols	$C_u = \frac{D_{60}}{D_{10}}$ Greater than 4 $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			Predominately one size or a range of sizes with some intermediate sizes missing			GP	Poorly graded gravels, gravel-sand mixtures, little or no fines			Not meeting all graduation requirements for GW
		GRAVELS WITH FINES (Appreciable amount of fines)	Non-plastic fines (for identification procedures see ML below)			GM	Silty gravels, poorly graded gravel-sand-silt mixtures			Atterberg limits below 'A' line or PI less than 4 — Above 'A' line with PI between 4 and 7 are *borderline* cases requiring use of dual symbols
			Plastic fines (for identification procedures see CL below)			GC	Clayey gravels, poorly graded gravel-sand-clay mixtures	For undisturbed soils add information on stratification, degree of compactness, cementation, moisture conditions and drainage characteristics.		Atterberg limits above 'A' line with PI greater than 7
	SANDS More than half of coarse fraction is smaller than No. 4 sieve size	CLEAN SANDS (Little or no fines)	Wide range in grain sizes and substantial amounts of all intermediate practicle sizes			SW	Well graded sands, gravelly sands, little or no fines			$C_u = \frac{D_{60}}{D_{10}}$ Greater than 6 $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			Predominately one size or a range of sizes with some intermediate sizes missing			SP	Poorly graded sands, gravelly sands, little or no lines.	EXAMPLE:–		Not meeting all gradation requirements for SW
		SANDS WITH FINES (Appreciable amount of fines)	Non-plastic fines (for identification procedures see ML below)			SM	Silty sands, poorly graded sand-silt mixtures	*Silty sand*, gravelly; about 20% hard, angular gravel particles ½ in. maximum size; rounded and subangular sand grains coarse to fine; about 15% non-plastic fines with low dry strength; well compacted and moist in place; alluvial sand; (SM)		Allerberg limits below 'A' line or PI less than 4 — Above 'A' line with PI between 4 and 7 are *borderline* cases requiring use of dual symbols
			Plastic fines (for identification procedures see CL below)			SC	Clayey sands, poorly graded sand-clay mixtures			Allerberg limits above 'A' line with PI greater than 7
FINE GRAINED SOILS More than half of material is *smaller* than No 200 sieve size	IDENTIFICATION PROCEDURES ON FRACTION SMALLER THAN No 40 SIEVE SIZE		DRY STRENGTH (CRUSHING CHARACTERISITICS)	DILATANCY (REACTION TO SHAKING)	TOUGHNESS (CONSISTENCY NEAR PLASTIC LIMIT)					
	SILTS AND CLAYS Liquid limit less than 50		None to slight	Quick to slow	None	ML	Inorganic silts and very fine sands, rock flour, silty or clayey fine sands with slight plasticity.	Give typical name; indicate degree and character of plasticity, amount and maximum size of coarse grains; colour in wet condition, odour if any, local or geological names, and other pertinent descriptive information, and symbol in parentheses.		
			Medium to high	None to very slow	Medium	CL	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays.			
			Slight to medium	Slow	Slight	OL	Organic silts and organic silt-clays of low plasticity.			
	SILTS AND CLAYS Liquid limit greater than 50		Slight to medium	Slow to none	Slight to medium	MH	Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts.	For undisturbed soils add information on structure, stratification, consistency in undisturbed and remolded states, moisture and drainage conditions.		
			High to very high	None	High	CH	Inorganic clays of high plasticity, fat clays.	EXAMPLE:–		
			Medium to high	None to very slow	Slight to medium	OH	Organic clays of medium to high plasticity.	*Clayey silt*, brown; slightly plastic; small percentage of fine sand; numerous vertical root holes; firm and dry in place; loess; (ML)		
HIGHLY ORGANIC SOILS			Readily identified by colour, odour, spongy feel and frequently by fibrous texture			Pt	Peat and other highly organic soils.			



PLASTICITY CHART

For laboratory classification of fine grained soils

Boundary classifications–Soils possessing characteristics of two groups are designed by combinations of group symbols. For example GW–GC, well graded gravel-sand mixture with clay binder.

All sieve sizes on this chart are US standard

Figure 11.8 Unified soil classification chart. (After *Earth Manual* (1974), US Department of Interior Bureau of Reclamation)

Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts.

Table 11.16 Definitions of terms used in description of rock

Description of cores and exposure carried out in general accordance with the recommendations set out in British Standards Institution Code of Practice for Site Investigations (BS 5930:1981). The description of weathered state is based on the Working Party Report on the Preparation of Maps and Plans in Terms of Engineering Geology in *Quarterly Journal of Engineering Geology*, **5**, 316–317, 1972.

Scheme of description
Colour/grain size*/texture and structure/weathered state/minor lithological characteristics/ROCK NAME/strength/other characteristics and properties.

* *Grain size often implied by the rock name.*

A. Definition of terms used to describe cores and exposures

Spacing of structural and lithological features

(Mean) spacing	Bedding plane spacing term*	Discontinuity spacing in one dimension
>2000 mm	Very thickly	Very widely
600–2000 mm	Thickly	Widely
200–600 mm	Medium	Moderately widely
60–200 mm	Thinly	Closely
20–60 mm	Very thinly	Very closely
6–20 mm	Thickly laminated (sedimentary and metamorphic rocks) Narrow (metamorphic rocks and igneous rocks)	Extremely closely
<6 mm	Thinly laminated (sedimentary and metamorphic rocks) Very narrow (metamorphic and igneous rocks)	Extremely closely

* *Bedding plane spacing describes thickness of lithological variations within rock. Frequency of 'bedding plane joints' described separately using discontinuity spacing terms.*

Weathered state

Term	Description
Residual soil	Rock is discoloured and completely changed to a soil. Original rock texture and structure is completely destroyed.
Completely weathered	Rock is discoloured and changed to a soil but original textures and structure is mainly preserved. There may be occasional corestones.
Highly weathered	Rock is discoloured; discontinuities may be open and stained. Rock texture and structure near discontinuities may be altered. Alteration may penetrate deeply, corestones are still present.
Moderately weathered	Rock is discoloured; discontinuities may be open and stained with alteration starting to penetrate inwards. Intact rock is noticeably weaker than fresh rock.
Slightly weathered	Rock may be slightly discoloured, particularly adjacent to discontinuities, which may be open and slightly stained. Intact rock is not noticeably weaker than fresh rock.
Fresh	Rock shows no discolouration, loss of strength or other weathering effects.

Additional term faintly weathered often used when staining is present but limited to surface of major discontinuities.
In moderately and highly weathered rocks, the ratio of original rock to weathered rock should be estimated where possible.

Strength classification

Term	Compressive strength (MN/m^2)	Field identification
Very weak	<1.25	Crumbles easily in hand
Weak	1.25–5	Thin slabs or edges broken by light hand pressure
Moderately weak	5–12.5	Thin slabs or edges broken by heavy hand pressure
Moderately strong	12.5–50	Broken by light hammer blows
Strong	50–100	Broken by heavy hammer blows
Very strong	100–200	Rock chipped only by heavy hammer blows
Extremely strong	>200	Rock rings on hammer blows

Table 11.16 *(continued)*

B. Additional terms used to describe rock cores

Fracture state (assessed natural fractures only unless otherwise specified)

Term	Definition	
Total core recovery (TCR%)	Percentage ratio of core recovered (both solid and non-intact) to the total length of core run.	
Solid core recovery (SCR%)	Percentage ratio of solid recovered to the total length of core run.	Solid core is here defined as pieces with at least one full diameter, but not necessarily with a full circumference measured along axis of the core.
Rock quality designation (RQD%)	Percentage ratio of total length of solid core pieces, each greater than or equal to 100 mm in length, to the total length of core run.	
Fracture spacing (I_s mm)	Average spacing of natural fractures over core lengths of reasonably uniform characteristics. Minimum/average/maximum of dimensions often quoted. The term non-intact (NI) is used when the core is recovered in a broken or fragmented state.	

C. Additional terms used to describe rock exposures

Size and shape of rock blocks

First term	Maximum dimension	Second term
Very large	>2000 mm	Blocky
Large	600–2000 mm	or
Medium	200–600 mm	Tabular
Small	60–200 mm	or
Very small	<60 mm	Columnar

In addition the following indices can be measured:	R_f (mm) the assessed dimension of individual blocks in geological units of reasonably uniform characteristics. I_f mm a linear measure of spacing of fractures usually along scan lines. (Similar to I_s as measured in cores.) Both indices are quoted as minimun/average/maximum dimensions.

Orientation of linear and planar features

Orientation described by direction of dip as three digit compass bearing relative to stated north point/two digit angle of dip from horizontal (e.g. 035°/15°).

Table 11.17 Aid to identification of rocks for engineering purposes

Grain size (mm)	*Bedded rocks (mostly sedimentary)*									
	Grain size description				At least 50% of grains are of carbonate				At least 50% of grains are of fine-grained volcanic rock	
20 6 2	RUDACEOUS		CONGLOMERATE Rounded boulders, cobbles and gravel cemented in a finer matrix Breccia Irregular rock fragments in a finer matrix				Calcirudite		Fragments of volcanic ejects in a finer matrix Rounded grains AGGLOMERATE Angular grains VOLCANIC BRECCIA	SALINE ROCKS Halite Anhydrite
0.6	ARENACEOUS	Coarse	SANDSTONE Angular or rounded grains, commonly cemented by clay, calcitic or iron minerals Quartzite Quartz grains and siliceous cement Arkose Many feldspar grains Greywacke Many rock chips			LIMESTONE and DOLOMITE (undifferentiated)	Calcarenite		Cemented volcanic ash TUFF	Gypsum
0.2		Medium								
0.06		Fine								
0.002	ARGILLACEOUS		MUDSTONE	SILTSTONE Mostly silt	Calcareous mudstone		Calcilu-tite	CHALK	Fine-grained TUFF	
			SHALE Fissile	CLAYSTONE Mostly clay			Calcilu-tite		Very fine-grained TUFF	
Amorphous or crypto-crystalline			Flint: occurs as bands of nodules in the Chalk Chert: occurs as nodules and beds in limestone and calcareous sandstone							COAL LIGNITE
			Granular cemented – except amorphous rocks							
			SILICEOUS				CALCAREOUS		SILICEOUS	CARBON-ACEOUS

SEDIMENTARY ROCKS

Granular cemented rocks vary greatly in strength, some sandstones are stronger than many igneous rocks. Bedding may not show in hand specimens and is best seen in outcrop. Only sedimentary rocks, and some metamorphic rocks derived from them, contain fossils.

Calcareous rocks contain calcite (calcium carbonate) which effervesces with dilute hydrochloric acid.

Grain size boundaries approximate

Igneous rocks: generally massive structure and crystalline texture

Grain size description (Increasing grain size ↑)				
	Pegmatite			Pyroxenite
COARSE	GRANITE[1]	Diorite[1,2]	GABBRO[1,2]	
MEDIUM	These rocks are sometimes porphyritic and are then described, for example, as porphyritic granite			Peridotite
FINE				
COARSE				
MEDIUM	Microgranite[1]	Microdiorite[1,2]	Dolerite[3,4]	
FINE	These rocks are sometimes porphyritic and are then described as porphyrites			
	RHYOLITE[4,5]	ANDESITE[4,5]	BASALT[4,5]	
	These rocks are sometimes porphyritic and are then described as porphyries			
	Obsidian[5]	Volcanic glass[6]		
	Pale ← Colour → Dark			
	ACID Much quartz	INTER-MEDIATE Some quartz	BASIC Little or no quartz	ULTRA BASIC

Metamorphic rocks

Foliated	Massive
GNEISS Well-developed but often widely spaced foliation sometimes with schistose bands. Migmatite Irregularly foliated; mixed schists and gneisses	MARBLE QUARTZITE Granulite HORNFELS Amphibolite Serpentinite
SCHIST Well-developed undulose foliation; generally much mica	
PHYLLITE Slightly undulose foliation; sometimes 'spotted'	
SLATE Well developed plane cleavage (foliation) Mylonite Found in fault zones, mainly in igneous and metamorphic areas	
CRYSTALLINE	
SILICEOUS	Mainly SILICEOUS

IGNEOUS ROCKS

Composed of closely interlocking mineral grains. Strong when fresh; not porous.

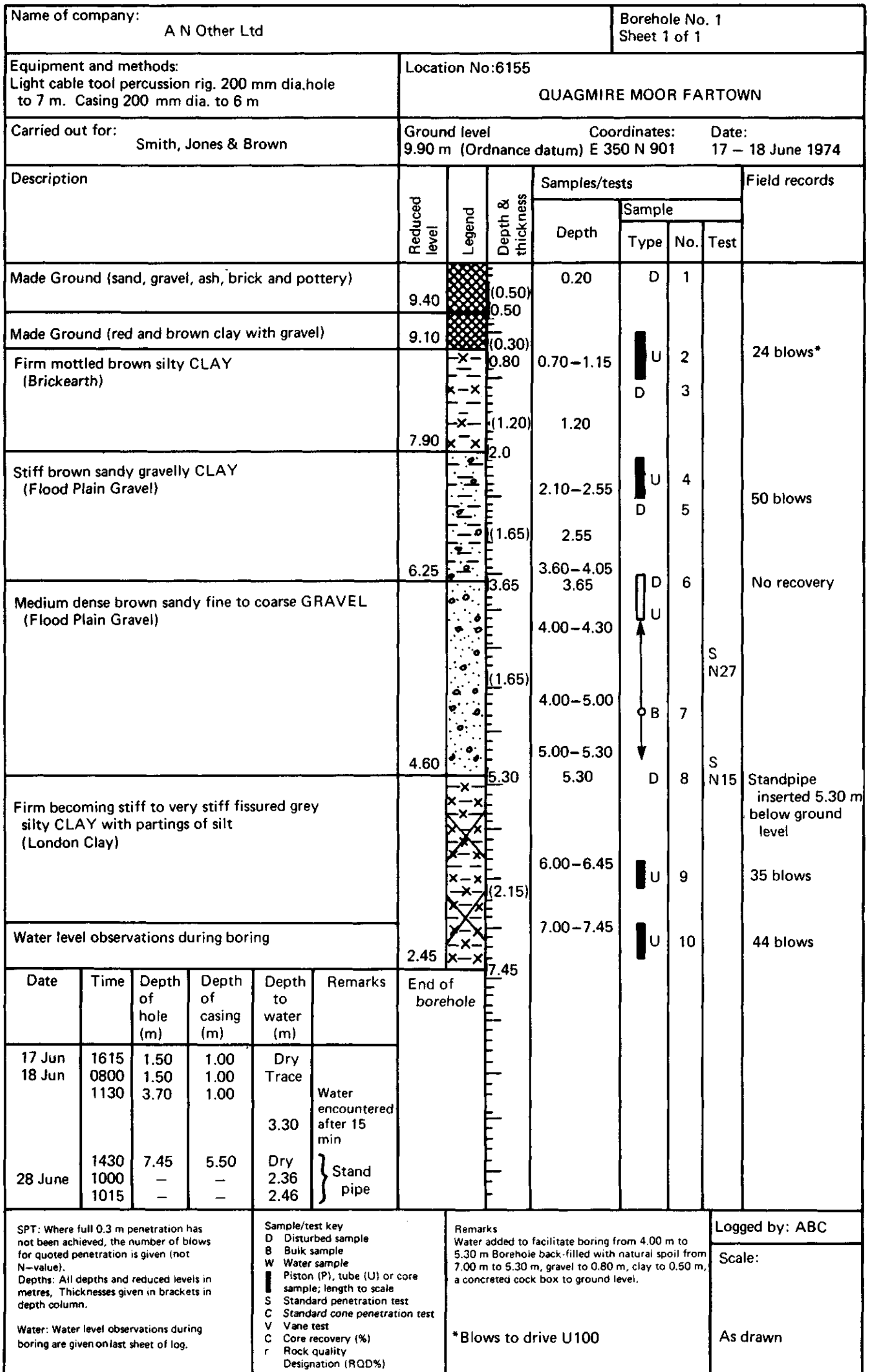
Mode of occurrence: (1) Batholiths; (2) Laccoliths; (3) Sills; (4) Dykes; (5) Lava flows; (6) Veins

METAMORPHIC ROCKS

Generally classified according to fabric and mineralogy rather than grain size.

Most metamorphic rocks are distinguished by foliation which may impart fissility. Foliation in gneisses is best observed in outcrop. Non-foliated metamorphics are difficult to recognize except by association.

Most fresh metamorphic rocks are strong although perhaps fissile.



Name of company: A N Other Ltd	Borehole No. 1 Sheet 1 of 1
Equipment and methods: Light cable tool percussion rig. 200 mm dia.hole to 7 m. Casing 200 mm dia. to 6 m	Location No:6155 QUAGMIRE MOOR FARTOWN
Carried out for: Smith, Jones & Brown	Ground level 9.90 m (Ordnance datum) Coordinates: E 350 N 901 Date: 17 – 18 June 1974

Description	Reduced level	Depth & thickness	Samples/tests: Depth	Sample Type	Sample No.	Test	Field records
Made Ground (sand, gravel, ash, brick and pottery)			0.20	D	1		
	9.40	(0.50) 0.50					
Made Ground (red and brown clay with gravel)	9.10	(0.30) 0.80					
Firm mottled brown silty CLAY (Brickearth)			0.70–1.15	U	2		24 blows*
				D	3		
		(1.20)	1.20				
	7.90	2.0					
Stiff brown sandy gravelly CLAY (Flood Plain Gravel)			2.10–2.55	U	4		50 blows
				D	5		
		(1.65)	2.55				
	6.25	3.65	3.60–4.05 3.65	D U	6		No recovery
Medium dense brown sandy fine to coarse GRAVEL (Flood Plain Gravel)			4.00–4.30			S N27	
		(1.65)	4.00–5.00	B	7		
			5.00–5.30				
	4.60	5.30	5.30	D	8	S N15	Standpipe inserted 5.30 m below ground level
Firm becoming stiff to very stiff fissured grey silty CLAY with partings of silt (London Clay)			6.00–6.45	U	9		35 blows
		(2.15)					
			7.00–7.45	U	10		44 blows
	2.45	7.45					
	End of borehole						

Water level observations during boring

Date	Time	Depth of hole (m)	Depth of casing (m)	Depth to water (m)	Remarks
17 Jun	1615	1.50	1.00	Dry	
18 Jun	0800	1.50	1.00	Trace	
	1130	3.70	1.00	3.30	Water encountered after 15 min
	1430	7.45	5.50	Dry	
28 June	1000	–	–	2.36	Stand pipe
	1015	–	–	2.46	Stand pipe

SPT: Where full 0.3 m penetration has not been achieved, the number of blows for quoted penetration is given (not N–value).
Depths: All depths and reduced levels in metres, Thicknesses given in brackets in depth column.
Water: Water level observations during boring are given on last sheet of log.

Sample/test key
D Disturbed sample
B Bulk sample
W Water sample
▌ Piston (P), tube (U) or core sample; length to scale
S Standard penetration test
C Standard cone penetration test
V Vane test
C Core recovery (%)
r Rock quality Designation (RQD%)

Remarks
Water added to facilitate boring from 4.00 m to 5.30 m Borehole back-filled with natural spoil from 7.00 m to 5.30 m, gravel to 0.80 m, clay to 0.50 m, a concreted cock box to ground level.

*Blows to drive U100

Logged by: ABC

Scale: As drawn

Figure 11.9 Typical log of data from a light cable percussion borehole. (After BS 5930:1981)

Name of company: A N Other Ltd	Borehole No. 14 Sheet 1 of 4
Equipment and methods: Rotary coring, water flush and with diamond bits. PWF bit to 8.8 m and HWF beyond.	Location No: 65117 LUKE STREET UPHILL
Carried out for: Smith, Jones & Brown	Ground level 125.3 mm O.D. / Coordinates: E 295 N 635 / Date: 8 – 18 March 1975

Main description	Detail	Reduced level	Legend	Depth & thickness	Samples/tests Depth	Sample Type	Sample No.	Test	Field records: Drilling & casing progress	Field records: Water recovery
							C	r		
Yellow brown clayey gravelly SAND (Glacial drift)		123.2		(2.1) 2.1	0–1.7		0	0		Flush return normal to 9 m, where it ceased. Normal flush restored when bore-hole cased to 8.8 m
Firm to stiff reddish brown sandy silty CLAY gravelly and with cobbles (Glacial drift)		122.1		(1.1) 3.4	1.7–4.6		93	0		
Black friable coaly SHALE (Glacial drift?)	Grey clay and mudstone at base	121.5		(0.6) 4.1						
Grey thinly bedded sandy MUDSTONE, moderately weak (Middle Coal Measures)	Thin bands of fine grained grey argillaceous sandstone and occasional ironstone nodules	119.7		(1.8) 5.6					8 Mar	
Light grey thinly bedded to medium bedded fresh fine grained SAND-STONE, moderately strong (Middle Coal Measures)	Cross bedded frequently fissured and with bands of sandy dark grey mudstone	115.6		(4.1) 9.7	4.6–7.2 7.2–8.8 8.8–10.3		93 100 93	17 32 25	9 Mar	
Grey friable CLAY - (old mine workings - Crank coal)	Some broken coal in mudstone									

SPT: Where full 0.3 m penetration has not been achieved, the number of blows for the quoted penetration is given (not N–value).

Depths: All depths and reduced levels in metres. Thickness given in brackets in depth column.

Water: Water level observations during boring are given on last sheet of log.

Sample/test key
- D Disturbed sample
- B Bulk sample
- W Water sample
- ▌ Piston (P), tube (U) or core sample; length to scale
- S Standard penetration test
- C Cone penetration test
- V Vane test
- C Core recovery (%)
- r Rock quality Designation (RQD%)

Remarks
Borehole cased to 19 m

Logged by: ABC

Scale: As drawn

Note: TCR, SCR, RQD and I_f recorded as required.

Figure 11.10 Typical log of data from a rotary drillhole. (After BS 5930:1981)

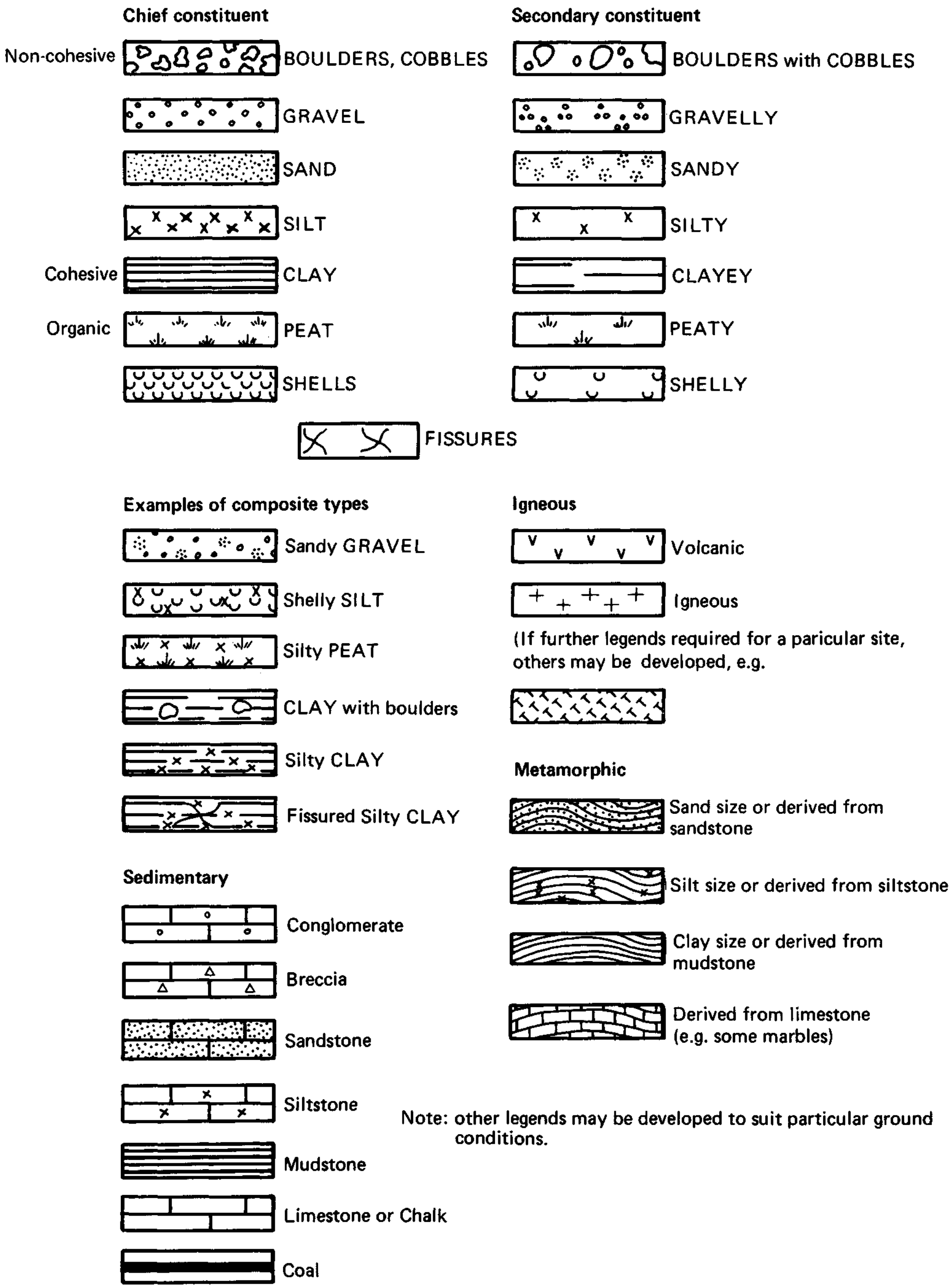
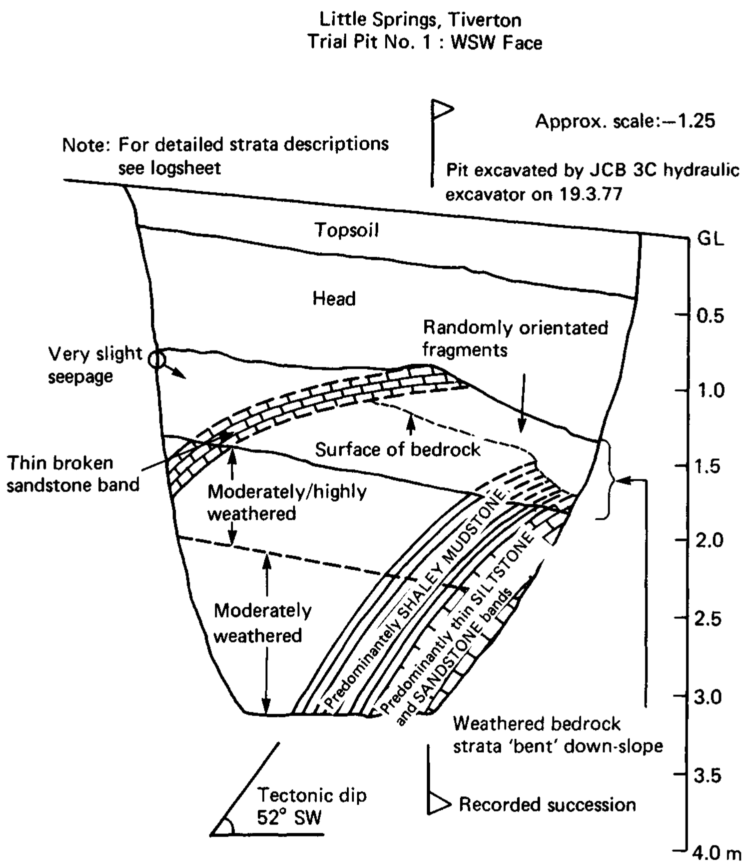


Figure 11.11 Standard legends



Trial pit no. 1 (GL:76.51 m AOD)

Depth (m)	Description
GL–0.30/0.40	Ashy TOPSOIL with a little brick gravel.
0.30/0.40 to 1.00/1.30	Firm to stiff becoming firm, red-brown becoming orange-brown silty CLAY with a little fine to coarse gravel sized fragments of various rock types including shaley mudstone, siltstone and subround quartz cobbles. (HEAD)
1.00/1.30–1.60/1.80	Stiff orange-brown and grey mottled shaley CLAY with some (25% increasing to 50%) gravel and angular cobbles and cobble sized blades of fine grained sandstone and shaley mudstone. Fragments random but striking 140/320 and dipping 37° SW at base. (COMPLETELY/HIGHLY WEATHERED BEDROCK)
1.60/1.80–2.40	Very weak dark grey, stongly stained orange brown, moderately to highly weathered thickly laminated shaley MUDSTONE and occasional very thin beds of fine grained sandstone. Degenerating to very stiff shaley clay in parts strike 115/295 and Dipping 56° SSW. (UPPER CARBONIFEROUS BEDROCK)
	–Transition–
2.40–3.10	Moderately weak to moderately strong, brownish-grey moderately weathered thinly interbedded shaley MUDSTONE and SILTSTONE. Bedding strikes 130/310° and dips 55° SW. (UPPER CARBONIFEROUS BEDROCK)

Notes:

(1) Pit orientated SSE–NNW and 3.5 m by 1 m in plan

(2) Very slight ground-water seepage at 1.3 m depth and southern end of pit and 'sweating' between approximately 1.5 m and 2.5 m depth. After 6 hours pit dry.

(3) Sides of pit shored and pit descended to examine strata

(4) Pit terminated in hard digging

(5) Recorded succession as shown in sketch.

(6) Hand Shear Vane Testing

At: 0.75 m 85 kN/m^2 1.50 m 120 kN/m^2
1.25 m 65 kN/m^2 2.00 m 115 kN/m^2

Could not penetrate pit sides below 2 m.

(7) Bulk disturbed samples recovered at 1.0 m, 1.7 m, and 2.3 m depth.

(8) Weather: fine and dry

Note on logging:
Where justified, each face may be shown separately and samples located on each face.

Figure 11.12 Typical log of data from a trial pit

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12

Reinforced and Prestressed Concrete Design

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Contents

12.1 Introduction

The design of reinforced and prestressed concrete has been increasingly codified during the past 40 years. Before the Second World War, recommendations for design had been published in the UK in a Code of Practice prepared by the Department of Scientific and Industrial Research, which was issued in 1934,[1] and in the Building By-laws of the London County Council of 1938.[2] After the war, the DSIR Code was revised and became the British Standard Code of Practice, CP 114, in 1948.[3]

The Institution of Structural Engineers published its First Report on Prestressed Concrete in 1951,[4] which gave design procedures for prestressed construction. This report was subsequently revised and issued as BS Code of Practice, CP 115, in 1959.[5] The BS Code of Practice for the Design of Precast Concrete, CP 116,[6] appeared in 1965 and supplemented the two earlier BS codes. By this time, a number of codes dealing with specialized forms of concrete construction were being prepared. Codes of Practice 114, 115 and 116 have been updated from time to time and are currently adopted as deemed-to-satisfy documents in the Building Regulations.

An important innovation took place in 1972 when a unified BS code, CP 110, for the structural use of concrete[7] was published. This code, which it was intended should supersede Codes 114, 115 and 116, introduced a new feature in design, namely limit state design in which account was taken directly of the possibility of failure or unserviceability occurring during the life of the structure being designed. The particular factors considered included the risks resulting from variability of the materials, inaccuracy in design assumptions and construction, variability of loading and the incidence of accidental damage. Whilst the approach to design was modified, many existing methods of analysis and calculation were retained. Provision was made for the incorporation of new data on loading and materials, and on structural performance and methods of construction as they became available. The basis for this approach had been developed by the European Committee for Concrete assisted by the International Federation for Prestressing who published a jointly prepared code[8] in 1978 having previously issued separate codes. This code was used in the production of a code for the European Economic Community.[9]

Code of Practice 110 did not replace the earlier Codes 114, 115 and 116, which still remain in force, but it has now been revised as BS 8110.[10] The approach adopted in CP 110 has been retained and the content has been brought up to date. In addition, a manual[11] has been prepared by the Institution of Structural Engineers conforming with its recommendations but presented in simpler form and dealing with a more limited range of construction. The guidance given in this chapter is related directly to the contents of BS 8110.

Whilst these developments were taking place in the UK, somewhat similar changes were occurring elsewhere. Some idea of the differences between the recommendations adopted in the UK and elsewhere are given in Table 12.1, which makes some comparisons between BS 8110, the American Building Code, ACI 318-83[12] and the EEC Code.

12.1.1 Definitions

This chapter is concerned with the basic approach to design of reinforced and prestressed concrete. It deals with both cast-in-place and precast concrete whether reinforced or prestressed. It includes information on the use of plain or deformed steel reinforcing bars and with tendons which may be either pretensioned or post-tensioned. In this context some definitions and an indication of limitations may be useful.

(1) Reinforcement which is used to provide the tensile component of internal forces in reinforced concrete, generally consists of one of three types of material: plain round mild-steel bar produced by hot-rolling; plain square or plain chamfered square twisted mild-steel bar which has had its yield stress raised by cold-working; ribbed bars, which may be hot-rolled from steel with high yield stress or cold-worked by twisting from hot-rolled mild-steel.

Since steel reinforcement can only develop an effective tensile force by extension of the concrete by cracking, there is a limit on the maximum strength of steel that can be used. In general the yield stress should not exceed 500 N/mm^2 although higher strength steels may be used if particular care is taken to avoid excessive cracking or deflection.

(2) Tendons are used to impart a prestress to concrete before service loads are applied which offsets the tensile stresses which will later result from the application of these loads. Tendons are usually comprised of plain, indented or deformed cold-drawn carbon steel wire, of seven-wire or nineteen-wire strand spun from one or two layers respectively of cold-drawn carbon steel wire around a core wire, or of high-tensile alloy steel bar. The strength of steel used must be high enough for it to be extended sufficiently to avoid excessive loss of tension due to elastic contraction, creep and shrinkage of the concrete. In general it is not of lower tensile strength than about 1000 N/mm^2.

(3) In prestressed concrete, prestressing may be effected by pretensioning or post-tensioning the tendons. Pretensioned tendons are stressed before the concrete is cast. They are stretched either between temporary anchorages placed sufficiently far apart for a number of moulds to be assembled in line around the tendons, i.e. the 'long-line' method, or between the ends of specially strong moulds, i.e. the 'individual' mould method; in each case, concrete is then cast and allowed to harden before the tendons are released from their temporary anchorages. The methods are best-suited to mass production in the factory and usually use wire or the smaller sizes of strand as tendons.

With post-tensioning, however, the tendons are stressed after the concrete has hardened and are usually accommodated in ducts within the concrete being held at their ends by anchorages, of which there are various proprietary types. Subsequently the ducts are grouted with cement grout to protect the tendons from corrosion. This method is mostly applied to site construction and tends to use tendons of relatively large size.

12.2 Behaviour of structural concrete

The characteristics of concrete that have conditioned its development as a structural material are its high compressive strength and relatively low tensile strength. In consequence its use for flexural members did not become practicable until it was discovered that steel reinforcement could be cast in the concrete to carry the bending tensile stresses whilst relying on the concrete to carry the bending compressive stresses. Experiment showed that mild steel, when present in the tension zone in relatively small amounts, provided a material with characteristics for deformation and strength which complemented those for concrete and provided a practical form of construction. Early research workers concluded that the presence of the steel increased the extensibility of the concrete. Later experiments showed, however, that this was not so. It then became clear that as the tensile stress in the steel of a beam increased beyond a small amount, which is appreciably less than that developed under service loading, cracks developed in the concrete. These cracks were controlled in width and numbers by the position of the reinforcement relative to the concrete surface and by the size

Table 12.1 Notes on different Codes of Practice (British, American and EEC)

(A) BS 8110 – *Structural use of concrete*[10]	(B) ACI 318 – *Building code requirements for reinforced concrete*[12]	(C) Eurocode No. 2 – 'Common unified rules for concrete structures'[9]
Status		
(A) This national code was prepared by the British Standards Institution, an organization with some direct support from Government, and accepted as providing conformity with British Building Regulations, but not in itself mandatory, other authenticated design procedures may be acceptable.	(B) This national code was prepared by the American Concrete Institute; it is used extensively in State regulations for building control. It is widely recognized internationally and is adopted in part or wholly in the codes of a number of other countries.	(C) The code has been prepared by the Commission of the European Community for use in member countries, and is one of a number now being produced to deal with all common materials and forms of construction. It is likely to be adopted for building control in those countries and will be recognized as satisfying the requirements of national regulations. The code has drawn on the work of international organizations, which are supported worldwide, and hence it is likely to have an important influence on the formulation and revision of codes in other countries outside as well as inside the Community.
Design procedure		
(A) Limit state procedures (described in this chapter) are adopted following closely the 1964 Recommendations of the European Committee for Concrete, which were used subsequently in developing the EEC Code; the basic approach in the two codes is therefore very similar. The ultimate limit states include strength and stability under dead and imposed loads, wind loads, and earth and water pressure for which partial safety factors are defined depending on load groupings, and the effects of accidental loading and damage. Durability and fire resistance are not treated as limit states but are included in the design process, the former being given more emphasis than in previous codes. As far as possible the analysis of structures is based on ultimate behaviour but, where methods have not been developed, elastic analysis is accepted. The strength of sections is based on the strength of the materials, as reduced by partial safety factors, and compatibility between stress and strain using idealized stress–strain relationships. Simplifying assumptions relating to these stress–strain relationships are allowable for many types of construction. The serviceability limit states include deflection and cracking under dead, imposed and wind loads, appropriate partial safety factors for combinations of loading being given with limitations on deflection and crack width. Where necessary, allowance is required for the effects of shrinkage and creep and of temperature change. For common types of construction, limits on deflections are imposed by placing limits on span:depth	(B) The objectives of the ACI Code are similar to those of (A) but the means of achieving them are different. Design requires consideration of ultimate strength but a single factor is defined for relating strength to the loads to be supported instead of adopting the combined effects of partial safety factors for both loading and strengths of materials, as in (A). The principles for calculating the strength of sections are otherwise similar in requiring compatibility between stress and strain. For flexure, the strength of concrete is defined in terms of 85% of the cylinder strength reducing as strength increases instead of 67% of the cube strength irrespective of strength, as in (A) and there are slight differences in the shape and extent of the stress–strain curve assumed; precautions are introduced to avoid brittle compression failures. Serviceability with respect to deflection is ensured either by limiting span:depth ratios or by checking that the long-term deflections do not exceed defined limiting values. Cracking is controlled by limitation of calculated crack width and provision of reinforcement for both reinforced and prestressed concrete. In (A), on the other hand, cracking of prestressed concrete may be controlled by limitation of tensile stress, the nominal tensile stress being related to amount and distribution of secondary reinforcement. The ACI Code has an appendix which gives an alternative method of design for reinforced concrete which is based on permissible stresses in the materials.	(C) Since the developments of the BS code and of the EEC code have drawn on a common source, the two codes, as already noted, have a common basis. However, the former was drafted by a committee with a British background in design and in the development of codes, whereas the latter has incorporated multi-national European experience and there are therefore a number of differences. Also in Britain, codes are generally regarded as advisory whereas in Europe they are mandatory. The main differences between the two codes are, however, in detail, the EEC Code tending to be more precise. Thus, the definition of limit states, the loads to be considered, and the strengths to be adopted with their relevant partial safety factors are closely similar. The EEC code is, however, based on cylinder strengths of concrete which gives rise to some differences when compared with the BS code. The simplified assumptions for calculation of flexural strength for each code, for example, show an apparent ratio of cylinder strength to cube strength of 0.89, which is appreciably higher than for most experimental data.

Table 12.1 Notes on different Codes of Practice (British, American and EEC)—*continued*

(A) BS 8110 – *Structural use of concrete*[10]	(B) ACI 318 – *Building code requirements for reinforced concrete*[12]	(C) Eurocode No. 2 – 'Common unified rules for concrete structures'[9]
ratios, and cracking may be controlled for reinforced concrete by defining the form that the reinforcement should take. Since many practical engineers have pressed for the retention of permissible stress methods of design, the previous code applicable for reinforced concrete[3] has been retained in use. It seems likely, however, that it will be withdrawn in the longer term.		

Concluding comment: Of necessity, these comparisons are very limited and superficial in character but should serve to show that developments in codes proceeding currently in different countries have much in common. This trend is likely to increase through the medium of the extensive international collaboration that now takes place.

of bars used. Thus with closely spaced bars near the surface, a large number of small cracks would develop, but with large widely spaced bars, the cracks would be fewer in number and much larger for the same stress in the steel. If the stress in the steel were increased the size of the cracks increased and their size was little influenced by the surface roughness of the steel, although at one time it was thought that roughening of the surface resulted in appreciably smaller cracks of larger numbers. It was eventually established that the main benefit of using bars with a roughened surface was in developing good end-anchorage.

Because steel needs to extend to develop stress and hence causes cracking and deformation of the concrete, there is a limit to the strength of steel that can be used efficiently for reinforcement, since unsightly cracking, which could lead to severe corrosion in adverse conditions and unacceptable deflections, must be avoided. The use of steel in prestressed concrete, where the stress in the steel is imposed before the concrete member is subjected to external load, avoids this problem, since the initial tensile force is developed without extending the concrete, and so no upper limit is imposed on the strength of steel that can be employed. This was not, however, appreciated in the early development of prestressed concrete. Then, steel of relatively low strength was used with a small initial tension. The experimenters found that, although this was effective at the start, the initial prestress disappeared with time. Eventually, however, it was established that this nonelastic behaviour was limited in extent and that if a sufficiently large elastic extension was imparted to the steel, the nonelastic effects of creep and shrinkage of the concrete did no more than reduce the prestress by an acceptable amount. Although for a time there was a tendency to underestimate the losses of prestress due to contraction of the concrete and to ignore creep in the steel tendons, research has now, however, clearly set the limits on what needs to be considered in design.

The performance of reinforced concrete and prestressed concrete beams under increasing load is characteristically different since cracking develops in different ways in each form of construction. This is illustrated by the results of tests on beams in each form of construction as illustrated in Figures 12.1 and 12.2.

Examined in more detail the deformation of the reinforced concrete beam under load is linear until cracking occurs; thereafter it approximates to a linear relationship until the steel yields as cracking becomes more extensive for beams of normal design. Subsequent deformation leads to the development of a hinge with continued yielding of the steel accompanied by damage to the concrete. This deformation continues at approximately constant moment until a stage is reached where the resistance reduces. The occurrence of this stage is influenced by the amount of transverse shear reinforcement in the section.

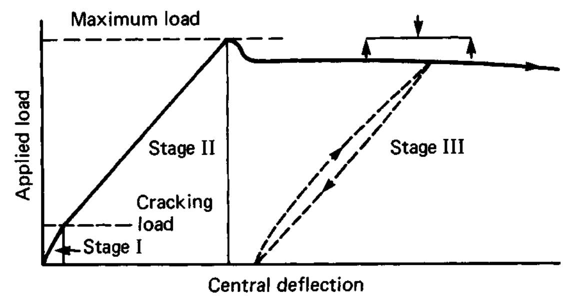


Figure 12.1 Relationship between applied load and deflection for a reinforced concrete beam showing recovery and reloading

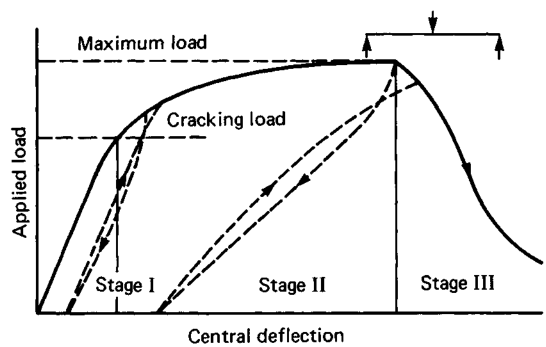


Figure 12.2 Relationship between applied load and deflection for a prestressed concrete beam showing recovery and reloading

The prestressed concrete beam, however, remains uncracked usually until the service load is exceeded, and in this range its deformation is elastic. Once cracking has occurred deformation increases disproportionately rapidly with increasing load as cracks widen until the maximum load is reached. Subsequently there is a rapid reduction in resistance. Since the prestressed concrete beam is usually uncracked under service conditions its stiffness is greater than that of reinforced concrete beams of the same overall depth.

In continuous construction subjected to applied loads of short duration, deformation of both reinforced concrete and prestressed concrete members is elastic or effectively elastic until service loads are exceeded. With further loading, as the applied moment at any section approaches the resistance moment at that section, there is a tendency for the moment to be relaxed and redistributed to sections that are less seriously stressed. Thus a loaded beam, built in at each end, may reach its maximum resistance moment at mid-span before the maximum resistance moments at the supports are attained; a hinge then forms at midspan with the applied moment there remaining sensibly constant whilst the applied moments at the supports increase until hinges form at the supports. The beam has then reached its maximum carrying capacity. The capability of reinforced and prestressed concrete beams for rotation at hinges is limited, however, and restrictions therefore need to be placed on allowances in design for redistribution of moment. These allowances are smaller for prestressed concrete sections than for reinforced concrete sections since their rotational capacities are smaller.

Under long-term loading, the deflection of reinforced concrete beams increases usually to about 2 or 3 times the initial deflection. Although the initial deflection is primarily influenced by the amount of steel in the section and its stress, the subsequent deflection is largely the result of creep of the concrete, breakdown of bond between the steel and the concrete in the tension zone between cracks which initially stiffens the beam, and the effect of the reinforcement in restraining the shrinkage of the concrete.

Since prestressed concrete is usually uncracked under long-term load the initial deflection is mainly due to the deformation of the concrete. The subsequent deflection results mainly from creep of the concrete and depends on the combined effects of the prestress and the stresses due to applied load. The former tend to deform the member in the opposite direction to the latter. In consequence, a loaded prestressed concrete member may initially have an upward deflection which can continue to develop upwards or downwards depending on how heavily it is loaded.

Under cyclic loading, reinforced concrete members usually fail in fatigue by fracture or yield of the reinforcement. The properties of most reinforcing steels, provided that they are free from welded connections, are, however, such that the ranges of stress experienced under service loading determined for static conditions are usually within the fatigue range. Cyclic loading leads to some increase in deflection of reinforced concrete members partly due to deformation of the concrete and partly due to breakdown of bond between cracks. Since prestressed concrete is uncracked under normal static service load conditions, the fluctuations of stress in the steel under cyclic loading are small. Fatigue failure of the steel only occurs when substantial cracks have developed and deflections are generally unacceptable. The effect of cyclic loading on prestressed concrete is to increase deflection by a small amount, i.e. 20 to 30% largely as a result of creep of the concrete. Large numbers of repetitions within the normal range of service loading do not reduce the ultimate strength of prestressed or reinforced concrete. Because of its freedom from cracking, prestressed concrete behaves better than reinforced concrete under severe cyclic loading and has therefore been used extensively for railway sleepers.

Resistance of beams to impact is indicated by the energy absorbed in deforming which is given by the area of the load deflection curves. Referring again to Figures 12.1 and 12.2, the deformation of prestressed and reinforced concrete beams has been defined in three stages. In stage I, deformation is elastic and largely recoverable; in stage II, deformation is in part elastic but accompanied by cracking and is partly recoverable; whilst in stage III, deformation is mainly due to permanent damage to the materials. Since stages I and II represent the largest amounts of absorbed energy for prestressed concrete, this material has a considerable capacity for recovery after impact. For reinforced concrete, the energy absorbed in stage III is substantially greater than in the other two stages. Thus, reinforced concrete does not show much recovery after impact but has a high ultimate impact resistance which is appreciably higher than that for prestressed beams designed for the same static loads. Prestressed concrete beams are, however, better in resisting repetitions of relatively light impacts with little residual damage.

So far, performance has been considered mainly in terms of bending conditions, but conditions of direct stress in compression exist in columns and walls. In such construction, unless high bending moments are also likely to occur, prestressed concrete would be unsuitable and reinforced concrete should be used with the steel acting in compression. For columns, transverse steel in the form of links is essential to contain the longitudinal steel and ensure ultimate resistance to strains in excess of those causing failure of plain concrete. Evidence from long-term tests also shows that the effect of creep of the concrete in a column under load is to raise the stress in the longitudinal steel to its yield stress and hence there is a need to retain it in its correct alignment. Walls when lightly reinforced are slightly weaker than walls without reinforcement and they can therefore only be treated as reinforced when the longitudinal reinforcement exceeds a specific minimum.

Other aspects of behaviour which are of importance are shear and torsion. In each case if these cause failure, the mode of failure tends to be brittle and less ductile than bending failures. Hence in design, the procedure is to avoid such failure by the inclusion of sufficient transverse reinforcement to ensure bending or compression failure in the event of severe overloading.

Members subjected solely to tension are relatively rare. If they are of reinforced concrete, then the role of the concrete is to protect the reinforcement which is designed to take the whole tensile force. In prestressed members, however, the precompressed concrete can sustain the tension until the load exceeds the cracking loading when the behaviour reverts to that of reinforced concrete with the steel carrying the whole of the tension, stiffened to some extent between cracks by the concrete.

For most building structures, the Building Regulations define fire resistance requirements, which are expressed in terms of a required endurance under service load when components are subjected to a standard heating regime. Both reinforced concrete and prestressed concrete are primarily influenced in their behaviour in fire by the behaviour of the steel at high temperature; as its temperature is raised its strength and yield characteristics are reduced. For reinforcing steels the rate of reduction in strength is lower than for steels used in tendons and hence greater amounts of protection are needed for prestressed concrete. This may take the form of concrete cover and the optional addition of insulating material. It is often easier, however, to provide the greater thicknesses of cover needed for tendons without loss of efficiency than that needed for reinforcement, since the positioning of tendons is governed by different requirements.

The need to provide adequate durability also affects the amount of cover required to the reinforcement or tendons. As concrete ages, carbon dioxide in the air causes carbonation of the concrete which, as it progresses, reduces its capacity for inhibiting rusting of the steel. For dense concrete the rate of progress is very low but, since defects exist, experience has shown that a greater thickness of concrete is required to prevent spalling of the concrete caused by expansion of the corrosion products on rusting. Cover requirements also affect the width of cracks that are likely to occur and hence need attention in dealing with serviceability.

These characteristics of the behaviour of both reinforced and prestressed concrete are considered in more detail in presenting design procedures.

12.3 Philosophy of design

The early developments of the design of reinforced concrete were crystallized in this country by the issue in 1934 of Recommendations for a Code of Practice[1] prepared by a committee set up by the Department of Scientific and Industrial Research. It was based on the premise that the stresses in the steel and concrete should not exceed certain permissible values, related to the strengths of the materials by safety factors, when the structure was subjected to the maximum loads that it would need to carry in service. The materials were assumed to behave elastically and compatability of strains between steel and concrete was ensured by assigning a value for the ratio of their moduli of elasticity. Some account was taken of the inelastic effects of creep of concrete by adopting a low value for the modulus of elasticity of concrete in determining the modular ratio for use in the design calculations. No account was taken of the effects of shrinkage and no estimate was made of the ultimate strength of the structure. When the British Standards Institution issued its first Code for Reinforced Concrete, CP 114,[3] in 1948, it followed the same general approach. In the revision in 1957, however, there was an alternative method for design in flexure which limited the stresses to the same permissible values as for elastic design but assumed that they were distributed as at failure and avoided the use of the modular ratio; this was therefore a form of ultimate strength design.

Limitations on the permissible stresses in the steel and on span:depth ratios were imposed to guard against excessive deflection or cracking. Thus it could be argued that CP 114 provided for safety against failure and for the avoidance of unserviceability.

The earliest formal presentation of a design procedure for prestressed concrete was contained in the First Report on Prestressed Concrete[4] published by the Institution of Structural Engineers in 1951. Many of the recommendations in that report found their way into the British Standard Code of Practice for Prestressed Concrete, CP 115,[5] issued in 1959. It conformed with CP 114 in the sense that it was based primarily on the limitation of stresses to permissible values related to the strengths of the materials with the object of preventing cracking and avoiding excessive deflection. It also provided for the calculation of ultimate strength and introduced separate requirements for minimum load factors for the dead and imposed loads.

Thus, when the drafting of CP 110[7] commenced in 1964 it had already been demonstrated that there were a number of limiting conditions or limit states which had to be considered by the designer in the overall conception of structural safety and adequacy. These were primarily limits of collapse, deformation and cracking, but other matters such as the effects of vibration, of fatigue, of deterioration with time or as a result of fire, needed attention in the design process.

A further major change in the content of structural codes first introduced in CP 110 in 1972 was the move towards considering the coordinated design of the structure as a whole for safety and serviceability rather than the separate design of its component parts with only limited appreciation of their interaction. This development has become necessary partly as a result of the evolution of design philosophy and partly because the utilization of the materials has become more onerous following the general increase in the levels of stress in both concrete and steel under service conditions.

12.3.1 Criteria for limit state design

The aim in limit state design is to codify the procedures normally adopted by engineers in the design of structures to provide safe, serviceable and economic construction with a reasonable degree of certainty, and to do this with a better appreciation of the margins of safety and of ignorance involved. As far as possible, it takes into account the variations likely to occur in the loads on the structure and in the strength of the materials of which it is comprised; it can allow for inadequacies of construction and methods of analysis, and should lead to design being more closely related to the risk of occurrence of specific conditions of failure and unserviceability.

For the purposes of design, both loads and strengths are expressed in terms of characteristic values. For loads, these are defined loads with a small but acceptable risk that they will be exceeded in service; they are given in the British Standard loadings for buildings,[13] in BS 5400 for highway bridges and in other standards for other construction. To meet the needs of limit state design, there has been a move in recent years away from specifying loads as maximum values and towards expressing them in terms of their likelihood of occurrence where possible determined from observations of their imposition on structures (see Chapter 19).

The characteristic values of loads allow for normally expected variations in loading but not for: (1) unforeseen loading effects; (2) lack of precision in design calculations; (3) inadequacies in the methods of analysis; and (4) dimensional errors in construction which alter the assumed positions or directions of loads and their effects, e.g. incorrect positioning of reinforcement and inaccurate alignment of columns in successive storeys. The values for loads used in design are therefore increased by partial safety factors to cater for these effects and to provide the margin of safety appropriate to the need for ensuring that a particular limit state is not reached. Thus, for conditions of failure, higher values are used than for those of serviceability. Where a combination of loads is assumed to be acting, the partial safety factors for each source of loading are smaller since the simultaneous occurrence of high values for each load is less likely. The loads for use in the design are therefore the sums of the products of the appropriate characteristic loads and their partial safety factors for the limit states and combination of loads being considered. For simplicity, the structural code for concrete, BS 8110,[10] reduces the number of situations needing consideration to a minimum, as will be seen later.

Characteristic values for the strengths of materials are usually given in the relevant standard or code. Research on materials shows that their strengths conform reasonably closely to a normal distribution, and their characteristic strengths can therefore be stated as follows:

Characteristic strength = mean strength $- k_1 \times$ standard deviation or:

$$f_k = f_m - k_1 \times \sigma_f \tag{12.1}$$

k_1 is usually given a value of 1.64, which ensures for a normal distribution that not more than 5% of strengths are less than the characteristic strength. This definition of strength has been adopted in British Standards for both steel and concrete.

The magnitude of the loads used in design is therefore increased by factors, partial safety factors for loads, to cater for these effects and to provide a margin of safety appropriate to the need for ensuring that any particular limit state is not reached. Thus, when envisaging conditions of failure, higher values for the factors are adopted than when considering serviceability.

The strengths of the materials used in the design calculations are those defined in the specification for the structure, which are checked by physical tests. The strengths of the materials as they exist in the structure, however, are likely to differ from those determined from test specimens and some allowance is also required for changes or deterioration with time. Partial safety factors for the materials are therefore introduced and the strengths taken for design are the characteristic strengths divided by a partial safety factor, γ_m, which has a value depending on the limit state being considered and the nature of the material, being less for steel than for concrete.

An idealized and simplified situation for a homogeneous material is illustrated in Figure 12.3. The provisions for safety outlined so far then require:

$$F_k \cdot \gamma_f \leqslant f_k/\gamma_m \tag{12.2}$$

where F_k = the characteristic load

This conforms reasonably closely with what has now become accepted practice in the recent revisions of British Standards codes, and was first adopted in CP 110 in 1972. Current thought, however, accepts the view that a further partial safety factor should be introduced to take account of the nature of the construction and its behaviour under overload conditions, e.g. whether it is capable of sustaining large deformations and so giving warning of the imminence of collapse, and of the seriousness of failure in terms of the risks to health, life and property. This factor, γ_c, might have a value of less than 1 for temporary construction not normally occupied by human beings but of more than 1 for buildings with large spans used for public assemblies. Thus design would then require:

$$F_k \cdot \gamma_f \cdot \gamma_c \leqslant f_k/\gamma_m \tag{12.3}$$

For an idealized situation, the global factor of safety relating characteristic loads to characteristic strength is then $\gamma_f \cdot \gamma_c \cdot \gamma_m$.

If the concept of relating the factors of safety to the nature of the construction is not followed, then the global factor is $\gamma_f \cdot \gamma_m$. Since reinforced concrete and prestressed concrete are composite materials, the value of the global factor for each limit state cannot be expressed as simply as this; it is dependent on the interaction between steel and concrete, each of which has a different value for γ_m. Also, γ_f cannot be given a single value for each limit state since the partial safety factors for dead, imposed, wind and other loads may differ and change with different combinations of loads. Hence, only upper and lower values for the global factor can be defined which makes comparison of the new Code with earlier or other codes imprecise. Nevertheless, in preparing the new Code, the aim has been to avoid substantial changes in the dimensions of the resulting structures whilst at the same time obtaining more consistent levels of safety and leaving room for development on more rational lines in the future.

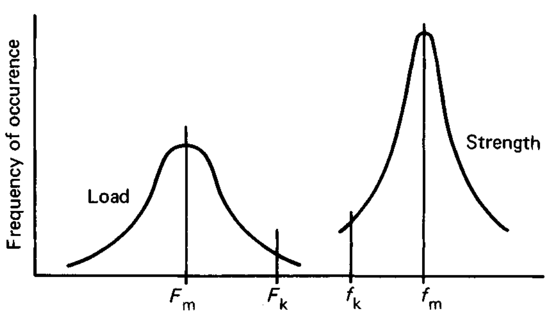


Figure 12.3 Idealized relationship between load and strength for a structure

It is convenient to divide the limit states to be considered in design into two kinds, namely those concerned with collapse and those concerned with serviceability. Limit states of collapse deal with overturning of the complete structure, failure of the whole or a large part of the structure as a result of overstressing of a number of sections or buckling of a number of compression members or as a result of a serious accident; the effects of fire and fatigue may also be included. Deflection, cracking, deterioration, corrosion and vibration are all aspects of serviceability and require limits of acceptability to be set for consideration. In the Code for Structural Concrete the limit states specifically dealt with are ultimate conditions in general, and deflection and cracking under the heading of serviceability. The criteria defining the serviceability limits are set out in Table 12.2.

The partial safety factors γ_f to be used with the characteristic loads for dead, imposed and wind loads obtained from CP 3, Chapter V, or other appropriate specification, are set out in Table 12.3 with notes on interpretation for ultimate and serviceability limit states. The combinations of loading to be taken are those which create the most severe conditions within the limits specified.

The partial safety factors for materials, γ_m, for the limit states considered are given in Table 12.4 also with notes on their interpretation.

The Code for Structural Concrete has special provisions to satisfy the requirement that, when a building suffers accidental damage, the amount of damage caused shall not be inconsistent with the original cause. It would seem reasonable to apply this same approach to other structures where safety and avoidance of excessive damage are necessary considerations in the event of accidents. To achieve this in buildings, attention should be given to the choice of an appropriate plan form since this may have a large influence on the mode of collapse as a result of an accident. When it is necessary to consider the effects of excessive loads outside those normally likely to be experienced or the residual strength of a structure after accidental damage the value of γ_f

Table 12.2 Limits for serviceability conditions

Limit state	*Reinforced concrete*	*Prestressed concrete*
Cracking	Controlled by detailing rules for sizing and spacing reinforcement (BS 8110, Part 1) For unusual structures or conditions, more specific recommendations are given in BS 8110, Part 2	Class 1: No flexural tensile stress Class 2: Flexural tensile stresses permitted but no cracking Class 3: Nominal flexural tensile stresses adopted to limit cracking to not more than 0.1 mm for severe exposures, e.g. sea water and otherwise not more than 0.2 mm (BS 8001, Part 1)
Deflection	Normally controlled by rules for span:depth ratio (BS 8110, Part 1). Exceptionally (BS 8110, Part 2), under vertical loads, not more than span/250: or for brittle finishes and partitions, not more than span/500 or 20 mm, or for nonbrittle materials, not more than span/350 or 20 mm	Normally controlled by limitations of stresses under service loadings for cracking considerations (BS 8001, Part 1) Exceptionally (BS 8001, Part 2), as for reinforced concrete but also applied to upward deflections

Table 12.3 Partial safety factors γ_f for loads and load effects

	Limit state design loads[a]		
	Ultimate	*Serviceability*	
Load combination			
Dead and imposed load	$\left\{\begin{matrix}1.4\,G_k+1.6\,Q_k\\1.0\,G_k\end{matrix}\right\}$	See note (1)	$\left\{\begin{matrix}1.0\,G_k+1.0\,Q_k\\1.0\,G_k\end{matrix}\right.$
Dead and wind load	$\left\{\begin{matrix}1.0\,G_k+1.4\,W_k\\1.4\,G_k+1.4\,W_k\end{matrix}\right\}$	See note (2)	$1.0\,G_k+1.0\,W_k$
Dead imposed and wind load	$1.2\,G_k+1.2\,Q_k+1.2\,W_k$		

[a]The figures given in the table are the values for the partial safety factors γ_f.

Notes:

(1) The minimum load for this combination should not be less than $1.0\,G_k$. When alternate spans are considered loaded in the design of continuous beams, for example, the loaded spans should be assumed to carry $1.4\,G_k+1.6\,Q_k$ and the 'unloaded' spans to carry $1.0\,G_k$.

(2) The most serious load condition will usually occur when the design dead load is taken as $1.0\,G_k$, but for certain cantilevered structures, for example, a more serious situation may exist when the design dead load for part of the structure is $1.4\,G_k$.

Table 12.4 Partial safety factors for materials, γ_m

Material	*Limit state values for* γ_m		
	Ultimate	*Serviceability*	
		Deflection	*Cracking*
Concrete[e] in bending and compression	1.5[a]	1.0[c]	1.3[d]
Steel	1.15[b]	1.0	1.0

[a]This value is related to the standards of workmanship and supervision advocated in the code for the production of concrete. If these standards are not applied, a higher value should be used. It relates primarily to compressive strength of concrete.

[b]This value is for reinforcement in tension or tendons. For reinforcement in compression it is increased to $1.15+f_y/2000$.

[c]Calculations of deflection are based on the characteristic strength of the material and therefore the modulus of elasticity of concrete derived for this strength is less than the mean value for the component or structure which strictly speaking would be more relevant. This slightly conservative approach is justified in the interests of simplicity.

[d]This higher value for γ_m is selected for all calculations of stress for class 2 prestressed concrete.

[e]For shear without reinforcement, γ_m should be 1.25 and for bond at the ultimate limit state should be 1.4.

can be taken as 1.05 for those loads likely to be experienced. In these circumstances also, the values for γ_m for steel and concrete may be taken as 1 and 1.3 respectively. The wind loading should be taken as one-third the characteristic wind load. These low values for the factors are acceptable because the loading considered will not be experienced by most buildings and it would therefore be uneconomic to design for it to be sustained without damage.

12.3.2 Characteristics of materials

The grades of concrete used for reinforced and prestressed concrete construction in the Structural Concrete Code are expressed as the characteristic strengths determined from 28-day tests on cubes; they are given in Table 12.5 with their application and properties relevant to design, including the increase in cube strength with age. No data are given for lightweight aggregate concrete since its properties are dependent on density in addition to strength as well as on the type of aggregate. The figures for flexural and indirect tensile strength refer to concretes made with smooth gravel aggregates; for crushed rock aggregates of tough texture, tensile strengths for the same grades of concrete would be somewhat higher. Generally, the minimum grade of concrete for reinforced concrete will be grade 25; there are, however, areas in Britain where the natural aggregates are not of high enough quality for concrete to meet this grade even though its cement content is sufficient to conform with requirements for durability. Unless there are special needs, grades stronger than grade 40 are unlikely to be used for reinforced concrete. When lightweight aggregate is used, a lower grade, grade 15, is acceptable for reinforced concrete but it is preferable to use a higher grade for the lightweight aggregates of higher strength. No upper limit needs to be set on the strength for prestressed concrete and higher grades than grade 60 may therefore be used, but only special circumstances would justify the much greater cost and need for control and supervision.

Calculations for conformity with ultimate and serviceability limit states require the strength and deformation characteristics for concrete to be defined in numerical terms. In particular, data are required on the relationships between stress and strain in compression under short-term loading and on creep and shrinkage when serviceability in the longer term is being considered. These aspects of behaviour are dealt with in section 12.4 and are simplified for design later in this section, but it must be recognized that there are substantial variations in the behaviour of concrete, depending on its constituent materials and environment, and that the values given for calculation should only be adopted if more reliable data are not available.

The strength properties of steel reinforcement and steel tendons are defined in British Standards which are summarized in Tables 12.6 and 12.7. For reinforcing bars of hot-rolled steel the characteristic strength is derived from the yield stress, but for cold-worked bars or wire reinforcement, it is derived from the 0.2% proof stress. The characteristic strength of tendons for prestressed concrete, however, is derived from their ultimate tensile strengths. In each case these are the relevant strengths for calculating ultimate strength for structural concrete members. Also in each case, the conformity with the specified characteristic strength is determined by ensuring that not more than two in forty consecutive results of tests made during the production of the steel falls below the specified value.

The design calculations for serviceability of structural concrete require information on the modulus of elasticity of steel.

Table 12.5 Grades and properties of structural concrete

Grade[a] *(characteristic strength 28 days)* (N/mm²)	*Cube strength*[a] (N/mm²) *at the age of:* 7 days	28 days	2 mths	3 mths	6 mths	1 year	*Flexural strength at 28 days* (N/mm²)	*Indirect tensile strength at 28 days* (N/mm²)	*Modulus*[a] *of elasticity at 28 days* (kN/mm²)	*Use*[a]
15	—	15	—	—	—	—		—	—	Reinforced concrete with lightweight aggregate
20	13.5	20	22	23	24	25	2.3	1.5	24	
25	16.5	25	27.5	29	30	31	2.7	1.8	25	Reinforced concrete with natural dense aggregates
30	20	30	33	35	36	37	3.1	2.1	26	Prestressed concrete for post-tensioning, $\nless 15$ N/mm² at transfer
40	28	40	44	45.5	47.5	50	3.7	2.5	30	Prestressed concrete for pretensioning, $\nless 15$ N/mm² at transfer
50	36	50	54	55.5	57.5	60	4.2	2.8	32	

[a]Recommendations in the Code of Practice for Structural Concrete.

The values adopted in the new code are: for reinforcement for all types of loading 200 kN/mm², and for short-term loading for wire and strand of small diameter 200 kN/mm² and for alloy bars and strand of large diameter 175 kN/mm².

In prestressed concrete, considerations of serviceability require allowance not only for the effects of creep and shrinkage of the concrete but also relaxation of the tendons which may modify the prestress conditions substantially. Appropriate requirements are incorporated in the standards which therefore provide guidance on values for relaxation to be used in design.

The stress–strain characteristics for concrete and steel may be needed for calculations of the deformation of structural members under short-term loading or for assessing ultimate strength. These are given in Figure 12.4 for concrete, in Figure 12.5 for reinforcement and in Figure 12.6 for tendons.

In interpreting these curves, the value of γ_m appropriate to the limit state being considered should be obtained from Table 12.4. The values for the modulus of elasticity given in these figures should not be used for estimating the required extension of tendons. These data should be obtained from stress–strain curves for actual material being stressed, which are supplied by the manufacturers.

The creep and shrinkage characteristics of concrete are considered in section 12.4. Where it is necessary to calculate long-term deformation, the effects of creep can be conveniently allowed for by adopting an effective modulus:

Table 12.6 British Standards for reinforcing bars for concrete

Type of steel[a]	*Specified characteristic strength*[b] (N/mm²)	*Elongation at fracture* (%)	*Diam. for 180° bend test* (no. of bar diam.)	*Upper limit for: carbon content* (%)	*sulphur content* (%)	*phosphorus content* (%)
BS 4449:1978 Hot-rolled steel bars for the reinforcement of concrete						
250 Grade	250	22	2	0.25	0.06	0.06
460 Grade	460	12	3	0.40	0.05	0.05
BS 4461:1978 Cold-worked steel bars for the reinforcement of concrete						
460 Grade	460	12	3	0.25	0.06	0.06
BS 4482:1969 Hard-drawn mild steel wire	485	—	Rebend test	0.25	0.06	0.06

Notes: [a]Preferred sizes: BS 4449, BS 4461 — 8, 10, 12, 16, 20, 25, 32 and 40 mm diameter, if a smaller size is required then 6 mm, if larger 50 mm diameter
BS 4482 – 5, 6, 7, 8, 10 and 12 mm diameter.
[b]The characteristic strength is the yield stress below which not more than 5% of results should fall

Table 12.7 British Standards for prestressing tendons for concrete

Type of steel	*Range of sizes available* (dia. in mm)	*Range of specified characteristic breaking load* (kN)	*Other information*
BS 5896: 1980 High-tensile steel wire and strand for the prestressing of concrete			
Cold-drawn steel wire in mill coils	3–5	12.2–30.8	Relaxation: 1000 h
			60% breaking load* 8%
			70% 10%
Stress relieved and may be crimped or indented and treated to reduce relaxation	4–7	21.0–64.3	Relaxation: 1000 h
			Class 1 2
			60% b.l. 4.5% 1.0%
			70% 8.0% 2.5%
			80% 12.0% 4.5%
Strand seven wire stress-relieved			Relaxation: 1000 h
			Class 1 2
standard	9.3–15.2	92–232	60% b.l. 4.5% 1.0%
super	8.0–15.7	70–265	70% 8.0% 2.5%
drawn	12.7–18.0	209–380	80% 12.0% 4.5%
BS 4757: 1971 Nineteen wire strand for the prestressing of concrete			
			Relaxation: 1000 h
as spun strand	25.4–31.8	659–979	60% b.l. 9%, 70% b.l. 14%
normal relaxation strand	18.0	370	60% 7.0%, 70% 12%
low-relaxation strand	18.0	370	60% 2.5%, 70% 3.5%
BS 4486: 1980 Hot-rolled and hot-rolled and processed high-tensile alloy steel bars for the prestressing of concrete			Relaxation: 1000 h all bars
hot-rolled	20–40	325–1300	60% b.l. 1.5%
hot-rolled and processed	20–32	385–990	70% 3.5%
			80% 6.0%
			*The breaking load for relaxation testing is the actual breaking load

Note: These values for relaxation at 1000 h apply in temperate climates and are those obtained at 20°C. When the prestressing steel is used at higher temperatures to prestress concrete, these percentage values should be increased. The increase may be as much as 2% for each 10°C increase in temperature.

$$E_{c.eff} = E_{ci}/(1 + \phi_c E_{ci}) \qquad (12.4)$$

where E_{ci} is the short-term modulus of elasticity of concrete and ϕ_c is the creep of concrete under a unit stress of 1 N/mm^2.

The effects of shrinkage may be treated by assuming that the concrete contracts without a change in stress except for that caused by the effect of the change in strain on the stress in the steel. Some readjustment of strains then becomes necessary to balance the forces in the cross-section by assuming that the concrete is stressed under this strain in proportion to the effective modulus.

12.4 Analytical and design procedures

12.4.1 Objectives

A recent trend in the approach to the initial design is to place much greater emphasis on the requirements for the durability of construction, since experience has shown that deterioration is a more serious cause of failure and of high maintenance costs than shortcomings in the structural calculations. It has therefore become more necessary to treat compliance with requirements for the quality of concrete, as placed in the construction and for

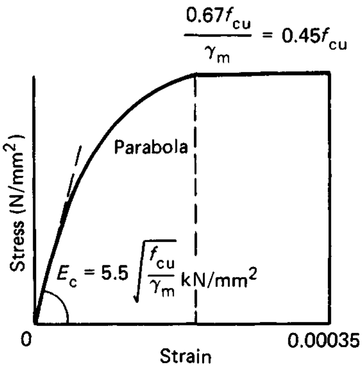


Figure 12.4 Short-term stress–strain curve for concrete

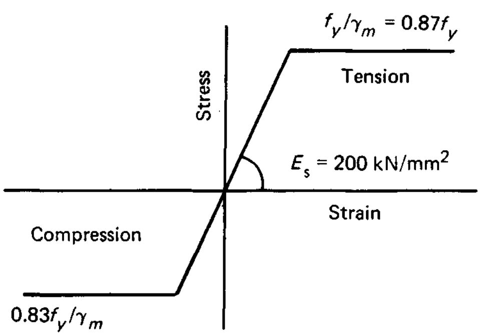
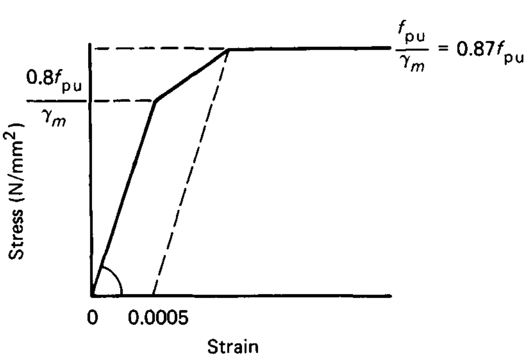


Figure 12.5 Short-term stress–strain curve for reinforcement



E_s=	205 kN/mm^2	for wire to BS 5896 section 2 Table 4.
	195 kN/mm^2	for strand to BS 5896 section 3.
	206 kN/mm^2	for rolled or rolled, stretched and tempered bars to BS 4486.
	165 kN/mm^2	for rolled and stretched bars to BS 4486

Figure 12.6 Short-term stress–strain curve for normal and low relaxation tendons

the protection of embedded steel by adequate concrete cover or other means, as being at least as important as compliance with requirements derived from design calculations. Whilst it is not practicable to define requirements for durability in terms of a limit state, it is nevertheless an aspect of the overall design process requiring primary attention.

For somewhat similar reasons, more care is now given to the requirements for fire resistance and information is presented in the Code (Part 2) which can be used for an analytical approach as an alternative to satisfying somewhat arbitrary requirements for concrete cover in order to obtain the necessary fire grading.

Inevitably, structural calculations continue to be a major part of design. Whilst the principles of limit state design require all possible limit states to be examined in the design of a particular structure, part of the purpose of the Code is to provide guidance on containing the effort required in design within reasonable limits without overlooking significant features, i.e. limit states. In doing this, the Code relies on the experience of the designer to ensure that the interpretation is sensible in each instance.

12.4.2 General assumptions

For most forms of concrete construction, with the possible exception of slabs, it is most convenient at the present time to base all design on elastic analysis of the structural system. The analysis would then apply directly to the serviceability limit states of deflection and cracking and, with some limited redistribution of moments and shear forces to the ultimate state. For slabs, other than one-way spanning slabs, it will usually be more satisfactory to use yield line methods or the strip method for ultimate design. For most construction it will usually be preferable to determine conformity with the serviceability limit states by using the arbitrary rules given in the Code for span:depth ratios and reinforcement detailing instead of calculating deflections and widths of cracks.

The Code recommends procedures in Part 1 for the detailed design of beams, solid slabs supported by beams or walls, flat slabs, columns, walls, staircases and bases, which are given at some length, and generally apply to both reinforced concrete and prestressed concrete. More information dealing with ultimate strength, serviceability and deformation due to creep, shrinkage and temperature effects is contained in Part 2. In this relatively brief summary, it is only possible to cover the more basic recommendations, and detailed design therefore requires reference to the main documents.

Other methods of analysis and design, where experimental procedures are used to develop the theoretical approach or to determine performance, are acceptable but will normally only be employed for specially complex structures or where repetition justifies more refinement than is obtained by established methods of calculation. The assessment of stresses in the region of load concentrations or of holes in continuous construction may be determined by photoelastic procedures. Model testing using special materials or scaled concrete has found applications in developing design methods, e.g. in the design of concrete box-girder bridges and pressure vessels for nuclear power stations. In precast concrete construction particularly, the behaviour of joints can only be established by tests on full-scale assemblies. It may also be economic to derive the dimensions of precast components for mass production by testing successively refined prototypes to obtain the final form; this approach applies particularly in dealing with the requirements for fire resistance. The interpretation of test data for design requires the special care of experienced engineers since tests cannot embrace all the loads and load effects that may need to be sustained and the circumstances that exist in actual structures cannot necessarily be fully reproduced experimentally. When test results are applied, therefore, there is a need to show convincingly the justification for departures from established practice, especially so, if these lead to less conservative design. If test data are applied in contexts for which they were not originally sought even more caution is necessary.

For the purpose of analysis, the Code offers three alternative methods for estimating beam and column stiffness: (1) the concrete section; (2) the gross section; and (3) the transformed section. The concrete section is the whole concrete section excluding the reinforcement, the gross section is the whole concrete section including the reinforcement allowing for the modular ratio, usually taken as 15, and the transformed section is the section of concrete in compression together with the reinforcement again allowing for the modular ratio. Generally, the concrete section is most convenient for use in design. For checking existing structures or for design in special circumstances, it would be more appropriate to use the transformed section for reinforced concrete; in construction where flexural cracking has occurred, however, the actual stiffnesses obtained by this assumption will be greater since the concrete exerts some tensile stiffening in the regions between cracks through bond with the reinforcement. The appropriate section for checking the design of existing or special prestressed concrete structures is the gross section since cracking does not usually occur with elastic deformation under service loads even for class-3 prestressed concrete.

12.4.3 Robustness

The Building Regulations require that all buildings of more than four storeys in height should be designed to resist accidental damage. The Regulations require that these buildings should be capable of sustaining removal of a structural member without excessive collapse resulting or should be able to withstand an internal pressure of 34 kN/m^2 without collapse.

The layout of the structure and its general form should not be sensitive to accidental damage whatever the cause. It is more realistic to interpret this as meaning that, in the event of an accident, the resulting damage should not be disproportionate to the magnitude of the cause. Where impact from vehicles is a possibility, buildings should be protected by barriers, such as bollards or earth banks. Greater margins should be allowed in design when the occupancy of a building may result in a greater than normal risk of accident, e.g. in flour mills and bonded stores.

Provisions envisaged in the Code go further in some respects in dealing with the effects of accidents than the Regulations require. The recommendations for robustness deal with both expected and accidental forms of loading, and include the following:

(1) All buildings should be so designed that all dead, imposed and wind loads are safely transmitted to the foundations.

(2) All buildings should be capable of withstanding a horizontal design ultimate load applied at roof and each floor level simultaneously corresponding to 1.5% of the dead-weight of the structure between the mid height of the storey below and mid height of the storey above for floors, and the surface for the roof. This, in effect, sets a lower limit for wind loading for the first two combinations of loading in Table 12.3.

(3) All buildings should be tied with effectively anchored and continuous reinforcement which is capable of withstanding the notional forces outlined in the following paragraphs. This reinforcement may consist of bars provided to resist stresses due to normal loads, which may be ignored for this purpose, and it may be assumed to be stressed up to its characteristic strength.

Buildings of four storeys or less require tying horizontally in two directions approximately at right angles with internal ties and peripheral ties.

Internal ties, which should be anchored to the peripheral ties and should be accommodated in the beams or slabs, should be capable of resisting a notional force of: $[(g_q + q_k)/7.5](\rho/5)F_t$ kN/m or $1F_t$ kN/m width, whichever is the greater, where $(g_q + q_k)$ is the sum of the average characteristic dead and imposed load in kilonewtons per square metre, and ρ is the greater of the distances in metres between the centres of supporting columns, frames or walls of any two adjacent floor spans parallel to the tie. F_t is the lesser of $(20 + 4n_0)$ or 60, n_0 being the number of storeys. The spacing of the ties should not be more than 1.5ρ.

Peripheral ties should be provided at each floor and roof level and be capable of withstanding a notional force of not less than $1F_t$ kN. They should be located within 1.2 m of the edge of the building.

Horizontal ties to external columns and walls should be provided for each external column, in two directions for corner columns, and for each metre length of external wall at each floor and at roof level. The notional force considered should be the greater of the following: $2F_t$ (or $(F_t/2.5) \times$ ceiling height m) kN, or 3% of the total design ultimate load carried by the column or wall at that level.

Buildings of five storeys or more require additional provision for robustness, which usually will be met by the inclusion of vertical ties in all walls and columns. These should be designed for a notional force corresponding to the maximum design ultimate dead and imposed load received by the column or wall from any one storey or roof.

(4) Where there are key elements in a building design, the failure of which might cause extensive collapse, their design should take their importance into account if their use cannot be avoided. Where vertical ties cannot be provided (see (3) above), provision should be made for bridging by the structure above in the event of their removal.

The purpose of these recommendations is to ensure that all structures are insensitive to damage from localized disturbances. It is therefore important in providing ties, for bridging or any other action, that the arrangements are sound engineering.

12.4.4 Beams and slabs

The effective span (l) of beams or slabs, which are simply supported, is taken as either the distance between the centres of bearings or the clear distance between supports plus the effective depth, whichever is the smaller. For continuous members, however, the effective span is the distance between the centres of the supports. Whilst for a cantilever which forms part of a continuous beam or slab, it is to the centre of the support, but for an isolated cantilever the effective span is to the centre of the support plus half the effective depth.

The effective width of a flange to a T-beam may be taken as the smaller of the width of the web plus one-fifth of the distance between points of zero moment or the actual width. Similarly, the effective width of flange for an L-beam is taken as the smaller of the width of the web plus one-tenth of the distance between points of zero moment; for continuous beams the distance between points of zero moment may be assumed to be $0.7\,L$.

The lateral stability of beams may need attention, usually by providing for adequate restraints and stiffness. The limits between lateral restraints for simply supported beams or continuous beams should not exceed $60b_c$, or $250b_c^2/d$, where d is the effective depth and b_c the breadth of the compression face midway between supports. For cantilevers restrained only at the support, its length should not exceed $25b_c$ or $100b_c^2d$.

The following loading conditions should usually be considered in the design of continuous beams and slabs: (1) the design ultimate load of $1.4G_k + 1.6Q_k$ on all spans; and (2) the design ultimate load as (1) on alternate spans with $1G_k$ on intermediate spans. When moments at sections are determined by elastic analysis, the maximum moment may be reduced by redistribution provided that the calculated depth of the neutral axis is not greater than $(\beta_b - 0.4)d$ where d is the effective depth and β_b is:

$$\frac{\text{moment at the section after redistribution}}{\text{moment at the section before redistribution}} \ngtr 1$$

and that the resistance moment at any section is not less than 70% of the moment at that section from elastic analysis.

12.4.5 Continuous and two-way solid slabs

Slabs which are continuous in extent in one or two directions may be designed as simply supported, provided that continuous ties that may be required for overall stability of the structure are incorporated in the construction. In such cases, cracking will develop in the top surface of the floors at their supports and some provision will be needed for dealing with this in applying floor finishes.

Where slabs are required to span in one direction over a number of supports, they should be designed for moments and shears, calculated in similar manner to those for continuous beams.

If solid slabs are required to span in two directions, yield line analysis or the strip method of design may be used. British Standard 8110, however, gives simple methods for the design of rectangular slabs for simply supported two-way panels and two-way continuous or restrained slabs.

12.4.6 Flat slab construction

Flat slab construction usually consists of a slab which spans between columns in two directions without supporting beams. Drops may be provided over the columns by increasing the depths of the slab and sometimes the column heads may be flared to reduce shear stresses. The slabs may be solid or ribbed in two directions.

British Standard 8110 offers a method of design but does not exclude the use of other methods such as finite element analysis or other procedures. In the BS method, it is assumed that the slab is supported by a rectangular grid of columns in which the ratio of the longer spans to the shorter spans is not greater than 2. The slabs are divided longitudinally and transversely into column strips and middle strips; the columns and column trips are designed as frames spanning in each direction. Each frame is then analysed elastically; a simplified method is given for the situation where the structure is braced against lateral loading and the column grid has a regular layout. Procedures are given for determining the widths of column strips and for the treatment of drops.

12.4.7 Frames

The loads to be adopted in the design of frames with their factors have already been given in Table 12.3. When considering the ultimate limit state, the forces, shears and moments calculated for design should be the worst combinations of loading regarded as feasible. British Standard 8110 gives some simplified procedures, which may be used for a number of common forms of construction. These analyse frameworks by breaking them down into subframes and make some provision for redistribution of moments. Two types of frame are dealt with – the no-sway frame, in which bracing, such as shear walls and lift or stair wells, are used to restrain sidesway, and sway frames, in which the frame itself provides the lateral restraint. For the latter, the amount of moment redistribution allowed is restricted with further restrictions on frames of four or more storeys in height to avoid excessive deflection and the possibility of frame instability.

12.4.8 Columns and walls

The determination of the loads and moments on columns is given in BS 8110 to which reference should be made for details. A column is described as slender when the ratio of the effective length to the corresponding breadth with respect to either axis is greater than 12 (10 for lightweight aggregate concrete); if the ratio is less than 12, the column is said to be short. The effective length is dependent on the length of the column and on the degree of restraint at the top and bottom connections with the structure. Generally, the slenderness ratio for a column should not be greater than 60. A distinction is made between braced and unbraced columns, a column being described as braced when the lateral stability of the whole structure is ensured by providing walls or bracing to resist all horizontal forces.

The procedures for dealing with walls in BS 8110 have much in common with those for columns. A concrete component is defined as a wall when the greater of the lateral dimensions is at least 4 times the smaller. For plain walls, however, the ratio may be less (since columns without reinforcement are not recognized) and reduction factors are then applied. To be described as a reinforced wall, the area of vertical reinforcement should not be less than 0.4% of the cross-sectional area of concrete; if the amount of reinforcement is less, the wall should be designed as a plain wall. Some reinforcement may be required in plain walls to control cracking. A stocky wall is one in which the ratio of effective length to thickness does not exceed 12 (10 for lightweight concrete), otherwise the wall should be treated as being slender. As for columns, the effective length is dependent on the height and conditions of end-restraint. Methods for calculating the loads and moments on walls (as for columns) are also given in some detail in BS 8110 to which reference should be made.

Provided the recommendations in the British Standards are followed, the deflections of columns and walls should not be excessive.

12.5 Reinforced concrete

12.5.1 General

In the design of reinforced concrete to meet the requirements of the Code, BS 8110, it will usually be most appropriate to consider the ultimate limit state first and then check the design against the requirements for cracking and deflection. This might be inappropriate in exceptional circumstances, e.g. where steels of characteristic strengths in excess of 500 N/mm² are being used or where spans were exceptionally long: in these cases cracking or deflection might govern design. In the sections that follow, design will be treated on the assumptions that normal conditions obtain. For these the Code gives simplified treatments for dealing with both cracking and deflection. It also gives methods more suited to the exceptional cases for which reference to the Code should be made.

12.5.2 Beams

12.5.2.1 Bending

Ultimate resistance in bending is calculated by assuming that:

The following loading conditions should usually be considered in the design of continuous beams and slabs: (1) the design ultimate load of $1.4G_k + 1.6Q_k$ on all spans; and (2) the design ultimate load as (1) on alternate spans with $1G_k$ on intermediate spans. When moments at sections are determined by elastic analysis, the maximum moment may be reduced by redistribution provided that the calculated depth of the neutral axis is not greater than $(\beta_b - 0.4)d$ where d is the effective depth and β_b is:

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Provided the recommendations in the British Standards are followed, the deflections of columns and walls should not be excessive.

12.5 Reinforced concrete

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12.5.2 Beams

12.5.2.1 Bending

Ultimate resistance in bending is calculated by assuming that:

(1) Sections which are plane before bending remain plane after bending.
(2) Stresses in the concrete may be determined using the stress–strain curve in Figure 12.4 (as assessed in the preparation of the design charts in Part 3 of the BS 8110), or may be taken as uniformly distributed across the most stressed 90% of the compression zone as indicated in Figure 12.7(a) with a value of $0.67f_{cu}/\gamma_m$, i.e. $0.45f_{cu}$ for deriving simplified formulae. Ultimate compressive strain in the concrete for analysis of sections is 0.0035.
(3) The strength of the concrete in tension is ignored.
(4) The stress in the steel is derived from the stress–strain relationships in Figure 12.5 with a value not greater than f_y/γ_m, i.e. $0.87f_y$ in tension and not greater than $0.83f_y/\gamma_m$ in compression, i.e. $0.72f_y$.

The simplified assumptions may be used to derive design formulae, which are shown in Figure 12.7(a–d). For beams reinforced in tension only:

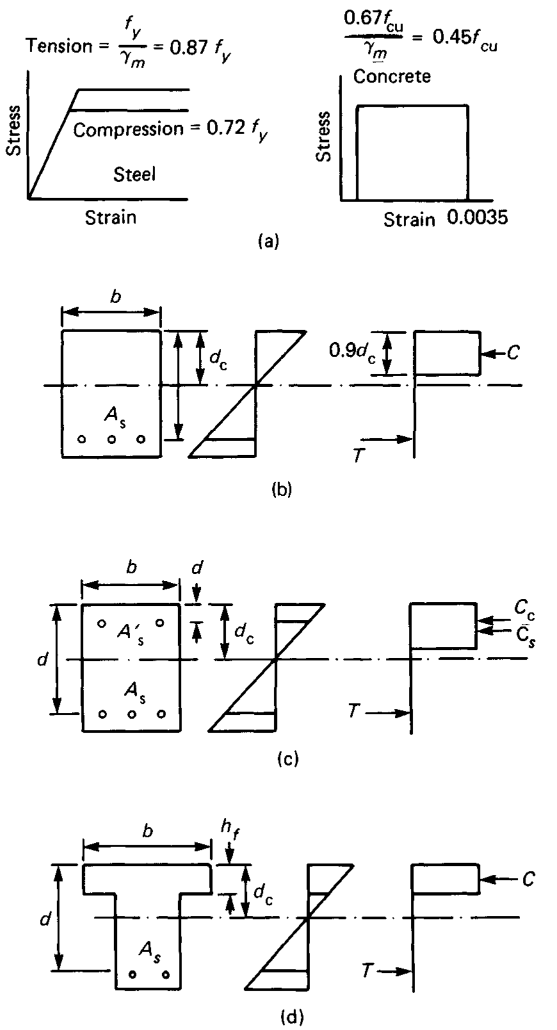


Figure 12.7 Flexural strength of beams – approximate methods. (a) stress–strain curves assumed; (b) beams reinforced in tension only; (c) beams reinforced in tension and compression; (d) flanged beams

$C = 0.4f_{cu}bd_c$ but not greater than $0.2f_{cu}bd$

T not greater than $0.87f_yA_s$ (12.5)

If d_c is not to exceed $0.5d$ as a practical limit, then:

$M_u = 0.87f_yA_sd(1-0.97f_yA_s/f_{cu}b_d)$

and not greater than $0.156f_{cu}bd^2$ (12.6)

For beams reinforced in tension and compression:

$C_c = 0.4f_{cu}bd_c$ but not greater than $0.2f_{cu}bd$ (12.7)

$C_s = 0.0035[(d_c-d_1)/d_c]A'_sE_s$ but not greater than $0.72A'_sf_y$ (12.8)

$T = 0.0035[(d-d_c)/d]A_sE_s$ but not greater than $0.87f_yA_s$ (12.9)

If d_c is not greater than $0.5d$ and d' is not greater than $0.5d_c$ where

$d_c = [(T-C_s)/C_c]d$ (12.10)

then:

$M_u = C_c(d-0.45d_c) + C_s(d-d_1)$ (12.11)

For flanged beams:

If $h_f < 0.9d_c < d/2$ then

$C = 0.45f_{cu}bh_f$ (12.12)

$T = 0.87f_yA_s$ (12.13)

$M_u = 0.87f_yA_s(d-h_f/2)$ but not greater than $0.45f_{cu}bh_f(d-h_f/2)$ (12.14)

provided that moment redistribution is restricted to not more than 10%. For full moment redistribution, considered earlier, either the more complex stress–strain relationships should be used in the calculations or, more readily, the Code design charts should be employed. This also applies when the form of section cannot be readily dealt with by the simple formulae.

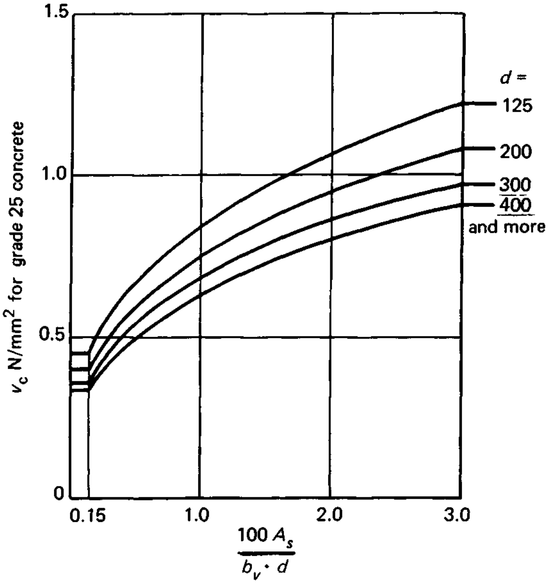
12.5.2.2 Shear

The resistance of beams in shear is calculated for the ultimate limit state. The procedure generally takes account of the contribution of the concrete as being additional to that of the shear reinforcement. The amount of shear reinforcement required is governed by the nature of the structural member and the level of shear stress in the concrete v in relation to the design shear strength of the concrete v_c. The shear stress, v, is given by

$v = V/(b_v \cdot d)$

where V = shear force due to ultimate loads, b_v = breadth of the section or the mean, breadth of the web for flanged beam, and d = effective depth (12.15)

The design shear strength of the concrete v_c is dependent on the strength of the concrete, the proportion of longitudinal reinforce-



For other grades of concrete multiply v_c for grade 25 concrete by: 1.06 for grade 30 concrete
1.12 for grade 35 concrete
1.17 for grade 40 concrete and stronger grades.

Figure 12.8 Design shear stress for concrete beams v_c

ment and the effective depth. Values for v_c for grade 25 concrete are given in Figure 12.8 with factors for determining v_c for other grades.

The situations considered in the Code are as follows:

(1) Where v is less than $\frac{1}{2}v_c$ and members are of no structural importance, no shear reinforcement is required. If the members are of structural importance, minimum shear reinforcement, as in (2) should be provided.
(2) Where v is greater than $\frac{1}{2}v_c$ but less than $v_c+0.4$, the area of reinforcement required A_{sv} should not be less than $(0.4b_v \cdot s_v)/0.87f_{yv}$

where s_v = spacing between links, and f_{yv} = characteristic strength of links $\not>$ 460 N/mm².

The links should be positioned throughout the length of the beam, spaced not further apart than in (3).

(3) Where v is greater than $v_c+0.4$ but less than $0.8\sqrt{f_{cu}}$ or 0.5 N/mm², whichever is the less (this limit is the limit for all beams), the amount of shear reinforcement required in the form of links is not less than:

$$A_{sv}=\frac{b_v v s_v(v-v_c)}{0.87f_{yv}} \tag{12.16}$$

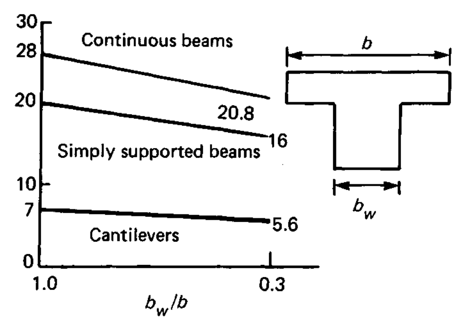
The spacing of the links longitudinally should not exceed 0.75d and transversely not more than d with no tensile reinforcement more than 150 mm from the vertical leg of a link. Alternatively, up to 50% of these links may be replaced by bent-up bars, which should be bent up at an angle of not less than 45° with a longitudinal spacing of not more than $1.5(d-d_1)$ reduced correspondingly if the angle is increased.

12.5.2.3 Deflection

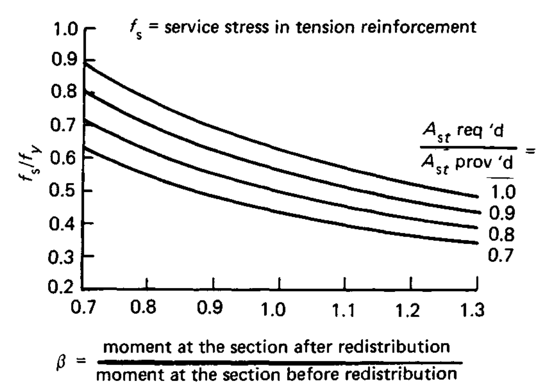
The accuracy of any calculation of deflection is dependent on the extent to which the conditions of loading are known both with respect to position and duration, and to which the assumptions made in design conform with the behaviour of the structure in reality. Apart from the dead load on the structure which may be known with reasonable accuracy, the imposed load that is actually applied may be unpredictable. The structure itself may have non-loadbearing components such as floor screeds and partitions which make a substantial contribution to its stiffness. The characteristics of the concrete may also not be known precisely since these are dependent on the different constituents actually used and provide additional uncertainty.

In most cases, therefore, it is not practical to calculate long-term deflections and this is recognized in the Code by giving a method of complying with the limit state requirements for deflection which take a number of features into account and define limits for span:depth ratios.

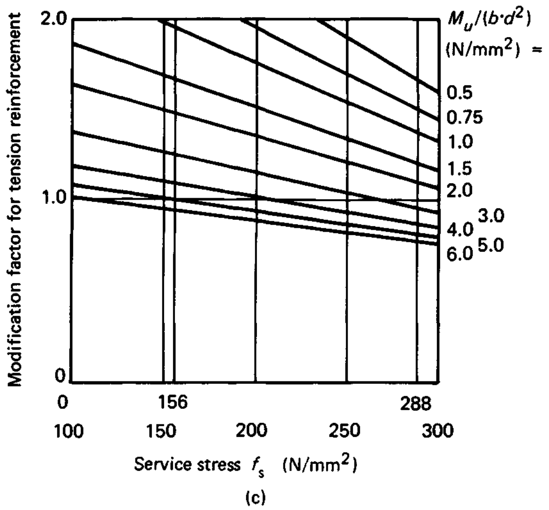
In defining these limits it is assumed that deflection of beams is primarily influenced by the conditions of support, the shape of the section, the proportions of tension and compression reinforcement in the section and their levels of stress under service loading. These features are dealt with by introducing modifying factors given with the basic span:depth ratios in Figure 12.9(a–d). To determine the limiting span:depth ratio for spans up to 10 m, a value for the ratio is obtained from (a), which is multiplied by the modification factor for tension reinforcement from (c) and, if appropriate, by the modification factor for compression reinforcement from (d). Figure 12.9(b) is used to derive the service stress in the steel required in the use of (c). The values given were developed for the Code to meet the requirement that the total deflection will not exceed span/250 and that the deflection after completion of finishes and partitions will not exceed span/350 or 20 mm, whichever is less.



(a)



(b)



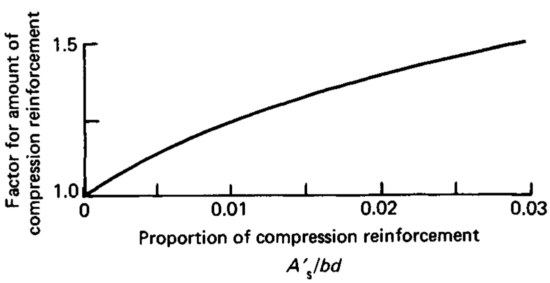


Figure 12.9 Factors for determining limiting span : depth ratios
(a) basic span: depth ratios;
(b) service stress for use in (c);
(c) modification factor for tension reinforcement;
(d) modification factor for compression reinforcement

For spans greater than 10 m where limitation of deflection is not necessary to avoid damage to finishes and partitions, the limiting span:depth ratios obtained above may still be used, but if such damage is not acceptable the limiting span:depth ratio should be reduced by multiplying by a factor 10/span. For a cantilever with a span greater than 10 m, the deflection should be calculated as indicated in Part 2 of the Code.

12.5.2.4 Control of cracking

The width of cracks at a particular location in a flexural member is dependent on a large number of parameters of which the following have been found by experimental investigations to be the most important:

(1) The distance from the nearest reinforcing bar spanning the crack.
(2) The distance from the neutral axis of the section.
(3) The mean strain at the level of the section considered.

These investigations, which showed that the surface characteristics of the bars have only a relatively small effect, have led to the derivation of the formulae recommended in Part 2 of BS 8110 for use in special circumstances when the calculation of crack width is necessary. For most construction, satisfaction of the requirements for the cracking limit state is provided by meeting the detailed needs for distribution of reinforcement in the concrete section with respect to location and spacing, which are dealt with later in sections 12.5.8 and 12.5.9.

In all construction, thorough moist curing of the concrete plays an important part in minimizing the extent of cracking due to drying shrinkage.

12.5.3 Slabs

The flexural strength of slab sections, including ribbed and flat slab sections, is treated in the same manner as for beam sections dealt with previously, design being based on derived moments. The moments, and also the shear forces, resulting from concentrated and distributed loads should be determined by elastic analysis or by yield-line or strip methods provided that these latter methods give a ratio of span-to-support moments similar to that obtained by elastic analysis. Rules are also given in the Code for the distribution of concentrated loads and for loading on slabs continuous over a number of bays when it is usually sufficient to assume that the most severe loading occurs with all spans fully loaded; this may not apply when cantilever spans are included. The design of two-way spanning slabs is covered in substantial detail with methods for calculating moments and shear forces and with requirements for the distribution of reinforcement between middle and edge strips and of reinforcement for resisting torsion at corners.

The shear stress in a solid slab should also be calculated as for a beam. The value of v for width b and an effective depth d is given by:

$$v = V/(b \cdot d) \tag{12.17}$$

should not exceed the lesser of 5 N/mm² or $\sqrt{f_{cu}}$ whatever shear reinforcement is provided. The recommendations for design shear stress for beams v_c shown in Figure 12.8 also apply for solid slabs and the following situations are considered:

(1) Where v is less than v_c, no shear reinforcement is required.
(2) Where v is greater than v_c, but less than $v_c + 0.4$, minimum links are required with a cross-sectional area of A_{sv} of $(0.46b \cdot s_v)/f_{yv}$ where s_v and f_{yv} are the spacing of the links and yield stress of steel as for beams.
(3) Where v is greater than $v_c + 0.4$, the amount of shear reinforcement required in the form of links is not less than

$$A_{sv} = [b \cdot s_v \cdot (v - v_c)]/0.87 f_{yv} \tag{12.18}$$

Alternatively, these links may be partly or completely replaced by bent-up bars. The spacing of links or bent-up bars need not be less than d.

Since it is difficult to bend and fix reinforcement for slabs with a depth of less than 200 mm, such slabs should be designed to avoid the need for shear reinforcement.

For most design, the deflection of solid slabs should be controlled by restrictions on span:depth ratio as for beams using the data in Figure 12.9(a–c). For two-way slabs, the span used in the calculations should be the shorter span.

Cracking is normally controlled by conforming with requirements for spacing reinforcement given on pages 12/19 to 12/21.

A convenient form of floor is the cast *in-situ* ribbed, hollow block or voided floor. The ribs may be connected by a structural topping of concrete of the same grade as that of the ribs or by a nonstructural topping not necessarily of the same grade. The ribs of floors with structural topping may be formed by solid or hollow blocks or formers, which can contribute to the structural strength provided that they are made of concrete or burnt clay (complying when appropriate with BS 3921) with a characteristic strength in the direction of compressive stress in the floor of 14 N/mm² or more. The spacing of *in-situ* ribs should not be greater than 1.5 m and their depth without topping should not be more than 4 times their width. The minimum thicknesses of structural topping required are related to the form of construction as follows:

(1) When the clear distance between ribs is not more than 0.5 m and permanent blocks are jointed with cement: sand mortar not leaner than 1:3 or weaker than 11 N/mm² – 25 mm.
(2) When the clear distance between ribs is not more than 0.5 m but the permanent blocks are not jointed with cement: sand mortar – 30 mm.
(3) All other slabs with permanent blocks – 40 mm or one-tenth of the clear distance between ribs whichever is more.
(4) All slabs without permanent blocks – 50 mm or one-tenth of the clear distance between ribs whichever is more.

If it is impracticable to provide sufficient reinforcement to develop the full support moment for continuous ribbed slabs, they may be designed as simply supported with not less than 25% of the mid-span reinforcement for the adjacent spans over the supports to restrict cracking; this reinforcement should extend for 15% of the span into the adjacent spans. When calculating the ultimate resistance moment of the section, the compressive stress in the blocks may be assumed to have a value of 0.3 times the specified characteristic strength. Design for shear follows that for solid slabs, the width of the section being taken as the width of the rib plus the thickness of the walls of hollow blocks or plus half the depth of the rib for solid blocks. The depth: span ratio of ribbed slabs should meet the requirements for the control of the deflection of beams; the thickness of the walls of hollow blocks may be added to the thickness of the ribs in making this check.

12.5.4 Columns

The Code draws particular attention to the need when commencing the design of columns to consider the dimensional requirements for cover for durability and for cover and minimum dimensions for fire resistance. Minimum amounts of reinforcement are given in Table 12.9.

Moments forces and shears in columns are derived by the analytical and design procedures considered earlier. For most construction with braced columns, i.e. where the structure is fully braced against lateral loading, the ratio of effective height to minimum breadth will not exceed 12 and the columns may be treated as short columns.

For short-braced axially loaded columns the ultimate load is derived from the assumptions made for beams and is given by:

$$N = 0.45 f_{cu} A_c + 0.72 A_{sc} f_y \qquad (12.19)$$

but to allow for inaccuracy in construction it is reduced by about 10% to give the relationship in the Code:

$$N = 0.4 f_{cu} A_c + 0.67 A_{sc} f_y \qquad (12.20)$$

where f_{cu} is the characteristic strength of concrete, f_y is the characteristic strength of steel in compression, A_c the area of concrete and A_{sc} the area of steel in compression.

If the braced short column has an approximately symmetrical arrangement (i.e. within 15% of span) of uniformly loaded beams, then loading may be treated as axial using a reduced value for N to deal with the small moments induced:

$$N = 0.35 f_{cu} A_c + 0.60 f_y A_{sc} \qquad (12.21)$$

This formula should not, however, be used for unsymmetrically loaded columns, e.g. corner columns.

When these simplified assumptions are applied to short columns subjected to combined axial loading and bending about one or two axes, the following recommendations are made in the Code for adjusting the moments for design:

$$\text{For } M_x/h,\ M_y/b, \quad M'_x = M_x + \beta(h/b)M_y \qquad (12.22)$$

$$\text{and for } M_y/b,\ M_x/h, \quad M'_y = M_y + \beta(b/h)M_x \qquad (12.23)$$

where M_x and M_y are the estimated design ultimate moments about the x and y axes respectively, and M'_x and M'_y are the corresponding ultimate design moments for use in the design calculations; h and b are the overall dimensions of the rectangular columns at right angles to the x and y axes respectively, and β is a factor with values given below in relation to the axial ultimate design load, N.

N/bhf_{cu}	0.2 or less	0.3	0.4	0.5	0.6	0.7 or more
β	0.90	0.65	0.53	0.40	0.28	0.15

The design of long columns receives extensive coverage and the details are not readily amenable to abbreviation. Two situations are, however, considered, namely braced and unbraced columns, and for each category effective lengths are defined in terms of the conditions of end-restraint and the corresponding additional moments for use in design are developed.

The deflection of short columns (and braced long columns) do not need to be checked since they will normally be within acceptable limits. Cracks are unlikely to occur in columns when the design ultimate axial load is greater than $0.2 f_{cu} \times$ the net cross-sectional area of the column: if bending predominates, cracking should be considered as for beams.

12.5.5 Walls

12.5.5.1 Reinforced concrete walls

A wall is usually defined as a vertical loadbearing member with a length exceeding 4 times its thickness. The method of design of of reinforced concrete walls is generally similar to that for columns, the treatment of stocky (effective length of 12 or less) and slender walls corresponding to that for short and long columns respectively.

Where a braced stocky wall cannot be subjected to significant moments, its ultimate load is given by:

$$N = 0.4 f_{cu} A_c + 0.67 A'_s f_y \qquad (12.24)$$

This is the same as the formula for columns and includes a reduction to allow for the effects of constructional tolerances.

If the spans on either side of a wall do not differ by more than 15% and are uniformly loaded, then it may be assumed that loading is axial and:

$$N = 0.35 f_{cu} A_c + 0.60 A'_s f_y \qquad (12.25)$$

12.5.5.2 Plain concrete walls

For stocky braced plain walls, the ultimate load per unit length, n_w is:

$$n_w = (h - 2e_x)_w f_{cu} \qquad (12.26)$$

where e_x is the resultant eccentricity of load at right angles to the plane of the wall, ^{w} is a coefficient with a value of 0.3 reduced by a factor varying linearly between 1.0 and 0.8 as the length reduces from 4 to 1 times its thickness.

Reinforcement may be needed in plain walls to control cracking due to flexure or drying shrinkage; it should not be less in each direction than 0.25% for 460-grade, nor 0.30% for 250-grade, steel.

12.5.6 Bond and anchorage

Bond and anchorage as distinct from cracking in reinforced concrete are affected substantially by the surface characteristics of the reinforcement. The BS Code recognizes three types of bar surface, i.e. plain round bars, type 1 deformed bars (which are usually twisted bars of square or chamfered square cross-section) and type 2 deformed bars which are usually of round cross-section with transverse ribs.

Where there are rapid changes in stress in the longitudinal direction over a short length or changes in the depth of the section, excessive local bond stresses at ultimate should be avoided by making sufficient provision for the anchorage of bars on each side of critical sections.

At the end of any bar, a sufficient length should be provided to anchor the tensile or compressive force in the bar. The length is found by dividing the force in the bar by the product of the ultimate average bond stress (f_{bu}) and the perimeter of the bar or group of bars; the perimeter of a group of bars is taken as that of a single bar of equal cross-sectional area. f_{bu} is assumed to be uniform along the bond length and it is obtained as follows:

$$f_{bu} = \beta\sqrt{f_{cu}} \qquad (12.27)$$

where β = the bond coefficient with the values given in Table 12.8.

Table 12.8 Values of the bond coefficient β

Bar type	*Bond coefficient* β *Tension*	*Compression*
Plain bars	0.28	0.35
Type 1 deformed bars	0.40	0.50
Type 2 deformed bars	0.50	0.63
Fabric	0.65	0.81

For beams where the minimum amount of link reinforcement for shear is not required, the values of β for plain bars should be used for deformed bars, too. This restriction does not apply to slabs.

For hooks conforming with BS 4466, the anchorage provided should be the smaller length of 24 times the bar size or 8 times the internal radius of the hook but not less than the length of the bar in the bend and the straight part of the hook. For 90° bends, the anchorage provided should be the smaller length of 12 times the bar size or 4 times the internal radius of the bend but not less than the length in the standard 90° bend. The radius of the bend is limited to twice the bend test radius in the appropriate British Standard or by the ultimate bearing stress in the concrete.

Bearing stress $= F_{bt}/r\phi$

and not exceed $1.5f_{cu}/(1+2\phi/a_b)$ (12.28)

where F_{bt} is the tensile force in the reinforcement, r is the internal radius of the bend, ϕ is the size of the bar or the equivalent size for a group of bars and a_b is the distance between bars perpendicular to the bend or the cover + ϕ for bars adjacent to the face of the member.

Links should be anchored by being passed through at least 90° round a longitudinal bar not less than its own size and continued for a length of at least 8 times its own size. Again, the internal radius of the bend should not be less than twice the bend test radius.

12.5.7 Cover

Concrete cover provides protection to the steel against corrosion and against too-rapid heating in the case of fire. Thus, the conditions of exposure and the requirements for fire resistance have a major influence in determining the amount of cover provided in design. Other factors are the dimensions of the reinforcing bars, the nature of the aggregate and the quality of the concrete.

For natural aggregate concretes, the nominal concrete cover to all reinforcement should not be less than that given for the appropriate condition of exposure and grade of concrete in Figure 12.10.

12.5.8 Spacing of reinforcing bars

The spacing and location of reinforcement must conform with the design requirements and must also allow proper compaction of the concrete to safeguard the protection of the steel against corrosion. The spacing must also be such that it tends to inhibit the spread of cracking and conforms with the needs for satisfying the criteria for the limit state of cracking.

In general, the maximum size of the aggregate governs the minimum spacing of bars, but when the size of the largest bars is 5 mm greater than that of the aggregate the spacing should not usually be less than the bar size. Bars or groups of bars should be located in horizontal layers with the gaps between the bars or groups in each layer in-line vertically.

Limitations on spacing are shown in Figure 12.11. The maximum distance between bars in tension is defined in the Code as a simple method of controlling crack width. For beams, the horizontal distance between bars is given in Figure 12.12(a). These requirements also apply to slabs except:

(1) When the slab is less than 200 mm thick, or 250 mm thick if f_y is less than 460 N/mm², or where the reinforcement is less than 0.3%.
(2) When the amount of reinforcement is less than 1% the spacing given in Figure 12.12(a) may be increased by dividing this spacing by the percentage of steel.

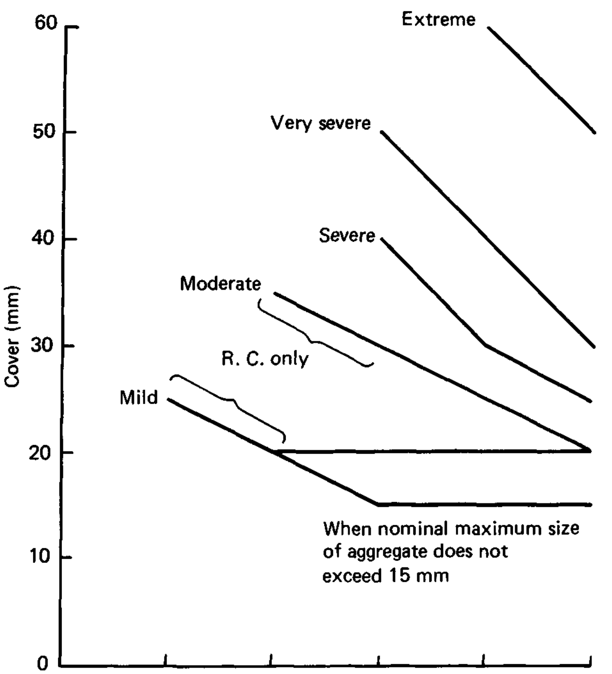
These recommendations relate to bars which, in size, are at least 0.45 times the size of the largest bar in the section and do not apply for particularly aggressive environments when f_y has a higher value than 300 N/mm².

The amount and spacing of side reinforcement required for beams of greater depth than 750 mm is illustrated in Figure 12.12(b).

12.5.9 Laps and joints

Bars can be lapped, welded or joined with mechanical devices to obtain continuity but joins should be located away from points of maximum stress. Load may be transferred in compression by cutting the ends of the bars square and holding them in direct alignment by a steel sleeve.

In general, the lap length should not be less than the greater of



	C30	C35	C40	C45	C50
Lowest grade of concrete	C30	C35	C40	C45	C50
Maximum free water : cement ratio	0.65	0.60	0.55	0.50	0.45
Minimum cement content – kg/m^3	275	300	325	350	400

Conditions of exposure

Mild – Concrete surfaces protected against weather or aggressive conditions

Moderate – Concrete surfaces sheltered from severe rain or freezing whilst wet; subject to condensation, continuously under water or in contact with soil (SO_3 content less than 0.2%)

Severe – Concrete surfaces exposed to driving rain, alternate wetting and drying and occasional freezing, severe condensation of flowing water

Very severe – Concrete surfaces exposed to sea water spray, directly or indirectly to de–icing salts, corrosive fumes or freezing conditions whilst wet

Extreme – Concrete surfaces exposed to the abrasive action of sea water or flowing water with PH equal to, or less than, 4.5

For concrete of C40 grade subjected to very severe exposures and for concrete of C45 grade subjected to extreme and very severe exposures and in each case to freezing whilst wet, air entrainment should be used

Note: Where low workability concrete is used in a precast factory, the minimum cement content may be reduced by up to 10% provided that the corresponding water : cement ratio is reduced by the same percentage.

Figure 12.10 Cover to reinforcement and tendons

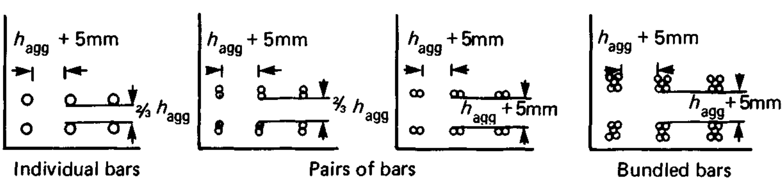


Figure 12.11 Bar spacing – h_{agg} is the maximum size of coarse aggregate

either 15 times the bar size or 300 mm. For tension reinforcement, the lap length should not be less than that required for anchorage in tension, but when the cover is less than 2ϕ and, either the bar is at the top of the member as cast or at the corner of a section, or the clear distance between adjacent laps is less than the greater of 75 mm or 6ϕ, the length should be multiplied by 1.4. If both conditions apply, the length should be multiplied by 2. For compression laps, the length should not be less than 1.25 times the required anchorage in compression. Lap lengths may be based on the size of the smaller bar when two sizes of bar are joined. If the size of a bar at a lap is greater than 20 and the cover is less than 1.5 times the size of the smaller bar, transverse links should be used; they should not be smaller than one-quarter the size of the smaller bar with a spacing of not less than 200 mm.

British Standard 8110 includes recommendations for joining bars by welding but stipulates that these should not be at bends and that, where possible, they should be staggered between parallel main bars. Where tests have shown that the strength of the welded bar is not less than that of the parent bar, joints in compression may be designed for the full strength of the joined bars and joints in tension for 80% of the strength of the joined bars. Welding should not normally be used when the stress in the bar is predominantly cyclic.

12.5.10 Curtailment and anchorage of bars

In principle, except at the ends of members, all reinforcement should extend beyond the point where it is no longer needed, i.e. where the resistance moment for the continuing reinforcement is equal to the design moment. The amount of this extension should not be less than either the effective depth of the member

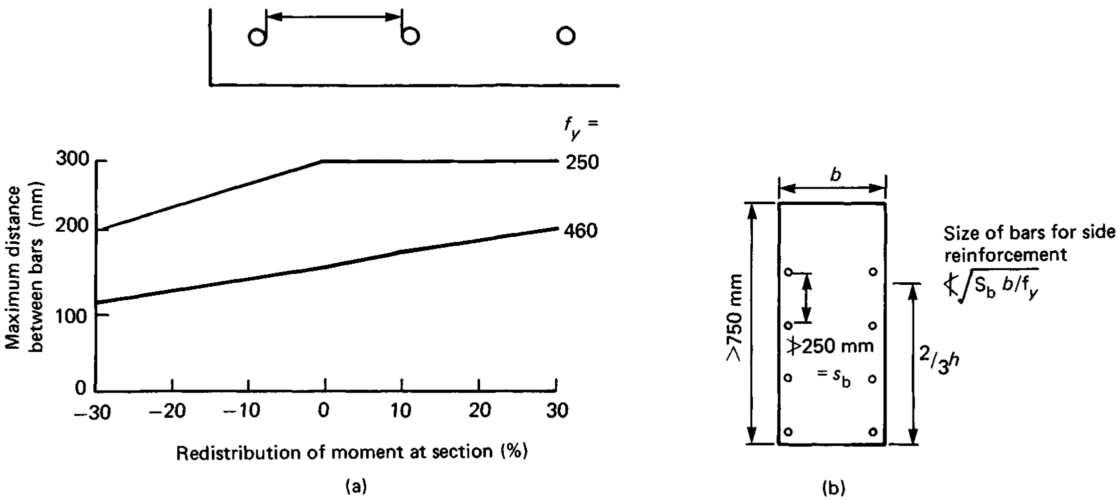


Figure 12.12 Reinforcement for crack control. (a) maximum distances between bars in beams and slabs; (b) minimum amounts and maximum spacing of side reinforcement for beams of 750 mm depth or more

or 12 times the bar diameter. Since the tension zone in the concrete may be cracked under service loading, special provisions for anchoring the bars in this region are necessary. These are met by one of the following:

(1) The extension should be an anchorage length.
(2) The shear capacity of the section where the bar stops should be twice that required.
(3) The continuing bars at this section provide twice the flexural strength required.

Each tension bar should be anchored at the end of a simply supported member by one or other of the following effective anchorage lengths:

(1) 12 times the bar diameter beyond the centre of the support, no bend or hook starting before $d/2$ from the face of the support.
(2) 12 times the bar diameter $+ d/2$ from the face of the support.
(3) For slabs, the greater in length of one-third the width of the support or 30 mm beyond the centre of the support provided that the design ultimate shear stress at the face of the support is not greater than half that allowed.

Items (1) and (2) above may be applied to hooked or bent bars whilst item (3) refers to straight bars. Simplified rules for application to the common cases of uniformly loaded beams and slabs are given in BS 8110 to which reference should be made.

In heavily reinforced members, curtailment of bars should be staggered to avoid undue cracking which might otherwise occur if a number of bars were stopped at almost the same position.

Despite these recommendations for anchorage and curtailment, the provision of ties required for the overall robustness of the structure should ensure their continuity and effective connection at changes of direction. At corners, the ties should extend 12 bar diameters beyond all the bars of the transverse ties or an effective anchorage length beyond their centreline.

12.5.11 Limits on the amount of reinforcement

For a concrete structure to be regarded as properly reinforced it is necessary to have reinforcement crossing all sections which could otherwise develop fracture planes and lead to failure. The Code only exempts certain columns and plain walls from this requirement. Generally it sets out lower limits for structural members which are listed in Table 12.9. These are supplementary to the amounts of steel required to provide stability in the event of partial damage (see page 12/13) and are contributory to these requirements.

Small amounts of vertical steel in walls do not contribute to the strength of the wall and hence the minimum percentage is 0.4% except when fire resistance is required when the minimum is 1%. For axially loaded reinforced walls the steel may be placed in one layer and in that case transverse links are not necessary, but if two layers are used the Code requires transverse links.

To avoid difficulty in compaction of concrete, upper limits are set on the amounts of steel in the section, and these are also shown in Table 12.9.

12.6 Prestressed concrete

12.6.1 General

The primary objective in prestressing is to avoid excessive cracking and deflection whilst at the same time enabling high-strength materials, particularly high-tensile steel, to be used efficiently in construction. The main criteria governing the design of prestressed concrete are therefore characteristics of serviceability rather than ultimate strength.

In setting out the criteria for serviceability of prestressed concrete in Table 12.2, three classes of structure have been identified but no indication was given, nor is it given in the Code, for what purposes these different classes of structure should be used. They nevertheless represent a logical progression from reinforced concrete construction which is likely to be cracked under service loading through class 3 and class 2 to class 1 prestressed concrete construction which is not only completely free from cracking but free from flexural tensile stresses under service conditions.

Where there are particularly adverse conditions of exposure or where cyclic or dynamic loading is severe, it may be appropri-

Table 12.9 Maximum and minimum requirements for reinforcement

Member	*Maximum*	*Minimum**
Tie	—	250G–0.80% 460G–0.45% of total area of concrete
Beams – tension reinforcement	4% of gross cross-sectional area of concrete	Rectangular sections – 250G–0.24% 460G–0.13% } of total area of concrete (applies to each direction in slabs) Flanged beams – webs $\frac{\text{breadth of web}}{\text{breadth of beam}}$: <0.4 / ≥0.4 250G – 0.32% / 0.24% 460G – 0.18% / 0.13% of area – breadth of web × effective depth Flanged beams – flanges at supports of continuous beams T-beams / L-beams 250G – 0.48% / 0.36% 460G – 0.26% / 0.20% of area – breadth of web × effective depth
Beams – compression reinforcement	4% of gross cross-sectional area of concrete	Rectangular sections – 0.2% of total area of concrete Flanged beams – flange in compression 0.4% of breadth × depth of flange Flanged beams – web in compression 0.2% of breadth of web × effective depth
Beams – shear	Limited by limit on shear in beams	Limitations on the amount of shear reinforcement are given in the sections shear in beams and slabs. Transverse reinforcement in flanges of flanged beams – 0.15% of the longitudinal cross-sectional area of the flange positioned near the top surface.
Columns	Vertically cast – 6% Horizontally cast – 8% At all laps – 10% of the gross cross-sectional area of concrete	Rectangular sections – 0.4% of total area of concrete. When all or part of the reinforcement is in compression in a column (or beam), ties or links are required at least one-quarter the size of the largest compression bar but not smaller than 6 mm at a spacing not more than 12 times the size of the smallest compression bar. Each corner bar and alternate bars should be tied by links and no bar should be more than 150 mm from a tied bar.
Walls	4% of total area of concrete	0.4% of total area of concrete. For up to 2% of compression bars, horizontal bars are needed not less than 6 mm size – 250G 0.30% or 460G 0.25% of concrete area – evenly spaced.

Table 12.9 Maximum and minimum requirements for reinforcement—*continued*

Member	*Maximum*	*Minimum**
		For more than 2% of compression bars, links should go through the wall not less than one-quarter size of largest compression bars or 6 mm size, at a spacing of not less than twice wall thickness horizontally and vertically and also vertically not more than 16 × the bar size. No compression bar not enclosed by a link should be more than 200 mm from an enclosed bar

*250G and 460G refer to the grade of steel, no grade is given for compression steel and either may be used.

ate to use class 1 structures. Where, on the other hand, these effects do not exist and cracking is acceptable, class 3 structures would be more appropriate. For general purposes, however, including water-retaining structures, class 2 structures offer most advantages being free from cracking but more economical than class 1 structures.

Since the serviceability requirements tend to dominate the design process rather than ultimate strength as in reinforced concrete (and in some prestressed concrete class 3 construction), the procedures for calculating stresses due to the prestress and likely service loadings in relation to serviceability will be given first. The main advantages of prestressing and its most important applications are seen in flexural members and main attention will therefore be given to beams and slabs with secondary attention to ties and columns subjected to bending; little advantage is gained by prestressing members subjected mainly to compression and such columns and walls are not therefore considered.

In dealing with serviceability, different sets of conditions need to be examined:

(1) During the imposition of the prestress, the stresses in the materials should not exceed certain values determined by the need to avoid failure during the transfer operations or excessive loss of prestress due to creep effects in the materials.
(2) After the losses of prestress have occurred due to creep and shrinkage of the concrete and relaxation of the steel, the remaining prestress should be sufficient to ensure that the limit state for cracking does not occur under the appropriate design loads.
(3) During none of these stages should the deflections exceed the limits set.

Additionally, attention has to be paid to the secondary effects that arise in anchoring prestressing tendons and finally to the need to meet the ultimate loading conditions.

12.6.2 Prestress and serviceability

12.6.2.1 General

In assessing the conditions of prestress, it is sufficient to assume that the concrete deforms elastically under short-term loading, that creep of concrete can be treated by adopting an effective modulus for the concrete (see page 12/10), and that shrinkage is uniform across the section.

12.6.2.2 Prismatic members

The stress conditions in a uniform member, subjected to external moment M and external force F with a prestressing force P with eccentricity to one axis only of e are shown in Figure 12.13.

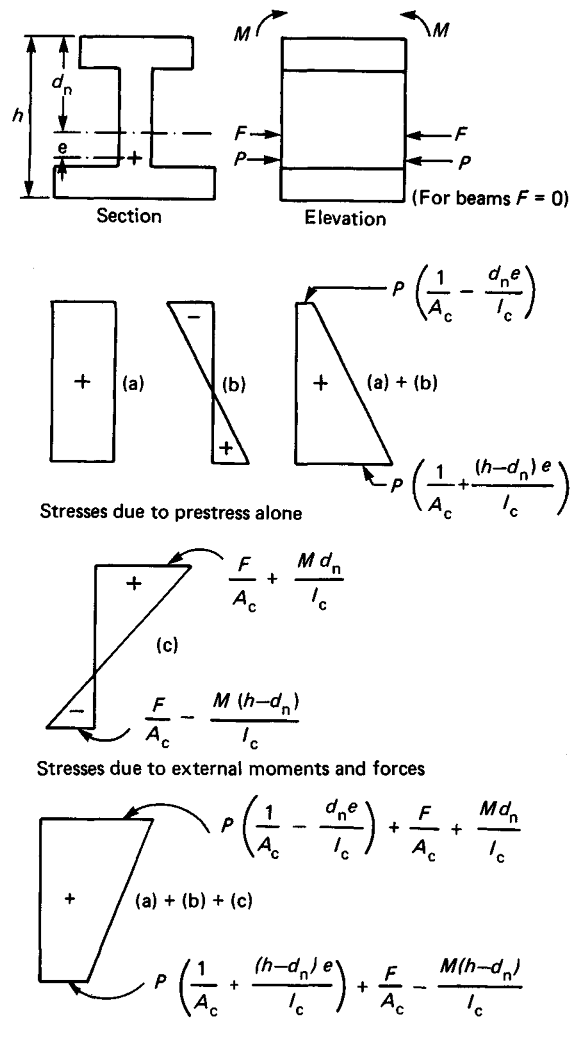


Figure 12.13 Elastic analysis of prestressed concrete sections. M, external moment; F, external force; P, prestressing force; A_c, area of concrete; I_c, second moment of concrete area; A_{ps}, area of tendons

Steel is normally used to impose the prestressing force although it can be applied by jacks or by the use of other materials. Such applications are, however, so rare that they are not considered further and this section is therefore concerned only with design for pretensioning and post-tensioning with steel tendons.

For pretensioning, the stress in the steel immediately after transfer, f_{p2}, is less than the initial stress, f_{pl}, by an amount corresponding to the relaxation of the steel at that stage and to the elastic contraction of the concrete adjacent to the steel; the stress is then:

$$f_{p2}=f_{pl}-\Delta f_{pl}-\frac{E_s}{E_{cl}}\left[f_{p2}A_{ps}\left(\frac{1}{A_c}+\frac{e^2}{I_c}\right)-\frac{M_g e}{I_c}\right] \qquad (12.29)$$

where Δf_{pl} is the relaxation of steel before transfer, M_g is the moment due to the proportion of dead load effective at transfer, E_s and E_{cl} are the moduli of elasticity of concrete and steel respectively, A_c and I_c are the area and second moment of area respectively of the concrete section and e is the eccentricity of the tendons.

The final term in the equation is the stress in the concrete adjacent to the steel multiplied by the modular ratio, to give the equivalent change in stress in the steel.

For beams of uniform section with steel at constant depth along the beam, the moment due to dead load, M_g, should be ignored, since the most severe conditions of prestress occur away from mid-span near the supports where M_g is small. Where the beams are of nonuniform section, the eccentricity of the steel is normally reduced towards the ends and checks on the stress conditions are required at several sections in the span.

In post-tensioning where a number of tendons are stressed successively, only the first tendon to be stressed contracts by the full amount of the elastic shortening of the concrete and the stress in the steel after transfer is then given by:

$$f_{p2}=f_{pl}-\frac{E_s}{2E_{cl}}\left[f_{p2}A_{ps}\left(\frac{1}{A_c}+\frac{e^2}{I_c}\right)-\frac{M_g e}{I_c}\right] \qquad (12.30)$$

Δf_{pl} is not included since there is little relaxation of steel between stressing and anchoring.

In structures with post-tensioned tendons, the dead-load moment effective at transfer may be large and may include dead load from additional superstructure which is temporarily propped but becomes effective on stressing the tendons. This dead-load moment has an important influence on design since it can, if properly manipulated, lead to improvements in efficiency.

The effect of time on the deformation of concrete and steel is taken into account in the following expressions which give the stresses in the tendons. For pretensioning:

$$f_{p3}=f_{pl}-\Delta f_p-SE_s-\left(\frac{E_s}{E_{cl}}+\phi E_s\right)\left[f_{p3}A_{ps}\left(\frac{1}{A_c}+\frac{e^2}{I_c}\right)-\frac{M_g e}{I_c}\right] \qquad (12.31)$$

For post-tensioning:

$$f_{p3}=f_{pl}-\Delta f_p-SE_s-\left(\frac{E_s}{2E_{cl}}+\phi E_s\right)\left[f_{p3}A_{ps}\left(\frac{1}{A_c}+\frac{e^2}{I_c}\right)-\frac{M_g e}{I_c}\right] \qquad (12.32)$$

where Δf_p is the total relaxation loss in the steel, S is the shrinkage strain and ϕ is the creep strain for unit stress.

Using these formulae, the value of P is obtained from:

$$P=f_p A_{ps} \qquad (12.33)$$

where f_p is the stress in the tendon at the stage considered. P is then substituted in the appropriate expressions in Figure 12.13 to obtain the required stress conditions immediately after transfer and subsequently under the loads for serviceability limit states.

In members subjected to bending in one direction only it will be normal to locate the centre of the prestressing tendons at an eccentricity which will provide maximum compression at what will become the tension face with a small amount of tension or compression at what will become the compression face under load. For members subjected to loads from any transverse direction, the tendons will be placed concentrically to give a uniform prestress. The formulae given apply to either case.

12.6.3 Losses of prestress

In making these calculations for the stress conditions during manufacture and in service, quantitative allowances must be made for the elastic contraction, shrinkage and creep of concrete, and relaxation of the steel. These characteristics are variable and are much influenced by the nature of the materials used, methods of production and the service conditions. Part 1 of BS 8110 gives values for the calculation of these losses for general use, while Part 2 amplifies this information for special circumstances; it is recommended that specialist literature should be consulted for very unusual conditions of temperature or exposure.

The shrinkage of concrete, so far as it affects the loss of prestress, depends on the quality of the concrete, the size of the component, the nature of the aggregate, age at transfer and the conditions of exposure. It is usually reasonable to assume that the shrinkage of concrete may be taken as 100×10^{-6} for external exposure in the UK and 300×10^{-6} for indoor conditions.

Experimental evidence shows that the creep of concrete is proportional to the applied stress for the stresses generally applied during transfer and, as for shrinkage, it is considerably affected by circumstances. For the calculation of the loss of prestress, it is convenient to define the amount of creep as a multiple of the elastic contraction at transfer, and to adopt a factor of 1.8 for transfer within 3 days, reducing to 1.4 for transfer at 28 days for outdoor exposure in the UK. These values may also be used for class 1 and 2 structures for internal conditions. Further advice for other conditions and for class 3 structures is given in Part 2 of BS 8110; for other problems specialist publications should be consulted.

If it is necessary to estimate the amount of creep at some intermediate stage in the life of a structure, it is often sufficient to assume that about half occurs during the first month after transfer and that about three-quarters of the total occurs during the first 6 months following transfer.

The relaxation of prestressing tendons due to creep of steel is dependent on the type of steel and method of manufacture. The relevant BS Standards (Table 12.7) require a 1000-h test for relaxation of tendons at different levels of initial stress as part of the acceptance test procedure. The relaxation loss used in design is obtained from the value in the manufacturers' test certificate corresponding to the initial prestress multiplied by a relaxation factor. The values in BS 8110 are quoted in Table 12.10. If, at the design stage, the steel supplier is not known, it will usually be sufficient to base calculations on specified values.

For many forms of repetitive construction, once the losses of prestress have been established, it may be possible to express the total loss as a percentage of the initial stress in the steel at transfer. Values for total loss, due to elastic contraction, shrinkage and creep of concrete, and relaxation of steel, of 20% for

Table 12.10 Relaxation factors for different types of tendons

Type of tendon	*Relaxation factor* *Pretensioning*	*Post-tensioning*
Wire and strand		
relaxation class 1	1.5	2.0
relaxation class 2	1.2	1.5
Bar	—	2.0

pretensioning and 15% for post-tensioning for an initial stress in tendons of 70% of their characteristic strength have been found to be appropriate. If such an assumption is made, however, detailed refinement in design should not be attempted.

Other sources of loss also need to be taken into account with post-tensioning. These arise through the movement of the tendons in the anchorage during the process of transfer, which needs to be determined by measurement and should be given by the manufacturer of the system, and through the development of friction between the tendon and its surroundings.

The profiles of the cables or bars may be curved to provide for counteracting variation of moment due to dead and imposed loads or due to continuity. As a result, friction develops during stressing between the cables or bars and the inner surfaces of ducts or tendon deflectors. The amount of friction depends on the construction of the cable, the materials in sliding contact and the angular displacement. For long cables, the actual profiles are likely to deviate from their correct position to such an extent that they have an effective additional curvature, which causes considerable frictional effects. Then the force in a tendon, P_x, at a distance x from the jacking point is given by:

$$P_x = P_o \exp - [(\mu x/r_{ps}) + Kx] \qquad (12.36)$$

where P_o is the force in the tendon at the jacking end, μ is the coefficient of friction from Table 12.11, r_{ps} is the radius of curvature, x/r_{ps} is the angle of deviation over length x and K is the constant and the form of tendon and duct which has a usual value of 33×10^{-4}/m but may be reduced to 17×10^{-4}/m for rigid sheaths or rigidly fixed duct formers or to 25×10^{-4}/m for greased strands in plastic sheaths.

Table 12.11 Values for coefficient of friction μ

Condition	μ
Lightly rusted strand on unlined concrete duct	0.55
Lightly rusted strand on lightly rusted steel duct	0.30
Lightly rusted strand on galvanized duct	0.25
Bright strand on galvanized duct	0.20
Greased strand on plastic sleeve	0.12

12.6.4 Stress limitations at transfer and for serviceability conditions

Limits need to be set on the stresses in the steel and concrete at transfer to ensure that the deformation of the materials is not excessive since this would lead to high losses of prestress, severe cracking and undue distortion of components. Limits also need to be imposed on the stresses likely to occur in service to keep deflections within acceptable bounds and to control cracking to required limits.

All calculations of stresses for these two sets of conditions are based on the assumptions that the section is uncracked and that the strains due to applied stresses are proportional to those stresses.

The stress in tendons during the initial stressing operations should not normally be more than 75% of the characteristic strength but may be as much as 80% provided special care is taken. The stress at transfer should not normally be more than 70% and never more than 75% of the characteristic strength.

The allowable maximum limit for compressive stress in concrete at transfer is $0.5f_{ci}$ at the extreme compression face or $0.4f_{ci}$ for a nearly uniform prestress, where f_{ci} is the strength at the time of transfer.

For the serviceability limit state, the compressive stress in the concrete should not be more than $0.33f_{cu}$ at the extreme compression face but for continuous construction this limit may be raised to $0.4f_{cu}$ within the negative moment zone. The stress in direct compression should not be greater than $0.25f_{cu}$.

Flexural tensile stresses in concrete are defined according to the class of structure decided at the outset of design. For class 1 structures, the maximum tensile stress at transfer is limited to 1 N/mm^2 and no tensile stresses are allowed for serviceability limit states.

For class 2 structures, in which some flexural tensile stresses are allowed up to the tensile strength of the concrete for pretensioning and up to 0.8 times the tensile strength of the concrete for post-tensioning, the tensile strength is assumed to be $0.45\sqrt{f_{ci}}$ for transfer and $0.45\sqrt{f_{cu}}$ for the serviceability limit states. Where a design service load is only likely to be rarely imposed and the concrete is normally stressed in compression so that any cracks that might occur are closed, the allowable tensile stress may be increased by 1.7 N/mm^2 provided that pretensioned tendons are well distributed throughout the concrete stressed in tension and post-tensioned tendons are supplemented by secondary reinforcement.

Although cracking is permitted in class 3 structures, the section is assumed to be uncracked and limits are set for notional tensile stresses for use in calculations for service loading to impose some restriction on the widths of cracks. At transfer, the limits set for tensile stresses are, however, the same as those for class 2 structures. The values for allowable notional tensile stresses are obtained from Table 12.12, which are multiplied by the factors in Table 12.13 to allow for the effect of the depth of section on cracking.

Table 12.12 Class 3 members – limits for notional tensile stresses

Group	*Limiting crack width* (mm)	*Design stress* (N/mm^2) *for concrete of grade* 30	40	50 and over
Pretensioned tendons	0.1	—	4.1	4.8
	0.2	—	5.0	5.8
Grouted post-tensioned tendons	0.1	3.2	4.1	4.8
	0.2	3.8	5.0	5.8
Pretensioned tendons distributed in tensile	0.1	—	5.3	6.3
zone and close to the concrete tension face	0.2	—	6.3	7.3

Table 12.13 Class 3 members – depth factors

Depth of member including composite members (mm)	*Factor*
200 and under	1.1
400	1.0
600	0.9
800	0.8
1000 and over	0.7

These stresses may be exceeded in certain circumstances for class 3 structures as indicated in BS 8110.

12.6.5 Beams

12.6.5.1 Flexural strength

The methods of calculation of the ultimate flexural strength of prestressed concrete beams are similar to those for reinforced concrete beams with the additional need that allowance must be made for the effect of the condition of prestress. The assumptions made are:

(1) Sections which are plane before remain plane after bending.
(2) The stresses in the concrete may be determined from the stress–strain curve in Figure 12.4 or, more normally, may be taken as uniformly distributed across the compression zone to a depth of 0.9 times that of the neutral axis with a value of $0.45f_{cu}$ for deriving simple formulae as in Figure 12.14(a). As for reinforced concrete, the ultimate compressive strain for the concrete is taken as 0.0035.
(3) The tensile strength of the concrete is ignored.
(4) The strains at ultimate in pretensioned tendons and in post-tensioned and bonded tendons are assumed to conform generally with the strains in the concrete so that the stresses may be determined from the stress–strain relationships for steel given in Figure 12.6 making allowance for the initial stress condition in the steel after all losses. As a simple alternative, the stresses at ultimate may be obtained from the curves in Figure 12.14(b) where allowance is made for the strength of the steel and the concrete, their respective proportions and the initial stress in the steel.
(5) The strains at ultimate in unbonded post-tensioned tendons do not conform directly with the compressive strains in the adjacent concrete but are directly influenced by the increase in separation between the end anchorages. A method of calculation of the stress in the steel is given in BS 8110 or the stress may be determined by test or analysis.
(6) Any additional steel reinforcement close to the tension face should be assumed to be stressed to its characteristic yield stress at ultimate. Such reinforcement in the compression zone should normally be ignored.

If the compression zone at failure is not rectangular in shape, the ultimate strength should be calculated from first principles using the stress–strain relationships shown in Figures 12.4 and 12.6.

12.6.5.2 Deflection

Control of deflection in the design of reinforced concrete beams is governed in BS 8110 by limitations on span:depth ratio, but the method is not appropriate for prestressed concrete beams. For normal construction no specific requirement is given, the limitations on stresses for service conditions usually being sufficient to avoid deflections becoming excessive.

For special construction, where checks are required however,

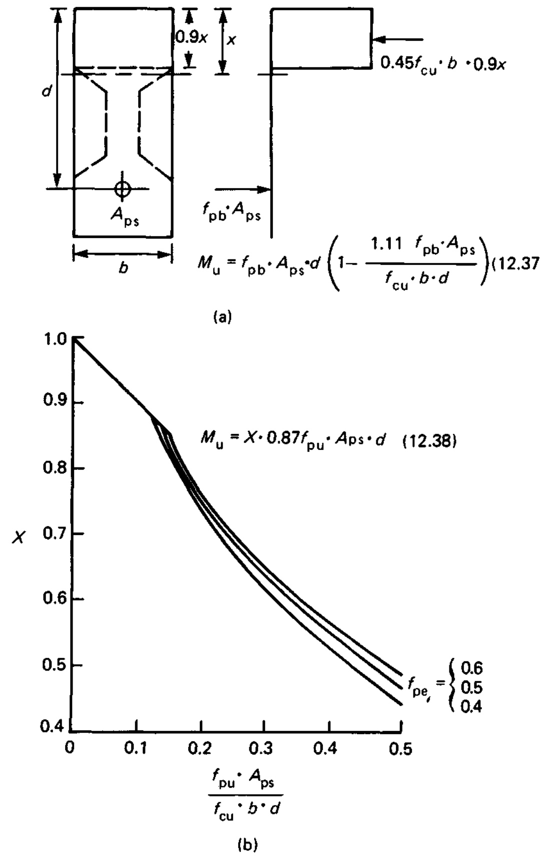


Figure 12.14 Flexural strength of beams – approximate method (pretensioning and post-tensioning with bond)

some guidance is provided in Part 2 of BS 8110. It should then be assumed in the calculation of the short- and long-term deflections of class 1 and class 2 prestressed beams that behaviour is elastic and that the properties of the section are those for the concrete with the deformation characteristics appropriate to the nature of the loading. For long-term loading, an effective modulus should be used in the calculations, which may be derived from the data given on creep. The same approach to the calculation of deflection may be adopted for class 3 prestressed beams provided that the section is not cracked. If, however, it is cracked under the load being considered, deflection is more likely to require limitation and it should then be calculated from the moment–curvature relationship determined from the properties of the materials and the characteristics of the section.

It should be noted that for members, such as precast mass-produced units with pretensioned steel with a uniform eccentric prestress along their length, the upward deflection at transfer and later in service, if the permanent loading is light, may need to be checked by calculation to ensure that it is not excessive. If such members are heavily loaded, it should be remembered that the regions near their ends are subjected to the effects of reversed bending due to the prestress which will tend to reduce the central deflection.

12.6.5.3 Shear

Shear in prestressed concrete beams needs only to be considered for ultimate conditions. Then, sections subjected to shear remote from regions of maximum bending are likely to be uncracked but those subjected to both bending and shear will usually be cracked in flexure. These two situations give rise to substantially different distributions of stress and therefore require different methods of analysis; each is dealt with in BS 8110. In each case, the contribution of the concrete to shear strength is calculated and may be taken into account when provision is made for shear reinforcement.

Firstly, irrespective of the situation and amount of shear reinforcement, a limitation is set on the maximum shear that a section may sustain. This maximum shear strength is defined by limiting the maximum shear stress ($V/b_v \cdot d$) for cracked or uncracked sections to the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm².

For uncracked sections, the ultimate resistance of the concrete (V_{co}) is given by:

$$V_{co} = v_{co} \cdot b_v \cdot h \qquad (12.39)$$

where v_{co} is the ultimate shear stress that can be sustained by the uncracked concrete and is given in Figure 12.15; it is expressed in terms of the grade of concrete and compressive stress at the centroid due to the prestress (f_{cp})$\cdot b_v$ is the breadth of the section or of the web for T- and I-sections and h is the overall depth of the section

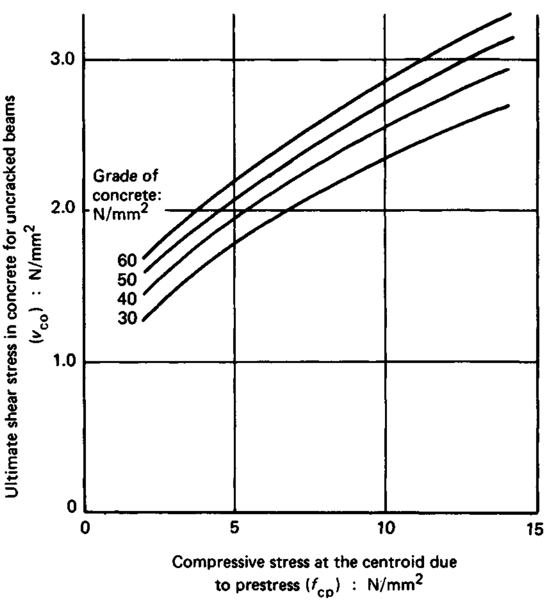


Figure 12.15 Ultimate shear stress in concrete for uncracked beams

The shear at the ends of units with pretensioned steel should be determined at a distance equal to the height of the centroid of the section above the soffit from the edge of the bearing. If this position is within the transmission length, the value of f_{cp} should be reduced by multiplying by the factor $r(2-r)$ where r is the distance of the section from the end of the unit as a fraction of the transmission length.

The ultimate shear strength of the cracked concrete section is given by:

$$V_{cr} = \left[1 - 0.55\left(\frac{f_{pe}}{f_{pu}}\right)\right] v_c \cdot b_v \cdot d + M_0 \cdot (V/M) \qquad (12.40)$$

but:

$$V_{cr} \nless 0.1 b_v \sqrt{f_{cu}} \qquad (12.41)$$

where v_c is given in Figure 12.8 and is the same as for reinforced concrete (the cross-sectional area of reinforcement should then be the sum of the areas of tendons and reinforcement in the tensile zone). f_{pe} is the effective prestress in the tendons but not more than $0.6f_{pu}$, M_0 is the moment to produce zero stress at depth d in the beam, and V and M are the shear and the moment at ultimate respectively

The calculated shear resistance of the concrete alone (V_c) for uncracked sections should be taken as that given by V_{co} above. For cracked sections, however, both V_{co} and V_{cr} above should be calculated and V_c assigned the lower value.

Since, if it were to occur, failure in shear of prestressed concrete beams without secondary reinforcement might take place suddenly with little warning, it is usually recommended that the provision of shear reinforcement should be conservative. Hence, BS 8110 recommends that reinforcement should only be omitted from prestressed concrete beams when the shear strength of the concrete is more than the required resistance. In such circumstances, reinforcement may be omitted from components of minor structural significance or from those which have been proved by tests. Shear reinforcement is not required in other beams when the shear strength provided by the concrete is calculated to be twice that required.

The Code gives the following guidance on the calculation of amount of shear reinforcement required. When the shear resistance required is less than $V_c + 0.4\, b_v \cdot d$, the shear reinforcement in the form of links should not be less than:

$$A_{sv}/s_v = (0.4b_v)/(0.87f_{yv}) \qquad (12.42)$$

If the shear resistance required (V) is greater than $V_c + 0.4\, b_v \cdot d$, the shear reinforcement required in the form of links should not be less than:

$$A_{sv}/s_v = (V - V_c)/(0.87f_{yv} \cdot dt) \qquad (12.43)$$

where A_{sv} is the cross-sectional area of two legs of a link, s_v is the link spacing, f_{yv} is the characteristic strength of the reinforcement but not more than 460 N/mm² and depth d_t is of the section from the compression face to the centroid of the tendons or to the longitudinal reinforcement if greater.

Links should pass round longitudinal bars or tendons of larger diameter than the link at the corners of the tensile zone and as close as the cover requirements permit to the tensile and compression faces. They should be anchored firmly and enclose all longitudinal tendons and reinforcement. The spacing of the links should not be more than $0.75d_t$ or 4 times the web thickness. The maximum spacing should be reduced to $0.5d_t$ when V exceeds 1.8 times V_c. Lateral spacing should not exceed d_t.

12.6.6 Other forms of member

12.6.6.1 Slabs

The design of slabs in prestressed concrete adopts the methods for dimensioning prestressed concrete beams. No shear reinforcement is required provided that V is less than V_c.

For two-way slabs, the stresses under service conditions should be determined by elastic analysis while the methods of design used for reinforced concrete slabs should be used for ultimate conditions.

12.6.6.2 Columns

It will usually only be appropriate to use prestressed concrete columns when eccentricities of load are high and so require high bending stresses to be resisted. British Standard 8110 recommends that columns with a mean precompression of less than 2 N/mm^2 should be treated as reinforced concrete columns.

The analysis of columns in prestressed concrete frameworks should, in general, be similar to that adopted for reinforced concrete but modified to take account of their different moment/curvature characteristics.

12.6.6.3 Tension members

Although prestressed concrete is seldom used for tension members, it is well suited for the purpose.

For serviceability limit state of cracking, the tensile stresses should be limited to possibly half those for class 2 construction. Ultimate strength should be calculated by assuming that both tendons and any secondary reinforcement are stressed to 0.87 times their respective characteristic strengths.

12.6.7 Requirements for tendons and reinforcement

12.6.7.1 Transmission lengths for pretensioning

With pretensioning, it is usual to anchor the tendons by bond with the concrete. In this case, the stress in the steel and the prestress in the concrete builds up along the length of the member near its ends. Consequently, in dealing with shear in this region some allowance must be made for this build-up, as already explained in the section on shear.

The transmission length is dependent on a number of factors including the strength of the concrete, the degree of compaction, the pretension in the tendons, the nature of the tendon, its size, and surface characteristics. Since the concrete near the top of a member is often less well compacted than that near the bottom, the transmission length for tendons near the top is usually greater. Values for the transmission length, obtained from measurements in the field and in the laboratory, show that very substantial variations do occur. Where possible, therefore, transmission lengths should be determined for the particular conditions of production that obtain in the precast works. Where such information is not available, the method of calculation recommended in BS 8110 should be used. The transmission length is calculated as follows:

$$l_t = (K_t \cdot \phi)/\mathrm{N}f_{ci} \tag{12.44}$$

where K_t is 600 for plain or indented wire or crimped wire with a small offset; 400 for crimped wire with an offset greater than 0.15 ϕ; 240 for standard or super seven-wire strand; 360 for drawn seven-wire strand; ϕ is the nominal diameter of the tendon; and f_{ci} is the strength of the concrete at transfer

Within the anchorage region, the pretensioned tendons should be distributed as uniformly as possible across the section to avoid the development of unnecessary bursting stresses between groups of tendons. Reinforcement of the end-regions may be necessary; it should enclose all the tendons and some guidance on the amount required may be obtained from that needed in end-blocks.

12.6.7.2 End-blocks for post-tensioning

The anchorages of post-tensioned tendons at the ends of members give rise to high bursting stresses in the concrete. Recommendations for the design of end-blocks may be made by the manufacturers and, if so, should be followed.

If they are not made the recommendations in the Code should be followed. These give a method of estimating the bursting forces for square blocks which may be applied also to rectangular blocks or combinations of rectangular blocks to cover irregular shapes of anchorage. The basic formula is:

$$F_{bst}/P_k = 0.32 - 0.30(y_{po}/y) \tag{12.45}$$

for:

$$0.3 < y_{po}/y < 0.7$$

where F_{bst} is the bursting force, P_k is the maximum prestressing force during jacking when tendons are grouted, y_{po} is the half-side of the loaded square end-plate when y is the half-side of the square block

The reinforcement required in each direction should be located transversely in the distance 0.2 y_{po} to 2 y_{po}. For rectangular end-blocks, the treatment should be similar giving different amounts of reinforcement in each direction. Circular bearing plates should be treated as square plates of the same area. The reinforcement should consist of spirals or closed links stressed to not more than 200 N/mm^2. Where the whole anchorage region is comprised of a number of individual anchorages additional reinforcement will be required to enclose the whole.

For unbonded tendons, the calculations should be based on the characteristic strength of the tendons and reinforcement assumed to be stressed to 0.87 times its characteristic strength.

12.6.7.3 Proportions of prestressing steel

The characteristics of prestressed concrete of being crack-free under normal conditions and of exhibiting extensive cracking and deflection under overloads have been emphasized as advantages, but these are not necessarily obtained with all prestressed concrete construction. Brittle fracture will occur if there is insufficient prestressing steel in the section to sustain the tensile stresses transferred to it on the development of cracking. To prevent this from taking place, it is recommended that the calculated ultimate moment of resistance should exceed the calculated moment of resistance of the uncracked section corresponding to a maximum flexural tensile stress of $0.6\sqrt{f_{cu}}$ after allowing for losses.

Brittle fracture can also occur by crushing of the concrete in 'over-reinforced' members. It may be experienced with precast members of inverted T-section which are designed for incorporation in composite construction but which are susceptible to this form of failure during erection. Difficulty can be avoided by consideration of erection stresses and proper supervision of construction.

12.6.7.4 Cover and spacing for tendons

In general the requirements for concrete cover for tendons and for their spacing are governed by the same needs as for reinforced concrete.

The cover of concrete necessary to protect steel against different conditions of exposure is given in Figure 12.10 (page 12/20) and applies to both tendons and reinforcement. The concrete should not be less than grade 30 and, for concretes of higher grades than 50, there should be no reduction in the thickness of the cover. The cover to ducts for post-tensioned tendons should not be less than 50 mm. Since prestressing steels are more sensitive to the effects of heat than reinforcing steels, the requirements for concrete cover for the protection of tendons in fire are more onerous and are dealt with later on pages 12/32 and 12/33.

Experience has shown that ends of individual pretensioned tendons do not require concrete cover for protection and may be left cut flush with the end of the member. Where post-tensioned tendons are positioned outside the member, they are normally protected with added concrete to provide the cover; some attention should be given, however, to the way in which this is done since the development of possible paths for penetration of moisture to the steel must be avoided.

There should be sufficient space between tendons to allow proper compaction of the concrete and the rules applied to reinforced concrete should therefore apply. Large ducts, however, provide more difficulties than are experienced with large reinforcing bars and careful attention is needed in detailing. If these ducts are required in thin diaphragms or webs then care must also be given to the avoidance of bursting the concrete and possibly to the provision of additional reinforcement. Additional reinforcement may also be required in members with pretensioned steel if the individual tendons are in separate groups to prevent longitudinal splitting at the ends where transmission of the prestress by bond is developed.

12.6.7.5 Secondary reinforcement

Reinforcement is required in prestressed concrete to meet requirements for resisting shear, to permit higher tensile stresses in class 3 structures, to prevent bursting in the region of end-anchorages, to reinforce thin webs and to retain concrete cover in place for longer periods of fire resistance. Any longitudinal reinforcement provided for these purposes can be taken as contributing to ultimate strength as can the ties needed to ensure stability in the event of accidental damage.

Reinforcement may also be desirable for members prestressed by post-tensioning to restrict cracking after casting caused by restraint of the formwork due to shrinkage or cooling of the concrete.

12.7 Precast and composite construction

12.7.1 General

The previous sections have dealt with concrete construction without specific reference to the method of making the concrete. Those sections are generally applicable when the concrete is cast *in situ*. Not uncommonly, however, it is economic or convenient to precast concrete in the factory and use it in construction with site-cast concrete to form composite members or structures. This section deals with the special needs of using precast concrete in construction which are generally additional to those already given for cast *in situ* construction.

Precast members must be handled, and possibly transported, stored and erected, at an early age. The strength of the concrete must therefore be sufficient, not only to satisfy the normal design requirements for the finished structure, but also to sustain these constructional operations without damage. It is therefore not unusual for precast members to be made with concrete of a much higher strength than would be used if they were to be cast *in situ*. Characteristic strengths as high as 50 or 60 N/mm^2 at 28 days may be obtained since the manufacturing requirement may be for 10 N/mm^2 or more at 24 h.

Since the strength and robustness of concrete construction depends mainly on the combined action of concrete in compression and steel in tension at all sections, the presence of joints between members requires special attention from the designer to provide continuity of reinforcement and between members in the development of overall stability. To help in achieving this aim, the responsibility for design should also be vested in one engineer to ensure that, not only are the individual units adequate for their purposes, but that the overall performance of the structure is also adequate.

12.7.2 Structural connections between units

The recommendations for the stability of precast construction conform generally with those for *in situ* construction. The first requirement in the design of structural joints is therefore to provide for transmitting the tie forces required for stability. In providing this reinforcement, it should be ensured that the arrangements for anchorage are realistic and can be obtained with normal site workmanship. Particular account needs to be taken of the requirements in BS 8110 with respect to location and anchorage. Bars must almost inevitably be lapped and anchored in cast *in situ* concrete or mortar and the Code gives detailed recommendations on conditions, relevant dimensions and form of secondary link reinforcement that should be provided. These details are not reproduced here and reference should be made to BS 8110.

When the joint is not required to transmit horizontal forces or moments, the detailing should be such that these effects are not, in fact, transmitted or, if that is not possible, that any unintended transmission should not lead to any untoward cracking or local damage. It must be recognized that shrinkage and creep effects can lead to the development of substantial restraints in structures and so cause cracking, and that these effects can be more serious in precast structures, particularly when the units are prestressed.

In dealing with the transmission of forces and moments at connections, the normal procedures for calculation for reinforced concrete, prestressed concrete or structural steel, should be used. Where special difficulties arise, tests should be made to assess both strength and mode of failure.

Attention needs to be paid to the protection of joints against the weather and corrosion and against fire to make sure that the performance of the joint under these conditions is not inferior to that of the rest of the structure. Emphasis has already been placed on the importance of detailing joints in such a way that they can be made properly on the site and effectively inspected. To achieve this, any projecting bars, ribs or fins should be sufficiently robust to avoid damage in transit and erection and sufficiently accurately located and dimensioned for ease of casting and assembly. The space between members being jointed should be sufficient to allow filling of the joint without difficulty and for subsequent checks on workmanship. Further, to make sure that joints are made properly, written instructions should be given to the site supervisor giving full details and sequence for jointing with information on what should be done in the event of misfits. The instructions should also contain information on the making of the joints in relation to progress of construction since stability during erection may well depend on the extent of the completion of joints in other parts of the structure.

Continuity of reinforcement in precast construction may require special consideration. If sockets or slots are left for lapping or anchoring reinforcement they should be sufficiently large and of a form likely to achieve their object with a suitable form of surface for developing bond between the infill concrete, mortar or grout and the precast units. Sleeves or threaded anchors for connecting reinforcement may be used as well as welded connections with structural steel sections. Test data for many of these types of connections are available.

12.7.3 Beams, slabs and frames

Where the continuity of beams, slabs and frames is obtained by the appropriate positioning and anchoring of steel reinforcement or tendons, the amount of redistribution of moments adopted for *in situ* construction may also be assumed for precast construction.

12.7.4 Floor slabs

Precast slabs are often used in floor construction. Units are usually placed side by side with an *in situ* topping and have provision for the transmission of transverse shear between units. Where these floors are needed to support partitions or other concentrated loads it may be assumed, subject to certain provisos in BS 8110, that the width of floor supporting the load may be up to 3 times the width of the units together with their joints for unreinforced topping and up to 4 times for reinforced toppings.

12.7.5 Bearings

The design of bearings in precast concrete construction has received detailed scrutiny in recent years, since a number of roof failures have been attributed to their inadequacy. The function of the bearing is to provide support for beams and slabs without cracking or spalling of the concrete under the relatively high stresses due to rotational effects needing to be sustained, whilst ensuring at the same time that unintended relative linear movement at the bearing does not take place. If rotational effects are likely to change the point of application of loading on a member or longitudinal restraint and likely to induce substantial stresses, these should be allowed for in design. Constructional inaccuracies are usually adjusted at the bearings and so it is also essential that proper allowance is made for accommodating building tolerances. A further requirement is that reinforcement in the supporting component should have an effective overlap of the member being supported. These aspects are dealt with in greater detail in BS 8110, but the necessary design calculations are based on the following limitations:

(1) The net bearing width, i.e. in the direction of the span of the supported member after allowing for inaccuracies, should not be less than 40 mm; for isolated members, it should not be less than 60 mm.
(2) The effective bearing length, i.e. the transverse direction to span of the supported member, should not be taken as greater than the effective length, half the effective length + 100 mm or 600 mm, whichever is the less.
(3) The ultimate bearing stress should not exceed $0.4f_{cu}$ for concrete directly to concrete, $0.6f_{cu}$ for a bedded bearing on concrete or $0.8f_{cu}$ for concrete bearing on steel where the dimension of the steel does not exceed 40% that of the concrete.

The corbel is a common form of support for beams on columns, which BS 8110 defines as a short cantilever beam, and provided the limiting dimensions given in Figure 12.16 are adopted it may be designed as an inclined strut. The tie reinforcement should be capable of sustaining an additional horizontal force equal to at least half the vertical force and compatibility between the strains in the concrete and the steel tie should be checked at the root of the corbel. The tie should be anchored at the front of the corbel by fabricating the reinforcement as a closed loop. The depth of the corbel at the face of the support should be determined from considerations of shear. Reinforcement should also be provided in the form of horizontal links with a total cross-sectional area of half that of the main tie over the upper two-thirds of the corbel at its root to control cracking under service conditions.

Where precast floor units or other units are supported by beams, a concrete nib may be required along the sides of the beams, which may need to be continuous. This may be designed

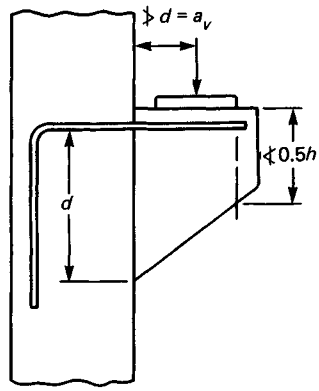


Figure 12.16 Dimensions of corbels

as a cantilever assuming that the load acts at the outer edge of the loaded area, i.e. the edge for a nib without a chamfer, the edge of the chamfer or the edge of a bearing pad. The eccentricity of the load is then taken as the distance to nearest vertical link reinforcement in the beam, which should be anchored in the compression zone of the beam. The reinforcement in the nib should extend as near to the front face of the nib as requirements for cover permit and it should be anchored there by being in the form of a closed loop. Shear should be checked and the shear stress should not exceed normal values for reinforced concrete multiplied by a factor equal to twice the effective depth divided by the eccentricity.

12.7.6 Composite concrete construction

Precast members of reinforced or prestressed concrete are commonly used in composite construction, particularly beams in combination with a structural cast *in situ* concrete topping to form floor slabs. Design of composite sections generally follows the methods developed for sections in reinforced or prestressed concrete with the exception that, when considering serviceability, some allowance should be made for differential shrinkage between the precast and cast *in situ* concrete.

With regard to serviceability, the recommendations for calculation of deflection and cracking normally apply with some additional provisions. For precast prestressed units in composite members, the compressive stress in the concrete may be up to 50% greater than for noncomposite conditions, provided that the ultimate mode of failure of the member is tensile in form and, when the member is effectively continuous, the compressive stress at the end of the prestressed unit within the transmission length for the tendons may be ignored. Also, for prestressed precast units with compositely cast *in situ* concrete, the flexural tensile stress under composite loading in this concrete should not exceed 3.2 N/mm^2 for 25 grade concrete, 3.6 N/mm^2 for 30 grade concrete, 4.4 N/mm^2 for 40 grade concrete and 5 N/mm^2 for 50 grade concrete; these values may be increased by up to 50% so long as there is a corresponding numerical reduction in the tensile stress permitted in the prestressed concrete unit. For normal conditions of use, it will usually be sufficient to assume that the differential shrinkage between cast *in situ* and precast concrete is 100×10^{-6}, otherwise the advice in Part 2 of the Code should be taken.

The flexural strength should be checked for ultimate loading conditions by the methods given for noncomposite sections. Particular attention, however, needs to be paid to the horizontal shear strength at the interface between the precast and the cast *in situ* concrete under ultimate loads. The Code limits the horizontal shear stress for design to values which depend on the nature of the interface and the amount of vertical reinforcement. The horizontal shear force is calculated from the ultimate bending moment and is taken as the total compressive force when the interface is in the tension zone or as the compressive force in that part of the section above the interface when this is in the compression zone. The ultimate shear stress, v_h, for use in design is then obtained by distributing the shear force as given above over the contact area between the point of maximum positive or negative moment and the point of zero moment and then adjusting the value for stress, so calculated, in proportion to the vertical shear stress in the member. Values for v_h are given in Table 12.14. When nominal links are provided across the interface, they should have a cross-sectional area of not less than 0.15% of the contact area. When the horizontal shear stress exceeds that in Table 12.14, the reinforcement required, which should be anchored on both sides of the interface, A_h, in equal millimetres is given by:

$$A_h = (1000b \cdot v_h)/0.87f_y \tag{12.46}$$

The thickness of the structural topping should, in general, be greater than 40 mm and nowhere less than 25 mm, and special care should be exercised in placing to obtain effective adhesion.

12.8 Structural testing

In design and construction in reinforced and prestressed concrete the testing of components and structures can play an important role. It may take the form of:

(1) Model or full-scale testing to aid in the evolution of improved analytical and design methods.
(2) Development testing of prototypes of components or ancillary equipment for performance or feasibility of construction procedure.
(3) Check testing of factory production as a control on quality of output.
(4) Investigations of structural adequacy of construction which for some reason may have become suspect.

Model or full-scale testing to obtain a better understanding of structural behaviour has been used for many types of construction, as already mentioned. The tests may relate to solving general structural problems or may deal with the design of specific structures. In either case, testing requires sophisticated backing of experimental facilities and staff and can be undertaken only by universities and technical colleges, government laboratories and established industrial research organizations. The planning of the experiments and the interpretation of the data are largely matters for those with the expert knowledge and experience.

The development testing of components possibly for mass production in the factory or of ancillary equipment such as bridge bearings, prestressing jacks and structural connections is the direct concern of designers and specialist contractors. The objects may be to produce economic design, to simplify procedures or to ensure that details in design give the performance required. Where concern is with repetitive production or procedures, statistical methods of analysis should be used; where it is with structural performance some care may be necessary to be sure that, in isolating the problem for experimental evaluation, it has not been so simplified that its solution is irrelevant.

Testing as a control procedure has been used very widely by industry for many years and has found application in the production of precast concrete components. It may take the form of nondestructive testing of a proportion of production possibly with a smaller proportion being tested to failure. The types of test and the procedures are covered by British Standards for a large number of products, such as kerbs, paving slabs, pipes and lamp standards.

The Code of Practice, BS 8110, deals with some of the aspects of testing concerned with checking concrete quality and refers to the cutting of cores and the use of gamma radiography, of ultrasonic tests, of covermeters and of rebound hammers. These methods provide some check on construction when the quality of the work is in doubt and are used to give guidance on the need for structural tests on components and structures. Such checks may become necessary because of faults in construction, because the structure has been damaged possibly by fire or because of a change of occupancy.

The Code contains recommendations for load tests on structures or more usually on parts of structures to provide a check on serviceability or strength. It should be noted that when a part of a structure, e.g. part of a floor, is to be tested, attention should be paid to the possibility that some proportion of the test load may be supported by other parts of the structure not being subjected to test. To avoid the effects of lateral transfer of load in tests on part of a floor, the width tested should be at least as great as the span. The deflections and strains on the centreline transverse to the direction of the span may then be assumed to be unaffected.

The test loading recommended in BS 8110 is given as not normally less than the characteristic dead and imposed loads combined, and not greater than either the characteristic dead load with 1.25 times the characteristic imposed load or 1.125 times the sum of the characteristic dead and imposed loads, whichever is the greater. This level of loading is sufficient for checks on serviceability limit states but insufficient for a direct check on ultimate strength. In general, it is seldom feasible to make a direct assessment of compliance with ultimate limit state requirements, since to do so would almost certainly render the

Table 12.14 Permissible design ultimate shear stress (v_h) (N/mm^2)

Grade of in situ *concrete*		25	30	40 and over
Vertical reinforcement	Type of surface			
No links	1	0.4	0.55	0.65
	2	0.6	0.65	0.75
	3	0.7	0.75	0.8
With nominal links projecting into cast *in situ* concrete	1	1.2	1.8	2.0
	2	1.8	2.0	2.2
	3	2.1	2.2	2.5

Notes: Surface type (1) As cast with a rough finish or open-textured as extruded.
(2) Brushed, screeded or rough tamped, i.e. surface deliberately roughened but not exposed aggregate.
(3) Laitance removed by washing or surface treated with retarder and cleaned.

structure unserviceable, but some information on ultimate strength may be inferred from ancillary measurements such as deflection or strain at loads somewhat greater than the characteristic dead and live loads. Some caution is needed, however, in interpreting such information since it may not be possible always to rule out the sudden development of an unexpected mode of failure, such as shear failure when bending failure is expected.

At least two tests should be made applying the load in increments, and an interval of at least 5 min should be allowed after each increment before measurements of deformation are taken with at least a gap of 1 h between tests. Sometimes, it may be preferred to leave the maximum load in position for 24 h. Whenever deformation measurements are to be made in tests, provision should be made for recording the environmental temperature and, if there are likely to be variations, the relative humidity, since changes in these conditions may affect the results.

Results should be compared with the design calculations, and it may therefore be necessary to establish the strengths of the materials, determined in the way described in the appropriate British Standard, and to allow for any features of the structure that differ from what was assumed in design. Cracking and deflection in the tests should be consistent with the design calculations and, after the second loading, the recovery should be at least 75% for reinforced concrete and class 3 prestressed concrete structure and at least 85% for other prestressed concrete construction. After the tests, the structure should be examined to determine whether there are any unexpected signs of distress.

For prestressed concrete structures any departure from linearity of the load–deflection curve when plotted will give some indication of the level of prestress existing in the structure. This information is particularly useful in the case of structures damaged by fire since the retention of a high proportion of the prestress indicates that the tendons have not lost strength.

Other forms of test may be necessary to establish the structural performance of possibly an unusual form of construction or of precast concrete components as part of a quality assurance scheme, when the test schedule should be agreed by all concerned. Should it be necessary as part of such a system to test components to failure, the Code recommends that the ultimate strength of the units should be not less than 5% greater than the design ultimate load and the deflection under the design ultimate load should not exceed 2.5% of the span.

12.9 Fire resistance

The Building Regulations define requirements for health and safety in general and set amongst other matters the provisions that must be made to ensure safety in buildings in the event of fire. The Regulations deal with the prevention of the spread of fire, means of escape and facilitating the fighting of fires. They set out the maximum size of buildings or compartments into which buildings should be divided according to the class of occupancy and height, and then define the required fire endurance for the size of building and occupancy. The endurance is determined by tests conforming with the requirements of BS 476.

Owing to lack of comprehensive information on overall behaviour of structures it is assumed that, if each component part of the structure, walls, floors, columns and beams, is designed for the required fire resistance, the whole structure will have the necessary fire resistance. In general, this assumption would appear to be conservative but for certain types of structure, e.g. unbraced tall framed buildings, it might be conceivable for a fire to become widespread in one storey and so lead to instability of the columns at that level as their stiffness reduces with increasing temperature. Isolated fires would probably not produce this effect since the requirements for ties to prevent excessive damage due to accidents would enable the structure to bridge over any individual failures. The possibilities should, however, be given some consideration in design.

Both the Building Regulations and the Code give requirements for the minimum dimensions of members for various periods of fire resistance for concrete floors, walls, beams and columns. These are based on the results of fire tests in which the members were tested in a furnace heated at a rate laid down as a standard time–temperature curve. Results of these tests show that a number of factors may have an important influence although not all can yet be taken into account in design: these include the effects of size and shape of member, the properties of the different types of steel and concrete that may be used, the protection afforded to the steel, the level of stress that must be sustained by the steel (which is dependent on the proportion of the service load carried at the time of the fire) and the degree of restraint provided by the rest of the structure.

Parts 1 and 2 of the Code offer three methods of designing for fire resistance, the simplest method being presented in Part 1 where brief requirements for cover and minimum dimensions of members are tabulated. Some of this information has been extracted in Tables 12.15 and 12.16. Both parts of the Code should, however, be consulted for further details.

The factors, which affect the limitations defined in the tables apart from cover and size, are the degree of exposure of the member, whether it is continuous or simply supported, the type of steel used, and the maximum size and nature of the aggregate. Clearly, if a column is incorporated in construction in such a way that the full effects of a possible fire would not be experienced, a smaller amount of concrete cover is needed to control the rise in temperature of the steel. Continuity of construction allows some redistribution of moments and forces from weakened regions to those less severely heated and, hence, the temperature rise in the steel is less critical in its effect on fire resistance.

The steels used for prestressing are more sensitive to loss of strength at higher temperatures than those used as reinforcement. For example, cold-drawn wire or strand starts to lose strength at about 150° C and is reduced to half strength at about

Table 12.15 Fire resistance – minimum dimensions of reinforced and prestressed concrete members

Fire resistance (h)	*Min. width of beams* (mm)	*Min. width of ribs* (mm)	*Min. thickness of floors* (mm)	*Min. width of columns* 100% *exposed* (mm)	50%
0.5	200	125	75	150	125
1.0	200	125	95	200	160
1.5	200	125	110	250	200
2.0	200	125	125	300	200

Table 12.16 Fire resistance – nominal cover requirements for all steel including links

Fire resistance (h)	*Nominal cover* (mm) *Beams* (SS)	(C)	*Floors* (SS)	(C)	*Ribs* (SS)	(C)	*Columns*
		(SS = Simply supported: C = continuous)					
Reinforced concrete							
0.5	20*	20*	20*	20*	20*	20*	20*
1.0	20*	20*	20	20	20	20*	20*
1.5	20	20*	25	20	35	20	20
2.0	40	30	35	25	45⁺	35	25
Prestressed concrete							
0.5	20*	20*	20	20	20	20	—
1.0	20	20*	25	20	35	20	—
1.5	35	20	30	25	45⁺	35	—
2.0	60⁺	35	40	35	55⁺	45⁺	—

Notes:
*These thicknesses of cover may be reduced to 15 mm provided that the maximum size of aggregate does not exceed 15 mm.
⁺Additional measures are required to avoid spalling referred to below.

400° C, whereas the corresponding temperatures for reduction in the yield stress for reinforcement are 300 and 550° C. Hence, the requirements for cover in prestressed concrete are more onerous than for reinforced concrete.

Spalling of concrete in fire may be experienced when the initial water content of the concrete is greater than 2–3% for dense aggregates and becomes increasingly likely when the thickness of concrete cover exceeds about 40–50 mm. It is most common with siliceous aggregates such as flint gravel, much less so with limestone aggregate and rare with lightweight aggregates, for which the requirements for concrete cover may be reduced. Spalling is also aggravated if thermal expansion of the member is restrained leading to high stresses in the concrete and if the shape of the cross-section leads to concentrations of stress under severe temperature gradients. Its effect is to expose the main steel to the full effects of the fire and so result in premature failure but its effect may be determined by full-scale fire-testing or may be offset by appropriate detailing in design.

The measures that can be taken to avoid spalling include the application of a plaster or vermiculite concrete finish, the provision of some form of cladding such as a suspended ceiling, and the incorporation of a wire fabric mesh within the cover but conforming with the cover requirements for durability or the use of lightweight aggregates.

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13

Practical Steelwork Design

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Contents

13.1 Standards for the design of structural steelwork

13.1.1 British Standards and codes of practice

The British Standards and codes of practice for the design of structural steelwork have been reviewed over the past two decades and new codes, based on limit-state philosophy, have been developed. Some of the new codes have been published since 1980 and others are due to be issued after the publication of this book.

The earlier standards, which have served the industry well for a considerable number of years, are based on elastic analysis. Before being withdrawn they will continue to be available for a transition period of some years from the date of issue of the new codes.

It should be noted that, whereas many current British Standard (BS) codes of practice have the prefix CP in front of the number, more recent issues have BS numbers which the British Standards Institution (BSI) intends to adopt for all future codes. The use of the separate CP numerical series will therefore gradually disappear.

The philosophies behind the new steelwork documents differ considerably from those which have gone before. In particular they will require limit-state design to be used, entailing predictions of collapse loads and limits of serviceability and the adoption of partial load factors for different classes of loading taking into account, perhaps somewhat crudely, the statistical probabilities of different classes of loading occurring simultaneously.

The reader may wonder what the difference is between a standard specification and a code of practice. According to the BSI (see the 1981 BS guide *A standard for standards*), a specification is a detailed set of requirements to be supplied by a product, material or process indicating, wherever appropriate, procedures for checking compliance with these requirements. The function of a specification is to provide a basis for understanding between the purchaser and supplier, and the text is usually written with this interface in mind.

The main function of a code of practice is to recommend good accepted practice as followed by competent practitioners. Codes bring together the results of practical experience and research in a form that enables immediate use to be made of proven developments and practices. Codes tend to be complex documents, in many cases almost resembling textbooks – not only do they recommend good practice but some also indicate practices to be avoided.

Specifications may be looked upon as mandatory documents and codes as only advisory ones. In the UK, however, the code requirements ultimately become mandatory too, since many structural codes are called up, or referred to, in the national Building Regulations.

The following lists some of the more important standard specifications and codes of practice currently in use (1988) for the design of steelwork for structural applications.

(1) BS 153:1972 *Steel girder bridges*:
Part 1, 'Materials and workmanship'.
Part 2, 'Weighing, shipping and erection'.
Part 3A, 'Loads'.
Part 3B, 'Stresses'.
Part 4, 'Design and construction'.

It should be noted that this standard is no longer cleared for use in the UK by the British Department of Transport, and BSI have withdrawn it; however, it must be recognized that some overseas countries may still refer to it.

(2) British Standard 449 *The use of structural steel in building*:
Part 1, 1970: 'Imperial units'
PD 3343, Supplement No. 1 to BS 449:1970, Part 1, 'Recommendations for design'.
PD 4064, Addendum No. 1 (1961) to BS 449:1970, Part 1, 'The use of cold-formed steel sections in building'.
Part 2, 1969: 'Metric units': Addendum No. 1 (1975) to BS 449:1969, Part 2, 'The use of cold-formed steel sections in building'.

Part 1 of this standard is, according to BSI, obsolete although it is still referred to both in the UK and, particularly, in overseas countries still using imperial units.

The Supplement No. 1, being an extract from the final report of the Steel Structures Research Committee in 1936, is now a rather dated document that is rarely referred to in present-day designs.

The Addendum No. 1 (1961) to BS 449:1970, Part 1, covers cold-formed sections and is another obsolescent imperial document which the BSI have not yet withdrawn.

Addendum No. 1 (1975) to BS 449:1969, Part 2 is simply the metricated equivalent of the imperial version. Ultimately, BS 5950, Part 5, when published, will supersede it.

British Standard BS 449:1969, Part 2 is a metricated version of Part 1 of 1970. This will ultimately be superseded by BS 5950, Parts 1 and 2; meanwhile the BSI have indicated that there should be a transition period prior to its withdrawal.

(3) Code of Practice 117 'Composite construction in structural steel and concrete'.

Part 1, 1965: 'Simply supported beams in building'.
Part 2, 1967: 'Beams for bridges'.

Both parts of this code of practice are in imperial units. Part 1 will ultimately be superseded by BS 5950, Part 3. Part 2 has been superseded by BS 5400:1979, Part 5, and has therefore been withdrawn by the BSI.

(4) British Standard 5400 *Steel, concrete and composite bridges*.

Part 1, 1978: 'General statement'.
Part 2, 1978: 'Specification for loads'.
Part 3, 1982: 'Code of practice for design of steel bridges'.
Part 5, 1979: 'Code of practice for design of composite bridges'.
Part 6, 1980: 'Specification for materials and workmanship: steel'.
Part 9, 1983: 'Bridge bearings': Section 9.1, 1983 'Code of practice for design of bridge bearings', and Section 9.2, 1983 'Specification for materials, manufacture and installation of bridge bearings'.
Part 10, 1980: 'Code of practice for fatigue'.
Part 10C, 1980: 'Charts for classification of details for fatigue (these are large wall charts of the details presented in Part 10)'.

This standard, which has been written in metric units, generally has the approval of the British Department of Transport. Part 5, as written, is not entirely acceptable and modifications to the requirements are to be published shortly. It supersedes BS 153 (all parts) and CP 117, Part 2.

In order to assess the effectiveness and soundness of the newly drafted standard, the Department of Transport financed an extensive calibration exercise from which much valuable experience was gained. These standards were therefore 'tried and tested' before publication.

(5) British Standard 5950 *The structural use of steelwork in building*.
Part 1, 1985: 'Code of practice for design in simple and continuous construction: hot-rolled sections'.
Part 2, 1985: 'Specification for materials, fabrication and erection: hot-rolled sections'.

Part 3, 'Code of practice for design in composite construction'.
Part 4, 1982: 'Code of practice for design of floors with profiled steel sheeting'.
Part 5, 'Code of practice for design in cold-formed sections'.
Part 6, 'Code of practice for design in light gauge sheeting, decking and cladding'.
Part 7, 'Specification for materials and workmanship: cold-formed sections'.
Part 8, 'Code of practice for design of fire protection for structural steelwork'.
Part 9, 'Code of practice for stressed skin design'.

Most parts of this standard are in the drafting stage. Only Parts 1, 2, 4 and 5 had been published by 1988.

British Standard 5950 is being written in metric units and will be based on limit-state philosophy where appropriate. Parts 1 and 2 will eventually supersede BS 449, Part 2. With a view to 'debugging' the newly drafted Parts 1 and 2, the UK Department of the Environment financed a large-scale calibration exercise to verify its suitability prior to finalization and publication.

Part 3 will supersede CP 117, Part 1, and Part 5 will replace Addendum No. 1 to BS 449:1969.

Parts 4, 6, 7, 8 and 9 cover subjects which have not previously been included in British Standards.

In the field of structures for agricultural purposes, the relevant standard is BS 5502:1978–86, Parts 1, 2 and 3 'Code of practice for the design of buildings and structures for agriculture'. Each part is published in a number of separate sections covering, for example, materials, design, fire protection, insulation, etc.

On the subject of overhead runway beams, the current standard, namely BS 2853:1957 *The design and testing of overhead runway beams*, is a very outdated document, written in imperial terms and containing information on steel material and some steel beam sizes long-since withdrawn from production. It does not include details of the parallel flanged universal I-shaped sections specified in BS 4, which present certain design problems that do not arise with the older tapered flange joist sections. Apart from the general design philosophy contained therein, only the section dealing with testing requirements is currently pertinent. For more detailed information on this subject, it is recommended that the reader refers to runway manufacturers' publications.

13.1.2 European and international standards and codes of practice

Eurocode 1: *Basic principles.*
Eurocode 3: *Steel structures.*
Eurocode 4: *Composite steel and concrete structures.*
International Organization for Standardization (ISO): *Steel and aluminium structures.*
Part 1: 'Steel – material and design'.
Part 2: 'Steel – fabrication and erection'.

These standards and codes have been published only in draft form. In many respects they will be similar to BS 5400 and 5950. As they are currently at the drafting stage, considerable committee work is still anticipated before the documents are finally issued. It is well known how long it takes and how difficult it is to reach agreement on the contents of standards and codes within any one country. The reader will readily appreciate, therefore, how much more difficult drafting will be, and how it will be extended over longer periods, before agreement is reached both within Europe and internationally.

The reader may be interested to know why the UK is so concerned with the drafting of Eurocodes and international standards for structural steelwork. Basically, they become harmonization documents for each country's national standards and regulations governing the construction industry both in Europe and worldwide. In the case of Europe it is anticipated that, once published, Eurocodes will be called up in EEC framework directives and thus override national standards.

13.2 Steel as a structural material

13.2.1 Fundamentals of the steelmaking process

It is axiomatic that the designer of any engineering undertaking ought to have a fair understanding of the nature of the material he proposes to use. This ideal does not always obtain in steelwork or other building materials. To provide some grasp of the varying characteristics of steel and the origin and nature of possible defects, it is necessary to consider the manufacturing processes by which steel plates and sectional shapes are made.

The production of structural plates and sections is a three-stage process, namely: ironmaking, steelmaking and rolling. Ironmaking is performed in a blast furnace and consists of chemically reducing iron ore, using coke and crushed limestone. It is essentially a continuous process. The resulting material, cast iron, is high in carbon, sulphur and phosphorus. Steelmaking, on the other hand, is a batch process and consists of refining the iron to reduce and control carbon, sulphur and phosphorus and also to add controlled proportions of manganese, chromium, nickel, vanadium, niobium, etc. where necessary, depending on the grade of material to be produced. During this century, the technique of steelmaking has undergone vast changes in scale and new processes have been developed continually to meet the demands of speed, quantity and quality. Today, however, there are only two major steelmaking processes: (1) electric arc; and (2) basic oxygen (BOS). The latter method is really an enlarged and refined development of the old basic Bessemer process, now generally obsolete, and is some 15 times faster than the open hearth process, which is also obsolescent.

Comparatively little iron is allowed to solidify, the metal mostly being tapped and transferred directly, in the liquid state, to the steelmaking furnace. The 'melt', so called, in a steel furnace may well be several hundred tonnes, economy deriving from bulk production. This in large measure explains why small quantities of specially alloyed steels are expensive.

The steelmaking process may last an hour or more, during which chemical change is taking place. Samples are taken at intervals and analysed for composition in a laboratory. During the minutes which this takes the chemical process continues, and it remains a matter of nice judgement when to stop it by cutting the oxygen and tapping the melt into a teeming ladle. Once tapped into the ladle, a further sample analysis is made, the 'ladle analysis', which is taken as a record of the whole melt; however, it must be recognized that there may be some variability in the dispersion throughout the melt, which explains why samples of a part of the product may show slight variations from the ladle analysis. At this stage most of the slag, being lighter than the steel, rises to the surface of the ladle or is left behind in the furnace.

Next, the steel is poured, or 'teemed', from the ladle into moulds to form ingots (Figure 13.1) into specially shaped castings, or directly into slabs, blooms or billets by the continuous casting process (Figure 13.2)

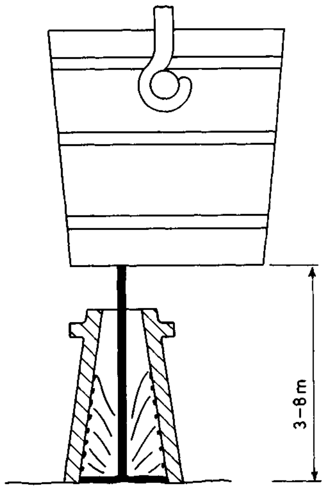

Figure 13.1 Teeming

13.2.1.1 Ingots

Not so long ago, a 10 t ingot was considered big; today, one of 40 t is common. Defects that concern the structural engineer may occur at this stage. In the first place, the steel may be poured from as high as 6 or 9 m into the bottom of the mould, splashing up the sides. Some drops, which 'freeze' instantly on contact with the relatively cold mould, may not remelt when the surface level rises to encompass them, and may even not fully forge into the body of the steel on rolling. This results in surface imperfections, which are mostly of little importance, apart from appearance. More seriously, oxidation inevitably occurs at the free surface of liquid metal, and some slag may still remain in the melt.

Most of this nonmetallic material floats to the surface of the ingot before solidification, but some may remain in the body of the steel, leading to internal laminations after rolling (Figure 13.3).

Ingots free of slag inclusions can be produced by special processes, such as uphill teeming (Figure 13.4) but inevitably are more expensive. For the bulk of heavy-engineering purposes, therefore, one must expect a small amount of laminations. As

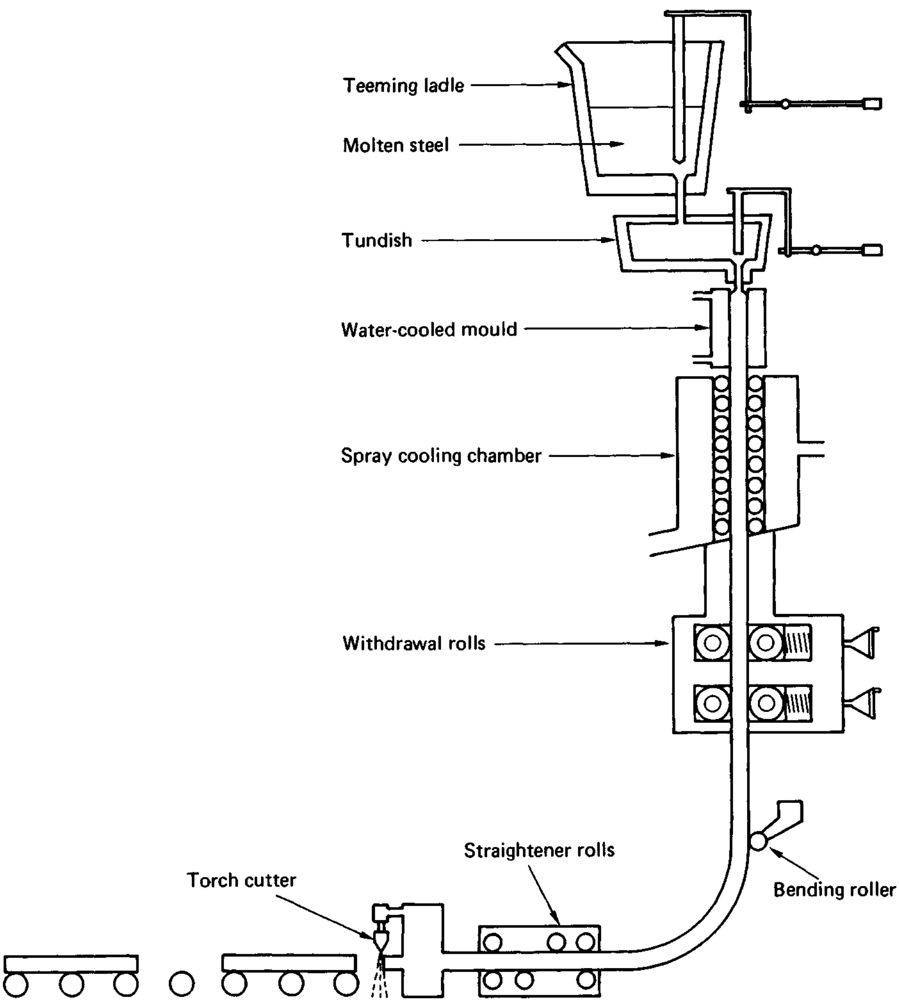

Figure 13.2 Continuous casting

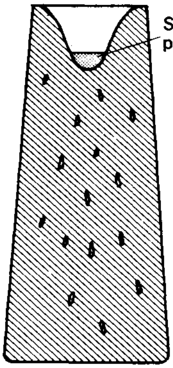

Figure 13.3 Inclusions

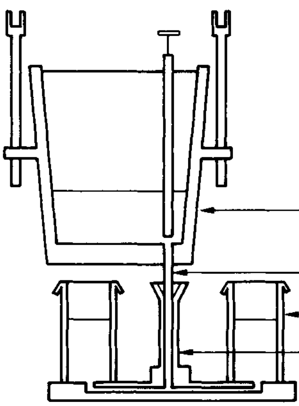

Figure 13.4 Uphill teeming

with knots in wood, what matters is where they are, how big they are and whether they render the material unfit for its intended purpose.

After the ingot has solidified and the mould has been stripped off, the ingot is transferred to a soaking pit, in which other hot ingots are stacked, to ensure even distribution of heat. After a period of time the red-hot ingot is removed from the pit and passed to the primary rolling mill. Here, the top end containing the slag puddle is cropped. The amount to be cropped is a matter of nice judgement on the part of the mill operator, the ingot passing towards him top end first. If he crops too much, the yield (i.e. the proportion of rollable product) is reduced; if too little, end piping or lamination will be present. Naturally, he errs on the safe side, but mistakes can occur sometimes leading to laminations in the end bar or two of a rolling. The ingot is now rolled down in a primary cogging or blooming mill by a series of passes to and fro between the rolls, which are closed slightly between each pass, reducing the girth of the metal and elongating it (Figure 13.5). The hot semi-finished blooms may be allowed to cool at this stage or be further rolled down to billets or narrow slabs. Some of these products may then be sold under the description of 'semi-finished products' for subsequent reheating and finish-rolling.

13.2.1.2 Special castings

Steel castings of various shapes for structural purposes vary in size and are usually required for special applications, such as bridge bearings, crane hooks, rope saddles, rope sockets and special jointing components for structural frames, etc. The molten steel is teemed into sand moulds which are broken up, when the steel has solidified, to remove the castings.

13.2.1.3 Continuous casting

Hitherto, before molten steel could be rolled or formed into plates, sheets, sectional shapes, bars, etc., it had to be cast into ingots which were then reheated in furnaces (or soaking pits) to bring them to a uniform temperature suitable for rolling semi-finished products in a primary mill.

In the continuous-casting process, the liquid metal is teemed direct into a casting machine (Figure 13.2 shows the process diagrammatically) which produces billets, blooms, slabs and dog-bone blooms for sectional shapes, instead of going through the ingot casting stage before being reheated and rolled in a primary mill.

The development of continuous casting has therefore eliminated much of the primary process and, whilst it has not yet superseded all steel-product making, it is used for a large proportion of the production of structural-steel products. Some of the larger-sized shapes, however, are still produced by the ingot route. A further economy of the continuous-casting process is the increased yield of usable product from a given weight of steel.

13.2.1.4 Finishing process

The semi-finished product, produced by either the conventional ingot route or the modern continuous-casting process, is then passed to a finishing mill, a universal beam mill, plate mill, etc. where it is further reduced in size by rolling through a series of reversals until it reaches its final shape which can, for example, be plate, sheet, joist, channel, universal beam, universal column, angle, square or round. Plate is rolled in a two-stand reversing mill which uses two rollers (Figure 13.5) and in which the gap between the rolls can be reduced progressively, the final gap determining the finished thickness. For very heavy work or for wide plates, two further back-up rollers may be incorporated to prevent the working rollers from bending (Figure 13.6). The maximum width available clearly depends upon the width of the widest mill which, in the UK, is currently 3.9 m.

Joists, channels and angles are also rolled in two-stand reversing mills similar to those for plates except, of course, that the desired shape is achieved by machined grooves in the rolls (Figure 13.7 shows the roll shape for forming channel sections). The sections are obtained by being passed through a succession of grooves, whose shapes change progressively from the billet

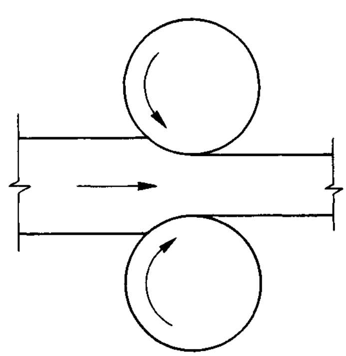

Figure 13.5 Two-stand reversing mill

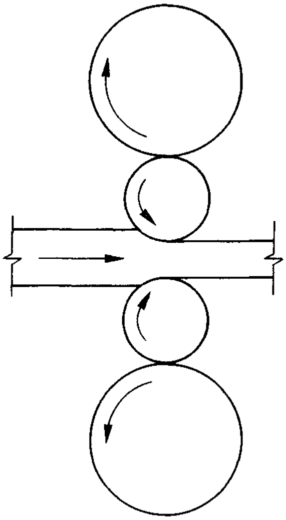

Figure 13.6 Use of back-up rollers

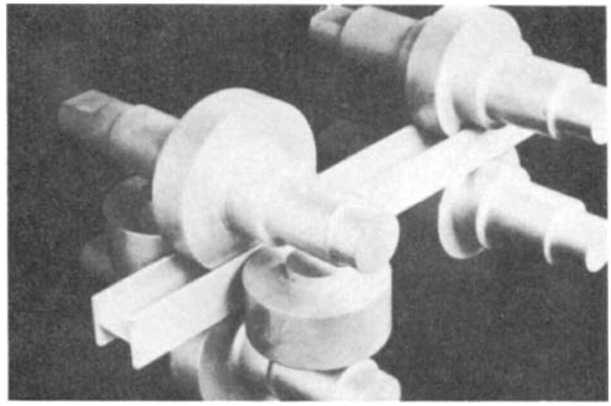

Figure 13.8 Universal mill

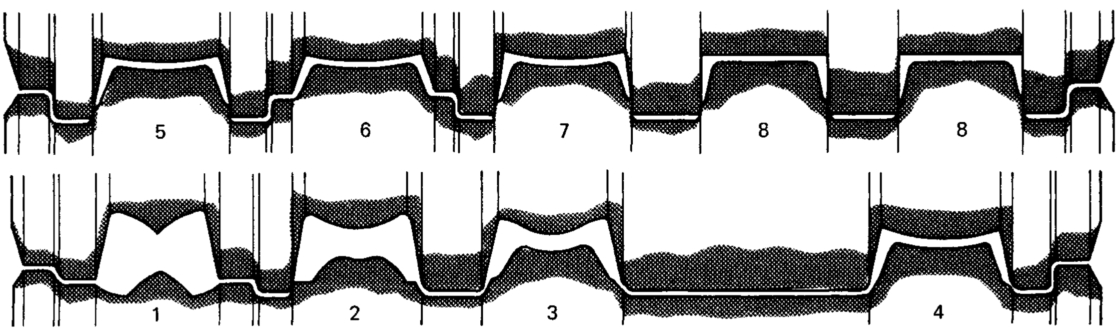

Figure 13.7 Roll shapes and forming sequence for channel sections

shape to that of the final section. Clearly, different sets of rolls are needed for each different sectional shape and size. For maximum economy, roll-changing must, however, be kept to a minimum and the tonnage output of any one rolling kept as high as possible.

Universal beams and columns are rolled in a different kind of mill, known as a universal mill, so called because for any one serial size of a sectional shape, it can roll a variety of final weights. Roll-changing is only necessary when the serial size needs to be changed. Figure 13.8 shows the principle of this type of mill from which it will be seen that different sizes within a serial range can be produced by varying the spacings between both the vertical and horizontal rolls. It is particularly important to note that for universal sections the dimension between the inside faces of the flanges is constant for any one serial size and not the overall dimensions as with rolled joists and channels. The web thicknesses, flange widths and thicknesses can all be varied.

Irrespective of the type of rolling mill employed, the total number of passes depends upon the final required thickness of product, and the cooling rate increases as the thickness is reduced. Thin sections, therefore, go through their final pass much cooler than thick sections, and in so doing take up a degree of work-hardening not evident in thick sections. Since, for any grade of steel, it is desirable to have as little variation in yield strength with thickness as possible, and as alloying elements are expensive, the steelmaker aims to keep additions to a minimum if thin sections are being rolled. Likewise, thick sections are likely to contain alloying elements near to the maximum for the particular specification. Consequently, the material of which thin sections are made is inherently easier to weld than thick sections, apart from thermal problems which are likely to arise with thick material.

After rolling, the hot bars or plates are sawn to length and transferred to a cooling bank. (It is to be noted that at this stage the steel is most unlikely to be straight or flat.) The rate of cooling of different parts of the plate or section will vary, depending upon its exposure. For instance, the toes of flanges and the centre portion of a deep thin web of an I-section will cool faster than the thicker material at the junction of the web and flanges, and it is this which creates the pattern of residual stresses (Figure 13.9a). The residual stress patterns for channels, angles and plates are shown in Figure 13.9(b)–(d). When the cold material is cold-straightened, sometimes by rolling and sometimes by pressing, dependent on the shape, this affects the level and distribution of residual stresses. The straightening process has a stress-relieving effect and so a bar which needs extensive treatment will finish with a low level of residual stress, while the rare bar which remains straight when cooled will have the highest level. The presence of residual stresses is most clearly seen when a member is cut longitudinally, as when an I-member is slit into two Ts, which curve noticeably on division owing to redistribution of longitudinal stresses. Plate material also shows similar behaviour to a smaller extent.

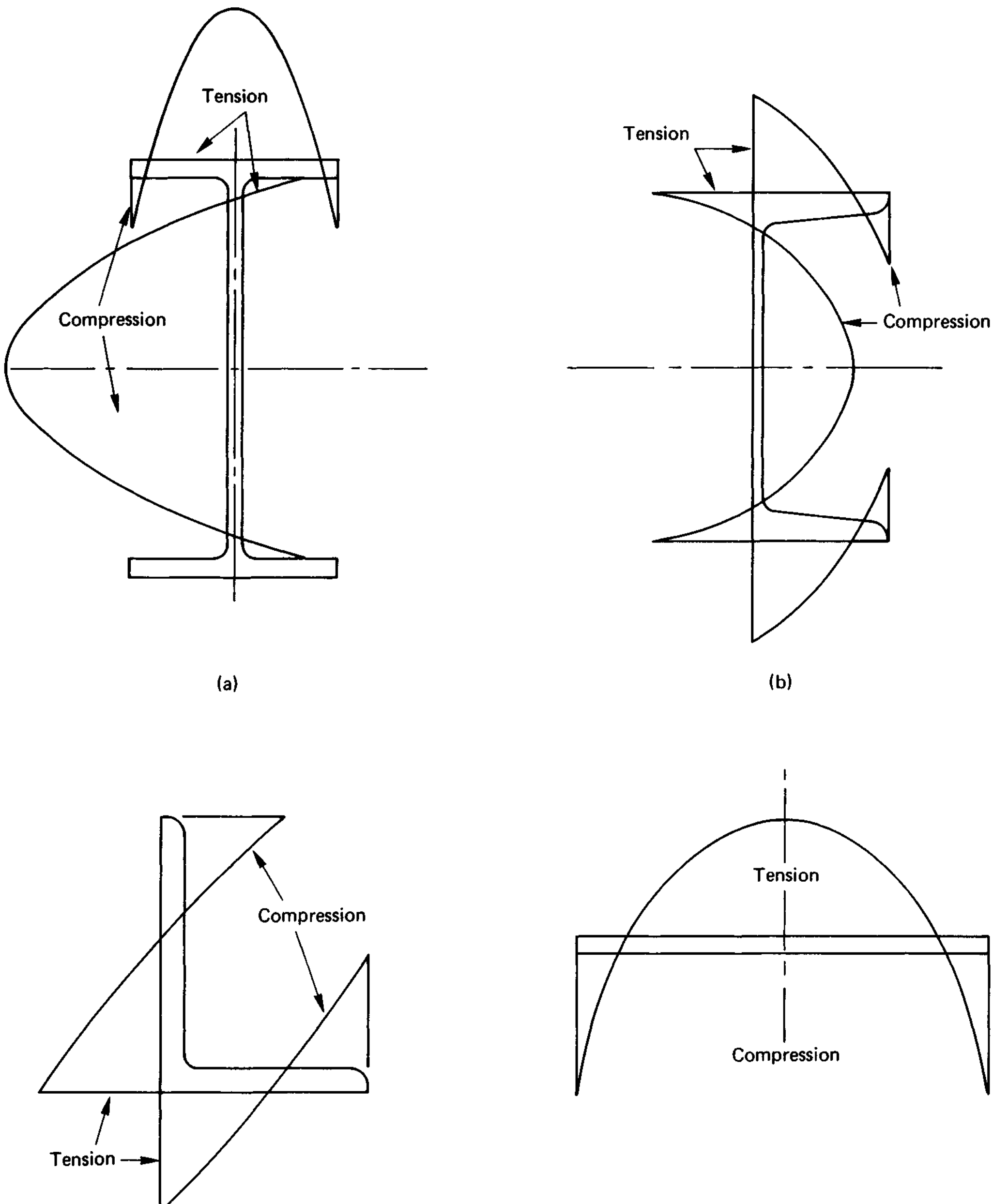

Figure 13.9 Residual stress patterns. (a) Universal sections and joists; (b) channels; (c) angles; (d) flats and plates

13.2.2 Fundamental properties of structural steel

Figure 13.10 indicates the familiar tensile stress–strain curve for steel. The properties of usual concern to the the design engineer are the value of the yield strength (R_e) and the gradient of the elastic portion, i.e. the modulus of elasticity, the value of which may be taken as between 200 and 210 kN/mm². Fortunately or otherwise (some think otherwise) the second property varies little from one grade of steel to another and cannot be controlled. It is to be noted that the finite value of the tensile strength (R_m) sometimes incorrectly referred to as the ultimate tensile strength (UTS) is of little significance in structural design, since a structure can have manifestly failed at a stress much lower than yield stress. The gradient of the line immediately after yield (i.e. the strain-hardening modulus) can be of significance in plastic design as it can affect moment/rotation behaviour. But the property which is at least as significant from a structural point of view as the yield strength is the ductility, expressed by the length of the horizontal portion of the curve. It is this plateau of ductility which enables the steel to relieve and redistribute residual stresses arising from cooling and welding.

Another factor to be recognized is that the yield and tensile strength values quoted in standards usually refers to a specimen cut in the rolling direction. A similar specimen cut transversely to the direction of rolling would show lower characteristics and, for thick plate, through-thickness strength can be markedly less.

Having referred to residual welding stresses some remarks concerning their mode of origin and magnitude seem appropriate. During welding, the heat put into the heat affected zone (HAZ) causes the zone to try to expand, but this expansion is prevented by the surrounding cold material. The HAZ therefore yields in compression and becomes effectively shorter and, when cooled, attempts to contract further causing stress reversal. The weld and the HAZ is then in a state of residual tensile stress, with a corresponding residual compressive stress in the main bulk of the material maintaining equilibrium. It can be assumed that, as welded, all weld metal and HAZ material is in a state of tensile stress equal to the yield stress. These residual stresses can be relieved, by heating in a stress-relieving furnace or by local heating with electric mats or by proof loading to a degree greater than that anticipated in service. It is here that the property of ductility comes into play. For the majority of structural purposes, stress-relieving is too expensive and occurs fortuitously on the first application of severe load, during which the 'peaks' of the residuals are cut off, so to speak. This explains the partly inelastic initial deflection of a heavily welded member. Recoveries of initial deflection of 80 to 90% have been observed, after which behaviour is fully elastic.

The foregoing underlines the necessity for achieving adequate ductility in the weld metal. The attainment of strength usually presents no problem, owing to the quenching effect of fairly rapid cooling. This quenching effect can be too severe tending to produce brittleness as, for example, in multipass welds in thick material. Therefore, preheating and, in extreme cases, postheating, are needed in order to slow the cooling rate in the interests of achieving ductility.

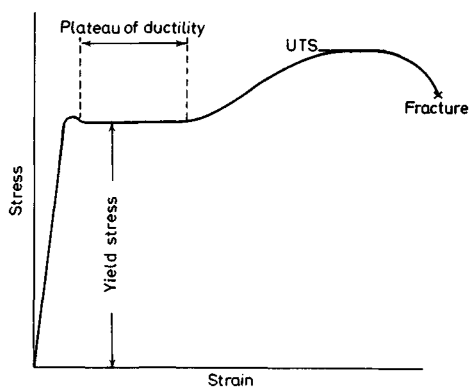

Figure 13.10 Tensile stress–strain diagram (mild steel)

13.2.3 Notch ductility

Let us now consider the mechanism of ductility. We recognize that uniaxial extension is accompanied by transverse contraction. If biaxial tensions are present, only one dimension remains to provide elongation, i.e. a reduction in thickness. If triaxial tensions are present, no distortions are possible, and apparent strength is much greater than the yield strength shown by a uniaxial tensile test piece. (This is the converse effect of triaxial compressions in soils or concrete.) However, a marked loss of ductility accompanies this increased tensile strength. We then have the situation where a material demonstrably ductile can behave in a brittle fashion under certain stress conditions. The material itself does not distinguish between the origins of different stresses, so that the situation where residual stresses in two mutually perpendicular directions exist together with applied stress in the third is to be avoided. This situation occurs in positions such as joints, where three-dimensional continuity is accompanied by restraint against contraction.

Stresses at the tip of a crack subjected to tension perpendicular to the plane of the crack are essentially triaxial which, as we have seen, leads to brittleness. The property of notch ductility or toughness is the ability to resist the propagation of such cracks under tensile stress.

One must recognize that, even with the closest of control over an industrial process, some defects will occur. It therefore follows that no weld can ever be perfect and may contain slag inclusions, porosity and cracking, and so for design purposes one must assume that welds contain cracks which are undetectable. We have therefore to face the fact that adequate notch ductility is vital in any welded structure. The problem, however, is one of measurement.

The generally accepted measure of notch ductility is given by the Charpy impact test in which a 10 mm square notched test piece 100 mm long is struck and broken by a pendulum and the loss of kinetic energy measured. The specimen is supported at both ends and struck in the centre opposite the notch, putting the notch tip into tension. The radius at the notch tip is 0.25 mm which is very much greater than that at a crack tip. The specimen is also quite small relative to actual structures and, of course, not subject to residual stresses.

If, however, standard material is tested over a range of temperatures, and energy value is plotted against temperature, a curve of the form shown in Figure 13.11 results. Steel shows the characteristic of high notch ductility at high temperatures and a marked fall in notch ductility at low temperatures. The temperature, or rather range of temperature, over which this transition takes place is known as the 'transition temperature'. The transition temperature can be altered by heat treatment of the steel, quenching tending to raise the level, and annealing, or normalizing, tending to lower it. For a given steel, strength can be increased at the expense of notch ductility and transition temperature, and vice versa. For any particular requirement a balance has to be struck.

It is important to recognize, though, that the Charpy test has its limitations. Firstly, the specimen is small and does not reveal the inherent increase of brittleness with thickness due to triaxial

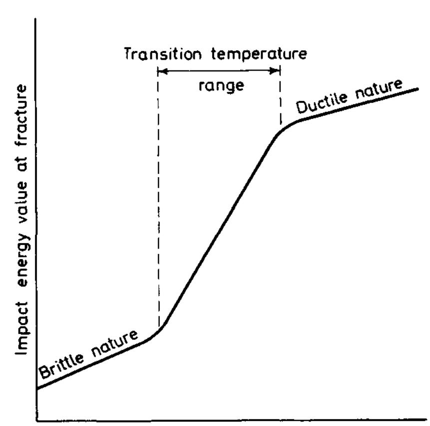

Figure 13.11 Impact–transition temperature curve

effects. Secondly, the notch tip is relatively blunt and, finally, the strain rate is high. High strain rate leads to loss of ductility, so the combination of these effects means that a finite value of a Charpy test does not give any accurate guide as to the actual in-service behaviour of a particular structure or detail. What it does provide is a quality control standard at the steelworks by which a batch of steel can be compared with another of known performance.

Other tests giving much closer correlation to in-service performance have been evolved, notably the Wells wide plate test, the crack opening displacement test and the Pellini drop weight tear test.

Factors leading to possible brittleness, as stated in the first paragraph of this section, include three-dimensional continuity or restraint, such as is usually present in welded components of thick material, coupled with loading giving high strain rates. But in addition to these, three features are essential for a brittle fracture to occur, namely: (1) a notch or severe stress concentration; (2) a tensile stress; and (3) a service temperature below the transition temperature. If any one of these three features is absent, brittle fracture cannot occur. The task of the designer, therefore, is to make a careful assessment of the particular situation in order to determine the degree of risk. In the majority of structural configurations, the risk is slight but where it is not, such factors as are under the designer's control, namely the detailed design and the material quality, in that order, must be modified. It is true to say that, of the failures that have occurred, more could have been prevented by good detailed design than by varying the steel quality.

For a fuller treatment of the subject of brittle fracture and testing procedures the reader should refer to material prepared by Richards[1] at the Welding Institute.

13.2.4 Fatigue

Failure from brittle fracture is sudden and temperature-dependent and can occur at any time in the life of a structure (though usually early owing to the effect of in-service stress relieving). Failure due to fatigue, however, is quite different. It arises principally due to the cumulative effect of pulsating or alternating stresses, and propagates slowly, often over a period of years. As one would expect, initiation occurs at a stress raiser – such as a sharp notch or crack – and its rate of growth is dependent both on the magnitude of the stress variation and the total number of cycles of stress. Whilst fatigue crack propagation occurs slowly it must be appreciated that it is nonductile in nature, i.e. no significant deformation takes place, and therefore detection of cracking can be difficult.

In considering the design of a structural element, the designer's first task must be to assess whether the live loading is essentially static or dynamic. In the majority of cases of building steelwork (with the notable exception of gantry girders and other lifting gear) the loading is essentially static and therefore fatigue presents no problems. Design may, therefore, proceed on the usual working stress basis. However, in bridgework, crane structures and their supports, and other structures subject to dynamic loading such as wind-induced vibrations, prevention of fatigue failure may determine the design.

As indicated earlier, stresses can be fluctuating above a certain minimum level or alternating or somewhere between the two. What matters is the stress range, or amplitude and the mean stress. It has been established that if one does a series of tests on identical specimens, varying the stress range S and recording the number N of cycles to failure and the results are plotted of S against log N, a straight line results as shown in Figure 13.12. This shows that, as the stress range is reduced, so the number of cycles to failure is increased. A family of curves can be derived for different levels of mean stress. If the proposed

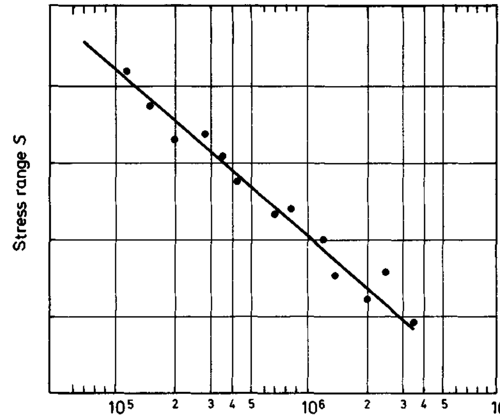

Figure 13.12 Fatigue curve

life is known, then a permissible stress range can be found and compared with that permitted under static conditions.

If we now study the effect of varying the form of specimen to include on the one hand plain rolled plate and on the other a plate with discontinuities, such as holes, notches or welded attachments, another phenomenon presents itself, i.e. that the presence of discontinuities significantly reduces the fatigue life – the more severe the discontinuity, the greater the effect. A plain-rolled plate at a stress of 250 N/mm^2 will give a fatigue life of 2×10^6 cycles in pulsating tension, i.e. at 2×10^6 cycles, plain grade 43 steel plate has a strength equal to the yield stress. (Note that there is no particular magic about 2×10^6 cycles. It may be a more than adequate life or it may be grossly insufficient in any particular case.)

A detail with a marked discontinuity can show a strength at 2×10^6 cycles, as low as 30 or 40 N/mm^2. As a result of extensive experimental work at the Welding Institute it has been possible to classify various structural details in terms of their severity as regards fatigue. These are clearly presented in the fatigue clauses of BS 5400:1980, Part 10, to which reference should be made.

For crane structures designed to BS 2573 *Rules for the design of cranes* (Part I, 1983 and Part 2, 1980), it should be noted that the fatigue rules are still based on BS 153 rules, not those in the new BS 5400, Part 10.

It may seem curious that fatigue can be a consideration when the applied loading is wholly compressive. This is because, as explained earlier, the presence of high residual stress due to welding can cause local reversals. Thus, a fatigue crack in a zone of high residual tensile stress, not overcome by the applied compressive stress, will propagate until it reaches a zone where the total stress is wholly compressive. Care must also be taken to see that zones of high local stress are not overlooked as, for example, the severe stress occurring in the top flange-to-web joint in a gantry girder immediately below a local wheel load.

In considering fatigue one recognizes that, in practice, stress variations are mostly random in nature and rarely of a uniform amplitude. If a picture of the stress spectrum can be arrived at, either by prediction or by measurement, cumulative damage can be assessed by applying Miner's rule, which states that the fatigue damage at any particular stress level is directly proportional to the number of cycles of that stress applied and accumulates linearly until failure occurs. This leads to the relation:

$$\sum \frac{n}{N} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots + \frac{n_r}{N_r} = 1 \qquad (13.1)$$

Failure is predicted in terms of n_r the number of cycles of a given magnitude and type actually occurring during a given period, and N_r the number of cycles of that type required to produce failure in a constant amplitude test. Failure is assumed to occur for k different types of loading when the summation:

$$\frac{k}{r} = 1 \frac{(n_r)}{(N_r)} = 1 \qquad (13.2)$$

Full details are given in a publication by the Welding Institute[2] and by Gray and Spence.[3]

It should also be emphasized that fatigue behaviour cannot be assessed with any great accuracy, there being a wide scatter evident in test results. This arises partly because the degree of severity at a discontinuity, such as a butt weld or fillet weld, depends upon the weld shape and the presence or otherwise of weld undercutting. Thus the skill of the individual welder enters into the picture, which is, of course, a variable.

The Welding Institute also publish a valuable booklet edited by Richards[4] which is worthy of study if fuller information is needed.

13.2.5 Engineering judgement

In the matter of fatigue and brittle fracture, it should be appreciated that in practice these factors are not dominant in the majority of structural configurations. When they are, however, they can be absolutely critical. The safety and life of the structure thus depends on the engineering judgement of the designer in his assessment of the various factors involved. This cannot be divorced from considerations of the consequences of failure, which may vary from simple inconvenience to catastrophic failure. Accordingly, the designer must therefore err on the optimistic or pessimistic side.

Published Document PD 6493:1980 *Guidance on some methods for the derivation of acceptance levels for defects in fusion welded joints* – provides guidance concerning the present state of knowledge on specifying acceptance levels for welds by making use of fracture mechanics methods. It covers brittle fracture, fatigue, yielding, corrosion fatigue, stress corrosion, creep, etc. and includes details on the simplified treatment of the use of fracture mechanics methods to establish acceptance levels based on fitness for purpose. The BSI issued the document as a published document rather than a British Standard because further research is required before the available information can be codified. Feedback on the published document is therefore encouraged to further such information.

13.3 Available structural steel material

13.3.1 British material standards for structural steels

In 1968 a new British Standard, BS 4360, *Weldable structural steels* was published, which superseded all the previous standards for structural steels. So important had welding become as a fabricating technique that it became necessary for all steels marketed for structural purposes to be of weldable quality. At the same time a new classification system was introduced whereby steels were graded according to their tensile strength and new higher-strength steels were introduced. Since 1968, BS 4360 had been revised on three occasions in 1972, 1979 and 1986; these revisions have not only incorporated changes resulting from developments in steelmaking practice but take cognizance of users' latest design requirements.

Tbe current version of BS 4360 is the 1986 edition. It presents compositions, tensile strengths, yield strengths and notch ductility (or Charpy impact) properties for five principal grades, namely 40, 43, 50, 55 and WR 50 (these numbers roughly corresponding to the tensile strengths expressed in kilograms force per square centimetre) for plates, rolled sections and hollow sections. It also shows the nearest equivalent Euronorm 25 and ISO 630 grades where applicable (see section 13.3.2).

The WR 50 group covers seven grades of weather-resistant weldable structural steels, generally based on the familiar 'Corten' steels, whose properties fall close to the grade 50 group as present specified.

The dimensional tolerances, i.e. rolling margins, for plate and flat material are given, and for all products details of all mills tests and their frequencies. These include tensile tests, bend tests, flattening tests for hollow sections and, where the specified properties require, Charpy impact tests. Other matters dealt with include identification, marking, provision of test certificates and inspection.

The three principal documents are:

(1) BS 4360:1986 *Specification for weldable structural steels.*
(2) BS 1449:1983 *Steel plate, sheet and strip.*
Part 1: 'Specification for carbon and carbon manganese plate, sheet and strip'.
(3) BS 2989:1982 *Specification for continuously hot-dip zinc coated and iron–zinc coated steel: wide strip, sheet/plate and slit wide strip.*

The latter specification is used primarily for light-gauge cold-formed sectional shapes such as profiled metal decking and cold-formed channel purlins and side rails. See sections 13.5.1 and 13.5.10 for further details.

13.3.2 European and international material standards for structural steels

13.3.2.1 European standards

Euronorm	EU 25-72	*Structural steels for general purposes.*
	EU 113-72	*Special-quality weldable structural steels grades and quality. General provisions*

Both EU 25 and EU 113 are now being reviewed by the relevant EEC working group to produce a single European standard (EN).

	EU 130-77	*Cold-rolled noncoated mild unalloyed steel flat products for cold forming – Quality standard.*
	EU 131-77	*Cold-rolled noncoated mild unalloyed steel flat products for cold-forming. Tolerances on dimensions and shape.*
	EU 137-83	*Plates and wide flats made of weldable fine-grained structural steels in the quenched and tempered condition.* Part 1, 'Technical delivery conditions – General requirements'.
	EU 139-81	*Cold-rolled uncoated nonalloy mild steel narrow strip for cold-forming. Quality standard.*
	EU 140-81	*Cold-rolled uncoated steel narrow strip. Dimensions, tolerances on dimensions, shape and mass.*

EU 142-79	*Continuous hot-dip zinc-coated unalloyed mild steel sheet and coil for cold-forming. Quality standard.*
EU 143-79	*Continuous hot-dip zinc-coated unalloyed mild-steel sheet and coil for cold-forming. Tolerances on dimensions and shape.*
EU 147-79	*Continuous hot-dip zinc-coated unalloyed steel sheet and coil with specified minimum yield strengths for structural purposes. Quality standard.*
EU 148-80	*Continuous hot-dip zinc-coated unalloyed steel sheet and coil with specified minimum yield strengths for structural purposes. Tolerances on dimensions and shape.*
EU 155-80	*Weathering steels for structural purposes. Quality standards.*

13.3.2.2 International standards

ISO 630:1980	*Structural steels.*
ISO 1035/1:1980	*Hot-rolled steel bars – dimensions.* Part 1, 'Dimensions of round bars'.
ISO 1035/2:1980	*Hot-rolled steel bars.* Part 2, 'Dimensions of square bars'.
ISO 1035/3:1980	*Hot-rolled steel bars.* Part 3, 'Dimensions of flat bars'.
ISO 1035/4:1982	*Hot-rolled steel bars.* Part 4, 'Tolerances'.
ISO 1052:1982	*Steels for general engineering purposes.*
ISO 4950/1:1981	*High-yield strength flat steel products.* Part 1, 'General requirements'.
ISO 4950/2:1981	*High-yield strength flat steel products.* Part 2, 'Products supplied in the normalized or controlled-rolled condition'.
ISO 4950/3:1981	*High-yield strength flat steel products.* Part 3, 'Products supplied in the heat-treated (quenched and tempered) condition.
ISO 4951:1979	*High-yield strength steel bars and sections.*
ISO 4952:1981	*Structural steels with improved atmospheric corrosion resistance.*
ISO 4995:1978	*Hot-rolled steel sheet of structural quality.*
ISO 4996:1978	*Hot-rolled steel sheet of high-yield stress structural quality.*
ISO 4997:1978	*Cold-reduced steel sheet of structural quality.*
ISO 4998:1977	*Continuous hot-dip zinc-coated carbon steel sheet of structural quality.*
ISO 5951:1980	*Hot-rolled steel sheet of higher yield strength with improved formability.*
ISO 5952:1983	*Continuously hot-rolled steel sheet of structural quality with improved atmospheric corrosion resistance.*
ISO 6316:1982	*Hot-rolled steel strip of structural quality.*
ISO 6930:1983	*High-yield strength flat steel products for cold-forming.*
ISO 7778:1983	*Steel plate with specified through-thickness characteristics.*

13.3.3 Economic considerations

One of the principal concerns of the designer is the matter of economy, and this must be considered when specifying the grade of steel to be used. Clear guidance on this issue is somewhat sparse. Not only is the pricing structure of the various grades complex and subject to periodic change but even if material costs are known, no clear-cut answer can be given in most instances. This is largely because more than half the cost of a finished steel structure derives from fabrication and erection costs. A paper by Needham[5] sets out in qualitative terms guidance for engineers in achieving the economic use of structural steelwork. The higher grades of steel require more carefully controlled, and therefore more costly, welding procedures, applied to a reduced total tonnage. Therefore, estimated rates per tonne can be misleading, total costs being the criteria. In particular, fabricators tend to load tender rates when steels with which they are not familiar are specified. Suffice it to say that in many structural applications the high-tensile steels possess potential which has not as yet been fully exploited.

Over the years, numerous attempts have been made to draw realistic cost comparisons between components and structures built from the various forms of structural material available. It has long been accepted that many steel-framed structures show considerable advantages over other forms of building both in economy and speed of construction. One of the most recent comparisons[6] is worthy of study.

If an engineer needs rather special properties outside those in the published standards, and the bulk tonnage required is likely to warrant the making of a special cast (say 250 t and upwards), he should explore the possibilities with the steelmakers.

Also worthy of mention is the possibility of supplying, for structural purposes, quenched and tempered (QT) steels of tensile properties much higher than grade 55. Such QT steels are only on the fringe of the structural market and are expensive but, for particular requirements, such as for offshore oil and gas platforms, these steels are available against a special order.

13.4 Available structural steel shapes

13.4.1 British Standards for structural shapes

As already mentioned in section 13.3, only the dimensional rolling tolerances for plates, wide flats, universal wide flats and flat, round and square bars are specified in BS 4360. For sectional shapes the sizes, sectional properties and dimensional rolling tolerances are specified in the standards published for the different profiles. These are BS 4 *Structural steel sections*, and BS 4848, *Specification for hot-rolled structural steel sections*, each of which is published in several parts.

BS 4:1980, Part 1 'Specification for hot-rolled sections' (as amended subsequently) covers universal sections (beams and columns), joists, structural Ts cut from universal beams and column sections, rolled Ts and channels.

BS 4:1969, Part 2, 'Hot-rolled hollow sections' was withdrawn in 1979 and replaced by BS 4848:1975. The sizes given in BS 4, Part 1, although in metric terms, are based on the old imperial sizes which have been in existence for many years. This range of sections is known as a 'soft' metric series. On the other hand a 'hard' metric series produced in true metric sizes (not simply by metricating imperial sizes) has been under consideration by the ISO for some 25 years, but to date agreement has not been reached between the world's numerous steelmaking companies. It seems, therefore, that the current ranges will be with us for some considerable time to come.

In the case of hollow sections, angles and bulb flats, these have been metricated, their sizes and properties have been published under BS 4848 *Hot-rolled structural steel sections*. The only parts issued so far are:

Part 2, 1975 'Hollow sections'.
Part 4, 1972 (1986) 'Equal and unequal angles'.
Part 5, 1980 'Bulb flats'.

These three parts are based on, and generally in accordance with, the equivalent international standards, namely ISO/R 657/14 for hollow sections, and ISO/R 657/1 and 657/2 for equal and unequal angles and Euronorms, namely EU 56-77 for hot-rolled equal angles, EU 57-78 for hot-rolled unequal angles and EU 67-78 for bulb flats. Although the ISO range of angles was revised in 1983, the UK steelmakers do not intend amending the range of sizes currently given in BS 4848: 1986, Part 4.

It should be noted that considerable variation in tolerance levels exists between the various national specifications with regard to cross-section length and area/mass tolerances of steel sections. For example, BS 4 and BS 4848 call for a ±2.5% tolerance on the specified mass per unit length, whereas on the continent of Europe and worldwide (see the equivalent Euronorms and international standards) the mass/area tolerance is usually not only greater (or wider) but is based on a total bar length, not a unit 'metre' length as in the UK. Generally, British steel plates and sections have tighter tolerances than most foreign products!

In the field of structural cold-formed steel products the two main uses are channel shapes for purlins and side rails, and profiled sheets for cladding, roofing and flooring. This type of product lends itself to many different shapes and sizes and, whilst standard ranges are produced by numerous cold-forming companies, individual 'one off' shapes can be obtained (at a cost!) if especially required for a project.

British Standard 2994:1976 *Specification for cold-rolled steel sections*, gives dimensions, properties and dimensional tolerances of a basic range of shapes, namely angles, channels, Ts, Zs, etc.

Euronorm 162-81 *Cold-rolled steel sections produced on machines for cold-forming sections* gives details of some six shapes available in Europe.

No standards are yet available on metal decking profile shapes.

Various hot-rolled structural shapes are available in accordance with the national standards of many countries throughout the world and the details of some of these are also covered in relevant Euronorms (for the ECSC) and international standards (worldwide). As an aid to identifying appropriate national structural steelwork specifications and to assist in the determination of equivalent sections where conditions dictate that sections other than those specified may be used, the British Constructional Steelwork Association published a summary[7] of various national steelwork design codes, steel material specifications and dimensions and properties of sections. This publication is currently under review for revision.

13.4.2 Euronorms for structural shapes

Euronorms are not yet widely used but the following lists the relevant documents.

EU 19:57	*IPE joists with parallel flanges. Dimensions.*
EU 24:62	*Standard beams and channels. Rolling tolerances.*
EU 29:81	*Hot-rolled plates 3 mm thick and above. Tolerances on dimensions, shape and mass.*
EU 34:62	*Broad beams with parallel sides. Rolling tolerances.*
EU 44:63	*Hot-rolled IPE joists. Rolling tolerances.*
EU 53:62	*Broad flanged beams with parallel sides. Dimensions.*
EU 54:80	*Small hot-rolled steel channels.*
EU 55:80	*Hot-rolled equal flange Ts with radiused root and toes in steel.*
EU 56:77	*Hot-rolled equal angles (with radiused root and toes).*
EU 57:78	*Hot-rolled unequal angles (with radiused root and toes).*
EU 58:78	*Hot-rolled flats for general purposes.*
EU 59:78	*Hot-rolled square bars for general purposes.*
EU 60:77	*Hot-rolled round bars for general purposes.*
EU 67:78	*Hot-rolled bulb flats.*
EU 162:81	*Cold-rolled steel sections (produced on machines for cold-forming). Technical conditions of delivery.*

13.4.3 International standards for structural shapes

A limited range of sizes from these ISO standards have been adopted in the UK and form the basis for the structural shapes given in BS 4 and BS 4848.

ISO/R 657/1:1968	*Dimensions of hot-rolled steel sections.* Part 1, 'Equal-leg angles – metric series – dimensions and sectional properties'.
ISO/R 657/2:1968	*Dimensions of hot-rolled steel sections.* Part 2, 'Unequal-leg angles – metric series – dimensions and sectional properties'.
ISO 657/5:1976	*Hot-rolled steel sections*, Part 5, 'Equal-leg angles and unequal-leg angles tolerances for metric and inch series'.
ISO 657/11:1980	*Hot-rolled steel sections.* Part 11, 'Sloping flange channel sections (metric series) – dimensions and sectional properties'.
ISO 657/13:1981	*Hot-rolled steel sections.* Part 13, 'Tolerances on sloping flange beam, column and channel sections'.
ISO 657/14:1982	*Hot-rolled steel sections.* Part 14, 'Hot-finished structural hollow sections – dimensions and sectional properties'.
ISO 657/15:1980	*Hot-rolled steel sections.* Part 15, 'Sloping flange beam sections (metric series) – dimensions and sectional properties'.
ISO 657/16:1980	*Hot-rolled steel sections.* Part 16, 'Sloping flange column sections (metric series) – dimensions and sectional properties'.
ISO 657/19:1980	*Hot-rolled steel sections.* Part 19, 'Bulb flats (metric series) – dimensions, sectional properties and tolerances'.
ISO 657/21:1983	*Hot-rolled steel sections.* Part 21, 'T-sections with equal depth and flange width – dimensions'.

13.5 Types of steel structure

It is helpful in this section to review the classes of structure in which structural steelwork finds its principal applications. Accurate statistics of use are elusive, the best available being recorded by the British Steel Corporation (BSC). However, the rolling mills produce a much greater tonnage of plates and sections than are revealed in construction statistics. Much of this difference is accounted for by sales to stockholders, the ultimate destiny being obscure, the motor industry for truck and trailer bodies, crane manufacturers, pressure vessels, scaffolding, etc. There remains, though, an inexplicable discrepancy in the figures.

Below is given a schedule of types of structure, followed by guidance on some of them which it is hoped will prove of assistance in the early stages of design.

(1) Light industrial buildings, e.g. factories, warehouses, etc. which are mostly single-storey structures.
(2) Heavy industrial buildings, e.g. power stations, steelworks, mill buildings, etc.
(3) Multistorey buildings, e.g. offices, hotels, flats and industrial structures.
(4) Industrial plant, e.g. bunkers, hoppers, conveyor structures, tanks, petrochemical plant, etc.
(5) Single-storey institutional and commercial buildings, e.g. hypermarkets, sports stadia, etc.
(6) Pressure vessels.
(7) Short- and medium-span bridges.
(8) Major bridges.
(9) Other miscellaneous structures, e.g. electricity transmission towers, television masts, railway electrification structures, temporary bridges, space frame roofs, offshore and maritime structures, including oil and gas platforms, etc.
(10) Use of profiled metal decking in some of the types of structure noted above.

It is to be noted that tubular structures have not been shown as a separate category since circular and rectangular hollow sections (CHS and RHS) have been widely used alongside more traditional sectional shapes for many years. Indeed, these shapes are now accepted throughout the industry as one of the arrows in the quiver, and their full versatility is being exploited more and more. The same principle applies to sectional shapes cold-rolled from strip.

13.5.1 Light industrial buildings

It is in this application that there exists the greatest competition between fabricators, which is reflected in design. There is plenty of scope for new ideas and changes in techniques as indicated by the swing from roof trusses to portal frames. Plastic design has been widely adopted for design of portals, but it is worth pointing out that if absolute economy is desired (and the roof space is not needed for stacking goods, etc.) tied portals show benefit, i.e. using a horizontal slung tie between the knees at eaves level. Controversy continues as to whether haunched or uniform section portals are most economic. Whilst material costs tend to increase, so too does the cost of labour, but at a faster rate. Therefore, in today's conditions and for the foreseeable future the amount of fabrication should be minimized even if this means an increase in material content. Tapered members are often used but, in the majority of cases, at the expense of overall economy.

Figure 13.13 Typical portal frame building

A paper by Horridge and Morris[8] gives details of an extensive investigation into the relative costs of the more popular structural forms and should be helpful to design engineers in determining the most appropriate arrangements.

Whilst not condemning the pursuit of economy in the design of frames it must be stressed that it is unwise to economize on wind and rafter bracing. There have been too many instances of frames moving or collapsing before the cladding has been fixed. In this context, interesting work has been done by Professor E. R. Bryan at Salford University on the stiffening and bracing effect of cladding.[9] Even when his methods of stressed skin design are contemplated, adequate reusable erection bracing must be included.

It is not often appreciated that the cost of purlins and side cladding rails form significant portions of the total. Two points should be considered here: (1) if the structural designer has freedom in fixing the spacing of trusses or frames it is worth designing the purlins and rails first and stressing them fully by adjusting their spans; (2) cold-formed sections almost invariably show cost advantages over hot-rolled angle or channel purlins or side rails, largely because most cold-formed sections are symmetrical and consequently show greater strength/weight than angle shapes.

Whilst cold-formed sections are almost universally used today, it is sometimes worth considering the use of hot-rolled symmetrical shapes, such as small-sized universal beams, joists or channel sections, particularly if the first procedure mentioned above is adopted, i.e. maximum spacing of supports to utilize maximum strength. In addition, purlin cleats can be dispensed with as these purlin sections can be attached directly to the frames.

In light factory construction, increasing emphasis is being put on the benefit to productivity of a pleasant environment, and this tends to influence design. Tubular space frames have much to offer here and should be considered. On the other hand, for high-bay warehouses and warehouses designed round the characteristics of the forklift truck, internal aesthetics are insignificant and structural capability is all-important. Benefit can be derived by integrating the roof and wall structure with the internal racking.

13.5.2 Heavy industrial buildings

In considering this group one must emphasize that great accuracy of calculation and sophisticated techniques of analysis are largely misplaced. Highly competitive designs which give rise to a host of minor troubles to the user are not preferred to relatively crudely designed structures of a high standard of robustness. The principal loading arises from the plant to be accommodated and, whilst in an ideal world the plant design should be finalized before the structural engineer starts, in practice this situation rarely obtains. Instead, one can expect loading figures given in the first instance to be subject to almost continuous and sometimes radical change during the development of the project, even up to the erection stage. Accordingly, the wisdom of providing an additional margin of stress in design to accommodate some of these probable loading changes will be evident. There is also the strong possibility of structural requirements changing during the lifetime of the building, and the designer will not be blessed if a succession of strengthening operations is needed subsequently.

In this class of structure there is not a great deal of scope for originality on the part of the designer. For the most part, plant requirements dominate and the structural layout is not decided – it occurs. It is important, nonetheless, to ensure adequate economy by studying actual needs. For example, a steel melting shop and a power station turbine house may demand cranes of similar capacity but, whereas the ladle crane may make a dozen passes a day at maximum load, the turbine house crane only needs its full capacity for the replacement of a turbo-alternator every few years. Clearly, different levels of fatigue are present in these instances, which results in quite different standards for structural adequacy.

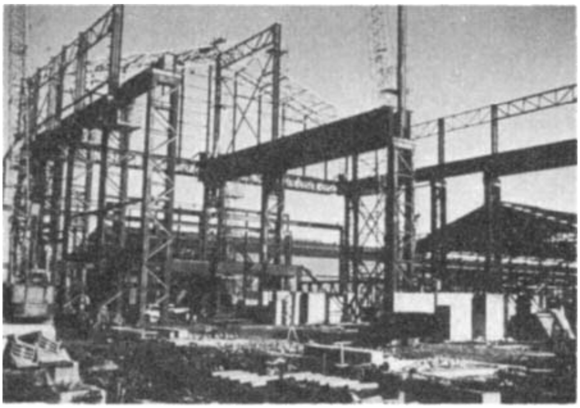

Figure 13.14 Anchor steelworks, Scunthorpe

13.5.3 Multistorey buildings

Steelwork for this application faces the severest competition from concrete, so much so that a decade or so ago it seemed as if it had almost been ousted from the scene, except for a few very special applications. This had largely come about as a result of fire regulations. There are signs, however, which indicate that with some of the newer concepts, particularly for very high-rise prestige office buildings, steelwork offers economic and practical solutions. Two factors have brought about this situation. Firstly, the fire regulations require the lift and stair well enclosures to be separated and protected from the spread of fire from the occupancy floors. Secondly, the current wind code CP3:1972, Part II, Chapter V requires larger transverse forces to be taken by the building than pertained a decade ago. What better solution could there be than to have a reinforced concrete core containing the lifts and services which is itself capable of resisting transverse forces and surrounding it with a simple steel frame supporting only vertical forces? In certain cases, the height and plan form of the core may require vertical post-tensioning to enable lateral forces to be resisted. This post-tensioning can be provided by the surrounding office floors if these are suspended from the top of the core. Several examples of such structures have been constructed; these have the added advantage, in city centre developments, of keeping all the foundation work within the plan area of the building.

Two developments have taken place in multistorey buildings in recent years. Firstly, the gas explosion at Ronan Point in 1968 has led to the general recognition that structures need to have adequate three-dimensional coherence if they are to be sufficiently resistant to accidental damage. This philosophy is now embodied in section A3 of the Building Regulations,[10] which states that in the event of misuse or accident, damage should not be disproportionate to the cause. In truth, with very few exceptions, any normal well-designed steel-framed multistorey building will be found, on analysis, to comply automatically with the statutory requirements for peripheral and transverse tying forces, although in design it is now necessary to check this. If difficulty is found in compliance, it should be taken as an indication that the basic structural anatomy is capable of improvement.

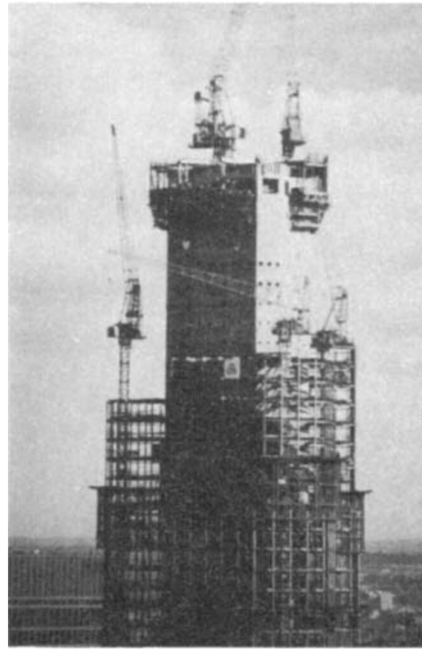

Figure 13.15 National Westminster Bank

Secondly, partly arising from the wide acceptance of sheet steel composite floors (treated in detail in section 13.5.10) and the savings in construction times thus brought about has come a wider recognition of the economic savings accruing from rapid construction generally. Given that immediate occupation is anticipated, high interest rates dictate that the fastest method is, in the long run, the cheapest. This recognition has led to some increase in the use of steelwork for multistorey buildings.

Having dealt shortly with the latest developments one must state that now, and in the future, the bulk of multistorey steel frames will be discontinuous column and beam frames designed on simple methods. That is to say that beams are assumed to be simply supported and columns are assumed to carry only such bending moments as arise from the end reaction of the beams acting at a given eccentricity from the column centreline. This method is easy and quick to apply and has stood the test of time in that failures in such structures are extremely rare indeed, and if they do occur it is usually due to instability during erection. However, it is important to realize that the stress values arrived at in calculations bear little relationship to those actually experienced by the frame. This is due primarily to the omission of any consideration of joint stiffness between beam and column. In practice a true simple support is very rare and, even with the simplest connection, some moment transference will take place between them. In the event of both beams and columns being encased in concrete, the degree of joint stiffness will be considerable. Thus, the simple design method tends to overestimate beam moments and underestimate column moments.

This does not matter, certainly so far as internal columns supporting beams of approximately equal span and loading are concerned, but outer columns (particularly corner columns) carrying asymmetrical moments deserve some thought, certainly if uncased. Therefore, in a highly competitive design it is quite safe to trim the design of beams and internal columns to the bone, even to accepting some degree of overstress, but leaving the face and corner columns conservatively designed.

The results of tests on full-scale structures (apart from tests on isolated elements) have indicated a very considerable margin of strength in the columns, well in excess of any predictions made by elastic design. Clearly, further research is needed before this can be codified into a simple design method.

13.5.4 Industrial plant

In many respects this group is a speciality and indeed has its own trade association. Design problems are likely to arise in many respects, notably in connection with working stresses and whether those specified in BS 449 and BS 5950 are applicable and appropriate. Judgement is required, particularly in respect of the possibility of overloading, accidental or deliberate. Clearly, tanks cannot be overloaded, and a slender load factor may be appropriate, whereas in some other applications normal load factors will result in structures insufficiently robust for their purpose.

In design, fatigue may be a consideration in dynamically loaded structures such as conveyor frames, gantry structures, etc. Reference to the BS 153, Part B or BS 5400, Part 10 rules will indicate whether this is a governing factor.

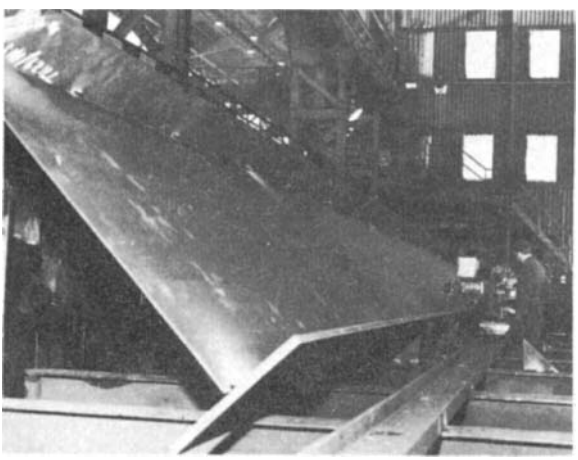

Figure 13.16 Plate girder for British Steel Corporation project, Scunthorpe, being welded

In such work, maintenance costs to prevent corrosion can be heavy. It is not sufficient to design and detail a steel structure and then ask oneself what means of protection should be applied. It may by then be too late since moisture traps may have been built in and certain areas may be inaccessible for painting after erection, e.g. back-to-back angles. Care must be taken in detailing to ensure that water cannot collect in puddles. If this situation is unavoidable, drain holes should be provided. The BSC publish two useful booklets[11,12] which give simple guidance. Chandler and Bayliss[13] provide a practical guide to current knowledge on coatings and processes involved in achieving sound protection of steelwork. It should be borne in mind that hot-dip zinc coating gives excellent protection at modest cost and that the trend is for longer baths enabling greater lengths and larger areas of steel to be treated.

13.5.5 Single-storey institutional and commercial buildings

In recent years a considerable increase in steel use has occurred in such buildings as leisure centres, swimming baths, cash and carry warehouses, sports stadia, hypermarkets, exhibition halls, etc. to the extent of becoming a new classification. A study reveals a very wide variety of different structural forms, including columns and beams, column and truss, arches, domes, space frames, etc. and it is clear that potential exists for the use of imagination and inventiveness. However, they mostly share two characteristics: (1) that they are single-storey; and (2) require the roofing of large areas, mostly uninterrupted by internal columns. As such, the latter presents common problems, including the disposal of large quantities of rainwater, the provision of services of considerable total weight often within the roof structure and, not least, the accommodation of temperature movements. This last is known to have caused problems in the plumbing of columns, when cladding sensitive to such movement was adopted. Provision must be made for movement joints in the cladding of both roofs and walls, and also possibly in the structure itself. In the absence of the latter in a large building it is futile to call for accuracy in plumb which is less than the anticipated total expansion.

Agreement as to total tolerance must be obtained at the design stage or disputes may subsequently arise. Further, lining and levelling of the final structure, before fixing the cladding, is best done at night, when the structure has settled to an even temperature with no differential between the shaded side and that exposed to strong sunlight.

13.5.6 Pressure vessels

The design of pressure vessels forms the subject of a separate British Standard BS 5500:1985 *Specification for unfired fusion welded pressure vessels*. The requirements laid down therein are considerably more stringent than for normal steelwork, both as regards material quality and fabrication, and inspection procedures are correspondingly more severe. That this is right is undeniable, having regard to the catastrophic consequences of possible failure, but it does lead to fabrication costs greatly in excess of the usual. Specialist plant is needed, such as that necessary to bend thick plates and spin dished ends.

In addition to the basic BS 5500 standard, the BSI also publish a series of *Enquiry cases* under the same number, which are supplied free of charge to subscribers of the BS 5500 updating service. These documents, which are numerous, provide information on a wide variety of pressure vessel problems.

In 1982, the Pressure Vessels Quality Assurance Board (PVQAB) came into being; its prime purpose is to provide the pressure vessel industry with a means of recognition, both nationally and internationally, of the quality of their products. The Board was originally set up under the aegis of the Institution of Mechanical Engineers with the object of providing a central UK authority capable of approving and certifying the quality assurance systems of pressure-vessel manufacturers, their products, and the material suppliers. It has now been taken over and is being run by Lloyd's. It is a similar scheme to that operated for many years by The American Society of Mechanical Engineers (ASME).

13.5.7 Short- and medium-span bridges

Competition with concrete is very severe. It will frequently be found that the most economic solution is arrived at by using steel beams or girders acting compositely with a reinforced concrete deck. In design it is no longer adequate to consider statical transverse distribution. An analysis in accordance with the theories of Hendry and Jaeger[14] or Morice and Little,[15] or a finite-element grillage analysis will demonstrate great economy in the sizes of beams at the expense of a minimum addition to the transverse reinforcement in the slab. If full transverse distribution is assumed it is usual to provide continuous transverse top steel in the slab.

A number of model tests have been carried out on composite bridges designed to these principles (see also section 13.7.4.5). Departure from strict linearity occurs at around twice working load, which indicates reasonable economy with adequate safety. Destructive tests, however, give enormous margins of strength at collapse, ultimate load factors of 10 being common.

Figure 13.17 Clunie Bridge, Pitlochry

In one such test which the authors witnessed, an ultimate load factor of 7 was achieved with a system in which the transverse top slab steel was discontinuous, i.e. over the beams only, calling into question the necessity for continuous top steel.

Girders may be rolled universal beams, or purpose-built plate girders, either with equal flanges or with a top flange smaller and narrower than the bottom. In the latter case, the intermediate design stage must be examined when the laterally unrestrained beam is supporting the weight of the wet concrete. It is rare for such systems to be designed as propped, and thus composite action is assumed for live load only.

The theoretical advantage of unequal flanges giving minimum weight of steel will prove largely illusory if a rolled beam could have been used as an alternative. Minimum fabrication leads to minimum total cost. A steel bridge design guide by Nash[16] illustrates the application of the new steel bridge code, BS 5400:1982, Part 3.

It is in the kind of bridge discussed, where the girders are largely protected from driving rain, that the 'Cor-ten' or weathering steels are becoming popular. At small extra cost, bridges can be built which will be effectively maintenance-free during their life. Depending on the corrosion environment, the Department of Transport require a sacrificial 'skin' of either 1 or 2 mm on the exposed surfaces. To facilitate the use of rolled beams, modified tables of properties, omitting this 'skin' have been published by Constrado.[17]

13.5.8 Major bridges

It is here that the designer has greatest scope for imagination and inventiveness, as is illustrated by the wide variety of elegant structures which have been built in recent years throughout the world. Steel dominates the scene here, largely due to its strength:weight ratio, since the greater part of the load sustained by a large-span bridge is its own self-weight. It is, unfortunately, given to few of us to have a hand in the design of such works but, with increasing complexity, design becomes a matter of teamwork, and no one name can any longer be attached to one structure. On the other hand, the hazards are great, as events over the years have shown.

The considerable effort put into box-girder research following the collapse of four box-girder bridges in the early 1970s has been distilled into the new steel bridge code, BS5400:1982, Part 3.

13.5.9 Other miscellaneous structures

Within this group one should include electricity transmission towers (themselves something of a speciality), lighting masts, television masts, railway electrification structures, temporary bridges, footbridges, offshore structures, etc.

Structures frequently are evolved which could not be visualized by the drafting committees responsible for British Standards, e.g. railway electrification structures supported only on one side of the track, gallows style. In this instance, tension in the conductor wires can produce considerable torsion in the column and severe moments arise when the track is on a curve. Accordingly, strict application of codes of practice for design needs to be tempered by engineering judgement, and it is arguable whether the existing British Standards are really applicable in such cases.

Likewise, difficulties have been known to arise in transmission towers. Even greater line voltages have necessitated taller and taller towers. Some years ago, 30 m was a tall tower; today, 75 m is not uncommon and for long river crossings even greater heights are needed. It is not sufficient for the designer merely to extrapolate from existing designs. There will always be a tendency for the designer to assume that what is flat on his drawing, will also be planar when erected. This may be a reasonable assumption up to a certain size, but ceases to be accurate to a larger scale. Self-weight deflections of long members of lattice structures can become significant, particularly if the member is intended to act as a strut.

There was an obvious need for a rational approach to the design of lattice towers and masts which led the BSI to set up a steering committee in 1970 with the task of preparing a code of practice. The result of the drafting committee's deliberation is the recently published BS 8100:1986 *Lattice towers and masts*, which interestingly does not supersede any current standard. It is published in two parts, namely:

Part 1, 'Code of practice for loading'.
Part 2, 'Guide to background and use of Part 1'.

This standard defines procedures for the determination of loading for the design or appraisal of free-standing towers of lattice construction up to 300 m in height.

Part 2 includes two worked examples cross-referenced to the relevant clauses of the code. Also available is a draft for development DD 133:1986, *Code of practice for strength assessment of members of lattice towers and masts* which has been issued in this form to enable industry to provide practical feedback in the light of experience so that ultimately a British Standard code under BS 5950 can be published.

Developments in the North Sea opened up a vast potential market for steel structures. The sizes of many of those built so far beggar description, and the limit has by no means been reached. Again, it was arguable whether BS 449 really applied and the design engineer was in many ways out on a limb and had to work from first principles. To start with, the assessment of loading was far from straightforward and much had to be learned about the effect of wave forces on structures of various shapes. Temperature also had to be considered and the principles of notch ductility and fatigue mentioned in sections 13.2.3

and 13.2.4 had to be observed. From this it follows that a structure satisfactorily designed for use in a tropical sea may not be suitable, indeed may be downright dangerous, in Arctic conditions. The problems, therefore, required a conservative approach, tempered in the light of experience.

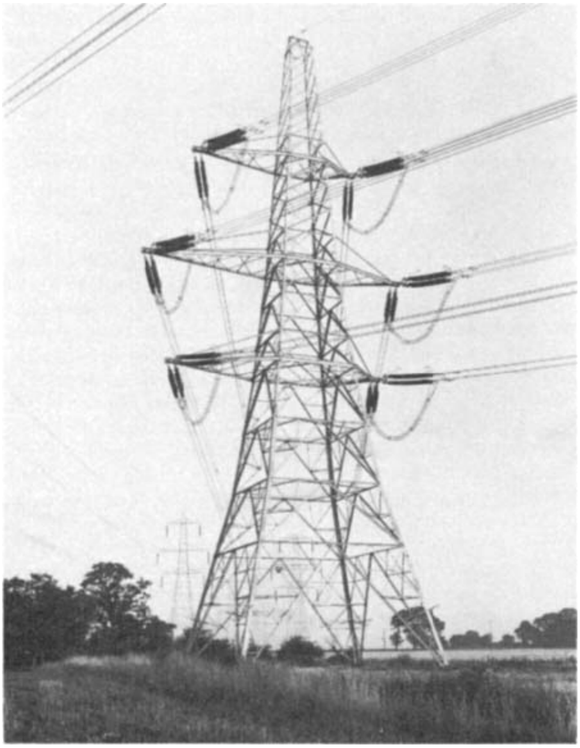

Figure 13.18 Electricity transmission tower. (*Courtesy*: Central Electricity Generating Board)

Actual failure of some offshore structures has provided much-needed information and, having spelt out some fundamental lessons, one hopes that the experience gained will result in safer structures in the future, although no doubt there are other traps awaiting the unwary. As designers tend to overlook stability questions on landbased structures in the part-erected condition, so also is there a danger of failing to recognize the situation when an offshore structure is being towed, usually on its side, to its final location. Buoyancy considerations arise which warrant more than cursory attention.

13.5.10 Use of profiled metal decking in some of the types of structure noted in sections 13.5.1 to 13.5.9

In 1983, the first part of the new BS 5950 *The use of structural steelwork in building* was published; it was, in fact, numbered Part 4, 'Code of practice for the design of floors with profiled steel sheeting'. This code is intended to assist both structural designers and the construction industry in general in the selection of steel deck floors for the most appropriate applications, and it is based on a recommendation made in 1970 by Committee 11 of the European Convention for Constructional Steelwork (ECCS). In North America, floors of the type described in BS 5950, Part 4 have been incorporated in the construction of multistorey buildings since the early 1930s. In the early 1960s this kind of floor was introduced into Western Europe where they are now widely used. Various interim recommendations have been produced in North America and Europe covering their analysis and design, but no standard specifications or codes of practice have yet been published, although in the US a standard has been in the drafting stage for some years.

Profiled decking can be used noncompositely in the form of permanent shuttering or, more usually, compositely with concrete for roof and floor slabs. This type of construction is being specified and used more each year because of its economic and performance advantages over other more traditional forms of floor construction. The deck provides the positive reinforcement for the slab and at the same time replaces the temporary shuttering usually associated with reinforced concrete construction. Once placed, the decking provides both an immediate working platform for subsequent trades on the level on which it is laid and cover for the operatives working on the floors below. Thus, construction can proceed on several levels simultaneously whilst conforming more readily to the requirements of the Health and Safety at Work Act 1974.

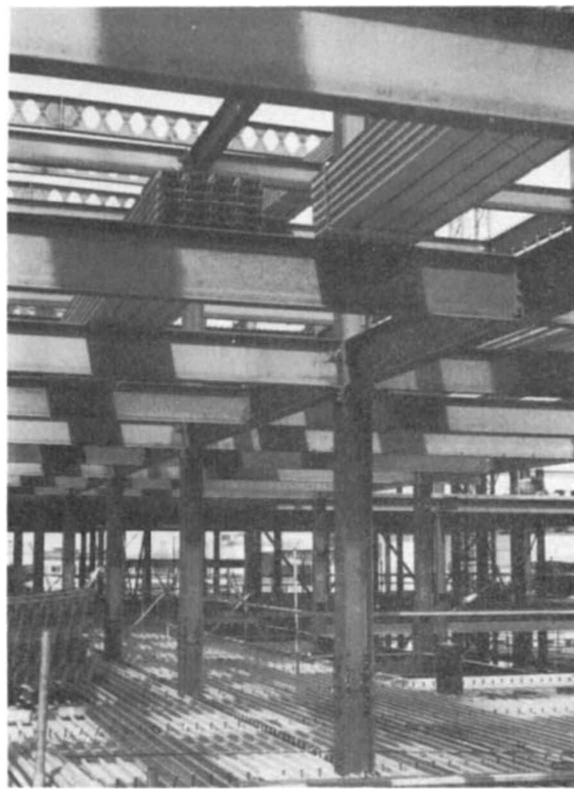

Figure 13.19 Metal decking

The use of this type of floor is not limited to steel structures only, since it is equally suitable for use in structures of reinforced and precast concrete, timber or masonry.

The designer should appreciate that it is misleading simply to compare costs of floors using profiled steel sheeting with other more conventional forms of floor construction, since changes in overall weight, stiffness and stability, composite action, speed of construction, etc. will influence the cost relating to other parts of the supporting structure and foundations.

Recent building projects have shown quite clearly that the overall building costs of structures using steel deck floors are considerably lower than those of structures utilizing other forms of construction. The reasons for these cost reductions have been investigated in depth and confirmed in the findings of a report by Walker and Gray.[6]

The advantages of using profiled steel decking may therefore be summarized for both composite and noncomposite floors as follows:

(1) Steel decks provide permanent formwork and do not normally require the use of additional temporary formwork or propping.
(2) The low weight of the steel deck unit means they can be manhandled with comparative ease, thus reducing or eliminating mechanical handling costs.
(3) Faster construction times are obtained with improved safety.
(4) Maximum efficiency and cost-effectiveness is obtained where regular grids are adopted, although the designer has freedom to arrange the structural support framing to accommodate irregular shapes.
(5) For floors acting compositely there is an efficient use of the two basic construction materials of steel and concrete, since the steel deck acts as both formwork and tensile reinforcement.

13.6 Overall structural behaviour

Having referred frequently to British Standards for design, i.e. BS 449, BS 153, BS 5400, BS 5500 and BS 5950, a word of caution is warranted. These documents, as all British Standards, are drafted by committees of experienced people drawn from all quarters, charged with the task of formulating design rules in the light of the state of knowledge at the time, incorporating the best modern practice and exploiting the latest research. This is no easy task. They recognize that many of those who would use the documents would not understand, indeed would not want to understand, the background theory behind many of the requirements. Inevitably, many areas of doubt and uncertainty are encountered and differences of opinion expressed. At the end of the day, they rightly tend on the conservative side in such situations though, in one or two instances, subsequent research has indicated that their rules were not sufficiently cautious in the earlier standards. However, these documents have stood the test of time and, by and large, when properly interpreted result in structures in which safety and economy are reasonably in balance. But they tended to lay undue emphasis on the determination of exact values of permissible stresses and insufficient attention (certainly in the case of BS 449) to questions of overall structural behaviour and stability, particularly during erection.

The young designer can be forgiven for supposing that, if all members and connections in a structure are assessed and designed strictly in accordance with the British Standard, then the complete structure will be adequate for its task. In the great majority of cases this will be so, but in rare instances it has been found to be a fatal error. In a number of cases the designer has, unconsciously perhaps, relied on the stiffening effect of the cladding to provide overall stability and a collapse has occurred, sometimes under the weight of an erection crane before the cladding has been fixed. What one does not know, and cannot assess, is how many structures are satisfactorily in service which were within a hair's breadth of collapse at some stage during erection. As an illustration, it may seem an economic solution in the design of a frame for a multistorey building to have a number of load-carrying plane frames, interconnected with light ties. In the absence of temporary bracing, such a structure relies for stability on the stiffness of the joints at the ends of the tie beams. Such joints, being on nominally unloaded or light members, are often regarded as being unimportant and are sometimes detailed as simple connections, instead of joints which have some moment capacity. Thus, in the unclad state the frame possesses insufficient longitudinal stability. This illustration is but one of many which could have been quoted but it is hoped that the point has been driven home. Overall supervision of the design of both members and connections by a competent engineer, and the intelligent assessment of the problems by the erection supervisor, should be sufficient to prevent such mishaps.

The stiffening effect of cladding, which has been referred to, is very great indeed but, except in the case of very simple structures, it is likely to remain impossible to assess in the foreseeable future. Because of this and because of the accidental and unquantifiable composite action which occurs between steel members and other materials, the actual stresses experienced in service are likely to be very different and usually much lower than those calculated. This fact, coupled with the fact that loadings which are specified, particularly wind loadings, are at best only general estimates, means that great nicety of calculation is largely misplaced. Thus, the pursuit of notional absolute economy is a cardinal error if it means that under pressure of time the consideration of overall behaviour is omitted.

A related problem frequently encountered and on which little guidance is available, concerns the amount of sway or horizontal deflection permitted at the top of a tall multistorey building under wind loading. It has been common to limit such sway to 1/500 of the height, calculated on the stiffness of the bare steel frame. This largely begs the question, since most of the wind force would be absent without cladding. With it in place, the deflection must be considerably reduced in consequence of its stiffening effect. What is being considered here is not structural safety – that is not in question – but serviceability. In the absence of any knowledge at all of cases where discomfort has been caused to occupants or damage to finishes through excessive sway, the authors conclude that the above notional limit is conservative. Clearly, long-term observations are called for which may take years to yield worthwhile data. Meanwhile, it is suggested that if the above limitation is deemed acceptable for dwellings with curtain walling, then relaxation of the limitation would seem appropriate in respect of two factors: (1) other uses; and (2) stiffer cladding. Again, in city centres, the sheltering effect of neighbouring buildings may be taken into account in assessing wind forces. Is one to take into account the possibility of some, or all, of them being demolished some time in the future? Such considerations need careful assessment, and authoritative advice.

13.7 Design of structural components

There would seem little point in including worked examples in this section, in view of the fact that the old design standards are unlikely to be used for most new projects, whilst designs to the new standards are doubtless covered in numerous recent publications such as that by the Steel Construction Institute.

13.7.1 Importance of correct loading assessment

Great care should be taken in the assessment of dead and live loading, as mistakes at this stage can make all subsequent calculations abortive and, in extreme cases, result in completed members being scrapped. This has been known to happen and unfortunately not all that infrequently. Some common causes of mistakes in this matter are: (1) failing to make adequate allowances for finishes as, for instance, in providing screeding to falls in a roof slab to provide adequate drainage; (2) failure to make proper allowance for surges in gantry structures; (3) incorrect interpretation of wind loading requirements; and (4) effect on pressure of flow in granular materials, etc. Safety requires that a investigated in depth and confirmed in the findings of a report by Walker and Gray.[6]

The advantages of using profiled steel decking may therefore be summarized for both composite and noncomposite floors as follows:

(1) Steel decks provide permanent formwork and do not normally require the use of additional temporary formwork or propping.
(2) The low weight of the steel deck unit means they can be manhandled with comparative ease, thus reducing or eliminating mechanical handling costs.
(3) Faster construction times are obtained with improved safety.
(4) Maximum efficiency and cost-effectiveness is obtained where regular grids are adopted, although the designer has freedom to arrange the structural support framing to accommodate irregular shapes.
(5) For floors acting compositely there is an efficient use of the two basic construction materials of steel and concrete, since the steel deck acts as both formwork and tensile reinforcement.

13.6 Overall structural behaviour

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13.7.2 Determination of structural layout

It is all too rarely, in building structures at least, that the structural engineer is summoned sufficiently early in the planning of a project to be able to contribute to the overall efficiency of the building by influencing the choice of layout. The steelwork designer more often is set an almost impossible problem by a form of building already finalized by others with little thought being given to structural needs. Between these two extremes fall the majority of problems. Thus, frequently it comes about that the structural layout merely 'occurs' and the steelwork design engineer has little room to manoeuvre in planning such matters as column layout, overall beam depth, pitch of roof, etc. Where such freedom does exist, however, and time permits, the structural designer should explore the effect upon overall economy of various arrangements before any one particular layout is finally selected.

13.7.3 Calculations

All calculations for a project should be neatly prepared in logical sequence, properly indexed and cross-referenced. It should be remembered that in most cases schemes including calculations have to be submitted to a local authority for approval. Incomprehensible papers invite criticism and do not smooth the road to acceptance. Additionally, mistakes are more easily spotted if the work is clearly presented, and the effect of structural alterations, either during the contract period or perhaps at a much later date, can be more readily followed.

13.7.4 Consideration of individual members

13.7.4.1 Angle members

Angle members, e.g. purlins, cladding or sheeting rails, bracing members, truss components, usually act as simple beams, over one or two spans, or pin-ended struts and ties. Within the stress rules laid down, having regard to deduction for holes and eccentricity of connections, etc. design is a matter of trial and error and simple arithmetic. Certain practical points, though, are worth bearing in mind. Typical mistakes made by designers in the past include: (1) the transport of double-angle rafters in too-long lengths, resulting in extensive damage in transit; (2) the design of diagonal bracing to resist longitudinal surge in a gantry, which although structurally adequate, was much too flimsy (in this case, when examined, the bracing looked far too light, and since the gantry served a scrapyard it was obvious that the bracing members would be quickly damaged); (3) long span purlins and side rails which, prior to placing cladding, required sag-rods; and (4) angles have sometimes been used for unloaded (tie) members in frameworks when heavier I or [sections, having greater inertia, would have been more sensible.

13.7.4.2 Simply supported beams

Such members fall into two categories: (1) laterally restrained; and (2) laterally unrestrained. In (1), direct design is possible and simple arithmetic will usually be enough to give the required section modulus, either elastic or plastic depending on the design method. The necessary lateral support is often provided either by the load itself or connecting load-bearing beams, a floor of steel chequer plate, profiled steel sheeting, *in situ* or precast concrete or timber suitably secured in some way to the beam. In the case of long spans, it is sometimes economic to provide restraint in the form of light tie beams rather than design the main beam as an unrestrained member. The laterally unrestrained beam is designed to a lower working stress depending on a number of factors: (1) section shape; (2) slenderness ratio; (3) effective length; and (4) torsional restraint at the supports. The designer must first assess the effective length of the compression flange in lateral buckling and both BS 449 and BS 5950 provide guidance which in turn depends partly on torsional restraint at the supports. A trial section is then selected, the slenderness ratio $l:r$ (length: radius of gyration) calculated, the shape parameter allowed for and a permissible stress derived. Thus, design is a trial-and-error method.

It must be understood that the assignment of an effective length is a somewhat arbitrary process and therefore it follows that permissible stresses based on this value are themselves somewhat notional. Again, great accuracy in calculating actual stresses is therefore unnecessary.

One word of warning: the safe load tables for beams refer to beams fully laterally restrained, and therefore beam sizes cannot be selected direct from these tables if there is any doubt whatever on this issue. Examples of unrestrained beams include wall-bearing beams not at a floor level (as, for instance, at the rear of a lift shaft), beams supporting runways or hoists and, of course, the worst possible case – gantry beams – where the load, far from restraining the compression flange, can impart disturbing horizontal forces due to surge and impact.

13.7.4.3 Plate girders

This is a subject on which whole books have been written. At one time most steel bridges were built of plate girders in one form or another but in recent years box girders have entered the scene in a big way. Plate girders have also themselves been the subject of rapid development and change, owing to the almost complete supersession of riveting in favour of welding. Nowadays most plate girders consist principally of three plates. Variations include asymmetrical girders with larger tension flanges than compression for use in composite action with a concrete deck, and girders which are the opposite, where the top compression flange is wider than the tension flange particularly for use in unrestrained conditions or to resist surge in the case of gantry girders. Plate girders can also have continuous or curtailed flanges. The wisdom of curtailing flanges depends on the size of the girder, and to an extent the form of the loading or rather the shape of the bending moment diagram or envelope. It can be said in general terms that, if the size of the girder is such that flanges can be supplied in one piece, it is never economic to curtail the flanges by introducing butt welds in them, with all that this entails in terms of machined preparations, possible preheating, welding and final nondestructive testing. It is most important for designers to recognize that minimum weight does not necessarily mean minimum cost, and it is in the latter that most clients are really interested. Lower weight constructions usually require a more expensive fabrication.

It is rare in the case of rolled beams that the web requires stiffening to resist buckling tendencies, and then only at the supports and points of concentrated loads. In plate girders, however, the web thickness is decided by the designer and is not simply presented as one of the dimensions of a section of adequate bending resistance. Thus, the web can be much thinner relative to the girder depth than in the case of rolled beams, bringing in its train the necessity to provide web stiffeners. Stiffeners do not have to be of great size, except in the case of bearing stiffeners, and those under a concentrated load. Quite small restraining forces are needed to keep an initially flat plate in that condition. Thus, if appearance requires it, stiffeners can be provided on one side of the web only.

Whilst many rolled beams are used as simply supported members, worthwhile economy can be derived particularly in plate girders by making them continuous or semicontinuous, i.e. alternate cantilevers and suspended spans. This creates a situation over the supports where maximum bending moment and maximum shear force act together, a circumstance which requires special consideration of combined stresses.

Plate girders with thin webs often display waves in the webs during fabrication owing to the shortening of the weld and HAZ in the web-to-flange joint. This can make the subsequent fitting of the stiffeners difficult. Where possible, therefore, the stiffeners should be welded to the web beforehand, but this precludes the use of automatic welding, which is a disadvantage. Thus, it can come about that it is not necessarily economic to design with the bare minimum web thickness, and additional web area does also add to the bending resistance.

If it is necessary to make a full-strength splice in a plate girder the web and flange welds should coincide. With the flanges butted for welding there should be a clearance in the web to allow for shrinkage when welding the flanges, otherwise the waviness mentioned above will again be evident. The reason the welds should coincide is that there will inevitably be a slight difference between the two web depths, making fit-up difficult. Only by great good luck will the fit be exact.

13.7.4.4 Columns

Columns can be continuous as, for example, in a multistorey building, or discontinuous, as in a column-and-truss shed. They may be laterally restrained at intervals by other members, or entirely unrestrained throughout their lengths (or heights). These factors are taken into account in the design of an axially loaded column in accordance with BS 449 by assigning an 'effective length' somewhat arbitrarily as a first step in design. Thereafter, a section is selected for trial, its slenderness ratio, i.e. effective length divided by the relevant radius of gyration, calculated, and from this a permissible stress found from a column-stress formula. Such a formula is usually presented in the form of tables or curves. In BS 5950, a similar procedure is adopted, with the exception that four different column curves are presented. The choice of curve will depend on the section shape, it having been established experimentally that upon this depends the level of residual stress, which influences column behaviour.

There are many stress formulae similar to those of BS 449 used in various codes throughout the world. Most of them incorporate some factor to cover notional initial imperfections, and none of them is exact, since they are all attempts to express mathematically what is in essence a naturally occurring situation. Further, all of them are based on the behaviour, under laboratory tests, of pin-ended struts, and it should be understood that a truly pin-ended strut is a very rare occurrence in structural engineering, since a special bearing would be necessary at each end to meet this requirement.

Almost equally rare is a truly axially loaded column, and some bending moment is almost invariably imparted by the connecting members, about one or both axes, at one or both ends. The values of these bending moments may be insignificant or they may be such that the stresses induced thereby dominate the situation and render the axial stress insignificant. In order to take account of this, some form of stress interaction formula is used. The permissible stress in bending is established in the same manner as for a laterally unrestrained beam. It can be argued that this manoeuvre is not accurate, as indeed it is not, but it has the virtue of simplicity and can be readily applied to a variety of cases. Suffice it to say that it has stood the test of time and results generally in safe columns.

Effective lengths, or lengths between points of contraflexure, have also been seen to vary with the magnitude of the axial load; they also vary with the loading pattern. Thus, in a multistorey frame, variations of loading can induce conditions ranging from full double curvature to full single curvature, i.e. effective lengths of $0.5l$ and $1.0l$ (Figure 13.13(a) and (b). One discounts the possibility of 'chequerboard' loading and adopts an intermediate value, say $0.7l$ or $0.85l$.

The foregoing refers to elastic design of columns and, as indicated, the accepted methods err on the side of conservatism. Plastic design methods cannot as yet be universally adopted, and form the subject of widespread research. The first practical design method, for columns in portal frames, was evolved by Professor M. R. Horne. An extended version of Professor Horne's method is given by Morris and Randall.[19] It is in column design that the greatest potential exists for achieving economy through research in pursuit of realistic design methods.

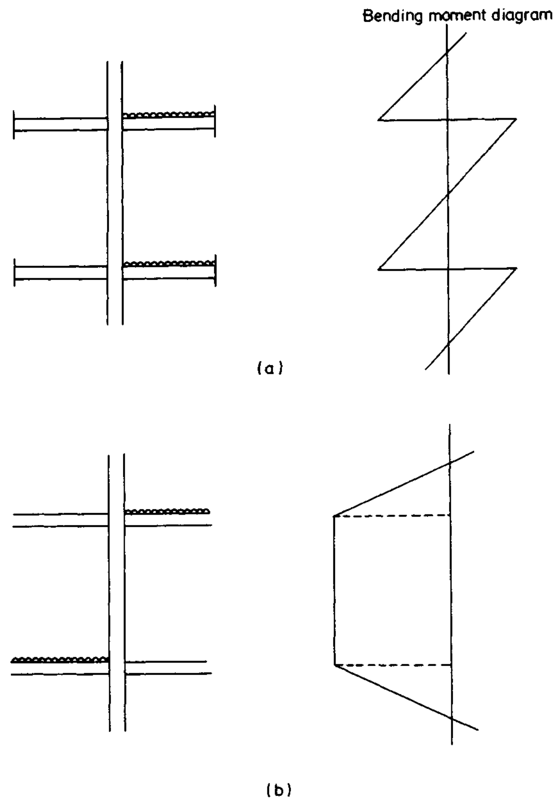

Figure 13.20 Column bending moments

13.7.4.5 Box girders

In the early 1970s nationwide research was undertaken at the behest of the Merrison Committee into a number of governing parameters in box-girder design in order to establish new design rules following the failures of certain bridges, as noted in section 13.5.7. Much of the information published in the public press was misleading and some was downright wrong. It may therefore be helpful to spell out some of the first principles in order to put matters into perspective.

Firstly, box girder construction is not new; some structures built in Victorian times incorporated steel box girders, albeit of riveted construction. The particular virtue of box construction is its much greater torsional resistance compared with normal single web girders. For this reason, welded box girders have been used for a number of years in crane construction, particularly heavy electric overhead travelling (EOT) cranes subjected to racking and surge forces. Of course, with the complete changeover from riveting to welding since the Second World War has come the greater ease with which one can tailor a member to suit a need. It is worth remembering that many hundreds of welded box-girder bridges have been successfully put into service, on the Continent and elsewhere, and none has failed in service.

Clearly, therefore, such structures could be designed successfully. Had it not been for the pressure to build the cheapest possible structure (which in the eyes of some wrongly equates to minimum weight[9] (see section 13.7.4.3)) there would have been no great problems. It was in pursuit of maximum economy, particularly in plate thicknesses, that the trouble arose.

With the exception of the long suspended span, it is true to say that box-girder construction is almost always more expensive than using discrete single web girders, with or without cross-beams. For a straight bridge this is always so – it is only when the structure is subject to high torsional forces as, for instance, in a curved bridge, that the particular virtue of high torsional resistance renders box construction economic. For the most part it has been the clean appearance and good corrosion performance of boxes which have determined the choice.

The Merrison Committee report led to the promulgation of the Interim Design and Workmanship Rules (IDWR)[20] which in turn were replaced by BS 5400:1982, Part 3. Experience has shown that when properly applied this code leads to cheaper box girders, albeit a bit heavier.

Turning to detailed design, one must accept that it has become a speciality of its own. The determination of approximate web and flange sizes to suit the calculated bending moments and shear forces is not of itself difficult, if a bit tedious in the absence of a computer program. But it is in the refinement of the outline design in regard to such matters as support diaphragms, web and flange stiffening, torsional stresses, the effect of shear lag, etc. that the process becomes complicated.

13.7.4.6 Simple bridges

In recent years, there has been some resurgence of the use of steelwork in bridges and viaducts of small and medium span. In some cases alternative designs submitted at the tender stage have demonstrated significant savings and have thus been adopted. In these situations, the cheapest solution has frequently been found to consist of continuous universal beams spaced at 2.5 to 3.0 m apart, acting compositely with the reinforced concrete deck. In all cases the transverse distribution capacity of the deck has been fully utilized by way of a grillage analysis. It is to be noted that design to the new bridge code, BS 5400:1982, Part 3 has increased somewhat the maximum spans for which rolled beams can be acceptable from 22 m to the order of 25 m. For greater spans, welded plate girders of constant depth, at somewhat greater spacing, have been adopted, again continuous and composite. The common characteristic in these successful alternatives has been great simplicity of detail, so as to reduce the workmanship content to the minimum.

Bridges with gracefully curved lower flanges are undoubtedly elegant, but do entail greater workmanship content. Splices have to be carefully detailed and made, and diaphragm bracings contain few common parts. However, only the client authority can indicate where the balance is to be drawn between elegance and economy. Current evidence suggests, rightly or wrongly, that greater emphasis is placed on lowest first cost. This is not to say, though, that parallel girders cannot of themselves be visually attractive, given cleanliness of detail.

13.7.4.7 Square hollow section (SHS) members

Circular hollow sections (CHS) have been available for many years, supplemented in the 1960s by square and rectangular hollow sections (RHS) in a large range of sizes. The use of these members is accelerating annually. Initially inhibited by a high price, ex-mill, compared with traditional sections, this factor is of decreasing importance having regard to the greater rise in fabrication labour rates and, architecturally, the cleaner lines presented.

Although it has long been appreciated that a CHS is technically the most efficient form of strut, the greater ease with which connections can be effected with the square and rectangular shapes has led to their wide acceptance. For example, to fabricate a lattice girder from circular sections requires the use of a profiling machine to generate the interpenetration curves necessary to give a fit-up adequate for welding, whereas only straight cuts are needed for square hollow sections (SHS) or RHS.

Care must be taken, however, in detailing the joints in such structures. It is unwise to use hollow sections for the secondary internal members which are significantly smaller than the main chords, unless some form of stiffening can be used, since the effect of applying a load to a small area of a flat face can lead to premature distortions in the main member. Such mistakes can lead to yielding occurring at or below working load.

Square hollow sections are widely used for space structures, which have aesthetic appeal, but the problems of jointing are considerable. Consequently, the cost of jointing dominates. In order to overcome this difficulty, the Tubes Division of the BSC developed and patented a special joint for space frames which is called the Nodus joint.

It is unfortunate that the way tubular fabrication has developed into a specialization has tended to lead to structures which are either all-tubular or all-traditional. Clearly, the greatest potential for both types of section will be realized only when they are fully blended.

The versatility of SHS is demonstrated by the very wide range of applications, from steel furniture to aircraft hangar roofs. On this topic, a notable success was achieved with the construction of the 'jumbo jet' hangar at Heathrow Airport in 1969 which broke new ground on two counts: (1) it was the largest space frame built at that time, the fascia girder carrying the doors spanning 130 m (the roof behind supports heavy loads from servicing equipment); (2) it was the first large structure in the UK to make use of BS 4360 Grade 55 steel of 448 to 463 N/mm^2 yield strength which was successfully shop- and site-welded in large thicknesses, notably in the main chords which were formed from curved plate into hollow sections 2.74 m in diameter. This structure clearly pointed the way for others and since its construction many equally imposing large-span structures have been built.

13.7.4.8 Cold-formed members

As indicated earlier, cold-rolled purlins have already established a wide market. Other shapes are used in the industrialized building field and in proprietary components such as light-weight lattice beams. These forms of structural sections have certain advantages: (1) when the loading is light, as in the case of purlins and cladding rails, the section can be structurally more efficient than a hot-rolled section owing to the limited number of smaller sizes of the latter; (2) they can be made from tight-coated galvanized strip which, with a subsequent paint coat, gives excellent corrosion protection.

It is not always appreciated that a cold-forming mill is a relatively cheap piece of equipment, only a fraction of the cost of a hot-forming mill. Thus, if a reasonable market is anticipated the designer has the freedom to design his own 'tailor-made' section to suit his particular requirement. This is not to say that it would be economic to design sections for a particular building but it would be for a range of standard buildings. There are several manufacturers willing to undertake such work and to give guidance.

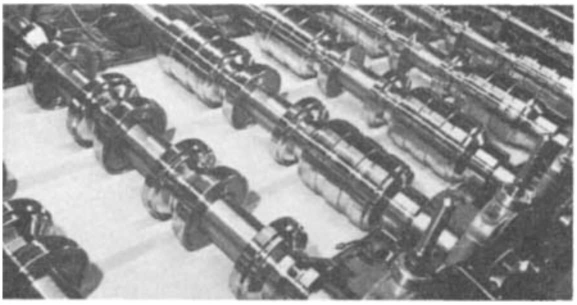

Figure 13.21 Cold-forming mill

In the design of sections, prevention of local buckling is the prime consideration, and the derivation of exact solutions can be most complex. To prevent local buckling the edges of members are frequently lipped, inwards or outwards, and comparatively slender webs are often formed with a ridge or groove, longitudinally. In deriving a section it is often helpful to ensure that sections will 'nest'. Apart from saving space in transport it also ensures that the minimum damage will occur in transit, to which cold-formed sections are otherwise somewhat sensitive.

It is worth pointing out that, in the manufacturing process, since it is done cold, a significant amount of work-hardening takes place, particularly in those zones bent to a small radius. Thus, a section tends to be stronger than calculated on the basis of the yield strength of the flat strip. This effect is not at present taken into account in design and, hence, one has more margin of stress than might be supposed. When published, BS 5950, Part 5 will give much-needed guidance on this subject.

13.7.4.9 Gantry girders

These members are being considered separately since they present unique problems, not found in other members. One must admit that there is a degree of irrationality in British Standards, as it would seem logical for gantry girders to be designed to the same stresses and safety factors as the cranes they support. However, this is not the case and cranes are designed with higher margins than the supporting structure. Of course, a line of demarcation has to be drawn somewhere and one can put up a good case for making this between the moving and the static structure, which is the situation which obtains. Nonetheless, this brings in its train certain difficulties:

(1) For insurance purposes EOT cranes are subjected to a test load higher than the normal crane capacity. If this should be done with the crane midway between columns, with the crab at the end of its cross-travel, flange stresses in the gantry girder may well be approaching yield stress.
(2) Because the operatives are aware of the test overload requirement, a blind eye is often turned on the specified safe working load (SWL) when a particularly heavy load has to be lifted, and again it is often not possible to estimate the weight of a complicated piece of machinery or other load accurately. These factors lead to occasional overloading of the gantries.
(3) In many applications, notably in steelworks, it is not uncommon for a crane to be uprated by retesting, with little thought being given to the supporting structure.

In view of all this, it is most unwise to design gantry girders, or indeed their supporting columns, to fine limits. Prudence suggests that a fair margin of stress be left, consistent with reasonable economy. These comments apply with particular force to the heavier types of crane supported by plate girders, and less so to lighter cranes carried on rolled beams.

Turning to the principles of design, BS 449 and BS 5950 lay down certain factors for longitudinal and transverse surge and vertical impact. These represent a reasonable average, although one suspects that the factors tend to be too conservative for heavy cranes and the opposite for light cranes. The treatment of load combinations is given, vertical loading being taken together with surge either along the rail or transverse to it.

In calculating vertical bending moments, one must know not only the load to be lifted but also the crane characteristics in terms of self-weight, crab weight, end-carriage wheel spacing, etc. Envelope diagrams of shear and bending moment should be derived. In this context it is worth bearing in mind that two cranes often run on the same track. One therefore has to take into account how closely they can be spaced. Sometimes long buffers are fixed to the cranes in order to prevent the two cranes from running on to one girder. But beware, it has been known for such buffers to be removed by the operatives because they are inconvenient!

The worst condition, which usually governs the design, is that with maximum vertical bending moment and transverse surge. In the calculation of stresses it is usual to consider that only the top flange resists the transverse surge, this being somewhat conservative. If a rolled beam can be used it will offer by far the cheapest solution. The next-best alternative is a rolled beam as a core section with either a flat plate or a toe-down channel connected to the top flange to accommodate surge stresses. If calculations suggest that the bottom flange also should be reinforced, one should then turn to a tailormade plate girder, since there will be no difference in the number of main weld runs and the section will be lighter. Finally, very heavy gantries are often built in two parts connected at intervals, i.e. main girder to resist vertical loads and horizontal surge girder for transverse loads, this latter often serving also as a maintenance walkway.

Final selection of section size is a trial-and-error process which can be very tedious. As a first trial one could assume a span:depth ratio of about 15 and a top flange with 30 to 50% greater cross-sectional area than the bottom. At this stage it is worth examining the shear situation at the supports in order to determine an appropriate web thickness, which will affect the overall moment of resistance. Ideally, one should arrive at a section where maximum permissible bending stresses are approached simultaneously in top and bottom flanges. This can only rarely be achieved and in any case it is an illusion to calculate stresses to a greater degree of accuracy than one's real knowledge of the loads.

Finally, certain detail points should be considered. Experience with the earliest welded gantry girders was unfortunate, since little regard had been paid to the local effect of rolling wheel loads. In the absence of accurate web-to-flange fit-up, the fillet welds were required to transmit the whole of the local compressive stresses under the wheel. This was sometimes compounded by a slightly eccentric rail, creating a rocking effect to the top flange. The result was early fatigue failure of the web-to-flange fillet welds. Several solutions have subsequently demonstrated their effectiveness: (1) the rail can be mounted on a resilient pad to give overall rather than point contact on the high spots; (2) where possible, the weld position can be moved to a zone of lower local stress by using a T-section top flange, formed by using a universal T-section, i.e. by splitting a universal beam, the web plate being butt-welded to the stalk; and (3) if this is not possible, the web-to-flange weld should be made by a full-strength double-V butt.

The effect of misalignment should also be considered. With the best of efforts, no foundation can be guaranteed free from some degree of settlement in the lifetime of a building and, if magnified by height, some realignment of the track may be needed. This is greatly facilitated if there is some means provided at the girder supports for transverse adjustment. If this is not provided, the only solution is to realign the rail on the top flange, leading to eccentricity of loading. Some degree of eccentricity is, however, inevitable, owing to lack of fit of the rail, even with a pad, and the wheels not running on the crown of the rail. For this reason, web stiffeners should not be spaced too widely apart and it may be desirable to introduce short intermediate stiffeners supporting the top flange.

Various means have been used for securing the rails on gantries. For light work, bolts through both rail and girder flanges suffice, but these should not be at too wide a spacing, otherwise each bolt will in turn suffer the benefit of full transverse surge. Rail clips are more popular, can be purpose-made, and offer the possibility of adjustment without the necessity to slot holes. Direct welding of rail to flange has been tried, often unsuccessfully. This is mostly due to the fact that rail steels have a high manganese content to give resistance to wear and require a welding technique foreign to many fabricating shops. Also, rail replacement is a most difficult operation!

13.7.4.10 Curved steel sections

Recent developments in section bending have opened up new scope for imaginative structural design using curved steel sections. There are several specialist section benders whose technical brochures give useful information. Briefly, some modern bending rolls are now capable of maintaining the geometric shape of sections during bending without web and flange buckling occurring. The entire range of UK structural sections, and most continental sizes, are now readily available curved about either the major or minor axes. There are obviously limits to the minimum radius for each section, and also cold-bending results in work-hardening, leading to greater strength but reduced ductility, whilst reducing substantially residual rolling stresses. These factors must be taken into account in structural design, e.g. elastic design must be adopted but, even so, such members allow great architectural and engineering freedom. Typical applications include whalings to support steel sheet piling; domed and vaulted roofs for atria, arcades, etc. arched lintels and strengthening or support for masonry arches. A striking example is the roof of the Lee Valley Ice Centre, in which $533 \times 210 \times 122$ kg/m universal beams, curve to 24-m radius and form nine 40-m span three-pinned arches.

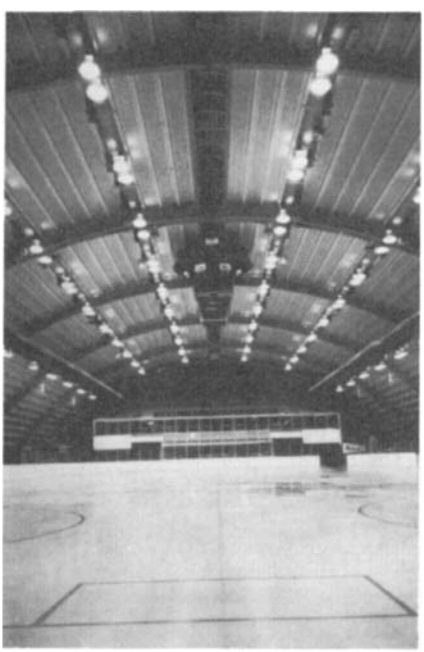

Figure 13.22 Lee Valley Ice Centre. (*Courtesy*: Angle Ring Co. Ltd)

13.8 Methods of design

The current British Standards for the design of structural steelwork, BS 449 and BS 5400, are based on elastic design principles, except that BS 449 has a 'let-out' clause which permits other proven methods of design, or design on an experimental basis. This allows plastic design to be adopted where this is possible, e.g. in continuous beams and single-storey portal frames, but does not lay down any design criteria or even a recommended overall load factor. The new BS 5950 is based on limit-state philosophy, and sets out values for partial safety factors for use in plastic design.

Briefly, elastic design is based on the philosophy of permissible working stresses for different situations, these being some proportion of yield stress in the case of tie members or laterally restrained beams, and some proportion of the critical buckling load in the cases of laterally unrestrained beams and columns. A similar principle applies to shear stresses. Unfortunately, these proportions or so-called 'safety factors' are not stated but are implicit in the permissible stresses specified. If one delves deeply enough one can establish that the safety factors differ from one member to another, which is irrational to say the least. It is this anomaly which the drafting committees for BS 5950 were anxious to eradicate.

If the basic aims of structural design are considered, it becomes clear that structures should have an adequate margin against collapse and not become unserviceable (due perhaps to excessive deflection) under normally anticipated service loading. The only real purpose in calculating levels of stress assumes that from this the margin against failure can be predicted. And here we must come to define failure. The principles of elastic design implicitly define failure as either the attainment of first yield in an extreme fibre in a tension situation, or the attainment of the critical buckling stress in a compression or shear situation. So far, this appears to be rational, and it is on such lines that the design of the majority of building steelwork and all bridge steelwork is based. It is argued that if a known factor against first yield is ensured, a safe structure results, as indeed it does, and that what happens under a greater load than that necessary to achieve yield is of no consequence.

Such philosophy has been upset with increasing knowledge of actual behaviour gained from practical research. Firstly, it is now widely recognized that although members may be elastically designed many parts of a structure may reach yield stress, and indeed physically yield, on the first application of working load. A typical example of such a situation is the traditional end seating bracket and top cleat beam-to-column connection. Simple design assumes this to be a pinned connection and that the top cleat is simply a stabilizing fixing, not intended to transmit a fixed-end-moment. For the design assumption to be realized, some end rotation must take place, causing permanent but local yielding in the top cleat. Yield stress is often attained in many places, particularly if account is taken of residual cooling and welding stresses. But this is not to say the structure is unsafe.

Secondly, it has been established experimentally that all structures possess a margin of strength considerably greater than that calculated on the elastic basis. That is to say, on yielding, stresses will be redistributed extensively before a structure shows permanent deformation such as to render it unserviceable. What now appears to make the elastic philosophy irrational is that this extra margin of strength differs from one situation to another as, for instance, a laterally restrained beam compared to a slender strut. Redefining failure as excessive and unacceptable permanent deformation, ultimate load philosophy (plastic design in the case of steelwork) attempts to quantify this additional margin of strength in different situations, and to utilize it as part of the overall margin against failure, with the aim of achieving economy.

The principles of plastic design were originated by Sir John Baker at Cambridge, developed there by him, with others, and later taken up extensively at Manchester and Lehigh Universities, and elsewhere. As a result, plastic design can now be readily applied to continuous beams and portal frames of various shapes. The majority of steel portal frames built in this country are now designed plastically. The justification, if any is needed, for the extensive research which is proceeding lies in the fact that plastic design is by far the most accurate method of predicting that load at which real structural failure will occur.

As stated, plastic design consists of recognizing, and quantifying, the additional margin of strength in bending, beyond the attainment of first yield in an I-section. It has been established experimentally that before a beam or structure can be made to collapse it must be transformed from a structure to a mechanism. This occurs due to the formation of 'plastic hinges' at points of severe moment, which occur only when the whole depth of a member has reached yield stress, compressive on one side of the neutral axis and tensile on the other, i.e. no portion of the member depth remains elastic. The principles of design of continuous beams and portal frames are clearly set forth in some Constrado publications and other works. The Constrado brochure by Morris and Randall,[19] last published in 1979, together with its supplement (which incorporates design charts extracted and metricated from earlier publications[21]) has been published in various editions during the past two decades. It has been an invaluable source of information on the subject of plastic design worldwide, and contains details of numerous structural forms and design processes including details of two design methods applicable to multistorey frames.

A Constrado monograph by Horne and Morris,[22] and a paper by Morris[23] provides the most up-to-date information available on the subject of plastic analysis.

Having established that a structure has a known margin against collapse one has satisfied the strength criterion, and the value of stresses in various parts under working load are of little significance provided overall and local stability requirements are satisfied. Serviceability conditions, i.e. elastic deflections under working load, may however, not have been satisfied, and in effect one has to do an elastic analysis in order to establish the position, which lengthens the design process somewhat. Computer programs now exist for portal frames which, for given loading conditions, will select a section found plastically and provide values for elastic deflections.

Fully rigid multistorey frames, however, present considerable problems. Under sway conditions a large frame may require very many hinges to form before a mechanism is created and at any intermediate stage the frame is part elastic and part plastic. Proper understanding and control of stability considerations is essential and as yet rules of thumb suitable for codes of practice cannot be produced.

As an intermediate step it was proposed, many years ago, that frames could be designed as 'semi-rigid', i.e. with certain connection requirements satisfied, part transference of moment from beam to column could be assumed. This never found favour, owing to a complicated design process, but a gross simplification of it is allowed in BS 449 and BS 5950, Part 1 whereby beam moments may be reduced by 10% provided the columns are designed to resist such extra moment and are structurally cased. This is a swings-and-roundabouts situation, not leading to any great economy.

13.9 Partial load factors

Alongside the development of ultimate load philosophy, i.e. plastic design, has come consideration of appropriate load factors, i.e. the determination of the right margin necessary against collapse. As indicated earlier, there are inconsistencies in present practice, using the principles given in BS 449.

Clearly, a structure must never closely approach collapse conditions in service, but have some margin. This margin is to allow for a number of uncertainties, including design inaccuracies, variations in material strengths, fabrication errors, lack-of-fit, foundation settlement, residual stresses and errors in assumed loading. Most of these uncertainties will never be quantified, but must be allowed for by a global factor, with two exceptions: (1) material strength which can be treated statistically; and (2) errors in assumed loading. Strength data sufficient for a rational statistical treatment has been accumulated over the past decade or so. This, together with loading data which has also been reviewed, has resulted in a new BS 6399 *Loading for buildings*. The BS 6399:1984, Part 1 'Code of practice for dead and imposed loads', which replaces CP 3:1967, Part 1, Chapter V, enables a more satisfactory design treatment to be adopted than hitherto according to BS 449, i.e. the concept of partial safety factors has been introduced. Different factors are to be applied to dead, imposed and wind loading, etc. by which the specified loads are to be multiplied. These factored loads are summated and the structure designed to be on the point of collapse under such factored loading. There is thus no fixed value for an overall safety factor, since it will depend on the proportions of the loadings from the several sources.

In the design of a building, for instance, one can calculate and control self-weight or dead load quite closely. The magnitude of applied load is very much less certain, and with possible change of use in the lifetime of a building, control is difficult, even by legislation. Further, naturally occurring loading such as wind loading and snow loading entail the consideration of the likely frequency of attainment and the probability of this occurring simultaneously with maximum service loading. One must also consider in continuous or semi-continuous structures that dead load may have a counterbalancing effect to certain live-load situations. Since it is possible for self-weight to be overestimated it is therefore necessary to apply minimum as well as maximum dead-load factors.

A rational approach, therefore, is to adopt maximum and minimum dead-load factors acting either alone or in combination with different factors for imposed and wind loads, the values of the latter depending on whether they act together or separately. This procedure has been adopted in BS 5950 and is fully documented in Part 1 of that standard.

The effect of this, however, will be to complicate the design process somewhat, but the benefit will be a much greater consistency in safety margins. When applied it will appear to make little difference to the run-of-the-mill structure previously designed to BS 449, but for the type of structure in which one kind of loading dominates, e.g. dead load or wind load, significant differences will be apparent.

13.10 Limit-state design

The fundamental objectives of structural design are to provide a structure which is safe and serviceable in use, economical to build and maintain, and which satisfactorily performs its intended function. All design rules, whatever the philosophy, aim to assist the designer to fulfil these basic requirements. However, as mentioned in section 13.9, it must be appreciated that design procedures are formulated to produce a satisfactory structure without necessarily representing its exact behaviour.

The design rules given in the various codes and standards represent the consensus of opinion of many experienced engineers. The rules, however, are not able to cover in detail every situation which designers may encounter, and so judgement must be exercised in their interpretation and application.

By using limit-state philosophy the design engineer has to consider two possible conditions of failure:

(1) *The serviceability limit state*, i.e. when a structure, although standing and experiencing safe stresses, will not be of use due to, say, excessive deformations.
(2) *The ultimate limit state*, i.e. when the structure has reached the point when it is unsafe for its intended purpose, and catastrophic failure is about to take place or already has taken place.

The appropriate factors of safety must therefore be determined, and whilst the primary object is to ensure that it is consistent throughout the structure, it must be recognized that parts of the framework are more sensitive than others to failure.

The change from working stress analysis, according to BS 449, to limit-state philosophy in accordance with BS 5950, Part 1, will produce certain changes in the structural design of a comparable framework. Hence, consideration must be given to economic as well as safety aspects by those responsible for establishing safety levels.

Safety can seldom, if ever, be absolute, but an increase in the level of safety will almost invariably be accompanied by an increase in the construction cost. The public, however, are extremely sensitive to the risk of failure and unserviceability but, whilst extreme care must be exercised, it should not be at the expense of pricing the structure such that it becomes uneconomic to build. An excellent résumé on this topic is given by Tordoff.[24]

13.11 Corrosion protection

The British Standard document giving guidance on this topic is BS 5493 *Code of practice for protective coating of iron and steel structures against corrosion.*. It gives details of how to specify a chosen protective system, how to ensure its correct application, and how to maintain it. Many textbooks have also been written on the subject, to which reference may be made.[13,25] For specialist advice the BSC runs a Corrosion Advice Bureau for dealing with ad hoc problems. The BSC also publishes various leaflets on the subject of corrosion protection of structures, which may be obtained from BSC sections and commercial steels.

Generally, one must admit that, at one time, corrosion protection was given far-too-scant attention, as regards the effect of both structural detail and protective treatment. In recent years great advances have been made in both directions. Welding has relieved a lot of detail difficulties, and sophisticated surface and paint treatments have been developed which promise long repaint lives.

Whilst major exposed structures such as big bridges, where repainting is expensive, rightly receive 'Rolls-Royce' treatment, it should be remembered that such may not be appropriate in each and every case. It seems to the authors that in some respects the pendulum has swung too far the other way, some engineers specifying treatments and inspection standards not warranted in many cases. Once again, judgement enters the picture and in any design it must be first assessed whether a problem exists at all. In the case of steelwork which can be guaranteed to be kept dry (say internal to a heated building) little problem exists. If exposed within the building, painting is usually carried out for cosmetic reasons only.

Steelwork exposed to the elements presents an entirely different problem which should receive attention before design commences, let alone detailing. Again, although account should be taken of the probable life of the structure, its use, atmospheric environment and location before coming to a judgement, structures in the public eye rightly demand the full treatment. Industrial structures of a temporary or semi-temporary nature do not warrant expensive treatment, particularly if they are likely to be subject to accidental damage.

The treatment appropriate to a particular case can thus vary from nothing to the blast-cleaned, metal spray and four-coat paint system. Further information on methods of treatment is given in Chapter 4.

As mentioned in section 13.5.7 it may sometimes be advantageous to consider the use of weathering steel to solve the corrosion problem as an alternative solution to protecting steel from corrosion. Whilst a sacrificial surface can sometimes be allowed, as required by the Department of Transport for bridge works, weathering grade steels painted subsequently usually provide a more durable protective system than ordinary grade steels.

When using universal sections requiring a sacrificial weathering allowance, the design properties are obviously different from those given in BS 4 and other informative documents. In this respect Constrado produced a useful brochure[26] which provides properties adjusted to give a 1 or 2 mm weathering allowance. The use of this brochure is not, however, necessarily confined to bridgework only.

13.12 Detailed design

Ideally, design should be carried out only by those familiar with shopfloor problems and procedures, but this is a counsel of perfection. Where doubt exists an approach to a fabricator is worth while.

What are referred to here are the difficulties which arise because a designer may sometimes interpret technical information literally without recognizing its limitations. The tables published in BS 4:1980 *Structural steel sections*, Part 1; BS 4848 *Specification for hot-rolled structural steel sections*, Parts 2, 4 and 5 and in various handbooks[18, 27–29] giving dimensions and web and flange thicknesses, are average values only. All steel members are subject to weight rolling margins of $\pm 2.5\%$. Sections can also be out of square and the limits of tolerance on shape are given in the relevant publications. Whilst it would be unfair to assume that all member sizes and shapes are at the tolerance limits, it is also unfair to suppose that all dimensions are strictly accurate to three significant figures. One must have some regard to the possibility of members being slightly out of true. No bar is ever completely straight, no plate flat, and no flange at right angles to its web. Fortunately, in most cases the work can be forced into alignment (albeit introducing locked-in stresses) but occasions can sometimes arise where this is not possible. When this occurs it becomes necessary to use packing pieces, occasionally needing costly machining which cannot be charged for. The designer's aim, therefore, must be to eliminate the need for accurate fit-up and to use machining only as a last resort.

Another point frequently overlooked is the matter of accessibility for welding. The easiest and therefore the best fillet weld results when the electrode, either manual or automatic, can be offered at 45°. The limits for satisfactory work are roughly 120 and 60° and outside these limits poor welds result since the electrode must be bent, breaking the flux coating and introducing many stops and starts. The designer cannot be expected to foresee all possible difficulties, but having made some attempt he should retain an open mind and be prepared to consider sequences and procedures suggested from the shop floor.

Handling and floor space also deserve some thought. For straightforward fabrication one should aim to keep complicated weldments small so that they can be handled readily to enable all welding to be done downhand. A complicated end detail to a long member makes this difficult if not impossible. If design can be effected such that all that long members need is to be cut to length and drilled, probably on an automatic machine, competitive tender sums will be offered. This is at least partly due to the fact that shop-floor space used is kept to a minimum and hence throughput can be high.

Where possible, repetition should be the aim. Money will not be saved if in the pursuit of imaginary economy a great variety of member sizes is used for broadly similar loading conditions. An example, for instance, occurred in a bridge consisting of 54 girders, all of which were different. Admittedly, this is an extreme case but some saving through repetition must have been possible. Since building structures generally offer the greatest scope for repetition, this should not be overlooked at the design concept stage.

13.13 Connections

The greater part of the cost of a steel structure is in the connections, whether they be bolted or welded. Simplicity therefore must be the keynote, with the greatest standardization possible if economy is to result. Typical examples of a great variety of connections are illustrated in the *Steel designers' manual*.[30] Suffice it to say here that the general trend is to use shop welding and site bolting. Site welding tends to be very expensive and should be considered only if extensive work is in hand as, for instance, in a big bridge or long pipeline, whilst shop bolting is usually more expensive than welding.

13.13.1 Welding

The British Standards for welding of greatest concern to the structural engineer are:

BS 5135:1984 *Specification for the process of arc welding of carbon and carbon manganese steels.* This standard supersedes BS 1856:1964 and BS 2642:1965 which have been withdrawn by the BSI.

BS 639:1976 *Covered electrodes for the manual metal-arc welding of carbon and carbon manganese steels.*

There are many other current British Standards covering various welding processes, inspection procedures, and welding of special alloy steels and other materials. Indeed, so extensive is the coverage that to the uninitiated great difficulty may be experienced in selecting the appropriate standard or most effective procedure. However, excellent guidance is available from the Welding Institute which publishes a series of booklets, some of which have already been mentioned, but Richards[4] is particularly recommended. It is couched in easily understood language and defines the fundamentals and points out pitfalls for the unwary. Armed with such guidance, an attempt can be made to propose details and procedures for a particular case, but an open mind should be retained for ideas and proposals from the welding engineers responsible for carrying out the work.

The PD 6493:1980 referred to in section 13.2.5 is also invaluable, as is BS 4870:1981 *Specification for approval testing of welding procedures*, Part 1, 'Fusion welding of steels'. This standard gives details of various processes, types of test weld and test pieces and recommended test procedures.

13.13.2 Bolting

The various types of bolts in structural use and the respective British Standard to which they are made, or governing their use, are as follows (the user must ensure that the version incorporating the latest amendments is used):

(1) *Black bolts*: BS 4190:1967 *Specification for ISO metric black hexagon bolts, screws and nuts.* Two obsolete standards covering imperial sizes, namely BS 325:1947 and BS 916:1953 have not yet been withdrawn by BSI.
(2) *High-tensile bolts*: BS 3692:1967 *Specification for ISO metric precision hexagon bolts, screws and nuts. Metric units.*
(3) *High strength friction grip (HSFG) bolts*: BS 4395:1969: *High strength friction grip bolts and associated nuts and washers for structural engineering*, Part 1, 'General grade'; Part 2, 'Higher-grade bolts and nuts and general-grade washers'; and Part 3, 'Higher-grade bolts (waisted shank) nuts and general grade washers'. BS 4604, *Specification for the use of high strength friction grip bolts in structural steelwork. Metric series*, Part 1, 'General grade'; Part 2, 'Higher grade (parallel shank)'; and Part 3, 'Higher grade (waisted shank)'.

Whilst at one time the most popular structural bolt was the black bolt to BS 4190 (Grade 4.6), in recent years there has been a move towards the much wider use of Grade 8.8 bolts covered by BS 3692. But since these bolts are mostly used in clearance holes, the shank diameter precision is not exploited. Consequently, manufacturers now supply bolts of Grade 8.8 strength grade to BS 4190 tolerances, specifically for structural purposes. These Grade 8.8 bolts are of material properties comparable to HSFG bolts to BS 4395, Part 1, but whereas the latter are supplied with Grade 10 nuts, the former come with Grade 8 nuts. Further, the HSFG nut is thicker in order to limit thread stresses during tightening. The result is that bearing and shearing values for Grade 8.8 bolts are greater than slip values for HSFG bolts, and the designer must decide whether some slight initial movement is admissible – usually it is in building structures, except where reversals of load may occur. Thus, in these cases Grade 8.8 bolts offer economy compared to HSFG bolts, particularly since they do not require controlled tightening or special tools. One would suggest, however, that the greater strength in tension offered by HSFG bolts, due to their thicker and harder nuts, justifies their adoption in wholly tension situations. An amendment in 1982 to BS 4604, Part 1 permits such bolts to be used without controlled tightening in these circumstances, which is quite acceptable for most building structures.

In the past, turned and fitted bolts were used only where accurate fit-up was essential. These are not now used having been superseded by HSFG bolts.

Turned barrel bolts are bolts in which the machined shank is of a larger diameter than the protruding threaded end, being shouldered at the spigot. They are for situations in which it is necessary to secure the bolt effectively without gripping the work and exerting pressure between the plies. Such a situation occurs in an expansion joint where one of the holes is slotted to allow for movement.

High strength friction grip bolts are now widely used, having already become popular in the mid 1960s. Both general- and higher-grade bolts, as the name implies, resist shear through the interface friction arising from bolt tension. The coefficient of friction to be assumed is called the slip factor, and the strength of a bolt is calculated as bolt tension × (slip factor/load factor) × number of interfaces. It is assumed that bolts are tightened-up to their proof load in tension and, to ensure this, alternative methods of tightening are specified in BS 4604. These are the part-turn method and the torque-control method. In the former the bolts are brought up hand-spanner tight, to bring the surfaces into contact, the nuts and threads marked, and tightening is then continued by a predetermined amount depending on dimensions. The latter method depends upon the use of either a manual or power-driven tool preset to slip at a particular torque value, which can be adjusted to suit the case. Of the two methods, the former is the more accurate and reliable, but tedious, whilst the latter is much quicker but less accurate owing to the fact that torque and bolt tension do not necessarily relate exactly, being dependent on thread fit.

Additionally, and not referred to in any British Standard, since there is only one manufacturer, is the use of 'Coronet' load indicating washers. These are washers, with raised nibs, to be inserted under the bolt heads. These raised nibs are flattened when the specified shank tension has been attained during nut-tightening. Since they do not require special calibrated tools, nor any marking procedure, they are very much simpler to use than either of the other two methods. Accordingly, their use now accounts for some 80% of cases. It is important, however, that pattern tightening of bolt groups is retained, or relaxation of the bolts tightened first will occur. Also, proper inspection during and after bolting is still essential.

The slip factor normally adopted is 0.45 for untreated surfaces, but BS 4604 *Specification for the use of high strength friction grip bolts in structural steelwork, metric series* gives details of a slip factor test to determine the value in other cases. It also recommends a load factor in design of 1.4. British Standard 449 refers to BS 4604 whereas BS 153 called for higher load factors depending on load combinations. In the new bridge and buildings codes (BS 5400 and 5950) the load factors are more closely related.

The reason for the apparently low load factor of 1.4 lies in the fact that a bolt possesses a margin of strength after slip has occurred, when the bolt starts to act in bearing as well as friction.

It might be supposed from the apparently full coverage in British Standards outlined above that all outstanding problems regarding HSFG bolts had been solved. Unfortunately, this is far from being the case. In the first place, post-slip strength is uncertain and clearly depends to some extent on the thickness of plies. Secondly, the test to determine slip factors consists of four bolts in line, two either side of the joint, with double cover plates putting the bolts into double shear. The effect of eccentricity arising in single-shear conditions is not determined. More particularly, though, it has been established that very large or long joints do not behave in the simple fashion assumed. For instance, a very long cover plate acting in tension transverse to its length also develops longitudinal tension. The Poisson's ratio effect in this biaxial tension situation brings about a reduction in thickness and, thus, bolt relaxation. This can reduce bolt strengths by as much as 20%. Similar biaxial tension effects can occur in large joints in lattice girders, particularly when more than three plies are involved. Indeed, the effect of a large number of plies is completely unknown.

Another difficulty arises in large joints, i.e. that if faces are not machined truly flat an indeterminate amount of bolt tension is used to bring the surfaces in contact, even with the best fabrication. An extreme case would occur in a splice in a box girder. If the overall widths and depths of the two lengths do not coincide exactly towards the corners, proper contact cannot be made without the use of machined packings. Proper contact would be made only towards the middle of the faces where biaxial tensions exist. Large joints therefore need careful thought and cannot be treated by applying rule-of-thumb methods. It should be noted that where HSFG bolts are used, machining of faying surfaces is detrimental since the slip factor will be considerably reduced due to the smoother surface; hence more or larger-sized bolts will be required in the joint.

Two papers by Needham[31,32] on the subject of connections are available. They cover in some detail the basic principles and design philosophy underlying the design of both welded and bolted joints. They emphasize the need for joint design to be consistent with assumed structural behaviour.

13.14 Inspection of structural steelwork during construction

Any inadequacies of materials and structural frameworks of building and bridge structures always causes concern, especially among owners who are frequently faced with the high costs of remedial work.

Many of the defects producing constructions not complying with the design specification may be found to originate from one or more of the following:

(1) *Material*: supply of incorrect grade or out-of-standard material – wrong sizes, etc.
(2) *Detail design*: poor and inaccurate details.
(3) *Fabrication*: incorrect material and size selected. Inadequate or incorrect assembly of joints – use of wrong-grade weld material or bolts.
(4) *Site erection*: misplacing of similar sized but different-grade sections. Wring-grade weld material or bolts. Lack of fit – overstraining of components. Poor foundation connections; omission of, or inadequate, bracing.

Thus, from the outset, the design concept and process should include considerations of:

(1) Availability and reliability of proposed materials.
(2) Special requirements for control of quality and speed of fabrication and erection.
(3) The degree of supervision and inspection likely to be present during fabrication and erection.
(4) The type of contract and its effect on the design/construction process.

It is important to recognize the contractual relationship as defined in the contract documents and that legal responsibility for satisfactory erection rests with the contractor. It is necessary to have good site management by way of planning at all stages, including delivery sequence, laying out of stock and positioning of cranes. Correct setting out is a prerequisite for satisfactory completion of a project, followed by inspection of steelwork on delivery and during erection, temporary and permanent bracing, lining and levelling and examination of all connections.

Two publications on this subject are by the Institution of Structural Engineers[33] and by Needham.[34]

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14

Aluminium

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Contents

14.1 Introduction

14.1.1 History

Although the most abundant metal in the Earth's crust, aluminium was ranked as a precious metal until 1890. In that year the modern electrolytic method of smelting was invented, which transformed the status of aluminium as an industrial metal. Today, it is second-cheapest, after steel, among the metals suitable for structural use. Its volume usage roughly equals that of all the other nonferrous metals put together.

The first strong alloy ('Duralumin') was developed in 1905, which made possible the structural use of aluminium in the German Zeppelins of the First World War. Between the wars its use was developed in aircraft, leading to a vast increase in aluminium production during the Second World War. This was accompanied by a dramatic decrease in cost relative to other metals. After 1945 there was great pressure to develop fresh outlets for aluminium and many new markets were found. By now it is well established in a wide range of industries. Aerospace accounts for a fairly small, but important, part of the total tonnage.

The use of aluminium for civil engineering structures was pioneered in the US during the early 1930s, the first epic example being several 45 m dragline jibs used on the Mississipi's levees. This was followed by the replacement of the steel deck of the Smith Street Bridge in Pittsburgh by an aluminium one in 1933, thus uprating its load capacity. This deck lasted for over 40 yr until replaced by a second aluminium one. Today, aluminium is acknowledged as a general structural material. It is chosen for main structures in situations where its special properties – low density and nonrustability – justify the extra metal cost compared with steel. Figure 14.1 shows a large aluminium roof structure built in Malaysia. This structure is mechanically jointed. Welding is also now widely used as, for example, in the aluminium military bridge shown in Figure 14.2.

A much greater tonnage of aluminium is consumed in secondary structural applications, such as maintenance gantries, glazing bars, window frames, curtain walling, shopfitting, prefabricated buildings, greenhouses, balustrades, crash-barriers, road signs and lamp posts. It is also used widely in the form of profiled sheeting for the cladding of buildings.

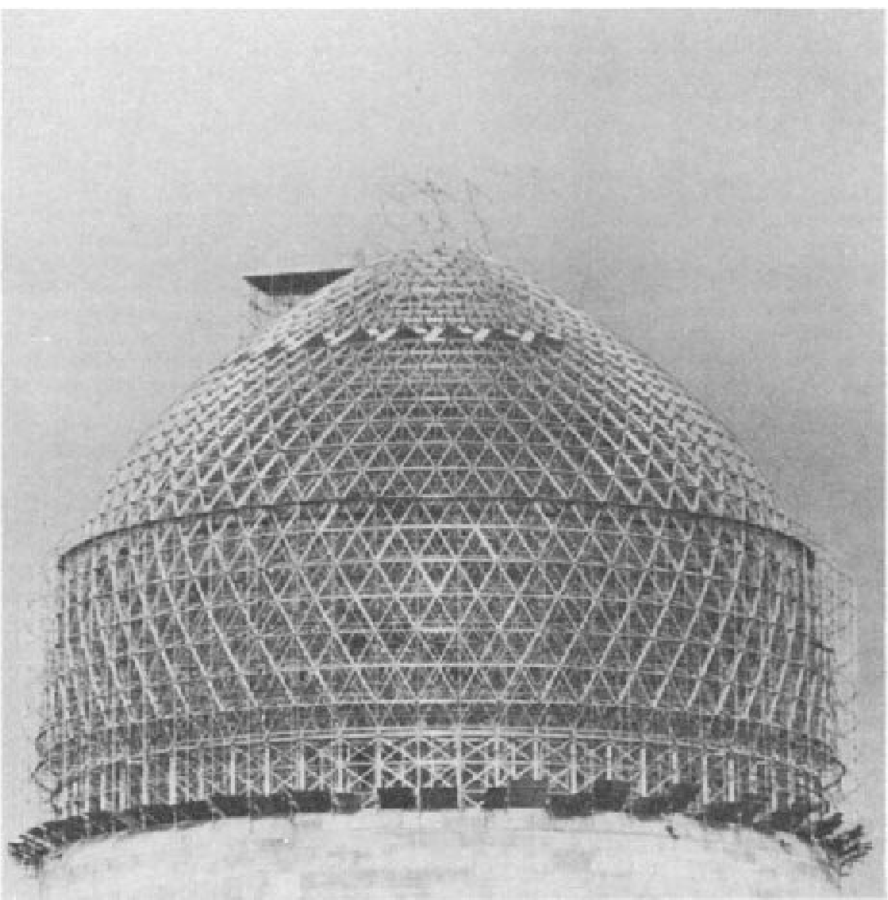

Figure 14.1 Aluminium roof of 50 m diameter structure for the Selangore State Mosque, Malaysia. Tubular construction, employing special extruded node units for the joints. Tubes in 6061-T6 alloy, and all other extrusions in 6082-T6. (Triodetic system. *Courtesy:* British Alcan Aluminium plc)

Figure 14.2 Rapidly erectable aluminium bridge for the British Army, built of specially designed extruded sections. The prefabricated units are of welded construction in 7019-T6 alloy. (Crown copyright)

14.1.2 Comparison with steel

The following is a crude statement of how aluminium differs from steel as a structural material (G = good, B = bad):

Light:	one-third the density	G
Nonrusting:	seldom needs painting	G
Extrusion process:	design your own sections	G
Fabrication:	generally easier	G
Brittle-fracture:	not susceptible	G
Expensive:	about 3 times the cost by volume	B
Deflections:	*E* one-third that of steel	B
Fatigue:	more susceptible	B
Buckling:	more critical	B
Ductility:	tends to be lower	B
Welded strength:	suffers from heat-affected zone (HAZ) softening	B

Other differences from steel are:

Thermal expansion:	twice that of steel
High conductivity (electrical and thermal)	

All aluminium alloys have a rounded stress–strain curve (Figure 14.3), in which they resemble cold-rolled rather than hot-finished steel. Yield is defined in terms of the 0.2% proof stress.

14.2 Production of structural material

14.2.1 Primary production

Aluminium is obtained from the ore bauxite, the first stage being to extract pure alumina. The smelter comprises lines of relatively small furnaces ('pots'), in which the alumina powder is dissolved in liquid cryolite and smelted electrolytically using big carbon electrodes. The output of the smelter is pure aluminium ingot, a major item in the cost of which is the electricity. The ingot is shipped to secondary plants where it is remelted and alloyed to produce wrought ('semi-fabricated') products, in the form of plate, strip, sheet, sections and tube.

14.2.2 Wrought products

14.2.2.1 Flat material

Mill practice for this is much as for steel. Continuously cast slabs are hot-rolled to produce plate, which may then be cold-reduced to make strip, from which sheets are cut. The coiled strip can be roll-formed to produce cold-rolled sections (less common than in steel) or profiled sheeting.

14.2.2.2 Sections

Hot-finished sections are made by a radically different technique from steel, namely extrusion. Cast billets are heated and inserted into a horizontal extrusion press, where they are forced through an aperture in a die, the emerging sections having a cross-section determined by the shape of the aperture. The process is highly flexible, and the die charge is minute compared with the cost of rolls for a steel section. It is common practice for a customer to order new sections to meet his special requirements. The complexity of possible profiles is almost unlimited. Hollow shapes are readily extruded.

14.2.2.3 Tubes

Normally tubes are produced as hollow extrusions. Thin tubing, e.g.irrigation pipe, can be formed from strip by welding. Alternatively, thin tubing for precision use may be produced as drawn tube, at greater cost.

14.2.3 Castings and forgings

Aluminium castings (see section 14.4.4.5) may be employed in conjunction with wrought material, typically for small fittings and attachments. They may be sand-cast, or else chill-cast (for larger quantities). Aluminium forgings can fill a similar role when better strength and ductility are needed.

14.3 Control of strength

14.3.1 Heat-treatable and nonheat-treatable material

Nearly all the aluminium alloys, unlike steel, are unacceptably weak in the hot-finished state. There are two main kinds of alloy: (1) 'heat-treatable'; and (2) 'nonheat-treatable'. The producer strengthens the former by heat treatment, and the latter by cold-working. The heat-treatable alloys are generally the stronger, but less tough, and are more often the choice for main structural use. The nonheat-treatable alloys typically appear in the form of sheet, for which the necessary cold-working is provided during manufacture by the reduction in the rolling operation. Either type can be softened again by annealing.

It is essential for extrusions to be in heat-treatable alloy, since there is no way of cold-reducing them during manufacture. Drawn tube can be of either type.

14.3.2 Heat treatment

The strengthening process for heat-treatable alloys consists of quenching ('solution treatment') followed by ageing. The quench has little immediate effect, but with time the metal will gradually harden at room temperature, reaching its final strength after several days. Such material is said to be 'quenched and naturally aged'.

The ageing process is speeded up usually by heating the metal in a furnace for some hours at about 150 to 200°C ('precipitation treatment'). Such material is stronger than if naturally aged. It can be described as 'quenched and artificially aged', or more commonly as 'fully heat-treated'.

The quench ideally takes place from a carefully controlled temperature in the region of 500°C. The resultant distortion has to be corrected, usually by stretching (before artificial ageing). With wide, thin, extrusions distortion is a major problem, and may well dictate the thickness and, hence, economy of a design. A common practice with the 6000-group alloys is to spray-quench thin extrusions as they emerge from the die, which is more economic and causes less distortion than if they were heated and quenched in a tank as a subsequent operation. For very slender profiles in the 6063 alloy it is even possible to turn off the water entirely and rely on an air-quench at the die, thereby reducing distortion even more. Ideal quenching is obviously not achieved with air-quenching and the resulting material has reduced properties.

14.3.3 Cold-working

Nonheat-treatable aluminium material is strengthened by means of cold-working applied during manufacture. This is possible for products that are cold-reduced in bringing them to their final thickness, i.e. sheet and drawn tube. It is also possible, to a lesser degree, for plate at the lower end of the thickness range ('cold-rolled plate').

The required properties are achieved by careful control of the reduction passes and of interpass annealing. It is common to refer to cold-worked material as being one-quarter, one-half, three-quarters or fully hard, as an indication of its 'temper', i.e. of the extent to which it has been strengthened. The strongest temper is fully hard, but a lower temper may be called for when formability is a factor. For the stronger of the nonheat-treatable alloys the fully-hard temper is not offered.

14.4 Alloys

14.4.1 Alloy numbering system

The specification of aluminium materials has been much simplified by the recent worldwide adoption of the US numbering system. Engineers should abide by this and use no other.

A given alloy, i.e. composition, is referred to by a four-digit number, the first digit of which indicates the alloy group to which it belongs. The alloys are grouped according to main alloying elements as follows, the groups of interest to civil engineers being given asterisks:

Heat-treatable alloys:	2000 group	Copper
	*6000 group	Magnesium plus silicon
	*7000 group	Zinc
Nonheat-treatable alloys:	1000 group	(Pure)
	*3000 group	Manganese
	4000 group	Silicon
	*5000 group	Magnesium

Apart from the alloy it is necessary to specify the condition (heat treatment or temper). This is done by means of appropriate symbols written after the alloy number. For heat-treatable alloys:

T6	Fully heat-treated, i.e. quenched and artificially aged
T5	Air-quenched and artificially aged (extrusions)
T4	Quenched and naturally aged

For nonheat-treatable alloys:

H12 or H22	Quarter-hard temper
H14 or H24	Half-hard temper
H16 or H26	Three-quarter-hard temper
H18 or H28	Fully hard temper

For all alloys:

O	Annealed
F	As-extruded or as-rolled

Thus, typical material specifications would read as follows:

6082-T6	An alloy with a particular aluminium–magnesium–silicon composition, in the fully heat-treated condition
3103-H14	An alloy with a particular aluminium–manganese composition, in the half-hard temper

In the temper designation for nonheat-treatable alloys the first digit after the H (1 or 2) is of academic interest to the average user; it merely shows whether the material has been cold-reduced to the final temper, or has been partly annealed after the last pass. What matters is the ensuing digit (2, 4, 6 or 8) which indicates the actual hardness.

The F-condition is ill-defined. It essentially refers to hot-finished material (extrusion, plate) that has received no further treatment, the properties of which cannot be specified closely.

14.4.2 Selected alloys – properties

14.4.2.1 Strength values

Table 14.1 gives a short list of structural aluminium materials that are of interest in civil engineering. The quoted mechanical properties are based on BS 1470[1] (flat products) and BS 1474[2] (extruded sections). The reader is urged to refer to these or other national standards for fuller information.

14.4.2.2 Physical properties

Approximate values roughly applicable to all aluminium alloys are as shown in Table 14.2. Weight of aluminium material may be estimated using the following formulae:

Weight of section (N/m) = 0.027 × area in square millimetres
Weight of plate (N/m^2) = 27 × thickness in millimetres

Table 14.2

Density	2.7 g/cm^3
Modulus of elasticity E	70 kN/mm^2
Shear modulus	26 kN/mm^2
Poisson's ratio μ	0.33
Linear expansion coefficient	24×10^{-6} per °C
Melting point	660°C

14.4.3 Heat-treatable alloys

14.4.3.1 2000-group

This group of alloys, sometimes referred to as 'Duralumin', is typified by a high copper content (around 4%). It includes most of the strong alloys used for aircraft, an example being 2014-T6 with a tensile strength approaching 450 N/mm^2. These alloys are seldom used outside the aerospace industry, because of their low ductility in the T6 condition, higher cost, inferior corrosion resistance and nonweldability. They have to be fabricated with great care. To reduce corrosion it is possible to use them in the form of 'clad sheet', a product with rolled-on pure aluminium facings.

14.4.3.2 6000-group

This very important group, covering aluminium alloyed with magnesium and silicon, essentially comprises two basic grades of alloy: (1) a stronger; and (2) a weaker grade. The stronger comes in slightly different versions in different parts of the world, the European version, 6082, being broadly similar to the 6061 more commonly used in North America. Material of this type in the T6 condition may be regarded as the 'mild steel' of aluminium, and is the commonest choice for general structural use. It has a 0.2% proof stress about equal to the yield of mild steel, although with a lower tensile strength and less ductility. It is readily welded, but with nearly 50% loss of strength in the heat-affected-zone (HAZ) (Figure 14.3). A particular feature is the ease with which these alloys can be extruded into thin intricate sections.

The second type of alloy in the group is 6063, which is considerably weaker. This is the extrusion alloy *par excellence*

Table 14.1 Selected structural alloys

Approximate % composition	*Alloy and condition*	*Form*	t_m (mm)	*Minimum properties* f_0 (N/mm^2)	f_u (N/mm^2)	*elong.* (%)	*Design stresses* p_o (N/mm^2)	p_a (N/mm^2)
Heat-treatable								
Zn4.0, Mg2.0	7019-T6	E, P	—	330	380	8	330	355
Mg0.9, Si1.0, Mn0.7	6082-T6	E, S	20	255	295	8	255	275
		P	25	240	295	8	240	265
Mg0.7, Si0.4	6063-T6	E	—	160	185	8	160	170
Mg0.7, Si0.4	6063-T5	E	25	110	150	8	110	125
Nonheat-treatable								
Mg4.5, Mn0.7	5083-0	P	—	125	275	14	105	145
Mg4.5, Mn0.7	5083-H22	P	6	235	310	8	235	270
Mg2.0, Mn0.3	5251-H24	S	6	175	225	5	175	200
Mn0.6, Mg0.5	3105-H18	S	3	190	215	1–2	190	200

Notes:
f_0 = 0.2% proof stress, f_u = tensile strength.
E = extrusion, P = plate, S = sheet
t_m = maximum thickness for which stresses are valid.
The following are similar to the 6082 alloy, but are slightly weaker: 6061, 6081, 6181, 6261, 6351.
The alloy 3103 is similar to 3105, but again slightly weaker.

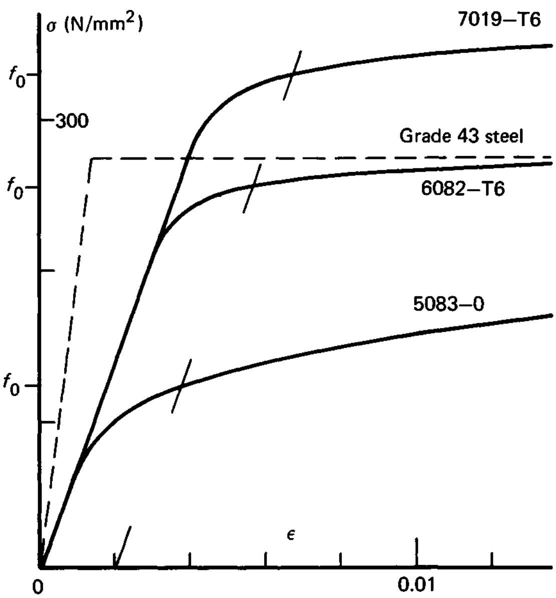

Figure 14.3 Typical stress–strain curves. f_0=0.2% proof stress

(even better than 6082) and is the automatic choice for slender complex architectural shapes, such as window sections and curtain-wall mullions, where stiffness rather than strength is important. Another feature is its smooth surface finish (with a well-made die). Extrusions in 6063 can be produced in the normal T6 or else in the weaker T5 condition (air-quenched), the latter being suitable for very slender profiles that would not otherwise be feasible; 6063 is not supplied in the form of plate or sheet.

14.4.3.3 7000-group

This group, comprising aluminium alloyed with zinc and magnesium, was originally developed in the form of ultra-strong materials for use in military aircraft, having tensile strengths exceeding 500 N/mm². Of value to the civil engineer are the less-strong alloys 7020 and 7019, which are of special interest for welded construction. With these, the heat-affected material adjacent to a weld gradually regains strength over a period at room temperature, and after a month gets back to 75% or more of the full T6 properties. Both alloys extrude nearly as well as 6082. The weaker version 7020 has comparable strength to 6082 in the T6 condition, while 7019 is up to 30% stronger.

Material of 7000-type is susceptible to stress corrosion, when the amount of the alloying elements exceeds a critical level. The alloy 7020 was developed and standardized in Europe with this danger taken into account. As a result it is safe, but hardly any stronger than the cheaper and more readily available 6082. The stronger 7019 version would seem more attractive to a designer. However, 7019, being more highly alloyed, is closer to the critical level for stress corrosion. It was developed in the UK for military bridges, in which form some 15 000 t have been used satisfactorily. But this success was only achieved by very careful control of fabrication procedures. It is essential for an intending user to realize that 7019 is not as simple to fabricate as 6082 or 6061, and to seek advice before doing so.

14.4.4 Nonheat-treatable alloys

14.4.4.1 1000-group

This comprises nominally pure aluminium with different levels of guaranteed purity, material too weak for serious structural use. The cheapest version is 1200 with a minimum aluminium content of 99.0%. Higher purities are available, and in the annealed condition these can provide a valid alternative to lead as a soft flashing material.

14.4.4.2 3000-group

This covers material with manganese as the main alloying element. The two common versions are 3103 and 3105, of which 3105 is slightly the stronger. Used in the fully hard H18 temper, they represent the standard type of material used for profiled aluminium sheeting as employed for cladding of buildings.

14.4.4.3 4000-group

The only interest here in this minor group (aluminium plus silicon) is that it includes one type of weld filler wire.

14.4.4.4 5000-group

This comprises a range of alloys having varying amounts of magnesium as the main alloying element. They are characterized by their ductility and toughness. They are generally unsuitable for use as extruded sections.

The most important in structural terms is the strongest (5083), a plate material. Until recently this was supplied either in the annealed 0 condition, or else in the indeterminate F condition (as-hot-rolled). Its use was confined to low-stress applications, where toughness rather than high yield was needed as, for example, for the entire superstructure of the liner *Queen Elizabeth II*. Material 5083-0 has too low a proof stress (only 125 N/mm²) for use in highly stressed structures; 5083-F will often have a proof stress 40% higher, but this cannot be guaranteed and the designer must still work to the low 0-condition properties. Recently, 5083 plate in thicknesses up to 6 mm has become available in the H22 temper; in this condition it becomes much more attractive, its properties matching those of 6082-T6 for which it is a valid replacement.

Other 5000-group alloys in decreasing order of strength are 5154A, 5454 and 5251. They are typically used in the medium tempers, where their combination of formability and toughness makes them suitable for boatbuilding and sheet metal fabrications generally.

14.4.4.5 Casting alloys

A useful casting alloy contains aluminium with a nominal 12% Si (known as LM6 in the UK). This has a tensile strength roughly comparable to that of 6063-T6, but with a proof stress 50% lower. It has excellent foundry characteristics and good ductility. An alternative is the Al–5% Mg alloy (known as LM5) which is better in terms of surface finish, but which can only be cast into simple shapes; it is less ductile and slightly weaker than LM6.

14.4.5 Alloy selection – summary

For highly stressed welded construction the ideal choice is 7019-T6, because of its good strength and less severe degree of HAZ softening. But it is vital to realize that this is not a material for amateurs in anything but the simplest fabrications because of the latent risk of stress corrosion if correct procedures are not followed.

A more common choice is 6082-T6 or 6061-T6, which is somewhat weaker but more straightforward to fabricate. HAZ softening at welds is more severe than with 7019, and this calls for ingenuity in the location and design of joints.

For welded stiffened plating where toughness is needed, the normal choice would be 5083-F plate welded to 6082 (or 6061)-T6 extruded stiffeners. At thicknesses up to 6 mm the plate is available as 5083-H22 with higher and more precise properties.

For triangulated structures (trusses, space frames) the normal choice is 6082 (or 6061)-T6. Mechanical joints are often used, instead of welding, to avoid the problem of HAZ softening. For extruded members whose design is governed by stiffness rather than strength, as for intricate architectural profiles, the natural choice is 6063-T6. If the section is on the limit of feasibility due to its slenderness, the weaker condition 6063-T5 has to be used instead.

Profiled sheet used for the cladding of buildings is normally supplied in 3105-H18 or the slightly weaker 3103-H18.

Sheet metal fabrications of secondary importance can be readily made in 5251-H14 which is ductile and formable. Pure aluminium in the form of 1200-H14 can be useful for unstressed sheet-metal work. Pure aluminium is also employed, in higher purities, for chemical plant (as an alternative to stainless steel) and for electrical conductors (busbars and transmission lines).

A good general-purpose casting alloy is the Al–12% Si (known as LM6).

14.5 Fabrication

14.5.1 Cutting and forming

14.5.1.1 Cutting and machining

Thin material can be sheared like steel, but more readily. For thicker material cold-sawing is used, with either a circular or band saw. Aluminium (except in the softest tempers) can be sawn faster than steel, especially if suitable coarse-toothed saws are used.

Aluminium is also more readily machined than steel, and it is not unusual in design to employ extrusions incorporating attachment flanges which are machined away over the greater length of the member. (Stiffened panels in aircraft are often machined out of the solid.)

Ordinary flame-cutting is unsuitable for aluminium, because of the ragged edge produced. Instead, one can employ plasma-arc cutting, an adaptation of the tungsten inert gas (TIG) welding process.

14.5.1.2 Bending

The heat-treatable alloys in the full strength T6 condition are less easily manipulated than steel. They will only accept a small deformation when bent cold, due to their lower ductility. Heating, on the other hand, disturbs the heat treatment and causes severe softening. One solution is to form the material in the more ductile T4 condition and then bring it up to the full T6 strength by subsequent artificial ageing in a low-temperature furnace.

With the nonheat-treatable alloys the practice for forming is more as for steel. Cold-bending is employed when possible, the temper of the material being selected to suit the severity of the bend. Springback is more than for steel. For severe manipulations it is possible to apply local heating with a gas flame, the necessary temperature being 450 to 500°C. Great care is necessary to avoid overheating the aluminium, since there is no colour change at this temperature. Temperature-sensitive crayons may be used; alternatively one can rub a pine stick on the heated area and see if it leaves a mark.

14.5.2 Mechanical joints

14.5.2.1 Riveting

For many years, riveting was the normal means of making shop joints in aluminium. More recently there has been a wholesale move to welding, even for structures in the 6000-group alloys which are severely affected by HAZ softening. Riveting is little used and rivets have become hard to get. One wonders if the swing to welding has not been overdone.

Small solid rivets would usually be in 5154A alloy with the small 'pan' head driven cold. Squeeze riveting is preferred to hammering. Larger rivets can be driven hot. Alternatively, one can use 6082-T4 rivets (or equivalent) which have been held in a refrigerator since quenching, to suppress natural ageing. These are readily driven cold, after which they age-harden in position to attain their proper T4 strength.

Proprietary fasteners such as 'Pop' and 'Chobert' rivets are available for joints in sheet-metalwork. These are suitable for blind riveting, i.e. from one side, and are quick to use.

14.5.2.2 Bolting

Aluminium structures can be assembled using either ordinary bolting (dowel action) or high-strength friction-grip (HSFG) bolting (friction action).

Ordinary bolding is used with clearance or close-fitting reamered holes as appropriate, possible bolt materials being: 6082-T6 aluminium (or equivalent), steel (suitably coated) or stainless steel (316S16 or 304S15). Aluminium bolts are none too good in tension, especially in fatigue. On the other hand it may be difficult to get steel bolts with a coating of sufficient durability to match that of the aluminium, unless they are painted. The ideal answer is stainless steel, which is usually worth paying for.

In recent years it has become acceptable to employ HSFG bolting for aluminium, taking care with the protection of the steel bolts. Bolt material (high yield steel) and torqueing procedures follow HSFG practice in steel. Proper attention must, of course, be paid to the condition of the contact surfaces, which should be grit-blasted. The slip resistance can be improved by applying epoxy resin (HSFG bolting is not recommended for use on plates having a 0.2% proof stress under 230 N/mm^2).

14.5.2.3 Screwing

Tapped holes in aluminium tend to be unsatisfactory. Patent stainless-steel thread-inserts are available, which give good service on parts that have to be screwed and unscrewed repeatedly.

14.5.3 Welding

14.5.3.1 Welding processes

Alloys in all groups except 2000 are readily welded. Unfortunately, welding is accompanied by local HAZ softening. This occurs to a greater or lesser degree depending on the parent alloy (see section 14.7.4), except with annealed material.

The standard arc-welding process is manual inert gas (MIG), using d.c. current. This is similar to CO_2 welding of steel except that the shielding gas is argon (or helium in North America). It is easy to operate and ideal for positional welds. It can be used on thicknesses down to about 2 mm. With the MIG process, aluminium can be welded as easily as steel, after an initial training period. Current settings are higher and deposit areas tend to be greater.

For thin work the TIG process is used instead of MIG. In this the arc is struck from a nonexpendable tungsten electrode, the filler wire being held in the left hand. This is an a.c. process which needs more skill than MIG. It is slower and causes more distortion.

Aluminium can be spot-welded, but with higher energy inputs than for steel.

14.5.3.2 Filler wire

Simplified recommendations for selection of arc welding filler wire material are shown in Table 14.3. For further information refer to BS 3019 or 3571.[3]

Table 14.3

Parent alloy group	*Filler composition*
6000*	5% Si (4043A), or 5% Mg (5056A, 5356)
5000 or 7000	5% Mg (5056A or 5356)
3000 or 1000	Parent composition

Note:
*When welding 5000 to 6000 use the 5% Mg wire.

14.5.4 Adhesive bonding

Aluminium is eminently suitable for glued joints using epoxy resin, a technique successfully used for lamp posts and other components. The epoxies are attractive because of their ability to tolerate poor fit-up. Shear strengths up to 15 N/mm^2 can be developed, but it is essential to guard against premature failure due to peeling from the end of a connection. An extruded tongue-and-groove feature is often a good way of preventing this.

The resin can be used cold or, alternatively, can be hot-cured to give improved strength. In the latter case the curing temperature is the same as that needed for artificial ageing. Thus, with heat-treatable alloys it is economic to order the material in the T4 condition, and rely on the hot-curing operation to harden the aluminium (up to T6).

14.5.5 Use of extruded sections

14.5.5.1 Availability

The relatively low cost of extrusion dies often makes it economic to design one's own section or 'suite' of sections to suit the job in hand. The use of such sections can reduce fabrication costs and produce an improved final product provided, of course, the quantities are sufficient.

Extrusion is mainly confined to the 6000 and 7000 alloy groups, the order of merit for extrudability being: (1) 6063; (2) 6082 or 6061; and (3) 7019 or 7020. Complex sections, including hollows, are produced in all of these. Extrusions are also possible in 2014 (high-strength) and 5083 (high-ductility), but with severe limitations on profile and at much higher cost.

Hollow sections are normally produced using a 'bridge die' in which a mandrel, defining the internal shape, is supported on feet locating on the body of the die (which defines the outer shape). Since the hot plastic metal has to flow around these feet and reunite, the final section contains longitudinal welds. These cannot be seen and, in the vast majority of applications, are quite acceptable. But there are some situations where they would be regarded as a potential danger. Hollow sections extrude more slowly than nonhollows, and thus cost more per kilogram; the die charge is also higher.

Apart from custom-made profiles, the designer has a wide range of conventional sections from existing dies to choose from, such as channels, angles, T- and I-sections and boxes. Stockists hold these, usually in 6082-T6 or equivalent.

Sections are extruded in long lengths and can be supplied up to 20 m long to meet special needs. The normal limit on length is much less than this and is dictated by handling and transport.

14.5.5.2 Limiting dimensions

Sections generally are available up to about 300 mm wide from small and medium extrusion presses. With large presses, using special die assemblies, it is possible to extrude sections up to 600 mm wide, depending on the shape. But relatively few mills contain such equipment.

The designer often wants a section to be as thin as possible, for economy. In 6063 alloy the lower limit on thickness can very roughly be taken as the lesser of 1.0 mm and width/120. In 6082 (or equivalent) the corresponding values are 1.5 mm and width/80, while in 7019 they are somewhat more. Sections of 6063 at the limit of slenderness can be supplied in the T5 condition (air-quenched) to reduce the amount of post-extrusion straightening needed to correct distortion.

14.5.5.3 Section design

Figure 14.4 shows a few of the devices that can be incorporated in the design of extruded shapes. Figure 14.4(a) shows a lipped channel space-frame chord, which is a more efficient shape than a plain (unlipped) channel, having greatly increased local buckling resistance. The planking section (Figure 14.4(b)) incorporates various features, including integral stiffeners, interlock, and anti-slip surface. Planking sections, first developed as flooring for trucks, have also been employed in bridge decks and (after piercing) for open-work flooring. Figure 14.4(c) shows a double-sided planking section, again interlocking.

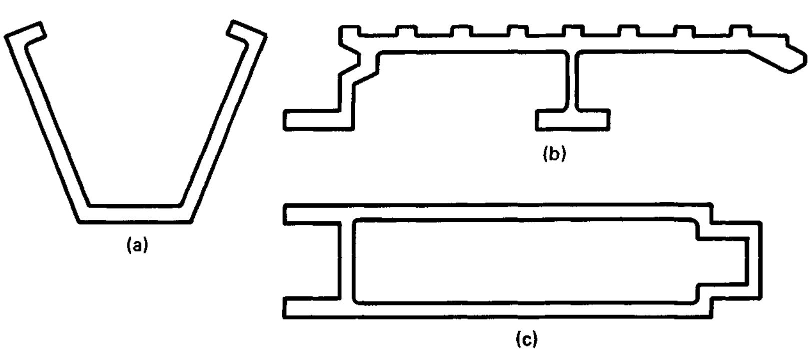

Figure 14.4 Examples of extruded sections

14.6 Durability and protection

14.6.1 Unpainted use of aluminium

14.6.1.1 The corrosion process

Atmospheric corrosion of unprotected aluminium proceeds by localized pitting, a radically different process from the rusting of steel. The oxide corrosion products formed at the pits are voluminous, giving an exaggerated impression of the actual damage. The rate of attack, defined by the depth of pitting, becomes stifled by the corrosion products and slows down after the first 2 or 3 yr. In outdoor sites the corrosion is less when the surface is regularly washed or rained on.

Corrosion failures, on the rare occasions that they happen with aluminium, usually stem from contact with other materials (see section 14.6.3).

14.6.1.2 Durability rating

Aluminium will usually last for ever unprotected, even out of doors. The decision whether or not to paint in an exposed environment depends on the durability rating of the alloy as shown in Table 14.4.

Table 14.4

Rating	*Alloy groups*	*Whether to paint*
A	1000, 3000, 5000	Usually no need
B	6000	Only necessary when exposed to severe industrial or marine environment
C	2000, 7000	Generally necessary, except in dry unpolluted situation

14.6.2 Protective systems

14.6.2.1 Conventional painting

When an aluminium structure has to be painted, it is important that the priming and subsequent coats contain no copper, mercury or graphite, and preferably no lead. A zinc chromate priming coat is recommended.

14.6.2.2 Powder coating

In recent years, powder coating has become an economic process for the coating of aluminium components, on a mass-production basis, and has to some extent replaced anodizing. The powder is sprayed on and stoved, the resulting coat having a more even thickness than with solvent-based paint. Components are often powder coated for purely decorative purposes.

14.6.2.3 Anodizing

This is a process whereby the inherent oxide film is artificially increased electrolytically, the minimum oxide thickness for 'architectural anodizing' being 25 μm. This gives a pleasant satin appearance, which will last for years *if regularly washed.* Colour anodizing is also available, but only in a limited number of shades.

14.6.3 Contact with other materials

When aluminium is in direct contact with certain other metals under moist conditions, the adjacent aluminium gets eaten away. This is known as 'electrolytic' or 'galvanic' corrosion. Failure to take suitable precautions is likely to cause serious trouble.

Such corrosion occurs when aluminium is in contact with steel (other than stainless) or cast iron and, more severely, with copper, brass and bronze. The attack can be stopped by preventing direct contact, either by means of bituminous paint, or preferably with an interposed tape or gasket. Electrolytic corrosion need not be a problem if suitable precautions are taken.

With copper the electrolytic effect is so strong that water dripping off a copper roof on to aluminium sheeting will quickly perforate the aluminium, because of dissolved copper ions. The action between aluminium and lead is only slight. When aluminium and zinc are in contact it is the zinc that suffers. Galvanized bolts in an exposed aluminium structure tend to lose their protection more quickly. Aluminium that is to be embedded in concrete should be protected with bituminous paint; otherwise it will suffer attack while the concrete is 'green'.

14.7 Structural calculations

14.7.1 Principles of design

14.7.1.1 Codes of practice

At the time of writing (1987) existing codes for aluminium design are in the process of being redrafted into limit state format. In the UK, BS 8118 *Structural use of aluminium* which is near to publication and due to replace CP 118:1969, will be in two parts. Part 1: 'Code of practice for design' and Part 2: 'Specification for materials, fabrication and protection'. The simplified design rules given below have been broadly based on the draft to Part 1, which is still subject to possible alteration.

14.7.1.2 Basic requirements

All structures should satisfy: (1) the ultimate, and (2) the serviceability limit state. Fatigue may also be a factor (see Section 14.7.7).

14.7.1.3 Ultimate limit state

Every component, i.e. member, joint, must satisfy the following:

Action under factored loading $\leqslant$ factored resistance

where *action* means moment or force, as appropriate, *resistance* means ability to withstand that action, *factored loading* is nominal loading $\times \gamma_f$ and *factored resistance* is calculated resistance/γ_m.

The partial factor γ_f, applied to the nominal working loads, has basic values as follows:

Dead loads	1.20
Imposed loads (except wind)	1.33
Wind loads	1.20

However, when more than one imposed or wind load acts simultaneously it is permissible, in the case of that which produces the second, third or fourth most severe action, to multiply the basic value by 0.8, 0.6 or 0.4 respectively.

The partial factor γ_m, applied to the ideal calculated resistance, is taken thus:

	Members	*Joints*
Nonwelded construction	1.20	1.25
Welded construction	1.25	1.30

14.7.1.4 Limiting design stresses

Resistance calculations (see sections 14.7.2 to 14.7.5) involve the use of limiting stresses p_o and p_a, listed in Table 14.1 for selected alloys. p_o is usually taken to be equal to the guaranteed 0.2% proof stress. However, a reduced value is taken for materials having a high ratio of ultimate to proof stress, such as 5083-0, for which the stress–strain curve tends to be more rounded. This is to prevent plastic deformation at working load.

14.7.1.5 Combined actions

When a member carries simultaneous axial load and moment, the ultimate limit state is satisfied if:

$$P/P_R + M/M_R \leqslant 1.0$$

where P and M are actions arising under factored load and P_R and M_R are the separate factored resistances.

14.7.1.6 Serviceability limit state

The requirement is that recoverable elastic deflection under nominal (unfactored) loading should not exceed the specified limiting value. In view of the lower modulus it is common to accept larger deflections in aluminium than those normal with steel.

14.7.2 Section classification

14.7.2.1 Compact and slender sections

The first step in checking a member for the ultimate limit state, except in simple tension, is to establish whether it has a *compact* cross-section. If, instead, it is of *slender* section, the resistance will be reduced by premature failure due to local buckling.

14.7.2.2 Classification for axial load or moment

The plate elements comprising a section are of two basic sorts: *outstand* and *internal* (Figure 14.5). The procedure for classifying the section is as follows:

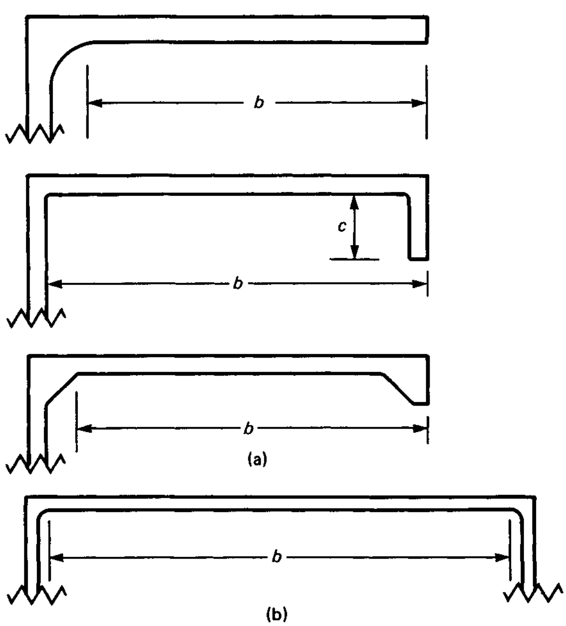

Figure 14.5 Plate elements as considered for local buckling. (a) Outstand (plain and reinforced): (b) internal element

(1) Determine the parameter β/ε for each of the elements comprising the section (except the tension flange of a beam). β depends on the width:thickness ratio $b:t$ as follows, with b measured to the toe of the root fillet (if any):

Plain outstand element, uniform compression	$\beta = b:t$
Internal element, uniform compression	$\beta = b:t$
Web of beam, neutral axis at centre	$\beta = 0.35b:t$

and $\varepsilon = \sqrt{250/p_o}$ with p_o in newtons per square millimetre (Table 14.1).

(2) Classify the individual elements, according to the value of β/ε, in Table 14.5.

Table 14.5

		Outstand	*Internal*
Element in strut	Compact	$\leqslant 7(6)$	$\leqslant 22(18)$
	Slender	$>7(6)$	$>22(18)$
Element in beam	Fully compact	$\leqslant 6(5)$	$\leqslant 18(15)$
	Semi-compact	$\leqslant 7(6)$	$\leqslant 22(18)$
	Slender	$>7(6)$	$>22(18)$

The values in brackets represent the tighter limits applicable to *welded* elements.

(3) The classification of the section is then taken as that of the least favourable element.

14.7.2.3 Reinforced outstand elements

The ability of outstands to resist local buckling can be increased by stiffening the free edge with a lip or bulb (Figure 14.5). For such an element, if reinforced by a standard lip of thickness t equal to that of the plate, a more favourable value of β may be taken as follows:

$$\beta = (b:t)\{1 + 0.03(c:t)^3\}^{-1/3} \qquad (14.1)$$

where c is the internal lip height (Figure 14.5).

If c is large, there is the chance of the lip itself buckling prematurely as a plain outstand, and this should be checked. With any other shape of reinforcement, β should be found by replacing it with an equivalent standard lip (thickness t), the inertia of which about the mid-plane of the plate is the same as that for the actual reinforcement.

(*Note*: In channel-section struts it is immaterial if lips face in or out. But in a beam any lip on the compression flange must be inward facing to be effective.)

14.7.2.4 Classification for shear force

This depends on the depth:thickness ratio $d:t$ of the web or webs and on ε (defined in section 14.7.2.2), as follows:

Compact	$d:t \leqslant 49\varepsilon$
Slender	$d:t > 49\varepsilon$

14.7.3 Resistance of the cross-section

14.7.3.1 Axial load resistance

The factored resistance P_R of an unwelded section is found as follows:

Tension: P_R = lesser of $p_a A_n/\gamma_m$ and $p_o A/\gamma_m$ (14.2)

Compression (with overall buckling prevented):

Compact, unwelded section $P_R = p_o A/\gamma_m$ (14.3a)

Other sections $P_R = p_o A_e/\gamma_m$ (14.3b)

where p_a, p_o = limiting design stresses (Table 14.1), A, A_n = gross and net section areas, A_e = area of *effective* section (see sections 14.7.3.3, 14.7.4.4) and γ_m = partial safety factor (see section 14.7.1.3).

14.7.3.2 Bending moment resistance

The moment resistance M_R of an unwelded section in the absence of lateral–torsional buckling, is found thus:

Fully compact, unwelded section $M_R = p_o S/\gamma_m$ (14.4a)

Fully compact, welded section $M_R = p_o S_e/\gamma_m$ (14.4b)

Semi-compact, unwelded section $M_R = p_o Z/\gamma_m$ (14.4c)

Other sections $M_R = p_o Z_e/\gamma_m$ (14.4d)

where S and Z are plastic and elastic section moduli and S_e, Z_e are the same for the *effective* section (see sections 14.7.3.3 and 14.7.4.4).

14.7.3.3 Effective section

For sections classified as slender (see section 14.7.2.2) the effect of local buckling is catered for by basing the section properties (A_e and Z_e) on an *effective* section, instead of the true one. In unwelded construction the effective section is found by taking a thickness of k_L times the true thickness for any slender element within the section. k_L is read from Figure 14.6, the quantities β and ε needed to enter which are as defined in sections 14.7.2.2 and 14.7.2.3.

The effective section to be used for welded members, to allow for HAZ softening at welds, is defined in section 14.7.4.4.

14.7.3.4 Shear force resistance

The factored shear resistance V_R is found thus for sections having unwelded compact webs, normally orientated:

$$V_R = 0.6 p_o A_v/\gamma_m \tag{14.5}$$

where A_v is the web area.

For unwelded webs classified as slender (see section 14.7.2.4) the following formula may be used:

$$V_R = \frac{600 \times 10^3 A_v}{(d:t)^2\,\gamma_m} \tag{14.6}$$

For welded webs, refer to section 14.7.4.4. Note that Equation (14.6) becomes oversafe if applied to very slender stiffened webs.

14.7.3.5 Moment and shear combined

The moment resistance is unaffected by the presence of a shear force V not exceeding half the value of V_R. For higher values of V, M_R becomes reduced as follows:

$$M_R = M_{R_0}\{1 - 8(V/V_R - 0.5)^3\} \tag{14.7}$$

where M_{R_0} is the factored resistance in the absence of shear.

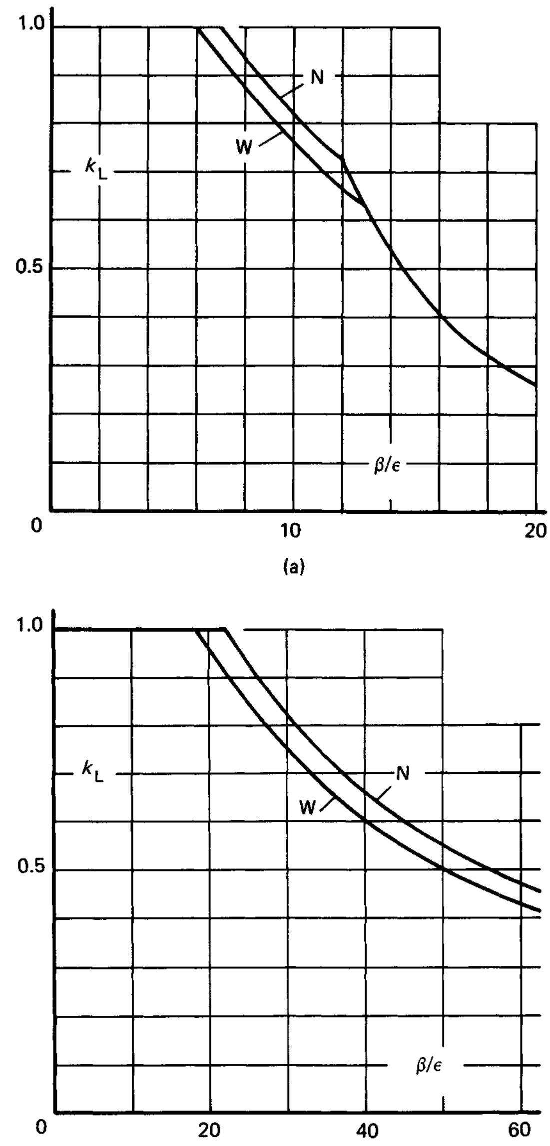

Figure 14.6 Local buckling factor k_L for slender plate elements. (a) Outstand: (b) internal. N=unwelded, W=welded

14.7.4 Softening at welds

14.7.4.1 Severity of softening

In welded construction the designer must allow for the local softening that occurs in the HAZ adjacent to welds, except when the parent metal is in the annealed (0) condition. It is assumed that within a certain distance of each weld the material properties are reduced to the parent properties multiplied by a softening factor k_z, which depends on the alloy as follows:

7000-group, T6 condition $k_z = 0.75$

6000-group, T6 condition $\quad k_z = 0.55$

5000-group $\quad k_z = \dfrac{\text{tensile strength in 0 condition}}{\text{tensile strength in temper used}}$

14.7.4.2 Extent of the HAZ

The area over which the material properties are thus reduced (the HAZ) is assumed to extend a distance z from a weld, measured: (1) transversely from the centre of a butt weld or the root of a fillet; and (2) longitudinally from the end of any weld. Provided welding is by the MIG process, with rigorous thermal control (see below), z may be generally taken as the lesser of two values found as in Table 14.6, where t_1 is the average thickness of the plates joined (but not exceeding $1.5t_2$), and t_2 is the thinnest plate thickness. But note that these values become unreliable if: (1) t_2 exceeds 25 mm; or (2) longitudinal welds have a total deposit area exceeding 3% of the gross area of the section for 7000- or 5000-group alloys, or 4% thereof for 6000-group.

Table 14.6

	Butt	*Fillet*
7000-, 5000-groups	$4.5t_1$ or 35 mm	$(4.5t_2^2/t_1)$ or 35 mm
6000-group	$3t_1$ or 25 mm	$(3t_2^2/t_1)$ or 25 mm

It is important to exercise rigorous thermal control during welding to limit the extent of the HAZ. The values of z given above are only valid if the metal temperature adjacent to a weld at the start of deposition, of any pass, does not exceed 40°C (for 7000- and 5000-group parent alloys) or 50°C (for 6000). If these temperatures are exceeded, the predictions in section 14.7.4.2 will underestimate the affected area. Also, with 7000-group material the softening factor k_z may drop below 0.75.

14.7.4.4 Effective section of welded members

To allow for the effects of HAZ softening, the true section is replaced by an effective one, which is assumed to have full parent properties throughout, but with reduced thickness in the HAZs. The resistance is then found generally as in section 14.7.3, using section properties based on the effective section:

(1) *Compact sections.* The effective section is obtained by taking an assumed thickness in the HAZ equal to $k_z t$ instead of the true thickness t.
(2) *Slender sections.* For any plate element that is both slender and affected by welding, the assumed thickness is taken as the lesser of $k_z t$ and $k_L t$ in the HAZ and as $k_L t$ elsewhere in that element. The rest of the section is treated according to (1) above or section 14.7.3.3 as appropriate.

The shear resistance of a welded web of slender proportions may be taken as the lower of two values: (1) based on Equation (14.5) with HAZ effects allowed for in the calculation of A_v using *compact sections* in (1) above; and (2) based on Equation (14.6) with HAZ effects ignored.

14.7.5 Buckling

14.7.5.1 Buckling of struts

There are two possible modes of overall buckling to be considered in axial compression: (1) *flexural*; and (2) *torsional.* Torsional buckling tends to become critical for thin open sections such as angles and channels.

The factored resistance for either mode is taken as the basic resistance P_R of the section (Equation (14.3a) or (14.3b)) times the factor k_c which is read from Figure 14.7. In order to enter the figure the quantity ε is found as follows, with p_o in newtons per square millimetre:

Compact, unwelded section $\quad \varepsilon = \sqrt{(250/p_o)} \quad$ (14.8a)

Other sections $\quad \varepsilon = \sqrt{(250A/P_R)} \quad$ (14.8b)

When considering ordinary flexural buckling, the slenderness parameter λ is simply the effective slenderness ratio $k_L{:}r$ as used in conventional steel design.

For torsional buckling, λ may be obtained from the general expression:

$$\lambda = \pi\sqrt{(EA/P_{cr})} \quad (14.9)$$

where P_{cr} is the elastic critical load for torsional buckling of a strut, as given in textbooks, allowing for interaction with flexure when necessary.

For struts of plain angle section (unreinforced) the torsional buckling check may be waived when the section is compact (section 14.7.2.2). If the angle section is slender, it can be assumed that torsion is adequately covered by simply taking P_R based on the effective section, with $k_c = 1$.

(*Note*: The use of Figure 14.7 may tend slightly to overestimate buckling strength of struts that are: (1) welded; or (2) of very asymmetric section (buckling axis much nearer to one edge than the other).)

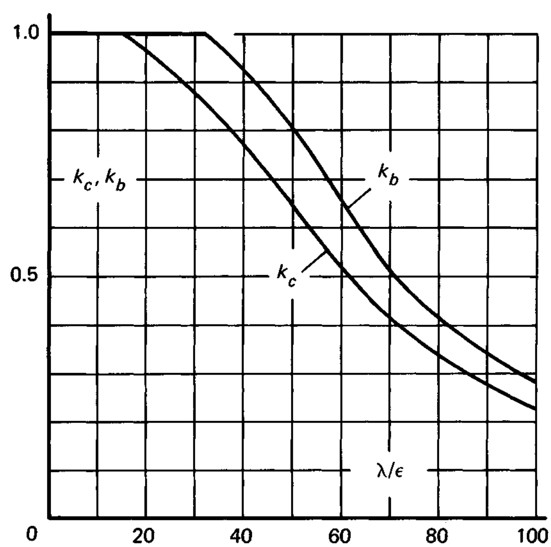

Figure 14.7 Overall buckling factors k_c (struts) and k_b (beams)

14.7.5.2 Lateral–torsional buckling of beams

The factored moment resistance for a member prone to lateral–torsional buckling is taken as the basic resistance M_R of the section (based on Equation (14.4a), (14.4b), (14.4c) or (14.4d) times the factor k_b which is again read from Figure 14.7. In entering the figure ε is now found as follows, again with p_o in newtons per square millimetre:

Fully compact, unwelded section $\varepsilon = \sqrt{(250/p_o)}$ (14.10a)

Other sections $\varepsilon = \sqrt{(250S/M_R)}$ (14.10b)

The slenderness parameter λ may be obtained from the following general expression:

$$\lambda = \pi\sqrt{(ES/M_{cr})}$$

where M_{cr} is the elastic critical moment for lateral–torsional buckling, as given in textbooks. Alternatively, for beams of conventional I-, channel, T-shapes, λ may be found using the appropriate steel data.

14.7.6 Connections

14.7.6.1 Ordinary riveting and bolting

Limiting stresses for aluminium fasteners, to be used in conjunction with an appropriate value of γ_m (see section 14.7.1.3), are given in Table 14.7 in newtons per square millimetre. The corresponding bearing stress on the ply is taken as $2p_a$ (see Table 14.1). Limiting stresses for steel fasteners should normally be taken as $0.7p_y$, $2p_y$ and p_y respectively for shear, bearing and tension, where p_y is the yield stress.

Table 14.7

Fastener	*Shear*	*Bearing*	*Tension*
5154A rivet	125	400	—
6082-T4 rivet	95	310	—
6082-T6 bolt	170	550	220

14.7.6.2 Friction grip bolting

The factored resistance in shear (depending on friction capacity), again with an appropriate γ_m, may be based on a slip factor of 0.3. This is valid provided: (1) the surfaces are grit-blasted; (2) the bolt diameter is not less than the combined ply thickness; and (3) the 0.2% proof stress of the ply material is not less than 230 N/mm^2.

14.7.6.3 Welded joints

Suitable limiting stresses for weld metal, used in conjunction with an appropriate value of γ_m (see section 14.7.1.3), are given in Table 14.8 in newtons per square millimetre. These assume that the welds are sound, and that the right filler wire is used (see section 14.5.3.2).

Table 14.8

Parent alloy	*Tension*	*Shear*
7019, 7020, 5083	240	170
6082, 6061, 5251	150	100

14.7.7 Fatigue

Fatigue calculations are generally based on stress arising under nominal working loads (unfactored). In the new aluminium codes (e.g. BS 8118) fatigue data will be presented in terms of stress range, following steel practice. Details are classified broadly as in steel. For a given detail the stress range to be used in design, corresponding to a given number of cycles, is about one-third of the corresponding stress range for steel. It is largely independent of the alloy used.

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15 Load–bearing Masonry

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Contents

15.1 Introduction

In this chapter the word 'masonry' has been used to describe either brickwork or blockwork as well as natural stone. Today, natural stone is seldom used except as a decorative facing and the advice which engineers need on masonry relates primarily to brickwork and concrete blockwork.

The use of brickwork in the UK has changed quite markedly in the 20 to 25 years following its rebirth as a structural material in the early 1960s. Full exploitation of its strength for slender load-bearing walls in high-rise flats has now been halted by the sharp social reaction against this form of building. In the domestic field what once looked like the greatest justification for 'engineered' brickwork has given way to the limited demands of traditional housing. However, there are now new challenges at least as great as those of high-rise housing.

The structural use of concrete blockwork largely dates from the 1960s and its fortunes have followed the same path as those of brickwork.

One of the new structural challenges for masonry lies in the construction of buildings for sport, education, manufacturing and storage. Here the economy of masonry is being used increasingly in unframed buildings, often single-storey, with larger spans than in domestic construction, taller walls and few partitions or returns to brace the whole assembly. As a result of these changes, today's engineering problems with masonry in buildings are largely wind resistance, overall stability and composite action with floors and roofs. Crushing strength takes second place.

Another field where masonry is finding increasing favour is in the cladding of framed construction especially in large industrial units built in steelwork. In this case not only are there problems of the lateral strength of large thin panels but there are complex questions of movement and of the compatibility of the different materials.

All these are very much engineering problems and not matters of architectural opinion.

In civil engineering, the once dominant place of masonry was taken about a century ago first by mass concrete and then by reinforced concrete. Concrete may be more in keeping with a mechanized age than a labour-dominated material like masonry but its appearance is increasingly being criticized and now doubts are arising as to its durability, especially when reinforced. What is more, with growing appreciation of the structural performance of masonry, especially when reinforced or prestressed, concrete has a rival both in slenderness and load-bearing capacity. This makes masonry particularly attractive for structural use in retaining walls, bridge abutments and other civil engineering works, particularly in areas which are visually sensitive. There is also a good case for the revival of the masonry arch.

Engineers need to keep in touch with developments in the use of masonry. Today it is not just a craft material for houses or decorative facings, as was thought 30 years ago, but a major structural element and one benefiting increasingly from engineering understanding.

15.2 Material properties

Before embarking on any structural design in masonry it is important to distinguish between the physical properties of the different materials of which the units are made and to appreciate the limitations of each.

Table 15.1[1] shows the types of masonry units normally available with their materials, sizes, unit strengths and approximate share of the UK market. It also gives the numbers of the current British Standards which define the acceptable quality of each.

Table 15.2 gives some indication of the dimensional stability of each type of masonry unit, i.e. its response to changes in temperature, load and moisture content. Equivalent figures are also given for other materials commonly used in construction. The most essential factors to note are the initial moisture movements:

(1) Clay units are fired at a high temperature and expand, for the most part irreversibly, as they take up moisture from the atmosphere. The expansion is greatest immediately after firing but continues at a diminishing rate for effectively about 10 to 20 years.
(2) Concrete units (bricks or blocks) are cast wet and shrink as they dry out, again largely irreversibly. The shrinkage is greatest immediately after casting but continues at a diminishing rate for effectively about 10 to 20 years.
(3) Calcium silicate bricks, which are of sand and lime, hydrated, pressed and then autoclaved, behave similarly to concrete units.

Not only are the initial moisture movements generally greater than any subsequent cyclic ones due to change of atmospheric conditions, but *those of clay and concrete are of comparable magnitude and in opposite directions.*

This simple distinction between the behaviour of clay and concrete has frequently been ignored in the past with results which have sometimes caused major disruption. Today, now that the different properties of the materials are better understood, there is a tendency to over-react to the problems of movement and sometimes to take precautions which are unnecessary and could even be harmful. The question of precautions against movement is discussed in section 15.7.

15.3 Codes of practice

In the UK, the accepted guidance on the way in which masonry should be designed is given in the British Standard Code of Practice BS 5628. The first part of this Code dealing with unreinforced masonry was issued in 1978.[2] This part is the successor to the greater part of the earlier code CP 111 and deals essentially with walls and piers.

The second part of BS 5628[3] which covers the structural use of reinforced and prestressed masonry was not published until 1985. It makes good the wholly inadequate treatment of reinforced masonry in CP 111 and also puts prestressed masonry on an 'official' basis for the first time. This part of the Code covers the design of all types of spanning structures in masonry as well as walls and piers.

The third part of BS 5628,[4] also published in 1985, gives advice on various aspects of detailing with masonry and on workmanship, durability and similar topics. It could be said to be more architecturally slanted than the first two parts of this Code and is the successor to the earlier Code CP 121.

Since the issue of all three parts of BS 5628, masonry in the UK has been on a parallel basis to concrete in up-to-date and officially recognized design methods. This does not mean that all an engineer needs to know about masonry is in the three parts of this Code – far from it. However, anyone designing masonry structures, in the UK at least, should be aware of the contents of this Code and, whether experienced in masonry or a newcomer to it, would do well to consult the handbooks to Parts 1 and 2. References to these handbooks and to a selection of other publications on the structural design of masonry are given at the end of this chapter.[5,6]

While BS 5628, together with the relevant material standards, will be used as anchor points for the advice in this chapter, this Code should be used for checking design rather than as a

Table 15.1 Types of masonry units normally available.

	Material and manufacture	*Normal (actual) dimensions of unit* (mm)	*Type of unit*	*Characteristic strength of unit* (N/mm²)		*Approximate share of UK market 1985* (10^6m² of wall)
				Range in British Standard	*Range commonly used*	
Clay brick (BS 3921)	Clay fired generally at > 1000°C to achieve ceramic bond	Standard 215 × 102.5 × 65 high	Solid, frogged or perforated	7–100	14–100	58*
Calcium silicate brick (BS 187)	Sand and lime; hydrated, pressed and autoclaved	Metric modular (small demand) 190 × 90 × 65 (BS 6649)	Solid, or frogged	14–48.5	20.5–34.5	1.75
Concrete (BS 6073)	Aggregate and cement hydrated and moulded with pressure and vibration		Solid or frogged	7–40	7–40	4.5
Aggregate Concrete block (BS 6073)		Varies widely: length 390–590 height 140–290 thickness 60–250	Solid or hollow	2.8–35	3.5–21	Dense 30.4 lightweight 22
Autoclaved (aerated) concrete block (BS 6073)	Cement and ground sand or PFA with aerating agent hydrated and moulded in large blocks and then cut		Solid only	> 2.8	2.8–7.0	23

* Equivalent based on 102.5 mm wall

starting-point. The wide variety of forms of structural brickwork and blockwork make their use even more of a design matter, needing individual judgement, than almost any other material.

It is the aim in this chapter to point to these design aspects and to emphasize both the great opportunities for the use of masonry and some of the pitfalls, rather than to provide another handbook to the BS Code.

Reference is made throughout this chapter to BS 5628.

Readers working in countries other than the UK will need to be aware of the local codes, which may differ quite markedly from BS 5628. The following information may be helpful in this respect.

US
The most widely used code in the US is the Uniform Building Code. Chapter 24 of the 1985 edition deals with masonry on a linear elastic (working stress) basis.

Canada
The current code CAN-S304-M84 issued by the Canadian Standards Association covers both design by rules and design by full engineering analysis. This is still on a working stress basis. A limit state code is planned for 1990.

Australia
A unified code incorporating AS1640-1974 (SAA Brickwork Code) and AS1475 (SAA Blockwork Code, Part 1: unreinforced blockwork and Part 2: reinforced blockwork) is about to be issued. This is written in ultimate strength format and will be converted to a full limit state form in the next edition.

New Zealand
References to existing codes may be misleading but two new codes are in draft DZ 4229 for masonry not requiring specific design and DZ 4210 for designed masonry.

The information given above is considered as a starting-point only. Readers are advised to check directly with the appropriate authority in each country.

15.4 Limit state principles

The design guidance in BS 5628: Parts 1 and 2 for unreinforced, reinforced and prestressed masonry follows the same limit state principles, with partial safety factors, as are used with reinforced

Table 15.2 Dimensional stability of masonry compared with reinforced concrete and steel

	Coefficient of thermal expansion (per °C × 10^{-6})	*Movement as result of 20°C change* (%)	*Unrestrained drying shrinkage (partly reversible)* (%)	*Unrestrained moisture expansion* (%)	*Elastic modulus* (kN/mm²)	*Creep with time: creep factor = final strain/ elastic strain* (for stress ⩽ *0.5 × ult.)*
Clay brickwork	5–8	0.010–0.016	Shrinkage of mortar allowed for in expansion figures (right)	Depends on type of clay and firing temperature. Probably 0.02–0.12%. Precise figures uncertain. Too few long-term tests	4–26	1.2–4.0
Calcium silicate brickwork	8–14	0.016–0.028	0.01–0.04 (BS limit 0.04)	—	14–18	Approximately 2.5
Aggregate concrete blockwork*	6–12	0.012–0.024	0.02–0.06 (BS limit 0.09 maximum)	—	4–25	2.0–7.0
Aerated concrete blockwork	Approximately 8	0.016	0.02–0.09 (BS limit 0.09 maximum)	—	1.5–4.0	No test results available
Reinforced concrete	7–14	0.017–0.028	0.02–0.10	—	15–36	1.0–4.0
Steelwork	Approximately 12	0.024	—	—	175–210	—

* Concrete bricks similar

and prestressed concrete. Most engineers in the UK are now familiar with these principles but, regrettably, there is still a lack of uniformity in the terminology used in the different BS material codes.

In BS 5628, the phrases 'design load' and 'design strength' are used to denote the factored loads and strengths which need to be compared to check adequacy. Thus, for the ultimate limit state, if γ_f is the partial factor of safety for loading and γ_m is the partial factor of safety for material strength, adequacy is achieved if:

$$\frac{\text{Ultimate (characteristic) strength}}{\gamma_m} \geqslant \text{Characteristic load} \times \gamma_f$$

In all cases (direct load, bending, shear, etc.) the partial factors of safety are expressed separately in BS 5628 and never lost within the characteristic values quoted.

The same principle and the same terminology are used in BS 5628 for the serviceability limit states and for precautions against disproportionate collapse following a major explosion or other accident, but in these cases the partial factors of safety are different.

Table 15.3 shows the principal factors for each limit state and how these compare with the factors of safety used in BS 8110[7] for concrete. With unreinforced masonry the serviceability limit states of deflection and cracking are seldom if ever relevant but, when considering the behaviour of reinforced or prestressed masonry sections in bending, they can be vital.

15.5 Unreinforced masonry

15.5.1 The mechanism of failure in compression

Rather than just accept the characteristic strengths and 'Code' factors of safety for masonry, designers are advised to consider what influences its strength and to try to visualize the actual mechanism of failure.

Table 15.4 lists some of the major factors affecting the strength of a masonry wall.

The mechanism of failure seems to be generally agreed. Because the mortar is almost always weaker than the masonry units it tends to be squeezed out of the joints. This movement of the mortar is restrained by the bricks or blocks, which are thus subjected to lateral tensile stresses which lead first to splitting and finally to collapse. This mechanism is shown diagrammatically in Figure 15.1.

Even under absolutely uniform downward loads, masonry walls – brick or block – fail first due to vertical splitting. This is virtually universal. With brickwork, the wall strength averages about 0.3 times (0.15 to 0.45 times) the brick strength and with

Table 15.3 Partial factors of safety (material) for masonry compared with concrete (BS 5628 and BS 8110)

BS 5628: 1978, Part 1	*(Unreinforced masonry)*	*Ultimate limit state*	*Accidental damage*	*Serviceability limit state*	*Notes*
γ_{mm} (compression)	*Control level*				
	Manufacturing and site special	2.5	1.25		
	Manufacturing normal and site special	2.8	1.4		
	Manufacturing special and site normal	3.1	1.55	—	
	Manufacturing and site normal	3.5	1.75		
γ_{mv} (shear)	—	2.5	1.25	—	
γ_m (wall ties)	—	3.0	1.5	—	
BS 5628:1985, Part 2 (Reinforced and prestressed masonry)					
γ_{mm} (compression)	*Control level*				
	Manufacturing and site special	2.0	1.0	1.5	
	Manufacturing normal and site special	2.3	1.15	1.5	
γ_{mv} (shear)	—	2.0	1.0	—	No shear reinforcement assumed
γ_{mb} (bond to steel)	—	1.5	1.0	—	
γ_{ms} (steel reinforcement)	—	1.15	1.0	1.0	
BS 8110:1985 Parts 1 and 2 (Concrete)					
γ_m (compression or bending	—	1.5	1.3	1.05	
γ_m (shear)		1.25	—	1.05	
γ_m (bond to steel)	—	1.4	—	1.05	
γ_m (steel reinforcement)	—	1.15	1.0	1.05	

Note: Partial factors of safety for load (γ_f) with masonry similar to those for concrete (basically 1.4 for dead load and 1.6 for superimposed load with variations for combinations and different limit states)

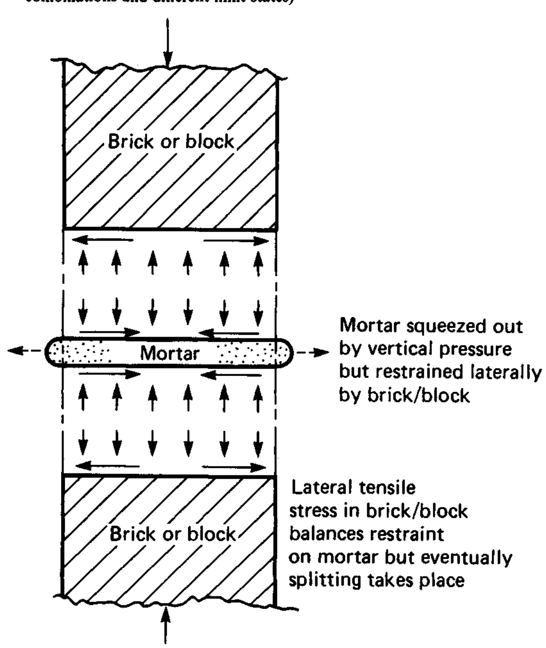


Figure 15.1 Simplified failure mechanism for vertical loads on masonry

concrete blockwork it averages about 0.8 times the block strength.[1]

The difference between the apparent performance of bricks and blocks in walls compared with their individual strengths is primarily due to the shape of the units. The 'cube' strength of the concrete in the blocks is generally well below the equivalent strength of the fired clay in bricks but, when tested as units, the taller blocks fail at a stress nearer to that in a wall than the squat bricks, which are more fully restrained laterally by the platens of the testing machine. This is shown in Figure 15.2. The logical climax is that a storey-high unit should fail at the same load as the wall into which it is built.

Figure 15.3 shows the relationship, as given in BS 5628, Part 1, of the characteristic compressive strength of different types of masonry to that of the individual masonry units. This follows the principles outlined above.

Other important factors affecting the capacity of a wall or pier to support vertical loads, apart from those shown in Table 15.4, are slenderness, eccentricity and concentration of loading.

Table 15.4 Major factors influencing the strength of masonry walls

Variable factor	*Effect on wall strength*		
	Brickwork		*Concrete blockwork*
Strength of masonry unit	Most dominant factor: wall strength proportional to square root of brick strength		Most dominant factor
Strength of mortar	Not very significant: wall strength proportional to cube root of mortar strength for middle range of brick strengths		Little effect on wall strength
Thickness of mortar bed	Fairly critical: 17 mm bed instead of 10 mm gives 30% reduction; with ground faces and no mortar, wall strength approaches brick strength		No experimental data; effect probably less significant than with bricks
Geometry of masonry units	Ratio of wall strength to brick strength little affected whether wirecut, deeply frogged or perforated		 Ratio of wall to block strength about 0.8  Ratio of wall to block strength reduced to about 0.5 with normal bond because cross-webs do not line up; higher with stack bond
Bond	English (50% cross-bonded material) Flemish (33% cross-bonded material)	No noticeable difference in strength	Seldom used other than in stretcher bond (or in stack bond with reinforcement in horizontal joints)
	Stretcher (100% cross-bonded material)	Up to 40% stronger than English or Flemish	
	Collar jointed (steel ties only between skins of stretcher bond but no cavity)	10–15% weaker than English or Flemish	
Poor filling of bed joints	Tests show 30% reduction in strength common, and more possible		No equivalent tests
Poor filling of perpendicular joints	No reduction found in tests even with wholly unfilled perpendiculars		Effect of not filling perpendicular joints at all is small

Source: Sutherland (1981) 'Bride and block masonry in engineering'. *Proc. Instr. Civ. Engrs*, **70**, Table 2.

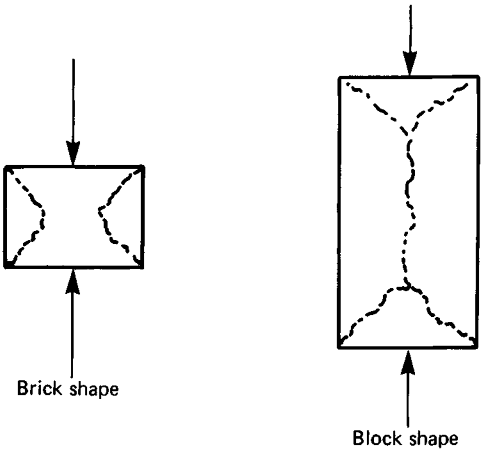


Figure 15.2 Effect of platen restraint on failure of bricks and concrete blocks under test

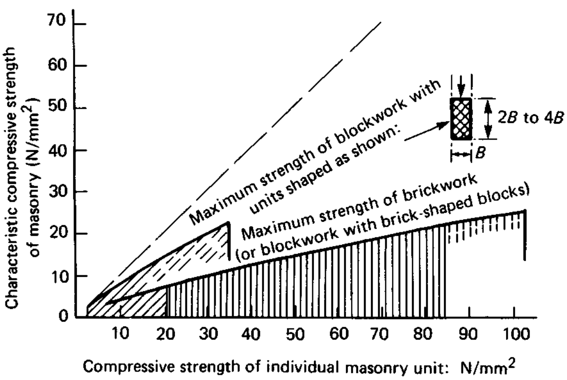


Figure 15.3 Relationship of characteristic compressive strength of masonry to the compressive strength of the units (BS 5628)

15.5.2 Slenderness

Tests on walls in recent years have shown that slenderness is less of a problem than it was thought to be 30 years ago and, in successive British Codes, reductions in load-carrying capacity for slenderness have tended to become smaller.

Reduction factors for slenderness tabulated in BS 5628 permit walls with a slenderness (effective height/thickness) of up to 27. Thus, for instance, allowing for end fixity a half-brick (102.5-mm thick) wall could be 3.7 m high and still carry 40% of the axial load appropriate for a low-height wall of the same thickness. Such a slim wall, as shown in cross-section in Figure 15.4, must lack robustness and most designers would prefer not to extend slenderness to this limit even with purely axial loading.

15.5.3 Eccentricity of loading

Simple eccentricity can be dealt with conservatively by using the appropriate reduction factor for slenderness and eccentricity in BS 5628 and assuming that the eccentricity at the top of any wall reduces to zero at the next level of lateral support below as shown in Figure 15.5(a). In practice, the eccentricity at the bottom of the wall is likely to be as in Figure 15.5(b) which is normally more favourable as it tends to counteract any further eccentricity applied at that level.

As an alternative to this simple procedure, the whole structure can be analysed rigorously as a frame.

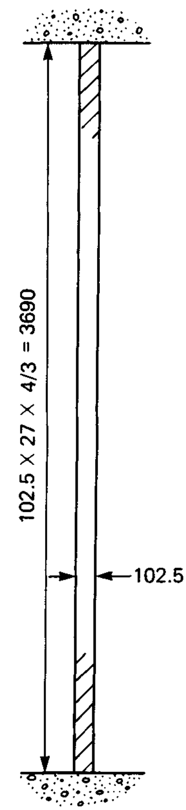


Figure 15.4 Maximum slenderness of wall (BS 5628)

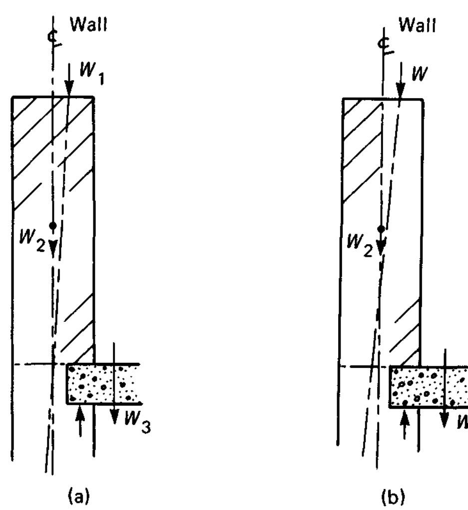
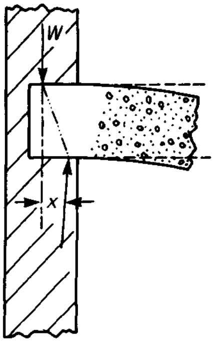
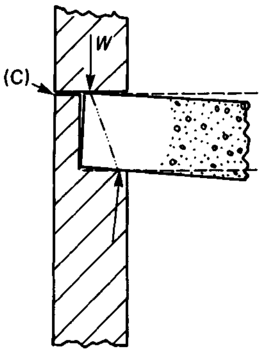


Figure 15.5 Line of thrust due to eccentric loading: (a) simplified assumption on line of thrust (BS 5628); (b) more likely line of thrust

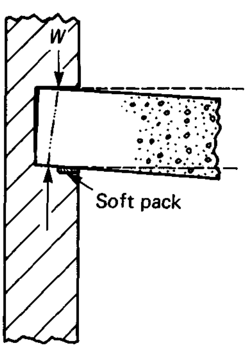
Unless loaded heavily enough to provide fixity as in Figure 15.6(a), the rotation of the ends of floor slabs can cause unsightly cracking as in Figure 15.6(b). This is particularly likely with long-span concrete roofs or upper-floor slabs supported on masonary walls. In such locations the cracking can usually be eliminated, or at least controlled, without over-stressing the masonry, by allowing rotation as shown in Figure 15.6(c).



(a)



(b)



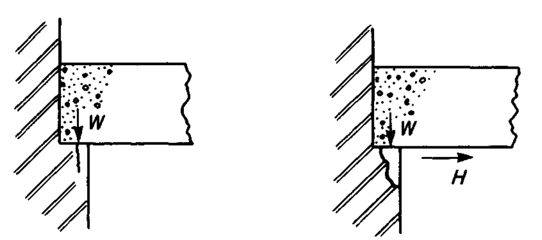
(c)

Figure 15.6 Avoidance of cracking of masonry walls due to rotation of floor slabs at supports. (a) Slab rotation resisted by couple Wx. No cracking of masonry. (b) Load W not great enough to resist slab rotation. Upper wall lifted by rotation of slab and crack induced at point (C). (c) Load W not great enough to resist slab rotation, but soft pack as shown permits rotation without cracking of masonry walls

15.5.4 Concentrated loads

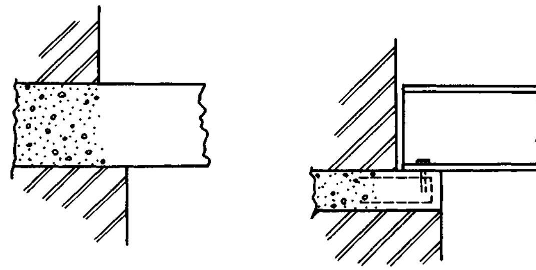
Traditionally, the need to spread concentrated loads from columns or the ends of beams has been met by using padstones of greater strength than the basic masonry of the walls or by laying local courses of stronger brick or stone. Today, this practice has been formalized in rules such as those in BS 5628. The problem is one of splitting and is most serious at the ends of walls or at piers where the splitting is most likely to lead to failure.

Such splitting could also be caused – or accentuated – by horizontal forces due to thermal or other movements. This is



(a) Splitting caused by vertical load from end of beam

Splitting dangerously accentuated by horizontal force due to shrinkage or thermal movement



(b) Beam ends carried well back along supporting wall

Figure 15.7 Concentrated loads on ends of walls or on stiffening piers

shown in Figure 15.7(a). It is something which is not explicitly covered in most Codes of Practice but which designers should consider. With long-span beams especially, the bearing should be carried back as far as practicable or a reinforced padstone should be used as indicated in Figure 15.7(b).

15.5.5 Lateral loads on masonry panels

The design of masonry walls so that they can be shown to resist wind forces is one of the major areas of doubt in the treatment of the material.

The uncertainty is greatest with panel walls with no vertical load except their own weight, but it is present with most thin or lightly loaded walls.

Following an extensive series of tests by the British Ceramics Association[8] some partly empirical and partly theoretical guidance has been incorporated in BS 5628, Part 1. This can be used to demonstrate adequacy in many of the most common situations but with vertical spanning in particular it is restrictive compared with what has been common practice for many years.

Where it can be shown to be appropriate, design for arching is a very good method of proving lateral stability; it, too, is recognized in BS 5628.

Real walls do sometimes blow out, or over, and those which do always seem to have a slenderness or lack of restraint far outside recommended limits. Some unlikely 'successes' may well be due to arching – even unexpected arching – and some to much higher tensile strengths in mortar joints than those generally assumed. In many cases the full 'Code' wind forces may never have occurred and may never occur in the future.

Much research is still being carried out on the resistance of masonry panels to lateral loads and it is hoped that further guidance may be given in future amendments to BS 5628. In the meantime, it is worth keeping in touch with the results of this research.

Two points of detail which designers would do well to remember are these:

(1) Whatever the published bending strengths, bending in a horizontal plane as in Figure 15.8(a) is much more reliable

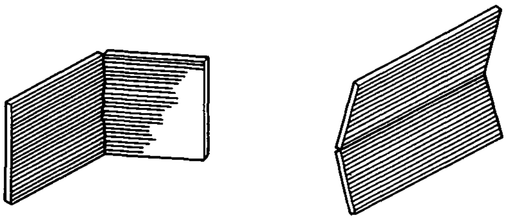


Figure 15.8 Resistance of masonry panels to lateral loads: (a) Strength in bending in horizontal plane; (b) greater and more certain than in vertical plane

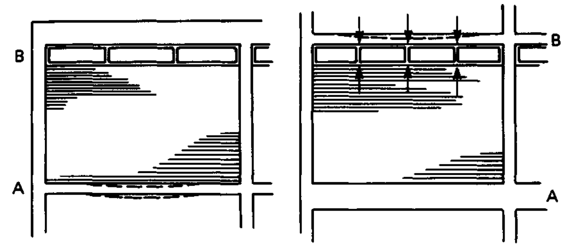


Figure 15.9 Possible effects of deformation on lateral support for masonry cladding

in practice than in a vertical plane as in Figure 15.8(b). Tensile strength across bed joints can easily be disrupted especially during construction after the mortar has set but before it has gained its full strength.

(2) The edge support to panels will often be altered – sometimes for better and sometimes for worse – by the deformation of the structure, as can be seen from Figure 15.9. It is important to consider such possible movements and to make sure that adequate fixings are used at all edges assumed to provide restraint, even if there are relative movements due to deflection, shrinkage or settlement.

15.5.6 Stability and robustness

The great bulk of the advice in the world's masonry codes is devoted to stresses and compressive strength with only passing exhortations to consider stability. This seems curious when one considers that almost all failures of masonry structures – few in practice – have been due, not to overstressing, but to some form of instability. The reason may be that attempts to codify stability, while helpful in some circumstances, have tended to lead to anomalies, unreasonable restrictions, or even new dangers, in others.

Stability and robustness are best seen as design matters. They need thought rather than rules.

Stability is a particularly important factor with masonry because of its low tensile strength. Unless reinforced or prestressed, masonry should generally be planned to rely for stability on gravity forces and on the friction induced by such forces.

With today's thin walls, this stability depends on the interaction of walls, floors and roofs. A lateral force acting on the face of a wall, such as that due to wind, is transferred to floors and roofs, which act as rigid horizontal plates and in turn transfer the force to the ground through shear walls running in the direction of the force. This is shown diagrammatically in Figure 15.10.

Designers should always follow the forces through this route to make sure that all the connections are adequate, that the floors and roofs are stiff and strong enough and that the 'racking' shear strength of the shear walls is great enough. Even if the structure is fully sheltered from wind it is important to consider a nominal lateral force acting in any direction and to

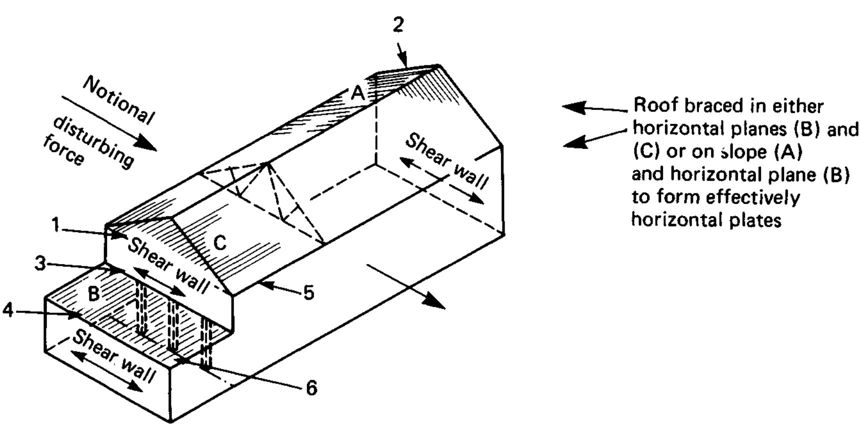


Figure 15.10 Diagrammatic representation of transfer of disturbing force on masonry structure to provide stability

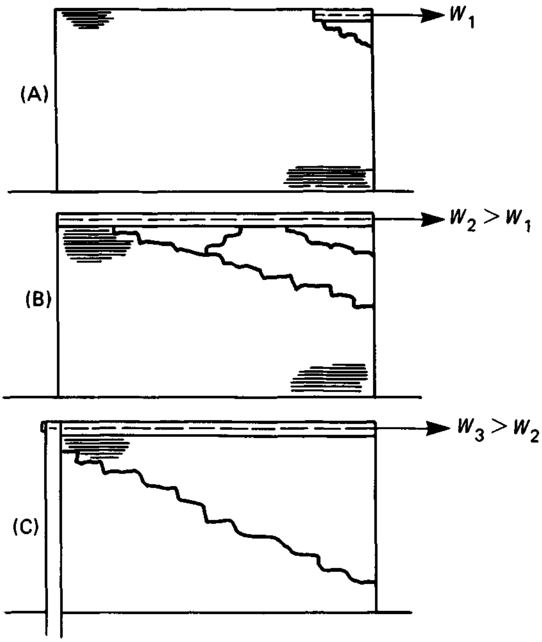


Figure 15.11 Relative strengths of three possible methods of connecting on to a shear wall

follow this to the ground. British Standard 5628 recommends a nominal lateral force of 1.5% of the dead load above the level being considered. A larger force may sometimes be preferable.

One of the most difficult questions with masonry is the assessment of the strength of connections. Figure 15.11 shows three means of connecting on to a shear wall. Designers would do well to opt for the strongest connection which is compatible with cost and other requirements rather than accept the lowest level of apparent adequacy.

There are few absolutes in design for stability. The skill lies in balancing requirements which are often in conflict. This is illustrated in Figure 15.12. There is no virtue in just achieving what appears to be adequate stability if an unnoticeable change can make this much more certain. Accidental forces and material defects are largely unpredictable.

15.5.7 Accidental forces

Design to allow for accidental forces is just a particular case of design for stability. Rules introduced following the partial collapse of the large panel concrete structure at Ronan Point in 1968 have largely blinded engineers to the broad issue of accidental forces. These rules, which apply only to buildings of five storeys and above, were made to guard against gas explosions and do not in themselves ensure safety against all hazards.

The thinking was rightly to limit damage but the tying forces introduced in the structural codes to satisfy the rules may in some cases actually spread the damage. This is illustrated in Figure 15.13 where continuous vertical ties could actually cause progressive collapse as was shown in tests on a quarter-scale model at the Building Research Establishment.[9]

Accidental damage can be limited by planned sacrifice or by greater structural strength. The choice is a design matter although the solution must satisfy the broad functional requirements of the Building Regulations in most practical cases.

15.6 Reinforced and prestressed masonry

15.6.1 General

Much of the previous section on unreinforced masonry still applies but, when reinforced or prestressed, the scope for the use of masonry is greatly extended. Reinforced or prestressed beams can be made, as well as walls, following exactly the same general principles as those used with concrete. This has been demonstrated in tests and in practice.[10] The question to be decided is when such forms are advantageous, and this can be done only with some knowledge of the possible structural performance of

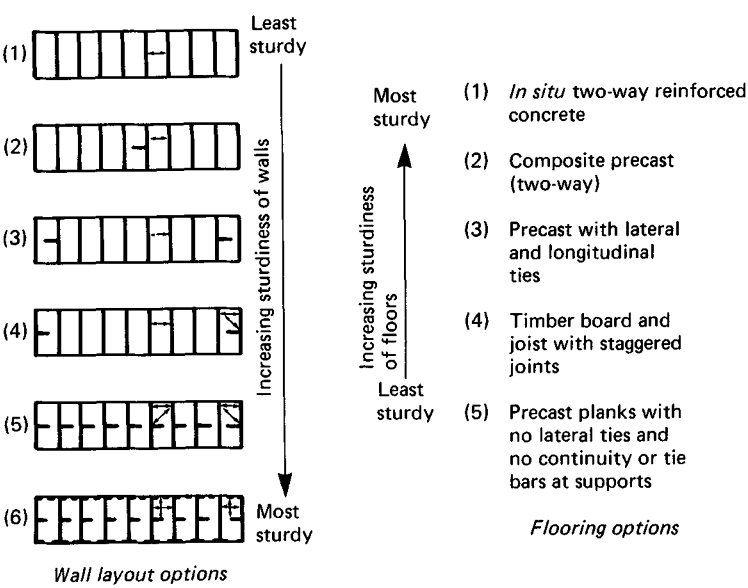


Figure 15.12 Comparison of wall and floor options for simple masonry cross-wall construction

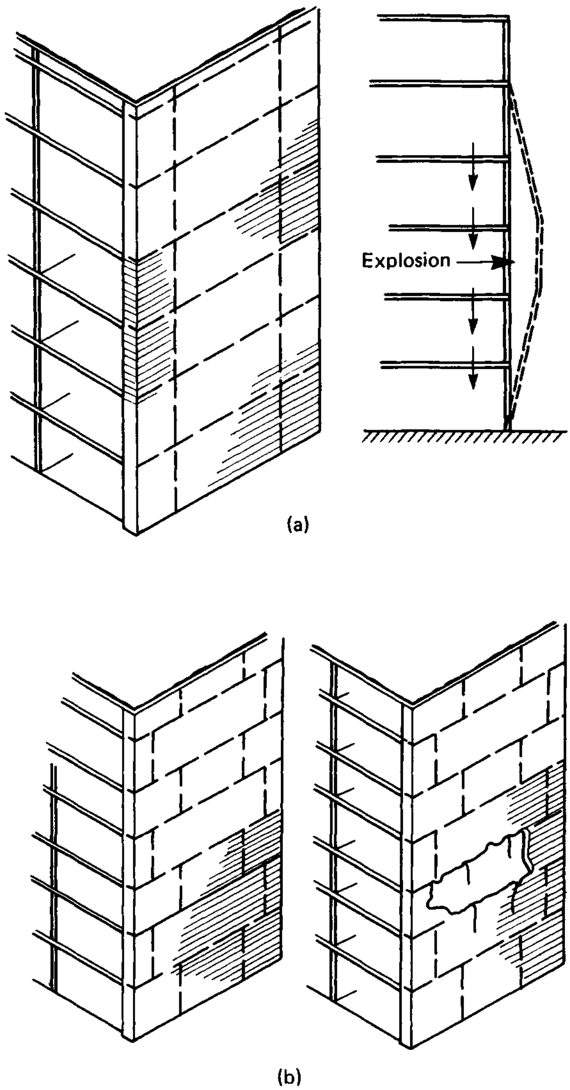


Figure 15.13 Diagrammatic representation of how simple code recommendations can spread accidental damage. (a) Horizontal ties broken by explosive force. Continuous vertical ties (as codes) keep well intact which bows out dropping ends of several floors above and below. (b) Independent staggered ties would allow local sacrifice and confine damage

reinforced or prestressed masonry. It is convenient, in this context, to compare the properties of masonry with those of concrete.

15.6.2 Structural performance of reinforced masonry

Figure 15.14 shows the relative bending strengths of reinforced brickwork (BS 5628, Part 2) and reinforced concrete in accordance with BS 8110. It can be seen that with moderate brick strengths the bending capacity of brickwork matches that required for most reinforced concrete and that, even with the lowest brick strength likely to be used (20 N/mm² unit strength), there is a very useful level of bending strength. A similar comparison can be made with concrete blockwork although the highest practical bending strength with blockwork is not so great.

With shear, the comparison is not so favourable to masonry as it is with bending. Figure 15.15 shows – again for brickwork – that without shear reinforcement the shear strength of all strengths of brickwork is well below that of reinforced concrete; the same is true of concrete blockwork. It is worth noting that the shear strength is virtually independent of the strength of the masonry units, being dependent on the tensile strength of the mortar and its bond to the units.

One advantage of prestressing over reinforcing with any material is that the prestress reduces the principal tensile stress and thus increases the shear capacity. With masonry this is a very real advantage but, curiously, no reference has been made to it in BS 5628, Part 2. The omission does not mean that designers cannot take advantage of this property of prestressing.

Design resistance moments

Brick (BS 5628:Part 2)			Concrete (BS 8110:1985)	
Mortar (i) : special mfg. control		$y/x = 0.6$		
$Md = 0.2\,bd^2 f_k$			$m = 0.156\,bd^2 f_{cu}$	
Unit strength (N/mm²)	f_k N/mm²	DESIGN R.M. M_d (N.mm)	CHAR. CUBE STRENGTH f_{cu} (N/mm²)	DESIGN R.M. m (N. mm)
7	3.4	0.7 x bd^2		
10	4.4	0.9 x bd^2		
20	7.4	1.5 x bd^2	10	1.6 x bd^2
35	11.4	2.3 x bd^2	15	2.3 x bd^2
50	1.5	3.0 x bd^2	20	3.1 x bd^2
100	2.4	4.8 x bd^2	30	4.7 x bd^2
			45	6.2 x bd^2
			50	7.8 x bd^2

Figure 15.14 *Comparison of bending strength of reinforced brickwork and reinforced blockwork*

Design shear strength				
Brick (BS 5628:Part 2)		Concrete (BS 8110)		*Notes:*
Unit strength (N/mm²)	Design shear strength (N/mm²)	Char. cube strength (N/mm²) f_{cu}	Design strength (N/mm²)	
All	0.35 → 0.7 (γ_m = 2.0) (γ_m = 2)			(1) No shear steel
	0.175 → 0.35	25	0.34 – 1.22	(2) Shear strength varies with proportion of main steel
	In some cases with short shear spans this can be increased to a max. of 0.87	30	0.36 – 1.27	
		40	0.40 – 1.43	
		Note: Factor of safety of 1.25 included for concrete		

Figure 15.15 Comparison of shear strength of reinforced brickwork and reinforced concrete

It has been an accepted feature of prestressed concrete for at least 40 years.

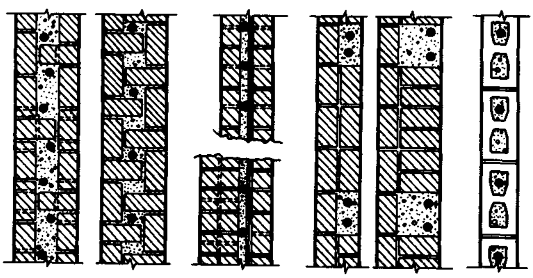
Compared with reinforced masonry, there has been little practical experience with prestressed masonry and there is a school of thought which considers that it would have been best to omit it altogether from BS 5628 at this time. Nevertheless, it has been used successfully, as is shown later in this chapter, and may be used more in the future.

15.6.3 Uses for reinforced masonry

Not only is the shear strength of masonry low, but it is very difficult to incorporate shear reinforcement in it to increase this. For both these reasons, masonry compares unfavourably with concrete for use in beams except possibly in special cases such as that of a deep beam within the plane of a wall. However, reinforced masonry comes into its own in laterally loaded walls where the low shear strength is seldom a major problem.

Masonry is essentially a wall material, or one for arches and vaults. Traditionally, it used to depend for stability on its mass but today, with reinforcement, laterally loaded walls can be made of comparable slenderness to those in reinforced concrete. Further, while concrete walls, if visible, are increasingly being faced with masonry, reinforced masonry has its elegance built into the structure. Figure 15.16 shows a number of ways of reinforcing masonry walls using standard bricks and blocks.

Vertical-spanning reinforced masonry



Modified quetta bond*	Quetta bond*	Grouted cavity†	Pocket wall*	Filled hollow block*
Bars set in fine concrete	Bars normally set in mortar	Bars set in fine concrete or 'grout'	Bars set in normal concrete	Bars set in fine concrete
Brick	Brick	Brick (or concrete block)	Brick	Concrete block
Used in air-raid shelters	First developed for earth-quake resist-ance	225 mm wall also possible with bars in mortar (stainless steel advisable if exposed)	Mainly used for earth retaining walls	Widely used in the US

* Essentially vertical span but secondary horizontal reinforcement often used in bed joints
† Full two-way span possible (or horizontal only). (Grout is the term used in the US for fine high slump concrete.)

HORIZONTAL-SPANNING REINFORCED MASONRY

Normally used for light loads only (except grouted cavity type).
Reinforcement normally small bars or mesh set in bedding mortar.
Equally suitable for brickwork and blockwork.

Figure 15.16 Typical methods of reinforcing masonry walls

15.6.4 Durability of reinforced masonry

As with reinforced concrete, durability is a factor which is receiving increasing attention today. British Standard 5628, Part 2 gives clear and full recommendations on cover to normal carbon steel and to galvanized reinforcement in masonry for different levels of exposure.

With stainless steel reinforcement there is no need for any cover to the steel specifically for durability. Although the supply cost of the steel is several times that of normal carbon steel, the percentage extra on the whole project tends to be very small once fixing and all the other costs of the construction are included.

Stainless steel is being used increasingly for ties and fixings in masonry as well as for reinforcement and is no longer the exorbitantly expensive material it used to be.

15.7 Dimensional stability of masonry

The order of unrestrained movements of clay and concrete products is shown in Table 15.2. Because of the restraints which exist to some extent in all real buildings these movements tend to be less than one would calculate from the tabulated figures. The vital question is how much less, and the answer must be that at present we do not know.

British Standard 5628, Part 3 recommends an allowance of 1 mm of movement per metre for clay brickwork, with movement joints a maximum of 15 m apart. For calcium silicate bricks joints are recommended at 7.5 to 10 m and at 6 m for concrete bricks and blocks.

It is worth remembering what these joints are for. In the case of clay brickwork they are needed primarily for expansion and thus, to be effective, must be wide enough and filled with something soft enough to allow the expansion to take place. In the case of calcium silicate bricks and all concrete units the joints are needed mainly for shrinkage and effective sealing becomes most important.

As in the case of movement joints in concrete structures, there is some evidence that the recommended cure for movement problems has not always been effective. The cure may even introduce new problems of maintenance. A recent study for the Construction Industry Research and Information Association (CIRIA)[11] showed that, both for clay and concrete units, adherence to the spacing then recommended in CP 121 failed to eliminate noticeable defects in a significant number of cases, while in quite a large proportion of other cases no noticeable defects were found even in walls well beyond recommended limits of unbroken length.

The problem of movement is too complex for simple rules. What is more, in many cases it may be most economical, and in the long term most satisfactory, to reduce the number of joints, risk some cracking and repoint after a few years.

Designers would do well to study case histories, observe real buildings and then try to recognize the situations where movement may be serious and those where damage, if it occurs, is only of a cosmetic nature. Figures 15.17 and 15.18 show some key factors but these should only be considered as examples.

15.8 Application of masonry and scope for future use

15.8.1 High-rise (small-cell) residential buildings

Unreinforced masonry has formed the sole vertical support to residential buildings of up to at least 18 storeys while with vertical reinforcement it has been used in blocks of over 20 storeys even in seismic zones. In most cases the design has been dominated by the need for resistance to lateral forces and the assessment of interaction between floors and walls to achieve this. Perhaps surprisingly, tension has often proved more of a problem than compression.

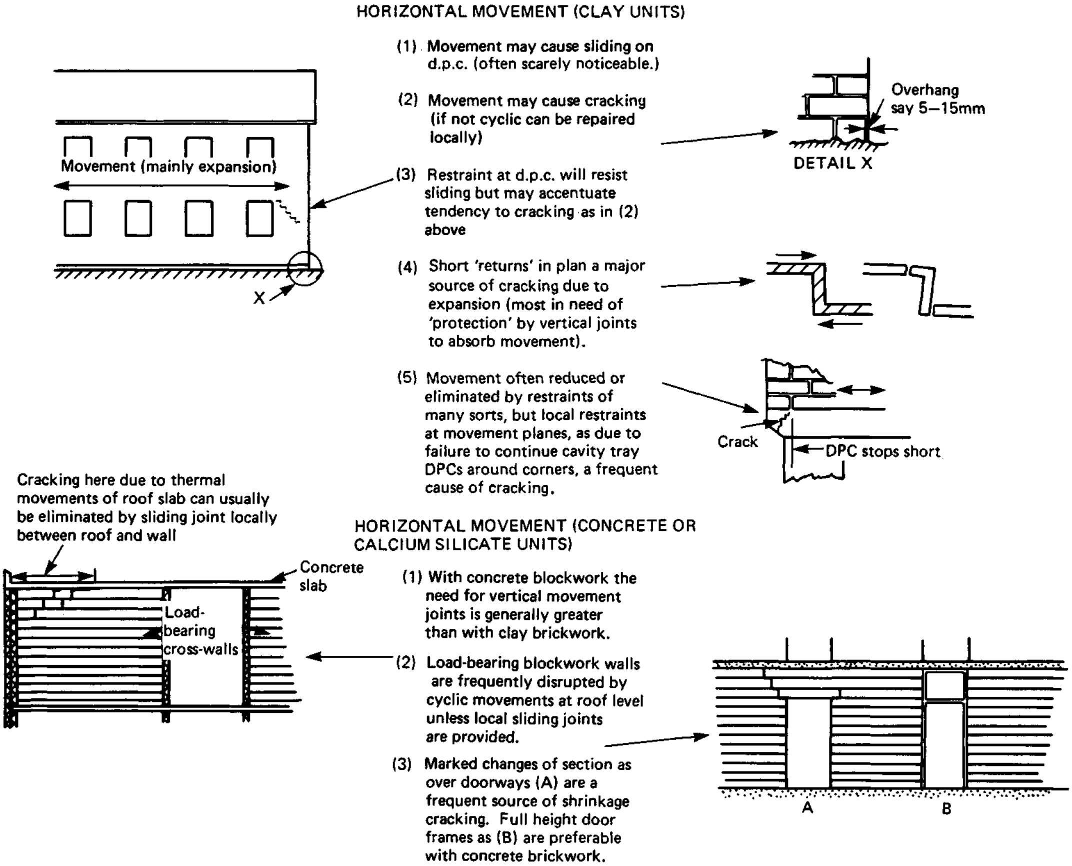


Figure 15.17 Typical serviceability problems due to horizontal movements in masonry

With the increased development of computer programs in the last 15 years the design of such buildings should be much easier than it was in their heyday in the mid 1960s. There are also some signs of a revival in the popularity of high-rise flats and hostels.

15.8.2 Low-rise (large-cell) buildings

The challenge of extending the economic use of masonry, as proved in domestic construction, to larger-cell buildings such as sports halls or warehouses has been mentioned at the beginning of this chapter.

Here the downward loads tend to be small but the walls may need to span vertically 2 or 3 times as far as in housing. This has been achieved in frameless construction with deep ribs at regular intervals, emphasized architecturally, or by making the walls cellular. In such cellular walls – popularly known as 'diaphragm' walls – the masonry is placed in the most efficient way to resist lateral forces (Figure 15.19a). The walls are usually designed to span from the ground to a roof which is braced to transfer the load to shear walls as already discussed. Sometimes such walls are prestressed with vertical cables either bonded or in voids as shown in Figure 15.19(b), or they can be reinforced. Care is needed during construction to make sure that the walls do not fail in wind before the bracing of the roof is provided.

15.8.3 Boundary walls

Masonry has proved itself for boundary walls over a very long period. Such walls, either of constant section or with wholly inadequate stiffening piers, frequently defy all probability of stability but have given good service for decades or centuries; some such walls survive even in spite of considerable bulges or tilts. Nevertheless, from time to time they do blow over and there is no justification for building unstable boundary walls today.

Stability can be achieved by an irregular planform or by reinforcing vertically or prestressing or by any combination of these. Reliance on the tensile strength of the masonry across the bed joints is unwise except on a very small scale. The forms of reinforcement shown in Figure 15.16 can be used or stable walls of very slender sections can be built with special or cut bricks as shown in Figure 15.20. The use of stainless steel is advisable in most boundary walls because of their extreme exposure.

Shear strength is virtually never a problem with boundary walls.

15.8.4 Retaining walls

Reinforced masonry has proved to be particularly suitable for retaining walls, either on a small scale associated with housing or on major civil engineering works. It is hard to understand why it has not been used more. Reinforced concrete walls are often faced in masonry for appearance. Why not use the masonry structurally? The answer to this must be that it is largely habit which prevents engineers from thinking of reinforced masonry, or fear which leads to unduly high pricing of

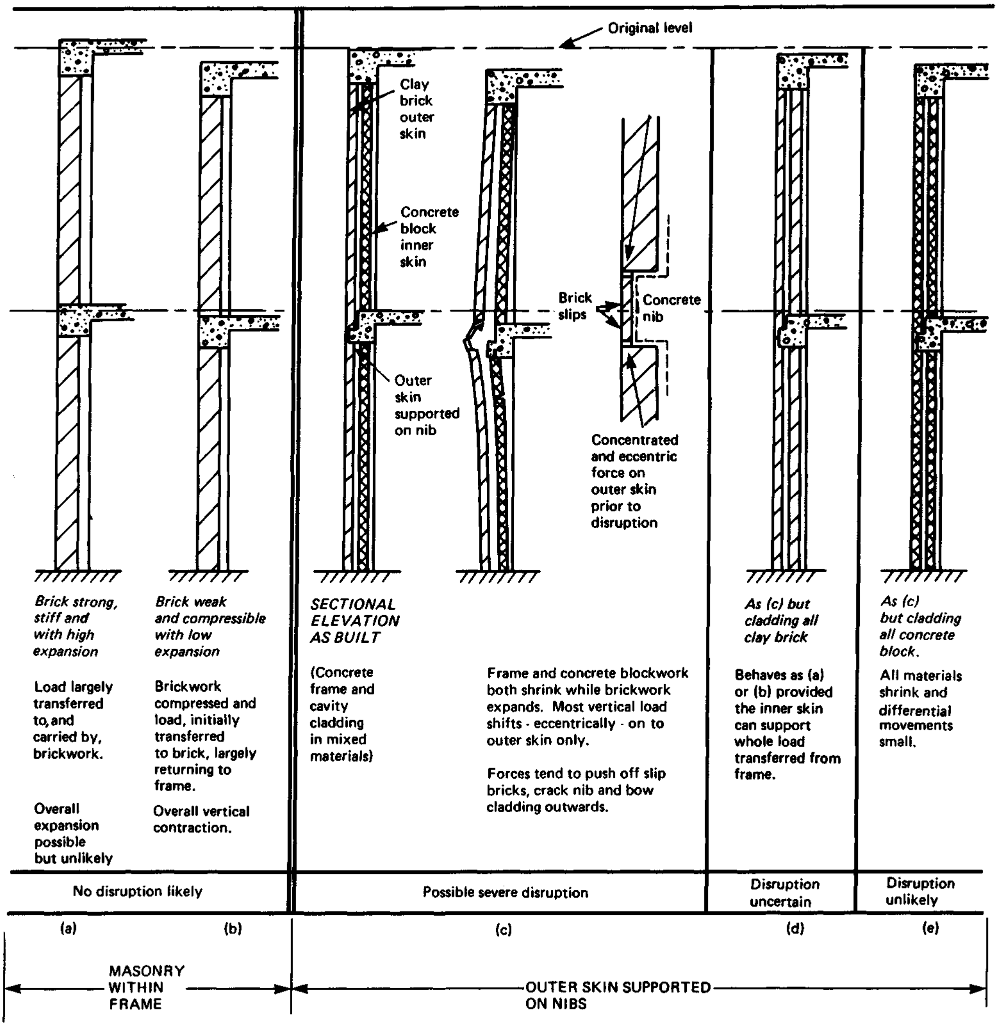


Figure 15.18 Relative movements of masonry cladding and concrete frames: diagrammatic representation based on two storeys. Real problems tend to be confined to multi-storey building

the slightly unfamiliar. However, today there is enough evidence of successful construction of retaining walls in masonry to prove that they are easy to build, and perform well.

Brick retaining walls have been built successfully in Quetta bond, in grouted cavity construction and using the 'pocket wall' technique. With concrete blockwork the fixing of reinforcing bars in the filled hollows in the blocks has become quite common. The planforms of such walls are shown in Figure 15.16.

Unlike boundary walls, retaining walls normally need only resist lateral forces in one direction. Thus, for cantilevers the reinforcement should be as near to the loaded face as possible. The pocket-type of wall is ideal in this respect. Figure 15.21 shows how such walls are formed, with the thickness increasing to match the increasing bending moment and the reinforcement as close to the rear face as practicable. In some cases the steps in thickness have been repeated two or three times.

In pocket-type walls the reinforcement is surrounded in dense concrete whose compaction can be checked once the small back shutter to the pocket is removed. Thus, the durability is equivalent to that of reinforced concrete but the compressive force is resisted not by the concrete but by the brickwork.

Pocket-type retaining walls have been built in the US with heights of up to 7.3 m.[12] There are now quite a few major walls of this type in the UK used for bridge wing walls and similar purposes. One British pocket-type retaining wall approximately 4 m high was monitored for 517 days after which the deflection was only 16 mm. As expected, the movement was apparently continuing but tailing off.[13]

The design of pocket-type walls is covered in BS 5628, Part 2 which deals with concrete cover, pocket spacing and workmanship as well, of course, as with structural design. In some circumstances, the characteristic shear strengths in BS 5628, Part 2 may prove a restriction on the performance of such walls, although tests on actual walls have almost all shown failure in bending. There is a good case for revising the shear clauses in BS 5628 especially in relation to retaining walls.

One objection which is sometimes raised to the use of masonry retaining walls on civil engineering projects is speed of building. This objection may or may not be real but with

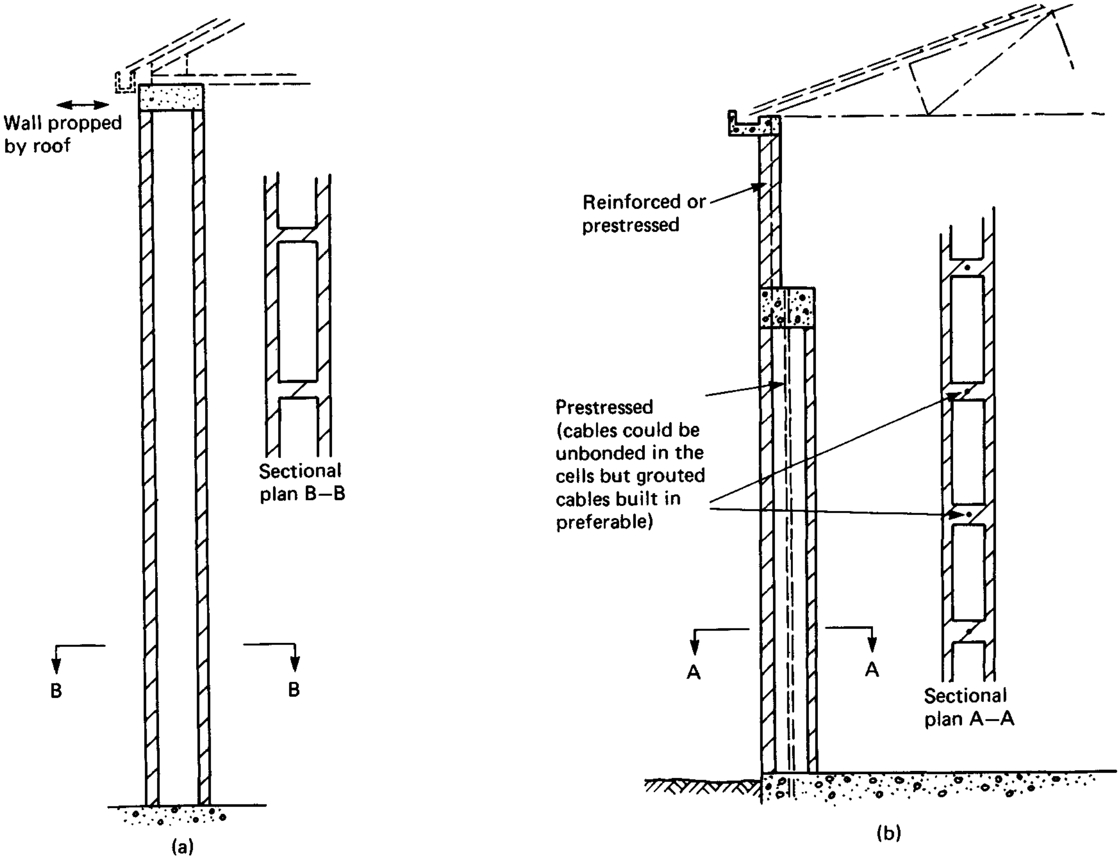


Figure 15.19 Typical forms of cellular (diaphragm) wall. (a) Unreinforced (wall spans from ground to roof). (b) Prestressed (or reinforced). Wall acts as vertical cantilever

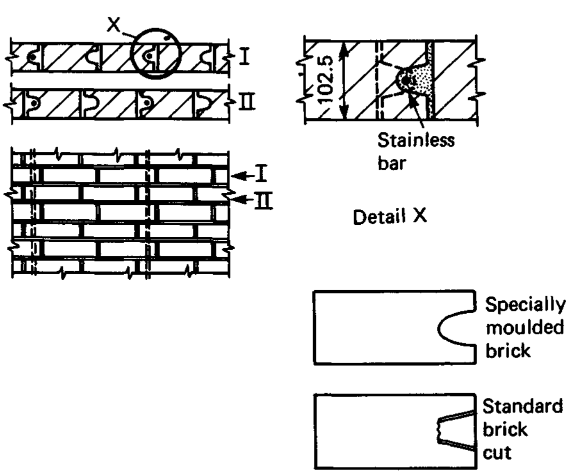


Figure 15.20 Thin but stable boundary walls formed with special or cut bricks (concrete blocks also as in Figure 15.16)

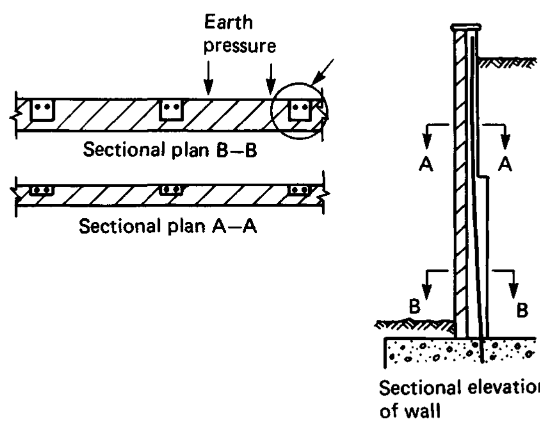


Figure 15.21 Typical pocket-type retaining wall

pocket-type walls, prefabrication is a very real possibility. This was demonstrated by a trial some years ago.[1]

15.8.5 Bridges

Masonry bridges have generally proved more durable than those of iron, steel or concrete. There are thousands of them under roads and railways and over canals which have had minimal maintenance during 100 years or more. They have adapted themselves to settlement without distress and still look attractive. Tests have been made in recent years, both in the laboratory[14] and by the Transport and Road Research Laboratory on actual arch bridges. Our understanding of the behaviour of masonry bridges is better today than it ever was but in spite of this we continue to use slab or beam bridges under highways which are subject to corrosion due to rain and de-icing salt.

In the future, there seems a clear case for using more masonry arches, possibly combining mass concrete with brickwork. Such arches would be particularly appropriate for medium spans where piped culverts are too small but major long spans are not needed. Useful guidance on the appraisal of existing masonry bridges is given in the Departmental Standard BD 21/84 and the associated Advice Note BA16/84 both issued by the Department of Transport.[15] These documents are also relevant, at least in part, to new construction.

15.9 Conclusions

With the publication of all three parts of BS 5628, the UK is probably leading the world in recognized guidance on masonry design. Further, in research, the UK has taken a leading role for 20 years. It is now up to design engineers to make full use of this rediscovered material.

15.10 Acknowledgements

Tables 15.1 to 15.3 were first published in the Author's paper[1] but have been brought up to date. Figures 15.1, 15.3, 15.16 and 15.21 and parts of Figures 15.19 and 15.21 have been adapted with the permission of Thomas Telford Ltd from those in this paper. Figures 15.10, 15.11, 15.12 and 15.13 have been adapted with the permission of the Institution of Structural Engineers from those already published in the Author's paper.[9] The Author wishes to thank those concerned for permission to reproduce this material.

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16

Timber Design

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Contents

16.1 Introduction

Timber is one of the finest structural materials: it has a high specific strength, can be easily worked and jointed and does not inhibit design. Like most other structural materials it suffers attack causing deterioration (corrosion, weathering and bio-deterioration) but once the material is known and the causes understood, effective preventative measures can be taken easily and economically.

Design is thus a confluence of specification, structural analysis, detailing and protection, each of which is of equal importance if an effective design is to be achieved.

Nowadays, structural design covers a wider range of components than ever before, for the intense wind loadings in high-rise building coupled with large glazed areas has meant that much window joinery is now subject to structural design. In addition, the effects of wind loadings together with the requirements of the Building Regulations and the newer building shapes has meant that even in low-rise buildings, components which once were built must now be designed.

Timber is thus used for a wide variety of structural purposes, either on its own or in combination with one of the 'heavier' materials. It can take extremely simple forms such as solid beams, joists and purlins or can be used in the more recent forms of glued-laminated construction or plywood panel construction. These latter forms allow the design of exciting and economic structural shapes, the variety of which may be judged from Tables 16.1 and 16.2.

Some of the characteristics of timber may be found in Table 16.3, whilst general properties are given in other publications.[1-3]

16.2 Design by specification

Essentially, this is a prescription of fitness for use under service conditions and requires the choice not only of an appropriate material but also of its condition, use and protection.

The success of the specification will depend upon its interpretation; standard glossaries are available for timber and woodwork,[4] nomenclature of timber[5] and preservative treatment.[6]

Functional and user needs will dictate the choice of material based on the following factors.

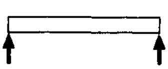
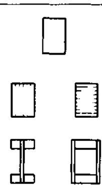
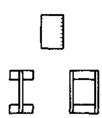
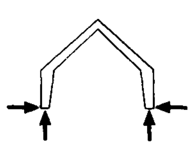
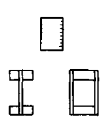
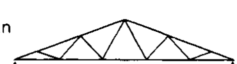
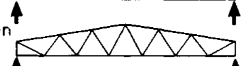
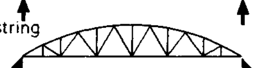
16.2.1 Species and use

Very many timbers are structurally useful, whereas usefulness for joinery purposes is often more restrictive. Where timber is used for structural joinery, the combination of requirements may be even more restrictive.

Table 16.3 lists most of the timbers and their characteristics for which working stresses are given in BS 5268: Part 2,[7] whilst BS 1186[8] indicates the joinery use of specific species. A comprehensive guide to West African species and their uses, both structural and joinery, is given in pamphlets issued by the United Africa Company.[9]

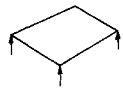
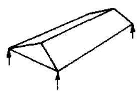
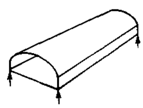
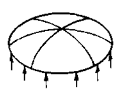
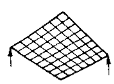
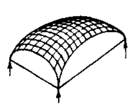
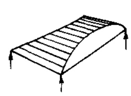
Flooring, particularly industrial flooring, has particular requirements and recommendations for suitable timbers for these

Table 16.1 Roof selection

Division	*Subdivision*		*Construction*	*Minimum support conditions*	*Maximum economic spans* (m)	*Fastenings*
Beams			Solid timber		6	None
			Laminated, either vertically or horizontally, depending on size	Vertical support at ends	24	Glue
			I or box sections: flanges solid or laminated. Webs plywood or diagonally boarded		30	Glue and/or nails
Arches			Laminated horizontally	Vertical and horizontal support at ends	46	Glue and/or nails for laminating
			I or box sections: flanges laminated horizontally. Webs diagonally boarded		46	Connectors for site joints
Portals			Laminated horizontally		24	Glue and/or nails for laminating
			I or box sections: flanges solid or laminated. Webs plywood or diagonally boarded	Vertical and horizontal support at ends	46	Connectors for site joints
Trusses	Belgian 		Solid timber	Vertical support at ends	12	Nails and/or glue
					24	Connectors
	Warren 		Solid timber		12	Nails and/or glue
					30	Connectors
	Bowstring 		Laminated chords. Solid webs		46	Glue and/or nails for laminating Connectors at joints

(After: L. G. Booth, *Engineering*, 25 March 1960)

Table 16.2 Roof selection

Division	*Subdivision*		*Construction*	*Minimum support conditions*	*Maximum economic sizes* (m)	*Fastenings*
Plates	Flat		Membrane formed from plywood or layers of diagonal boarding A single-skin structure may have stiffening ribs	Vertical support at corners	12 × 12	Nails and/or glue
	Folded		A double-skin structure will have spacing ribs Edge beams and end diaphragms required	Vertical support at corners	18 × 9	Membrane with nails and/or glue Diaphragms with nails or connectors
Singly curved shells	Circular cylindrical		Membrane formed with layers of diagonal boarding. May have stiffening ribs Edge beams required End diaphragms required	Vertical support at corners	30 × 12	Membrane with nails and/or glue Beams (see Table 16.1) Diaphragms with nails or connectors
Doubly curved shells	Spherical dome		Boarded membrane with or without laminated ribs Laminated ring beam	Ring beam to be supported at intervals	30 dia.	Membrane with nails and/or glue Ribs and ring beam glued
	Hyperbolic paraboloid		Boarded membrane with laminated edge beams	Vertical support only at low points, if columns tied together. Otherwise buttresses at low points	21 × 21	Membrane with nails and/or glue Edge beams glued
	Elliptical paraboloid		Boarded membrane with laminated tied arches along edges	Vertical support at corners	24 × 24	Membrane with nails and/or glue Tied arches with glue and connectors
	Conoid		Boarded membrane with laminated tied arches on ends Edge beams required	Vertical support at corners	30 × 9	membrane with nails and/or glue Beams (see Table 16.1) Tied arches with glue and connectors

(After: L. G. Booth, *Engineering*, 25 March 1960)

requirements are given in Princes Risborough Laboratory (PRL) Technical Note No. 49.[10]

16.2.2 Availability and sectional properties

Availability is equally important, and Table 16.3 indicates the availability of the structural timbers from the viewpoints of supply and length. The geometric properties of sawn and precision timber to be used in design are also given in BS 5268: Part 2. Guidance on the usefulness of worldwide species may be found,[11] whilst the available sizes for hardwoods are given in BS 5450.[12]

16.2.3 The movement of timber

Even with dried timber, changes in atmospheric conditions will result in a varying moisture content which will induce fluctuating dimensional changes in the timber, known as 'movement'. The variation can be designed-for quite simply but some knowledge of the degree of possible movement is helpful. Some indication can be obtained from Table 16.3, whilst further information can be obtained from the publications of the PRL.[2,3,13,14]

16.2.4 Moisture content and end use

Every species of timber will achieve a fairly steady moisture content for a particular environment – the equilibrium moisture content. The PRL has established moisture contents for various environments.[12] Greater reliability can be achieved by drying timbers to these moisture contents before construction.

Table 16.3 Characteristics and availability of some structural timbers

Standard name	*Approx. density at* M/C 18% (kg/m³)	*Natural durability*	*Resistance to preservative treatment*	*Moisture movement*	*Working quality*	*Availability*		*Relative price*
						Supply	*Normal length* (m)	
SOFTWOODS (IMPORTED)								
Douglas fir–larch	590	Moderately	Resistant	Small	Good	Good	4.20–4.80	Medium
Hem–fir	530	Not	Resistant	Medium	Good	Good	4.20–4.50	Low
Parana pine	560	Not	Moderately	Medium	Good	Good	3.60–3.90	Low
Pitch pine	720	Durable	Resistant	Medium	Good	Good	4.50–9.00	Medium
E. redwood	540	Not	Moderately	Medium	Good	Good	1.50–7.00	Low
E. whitewood	510	Not	Resistant	Small	Good	Good	1.50–7.00	Low
Canadian spruce–pine–fir	450	Not	Very	Medium	Good	Good	2.40–5.10	Low
W. red cedar	380	Durable	Resistant	Small	Good	Good	2.40–7.30	Low
SOFTWOODS (HOME GROWN)								
Douglas fir	560	Moderately	Resistant	Small	Good	Fair	1.80–4.50	Low
Larch (E—Japan)	560	Moderately	Resistant	Medium	Good	Good	1.80–3.60	Medium
Scots pine	540	Not	Moderately	Medium	Good	Good	1.80–3.60	Low
European spruce	380	Not	Resistant	Small	Good	Good	1.80–3.60	Low
Sitka spruce	400	Not	Resistant	Small	Good	Good	1.80–3.60	Medium
Corsican pine	510	Not	Moderately	Small	Good	Fair	1.80–3.60	Medium
HARDWOODS (IMPORTED)								
Abura	590	Perishable	Moderately	Small	Good	Good	1.80–6.00	Low
African mahogany	590	Moderately	Extremely	Small	Medium	Good	1.80–7.30	Medium
Afrormosia	720	Very	Extremely	Small	Medium	Good	2.40–7.30	Med high
Greenheart	1 060	Very	Extremely	Medium	Difficult	Good	4.80–9.00	High
Gurjun/Keruing	720	Moderately	Resistant	Large	Medium	Good	1.80–7.30	Low
Iroko	690	Very	Extremely	Small	Medium	Good	1.80–6.00	Medium
Jarrah	910	Very	Extremely	Medium	Difficult	Good	1.80–8.40	Med high
Karri	930	Durable	Impermeable	Large	Difficult	Good	1.80 up	Med high
Opepe	780	Very	Moderately	Small	Medium	Good	1.80–6.00	Medium
Red meranti	540	Moderately	Resistant	Small	Good	Good	1.80–7.30	Low
Sapele	690	Moderately	Resistant	Medium	Good	Good	1.80 up	Medium
Teak	720	Very	Extremely	Small	Medium	Good	1.80 up	High
HARDWOODS (HOME GROWN)								
European ash	720	Perishable	Moderately	Medium	Good	Fair	1.80 up	Low
European beech	720	Perishable	Permeable	Large	Good	Good	1.80 up	Medium
European oak	720	Durable	Extremely	Medium	Medium	Good	1.80 up	Medium

16.2.5 Working properties

Ease of fabrication is indicated in Table 16.3, although more detailed information may be found in PRL publications.[2,3]

16.2.6 Natural resistance to attack

Timber has a widely varying resistance to attack by fungi, insects, marine borers and termites. Fungi will not normally attack timber having a moisture content lower than 20% but a timber's ability to resist fungal attack is classified according to Table 16.4.

The natural durability of some structural timbers is given in Table 16.3. Information on further timbers will be found in PRL

Table 16.4 Durability classification of the heartwood of untreated timbers

Grade of durability	*Approximate life in ground contact* (yr)
Very durable	More than 25
Durable	15–25
Moderately durable	10–15
Nondurable	5–10
Perishable	Less than 10

Technical Note No. 40[15] and the *Handbooks* on softwood and hardwoods[2,3] whilst further advice on the control of decay will be found in PRL Technical Notes 29, 44 and 57.[16-18]

Termite attack and its prevention are dealt with by the PRL[19] in which the following timbers are mentioned as being generally resistant: iroko, opepe, Californian redwood and teak. Other timbers are given in the *Handbooks* on softwood and hardwoods.[2,3]

Marine borers are a hazard in the sea or brackish waters and PRL Leaflet No. 46[20] gives advice on the protection of timbers against this attack. Highly resistant timbers suitable for marine works are: greenheart, pyinkado, turpentine, totara, jarrah, basralocus and manbarklak.

16.2.7 Preservative treatment

The sapwood of all timbers is liable to attack by fungi and insects but it is often possible to obtain a more attack-resistant structure by pressure-impregnating nondurable or perishable timbers than by using durable species. Indeed, it is sometimes more economic also.

The amenability of timbers to preservative treatment is given in Table 16.3 and is related to the following classification:

Permeable:	Easily treated by either pressure or open tank.
Moderately resistant:	Fairly easy to treat by pressure, penetration 6 to 20 mm in 2 to 3 h.
Resistant:	Difficult to impregnate, incising often used. Penetration often little more than 6 mm.
Extremely resistant:	Very little penetration can be achieved even after prolonged treatment.

Further information and guidance on satisfactory types and methods of treatment may be found in publications of the British Standards Institute (BSI)[21-23] and the Timber Research and Development Association (TRADA). The economics of timber preservation is discussed in *Timberlab 17*.[24]

16.2.8 Fire resistance

Although timber ignites spontaneously at about 250°C, ignition is a function of the external surface area to the total volume of timber and the rate of charring does not significantly increase with a rise of temperature. The rate of charring is generally taken as about 0.5 mm/min (western red cedar 0.85 mm/min, dense hardwood 0.42 mm/min), but perhaps the most important factor is that the structural properties of uncharred material remain virtually unchanged.

Thus, if adequate protection against combustion is provided, timber is one of the safest structural materials in a severe fire. These measures are usually: (1) the provision of sacrificial material; (2) chemical impregnation; and (3) protective covering.[25-27]

16.3 Stress grading and permissible stresses

Timber is a natural organic material and therefore is subject to wide variability because of environmental, species and genetic effects. This variability affects both visible quality and strength.

If, for any particular property and species only one design stress were specified, this would have to be set so low (to allow for variability) that the material would have a very limited structural application. In consequence, a number of stress grades have been adopted, leading not only to a more economic use of the material but also to a higher yield of structurally useful material.

There are two main methods of stress grading for solid timber: (1) visual grading; and (2) mechanical grading. Each requires a different procedure.

16.3.1 Visual stress grading

In visual grading, data obtained from clear material (straight grained and free from knots and fissures) are analysed statistically for each species and basic stresses for each property are devised. These basic stresses are then reduced by factors which take account of the strength-reducing effects of the permissible growth characteristics for each stress grade.

At the present time, there are two sets of quality requirements for visual stress grading in this country, one for softwoods and one for hardwoods.

The first set is given in BS 4978.[28] Besides setting the requirements for two grades for solid softwood timber construction (SS and GS), this standard restates the requirements for laminating timber grades.

The second set is given in BS 5756[29] for tropical hardwoods (HS grade). In the current edition of BS 5268:Part 2 home-grown hardwoods have been deleted, but it is probable that these will be reintroduced.

16.3.2 Mechanical stress grading

Mechanical stress grading[30] is a method of non-destructive testing each piece to be graded. The piece is bent under a constant central load over a constant short span. The strength of the material can then be calculated accurately from the resultant deflection. Four grades are presented (M75, M50, MSS and MGS) and the grade stresses for the dry condition only are tabulated in BS 5268:Part 2. At present, machine-grade stresses are limited to six softwood species for which control information is available. However, it will be possible to machine-stress-grade other timbers in accordance with BS 4978.[28]

16.3.3 Glued–laminated timber grades

In glued–laminated members, the presence of strength-reducing characteristics will have a smaller effect than in solid timber, since the probability of identical structural defects appearing in identical positions in adjacent laminations is very small. British Standard 5268:Part 2, therefore allows higher grade stresses for glued–laminated timbers, these being obtained by applying tabulated modification factors to the grade stresses for each species.

16.3.4 Strength classes of timber

For the first time, BS 5268:Part 2 introduces the concept of strength classes for timber. Softwood species–grade combinations for strength classes, graded to BS 4978 are tabulated in Tables 3, 4 and 5 of that standard, whilst species groupings of hardwoods graded to BS 5756 are tabulated in Table 7 of that standard for the higher strength classes.

The concept, similar to the older species groupings, is that a strength class rather than a species may be specified. However, sometimes there are advantages in specifying a particular species and grade where the grade stress is higher than the strength class stress.

16.3.5 Permissible stresses

Permissible design stresses for both solid and laminated timber components are governed by the type of component, the conditions of service and the type of loading. They are obtained from grade stresses by applying the appropriate modification factors.

16.4 Design – general

Design in timber is similar to that in any other structural material as long as timber's peculiar qualities are acknowledged; indeed, these qualities can be exploited by resourceful designers. Timber is idealized as an orthotropic material, but in practice, only two directions need be considered: that parallel to the grain (along the trunk) and that perpendicular to the grain. Most strength properties, of both timber and joint fasteners, vary according to these directions and the variation has been found to follow the Hankinson relationship:

$$N=\frac{PQ}{P\sin^2\theta+Q\cos^2\theta}$$

where θ is the angle between directions of load and grain, N the stress at θ to the grain, P the stress parallel to the grain, and Q the stress perpendicular to the grain

from which intermediate stress or strength values can be calculated. This is not normally required for solid beams, joists and columns where only the major directions are used, but is often met where members intersect at joints. The stresses given in BS 5268:Part 2 are for permanent loading and increased values are allowed for short- and medium-term loads. This Code of Practice governs general design, but additional information is available.[31–41]

In the past, design has been inhibited by the relatively short lengths of timber available (Table 16.3 indicates availability). However, the production of durable resin adhesives has led to new construction techniques and structural forms being developed. Glued–laminated timber in which thin laminae are glued together to form structural components of almost any shape or length is a common reality, whilst structural plywood can be combined with either solid or glued–laminated timber to produce composite components which are lightweight, reliable and pleasing. The design in these forms is more complex than in solid timber but information on a wide variety of structural forms can be found,[42–70] whilst advice on the selection of a particular form is given in Tables 16.1 and 16.2. General design advice is provided by TRADA.[71]

16.5 Design in solid timber

Since permissible stresses are maxima there may be some advantage in using structural hardwoods or the higher-grade-stress softwoods whenever stress governs design. However, if deflection governs, there will be no advantage in using these more expensive materials unless the moduli of elasticity are sufficiently high. A possible exception is keruing (*dipterocarpus* spp.) whose current cost is roughly similar to that of softwoods. Some indication of price is given in Table 16.3.

As design in solid timber is limited by the maximum size of timber available, this has led to the development of many types of girder framework: however, where there is sufficient headroom, trussed beams can give an economic solution for heavy loads and long spans.[31,35,41]

In BS 5268:Part 2, minimum sizes are specified and the geometric properties tabulated in Appendix D of BS 5268:Part 2 are based on those minimum sizes.

Further reductions in section should be made for notches, mortices and bolt, screw and connector holes. Modification factors may also be required for the length and position of bearing, the shape of a beam and its depth if greater than 300 mm, whilst for compression members, combined factors are given for both slenderness and loading.

Lateral stability is important both for deep beams and for compression members, and in built-up members web stiffeners should be provided wherever concentrated loads occur.

General design data are available[71–73] applicable to particular structural forms[45,50–61,65–67] whilst design aids have been published for solid beams, portal frames and trussed rafters.

16.6 Glued–laminated timber assemblies

Glued–laminated timber is essentially a built-up section of two or more pieces of timber whose grains are approximately parallel and which are fastened together with glue throughout their length. This enables the properties of timber to be regulated to some degree and provides structural sizes and shapes which would not be possible in solid timber. Variation in section is possible, whilst high-grade material can be placed in zones of high stress and low-grade material in zones of low stress. All softwoods glue well and are generally preferred in the UK, although occasionally there can be some advantage in using wholly hardwood laminae.

Construction may use either vertical or horizontal laminations.

With vertical laminations, the zones of equal stress are shared between the laminations so that the strength of a beam can be said to be the sum of the individual laminations. This load-sharing concept has led to the grade–stress modification factors tabled in BS 5268:Part 2 which give higher permissible stresses for the laminated beam.

Horizontally laminated beams have been permitted since 1967 but the philosophy for behaviour is entirely different from that for vertical laminations. A beam will consist of material containing knots whose presence will affect the strength ratio of the beam. Since the knot effect will vary according to the sizes of knots and the number of laminations, BS 5268:Part 2 tables *basic stress* modification factors according to these variables.

Since curved laminated beams are fabricated by bending the individual laminations, fabrication stresses are induced which depend upon the degree of curvature, the thickness of the lamination and the species of timber. Therefore, modification factors to be applied to the grade bending stresses for different values of t/R are specified in BS 5268:Part 2.

The production of long laminations depends upon the use of efficient methods of end jointing. Where the efficiency of an end joint is known, the laminations containing them can be included when calculating the section properties, but where efficiencies are not known, the laminations containing the end joints must be omitted when calculating section properties. Efficiency ratings for plain scarf joints and for finger joints are given in BS 5268:Part 2: these are used to modify the basic stresses to give the maximum stresses to which any particular lamination may be subjected. British Standard 5291[74] governs finger joints in structural softwood. Butt joints do not transmit load and should only be used in zones of zero or very low stress.

Apart from the consideration of end joints and curvature, design is similar to that for solid timber,[46,64,75,76] whilst design aids are noted for glulam beams.

16.7 Plywood and tempered hardboard assemblies

Plywood is a type of glued–laminated construction in which the laminae are formed from thin flat veneers of timber. These veneers are produced by the rotary cutting of logs and are laid alternately at right angles in an odd number of layers. Since both the shrinkage and strength of timber differ according to the grain direction, the type of construction gives greater dimensional stability and tends to equalize the strength properties in both major directions of the plywood sheet.

There are two distinct design philosophies: (1) the North American approach which only considers the 'parallel plies', i.e. those plies whose grain lies in the direction of the load (this approach is based on the basic stresses and moduli for solid timber); and (2) the Finnish and British approach, known as the 'full cross-section' approach, in which grade stresses and moduli for the sheet materials have been determined from tests. In BS 5268:Part 2 all the grade stresses and moduli are for full cross-section, but it is well to remember that many North American design manuals will be based on the 'parallel-plies' approach. Plywood is a strong, durable and lightweight structural material which can be used to produce exciting structural shapes.[44,47–49,61–63] Design data are available for a variety of constructions[77–79] whilst design aids are available for stressed skin panels and portal frames.

Perhaps plywood's most useful property is that of providing excellent shear resistance for a small cross-section, although it is well to remember that lateral stability constraints may be required.

Tempered hardboard is a durable compressed fibreboard for which BS 5268:Part 2 now gives grade stresses for use in structural components instead of plywood.

16.8 Timber fastenings

Available methods of jointing are perhaps the most important criteria for the design of structural components. This is particularly true in timber for which highly efficient methods of transferring tensile loads have been developed only during the past 50 yr. Split-ring and tooth-plate connectors are now available which have load capacities much greater than those for nailed and bolted joints. A comparative indicator of fastener efficiency and the required member sizes is given in Table 16.5.

The strength of mechanical fasteners depends upon member size and thickness and the spacing of the fasteners. British Standard 5268:Part 2 tabulates permissible values of a wide range of variables, whereas some manuals prefer a presentation as a series of design curves.[31,34,35,40]

However, the major advance in fastening techniques has been in glued joints. Early glues were unreliable, deteriorating quickly, but the present phenolic and resorcinol resins are so durable that the risk of delamination has been almost entirely eliminated, even under extreme exposure. Unfortunately, gluing still requires controlled conditions and its application to site work is still limited.

Since the shear strength of adhesives is usually higher than that of timber, a fastener efficiency of 100% can be achieved. Nevertheless, it is important to remember that glues seldom have a good tensile strength, so that they should be stressed in shear as much as possible.

Information is available on gluing,[80] the requirements for adhesives,[81] and the compatibility between glues and preservatives.[82] The permissible stresses for glued joints are the shear stresses for the timber;[7] however, regard must be paid not only to the variation of that shear strength but also to the possibility of differential shrinkage and stress concentrations in the joint.

The type of fastener chosen will depend upon the skills and equipment available, possible fabrication conditions, relative costs and whether or not it is necessary to take down and reassemble the structural components.

Table 16.5 The relative strengths of timber joints

Comparison – axial compression in GS/M50 (SC3) European redwood					
	FASTENER		*TIMBER*		
Type & dia. (mm)	*No.*	*Capacity* (kN)	*Size* (mm)	*Effective Area* (mm^2)	*Capacity* (kN)
NAILS					
3.75	63	40.3	47 × 145	5581	40.7
8.00	21	42.7	72 × 145	8712	63.6
SCREWS					
8.43	16	42.2	60 × 145	6677	48.7
BOLTS					
M8	30	41.3	44 × 169	5676	41.4
M12	13	40.3	60 × 145	6540	47.7
TOOTH PLATE					
2/51 mm dia. + M12 bolt	5	40.5	60 × 145	6540	47.7
2/64 mm dia. + M12 bolt	5	43.5	60 × 169	7980	58.3
SPLIT RING					
2/64 mm dia. + M12 bolt	3	49.7	60 × 145 or 72 × 120	6800 6740	49.6 49.2

Assumptions: (1) three member joints loaded to 40 kN in axial compression parallel to grain
(2) timber to timber joints
(3) GS/M50 European redwood

Grade stress Table 8 SC3 6.8 N/mm^2. Grade stress Table 9 M50 7.3 N/mm^2. Using Table 9 value, required timber area = 5479 mm^2

EXAMPLES OF THE DESIGN OF A SIMPLE TENSION SPLICE JOINT

LOAD CAPACITY: 25 kN DURATION: MEDIUM TERM

TIMBER: EUROPEAN REDWOOD GRADE: M50

EXPOSURE CONDITION: DRY

(1) *REQUIRED TIMBER SECTION*

Dry exposure condition grade stresses, Tension //g

SC3 (**TABLE 8**): 3.2 N/mm^2 × 1.25 = 4.0 N/mm^2

European redwood (**TABLE 9**): 4.0 N/mm^2* × 1.25 = 5.0 N/mm^2

Maximum permissible timber stress = 5.0 N/mm^2

Section $= \frac{25000}{5} = 5000$ mm^2

allow 10% reduction in effective section at joint

SAY 41 × 145 mm planed = 5950 mm^2 (**TABLE 99**)

(2) *NAILED JOINT*

CHOICE OF NAIL DIAMETER

Possible joint thickness = 3 × 41 mm = 123 mm

Maximum available stock lengths:

4 and 4.5 mm ϕ: 100 mm 5 mm ϕ: 125 mm

Standard thicknesses for members in double shear:

4 mm	ϕ: 0.7 × 44 = 31 mm	(splice 35 × 145)
4.5 mm	ϕ: 0.7 × 51 = 36 mm	(splice 35 × 145)
5 mm	ϕ: 0.7 × 57 = 40 mm	(splice 41 × 145)

CLAUSE 41.4.2 **TABLE 57**

Required nail lengths:

4 mm ϕ : 2 × 35 + 41 = 111 mm

4.5 mm ϕ : 2 × 35 + 41 = 111 mm

lengths not available

5 mm ϕ : 2 × 41 + 41+ = 123 mm*

DESIGN OF JOINT (5 mm nails)

Basic single shear lateral load capacity, dry exposure:

5 mm, SC3: 635 N (**TABLE 57**)

Multiple shear factor (**CLAUSE 41.42**)

0.9 × number of shear planes provided each member is thicker than 0.7 × standard point size penetration

Permissible joint load (**CLAUSE 41.8**)

= basic × K_{48} × K_{49} × K_{50}

duration of load (medium term: 1.12) moisture content (dry: 1.0) number of nails (< 10 'line': 1.0)

= 635 × (2 × 0.9) × 1.12 × 1.0 × 1.0 – (assumed)

= 1280.16 N.

Number of 5 mm nails required $= \frac{25}{1.28} = 20$

TABLE 56 SPACING, modified by 0.8 (**CLAUSE 41.3**)

TRY 4 × 5 pattern (20 nails)

```
|25 × 40 × 40 × 40 × 25|   undrilled width ≥ 170 mm
    ×    ×    ×    ×
    ×    ×    ×    ×     joint length 4 × 40 + 2 × 40 = 240 mm
    ×    ×    ×    ×
|25 × 12 × 12 × 12 × 25|   drilled width ≥ 86 mm*
```

Effective area = (145 – 4 × 5) 41 mm^2 (**CLAUSE 41.2**)

= 5125 mm^2

Effective timber load capacity = 25.625 kN

Therefore the nailing pattern is acceptable, but requires predrilling

Alternatively, try 3 × 7 pattern (21 nails, but easier to control), to avoid cost of pre-drilling

```
|    ×    ×    ×     |
|25 × 40 × 40 × 25   |   undrilled width ≥ 130 mm*
|    ×    ×    ×     |
|    ×    ×    ×     |   joint length 5 × 40 + 2 × 40 = 320 mm
|    ×    ×    ×     |
|    ×    ×    ×     |
|    ×    ×    ×     |
```

Effective area = (145 – 3 × 5) 41 = 5330 mm^2 (**CLAUSE 41.2**)

Effective timber load capacity = 26.65 kN

Fastener capacity = 21 × (2 × 0.9) × 1.12 × 1.0 × 635 kN
= 26.9 kN

(3) *BOLTED JOINT*

CHOICE OF BOLT DIAMETER

Number of lines of bolts (n) possible in a 145 mm wide member

$= \frac{\text{required timber capacity}}{\text{effective area} \times \text{permissible stress}}$

M10 bolt, n = 2.3 lines

For any larger diameter bolts, only one line of bolts would be possible.

DESIGN OF JOINT

Basic single shear lateral load parallel to grain, (**TABLE 67**), member 41 mm thick.

M10, 1.28 kN: M12, 1.84 kN: M16, 3.15 kN (interpolated)

Double shear factor (**CLAUSE 43.4.2**) 2.0

Permissible joint load (double shear)

= 2 basic × K_{55} × K_{56} × K_{57}

medium term = 1.25 number 'in line'

m/c dry = 1.0

Number of bolts required

M10, 7.8: M12, 5.44: M16, 3.17:

bolts 'in line' and $(K_{57}) = [1 - \frac{3(n-1)}{100}]$ for $n < 10$

M10, 4 (.91): M12, 6 (.85): M16, 4 (.91)

revised number of bolts $= \frac{\text{original number}}{K_{57}}$

M10, 8.6: M12, 6.4: M16, 3.5:

Effective timber capacity = effective area × permissible stress

M10, 25.63 kN: M12, 27.26 kN: M16, 26.45 kN

BOLTS REQUIRED

M10, 9 (in two lines): M12, 7 (one line): M16, 4 (one line)

Compare with Timber Research and Development Association (1986) *Design aid DA1*. p. 25.

16.9 Timber-frame construction

It is estimated that the major use of structural timber will be in the housing field.

In high-rise construction, timber will play a supplementary role to the heavy material, being used for partitions, infill panels and floor and roofing systems. In this role, timber's ready adaptability to prefabrication is a great benefit.

In low-rise construction, on the other hand, timber is increasingly being used to provide the structural skeleton for the building; indeed, at the present time, timber-frame construction constitutes some 20% of all house construction. The method of construction is a simplification and refinement of that which has been used successfully for many centuries, but which is equally well applicable to many other uses besides housing.

The structural form is that of a free-standing skeleton for which standard details have been produced.[50–52,56,57,59] The basis of the skeleton is the stud-framed panel for which designs are described.[53,55,60]

Of especial importance is the structure's ability to withstand

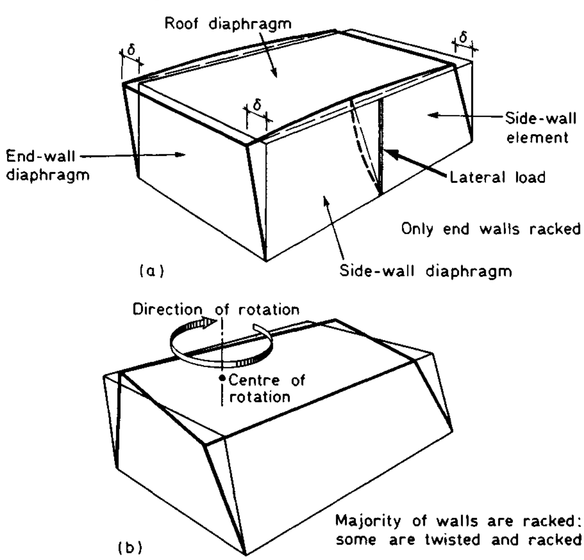


Figure 16.1 Deformation of idealized house structure under lateral loading. (a) Uniform translation of top wall parallel to the lateral load; (b) rotation after lateral translation, caused by lack of symmetry

lateral loading and, particularly, that resistance to planar deformation of a wall panel known as its racking resistance. Figure 16.1 shows the deformation of an idealized frame structure under lateral loading. Figure 16.1(a) shows the uniform translation of the walls which occurs when there is complete symmetry of both structure and loading. This hardly ever occurs and this lack of symmetry causes a rotation in addition to the translation (Figure 16.2(b)). The calculation of racking resistance is described by TRADA[58] and will also be dealt with by BS 5268:Part 6 (currently being written).

16.10 Repair and restoration

An increasing amount of work is being carried out on the repair and restoration of old timber-framed buildings. The work is specialized and requires not only sound structural assessment of the existing building but also a good understanding of the methods used in its construction and of acceptable methods of repair and restoration.

The methods of repair may be either by replacement of the damaged joints or members in the traditional manner or by the use of resin or stainless steel rods and resin fillers.

Brunskill[72] describes the traditional methods of timber-frame construction whilst Charles and Charles[83] give restoration case studies. Conservation and restoration are reported on by Fielden,[84] Gifford,[85] Ministry of Public Buildings and Works,[86] and repair by Avent *et al.*,[87–88] Oates and Richards[89] and Powys.[90]

16.11 Termite-resistant construction

There are two basic kinds of termite, which are mainly found in tropical and sub-tropical areas. The subterranean termite (white ant or wet-wood termite) has the larger distribution, builds huge nests, farms fungi and needs to maintain contact with the damp earth. Attack is from the ground.

There are many thousands of species of subterranean termites and a species of timber providing good termite resistance in one area may not be resistant in another. Resistive construction depends upon correct detailing, ground poisoning and preservative treatment.

The dry-wood termite, on the other hand, is less widely distributed, has fewer species, and flies, mates and deposits its eggs in timber to continue the life cycle.

Fly screens and preservative treatment give the best protection for new structures, whilst fumigation may be required when infestation is found in existing buildings. The recognition, control and detailing required are given by Harris[91] and Sperling.[92]

16.12 Storm-resistant construction

Wind loading is one of the commonest and most variable types of loading that can occur and ranges from normal wind loading to tornadoes.

Tropical storms have wind velocities in the range 55 to 117 km/h and damage is usually caused by flooding, including that produced by waterborne detritus.

Hurricanes are more severe and damage is caused by wind action, flooding and flying debris. Air circulation is counterclockwise with a damage area having a dimeter of 50 to 160 km. Wind speeds are commonly 120 to 190 km/h with gusting up to 320 km/h. Rainfall, normally 125 to 250 mm, occasionally reaches 750 mm.

Tornadoes are unquestionably the most devastating of wind storms. The vortex is much smaller than that of a hurricane, causing intense damane from wind action and large pieces of flying debris.

Generally, storm-resistant structures require strength linked to rigidity with interconnected members and components securely fastened together. The Southern Pine Association gives valuable advice.[93]

16.13 Earthquake-resistant construction

Horizontal and vertical movement of the Earth's surface, caused by earthquakes, results in forces being generated by the inertia of the mass of the structure. The magnitude of these inertial forces varies directly as the mass of the structure and in consequence heavy structures are more severely loaded than are lightweight ones. Indeed, for timber structures, the forces may be little higher than those produced by normal wind loading. However, unlike storm-resistant structures, those for earthquake-resistant construction require strength linked to flexibility. Several publications give outlines of sound practice.[94–96]

16.14 Design aids

There are three areas in which design aids can make a valuable contribution to the design process. These are: (1) rapid preliminary design either for comparison or estimation of cost; (2) routine elementary design; and (3) complex design processes for which the design time can be reduced drastically.

Nomograms and load–span tables have been used for many years, but the application of computer programming has extended considerably the use of design aids.

The bibliography to this chapter indicates some of the design aids which are now available for structural design in timber.

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Abbreviations and useful addresses

AITC	American Institute of Timber Construction, 1100, 17th Street NW, Washington DC 20036, USA.
BRE	Building Research Establishment, Garston, Watford, Hertfordshire.
BSI	British Standards Institution, 2 Park Street, London, W1A 2BS.
BWF	British Woodworking Federation, 82 New Cavendish Street, London, W1M 8AD.
BWPA	British Wood Preserving Association, 62 Oxford Street, London, W1N 9WD.
CITC	Canadian Institute of Timber Construction, 100 Bronfon Avenue, Ontario, Canada.
CPA	Chipboard Promotion Association, 50 Station Road, Marlow, Bucks, SL7 1NN
COFI	Council of Forest Industries of British Columbia, Tileman House, 131–133 Upper Richmond Road, Putney, London, SW15 2TR.
CWC	Canadian Wood Council, 701–710 Laurier Avenue West, Ottawa, Canada K1P SV5
FIDOR	Fibreboard Development Association, 1 Hanworth Road, Feltham, Middlesex, TW13 5AF.
FINPLY	Finnish Plywood Development Association, P.O. Box 99, Welwyn Garden City, Herts, A16 0HJ.
PRL	Princes Risborough Laboratory (Timberlab), Building Research Establishment, Aylesbury, Bucks, and at Building Research Station, Garston, Watford, Herts.
S/FTC	Swedish/Finnish Timber Council, 21 Carolgate, Retford, Notts, DN22 6BZ.
TRADA	Timber Research and Development Association, Stocking Lane, Hughenden Valley, High Wycombe, Bucks.
UAC	United Africa Company (Timber) Ltd., United Africa House, Blackfriars Road, London, SE1.

17 Foundations Design

M J Tomlinson
FICE, FIStructE, MConsE

Contents

17.1 General principles

17.1.1 The function of foundations

Foundations have the function of spreading the load from the superstructure so that the pressure transmitted to the ground is not of a magnitude such as to cause the ground to fail in shear, or to induce settlement of the ground that will cause distortion and structural failure or unacceptable architectural damage. In fulfilling these functions the foundation, substructure and superstructure should be considered as one unit. The tolerable total and differential settlement must be related to the type and use of the structure and its relationship to the surroundings. Foundations should be designed to be capable of being constructed economically and without risk of protracted delays. The construction stage of foundation work is not infrequently subjected to delays arising from unforeseen ground conditions. The latter cannot always be eliminated even after making detailed site investigations. Thus, elaborate and sophisticated designs and construction techniques which depend on an exact foreknowledge of the soil strata should be avoided. Designs should be capable of easy adjustment in depth or lateral extent to allow for variations in ground conditions and should take account of the need for dealing with groundwater.

Foundation designs must take into account the effects of construction on adjacent property, and the effects on the environment of such factors as piledriving vibrations, pumping and discharge of groundwater, the disposal of waste materials and the operation of heavy mechanical plant.

Foundations must be durable to resist attack by aggressive substances in the sea and rivers, in soils and rocks and in groundwaters. They must also be designed to resist or to accommodate movement from external causes such as seasonal moisture changes in the soil, frost heave, erosion and seepage, landslides, earthquakes and mining subsidence.

17.1.2 General procedure in foundation design

The various steps which should be followed in the design of foundations are as follows.

(1) A *site investigation* should be undertaken to determine the physical and chemical characteristics of the soils and rocks beneath the site, to observe groundwater levels and to obtain information relevant to the design of the foundations and their behaviour in service. The general principles and procedures described in Chapter 11 should be followed.
(2) The *magnitude and distribution of loading* from the superstructure should be established and placed in the various categories, namely:
   (a) dead loading (permanent structure and self-weight of foundations);
   (b) 'permanent' live loading, e.g. materials stored in silos, bunkers or warehouses;
   (c) intermittent live loading, e.g. human occupancy of buildings, vehicular traffic, wind pressures;
   (d) dynamic loading, e.g. traffic and machinery vibrations, wind gusts, earthquakes.
(3) The *total and differential settlements* which can be tolerated by the structure should be established. The tolerable limits depend on the allowable stresses in the superstructure, the need to avoid 'architectural' damage to claddings and finishes, and the effects on surrounding works such as damage to piped connections or reversal of fall in drainage outlets. Acceptable differential settlements depend on the type of structure; a framed industrial shedding with pin-jointed steel or precast concrete elements and sheet metal cladding, for example, can withstand a much greater degree of differential settlement than a 'prestige' office building with plastered finishes and tiled floors.
(4) The most suitable *type* of foundation and its *depth* below ground level should be established having regard to the information obtained from the site investigation and taking into consideration the functional requirements of the substructure, e.g. a basement may be needed for storage purposes or for parking cars.
(5) Preliminary values of the *allowable bearing pressures (or pile loadings)* appropriate to the type of foundation should be determined from a knowledge of the ground conditions and the tolerable settlements.
(6) The *pressure distribution* beneath the foundations should be calculated based on an assessment of foundation widths corresponding to the preliminary bearing pressures or pile loadings, and taking into account eccentric or inclined loading.
(7) A *settlement analysis* should be made, and from the results the preliminary bearing pressures or foundation depths may need to be adjusted to ensure that total and differential settlements are within acceptable limits. The settlement analysis may be based on simple empirical rules (see Chapter 9) or a mathematical analysis taking into account the measured compressibility of the soil.
(8) Approximate *cost estimates* should be made of alternative designs, from which the final design should be selected.
(9) *Materials* for foundations should be selected and concrete mixes designed taking into account any aggressive substances which may be present in the soil or groundwater, or in the overlying water in submerged foundations.
(10) The *structural design* should be prepared.
(11) The *working drawings* should be made. These should take into account the constructional problems involved and, where necessary, should be accompanied by drawings showing the various stages of construction and the design of temporary works such as cofferdams, shoring or underpinning.

17.1.3 Foundation loading

A foundation is required to support the dead load of the superstructure and substructure, the live load resulting from the materials stored in the structure or its occupancy, the weight of any materials used in backfilling above the foundations, and wind loading.

When considering the factor of safety against shear failure of the soil (see Chapter 9) the dead loading together with the maximum live load may be either a statutory or code of practice requirement, e.g. the requirements of the BS *Code of practice for loading*, BS 6399, or it may be directly calculated if the loads to be applied are known with some precision.

With regard to wind loading the BS *Code of practice for foundations*, BS 8004 states:

> Where the foundation loading beneath a structure due to wind is a relatively small proportion of the total loading, it may be permissible to ignore the wind loading in the assessment of allowable bearing pressure, provided the overall factor of safety against shear failure is adequate. For example, where individual foundation loads due to wind are less than 25% of the loadings due to dead and live loads, the wind loads may be neglected in this assessment. Where this ratio exceeds 25%, foundations may be so proportioned that the pressure due to combined dead, live and wind loads does not exceed the allowable bearing pressure by more than 25%.

When considering the long-term settlement of foundations, the live load should be taken as the likely realistic applied load over the early years of occupancy of the structure. Consolidation settlements should not necessarily be calculated on the basis of the maximum live load.

Loadings on foundations from machinery are a special case which will be discussed in section 17.6.

17.1.4 The design of foundations to eliminate or reduce total and differential settlements

The amount of differential settlement which is experienced by a structure depends on the variation in compressibility of the ground and the variation in thickness of the compressible material below foundation level. It also depends on the stiffness of the combined foundation and superstructure. Excessive differential settlement results in cracking of claddings and finishes and, in severe cases, to structural damage. Where the total settlements are expected to be small, cracking and structural damage can be avoided by limiting the total settlement. For example, if the total settlement of buildings on isolated pad foundations is limited to about 25 mm the differential settlement is unlikely to cause any significant damage. Buildings on rafts can usually tolerate somewhat greater total settlements. Where total settlements are expected to be appreciably greater than 25 mm the effects of differential settlement should be considered in relation to the type and function of the structure. These effects are discussed comprehensively by Padfield and Sharrock[1] who tabulate acceptable deflection limits as shown in Table 17.1.[2]

Differential settlement may be eliminated or reduced to a tolerable degree by one or a combination of the following measures:

(1) Provision of a rigid raft either as a thick slab, or with deep beams in two directions, or in cellular construction.
(2) Provision of deep basements or buoyancy rafts to reduce the net bearing pressure on the soil (see sections 17.3.2.1 and 17.3.3).
(3) Transference of foundation loading to deeper and less compressible soil by basements, caissons, shafts or piles (as described in sections 17.3 and 17.4).
(4) Provision of jacking pockets within the substructure, or brackets on columns from which to re-level the superstructure by jacking.
(5) Provision of additional loading on lightly loaded areas by ballasting with kentledge or soil.
(6) Ground treatment processes to reduce the compressibility of the soil.

17.2 Shallow foundations

17.2.1 Definitions

British Standard 8004 defines shallow foundations as those where the depth below finished ground level is less than 3 m and which include many strip, pad and raft foundations. The code states that the choice of 3 m is arbitrary, and shallow foundations where the depth: breadth ratio is high may need to be designed as deep foundations.

(1) *A pad foundation* is an isolated foundation to spread a concentrated load (Figure 17.1).
(2) *A strip foundation* is a foundation providing a continuous longitudinal bearing (Figure 17.2).
(3) *A raft foundation* is a foundation continuous in two directions, usually covering an area equal to or greater than the base area of the structure (Figure 17.3).

17.2.2 Foundation depths

The first consideration is, of course, that the foundation should be taken down to a depth where the bearing capacity of the soil is adequate to support the foundation loading without failure of the soil in shear or excessive consolidation of the soil. The minimum requirement is thus to take the foundations below loose or disturbed topsoil, or soil liable to erosion by wind or flood. Provided these considerations are met the object should then be to avoid too great a depth to foundation level. A depth greater than 1.2 m will probably require support of the excavation to ensure safe working conditions for operatives fixing

Table 17.1 Limiting values of distortion and deflection of structures. (After Tomlinson (1986) *Foundation design and construction* (5th edn.). Longman Scientific and Technical)

Type of structure	*Type of damage*	*Limiting values*			
		Values of relative rotation (angular distortion), β			
		Skempton and MacDonald[3]	Meyerhof[4]	Polshin and Tokar[5]	Bjerrum[6]
Framed buildings and reinforced load-bearing walls	Structural damage	1/150	1/250	1/200	1/150
	Cracking in walls and partitions	1/300 (but 1/500 recommended)	1/500	1/500 (0.7/1000 to 1/1000 for end bays)	1/500
		Values for deflection ratio Δ/L			
		Meyerhoff[4]	Polshin and Tokar[5]	Burland and Wroth[7]	
Unreinforced load-bearing walls	Cracking by sagging	0.4×10^{-3}	$L/H = 3$:0.3 to 0.4×10^{-3}	At $L/H = 1$: 0.4×10^{-3} At $L/H = 5$: 0.8×10^{-3}	
	Cracking by hogging	—	—	At $L/H = 1$: 0.2×10^{-3} At $L/H = 5$: 0.4×10^{-3}	

Note: The limiting values for framed buildings are for structural members of average dimensions. Values may be much less for exceptionally large and stiff beams, or columns for which the limiting values of angular distortion should be obtained by structural analysis.

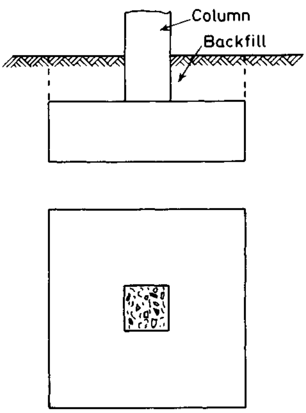

Figure 17.1 Pad foundation

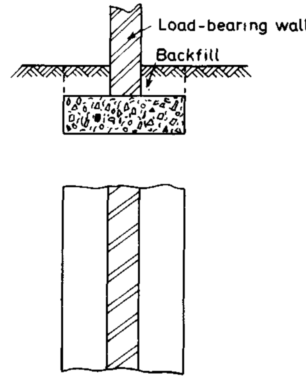

Figure 17.2 Strip foundation

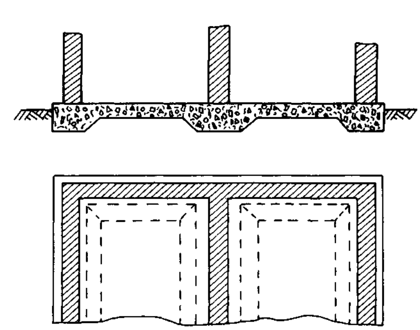

Figure 17.3 Raft foundation

reinforcing steel or formwork, which adds to the cost of the work. If at all possible the foundations should be kept above groundwater level in order to avoid the costs of pumping, and possible instability of the soil due to seepage of water into the bottom of an excavation. It is usually more economical to adopt wide foundations at a comparatively low bearing pressure, or even to adopt the alternative of piled foundations, than to excavate below groundwater level in a water-bearing gravel, sand or silt.

Apart from considerations of allowable bearing pressures, shallow foundations in clay soils are subject to the influences of ground movements caused by swelling and shrinkage (due to seasonal moisture changes or tree root action), in cohesive soils and weak rocks to frost action, and in most ground conditions to the effects of adjacent construction operations such as excavations or pile-driving.

It is usual to provide a minimum depth of 500 mm for strip or pad foundations as a safeguard against minor soil erosion, the burrowing of insects or animals, frost heave (in British climatic conditions other than those sites subject to severe frost exposure), and minor local excavations and soil cultivation. This minimum depth is inadequate for foundations on shrinkable clays where swelling and shrinkage of the soil due to seasonal moisture changes may cause appreciable movements of foundations placed at a depth of 1.2 m or less below the ground surface. A depth of 0.9 to 1 m is regarded as a minimum at which some seasonal movement will occur but is unlikely to be of a magnitude sufficient to cause damage to the superstructure or ordinary building finishes.[8]

Movements of clay soils can take place to much greater depths where the soil is affected by the drying action of trees and hedges, and in countries where there is a wide difference between the rainfall in the dry season and wet season.[9] Permafrost (permanently frozen ground) has a considerable influence on foundation depths.

Consideration should be given to the stability of shallow foundations on stepped or sloping ground. Analyses as described in Chapter 9 should be made to ensure that there is an adequate safety factor against a shear slide due to loading transmitted to the slope from the foundations.

The depth of foundations in relation to mining subsidence problems is discussed in section 17.7.2.

17.2.3 Allowable bearing pressures

Allowable bearing pressures (see definition in Chapter 9) for shallow foundations may be based on experience, or for preliminary design purposes on simple tables of presumed bearing values for a standard range of soil and rock conditions.

Where appropriate, more precise allowable bearing pressures for shallow foundations on cohesionless soils may be obtained from empirical relationships based on the results of *in situ* tests made on the soils (Chapter 11). In the case of shallow foundations on cohesive soils, the allowable bearing pressures may be obtained by applying an arbitrary safety factor to the ultimate bearing capacity calculated from shear strength determinations on the soil (Chapter 9). Where settlements are a critical factor in the design of foundations, detailed settlement analyses will be required based on the measured compressibility of the soil (Chapter 9).

17.2.4 Description of types of shallow foundations

17.2.4.1 Pad foundations

Pad foundations (Figure 17.1) are suitable to support the columns of framed structures. Pad foundations supporting lightly loaded columns can be constructed using unreinforced concrete, in which case the depth is proportioned so that the angle of spread from the base of the column to the outer edge of the ground bearing does not exceed 1 vertical:1 horizontal (Figure 17.4). The thickness of the foundation should not be less than the projection from the base of the column to its outer edge, and it should not be less than 150 mm.

Pad foundations to be excavated by a powered rotary auger should be circular in plan, so providing a self-supporting excavation in firm to stiff cohesive soils and weak rocks. Square or rectangular foundations can be excavated by mechanical grabs or backacters. The designs should not require the bottom to be trimmed by hand to a regular profile (Figure 17.4). This necessitates operatives working at the bottom of excavations in confined conditions, and for safety reasons the sides of excavations deeper than 1.2 m may have to be supported.

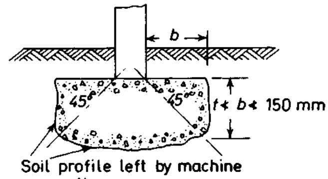

Figure 17.4 Proportioning of unreinforced concrete foundations

Savings in the volume of concrete can be obtained by providing steel reinforcement for pad foundations where heavy column loads are to be carried, and it may be advantageous to save depth of excavation by adopting a relatively thin base slab section (Figure 17.5). Reinforcement is also necessary for foundations carrying eccentric loading which may induce heavy bending moments and shear forces in the base slab. The procedure for reinforced concrete design is described in section 17.2.6.

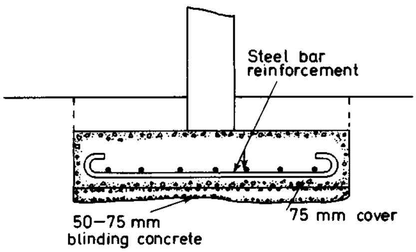

Figure 17.5 Reinforced concrete strip foundation

17.2.4.2 Strip foundations

Strip foundations are suitable for supporting load-bearing walls in brickwork or blockwork. The traditional form of strip foundation is shown in Figure 17.6(a). The concrete-filled trench foundation (Figure 17.6(b)) is suitable for stable soils in level ground conditions but should not be used where substantial swelling of clay soils may occur owing, say, to removal of trees or hedges. The swelling is accompanied by horizontal thrust on the foundation followed by movement of the foundation and superstructure. Strip foundations are also an economical method of supporting a row of closely spaced columns (Figure 17.7).

As a general rule, the thickness of unreinforced strip foundations should not be less than the projection from the base of the wall and not less than 150 mm. Where foundations are laid at more than one level, at each change of level the higher foundation should extend over and unite with the lower one for a distance of not less than the thickness of the foundation and not less than 300 mm (Figure 17.8).

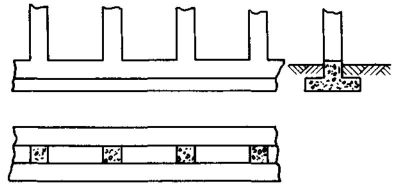

Figure 17.7 Strip foundation for closely spaced columns

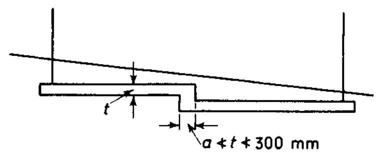

Figure 17.8 Stepping of strip foundations

The excavations for strip foundations are normally undertaken by a backacter machine, and it is usually possible to trim by the machine bucket to a rectangular bottom profile.

Reinforcement can be provided to strip foundations to enable savings to be made in the volume of concrete and also in foundation depths owing to the lesser required thickness of the base slab. Reinforcement is also necessary to enable the foundations to bridge over weak pockets of soil to minimize differential settlement due to variable loading conditions, e.g. when a strip foundation is provided to support a row of columns carrying different loads.

The procedure for the design of reinforced concrete foundations is described in section 17.2.6. In nonaggressive soil conditions a concrete mix consisting of 1 part of ordinary Portland cement to 9 parts of combined aggregate is suitable for unreinforced concrete strip foundations. The design of concrete mixes suitable for aggressive soil conditions is described in section 17.8.4.

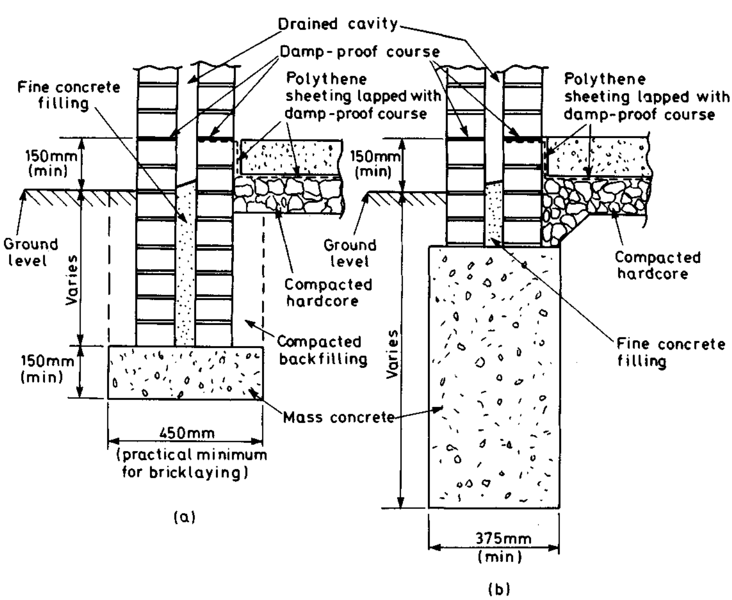

Figure 17.6 Unreinforced concrete strip foundations for load-bearing walls. (a) Traditional; (b) concrete-filled trench

17.2.4.3 Raft foundations

Raft foundations are a means of spreading foundation loads over a wide area thus minimizing bearing pressures and limiting settlement. By stiffening the rafts with beams and providing reinforcement in two directions the differential settlements can be reduced to a minimum.

Edge beams and internal beams can be designed as 'upstand' or 'downstand' projections (Figure 17.9). Downstand beams save formwork and allow the rafts to be concreted in one pour. However, the required trench excavations may not be self-supporting in loose soils and there are difficulties in maintaining the required profile in water-bearing ground. Upstand beams are required where rafts are designed to allow horizontal ground movements to take place beneath them, as in mining subsidence areas (section 17.7.2.3).

Raft foundations, in order to function as load-spreading substructures, must be reinforced and concrete mixes must be in accordance with code of practice requirements for reinforced concrete (BS 8110). Special mixes may be required in aggressive soil conditions.

17.2.5 Shallow foundations carrying eccentric loading

The soil adjacent to the sides of shallow foundations cannot be relied on to provide resistance to overturning moments caused by eccentric loading on the foundations. This is because in clays the soil is likely to shrink away from the foundation in dry weather and, in the case of cohesionless soils, excavation and subsequent backfilling will cause loose conditions around the sides. It is therefore necessary to check that the soil beneath the foundation will not be overstressed or suffer excessive compression under the unequal bearing pressures induced by the eccentric loading.

The pressure distribution beneath an eccentrically loaded foundation is assumed to be linear. For the pad foundation shown in Figure 17.10(a) where the resultant of the overturning moment M and the vertical load W falls within the middle third of the base:

Maximum pressure

$$q_{max} = \frac{W}{BL} + \frac{My}{I} \tag{17.1}$$

For a centrally loaded pad foundation this becomes:

$$q_{max} = \frac{W}{BL} + \frac{6M}{B^2L} \tag{17.2}$$

The minimum bearing pressure is given by:

$$q_{min} = \frac{W}{BL} - \frac{6M}{B^2L} \tag{17.3}$$

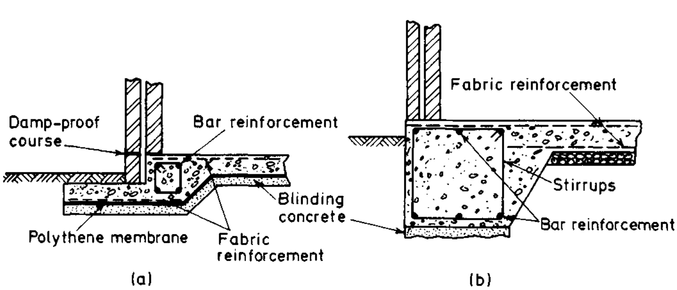

Figure 17.9 Reinforced concrete raft foundations. (a) With upstand beam; (b) with downstand beam

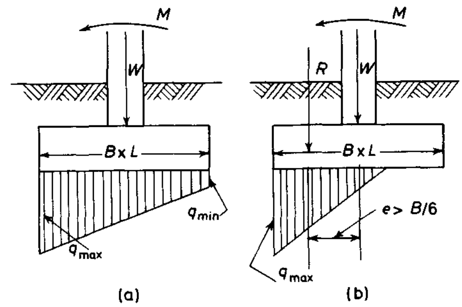

Figure 17.10 Eccentrically loaded foundations. (a) Resultant within middle third; (b) resultant outside middle third

When the resultant W and M falls outside the middle third of the base, Equation (17.3) indicates that tension theoretically occurs beneath the base. However, tension cannot develop and redistribution of bearing pressure will occur as shown in Figure 17.10(b). The maximum bearing pressure is then given by:

$$q_{max} = \frac{4W}{3L(B-2e)} \tag{17.4}$$

In Equations (17.1) to (17.4) W is the total axial load on the column, M is the bending moment on the column, y is the distance from the centroid of the pad to the edge, I is the moment of inertia of the plan dimensions of the pad, e is the distance from the centroid of the pad to the line of action of the resultant loading.

The maximum bearing pressure q_{max} should not exceed the allowable bearing pressure appropriate to the depth and width of the foundation, but the effective width for consideration of settlement in cohesionless soils (see Chapter 9) can be taken as one-third of the overall width for the pressure distribution shown in Figure 17.10(b) for a triangular distribution of pressure.

17.2.6 The structural design of shallow foundations

17.2.6.1 Pad and strip foundations

The following steps should be taken in the structural design of a pad foundation.

(1) Calculate the base area of the foundation by dividing the total net load by the allowable bearing pressure on the soil, taking into account any eccentric loading.
(2) Calculate the required overall depth of the base slab at the point of maximum bending moment.

(3) Decide on either a simple slab base with horizontal upper surface or a sloping upper surface, depending on the economics of construction.
(4) Check the calculated depth of the slab by computing the beam shear stress at critical sections on the assumption that diagonal shear reinforcements should not be provided.
(5) Design the reinforcement.
(6) Check the bond stress in the steel.

The main reinforcement, consisting of bars at the bottom of the base slab, is designed on the assumption that the projection behaves as a cantilever with its critical section on the face of the column (Line X–X in Figure 17.11), and with a loading on the underside of the cantilever equal to net bearing pressure under the worst conditions of loading, i.e. maximum eccentricity if the loading is not wholly axial. In Figure 17.11, the bending moment at the face of the column is given by:

$$M_b = \frac{q \times b^2 \times L}{2} \tag{17.5}$$

For pads of uniform thickness, the critical section of shear is along a vertical section Y–Y extending across the full width of the pad at a distance from the face of the column as defined in clause 3.4.5.8 of BS 8110. It is also necessary to check the punching shear along a critical peripheral section at a distance 1.5 times the thickness of the pad from the faces of the column. If the shear stress or punching shear stress exceed permissible limits they should be reduced by increasing the effective depth of the pad. Shear reinforcement in the form of stirrups or inclined bars should be avoided if at all possible.

Strip foundations are designed in the same manner, the critical sections for bending moment and shear being as shown in Figure 17.11.

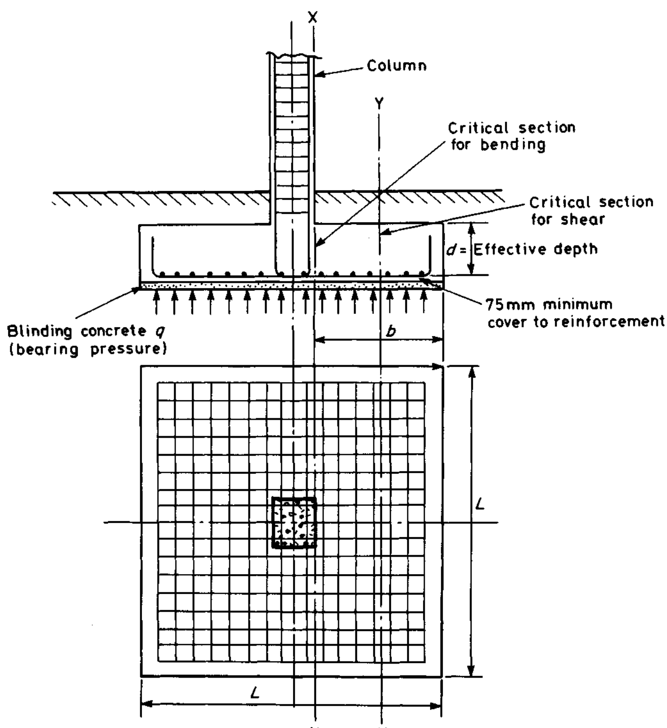

Figure 17.11 Reinforced concrete pad foundations

17.2.6.2 Raft foundations

Rafts are provided on compressible soils, and particularly on soils of variable compressibility. Thus, wherever rafts are needed from the aspect of soil compressibility, some settlement is inevitable, either in the form of dishing (on soils of uniform compressibility) or hogging (where the compressibility of the soil or the thickness of the compressible layer varies across the raft) or twisting where the compressibility conditions are irregular.

Distortion of a raft will also occur as a result of variation in the superimposed loading. The magnitude of dishing, hogging or twisting, i.e. the angular distortion of the raft, will depend on the stiffness of the raft and of the superstructure. Only in the case of a uniformly loaded raft on a soil of uniform compressibility can the raft be designed as an inverted floor, either in slab and beam construction or as a stiff slab (Figure 17.3). In all other cases the design is a complex process of redistributing column load bending moments and shears by the amount calculated from a consideration of the stiffness of the substructure and superstructure and the settlement of the soil. The starting point is always the theoretical total and differential settlements calculated by the soil mechanics engineer on the assumption of a fully flexible foundation. Flexibility of the raft is desirable to keep bending moments and shears to a minimum, but if the raft is too flexible there will be excessive distortion of the superstructure.

Analysis of the complex interaction between the raft structure and a subgrade soil undergoing elastic or plastic deformation lends itself to computer methods for solution. A report by the Institution of Structural Engineers[10] discusses the problems involved in computer analysis. Reference may also be made to the work of Hooper[11] and Poulos and Davis.[12] Where settlements are expected to be fairly small, the complexities of raft design can be avoided by designing the substructure as a series of touching but not interconnected pad or strip foundations.

This will greatly reduce the amount of reinforcement required to resist the high bending moments and shears which occur in the short stiff members of a raft with close-spaced columns.

17.2.7 Ground treatment beneath shallow foundations

If the ground beneath a proposed structure is highly compressible it may be economical to adopt shallow foundations in conjunction with a geotechnical process to reduce the compressibility of the ground as an alternative to deep foundations taken down to a stratum of lower compressibility. Geotechnical processes which may be considered are:

(1) Preloading.
(2) Injection of cement or chemicals.
(3) Deep vibration.
(4) Dynamic compaction.

(See also Chapter 9.)

Preloading Preloading consists of applying a load to the ground equal to, or greater than, the proposed foundation loading so that settlement of the ground will be complete before the structure is erected. The method is applicable to loose granular soils or granular fills, where the settlement will be rapid. It is generally unsuitable for soft clays where shear failure may occur under rapid application of preload and, because of the long-term character of consolidation settlement, the preloading would have to be sustained over a long period to be effective. Preloading is most economical over a large area where the granular material such as gravel or colliery waste can be provided in bulk and moved progressively across a site using earthmoving machinery.

The injection of cement or chemicals Injection of cement or chemicals is suitable for treatment of loose granular soils or fills where the particle size distribution of the materials is suitable for the acceptance of grouts. The effect of injecting cement or chemicals is to replace the void spaces by relatively incompressible material, thus greatly reducing the overall compressibility of the ground mass.

Cement or chemicals used for injection are costly and the process is not normally recommended for dealing with large foundation areas or deep compressible strata. The process is usually restricted to small-scale application beneath important structures such as complex machinery installations. It is also employed as a remedial treatment to arrest the excessive settlement of foundations.

Unslaked lime can be mixed with soft clays by rotary drilling equipment to form load-bearing columns of stabilized soil.[13] These are suitable for the foundations of light buildings provided that minor settlements are acceptable.

Deep vibration Deep vibration methods comprise the insertion of a large vibrating unit into the soil for the full depth required followed by its slow withdrawal. Granular material is fed into the depression surrounding the vibration unit as it is withdrawn, and the unit is re-inserted several times to form a cylinder of densely compacted soil mixed with the imported material. By adopting close-spaced insertions on a grid pattern beneath loaded areas or in single or double rows beneath strip foundations, the whole mass of compressible soil can be compacted to a reasonably uniform state, thus reducing the total and differential settlements beneath the applied loading.

In the 'vibroflotation' process the vibratory unit is assisted in its insertion by water jetting. During withdrawal the direction of the jets is reversed to consolidate the added materials. In the 'vibro-replacement' process no water jetting is used, the vibratory unit resembling a large poker vibrator. Compressed air is used to assist penetration of the vibratory unit in the vibro-displacement process.

The depth of treatment is limited to the maximum depth to which the vibratory unit can be inserted which, with the most powerful units assisted by water jetting, is about 20 to 30 m. The process has been used to advantage in compacting very loosely placed brick rubble and building debris filling on urban redevelopment sites. Houses can then be built on conventional strip foundations on the fill which has been compacted to a reasonably uniform state of density. The process may not be suitable if the debris contains a high proportion of timber or other organic or soluble materials which may decay or dissolve over a period of years, resulting in further settlement of the fill.

Dynamic compaction This consists of dropping a heavy weight on to the surface of the soil to compact and consolidate the weaker upper layers. Commonly weights of 15 to 20 t are dropped from heights of about 20 m to achieve useful compaction of the soil over a depth of about 10 m. Tamping is usually undertaken on a rectangular grid at points spaced 5 to 10 m apart. About five to ten blows of the tamper are applied to each grid point and the resulting craters are backfilled with granular material. Successive passes are then applied to the same or intermediate grid points until the desired standard of compaction has been achieved. The process is suitable for free-draining coarse granular soils, rockfill, refuse tips and industrial waste tips. Fill material in waste tips should not contain appreciable quantities of biodegradable or soluble substances.

The deep vibration and dynamic compaction processes have been reviewed comprehensively by Greenwood and Kirsch.[14]

17.3 Deep foundations

17.3 Definitions

Deep foundations are required to carry loads from a structure through weak compressible soils or fills on to stronger and less compressible soils or rocks at depth, or for functional reasons. The types of deep foundations in general use are as follows.

(1) Basements.
(2) Buoyancy rafts (hollow box foundations).
(3) Caissons.
(4) Cylinders.
(5) Shaft foundations.
(6) Piles.

Basements These are hollow substructures designed to provide working or storage space below ground level. The structural design is governed by their functional requirements rather than from considerations of the most efficient method of resisting external earth and hydrostatic pressures. They are constructed in place in open excavations.

Buoyancy rafts (hollow box foundations) Buoyancy rafts are hollow substructures designed to provide a buoyant or semi-buoyant substructure beneath which the net loading on the soil is reduced to the desired low intensity. Buoyancy rafts can be designed to be sunk as caissons (see below): they can also be constructed in place in open excavations.

Caissons Caissons are hollow substructures designed to be constructed on or near the surface and then sunk as a single unit to their required level.

Cylinders Cylinders are small single-cell caissons.

Shaft foundations These are constructed within deep excavations supported by lining constructed in place and subsequently filled with concrete or other prefabricated load-bearing units.

Piles Piles are relatively long and slender members constructed by driving preformed units to the desired founding level, or by driving or drilling-in tubes to the required depth – the tubes being filled with concrete before or during withdrawal – or by drilling unlined or wholly or partly lined boreholes which are then filled with concrete. Piles form a large group within the general classification of deep foundations and will be described separately in section 17.4.

17.3.2 The design of basements

17.3.2.1 General

Basements are constructed in place in open excavations. The latter can be excavated with sloping sides, or with ground support in the form of sheeting or sheet piling. The choice of either excavation method depends on the clear space available around the substructure and the need to safeguard existing structures adjacent to the excavation. It may be economical to use the permanent retaining walls as the means of ground support as described in section 17.3.2.3. A circular shape to a basement can save construction costs where ground support is required, as cross-bracing to support the sheeted sides may not be needed. A circular plan should always be considered for structures such as underground pumping stations.

The walls of basements are designed as retaining walls subjected to external earth pressure and water pressure. The methods of calculating earth pressure on retaining walls are described in Chapter 9. If no groundwater is encountered in site investigation boreholes it must not be assumed that there will not be any water pressure. For example, where backfill is placed between the walls of a basement and the sides of an excavation in clay soil a reservoir will be formed in which surface water running across the site will collect and a head of water will progressively rise around the walls. Such accumulations of water will not occur in permeable soil or rock formations in which the rate of downward seepage exceeds the inflow from surface water.

The floors of basements are designed to resist the upward earth pressure and any water pressure. The basement slabs span between the external walls or cross-walls or between ground beams placed along the lines of the interior columns. Alternatively, they can be designed as flat slabs propped at column and wall positions. They act as raft foundations subjected to bending moments and shears induced by differential settlements. The results of the site investigation will normally provide estimates of total and differential settlement on the alternative assumptions of a rigid raft (heavy beam and slab construction) or a fully flexible raft (thin flat slab construction). It is then a matter for the structural designer's judgement to assess the degree of flexibility of the raft and its interaction with the superstructure for the particular design under consideration. The complexities of this assessment have already been discussed in section 17.2.6.2. Particular points to be taken into consideration with basement floor designs are noted below.

Basements constructed in water-bearing strata may become buoyant if the groundwater level in the excavation around the completed (or partly completed) structure is allowed to rise to its normal rest level. At this stage there may not be sufficient loading from the superstructure to prevent uplift occurring. Therefore care should be taken to keep the excavation pumped down until the structural loads have reached the stage when uplift cannot occur.

17.3.2.2 Design of basement floors

Basement floors founded on rock or other relatively incompressible soils will not undergo appreciable downward movement due to elastic or consolidation settlement of the subgrade material. Then differential settlements will be negligible and it will be necessary only to design the floor to resist upward water pressure. If no water table exists or cannot develop in the future then columns and walls can be designed with independent foundations, the floor slab being only of nominal thickness (Figure 17.12).

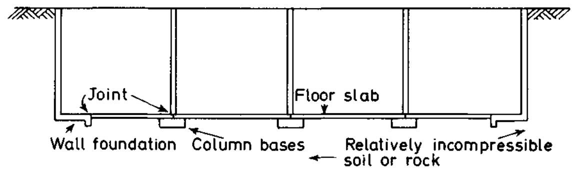

Figure 17.12 Basement floor founded on relatively incompressible stratum

Where appreciable total and differential settlements of the substructure can occur the basement floor should be designed as a stiff raft, either in slab and beam construction (Figure 17.13(a)) or as a flat slab (Figure 17.13(b)). Design practices are similar to those described in section 17.2.6.2 for surface rafts.

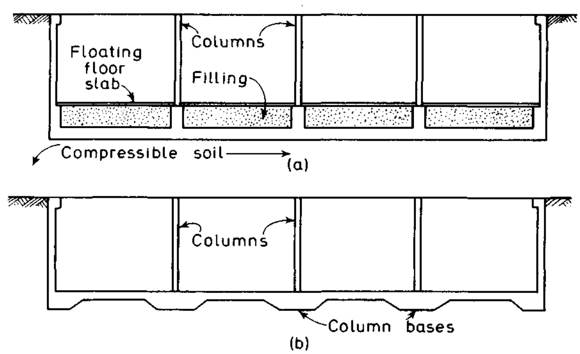

Figure 17.13 Basement floor founded on compressible stratum

When basements are supported on piles and settlements are expected in the pile group, i.e. where the piles terminate on compressible soils, some loading will be transferred to the underside of the floor slab. The magnitude of the pressure which develops will depend on the amount of settlement of the piles, the amount of heave of the base of the excavation due to relief of overburden pressure, the amount of heave and reconsolidation of the soil due to the installation of the piles and the time interval between completion of the excavation (including final trimming and removal of heaved soil) and the time when yielding of the piles commences due to superstructure loading. In all cases where there is potential transfer to the underside of the floor slab, or where hydrostatic pressure has to be resisted, the piled raft (Figure 17.14(a)) is the appropriate form of construction. The problems of load sharing between the piles and basement slab of a piled raft have been reviewed by Padfield and Sharrock[1] and by Hooper.[15]

Where the piles are terminated on rock or other relatively incompressible material and there is no hydrostatic pressure, there will be no load transfer to the floor slab, the latter being only of nominal thickness (Figure 17.14(b)). This assumes that ground heave causing uplift on the underside of the slab has ceased and that the heaved soil has been stripped off before placing the floor concrete.

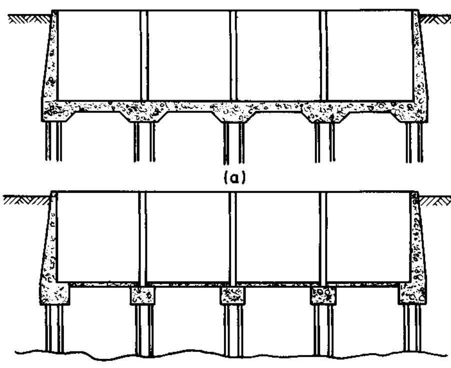

Figure 17.14 Piled basement floors. (a) With load transfer to floor slab; (b) with no load transfer to floor slab

17.3.2.3 Design of basement walls

Although the exterior walls of basements are supported by the ground-floor slab of the main structure and any intermediate subfloors in deep basements, they should be designed as free-standing cantilever retaining walls (Figure 17.15). This is because the supporting floors are not usually constructed until the final stage of the work (a special method of supporting the external walls of deep basements is shown in Figure 17.21, page 17/13). Similarly, the foundation slab of the retaining wall should not be dependent on its connection to the basement floor slab for stability.

The structural form of the retaining wall is governed to some extent by the ground conditions and by the need or otherwise for waterproofing treatment (see below). Thus, the sloping back and projecting heel shown in Figure 17.15(a) require additional width of excavation, the cost of which may outweigh the increase in concrete volume required by a wall of uniform thickness (Figure 17.15(b)). In stable ground it may be possible to undercut the excavated face to form the heel enlargement. The wider excavation required for the sloping back wall (Figure 17.15(a)) may be needed in any case to allow room for applying a waterproof asphalt layer, whereas the vertical back requires either an enlarged excavation or the construction of a separate vertical backing wall on which to apply asphalt.

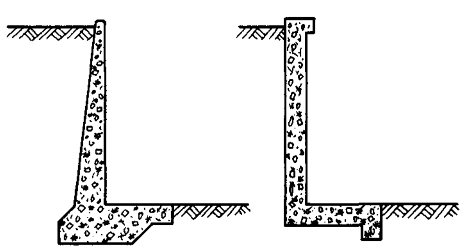

Figure 17.15 Basement floors. (a) With sloping back and heel; (b) with vertical back and no heel

The basement walls can be constructed as diaphragm walls by excavating a narrow trench by a mechanical grab using bentonite to support the excavation (Figure 17.16). The excavation is taken out in alternate panels 3 to 6 m long between guide walls. The level of the guide walls should be such that there is at least a 1-m head of bentonite slurry above the highest groundwater level. A preassembled reinforcing cage is lowered into the bentonite-filled trench and then concrete is placed by tremie pipe. The intermediate panels are then constructed in a similar manner. Diaphragm walls are designed as retaining walls using conventional methods for calculating earth pressure (Chapter 9). However, they cannot usually be designed to act as cantilever walls at the final stage of excavation, and they require to be propped by shores (or held at the top or intermediate levels by ground anchors) as described in section 17.3.2.5.

Contiguous bored pile walls faced with reinforced concrete can also be used for basements (see Figure 17.43(f), page **17**/24).

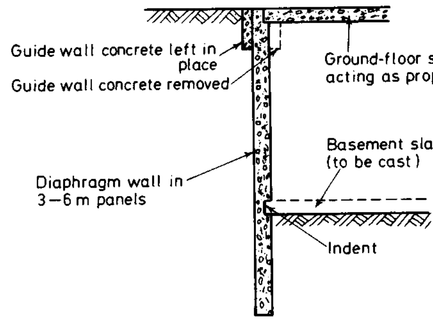

Figure 17.16 Diaphragm wall construction

17.3.2.4 Waterproofing basements

Watertightness of a basement can be obtained either by relying on impervious concrete and leaktight joints, or by providing an impermeable membrane in the form of trowelled-on asphalt tanking or preformed sheathing material. Neither method is entirely satisfactory.

If complete watertightness is required for functional reasons in a basement it is probable that the asphalt tanking method has a slight advantage compared with relying on the concrete alone, as tanking is a distinct operation carried out by skilled operatives, and the work can be restricted to favourable weather conditions and subjected to intensive supervision; whereas if the concrete alone is to be relied upon for watertightness, the concreting operations proceed in stages over a long construction period, in all weathers, with comparatively unskilled labour, and in congested situations, thus making close supervision difficult at all times.

Asphalt tanking or self-adhesive plastics sheathing is laid on blinding concrete beneath the basement floor and may be applied either to the exterior of the retaining walls if space is available around the excavations or, in restricted space conditions, it can be applied to a vertical backing wall before constructing the main wall (Figure 17.17). It is useless to apply tanking to the interior of the structural wall as the water pressure will merely force it off. Tanking applied to the exterior of the retaining wall should be protected by a 100-mm thick backing wall (in a manner similar to that shown in Figure 17.17) to prevent damage by sharp objects in the backfill materials.

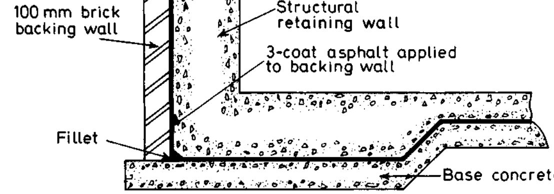

Figure 17.17 Asphalt tanking to basement

Asphalt tanking is covered by BS 988 and BS 1162 for limestone aggregate and natural rock asphalt aggregate respectively. The tanking should be applied in three coats to a total thickness of not less than 27 mm for horizontal work and 20 mm for vertical work. Other points of workmanship are covered in CP 102. An alternative to asphalt tanking is the use of Volclay panels. These consist of fluted cardboard slabs. The flutes are filled with bentonite which swells when wetted to form a permanent flexible gel.

Pumps keeping down the groundwater level around the excavation should not be shut down until the structural concrete walls have been concreted and have attained their design strength.

17.3.2.5 Construction of basements

If space around the substructure permits, the most economical method of constructing a basement is to form the excavation with sloping sides, followed by concreting the floor slab and then the retaining walls. If the space is restricted it will be necessary to support the vertical face of the excavation with steel sheet piling (Figure 17.18) or by horizontal timber sheeting in conjunction with vertical soldier piles (Figure 17.19). The sheet piling method is suitable for soft or water-bearing ground where continuous support is necessary and where it is desired to maintain the surrounding groundwater table at its normal level to safeguard existing structures. Horizontal sheeting can be used in 'dry' ground conditions, or where drainage towards the excavation can be permitted. In the latter case, hydrostatic pressures do not develop with correspondingly reduced loads to be carried by the bracing system.

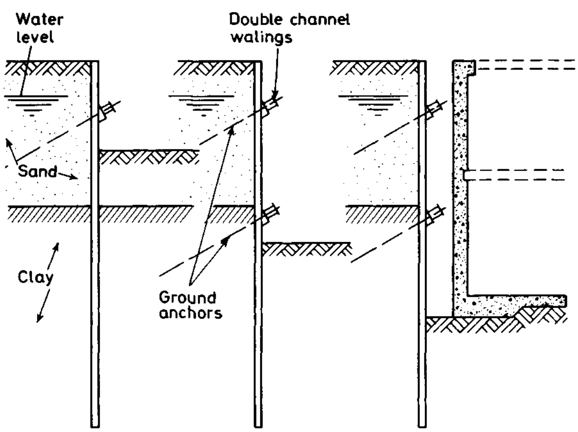

Figure 17.18 Excavation supported by tied-back sheet piling

The bracing system required to support sheeting to excavations of moderate width (say up to 30 m) can be in the form of horizontal struts and walings restrained against buckling by king piles and vertical cross-bracing (Figure 17.20). The struts can be preloaded by jacking to minimize inward movement of the sides. Where wide excavations have to be supported it is preferable to use a system of ground anchors (shown in various stages of construction in conjunction with sheet piling in Figure 17.18) or raking shores (shown in conjunction with horizontal sheeting in Figure 17.19).

Ground anchors have the advantage of providing a clear working space within the excavation and they can conveniently provide a preloading force to minimize inward movement, but there may be problems with existing sewers or other obstructions preventing their installation; also, it may be impossible to obtain wayleaves from surrounding property owners. Raking shores obstruct the working space and require substantial bearing blocks at the toe. These may give difficulties with maintaining waterproofing in thin basement slabs.

Inward movement of the sheeted sides of an excavation will take place inevitably owing to relief of lateral pressure on removal of the excavation, the compression of the supporting struts (or stretch and creep of ground anchors) and the thermal movements of the support system if the work is properly designed and carefully executed. The inward movement is proportional to the depth of the excavation and appears to be independent of the type of soil and the particular support system.

The inward movements of strutted or anchored diaphragm walls in a wide range of soil types have been shown by observation to be in the general range of 0.05 to 0.6% of the excavation depth.[2] The inward movement is accompanied by a vertical settlement of the same magnitude of the ground surface close to the perimeter of the excavation. The settlement is about half this maximum value at half the excavation depth from the face and falls to a negligible amount at a distance of 3 or 4 times the excavation depth from the face.

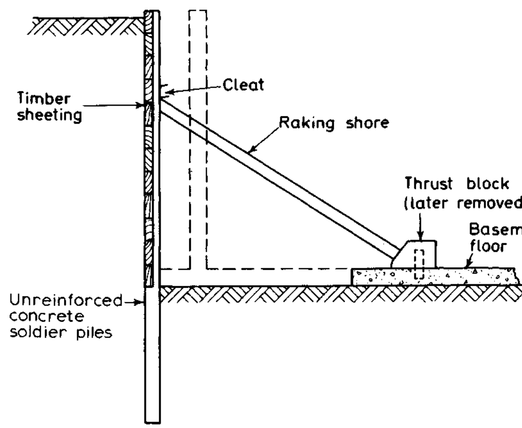

Figure 17.19 Excavation supported by soldier piles and sheeting

Where sheet piling is supported by berms of soft clay sloping not steeper than 2 horizontal: 1 vertical, observations have shown a maximum inward deflection of about 2% of the excavation depth.[2]

If there are existing structures within a distance of 3 times the excavation depth from the excavation line then consideration will have to be given to the need for underpinning them before excavation commences. For reasonably good ground conditions, underpinning is unlikely to be needed if the existing structures are not nearer than a distance equal to the excavation depth. For example, Figure 17.20 shows the order of settlements of the ground around a 10-m deep basement. A building in the

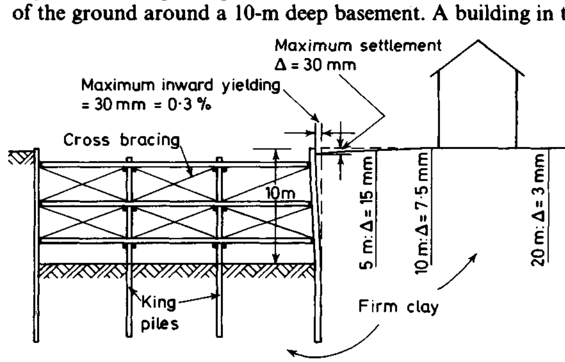

Figure 17.20 Bracing to wide excavation (also showing inward movement)

position indicated would not need to be underpinned. Consideration should be given to the comparative cost of repairs to make good cracking caused by small settlements and that of underpinning, bearing in mind that underpinning operations are themselves usually accompanied by some small settlement.

The various stages of excavation of a four-level deep basement using ground anchors to support the upper two levels and the basement floors to support the lower levels of a diaphragm wall are shown in Figure 17.21. Excavation is undertaken beneath the completed floors and openings are left for removal of spoil. The permanent columns supporting the basement floors are set in drilled holes before commencing the excavation. The inherent stiffness of a diaphragm wall combined with preloading of ground anchors, say to 50% higher than the calculated working load, reduces to a minimum (but does not eliminate) inward yielding of the wall.

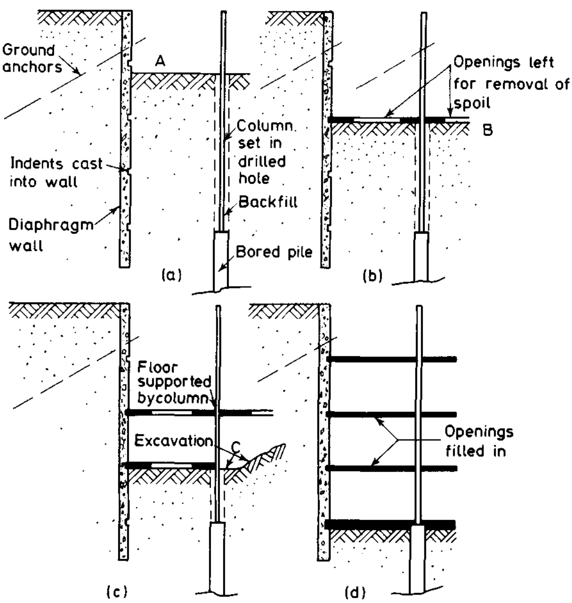

Figure 17.21 Construction of deep basement. (a) Excavation to level A and ground anchors installed; (b) excavation to level B and floor slab cast; (c) excavation to level C and further floor slab cast; (d) completed excavation with all basement floor slabs cast

17.3.3 Buoyancy rafts (hollow box foundations)

The substructure should be as light as possible consistent with the requirement of stiffness. A cellular ('egg box') construction is suitable. This structural form does not normally allow the substructure to be used for any purpose other than its function as a foundation element.

A cellular buoyancy raft may be designed as a caisson (Figure 17.22) which is an economical method of sinking for soft ground conditions, but ground disturbance during sinking can result in some settlement. A buoyancy raft should preferably be constructed within an open excavation. If necessary, the cells may be constructed in individual small areas or strips which are subsequently bonded together. By limiting the area of the excavation in this way, the heave and subsequent reconsolidation of a soft clay can be minimized to a marked degree.

Although considerable gain in uplift can be obtained if buoyancy rafts are designed as watertight structures, there are practical difficulties in achieving this. The space within the cells of a buoyancy raft is normally unoccupied and, if leaks occur, either through the substructure or from fracture of water pipes within the structure, the flooding of the cells may remain undetected. While the cells can be interconnected and provided with a drainage sump and automatic pumping arrangements there can be no certainty that these arrangements will be maintained in a sound working condition throughout the life of the supported structure. Therefore, unless drainage by gravity to an existing piped system is possible, the net bearing pressures beneath the buoyancy raft should be calculated on the assumption that the cells will become flooded to the level at which gravity drainage can be assured. As noted in section 17.3.2.4, the tanking of a buoyancy raft with asphalt does not give any guarantee of lasting watertightness.

Pipes carrying potentially explosive gases should not be routed through the cells of a buoyancy raft. Leakage of gas into the unventilated cells could remain undetected with a consequent risk of an explosion from accidental ignition.

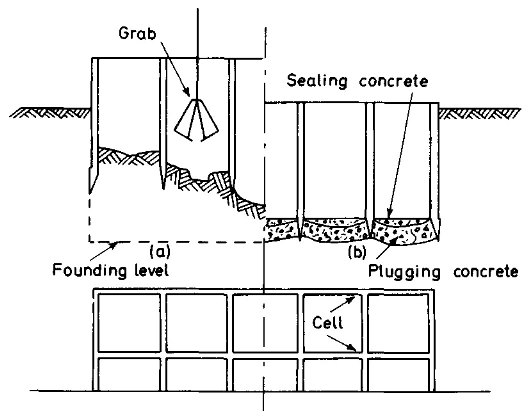

Figure 17.22 Caisson-type cellular buoyancy raft

17.3.4 Caisson foundations

17.3.4.1 General

The types of caisson foundation are:

(1) *A box caisson*, which is closed at the bottom but open to atmosphere at the top.
(2) An *open caisson*, which is open both at the top and bottom.
(3) A *compressed air* or *pneumatic caisson*, which has a working chamber in which air is maintained above atmospheric pressure to prevent the entry of water and soil into the excavation.
(4) A *monolith*, which is an open caisson of heavy mass concrete or masonry construction containing one or more wells for excavation.

The allowable bearing pressures beneath caissons are calculated by the methods described in Chapter 9. However, allowance must be made for the disturbance which may occur during the installation of the foundation. These factors are noted in the following subsections which describe the design and construction methods for the various types.

Caissons are often required to carry horizontal or inclined loads in addition to the vertical loading. As examples, caisson piers to river bridges have to carry lateral loading from wind forces on the superstructure, traction of vehicles on the bridge, river currents, wave forces and sometimes floating ice or debris. Caissons in berthing structures have to be designed to withstand impact forces from ships, mooring-rope pull, and wave forces. Methods of calculating the bearing pressures beneath eccentrically loaded foundations are described in section 17.2.5. A caisson will be safe against overturning provided that the bearing pressure beneath its edge does not exceed the safe bearing capacity of the foundation material, but it is also necessary to ensure that tilting due to elastic compression and consolidation of the foundation soil or rock does not exceed tolerable limits.

The walls of caissons are frequently subjected to severe stresses during construction. These stresses may arise from launching operations (when caissons are constructed on a slipway and allowed to slide into the water), from: (1) wave forces when floating under tow or during sinking; (2) racking due to uneven support whilst excavating individual cells; (3) superimposed kentledge; and (4) the drag effects of skin friction.

Lateral pressures on the external walls of caissons initially may be relatively low, corresponding to active pressure of soil loosened by the sinking process. However, with time the loosened soil will reconsolidate and, because the walls may be rigid and unyielding the conditions of earth pressure 'at rest' may develop (the coefficients appropriate to 'active' or 'at rest' earth pressure conditions are stated in Chapter 9). Where caissons are sunk through stiff over-consolidated clays or shales it may be necessary to cut the excavation larger than the plan dimensions of the foundation. With time the soil will swell to fill the gap and substantial swelling pressures may develop on the external walls.

17.3.4.2 Box caissons

Box caissons are designed to be floated in water and sunk on to a prepared foundation bed. The stages of sinking are shown in Figure 17.23. The foundation bed is prepared under water by divers, and the caisson is lowered by opening flood valves to allow the unit to sink at a controlled rate. Box caissons are suitable for site conditions where the bed can be prepared with little or no excavation below the sea- or river-bed. Thus, they are unsuitable for conditions where scour can undermine a shallow foundation. They are also unsuitable for conditions where scour can occur during the final stages of sinking by the action of eddies and currents in the gap between the base of the caisson and the bed material as the gap diminishes. For founding on soft clay or in scouring conditions, box caissons can be sunk on to a piled raft constructed underwater, but this method is normally more expensive than adopting an open-well caisson.

Box caissons can be of relatively light reinforced-concrete construction, since they are not subjected to severe stresses during sinking. Light construction is desirable to give the required freeboard whilst floating. After sinking they can be filled with mass concrete or sand if dead weight is required for the purpose of increasing the resistance to overturning or lateral forces.

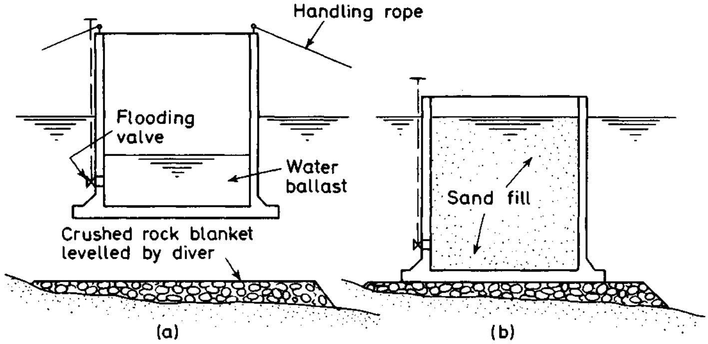

Figure 17.23 Stages in sinking a buoyancy raft. (a) Flooding valve opened to admit water ballast; (b)caisson sunk in final position

17.3.4.3 Open caissons

Open caissons are designed to be sunk by excavating while removing soil beneath them through the open cells. They are designed in such a manner that the dead weight of the caisson together with any kentledge which may be placed upon it exceeds the skin friction of the soil around the walls and the resistance of the soil beneath the bottom (cutting) edges of the walls. To aid sinking, the soil may be excavated from beneath the cutting edges, or kentledge may be placed on the top of the walls to increase the dead weight. The skin friction around the external walls can be reduced considerably by injecting a bentonite slurry above the cutting edge between the walls and the soil. On reaching founding level, mass concrete is placed to plug each cell after which any water in the cells can be pumped out and further concrete placed to form the final seal. The portions of the cells above the sealing plugs can be left empty, or they can be filled with mass concrete, sand, or fresh water depending on the function of the unit and the allowable net bearing pressure. The stages of sinking are shown in Figure 17.24.

The lower part of an open caisson is known as the *shoe*. This is usually of thin mild steel plating stiffened at the edges with steel tees or angles and provided with internal bracing members. Concrete is placed in the space between the skin plates of the shoe to provide ballast for sinking through water and thereafter more concrete and further strakes of skin plating are added to obtain the required downward forces to overcome skin friction and the bearing resistance of the soil beneath the cutting edges. While the top of the shoe is still above water level, formwork is assembled and the walls extended above the shoe in reinforced concrete. The formwork is usually arranged in lifts of about 1.5 m and a 24-h cycle of operations comprises grabbing to sink 1.5 m, erecting steel skin plating or formwork in the walls, placing the concrete and striking the formwork. Sinking proceeds steadily throughout this cycle. Thick walls are needed for rigidity and to provide dead weight. As well as being reinforced to withstand external earth and hydrostatic pressures, they must resist racking stresses and vertical tension stresses. The latter may occur when the upper part of the caisson is held by skin friction and the lower part tends to fall into the undercut and loosened zone beneath the shoe.

The form of construction, incorporating a shoe fabricated in steel plating, is the traditional method of design, which provides optimum conditions for control of sinking at all stages. However, the introduction of bentonite injection techniques to aid sinking has improved the control conditions making it possible to design caissons entirely in reinforced concrete and enabling them to be sunk to great depths. Circular caissons were sunk to depths of as much as 105 m below the bed of the Jamuna River.[16]

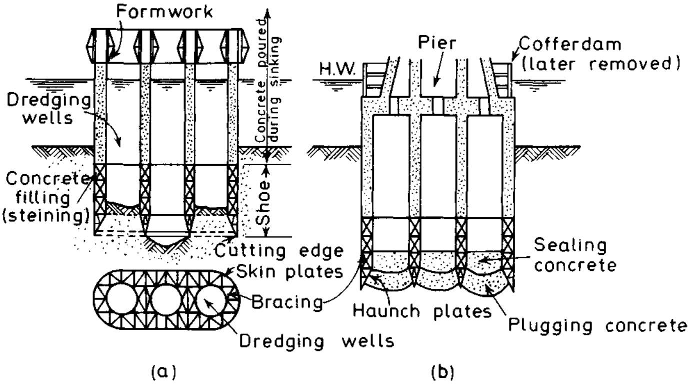

Figure 17.24 Stages in sinking an open caisson. (a) Grabbing from cells and concreting in walls; (b) plugging and sealing concrete in place with caisson at final level

Some typical values used to give a rough guide to skin friction are shown in Table 17.2.[17]

Table 17.2 (After Terzaghi and Peck (1967) *Soil mechanics in engineering practice*. Wiley)

Type of soil	*Skin friction* (kN/m^2)
Silt and soft clay	7–30
Very stiff clay	50–200
Loose sand	10–35
Dense sand	30–70
Dense gravel	50–100

The soil is excavated from within the cells and, where necessary, from below cutting edge level by mechanical grab. In uncemented granular soils, the spoil can be removed by an airlift pump. On reaching founding level any kentledge placed on the walls is removed to arrest sinking and mass concrete is quickly placed at and below cutting edge level in the corner cells to provide a bearing on which the caisson comes to rest. The remaining outer cells are then plugged with concrete followed by completion of excavating and plugging of the inner cells. The concrete plugs are placed under water and after the concrete has hardened the cells are pumped out and further sealing concrete is placed.

Accuracy in the positioning of caissons and control of verticality while sinking are necessary. Various methods of achieving these are:

(1) Sinking between moored pontoons (Figure 17.25).
(2) Sinking within a piled enclosure (Figure 17.26).
(3) Sinking through a sand island (Figure 17.27).

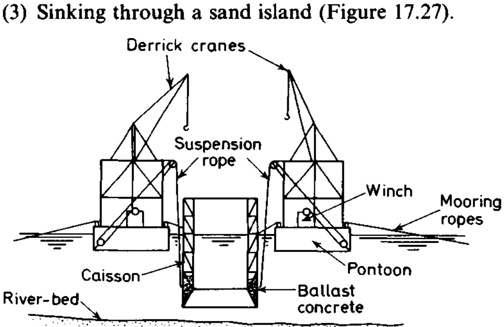

Figure 17.25 Lowering caisson from pontoons

The choice of method depends on the site conditions, i.e. the depth of water, degree of exposure, and velocity of sea or river currents. It also depends on the number of caissons to be sunk on any particular project. The cost of an elaborate floating sinking set as shown in Figure 17.25 is justified if spread over a number of sinking sites. Lowering during sinking can be achieved by using suspension links and jacks (Figure 17.26) by lowering from block and tackle (Figure 17.25) by free sinking with the use of guides (Figure 17.27) or by the controlled expulsion of air from the cells in conjunction with air domes (Figure 17.28).

Open-well caissons are best suited to sinking in soft or loose soils to reach a founding level on stiff or compact material, i.e.

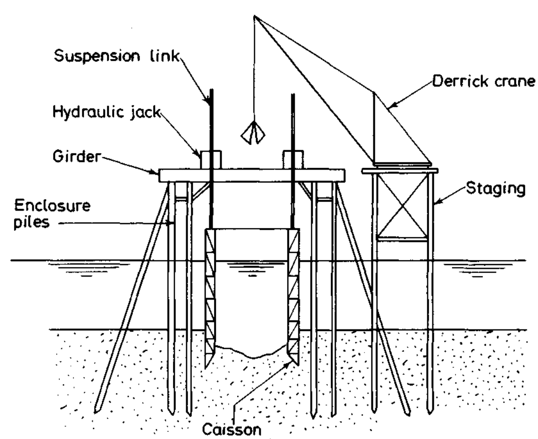

Figure 17.26 Lowering caisson from piled staging

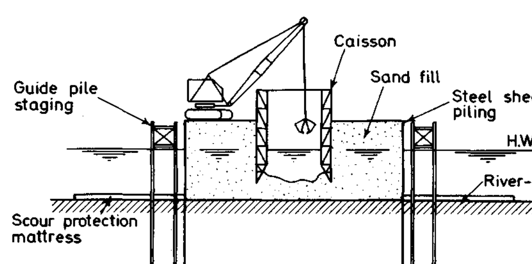

Figure 17.27 Sinking caisson through a sand island

through materials which can be dredged readily and are free of obstruction such as boulders, tree trunks or sunken vessels. They are unsuitable for ground containing obstructions which cannot be broken out from beneath the cutting edge, and are also unsuitable for sinking on to an irregular rock surface. Problems also arise when founding on weak rocks. Grabbing through water causes softening and breakdown of the rock, making it difficult to judge when a satisfactory bearing stratum has been reached and to clean the rock surface to receive the concrete plug.

Removal of soil from within or below the cells of an open caisson causes quite appreciable loss of ground, i.e. the total volume of soil excavated exceeds the volume displaced by the caisson. Open caissons are therefore unsuitable for sinking close to existing structures.

Some of the difficulties mentioned above can be overcome by providing an open caisson with air domes. These are provided with airlocks and are designed to be placed over individual cells as required. Having placed a dome on top of a cell, compressed air is introduced to expel water, after which workmen can enter through an airlock to remove obstructions or to prepare the bottom to receive the sealing concrete. There are limits to the air pressure under which operatives can work in this manner (see section 17.3.4.4). Air domes provided on all cells can be used as the means of floating an open caisson to the sinking site and for controlling its vertical aspect during sinking by varying the rate of expulsion of air from individual cells. Caissons designed in this way are known as flotation caissons. A design used for the Tagus River bridge[18] is shown in Figure 17.28. The cutting edge of this caisson was 'tailored' to suit the profile of the rock surface on which the caisson was landed. The domes of flotation caissons are not normally provided with an airlock. After they have been removed, grabbing proceeds in the normal way for open well caissons.

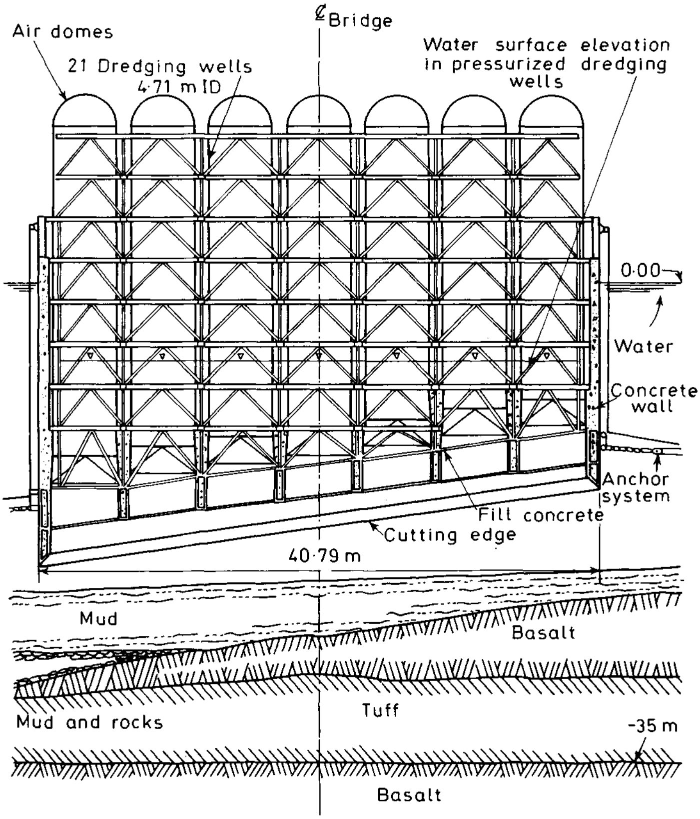

Figure 17.28 Flotation caisson for the Tagus River bridge. (After Riggs (1965) 'Tagus River Bridge – tower piers', *Civ. Engng* (USA) (Feb.) 41–45)

17.3.4.4 Pneumatic caissons

Pneumatic caissons are designed to be sunk with the assistance of compressed air to obtain a 'dry' working chamber. The general arrangement is shown in Figure 17.29. The caisson consists of a single working chamber surrounded by the shoe with its cutting edge, and a heavy roof. Walls are extended above the shoe in the form of double steel skin plating with mass concrete infilling. The height of the walls depends on the weight required to provide sinking effort and the need to provide freeboard when sinking through water. The airshaft extends from the working chamber to the full height of the caisson and it is surmounted by a combined manlock and mucklock. As the names imply, the former is used for access and egress by operatives and the latter for removal of spoil in crane buckets. The manlocks must at all times be above the highest tide or river flood levels, with due allowance being made for rapid sinking in soft or loose soils.[19]

Work in pneumatic caissons is regulated by the statutory regulations governing working conditions in compressed air. The regulations require 0.3 m^3 of fresh air per minute per person in the working chamber *at the pressure in the chamber*. The air is supplied from stationary compressors powered by diesel or electric motors. Standby power must be available if the site conditions are such as to endanger life or property if the main supply fails. To improve working conditions and to reduce the incidence of caisson-sickness the air supply should be treated to

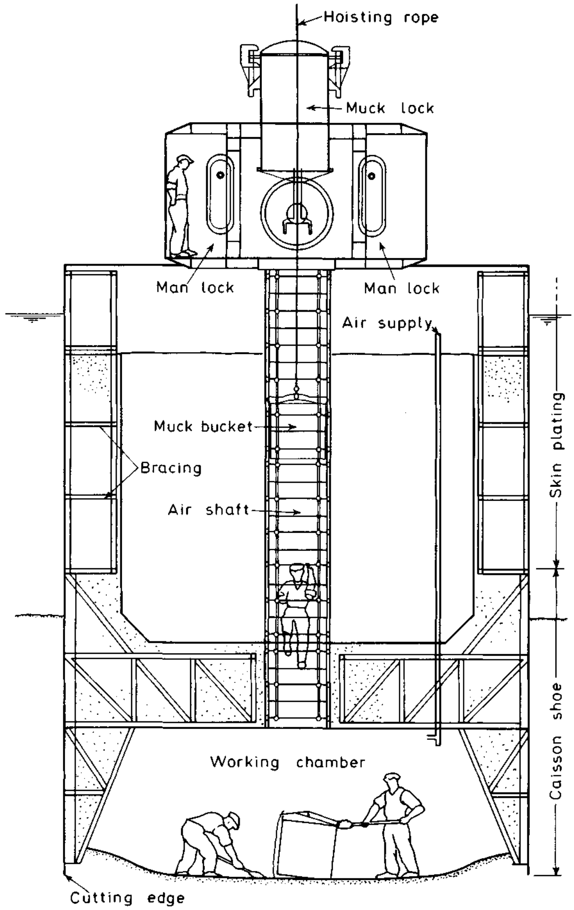

Figure 17.29 Compressed-air caisson. (After Wilson and Sully (1949) *Compressed air caisson foundations.* Works Construction Paper Number 13, Institution Civil Engineers)

warm it for working in cold weather and to cool it for hot-weather working. In tropical climates the air should be dehumidified to keep the wet bulb temperature at less than 25° C. In very permeable ground the escape of air into the soil beneath the working chamber may cause too great a demand on the air supply. This can be reduced by pregrouting the ground with clay, cement or chemicals.

If the dead weight of the caisson, together with any added kentledge, is insufficient to overcome the skin friction, the effective sinking weight can be increased temporarily by 'blowing down' the caisson. This involves removing the operatives from the working chamber, then reducing the air pressure by about one-quarter of the gauge pressure.

On nearing founding level, concrete blocks are placed on the floor of the working chamber and the roof is allowed to come to rest on them. The working chamber is then filled with concrete and the airshaft and airlocks removed.

The pneumatic caisson is suitable for sinking close to existing structures since the excavation is not accompanied by loss of ground. It is also suitable for sinking in ground containing obstructions, and for founding on an irregular rock bed. Pneumatic caissons have the severe limitation that the depth of sinking cannot exceed a level at which the required air pressure to exclude water from the working chamber exceeds the limit at which operatives can work without danger to their health. A pressure of 345 kN/m^2 is considered generally to be a safe maximum but stringent medical precautions and supervision are required at all stages of the work.[20] The high cost of compressed-air sinking generally precludes pneumatic caissons for all but special foundations where no alternatives are feasible or economically possible.

17.3.4.5 Monoliths and cylinders

Monoliths are open caissons of reinforced concrete or mass concrete construction (Figure 17.30) and are mainly used for quay walls where their heavy weight and massive construction are favourable for resisting the thrust of the filling behind the wall and for withstanding the impact forces from berthing ships. Because of their weight they are unsuitable for sinking through deep soft deposits. Their design and method of construction generally follow the same principles as those for open caissons in section 17.3.4.3.

Open caissons of cylindrical form and having a single cell are sometimes referred to as cylinder foundations.

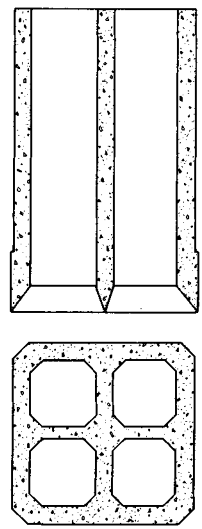

Figure 17.30 Concrete monolith

17.3.4.6 Shaft foundations

Where deep foundations are required for the heavily loaded columns of a structure it may be desirable to sink the foundation in the form of a lined shaft excavated by hand or by mechanical grab. This type of foundation is similar to the large bored pile as described in section 17.4.3.1 but its distinguishing characteristic is the construction of the lining in place, taken down stage-by-stage as the shaft is deepened. The shaft foundation would be selected in cases where the required diameter was larger than the capacity of the large-bored-pile drilling machine, in ground containing boulders or other obstructions which could prevent machine drilling or caisson sinking, and in localities where specialist pile-drilling plant is not available but where labour for hand excavation can be provided from local resources.

Shaft foundations can be of any desired shape but the cylindrical form is the most convenient since internal bracing is not required. The lining can consist of mass concrete placed *in situ* behind formwork (Figure 17.31(a)) or bolted precast concrete, steel or cast-iron segments (Figure 17.31(b)). The *in situ* concrete lining is suitable for relatively dry ground which can stand without support for a height of about 1.5 m. Segmental lining can be used in water-bearing ground which can stand unsupported for the height of a segment. Cement grout must be injected at intervals into the space between the back of the segments and the soil. This is necessary to prevent excessive flow of water down the back of the lining, and also to support the segments from dropping under their own weight augmented by downdrag forces from the loosened soil. The collar at the top of the shaft is also required to support the lining.

Shaft foundations may be constructed as a second stage after first sinking through soft or loose ground as a caisson (Figure 17.31(a)) or at the base of a sheet piled cofferdam (Figure 17.31(b)).

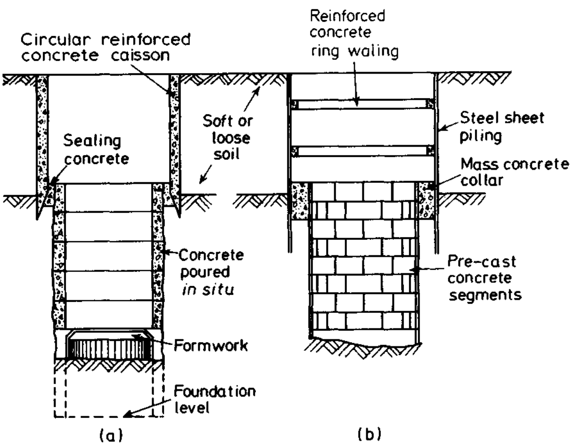

Figure 17.31 Shaft foundations. (a) With mass concrete lining constructed below caisson; (b) with precast concrete segmental lining constructed below a sheet-piled cofferdam

17.4 Piled foundations

17.4.1 General descriptions of pile types

There is a large variety of types of pile used for foundation work.[21] The choice depends on the environmental and ground conditions, the presence or absence of groundwater, the function of the pile, i.e. whether compression, uplift or lateral loads are to be carried, the desired speed of construction and consideration of relative cost. The ability of the pile to resist aggressive substances or organisms in the ground or in surrounding water must also be considered.

In BS 8004, piles are grouped into three categories:

(1) *Large displacement piles*: these include all solid piles, including timber and precast concrete and steel or concrete tubes closed at the lower end by a shoe or plug, which may be either left in place or extruded to form an enlarged foot.
(2) *Small displacement piles*: these include rolled-steel sections, open-ended tubes and hollow sections if the ground enters freely during driving.
(3) *Replacement piles*: these are formed by boring or other methods of excavation; the borehole may be lined with a casing or tube that is either left in place or extracted as the hole is filled.

Large or small displacement piles In preformed sections these are suitable for open sites where large numbers of piles are required. They can be precast or fabricated by mass-production methods and driven at a fast rate by mobile rigs. They are suitable for soft and aggressive soil conditions when the whole material of the pile can be checked for soundness before being driven. Preformed piles are not damaged by the driving of adjacent piles, nor is their installation affected by groundwater. They are normally selected for river and marine works where they can be driven through water and in sections suitable for resisting lateral and uplift loads. They can also be driven in very long lengths.

Displacement piles in preformed sections cannot be varied readily in length to suit the varying level of the bearing stratum, but certain types of precast concrete piles can be assembled from short sections jointed to form assemblies of variable length. In hard driving conditions preformed piles may break causing delays when the broken units are withdrawn or replacement piles driven. A worse feature is unseen damage particularly when driving slender units in long lengths which may be deflected from the correct alignment to the extent that the bending stresses cause fracture of the pile.

When solid pile sections are driven in large groups the resulting displacement of the ground may lift piles already driven from their seating on the bearing stratum, or may damage existing underground structures or services. Problems of ground heave can be overcome or partially overcome in some circumstances by redriving risen piles, or by inserting the piles in prebored holes. Small-displacement piles are advantageous for soil conditions giving rise to ground heave.

Displacement piles suffer a major disadvantage when used in urban areas where the noise and vibration caused by driving them can cause a nuisance to the public and damage to existing structures. Other disadvantages are the inability to drive them in very large diameters, and they cannot be used where the available headroom is insufficient to accommodate the driving rig.

Driven and cast-in-place piles These are widely used in the displacement pile group. A tube closed at its lower end by a detachable shoe or by a plug of gravel or dry concrete is driven to the desired penetration. Steel reinforcement is lowered down the tube and the latter is then withdrawn during or after placing the concrete. These types have the advantages that: (1) the length can be varied readily to suit variation in the level of the bearing stratum; (2) the closed end excludes groundwater; (3) an enlarged base can be formed by hammering out the concrete placed at the toe; (4) the reinforcement is required only for the function of the pile as a foundation element, i.e. not from considerations of lifting and driving as for the precast concrete pile; and (5) the noise and vibration are not severe when the piles are driven by a drop hammer operating within the drive tube.

Driven and cast-in-place piles may not be suitable for very soft soil conditions where the newly placed concrete can be squeezed inwards as the drive tube is withdrawn causing 'necking' of the pile shaft, nor is the uncased shaft suitable for ground where water is encountered under artesian head which washes out the cement from the unset concrete. These problems can be overcome by providing a permanent casing. Ground heave can damage adjacent piles before the concrete has hardened, and heaved piles cannot easily be redriven. However, this problem can be overcome either by preboring or by driving a number of tubes in a group in advance of placing the concrete. The latter is delayed until pile driving has proceeded to a distance of at least 6.5 pile diameters from the one being concreted if small (up to 3 mm) uplift is permitted, or 8 diameters away if negligible (less than 3 mm) uplift must be achieved.[22] The lengths of driven and cast-in-place piles are limited by the ability of the driving rigs to extract the drive tube and they cannot be installed in very large diameters. They are unsuitable for river or marine works unless specially adapted for extending them through water and cannot be driven in situations of low headroom.

Replacement piles or bored piles These are formed by drilling a borehole to the desired depth, followed by placing a cage of steel injected at intervals into the space between the back of the segments and the soil. This is necessary to prevent excessive flow of water down the back of the lining, and also to support the segments from dropping under their own weight augmented by downdrag forces from the loosened soil. The collar at the top of the shaft is also required to support the lining.

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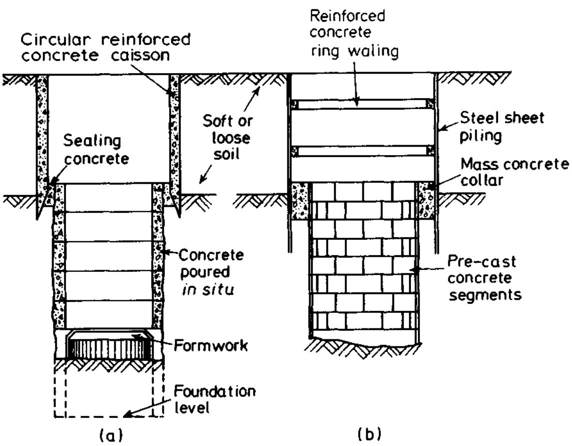

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Displacement piles suffer a major disadvantage when used in urban areas where the noise and vibration caused by driving them can cause a nuisance to the public and damage to existing structures. Other disadvantages are the inability to drive them in very large diameters, and they cannot be used where the available headroom is insufficient to accommodate the driving rig.

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Driven and cast-in-place piles may not be suitable for very soft soil conditions where the newly placed concrete can be squeezed inwards as the drive tube is withdrawn causing 'necking' of the pile shaft, nor is the uncased shaft suitable for ground where water is encountered under artesian head which washes out the cement from the unset concrete. These problems can be overcome by providing a permanent casing. Ground heave can damage adjacent piles before the concrete has hardened, and heaved piles cannot easily be redriven. However, this problem can be overcome either by preboring or by driving a number of tubes in a group in advance of placing the concrete. The latter is delayed until pile driving has proceeded to a distance of at least 6.5 pile diameters from the one being concreted if small (up to 3 mm) uplift is permitted, or 8 diameters away if negligible (less than 3 mm) uplift must be achieved.[22] The lengths of driven and cast-in-place piles are limited by the ability of the driving rigs to extract the drive tube and they cannot be installed in very large diameters. They are unsuitable for river or marine works unless specially adapted for extending them through water and cannot be driven in situations of low headroom.

Replacement piles or bored piles These are formed by drilling a borehole to the desired depth, followed by placing a cage of steel reinforcement and then placing concrete. It may be necessary to support the borehole by steel tubing (or casing) which is driven down or allowed to sink under its own weight as the borehole is drilled. Normally the casing is filled completely with easily workable concrete before it is extracted, when the concrete slumps outwards to fill the void so formed.

In stiff cohesive soils or weak rocks it is possible to use a rotary tool to form an enlarged base to the piles which greatly increases the end-bearing resistance. Alternatively, men can descend the shafts of large-diameter piles to form an enlarged base by hand excavation. Reasonably dry conditions are essential to enable the enlarged bases to be formed without risk of collapse.

Care is needed in placing concrete in bored piles. In very soft ground there is a tendency to squeeze of the unset concrete, and if water is met under artesian head it may wash out the cement from the unset concrete. If water cannot be excluded from the pile borehole by the casing, no attempt should be made to pump it out before placing concrete. In these circumstances the concrete should be placed under water by tremie pipe. A bottom-opening skip should not be used. Breaks in the concrete shafts of bored piles may occur if the concrete is lifted when withdrawing the casing, or if soil falls into the space above the concrete due to premature withdrawal of the casing.

Bored piles have the advantages that their length can be readily altered to suit varying ground conditions, the soil or rock removed during boring can be inspected and if necessary subjected to tests, and very large shaft diameters are possible, with enlarged base diameters up to 6 m. Bored piles can be drilled to any desired depth and in any soil or rock conditions. They can be installed without appreciable noise or vibration in conditions of low headroom and without risk of ground heave.

Bored piles are unsuitable for obtaining economical skin friction and end bearing values in granular soils because of loosening of these soils by drilling. However, stable conditions can be achieved if the pile borehole is supported during the drilling operation by a bentonite slurry. Boring in soft or loose soils results in loss of ground which may cause excessive settlement of adjacent structures. They are also unsuitable for marine works.

17.4.2 Details of some types of displacement piles

17.4.2.1 Timber piles

In countries where timber is readily available, timber piles are suitable for light to moderate loadings (up to 300 kN). Softwoods require preservation by creosote in accordance with BS 913. If this is done they will have a long life below groundwater level but are subject to decay above this level. Where possible, pile caps in concrete should be taken down to water level (Figure 17.32(a)). If this is too deep, a composite pile may be installed, the upper part above water level being in precast concrete or concrete cast-in-place jointed to a timber section (Figure 17.32).

To prevent damage to timber piles during driving, the head should be protected by a steel or iron ring, and the toe by a cast-iron shoe (Figure 17.33(b)).

British Standard 8004 requires that the working stresses in compression on a timber pile do not exceed those tabulated in BS 5268 for compression parallel to the grain for the species and grade of timber used, due allowances being made for eccentricity of loading, nonverticality of driving, bending stresses due to lateral loads, and reductions in section due to drilling lifting holes or notching the piles. The working stresses of BS 5268 may be exceeded while the pile is being driven.

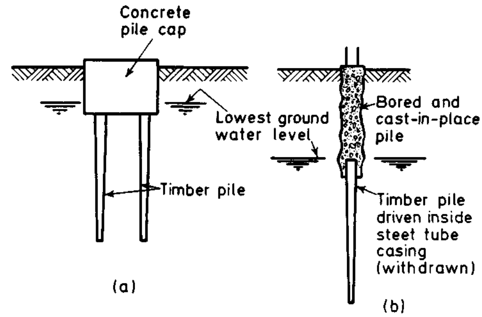

Figure 17.32 Methods of avoiding decay in timber piles

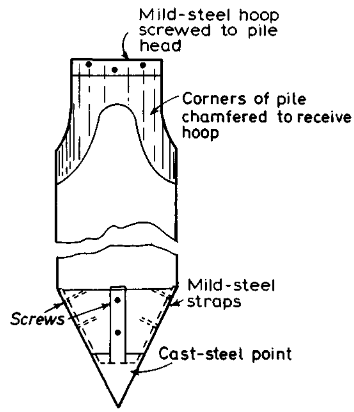

Figure 17.33 Protecting the head and toe of a timber pile

17.4.2.2 Precast and prestressed concrete piles

Precast reinforced concrete piles may not be economical for use in land structure because a considerable amount of steel reinforcement is needed to withstand bending stresses during lifting and subsequent compressive and tensile stresses during driving. Precast concrete piles are also liable to damage on handling and during driving in hard ground. However, the reinforcement may be needed for resisting lateral forces on the pile, e.g. for resisting impact forces on wharves or jetty piling. Much of this reinforcement is not required once the pile is in the ground.

The effect of prestressing of solid or hollow concrete piles in conjunction with high-quality concrete is to produce a unit which should not suffer hair cracks while being lifted or transported and therefore should produce a more durable foundation element than the ordinary precast concrete pile. This is advantageous in aggressive ground conditions. However, prestressed concrete piles are liable to crack during driving and require careful detailing of reinforcement and precautionary measures during driving to ensure concentric blows of the hammer and accurate alignment in the leaders of the pile frame.

The maximum pile lengths for main reinforcement of various diameters are listed in Table 17.3. These lengths allow for the pile to be lifted at the head and toe.

The pile lengths were based on a characteristic stress in the steel of 250 N/mm^2 and concrete having a characteristic strength of 40 N/mm^2. British Standard 8004 requires lateral reinforcement in the form of hoops or links to resist driving stresses, the diameter of which shall not be less than 6 mm. For a distance of 3 times the width of the pile from each end the volume of the lateral reinforcement should not be less than 0.6% of the gross volume. In the body of the pile the lateral reinforcement should not be less than 0.2% of the gross volume spaced at a distance of

Table 17.3 Maximum pile lengths for given reinforcement

Bar diameter for 4 bars (mm)	300 mm *pile* (m)	350 mm *pile* (m)	400 mm *pile* (m)	450 mm *pile* (m)
20	9.0	8.5	—	—
25	11.0	10.5	10.0	9.5
32	—	13.0	12.5	12.0
40	—	—	15.5	15.0

not more than half the pile width. The transition between close spacing at the ends and the maximum spacing should be made gradually over a length of about 3 times the width. A typical precast concrete pile of solid section designed for fairly easy driving conditions and the minimum transverse reinforcement required by BS 8004 is shown in Figure 17.34. Other recommendations are:

(1) *Reinforcement*: to comply with BS 4449 and 4461.
(2) *Concrete mixes*: for hard to very hard driving conditions and all marine work use cement content of 400 kg/m^3. For normal or easy driving use cement content of 300 kg/m^3.
(3) *Concrete design*: stresses due to working load, handling and driving not to exceed those in BS 8110 or CP 116.
(4) *Cover to reinforcement*: to comply with BS 8110:Part 1, Table 3.4.

Where piles are driven through hard ground which must be split to achieve penetration or ground containing obstructions liable to damage the toe of a pile, a cast-steel or cast-iron shoe should be provided as shown in Figure 17.35(a). For driving on to a sloping hard rock surface a rock point should be provided as shown in Figure 17.35(b) to prevent the toe skidding down the slope. A shoe need not be provided for easy to fairly hard driving in clays and sands when the pile may have a flat end or be terminated as shown in Figure 17.35(c).

The recommendations of BS 8004 for prestressed concrete piles are as follows:

(1) *Materials*: to be in accordance with BS 8110 or CP 115.
(2) *Design*: maximum axial stress 0.25 × (28-day works cube stress less prestress after losses). The stress should be reduced if the ratio of effective length: least lateral dimension is greater than 15.

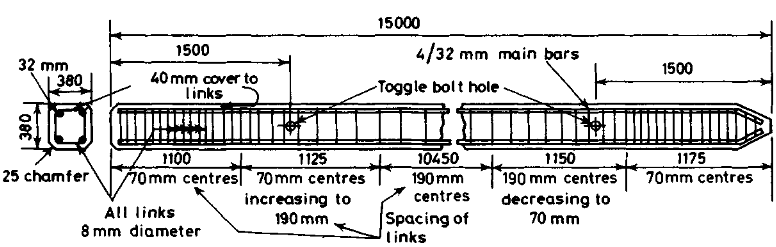

Figure 17.34 Design of precast concrete pile suitable for fairly easy driving conditions and for lifting at third point from one end or at positions shown

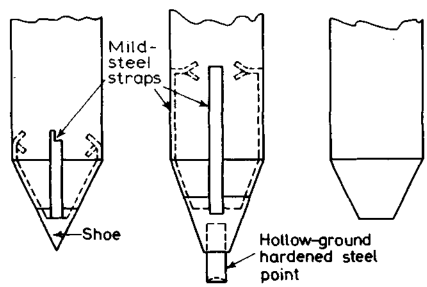

Figure 17.35 Design of toe for precast or prestressed concrete pile

Static stresses produced by lifting and pitching not to exceed values given in Tables 1 and 2 of CP 115 using in Table 2 of that code the values relating to loads of short duration.

(3) *Prestress*: minimum prestress is related to ratio of weight of hammer: weight of pile thus:
Ratio 0.9 0.8 0.7 0.6
Minimum prestress for normal driving (N/mm^2) 2.0 3.5 5.0 6.0
Minimum prestress for easy driving (N/mm^2) 3.5 4.0 5.0 6.0
The minimum prestress for diesel hammers should be 5.0 N/mm^2
(4) *Lateral reinforcement*: mild steel stirrups not less than 6 mm diameter spaced at a pitch of not more than side dimensions less 50 mm. At top and bottom for length of 3 times side dimension stirrup volume not less than 0.6% of pile volume.
(5) *Cover*: as for precast concrete piles (see section 17.4.2.2).

To minimize damage to pile heads during driving, precast concrete or prestressed concrete piles should be driven with timber or plastic packing between the helmet and the hammer. The hammer weight should be roughly equal to the weight of the pile and never less than half its weight. The drop should be 1 to 1.25 m. Particular care is necessary when driving with a diesel hammer when an uncontrollable sharp impact can break the pile if the toe meets a hard layer. Drop hammers or single-acting hammers are preferable for these ground conditions.

A typical prestressed concrete pile designed to the above recommendations is shown in Figure 17.36.

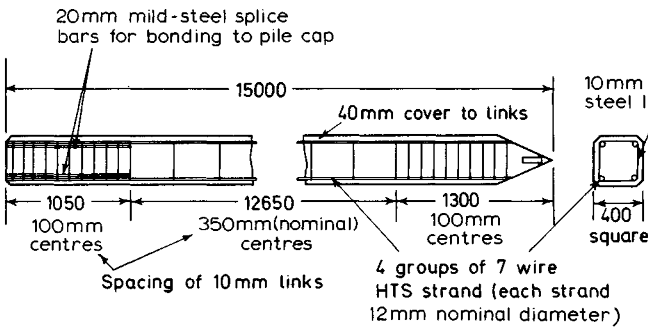

Figure 17.36 Design of prestressed reinforced concrete pile

lateral forces and to buckling. They are advantageous for marine work. They can be lengthened by welding on additional lengths as required and cut-off sections have scrap value. If a small displacement is needed to minimize ground heave the H-section can be used or tubular piles can be driven with open ends and the soil removed by a drilling rig.

Various types of steel pile are shown in Figure 17.38. Reference should be made to the British Steel Corporation's handbook for dimensions and properties of the various sections. British Standard 8004 requires steel piles to conform to BS 4360, grades 43A, 50B or other grades to the approval of the engineer.

17.4.2.3 Jointed precast concrete piles

One of the drawbacks of ordinary precast or prestressed concrete piles is that they cannot be readily adjusted in length to suit the varying level of a hard-bearing stratum. Where the bearing stratum is shallow a length of pile must be cut off and is wasted. Where it is deep the pile must be lengthened with an inevitable delay in the process of splicing on a new length. This drawback can be overcome by the use of precast concrete piles assembled from short units. Two principal types are available. The West's pile (Figure 17.37(a)) consists of short cylindrical hollow shells made in 380, 405, 445, 510, 535 and 610 mm outside diameters. The shells are threaded on to a steel mandrel which carries a shoe at the lower end. The driving head is designed to allow the full weight of the drop hammer to fall on the mandrel while the shells take a cushioned blow. Shells can be added or taken away from the mandrel to suit the varying penetration depths of the piles. On completion of driving, the mandrel is withdrawn, a reinforcing cage is lowered down the shells and the interior space filled with concrete. Care is needed with this type of pile in driving through ground containing obstructions. If the mandrel goes out of line there is difficulty in withdrawing it and the shells may be displaced. The shells are also liable to be lifted due to ground heave in firm to stiff clays. Piles driven in groups should be prebored for part of their length or the order of driving arranged to minimize ground heave.

The other type comprises solid square or hexagonal section precast units with locking joints which are stronger than the concrete section. The joints are capable of withstanding uplift caused by ground heave. The lengths are manufactured to suit the requirements of the particular job and additional short lengths are locked on if deeper penetrations are required. Piles of this type include the West's Hardrive, the Herkules and Balken sections.

17.4.2.4 Steel piles

Steel piles of tubular, box, and H-section have the advantages of being robust and easy to handle and can withstand hard driving. They can be driven in long lengths and have a good resistance to

The stress under the working load should be limited to 30% of the yield stress except where piles are driven through relatively soft soils to an end bearing on dense soils or sound rock, when the allowable axial working stress may be increased to 50% of the yield stress.

Slender-section steel piles driven in long lengths are liable to go off-line during driving. It is desirable to check them for

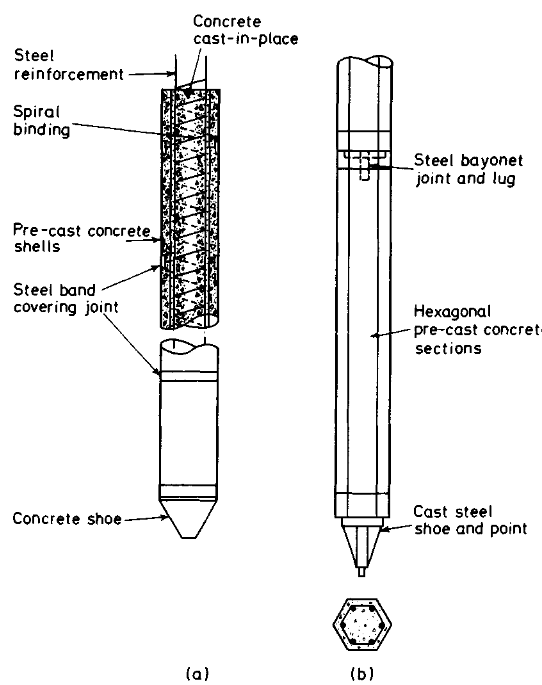

Figure 17.37 Jointed precast concrete piles. (a) West's shell pile; (b) Herkules pile

curvature after driving by inclinometer (a small-diameter tube can be welded to the web of an H-section pile for this purpose). If H-piles or unfilled tubular piles have a curvature of less than 360 m they should be rejected. Tubular piles need not be rejected if they are designed to be filled with concrete capable of carrying the full working load.

Steel piles are liable to corrosion where oxygen is available, e.g. above the soil line or above water level, but allowance can be made for corrosion losses within the useful life of the structure or special protection can be provided (see section 17.8.3).

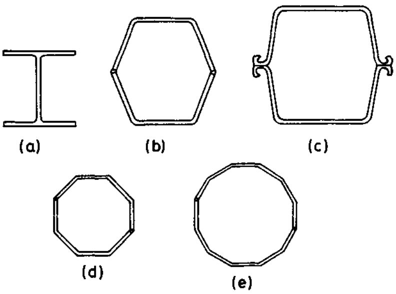

Figure 17.38 Steel-bearing piles of various types. (a) Universal bearing pile (UBP); (b) Rendhex foundation column (obsolete); (c) Larssen box pile; (d) Frodingham octagonal pile; (e) Frodingham duodecagonal pile

17.4.2.5 Driven and cast-in-place piles

There is a wide range of types of proprietary driven and cast-in-place piles in which a steel tube is driven to the required penetration depth and filled with concrete. In some types the tube is withdrawn during or after placing the concrete. In other types the tube of a light steel shell is left permanently in place.

In one type (Figure 17.39) a drop hammer acts on a plug of gravel at the bottom of the tube. This carries down the tube and, on reaching the bearing stratum, further concrete is added and the plug is hammered out to form an enlarged base. The drop hammer is also used to compact the concrete in the shaft as the tube is withdrawn. This type of pile can be provided with a light-section steel shell which is placed in the tube before filling with concrete to provide a permanent casing to withstand 'squeezing' ground conditions.

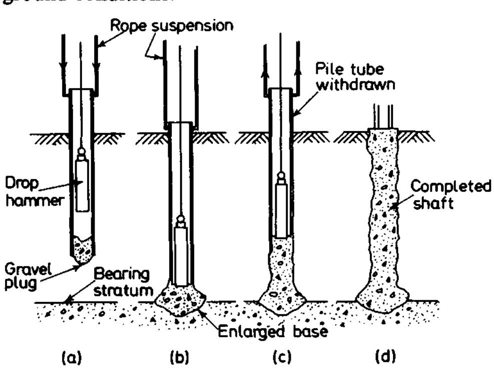

Figure 17.39 Driven and cast-in-place pile (end closed by gravel plug)

In another type a steel drive tube (Figure 17.40) is provided with a detachable steel shoe and is driven to the required penetration by a drop hammer or diesel hammer acting on top of the tube. A reinforcing cage is then placed in the tube and concrete is poured before or during withdrawal of the tube.

Driven and cast-in-place piles of the types described above are cast to nominal outside diameters ranging from 250 to 750 mm. Their lengths are limited by the capacity of the rig to pull out the drive tube to a maximum of about 40 m.

In the Raymond Step Taper Pile light gauge steel shells of progressively reducing diameter are driven to the required depth on a mandrel. The latter is then withdrawn and the shells are filled with concrete. Placing concrete in the shells should be delayed until ground heave has ceased when driving these piles in groups. Ground heave can be reduced by preboring. When the required pile length exceeds the limits of the available equipment to drive an all-shell pile, a pipe step-taper pile may be used. With this type the bottom unit consists of a pipe of constant 273 mm section of the required length.

The BSP cased pile system consists of driving a fairly light spirally welded steel tube either by a hammer on top of the pile or by a drop hammer acting on a plug of dry concrete at the bottom of the closed-end pile. On reaching founding level the whole pile is filled with concrete. This type of pile can be used for marine works. Inside tube diameters range from 245 to 508 mm. The BSP cased pile is unsuitable if hard layers must be penetrated to reach the required toe level. Prolonged driving on to the concrete plug can fracture the enclosing tube.

British Standard 8004 requires the concrete of all driven and cast-in-place types to have a cement content of not less than 300 kg/m^3. The average compressive strength under working loads shall not exceed 25% of the specified 28-day works cube strength. Care should be taken to ensure that the volume of concrete placed fills the volume of the soil displaced by the drive tube or the volume of shells left in place. This is a safeguard against caving of the ground while withdrawing the tube or collapse of shells.

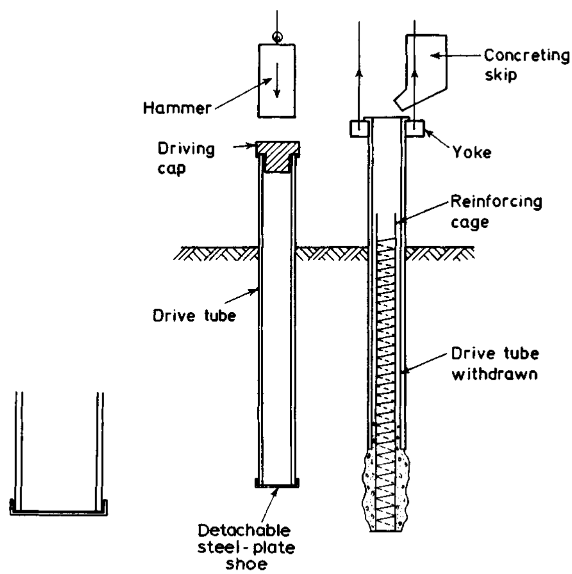

Figure 17.40 Driven and cast-in-place pile with detachable shoe

17.4.3 Types of replacement piles

17.4.3.1 Rotary bored piles

If the soil is capable of remaining unsupported for a short time the pile borehole can be drilled by a rotary spiral plate or bucket auger. Support to soft, loose or water-bearing superficial soil deposits in the upper part of the pile borehole can be provided by a length of temporary casing which is driven down to seal into a stiff cohesive soil in advance of the drilling operation. The borehole is continued in the stiff cohesive soil or weak rock without support by temporary casing unless it is desired to enter the hole for visual inspection of the base or to enlarge the base by manual excavation. In these cases it is necessary to give temporary support by full-length lining tubes which are suspended from the ground surface. After completion of drilling and cleaning the bottom of the borehole the reinforcing cage is inserted and concrete is placed by discharging it from a hopper at the mouth of the hole. An easily workable self-compacting mix with a slump of 125 to 150 mm is used.

Base enlargements can be formed in stiff cohesive soils and weak rocks by a rotary under-reaming tool provided that the borehole is reasonably dry.

Where groundwater seepages enter the borehole below the level of the temporary casing in quantities which cause accumulations at the bottom of the hole of more than a few centimetres in 5 min, no attempt should be made to bale out the water which should be allowed to rise to its standing level. The concrete should then be placed under water through a tremie pipe. The mix should have a slump of 175 mm or more and a minimum cement content of 400 kg/m^3.

In 'squeezing' soils or in ground contaminated by substances aggressive to concrete, light steel or plastic tubing can be used as a permanent sheathing to the concrete in the pile shaft.

In water-bearing soils and rocks and in cohesionless soils, support to the pile boreholes can be provided by a bentonite slurry. The concrete in the pile shaft is placed through the slurry by tremie pipe.

Rotary augers can drill to depths of up to 60 m with shaft and base diameters up to 5 and 6 m respectively.

Safety precautions in bored piling work are covered by BS 5573.

17.4.3.2 Percussion-bored piles

In ground which collapses during drilling, requiring continuous support by casing, the pile boring is undertaken by baling or grabbing. For small-diameter (up to 600 mm) piles the tripod rig is used to handle the drilling tools and to extract the casing. For large-diameter piles a powered rig which combines a casing oscillator and a winch for handling grabbing and chiselling tools is used to drill to diameters of up to 1.5 m and depths of 50 m or more. *Barrettes* are rectangular- or cruciform-shaped piles formed by excavating under a bentonite slurry by a trenching grab, followed by placing the concrete through the slurry by tremie pipe. Barrettes are suitable for deep foundations carrying high lateral forces, e.g. in retaining walls.

Problems of placing concrete in difficult conditions, e.g. in 'squeezing' ground, can be overcome in special cases by placing concrete under compressed air with the assistance of an airlock on top of the casing, i.e. the Pressure pile, or by placing precast concrete sections in the casing and injecting cement grout to fill the joints between and around the sections while withdrawing the casing (the Prestcore pile).

17.4.3.3 Auger-injected piles

A continuous-flight auger is used to drill the pile borehole to the required depth. A sand–cement grout or concrete is then pumped down the hollow stem of the auger as it is being withdrawn. The reinforcement cage is lowered down the shaft after the auger has been fully withdrawn. Presently available rigs can drill to diameters in the range of 300 to 750 mm and to depths of up to 25 m. The auger-injected pile is suitable for most soils. The process is virtually vibration-free which makes it suitable for use close to existing structures.

17.4.3.4 Concrete for replacement piles

The cement content should not be leaner than 300 kg/m^3. The average compressive stress under the working load should not exceed 25% of the specified works cube strength at 28 days. British Standard 8004 permits a higher allowable stress if the pile has a permanent casing of suitable shape.

The concrete should be easily workable and capable of slumping to fill all voids as the casing is being withdrawn without being lifted by the casing. If a tremie pipe is necessary for placing concrete under water the mix should not be leaner than 400 kg of cement per cubic metre of concrete and a slump of 175 mm is suitable.

17.4.4 Raking piles to resist lateral loads

Where lateral forces are large it may be necessary to provide raking piles to carry lateral loading in compression or tension axially along the piles. Arrangements of raking pile foundations for a retaining wall and a berthing structure are shown in Figures 17.41(a) and (b) respectively.

Raking piles should not have a rake flatter than 1 in 3 if difficulties in driving are to be avoided, but flatter rakes are possible with short piles. It is not easy to install driven and cast-in-place or bored piles on a rake.

Methods of calculating the ultimate capacity and deflection of piles under horizontal loading are given by Tomlinson[23] and Elson,[24] but load testing is necessary if deflections are critical.

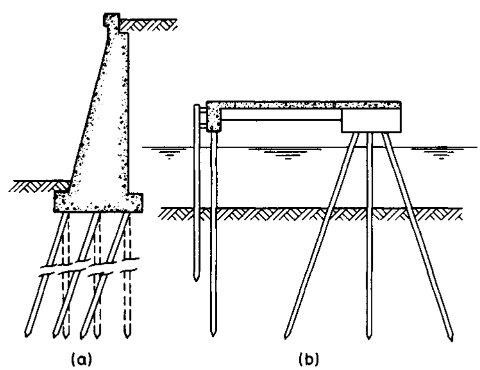

Figure 17.41 Raking piles to resist lateral loads. (a) Beneath retaining wall; (b) in a marine berthing structure

17.4.5 Anchoring piles to resist uplift loads

Piles can be anchored to rock by drilling in a steel tube with an expendable bit at its lower end. Grout is injected through the tube to fill the annulus to form an unstressed or 'dead' anchor. Alternatively, a high-tensile steel rod or cable can be fed into a predrilled hole. It is stressed by jacking from the top of the pile. In the second method the upper part of the anchor should be prevented from bonding to the grout by surrounding the greased metal with a plastic sheath. This is to ensure mobilization of the uplift resistance of the complete mass of rock down to the bottom of the anchorage. Methods of calculating this resistance are described in Chapter 10.

17.4.6 Pile caps and ground beams

A pile cap is necessary to distribute loading from a structural member, e.g. a building column, on to the heads of a group of bearing piles. The cap should be generous in dimensions to accommodate deviation in the true position of the pile heads. It is usual to permit piles to be driven out of position by up to 75 mm and the positioning of reinforcement which ties in to the projecting bars from the pile heads should allow for this deviation. Caps are designed as trusses or beams spanning the pile heads and carrying concentrated loads from the superimposed structural member.[25] The heads of concrete piles should be broken down to expose the reinforcing steel which should be bonded into the pile cap reinforcement. The loading on to steel piles can be spread into the cap by welding capping plates to the pile heads or by welding on projecting bars or lugs as shear keys. A three-pile cap is the smallest which can be permitted to act as an isolated unit. Single- or two-pile caps should be connected to their neighbours by ground beams in two directions or by a ground slab. A system for the standardization of pile-cap dimensions has been described by Whittle and Beattie.[26]

Piles placed in rows beneath load-bearing walls are connected by a continuous cap in the form of a ground beam (Figure 17.42). In the illustration the ground beam is shown as constructed over a compressible layer such as cellular cardboard designed to prevent uplift on the beam due to swelling of the soil, and the pile is sleeved over its upper part to prevent uplift within the zone of swelling. The ground beam should be designed to resist horizontal thrust from the swelling clay. As an alternative to sleeving the upper part of the pile it may be preferable to provide for uplift by increasing the length of the shaft.

17.4.7 Testing of piles

Tests to determine the integrity of the shafts of concrete piles can be made by nondestructive methods described by Weltman.[27] In soils where time effects are not significant in determination of bearing capacity a reasonably accurate prediction of ultimate bearing capacity and settlement can be made by measurements of strain and acceleration under hammer impact at the time of driving.[28]

Loading tests on piles may be needed at two stages: (1) to verify the carrying capacity of the piles in compression, uplift or lateral loading; and (2) to act as a proof load to verify the soundness of workmanship or adequacy of penetration of working piles.

For first-stage testing either the constant rate of penetration (CRP) test or the maintained load (ML) method may be used. The latter is to be preferred if information on the deflection of the pile under the working load, or at some multiple of this load, is needed.

For proof loading of working piles the ML test should be made. It is not usual to apply a load of more than 1.5 times the working load in order to avoid overstressing the pile.

The procedures for the CRP and ML tests are described in BS 8004.

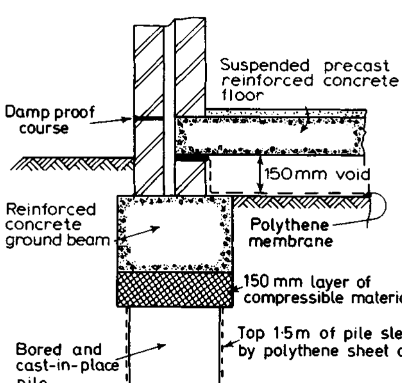

Figure 17.42 Ground beam for piles carrying a load-bearing wall

17.5 Retaining walls

17.5.1 General

This section covers the design and construction of free-standing or tied-back retaining walls. The design of retaining walls for basements, bridge abutments and wharves is described in section 17.3.2.3, in Barry[29] and in Chapters 23 and 26 respectively.

Free-standing or tied-back retaining walls can be grouped for design purposes as follows:

(1) Gravity walls which rely on the mass of the structure to resist overturning (Figure 17.43(a)).
(2) Cantilever walls which rely on the bending strength of the cantilevered slab above the base (Figure 17.43(b)).
(3) Counterfort walls which are restrained from overturning by the force exerted by the mass of earth behind the wall (Figure 17.43(c)).
(4) Buttressed walls which transmit their thrust to the soil through buttresses projecting from the front of the wall (Figure 17.43(d)).
(5) Tied-back diaphragm walls which are restrained from overturning by anchors at one or more levels (Figure 17.43(e)).
(6) Contiguous bored pile walls (Figure 17.43(f)).

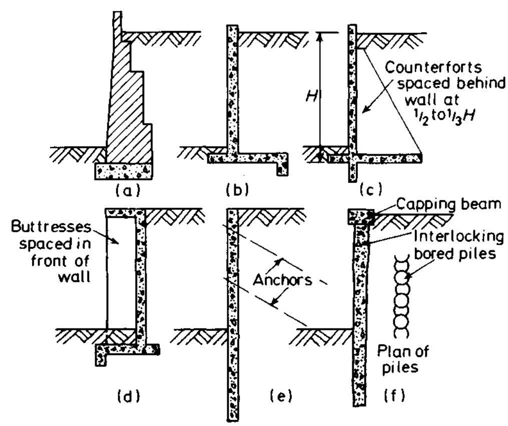

Figure 17.43 Types of retaining wall. (a) Gravity wall; (b) cantilever wall; (c) counterfort wall; (d) buttressed wall; (e) tied-back diaphragm wall; (f) cantilevered wall contiguous bored pile wall

It is assumed that sufficient forward movement of free-standing walls takes place to allow the earth pressure behind the walls to be calculated as the 'active pressure' case (see Chapter 9). Where the foundation of the wall is at a shallow depth below the lower ground level, the passive resistance to overturning or sliding is neglected since it may be destroyed by trenching in front of the wall at some future time.

The forces acting on a gravity and simple cantilever wall are shown in Figures 17.44(a) and (b). The force R is the resultant of the active earth pressure P_A and the weight of the wall W and backfill above the wall foundation. The surcharge on the fill behind the wall is allowed for when calculating P_A but is not included in the weight W. To prevent overturning of the wall the resultant R should cut the base of the wall foundation within its middle third, i.e. the eccentricity must not exceed $B/6$.

Having determined the position and magnitude of R, the bearing pressures at the toe and heel of the base are determined as described in section 17.2.5. These should not exceed the allowable bearing pressure of the ground, and the settlement at the toe should be within tolerable limits. Then the resistance to sliding of the base should be determined. If this is inadequate the base should be widened or taken down to a depth where the passive resistance in front of the wall may be safely mobilized (Figure 17.45).

Hydrostatic pressure behind the retaining walls should be avoided by the provision of a drainage layer behind the wall combined with weepholes and a collector drain (as shown in Figure 17.47).

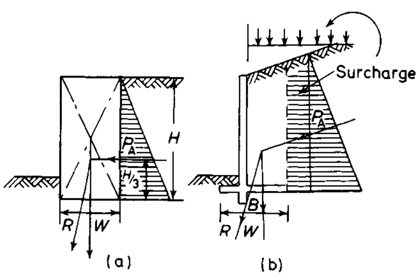

Figure 17.44 Forces acting on a free-standing retaining wall. (a) Simple gravity wall; (b) cantilever wall

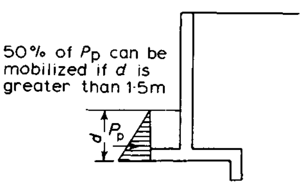

Figure 17.45 Passive resistance at toe of retaining wall

17.5.2 Gravity walls

Typical designs for gravity walls in brickwork, mass concrete and cribwork, are shown in Figure 17.46(a) and (b). Walls of these types are economical for retained heights of up to 2 to 3 m, or up to 5 m for cribwork walls. The width of the base should be about 0.40 to 0.65 times the overall height. For walls designed to present a 'vertical' appearance the front face should be battered back slightly say to 1 in 24 to allow for the inevitable slight forward rotation. The sloping wall and base (Figure 17.46(b)) provides the best alignment to resist earth pressure. Vertical joints in brick walls should be at 5 to 18 m and in concrete walls at 20 m centres or at some convenient length for a day's 'pour' of concrete. A preformed joint filler strip in bituminized fibre or PVC may be used.

Gravity walls of a type similar to that shown in Figure 17.46(b) can be built up from gabions (rectangular wire baskets filled with graded stone).

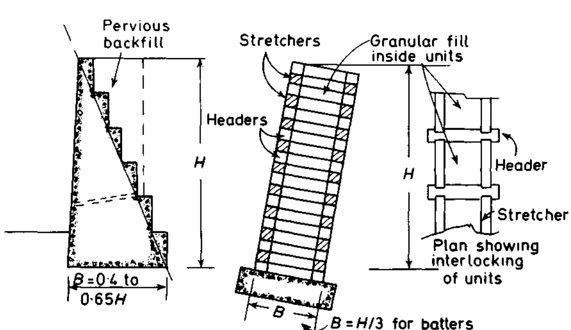

Figure 17.46 Designs for gravity retaining walls. (a) Mass concrete; (b) cribwork

17.5.3 Cantilevered reinforced concrete walls

A typical design for a cantilevered wall is shown in Figure 17.47. The projection of the base slab in front of the wall may be omitted if the wall face forms the boundary of the property but this arrangement should be avoided if at all possible because of the high pressure on the soil at the toe and the consequent risk of excessive forward rotation.

The design shown in Figure 17.47 is economical for heights of 4.5 to 6 m. The counterfort or buttressed types (see sections 17.5.4 and 17.5.5 respectively) should be used for higher walls.

The width of the base should be from 0.40 to 0.65 times the overall height of the wall. The minimum wall thickness should be 150 mm for single-layer reinforcement and 230 mm for front and back reinforcement. Although economy of concrete can result from progressive reduction in thickness of the wall section from the base to the top, a uniform thickness will give the lowest overall cost for walls up to 6 m high. A sloping or stepped-back face may show savings for higher walls.

The base slab thickness should equal the wall thickness at the stem of the latter. The projection in front of the wall should be about one-third the base width.

Expansion joints should be provided at spacings determined by the estimated thermal movement. A spacing of from 20 to 30 m is suitable for British conditions. The reinforcement should not be carried through these joints. Vertical contraction joints are required at 5 to 10 m spacing. The reinforcement may be carried through the contraction joints or stopped on either side. Where possible, construction (daywork) joints should coincide with expansion or contraction joints. The minimum cover to the reinforcing steel, appropriate in each case to the exposure conditions, is shown in Figure 17.47.

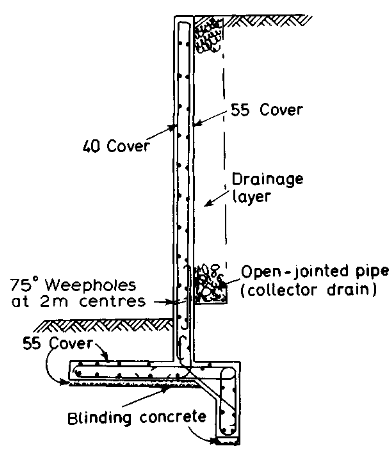

Figure 17.47 Design for reinforced concrete cantilever wall

17.5.4 Counterfort walls

The wall slab of counterfort retaining walls spans horizontally between the counterforts except for the bottom 1 m which cantilevers from the base slab. The counterforts are designed as T-beams of tapering section, and they are usually spaced at distances of one-third to one-half the height of the wall. The base of the counterfort must be well tied into the base slab. The latter acts as a horizontal beam carrying the surcharge load of the backfill and spanning between counterforts or from back beam to front beam. The counterforts transmit high bearing pressures to the ground at their front ends and may require piled foundations or a stiff front beam to distribute the pressure along the front of the wall.

17.5.5 Buttressed walls

Buttressed walls are economical for walls higher than 6 m designed to be cast against an excavated face, whereas the counterfort wall is more suitable where the ground behind the wall is to be raised by filling. The wall slab spans horizontally between the buttresses except for the bottom 1 m which cantilevers from the base slab. The buttresses act as compression members transmitting loading to the base slab or to piles on weak ground.

17.5.6 Tied-back diaphragm walls

The stages in constructing a tied-back wall in the form of a diaphragm wall are shown in Figure 17.48. In a stage I excavation the wall must be designed to cantilever from the stage I excavation level. For stage II excavations the wall spans between the anchorage level and the soil at the excavation line, similarly at stage III. At the latter stage the passive resistance of the soil in front of the buried portion must be adequate to prevent the wall moving forward at the toe, and the pressure beneath the base of the wall due to the vertical component of the anchor stress must not exceed the allowable bearing pressure of the soil.

The use of the tied-back wall as a basement retaining wall is described in section 17.3.2.5 and the design of ground anchors is discussed in Chapter 9. Guidance on the design of retaining walls of this type is given by Padfield.[30]

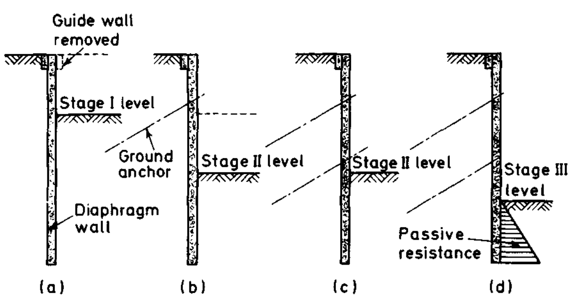

Figure 17.48 Stages in constructing a tied-back diaphragm wall. (a) Excavating to first stage in preparation for installing top-level ground anchors; (b) top-level anchors installed, excavation to second stage in preparation for installing bottom-level anchors; (c) bottom-level anchors installed; (d) excavation for third (final) stage

17.5.7 Contiguous bored pile walls

Retaining walls formed by a continuous line of bored piles can be designed as simple cantilever structures (Figure 17.43(f)) or as tied-back walls. Walls of this type are economical to construct by rotary auger drilling methods (see section 17.4.3.1) in self-supporting ground above the water table. In these conditions the piles can be installed merely as abutting units.

In water-bearing cohesionless soils the piles must interlock. If this is not done water and soil will bleed through the gaps causing loss of ground behind the wall. Interlocking is done by drilling and concreting alternate piles; then, by using a chisel to drill in the space between these piles, forming a deep groove in each of the latter. The drilled-out space is then filled with concrete to form the continuous wall. Construction in this manner is likely to cost more than the diaphragm wall.

17.5.8 Materials and working stresses

Concrete mixes and the quality of bricks or blocks should be selected as suitable for the conditions of exposure, attention being paid to frost resistance. Information on the durability of these materials in aggressive conditions is given in section 17.8.

The materials and working stresses for reinforced concrete should be in general accordance with BS 8110.

17.5.9 Reinforced soil retaining walls

Retaining walls can be constructed from soil which is reinforced to resist the internal tensile stresses which are induced by the horizontal movement towards the retained face of the wall.

There are two principal types of reinforced soil wall. In Figure 17.49(a), granular fill is brought up in compacted layers, each layer being reinforced by horizontal metal or plastic ties spaced at predetermined horizontal and vertical intervals. The vertical or steeply inclined face of the soil wall is retained by cladding panels which are secured to the ends of the ties. These panels may be constructed in precast concrete, metal or plastics and they can be preformed to a patterned profile to give a decorative effect to the finished wall.

In Figure 17.49(b), granular fill is placed on sheets of woven plastic mesh and compacted to form a thick bottom layer. The leading edge of the mesh is then folded back over the fill layer and a second sheet is placed on it followed by a second and successive layers of fill, each layer being partly wrapped by the sheets of mesh. The latter act as horizontal reinforcement restraining the fill from spreading outwards and as a means of retaining the steep outer face of the wall. Protection to the face can be given by precast concrete blocks, hand-placed stone pitching or turf. The mesh is designed to have tensile strength principally in the direction of horizontal forces induced by earth movements.

Reinforced soil walls for temporary works have been constructed by layers of scrap motor tyres lashed together by wire rope with granular fill placed in layers in the interstices between the tyres.

Reinforced soil retaining walls have the advantage of a high degree of flexibility which makes them suitable for retaining the face of deep cuttings where considerable heave and lateral movement may take place as a result of stress relief after excavating for the cutting. Walls of this type are also suitable for use in mining subsidence areas and in retaining the toe of embankments built on sloping ground.

The principles of reinforced soil have been stated by Jones.[31]

17.6 Foundations for machinery

17.6.1 General

In addition to their function of transmitting the dead loading of the installation to the ground, machinery foundations are subjected to dynamic loading in the form of thrusts transmitted by the torque of rotating machinery or reactions from reciprocating engines. Foundations of presses or forging hammers are subjected to high impact loading and rotating machinery induces vibrations due to out-of-balance components vibrating at a frequency equal to the rotational speed of the machine. Thermal stresses in the foundation may be high as a result of fuel combustion, exhaust gases or steam, or from manufacturing processes. Foundation machinery should have sufficient mass to absorb vibrations within the foundation block, thus eliminating or reducing the transmission of vibration energy to surroundings; they should spread the load to the ground so that excessive settlement does not occur under dead weight or impact forces and should have adequate structural strength to resist internal stresses due to loading and thermal movements.

Machinery foundation blocks are frequently required to have large openings or changes of section to accommodate pipework or other components below bedplate level. These openings can induce high stresses in the foundation block due to shrinkage

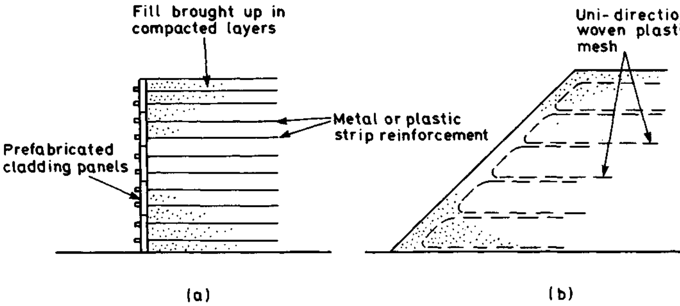

Figure 17.49 Reinforced soil construction. (a) Gravity-type retaining wall reinforced with strips of metal or plastic; (b) embankment reinforced wit woven plastics mesh

combined with other effects. Abrupt changes of section should be avoided, and openings should be adequately reinforced.

17.6.2 Foundations for vibrating machinery

When the frequency of a foundation block carrying vibrating machinery approaches the natural frequency of the soil, resonance will occur and the amplitude may be such as to cause excessive settlement of the soil beneath the foundation, or beneath other foundations affected by the transmitted wave energy. This is particularly liable to occur with foundations on loose granular soils. Knowing the weight of the machine and its foundation and the vibration characteristics of the soil, it is possible to calculate the resonant frequency of the machine–foundation–soil system. The frequency of the applied forces ideally should not exceed half of this resonant frequency for most reciprocating machines and should be at least 1.5 times the resonant frequency for machinery having frequencies greater than the natural frequency. If the applied frequencies are within this range there is a danger of resonance and excessive amplitude. These criteria are recommended by Converse[32] who describes various mathematical theories for calculating natural frequency and amplitude, and tabulates recommended ratios of foundation weight: engine weight for various types of machinery. The aim in design generally is to provide sufficient mass to absorb as much of the energy as possible within the foundation block and to proportion the block in such a manner that energy waves are reflected within the mass of the block or transmitted downwards rather than transversely in order not to affect adjacent property. In some cases it may be advantageous to mount the foundation block on special mountings such as rubber carpets or rubber–steel sandwich blocks.

17.6.3 Foundations for turbo-generators

The foundation blocks for large turbo-generators are complex structures subjected to periodic reversing movements due to differential heating and cooling of the concrete structures, moisture movements related to ambient humidity, steam and water leakage and to dynamic strains within the elastic range. They are also subjected to progressive movements resulting from long-term settlements of the foundation soil and from shrinkage and creep of concrete. These movements may be of sufficient magnitude to cause misalignment of the shafts of the machinery.[33]

17.7 Foundations in special conditions

17.7.1 Foundations on fill

If granular fill can be placed in layers with careful control of compaction, the resulting settlement due to the foundation loading and the settlement of the fill under its own weight will be small. Provided the fill has been placed on a relatively incompressible stratum the settlement of the structure will be little if anything greater than would occur with a foundation on a reasonably stiff or compact natural soil.

However, in most cases of construction on filling, the material has probably not been placed under conditions of controlled compaction but has been loosely end-tipped, and the age of the fill may not be known with certainty. However, it is usually possible to obtain a good indication of the constituents of the fill and its state of compaction from observations in boreholes and trial pits (preferably the latter). From these observations an estimate can be made of the likely remaining settlement due to consolidation of the fill under its own weight and that of the superimposed loading. Reference should be made to *Building Research Establishment Digest* Number 274[34] for information on the amount and rate of settlement of various types of fill material.

For shallow granular fills, strip or pad foundations are suitable for most types of structure. For deeper granular fills which have not had special compaction, it will be necessary to use raft foundations for structures which are not very sensitive to differential settlement, or piled foundations for structures for which small settlements must be avoided.

Ordinary shallow foundations can be used on hydraulically placed sand fill where this can consolidate by drainage but not when the fill has been allowed to settle through water. Piled foundations are necessary for structures on hydraulically placed clay fill or on domestic refuse.

Raft or piled foundations can be avoided on loose granular fills if one of the ground treatment processes described in section 17.2.7 is adopted.

Where piled foundations are used in fill areas, consolidation of the fill and of any underlying natural compressible soil will cause dragdown forces on the pile shafts which must be added to the working load from the superstructure.

Where bored piles are used through the fill the dragdown or negative skin friction forces may be very high, and for economy it may be desirable to minimize the dragdown by adoption of slender preformed sections, e.g. high-strength precast concrete, or to surround the pile shaft with a sleeve or a layer of soft bitumen.

Fills consisting of industrial wastes may contain substances which are highly aggressive to buried concrete or steelwork.

17.7.2 Foundations in areas of mining subsidence

17.7.2.1 General

Cavities are formed where minerals are extracted from the ground by deep mining or pumping. In time, the ground over the cavities will collapse wholly or partly filling the void. This leads to subsidence of the ground surface. Movements of the surface may be large both in a vertical direction and in the form of horizontal ground strains and the foundations of structures require special consideration to accommodate these movements without resulting damage to the superstructure. The majority of foundation problems in the UK are due to coalmining, but subsidence can occur due to extraction of other minerals such as brine.

In the nineteenth century and earlier, coal was extracted by methods known variously as 'pillar-and-stall', 'room-and-pillar' and 'bord-and-pillar'. The galleries were mined in various directions from the shaft followed by cross-galleries leaving rectangular or triangular pillars of coal to support the roof above the workings (Figure 17.50).

The current method of coalmining is by 'longwall' methods in which the coal seam is extracted completely on an advancing face (Figure 17.51). The amount of subsidence at ground level is less than the thickness of coal extracted owing to bulking of the collapsed strata.

The problems of foundations of buildings on old mine workings are discussed by Healy and Head.[35]

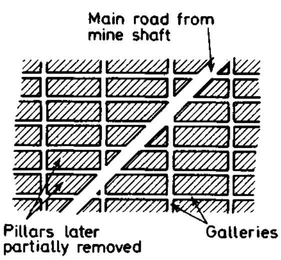

Figure 17.50 'Pillar-and-stall' mineworkings

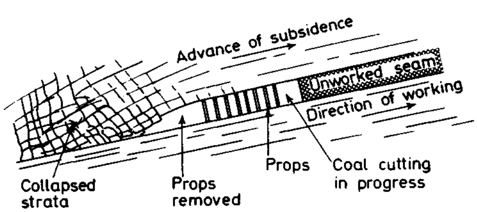

Figure 17.51 Extraction of coal by the longwall method

17.7.2.2 Foundation design in areas of pillar-and-stall workings

The risk of collapse depends on the conditions of the 'roof' over the workings. Where this consists of weak or broken rock, stage collapse will occur at some time and the void formed will gradually work its way up to the ground surface to form a 'crown hole' (Figure 17.52(a)). If, however, the roof is a massive sandstone it will bridge over the cavity for an unlimited period of years (Figure 17.52(b)). However, the pillars of coal may suffer slow deterioration at an unpredictable rate.

In considering the design of foundations over workings of this type an appraisal is made of the general geological conditions. Where the collapse of overburden strata or coal pillars could result in severe local surface subsidence, precautions against these effects must be taken. Methods which may be considered are:

(1) Filling the workings by injection techniques; or
(2) Constructing piled or deep shaft foundations to a founding level below the workings.

Method (1) is used where the workings are at such a depth that method (2) is uneconomical. No attempt is made to locate individual galleries but the area of the structure is ringed by a double row of injection holes at close spacing. Gravel or a stiff sand–cement grout is fed down these holes to form a barrier in the voids of the worked seam. Holes are then drilled on a nominal grid in the space within the barrier and low-cost materials are fed down these holes to fill all accessible voids. These materials may consist of sand–pulverized fuel ash–water slurry, or a lean sand–pulverized fuel ash–cement grout.

Where deep shaft or piled foundations (method (2)) are used the shaft is sleeved where it passes through the overburden to prevent transference of load to the foundation in the event of subsidence. The outer lining forming the sleeve must be strong enough to resist lateral movement caused by subsidence. Where structures are to be built on soft compressible soils overlying mine workings, piled foundations bearing on a thin cover of rock strata above the workings must not be used since the toe loading from the piles may initiate subsidence. Buoyancy raft foundations should be used (see section 17.3.3).

17.7.2.3 Foundation design in areas of longwall workings

In the case of current or future workings, subsidence is inevitable and the degree to which precautions are taken in foundation design depends on the type and importance of the structure under consideration.

It will be seen from Figure 17.53 that as the subsidence wave crosses a site the ground surface is first in tension and then in compression. As subsidence ceases, the residual compression strains die out near the surface. The simplest form of construction is a shallow reinforced concrete raft. This is usually adopted for houses for which the cost of repairs due to distortion of the raft can be kept to a reasonable figure.

Points to note in the design of raft foundations are:

(1) The underside of the raft should be flat, i.e. it should not be keyed into the ground.
(2) A slip membrane is provided beneath the raft to allow ground strains to take place without severe compression or tension forces developing in the substructure.
(3) Reinforcement is provided in the centre of the slab to resist bending stresses caused either by hogging or sagging.

A raft may not be suitable for heavy structures such as bridges or factories. In these cases, the principle to be adopted is to use bearing pressures as *high* as possible, so minimizing the foundation area and, hence, the horizontal tension and compression forces transmitted to the superstructure. If the layout permits, the structure should be supported on only three bases to allow it to tilt without distortion.

Trenching around a structure can be used to reduce compressive strain but this method is ineffective in countering tension strains.

17.7.2.4 Foundations adjacent to existing shafts

Foundation problems may arise owing to the collapse of deteriorated shaft linings followed by surface subsidence. The type of material for filling the shaft should be ascertained. If it is granular, the loose material and any cavities can be consolidated by injection of a low-cost grout. If the infill consists of clay, grouting may be ineffective. However, in this case the shaft may be capped with a reinforced-concrete slab. The latter method should be adopted only if the shaft lining is sound and durable over its full depth. If not, or if for reasons of safety the condition of the lining cannot be ascertained, the shaft should be surrounded by a ring of bored piles or by a diaphragm wall taken down to a stable stratum.

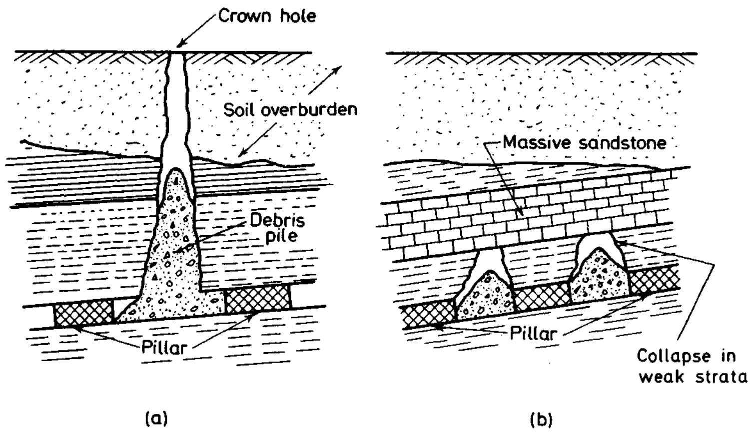

Figure 17.52 Subsidence due to collapse of cavities in mineworkings. (a) Weak strata over coal seam; (b) strong 'roof' over coal seam

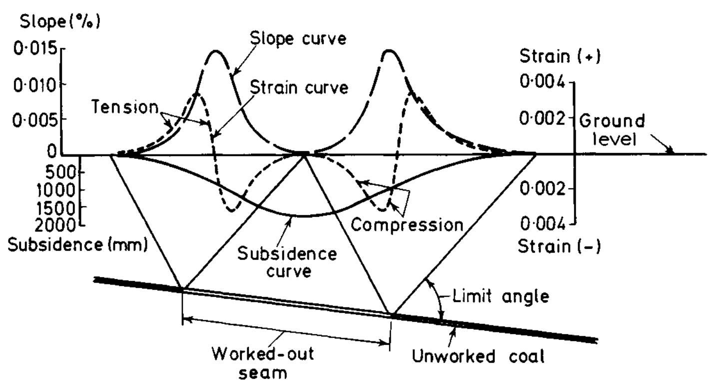

Figure 17.53 A form of subsidence above longwall workings

17.8 The durability of foundations

17.8.1 General

Foundation materials are subjected to attack by aggressive compounds in the soil or groundwater, living organisms and mechanical abrasion or erosion. The severity of the attack depends on the concentration of aggressive compounds, the level of and fluctuations in the groundwater table or the variation in tidal and river levels and on climatic conditions. Immunity against deterioration of foundations can be provided to a varying degree by protective measures. The protection adopted is usually a compromise between complete protection over the life of the structure, and the cheaper partial protection while accepting the possible need for periodic repairs or renewals. Problems of durability of a wide range of materials and the appropriate protective measures have been reviewed by Barry.[29] Methods of protection of some foundation structures are described in the following sections.

17.8.2 Timber

Timber piles are liable to fungal decay if they are kept in moist conditions, i.e. above the groundwater level. Piles wholly below the water level, if given suitable preservative treatment, can perform satisfactorily for a very long period of years. Properly air-seasoned timber, if kept wholly dry, i.e. moisture content less than 22%, will also remain free of decay for an indefinitely long period. The best form of protection against fungal attack and termites is pressure treatment with coaltar creosote or the copper (chrome) arsenic-type waterborne preservative. Creosote protection should be applied in accordance with BS 913 and the waterborne type to BS 4072.

Timber piles in marine structures are liable to destruction by molluscan and crustacean borers which inhabit saline or brackish waters. Although preservative treatment gives some protection against these organisms, the longest life is given by a timber known to be resistant to their depredations. Greenheart, jarrah and blue gum are suitable for cold European waters. In other countries, billian in the China seas, turpentine in New South Wales, black cypress and ti-tree in Queensland, spotted gum in Tasmania and teak in India have been found to have some immunity.

Protection can be given by jacketing piles in concrete before driving, or by gunite mortar after installation. Concrete can also be used in land foundations either in composite concrete–timber piles, or in deep pile caps down to groundwater level (see Figure je17.32, page 17/19).

17.8.3 Metals

Protection can be given to steel piles by impervious coatings of bitumen, coaltar, pitch or synthetic resins but these treatments are not effective for piles driven into the ground since the coatings are partly stripped off. It is the normal practice to provide sufficient cross-sectional area of steel to allow for wastage over the useful life of the structure while still leaving enough steel to keep the working stresses within safe limits.

In undisturbed cohesive soils corrosion is negligible because of the absence of oxygen. There may be some local pitting corrosion near the ground surface where the capillary moisture zone is mobile and replenished with oxygenated waters. Corrosion generally is low or negligible below groundwater level in natural soils, again because of the absence of oxygen.

Morley[36] quotes a corrosion rate of 0.08 mm/yr for unprotected low-alloy steel in static sea-water, and 0.1 to 0.25 mm/yr in the splash zone. He quotes average corrosion rates of 0.1 to 0.2 mm for corrosion in an industrial atmosphere in the UK.

In severe conditions, i.e. in polluted ground, it may be necessary to adopt a system of cathodic protection. Steel piles in river and marine structures can be protected above the soil line by heavy coatings of coaltar, bituminous enamel, epoxy pitch and vinyl pitch. However, these coatings are liable to damage by floating objects or barnacle growth and cathodic protection is necessary in marine structures if a long life is desired.

Cast iron has a similar corrosion resistance to mild steel and protective coatings provide the best method of treatment of substructures such as cylinder foundations constructed from cast-iron segments.

17.8.4 Concrete

The principal cause of deterioration of concrete in structures below ground level is attack by sulphates in the soil or groundwater. Sulphates occur naturally in some soils and in peats. They occur in sea-water at a concentration of about 230 parts per 100 000 which is greatly in excess of the figure regarded as marginal between nonaggressive and aggressive. However, because of the inhibiting effect of the chlorides in sea-water the sulphates do not cause an expansive reaction to normal Portland cement concrete if it is of good quality and well compacted. However, it is a good idea as a precaution to use sulphate-resisting cement or Portland blast-furnace cement in *reinforced* concrete structures immersed in sea-water.

Concentrations of sulphates may be high in industrial wastes, particularly in colliery wastes and some blast-furnace slags. Where fill material contains industrial wastes a full chemical analysis should be made to identify potentially aggressive compounds.

The precautions to be taken to protect concrete substructures are listed in Building Research Establishment Digest Number 250[37] and also in BS 8004 but these recommendations do not give much consideration to the workability required for the recommended concrete mixes for the particular placing conditions. Guidance on this aspect is given by Tomlinson.[2]

In normal climatic conditions in the UK, concrete at a depth greater than 300 mm is unlikely to suffer disintegration due to frost expansion. In severe conditions of exposure a dense concrete mix should be used with a water:cement ratio of less than 0.5. If the ratio is between 0.5 and 0.6 there is a risk of frost attack and above 0.6 the risk becomes progressively greater.

The required cover of steel reinforcement to prevent corrosion of the steel for various exposure conditions is listed in BS 8110.

17.8.5 Brickwork

Bricks with a high absorption should be avoided since they are liable to frost disintegration, and they can absorb sulphates or other aggressive substances from the soil or from filling-in contact with the brickwork.

In sulphate-bearing soils or groundwater the brickwork mortar should be a 1:3 cement:sand mix made with sulphate-resisting cement or in severe conditions with supersulphated cement.

Concrete bricks or blocks may be used for foundations if they are in accordance with British Standard 1180. Precautions should be taken against sulphate attack by specifying the type of cement and the quality of concrete to be resistant to the concentration of sulphates as determined by chemical analysis.

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BS 8004, *Foundations*
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BS 449, *The use of structural steel in building*
BS 913, *Pressure creosoting of timber*
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BS 1162, 1410; 1418, *Mastic asphalt for building (natural rock asphalt aggregates)*
BS 1180, *Concrete bricks and fixing bricks*
BS 4072, *Wood preservation by means of waterborne copper/chrome/arsenic compositions*
BS 4360, *Weldable structural steels*
BS 4449, *Hot-rolled steel bars for the reinforcement of concrete*
BS 4461, *Cold-worked steel bars for the reinforcement of concrete*
BS 5930, *Code of practice for site investigations*
BS 6031, *Code of practice for earthworks*

18

Dams

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Contents

18.1 Definition

In the UK, the name 'dam' is given to a civil engineering structure built across a valley to form an artificial lake as a reservoir of water. There are numerous variants. Some reservoirs are formed on relatively flat land by building long dams to encircle the required areas. Others are built to store materials other than water. In South Africa and some other countries the word 'dam' is used for the reservoir which is retained by a 'wall' or 'dam wall'.

18.1.1 Types of dam

Dams are separated into two main types by the choice of material used for their construction: (1) embankment; and (2) concrete dams.

(1) *Embankment dams* are made from nonorganic particulate material excavated from the Earth's surface local to the dam site and used more or less as excavated. They are subdivided into earthfill and rockfill dams, although many embankment dams contain both types of fill. Further subdivisions can be made, according to the material used, to make the waterproof element, e.g. central clay core, sloping clay core or upstream membrane of asphalt or reinforced concrete.

(2) *Concrete dams* are made from a carefully selected and processed harder fraction of this material, bound together and strengthened by an hydraulic cement. They are subdivided according to their mechanism for remaining stable.
   (a) *gravity dams*: these are the simplest because they rely on gravitational force to oppose the overturning moment caused by the pressure of the reservoir water on their upstream faces.
   (b) *hollow gravity dams*: these require less concrete and therefore cost less to construct. Foundation requirements are more critical.
   (c) *buttress dams*: these also require less concrete than gravity dams. The buttresses support the upstream face of a buttress dam. The upstream edges of the buttresses are commonly widened so that they join, forming the contiguous buttress dam. As an alternative, the upstream face may consist of small arches between buttresses, *forming a multi-arch dam*.
   (d) *arch dams*: these may be constructed as a whole in one large arch, spanning the valley sides and relying on them to carry the very large thrusts caused by reservoir water pressure. This type is the most sophisticated of the concrete dams and may be subdivided into single-curvature and double-curvature, according to whether the vertical section is straight, or is curved to further reduce bending moments in the concrete.

18.2 Brief history

Dams have made a major contribution to the development of our civilization. Their earliest role was to provide storage for irrigation water; now, they also provide hydro-electric power and water for industry and large cities.

Early dams were all of the embankment type, of necessity built from the earth and stones found at the site. Helms (quoted by Kerisel)[1] states that the oldest dam in the world is at Jawa in Jordan, dating from about 4000 B.C.and was built of earth with a masonry facing. Perhaps the second-oldest is the Sadd el-Kafara on the Wadi el-Garawi near Helwan in Egypt, built about 2900 B.C. It was 11 m high with upstream and downstream rubble masonry walls, each 24 m wide at their bases, separated by a central earthfill section 36 m wide.

Rao[2] reports that there was a tradition of dam building in India where it was once considered as one of the seven meritorious acts which a man ought to perform during his lifetime. During the period of British tenure, many embankment dams were constructed by traditional methods and were accepted as a means of famine relief, giving employment to thousands. According to Buckley,[3] the completed schemes were not only profitable, but brought happiness and contentment to the people by ensuring reliable crop production.

In the UK the industrial revolution required water for transport, industrial processes and a growing population. In the eighteenth century, dams were built to store water for canals; during the nineteenth, the majority were for water supply, and early in the twentieth century, dams were built specifically to provide power for aluminium smelting in Scotland and Wales.

Before 1800, almost all the world's dams were of the embankment type. During the nineteenth century, concrete technology and methods of structural analysis were developed. The many sites then available with sound rock foundations at shallow depth created increasing interest in concrete dams.

The increasing size of the human population of the world, together with ever-rising demands for irrigation water and power, caused rapid increase in the number of dams built. Figure 18.1 shows, from 1800 to the present, the increase in world population, together with the number and height of embankment dams. Although the numbers of dams increased in response to the rise in population, the number of embankment dams fell during the second half of the nineteenth and early twentieth centuries due to the number of concrete dams being built in that period. The proportion of the total that were embankment dams being built during any 5-yr period fell to a minimum of about 30% by the end of the first quarter of this century.

Since that period, the proportion has increased until currently more than 80% of dams being built are embankment dams. The highest dam in the world (Nurek, 300 m) is of the embankment type and is soon to be exceeded by another (Rogan) which will be 325 m high when complete. Both are in the Soviet Union.

This reversal of trend can be attributed to:

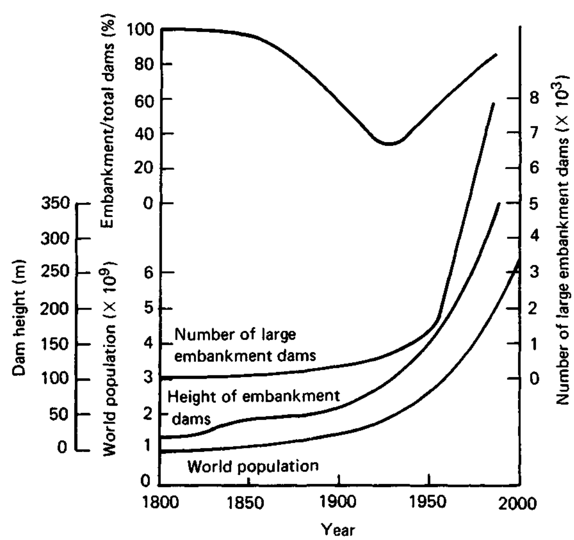


Figure 18.1 Numbers of people and embankment dams, 1800–1985. Curves also show heights of embankment dams and their proportion of total world dams

(1) Improved understanding of the behaviour of embankment dams due to advances in the art and science of soil mechanics since publication of Terzaghi's *Erdbaumechanik* in 1925.
(2) Increased capacity of earthmoving machinery after the introduction of the internal combustion engine and caterpillar tracks.
(3) Reduction in the number of sites with bedrock at shallow depth suitable for concrete dams.
(4) Increasing cost of labour.

At present, even at sites with strong rock near the surface suitable for concrete dams, e.g. the Lesotho Highlands schemes, embankment dams are often found to be cheaper. They can tolerate relatively poor foundation conditions and, when combined with their low price, this makes them a most attractive option.

18.3 Embankment dams

18.3.1 Introduction

Embankment dams can be subdivided according to type and position of the waterproof element. In this section, a description of the salient features of each type will precede an actual example.

18.3.2 Rockfill dam with upstream reinforced concrete membrane

This type is currently rising in prominence and so will be described first. (This does not mean that it is more important than other types of embankment dam.)

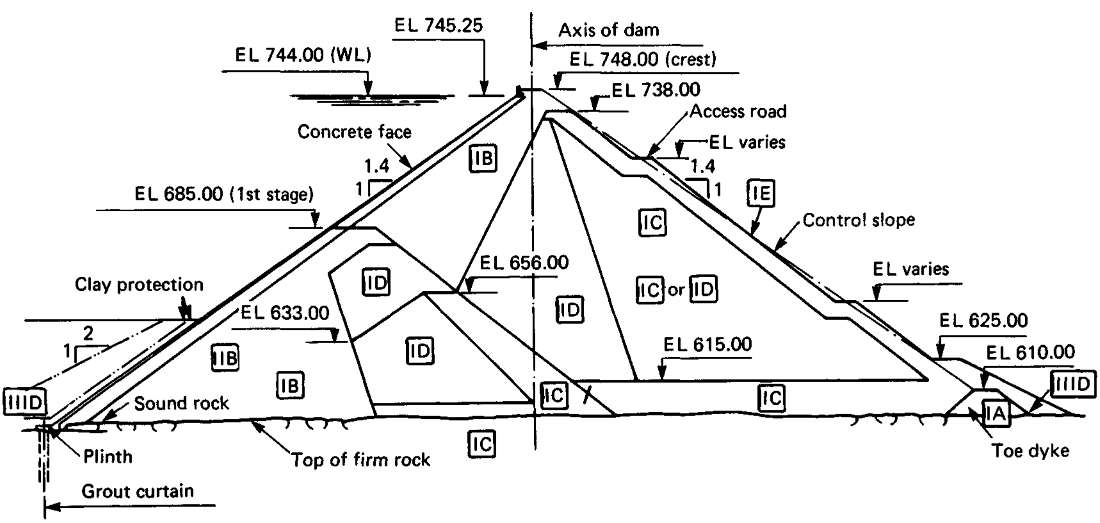
The Foz do Areia (160 m) on the River Iguaçu in Brazil is the world's highest dam of this type and is a typical example (Figures 18.2 and 18.3). It was required for hydro-electric power and river control. The region of the dam site is made up of medium to thick basaltic flows of 25 to 55 m depth. As is common with volcanic rocks, the interface between successive flows forms a zone more pervious than the massive basalt. The site investigation showed that about 70% of total volume was predominantly dense basalt with about 30% consisting mostly of basaltic breccia. Table 18.1 gives some geomechanical properties of the rocks.

More than 14 million m^3 of rock excavation was required for tunnels, power station and spillway chute. This excavated rock was used as fill for the dam. It was placed in layers up to 1.6 m thick, sluiced with water at a rate of 25% volume of placed rock and compacted by four passes of a 10-t vibrating roller. Details of layer thickness and zones of rockfill are given in Figure 18.2, showing a cross-section of the dam.

Fragmentation of the hard rock by blasting produced a fill that was rather too uniform in size. To reduce compressibility under the upstream membrane, a transition zone of rockfill crushed to a specified grading with a maximum particle size of 150 mm was placed in 400 mm layers, compacted by the main 10-t vibrating rollers. The upstream face of this zone was smoothed and coated with a bitumen emulsion covered with sprayed sand to prevent erosion and facilitate compaction. This was achieved by six passes with the smooth 10-t roller pulled by cable up and down the slope, vibration only being used while moving upwards.

Great care was exercised in constructing the plinth to ensure a satisfactory water-tight connection between upstream membrane and valley sides. The plinth section is shown in Figure 18.4.

The reinforced concrete membrane was made 800 mm thick at the base, tapering to 300 mm at the top of the dam. The main part of it was cast as slabs 16 m wide, placed in slipforms. Before slipforming could start, the bottom piece of each strip, at its junction with the plinth, had to be formed in fixed shutters to produce a square end for the slipforms.



1C Basalt rockfill. 1.6 m layers compacted four passes 10 t vibrating roller water 0.25 fill volume
1B Basalt rockfill. 0.8 m layers compacted four passes 10 t vibrating roller water 0.25 fill volume
1E Basalt rockfill. selected pieces placed
1A Basalt rockfill. dumped
1D Basalt and breccia 0.8 m layers compacted four passes 10 t vibrating roller water 0.25 fill volume
11B Well-graded crushed basalt maximum size 150 mm 400 mm layers
111D Impervious earthfill 300 mm layers

Figure 18.2 Foz do Areia: cross-section

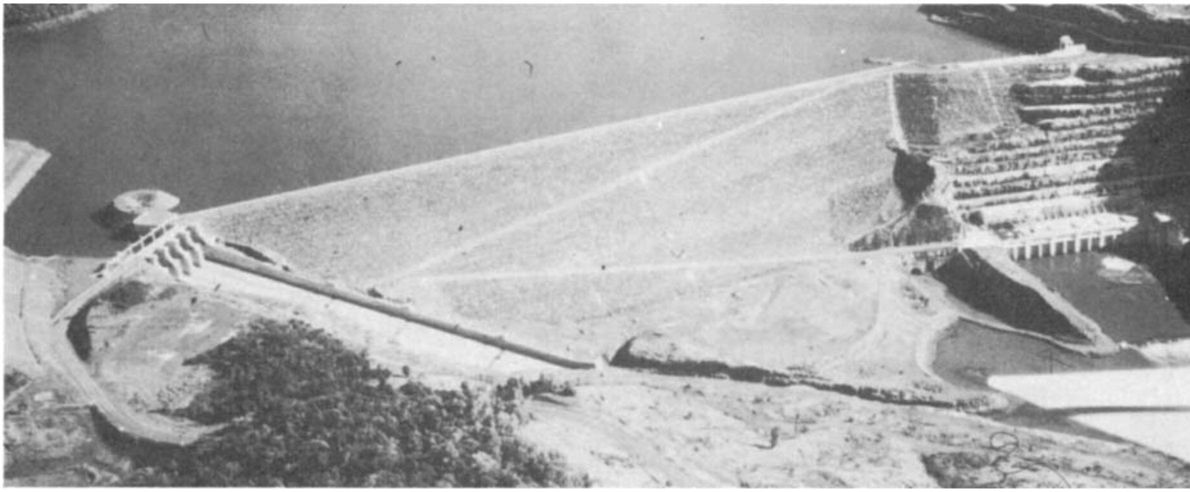


Figure 18.3 Foz do Areia

Table 18.1 Foz do Areia: geomechanical properties of bedrock

		Dense basalt	*Basaltic breccia*
Specific gravity		2.80	2.30
Porosity (%)		1.30	11.80
Compressive strength (MN/m²)	Dry	233	37
	Saturated	190	25
Modulus of elasticity (MN/m²)	Dry	66 690	25 500
	Saturated	64 725	23 550
Soundness test			
Sodium sulphate (% of loss)	Coarse aggregate	2	50
	Fine aggregate	5	35
Los Angeles abrasion test (Type E grading, 1000 rev.) (%)		11	20

Construction of the dam began early in 1977 and the dam had reached full height in 1979. Impounding began in March 1980. Details of the design and performance of this dam have been given by Pinto, Materon and Marques.[4]

A list of six other concrete-faced, compacted-rockfill dams and four proposed dams is given in Table 18.2.

18.3.3 Rockfill dam with upstream asphaltic membrane

The superior flexibility of asphalt makes it appear attractive as material for an upstream membrane. Its use avoids the construction of contraction joints and the reinforcement of a concrete membrane, but considerable care has still to be exercised at the plinth, where excessive relative movements can tear an asphaltic membrane and permit leakage. Asphaltic membranes have been used on dams up to 83 m high. Slopes vary from about 1 : 1.5 to 1 : 2 (Figures 18.5 and 18.6).

Winscar dam (54 m) was constructed on the River Don in South Yorkshire between 1972 and 1975. The site is in the Millstone Grit series which includes alternating bands of sandstone and shale. The shale was moderately strong *in situ*, but deteriorated rapidly on exposure and had to be immediately covered with a fine granular fill where it was encountered at formation level. The sandstone, particularly those seams known as the Huddersfield White rock, was most stable and was used for the rockfill.

The pronounced fissure structure of this rock tended to control the shape and size of the pieces produced in the quarry by blasting. Fragmentation produced plenty of the finer sizes and the resulting rockfill was relatively well graded. It was compacted in 1.7 m layers by four passes of a 13.5 t vibrating smooth roller. Water was added to the rockfill when necessary to bring the water content up to 6%.

A selected finer fraction of the fill was placed under the 1 : 1.7 upstream slope and rolled with the 13.5-t roller, hauled by cable from the crest, but without use of vibration. The surface was sprayed with a tackcoat of bitumen before being covered with a levelling layer of porous asphalt placed by a paving machine. It was followed by two layers of dense asphalt, placed with staggered joints and compacted by small, smooth vibrating rollers. It was covered with a sealcoat of bitumen and treated with a white finish above low-water level as a protection from sunshine. Figure 18.6 shows conditions while the membrane was being placed.

18.3.4 Rockfill dam with central asphaltic core

In the absence of any suitably impervious fill that could be used for a core, asphalt has been used to construct a central waterproof element.

The practice may be said to have started with the construction of the 45 m high Vale de Gaio dam in Portugal in 1948. A tightly packed rubble stone wall was formed to a slope of 1 (verti-

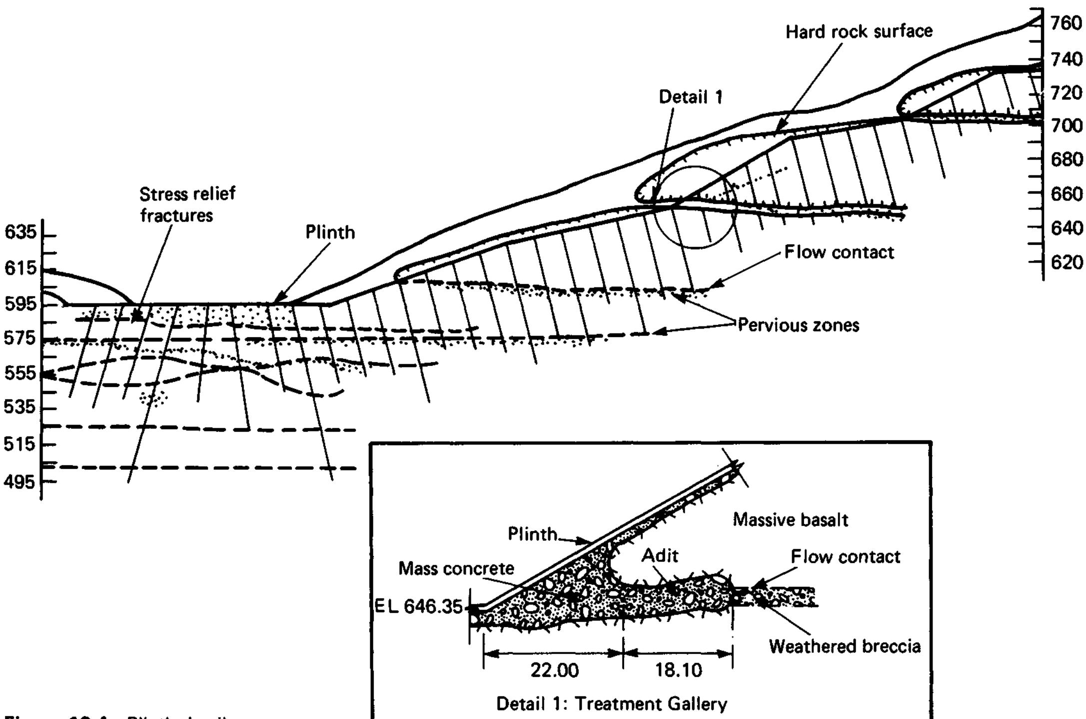


Figure 18.4 Plinth detail

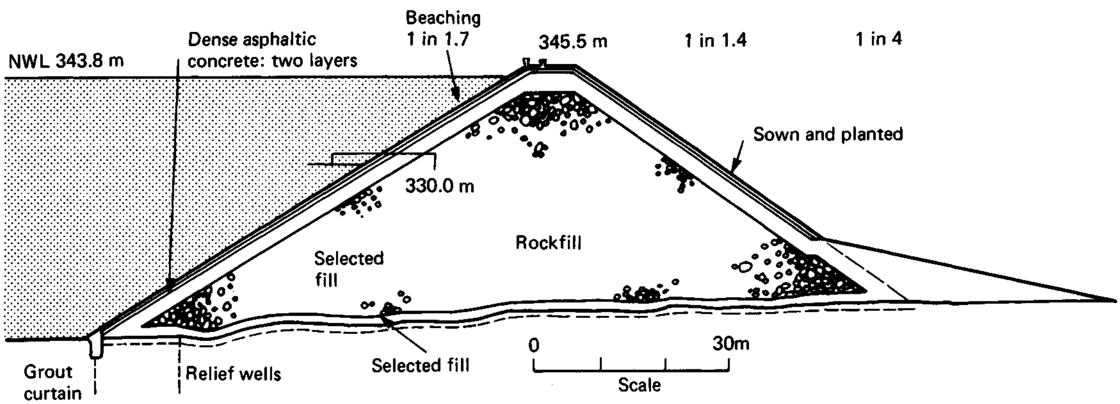


Figure 18.5 Winscar dam: cross-section

Table 18.2 Concrete-faced compacted-rockfill dams

Name	*Country*	*Height* (m)	*Year completed*
Foz do Areia	Brazil	160	1980
New Exchequer	USA	150	1966
Salvajina	Colombia	148	1984
Alto Anchicaya	Colombia	140	1974
Khao Laem	Thailand	130	1984
Shiroro	Nigeria	125	1984
Miel I	Colombia	185	Proposed
Segredo	Brazil	145	Proposed
Xingo	Brazil	140	Proposed
Ita	Brazil	125	Proposed

Figure 18.6 Construction of asphaltic membrane: Winscar dam

cally):0.8 (horizontally) against the downstream fill and smoothed with a lean concrete that acted as a drainage layer. Asphalt with a maximum particle size of 9 mm was then placed under timber shuttering to a thickness of 200 mm, tapering to 100 mm near the top of the dam. As the upstream shoulder was raised, so the shuttering was moved up to form the sloping core.

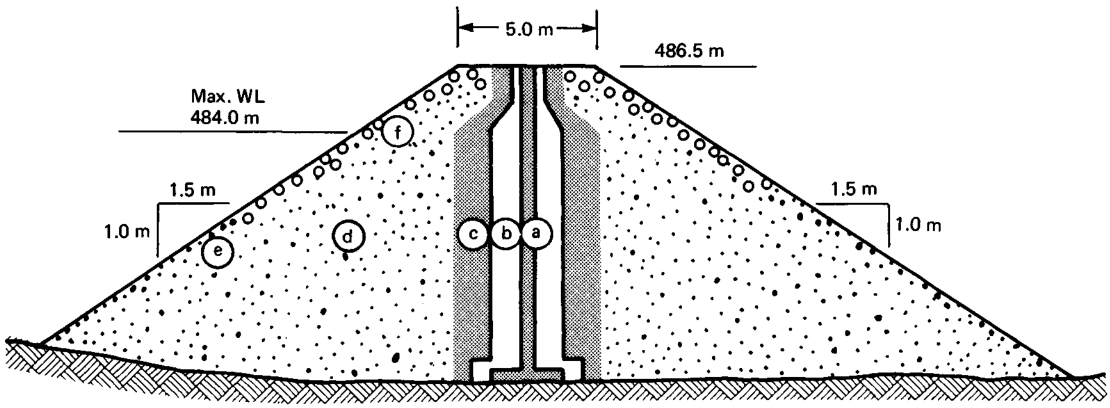
According to Steffen,[5] this method of construction was not popular. In 1954, the 58 m high Henne dam in Germany was built with a 1 m wide sloping core formed between shutters. A soft bituminous concrete overfilled with bitumen and filler was stiffened by adding clean, dry stones that were vibrated into the mix. The shutters were moved up at each lift. This technique was used to form cores for several dams in Germany, Austria and France until the end of the 1960s.

A similar, but perhaps simpler, approach was used in Norway in 1969 for building a vertical asphaltic core without the use of special plant. The idea was to construct the core of aggregate and then fill the voids with bitumen. To economize on bitumen the voids should be small, but to obtain deep penetration they should be large and, preferably the aggregate should be warm. Laboratory tests showed that a rounded aggregate graded from 19 to 76 mm, not colder than 3° C, was penetrated to a depth of 20 to 30 cm by bitumen poured at 160° C.

A core was built by this method in the 12 m high approach dam at the south end of the spillway structure of the 67 m high Grasjø dam near Trondheim. Timber shutters were used to produce a core width of 0.5 m and the aggregate was compacted with adjoining filter material. A 180- to 200-penetration bitumen at 170° C was poured into the voids in the core aggregate. The core, as shown in Figure 18.7a and b, was extended to the very top of the dam to ensure that the head of bitumen was higher than the head of water in the reservoir to prevent it from being displaced by the water pressure (hydraulic fracture) and so that the bitumen could be topped-up if required. Electrical bitumen pressure gauges were installed in the core during construction. Measured pressures were less than the hydrostatic bitumen pressure, as shown in Figure 18.7c. As the reservoir rose to top water-level, the bitumen pressure increased to values shown in Figure 18.7d and remained above water pressure. In a description of this work, Kjaernsli and Sande[6] say that the performance of the core during the first 2 yr of operation was satisfactory with negligible leakage through the dam.

Machines were developed in Germany in the early 1960s to place narrow cores of dense bitumen–concrete. Since 1962, at least 23 dams have been built with machine-placed bitumen–concrete central cores. Those higher than 50 m are listed in height order in Table 18.3.

The first, completed in 1962, was the Dhünn valley rockfill dam (35 m high), described by Breth.[27] It was founded on river gravel over a bedrock of greywacke sandstone and argillaceous slate: the shoulder fill was crushed slate placed in 0.6 m layers and compacted by a 7 t sliding vibrator. Concern was expressed that the asphaltic core should be compatible with the slate rockfill. A system of measurement was therefore devised (Figure 18.8) consisting of a vertical inspection shaft lined with concrete rings each separated by a few centimetres to allow for settlement, built with the dam and situated 6 m downstream of the core at about the major section. Six horizontal cross-pipes, spaced 5.5 m vertically, facilitated observation of the downstream face of the core.



a, Asphaltic core; b, filters; c, transition zone; d, rockfill placed in 1.5 m layers and sluiced; e, riprap; f, larger-size riprap for wave protection

(a)

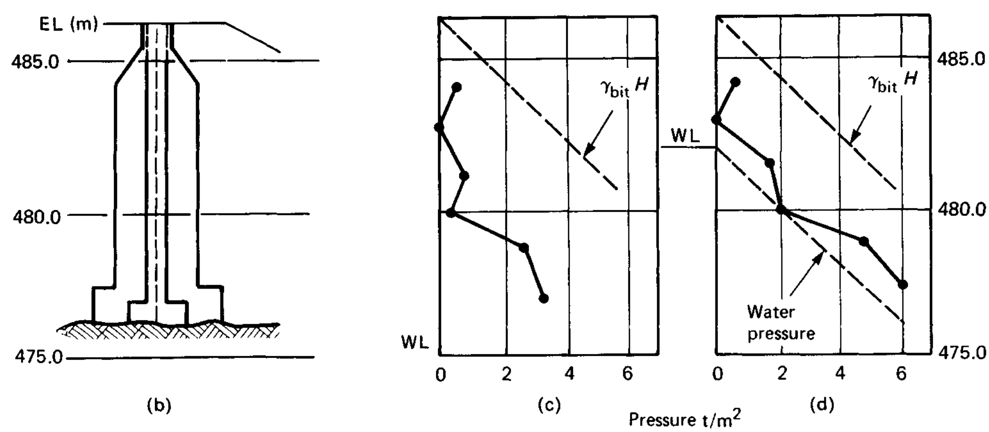


Figure 18.7 (a) Dam section; (b) Asphaltic core and filter; (c) and (d) bitumen pressures in the core wall

Table 18.3 Dams (> 50 m) with machine-placed bitumen–concrete central cores

Dam	*Height* (m)	*Core thickness* (cm) (max.) (min.)	*Completion date*	*Country*
High Island – east	105.0	120–80	1977	Hong Kong
High Island – west	96.0	120–80	1977	Hong Kong
Finstertal	92.0	70–50	1981	Austria
Kleine Kinzig	67.5	65–50	1981	West Germany
Dhünn main dam	62.5	60	1981	West Germany
Megget	58.0	90–60	1983	UK
Wiehl main dam	54.0	60–50	1971	West Germany

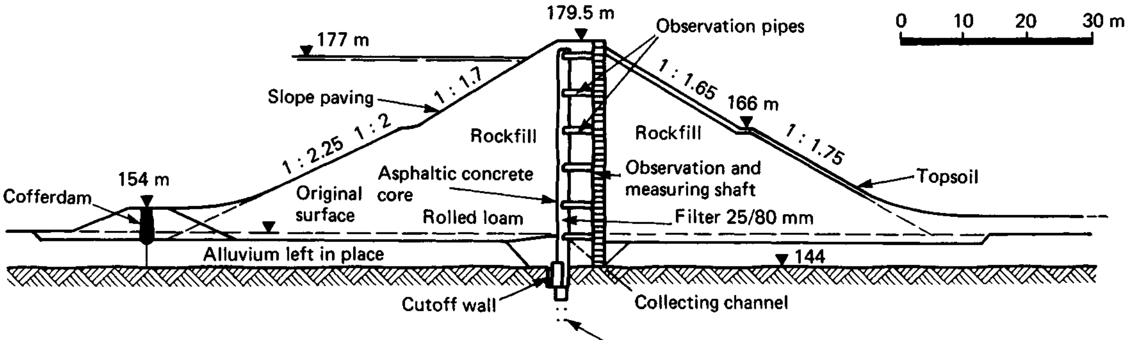


Figure 18.8 Arrangement for core measurements in the Dhünn dam

The very bottom of the core was made 1 m wide where it was in contact with a concrete groutcap, but the main core was 700 mm wide over the lower part, 600 mm in the middle section and 500 mm at the top. The results of measurements published by Lohr and Feiner[7] are given in Table 18.4. As the reservoir was filling for the first time, the upper part of the core moved upstream presumably because of the effect of wetting the upstream rockfill, but subsequently the horizontal movements have been in a downstream direction. Some part of this downstream movement could be caused by spread of the core under vertical loading. Core settlement at each measuring point tended to exceed settlements of the fill slightly, indicating that the core settlements were caused by self-weight rather than downdrag by the fill.

Example 18.1

In the UK, two dams have been built with this type of core; (1) the Sulby rockfill dam (60 m) on the Isle of Man, completed in 1982; and (2) the gravel-fill Megget dam (56 m) in Scotland, completed in 1983.

The Sulby dam. (Figure 18.9) was originally intended to be built in two stages. The first was to be a 35 m high structure with a central core wall of machine-placed bituminous concrete. The second stage, which was expected to be built at some future date to raise the height to 60 m, was to consist of rockfill placed downstream of the first dam and fitted with an upstream membrane of bituminous concrete (b) connected to the top of the central core wall (a).

While the 35 m high dam was under construction, it was realized that it made economic sense to build both stages at the same time, and so the contractor continued with the 60 m high dam (68 m above lowest foundation) using an upstream membrane for the upper part.

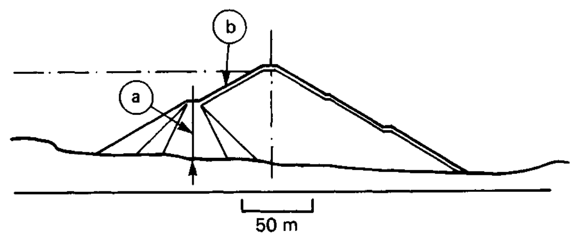


Figure 18.9 Sulby dam: cross-section.
a, Central core of bituminous concrete; b, membrane of bituminous concrete

The vertical core was placed by a Teerbau machine, which used steel sideplate shutters to form the core to the required width of 750 mm with vertical sides. Hot asphaltic mix was dropped into the hopper and fed down to the shutters as the machine advanced along the dam, the machine's tracks bearing on the transition material, adding a lift of 200 to 250 mm to the core. A second part of the machine, linked to the first, placed transition material on either side over a width of about 1.5 m. After it had passed, the machine left a level compacted surface of transition material with the asphaltic core in the middle. Dam fill was brought up on either side to support the transition material.

Example 18.2

The Megget dam. The vertical core of this dam was formed with a Strabag machine which also travelled with its tracks bearing on a 1.5 m width of transition material on either side of the core. The machine (Figure 18.10) had a long steel nose which projected over the core and contained preheating equipment. Hot asphaltic mix passed from a hopper through an adjustable

Table 18.4 Measured movemements of the asphaltic core of Dhünn main dam. (After Lohr and Feinner (1973) 'Asphaltic concrete cores: experiences and developments'. *Transactions, 11th international congress on large dams,* Madrid)

Position	*Base*	*1*	*2*	*3*	*4*	*5*	*6*	*Crest*
Height of measuring position above base (m)		3.4	8.9	14.4	19.9	25.4	30.9	35
Settlement during construction to 1962 (mm)		35.0	60.0	130.00	170.0	100.0	20.0	
Movements during operation to 1970 – vertical (mm)		50.0	78.0	200.0	290.0	253.0	202.0	
Horizontal (mm) maximum upstream on impounding		0.0	0.0	0.0	8.0	14.0	20.0	
Total downstream to 1970		33	39	81	71	63	52	

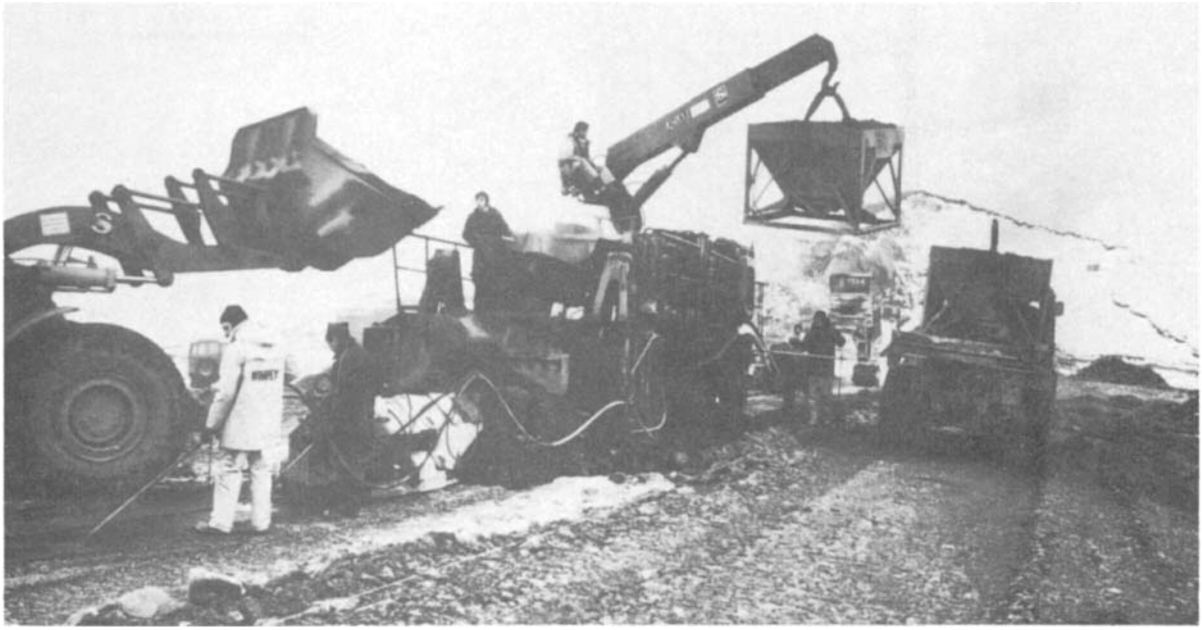


Figure 18.10 Strabag machine placing core at Megget dam

aperture to give the desired width of core and was immediately supported by transition material that had been dumped over the nose. As the machine advanced, guide plates spread the fine granular transition material which was compacted, together with the core leaving, as with the Teerbau machine, a firm level surface to the 200 to 250 mm lift of core plus transition.

This method of construction gave an inverted 'fir tree'-shaped edge to the core. Although the aperture gave the design width to the bottom of the lift, compaction caused the upper part to spread slightly and press into the transition fill. Thus, the design width of 900 mm at the base, reducing in steps to 600 mm in the upper part of the dam, was never decreased: it was slightly exceeded at the top of each lift.

The surface of each lift was kept clean by a strip of tarpaulin laid over it. This was very effective and even allowed core placement to continue immediately after snowfall. The machine was guided by a string pegged out at a specified distance upstream of the core. The Megget core was curved in plan and the pegs were positioned with the aid of a theodolite-mounted electronic distance-measuring device stationed over a reference point above crest level on the left abutment.

Shoulder fill was a well-graded gravel that was placed in 400 mm layers and compacted with four passes of a 5.5 t vibrating roller. This produced a very high density and a very stiff material. Numerous instruments installed in the dam during construction included horizontal plate gauges taken through the downstream fill to touch the core and thereby measure any movements that occurred. Figure 18.11 is a section of the dam showing the positions of these gauges, and the movements observed during first filling of the reservoir are given in Figure 18.12. A comparison between observed and predicted deformations of this dam have been given by Penman and Charles,[8] who conclude that the asphaltic concrete core acted as a thin diaphragm. It had little effect on construction deformations and during reservoir impounding simply transmitted the increased lateral thrust caused by the impounded water on to the fill of the downstream shoulder. During dam construction, there were virtually no downstream movements of the downstream face of the core, indicating that, unlike a clay core, it did not exert a large lateral thrust on the gravel fill because of its own weight.

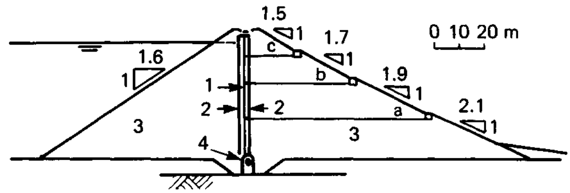


Figure 18.11 Major section of Megget dam, showing the positions of horizontal plate gauges a, b and c where (1) is the asphaltic concrete core, (2) the transition zones, (3) gravel fill, and (4) the control gallery

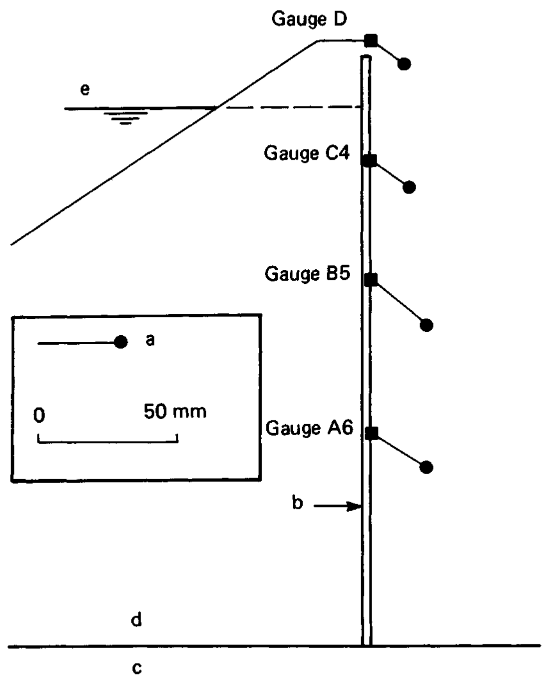


Figure 18.12 Downstream movements of the core on impounding. a, Observed movements; b, asphaltic concrete core; c, foundation; d, gravel fill; e, reservoir water level

18.3.5 Rockfill dam with central clay core

At sites where there is a source of suitable clay as well as rock, this type of dam may prove more economic than a rockfill dam with an upstream membrane, particularly if delivery of cement or asphalt to the site would be expensive. This design avoids the detailed handwork and special machinery required for slipform work or placing and compacting asphalt. It also obviates the dangers of damaging, concentrated movements near the plinth of upstream membranes, and the very high hydraulic gradient under the plinth that could cause internal erosion in some bedrock formations.

The width of the rolled clay core is usually between $0.5H+C$ to $0.33H+C$ where H represents height and C is the minimum width at dam crest. This provides a theoretical hydraulic gradient of 2 or 3 along the contact between the core and the foundation or abutment on which it rests.

In order to ensure good contact and an absence of cracks or fissures which would allow a passage for water under the clay, the cleaned formation is often coated with a layer of concrete over the area of contact with the clay core. The foundation is often sealed by grout injected through boreholes to form a grout curtain under the contact area to act as a below-ground cutoff.

Example 18.3

The Llyn Brianne dam (91 m) is an example of this type of dam (Figures 18.13 and 18.14). The dam site, on the River Towy in central Wales, is in a slatey, argillaceous rock which fragmented into plate-shaped pieces when won by blasting in the quarry. A trial embankment showed that placing and compaction prevented any preferred orientation, broke up some of the pieces and produced a dense fill.

The stripped bedrock was blanket-grouted to 10 to 15 m depth over the core contact area and coated with a pneumatically applied mortar skin 50 mm thick. In addition, a grout curtain was formed to a depth of 45 to 75 m under the centreline.

The rockfill to form the shoulders was spread in 0.5 m layers and compacted on every second layer, i.e. on a 1 m layer, by four passes of either an 8.6 or 13.5 t smooth vibrating roller.

Riprap on the upstream slope consisted of oversize pieces of the rockfill 1 to 1.5 m size, bulldozed out from the general fill.

18.3.6 Earthfill dam – homogeneous section

Dams built almost entirely from one type of fill, without

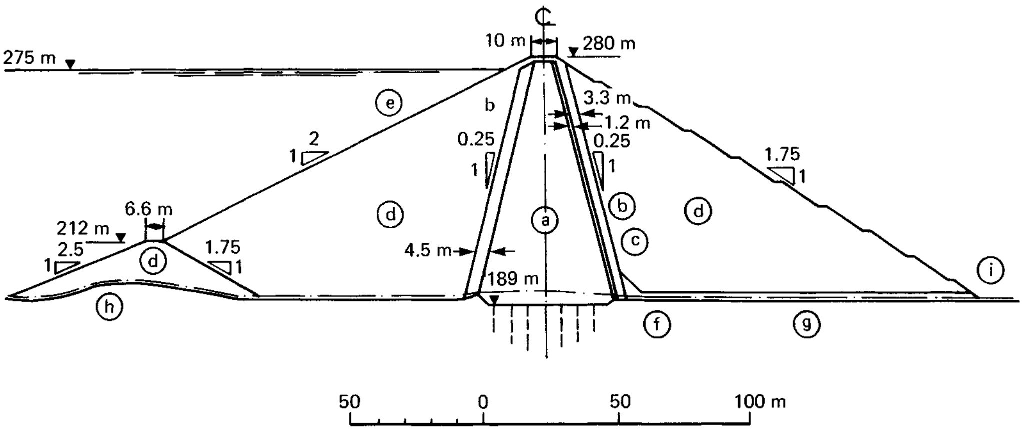


Figure 18.13 Llyn Brianne: cross-section.
(a) Clay core; (b) transition; (c) filter; (d) rockfill; (e) riprap; (f) rockfill drain in river channel; (g) excavated level bedrock; (h) cofferdam; (i) original ground level

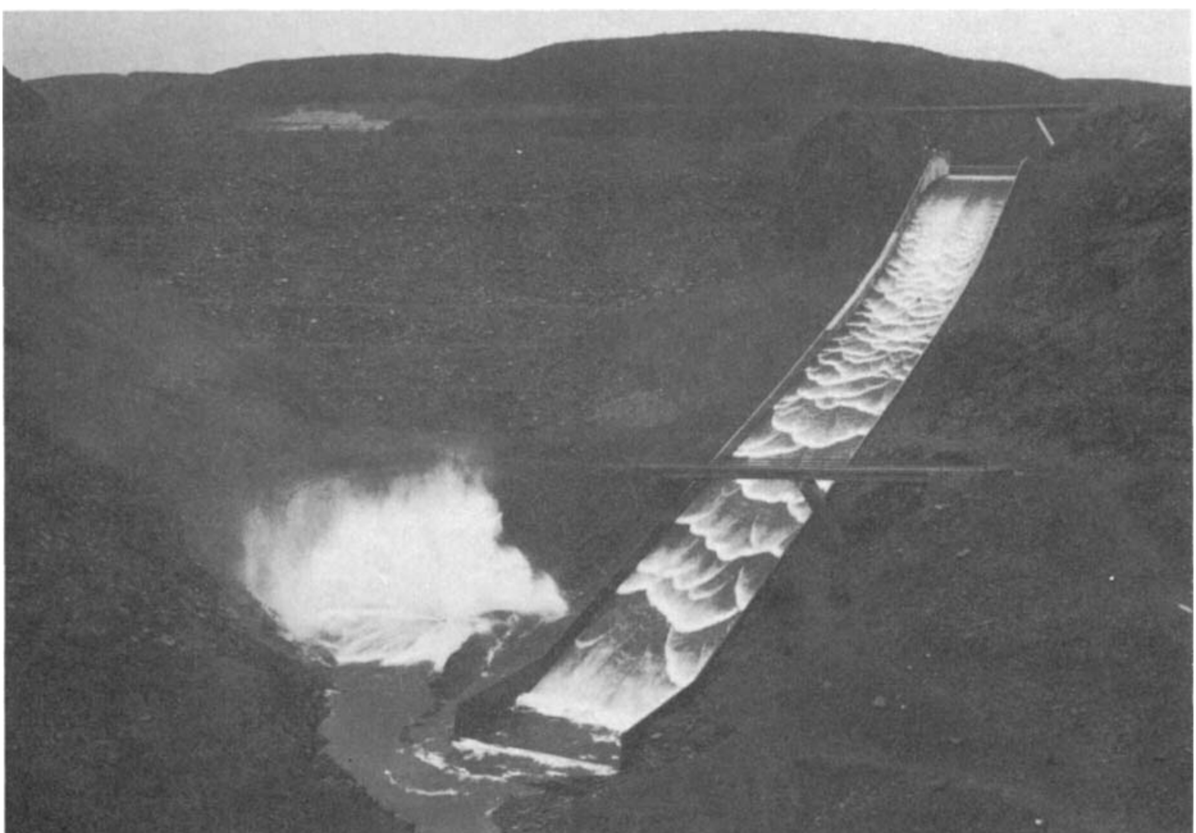


Figure 18.14 Llyn Brianne dam

provision for neither a less pervious core nor more stable shoulders, became popular in the Americas as the size and power of earthmoving machinery developed in the 1920s and 1930s.

A danger quickly recognized was that, if under full reservoir, the phreatic line could cut the downstream slope and local slips develop, leading to backsapping and eventual destruction of the dam.

The US Bureau of Reclamation installed standpipe piezometers in the mid 1930s to check on the positions of phreatic surfaces and in this way revealed the presence of construction pore pressures. Placement at water contents below Proctor optimum substantially reduced construction pore pressure but produced a relatively stiff fill. Differential settlements could cause cracking: examples of cracked and failed dams have been given by Sherard.[9]

A solution was provided by Terzaghi in his design of the Vigario dam in 1947 by using a central vertical core of filter material to drain leakage and prevent the phreatic surface reaching the downstream slope. Examples of this design are shown in Figure 18.15.

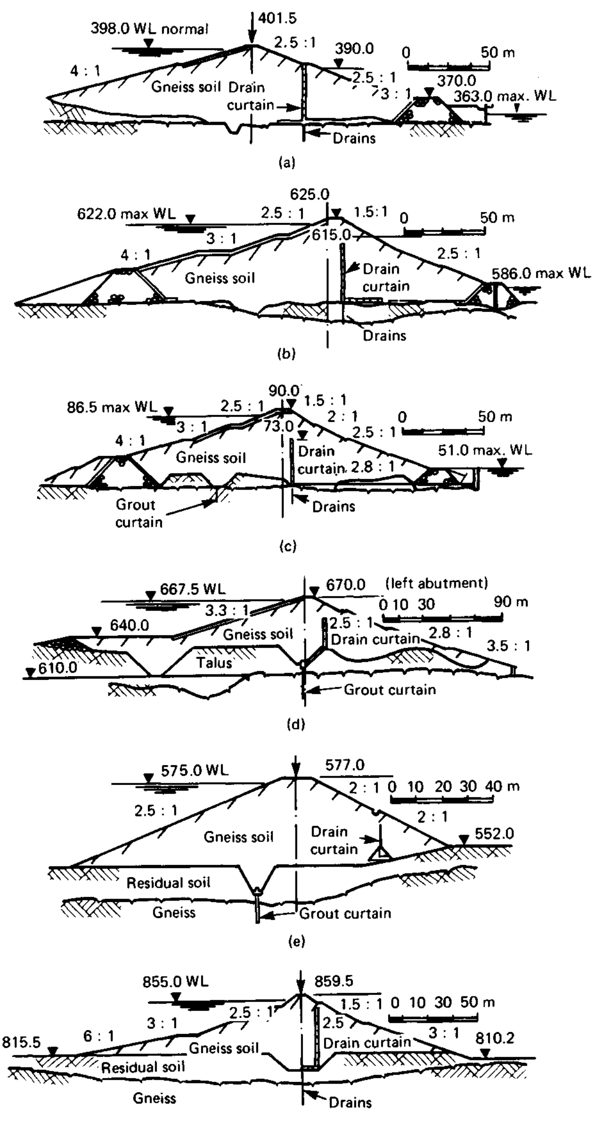


Figure 18.15 Homogeneous earth dams with Brazilian section. (a) Vigario Dike; (b) Santa Branca; (c) Ponte Coberta; (d) Euclides da Cunha; (e) Limoeira; (f) Graminha

18.3.7 Earthfill dam with central clay core

Central cores of puddled clay were used in the traditional British dam in the nineteenth century. It had an upstream slope of 1 in 3 and a downstream slope of 1 in 2.5. The puddled clay core was usually taken down in trench to form a below-ground water-stop.

The fissures, bedding planes, silt layers, etc. found in deposits of clay can give it a relatively high *in-situ* permeability. The action of puddling destroys this fabric and, by addition of water where necessary, the strength was usually reduced sufficiently ($c_u = 10$ to 15 kN/m^2) to enable the puddled clay to be compacted down to an air void of about 5% by the heeling of the puddling gang.

Increasing cost of labour and unsuitability of compacting machinery of the time for working with puddled clay caused the change to rolled clay cores. The era of British puddled clay cores came to an end during the 1950s, although there were still dams occasionally built with these cores until 1970.

The use of earth rather than rockfill for the shoulders poses the problem of construction pore pressures. In areas of high rainfall and/or when the borrow material is wet, compression of the fill under self-weight as height is increased can produce undesirable pore pressure which, in the extreme, may endanger stability by preventing the required increase of effective stresses within the fill.

This problem is overcome by use of drainage layers placed in the fill during construction. These reduce the length of the drainage path which the pore water must traverse to escape. The time required for dissipation of pore pressure is proportional to the square of the length of the drainage path, so drainage layers are particularly effective in reducing pore pressures in shoulder fill during construction.

Granular material for the drainage layers is often graded so that it acts as a filter to prevent loss of fines from the fill, but this aspect is not of prime importance in the downstream shoulder, where volume of escaping pore water is unlikely to cause significant particle migration. In the upstream shoulder, however, there is a danger that fluctuations of reservoir level could produce damaging flows if the layers do not act as effective filters.

Example 18.4

The Backwater dam (43 m) was constructed during the period 1964–69 (Figure 18.16). At the site, boulder clay overlies a schistose grit and micaceous schist. The embankment was built from compacted boulder clay placed in layers with a slight outward fall to help shed rainwater. Granular drainage layers were placed at about 7 m vertical intervals.

The core was also constructed of boulder clay, carefully selected in the borrow pits to contain least stones and most clay. It was placed over a grout curtain cutoff on to a prepared surface. It was separated from the downstream shoulder by a drainage filter which connected to the drainage layers and the main underdrainage layer separating shoulder fill from foundation.

18.4 Concrete dams

18.4.1 Introduction

Use of certain volcanic ashes by the Romans to cement together sand and gravel into a reconstituted rock is often regarded as the first concrete. The aggregates used for modern concrete, such as well-graded sands and gravels, and hard rock that has been crushed, sieved and graded to a designed grading, would form excellent fill, even without the addition of cement. The additional strength imparted to it by the cement enables less volume to be used, so redressing the high cost of production.

18.4.2 Gravity dams

The simplest type of concrete dam has a gravity section, i.e. it is heavy enough not to be overturned by horizontal thrust from the reservoir water. The foundations have to be relatively strong to support the large weight and they should not be subject to

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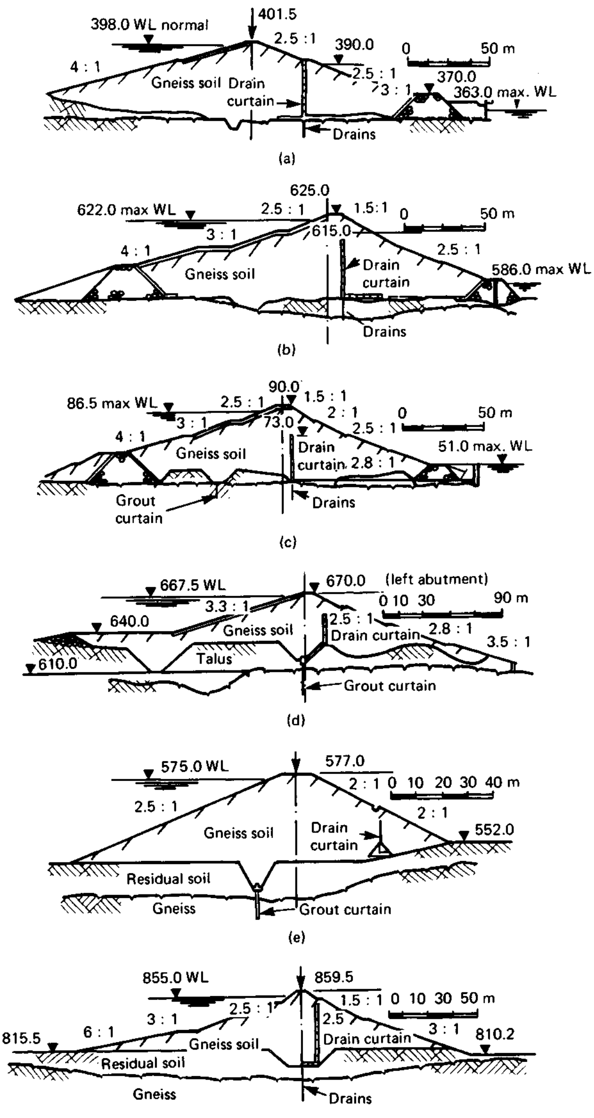

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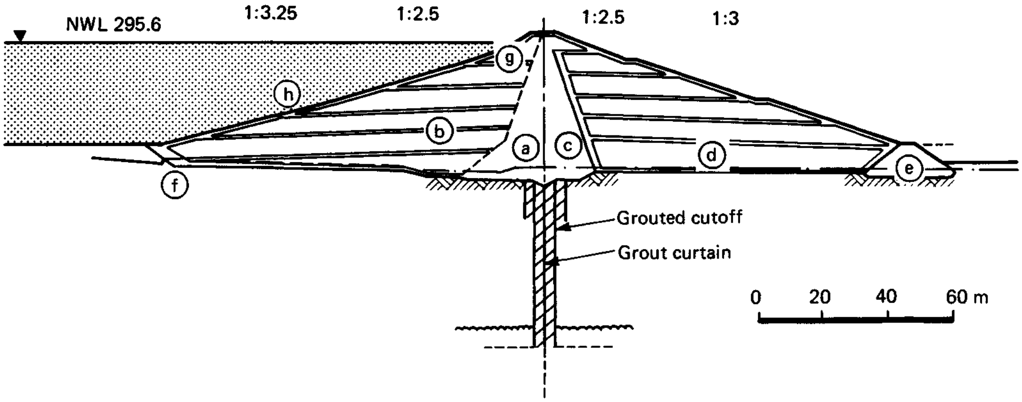


Figure 18.16 Backwater dam: section showing drainage layers. (a) Glacial till core; (b) glacial till shoulder; (c) chimney drain; (d) drainage layers; (e) rubble toe; (f) spoil; (g) concrete block; (h) riprap

long-term settlements that could be caused by consolidation of an underlying clay.

The heat liberated by hydration of the cement in concrete can produce damaging temperature rises in large masses of concrete. As with the problem of dissipating construction pore pressures in earthfill shoulders, so consideration has to be given to limiting the temperature rise which will occur in a gravity dam as it is being built. The counterparts of drainage layers are layers of cooling pipes placed in the mass concrete as it is being cast. A refrigeration plant is used to lower the temperature of brine circulated through the cooling pipes.

It is usual to construct a gravity dam as several separate sections or, sometimes, as large monoliths. When temperatures have fallen sufficiently so that little more cooling shrinkage is likely to occur, the joints between the sections are grouted or the monoliths are connected with infilling sections to form the complete dam.

Temperature rise can be reduced by slow rates of construction, use of slow-setting, coarse-ground cement and use of relatively inert cement replacements such as power station fly ash.

Drainage is essential in a gravity dam to limit uplift pressures. In effect, the upstream face forms the waterproof element, supported by the remainder of the massive dam. Reservoir water percolating along the interface between dam and foundation or along any of the horizontal lift joints could develop destabilizing uplift pressures if not safely drained away. Drainage galleries and shafts are formed in the body of the dam. A gallery is usually provided close to the foundation on the upstream side from which either additional drainage or grouting holes can be drilled into the foundation if found to be necessary during reservoir operation.

Compressive stress in the concrete forming the upstream face should always exceed reservoir water pressure to avoid tensile cracking.

Example 18.5

The Grand Dixence dam (285 m) was constructed during the period 1953–62 (Figures 18.17 and 18.18). This remained the world's highest dam for 18 yr until it was exceeded by the Russian Nurek (300 m) embankment dam.

The Grand Dixence dam was built in the Swiss Alps across the River Dix on sound bedrock. Its 695 m crest length was divided into 16 m blocks connected by two copper sealing strips to form a continuous, watertight upstream face. The blocks, particularly in the lower part of the dam where it was very wide (201 m maximum) were further divided by joints to allow for some shrinkage movements on cooling. The joints were grouted up only after the concrete temperature had fallen below 6° C.

Cooling was effected with layers of pipes placed on every 3.2 m lift. About 1000 km of 20 mm bore pipes were installed to circulate the cooling water.

The waterproof, upstream face was made with concrete containing 250 kg of cement per cubic metre so as to be watertight and frost-resistant. Concrete in other parts of the dam contained 140 to 300 kg/m^3 of cement according to calculated stresses. Air-entraining agent was used to produce 3 to 4% air void in order to improve workability. Maximum aggregate size was 120 mm.

The concrete was placed with the aid of four cableways spanning the valley over the dam. On the right bank they were attached to mobile carriages anchored to rails so that the cables could be brought over almost every part of the dam. The cableway buckets could carry 6 m^3 of concrete. After discharge from the buckets, the concrete was spread by small bulldozers. It was compacted with vibrating pokers: frames of five pokers were attached to the blades of other bulldozers so that they could be moved about and lowered into the concrete. At the time, this was considered to be the first use of earthmoving machinery on a concrete dam.

18.4.3 Rollcrete dams

The term 'rollcrete' was coined by Lowe[10] to name a new approach to concrete placement. Increasing labour costs were making the labour-intensive concrete dams less competitive than the embankment dam, even on sites suitable for a concrete dam. To help redress the balance, Lowe proposed using a relatively dry, lean concrete placed by earthmoving machinery and compacted by smooth vibrating rollers. His first use of the material was to construct the core of the cofferdam for Shihmen dam, Taiwan in 1961–62.

At the same time, a similar approach was being used in Italy. Gentile[11] designed the 175 m high Alpe Gera dam for placement by earthmoving machines. He used a blastfurnace cement to reduce heat of hydration, but also provided an adequate number of open contraction joints to allow for shrinkage. The gravity section, shown in Figure 18.19, was designed only to support the forces imposed by the reservoir; it was not intended to be watertight. A 3 mm thick steel plate was used to form the waterproof element on the upstream face of the dam. Maximum

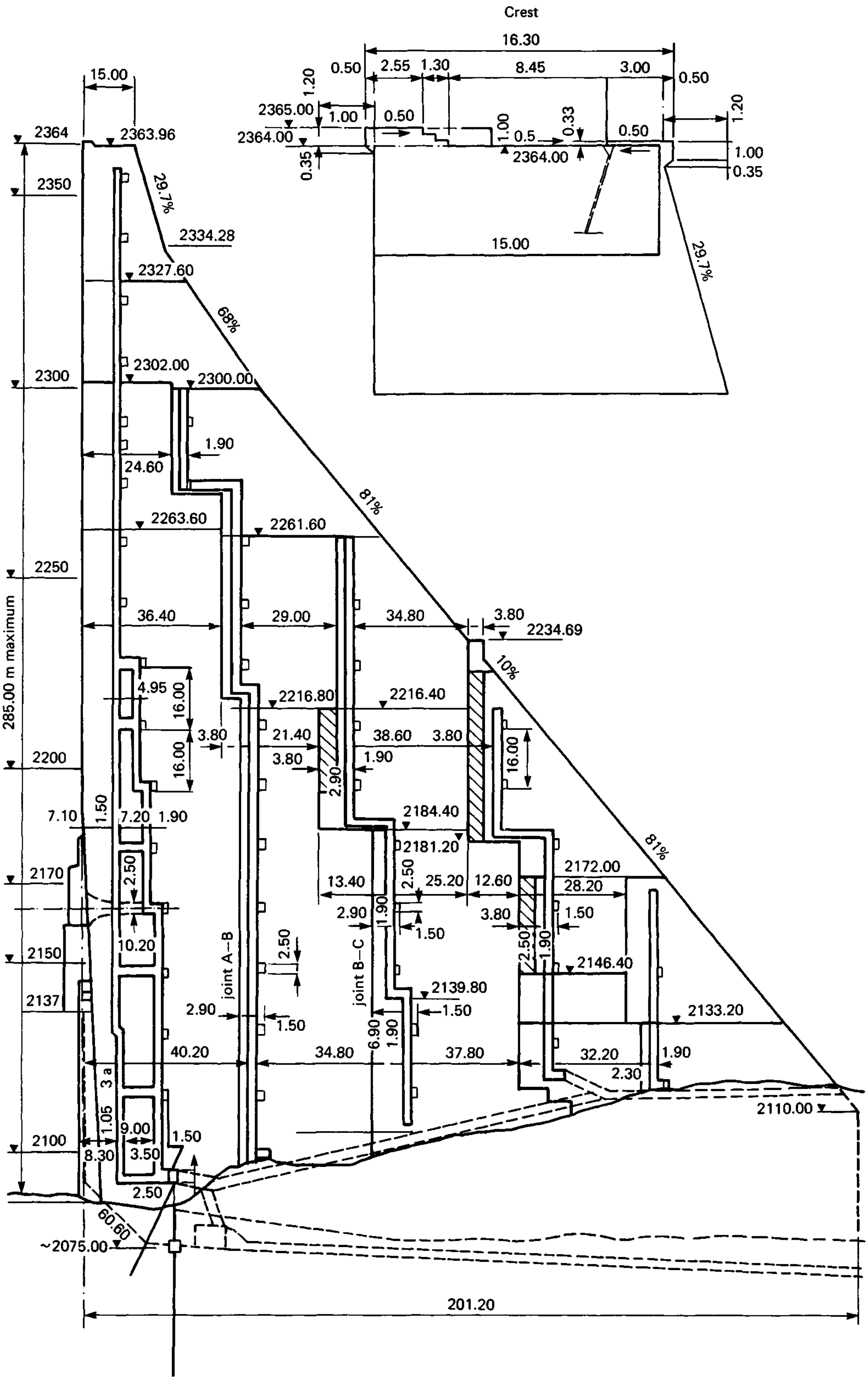


Figure 18.17 Grand Dixence dam: cross-section

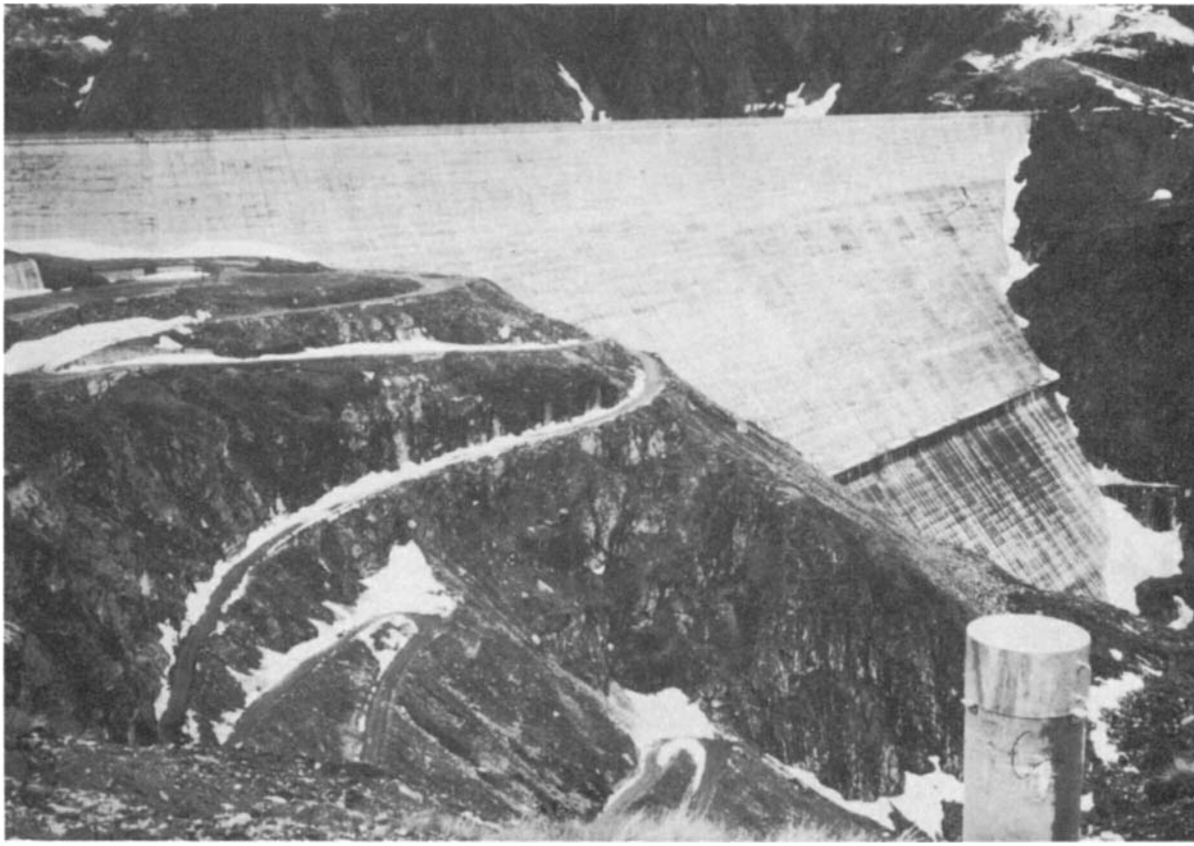


Figure 18.18 Grand Dixence dam

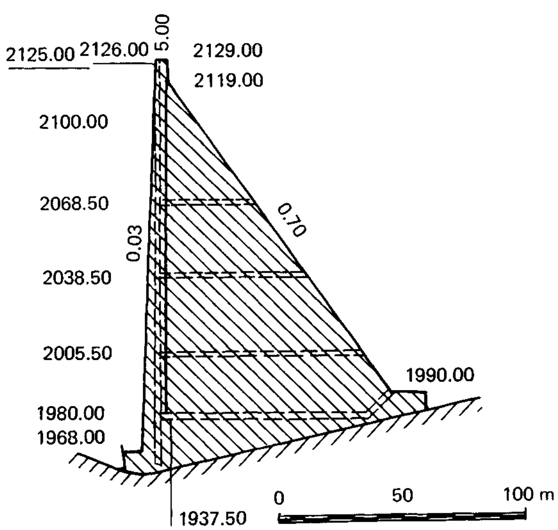


Figure 18.19 Alpe Gera dam; rollcrete with sheet steel waterproof element

compressive stresses in the concrete were calculated to be in the range 3.25 to 5.60 MN/m^2.

The concrete was made from an alluvial sand and gravel with a cement content of 115 to 300 kg/m^3 according to expected stresses in the dam. It was transported from the batching plant by funicular railway to placement level and carried across the surface of the new concrete in 6 m^3 dumpers, spread to 0.8 m thickness by angle dozers and compacted by tractor-mounted vibrating pokers. Contraction joints were cut through the freshly placed concrete by a manganese steel blade 3 m long and 1 m high carried on a wheeled chassis and pressed down with a force of 2.5 t by two hydraulic jacks while being vibrated at a frequency of 50 Hz.

No cooling was employed and maximum measured temperature in concrete with a cement content of 150 kg/m^3 reached 35° C. Concrete containing 115 kg/m^3 cement reached a maximum of 30° C. The dam was built during three-and-a-half 6-month summer periods from August 1961 to the end of September 1964.

Wallingford[12] also proposed the use of rolled concrete to reduce the cost of a gravity section. This concept was followed by research studies, described by Moffat,[13] at the University of Newcastle into the properties of dry lean concrete as a potential material for dam construction by earthmoving machinery. The dam section proposed by Moffat (Figure 18.20) used a poured-bitumen sheet as the waterproof element in the upstream facing made of precast units. Following a study of temperature rises in the Upper Tamar dam, Dunstan[14,15] outlined a dam section containing low cement content concrete placed in continuous 0.6 m layers between upstream and downstream facings. These facings were to be laid as horizontally slipformed kerbs, with a sheet of reinforced butyl rubber inside the upstream facing as the waterproof element. Dunstan's further work[16] led him to use almost an excess of fly ash to ensure that voids between larger particles were completely filled, and in order to improve workability, reduce water content to very small amounts and produce a concrete of low permeability.

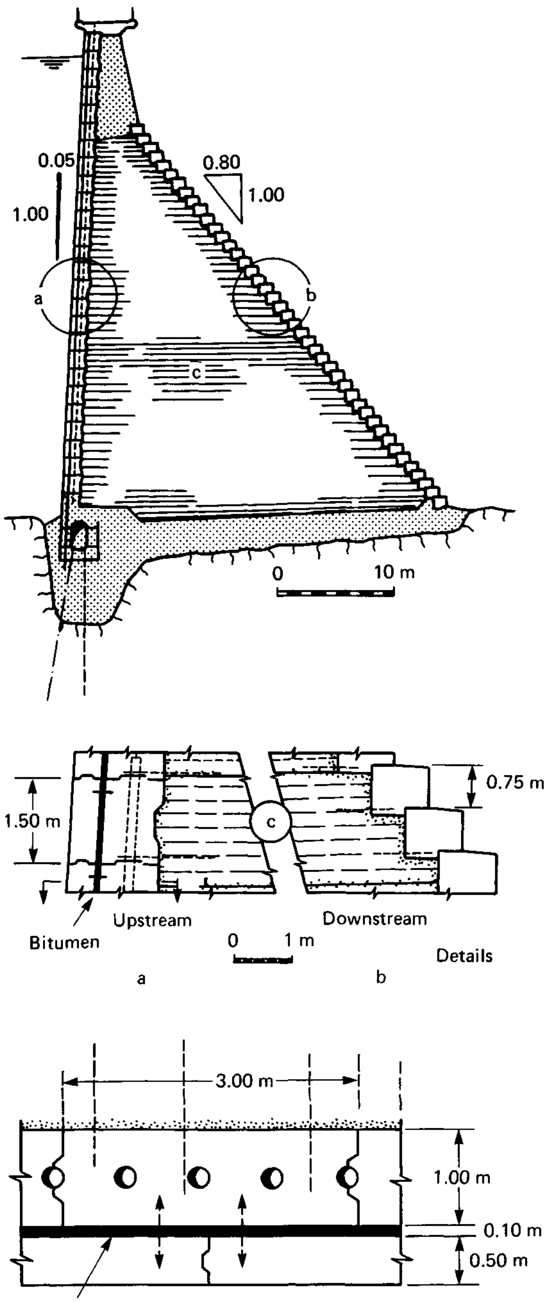


Figure 18.20 Proposed dry lean concrete dam with bituminous waterproof element. (After: Moffat (1973) 'A study of dry lean concrete applied to the construction of gravity dams'. *Transactions, 11th international congress on large dams,* Madrid)

Dunstan supervised construction of a trial bank at the site of Wimbleball dam in 1979. The bank was contained between slipformed kerbs (Figure 18.21). The cementitious content of the concrete was 0.75 fly ash and 0.25 cement. Each cubic metre of concrete contained 85 kg cement. It was compacted in 0.3 m layers by a 7 t duplex vibrating roller. The very cohesive mix did not segregate, but flowed under the vibrating action of the roller. This produced a hard surface on which the laser-guided, offset slipformer could immediately run to lay a further lift of kerb. Several different time intervals were used between lifts, and subsequent tests on cores which were axially drilled for water-pressure tests showed no leakage at the lift joints. The work was in preparation for the Milton Brook dam but financial restrictions caused postponement of construction.

In the US, roller compacted concrete has been used to build several dams (Table 18.5). One of the first was the 37 m high Willow Creek dam, completed in 1982. The construction joints proved to be fairly porous, causing the downstream face to be fairly wet, giving a clear indication of reservoir level. The joints have since been grouted to reduce leakage.

Rapid construction is one of the attractive features of roller-compacted concrete dams: some times of construction are given in Table 18.5. As an example of the concrete mix, that used for the Upper Stillwater dam contained only 77 kg cement per cubic metre, with 170 kg fly ash and 107 kg water.

In Japan, roller compaction was used to build the 89 m high Shimajigawa dam completed in 1980. Previously a base pad over the fractured rock foundation for the concrete gravity Okawa dam (75 m) used the roller-compacted dam method in 1978. The mat had an average thickness of 25 m and its volume of 300 000 m^3 was placed in 9 months.

The use of roller compaction for these dams has been followed by its use for the construction of the 100 m high Tamagawa dam which has the fairly large (for a gravity dam) volume of $1.14 \times 10^6\,m^3$. The maximum size of aggregate, previously limited to 80 mm, has been increased to 150 mm. A section of the dam is shown in Figure 18.22 and the mixes used in the various parts of the dam are given in Table 18.6. The rolled concrete was placed in 0.75 m thick layers and compacted by twelve passes of a Bomag BW-200 vibrating smooth roller weighing 7 t.

Other Japanese roller compacted dams are listed in Table 18.7.

In South Africa, rollcrete was used for the first time to construct the lower half of the concrete section of the 52 m high Braam Raubenheimer dam in eastern Transvaal. The concrete gravity section is 33 m high and contains 20 000 m^3 of rollcrete placed in 1984.

It has also been used to construct the Zaaihoek dam and the 30 m high De Mist Kraal weir on the Little Fish River in the Eastern Cape. Figure 18.23 shows a section of this weir. Rapid construction was achieved, with 34 000 m^3 of rollcrete being placed in 26 days during the winter of 1986. The contents of each cubic metre are given in Table 18.8.

The 36 m high gravity section of the Arabie dam on the Olifants River, 27 km north of Marble Hall was constructed of rollcrete, completed in 1987.

18.4.4 Buttress dams

The weight and, therefore, volume (and cost) of a gravity dam can be greatly reduced by retaining only the upstream face and supporting it by buttresses instead of the whole mass of the gravity section. To utilize some of the reservoir water pressure to resist overturning moment, the upstream face is usually sloped. The water pressure acting on it has a vertical component that replaces some of the weight of a gravity section as well as the horizontal component producing the overturning moment.

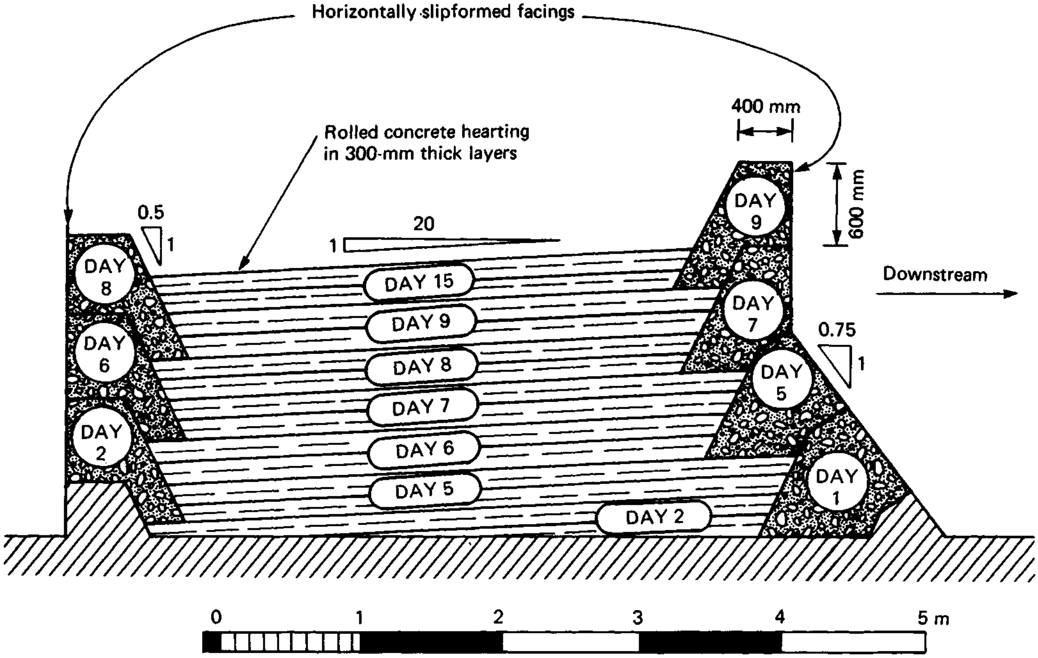


Figure 18.21 Milton Brook trial bank: roller-compacted concrete between slipformed kerbs

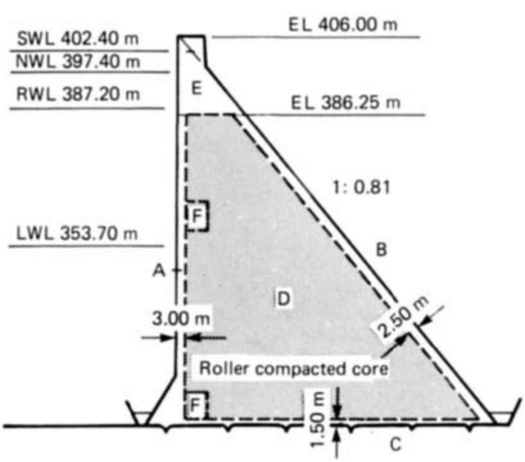


Figure 18.22 Tamagawa roller-compacted dam

A plane upstream face, spanning between buttresses, would have to resist bending moments. Some early milldams in timber had a plane deck at a relatively flat slope, e.g. 1 vertical:3 horizontal supported on wedge-shaped timber frames. The introduction of reinforced concrete at the beginning of the twentieth century enabled larger, plane-faced dams to be built. Ambursen designed many dams of this type in the US, including the record-breaking 41 m high La Prele dam in Wyoming in 1909. Schnitter[17] reports that the dam contained only 43% of the concrete needed for an equivalent gravity section. The dam is at an elevation of 1600 m and the severe climate had disintegrated more than 20% of face thickness when a new slab was built in 1977–79. The highest flat slab buttress dam (83 m) was completed in 1948 at Escaba in Argentina.

A more usual approach is to use an arch between buttresses, producing the multiarch buttress dam (Example 18.6).

Table 18.5 Roller-compacted concrete dams in the USA

Name of dam	*Height* (m)	*State*	*Construction time* (weeks)	*Date completed*
Upper Stillwater	87	Utah	18	1987
Pamo Valley	80	California	—	*c.* 1987
Elk Creek	76	Oregon	—	Designed 1983
Willow Creek	52	Oregon	21	1982
Galesville	51	Orgeon	6	1985
Monksville	46	North East	—	
Middle Fork	38	Western Colorado	7	1984

Table 18.6 Tamagawa dam – concrete mix designs

Position	*Grade*	*Aggregate size* (max.) (mm)	*Cement* (kg/m³)	*Fly ash* (kg/m³)	*Water* (kg/m³)
Upstream face	A	150	168	72	115
Downstream face	B	150	154	66	112
Foundation contact	C	150	126	54	108
Main body – rolled	D	150	91	39	95
Crest section	E	150	112	48	106
Reinforced openings	F	80	189	81	138

Table 18.7 Roller-compacted dams in Japan

Name of dam	*Height* (m)	*Volume* ($m^3 \times 10^3$)	*River*	*Construction*
Sakaigawa	115.0	626	Sakai	1988–92
Tamagawa	100.0	1140	Tama	1983–87
Shimajigawa	90.0	324	Shimaji	1977–80
Asahi Ogawa	84.0	350	Ogawa	1986–89
*Shin-Nakano	74.9	201	Kameda	1979–82
Mano	69.0	212	Mano	1985–88
Shiromizugawa	54.5	312	Shiroizu	Under construction
†Pirica	40.0	360	Shiribeshi	1982–88

*Dam raised with roller-compacted concrete
†Composite with rockfill

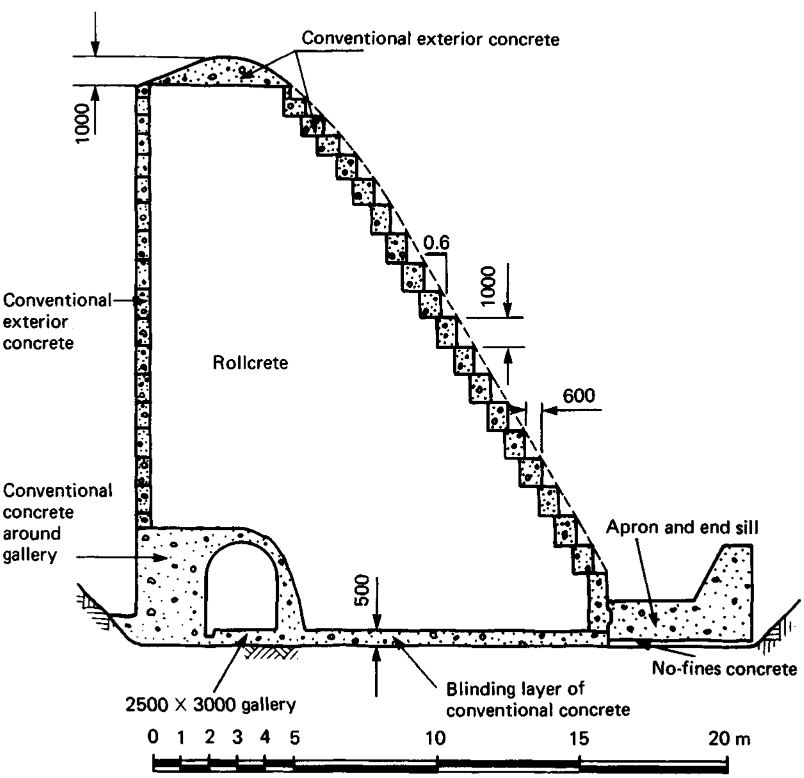


Figure 18.23 De Mist Kraal weir. Typical section through spillway

Table 18.8 Rollcrete mix for de Mist Kraal

Material	*Quantity* (kg)
Water	105
Portland cement	58
Fly ash	58
Sand	736
Aggregate (75–37.5 mm)	805
Aggregate (37.5–19 mm)	537
Aggregate (19–9.5 mm)	268
Aggregate (9.5–4.75 mm)	121
Conplast air-entraining agent	99 cm^3
Compressive strengths	(MN/m^3)
7 days	8.1
28 days	13.3
1 yr	25.0

Example 18.6

The Meicende dam northwest Spain (20 m) completed 1961 (Figures 18.24 and 18.25). Bedrock is granite and valley floor contained outcrops of sound granite but some decomposed rock and a lode of feldspar aplite. Use of the multiarch design enabled buttresses to be positioned off areas of poorest quality.

The final design provided a maximum height of 20 m with: 11 circular arches, each of 11 m internal radius and 1 m thick; 12 buttresses, 2.5 m thick at 22 m centres, with arch springing face at a slope of 1 vertical:0.577 horizontal.

In contiguous buttress dams, the upstream face may be formed by widening the upstream edges of each buttress so that they touch and are sealed by suitable waterstops, instead of using a flat slab or multiple arches.

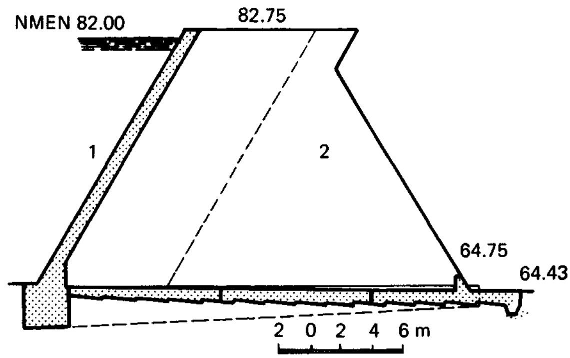


Figure 18.25 Meicende dam: cross-section, where (1) is the arch thickness, and (2) is the abutment between arches

Example 18.7

The Itaipu dam, Parana river (196 m) constructed 1975–82 (Figure 18.26, showing the double buttress of hollow gravity block in river channel, and Figure 18.27, showing the diamond-headed buttress of the main dam).

This is an outstanding example of a contiguous buttress concrete dam. It crosses the frontier between Brazil and Paraguay, storing water and providing head for the world's largest hydro-electric installation, designed for an output of 12 600 MW.

The river section hollow-gravity dam, composed of double-buttress monoliths and the diamond-headed buttresses, is founded on basaltic flows 15 to 50 m thick. There are sub-horizontal discontinuities at different levels and special treatment was required to increase shear strength and reduce compressibility. Treatment included consolidation and contact grouting, special drainage systems, as well as concrete keying at some weak points.

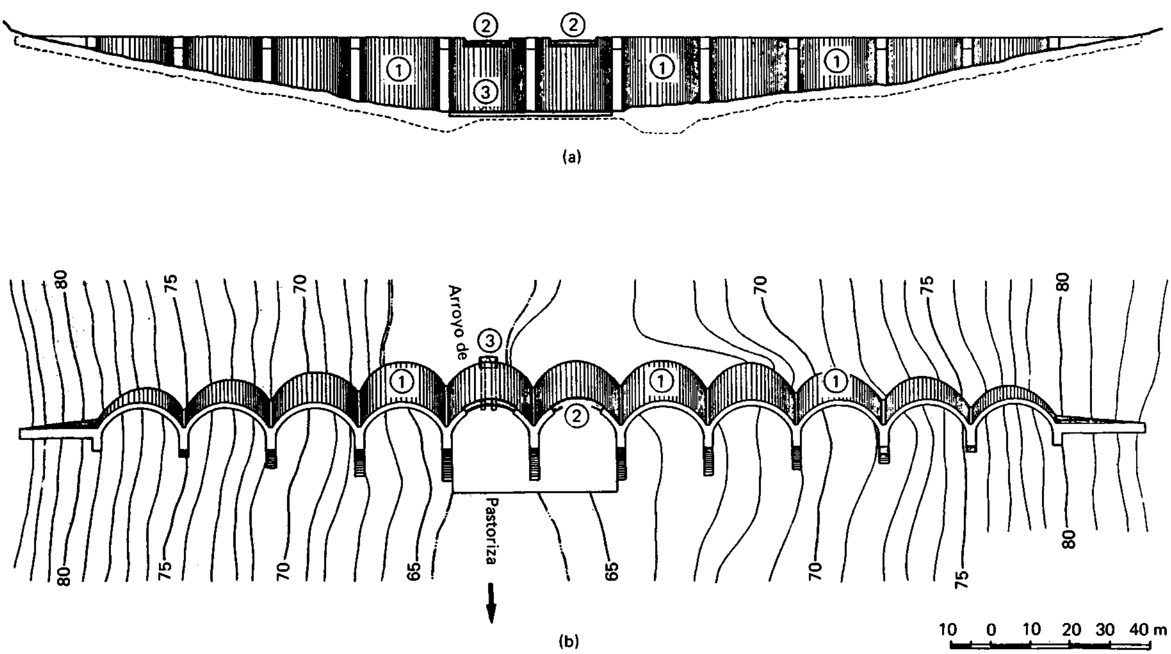


Figure 18.24 Meicende dam. (a) Elevation; (b) plan, where (1) is an arch, (2) are spillways, and (3) is the intake

A preloading test was made by flooding the space between the upstream coffer dam and the main dam. This enabled water to be raised almost 100 m against the major section of the dam. Good agreement was found between observed and predicted deformations. It lent reassurance that the structure would behave, as it did, in a satisfactory manner during the very rapid, irreversible impounding which took only 14 days. The main concrete section of the dam is flanked by embankment dams, forming a total length of 7772 m. Individual lengths are given in Table 18.9.

Table 18.9

	(m)
Hollow gravity of double buttress monoliths	612
Diamond-headed buttress	1450
Mass gravity section	532
Rockfill embankment	1984
Earthfill embankment	3194

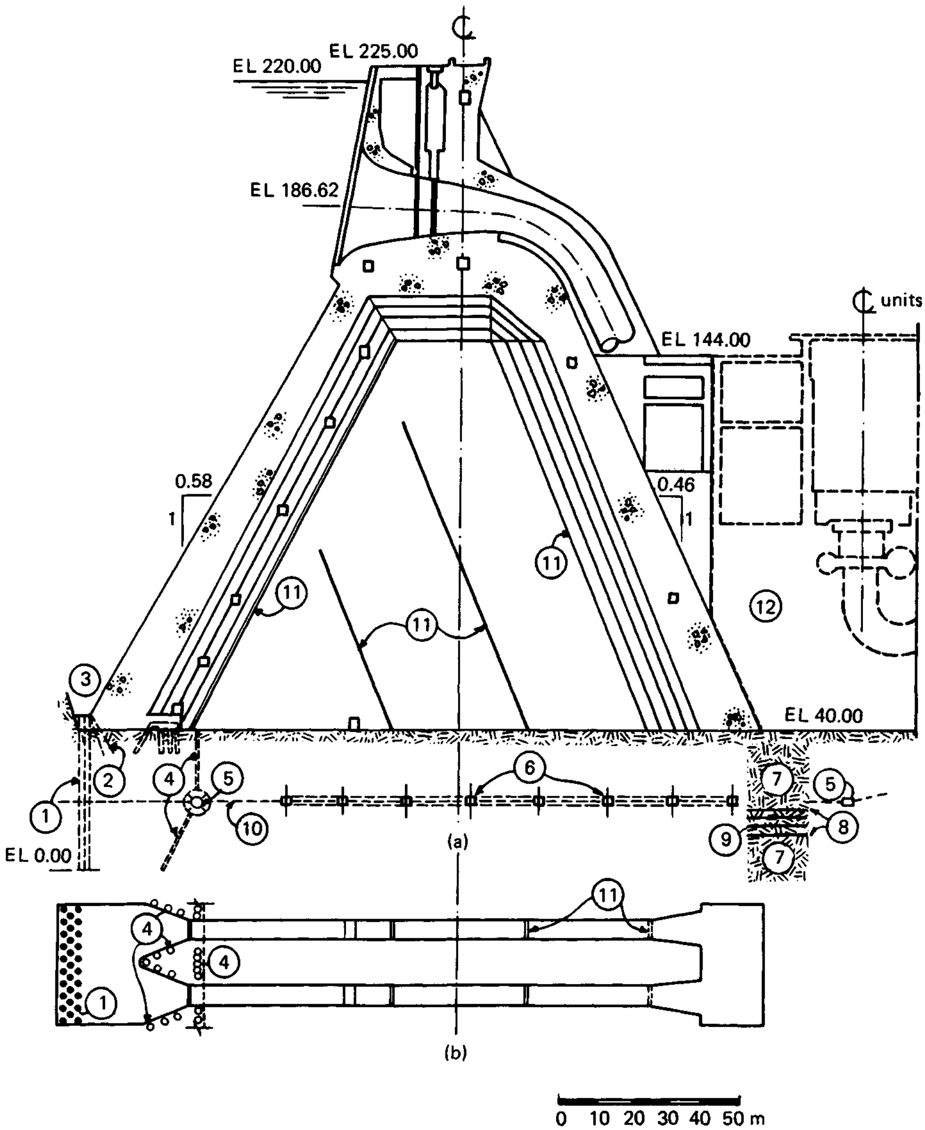


Figure 18.26 Itaipu dam: double buttress monolith of hollow gravity section in river channel. (a) Section; (b) plan, where (1) is the grout curtain, (2) contact grouting, (3) consolidation and contact grouting, (4) drainage holes, (5) drainage tunnel, (6) shear keys, (7) dense basalt, (8) breccia, (9) vesicular amygdaloidal basalt, (10) discontinuities, (11) contraction joints, and (12) is the powerhouse

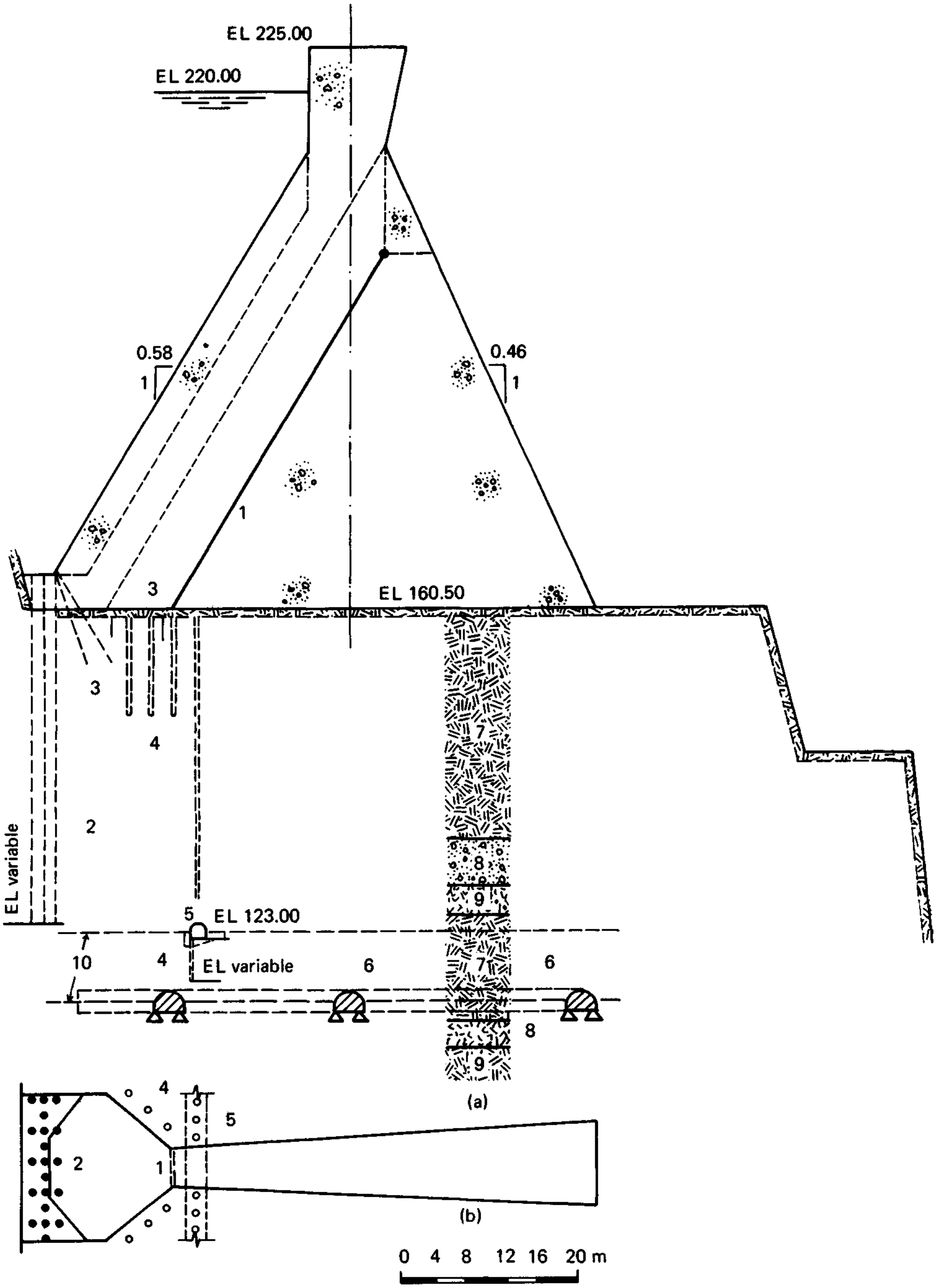
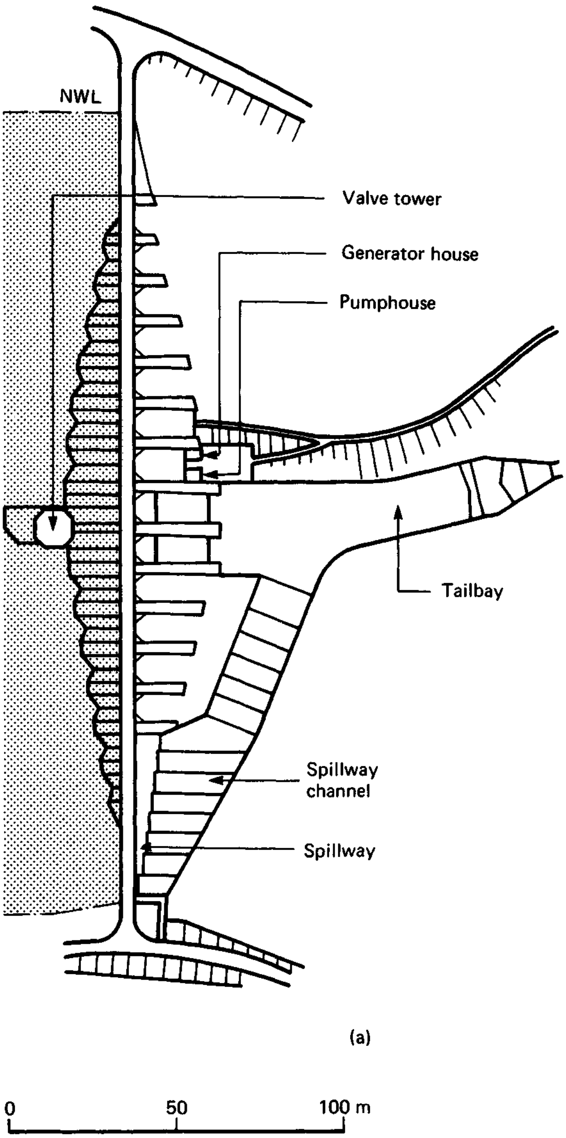


Figure 18.27 Itaipu dam: diamond-headed buttress of main dam. (a) Section; (b) plan, where (1) is the contraction joint, (2) grout curtain, (3) consolidation grouting, (4) drainage holes, (5) drainage tunnel, (6) shear keys, (7) dense basalt, (8) breccia, (9) vescicular amygdaloidal basalt, and (10) represents discontinuities EL 125 and EL 112



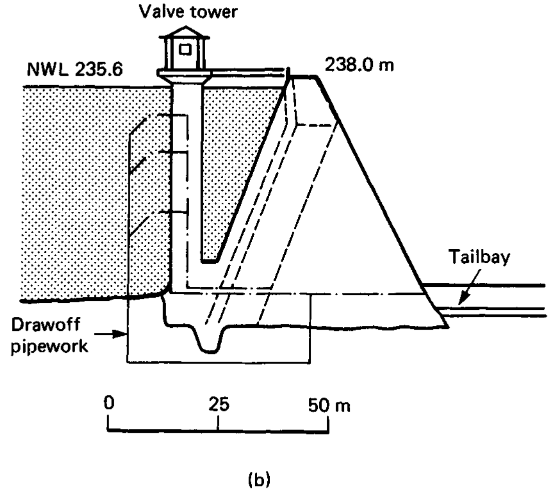


Figure 18.28 Wimbleball dam: a diamond-headed buttress. (a) General arrangement; (b) section through dam and valve tower

Example 18.8

The diamond-headed buttress – Wimbleball dam, southwest England (63 m) built 1975–79 (Figure 18.28).

The foundation consists of highly folded and contorted sandstones and siltstones with intercalated beds of slatey shales and mudstones. To limit underseepage, grouting was extended to 40 m below formation level and relief wells were drilled between the buttresses. Fly ash was used in the concrete to reduce the heat of hydration. Contraction joints between diamond heads were sealed by a 300 mm rubber waterbar upstream of a 200 mm Paracore plug.

The diamond heads, forming the upstream face of the dam, sloped at 1 vertical:0.383 horizontal.

The heights achieved by the various types of buttress dam are shown in Figure 18.29.

18.4.5 Arch dams

An arch dam can be likened to an arch bridge lying on its side with the abutments acting as springings for the arch. The arch transfers pressure from the reservoir water to thrust on the abutments, which must be strong enough to carry the thrust without permitting unacceptable deformations. The joints and bedding planes in many bedrocks provide potential weaknesses. Valley formation releases horizontal stresses and can be accompanied by strains and movements such as cambering and valley bulging which can loosen the rock mass. Great care is needed to ensure that the rock forming the abutments can accept both the magnitude and direction of the thrust that will come from the arch dam. As the reservoir filled for the first time, the inability of the left abutment rock structure to support the thrust from the 61 m high Malpasset dam brought about its collapse on 2 December 1959 causing serious damage to the downstream town of Fréjus, and killing 421 people.

18.4.5.1 Arch gravity dam

A mass gravity dam requires much less weight to keep it stable if it is built curved in plan (just as a piece of cardboard will stand on its edge if it is curved). Stresses thrown on to the abutments are less than with a thin arch dam but, unlike a true gravity dam, no separate section would be able to support the reservoir water pressure without the benefit of the arch action.

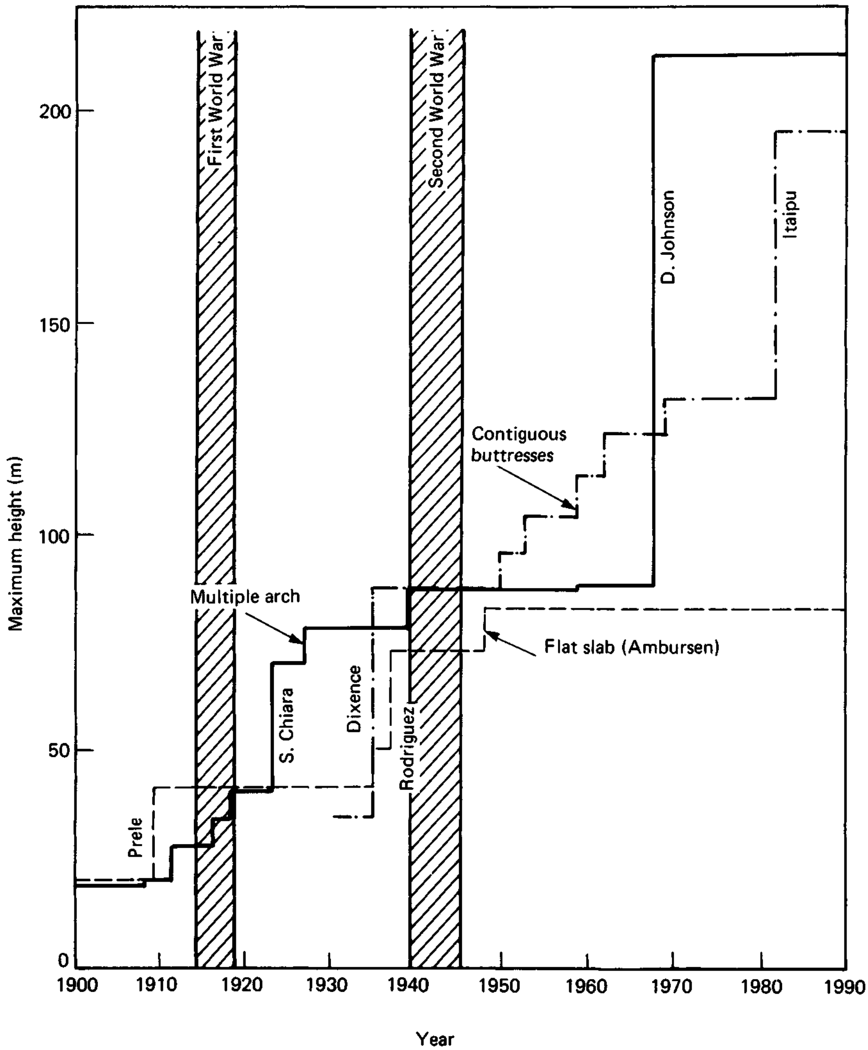


Figure 18.29 Maximum heights of buttress dams since 1900

Example 18.9

The Pieve di Cadore dam, Italy (112 m) constructed 1946–49 (Figures 18.30 and 18.31). This dam is founded in dolomitic limestones of the upper Trias. The modulus of elasticity of the rock lay in the range 2000 to 3000 MN/m^2: this was improved by grouting to 5000 to 6000 MN/m^2. The lowest part of the gorge was filled in with a mass-concrete plug to a height of 57 m. The arch gravity dam above this level was constructed in 33 monoliths, each about 12 m wide.

The upstream face, varying in thickness from 1.5 m at the top to 4 m at the base, was made with concrete containing 250 kg of cement per cubic metre. The remaining major part contained 200 kg/m^3.

At the interface between the two concretes, there was a drainage system of 300 mm diameter pipes over the whole height of the dam.

To allow cooling and shrinkage, alternate monoliths were constructed ahead of the others. On completion, and after sufficient cooling, the 34 vertical construction joints were grouted and coated with bituminous material to protect any tensile cracking that might develop.

18.4.5.2 Double curvature arch

To avoid tensile stresses in an arch resisting reservoir water pressure, some curvature is desirable in the vertical section. With a perfect shape, putting the concrete into pure compression, the section could be relatively thin without exceeding the safe working compressive strength of the concrete.

A model using an elastic sheet in pure tension, bulging under the water pressure, gives the shape in mirror image for pure compression. In practice, some abutment deformation, temperature stresses and uplift pressures from water in the foundation interface and joints in the concrete affect the pure compressive stress ideal and require greater thickness to be used.

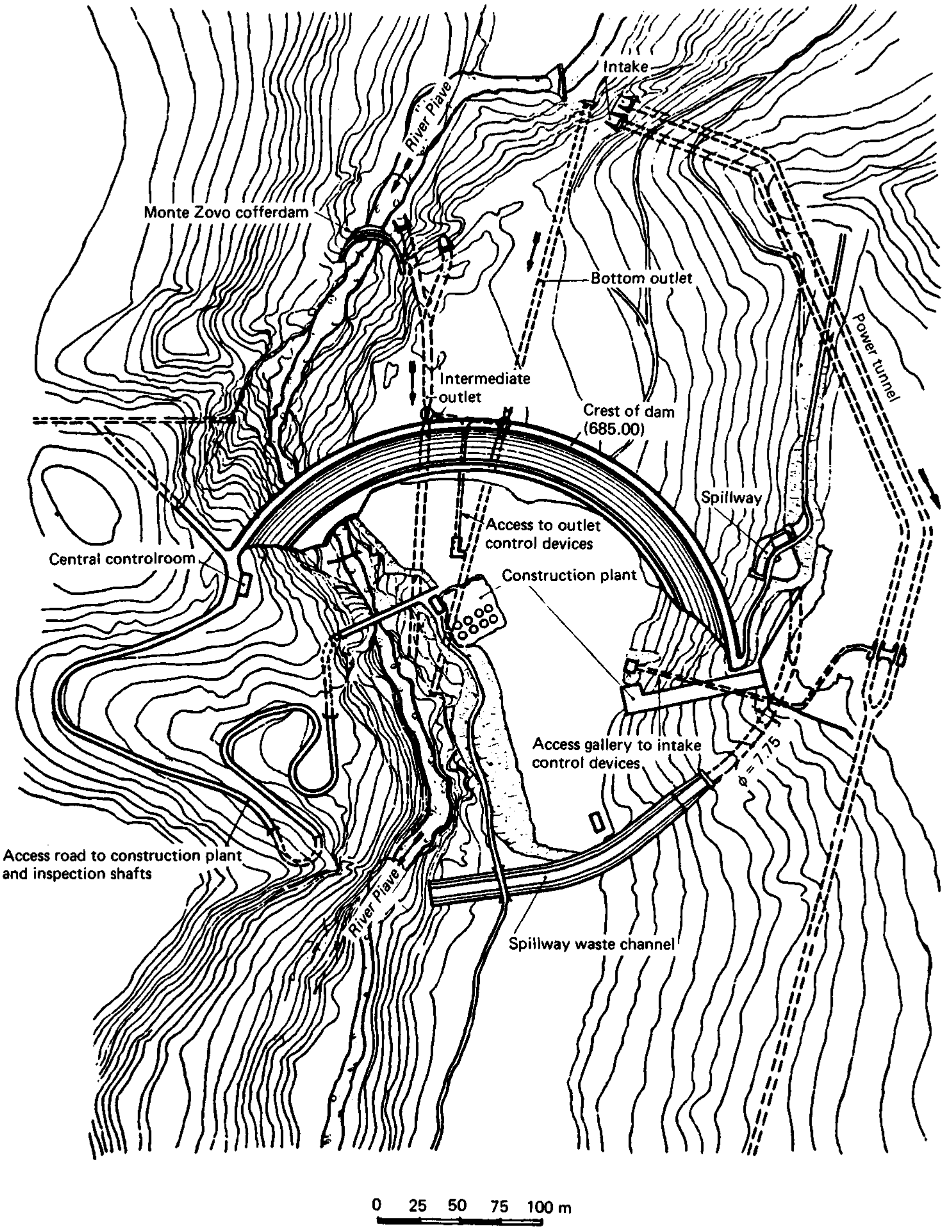


Figure 18.30 Pieve di Cadore dam: plan

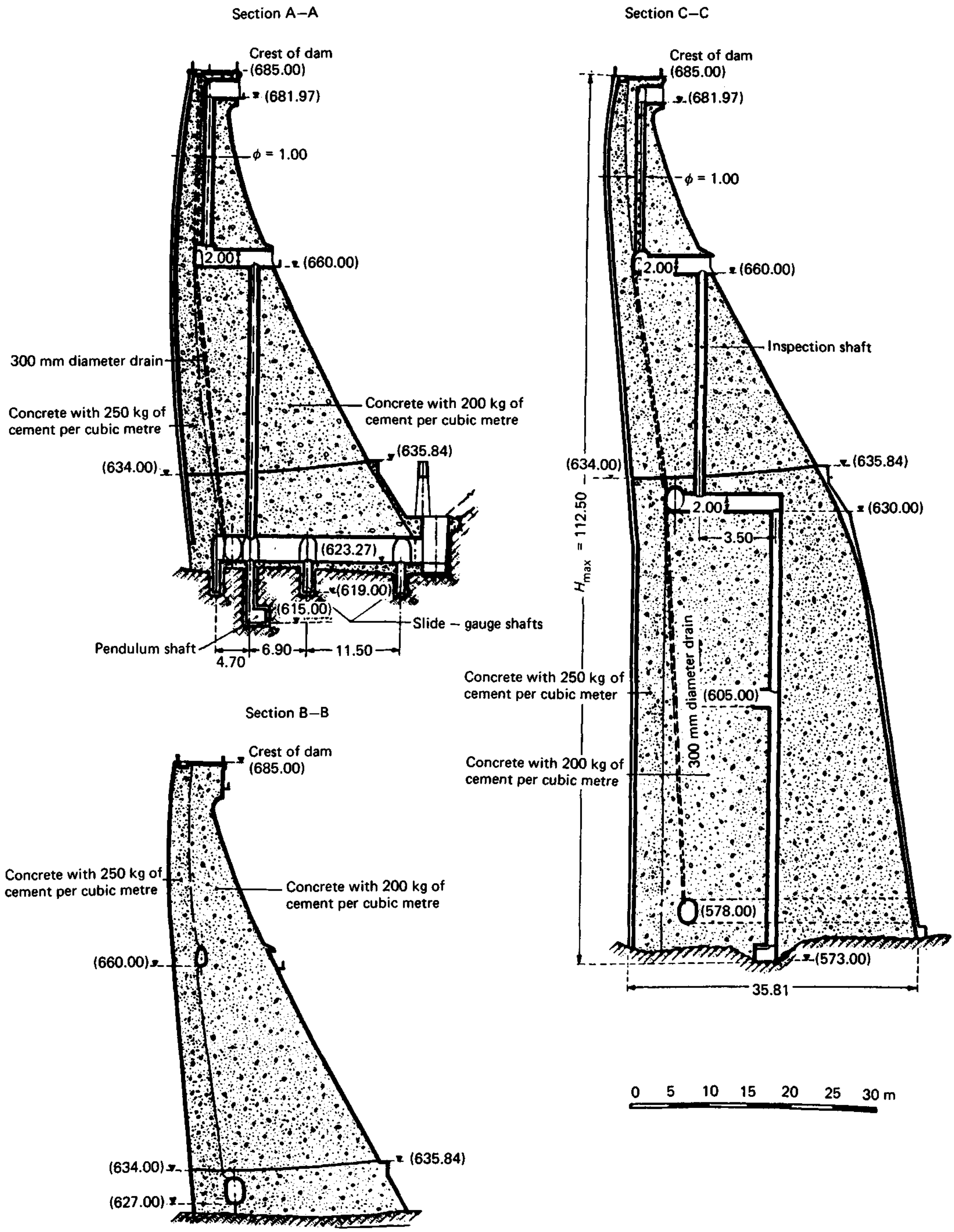


Figure 18.31 Pieve di Cadore dam: cross-sections

Example 18.10

Neves dam, Italy (94.7 m) constructed 1960–63 (Figures 18.32 and 18.33). Neves dam is founded in a gneissic granite formation. Design, based on a mathematical model using finite element techniques, was based on a modulus of elasticity for the rock of 14 000 MN/m² and a value of 28 000 MN/m² for the concrete in the lower third of the dam, 30 000 MN/m² for the upper part. The radius of the arch increased with height as shown in Figure 18.34.

The dam was built in 12 sections, each about 15 m long with concrete containing 240 kg/m³ of pozzolanic cement.

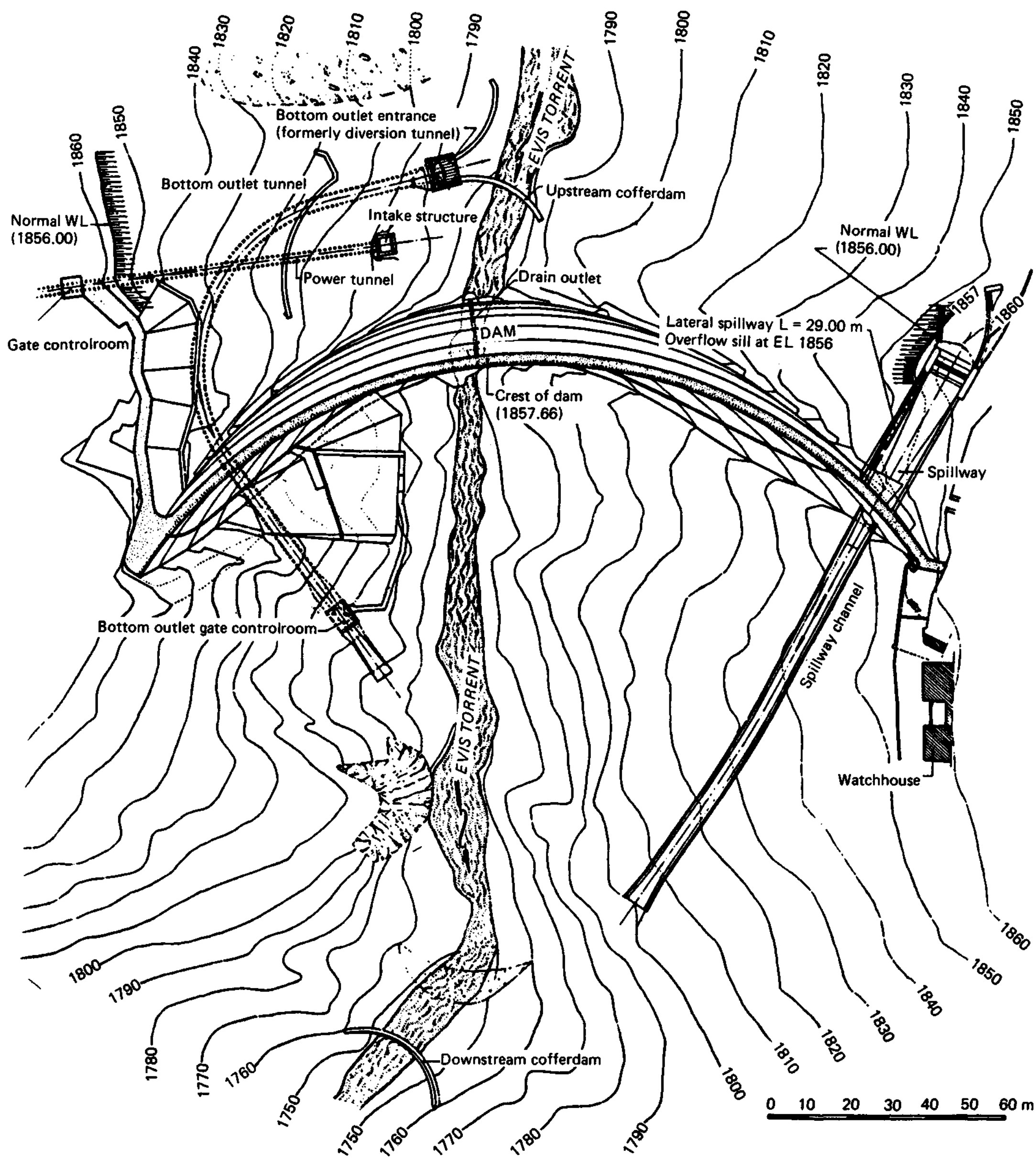
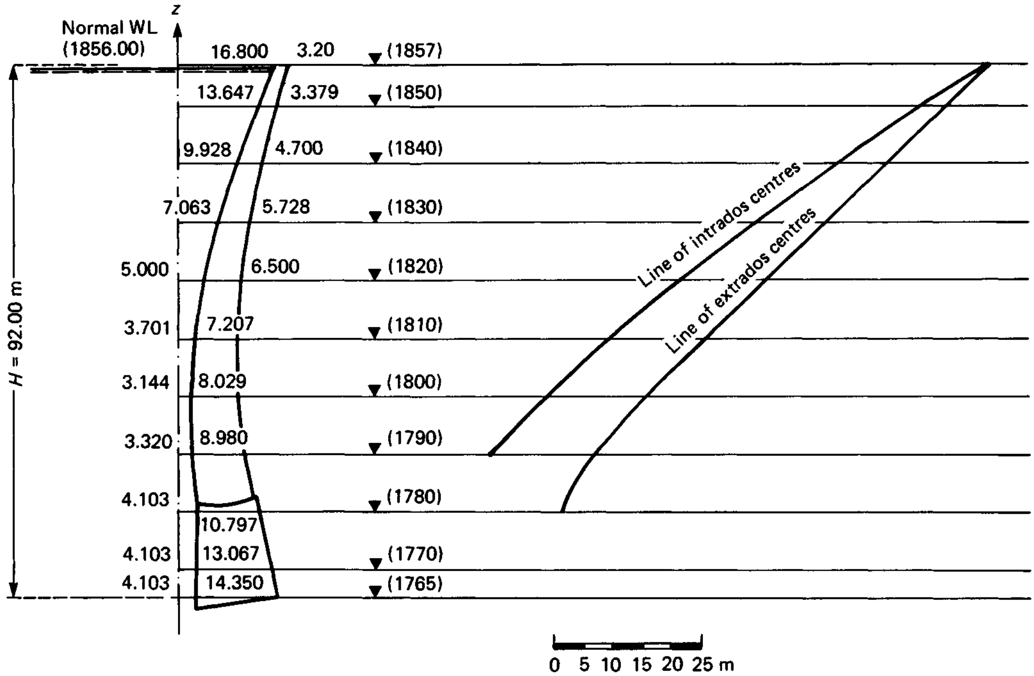
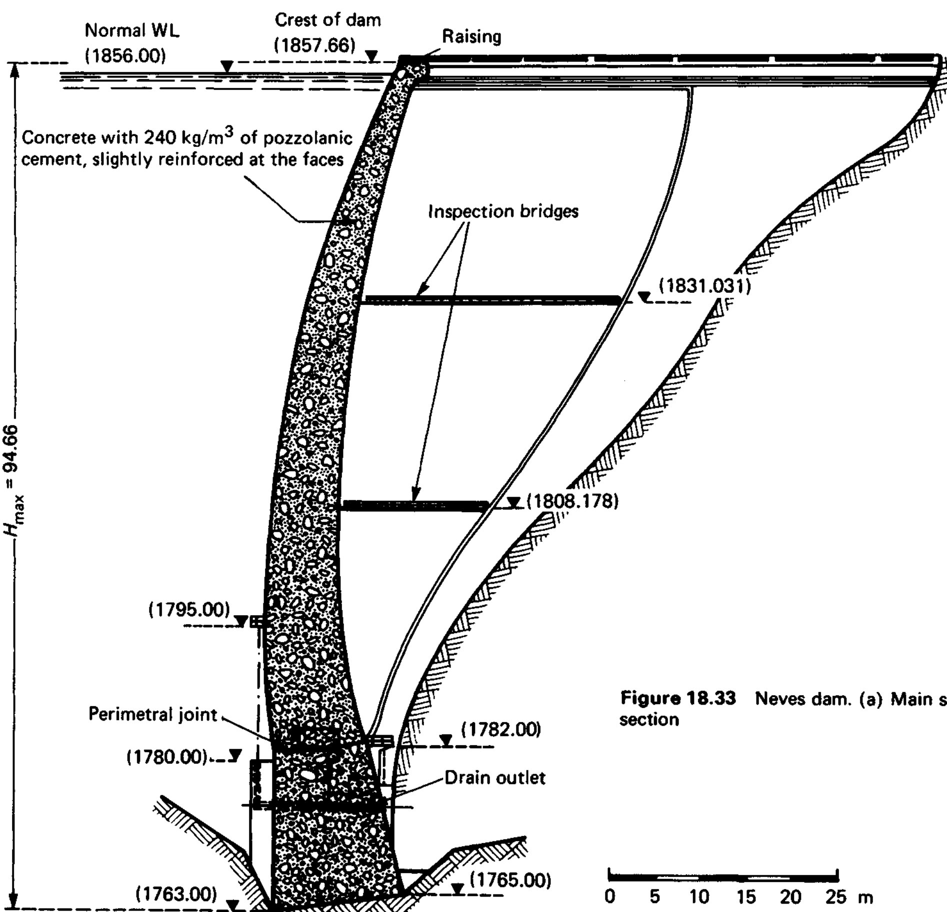


Figure 18.32 Neves dam: layout





(b)

Figure 18.33 Neves dam. (a) Main section with data; (b) main section

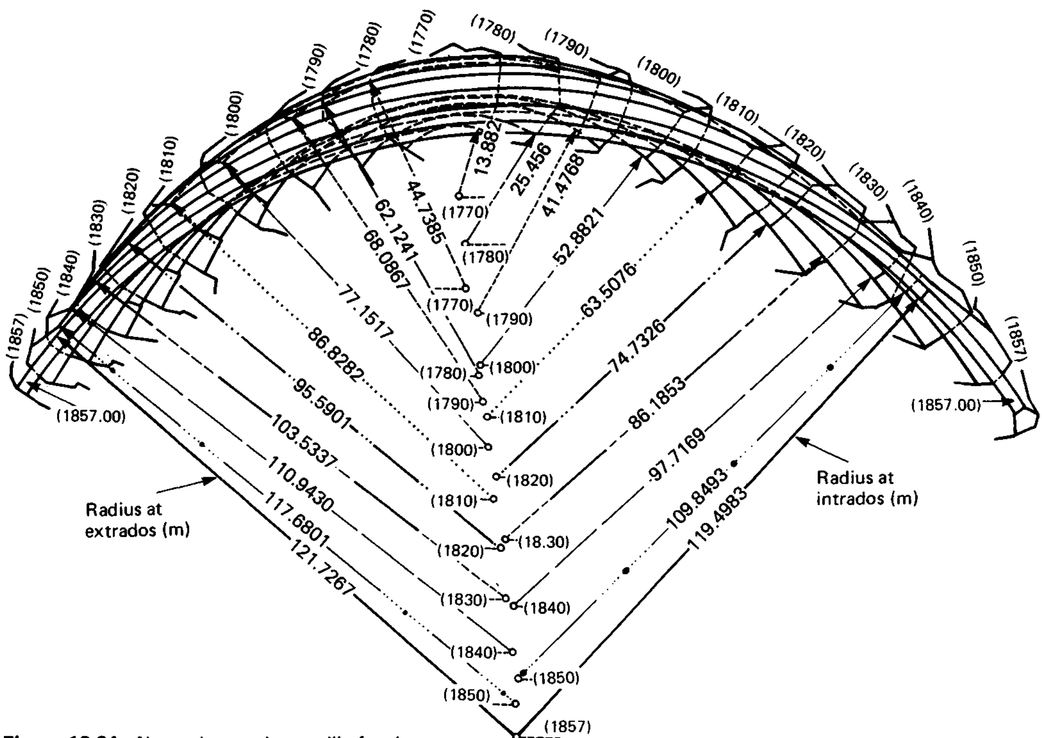


Figure 18.34 Neves dam: various radii of arches

18.5 Design concepts

18.5.1 Embankment dams – homogeneous section

Two important points must be borne in mind:

(1) It is essential to prevent the phreatic surface from reaching the downstream slope. This is most effectively achieved by placing a filter drain to separate the upstream and downstream shoulders (see Figure 18.15, page **18**/13).
(2) Foundation deformations and settlements over abutment irregularities such as those indicated in Figure 18.35 may create tensile strains that can cause visible cracks and may permit leakage to the central drain.

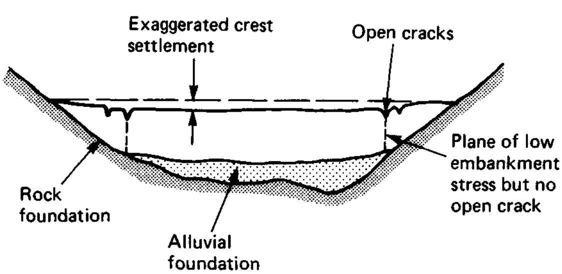


Figure 18.35 Zones of tensile strain

18.5.2 Embankment dams with central clay cores

The traditional British section had upstream slopes of 1:3 and downstream slopes of 1:2.5 (Figure 18.36).

18.5.2.1 Shoulders

The function of shoulders is to support the clay core. They must resist sliding along the foundation and any plane through the fill against horizontal thrust from the core.

Slope stability should be checked along a slip surface that might pass through core and foundation, or within the fill (see Chapter 9).

High construction pore pressures in the shoulders should be avoided by:

(1) Use of permeable fill.
(2) Provision of drainage layers.
(3) Placement of fill at a water content which allows a maximum dry density to be developed by the compacting machines to be used.

A transition is required between permeable shoulder fill and semi-impervious core material. This is often achieved by selecting borrow material so that the coarsest is placed in the outer portions of the shoulders, with the finest fraction adjacent to the core. It is good practice to place a graded filter between core and shoulder fill.

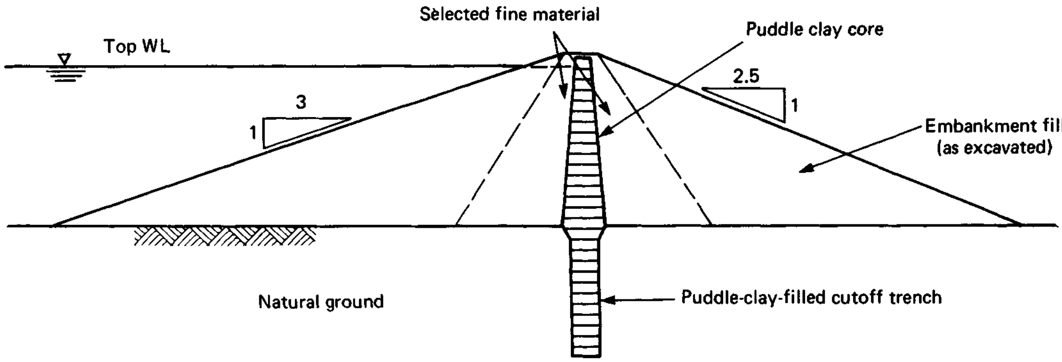


Figure 18.36 Traditional puddled clay core dam

18.5.2.2 Filters

The aim is to use a well-graded material, with pores small enough to prevent entry by particles from the core. At the same time, the filter material must not be so fine that it would be lost in voids in the shoulder fill. To satisfy these requirements, it may be necessary to use several different grades of material in a composite filter with finest next to the core graduating to coarsest in contact with the shoulder fill.

Traditional filter design is based on the concept that the pores in a granular material are only about one-fifth the diameter of the particles. Thus, the grain size of a filter can be about 5 times that of core material. In practice, filter material is well graded and the controlling size is often taken as the maximum of the smallest 15% of the mixture. When a sieving analysis has been made, producing a grading curve of the type shown in Figure 18.37, the controlling size is D_{15}.

A filter rule is:

(1) D_{15} maximum of the filter must be equal to or less than 5 times D_{85} minimum of the core (or smaller-size filter zone).
(2) D_{15} minimum of the filter must be equal to or greater than 5 times D_{15} maximum of the material to be protected to provide adequate permeability.

Specifications usually contain other restraints such as maximum and minimum sizes, absence of gap grading, etc.

The smallest particle in the clay core may be of clay size, i.e. < 0.002 mm and it is unlikely to be practical to provide a theoretically correct filter to trap this size of particle. Fortunately, most clayey fills that are used for cores are well-graded materials containing coarse sand or even pebbles. The clay particles tend to floc (unless they are dispersed by the chemical composition of the reservoir water) so that it is only necessary to filter the floc size.

Even though, initially, some clay flocs may pass into the pores of the filter, the slightly larger sizes of the clay core cannot pass into the pores and, after an initial slight loss of material, a finer filter of core material builds up against the provided filter. It is necessary to ensure that the grading of the core material will allow this to happen and that the various zones of the filter will prevent loss of filter material into the shoulder fill.

Vaughan and Soares[18] have developed a method for designing filters by using their permeability to indicate pore size. The permeability of a filter is proportional to the square of its pore size and if retained particle size is used to represent this, then:

$$k = Ad^x \tag{18.1}$$

where k represents filter permeability, d, particle size, A is a constant depending on other geometric factors, and x is a power of about 2.

Tests with a number of clays used for the cores of British dams gave $A = 6.1 \times 10^{-6}$ and $x = 1.42$.

The floc size d can be found from hydrometer sedimentation tests, using the local, natural water with a soil concentration of 25 g/l. It was found that floc size could be determined with equal accuracy by observing the settling velocity of the clearing front of the suspension.

Filters for the small Ardingly dam in Sussex were designed by this method. Sedimentation tests with river water gave a floc size of 6 to 15 µm, with an average of 10 µm. The above values of A and x gave a required permeability for the filter $k = 1.6 \times 10^{-4}$ m/s. A natural, medium-sized sand was found with $D_{50} = 0.4$ mm and $D_{15} = 0.23$ mm. Its permeability was 0.9×10^{-4} m/s and it has been used successfully.

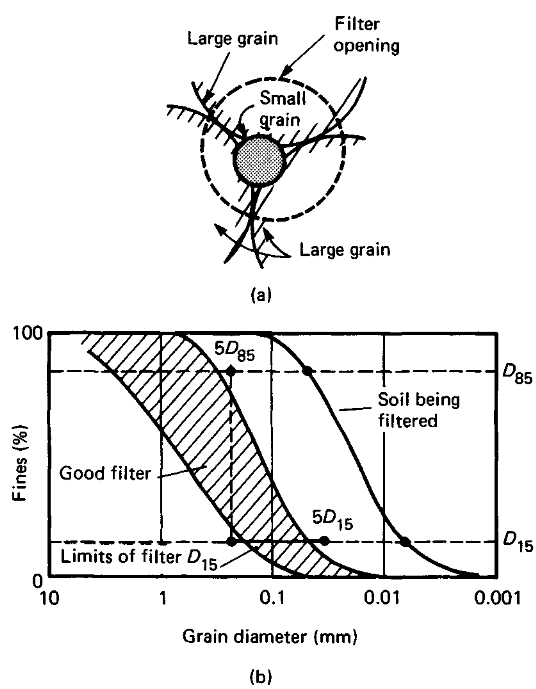


Figure 18.37 Grading curves showing filter rule. (a) Large grains screen small grains at filter opening; (b) grain size criteria for soils used as filters

Further work by Vaughan and Soares indicated values as high as $A=6.7\times10^{-6}$ and $x=1.52$. The small difference between these sets of values may be regarded as the current latitude in this new design method.

Drainage layers under and in the shoulder fill can be designed as filters to prevent fill finding its way into them. This aspect is not important on the downstream side where water flows are likely to be small. In the upstream side, however, it may be desirable to design the drains as filters, particularly when the reservoir will be subjected to repeated large fluctuations, as in the case of a pumped storage hydro-electric scheme.

18.5.2.3 Clay core

The permeability of particulate materials is dependent on stress history, as well as, for example, grain shape, size and grading.

A heavily overconsolidated clay may show a low permeability, e.g. 1×10^{-9} cm/s when an intact sample is tested in the laboratory. Field tests from piezometers usually show much higher values due to the joints, fractures, bedding planes, silt inclusions, etc. which form the fabric of the clay.

When excavated and compacted into a rolled clay core, intact fragments may retain the low permeability of the intact sample, but the mass permeability of the core may be controlled more by the joint surfaces between fragments. Compaction to optimum density will ensure that these joints are not only tightly closed, but carry across them a prestress induced by the compaction machinery – a stress which will be increased as the height of fill rises.

Hydraulic fracture. Water from the reservoir tends to percolate along a myriad joints. If its pressure exceeds the total stress acting across the joints, they may be forced apart, producing a fracture that will penetrate into the core until it meets a zone of higher stresses.

The amount by which the joints are forced open depends on confining conditions and the compressibility of the compacted clay. The fissure may not become very wide and as water migrates into the pores of the clay forming its walls, reducing the effective stresses to zero, the clay will swell and may close the fissure, if the reservoir water pressure ceases to rise.

A fissure which extends until it finds an outlet may permit sufficient flow to cause erosion which will increase fissure width, thereby further increasing flow and leading to piping. Rate of erosion may depend on how well the clay will disperse into the reservoir water. The degree of dispersion of a clay may be measured by the Volk[19] double hydrometer test or the Sherard *et al.*[20] pinhole test. A fairly clear indication may be obtained, however, from the ball test which used to be carried out on clay samples to check their suitability for use in a puddled clay core.

Ball test. Samples of the clay at the proposed placement water content were rolled by hand into balls about 50 mm diameter. These were carefully placed in a bucket of river water (to be the reservoir water) and left for 24 h. The degree of dispersion could be judged from the amount and fineness of the material that had fallen away from each ball. In the extreme, a ball could disintegrate into a loose heap and produce a muddy suspension covering the bottom of the bucket. A ball of nondispersive clay would remain intact in clear water.

Wet seams. Materials of low plasticity, such as silts, erode readily. If a fissure develops slowly, without appreciable flow, i.e. before it has reached an outlet, then, under the release of effective stress, the silt can expand into the water to loosely fill the fissure. As it progresses through a wide core, under the action of rising reservoir pressure, it may fill up with loose silt until, when it reaches an outlet, the permeability of the loose silt may be sufficiently low to limit flow to nonerodable amounts. The resulting seam of loose, saturated core material may be found if dry excavations are made into the core. Boreholes drilled with flush water will not disclose such seams, although when they are reached, loss of washwater can be expected.

Sherard[21] has described wet seams found in the cores of Manacouagan 3, Yard's Creek and Teton dams. He believed they originated in hydraulic fractures and proposed the above mechanism to account for their development. A more detailed discussion about the wet seams found in the Teton core during the post-failure investigation were given by Sherard.[22] The wet seams in the main body of the Teton core were not the cause of failure.

Total pressures in a core. The risk of hydraulic fracture can be reduced by increasing total stresses in the core. The maximum total stress on a horizontal plane could not be expected to exceed the overburden pressure. More usually it is much less.

It requires relatively little differential settlement between core and shoulders to develop the full shear strength of the clay. A narrow core can resemble material in a silo and its attachment to the silo walls supports some of its weight. This silo, or arching action, reduces the vertical total stress in the core, below its potential value of $\sigma_v=\gamma h=\sigma_o$ the overburden pressure, where σ_v represents the vertical total stress, γ the bulk density of core material, and h the height from considered point to crest.

If the core is assumed to have vertical sides, as indicated in Figure 18.38, then a guide to the vertical total stress at any level in the core can be obtained from:

$$\sigma_v=h(\gamma-\frac{c_u}{a})$$

where a represents the half width of the core and c_u the undrained shear strength of the core.

In an axial direction, where the core is restrained by the valley sides, the total pressure σ_a, which may control hydraulic fracture, is given by:

$$\sigma_a=K_0\,(\sigma_v-u)+u$$

where K_0 represents the coefficient of earth pressure at rest, and u the pore pressure in the core.

In order to maximize σ_v, a dense material should be used for the core and adjustments made by reducing c_u and increasing a to obtain desired values.

In general, reducing c_u increases K_0 so that maximum values of σ_a can be obtained with suitably low values of c_u.

An increased water content will reduce c_u and increase $\bar{B}$ (see Chapter 9), thereby increasing construction pore pressure u. This increase in the value of u also helps to increase σ_a.

It is desirable to have the clayey core material in such a condition that, at the end of construction, piezometric level in the core is above crest level. This may fall by dissipation as reservoir level rises and an ideal solution would be for the piezometric level to have fallen to top-water level in time to meet the reservoir water at that level.

18.5.2.4 Below-ground cutoff and core contact area

Traditional British treatment at a site on alluvium was to excavate a below-ground trench along the core centreline, taken down to a suitably impervious strata.

The trench was filled with concrete to form an impervious cutoff and this was taken up a short distance into the clay core. The top of the wall was sometimes shaped like a spearhead so that it would push up into the clay as the core settled. To avoid flow of water along the interface between concrete and clay it

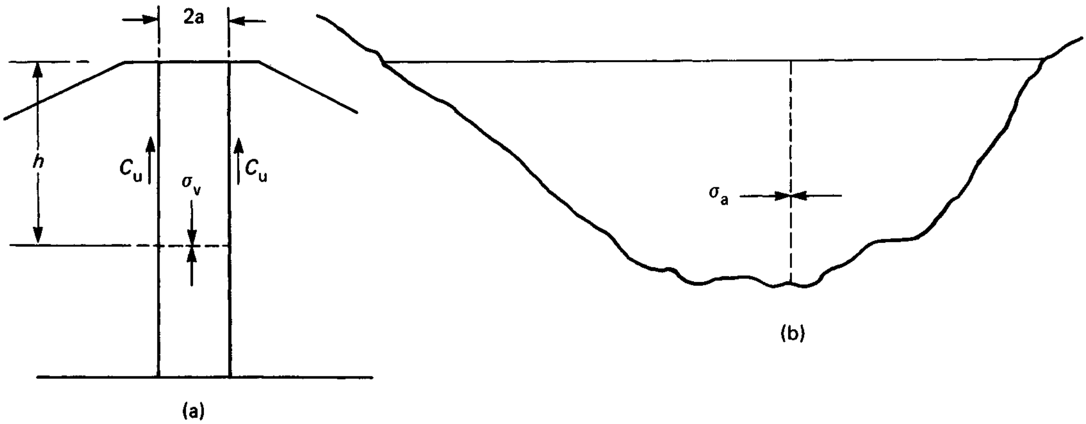


Figure 18.38 Silo or arching action in a clay core. (a) Total vertical stress at any level in a vertical core; (b) core restrained by valley sides

was necessary to have a total pressure across the interface in excess of reservoir pressure.

The deep trench required a strong support system which made the whole operation expensive. A more modern alternative is to support the trench with a slurry during excavation and place the concrete by tremie-pipe, displacing the slurry from the bottom upwards.

Since the purpose is to stop water flow under the dam, there is no need of a strong concrete wall. To allow for consolidation of the ground under dam weight, a compressible concrete is sometimes used to fill the trench.

Another method is to form the wall with contiguous bored piles constructed of concrete. This can be made by boring holes for alternate piles and then, after they have been cast, bore between them, cutting a small crescent from each.

The most common below-ground treatment is the injection of grout from small-diameter boreholes with the aim of forming a continuous wall of ground, made watertight by the grout. The type and extent of treatment depends on ground conditions. Sometimes a grout curtain is formed from one line of holes along the core centreline, but it is more usual to have three lines. The grouted width is usually increased near ground surface to at least core base width by blanket grouting from shallow holes, as indicated in Figure 18.39.

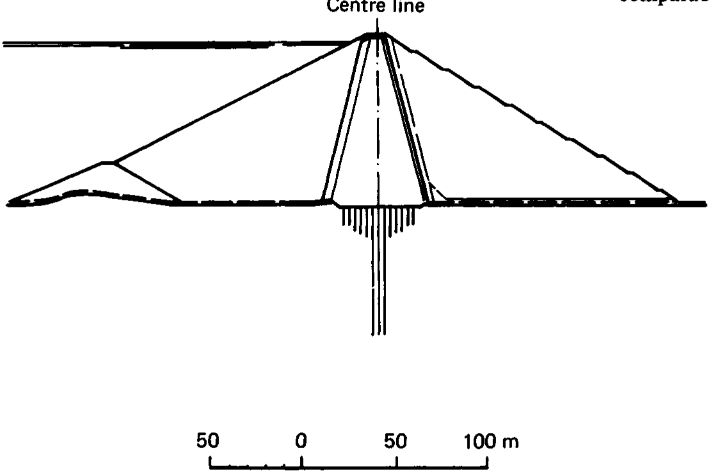


Figure 18.39 Grout holes under a clay core

Grout can be injected as the hole is drilled, but it is more common to drill to full depth and grout in stages from the bottom. It is now usual practice to use perforated tubes covered with rubber tubes (*tube à manchette*) which are grouted into the bored hole with a weak grout. A double packer is then passed into the steel tube so that grout can be injected horizontally at any level. The rubber tube acts as a flap valve and allows the perforated steel tube to be washed out, so that further grouting can be carried out if necessary.

A test for determining whether grouting is necessary and to check the effectiveness of grouting was devised by Lugeon.[23] This requires injection of water in a borehole at a pressure of 10 bar. Flow is measured and expressed as litres per metre length of hole. A flow of 1 litre per metre per minute (referred to as a Lugeon) is regarded as the limit before grouting is needed.

This test, intended for bedrock, is dangerous because the top-of-hole pressure of 10 bar will exceed overburden pressure to a depth of about 50 m. Many sites show much higher Lugeon values over the upper 20 to 30 m, and this is often taken to indicate the degree of weathering, but it can also be due to excessively high water-pressure. Borehole diameter was not specified by Lugeon so it is difficult to convert Lugeon values to *in situ* permeability. In bedrock, water loss is along fissures, so that a determination of *in situ* permeability which would be comparable with a sand or gravel is not possible.

Today, the danger of causing hydraulic fracture in the ground through use of excessive pressures is generally recognized, so although tests carried out are still referred to as Lugeon tests, water pressures are related to overburden pressure. This also applies to grout pressures. It is very easy to discharge large volumes of grout into the ground, but it may be opening fissures rather than sealing them.

A great deal of money can be wasted on extensive preconstruction grouting programmes on a dam site. Tests to measure *in situ* permeability should be designed on soil mechanics principles (see Chapters 9 and 11) and be carried out during the site investigation. If the results indicate that dam underflow will not cause unacceptable water loss or dangerous build-up of pressure under the downstream shoulder, consideration should be given to omitting a grout curtain.

In the interests of security, an inspection/drainage gallery should be built in a trench at ground surface to form part of the base contact area for the core.

On completion of the dam, the underflow condition can be assessed from instrument readings as the reservoir fills, when any necessary extra drainage or grouting can be carried out from the gallery. Much higher grouting pressures can then be used because the foundation is held down by the weight of the dam.

It is unfortunate that financial arrangements can seldom be made that will allow construction of a below-ground grout curtain after dam completion. If the cost of the grouting programme is not included in the estimated cost of the dam, it is extremely difficult to obtain money at the end of construction. It is also impossible to avoid the notion that the design has been a failure if grouting is found to be necessary as the reservoir fills for the first time.

18.5.2.5 Rockfill

Earthmoving machinery can handle rockfill containing pieces of 1.5 to 2 m. Heavy vibrating rollers can compact layers of 2 m thickness and these sizes have commonly been used in rockfill dams.

The rockfill should be well graded with a tendency towards an excess of the smaller sizes to ensure that the large pieces are fully bedded and do not touch. In general, most of the material from a quarry can be accepted. A limit to the amount and size of fines is when the *in situ* permeability of the rockfill is reduced to 1×10^{-5} m/s. A soft rock such as a sandstone may produce an ample quantity of the finer material, whereas a very hard rock such as basalt may be short of fines. Because of this, the situation often arises in which a rockfill of soft rock suffers less deformation than one composed of hard rock.

Placement. Segregation may be reduced by tipping the rockfill 4 or 5 m back from the advancing edge of the layer, then bulldozing it over to the level of the new layer. The larger pieces fall to the bottom and are covered by fines which produce a smooth surface that is kind to the placing machinery and makes good contact with a smooth vibrating roller, transmitting the compaction energy into the fill. Provided there are sufficient of the small fines, the voids between large pieces become completely filled and pressed into the lower surface so that surfaces between layers cannot be detected except when markers (such as coloured sand) are used. Water content should be sufficient for workability, typically 5 to 10%. An excess of water is not harmful because the rockfill should be sufficiently permeable for it to drain freely.

Compaction control. It is not practical to control rockfill compaction by *in situ* density measurements. Specification is usually of method (size of roller, number of passes, etc.) but roller performance can be measured with a compactionmeter which will show when a desired compaction has been achieved. An apparatus designed by Geodynamik in conjunction with Dynapac Research has been described by Forssblad[24] and Thurner and Sandstrom.[25]

Stability of rockfill. The failure envelope for rockfill is curved. A typical example is shown in Figure 18.40.[27] The value of shear strength can be expressed by:

$$\tau = A(\sigma')^b$$

where τ represents the shear strength of the rockfill in kilonewtons per square metre, σ' represents the normal effective stress in kilonewtons per square metre and A and b are constants.

Values of A and b found from tests on several rockfills are given in Table 18.10.

Testing equipment used to obtain the parameters A and b requires to be fairly large, but clearly it is impractical to test the full-size rockfill. It has been found that samples obtained by sieving off the larger sizes give results more representative of the whole rockfill than samples with parallel, scaled-down grading curves. Test samples of 300 mm diameter and 700 mm height should be regarded as a minimum. Density and water content should be as close as possible to those expected in the field.

Design of slopes can be simplified by use of stability numbers given by Charles and Soares.[26] They have shown that the factor of safety:

$$F = \frac{\Gamma A}{(\gamma H)^{(1-b)}}$$

where Γ represents the stability number, H the height of the slope, and γ the bulk density of the rockfill.

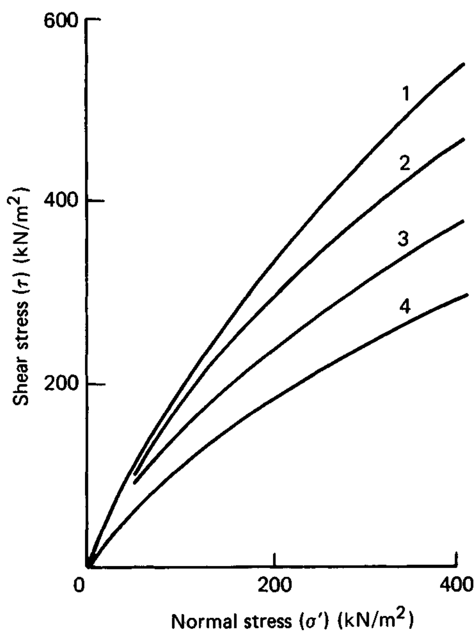


Figure 18.40 Curved failure envelope: rockfill, where (1) is sandstone, (2) slate, (3) is also slate, and (4) is basalt. (After: Charles and Watts (1980) 'The influence of confining pressure on the shear strength of compacted rockfill'. *Géotechnique,* **30,** 4)

The stability number for a given slope (1 vertical : x horizontal) is given in Figure 18.41.

As a guide to the confining pressures to be used in the tests for the determination of A and b, Charles and Soares have provided design curves to give values for σ'_m, the maximum normal stress on the critical failure surface. The curves are reproduced in Figure 18.42.

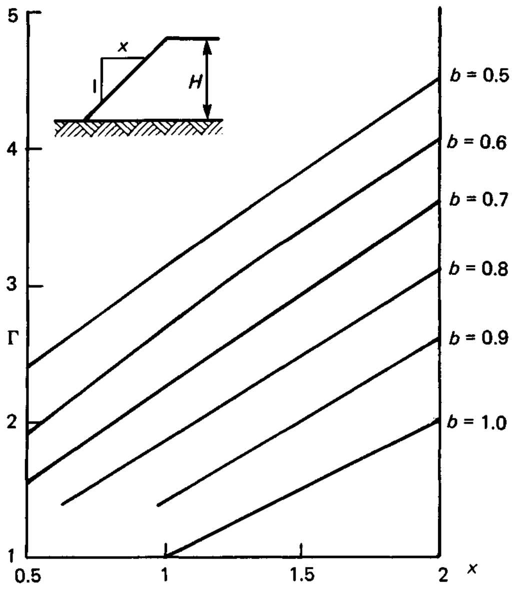


Figure 18.41 Stability numbers: rockfill slopes

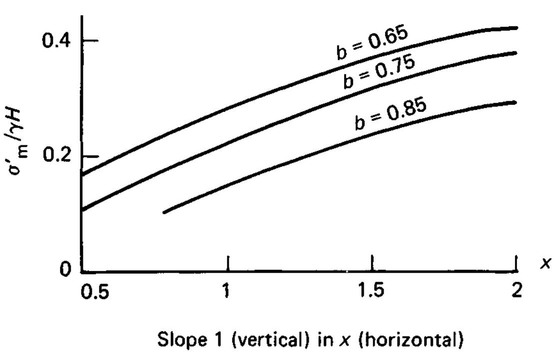


Figure 18.42 Maximum normal stress on critical failure surface

18.5.3 Roller-compacted concrete

A well-graded sandy gravel composed of rounded particles forms an excellent shoulder fill for embankment dams. It readily compacts to a maximum density and is usually sufficiently permeable to be classed as rockfill. Addition of some cementitious materials can both increase its strength so that it can be placed at steeper slopes, and reduce its permeability so that it becomes suitable for the construction of gravity section dams. By restricting heat liberation during hydration, temperature rises can be kept low enough so that long, continuous lengths will not develop shrinkage cracks. This enables it to be placed like earthfill. For a given height of dam, less volume of fill is required than with earth or rockfill, so more rapid construction can be achieved.

Mixes. Cement-stabilized earth and lean mix have been used to construct road sub-bases for half a century. Rollcrete used by Lowe[10] for the core of the Shihmen cofferdam in Taiwan was made from a concrete-type aggregate to a grading as shown in Figure 18.43. It had a maximum size of 76 mm and contained only 53 kg Portland cement and 53 kg fly ash per cubic metre (traditional concrete would contain about 300 kg Portland cement per cubic metre). Lowe used the modified American Association of State Highway Officials (AASHO) test (see Chapter 9) to determine optimum placement water content. He found this also gave maximum compressive strength, as shown in Figure 18.44.

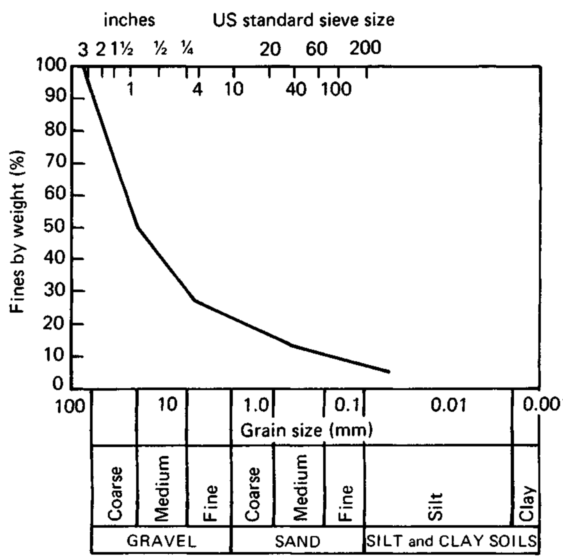


Figure 18.43 Rollcrete for Shihmen core: grading curve

Table 18.10 Values for the parameters *A* and *b* for several compacted rockfills

Rock type	*A*	*b*	*Reference*
Diorite	2.0	0.870	Marsal[28, 29]
Silicified conglomerate	2.6	0.846	Marsal[28,29]
Pizandarau sand and gravel	2.2	0.876	Marsal[28]
Nelzahualcoyotl conglomerate	2.1	0.881	Gamboa and Benassini[30]
Malpaso conglomerate	3.8	0.808	Marsal[29]
Carboniferous sandstone	6.8	0.670	Charles and Watts[27]
Palaeozoic slate	5.3	0.750	Charles and Watts[27]
Palaeozoic slate (weathered)	3.0	0.770	Charles and Watts[27]
Basalt	4.4	0.810	Charles and Watts[27]

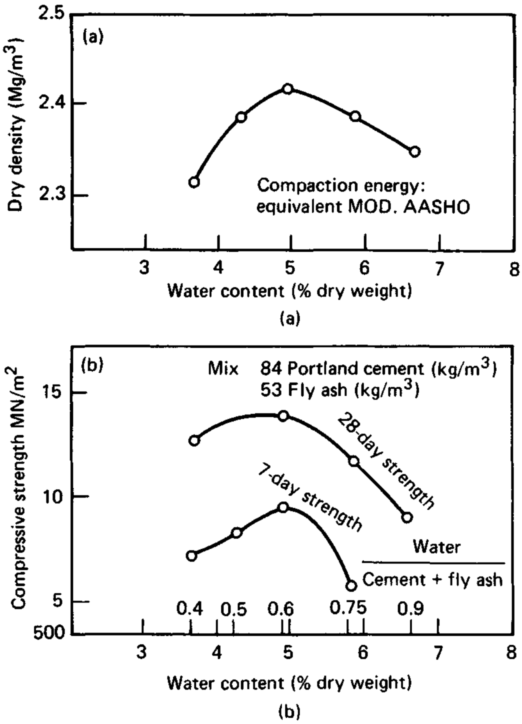


Figure 18.44 Rollcrete and water content. (a) Density; (b) compression strength

The mix used for Willow Creek dam (51.5 m), the first American dam built with roller compacted concrete and completed in 1982, contained 36.3 kg Portland cement and 14.5 kg fly ash per cubic metre. Segregation at the base of each layer produced porous joints which had to be grouted at a later stage.

The British approach outlined by Dunstan[16] has been to fill completely the voids in the aggregate to both minimize permeability and improve workability at low water contents. The air:voids ratio in a compacted fine aggregate passing a 5 mm sieve is typically 0.35:0.38.

A paste of this volume is therefore required to fill the voids completely. A study of the behaviour of joints between successive layers showed that even higher proportions of paste were advantageous.

The Milton Brook trial bank used a crushed limestone aggregate and an excess of paste with very low water content.

In Japan, the roller compacted dam method of construction has tended to follow the American roller compacted concrete approach, namely of a dry, lean-mix concrete. Fly ash is used to reduce heat of hydration but in general much less has been used than at the Milton Brook trial. Mixes that have been used for the hearting of several roller-compacted dams are given in Table 18.11. To allow for some shrinkage, transverse contraction joints, made by vibrating a cutting plate into freshly placed layers, have been spaced typically at 15 m.

Consistency tests. Concrete for roller compaction is too dry for the traditional slump test. Instead, vibrating equipment is used. The Japanese consistency meter is a steel cylinder of 480 mm diameter and 400 mm height which is filled with the mix and mounted on a vibrating table. It has a transparent plastic plate/piston, loaded to 200 kN, on the surface of the mix. The table vibrates with an amplitude of 1 mm at a frequency of 4000 c/min. The vibrating compaction value is the time, in seconds, for the cementitious paste to have risen so as to make complete contact with the underside of the transparent plastic plate.

There is a close relation between number of passes required to compact the mix with a vibrating roller and the vibration compaction value.

In designing a mix, the test can be used to adjust sand content, water content, etc. to give optimum values. The effect of varying sand content is shown in Figure 18.45.

Dam section. In a dam section, two approaches may be used:

(1) Placing a waterproof element i.e. steel sheet, bituminous membrane, or dense concrete with contraction joints containing waterstops, on the upstream face. With this arrangement, the main body of rollcrete does not have to have low permeability and can be provided with open contraction joints.
(2) In the homogeneous section, upstream and downstream faces may be formed by horizontally slipformed kerbs, precast units being held on by reinforcement laid into the rollcrete, etc. The rollcrete mix must be given low permeability by use of excess paste content – this will also enable tight joints to develop between successive layers. As with homogeneous embankment dams, drainage should be provided to prevent water reaching the downstream face.

There may be an advantage in sloping both faces: this may simplify provision of finishes to the faces and will assist stability by giving a vertical component to the thrust from the reservoir water.

Galleries. Drainage/inspection galleries can be formed:

(1) With traditional concrete, using formwork.

Table 18.11 Mixes for the hearting of several roller-compacted dams

Name	*Aggregate size (max.)* (mm)	*Coarse aggregate* (> 5mm)	*Fine aggregate* (< 5mm)	*Cement*	*Fly ash*	*Water*	*Date*
			(kg/m^3)				
Shimajigawa	80	1476	749	91	39	105	1977–80
Ohkawa	80	1500	686	96	24	102	1978–79
Shin-Nakano	150	1468	685	84	36	90	1979–82
Pirica	80	1588	668	84	36	90	1982–88
Tamagawa	150	1544	657	91	39	95	1983–87
Mano	80	1552	726	96	24	102	1985–88
Asahiogawa	80	1500	706	96	24	102	1986–89
Sakaigawa	80	1182	752	84	36	105	1988–92

(2) As precast units, placed by crane.
(3) As sandbags and loose fill.

The first two systems interfere with placement of rollcrete and cause time delays. In the third system, the shapes of the galleries are defined by laying sandbags in the rollcrete and filling the space between with loose fill (sand and gravel or other suitable fill). In this way, the placing machinery can work over the galleries without interruption. When the rollcrete work is complete, the uncemented fill is dug out to form the galleries.

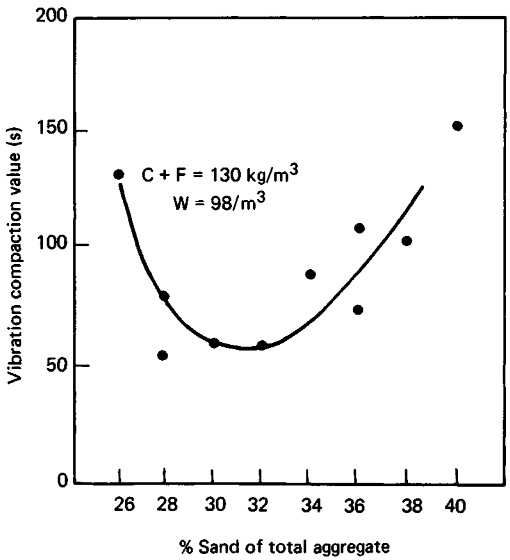


Figure 18.45 Sand : aggregate ratio and vibration compaction values

18.6 Legislation

Dams are subject to legislation in most countries. In the interests of public safety, there are usually conditions imposed on the qualifications of engineers permitted to design and supervise the construction of dams. There is also often a system of inspection of dams during their operation.

In the UK, dams retaining a reservoir of more than 24 000 m^3 are controlled by the Reservoirs Act 1975. This supersedes the Reservoirs (Safety Provisions) Act 1930, which required engineers to be approved and registered by the Secretary of State before they were permitted to design and supervise dam construction. Dams had to be inspected at certain times and at least every 10 years, when a report had to be made available to interested parties.

The new Act continues this general principle, but provides powers to implement recommendations for repairs or modifications contained in the Inspector's report. Appointments to a panel are for 5 years only and all reservoirs must be registered and continuously supervised by a qualified civil engineer, in addition to being inspected periodically by an independent engineer.

18.7 Further reading

The International Commission on Large Dams (ICOLD), holds congresses every 3 yr in various parts of the world. The transactions of these congresses contain a wealth of information and form milestones along the path of developing technology. The breadth of subject discussed at each congress is kept within reasonable bounds by being addressed to only four questions chosen internationally prior to each congress. Abstracts of ICOLD publications covering the contents of the transactions are available in two volumes. A list of all ICOLD publications, which includes special volumes such as *Lessons from dam incidents* and *World register of dams* as well as numerous bulletins covering specific subjects, is available from the central office of ICOLD in Paris. In the UK, information may be obtained from the Institution of Civil Engineers, where meetings are held by the British National Committee of ICOLD.

A few of the numerous publications on dams are listed in the bibliography. Innumerable papers relating to dams can be found in the *Proceedings* of civil engineering institutions throughout the world.

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19

Loadings

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19.1 Loading

The process of structural design usually leads to some criterion of acceptability based on comparing the maximum predicted action of loads with an assured value of structural resistance. The assessment of the loading is thus as important as the structural analysis proper, although it has tended in the past to receive much less critical attention. This lack of attention has been fostered by a tendency for design loadings to be specified by clients or by governmental authority in broad terms to a degree of rigidity that leaves little freedom of choice to the designer.

Virtually all structural loadings are subject to some degree to statistical uncertainty; in other words, the maximum load that will act on any given structure during its life cannot be precisely known in advance, even if the probability of exceeding any particular value is known. In conjunction with the statistical uncertainty in the actual strength of any structure, the problem of safety is essentially probabilistic: a satisfactory design is one that limits the chance of occurrence of a load exceeding the actual strength to an acceptably low value (indeed, probably very small indeed, so that both its calculation and assessment of its significance are rather difficult), but does not make this event strictly impossible.

The format advocated by the International Standards Organization[1] suggests identification of a 'characteristic load' based on a statistical description of the maximum value of the load to occur in the design lifetime. The characteristic load may be taken as the expected maximum value (i.e. the mean of the values that might be observed in the life of an 'ensemble' of structures of the given type), or preferably as a value having a lower chance of occurrence, such as the expected maximum plus (say) one standard deviation of the variation across the ensemble. The characteristic value is then augmented by a partial safety factor to produce the 'design load' (F_d, say). It is often not easy to assess the relationship of these values to a 'nominal load', e.g. the value to be quoted on a notice of permitted loading. Indeed, the term 'nominal load' is best avoided, as international usage permits it to be applied to a very arbitrarily-assessed base value.

The check relevant to any limit state is then applied, taking account of the design load, a similarly-defined design material strength (f_d, say; characteristic value divided by a partial factor for the given material), and other factors. Typically, for the ultimate limit state the design condition can be separated into an action effect function S and a resistance function R and expressed by:

$$\gamma_n S(F_d, a_d, \gamma_{Sd}) \leqslant R(f_d, a_d, \gamma_{Rd}, C) \quad (19.1)$$

where γ_n is a factor reflecting the importance of the structure and the consequences of failure; γ_{Sd}, γ_{Rd} are coefficients reflecting uncertainty in the relationship of the loads effect to the loads, and the structural resistance to the material strength ('model uncertainties'); a_d are geometric parameters of the structure; and C are any additional constraints operative. (Subscript d signifies 'design' values.)

Unfortunately, these definitions are often difficult to apply, and there are considerable variations of interpretation and application among current codes of practice. The British Standard for dead and imposed loads on buildings,[2] which covers an extremely wide range of occupancy loadings for buildings, including stadia and car parks, does not explicitly consider the probability level, or frequency of occurrence, of the loadings specified. The corresponding standard for wind loading on buildings[3] gives the parameters of a statistical extreme-value distribution that can be used to estimate the strength of the storm having any required low probability of being exceeded during the design life of the structure. It should be noted, however, that this is not the only source of variability or uncertainty in wind loading; further uncertainty is introduced in making allowance for the effect of terrain (ground roughness) and topography (ground contours) modifying the basic storm wind speed, for the various effects of gusts and in estimation of force coefficients, etc. It is common to base design on the storm having a return period equal to the design life of the structure. It is a property of the postulated extreme-value distribution that the probability of this value being exceeded at least once in the design life is 0.63; the load defined in this way is thus a statistically based characteristic value, but one having a rather high probability level. This approach is followed in the British Standard specification for loading on lattice towers[4] which has the additional feature of indicating variation of the associated load factor according to the function and consequences of failure of the tower.

Recent UK practice for traffic loads on bridges has been to specify[5] values having a relatively low probability of exceedance, although other countries (notably the North American specifications, which also have wide international influence) commonly refer to 'nominal' vehicles or arrays of vehicles. Thus, despite efforts at harmonization,[1] it remains necessary to caution the reader that load specifications are not freely interchangeable between the structural design specifications with which they interact, and great caution must be used to interpret the probability level (sometimes explicit, but still more commonly not explicitly stated) associated with each loading specification.

The objective in this chapter is to give some guidance on the fundamental characteristics of various types of loadings. No attempt is made to summarize specifications in detail, nor to give densities of building or other materials such as would normally be found in a data handbook, but rather to present background material and to highlight features where a lack of appreciation of fundamental characteristics could lead to misuse of specified values. Shortage of space has prevented discussion of certain difficult specific problems, such as the probability of simultaneous occurrence of high wind loading and ice accretion on slender structures. On the problem of snow loading, a comprehensive British Standard[6] has only recently appeared, which has a format compatible with the wind loading.[3] The British Standard for agricultural buildings[7] also makes useful recommendations in this field, and the French specification (Règles Neige–Vent) includes a helpful commentary.

19.2 Occupancy loads on buildings

The floor loadings specified for office or residential buildings have remained little changed for many years and are undoubtedly based as much on experience that the accepted values lead to a satisfactory level of safety as on detailed knowledge of actual loads in service. Two major surveys, covering office and retail premises respectively, are, however, now available.[8] The raw observations of actual weight loadings in the office premises were first used to determine the actual average load over notional 'bays' of various sizes (irrespective of the real structural systems of the floors concerned); the relative frequency of finding a 'bay' to be subject to any given load is shown by Figure 19.1, for three selected sizes of bay. The most remarkable characteristic is the wide range of loads observed, even when the average is taken over quite a large region of floor. The average observed value (including personnel or other 'mobile' loads) was about 0.62 kN/m^2, leaving lowest basement floors out of account, but loads considerably in excess of 2.5 kN/m^2 were observed, even among values averaged over bays of 100 m^2.

Values of load having 99% and 99.9% probability of not

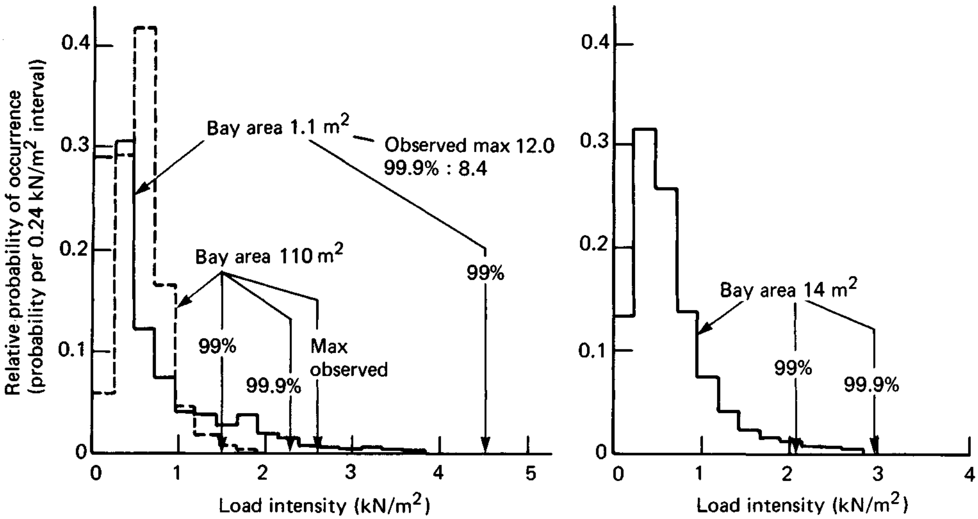

Figure 19.1 Observed local intensities in office buildings (ground floors and basements excepted)

being exceeded in the design life (which has been taken as equivalent to twelve complete changes of load such as would occur on a change of occupancy of the premises) for office premises taken from the International Standards Organization, document DIS 2394, are given in Table 19.1.

Table 19.1 Average load intensities in office premises corresponding to 99% and 99.9% probabilities of not being exceeded with 12 changes of occupancy (excluding ground floors and lowest basements)

Area of bay (m²)	1.1	5.2	14	31	111	192
Load for 99% probability (kN/m²)	9.4	4.3	3.2	2.6	2.15	1.7
Load for 99.9% probability (kN/m²)	17.4	5.3	4.3	3.5	3.2	2.3

The UK Code BS 6399 Part 1 (1984)[2] specified 2.5 kN/m² for general office premises, with a moderate reduction permissible when designing beams or further members supporting areas greater than 40 m². It is difficult to relate the above results to the standard format, as the sensitivity of the values to the probability considered is such that the usual partial load factors applied to *strengths* would no longer adequately fulfil their role of assuring a consistent level of safety between different structural types or materials. Furthermore, the dependence of this sensitivity on the size of bay suggests that the partial factor applied to the load would have to vary with the area. This problem may prove to be better treated by a more advanced probabilistic specification format.

Broadly, however, simple replacement of the existing code values by the 99% probability values with an appropriate reduction of partial load factor (say 1.25 in place of 1.6) would give a more rational balance of safety against size. It should be noted that a moderate improvement of safety with increasing size is desirable, in view of the likelihood of more serious consequences following the failure of a large bay. The variation of design load with bay size would thus be much more than hitherto accepted. The extent to which the average load intensity on any bay should be modified according to the shape of the influence function for the structural effect under consideration, to allow for local concentrations of high intensity within the bay area, is also discussed in the reference quoted.

The office occupancy load survey also permits some general observations on the nature of the load. The occurrence of high values of loading was commonly associated with shelving, often in conjunction with filing cabinets. The relatively frequent change of building occupation is an important factor, and it is suggested that it is unwise to assume that these heavy items will be restricted to particular floor zones throughout the life of a building. The loads resulting from computer equipment have been shown not to require special consideration.

The survey of retail premises by Mitchell and Woodgate mentioned above shows a clear distinction between sales zones and non-sales zones; the latter were particularly important in food retailing, amounting to roughly half the area of such premises and subject to much heavier loading than the actual sales zones. Books and ironmongery also showed heavily laden storage areas, but with these exceptions the distinctions between trades were not very important. Taking all trades together, the result obtained for the load intensity having 99.9% probability of not being exceeded in fourteen changes of the 'fixed' loads, including an allowance for the weight of persons in the bays concerned, is shown in Table 19.2.

Table 19.2 Average load intensities in retail premises corresponding to 99.9% probability of not being exceeded with fourteen changes of occupancy irrespective of trade

Area of bay (m²)	1.1	5.6	15	28
Load on sales areas (kN/m²)	9.1	5.4	4.0	3.3
Load on non-sales areas (kN/m²)	18.2	10.8	7.7	6.3

The statistical variability was rather less wide than in the case of office premises, and the reduction of load intensity as the area increases was also less marked; for sales zones there was virtually no further reduction beyond 28 m² area, but insufficient evidence was available for larger areas in non sales zones.

For buildings for special purposes the lessons of the possibility of wide statistical variation should be borne in mind and in particular the possibility of change in use, unless the structural layout imposes positive constraints on the use that would ensure qualified professional consideration being given prior to any major change. One important practical example of such constraint is the multi-storey (or any other roofed) car park, where restricting headroom to approximately 2 m effectively ensures that vehicles heavier than private cars are excluded.

19.3 Containers for granular solids

The general problem of forces in a body of granular material is more appropriately classified in the field of soil mechanics rather than loading, but some important special factors can occur, particularly in the form of large transient forces or dynamic effects during the discharge of material from bunkers or silos. The terminology is somewhat imprecise, with no strict distinction between these two terms; both can be described as bins (a common usage in the US). A hopper may be a container with inclined walls only (i.e. an inverted cone or pyramid), or the section with inclined walls forming the base of a parallel-sided bin.

It is also necessary to distinguish between 'mass flow' and 'funnel flow' when discharging. In mass flow the movement of material towards the outlet is uniform across the cross-section with the exception of a fairly localized 'boundary layer' adjacent to the walls, whereas in funnel flow movement is localized in a relatively narrow pipe or core which is replenished from the top. The former behaviour is often called for when storing perishable material to ensure that material is discharged in approximately the same order as it was loaded, but has the disadvantage of being associated with considerable increases during discharge in the loads acting on the walls of deep bins.[9] These increases are imperfectly understood; they appear to be rather inconsistent, and although often referred to as dynamic loads they are not generally true inertial effects.

The most common basis for design of deep bins is the theory of Janssen, the horizontal load p_h at a depth h below the free surface being related to a material parameter k by the equation:

$$p_h = \frac{\rho R}{\mu'}(1 - e^{-h/c}) \qquad (19.2)$$

where ρ is the density of material, R the ratio of area to perimeter of bin cross-section (one-quarter of diameter for circular bin), μ' the coefficient of friction of material on wall, and $c = R/k\mu'$.

In Janssen's derivation, k is the ratio of the horizontal pressure to the vertical pressure in the active state, given by $k = (1 - \sin\phi)/(1 + \sin\phi)$, ϕ being the angle of internal friction of the material. This angle may in practice be somewhat less than the angle of repose; some values are given in Table 19.3. The coefficient of friction of these materials on a concrete wall is about 0.5, rather less on a steel wall. Experimental results generally imply a rather lower value of k than given by the above (i.e. smaller loads near the top but asymptotically the same at greater depths). The larger value $k = 0.5$ suggested by Reynolds would seem to include some allowance for the dynamic effects, although the increase of load thus predicted does not entirely conform to the description that follows.

Table 19.3 Granular materials

	Angle of friction, ϕ	*Density, ρ (10^3 kg/m^3)*
Gravel	35°–45°	1.6–2.2
Coal	20° (fines) to 40° (washed coal)	0.9
Grain	30°	0.5 (oats) to 0.8 (wheat)
Cement	10°–18°	1.4

The dynamic effects during discharge may increase the effective horizontal loading by a factor as great as 3, and many failures have been reported as thus caused. According to Jenike[10] (who also gives an excellent bibliography) the explanation is that the condition at rest is approximated by the 'active' state with the major pressure nearly vertical (as in Janssen's theory), but that on withdrawal of material from below there is vertical expansion producing a 'switch' to a 'flow' condition with the major pressure nearly horizontal. This approximates to the passive pressure state, corresponding to arching across the bin. Once this state is established each 'arch' has only to support its own weight and the horizontal loading is again similar to the Janssen theory, but at the instant of 'switch' the top of the arching region has to give some support to the 'active state' material above, producing a very high horizontal loading locally. The vertical expansion required to cause the switch is very small, so that the switch generally propagates rapidly upwards and the strength provided must at all points cater for the corresponding concentrated load. At the time of writing these theories had not been fully verified and demonstrated. The Russian specification (see Jenike[10]) suggests that the basic value given by the Janssen formula should be doubled over the lower 65% of the height of deep bins to allow for this dynamic effect. The lower 15% of height of circular bins with flat floors are also exempted from the dynamic loading, because of the formation of a dead zone of inert material. The specification of the American Concrete Institute for grain silos requires allowance only where the outlet is markedly eccentric, a condition which can give rise to severe 'ovalization' loading; qualitative warning is given about dynamic effects in other materials.

Dynamic effects are relatively small in 'funnel flow', but the design features necessary to ensure funnel flow are not fully established. Funnel flow is assured if the depth does not exceed 1.3 to 1.5 diameters, the lower limit applying to grain silos. A perforated tube or a lattice tower placed over the discharge orifice also prevents the type of mass flow that can cause dynamic loading. Projecting circumferential fins on the inside face of the wall are of rather less certain action. The problem of very fine materials such as cement has been discussed by Leonhardt *et al.*[11]

19.4 Road bridges

Probabilistic considerations are also important in traffic loading on road bridges. Where the loaded length is sufficient to admit more than one loaded vehicle, the governing condition will occur when the traffic is brought to a standstill, minimizing the separation between vehicles. The maximum load intensity of heavy goods vehicles is very much larger than the load intensity of light goods vehicles and private cars, so the effective loading is strongly influenced by the degree of 'dilution' of the heavy vehicles. There is generally a clear gap of vehicle weights between about 25 kN and 50 kN; for census purposes, the UK licensing limit of 3 t unladen is convenient (above this limit, 'heavy goods vehicles', 'HGV'), and 12 000 lb laden (53 kN) is common in North America.

Recognition of these factors led to the British HA loading specification. When formulated in 1954,[12] it was presumed on the basis of common experience that stationary traffic implied traffic congestion and thus in turn a time of high traffic demand

and consequently a high proportion of light vehicles. Three maximum load-intensity vehicles (at that time, 24 t four-axle vehicles about 7.2 m long for bulk loads such as heavy liquids or powders) were considered to occur consecutively followed by a greatly diluted vehicle sequence, giving a lane load falling from about 30 kN/m to about 5 kN/m. Specifications in other countries followed similar lines, albeit with a less dramatic decrease of load intensity with increasing span.

When the Motor Vehicle (Construction and Use) Regulations were amended in 1964 to permit total vehicle weights exceeding 24 t, *minimum* permissible values of wheelbase were specified for such vehicles, such that the heaviest (32 t) vehicles would give a load intensity of about 25 kN/m. This was designed to protect bridges against overloading. However, it rapidly became apparent that very large numbers of maximum-weight vehicles were being put into service, and the totally revised British Standard for bridges issued in 1978[5] included a modification to the HA loading such that the lane load for very long spans did not fall below 9 kN/m.

The Construction and Use Regulations were further eased in 1978 and, more significantly in terms of loading, in 1982 (effective 1983) when the maximum permitted vehicle weight became 370 kN (38 t) for a vehicle of length between 12 and 15.5 m. It was no longer possible to require (or, indeed, to permit) a corresponding increase of vehicle length. A substantial proportion of the total traffic can now be expected to contribute 25 kN/m, and several vehicle configurations are permitted to reach 30 kN/m, to which allowance may be added for overloading.

Recent re-examination of the bridge specifications, backed up by quantified statistical reliability analysis, has drawn attention to the possible importance of obstructions to traffic flow that may occur even at times of low flow, e.g. due to inadequately secured loads. The increased journey speeds permitted by motorways have greatly reduced the motivation to night-time travel, given the relatively short travel distances within the UK, and the residual night-time traffic commonly has a very high proportion of large heavy vehicles. Examples have been recorded on British motorways of 1-h traffic counts in the very early morning with well over 80% heavy goods vehicles, and thus over 90% heavy goods vehicles by length in any stationary queue that might have formed from this traffic. Furthermore, such 'off-peak' traffic has a significantly higher proportion of loaded vehicles than the long-term average. The current *average* weight of a four-axle articulated vehicle in these circumstances is over 20 kN/m. Taking both traffic constitution and the probability of blockage into account, attention has now focused on the period around 6 a.m., when larger flows occur, but still with typically 60 to 70% heavy goods vehicles.

As a result of this reappraisal, the basic unit lane load

$$W = 36/L^{0.1}\ (\mathrm{kN/m}) \tag{19.3}$$

has been put forward for loaded lengths exceeding about 50 m, where L is the loaded length in metres. At (for example) $L = 400$ m, this gives 19.8 kN/m, which is more than twice the value given by the existing BS 5400 or the preceding BS 153. A coexistent point load ('knife-edge load') of 120 kN is proposed (unchanged), which now has rather little influence on the total load effect. A value of this order would be specified for new designs for the British Department of Transport. However, the probabilistic description of the maximum load events now envisaged differs considerably from that considered when the various load factors were selected for the existing specification, and the final outcome remains to be seen.

For short loaded lengths, the governing case in the UK has hitherto generally been the HB loading, representing special-purpose vehicles which travel under supervision. It may, however, prove necessary to include a new approach to the HA loadings for short lengths in future specifications.

Recent thinking in North America has recognized similar trends, although the result has not yet been as severe as outlined above. Vehicle weights have shown the same inexorable upwards trend, and great concern has also been expressed about vehicle overloading. The latter is perhaps exacerbated by discrepancies between individual state/province regulations, and widespread use of 'citizens' band' radio has impeded enforcement. It has been suggested by Buckland *et al.*[13] that a maximum truck weight of 530 kN should currently be considered. In terms of current influence and importance, the 1979 Ontario Bridge Code should be cited, together with the AASHTO Standard Specification for Highway Bridges as revised in 1977. Buckland *et al.* give much background to recent proposals by the American Society of Civil Engineers.[14] It appears, however, that the proportion of vehicles approaching maximum weight is relatively low in North America, and the postulated overall average weight of the heavy vehicle component of traffic is only about 10 kN/m. The overall average of heavy goods vehicles in the UK is similar, but the weights in critical conditions range much higher, as noted above.

Buckland *et al.* also define clearly their assumptions concerning the frequency of traffic blockage and on the behaviour of traffic when only a fraction of the carriageway width is obstructed. These factors have a strong influence on the appropriate value of loading for design (including the relevant load factors), especially when the diurnal pattern of variation of traffic is taken into account, but there is very little published information. In the authors' opinion, this is the aspect of bridge loading most worthy of further studies.

Typical vehicle configurations in the UK together with the idealized vehicle used in the AASHTO specification, are shown in Figure 19.2. An international comparison of specifications has been published by the Transport and Road Research Laboratory.[15]

The fatigue-check count of load cycles, being to a first approximation a long-term average, is relatively insensitive to the time-of-day factors which have emerged in the reassessment of maximum loading discussed above. Excellent guidance on the cycle count for British traffic conditions is given in appendices to the British specification.[16]

Concern over the dynamic effects of traffic on highway bridges has perhaps receded in recent years. As already noted, the occurrence of the maximum total load on any span greater than about 20 m requires that the traffic shall be stationary. For shorter spans, allowance should be made for dynamic augmentation of the load effect. Some consideration of public reaction to perceived motion of bridges caused by the passage of traffic may also be desirable.

The dynamic action of the load is predominantly a question of the movement of the vehicle on its springs (or of the 'unsprung weights' of wheels and axles on the tyres); the dynamic effect of the addition of the weight of the vehicle *per se* arising from the time it takes to travel from the end of the span to somewhere near midspan is negligible, presuming the vehicle to be running smoothly on a smooth road. The excitation is therefore at a vehicle natural frequency; the fundamental is typically about 1.4 Hz for commercial vehicles, which corresponds to a suspension having a static deflection of about 150 mm. Only rarely is the fundamental frequency significantly lower than this, although this may not remain true if the trend continues towards self-levelling suspensions that can have a much larger equivalent static deflection; it might be considered possible for the vehicle to be resonant with a structural frequency within the range from 1 to 3 Hz, the latter value being limited to older or only part-laden vehicles. The natural frequency of the unsprung masses on the tyres is in the range from

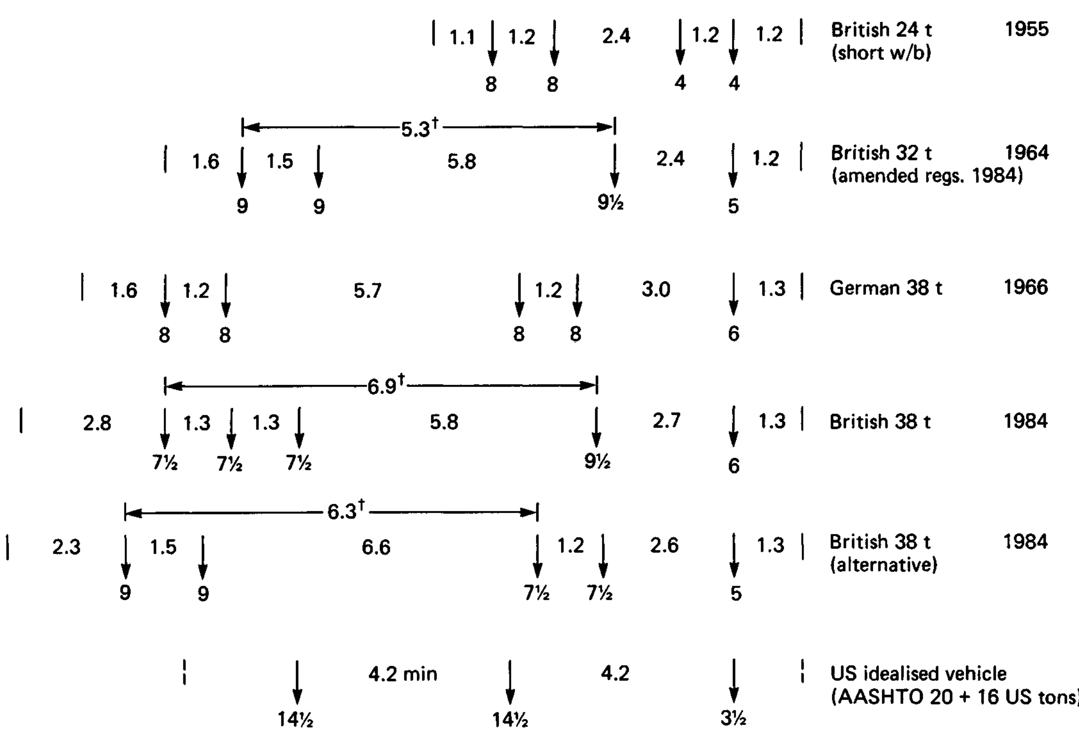

Figure 19.2 Typical road vehicles. Masses in tonnes. Dimensions in metres. Dimensions marked thus † are critical minimum values in regulations effective May 1984

8 to 14 Hz, above the range of basic frequencies of the whole bridge, but possibly significant for deck units.

It is useful to distinguish two classes of possible oscillation of the vehicle which then leads to excitation of the structure: (1) passage over a single severe road surface irregularity leading to a large vehicle motion which is then damped out by the vehicle dampers; or (2) a more random motion caused by the succession of small imperfections in the surface. The former is believed to be the governing factor on most bridges, associated usually with the joint between abutment and bridge. The first pulse (half cycle) of excitation to the bridge can be taken as applied at a distance from the bump equal to the distance travelled by the vehicle in one-quarter of the natural period, and subsequent pulses clearly progress across the span but rapidly diminish in amplitude; a vehicle damping of 15% of critical damping can be assumed, so that each half-cycle has an amplitude 0.6 of the preceding one. The amplitude of the first pulse can conservatively be taken from Table 19.4.

Table 19.4 Dynamic loading caused by single major surface irregularity

Speed of vehicle (m/s)	10	20	30
$\dfrac{\text{Amplitude of first load pulse}}{\text{Weight of vehicle}}$	0.25	0.4	0.6

This description of the dynamic excitation may be useful when it is desirable to assess a design with unusual dynamic parameters, or to assess the resonance effects on spans exceeding 20 m from the viewpoint of user perception. For the governing maximum total load effect on short spans, specifications are commonly based on a simple impact factor approach based on a generalization of practical experience ignoring the specific dynamic characteristics of the structure. The British specifications have hitherto included an allowance of 25% (impact factor 1.25) on the maximum effect of one axle load. A recent TRRL report[17] on measurements of short-span motorway bridges has indicated higher values, which are likely to be imposed on updating the short-span HA loading. In view of the sensitivity to vehicle speed (cf. Table 19.4), the HB load is not considered to be affected.

It remains to consider the possible reaction of users to any noticeable oscillation, especially if pedestrians have access to the bridge. A suggested approach is to consider the response of the system to a unit sequence of load pulses corresponding to a vehicle with suspension resonant with the bridge and thus to deduce the magnitude of excitation (measured by the amplitude of the first pulse) necessary to induce a response that would be considered unwelcome by a typical pedestrian, say, an oscillation building rapidly to an acceleration amplitude of 0.1 g, subsequently decaying at the relatively slow natural damping of the bridge. By comparison of the critical excitation with the nature of the expected commercial traffic density and its speed, Table 19.4 will enable an estimate to be made of the proportion of pedestrians using the structure who would regard the motion as unpleasant.

The kinematics of a person walking are such that the centre of mass of the body moves vertically over a range of some 30 mm during each pace. This clearly results in a cyclic variation of the vertical load imposed on the bridge deck, with an important Fourier component at the walking-pace frequency. This is not large enough to have a serious effect on a massive highway bridge, but requires serious consideration for light, long-span footbridges. As there is a positive tendency for walkers to synchronize their pace with perceived motion of the deck such as to augment the motion, if unfavourable response is possible it is likely to be developed fairly often and to constitute a significant problem of acceptability to the user.

A good design check procedure is given as Appendix C to the current British Standard.[5] This is based on an exciting force of amplitude 180 N irrespective of frequency: the author believes that it would be worth taking account of the typical variation of walking kinematics with pace as suggested in Table 19.5.

Table 19.5 Footbridge dynamic excitation

Walking description	*Frequency* (Hz)	*Pace* (mm)	*Force amplitude* (N)
Leisurely	1.6	870	140
Brisk	1.9	970	240
Very hard	2.2	1040	370

Because this problem is associated with substantial resonant build-up of response, adding damping may be an effective and economical counter-measure. Friction dampers have been found to be useful, probably because they result in a change of resonant frequency with amplitude which may not be followed by the walker, losing synchronism. The possibility of wilful stronger excitation should also be considered, particularly to ensure that the structure is adequately located on its supports.

19.5 Railway bridges

Train weights and loadings are generally closely under the control of the owner, and a single train can extend to cover almost any loaded length of practical interest. The statistical variability of the estimate of the maximum loading that will occur on any specific structure is thus relatively small, but because trains crossing the bridge at speed may frequently approach close to the maximum weight, dynamic effects are important.

Design loadings may be in the form of an idealized train, specifying axle loadings and spacings (the body of the train apart from the locomotives is commonly taken as a uniformly distributed load (UDL)); or as an equivalent UDL tabulated as a function of span. In either case, most existing specifications are based on steam-locomotive practice and somewhat outdated freight vehicle types, with the weight per unit length of the locomotives about twice that of the trailing vehicles. With modern traction, the disparity between locomotive and trailing load intensities is much less, although when two diesel locomotives run coupled together a very sharp load concentration arises from the two bogies coming adjacent to the coupling. Typical modern rolling stock for European standard-gauge railways is illustrated in Figure 19.3.

The UK specification[5] now includes, as the RU railway loading, the recommendations of the International Union of Railways (UIC) for the European Region. This is clearly described, with diagrams of the governing vehicles, in Appendix D to the specification. Loadings for urban systems, including light rail transit (LRT), are usually prepared on the basis of the anticipated rolling stock. Allowance should be made for maintenance traffic and for future changes. The British Standard gives an RL loading based on London practice as an illustration.

Dynamic effects on railway bridges were fully studied in Britain during the 1920s, with particular reference to the 'hammer blow' caused by steam locomotives. The report[18] is an excellent exposition of the factors involved, although large unbalanced reciprocating masses are a thing of the past and big changes have also affected the relevant bridge parameters; natural damping is commonly much lower, and resonant (natural) frequencies have also fallen.

A more recent investigation focused on the ratio of the peak measured bending stress to the corresponding value calculated statically from the nominal train weights, for a large number of short spans in Britain,[19] gave results summarized in Figure 19.4. The most important dynamic excitation here was probably bouncing or other oscillation of the rolling stock on its suspensions; some departures of the actual train weights from the nominal values are presumably included in the results shown.

Another dynamic excitation that has received much attention in recent years as train speeds have increased but bridge natural frequencies have fallen is the rate of application of the load to the span; it may be that somewhat excessive attention has been paid to this factor, taking note that the trend to increase train speeds appears now to have fallen away. On the other hand,

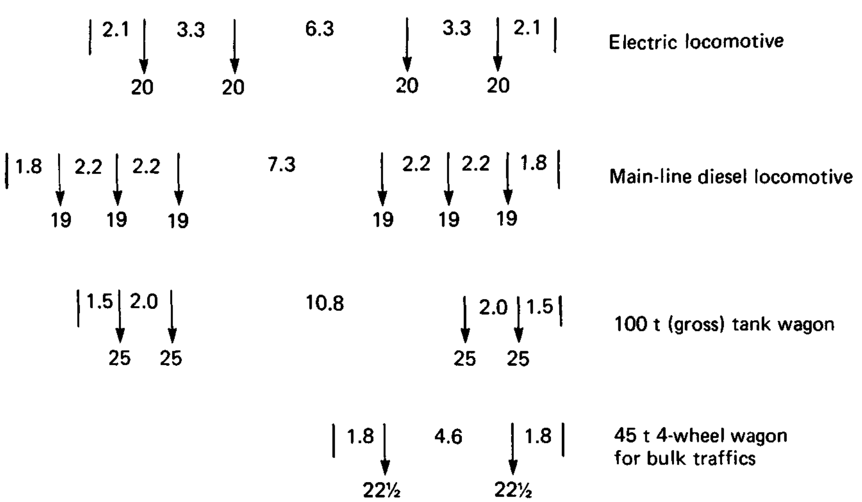

Figure 19.3 Railway vehicles. Masses in tonnes. Length and spacing in metres

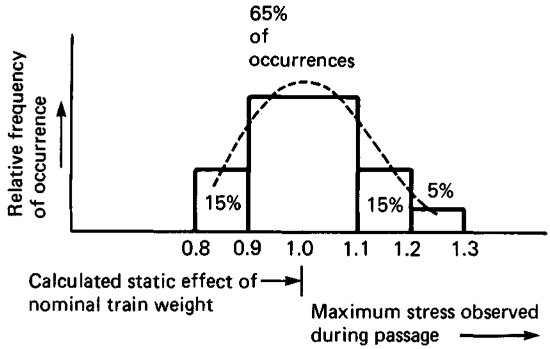

Figure 19.4 Histogram of measurements of maximum stress caused by passage of train (or single vehicle) at speed, compared with the nominal effect of the load

assumptions of improved track and vehicle suspension may have been overoptimistic, when the whole stock over a long period of time must be covered. The normalized speed parameter

$$K = V/2nL$$

where V is the train speed, n is the bridge natural frequency and L is the span, compares the time over which the load builds up (presuming the train to be longer than the span) with K being one-half of the natural period of oscillation. The UIC has recommended[20] the formula:

$$\phi' = K/(1 - K + K^4)$$

as an empirical bound to this effect, where ϕ' is an impact factor such that the maximum total bending moment is $(1+\phi')$ times the maximum static value. Two-thirds of this value is taken for design shear force. The gross loading recommended by the UIC is an envelope comprising this impact factor, with the addition of a term making allowance for vehicle dynamic response to track irregularities (in two grades), for a variety of trains having different maximum speeds according to type, and bridge natural frequencies in the practical range of current construction. The British Standard RU loading is based thereon.

19.6 Wind loading

Wind is by its very nature a dynamic loading. It is obvious that the gustiness always noticeable in strong winds will cause a fluctuation of the loading; but in addition to this action, aerodynamic instability of the flow pattern round the structure, or interaction between motion of the structure and the flow, may cause periodic fluctuation of the loading that can result in serious oscillation of the structure. The gust action is most important as regards excitation in the downwind direction and increases rapidly with wind speed, so that this is normally the action governing the 'static' strength required. The instabilities usually cause maximum motion perpendicular to the flow, and may have their most serious effect at relatively moderate wind speeds, so that these may be most important in respect of fatigue damage, comfort of occupants, or in some cases deflection serviceability criteria. The instability excitation is usually only significant on slender structures, but in both cases it is generally true that reducing either the natural frequency or the natural damping markedly increases the risk of serious dynamic response. Current trends in design are thus forcing designers to pay much more attention to these problems.

The basis for calculation of the required strength is the equation:

$$p = \tfrac{1}{2} C_p \rho V^2 \qquad (19.4)$$

in which p is the pressure on the structure (e.g. N/m^2), ρ is the density of air, 1.23 kg/m^3, and V is the wind speed (e.g. m/s).

C_p, the 'pressure coefficient', is dependent on the shape of the body. A complete analysis thus requires: (1) analysis of local meteorological records and extrapolation to determine the strength of wind having a given low probability of being exceeded; (2) a 'model' of the gusts, involving definition of their fluctuation in space (area of influence of any one gust) and time (dynamic effects); (3) knowledge of the pressure coefficient for the given shape; and (4) dynamic analysis of the structure to determine the maximum value of stress in any selected structural element.

19.6.1 Meteorological data

The strength of the wind is usually conveniently expressed by its hourly mean value ($\bar{V}$, say), because at the peak of a major storm (very localized tornado phenomena excepted) the gusts can be treated as a 'stochastic' (random) process that is 'stationary' (having constant statistical properties although the instantaneous values at any point are changing) over such a period. Furthermore, in step (4) the fluctuations in the response prove to be sufficiently rapid that an hour provides a large sample, and the maximum reached in the sample is then relatively insensitive to the actions of chance: thus, to estimate the overall maximum response having a given probability of occurrence, take the hourly mean wind having that probability and multiply the mean response by the 'expected' value (average that would be found from a number of statistically similar samples) of the ratio of the peak gust response to the mean. The probability of a worse condition arising from a particularly adverse low-probability gust action in an hour of lower mean speed can be neglected.

The method usually adopted for extrapolation of meteorological records to predict the wind speed having the selected low probability of occurrence is to take the maximum values from each year of the available record, and fit a Fisher-Tippett Type-I extreme value distribution. Shellard[21] initiated the application of this method in the UK. The main difficulty is that a long run of reliable and consistent records is required, and serious distortion can occur if there is a systematic change within the duration of the record, e.g. due to change or resiting of the anemometer or even to change of its exposure.

Important progress has recently been made in improving the application of the extreme value distribution by careful correction of the raw data from meteorological stations for variation of the terrain and/or topography as a function of wind direction, by optimization of the number of storms that can be regarded as contributing usefully to the extreme value distribution (i.e. including more than simply the single biggest value each year) and by application of Lieblein's method of parameter estimation.[21]

The high-level winds which are basically dependent on the synoptic meteorology are greatly modified in their influence on any practical civil engineering structure by the roughness of the ground, averaged over many kilometres of the approach of the wind. Indeed, in strong winds the gustiness is regarded as wholly caused by the mechanical disturbance of the flow by ground obstructions.[22] Ground roughness is assessed empirically by reference to qualitative descriptions of the terrain; the usual parameter by which this is expressed is now generally the *roughness length* z_0, although this can readily be related approximately to the power law index α which is more familiar to most practising engineers (see below).

Meteorological records are usually corrected to the value applicable at a height of 10 m above ground, and are most often from sites in open terrain. British specifications are now being based on a typical inland open terrain with hedgerows and scattered trees, $z_0 = 0.03$ m. A logarithmic formulation is preferred for scientific purposes to describe the variation of wind speed with height, but the simple empirical power law remains useful and convenient:

$$\bar{V}_z = \bar{V}_{10}(z/10)^{\alpha} \tag{19.5}$$

in which $\bar{V}_z$ is the hourly mean speed at z (m) above ground and α is the power law index.

The wind speed at height 10 m as a function of terrain can be expressed by a factor R, such that:

$$\bar{V}_{10} = R\bar{V}_B \tag{19.6}$$

where $\bar{V}_B$ is the value for the basic open terrain. Gustiness is expressed by the root mean square value $\sigma(V)$, or by the intensity of turbulence $I = \sigma(V)/\bar{V}$; suffices are used to indicate height above ground.

A concise summary of these terrain-dependent parameters is given in Table 19.6. Further guidance on terrain classification is given in British Standard Specification 8100,[4] Engineering Sciences Data Unit items 82026 and 83045,[23] and items 74030/1 and 75001.[24] For terrain Classes IV and V the height should be measured from a substitute datum, 2 and 10 m respectively above the actual general level of the ground surface. It is the author's opinion that designers should not assume values based on any greater roughness (e.g. for city centres), because the terrain in such locations is inevitably 'heterogeneous', i.e. the roughness cannot be regarded as uniform over sufficient distances to achieve the equilibrium statistical pattern of turbulence and the flow incident to any particular structure will be influenced by specific neighbouring features.

Table 19.6 Terrain parameters

	Category and description	z_0 (m)	α	R	I_{10}
(I)	Sea coasts	0.003	0.13	1.2	0.15
(II)	Open country, exceptionally few obstacles	0.01	0.14	1.1	0.17
(III)	Basic British terrain; arable farmland with some hedges and isolated trees	0.03	0.165	1.0	0.19
(IV)	Farms with small fields, many hedgerows, trees	0.1	1.19	0.86	0.21
(V)	Extensive suburbs or mixed forest	0.3	0.23	0.72	0.25

Early descriptions of gust structure[25,26] suggested that $\sigma(V)$ was invariant with height. The most systematic investigation to date of gust structure, sponsored in the UK Construction Industry Research and Information Association (CIRIA) with results published in 1981,[27] showed that over a substantial part of the height range of practical importance, $\sigma(V)$ decreases with increasing height above ground. For very tall structures this is worth taking into account by use of the detailed results cited. The values shown above apply when the wind is sufficiently strong that mechanical mixing breaks down any thermal effects, say $\bar{V}_{10} > 10$ m/s in British latitudes although possibly higher where solar radiation is stronger.

Topographic features have a substantial effect on wind speeds near the ground. Near the crest of hills (ridges, escarpments) the rate of decrease of wind speed with decreasing height (cf. parameter α) is greatly reduced. This may considerably increase wind loads in such locations, and reference should be made to the 1985 amendment to the general UK Code[3] or to the UK specification for towers.[4] Broadly, these presume that the hourly mean speeds are increased but that the superimposed gusts are unchanged by the topography. Earlier design proposals giving only a modified effective ground-level are non-conservative and should not be used.

The maximum instantaneous value of wind load on a large structure is dependent on the correlation of gust speeds over the area of the structure, and possibly on some time-averaging to allow for an approximation to the steady flow pattern to be established. The effective correlation of gust speeds is a function of the size of the structure relative to the cross-wind integral scale(s) of the turbulence, which express the effective dimensions of gusts in the horizontal and vertical directions. The corresponding along-wind scale parameter is also important; this can be expressed as a length (L_1 is a common notation) or as a timescale (T, say) on the basis

$$L_1 = \bar{V}T \tag{19.7}$$

The appropriate practical values of these scale parameters are still subject to argument. There has been a progressive increase in the values put forward as representative of very strong winds, and it seems likely that consensus may be reached on values not greatly different from those discussed in Papers 4 and 7 of a report by CIRIA.[27] The timescale (T) is thus about 8 s at 10 m above ground, increasing upwards approximately in proportion to $\bar{V}$ for the terrain roughness in question.

The computation of the effective correlation of gust action should also take account of the shape of the structural influence line for the load effect in question, which expresses the relative sensitivity to gusts affecting various parts of the structure. The result is an 'aerodynamic admittance' (J, say) which in its simplest form expresses the root mean square (r.m.s.) fluctuation $\sigma(F)$ of some load effect F by reference to $\sigma(V)$ and the respective mean values $\bar{F}$, $\bar{V}$:

$$\frac{\sigma(F)}{\bar{F}} = 2J\frac{\sigma(V)}{\bar{V}} \tag{19.8}$$

The factor 2 reflects the quadratic relationship between wind force and wind speed. This relationship becomes more complicated for vertical structures because of the changes in the respective parameters with height, but the additions have little effect on the practical results. Results are given in Paper 7 of the report by CIRIA.[27]

Such complete solutions may be required for structures with irregular or otherwise special influence lines, e.g. where the wind on some parts of the structure normally acts to reduce the net wind load effect, as may occur in the torsion of tall buildings, and in some bracings of towers of 'Eiffelized' profile. The UK specification for towers[4] provides design charts based on these complete solutions. For many cases, simpler generalized design formulations are sufficient,[28,29] commonly based on the assumption of a 'triangular' influence line, such as applies for the wind moment on a cantilever structure. The static response considered thus far constitutes the 'background excitation' in the terminology of these references.

A further simplification can be made by visualizing gusts as eddies carried along by the mean flow. The size of a gust will thus be in proportion to its duration measured at a point in free flow. The UK Code[3] has used this basis for many years, computing the equivalent loading for structures of dimension exceeding 50 m by using the 15 s gust wind speed. The 15 s gust is approximately equal to the mean speed plus $2.2\sigma(\bar{V})$. The increase of the consensus estimate of the gust scale parameters referred to above means that this may give a somewhat low estimate for slender lattice or 'line-like' structures, although remaining acceptable for 'solid' structures where the correlation effects are multi-dimensional.

To include the effect of fluctuation with time, recourse is made to power spectrum analysis, and the wind speed is subjected to a form of Fourier analysis, using an integral transform in place of the familiar Fourier series. In this way, the speed is represented not as a series of discrete harmonic components each having an identifiable amplitude, but as an integral of infinitesimal components over a continuous range of increments of frequency. This does not imply that identifiable periodicity exists in the wind, but does give a measure of the extent to which a structure of any given natural frequency would pick-up excitation. The wind speed spectrum $S^{v}(n)$ is found to have a universal shape (in the appropriate nondimensional form) for any height or terrain, as shown in Figure 19.5. A power spectrum portrays the distribution of the *square* of the quantity considered with frequency, so the units of $S^{v}(n)$ are $(m/s)^2$/Hz: to cover the wide frequency range of the natural wind a logarithmic scale of frequency (n, say) is preferred and $nS^{v}(n)$ is then plotted so that areas on the plot retain their significance as the distribution of the square of the gust speed fluctuations, $nS^{v}\,d(\log n) = S^{v}\,dn$. The use of the spectrum has been explained by Davenport[25,28] and space here permits only to point out that the spectrum can be operated upon by frequency-dependent functions expressing the correlation of the gusts over the structure and the dynamic magnification in terms of response in each natural mode of the structure. The correlation of any infinitesimal frequency component (measured by the 'normalized co-spectrum') follows similar rules to the correlation of the gross gust speed except that the characteristic longitudinal dimension is now the wavelength corresponding to the given frequency, $\bar{V}/n$. The effective crosswind dimension of frequency component n (twice 'lateral scale') can be taken as about $2\bar{V}/9n$.

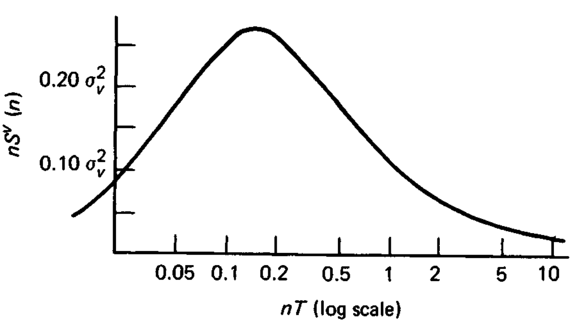

Figure 19.5 Wind gust speed spectrum. Universal nondimensional form for strong winds. S^{V}, power spectrum of wind; σ_v^2, variance of wind speed due to gustiness; n, frequency (Hz); T, time-scale of turbulence

Inspection of Figure 19.6 and the substitution of dimensions, such as for a typical example $\bar{V} = 25$ m/s, lowest natural frequency $n_1 = 0.5$ Hz, size of structure 50 m, shows that the resonant frequencies lie well on the diminishing upward tail of the gust spectrum and that the 'resonant gust' is small (here $2\bar{V}/9n = 11$ m) compared with the size of the structure. For most cases the dynamic effect is not large and is deemed to be covered by the load factor, but it is clear that the effect is sensitive to the value of frequency in relation to size, as well as to structural damping, and investigation is advisable if either of these factors is suspected to be lower than is usual. If such study is made it is the opinion of the writer that a 10 to 20% trade-off from the load factor is justifiable. A study of this kind (or better) is obligatory in Canada for buildings exceeding 120 m in height. For structures of simple shape and simple dynamic first mode shape, tabulated solutions are available;[28,29] these are based on a marginally different algebraic form for the spectrum, but also imply a smaller scale of turbulence than the values discussed above (it may be noted that Davenport's normalized independent variable $\tilde{n} = n\mathscr{L}/\bar{V}_{10}$ is equivalent to $12nT$ in Figure 19.6). This has the effect of increasing the predicted dynamic response.

The small size of the 'resonant gust' also permits drastic simplification of the 'exact' procedure, giving a straightforward explicit formulation for the variance of the resonant contribution to structural response.[27] The variances (mean square) of the static and resonant components are additive.

The power-spectrum method of analysis is directly applicable to some problems involving deflection or vibration amplitude criteria, although consideration should also be given to the possibility of oscillation perpendicular to the wind direction as discussed below, particularly for slender solid prismatic bodies, such as very tall buildings having a uniform cross-section in plan.

19.6.2 Force and pressure coefficients

Little has been said in the above about the pressure coefficient C_p or the corresponding force coefficients C_D, C_L that express the total force resolved into components parallel and perpendicular to the wind direction (or the 'body axes' $C_xC_yC_z$ maintaining constant direction as the wind direction varies). Several collections of these factors can be used for reference, including Hoerner's wide-ranging book,[30] the current UK and other national codes, Cowdrey[31] for modern bridge sections, Cohen and Perrin[32] for lattice masts, and the present author[33] for antenna structures. Very large numbers of ad hoc tests have been reported, notably by the National Physical Laboratory (Teddington, Middlesex, UK).

For prismatic shapes having sharp edges the coefficients are truly independent of wind speed in the practical range, whereas for rounded sections there is generally a sharp change in the flow pattern when the speed reaches a 'critical Reynolds number', which is usually about 10^5: in normal environmental conditions Reynolds number $= 6 \times 10^4 Vd$, where V is the speed in m/s and d the diameter in metres. The essential feature of the change is that the point at which the flow separates sharply from the surface moves towards the back of the body, accompanied by a reduction of drag in the so-called supercritical range. This change is provoked at a lower Reynolds number by roughness of the body surface, and by turbulence in the incident flow. At still higher 'postcritical' or 'transcritical' Reynolds numbers, notably corresponding to the design conditions for large cylindrical structures including chimneys, the drag coefficient increases once more, and this effect also is sensitive to roughness. The most refined procedure is to relate the force coefficients to an 'effective Reynolds number', computed to take account of these effects.[34]

Most wind-tunnel tests in the past have been conducted in a smooth flow, with the 'gustiness' reduced as far as possible. Turbulence has deliberately been introduced in relatively few cases, and its effect is by no means fully understood. The rear-face suction ('base pressure') is principally affected and in the case of prisms of substantial 'depth' in the downwind direction (square, or downwind greater than crosswind dimension) the result is to reduce the loading (care is necessary when reading test reports as an *increase* in the absolute value of base pressure

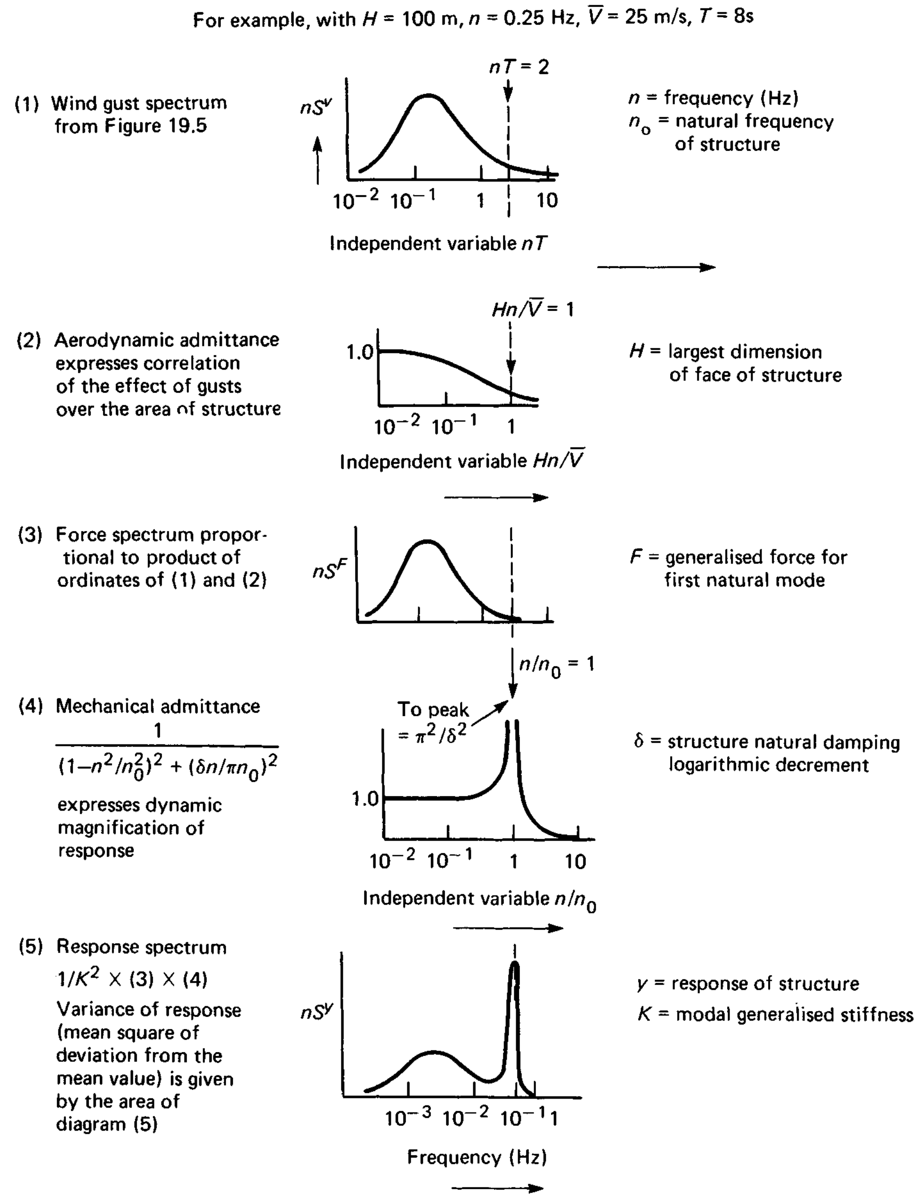

Figure 19.6 Illustration of steps in power spectrum analysis of gust action

corresponds to decrease of load): the substantial reduction in the force coefficients specified for *buildings* in the current UK Code[3] by comparison with earlier editions is in recognition of this. Conversely, for prisms of small downwind dimension (approaching a plate) the load may be slightly increased. These effects are unfortunately scale-dependent, as they are caused by the small rapid fluctuations corresponding to 'gust wavelengths' smaller than the cross-section of the structure. For small bodies such as members in lattice structures the corresponding components of turbulence in the natural wind may not then be sufficient to cause the reduction of coefficient referred to above, but it is not possible to reproduce the natural turbulence to a sufficiently large scale in a wind tunnel to determine the limiting conditions. A further reason for distrusting many early wind-tunnel results is that the effect of 'blockage' is much greater than was appreciated until about 1950; this is the increase in the apparent drag coefficient resulting from the constriction of the flow in the tunnel where it must pass the test body.

An important factor which is often misunderstood is the 'shielding effect', the fact that the drag force on (for example) a square lattice tower is less than the sum of the forces that would be predicted for the faces considered each in isolation. This is related to the flow 'seeing' the shape of the structure as a whole, and thus as the total 'density' of the structure increases, the incident velocities local to individual elements are reduced. It is thus not necessary that elements should be exactly 'shadowed'. Formulae for shielding factors commonly give the total load on two parallel frames as $(1+\eta)$ times the load for one frame independently, although the total load is in fact more nearly equally divided; for example:[35]

$$\eta = 1.0 - 1.7(\phi - 0.1\sqrt{(B/D)}) \text{ (with } B/12D < \eta < 1.0) \qquad (19.9)$$

where ϕ is the solidity ('shadow fraction') of one frame and B, D are the width and spacing of frames respectively (e.g. $B/D = 1$ for a square tower).

For lattice towers it is generally satisfactory to neglect the bracings in planes parallel to the wind direction when calculating the load for the case of wind direction perpendicular to one face. For square towers the maximum load component resolved perpendicular to a face is about 10% higher, and the diagonal-incidence load about 20% higher, than the normal-incidence value. The principal difficulty lies in assessing the 'ancillaries', the ladders, cable runs, etc. which commonly substantially increase the wind resistance. To a first approximation, if the tower face solidity exceeds 0.15, judgement can be used to estimate the fraction of the area of such elements that should be added to the basic windward face area. Detailed guidance is given in the UK specification.[4]

The suction on the rear face referred to above is much less in cases with a three-dimensional flow pattern (such as short prismatic structures where flow can pass over the end as well as the sides) than strictly two-dimensional flow past a very long prism. This can be expressed by an 'aspect-ratio factor', the aspect ratio being defined as the ratio of the longest dimension of the face presented to the wind to the lesser dimension (twice this value if flow can pass one end only, as in the case of a chimney) and in the absence of specific information this can be taken as reducing the overall drag coefficient by the ratio k_1, shown in Table 19.7.

Table 19.7

Aspect ratio	∞	40	20	10	5	2	1
Reduction factor k_1	1.00	0.90	0.80	0.72	0.67	0.63	0.60

It will be noted that the effect is important even for quite large aspect ratios. A small obstruction to end flow, such as the supporting post of a signboard, is not significant, but a larger obstruction such as the deck of a bridge covering the end of a supporting pier destroys the effect.

19.6.3 Wind-excited oscillation

The flow pattern passing a slender prismatic body commonly has unsteady features, often leading to the regular build-up of vortices in the wake which, on reaching a limiting size, are swept away by the stream. Unless this occurs simultaneously symmetrically on both sides (which is unusual) the result is a periodic fluctuation of the aerodynamic force in the crosswind direction. The frequency at which this process takes place depends on the cross-section shape, and is expressed nondimensionally by the Strouhal number, S:

$$S = nD/V \qquad (19.10)$$

where n is the frequency (Hz), D the cross-section dimension (usually that perpendicular to flow) and V the flow velocity.

Thus, as the velocity increases, so does the vortex-shedding frequency, and a critical condition is likely to arise when the frequency is resonant with the structure natural frequency N (say). For this reason the velocity may be expressed nondimensionally as the 'reduced velocity', V_R:

$$V_R = V/ND \qquad (19.11)$$

The critical condition is clearly $V_R = 1/S$. For most cross-section shapes giving trouble in practice V_R lies between 5 and 8; 5 for circular or roughly circular cross-sections (decagon, etc.), 6 to 8 for rectangular sections, up to higher values for rectangular sections where the downwind dimension is large (e.g. wind perpendicular to the *narrow* face of a slab block) although the excitation then becomes relatively weak. Sufficient data on Strouhal numbers, or critical values of reduced velocity, are available to permit prediction of the critical speed for most cases, but the strength of excitation is much less regular and often requires ad hoc wind-tunnel testing if the critical speed falls within the practical range of speed for the site. The strength of excitation is often sensitive to detail in the cross-section shape, e.g. details at the leading edge of prismatic bridge cross-sections. Test results are conveniently expressed by relating the steady-state amplitude to the damping nondimensional parameter $2m\delta_s/\rho D^2$, known as the Scruton number, in which m is the mass per unit length, δ_s the structure damping as a logarithmic decrement, and ρ the density of air.

The problem is often important in the case of circular sections, such as chimneys. Excitation is reduced by presence of the free end and is thus not normally significant for chimneys of height less than eight diameters. The excitation is also Reynolds number-dependent; below the critical Reynolds number the response is typically very steady as long as the wind speed is critical. In the region of the critical Reynolds number (2×10^5 to 4×10^5) the excitation is very weak. At higher values, excitation is again present but is more random, vortices of relatively short length along the body forming and shedding with poor correlation along the length, producing a response in which the natural frequency predominates but the amplitude is continuously modulated. In this region the test results are usually quoted as an r.m.s. value, either as r.m.s. displacement, or the r.m.s. amplitude which is $\sqrt{2}$ larger. The random modulation means that allowance must be made for values significantly larger than the r.m.s.; it may be assumed that the amplitude has a Rayleigh probability distribution. In smooth-flow tests, if the r.m.s. response amplitude exceeds about $0.015D$, the motion acts dramatically to improve the correlation of shedding along the length of the structure, and the response increases sharply;[36] the values thus predicted would be unacceptable to most massive structures. In turbulent flow this effect still applies, but the transition is much less sharp.

In cases such that resonance would occur at a low wind speed such that the standard mechanically induced turbulence cannot be presumed with complete confidence (limiting value perhaps $\bar{V}_{10} = 10$ m/s, see above), smooth flow should be considered. In such cases, and also where the critical condition would give a sub-critical Reynolds number (e.g. tubular structural members), or amplitudes exceeding (say) 5% of the diameter (e.g. slender steel chimneys), prediction may be based on a postulated regular harmonic excitation, characterized by a coefficient of fluctuating lift $\tilde{C}_L$. $\tilde{C}_L$ is generally taken in the range 0.25 (post-critical) to 0.4 (sub-critical).

For large chimneys the critical resonance is commonly at a speed such that turbulent flow is assured, in the post-critical Reynolds number range. Practical experience is gradually accumulating to support the use of a power-spectrum representation of the basic exciting force (characterized by the r.m.s. value $\sigma(\tilde{C}_L)$) and a negative aerodynamic damping substituted for the lock-on effect of motion of the structure.[37] Simple design formulae can be derived from this model.[38,39]

The analogy between the lock-on effect and negative aerodynamic damping has the effect that response may be very sensitive to the structural damping. It has long been recognized that unsatisfactory behaviour is unlikely in the post-critical range if the Scruton number is greater than 20.

Methods have been developed to reduce the excitation by aerodynamic means. The addition of helical 'strakes' to chimneys is now quite widely practised,[40] and can effectively eliminate response but at the expense of considerably increased drag. An alternative[40] is the 'perforated shroud', rather more expensive to make, but minimizing the drag penalty. Responses in modes other than the fundamental have rarely been reported for massive structures, but should be considered. They have been observed on guyed cylinders and are common on tensioned cables; the singing of telegraph wires is of this type, and has led to the whole class of vortex-induced oscillation being referred to as 'Aeolian'.

The shedding of vortices from a structure can also have very serious effects on other structures nearby downwind. This effect can extend over quite substantial separations,[41] up to at least $10D$. For simple cases, such as pairs of similar chimneys, the ad hoc test technique is straightforward, simulating the upwind element by a fixed model in the wind tunnel. For complex cases, such as slender tower-type buildings in a city-centre environment, very complex procedures are required, including simulation of the general incident turbulence, which can be undertaken only at a very few specialist research centres.

There are several other forms of aerodynamic excitation of oscillation in a crosswind direction. The best known arises from a 'negative lift slope' condition, and is often referred to as 'galloping' from the very large amplitude oscillations suffered by electricity transmission lines when their aerodynamic coefficients are modified by the shape of ice accretions. This behaviour can briefly be explained as follows: downwards motion (say) of a horizontal prismatic structure causes the incidence of a horizontal wind to appear as if inclined upwards when viewed from the structure, and if this causes a *decrease* in the lift force (positive upwards), the motion is reinforced. In most cases this is not the case, but it does occur over a limited range of incidence angle for a square prism (and a range of other rectangular prisms)[42] and over quite a wide range of angle for D-section or pear-shape cross-sections as on power lines with ice accretion. Clearly, a true circular section is immune. In the simplest case, where a principal elastic plane of the structure coincides with motion perpendicular to the wind (i.e. excitation in that direction causes motion exactly in that direction) and the variation of lift coefficient (C_L) with angle of incidence (α) is closely linear in the region of the mean incidence, the critical wind speed for the onset of oscillation is given by:

$$V = \frac{4mN\delta_s}{\rho D_0(\mathrm{d}C_L/\mathrm{d}\alpha)} \tag{19.12}$$

where D_0 is the dimension used in defining C_L. The other variables are as previously defined.

Galloping is distinguished by the direct proportionality of the critical wind speed to the structural damping. If this speed is exceeded, the amplitude grows to a large value, limited only by the curvature of the C_L versus α relationship. Methods for taking this into account are given in the references quoted. In contrast, vortex-shedding excitation is distinguished by a peak of response at the speed corresponding to $V_R = 1/S$, although the amplitude may increase again to higher values at substantially higher wind speeds in a turbulent wind. In response to vortex shedding, the amplitude (rather than critical speed) is damping-dependent, commonly roughly inversely proportional; although there may be a controlling damping value above which oscillation is virtually entirely suppressed, or marking a sharp change in amplitude as related to the effect of motion on correlation of shedding discussed above.

More complicated behaviour, possibly involving the phenomenon of flutter, with or without interaction with the two mechanisms already described, can lead to strong oscillation of sections akin to airfoils (having relatively large dimensions in the plane of the wind), such as slender bridges. Interim design rules sponsored by the British Department of Transport, together with extensive background discussion and commentary, are given by the Institution of Civil Engineers.[43]

19.7 Earthquake effects

The effect of earthquakes on civil engineering structures is primarily a question of the dynamic response of the structure excited by motion of the ground; in general, it is the horizontal components of ground acceleration that govern, although increasing attention is being paid to the effect of the vertical component of ground motion on such cases as large-span sheds having only small live load effects from other causes. The ground motion is normally assumed to be the same at all points on the foundation of the structure; it is not within the practical power of the engineer to deal with the possibility of major relative movement on some fault-line passing within the foundation, apart from site investigation to minimize the risk of building over an existing or incipient fault where movement can be expected.

The equations of motion of the masses of a structure excited by ground motion can be manipulated to give exactly the same differential equations for the displacements *relative to the ground* (and thus the strains in the structure) as for the case of the structure on a fixed base, subjected to horizontal loads applied to every mass equal to the product of the mass and the ground acceleration. A simple basis for design is thus to express an acceleration as a fraction of the acceleration of gravity (g) and to design for this fraction of the weight of the system, treated as a horizontal loading. Due to the dynamic nature of the problem, however, this equivalent acceleration is not simply equal to the maximum ground acceleration but will depend on the natural frequency (or natural period) of the structure and on the history of the ground motion extending over some time prior to the instant when maximum relative displacement is found to occur. For a given ground motion it is a straightforward matter to solve the equations of motion numerically and record the maximum response; repeating this process for single-degree of freedom structures of varying natural frequency (or period) leads to the *spectrum* of the earthquake. The so-called velocity spectrum, S_v, is the most commonly given form; the equivalent acceleration for design is ωS_v, where ω is the 'circular' natural frequency, rad/s. The spectrum is also dependent on the natural damping of the structure. The maximum of ωS_v typically occurs at a frequency of the order of 3 Hz; the structures of fundamental natural frequency below 3 Hz are progressively relatively less sensitive to earthquakes, although the effect of higher modes may become significant.

Unfortunately, the prediction of a ground motion to form a reasonable design basis for any specific structure is subject to many uncertainties. An earthquake occurs when strain energy gradually built up in the Earth's crust is suddenly released by movement on some fault plane. The energy released is measured by the *magnitude* of the earthquake, whereas its effect at some point on the ground is the *intensity* at that point. A rather crude single-parameter measure of intensity is given by scales such as the Modified Mercalli or Rossi-Forel ratings, which are based mainly on an only roughly quantified description of the human sensation or structural damage experienced (or expected).[44] The intensity experienced at a given distance from an earthquake of given magnitude depends greatly on the subsoil or shallow-rock conditions and considerably worse ground motion can be experienced where a thick layer of low-density low-stiffness material overlies heavier, stiffer, material. The duration and frequency content (and thus the shape of the spectrum) can also vary greatly even for cases where the overall intensity rating would be similar; a motion of given intensity recorded close to a low-magnitude shock would be shorter and have higher predominant frequencies by comparison with motion of the same intensity recorded distant from a high-magnitude shock. In the case of energy release from long faults the movements may be progressive along the fault, again leading to considerable differences in duration and frequency content from point to point.

The final factor to be introduced before describing the most useful approaches to design is that experience has shown that for most structures and in most regions where earthquake is a major design consideration, it would be highly uneconomic to base design on an 'elastic' or 'no significant damage' criterion. For most structures the aim must be to prevent major failures causing collapse and loss of life, while making use to the full of the possibility of inelastic structural behaviour resulting in dissipation of energy that is to a substantial degree analogous to increased structural damping. The obvious exceptions to the application of this principle are cases where even moderate damage must be prevented, such as nuclear reactor containment vessels, or buildings housing vital post-disaster services.

The most widely used format for a design code incorporating the factors described above is exemplified by the Unified Building Code of the US. The total horizontal load (base shear) V is given by:

$$V = ZISKAW \tag{19.13}$$

in which W is the weight of the structure. Z, the zoning factor, reflects the basic seismicity of the region, modified where necessary by the soil effect factor, S. Factor I permits allowance to be made for the significance of possible failure of the structure, whether as a result of the importance of the structure to post-earthquake services, or the severity of consequential risks in the event of failure. Factor K expresses the capacity of the structure for inelastic energy dissipation, varying from 0.67 to 1.33 (1.5 for exceptional cases), with low values for 'brittle' structural forms. Factor A represents the spectrum ($2\pi nS_v$), a simple, perhaps crude, approximation is generally specified, allowing a reduction as a function of predicted fundamental frequency where this is less than about 0.5 Hz. The UBC currently suggests $A = 0.07n^{1/2}$ (but not more than 0.12) for the US.

The total force (V) is then distributed over the structure in proportion to the product of the mass and the mode shape function for the first mode (the latter is often approximated by direct proportionality to the height above the ground). It has been noted above that slender tall structures may also show significant higher-mode response, and this is most liable to increase stresses near the top (a so-called 'whiplash' effect); an added proportion of the total load, perhaps 15%, may thus be required to be applied at the highest point.

When it is desired to give more detailed consideration to the behaviour of the structure in the inelastic range, the 'reserve energy' technique is simple to apply and can quickly give very useful guidance and economy in design. To proceed to greater detail requires ad hoc computer step-by-step solution of the response to a given ground motion; this is increasingly commonly done in both US and Japan, and is general practice in the latter country for buildings exceeding fifteen storeys. Two important points must be noted. Firstly, that most of the available ground motion records to input to this procedure were obtained at a substantial distance from a large shock, so that special consideration is necessary for sites in a region where more localized energy release is typical (producing a higher characteristic frequency in ground motion) as well as sites on soft subsoil (possibility of lower frequencies as well as overall magnification). Secondly, any one record is but one chance example of the superposition of ground-wave motions of considerable complexity. Although the broad statistical properties of the ground motion are thus generally representative, the actual net peak response of one specific structure will vary greatly owing to the random factors in this superposition. One technique is to generate artificial ground motion sequences, all having the same broad statistical properties, so that the calculated maximum responses can be averaged (or the value for any given probability of occurrence selected). A somewhat more crude method to make use of a single record is to repeat analysis with a scale factor applied to the mass of the structure to modify the natural frequency. Averaging the responses obtained over a range of (say) $\pm 30\%$ of frequency greatly reduces the probable error due to the random factors.

Finally, it is worth repeating that design to ensure ductility can give much more benefit for a given cost than directly increasing strength. Good design keeps to simple shapes and simple structural forms to reduce the risk of large-scale 'stress concentrations' which would arise, for example, between two wings of a building having different natural frequencies. The conference proceedings that include Blume's analysis of response[45] is strongly recommended for further reading; this has been followed by a consensus guide.[46] A wider reference handbook is also available.[47]

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20 Bridges

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Contents

20.1 Plan of work

The design of bridges requires the collection of extensive data and from this the selection of possible options. From such a review the choice is narrowed down to a shortlist of potential bridge designs. A sensible work plan should be devised for the marshalling and deployment of information throughout the project from conception to completion. Such a checklist will vary from project to project but a typical example might be drawn up on the following lines.

(1) *Feasibility phase*:

(a) data collection;
(b) topographical and hydrographical surveys;
(c) hydrological information;
(d) geological and geotechnical information;
(e) site investigation requirements for soil and rock evaluation;
(f) Meteorological and aerodynamic data;
(g) assembly of basic criteria;
(h) likely budget.

(2) *Assembly of design criteria*:

(a) data and properties on the material to be used including steel, concrete, aluminium, timber, masonry, etc.;
(b) foundation considerations;
(c) hydraulic considerations, flood, scour;
(d) loading and design criteria;
(e) clearances height and width (such as for navigation, traffic);
(f) criteria for gradients, alignment, etc.;
(g) hazards such as impact, accident;
(h) proximity to other engineering works, etc.;
(i) functional requirements;
(j) transportation and traffic planning;
(k) highway and/or railway engineering aspects;
(l) drainage requirements;
(m) provision for services (water, sewage, power, electricity, telephone, gas, communications links, etc.);
(n) design life and durability considerations.

(3) *Design phase*:

(a) choice of bridge;
(b) detailed design of bridge including foundations, substructure and superstructure;
(c) production of drawings and documentation, etc.;
(d) preparation of quality assurance plan;
(e) estimation of cost and programme.

(4) *Construction phase*:

(a) contractual matters;
(b) construction methods;
(c) budget and financial control;
(d) quality control;
(e) supervision of construction;
(f) commissioning;
(g) operating, inspection and maintenance schedules for each part of the work.

(5) *Performance phase*:

(a) obligations of owner;
(b) management of facility;
(c) inspection, maintenance and repair;
(d) rehabilitation and refurbishment requirements (change of loading, widening, change of use and durability aspects);
(e) decommissioning and demolition.

Such a project list serves to highlight the various and sometimes conflicting requirements of a bridge project, and those aspects where the bridge designer should seek the approval of the client throughout all the stages of a project for a truly successful collaboration.

This chapter covers the selection and analysis of bridge superstructures and attempts to relate the most frequently used bridging materials – steel and concrete.

As extensive treatment as possible is given to box girder analysis, an important aspect of modern bridge construction. Information about individual bridges will be found in the bibliography. Reference to these specific examples will assist an understanding of the historical background and the existing state of the art. A good general review of the structural form of bridges is given by Beckett,[1] whilst a sensitive aesthetic assessment is provided by Mock.[2]

Masonry arches and steel trusses have not been dealt with but interesting examples of these types of bridges are contained in the reference list.

The principles developed in this chapter for open or closed sections are applicable to trussed structures if suitable modifications are made to allow for shear behaviour of the truss system.

Thus, the authors hope that there is adequate information in this chapter to make preliminary assessments for most modern bridge designs by methods which enable the essential natures of structural behaviours to be perceived and which can be developed to detailed analyses without the necessity of revising basic principles.

20.2 Economics and choice of structural system

Cost comparisons which would make it possible to arrive at the most economical choice of material, structural form, span, etc. have been sought for many years by bridge engineers, but since the costs of any one bridge depend on the circumstances prevailing at that time, the information is always imprecise. Cost data must be up to date and sufficiently detailed to allow adjustments to be made for changed circumstances. It is the changes in these factors which lead to new methods of construction and new structural systems; a major change of this kind has been that involving box girders, plate girders and trusses.

A very early steel box girder bridge, the Britannia Bridge,[3] built by Stephenson over the Menai Straits (main spans 140 m, completed in 1850) was very successful and was in regular use for railway trains until it was damaged by fire. Each span was lifted into place in its entirety by hydraulic jacks. The advantages of truss construction were, however, sufficient to convince engineers for the next 100 years that box structures were not economical, though plated structures were used in the form of I-beams for smaller spans and lighter loads. The steel box girder re-emerged as a structural system for bridges after the Second World War, although short-span multicellular bridges in reinforced concrete had been used for short spans in the 1930s. In 1965 a large proportion of structures other than short spans were built as box structures of one form or another. A greater degree of selectivity then began to emerge and open cross-sections, even for substantial spans, were again being used provided no problems of aerodynamic stability arose. The use of plate girders has been further encouraged by the reaction caused by failures of steel box girder bridges but it seems likely that a balanced view of the merits of various forms of construction will prevail.

Figure 20.1[4] shows the possible cross-sections for bridge structures which can include truss systems if the plane of each triangulated panel is represented by either a web or flange member. The significance of box structures in a more general sense now becomes clear. It is the open cross-section that is a particular, although important, form of construction, whereas

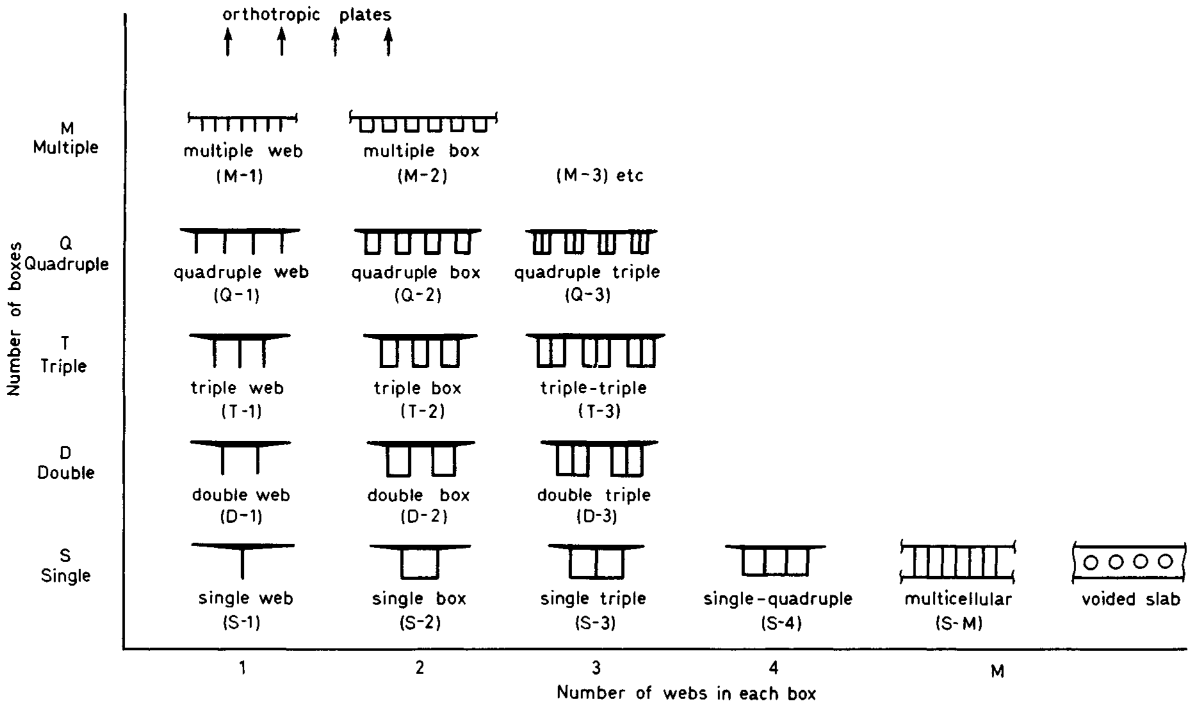


Figure 20.1 Classification of bridge-deck classifications. (After Lee (1971) 'The selection of box-beam arrangements in bridge design', *Developments in bridge design and construction.* (Crosby Lockwood).

the box system is perhaps a misleadingly simple description of the general range of structures.

The most basic structural dimension for a given span affecting both the least-cost and the least-weight methods of measuring efficiency is the effective lever arm of the structure for resisting bending moments resulting from the vertically acting forces from self-weight and imposed loads and vertical components of the support reactions. In bridges which depend on horizontal reactions from the ground, this distance is the rise of an arch above its foundations, or the dip of a suspension cable between towers. If the supports are at different levels, the dip or rise is measured vertically from the chord joining the supports.

The high strength:weight ratio of steel wire and favourable price:strength ratio results in dip:span ratios of 0.1 being suitable for even the longest suspension bridges (Table 20.1). The shallow cable has a higher tension which improves its capacity for carrying uneven loads without large deflection and increases its natural frequency of vibration. The cost of the cable alone is not, however, sufficient to reach conclusions on economics, since the cost of foundations to anchor the cables is substantial and varies with the ground conditions.

The lower strength:weight ratios of steel in compression and concrete combined with the destabilizing effect of the compressive force of the thrust lead to the rise:span ratios being considerably higher on average (Tables 20.3 and 20.4). Good foundations and the requirements of local topography may lead to reduced ratios, and arches – such as at Gladesville,[5] which are in flat country and yet have the roadway running above the arch rib – and the requirement for a low rise to minimize the cost of approach embankments.

The depth between compression and tension flanges is the lever arm of a simply supported beam structure, such as a truss, plate girder or box girder. If the structure is continuous at both ends, the sum of the depths at the centre span and one of the supports is the lever arm (Tables 20.6 and 20.7).

Table 20.1 The world's leading suspension bridges

Name of bridge	*Year*	*Main span* (m)	*Cable sag* (m)	*Span/sag*	*Location*
Humber	1981	1410	125	11.3	Humber River
Verrazano Narrows	1964	1298	117	11.0	New York Harbor
Golden Gate	1937	1280	145	8.8	San Francisco
Mackinac Straits	1957	1158	108	10.76	Michigan
Minami Bisan-Seto (Road/Rail)	u.c. (1988)	1100			Inland Sea of Japan
2nd Bosphorus	1988	1090			
Bosphorus	1973	1074	93.4	11.5	Ortakoy, Turkey
George Washington	1932	1067	96	11.1	Hudson River, New York state
Tagus	1966	1013	106	9.5	Lisbon
Forth	1964	1006	91	11.0	Queensferry
Kita Bisan-Seto (Road/Rail)	u.c. (1988)	990			Inland Sea of Japan
Severn	1966	988	82	12.0	Beachley, UK
Ohnaruto		876			Naruto, Japan
Tacoma Narrows II	1950	853	87	9.8	Puget Sound, Washington
Lions Gate	1938	846			Vancouver

u.c. = under construction

Table 20.2 Leading cable-stayed bridges

Name	*Location*	*Year*	*Main span length* (m)	*Span arrangement*	*Cables*		*Material*	*Function*	*Special notes*
					Planes	*Arrangement*			
Annacis	Vancouver, Canada	1986	465	Sym	2	MF	St/C	Road	
Hooghly	Calcutta, India	u.c.	457	Sym	2	F	St/C		
Barrios de Luna	Sierra Cantabrica, Spain	w	440	Sym	2	F	C	Road	
Hitsuishijima	Iwakurojima (two bridges)	(u.c. 1987)	420	Sym	2	MF	St	Road and Rail	Part of Kojma-Sokaido
Saint Nazaire	Loire estuary, Brittany France	1974	404	Sym	2	F	St	Road	
St Johns River	Jacksonville, Florida, US		400			F	C		
Rande	Vigo Estuary, Spain	1978	400	Sym	2	F	St	Road	
Luling	Mississippi River, Louisiana, US	1982	372	Sym	2	F	St	Road	
Dusseldorf Flehe	W. Germany	1979	368	Ass	1	H side span, MF main span	St/C	Road	Multiple side span anchor piers
Tjörn	Askeröfjord, Sweden	1981	366	Sym	2	MF	St/C	Road	Replaced steel arch demolished by ship collision
Sunshine Skyway	Florida, US	1987	366		1		C	Road	
Yamatogawa	Osaka, Japan		355	Sym	1	H	St	Road	
Duisberg–Neuenkamp	Rhine River, Duisberg–Moers, W. Germany	1970	350	Sym	1	MF	St	Road	
Jindo	S. Korea	1985	345	Sym	2	F	St	Road	
Westgate	Yarra River, Melbourne, Australia	1978	336	Sym	1	DF	St/C	Road	
Brazo Largo	Guagu, Argentina	1977	330	Sym	2	F	St	Road and Rail	Connected by long embankment
Zarate	Palmas, Argentina	1977	330	Sym	2	F	St	Road and Rail	
Posadas–Encarnacion	Paraguay, Argentina		330				C	Road and Rail	
Kohlbrand	Hamburg, W. Germany	1974	325	Sym	2	MF	St		
Knie	Rhine River, Dusseldorf, W. Germany	1969	320	Ass	2	H	St	Road	Multiple side span anchor piers
Brotonne	Seine River, Rouen, France	1977	320	Sym	1	MF	C	Road	
Bratislava	Danube River, Czechoslovakia	1971	316	Ass	1	S side span, F main span	St	Road	Two unequal spans, backward leaning tower
Erskine	Clyde River, Scotland	1971	305	Sym	1	S	St	Road	
Severins	Cologne, W. Germany	1959	302	Ass	2	F	St	Road	
Dnieper	Kiev, Soviet Union	1976	300	Ass	2	MF	St/C	Road	Two unequal spans
Pasco Kennewick	Washington State, US	1978	299	Sym	2	F	C	Road	
Neïwied	Rhine River, W. Germany		292	Ass	1	MF	St	Road	Two unequal spans with longitudinal A frame tower
Deggenau	Danube River, W. Germany	1975	290	Ass	1	F	St	Road	Two unequal spans
Coatzacoalcos II	Mexico	1984	288	Sym	1	MF	C	Road	
Kurt Schuhmacher	Rhine River, Mannheim Nord, W. Germany	1971	287	Ass	2	F	St	Road and tram-way	Two side span anchor piers

Table 20.2 (cont)

Name	*Location*	*Year*	*Main span length* (m)	*Span arrangement*	*Cables* *Planes*	*Arrangement*	*Material*	*Function*	*Special notes*
Wadi-el-Kuf	Beida, Libya, N. Africa	1971	282	Sym	2	S	C	Road	Articulated
Leverkusen	Rhine River, W. Germany	1965	281	Sym	1	H	St	Road	
Friedrich-Ebert	Rhine River, Bonn Nord, W. Germany	1967	280	Sym	1	MF	St	Road	
Dolsan	S. Korea	u.c.	280	Sym	2	F	St		
Speyer	Rhine River, W. Germany	1975	275	Ass	1	S side span, F main span	St	Road	
East Huntingdon	Ohio River, US	u.c.	274	Ass	2	MF	C	Road	Two unequal spans
Tiel	Waal River, Holland	1972	267	Sym	2	F	C	Road	
Theodor Heuss	Dusseldorf, W. Germany	1958	260	Sym	2	H	St	Road	
Oberkassel	Dusseldorf, W. Germany	1976	258	Ass	1	H	St	Road and streetcar	Multiple side-span anchor piers
Rees	W. Germany	1967	255	Sym	2	H	St	Road	
Save	Belgrade, Yugoslavia	1978	254	Sym	2	MF	St	Railway	
Papineau	Montreal, Canada	1969	251	Sym	1	F	St	Road	
Suchiro	Tokushima, Japan	1976	250	Sym	1	MF	St		
Manuel Belgrano	Parana River, Corrientes, Argentina	1972	245	Sym	2	F	C	Road	Articulated
Kessock	Inverness, Scotland	1982	240	Sym	2	H	St	Road	
General Rafael Urdaneta	Lake Maracaibo, Venezuela	1962	235	Sym	2	S	C	Road	Multiple spans. Articulated
Wye	Beachley, Wales	1966	235	Sym	1	S	St	Road	
Penang Crossing	Malaysia	1980	225	Sym	2	H	C	Road	
Luangwa	Zambia		223						
Rokko Island Double-decked	Kobe, Japan	1977	220	Sym	2	MF	St	Road	
Hawkshaw	New Brunswick, Canada	1969	217						
Toyosato	Yodo River, Osaka, Japan	1970	216	Sym	1	MF	St	Road	
Onomichi	Hiroshima Pref., Japan	1968	215	Sym	2	F	St	Road	
Polcevera Creek	Genoa, Italy	1967	210	Sym	2	S	C	Road	Multiple spans. Articulated
Albert Canal	Godsheide, Belgium	1977	210	Sym	2	MF	St	Road	
Batman	Tamar River, Tasmania	1968	206	Ass	2	S side span, F main span	St	Road	Two unequal spans. Forward leaning tower
Arno	Florence, Italy	1977	206	Sym	2	S side span, F main span		Road	Towers lean backwards
Stromsund	Sweden	1955	183	Sym	2	DF	St/C	Road	
Adhamiyah	Baghdad, Iraq	1984	182	Ass	1	H	St/C	Road	Two side-span anchor piers
New Galecopper	Rhine Canal, Amsterdam, Holland	1971	180	Sym	1	S	St	Road	Twin bridges skew spans
Maxau	Rhine River, W. Germany	1967	175	Ass	1	MF	St	Road	Two unequal spans
Ganter	Simplon Pass, Valais, Switzerland	1980	174	Sym	2	S	C	Road	Cables enclosed in web extensions. Curved side spans

Table 20.2 (cont)

Name	*Location*	*Year*	*Main span length* (m)	*Span arrangement*	*Cables*		*Material*	*Function*	*Special notes*
					Planes	*Arrangement*			
North Elbe	Hamburg, W. Germany	1962	172	Sym	1	ST	St	Road	
Daikoku	Yokohama, Japan	1974	165	Ass	2	MF	St		
Massena	Paris, France	1971	162	Sym	1	MF	St	Road	
Steyregger Donau	Linz, Austria	1979	161	Ass	2	S	St/C	Road	Two unequal spans
Kamatsugawa	Japan	1971	160	Sym	1	H	St		
Ishikara-Kako	Hokkaido, Japan	1975	160	Sym	2	F	St	Road	
Arakawa	Tokyo, Japan	1970	160	Sym	1	H	St	Road	
George Street	River Usk, Newport, Wales	1964	152	Sym	2	H	St/C	Road	
Sancho el Major	Rio Ebro, Castejon, Spain		146			MF	C		
Metten	Danube River, W. Germany		145		1	S	C		
Magliana	Tiber River, Rome, Italy	1967	145	Ass	2	S	C	Road	Curved in plan. Two unequal spans. Backward leaning towers
Dnieper	Kiev, Soviet Union	1964	144	Sym	2	F	C	Road	
Maya	Kobe, Japan	1966	139	Ass	1	MF	St		Two unequal spans
Ludwigshafen	W. Germany	1968	138	Eq	2	F	St	Road	Four-leg A-frame tower
Sitka Harbour	Alaska, US	1972	137	Sym	2	S	St/C	Road	
Danube Canal	Vienna, Austria	1975	119	Sym	2	S	C		
Second Main Bridge	Frankfurt, W. Germany	1972	148	Ass	2	H	C	Road and rail	Articulated main span connects to fin back. Three anchored side spans
Tarano	Alba, Italy	1983	114	Ass	1	S side span, F main span	St	Road	Two unequal spans. Backward sloping towers
Harmsen	Rotterdam, Holland	1968	108						
Bridge of the Isles	Montreal, Canada	1967	105	Eq	2	S	St/C	Road and rail	
St Florent	River Loire, France	1969	104	Eq	2	F	St/C		
Julicherstrasse	Dusseldorf, W. Germany	1963	99	Sym	1	S	St	Road	

Ass – asymmetric; C – concrete; DF – double fan; Eq – two equal; F – multiple fan; H – harp; MF – modified fan; S – single; St – steel; ST – star; St/C – composite steel and concrete; Sym – symmetric; u.c. – under construction

Table 20.3 The world's leading steel arch bridges

Name of bridge	*Span* (m)	*Rise* (m)	*Rise/span*	*Year*	*Location*
River Gorge	518			1977	West Virginia, US
Bayonne	504	81	0.161	1931	New York, New York, US
Sydney Harbour	503	107	0.212	1932	Sydney, Australia
Fremont*	383			u.c.	Portland, Oregon, US
Port Mann*	366	76	0.208	1964	Vancouver, Canada
Thatcher†	344			1962	Balboa, Panama
Laviolette†	335			1967	Trois Rivières, Canada
Zd'ákov	330	42.5	0.129	1967	Lake Orlik, Czechoslovakia
Runcorn–Widnes	330	66.4	0.202	1961	Mersey River, England
Birchenough	329	65.8	0.200	1935	Sabi River, Rhodesia
Glen Canyon	313			1959	Arizona, US
Lewiston–Queenston	305	48.4	0.159	1962	Niagara River, N. America
Hell Gate	298			1917	New York, New York, US
Other steel arch bridges of interest					
Rainbow	289	45.7	0.158	1941	Niagara Falls, N. America
Fehmarnsund*	249	43.6	0.175	1963	Fehmarnsund, W. Germany
Adomi (Volta)	245	57.4	0.234	1957	Adomi, Ghana
Kaiserlei*	220			1964	Frankfurt-am-Main, W. Germany

u.c. = under construction
*Tied arch †Cantilever arch

Table 20.4 The world's leading concrete arch bridges

Name of bridge	*Span* (m)	*Rise* (m)	*Rise/span*	*Year*	*Location*
Krk II	390			1980	Adria, Yugoslavia
Gladesville	305	40.8	0.134	1964	Sydney, Australia
Rio Paraná	290	53.0	0.183	1965	Paraná River, Brazil–Paraguay
Bloukrans	272			1983	Cape Province, South Africa
Arrabida	270	51.9	0.192	1963	Portugal
Sandö	264	40.0	0.151	1943	Angerman River, Sweden
Shibenik	246			1967	Krka River, Yugoslavia
Fiumarella	231	66.1	0.286	1961	Catanzaro, Italy
Novi Sad	211			1961	Danube River, Yugoslavia
Linenau	210			1967	Bregenz, Austria
Van Stadens	200			1971	Van Stadens Gorge, S. Africa
Esla	192			1942	Esla River, Spain
Groot River	189			1983	Cape Province, South Africa
Rio das Antas	180	28.0	0.156	1953	Brazil
Traneberg	178	26.2	0.147	1934	Stockholm, Sweden
Plougastel (Albert Louppe)	173	33	0.190	1930	Elorn River, France
Selah Creek	168			1971	Yakima, Washington, US
Bobbejaans	165			1983	Cape Province, South Africa
La Roche–Guyon	161	23.0	0.143	1934	France
Cowlitz River Bridge	158				Mossyrock, Washington, US
Caracas–La Guaira	152	39.0	0.257	1952	Caracas, Venezuela
Puddefjord	145			1956	Norway
Podolska	145			1942	Czechoslovakia
Other concrete arch bridges of interest					
Revin–Orzy	120	10.0	0.083		Meuse River, France
Glemstal	114	27.1	0.238		Stuttgart, W. Germany
Slängsboda	111	12.0	0.108	1961	Stockholm, Sweden

u.c. = under construction

Table 20.5 The world's leading truss bridges

Name of bridge	*Span* (m)	*Year*	*Location*
Quebec Railway	549	1918	Quebec, Canada
Forth Railway	2 × 521	1890	Queensferry, Scotland
Minato	510	1974	Japan
Delaware River	501		Chester, Penn–Bridgeport, New Jersey, US
Greater New Orleans	480	1958	New Orleans, Louisiana, US
Howrah	457	1943	Calcutta, India
Transbay	427	1936	San Francisco, California, US
Baton Rouge	376	1968	Baton Rouge, Louisiana, US
Tappan Zee	369	1955	Tarrytown, New York, US
Longview	366	1930	Columbia River, Washington, US
Queensboro	360	1909	New York, US
I Carquinez Strait	2 × 335	1927	San Francisco, California, US
II Carquinez Strait	2 × 335	1958	San Francisco, California, US
Second Narrows	335	1960	Vancouver, Canada
Jacques Cartier	334	1930	Montreal, Canada
Isaiah D. Hart	332	1967	Jacksonville, Florida, US
Richmond–San Rafael	2 × 326	1956	San Pablo Bay, California, US
Grace Memorial	320	1929	Cooper River, South Carolina, US
Newburgh–Beacon	305	1963	Hudson River, New York, US
Auckland Harbour	244	1959	Auckland, New Zealand

Table 20.6 Some of the world's leading steel girder bridges

Name of bridge	*Span* (m)	*Depth* (d) *at midspan* (m)	*Depth* (d_2) *at pier* (m)	$\frac{d_1+d_2}{Span}$	*Year*	*Type*	*Location*
Niteroi	300	7.4	12.9	0.068	1974	B	Rio de Janeiro, Brazil
Sava I	261	4.6	9.8	0.055	1956	P	Belgrade, Yugoslavia
Zoo	259	4.5	10.0	0.056	1966	B	Cologne, W. Germany
Sava II	250				1969	B	Belgrade, Yugoslavia
Koblenz	235					B	Rhine River, W. Germany
Foyle	234				1984	B	Londonderry, N. Ireland
San Mateo–Hayward	228	4.6	9.2	0.060	1967	B	California, US
Hochbrücke 'Radar Insel'	221	5	9.5	0.066		P	Nord–Ost see Canal, W. Germany
Moselle	219					B	Moselle Valley, W. Germany
Milford Haven	213	5.9	5.9	0.055		B	Pembroke Dock, Wales
Fourth Danube	210				1970	B	Vienna, Austria
Martigues	210				1976	Portal B	France
Düsseldorf–Neuss	206	3.3	7.8	0.054	1951	B	Düsseldorf, W. Germany
Wiesbaden–Schierstein	205	4.4	7.4	0.057		P	Rhine River, W. Germany
Europa	198	7.7	7.7	0.078	1964	B	Sill Valley, Austria
Köln–Deutz	185				1948	B	Rhine River, W. Germany
Poplar Street	183	6.2	7.6	0.070	1967	B	St Louis, Mississippi, US
Italia	175	8.5	8.5		1969	B	Lao River, Italy
Avonmouth	174	2.6	7.6	0.059	1974	B	Gloucestershire, England
Friarton	174	2.7	7.5	0.059	1978	B	Perth, Scotland
Gemersheim	165	9.1	5.4	0.058	1971	B	Rhine River, W. Germany
Speyer	163	3.4	6.40	0.060	1956	B	Rhine River, W. Germany
Concordia	160	4.9	4.9	0.060	1967	B	Montreal, Canada
New Temerloh	151	3.7	5.9	0.064	1974	B	Temerloh, Malaysia
			Other steel girder bridges of interest				
Calcasieu River	137	2.1	7.0	0.078	1963	P	Louisiana, US
St Alban	135	2.8	9.3	0.062	1955	P	Basel, Switzerland
Amara	82	3.7	12.1	0.087	1958	B	Tigris River, Iraq

u,c. = under construction
Bridge type: B box girder, P plate girder

Table 20.7 Some of the world's leading concrete girder bridges

Name of bridge	*Span* (m)	*Depth* (d) *at midspan* (m)	*Depth* (d_2) *at pier* (m)	$\frac{d_1+d_2}{Span}$	*Year*	*Type*	*Location*
Gateway	260	4.0	14.0	0.069	1986	C	Brisbane, Australia
Hikoshima	236					C	
Urato	230	4.0	12.5	0.072	1972	C	Shikoku, Japan
Three Sisters	229				u.c.	C	Potomac River, Washington, DC, US
Bendorf	208	4.4	10.4	0.071	1965	C	Bendorf, W. Germany
Orwell	190					C	Ipswich, England
Manazuru	185	3.1	10.0		u.c.	C	Japan
Brisbane Water	183					C + SS	New South Wales, Australia
Gardens Point	183					C	Brisbane Australia
Redheugh	160						Newcastle upon Tyne, England
Amakusa Nakana	160	3.0	10.0				Japan
Medway	152	2.2	10.8	0.086	1963	C + SS	Rochester, England
Neckarsulm	151	4.2	7.4	0.078	1968	C	Neckarsulm, W. Germany
Moscow River	148				1957	CG	Soviet Union
Amakusa	146				1966	C	Japan
Kingston	143	2.4	10.0	0.087	1970	C	Glasgow, Scotland
Victoria	142				1970	C + SS	Brisbane, Australia
Tocantins	142				1961	C	Tocantins River, Brazil
Bettingen	140	3.0	7.0	0.089		C	Main River, W. Germany
Don	139				1964	C	Rostow, Soviet Union

Table 20.7 (cont.)

Name of bridge	*Span* (m)	*Depth* (d) *at midspan* (m)	*Depth* (d_2) *at pier* (m)	$\frac{d_1+d_2}{Span}$	*Year*	*Type*	*Location*
Pine Valley	137				u.c.	CG	California, US
Alnö	134				1964	C	Alnösund, Sweden
Öland	130				1972	C	Kalmar Sound, Sweden
			Other concrete girder bridges of interest				
Worms	114	2.5	6.5	0.079	1952	C	Rhine River, W. Germany
Koblenz	114	2.7	7.2	0.087	1954	C	Moselle River, W. Germany
Nötesund	110	2.2	5.7	0.072	1966	C	Orust, Sweden
Siegtal	105	5.8	5.8	0.110	1969	CG	Eiserfeld, W. Germany
Chillon Viaduct	104	2.2	5.6	0.072	1973	CG	Chillon, Switzerland
Narrows	97	2.2	4.2	0.068	1959	C+SS	Perth, Australia
Benjamin Sheares	84				1981	C+SS	Singapore
Oleron	79	2.5	4.5	0.089	1966	CG	Rochefort, France

u.c. = under construction
C – concrete; C+SS – concrete with suspended span; CG – continuous girder

The cable-supported bridge can be seen as either a suspension bridge or a continuous beam with the effective depth at the supports equal to the height of the tower. Figure 20.2 shows the various arrangements of cables that are used, and various finished bridges are shown in Figures 20.3 to 20.9.

The choice of span depends on the foundations, depth of water and height of the deck but, in many cases, other requirements – such as navigation clearances – dictate the minimum span. It is usually only shorter spans where, proportionately at any rate, there is considerable variation possible. It has been claimed in the past that at the most economic span of a multispan structure, the cost of foundations equals the cost of the superstructure less the basic deck structure costs. The assumptions necessary for this to be valid are that the cost of superstructure per unit length should increase linearly with span and that that of the substructure should vary inversely with span. The slopes of the respective cost–span curves are then equal and opposite at the point of intersection of the curves provided any constant costs in both foundations and superstructure are first subtracted. If the cost of the superstructure is assumed to increase proportionately to the square root of the span, however, the same approach requires that half the superstructure cost should equal the foundation cost. In modern structures it is difficult to separate the costs of the basic deck system from the total of the multispan structure.

The well-known rule – that for maximum economy the total area of the flanges of a beam should equal the area of the web – is a more useful guide. Table 20.8 shows that for a given web thickness and a total area of cross-section of $1.0t$ the maximum section modulus is at a depth of 0.75 where the total flange area is one-third the web area, but at a depth of 0.5 where the flange and web areas are equal the section modulus is only 11% lower. A shallower beam is usually more economical because a simpler web is then possible provided the shear force can be carried. Fabrication, transportation and erection are also less costly.

Table 20.9 shows the types of standardized precast concrete beams that are appropriate to various parts of the short-span range. Apart from the cost advantages of standardization and factory production, which may be offset by higher overheads and transport costs, there are the following advantages.

(1) Estimates of cost more reliable.
(2) Speed of construction.
(3) No temporary staging required.
(4) Sample beams can be tested to demonstrate level of prestress and ultimate strength.

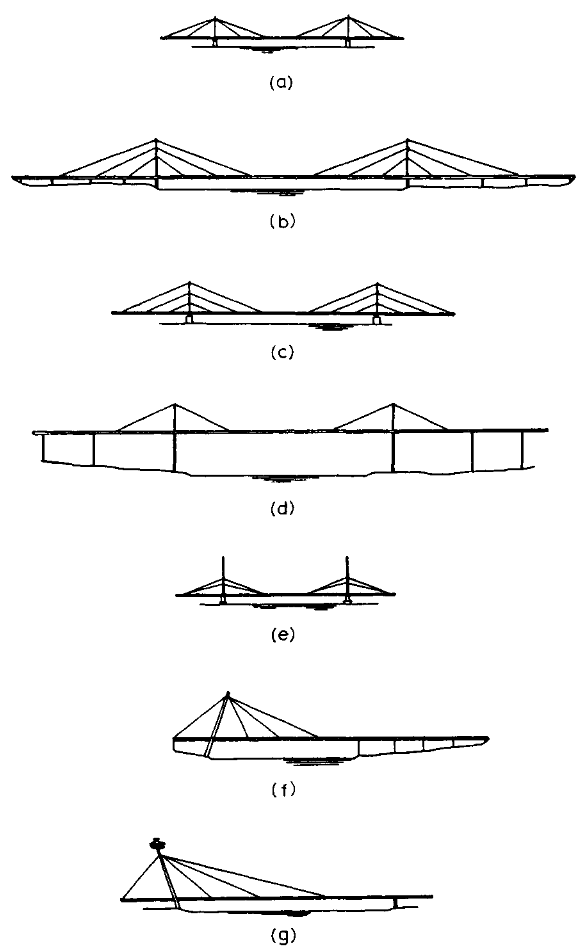


Figure 20.2 Examples of different cable systems (scale: approximately 1/10 000). (a) Fan (Stromsünd); (b) modified fan (Duisberg–Neuenkamp); (c) harp (Theodor Heuss); (d) single cable (Erskine); (e) star (Norderelbe); (f) asymmetric systems (Batman); (g) Bratislava). (*Courtesy*: Polensky and Zöllner)

Figure 20.3 Concrete girder bridge, Bettingen, Frankfurt-am-Main

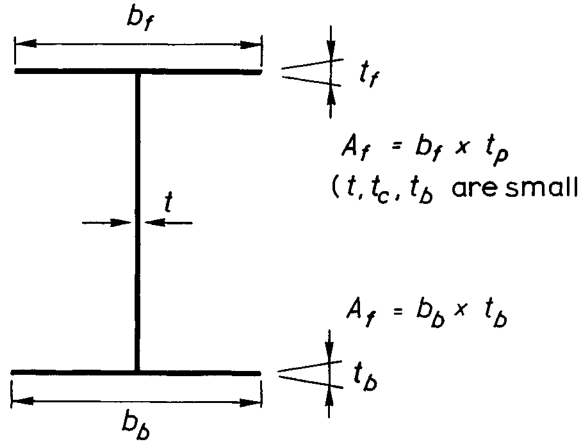
Figure 20.4 Steel girder bridge, Rio-Niteroi, Brazil. (*Courtesy*: Redpath Dorman Long and the Cleveland Bridge and Engineering Co. Ltd)

Figure 20.5 Concrete cable-stayed bridge, Tempul Aqueduct, Spain. (*Courtesy*: Torroja Institute, Madrid)

Figure 20.6 Steel trussed cable-stayed bridge. Batman Bridge, Tasmania. (*Courtesy*: Maunsell and Partners)

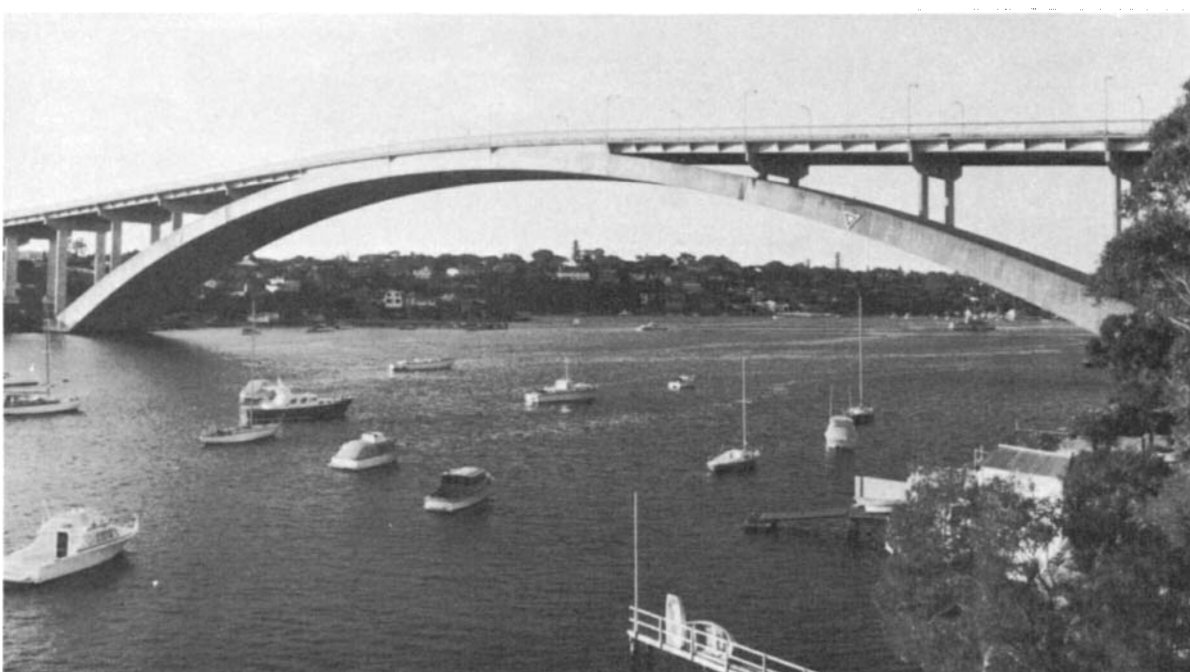
Table 20.8

Depth	0.5	0.6	0.7	0.75	0.8
Af	$0.25t$	$0.2t$	$0.15t$	$0.125t$	$0.1t$
Section modulus Z	$0.167t$	$0.180t$	$0.187t$	$0.1875t$	$0.187t$



Note: Total cross-section throughout = $1.0t$

Figure 20.7 Concrete-arched bridge, Gladesville, Sydney. (*Courtesy*: G. Maunsell and Partners)



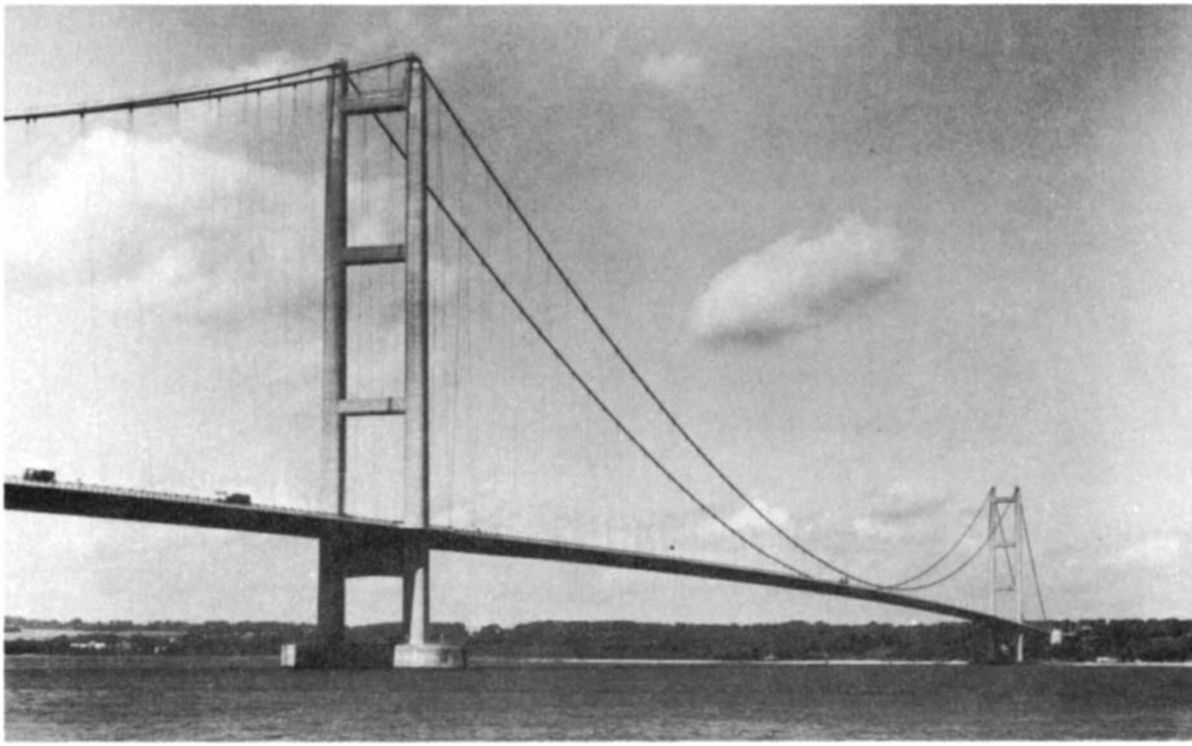


Figure 20.8 Humber suspension bridge. (*Courtesy*: Freeman Fox and Partners)

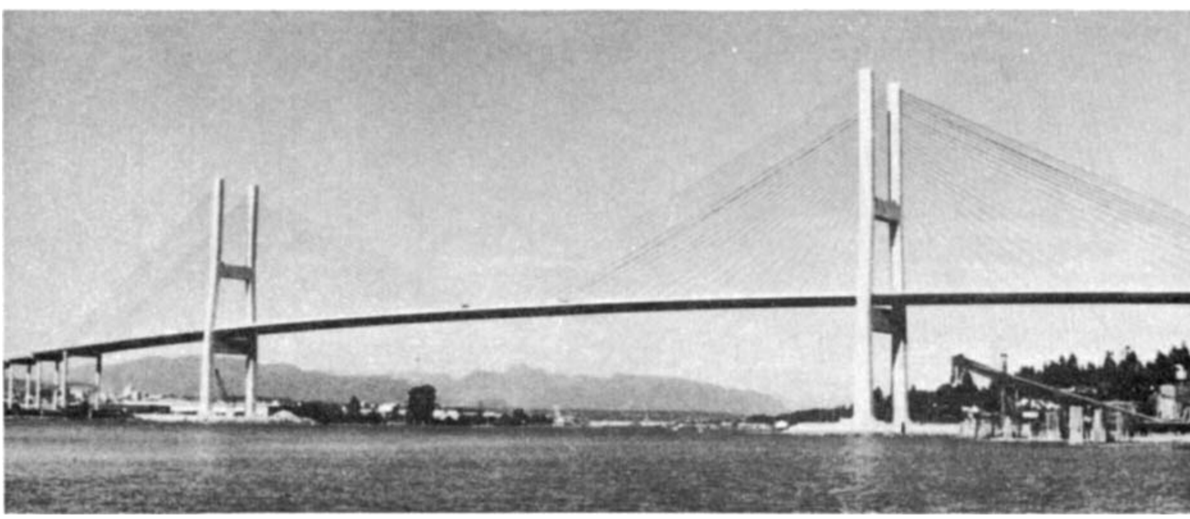


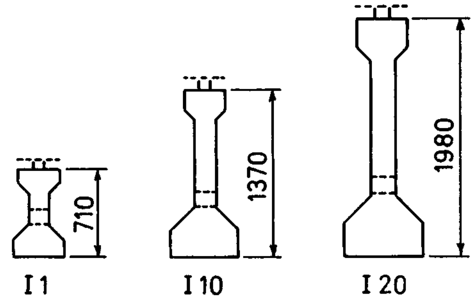
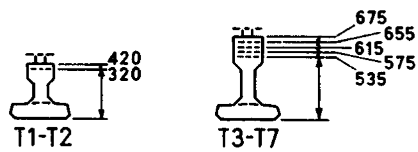
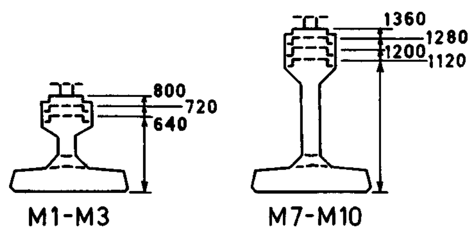
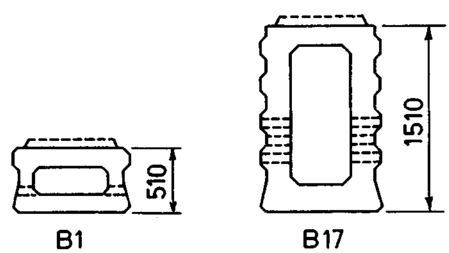
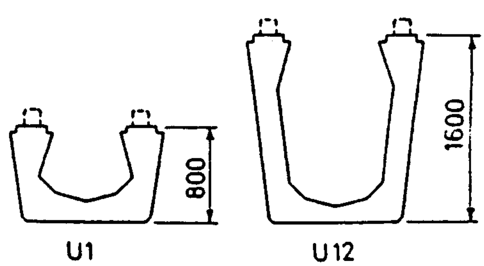
Figure 20.9 Annacis cable-stayed bridge, Vancouver. (*Courtesy*: Buckland and Taylor Ltd)

In simple right spans, the system chosen, apart from span, depends on construction depth limitations, difficulties of access and, of course, prevailing prices. For example, the top hat beam system[6] is suitable for restricted access and small construction depths. The U-beam system[7] is suitable for similar conditions but requires an increased depth. At the greater depth it is more economical. An advantage of torsionally stiff structures of this type, particularly when they are designed to be spaced apart in the transverse direction, is that they can readily be fanned out to support the structures with complex plan forms that are now common.

The standard concrete beams are essentially a series of elements that can be placed across the complete span, requiring only simple shuttering to support the transversely spanning top slab. Diaphragm beams at the supports are required and occasionally intermediate diaphragms may be provided.

Steel beams can be used as an alternative form of construction in the same span range. Either a series of I-sections or small box girders can be used.

Table 20.9 Precast concrete bridge beams

Type of beam	*Name of beam*	*Classification (as Figure 20.1)*	*Span* (m)	*Beam section*
I	C & CA I-section beam	M-1	12–36	
Inverted T	C & CA inverted T-beam for spans from 7–16 m	Orthotropic slab	7–16 m	
Inverted T (M range)	MoT/C & CA prestressed inverted T-beam for spans from 15 to 29 m	(a) T-beam M-1 (b) Pseudo box S-M	15–29	
Box	C & CA box section beam	S-M	12–36	
U	U-beam	M-2	15–36	

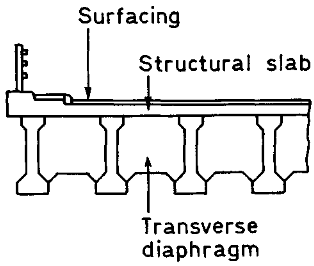
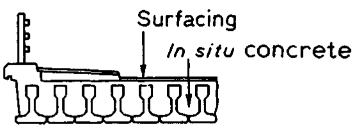
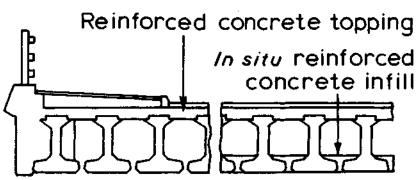
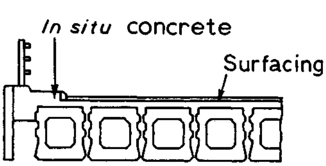
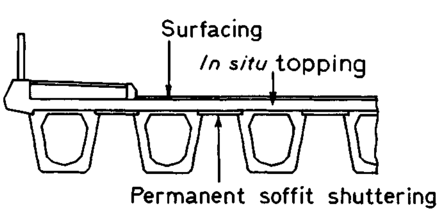
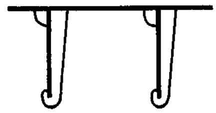
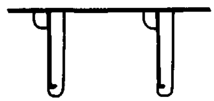
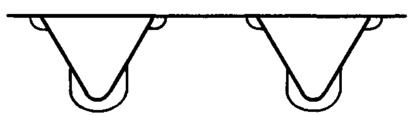
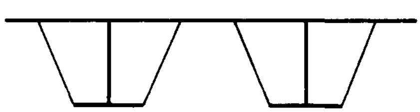
Section through part of typical deck	*Remarks*
	20 standard sections (I1–I20) Holes for transverse reinforcement provided at 30–50 mm centres
	7 standard sections (T1–T7)
 (a) T-beam (b) Pseudo box	10 standard sections (M1–M10)
	17 standard sections (B1–B17) Transverse prestress used to give optimum load distribution
	12 standard sections (U1–U12)

Table 20.10 Longitudinal stiffeners for orthotropic decks

Type of stiffener	*Classification (as Figure 18.1)*		*Remarks*
Flat	M-1, etc.		Torsionally weak. Easily spliced. Poor transverse load distribution. Earliest form.
Bulb flat	M-1		Torsionally weak. Easily spliced. Poor transverse load distribution. Bulb flats difficult to obtain. Out of date.
Trapezoidal trough	M-2, etc.		Torsionally stiff. Fabrication difficult through cross-frames. Relatively popular.
V-trough	M-2		Torsionally stiff. Fabrication difficult through cross-frames. Small effective lower flange area. Popular but less efficient than trapezoidal trough
Wineglass	M-2		Torsionally stiff. Very complicated fabrication. Expensive.
(cut from universal beam)	M-1		Easily spliced. Requires large cutout in cross-frame. Torsionally weak. Inefficient.

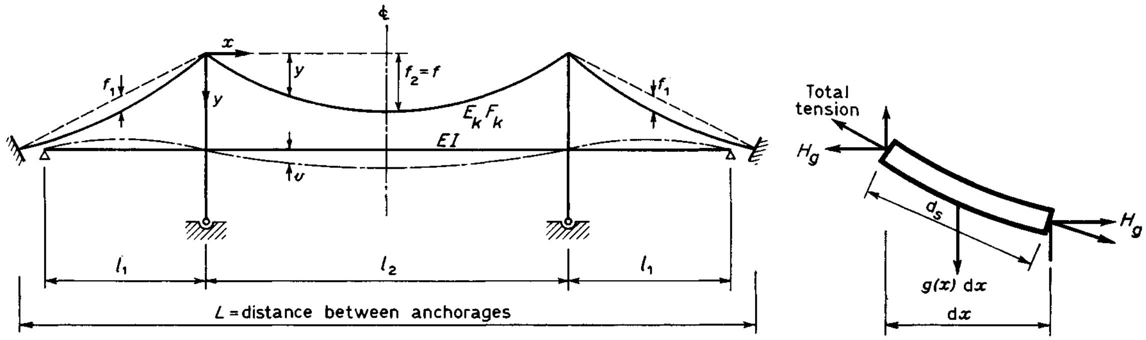


Figure 20.10 Suspension-bridge notation

Precast or prefabricated elements can be made as transverse rather than longitudinal elements and then joined together on site by prestressing in concrete structures or welding or bolting in steel structures. This approach, sometimes known as segmental construction, was used for the structures of Figure 20.11(d), (e), (f), (h), (i), (j) and (k). It was also used for the steel structures of Figure 20.11 (l) and (m). The remaining steel structures shown in Figure 20.11(n) to (r) were constructed by a similar process but with the subdivision taken a stage further. Each transverse slice was built up on the end of the cantilevering structure from several stiffened panels.

In situ concrete, reinforced or prestressed, can be used to form complete spans in one operation or else the cantilevering approach can be used. In the latter case, the speed of construction is limited by the time required for the concrete to reach a cube strength adequate for the degree of prestress necessary to support the next section of the cantilever and the erection equipment. Segmental methods of construction[8] avoid such delays. In shorter spans, provided that the restrictions on construction depth are not too severe, *in situ* concrete structures can be built economically using the cross-section of Figure 20.11(c). The simple cross-section[9] was developed to suit the use of formwork which, after supporting a complete span, could be moved rapidly to the next span. The resulting machine is only economical for multispan structures.

The stiffened steel plates (Table 20.10) are used for deck systems of long-span, and movable, bridges in order to reduce the self-weight of the structure.

20.3 Characteristics of bridge structures

The following theories have been chosen and developed for their value in demonstrating the principal characteristics of various types of bridge structure. Other methods of calculation, based on finite elements, for example, may be more accurate and more economical in certain circumstances. The theories are, however, linked to the main structural properties of the bridge types considered and are meant to assist the process of synthesis necessary before detailed calculations begin. The concepts described are also useful for idealizing structures when using computer programs and for interpreting and checking the computer output.

20.3.1 Theory of suspension bridges and arch bridges

The basic theory of arch and suspension bridges is the same and the equation derived below for suspension bridges is applicable to arches if a change in sign of H and y is made.

20.3.1.1 Suspension bridges with external anchorages

The dead load of the cable and stiffening girder is supported by the force per unit length of span produced by the horizontal component of the cable force and the rate of change of slope of the cable:

$$H_g y''(x) + g = 0 \tag{20.1}$$

where y, etc. are shown in Figure 20.10.

For a parabolic shape of cable corresponding to constant intensity of load across the span l, $y''(x) = -8f/l^2$ and:

$$H_g = gl^2/8f \tag{20.2}$$

The cable tension increases under live load $p(x)$ to:

$$H = H_g + H_p \tag{20.3}$$

The increase in support from the cable is $-[Hv''(x) + H_p y''(x)]$ where $v(x)$ is the vertical deflection of the cable and stiffening girder. The stiffening girder contributes a supporting reaction per unit length of $[EIv''(x)]''$ and adding the cable and stiffening girder contributions and equating them to the intensity of the applied load gives:

$$[EIv''(x)]'' - Hv''(x) = p(x) + H_p y'' \tag{20.4}$$

The term $H_p y''$ is added to the live load in order to show that the equation can be represented physically by the substitute structure of Figure 20.12. y'' is $-8f/l^2$ and therefore represents a force in the opposite direction to the live load.

H_p depends on the change in length of the cable and if Δdx is the horizontal projection of the change in length of an element ds then for fixed anchorages:

$$\int_0^L \Delta dx = 0 \tag{20.5}$$

Integrating along the cable and allowing for a change in temperature of ΔT gives:

$$\int_0^L \Delta dx = H_p \frac{L_k}{E_k F_k} \pm a_T \Delta T L_T + y'' \int_0^L v(x) dx = 0 \tag{20.6}$$

Approximate values of L_k and L_T are (see Figure 20.13):

$$L_k \simeq \left(1 + 8\frac{f^2}{l^2} + \frac{3}{2}\tan^2 v_0\right) + \frac{S_1}{\cos^2 v_1} + \frac{S_2}{\cos^2 v_2}$$

$$L_T \simeq \left(1 + \frac{16}{3}\frac{f^2}{l^2} + \tan^2 v_0\right) + \frac{S_1}{\cos v_1} + \frac{S_2}{\cos v_2} \tag{20.7}$$

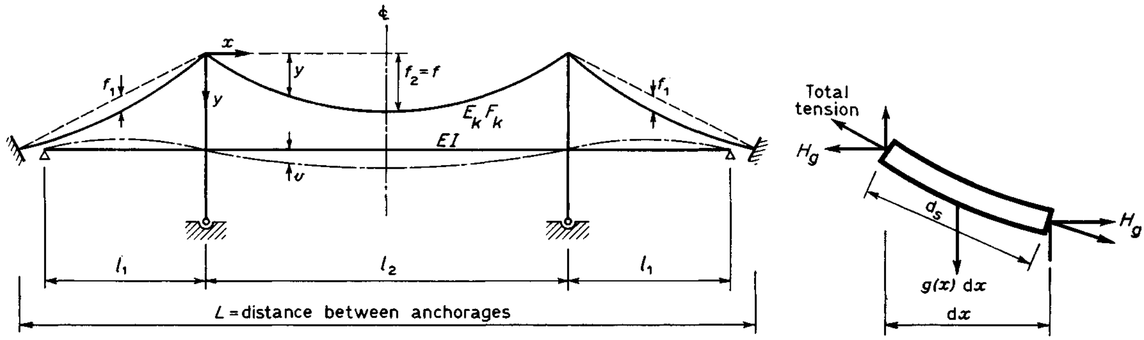


Figure 20.10 Suspension-bridge notation

Precast or prefabricated elements can be made as transverse rather than longitudinal elements and then joined together on site by prestressing in concrete structures or welding or bolting in steel structures. This approach, sometimes known as segmental construction, was used for the structures of Figure 20.11(d), (e), (f), (h), (i), (j) and (k). It was also used for the steel structures of Figure 20.11 (l) and (m). The remaining steel structures shown in Figure 20.11(n) to (r) were constructed by a similar process but with the subdivision taken a stage further. Each transverse slice was built up on the end of the cantilevering structure from several stiffened panels.

In situ concrete, reinforced or prestressed, can be used to form complete spans in one operation or else the cantilevering approach can be used. In the latter case, the speed of construction is limited by the time required for the concrete to reach a cube strength adequate for the degree of prestress necessary to support the next section of the cantilever and the erection equipment. Segmental methods of construction[8] avoid such delays. In shorter spans, provided that the restrictions on construction depth are not too severe, *in situ* concrete structures can be built economically using the cross-section of Figure 20.11(c). The simple cross-section[9] was developed to suit the use of formwork which, after supporting a complete span, could be moved rapidly to the next span. The resulting machine is only economical for multispan structures.

The stiffened steel plates (Table 20.10) are used for deck systems of long-span, and movable, bridges in order to reduce the self-weight of the structure.

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Approximate values of L_k and L_T are (see Figure 20.13):

$$L_k \simeq \left(1 + 8\frac{f^2}{l^2} + \frac{3}{2}\tan^2 v_0\right) + \frac{S_1}{\cos^2 v_1} + \frac{S_2}{\cos^2 v_2}$$

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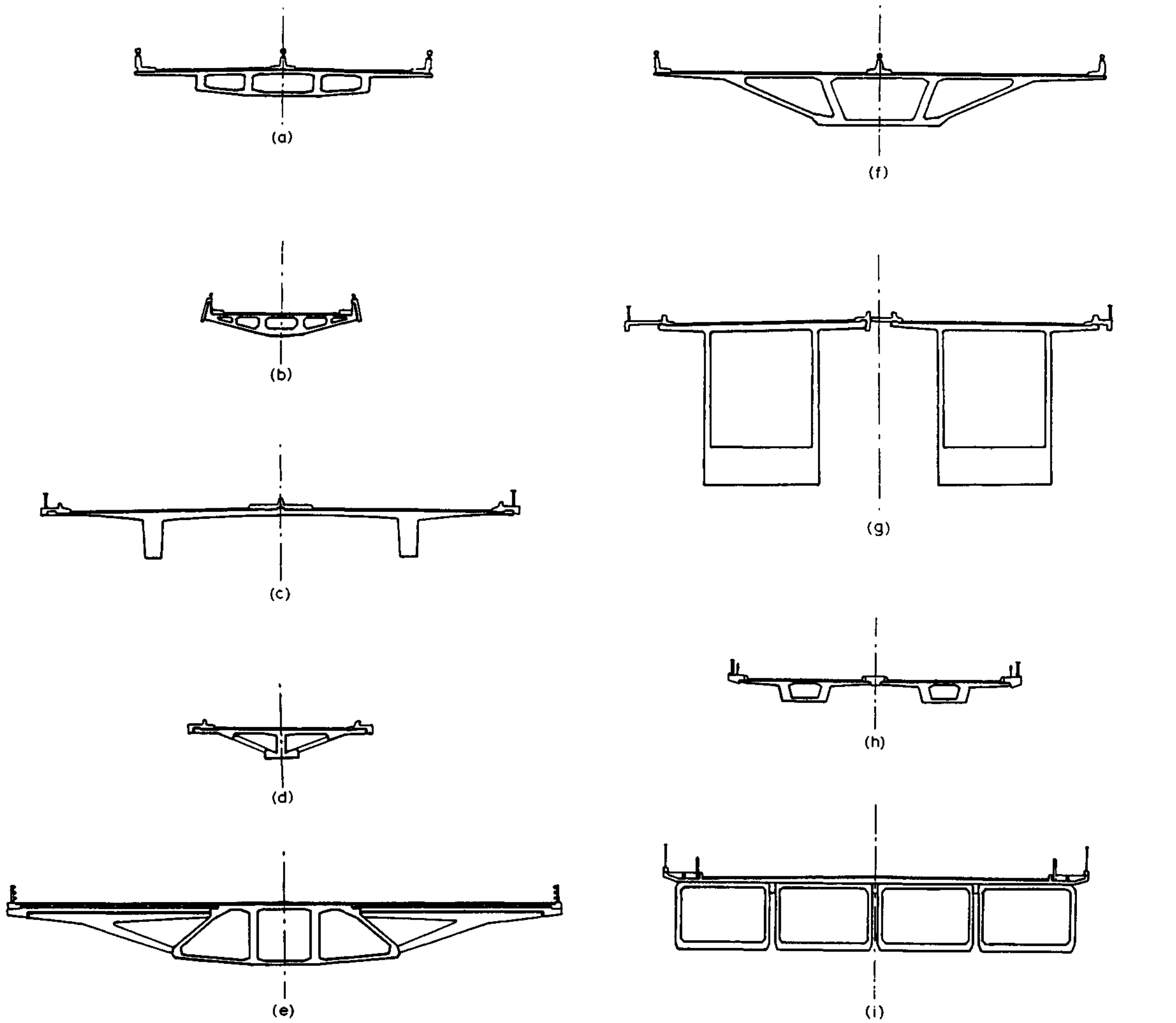


Figure 20.11 Elevated roadways. (a) Westway, Section One; (b) Tunnel relief flyover, Liverpool; (c) Vorlandbrucke Obereisesheim; (d) Illtal; (e) West Gate approach viaducts; (f) Westway, section five; (g) Bendorf, section at pier; (h) Mancunian Way; (i) Gladesville; (j) London; (k) Narrows; (l) Annacis; (m) Severn; (n) Europa; (o) Duisberg-Neuenkamp; (p) Concordia; (q) Kniebrücke; (r) Sava I; (s) Zoo

Equations (20.4) and (20.6) must be satisfied simultaneously and, although this makes the problem nonlinear, the correct value can be satisfactorily determined by interpolation by solving for two assumed values of H. Each assumed H gives an incorrect solution to Equation (20.6) and, assuming the error varies linearly, the correct value of H can be found. For each assumed value of H, the structure behaves as a simple beam and influence lines can be constructed for bending moments, etc., and for $\int_0^l v(x)\mathrm{d}x$. Hawranek and Steinhardt[10] suggest that for a particular loading case the bending moment and shear forces be found from both sets of influence lines as well as the $\int_0^l v(x)\mathrm{d}x$ values. H is found by interpolation and then the final bending moments and shears are found by interpolating between the two sets of values already found from the influence lines.

Typical results for a continuous stiffening girder are shown in Figure 20.14.

The above treatment follows that given by Hawranek and Steinhardt[10] who also give a comprehensive set of standard solutions for the substitute girder. The result quoted below illustrates the form the solutions take. Using:

$$\mu^2 = H/EI$$

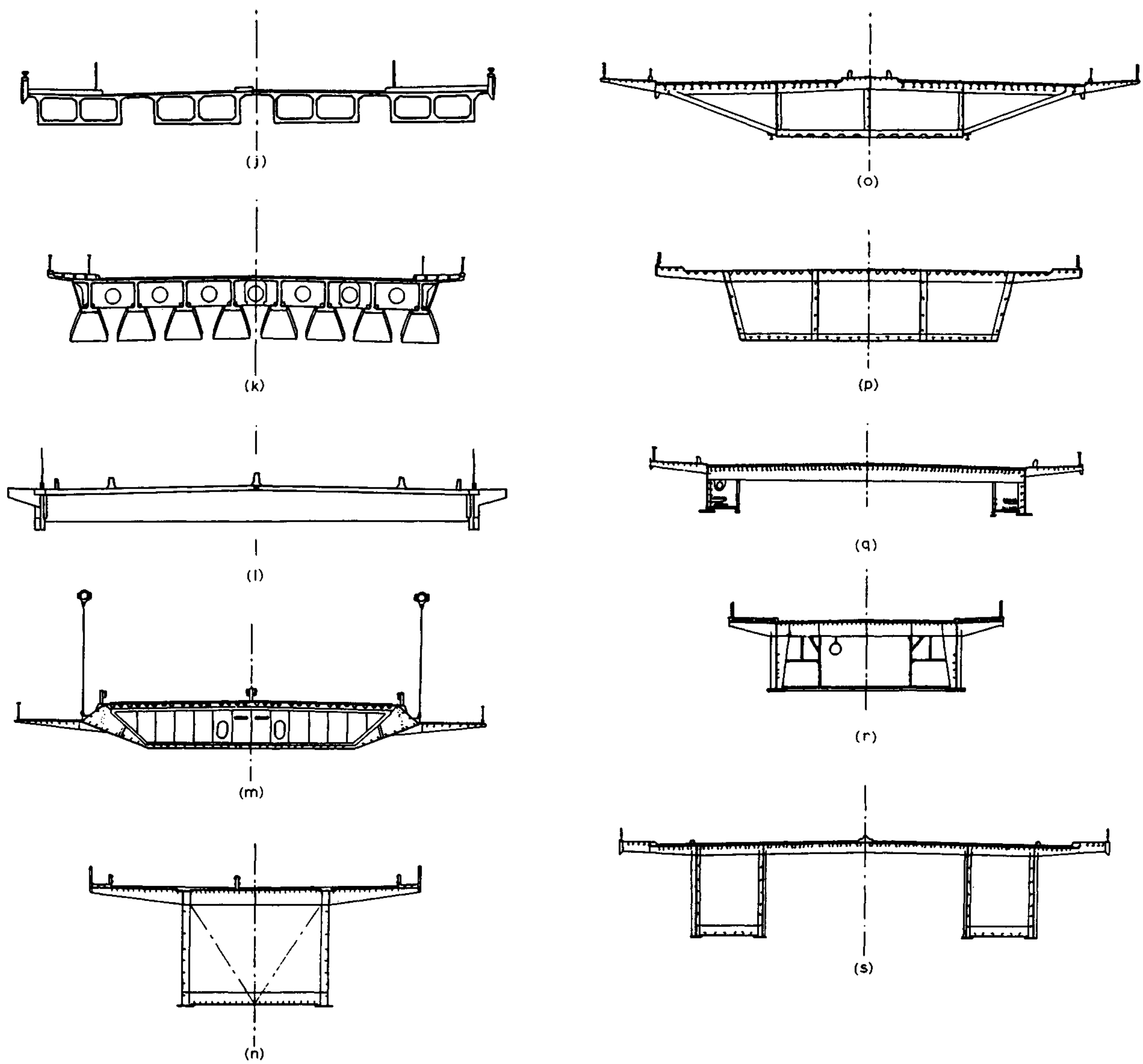
For the load case of Figure 20.15, deflections as a function of x are given by:

$$v(x, \xi) = PG(x, \xi)$$

$$= P\frac{l}{H}\left[\frac{x}{l}\left(1 - \frac{\xi}{l}\right) - \frac{\sinh \mu x \sinh \mu(l-\xi)}{\mu l \sinh \mu l}\right] \quad \text{for } \xi \geqslant x \tag{20.8}$$

$$v(x, \xi) = PG(x, \xi)$$

$$= P\frac{l}{H}\left[\frac{\xi}{l}\left(1 - \frac{x}{l}\right) - \frac{\sinh \mu\xi \sinh \mu(l-x)}{\mu l \sinh \mu l}\right] \quad \text{for } \xi \leqslant x$$



G(x, ξ) is known as a Green's function.

And:

$$F(\xi) = \int_0^L v(x)\mathrm{d}x$$

$$= P\frac{l^2}{H}\left[\frac{\xi(l-\xi)}{2l^2} - \frac{1}{(\mu l)^2}\left(1 - \frac{\cosh\mu(l/2-\xi)}{\cosh\mu(l/2)}\right)\right] \quad (20.9)$$

Computers can be used to analyse suspension bridges either by following the above approach or by means of standard framework programs provided the interaction of axial loads and deflections is allowed for. In other words, the change in geometry of the cable is considered. In some programs the axial loads must be stated as part of the data in the same way that H is assumed in obtaining a solution to Equation (20.4). In others, an interactive process produces the correct axial forces. The structure solved can include the actual system of suspenders, tower properties, etc. or can be a very simple solution of the substitute structure of Figure 20.12.

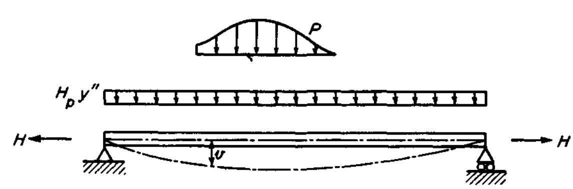


Figure 20.12 Substitute girder

20.3.1.2 Self-anchored suspension bridges

The horizontal component of the cable tension can be resisted by the stiffening girder which then acts as a laterally loaded compression member between suspenders. The net tension on the structure is therefore zero and the substitute girder has a zero axial load acting on it. The structure is substantially linear in its response to live load whereas the externally anchored bridge has an increasing stiffness with increasing deflection.

20.3.1.3 Arch bridges

The design of arches is based on the thrust line following the shape of the arch so that there is either no bending moment or a reduced bending moment in the arch member.

The shape of arch can only satisfy one condition of loading without bending moments being developed. Temperature changes, creep, foundation movements and imperfections must, however, introduce some bending in all but the three-hinged arch. In a bridge structure, live loading will produce a varying distribution of loading which will introduce bending. Clearly, the higher the proportion of dead load the more nearly can the arch be designed to be in pure compression.

The most common shapes are the circular arch, the parabolic arch and more recently the inclined leg frame (Figure 20.16). Loadings over the whole of (c) can be examined in two stages, which enables a design to be produced before detailed dimensions are known (Figure 20.17).

Arches for bridges frequently have continuous beams supporting the deck as in Figure 20.18.

At the design stage, since the dead load carried by the deck will depend on the construction method and the thrust jacked into the arch, the structure is to some extent determinate but, clearly, some bending of the deck beams between supports is introduced. The system can be represented as in Figure 20.19 where $H \simeq wl^2/8f$.

At the preliminary design stage, live loading can be examined by splitting it into symmetrical and antisymmetrical components (Figure 20.20). The symmetrical system will produce to a first approximation small bending moments and the asymmetrical system is equivalent to a simple beam with half the arch span (zero thrust due to opposite effects of load).

The local effects of loading on the deck can always be examined as a beam between columns and the overall behaviour can be seen as that of an arch with a total EI of $EI_{deck} + EI_{arch}$. The bending moments produced in the parts will be in proportion to stiffness.

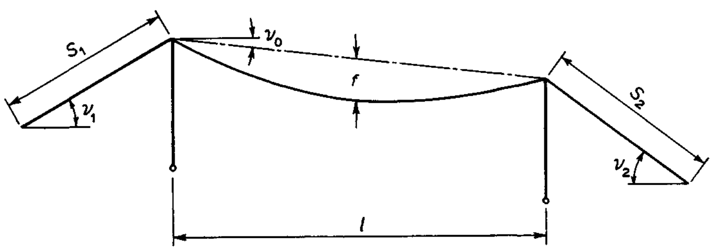


Figure 20.13 (After Hawranek and Steinhardt (1958) *Theorie und Berechnung der Stahlbrücken*. Springer-Verlag)

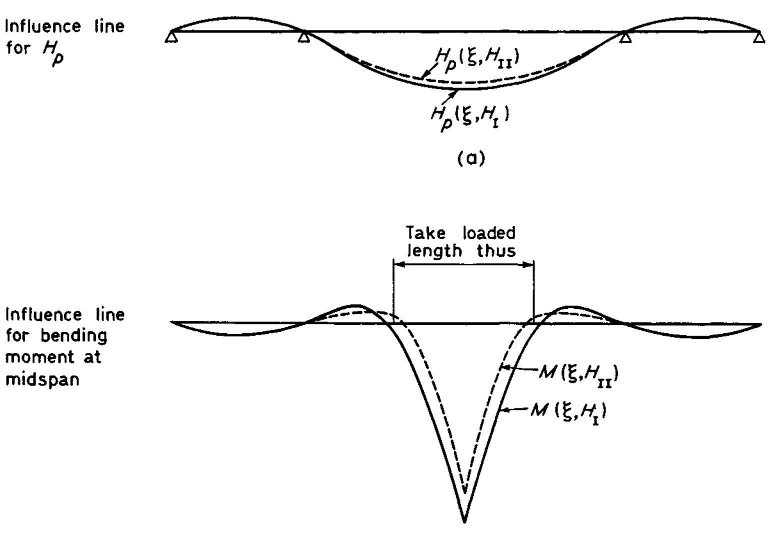


Figure 20.14 Influence lines. (a) Influence line for H_p; (b) for bending moment at midspan. (After Hawranek and Steinhardt (1958) *Theorie und Berechnung der Stahlbrücken*. Springer-Verlag)

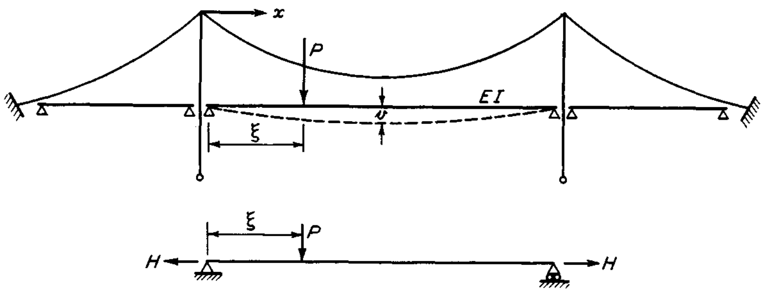


Figure 20.15 (After Hawranek and Steinhardt (1958) *Theorie und Berechnung der Stahlbrücken*. Springer-Verlag)

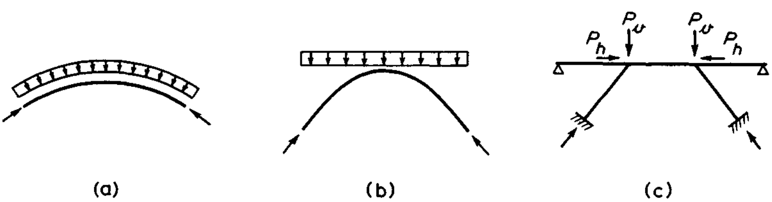


Figure 20.16 Arches with the loading that produces no arch member-bending. (a) Circular arch; (b) parabolic arch; (c) inclined leg frame

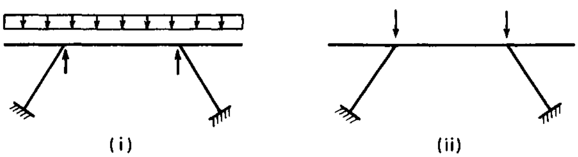


Figure 20.17 Two-stage analysis of inclined leg frame

Table 20.11

f_t	0	0.1	0.2	0.3	0.4	0.5	
For a two hinged arch	39.4	35.6	28.4	19.4	13.7	9.6	$H_{cr}=\frac{kEI}{l^2}$
For a fixed arch	80.8	75.8	63.1	47.9	34.8	24.4	

Arches may require correction for deflections since the thrust will magnify these in the same way that a strut is affected by end load. A two-hinged arch with a uniform loading buckles as in Figure 20.21 and to a first approximation the effective length of the strut is $l/2$. The magnification of moments can be found as for a strut using $1/(1-H/H_{cr})$ as the factor. Critical loads are available for a variety of cases as in Table 20.11.

Equation (20.4) for suspension bridges is applicable to arches with changes of sign in H and y. The nonlinear effects of deflections can be determined using that equation or the substitute beam in compression instead of tension. Computer programs can be used to analyse arches in the way already described for suspension bridges but if H/H_{cr} is small it is unnecessary to use programs which allow for changes of geometry.

Out-of-plane deflections can cause buckling or significant stresses in single arches or arches not braced laterally. This effect can be investigated readily using a grillage programme which allows for the interaction of axial loads and transverse deflections. The plane of the grillage must be considered as the plane of the arch for that purpose.

20.3.1.4 Tied arches

In the tied arch, the thrust is balanced by tensile forces in the stiffening girder which simplifies the in-plane behaviour of the structure since there is no net thrust on the structure. In-plane buckling is thus prevented but out-of-plane buckling is still possible.

20.3.2 Bridge girders of open section

The cross-girder and bracing members are shown as broken lines in Figure 20.22 since they do not affect the girder under twisting loads unless it has torsionally stiff members.

To find q (Figure 20.23) it is necessary to consider the equilibrium equation of the top flange (cf. simple beam theory). It is assumed that the shear force on the top flange is zero as shown and that the second moment of area of the top flange about a vertical axis is I_T.

Taking moments in a horizontal plane:

$$bq\,dx=\frac{(\sigma+d\sigma)}{b/2}I_T-\frac{\sigma}{b/2}I_T$$

$$=d\sigma\frac{2I_T}{b}$$

Therefore:

$$q\,dx=d\sigma\frac{2I_T}{b^2}$$

This is the same equation as that used for simple beams if their top flange area A is made equal to $2I_T/b^2$. It follows that q can be found by considering an equivalent simple beam with A_T replacing the deck and acted on by shear forces Q (Figure 20.24). Note that Q is not altered by horizontal shears and is therefore the same as in a simple beam loaded by W.

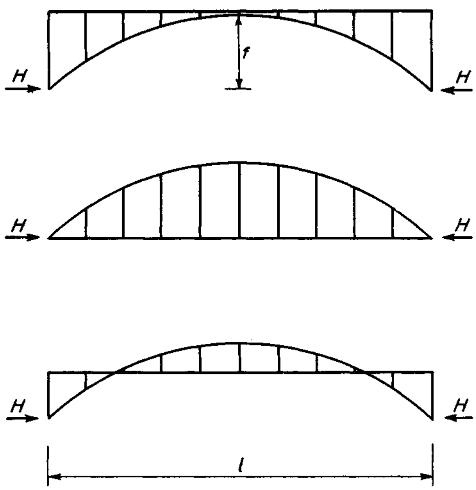


Figure 20.18

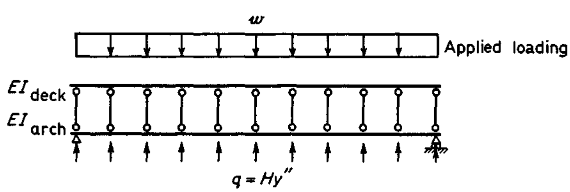


Figure 20.19 Application of substitute girder

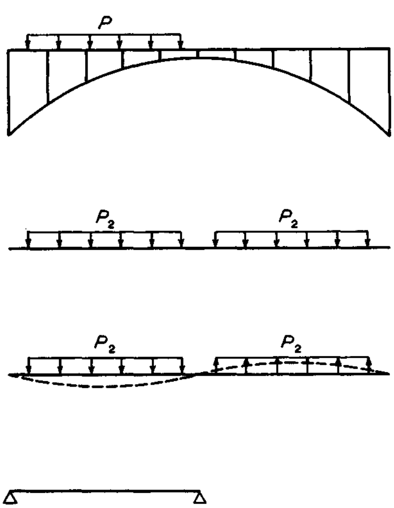


Figure 20.20 Arch live loading

Figure 20.21 Buckling of a two-hinged arch

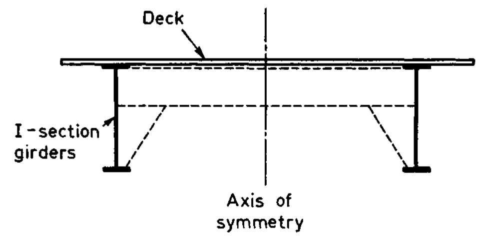


Figure 20.22 Girder of thin-walled open section

Any twisting load can be referred to the web positions giving the loads to be applied to the effective girder of Figure 20.24. The bending moments produced in the effective girder, acting as a single beam with the span of the actual structure, are applied to the effective girder cross-section. The second moment of area I^r_{eff} is used to find the longitudinal stresses between the top and bottom flanges. The remainder of the top flange stresses can be found by assuming a linear variation of stress between the web–flange junctions.

More complex loads (Figure 20.25) require consideration of horizontal forces and it is convenient to note that the centre of rotation (or shear centre) of the cross-section under purely twisting loads is a distance y_1 (Figure 20.26) above the top flange. This can be seen if it is recognized that longitudinal strain distribution and, hence, curvature results in the ratio of deflections in Figure 20.26 to be $w/v=2y_1/b$ or $w=2(y_1/b)v$. Therefore, at a height above deck of y_1 the normal to the midpoint of the deck must cut the vertical axis.

The vertical members of the cross-section may be individual boxes or thick-walled concrete webs with significant torsional stiffness in both cases. If transverse bracing is provided, thus preventing the shape of the cross-section from changing, the rotation of the boxes will equal that of the deck. This effect can be included to give the following governing equations with the deflections shown in Figure 20.25 which include a vertical displacement of the whole cross-section z. I_z and I_y are the usual second moments of area about a horizontal and vertical axis respectively. GK is the sum of the torsional stiffness of the whole cross-section:

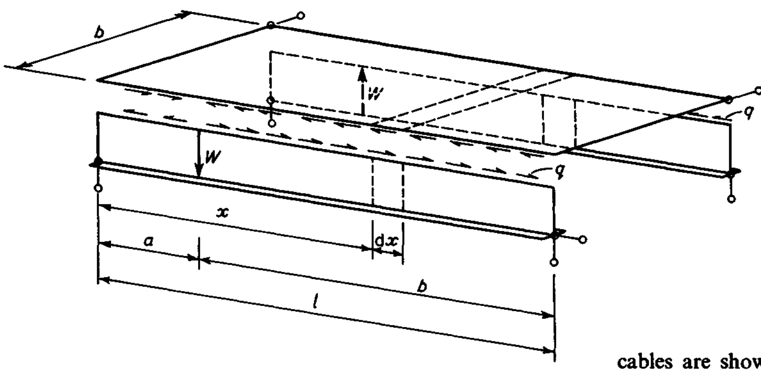
$$\left.\begin{aligned} &EI_z z^{iv}(x) = P_1(x) \\ &EI^r_{eff} v^{iv} - GK\frac{2}{b^2}v'' = P_1\frac{e_2}{b} + P_2\frac{(e_1+y_1)}{b} \\ &EI_y\left(w^{iv} - \frac{2y_1}{b}v^{iv}\right) = P_2 \end{aligned}\right\} \qquad (20.10)$$

Equations (20.10) are not put forward for solution as a set of differential equations but as a description of the various mechanisms involved. The second is similar to the suspension bridge Equation (20.4), since $GK(2/b^2)v''$ can be compared with the Hv'' term. There is no term corresponding to $H_p y''$. The equivalent structure is, therefore, a beam of stiffness EI^r_{eff} under axial tension $GK(2/b^2)$.

The deflections and bending moments will be duplicated correctly by such a structure but the stresses do not, of course, require a direct contribution from the imaginary tensile force.

20.3.3 More general behaviour of suspension bridges and arches

The above treatment of a girder can be extended to include the usual twin cables of a suspension bridge. The positions of the



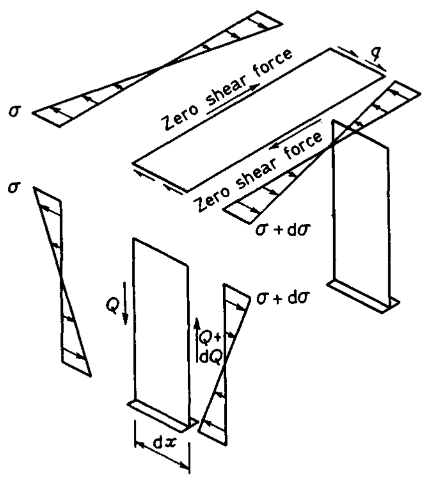


Figure 20.23 Loading and stresses on thin-walled open section

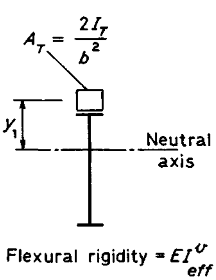


Figure 20.24

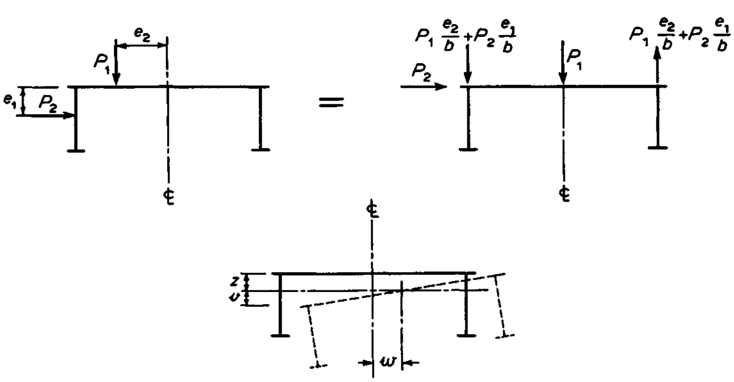


Figure 20.25 Overall loading and displacement

cables are shown in Figure 20.27. Equations (20.10) become extended into:

$$(1)\quad EI_z z^{iv}(x) - (H_1 + H_2)z''(x) - \underline{(H_1 - H_2)\frac{2e}{b}v''(x)}$$

$$= P_1(x) + (H_{p1} + H_{p2})y''$$

$$(2)\quad EI^v_{eff}v^{iv}(x) - GK\frac{2}{b^2}v''(x) - (H_1 + H_2)\frac{2e^2}{b^2}v''(x) - \underline{(H_1 - H_2)\frac{e}{b}z''(x)}$$

$$= P_1\frac{e_2}{b} + P_2\frac{e_1}{b} + P_2\frac{y_1}{b} + (H_{p1} + H_{p2})y''\frac{e}{b} \qquad (20.11)$$

$$(3)\quad EI_y\left(w^{iv}(x) - \frac{2y_1}{b}v^{iv}(x)\right) = P_w$$

The terms underlined are fairly small and can be neglected, which leads to (1) and (2) being independent equations in z and v.

Further, if purely torsional loading is assumed, $H_{p1} \simeq H_{p2} = H_p$ and writing $H_1 + H_2 = 2H$ the second equation becomes:

$$EI^{eff}_v v^{iv}(x) - \left(GK\frac{2}{b^2} + 2H\frac{2e_2}{b^2}\right)v''(x)$$

$$= P_1\frac{e_2}{b} + P_2\frac{(e_1 + y_1)}{b} + 2H_p\frac{e}{b}y'' \qquad (20.12)$$

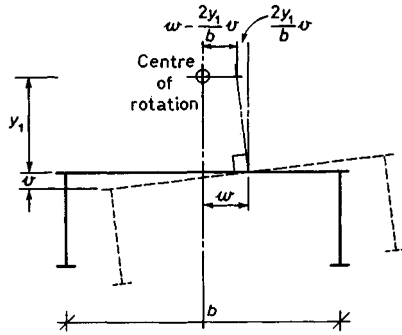


Figure 20.26 Twisting about centre of rotation

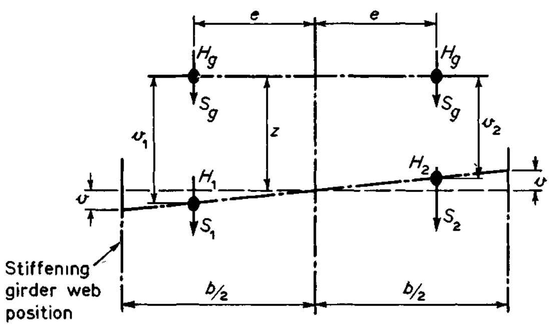


Figure 20.27 Change in cable position

For each cable the condition $\int_0^L \Delta \, dx = 0$ must be satisfied leading as before to the correct value of H_p. Equation (20.12) is clearly of the same form as the ordinary suspension bridge equation for vertical deflections except that EI^{τ}_{eff} replaces EI and $GK(2/b^2)+2H(e^2/b^2)$ replaces H. Therefore, the same results can be used to solve this equation.

The above equations may be used to investigate aerodynamic and other vibrational effects as well as live loading. The above treatment follows that of Hawranek,[10] but the effective beam concept has been used in order to give the equations more physical significance. Hawranek works in terms of a warping function and relates loads to the shear centre. Hawranek and Steinhardt give, however, a number of useful results for the natural frequency of various systems.

Horizontal deflections due to wind may produce a significant lateral component of the cable force but the complete properties of the structure must be known before the effect can be determined, e.g. the tower stiffness influences this type of behaviour. It is not considered in the above equations. For a given system, approximate results can be estimated by simple calculations but rigorous results can be obtained using, for example, a grillage programme which includes the interaction of axial forces and deflections. The plane of the grillage must be assumed to be the plane of the cable for this purpose.

20.3.4 Single-cell box girder

The full torsional stiffness of a single cell, $4A^2G/\oint(1/t)\,ds$ is only mobilized if the twisting forces are applied in the cross-section in a distribution corresponding to a constant shear flow around the box. A structure with effective diaphragms at the supports only (Figure 20.28(a)) and with hinges at the long edges, has no torsional stiffness under the twisting loads shown. They are carried in differential bending. The associated stresses are warping stresses (Figure 20.29). Figure 20.28(a) shows how the warping moments M_0 are found and Figure 20.28(c) shows the effective beams carrying equal and opposite values of M_0. The properties of one of the effective beams can be calculated from the following values:

Top flange area $\qquad A_T = \dfrac{2I_T}{b_T^2}$

Web as in actual box beam

Bottom flange area $\qquad A_B = \dfrac{2I_B}{b_B^2}$

where I_T and I_B are, respectively, the second moments of area of the top flange assembly and the bottom flange assembly about the vertical axis of symmetry.

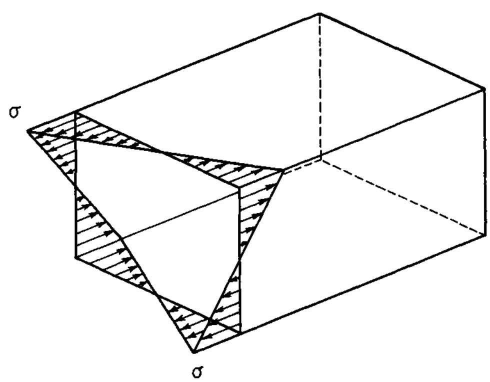


Figure 20.29 Warping stress distribution

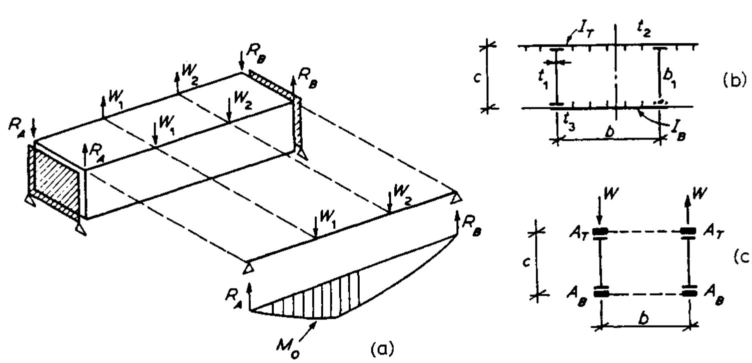


Figure 20.28 Equivalent I-beam

The warping stresses in the effective beam are determined as in normal beam theory and the remaining warping stresses in the flanges can be found by assuming a linear variation of stress across the top and bottom flanges.

Complex box structures can be solved most economically by suitable finite-element programs. Elaborate calculations by alternative means are not justified, but in order to understand the behaviour of box structures or for structures where an approximate result shows that further investigation is unnecessary, the following methods are of value.

20.3.5 Boxes with discrete diaphragms

Steel boxes usually have a number of diaphragms formed from a lattice system or solid plate and sometimes unbraced frames. The following influence-coefficient approach is one of several methods of dealing with such structures acted upon by twisting loads.[11,12]

The structure is rendered statically determinate for twisting loads by releasing the warping moments at each diaphragm thus permitting relative warping rotations to occur (Figure 20.30). Each bay of the box between diaphragms acts in a similar way to the box of Figure 20.28. At each diaphragm, however, instead of the twisting moments being taken by the supports, they are fed through the diaphragms into the box as pure torsion. The diaphragms then act as a series of elastic supports to the equivalent beam as shown in Figure 20.31 except that at the initial stage the beam acts as if hinged at each spring and diaphragm and is therefore determinate.

The stiffness of the diaphragms k is found from the relationship between the distortional force system, Figure 20.32(a), which is the distortional component of Figure 20.32(b), and the distortional deflection system shown in Figure 20.33(a).

$$\frac{P}{v} = \frac{\text{applied distortional forces}}{\text{distortional deflection}}$$

The deflections of the diaphragms or springs induce relative rotations at each hinge. The rotations at the releases are increased by the warping produced by the torque fed into the box at each diaphragm and the effect of the load between diaphragms. Denoting θ_1 as the relative rotation due to spring deflections and local loads, the total relative rotation is:

$$\theta_r = \theta_1 + \frac{Q_{r+1}}{\bar{f}_{r+1}} - \frac{Q_r}{\bar{f}_r} \qquad (20.13)$$

where Q is the torque T divided by b, and $\bar{f}$ is the shear stiffness linking torsion and warping rotation:

$$\bar{f} = \frac{G}{E}\frac{8c^2}{b/t_3 + b/t_2 - 2c/t_1} \qquad (20.14)$$

where r and $r+1$ refer to bays between diaphragms as in Figure 20.30.

The influence coefficients for solving the series of compatibility equations are:

$$E\theta_{r,r-2} = \frac{1}{a_{r-1}a_r k_{r-1}}$$

$$E\theta_{r,r-1} = \frac{a}{6I_r} - \frac{1}{a_r f^*_r} - \frac{1}{a_r}\left[\left(\frac{1}{a_r} + \frac{1}{a_{r-1}}\right)\frac{1}{k_{r-1}} + \left(\frac{1}{a_r} + \frac{1}{a_{r+1}}\right)\frac{1}{k_r}\right]$$

$$E\theta_{r,r} = \frac{a}{3I_r} + \frac{a_{r+1}}{3I_{r+1}} + \frac{1}{a_r f^*_r} + \frac{1}{a_{r+1} f^*_{r+1}} + \frac{1}{a_r^2 k_{r-1}} + \left(\frac{1}{a_r} + \frac{1}{a_{r+1}}\right)^2 \frac{1}{k_r} + \frac{1}{a_{r+1}^2 k_{r+1}}$$

$$E\theta_{r,r+1} = \frac{a_{r+1}}{6I_{r+1}} - \frac{1}{a_{r+1} f^*_{r+1}} - \frac{1}{a_{r+1}}\left[\left(\frac{1}{a_{r+2}} + \frac{1}{a_{r+1}}\right)\frac{1}{k_{r+1}} + \left(\frac{1}{a_{r+1}} + \frac{1}{a_r}\right)\frac{1}{k_r}\right] \qquad (20.15)$$

$$E\theta_{r,r+2} = \frac{1}{a_{r+2}a_{r+1}k_{r+1}}$$

where the effective distortional bending stiffness of the box in bay r is EI_r and an additional f^* is a shear stiffness for the warping rotation produced by distortional shears in the box:

$$f^* = \frac{G}{E}\frac{8c^2}{b/t_3 + b/t_2 + 2c/t_1} \qquad (20.16)$$

The various results given above can be found from concepts of virtual work using the mechanism shown in Figure 20.34 consisting of a series of shear webs and booms of axial stiffness. It can be used to obtain more general results such as those for boxes of trapezoidal cross-section.[11,13]

The warping produced by torsion results in stresses only if there is a change in torsion and therefore incompatible warping. Longitudinal stresses act to remove the lack of continuity. An upper bound estimate of these stresses can be made by assuming that the cross-section cannot deform. The warping moments produced by a change in torque $T = Pb$ is:

$$X = \frac{P}{2}\frac{\sqrt{f^* I_\theta}}{f}\exp[-\sqrt{(f^*/I_\theta)}x] \qquad (20.17)$$

where x is the distance from the cross-section at which the change occurs.

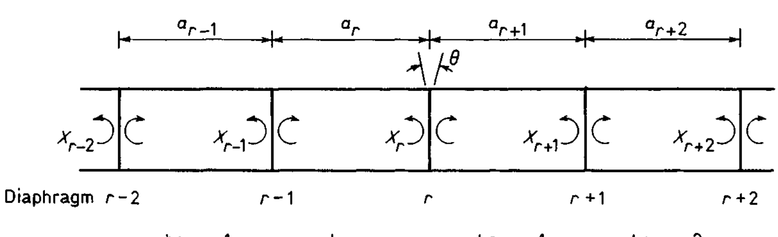


Figure 20.30 Box with discrete diaphragms

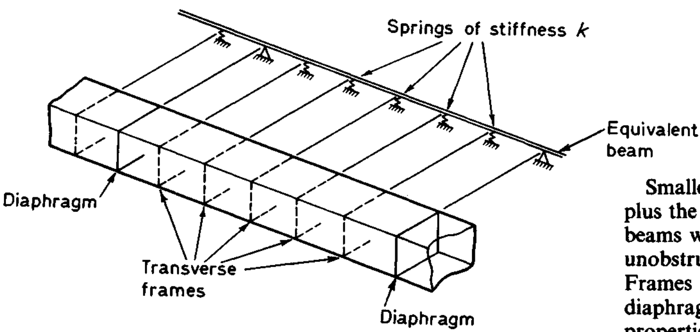


Figure 20.31 Equivalent beam on elastic supports

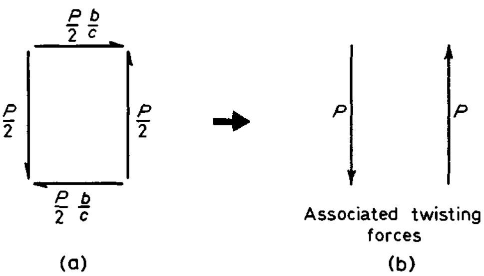


Figure 20.32 Distortional force system

The distribution of shear forces associated with warping stresses can be found from the warping stress distribution (Figure 20.35). Starting from the edge of the cantilever and assuming σ is the change in longitudinal stress over a unit length:

$$q=\int_0^s \sigma t \, ds$$

where q is the shear flow τt.

The complementary shear to q is on the face of the cross-section. The shear at the centre of the top flange can be assumed to be zero at the first stage of the calculation, which enables a simple shear system to be found. A pure torsional shear flow must be added to remove any component of pure torsion acting on the cross-section.

20.3.6 Box beams with continuous diaphragms

The frame action of the webs and flanges in concrete boxes provides a continuous resistance to distortion; consequently special diaphragms are not usually necessary except at disturbances such as bearings and other support points.

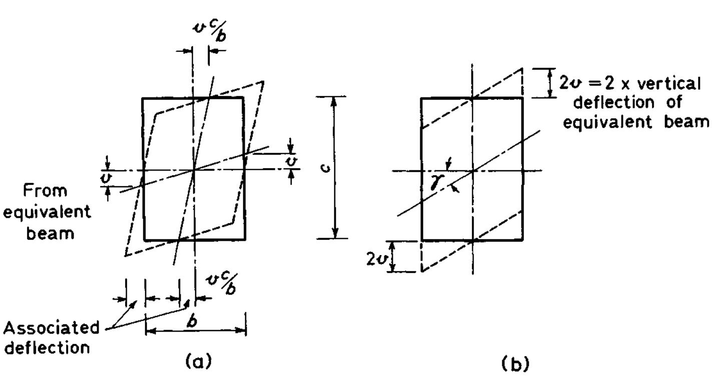


Figure 20.33 Cross-section distortion

Smaller steel box beams which rely on the stiffness of the sides plus the frame action of web and flange stiffness or larger box beams with special frames which leave the interior of the box unobstructed, have similar characteristics to the concrete boxes. Frames are generally flexible compared with braced or plate diaphragms but stiff frames can be made which will have properties that can only be explored fully by a treatment which is suitable for discrete diaphragms. The validity of the following approach for steel boxes can be determined from the half wavelength which should be greater than twice the spacing of the frames.

The effect of twisting loads P applied at the corners of the web on the vertical deflections caused by distortional bending of the box and allowing for the diaphragm action of the cross-section is:[14]

$$v=\frac{P}{2\alpha\beta}e^{-\alpha x}\left\{\beta\left[\frac{\lambda^2}{k}+\frac{1}{2E}\left(\frac{1}{f^*}-\frac{1}{f}\right)\right]\cos\beta x\right.$$
$$\left.+\alpha\left[\frac{\lambda^2}{k}-\frac{1}{2E}\left(\frac{1}{f^*}-\frac{1}{f}\right)\right]\sin\beta x\right\} \qquad (20.18)$$

assuming that the distance to a support is infinitely long. EI_θ is the effective distortional bending stiffness of the box, and k is the diaphragm stiffness per unit length. λ is defined by:

$$\lambda^4=\frac{k}{4EI_\theta}$$

and:

$$\alpha=(\lambda^2+k/4Ef^*)^{1/2}, \quad \beta=(\lambda^2-k/4Ef^*)^{1/2}$$

The generalized warping stress resultant is:

$$x=\frac{P}{2\alpha\beta}e^{-\alpha x}\left[\beta\left(\frac{1}{2}+\frac{\lambda^2 I_\theta}{f}\right)\cos\beta x\right.$$
$$\left.+\alpha\left(-\frac{1}{2}+\frac{\lambda^2 I_\theta}{f}\right)\sin\beta x\right] \qquad (20.19)$$

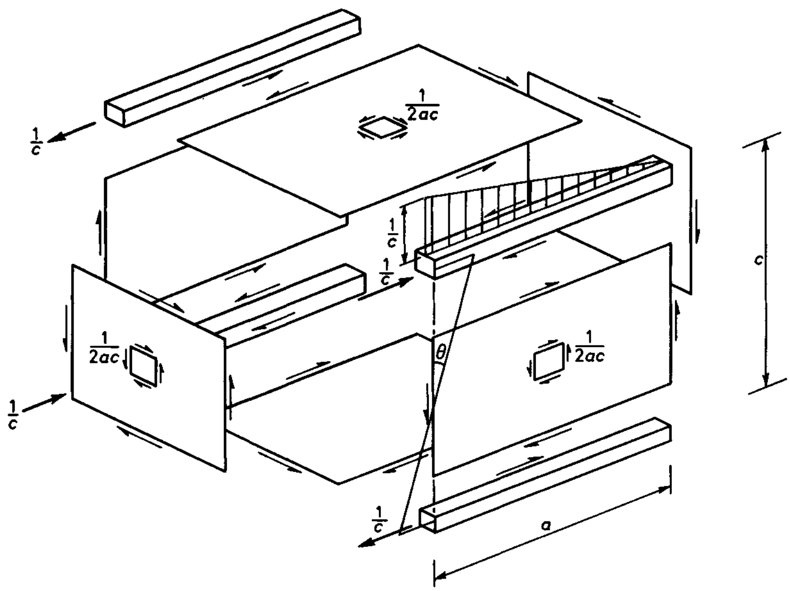


Figure 20.34 Exploded box girder

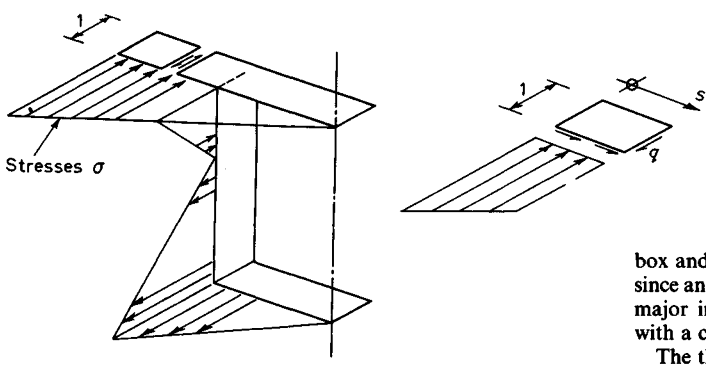


Figure 20.35 Half box

If the properties of the box are within the range $\lambda^2-(k/4Ef^*)<0$ then $\bar{\beta}=(k/4Ef^*-\lambda^2)^{1/2}$ replaces β and hyperbolic functions are used.

The half wavelength is $\pi/2\beta$.

In many cases, the effect of shear is not considerable and $a=\beta=\lambda$. The equations then simplify into the standard beam on elastic foundation results. The above equations include, however, the effect of the change in torque due to the twisting loads P and in order to correct the results of beam on elastic foundation theory the warping moment of Equation (20.17) should be added. The correction is likely to be of most significance in boxes which are much wider than their depth.

The above equations are valid for boxes of trapezoidal cross-sections if the concepts are generalized as was illustrated by Dalton and Richmond.[13]

20.3.7 Box girders with cantilevers

The advantages of box girders over structures of open cross-section are sometimes only marginally linked to structural efficiency. However, where a compact structure is to carry a much wider deck, producing a large cantilevered section, the torsional stiffness and strength of the box is of primary importance. The importance of the interaction of the cantilever and the box and the magnitude of the stresses is correspondingly great since any loss of torsional strength and stiffness could result in a major increase in the shear stresses and longitudinal stresses with a corresponding reduction in the load factor.

The three types of cantilever in Figure 20.36 show:[15]

(1) Transmission of torque Pe into twisting couple which must be resisted by diaphragm action.
(2) Cantilever bracket which, in the position shown, produces horizontal loads which are in the correct ratio to the vertical shear P to give a torsional shear flow without diaphragm action. In a concrete box the transverse moments in walls of the structure would be small.
(3) The relative stiffness of the web results in most of the cantilever moment being taken by the web which produces horizontal forces corresponding to a brace at any position of the load. The transverse moments in the walls are therefore reduced rather than increased as the eccentricity increases. If $e=b_T/2+b_B/2$ they are practically zero.

In concrete boxes where the only diaphragm stiffness is due to transverse bending of the walls, except at bearings, the beneficial effects described in (2) and (3) are of considerable importance. The reduction in transverse moments has been described but another important benefit is the reduction in the shear forces in the outer web.

20.3.8 Multiple web girders of open cross-section

The choice of an individual main girder or beam as the member to carry say a concentrated load applied to it with the distribution of that load as the next, correcting, operation is more appropriate to smaller spans where typically there are large numbers of main beams and the width of the bridge is comparable with the span. In most shorter span structures the width is in fact more than the span which lends further weight to the argument. The following section on harmonic analysis adopts this approach.

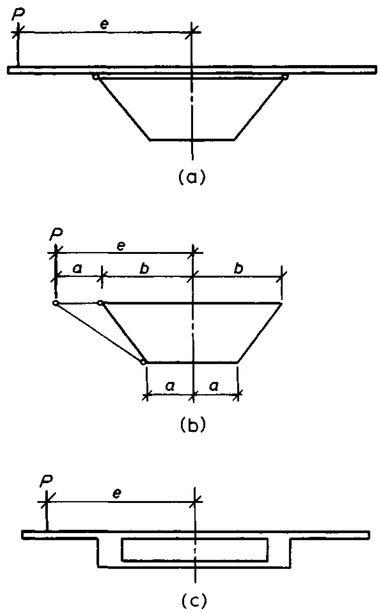


Figure 20.36

20.3.8.1 Harmonic analysis

The interaction between a series of separate simply supported beams of constant cross-section can be investigated for arbitrary loading using harmonic analysis. The sine series is the most suitable approach because any load which varies sinusoidally over the span in a complete number of halfwaves produces a deflected profile for each beam of the same form but of varying magnitudes. This result is justified provided that it is recognized that the interacting forces between the beams will be proportional to the transverse deflected form and will also vary sinusoidally.

The interaction between the beams is dependent on the ratio of the transverse to longitudinal stiffness:

$$a=\frac{12}{\pi^4}\left(\frac{L}{h}\right)^4\frac{D_y}{D_x} \qquad (20.20)$$

where D_y and D_x are stiffnesses of the equivalent orthotropic plate in the transverse and longitudinal directions.

For a single half-wave loading on one beam the distribution coefficients giving the fraction of the load taken by each beam have been calculated for a number of different systems by Hendry and Jaeger.[16] Figure 20.37 shows the coefficients for a five-beam bridge with beam 2 loaded (Figure 20.38).

The coefficients given are for the first harmonic only. Coefficients for subsequent harmonics can be found by varying a as appropriate for the shorter wavelength. Alternatively, if a sufficiently close approximation is given by the first harmonic alone for distribution to the unloaded beams, the behaviour of the loaded beam is given by its 'free deflection' curve less that which has been distributed to the other beams.

The results in Figure 20.37 are for a system with zero torsional rigidity; Hendry and Jaeger also give results for a torsionally rigid system.[16] Intermediate torsional stiffnesses can be analysed by interpolation.

Fixed-ended and continuous beams which they also consider by this method may be more easily solved by an influence coefficient method. Hinge releases at the supports can convert a continuous system of beams into two or more simply supported spans. The behaviour of the released structure and the influence coefficients can be found using the above approach for the loading applied and each influence coefficient.

20.3.8.2 Eigenvalue methods

The above method of analysing a grillage of beams under out-of-plane loading is a particular example of a more general method, i.e. the eigenvalue approach. Whereas a sinusoidal variation of loading of simply supported beams of constant cross-section produces similar deflection forms, there are eigenload systems for beams of all forms which produce deflections with a deflected form similar to the load-intensity curve. Thus, continuous beams of varying cross-section can be investigated by such an eigenvalue approach. Where the transverse member is a continuous concrete slab or steel plate or comprises a large number of transverse beams it may be assumed that a continuous transverse medium is the most appropriate physical model. In some circumstances – where there are a small number of transverse members or other significant variations from a constant transverse medium – the eigenload system becomes a series of discrete loads on the main girder corresponding to the positions of the transverse members. The discrete approach may also be the most appropriate representation of a continuous transverse system in order to facilitate the analysis of more complex systems by numerical techniques.

Longer span bridge superstructures are nearer in behaviour to a single beam formed from the aggregate of the individual longitudinal main beams, slabs and plates of the complete bridge cross-section. Consequently, there are both conceptual and numerical advantages in analysing such structures as single aggregate beams with subsequent correcting operations to allow for the deformations of the transverse beams, slabs and diaphragms which contribute to the transverse stiffness of the superstructure.

The eigenvalue approach can be used to determine the necessary corrections by considering the characteristics of the transverse structural system of the bridge thereby turning the method described above in section 20.3.8.1 through 90°. The five-beam system for which the distribution coefficients were given in Figure 20.36 for beams of zero torsional stiffness can be used as an example of this approach.

Figure 20.39(a) shows a point load A acting at any spanwise position on the central beam of a set of girders which form any system of spans and have any support condition provided that both spans and supports are the same for all girders. Figure 20.39(b) is then the load distribution required to produce the effect of all beams acting as an aggregate beam to carry the point load.

The correcting systems are shown in Figure 20.39(c) and (d) and are eigenloads or vectors $\mathbf{p}_1$ and $\mathbf{p}_2$, of the transverse beams or other transverse system. The particular system shown is valid for a transverse beam or slab system that is constant in its flexural properties across the width of the five beams. The eigenload systems are both self-equilibrating and $\mathbf{p}_1$ and $\mathbf{p}_2$ can be calculated readily from statics such that (b), (c) and (d) are equivalent to (a). The eigenloads themselves are found by choosing a pattern of point loads that produces the same ratio of deflection of the transverse system at each beam position relative to the respective component of the eigenload system.

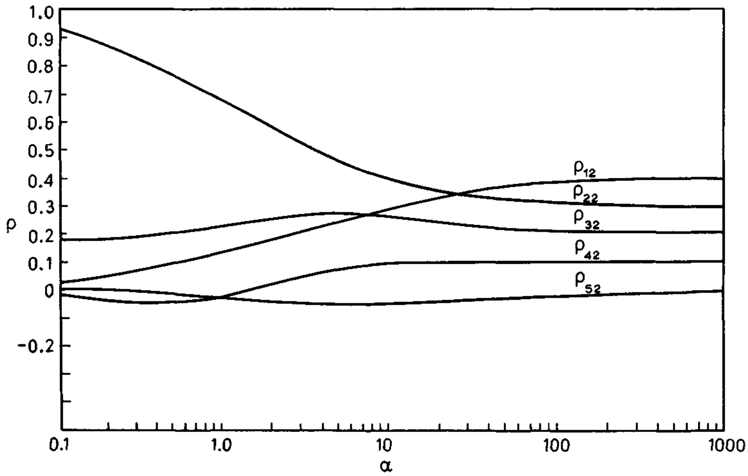


Figure 20.37 Distribution coefficients for a five-beam bridge, beam two loaded. (After Hendry and Jaeger (1958) *The analysis of grid frameworks and related structures.* Chatto and Windus)

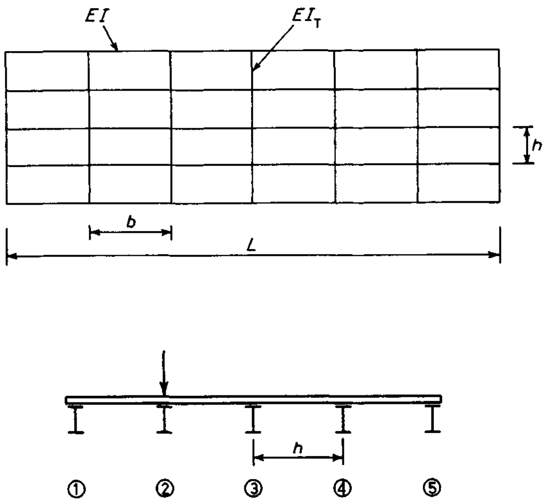


Figure 20.38 Five-beam bridge. (After Hendry and Jaeger (1958) *The analysis of grid frameworks and related structures.* Chatto and Windus)

Each beam is thereby effectively supported by a spring of equal stiffness when the complete system is loaded by one of the eigensystems. The spring stiffnesses for a transverse medium of flexural stiffness D per unit length for each beam is:

$$\frac{12\omega_1 D}{h^3} \quad \text{for eigenload system } \mathbf{p}_1 \tag{20.21}$$

and:

$$\frac{12\omega_2 D}{h^3} \quad \text{for eigenload system } \mathbf{p}_2 \tag{20.22}$$

where $\omega_1 = 0.0759016$ and $\omega_2 = 2.35267$

Thus, the beam systems under eigenload systems $\mathbf{p}_1$ and $\mathbf{p}_2$ can be analysed by considering for each load system the behaviour of the appropriate elastically supported beam. The essential feature of the method is, of course, that only one beam has to be investigated for each load system. Where the transverse medium of stiffness D is appropriate, that single beam can be analysed using beam-on-elastic foundation theory. The supports and continuity can be allowed for as appropriate. Consequently, the two correcting solutions are readily produced either as exact solutions or as a means of assessing the characteristics of the structural system.

Grillage programs enable solutions to be obtained by computer with the advantage that complex geometries can be simulated without difficulty. In both cases, it is necessary to estimate the effective top flange unless the more complex form of beam and slab program using finite elements is used.

20.3.9 Multiple single-cell box beams

A series of box beams connected by a top deck and, in some cases, cross-beams and stiffening diaphragms, can be analysed by various approaches. The grillage approach using a computer is not necessarily suited to all problems of this type but it is discussed first because, in determining the properties of the members, the essential mode of behaviour of this form of structure emerges.

Figure 20.40 shows part of a typical cross-section which could represent a series of concrete main longitudinal beams spanning 20 and 60 m or the trough stiffeners on a steel deck system spanning 4 m.

In such systems, the interaction between the beams is through the deck slab or deck plate only if no special cross-beams, etc. are provided. The magnitude of the interacting forces will be mainly dependent on the overall deflections of the beams and so the distortional stiffness of the individual boxes will almost equal the frame stiffness of the sides. The distortional bending or warping stiffnesses of the boxes may be assumed to be nil except for local wheel loads, which will be mentioned later.

Through isolating one of the boxes and its share of the deck slab, its behaviour can be considered further (Figure 20.41). A unit value of the antisymmetrical component of the vertical shearing forces will act as shown. The twisting effect will be resisted by pure torsion if there is no significant distortional resistance of the box beam except for diaphragm action. The cross-section, acting as a frame, is loaded by the distortional component of the twisting load. The net effect can be obtained by the device illustrated. The pin-jointed bars are placed so as to apply a pure torsional shear flow to the cross-section. They also prevent only a pure rotation of the cross-section since distor-

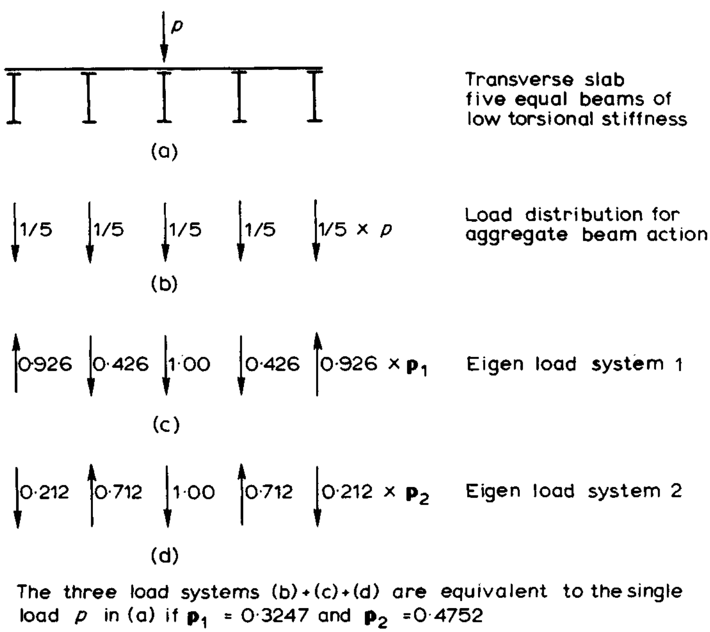


Figure 20.39 (a) Point load *A* acting on central beam; (b) load distribution acting as aggregate for point load *p*; (c) eigenloads for system 1; (d) eigenloads for system 2

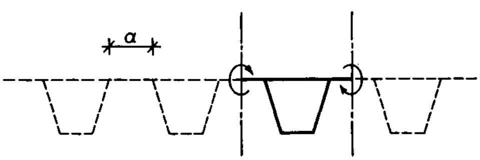


Figure 20.40 Interconnected multiple box girders

tional deflections do not have components in the direction of the restraints. A simple plane-frame analysis of the system gives the deflections and, hence, effective stiffness of a transverse member cantilevering out a distance $(a+b)/2$ from a grillage element with the torsional and flexural stiffnesses of the box. The antisymmetrical moments and symmetrical shears and moments can be applied to find the appropriate stiffnesses and, if the simplest form of grillage is used, a single compromise value must be chosen. The one based on antisymmetrical shears alone has been found to give results which compared well with a three-dimensional finite element simulation of a series of concrete boxes at 2-m centres.

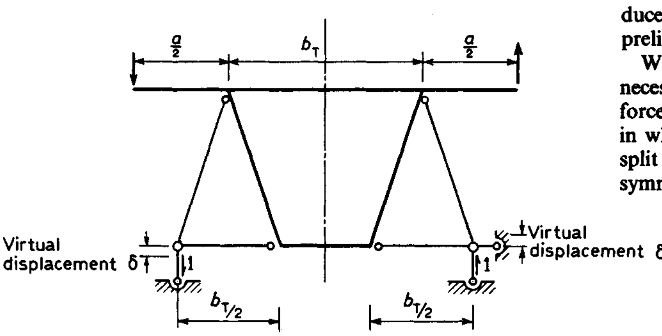


Figure 20.41 Torsional support system for slice of beam to give frame stiffness

The effect of local wheel loads on the transverse moments must be added to the above transverse moments by assuming that the boxes provide rigid supports. The maximum moments in the slab were, in the case mentioned, unaffected by the local slab loading.

Another effect already mentioned is the distortion of the box due to wheel loads applied on one side only. In fact, for boxes that are spaced at up to 2-m centres, the wheel loads are spread over a width sufficient to make the highest loads fairly symmetrically disposed about an individual box. An allowance can be made, however, by using the single cell theory to calculate stresses which are superimposed on those described above.

A similar approach has been used for steel deck systems, assuming points of contraflexure halfway between stringers but, instead of treating the structure as a series of discrete beams, it is transformed into an orthotropic plate. The transverse flexural stiffness is included in the torsional stiffness of the plate and is, therefore, taken as zero in the plate. The longitudinal flexural stiffness is determined in the usual way. Transversely, the deflections of the plate are represented as a sine series in order to solve the plate equation for wheel loading. A large number of terms in the series are required because of poor convergence which, together with the difficulties in obtaining detailed stress values other than longitudinal ones, from the solution, make the method of limited value. Graphs have, however, been produced[17] for the wheel loading used in the US which are useful for preliminary estimates.

Where a small number of large box girders are used, it may be necessary to allow for the various components of the interacting forces more exactly. An example of this is shown in Figure 20.42 in which the nonuniform component of load on two boxes is split into three load systems with the properties of either symmetry or antisymmetry. Releasing the forces at the centre of the connecting slab or cross-girder produces a lack of compatibility in each case. The influence coefficients are found from the unit forces of Figure 20.43(a) which relate to the compatibility equations including u_a and u_c and Figure 20.43(b) for u_b. δ represents the overall bending deflection of the box beam, $a\theta$ is the deflection produced by torsional rotation and δ_c is the deflection of the cross-girder or deflection of the slab. δ_c will be found from Figure 20.43(c) if it is a concrete box of the type already discussed, and θ_c similarly from Figure 20.43(d). Sinusoidally varying forces can be used for boxes without discrete diaphragms except at the supports, otherwise the influence coefficients can be related to individual cross-members.

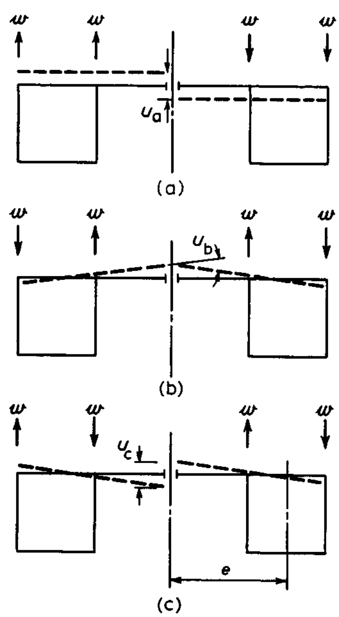


Figure 20.42 Interconnected boxes

20.3.10 Multicellular bridge decks

Bridge structures similar to the top-hat beam deck[6] which has 115 mm thick webs and no diaphragms between supports, have cross-sections which are relatively flexible in transverse shear. The usual grillage or orthotropic plate approach in which shear deformations are neglected is consequently invalid. The following treatment[6] is also relevant to cellular steel decks which may be even more flexible owing to higher web depth: thickness ratios.

Transverse shears are carried by the Vierendeel frame action (Figure 20.44) and the flexibility of the frame can be simulated by an equivalent web area of the transverse beams:

$$A_w = (E/G)\frac{12h/d}{(dh/2I_1)+(h^2/I_2)} \tag{20.21}$$

The grillage program used must include the effects of deflections due to shear strains. It is necessary to differentiate between rotations of initially horizontal and initially vertical lines when shear deformations are considered. In the grillage program used for Table 18.9 structure the rotation of vertical lines was the variable used. The flexural parameters are derived in the usual way but the torsional stiffnesses of the grillage members require further consideration.

20.3.11 Symmetrical loading

Loads disposed symmetrically in the transverse sense produce only relatively small transverse movements. True torsion is absent but Figure 20.45 shows that transverse members of grillage will be subjected to twisting which in the actual structure is simply a set of shear strains leading to shear transfer between the beams in a horizontal plane. This is sometimes referred to as shear lag. If the shear lag is small the shear transfer is high and the whole flange will be stressed uniformly. If the transverse members are assigned a stiffness per unit longitudinal distance of $h^2t/2$, this effect will be simulated. The torsional stiffness of the longitudinals is largely immaterial since they do not rotate significantly.

20.3.12 Antisymmetrical loading

Loads which cause twisting produce rotations of the cross-section which are the mean of a rotation of a vertical and horizontal line. The vertical line component is given directly by the grillage. The rate of change of rotation of a horizontal line is

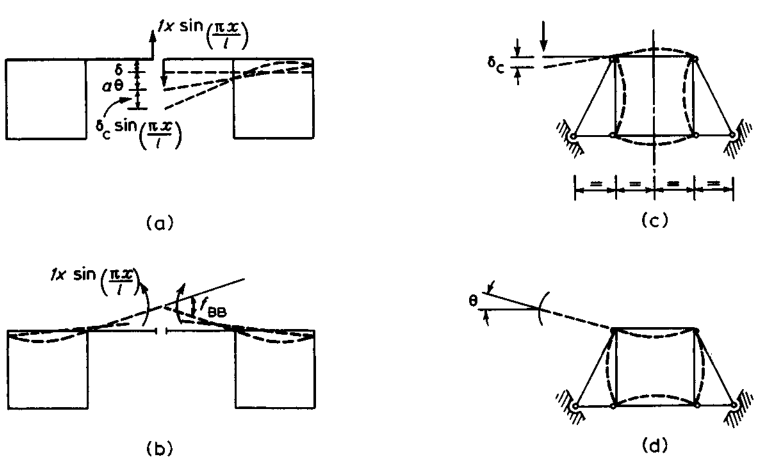


Figure 20.43 Unit loads and couples applied at releases

almost equal to the rate of change of rotation of a vertical line considered in the longitudinal and transverse direction respectively, since the true webs only undergo small shear strains compared with the frame of the cross-section.

Therefore, by allocating half the torsional stiffness GK of the cross-section to longitudinals and the remainder to transverse members, the mean rotation will be used as required.

GK for the whole structure is: $4GA^2/\oint(\mathrm{d}s/t)$. Therefore, the torsional stiffness of members divided by the spacing should be $(4GA^2/\oint(1/t)\mathrm{d}s)/2b$ for both sets of members.

Comparisons have shown that solving the structure in two stages using the different torsional properties described above does give good agreement, but it is clearly preferable to have one set of properties for any load case. It has been found that adopting $h^2t/2$ as the torsional stiffness for transverse members and dividing the remainder, $4GA^2/\oint(1/t)\mathrm{d}s-(h^2t/2)b$ amongst the longitudinals is a satisfactory compromise for all loading cases.

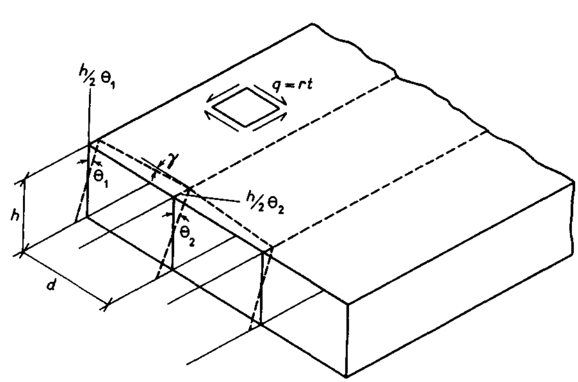


Figure 20.45 Shear strain due to differential longitudinal deflections

20.3.13 Design curves

Design curves have been obtained using the above approach for cellular decks constructed using precast 'top-hat' beams. They are of value for other cellular decks of similar proportions for preliminary design studies.

Figure 20.46 gives curves for HB coefficients (see Chapter 19). The range of decks covered is for spans from 60 ft to 120 ft (18.3 to 36.6 m) and deck widths from 30 ft to 90 ft (9.1 to 27.4 m), but the curves can be extrapolated to include values outside these figures and approximate solutions can also be derived for skew decks.

It is important to note that the curves have been derived by means of a grillage representation. Consequently, the application of the curves and the results obtained from them relate to the grillage solution and are subject to its conditions and limitations. The curves provide a coefficient per foot width of beam of the total moment M_L. The coefficients are plotted against the breadth: span ratio $B:L$ and are dependent upon the span L in conjunction with the edge stiffness ratio I and also upon the distance D from the centre of the outer wheels to the edge of the deck.

The values of k_L and k_I are derived from the intersection of $B:L$ and L, and k_p from the intersection of $B:L$ and D. The design live load moment for a composite top-hat beam is then given by:

$$M=k_Lk_p[1-k_I(I-1)]bM_L \tag{20.22}$$

where b is the beam width of either the edge beam, which may be asymmetrical, or the adjacent inner beam.

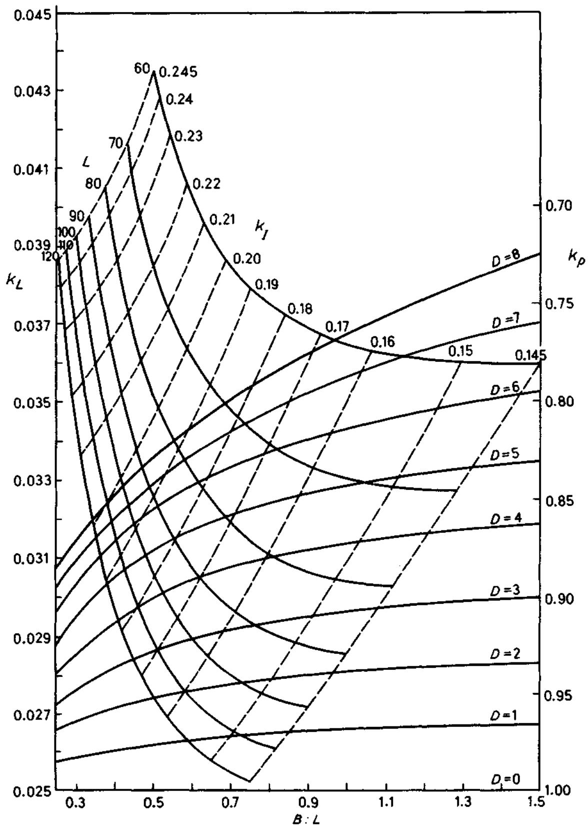


Figure 20.46 Design curves, HB coefficients

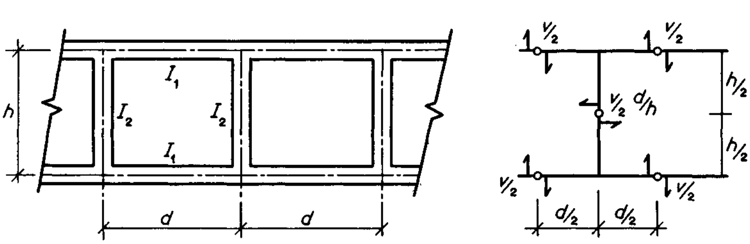


Figure 20.44 Transverse frame

20.4 Stress concentrations

Sudden changes in loads or in the shape of a structure produce stresses that cannot be calculated by normal beam theory. Concentrated loads such as the reactions at bearings and holes cut out of flanges are obvious examples of such changes but alterations in the direction of flanges, variations in thickness or width may produce significant effects. The introduction of strengthening members such as diaphragms or stiffening around holes are examples of situations in which the stress concentrations may weaken the structure instead of adding strength, if the full implications of the addition are not considered.

20.4.1 Shear lag due to concentrated loads

Beams with wide flanges are subject to shear lag effects, since the longitudinal stresses at points remote from the webs must be generated by the shear stress field across the flange. At sudden changes in shear, due to concentrated loads, the necessary changes in the longitudinal stresses in the flange require differential longitudinal movements in the transverse direction which produces the shear stress field in the flange. In other words, a longitudinal strain variation and therefore a stress variation across the flange is produced which is called the shear lag effect. It is important to note that transverse stresses are associated with shear lag as can be demonstrated by a consideration of the statics of the shear stress field alone.

Shear lag is most pronounced in girders of rectangular cross-section. The effect of cambering the flanges in the convex sense is to reduce shear lag. In the extreme case of a circular cross-section, shear lag does not occur, provided that all cross-sections remain circular. This is because the web of a rectangular box can deform in shear without shear deformations of the flange being necessary, whereas the circular beam can only deform in shear if all parts of the beam are subjected to shear strains. The distribution of shear strain in the latter case, which depends on purely geometrical considerations, agrees with the shear stress distribution produced by the longitudinal bending stresses of normal beam theory. Differential longitudinal movements are not required and no shear lag occurs. Figure 20.47 shows a substitute structure which enables shear lag effects to be found from standard formulae or relatively simple calculation. It is also useful for evaluating more precise results. The area A_F is the area of the edge member plus one-third of the web area between the flange and neutral axis. The area A_L is equal to half the area of the plate plus longitudinal stiffeners and b_s is given by Kuhn[18] as:

$$b_s = [0.55 + (0.45/n^2)] \qquad (20.23)$$

where n is the number of stringers in one-half of the flange. The actual distance of the centroid of the half-flange to the web is b_c whereas b_s is the distance which, together with the shear stiffness of the actual plate per unit width, simulates the shear lag property of the flange relative to the webs.

The above calculation gives the increase in stress at the web flange junction, $\Delta\sigma_F$, above normal beam theory. Figure 20.48 shows how the stress across the flange may be found assuming a cubic law variation.

The effects of shear lag are usually more important in steel box girders as the webs are more widely spaced than in concrete structures. Also, the thick diaphragms at bearings in concrete structures reduce the longitudinal stresses.

20.4.2 Changes in thickness and cut-outs

The change in thickness of a whole flange causes changes in the shear stresses between web and flange as well as local effects at the interface between the different flanges. In steel it is usual to taper the thicker plate to reduce the effects of fatigue and the possibility of brittle fracture. In some cases only part of the flange is thickened in areas of concentrated load such as forces from supporting cables or prestressing cables. The effect of such thickening is to tend to concentrate all flange forces in that part of the flange which must be allowed for by either gradually tapering out the increased area or by carrying the greater thickness through to a more lowly stressed region. A premature end to the reinforcement may overload the connecting unreinforced section.

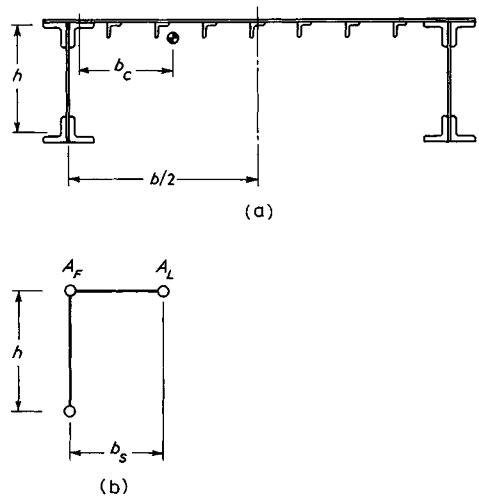


Figure 20.47 Transformation of actual into substitute beam cross-section. (After Kuhn (1956) *Stresses in aircraft and shell structures.* McGraw-Hill)

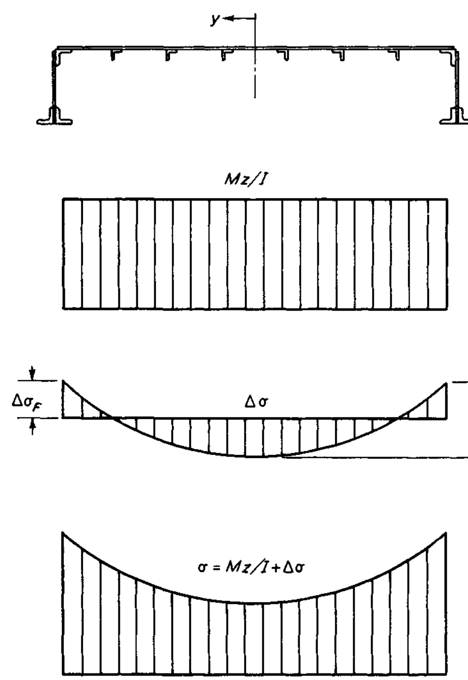


Figure 20.48 (After Kuhn (1956) *Stresses in aircraft and shell structures.* McGraw-Hill)

The reinforcement required for cut-outs must be continued or tapered for similar reasons but the need to do so is more obvious. The transverse and shear stresses associated with cut-outs are, however, also of considerable importance. In steel structures they are likely to cause buckling, fatigue and brittle fracture problems, whereas in concrete structures they cause cracking. It is, therefore, necessary to connect diaphragms, etc. to the structure by much more than nominal amounts of reinforcing steel. It is instructive to note with reference to steel structures that a number of large tankers have experienced local failures at cut-outs owing to the effects described above. It seems likely that the extrapolation of design knowledge from smaller structures was not backed-up with sufficient research into the complex stress systems produced and the associated buckling phenomenon. Similarly, the causes of failure of several steel box girder bridges have mainly been due to stress concentrations due to cut-outs in stiffeners, support reactions, and a cut-out produced by partly unbolting a main compression flange splice. An earlier failure of a plate girder bridge due to the stress concentrations produced by a flange cover plate completes an unanswerable case for the importance of allowing for stress concentrations in structural design. The basic engineering solution to this problem is to avoid severe stress concentrations and all those mentioned could have been avoided without significant cost or difficulty. Some degree of stress concentration is, however, unavoidable and only by using test data can the distinction be drawn between the acceptable and unsafe forms of structures, structural details, and associated stress levels.

The stress levels themselves due to known loads can be found with considerable accuracy using, for example, two- and three-dimensional finite element methods. Kuhn[18] describes ingenious methods for the approximate analysis of shell structures with cut-outs as well as the shear lag approach described above. They give a valuable insight into the structural behaviour of such systems but are more expensive to use than computer-based techniques using finite elements.

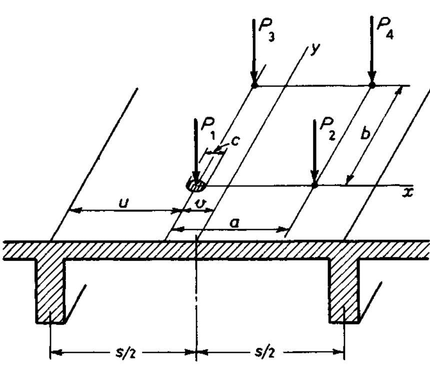


Figure 20.49 Slab supporting wheel loads

20.5 Concrete deck slabs

The usual form of deck system is a concrete slab spanning transversely between longitudinal beams which are often the main members of the bridge. Cross-girders can be used to produce a longitudinally spanning slab or the slab can be supported on a series of stringers spanning cross-girders. The simplest system is generally the most economical and only where there are special requirements are stringers and cross-girders used. In large steel trusses, for example, the loads must be carried to intersection points requiring more complex systems.

The slab must be designed for three different modes of behaviour:

(1) Local flexure due to the transfer of wheel loads to the adjacent beam members.
(2) Flexure due to relative movements of various parts of structure.
(3) In-plane stresses due to beam action of main and secondary members of structure.

Local stresses can be found by assuming that all supporting members are rigid when evaluating the slab moments and shears due to wheel loading. The remaining effects can then be found by applying loads equal to the reactions of the supporting members to those members. It is important that the latter loads should be statically equivalent to the vehicle loading, but the exact spanwise distribution is not usually required.

There are several publications giving influence surfaces for local slab bending moments for several types of support, i.e. simply supported on four sides, cantilever slabs, fixed on four sides.[19,20] A well-known treatment is by Westergaard[21] which uses Nádai's equations to obtain the bending moment under a wheel load:

$$\left.\begin{matrix} M_x \\ M_y \end{matrix}\right\} = \frac{(1+\mu)P}{4\pi}\left[\ln\left(\frac{4s}{\pi c_1}\cos\frac{\pi v}{s}\right)+\frac{1}{2}\right] \pm \frac{(1+\mu)P}{8\pi} \quad (20.24)$$

The meaning of the symbols is given in Figure 20.49 except for c_1 which is the equivalent diameter of the loaded area. According to Westergaard, the equivalent diameter c_1 is expressed with satisfactory approximation by the following formula, applicable when $c < 3.45sh$:

$$c_1 = 2[(0.4c^2 + h^2)^{1/2} - 0.675h] \quad (20.25)$$

He derived Figure 20.50 for the case of a wheel at the centre of a simply supported span, for a Poisson's ratio of 0.15, using Equation (20.25). The fixed edge values of M_x and M_y, respectively M_x' and M_y', are given in the same figure. A conservative value of the equivalent width of slab carrying the load is:

$$b_e = 0.58s + 2c$$

which was used to derive the result:

$$M_{0_x} = \frac{Ps}{2.32s + 8c} \quad (20.26)$$

The above result was used by Henderson[22] to show that the abnormal vehicle loading of 90 t on two axles 1.83 m apart, does not exceed the design uniformly distributed loading curve moments by more than 20%, which is within the overstress allowance permitted at that time.

The bending moments due to wheels at other points on the slab can be found from influence surfaces of the kind shown in Figure 20.51 which are also taken from Westergaard's paper.[21]

Influence surfaces for cantilever slabs of varying depth have been produced by Homberg and Ropers.[23] These show that longitudinal and transverse sagging moments under the wheel may be as large as the root moment. It should be noted that the root moment is often at the thickest part of a cantilever slab.

The assumption of a fixed edge at the root and a simple free edge at the tip of the cantilever are generally made in influence surfaces. In fact, edge beams are usually provided and the root may be supported by a deck slab and a web with significant flexibility. The root moment is nonuniform and its distribution is affected by that flexibility. Treating the first harmonic of the fixed edge moment as a fixing moment, a moment distribution process can be used to allow for the flexibility, but it is usually sufficiently accurate to consider one joint. The stiffness of the

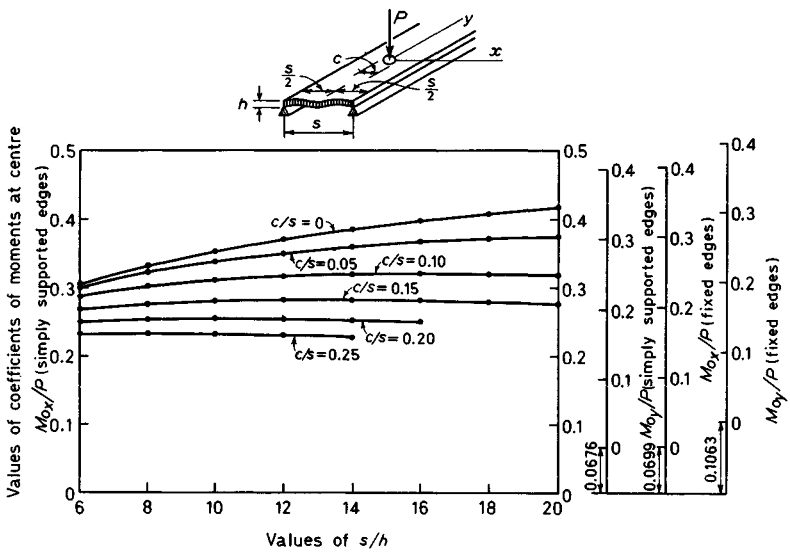


Figure 20.50 Coefficients of bending moments M_{0_x} and M_{0_y} in directions x and y respectively, produced at centre of slab by a central load P distributed uniformly over the area of a small circle with diameter c

flange, web and cantilever under sinusoidal edge moments must be used. The edge beam may also produce a significant change in the distribution of moments at the root. Both effects produce changes in longitudinal and transverse sagging moments in the cantilever.

20.6 Skew and curved bridges

Modern highway alignments result in many bridges with skewed supports, curvature in plan, and variation in width.

20.6.1 Skew

The primary shears and moments in bridge structures with angles of skew up to 15° approximate to those in a similar system with zero skew. A wider range of structures, provided that they approximate to slabs, are covered by the influence surfaces given by Rüsch and Hergenröder.[24] Most structures outside the range of the above approaches can be analysed by the grillage methods such as those mentioned in the previous sections.

Even where the skew is small, the behaviour of the structure near the bearings, particularly at the obtuse corner, requires special consideration, e.g. the interaction of the bearing diaphragm with the structure, uplift at bearings, transverse moments in the deck and shear distribution. These effects depend on the detailed form of the structure and the articulation of stiffness of the bearings and piers. In torsionally stiff structures, with twin bearings at both ends of a span, skew may produce significant end fixity with correspondingly high loads on the inner bearings (obtuse corners) and low loads on the others.

20.6.2 Curved in plan

Curved, torsionally stiff, structures supported against torsion at each vertical support can be analysed for the effects of bending continuity as if they were straight beams provided that the following requirements are satisfied:[25]

$\gamma < 1$ and $\alpha < 30°$
$\gamma < 5$ and $\alpha < 20°$
$\gamma < 10$ and $\alpha < 15°$

where $\gamma = EI/GK$ and α is the angle subtended by one span. The resulting bending moments are within 6% of curved beam solutions.

The solution found by that approach allows only for the vertical loads. The twisting moments produced by eccentricity of the loads relative to the shear centre must be considered separately. The vertical loads alone, however, produce torsional moments in the structure and graphs by Garret and Cochrane[25] enable these to be found from the continuity moments. The effects of applied torques for the ranges of structures given above can be estimated from considerations of statics and relative stiffnesses. More generally, the standard theory of curved beams can be applied to curved bridge structures or grillage programs can be used.

In all curved structures the longitudinal stresses produce lateral loads due to the curved path they follow. The effect of these loads, including the local loads caused by the curvature of prestressing cables, must always be given careful consideration. The resulting forces are resisted by the frame action of the cross-section producing deformations of the cross-section. In exceptionally thin-walled concrete box structures or steel box structures without special frames or diaphragms, these deformations can result in significant changes in the primary bending moments.[26] In the usual type of box structure, however, it is possible to consider this effect separately and, provided that the forces acting on the cross-section allow for curvature, reasonable results can be obtained by assuming the box is straight and using theories such as those described above.

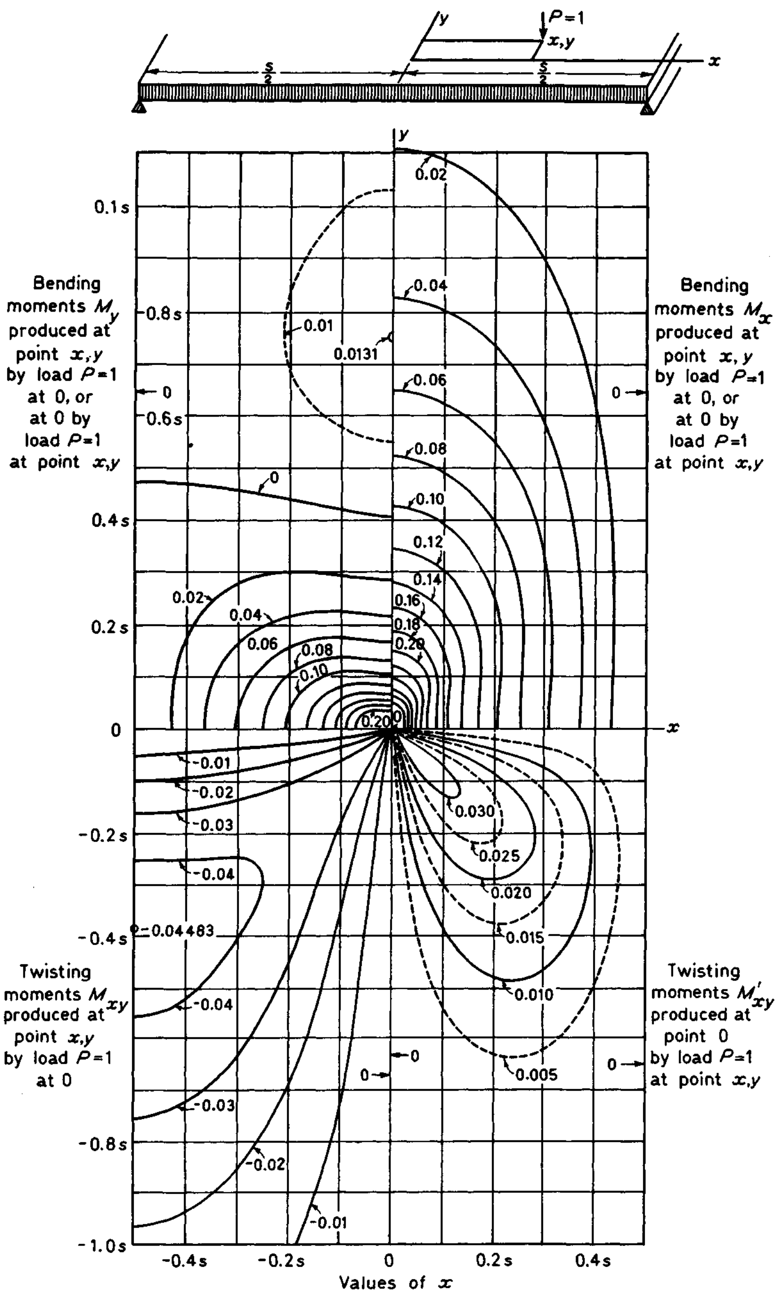


Figure 20.51 Contour lines of surfaces representing moments. Poisson's ratio $v=0.15$

20.7 Dynamic response

The stresses produced by the dynamic response of a bridge to a vehicle traversing it are covered by loading specifications. Unusual structures may require a special investigation but the main problem will be that of defining the vehicle size, speed and frequency of occurrence. Damping, though difficult to quantify, is not likely to be important in this type of problem since the main effects are usually short-term ones where damping is of little significance.[27] The prolonged oscillations produced by a series of vehicles acting in phase with each other is highly improbable at high live-load stress levels.

The effect of vibration on the user of a bridge is, potentially, more important since small accelerations, 0.02*g*, can produce discomfort in pedestrians and occupants of stationary vehicles. Occupants of moving vehicles cannot distinguish bridge movements as they are masked by the normal oscillations of the vehicle. Figure 20.52 from Leonard[28] shows limits proposed by various workers in the industrial field compared with those found by the Road Research Laboratory from the reactions of

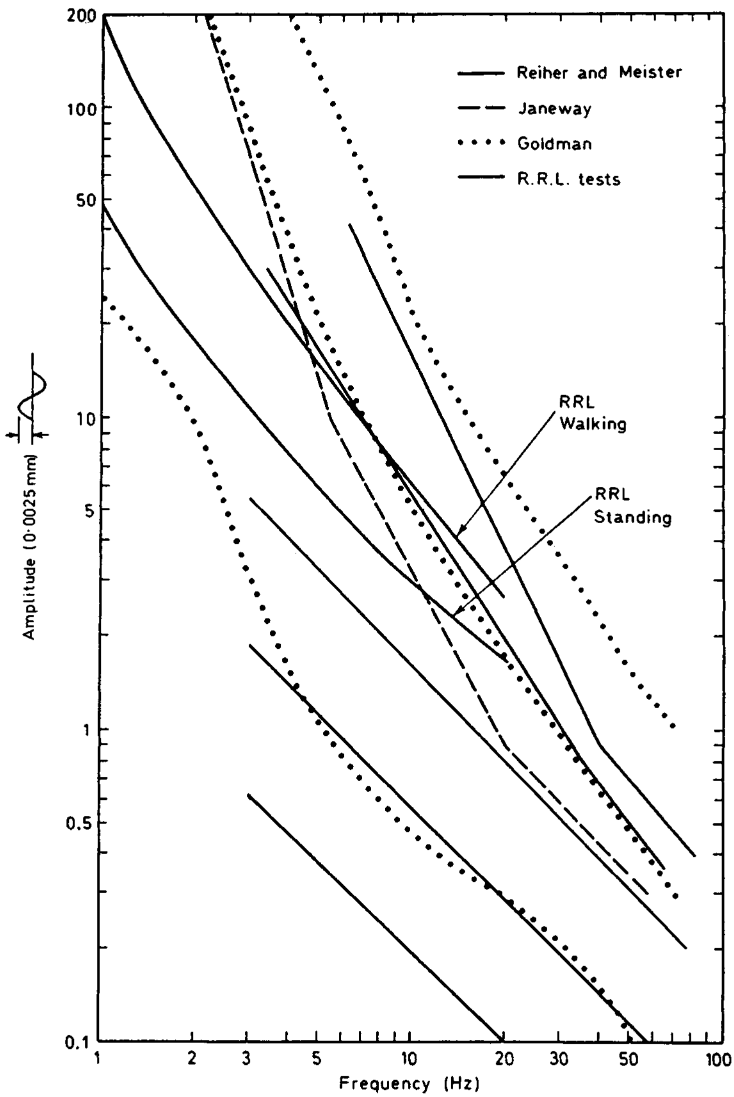


Figure 20.52 Comparison of tolerance levels to vibration

people using a footbridge which has been excited to a steady-state resonant motion. Short periods of oscillation much greater than the limits in the figure are likely to be acceptable to bridge users if they understand that it is part of the normal behaviour of the structure. The stationary car occupant is a different case, since resonance can produce oscillations of increased amplitude if the natural frequency of the vehicle, 1–3 Hz, and the bridge are nearly the same. Wyatt has commented, however, that for spans over 30 m:

> ... an obvious design aim would be to avoid the frequency range of 1–3 Hz. It has, however, also been shown that as a result of 40 years' design progress the economic structure is likely to have a frequency in this range and it is tantamount to throwing away this progress if higher frequencies are specified. Any specification which imposes a limit on frequency is thus strongly to be deplored.

Wind-excited oscillations are known to be of primary importance in long-span structures of the suspension bridge and cable-braced type. What is not clear is the significance of this phenomenon in more modest spans with or without cable supports. Theoretical investigations of the problem must allow for effects of vortex shedding on the particular cross-section in question; similarly, the changes in lift characteristics with rotation and deflection. Structural and aerodynamic damping are both important and, of course, the elastic characteristics of the structure must be known.

The behaviour of long-span structures is usually determined with the aid of wind-tunnel tests on models of segments of the deck and stiffening girder. Spring supports can reproduce to scale the natural frequency calculated for the prototype. The use of wind tunnels will continue to be important for major bridges where the economic advantages of determining the most suitable deck system are considerable.

In the intermediate range of spans the data for various shapes of cross-section from wind-tunnel tests are likely to be sufficient in the near future to permit an analytical estimate of the effect of wind without being unduly conservative.

20.8 Movable bridges

The principles associated with the various types of movable bridge are well established but increasing knowledge, particularly in the fields of control machinery and materials technology, has made new applications possible. The selection of the type of movable bridge is largely governed by the nature of the site. The relative priority of the two thoroughfares, usually a road or railway over a waterway, will help to establish the acceptable closed position headroom and also the speed and conditions under which the bridge will be required to operate.

Where an existing bridge is to be replaced, the condition of the existing foundations will influence the choice of type for the new bridge. For example, it would normally be uneconomic to excavate a tail chamber for a trunnion bascule bridge out of existing heavy concrete foundations.

There is a need for speed during those phases of construction that will obstruct navigation channels. This obstruction can be minimized by finally assembling the bridge as a few large items; the assembly of the leaves of one modern bascule bridge has been done in the bridge open position.

Minimum deadweight has the obvious advantage of reducing the required capacity of the bridge machinery; an orthotropic steel deck is used on most modern bridges although care is needed to ensure adequate adhesion between the wearing surface and the steel deck. A flexible PVC surface has been used.

The machinery to operate a movable bridge is as important as the structure itself. Particular requirements are that it should provide high torque at low speeds for starting, very slow inching speeds – to give final alignment for swing bridges and for landing on bearings for other types – and precise control at all times. The final drive to the movable structure has traditionally been rack-and-pinion, but many modern continental examples use a hydraulic linear actuating cylinder. A factor which may continue to favour the rack and pinion is the need for a very precise and rigid control of travel; also the braking system must be completely separated from the hydraulics and it is more difficult to arrange brakes for a sliding system than for rotary pinions.

A bascule bridge must be landed without violent impact and it is desirable to ensure a positive reaction in doing so. It has been customary to provide special inching motors which come into operation at the end of travel, with limit switches to stop the motors just before the bearing pads are reached. However, many modern examples are now using infinitely variable speed systems incorporating hydraulic motors or hydraulic transmission to remove the need for separate motors. Several modern Dutch bascule bridges incorporate the so-called 'snail' feature which is a mechanical device fitted on the rack-and-pinion drive used for slowing-down the movement of the bridge and locking it.

The normal requirement that a movable bridge should continue to function in all circumstances necessitates complete reliability of its machinery. Extensive duplication of machinery is incorporated, together with the provision of auxiliary motors so that the bridge can still operate, at reduced speed if necessary, with some of its machinery out of action for servicing or due to failure. In the event of failure of the main power supply, a standby power source, such as a diesel generator, may be provided; in some instances provision is made for hand operation.

A movable bridge out of control can do enormous damage; a braking system must be provided to ensure that this never occurs. This system, too, will require extensive duplication as well as being of the 'fail-safe' type. It must be adequate for the strongest anticipated wind loading for, even if the wind is sufficient to prevent the bridge from being moved, the brakes must be able to hold it with complete safety. Some form of emergency stop must be provided should a sudden danger to shipping occur while the bridge is moving. Another safety measure is an overspeed switch which engages the brakes should the moving bridge speed-up unacceptably.

While it is obviously desirable that the bridge should open and close as quickly as possible, attention must also be given to the speed of operation of crash barriers and warning lights and hooters as these operations occupy a major part of the road-closure time. Machinery must be provided to operate bridge locking systems such as nose-and-tail locks for bascule bridges and nose support jacks and centre wedges for swing-bridges.

The major types of movable bridge are shown in Figure 20.53. The Strauss, overhead counterweight Scherzer, and drawbridge have all their parts above ground. This avoids the need to provide a tail chamber, but care must be taken to avoid obtrusiveness. This is particularly true of the Strauss bascule which is rather inelegant and requires greater depth behind the quayside than the fixed trunnion bascule.

The rolling lift type has the problem of rolling tracks deteriorating with age. The very high bearing pressures at the points of contact of the curved rollers may lead to local crushing. This can be overcome by using wider tracks on heavier support girders.

Designing for wind loading on the opened bridge sets an economic limit to the single-leaf bascule span. With the double-leaf type, considerable care must be taken in the nose-locking arrangements between the two leaves; if the bridge is to carry a railway track it may be difficult to obtain a satisfactory joint. The trunnion bascule is the type used in many modern examples and may be driven in several ways. The rolling lift bascule gives a wider clearance with the bridge in the open position than a fixed trunnion bridge of the same span, although it therefore requires a greater depth behind the quay. The leaf of a bascule bridge is normally designed to be sufficiently rigid torsionally to be able to be opened with the drive applied to only one of its main girders, when the other set of machinery is inoperative.

The vertical lift-bridge sets a headroom limit and is expensive for narrow crossings. However, it can be used for very long spans without the nose-locking problems of the double-leaf bascule. The lifting machinery may be either at the top of the lifting towers or in the piers; a mechanical or electrical linkage connects the separate motors to ensure synchronized parallel motion of the corners of the lifting section.

The swing-bridge often provides the cheapest solution for a given span. It may be of the balanced cantilever type or have a shorter tail span ('bobtail' type), and may turn on a rim bearing or a central pivot bearing. Its main disadvantage is the need to protect the bridge in the open position. Space must be provided at the quayside to lay the span in the open position; this may not be possible, especially where there are adjacent locks requiring a clear quay for handling mooring ropes. The retractable bridge is not often used as it requires a suitable approach to accommodate the span in the open position and heavy rolling or sliding ways.

20.9 Items requiring special consideration

This chapter has described in general terms the types of bridges used throughout the world, primarily of steel or composite construction or a combination of both types, together with general principles of analysis which enable the preparation of the main members of the design to be established correctly. Space precludes treatment of many interesting aspects of bridge design.

The authors would like to draw attention to several points which, from experience, are likely to give rise to difficulties

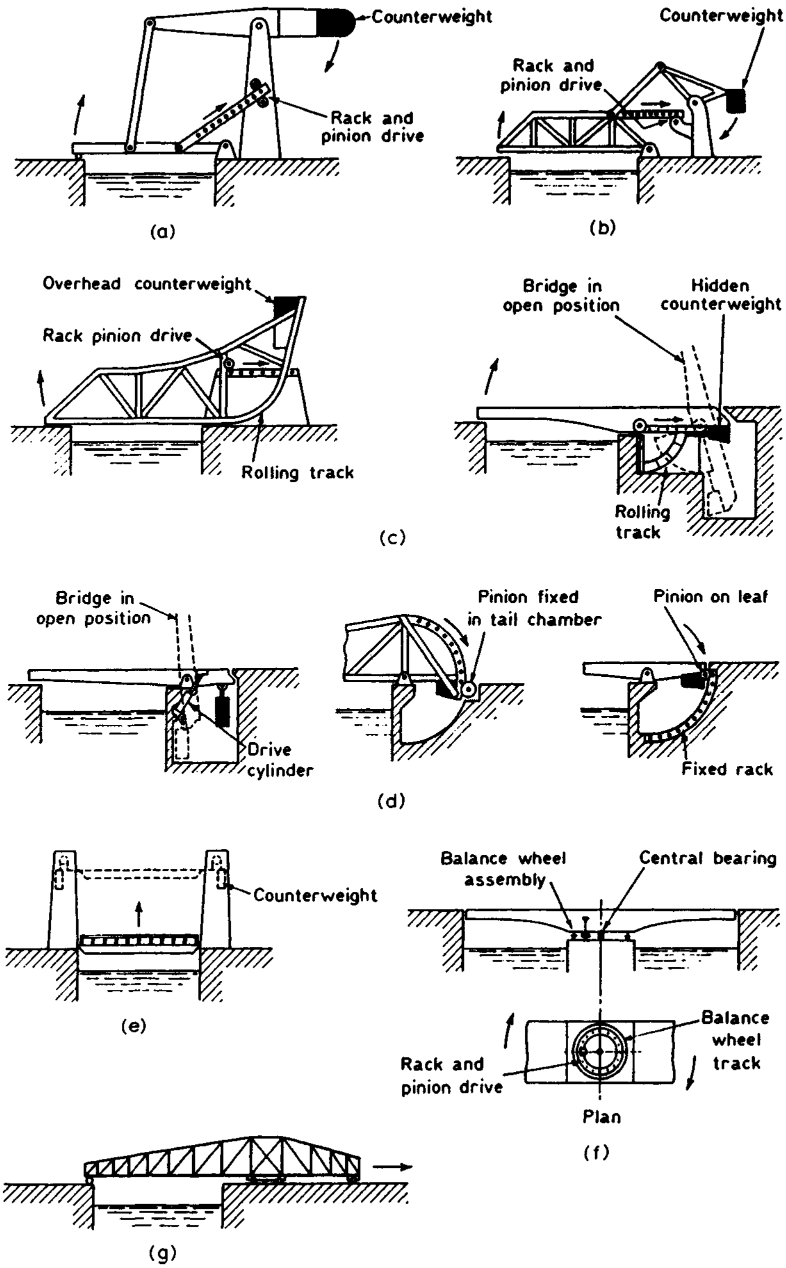


Figure 20.53 Types of movable bridge. (a) Drawbridge; (b) Strauss; (c) rolling lift (Scherzer) bascules; (d) trunnion bascules; (e) vertical lift; (f) swing; (g) retractable

unless adequate consideration is given to them. Many are obvious but it is a regrettable fact that it is obvious points which give rise to recurring difficulty.

It is essential that adequate time and effort is devoted to the checking of any bridge design by a senior engineer fully experienced in the type of work involved. Much time can be wasted in elaborate analysis either by slide-rule or computer when the simplest elementary assessment of some details will demonstrate inadequacies or inaccurate conception of the mode of behaviour. It is important for bridge engineers to have some grasp of three-dimensional actions of structures rather than a preoccupation with cross-sections, plans and elevations.

To this end, the engineer is encouraged to make simple models of paper or cardboard so that the implications of what is apparently an obvious decision can be grasped and interpreted. The relationship of frame members in space, the junctions of beams and columns, the implications of running bridge girders into diaphragms and end blocks are typical examples.

The following checklist may perhaps reinforce the plan of work and help bridge engineers to reduce the number of errors or difficulties on site.

(1) Check all the detailed items of the bridge structure and not just the main structural members.

(2) Check the basic principles of reinforced concrete design used. The same applies to steelwork details for welding and bolting.
(3) Can the reinforcement be fixed by a man of normal size?
(4) Can the concrete be placed and vibrated properly?
(5) Can welds be made in a suitable sequence and do bolt spanners, etc. foul other completed parts of the construction?
(6) The geometric calculations should be checked. The leading dimensions on the general arrangement drawings cause considerable confusion if they are not correct.
(7) Secondary forces and stresses should not be forgotten. It is better to consider them by elementary methods than to omit taking them into account.
(8) Do not forget constructional tolerances which will cause lack of fit and possibly additional loads and stresses due to eccentricity.
(9) Particular attention should be paid to bearing design and details for drainage and expansion joints. The specifications for these items should not be skimped.
(10) Remember that lateral loads are generated from lack of straightness in tension. These are often forgotten and include things as widely diverse as:

 (a) substantial lateral loading on elements of cellular construction;
 (b) change of direction of thrust members as at portal frame joints;
 (c) haunches on the soffit of a beam;
 (d) overall curvature of an arch;
 (e) the behaviour of prestressing cables in plan and elevation imposes a local loading as well as the effect of the line of pressure.

(11) Post-tensioned prestressed concrete anchorages should not only be considered in terms of bursting, splitting and the length of transmission of force for the single anchorages, but in terms of sets or the whole group.
(12) Openings for services and access for maintenance and inspection.
(13) Anchors require careful detailing. Holes, ties, etc. become stress raisers and they must be detailed to suit.
(14) Temporary openings and drain-holes necessary during construction should receive adequate consideration at an early stage, for the removal of items of plant, jacks, falsework, shuttering, lifting gear, etc.
(15) Engineers sometimes apply normal beam theory without considering whether this is applicable. Many elements act as deep beams, i.e. brackets, corbels, halved joints, anchorages, tiebacks. They require application of a specific approach which adequately caters for behaviour which would actually occur in service either in bending or in shear or torsion.
(16) Consideration should be given to the weathering of a structure in terms of orientation appearance and durability.
(17) The designer should consider whether the concept of erection is clear in his mind and what the effect would be of lack of compliance by the contractors.
(18) Are there any unforeseen events which would provoke an unfortunate chain of circumstances such as traffic collision on any parts of the structure, crane booms and dump trucks inadvertently striking part of the bridge during construction, fires, etc?
(19) When inspecting bridges under construction, keep a lookout for things which are not in accordance with the intention of the design from a safety point of view.
(20) Is the articulation of the structure as the designer intended?
(21) Is reinforcement in the bottom of cantilevers rather than the top?
(22) Has distribution or secondary reinforcement been omitted by mistake at the detailing stage?

These are some of the many items included in a quality assurance scheme and which should be extended to all the items in the plan of work.

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21

Buildings

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Building Design Partnership

Contents

The design of the total building including its internal and external environment has traditionally been the responsibility of the architect but this is now so complex a task that, except for the simplest of buildings, a multidisciplinary involvement is necessary whereby engineering, surveying and other specialist skills are integrated with those of the architect to achieve consistent quality throughout the project.

Internal form and environment will be determined by the functional requirements of the occupying organization, the space needed to meet these functional requirements and the required comfort levels in regard to such items as noise, temperature, humidity and lighting. The external form and environment will be determined by the characteristics of the site and adjacent buildings. Influencing all aspects will be the constraints arising from time and cost, town planning and building regulations.

21.1 Background

Architects look to the civil and structural engineer for a positive contribution to the building design; from concept to completion, with understanding of the basic objectives of the project and with sensitivity and inspiration in their realization. The technical and economic solution to a predetermined problem, on its own, is no longer sufficient. The primary responsibility of the civil and structural engineer will be to ensure the safety and rigidity of the building but, with the architect, he can make a creative contribution to the building form, the spaces within it and its impact, visually and psychologically.

The potential freedom of building layout and expression which stemmed from the development of the structural frame took a surprisingly long time to be understood and put into practice. Framing techniques were available from the mid nineteenth century but, in the main, they were used simply to support buildings of predetermined form and style into which the required functions had to fit. It was not until a few leaders of architectural thought and practice adopted a more rational approach to design that the potential of the structural frame was grasped and put to good effect. For those who understood and wanted it, there was now much greater freedom of internal planning and external expression; and for the rationalists, form could more easily follow function.

The design of the Bauhaus building in Dessau by its founder Walter Gropius was a turning point in bringing logic into the design of buildings. It was the first major building to derive its form not from the irrational imposition of style, symmetry and proportion, but from the requirements of function and structure. Its character came from intrinsic materials and design detail, not from applied decoration; its subtlety of form and space from an ordered solution to the planning problems, not from some preconceived design formula or style. It was a major demonstration of a rational design approach founded upon the working out of solutions from first principles. The architectural features of the Bauhaus building became popular among progressive architects: assymetry, rectangular forms, lightness of the external wall, space and precision, all, in a way, reflections of the contradictory combination of freedom and discipline afforded by the sensible use of structure.

For form to follow function became the natural starting point for design; indeed, it seemed strange that it could ever have been thought otherwise. Later experience showed that a too-rigid adherence to this principle leads to a too-'tailored' building unable to respond to changing need and that a loose-fit approach is advantageous. The introduction of the structural frame allows building expression to be whatever is wanted and acceptable. Structure and building services may be expressed or hidden. Height is no longer a problem if it is acceptable to the planners and is economically viable. Almost any clear span is achievable. In short, the constraints are no longer technical; given the resources, design options are now almost limitless.

The more significant question has become: How is this technical freedom to be applied? Buildings are for people, to provide them with shelter, comfort, spiritual uplift and psychological support, and to accommodate the sophisticated processes that are part of modern life. Changing expectations of people and social relationships have become major determinants of the volume and nature of building. Communication systems of all types have changed remarkably. Science-based industries of unimagined complexity now exist requiring extreme levels of environmental control. These changes have led to the need for completely new types of building.

The design and construction itself is complex, requiring great skills of co-ordination and management. Functional requirements in many building projects are now so diverse that specialist input and understanding are required to establish the brief before building design can commence. Building materials, methods and forms of contract are diverse and changing, as are the constraints of cost and time, town planning and building regulation. The finished building itself is complex and highly serviced; and it often requires sophisticated building control and security systems to ensure satisfactory and safe performance. Cost in use, maintenance and energy consumption, have become as important considerations as first cost.

The diversity and depth of these aspects of building design and construction cannot be covered in a single chapter of a book devoted primarily to civil engineering practice. What follows is an introduction to the subject, to help the civil and structural engineer see his contribution better in the context of building design and construction as a whole.

References are in the main to UK practice but most aspects are, in principle, applicable generally.

21.2 General management

The procedures for handling large-scale building projects as opposed to civil engineering projects are complicated by the larger number of individual professional parties involved and by the large amount of legislation on permissions and approvals. The handling of such projects in the UK has been studied by the Royal Institute of British Architects (RIBA).[1] A similar publication relating to US practice has been produced by the American Institute of Architects.[2]

The overall procedures for the organization of building projects are covered in another publication produced by the RIBA.[3] Table 21.1, taken from that publication, shows the twelve discrete stages into which the project can be divided and briefly indicates the contents of each stage and the parties directly involved. Full details of the work required from each of the several professions and contractors at each stage are shown in separate diagrams. For example the detailed breakdown of Stage C, Outline Proposals, is shown in Table 21.2 in which column 5 details the input required from the civil and structural engineer.

21.3 Brief

Buildings are either purpose-built for a particular user or are speculative. In either case, the first step is to compile an agreed brief setting out the basic requirements of the project covering:

Table 21.1 Outline plan of work. (After Royal Institute of British Architects (1973) *Plan of work*. RIBA, London).

Stage	*Purpose of work and decisions to be reached*	*Tasks to be done*	*People directly involved*	*Usual terminology*
(1) INCEPTION	To prepare general outline of requirement and plan future action.	Set up client organization for briefing. Consider requirements, appoint architect.	All client interests, architect.	BRIEFING
(2) FEASIBILITY	To provide the client with an appraisal and recommendation in order that he may determine the form in which the project is to proceed, ensuring that it is feasible, functionally, technically and financially.	Carry out studies of user requirements, site conditions, planning, design, and cost, etc. as necessary to reach decisions.	Clients' representatives, architects, engineers, and quantity surveyor according to nature of project.	
(3) OUTLINE PROPOSALS	To determine general approach to layout, design and construction in order to obtain approval of client on outline proposals and accompanying report.	Develop the brief further. Carry out studies on user requirements, technical problems, planning, design and costs, as necessary to reach decisions.	All client interests, architects, engineers, quantity surveyor and specialists as required	SKETCH PLANS
(4) SCHEME DESIGN	To complete the brief and decide on particular proposals, including planning arrangement appearance, constructional method, outline specification, and cost, and to obtain all approvals.	Final development of the brief, full design of the project by architect, preliminary design by engineers, preparation of cost plan and full explanatory report. Submission of proposals for all approvals.	All client interests, architects, engineers, quantity surveyor and specialists and all statutory and other approving authorities.	
Brief should not be modified after this point.				
(5) DETAIL DESIGN	To obtain final decision on every matter related to design, specification, construction and cost.	Full design of every part and component by collaboration of all concerned. Complete cost checking of designs.	Architects, quantity surveyor, engineers and specialists, contractor (if appointed).	WORKING DRAWINGS

Table 21.1 (*continued*)

Stage	*Purpose of work and decisions to be reached*	*Tasks to be done*	*People directly involved*	*Usual terminology*
Any further change in location, size, shape, or cost after this time will result in abortive work.				
(6) PRODUCTION INFORMATION	To prepare production information and make final detailed decisions to carry out work.	Preparation of final production information, i.e. drawings, schedules and specifications.	Architects, engineers and specialists, contractor (if appointed).	
(7) BILLS OF QUANTITIES	To prepare and complete all information and arrangements for obtaining tender.	Preparation of bills of quantities and tender documents.	Architects, quantity surveyor, contractor (if appointed).	
(8) TENDER ACTION	Action as recommended in paras 7–14 inclusive of *Selective tendering**	Action as recommended in paras 7–14 inclusive of *Selective tendering**	Architects, quantity surveyor, engineers, contractor, client.	
(9) PROJECT PLANNING	Action in accordance with paras 5–10 inclusive of *Project management**	Action in accordance with paras 5–10 inclusive of *Project management**	Contractor, subcontractors.	SITE OPERATIONS
(10) OPERATIONS ON SITE	Action in accordance with paras 11–14 inclusive of *Project management**	Action in accordance with paras 11–14 inclusive of *Project management**	Architects, engineers, contractors, subcontractors, quantity surveyor, client.	
(11) COMPLETION	Action in accordance with paras 15–18 inclusive of *Project management**	Action in accordance with paras 15–18 inclusive of *Project management**	Architects, engineers, contractor, quantity surveyor, client	
(12) FEEDBACK	To analyse the management, construction and performance of the project.	Analysis of job records. Inspections of completed building. Studies of building in use.	Architects, engineers, quantity surveyor, contractor, client.	

*Publication of National Joint Consultative Council of Architects, Quantity Surveyors and Builders.

Table 21.2 Stage C: Outline proposals – plan of work for design team operation
(To determine general approach to layout, design and construction, in order to obtain authoritative approval of the client on the outline proposals and accompanying report.)

Col. 1 Client function	*Col. 2 Architect management function*	*Col. 3 Architect design function*	*Col. 4 Quantity surveyor function*	*Col. 5 Engineer, civil and structural, functions*	*Col. 6 Engineer, mechanical and electrical, functions*	*Col. 7 Contractor (if appointed) function*	*Col. 8 Remarks*
(1) Contribute to meeting: note items on agenda in col. 8.	(1) Organize design team. Call meeting to discuss directive prepared in stage B, action 9 (col. 2): establish responsibilities, prepare plan of work and timetable for stage C. (See col. 8 for items for agenda for meeting.)	(1) Contribute to meeting: note items on agenda in col. 8.	(1) Contribute to meeting: note items on agenda in col. 8.	(1) Contribute to meeting: note items on agenda in col. 8.	(1) Contribute to meeting: note items on agenda in col. 8.	(1) Contribute to meeting, note items on agenda in col. 8.	ITEMS FOR AGENDA FOR MEETING: (1) *State objectives and provide information:* *(a) brief as far as developed;* *(b) site plans and other site data;* *(c) restate cost limits or cost range, based on client's brief;* *(d) timetable; and* *(e) agree dimensional method.* (2) *Determine priorities.* (3) *Define roles and responsibilities of team members and methods communication and reporting* (4) *Define method of work, tender procedure and contract arrangements.* (5) *Agree drawing techniques.* (6) *Agree systems of cost and engineering checks on design.* (7) *Agree type of bill of quantities.* (8) *Agree check list of actions to be taken.* (9) *Agree programming and progressing techniques.*
(2) Provide all further information required by architect. Assist as required in all studies carried out by members of design team. Initiate and conclude according to timetable, any studies that are required within own organizations. Make decisions on all matters submitted for decision relevant to stage C.	(2) Elicit all information relevant to stage C by questionnaire, discussion, visits, observations, user studies, etc. Initiate studies by consultants and client as required. Maintain and coordinate progress throughout this stage.	(2) Carry out studies relevant to stage C, e.g.: (a) study published analyses of similar projects, visit if possible; (b) study circulation and space association problems; and (c) try out detail planning solutions and study effect of planning and other controls.	(2) Carry out studies relevant to stage C, e.g.: (a) Obtain all significant details of client's requirements relevant to cost and contract information on site problems, etc.; and (b) re-examine, supplement and confirm cost information assembled in stage B.	(2) Carry out studies relevant to stage C, e.g.: (a) site surveys, soil investigation; and (b) complete questionnaires on structural and civil requirements.	(2) Carry out initial studies relevant to stage C, e.g.: (a) environmental conditions, user and services requirements, appraise M and E loadings on an area or cube basis; and (b) consider possible types of installation and analyse capital and running costs, possible sizes and effects of major services installations, main services supply requirements.	(2) Carry out studies relevant to stage C, e.g visit site and investigate: (a) ground conditions, access and availability of services for construction; (b) local labour situation; and (c) local sub-contractors and suppliers to assess quality reliability, production potential and price level, etc.	
		(3) In consultation with team assimilate information obtained in action 2, and produce diagrammatic analyses, discuss problems.	(3) Outline design implications of cost range or cost limit.	(3) Advise architect on, for example: (a) types of structure; (b) methods of building; (c) types of foundation; and (d) roads, drainage, water supply, etc.	(3) Advise architect on design implications of studies made, e.g.: (a) factors which would influence efficiency, and cost of engineering elements, i.e. site utilization,	(3) Advise architect on findings and also on: (a) approximate times for construction of alternative methods; and (b) effect of construction times on cost, etc.	

Table 21.2 *(continued)*

Col. 1 Client function	*Col. 2 Architect management function*	*Col. 3 Architect design function*	*Col. 4 Quantity surveyor function*	*Col. 5 Engineer, civil and structural, functions*	*Col. 6 Engineer, mechanical and electrical, functions*	*Col. 7 Contractor (if appointed) function*	*Col. 8 Remarks*
					building aspect and grouping, optimum construction parameters, etc.; (b) possible services solutions and ramifications of them; and (c) regulations and views of statutory authorities.		
		(4) Try out various general solutions; discuss with team; modify as necessary, and decide on one general approach. Prepare outline scheme, indicating, for example, critical dimensions, main space locations and uses and pass to team.	(4) Collaborate in preparation of outline scheme. Prepare quick cost studies of alternative structural and services solutions, and advise on economic aspects of solutions.	(4) Collaborate in preparation of outline scheme, prepare notes and sketches, consider alternatives, agree decision on general approach, and record details of alternative plans and assumptions.	(4) Collaborate in preparation of outline scheme, check that services decisions remain valid; record details of alternative plans and assumptions.	(4) Collaborate in preparation of outline scheme: continue to advise on time and cost implications of alternative designs or methods. Record details of proposals and assumptions.	
		(5) Assist quantity surveyor in preparation of outline cost plan; discuss and decide on cost ranges for main elements, and method of presentation of estimate to client.	(5) Confirm cost limit or give firm estimate based upon user requirements and outline designs and proposals. Prepare outline cost plan in consultation with team, either from	(5) Provide quantity surveyor with information for outline cost plan, with sketches on which to base estimate, and agree quantity surveyor proposals	(5) Provide quantity surveyor with cost range information for outline cost plan, and agree quantity surveyor proposals: interpret agreed standards by illustration.	(5) Provide quantity surveyor with information affecting price levels, for outline cost plan and agree quantity surveyor proposals.	

Table 21.1 *(continued)*

Col. 1 Client function	*Col. 2 Architect management function*	*Col. 3 Architect design function*	*Col. 4 Quantity surveyor function*	*Col. 5 Engineer, civil and structural, functions*	*Col. 6 Engineer, mechanical and electrical, functions*	*Col. 7 Contractor (if appointed) function*	*Col. 8 Remarks*
			comparison of requirements with analytical costs of previous projects or from approximate quantities based on assumed specification.				
	(6) Compile dossiers provided by team members on final (or alternative) sketch designs, recording all assumptions, and issue to all members of the team.	(6) Contribute to design dossiers, assemble all sketches and note relevant assumptions.	(6) Record basis of estimate to contribute to design dossiers.	(6) Compile dossier of essential data collected in actions (2) to (5) above.	(6) Compile dossier of essential data collected in actions (2) to (5) above.	(6) Compile dossier of basic cost information agreed with quantity surveyor and architect.	The report includes: (a) the brief as far as it has been developed; (b) an explanation of the major design decisions; and (c) firm estimate with outline cost plan.
	(7) Prepare report as coordinated version of all members' reports, including fully developed brief.	(7) Contribute to preparation of report.	(7) Contribute to preparation of report.	(7) Contribute to preparation of report.	(7) Contribute to preparation of report.	(7) Contribute to preparation of report.	
(8) Receive architect's report; consider, discuss and decide outstanding issues. Give instructions for further action.	(8) Present report to client; discuss and obtain decisions and further instructions.						

(1) Purpose, function and scope including limitations of cost and time; proposed activities and organization including numbers and types of people concerned, internal and external service requirements, particular systems such as document retrieval, special functional requirements such as security.
(2) Design factors and required standards covering internal and external environment; spatial requirements, organizational relationships and required groupings affecting layout.
(3) Internal and external traffic and required access for pedestrians, vehicles and materials.
(4) Factors affecting type of construction, expansion, alteration, change of use, life.
(5) Phasing required.
(6) Special sensitivities or critical functions.

Of primary importance is the building use and the associated schedule of basic accommodation including the number and nature of the intended occupants. By adding allowances for circulation, services, plant, toilet and ancillary accommodation, a close assessment of the gross floor area can be made and thereby the size of building determined. By considering the relationships between the different activities, the optimum grouping of the spaces provided for them can be analysed in preparation for their translation into a physical plan to suit the particular site.

A user client may have special requirements: most buildings are expected to have a useful life of 60 to 100 years, but in some cases, a more limited life span may be envisaged dictating a light form of construction which can be demolished and replaced easily and cheaply. Alternatively, a client may require a robust building shell of long life in which internal adaptation can be carried out to suit a later, and perhaps unknown, alternative use. Substantial mechanical and electrical service requirements, as occur in hospitals and some specialist laboratories and factories, may dominate the design leading, perhaps, to the incorporation of near-storey-height service floors alternating with the functional floors.

21.4 The site

Early site appraisal is vital. Suitability for the purpose intended requires consultation with various planning authorities to confirm zoning and land use definition. Access for vehicles, people and goods must be checked and the availability of public transport and future road or transport links determined. Increasingly, good access to major international air and rail termini or proximity to the national road network is a prerequisite of a site.

Subsoil deficiencies and underground service easements may present difficulties in development. Investigation of old mineral workings (e.g. brick clay, salt, sand and gravel extraction), coalmines, shafts and wells should be undertaken, particularly if such work is known to have occurred in the vicinity. Evidence of filling should be investigated and dated. A subsoil survey should be recommended to the client (together with a cost estimate) early in the life of the project to identify the underlying conditions which may ultimately influence the building location, arrangement and cost.

The local climate requires early checking: high wind speeds will involve special stiffening; atmospheric pollution or salt-laden coastal winds will require the selection of suitable materials and careful detailing of exposed building elements. Excessive external noise from major roads, railways or airports may necessitate soundproofing in the building or sound screening between the building and the noise source.

Confined city sites introduce problems such as: (1) delivery and storage of building materials and components; (2) the threat of restrictions or stoppages arising from local objection to construction noise; and (3) protection of adjoining property which may need underpinning and should be surveyed for dilapidations before work commences on site.

21.5 Landscape

The landscape is the setting to which new developments must relate, therefore its consideration is vital at the outset of each project. Landscape and civil engineering bear a close affinity, due to a mutual and direct concern with land form and natural resources. All but the most cosmetic landscape treatment involves civil engineering considerations. Landscape considerations include feasibility studies, environmental assessments, public inquiries, erosion control, reclamation, restoration, conservation, transportation, industry, commerce, natural heritage and the landscape related to all types of buildings both exterior and interior.

The quality of the landscape is now an essential constituent of the planning consent process. Early site appraisal should include an analysis of the landscape or urban space. Among the factors to be considered are geology, topography, soil, microclimate, drainage, land use, artefacts, vegetation and visual analysis. The effects of the interaction of these factors should be considered in relation to the development. A skilful appraisal will lead to the establishment of sound principles, which will enhance the less favourable aspects of the site whilst conserving the best.

On a yet broader scale, a full environmental assessment leading to designs which cause the least damage socially, aesthetically and to our natural resources, would extend to a large team including other specialists.

Reclamation of abandoned industrial and domestic wasteland is an area in the re-creation of the environment where engineering and landscape are inseparably combined. Such operations can restore the form of the landscape, provide new sites for housing, industry and recreation, and create new habitats, from wetlands to woodlands.

There are numerous factors concerning planning and design which will be important to the landscape architect, the civil engineer and the architect. These factors include planning for vehicles, finished levels and materials, economic cut-and-fill and the integration between the hard paved areas of the scheme and its immediate environment. Close collaboration between the professions is therefore needed to achieve an economic and sympathetic design.

As part of the site investigations, soil tests should be taken to assess biological qualities and should include horizon depth, soil type, texture, moisture content and pH. It is advisable to obtain a chemical analysis from an approved laboratory to assess deficiencies.

Planning the site operations to achieve the best results involves many decisions related to the landscape. Vegetation and topsoil are delicate natural resources which are easily damaged by thoughtless construction techniques. Their value must be assessed at the outset by a specialist, and if considered of value they should be protected carefully and retained. Topsoil must not be mishandled, as compaction and poor storage can render it useless as a growing medium. Drainage and grading should also be considered regarding any vegetation to be retained.

Working areas should be kept to a minimum in order to leave the maximum undisturbed area and avoid the replacement or restoration of topsoil and subsoil. Excavation, compaction, changes in water table and in finished level within the root spread of trees, should be avoided. The canopy will suffer in proportion to the amount of root damage sustained and similarly the stability, appearance and life expectancy of the tree will also be affected. On no account should the level of soil adjacent to the trunk be changed. Where trenching is unavoidable within the root spread, hand digging and retention of roots will be advantageous.

Where trees are an important feature close to buildings, roads or drainage foundations should be designed to withstand the effects of root growth or moisture movement.

The structural requirements and economic viability of planted areas within the building which are conceived as roof gardens, terraces or interior gardens, must be considered early in the design process together with the client's understanding of the long-term maintenance commitment. In addition, drainage, access and the sequence of construction are significant factors. The growing medium and planting should either be installed as the very last of the building operations, or adequately isolated and protected from further construction activities. All planting, but especially interior planting, can be affected adversely and even destroyed by subsequent operations such as the repair of faulty tanking, the installation of lighting and irrigation or the grinding of materials such as marble and terrazzo. It is clear, therefore, that where planting is part of a design concept it needs careful integration into the building process.

Management of the landscape in the long term is essential and should be discussed at the earliest opportunity, preferably when the brief is being formulated to ensure the wellbeing of the newly created environment and that a succession of planting is provided for the future.

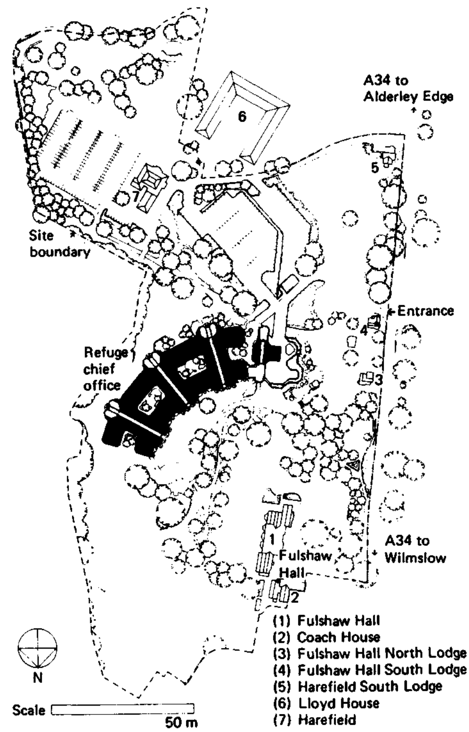
Figure 21.1 and the accompanying text describe the careful integration of an important headquarter building within a beautiful parkland setting. The aim was to provide a headquarters which would give high quality conditions for work, training and recreation. The new building was planned to have minimal impact upon the local environment, and to ensure that its landscaped surroundings would enhance working conditions. At the same time it had to cater for the latest demands of information technology, ensuring that the layout and fabric of the building were flexible enough to accept inevitable future change.

While the briefing process was underway, surveys were carried out on site conditions, tree planting and acoustic aspects of the location. It was seen as vital to respond to the exceptional natural quality of the site, and to the architectural qualities of the two main existing buildings there – the listed Fulshaw Hall, and Harefield House.

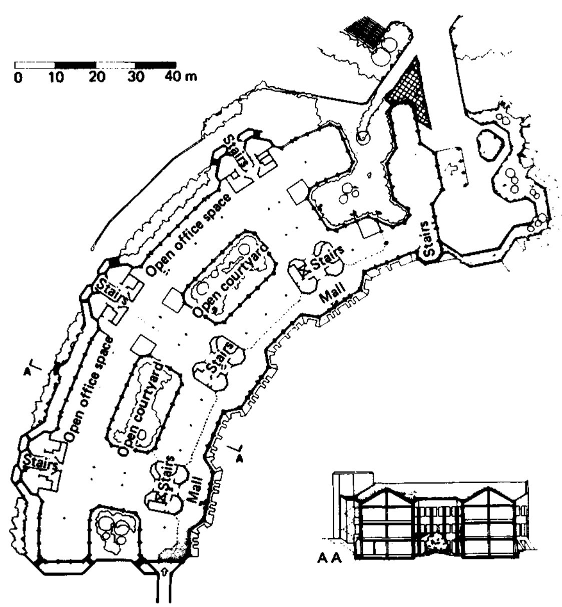
The form of the new headquarters evolved from these considerations as a low-lying building, tucked into the landscape on a slope of land across the lake from Fulshaw Hall. Car parking is provided discreetly to the south. The three-storey construction, pitched roofs, and brick and slate materials of the building help further to establish it as a worthy neighbour to the hall.

The plan provides outer and inner bands of office space, linked at intervals to create enclosed courtyards. Its external appearance is of a series of linked pavilions, sweeping round in a gentle curve that focuses upon the hall itself. The western end of the building surmounts a landscaped terrace facing the entrance from the A34 road.

Inside, circulation is provided by a pedestrian mall on the middle level of the inner band, facing the park. Vertical access is via stair towers at back and front. All the offices are fitted out with a raised floor to accommodate all cabling and air handling needs. The offices are 12 m wide along the bands, and 15 m wide along the links, so providing a good level of natural light. The planning module is a highly flexible 1.5 m allowing practically any type of interior fit-out. Uplighters bounce light off an



(a)



(b)

Figure 21.1 Integration of building and landscape. (a) Plan of the Fulshaw Hall site; (b) disposition of spaces acoustically treated structural ceiling, providing glare-free conditions and easy replanning of working spaces.

A 1000 m^2 computer suite is provided, together with dining room, coffee lounge and kitchens. Although the main spaces are left open, cellular offices may be provided as required throughout the plan.

21.6 Town planning

Most development and construction work is governed by the Town and Country Planning Act 1971. Section 22 of the Act defines development as: 'the carrying out of building, engineering, mining or other operations or the making of any material change of use of buildings or other land'. With some exceptions (mostly under the General Development Orders 1977 and 1981) permission to undertake any development is required from the local planning authority.

Other planning powers are concerned with individual buildings listed as of special architectural or historic interest (where consent is required for any works of demolition or alteration), conservation areas, advertisements, caravan sites, tree preservation, national parks and the countryside.

County and district planning authorities prepare structure (broad policy) and local plans against which applications for planning permission are judged. If planning permission is refused or conditions are imposed upon the permission, the applicant has the right of appeal to the Secretary of State for the Environment, and such appeals may be heard at a local public inquiry.

Early consultation with the local planning authority (the district or borough council) is recommended when advice will be given on the need to obtain planning permission, the scale of fees charged and the adopted planning policies which should be taken into account.

In some cases the local authority may provide access to grants available for special types of development. These include derelict land grant for approved ground restoration works and urban development grants for joint public/private sector funding of approved inner areas projects. In addition, most authorities offer grant, loan, site and premises assistance to encourage economic development in their area.

Major civil engineering projects such as oil refineries, power stations, radioactive toxic and dangerous waste treatment and disposal, iron and steelworks, asbestos extraction, chemical plants, motorways, ports and airports are all listed in an EEC Directive as likely to require an environmental impact assessment.

Further details of the planning legislation will be found in a work by Telling.[4]

21.7 Public utility

Once an outline brief exists and a site is under consideration the various public utility organizations (PO, gas, electricity, water authorities) should be consulted to determine the availability of their various services.

21.8 Feasibility

The compatibility of brief and site with the external constraints in their varying forms logically leads to the preparation of a feasibility study. This is normally the first design exercise and provides the design team with an opportunity to explore the problem, propose solutions, cost the alternatives and identify options for the client. Presentation of a preferred option with objective data supporting the preference completes the first stage and forms the basis for the final design.

21.9 Cost

Cost is an important factor at all stages of the design process. Alternative design solutions or materials must be considered carefully to ensure that cost is within budget, that money is allocated in a balanced way to best suit the client's needs and that, throughout the project, good value is obtained for the money spent. The most significant decisions affecting cost occur in the concept and outline planning stage.

Of first importance is the economic use of space in the proposed building. Although the basic range of accommodation is fixed, considerable additional space is required for circulation and access, stores, plant rooms and toilet facilities. This additional space, sometimes called 'balance area', can vary considerably according to the layout adopted and should be kept to the minimum by efficient planning of staircases and service ducts, grouping of toilet facilities and a restriction on the area of circulation routes. The economic planform will also aim at reducing the ratio of external wall area to total floor area thus saving expensive wall materials and reducing heat losses (or gains) and, hence, minimising the installation and running costs of the heating, ventilation or air-conditioning systems. The reduction of storey heights to a minimum will have similar cost benefits but could affect significantly the building's future adaptability.

It is usual to prepare a cost plan for the project in elemental form. Initially it is a cost estimate based on the preferred scheme and structural system together with a specification covering the main building elements. In the long term it forms a cost structure for monitoring the cost effect of changes and the detailed development of the design. The cost plan should state whether it provides for price inflation to tender stage or building completion, or is based upon rates current at date of estimate.

Major elements should be kept in reasonable balance, e.g. the use of an expensive cladding material could leave too little money for the remainder of the work resulting in a visually pleasing but operationally unsuccessful building. The cost plan is an excellent means of checking the balance between the different elements of structure, finishes and services though the relative percentages of the overall cost will vary from case to case according to the type of building and its user requirements.

While the capital construction cost of a building is of primary importance, other costs will also be significant and could affect design. The annual running cost is one such item and services installations, particularly, should be considered in terms of operational as well as initial cost. Similarly, the use of an expensive but hardwearing material may be justified in terms of subsequently reduced outlay on cleaning or maintenance. Discounting techniques and, possibly, tax considerations are necessary to make true cost assessments of such comparisons.

The total cost of a building project will also include expenditure on land, borrowed capital and the fitting out of the completed building, compensation to adjoining owners and other associated costs as well as legal and design consultant's fees and expenses. In some cases, the earlier a development can be occupied the better the cost advantage to the client. The construction method and programme are then significant and may affect the design form. It is often possible to assess the financial advantage of early completion and by comparative financial analysis to justify additional construction cost to shorten the construction period. Similarly, value engineering can be applied to ensure that optimum arrangements are adopted to meet the client's objectives.

21.10 Internal environment

21.10.1 Thermal environment

The required comfort conditions and tolerances are determined by the intended function of the space concerned.

Thermal comfort depends on a complex of inter-related factors: air temperature, ventilation rates, relative humidity and mean radiant temperature of the enclosing space. Mean radiant temperature is generally a function of enclosure construction, although the form of heating can have an influence. All other factors are determined by the air-conditioning system. Many attempts have been made to devise indices which will represent in one figure the composite effect of the different variables, such as equivalent temperature (T_{eq}) and corrected effective temperature (CET). The former incorporates three of the basic variables: (1) air temperature; (2) mean radiant temperature; and (3) rate of air movement; the CET adds relative humidity. For the purpose of design calculations, however, the generally accepted index is resultant temperature, which is the mean of the air temperature and the mean radiant temperature.

Internal design temperatures for air-conditioned buildings in this country are usually of the order of 20° C in winter and 22° C in the summer; relative humidity values are usually kept within limits depending upon the spaces served, the types of system, condensation considerations and the enclosure construction. Glass area and type, especially large single glazed windows, has an appreciable effect on mean radiant temperature and also restricts the permitted humidity level in cold weather.

21.10.1.1 Site and climate

Internal thermal control will also be influenced by external seasonal temperatures, relative humidity, wind velocities and direction, air quality (industrial smoke pollution, etc.), solar orientation and latitude and relation of the site to surrounding locality and adjacent buildings.

In other than air-conditioned buildings, external temperature related to occupancy levels and internal heat gains determine the amount of external ventilation air to be introduced. Where windows can be opened, however, occupant behaviour tends to be the dominant influence. In air-conditioned buildings ventilation air quantity can be related to external temperature and relative humidity, but this is dependent on the type of air-conditioning system. In warm summer conditions, the amount of ventilation air has a direct effect on refrigeration loads, but at other times of the year, cool outside air can be introduced beneficially to offset internal heat gains.

Excessive infiltration through openings such as doors, window gaps, etc. can reduce performance seriously and increase operating costs; satisfactory sealing is necessary as are effective measures to reduce the stack effect (flow of air up stair and lift areas) which grows in significance with increasing building height.

Solar penetration into the building is determined by latitude and season and the resulting heat gain can be serious. Methods of control include internal or external louvres and blinds, special heat-absorbing and reflecting glasses, small glass areas and various forms of external shading structure.

21.10.1.2 Building function and form

Thermal design is affected by the energy-producing elements within the building: human, mechanical and electrical. Building configuration, size and proportion and construction of the building shell influence the adaptability and capacity of the system to cope with external environmental changes. The proportion between interior space which is independent of external effects and perimeter space which is not, is important. External conditions penetrate a building to approximately 6 m: this perimeter zone will require a system which can quickly adapt to rapid variations in the heating or cooling loads. In contrast, load changes in interior spaces are usually less rapid and represent a predominantly cooling requirement.

21.10.2 Air-conditioning

Natural ventilation has certain potential drawbacks: (1) noise infiltration through open windows; (2) overheating during summer due to solar and internal heat gains; (3) excessive infiltration of outside air resulting in uncontrollable internal air movement; and (4) ineffective ventilation beyond about 5 m from the perimeter with attendant overheating.

Mechanical ventilation solves only a few of these problems. Noise and outside air infiltration are reduced as windows are opened less frequently. Increased air movement during warmer weather can alleviate discomfort to some degree.

Overheating and high humidity can, however, occur due to the inability of the system to supply air at the correct thermal condition. This inability is overcome by the inclusion of refrigeration, thereby changing the system from mechanical ventilation to air-conditioning.

Air-conditioning provides a controlled internal thermal environment which is largely independent of the external conditions or of any changes in the internal load conditions. Planning and configuration of the building will be influenced by the provision of air-conditioning. Deep space can be created with the knowledge that a satisfactory internal thermal environment will be achieved. Similarly, nonopening windows avoid infiltration problems which are accentuated with increased building height.

Moisture control and filtration of the incoming air are integral parts of full air-conditioning giving a cleaner, healthier and more comfortable atmosphere compared with ventilation by natural methods. Redecorating costs and absenteeism may be reduced and working efficiency increased.

21.10.2.1 Air-conditioning systems

Many types of air-conditioning systems are available and can be classified into three basic groups: (1) 'centralized'; (2) 'decentralized'; and (3) 'self-contained' systems; some solutions are combinations of these three.

Centralized systems. Centralized systems are:

(1) Systems where air is processed at a central plant and distributed for use without further treatment:
  - (a) single-duct all-air systems using high-, medium- or low-velocity distribution;
  - (b) double-duct all-air systems using high-, medium- or low-velocity distribution with local terminal mixing units (referred to as dual-duct systems).

(2) Systems where air is processed at a central plant, but with final heat addition or subtraction at the point of use:
  - (a) single-duct all-air reheat/recool systems, using high-, medium- or low-velocity air distribution with associated heating and/or cooling water distribution;
  - (b) perimeter induction air/water systems using high-, medium- or low-velocity primary air distribution with secondary heating and/or cooling water distribution on a two-, three- or four-pipe principle.

Decentralized systems. Decentralized systems are:

(1) Systems where a liquid medium is distributed from a central

point to units which condition air locally: some such systems also have a supplementary primary air supply from a central plant to the unit or space:

(a) room fan coil unit air/water system with two-, three- or four-pipe water distribution and local outside air connections;
(b) as for (a) but with supplementary primary air from central plant;
(c) localized zone air-handling unit all-air systems with associated heating/cooling water distributions and with low-velocity air distribution to conditioned spaces from the units;
(d) radiant ceiling systems supplied with heating/cooling water distribution and supplemented with separate single-duct all-air system.

Self-contained systems. Self-contained systems are systems where self-contained air-conditioners process and supply air at the point of use.

Each system has merits and limitations. The simpler low-velocity all-air single-duct systems require a large amount of duct space and are not a practical solution where a large number of zones of varying use are to be served. In these cases a system which can respond to these variations is required. One of the following systems would be appropriate. Double-duct all-air systems mix air from separate hot and cold distribution ducts using ceiling- or sill-mounted terminal mixing boxes. This system is very adaptable, but the combination of two supply ducts plus a return air duct requires considerable service space, even when using high- and medium-velocity distribution.

The induction unit discharges primary air supplied from the central plant through high-pressure nozzles and this induces air from the space into the unit which then mixes with the primary air before discharging back to the space; temperature control is achieved by a heating/cooling coil. Space is saved because the air is distributed at high velocity. The basic difference between two-, three- and four-pipe associated water distribution systems is that the latter two can provide, at the point of use, the simultaneous facility for either heating or cooling, while the two-pipe system is restricted at any one time to one or the other.

Fan coil systems incorporate a heating/cooling coil and a circulating fan. Primary air can be ducted direct to the units from a central system or discharged to the space independently or alternatively, each unit can draw in air direct from outside.

Radiant heating/cooling ceilings, when used with a supplementary air system, can provide an effective environment although their adaptability to meet rapid fluctuations in heating and cooling loads is limited.

Self-contained packaged air-conditioning units are usually restricted to smaller specialized projects.

21.10.2.2 Air-conditioning – distribution and integration

Considerable duct distribution space is required and air outlets and extracts are often incorporated in the detailing of light fittings and suspended ceilings. From the earliest stages, therefore, the air-conditioning system should be integrated into the total planning and detail design process of both the building elements and the structure.

Perimeter units can be served from a network of air ducts or water pipes concentrated in zones near the outer wall, within the under-sill or ceiling void for horizontal piping or ducts and within structural column enclosures for vertical distribution. Alternatively, the perimeter area may be served from the central core with ducts and pipes accommodated above a false ceiling, within a structural hollow floor or beneath a raised floor.

In areas where little flexibility for changing use is required, a totally integrated solution using the structure to accommodate air and water distribution may produce some economies including reduced storey height. Where a high degree of flexibility is required as, for example, in open-plan buildings, ceiling distribution on a modular basis for interior zones and sill or ceiling distribution for the perimeter becomes essential and a false ceiling is required, the ceiling space being used to accommodate the ducts and pipes.

The above systems can be described as fully ducted. There are two other basic air-supply and exhaust methods using the ceiling space as a large duct or plenum:

(1) *Negative plenum*: air is extracted into the plenum through outlets in the false ceiling which are usually part of the light fittings. Air supply is ducted to diffusers or slots incorporated in the ceiling design.
(2) *Positive plenum*: the plenum is used as the supply duct, air being forced through ports in the false ceiling. Extracted air is ducted from terminals usually incorporated in the light fittings.

When the air is exhausted through the light fittings it cools and, hence, increases the efficiency of the light source: it also removes excess heat (arising from high light levels) which can be transferred for use elsewhere, e.g. the perimeter area, but is more commonly vented to the exterior. The outlets require careful design coupled with adequate ceiling height, 4 to 5 m if possible, to prevent downdraughts.

The completely ducted system has fewer thermal problems, but occupies more space and is more expensive. The plenum systems substantially reduce duct requirements, but are less efficient; they also require careful control of temperature to prevent condensation and, sometimes, the incorporation of insulation on the underside of the structural floor to confine the plenum effects to the storey intended.

21.10.3 Accommodation of building services

Services can occupy 15% or more of the volume of a building and their distribution through the building is critical to its performance and flexibility. The organizing of space for services is thus of vital importance both in the strategic planning and detail design stages of the building. The servicing systems may be given direct expression or be entirely hidden within the overall form and finishes of the building.

The strategic planning of the services installations involves the optimization of the location and size of plant room spaces and the distribution systems linking them with the building areas being serviced, coupled with their integration with the structural and architectural elements. Frequently, there is pressure on the design team to minimize the space occupied by the services as the result of planning height restrictions or on grounds of economics. This can prove a false economy as such an approach can affect significantly future flexibility in the use of the building.

Plant rooms should be positioned as close as possible to the centre of gravity of the areas they serve to keep maximum duct sizes to a minimum and should be readily accessible to connecting ductwork without impediment from adjacent structure. The impact of weight, noise or vibration on adjacent elements or building functions should be considered. In general, service runs should not be more than 25 m from the point of origin and, vertically, plant rooms should not serve floors more than ten storeys away. Plant rooms should be sensibly proportioned avoiding L-shapes and long thin spaces. Clear height generally has to be to the underside of structural beams and if possible the plant room space should be column-free.

Frequently several plant rooms are required covering the following items.

(1) *Boilers and refrigerators*: commonly referred to as the energy centre.
(2) *Air handling*: fans, heating and cooling coils and filters.
(3) *Water*: storage tanks.
(4) *Sprinklers*: storage tanks.
(5) *Cooling towers*: serving the cooling plant.
(6) *Lifts*: motors, winding gear or pumps.
(7) *Electrical*: switchroom or substation or standby generator.
(8) *Telecommunications*: telephone and data transmission equipment.

Some of these items must not be incorporated in the same plant space. Examples are water and electricity, refrigeration machines (chillers) and boilers to avoid toxic fumes from refrigerator gas coming into contact with boiler flames.

Access for installations, repair and maintenance must be incorporated and construction problems including speed should be taken into account in the siting of these elements especially when the installation of plant is on the critical path to completion.

In air-conditioned buildings, the air intake must be separated from the air discharge. The top of the building is often the preferred location for the air-handling plant particularly with all air systems involving recirculated air. In very large buildings, a number of air-handling plant rooms distributed through the building provide greater flexibility and less inroad into usable building volume.

The combined plant room area typically ranges from 4 to 15% of total floor area depending upon building type. Some plant area requirements are as follows:

	(%)
Hospitals/laboratories	9–15
Swimming pools/ice rinks	5–12
Shopping centres	5–8
Theatres/concert halls	9–11
Air-conditioned speculative offices	6–9
Residential/hotels	4–5
Factories/warehouses	3–4

Special cases can lead to even greater plant areas when environmental control is required to extremely fine limits, e.g. in pharmaceutical or semiconductor production facilities.

The incorporation of the horizontal services within the ceiling and floor construction is a vital element in the efficient design of the building particularly when overall floor depth is critical. Many different arrangements have been developed around both steel and concrete structural elements with the objectives of keeping floor depth to a minimum yet providing easy access and flexibility for future change. Some typical arrangements and corresponding floor depths are shown in Figure 21.2. In some cases complete storeys may be given over to services distribution.

The location of the plant rooms and the location of pipe and duct runs can have a critical impact on structural arrangement and detail. The co-ordination of structural penetrations is an important task for the structural engineer and timely receipt of relevant information from the services engineers is vital. In certain cases, duct and plant room walls may be subjected to positive or negative pressure which the structural engineer may need to take into account. Enclosure materials and construction would need to be appropriately airtight.

Modern office design has to cater for widespread use of the computer. Space for cabling and easy access for modification or extension are essential ingredients for good design catering for both immediate and long-term requirements. At the same time, increased space is needed for air ducting to deal with the higher heat loads generated. The growing impact of this heat-generating equipment on the total heat load that has to be dealt with by the air-conditioning system is illustrated in Figure 21.3. Figures 21.4 (a)–(c) show diagrammatically three methods used for the incorporation of air-conditioning and cabling in the present-day electronic office.

Figure 21.4(a) shows a conventional-sandwich ceiling and raised floor. In this arrangement, which is favoured by most speculative developers, the air supply and removal and the general lighting are incorporated in the space between the structure and a suspended ceiling. All cable services are in the elevated floor usually between 75 and 150 mm deep. In some cases, cables are run in hollow cells in the structural floor deck. This arrangement separates service systems cleanly but costs more.

Figure 21.4(b) shows a total ceiling servicing using 'stalactites'. In this arrangement, a little more depth is added to the ceiling space and facilities provided for easy and frequent access so that heavy cabling can be accommodated in the ceiling space. The cabling is brought to the workstation down partitions, columns or free standing 'power poles'. This is the lowest cost option but is not much used in new design outside the high-technology industries. However, the increasing shift back to cellular offices coupled with the arrival of slimmer, more flexible, data cabling could make this solution more acceptable.

Figure 21.4(c) shows the total floor servicing using 'stalagmites'. In this arrangement, all the services and cabling are incorporated in the floor void. Uplighting is bounced off the ceiling helping to provide glare-free background lighting for visual display unit (VDU) working, and is augmented where required by task lighting. Air can be supplied, under occupants' control, through desks and removed through heat-producing equipment and light fittings. Partitions sit between the heavy floor and the solid ceiling giving better sound insulation. The exposed structural ceiling acts favourably as a heat sink helping to even-out internal temperatures. Removal of a small proportion of overhead stale air can be effected through uplighter units or in voids at walls or around structural columns.

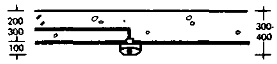
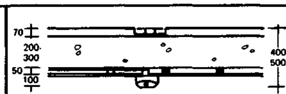
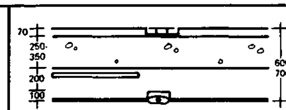
21.10.4 Heating/cooling generation

Arrangements for the heating and cooling generating plant will depend on a number of general and localized factors: (1) availability, suitability, and economic costs associated with the utilization of fuel and power; (2) resources peculiar to the site; and (3) utilization of recoverable energy associated with the heating and cooling systems installed within the building.

Fuel and power considerations are complex and include a detailed appraisal of operating and capital costs for various fuel alternatives (coal, gas and oil) and power. Boiler plants incorporating combined dual-firing burners suitable for gas (town or natural) and oil can offer attractive capital and operating cost characteristics combined with greater flexibility.

Heat-recovery systems have been gaining popularity. A common arrangement is to utilize low-grade heat being rejected from refrigeration machines. Another is to transfer heat extracted from the interior of deeply planned areas, which have to be cooled, to spaces requiring a heating load, such as perimeter zones, during winter and certain mid-season periods.

On larger specialized projects, total energy is finding an application. This is based on the concept that the total energy requirement, in all its forms, can be provided from a single fuel source. These systems incorporate electrical generation with heat being produced as a byproduct. Refrigeration, which can be met by either electricity or heat, is usually a complementary part of such an integrated energy system.

Floor ceiling construction	 (a)	 (b)	 (c)
Description	Solid concrete slab–power floated finish Cast-in electrical conduits Surface fixed lighting	Solid concrete slab, screeded finish No access false ceiling Surface fixed lighting	Solid concrete slab, screeded finish Limited access false ceiling recessed light fittings
Services implications	Lighting position fixed Perimeter power, data, telecom trunking Perimeter mechanical systems typically radiators or convectors, visible pipework	Light position may be altered by stripping out and replacing false ceiling Screed can accommodate flush floor but unlikely to have electrical trunking as an alternative to perimeter trunking Perimeter mechanical systems Visible pipework	Light position may be altered by local modification of false ceiling Ceiling zone can accommodate pipework runs that serve perimeter mechanical systems (concealed pipework) Flush floor trunking possible (and likely) with floor screed
Typical applications	Heated and naturally ventilated office (simple), hotel bedrooms, multiple housing and dormitories	Heated and naturally ventilated office (simple)	Most naturally ventilated and heated buildings with a 'simple' services content
Typical floor to floor height	2.7 m	3.0 m	3.3 m

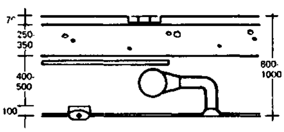
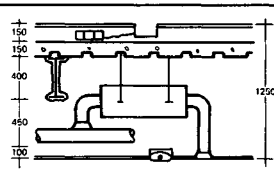
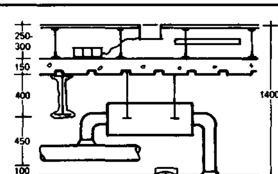
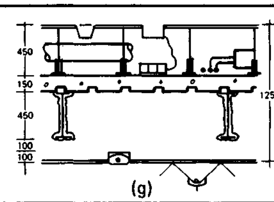
 (d)	 (e)	 (f)	 (g)
Solid concrete slab, screeded finish Full access false ceiling recessed light fittings	Service void and ceiling as for (d) Structure change from concrete slab to steel frame metal deck, small raised floor	As for (e) false floor depth increased	False floor depth increased, services removed from ceiling zone–transferred to floor zone
Changes to ceiling layout easy service zone sufficient size for air ducts serving air/water air-conditioning systems - but not all-air systems Perimeter terminal units for 400 mm void Some ceiling-mounted terminal units (such as, fan coils) for 500 mm void No space for duct crossovers	Changes to ceiling layout easily accommodated False floor allows easy location relocation/addition to electrical services and outlets All-air systems possible with large terminal units in structural zone between beams Large duct crossovers possible within structural zone	Increase in depth of false floor allows pipework to be located in the floor zone Flexibility due to ease of addition of piped services Allows flexible location of computer rooms Easy to upgrade cooling capacity on floors for tenants with high floor heat loads	Increase in depth of false floor allows pipework ductwork and electrical services to be located Allows flexible location of all services
Average-quality office, refurbished office all areas with an average servicing requirement	Minimal perimeter systems high-quality office; areas with average to high services content	High-quality office with high information technology content office where frequent internal replanning/changes occur	High-quality office with high information technology content office where frequent internal replanning/changes occur
3.6 m	3.9 m	4.2 m	3.9 m

Figure 21.2 Options for horizontal service distribution showing increasing size and complexity of service zone planning as sophistication increases provision. (After *Architect's Journal* **183**, 9, p.62 (1986))

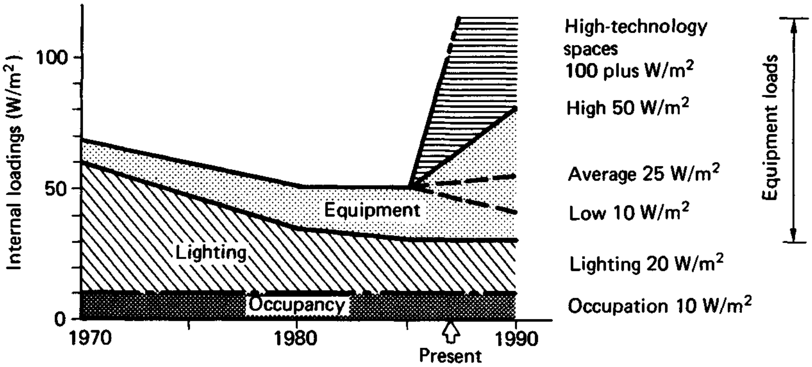


Figure 21.3 Trend in office space internal heat gains from equipment lighting and occupancy

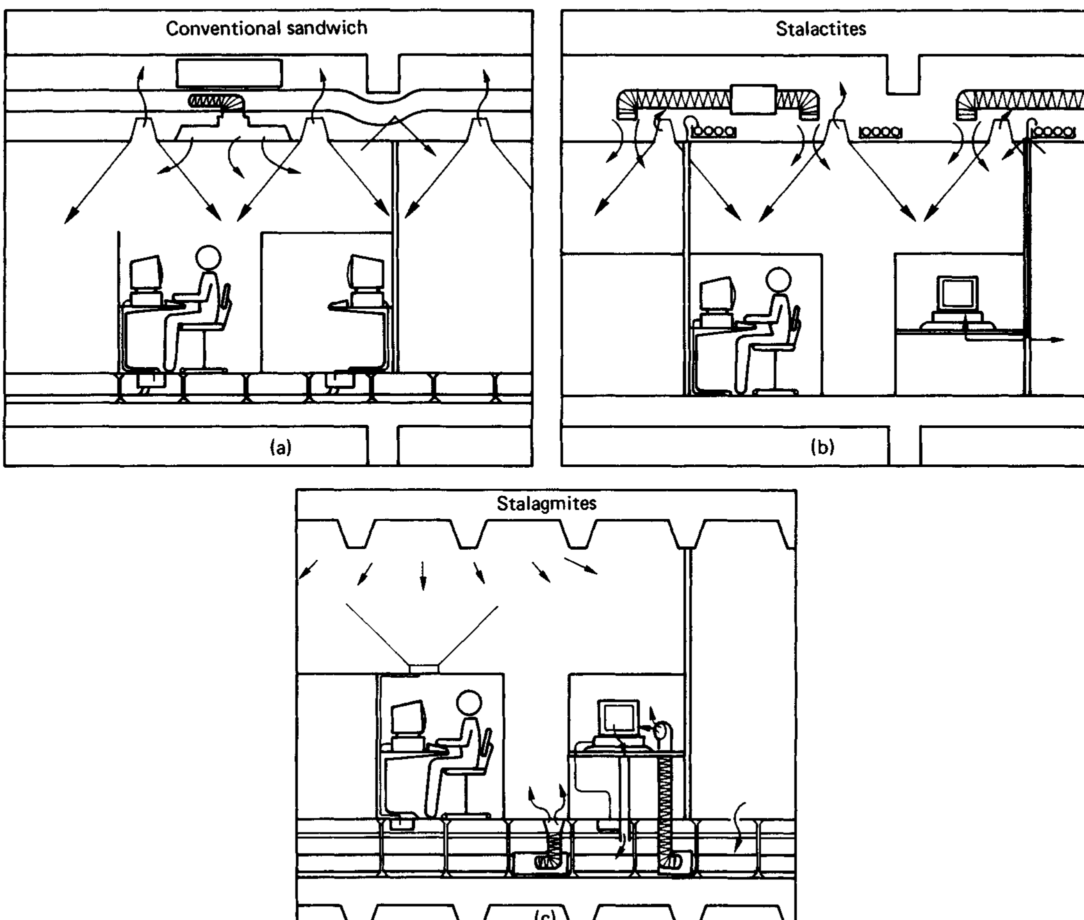


Figure 21.4 Incorporation of air-conditioning and cabling in the electronic office. (a) Conventional sandwich; (b) stalactites; (c) stalacmites

21.10.5 Thermal insulation

The object of thermal insulation, together with heating, is to obtain, irrespective of the prevailing weather conditions, a near-constant internal temperature determined by requirements of human comfort and satisfactory conditions for manufacturing processes or storage of goods. Adequate insulation is needed to avoid excessive expenditure on heating plant and fuel. The insulation and heating of buildings for human occupation are normally designed to maintain a temperature of 16 to 21°C according to use when the outside temperature is -1°C. Good thermal insulating materials generally are those which entrap air, such as lightweight concrete, wood-wool slabs, glass or mineral fibre wool.

Calculation of the thermal transmittance, or U value, of a wall, floor or roof is carried out by adding together the thermal resistances of the materials and surface coefficients and taking the reciprocal of the answer to provide the thermal transmittance of the composite construction:

$$U = 1/R \tag{21.1}$$

$$R = R_{s_1} + R_{s_2} + \frac{(L_1)}{(K_1)} + \frac{(L_2)}{(K_2)} \text{ etc.} + R_a + R_h \tag{21.2}$$

where R is the total thermal resistance of the structure, R_{s_1} is the surface resistance of the inner face, R_{s_2} is the surface resistance of the outer face, R_a is the resistance of the air cavity if present, R_h is the resistance of unit material such as hollow block where resistance per unit thickness does not apply and L/K is the resistance of one layer of material or thermal conductivity K and thickness L.

To evaluate the total heat loss from a room or building the thermal transmittance of the walls, windows, floor and ceiling must be calculated and allowance made for the losses involved in heating-up the ventilating air and the structure when heating is intermittent.

Structural members penetrating the full thickness of a wall produce 'cold bridges', locally reducing the thermal resistance and internal surface temperatures, with consequent added risk of condensation. In such cases the cold bridge should be reduced in width or eliminated by appropriate insulation.

Ureaformaldehyde foam is sometimes used to fill the cavity in cavity-wall construction resulting in an almost 80% reduction in heat loss through the walls. Double glazing, as well as reducing heat loss, has advantages in increasing the temperature on the inside window surface and may improve internal comfort conditions.

Condensation problems have increased due to new methods of building, standards of heating and control of ventilation, and changing family habits which have led to intermittent heating coupled with the generation of more moisture inside the dwelling. Old buildings, particularly domestic ones, usually had open fires and flues and windows were generally less well-fitting resulting in natural, if draughty, ventilation which got rid of moisture-laden air and avoided condensation on cold walls and windows. Condensation in modern buildings can be avoided by adequate combination of insulation, heating and ventilation.

The amount of moisture which air can hold, increases with the temperature and when it can hold no more water it is said to be saturated and the relative humidity is 100%. The temperature at which air with any particular moisture content is saturated is called the dewpoint and if that air falls on a surface which is colder than the dewpoint, condensation will occur. Another object of thermal insulation, in conjunction with heating and ventilation, is to ensure that the inside surfaces of walls, floors, ceilings, roof and, if possible, windows, are kept above the dewpoint.

Moisture-laden air can pass through a porous wall or roof construction and condense inside where it meets a temperature below the dewpoint. Figure 21.5,[5] shows the relationship between the local material temperature and dewpoint through the cross-section of varying arrangements of a composite external wall, for given internal and external air temperatures and moisture contents. By appropriate positioning of a vapour barrier and combination of materials forming the wall, the local dewpoint can be kept above the local temperature and condensation avoided. Temperature drops across the section are determined by the proportional thermal resistances of the materials, surfaces and airgap; dewpoints are obtained by first determining the local vapour pressures from the proportional vapour resistances and then converting these to their respective dewpoint temperatures.

21.10.5.1 Estimation of condensation risk

At any point where the computed temperature is lower than the computed dewpoint temperature, condensation can occur in the conditions assumed. In the worked example, liquid may form in a position where, clearly, it can reduce the effectiveness of insulation and it is likely also to put the nearby timber at risk of rot. As in illustration of the effect of structural detailing, Figure 21.5(b) shows the construction reversed and free from risk in the same surrounding conditions. Slight modifications shown in Figure 21.5(c) and (d) are sufficient, however, to limit the potential risk by using materials that modify the vapour pressure gradient.

21.10.6 Lighting

Three types of lighting are used: (1) daylight; (2) daylight integrated with electric lighting; and (3) electric lighting. Good daylighting is more than the provision simply of large windows. Optimum size, shape and position of windows is a function not only of the required lighting levels, but also of the resulting eye adaptation conditions, sky glare and external view. In addition, heat loss or solar gain, ventilation, noise transmission, privacy and the shading effects of adjacent buildings, present or future, must be taken into account. Side-lit rooms often appear badly illuminated because of the contrast between the areas adjacent to and those remote from the windows, even though working illumination levels may be adequate throughout.

At one time, daylighting appeared cheap and its real cost went unquestioned. The present position is different: modern light sources cost less and are more efficient while the true cost of daylight is recognized in terms of added cost in construction, maintenance, heat loss or gain and, in urban areas, the inefficient use of the available site area. Simultaneously, the expected standards have increased in both quantity and quality and, in modern buildings, daylighting would not be relied upon as the sole source of light even during periods of good outdoor light.

By introducing electric lighting of a colour to blend with daylight it is possible to provide adequate illumination over the whole working area without a sense of deprivation of daylight. Moreover, such arrangements – known as permanent supplementary artificial lighting of interiors (PSALI) – can be applied without visual discomfort over areas much greater than can be lit by daylight alone, irrespective of the prevailing outdoor light; its added cost must be weighed against the direct and indirect costs of higher ceilings and bigger windows, reduced floor space for lightwells, and/or restricted useful depth of rooms.

The current quest for saving energy has stimulated research into methods of securing greater penetration of daylight into buildings. One such method involves the use of carefully machined acrylic prisms sandwiched between sheets of glass

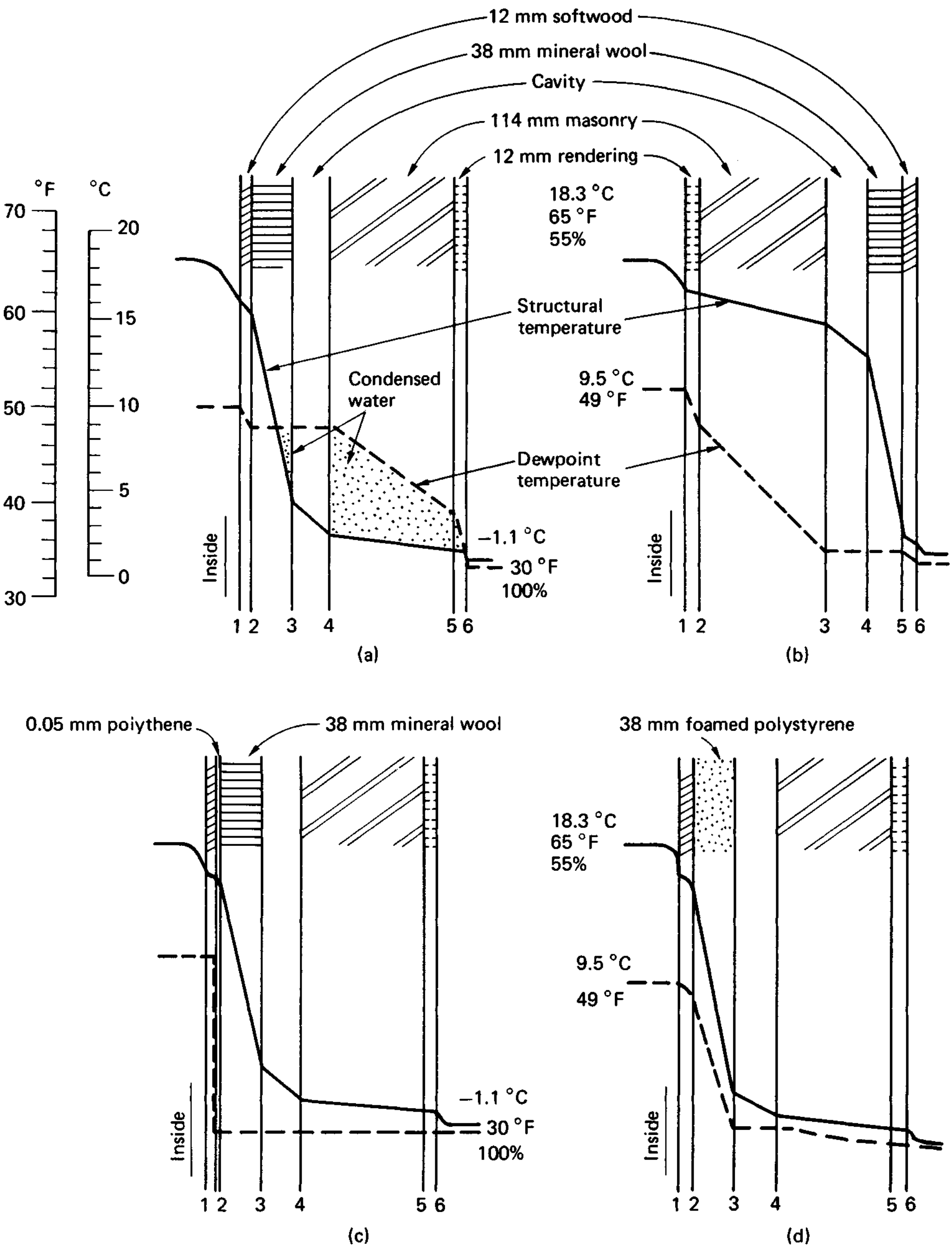


Figure 21.5 Prevention of condensation in wall cavities. (After Building Research Establishment (1979) *Thermal, visual and acoustic requirements in buildings.* Digest No. 91 (2nd edn). BRE, Watford)

attached to the exterior of the building as a form of shading. The prisms redirect the Sun's rays parallel to the ceilings within the building whilst blocking sky glare. The ceilings are specially shaped to divert the parallel beams and provide the uniform illumination on the working plane. Another approach uses heliostats to direct the Sun's rays into a hollow square acrylic pipe which, using the principle of total internal reflection in a manner similar to fibre optics, can feed light down the risers of a building to illuminate the inside. This method has the advantage that as daylight fades, artificial light sources can irradiate the same piping.

The use of external automatic sunblinds has had limited acceptance, mainly due to high costs both initial and subsequent. Inevitably, the continued operation of electro-mechanical devices such as these, subject to external forces, is difficult. More promising is the development of special glasses similar to the familiar photochromic but whose light transmission may be varied reliably by the application of an electrical potential.

Quality of the electric light is as important as quantity and design should take into account: (1) brightness and colour patterns; (2) directional lighting where appropriate; (3) control of direct or reflected glare from light sources; (4) colour rendering; and (5) prevention of excessive contrast between adjacent areas.

The most common light sources are tungsten lamps and fluorescent tubes with a growing acceptance of high-pressure discharge lamps. Tungsten lamps are common in domestic and decorative installations, but are inefficient in their light output and are generally uneconomic for the lighting levels required in most modern buildings. However, at a time when the new compact-source fluorescent lamps seemed likely to oust tungsten lamps even in the home, a specialized form – the low-voltage reflector tungsten halogen lamp – is growing in popularity, especially for display purposes. Their small size, longer life, improved efficacy and excellent colour rendition compared with standard tungsten lamps have tended to outweigh high capital cost and the inconvenience of the stepdown transformers and heavy cabling.

Fluorescent tubes are the most commonly used, but can take up considerable amounts of ceiling space. High pressure discharge lamps provide similar benefits of efficiency and long life, but more closely approach a point source, permitting greater freedom in ceiling design. The ability to accommodate an economic light fitting will depend upon the planning and structural grids. When these are not appropriate to the light fitting, the lighting system will be expensive in itself and may also cause extra cost in removing the unwanted heat.

The light fittings have to be spaced carefully to provide adequate lighting levels over the whole working plane. Due to the physical discomfort which can be caused by the brightness of the light source, careful attention must be given to the prevention of direct or reflected glare. Glare standards exist for most types of working environments and the glare characteristics of lamp fittings and control diffusers are readily available.

The rationale behind such lighting layouts has always been the ensurance of a high degree of uniformity so that any location of the working plane will be served adequately. The basic inhumanity of such schemes, together with the absurdity of lighting circulation spaces to the same level as the task, has resulted in the growing popularity of uplighting where the lighting plane is illuminated indirectly by light bounced off a reflecting surface, usually the ceiling. As in most spheres of life, high quality is difficult to reconcile with efficiency and the cost of a superior working environment is increased consumption, typically 16 W/m^2 for 400 lux on the desk. Possibly the greatest single factor behind the popularity of uplighting is the expansion of the use of VDUs and word processors where, unlike most other forms of lighting, a correctly designed indirect scheme can limit tiring and distracting reflections from the screen.

The varying colour qualities and corresponding luminance efficiencies of the available light sources have an important bearing, not only on the visual environment, but also on the degree of heating or air-conditioning that may be required. The colour appearance of a light source is always cause for much subjective judgement and prejudice. Daylight cannot be used as a reference value since its spectral composition shifts throughout the day. Indeed, what is wrong with light sources, the purists insist, is that their colour appearance does not noticeably change. Fluorescent tubes can now be had in a bewildering range of phosphors equally able to imitate tungsten lamps or cold north light. The triphosphor tubes now make it possible to have both excellent colour rendition and high efficacy. The most promising light source for commercial interiors is the high-pressure sodium lamp, which is able to better the fluorescent tube on most counts. However, its colour appearance even in the de luxe form remains controversial.

In some buildings, the energy for lighting can be a substantial part of the total required for all purposes. Since most of that provided for light appears as heat the possibility exists of using this as a major, and perhaps the only, source of internal heating; alternatively, the extra heat load may prove an embarrassment to the air-conditioning system. In either case the lighting must be treated as an integral part of the total environmental design.

Having selected the most efficient light source and used it in the most effective luminaire, the remaining part of the energy equation is control of the running hours. In many situations, people switch lighting on but never off so the advent of remote controls providing automatic switching is beneficial. Generally, such controllers operate either on a time basis or in response to some local stimulus. Their switching programmes may be held in their memories for as much as a year ahead with all holidays and weekends catered for. The instructions in the form of codes are transmitted along dedicated hard wiring or even over the supply cables themselves to the luminaires which are equipped with decoders enabling them to respond to one or several instructions. Local overriders often in the form of hand-held infra-red transmitters enable the central instructions to be modified. Less extensive forms of automatic lighting control take the form of presence detectors which switch off after a preset period, as the result of high daylight levels, or in the absence of people. The detecting principle may be either acoustic or infra-red.

21.10.6.1 Lighting for various categories of building

Speculative offices. Such buildings are generally leased without lighting fittings to avoid inhibiting either the letting pattern or the tenant's partition layout. Where lighting fittings are supplied, the preference is often for surface-mounted hot-cathode fluorescent tube units with prismatic light controllers. Lighting levels are currently in the region of 400 lux.

Offices: purpose design. In keeping with the design standards recommended in the Chartered Institute of Building Services Engineers (CIBSE) code for interior lighting,[6] average levels of 500 to 750 lux are usual, depending on the task. Such levels using combinations of light controllers with 'batwing' and asymmetric distributions may be had for as low as 10 W/m^2 but at the cost of inflexible and regimented workstation layouts. It is now possible to simulate lighting effects by means of models and artificial skies but this is best used where the budget will permit the purchase of purpose-designed luminaires. Much interest is being focused on the introduction of high-frequency control gear for fluorescent tubes which, for example, would reduce the loss on a 1500 mm tube from 13 to 5 W with gains in freedom from flicker and with silent operation.

Offices: burolandschaft. The gentle modulation of light and shadow produced by uplighting is particularly apt for this form of office. Using either metal halide or high-pressure sodium discharge lamps, uplighting brings good colour rendition, high efficacy, low maintenance and lack of glare, either direct or reflected. It saves the cost of a discrete lighting circuit since uplighting is usually fed from the small power points installed in the floor. The design process involved in an uplighting scheme is still unfamiliar to many, being task-related rather than building-related, and this unfamiliarity has tended to limit its more general acceptance.

Hospitals. The difficulties of reconciling the lighting needs in wards of patients who may either be lying supine or sitting up in their beds has led to separate systems being installed. In the latter case, wall-mounted units are preferred and these are often incorporated into continuous horizontal trunking runs which may contain other services such as oxygen, sound broadcasting, nurse call systems, etc. The former requirement is met by fluorescent fittings generally of the suspended pattern. There are many specialized considerations, such as operating theatres and anaesthetics rooms where totally enclosed, noise-proof fluorescent fittings sealed into the ceiling structure provide general illumination whilst shadowless operating-table lighting fittings incorporating tungsten light sources produce intensities up to 10 000 lux in the operating area.

Housing. Whilst tungsten fittings are still the norm for the home, the advent of compact fluorescent lamps with their significant economic advantages and tungsten-like colour appearance may change this. More sophisticated forms of lighting control, such as touch dimmers and infra-red switching, are now available and are beginning to be installed.

Schools. Cost considerations usually dictate surface-mounted fluorescent fittings with prismatic light controllers with levels in the region of 600 lux. In rooms where the seating has a fixed orientation, directional fittings may be used.

Industrial buildings. When ceiling heights are below about 4 m, fluorescent fittings are still the most-used light source. Above this, high-pressure mercury or sodium discharge lamps in reflector fittings are used with a wide range of distribution curves, both symmetrical and asymmetrical. The colour rendition of mercury fluorescent, mercury halide or high-pressure sodium light sources are satisfactory, but care has to be exercised in machine shops because of stroboscopic effects.

Car parks. The majority of multideck car parks use bare fluorescent tubes in fittings with moisture-proof lampholders and glassfibre or PVC-coated bodies. In the larger open car parks, increasing use is being made of high mast lighting.

Museums and art galleries. The lighting of museums and art galleries should be designed principally to meet the requirements of conservation, display and specialized study. Apart from atmospheric pollution, the main destructive agents will be the ultraviolet and infra-red content of light. Natural light is the worst offender with discharge sources such as fluorescent tubes, with high-pressure sources coming second. All three require careful filtering before they can be used to illuminate any exhibits containing organic materials or pigments.

Even tungsten halogen sources are suspect because of ultraviolet energy. The usual formula is a blend of tungsten display fittings giving a restrained average illumination plus fluorescent tubes with ultraviolet filtering. Deterioration of organic materials is a product of the intensity of the harmful wavebands and the length of exposure. The use of presence detectors – which ensure that exhibits are only illuminated for the period when there are people to see them – would be of value.

21.10.7 Noise

The control of noise requires consideration of its nature, source and mode of transmission. Typically, the main problems are: (1) reduction of noise to an acceptable level for efficient working; and (2) effective noise barriers for privacy. Problems of sound insulation and sound absorption are involved.

The main source of external noise is air or road traffic; penetration is reduced by double glazing (cavity preferably not less than 200 mm), minimum window area and heavy wall construction. In extreme cases, windows must be kept permanently closed and the building air-conditioned.

Internally, structural walls and floors are generally of sufficient mass to provide effective barriers against airborne sound but impact sound is not reduced by mass alone and a resilient material must be added to provide adequate total sound insulation. The lighter building elements, such as suspended ceilings or demountable partitions, do not provide good sound insulation. Continuity of sound insulation, where it is required, is important; a sound-insulating wall would need to extend through the void above a suspended ceiling, for example, unless the ceiling is itself a good sound insulator.

The use of sound-absorbing surface materials and shapes is effective in reducing the ambient noise level and may be so successful in *burolandschaft* offices that a degree of manufactured ambient sound may be needed to mask and, hence, reduce the disturbance from local intermittent noise.

Appropriate planning and detailing of the building is vital to the elimination of noise problems and the establishment of privacy. Wherever possible, areas requiring low noise levels should be divorced from noisy areas such as plant rooms, loading bays and lift motor rooms. Many items of mechanical and electrical equipment produce airborne noise which can pass along air-conditioning or ventilation ducts which then require silencer units. Equipment located in occupied rooms must be selected with appropriate low noise characteristics; in certain cases, especially on high-pressure systems, secondary silencer units are required. Rotating or reciprocating plant should be isolated from the structure to prevent structure-borne noise or vibrations. The increase in plant noise within buildings is increasingly a factor in modern design, requiring specialist advice.

Rooms with a high level of sound within them do not require such a good standard of insulation from adjoining rooms of similar level, but low-tolerance rooms will require a high standard. Figure 21.6 gives an indication of sound reduction levels for different room tolerances.

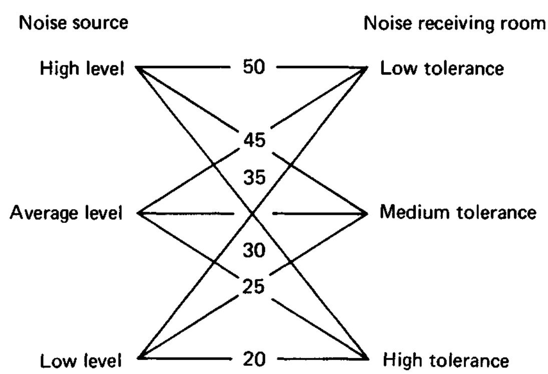


Figure 21.6 Sound reduction levels for various room tolerances. (After Parkins, Humphreys and Cowell (1979) *Acoustics, noise and building* (4th edn). Faber and Faber)

The sound reduction of dense walls varies with the sound frequency and with the weight of wall. At 550 Hz, the sound reduction is as follows:

Weight (kg/m^2)	3	6	12	25	50	100	200	400	800	1000
Sound reduction (dB)	20	24	28	32	36	40	44	49	54	55

For a cavity wall, a reduction value corresponding to the combined weight of the two leaves is used and to this is added the additional assistance provided by the cavity which varies with its width as follows:

Air space (mm)	30	40	50	60	80–100	150	200
Added sound reduction (dB)	6	8	9	10	12	10	6

If the wall contains a door, the equivalent resistance is an intermediate value between those for the wall and door, dependent upon the relative decibel values and areas. For a brick wall of say 46 dB and door of 20 dB and the wall 10 times the area of the door, the equivalent sound reduction values (obtained from charts, e.g. Neufert[7]) is 30 dB. The full insulation value is obtained only if all holes, e.g. for services, are sealed; even very small openings such as keyholes and open joints represent serious sound leaks and must be taken into account in the design if good insulation is to be achieved.

21.11 Water supply, drainage and public health

21.11.1 Water supply

Potable water supplies are generally supplied from the local water undertaking's mains, the local water companies being required under the Water Act, 1945 and EEC directives to supply consumers with a potable supply. The conditions are based on the Model Water Byelaws of 1982, the purpose of which is to prevent waste, undue consumption, misuse and contamination. Water charges may be based on rateable value, assessed annual consumption or on metered consumption.

In the UK, storage provision for cold water for purposes other than drinking is normal and is provided for convenience in the event of mains failure. British Standard Code of Practice 310 schedules the amount of water storage required based upon occupancy (or number of fittings) and building type. Water storage is ideally located at roof level and below the available mains head to minimize operating and maintenance costs and to avoid pumping. A major revision to the various existing water services codes of practice is BS 6700.

It is increasingly found that water mains have insufficient head to deliver water to the upper levels of buildings without the aid of supplementary boosting. The method of boosting should take into account the location of storage, the possible need for any intermediate storage, pressure limitations or requirements in the distribution system, routing, quantities and usage of water. The two most common methods are direct centrifugal pumps serving high-level storage tanks or pneumatic pressure cylinders to boost the available mains pressure; the latter avoids the need for, but does not preclude, the use of high-level storage tanks. Break-cisterns are often required at ground level to cushion demand and very high buildings require break-pressure cisterns restricting gravity drops to about 30 m.

The distribution pipework generally separates cold- and hot-water service feeds and is preferably arranged to provide hot and cold water to the fitments at equal pressures. The routing should take into consideration maintenance, the requirements for draining down, protection against back siphonage and insulation against freezing and condensation.

Most large buildings have extended hot-water distribution systems served by a central heating plant which generally also provides the space heating. A central plant offers economies of scale and uses less fuel than a system of dispersed boilers. The boiler water is kept separate, the hot-water supply being heated by means of heat-exchange coils in calorifiers located in proximity to the outlets being served. Deadlegs need to be avoided wherever possible. Intermediate calorifiers can be located to act as break-pressure cisterns.

21.11.2 Fire installations

Water for fire-fighting purposes in buildings is separated from general water usage and is required for the hose reels, wet risers and sprinklers.

Consultations with the local fire authorities are required to ensure that storage and system duties are met. A number of packaged pumping units are available on the market for hydraulic hose reel installations. Wet risers are a fire authority requirement in tall and large-volume buildings. Sprinklers may be a requirement of the local fire authority or the building owner's insurance company. In the UK, most installations are required to comply with the 29th edition of the Fire Officers' Committee *Rules*[8] which have very specific water flow/pressure requirements and can involve large bulk water-storage requirements, dependent upon the fire risk hazard category. Specialist advice should be sought on these installations.

21.11.3 Water treatment

The growth of the electronics and pharmaceutical industries has expanded the need for water-quality levels far in excess of those supplied by the statutory authorities and special advice should be sought. In hospitals, additional chemical treatment may be required to reduce the rise of disease transmission through the water system.

21.11.4 Drainage

The aim of a well-designed building drainage, sanitation and rainwater installation is to convey foul waste and rainwater efficiently to the sewer or outfall without nuisance or risk to health and self-cleansing. The layout should be as simple and direct as possible and in accordance with the requirements of BS Code of Practice 8301:1985 'Building drainage', BS 572:1978 *Sanitary pipework*, and BS Code of Practice 6367:1985 'Drainage of roofs and paved areas'.

21.11.4.1 Design considerations

The practice of combining soil and rainwater pipes within a building is extremely unwise and the connection of the two systems, even with a combined sewer system, should be located externally, preferably at the last manhole before discharging to the sewer. Soil and waste stacks should be as vertical as possible with the minimum number of offsets. Particular care should be taken with discharges from kitchens, laboratories and disposal units. Separate systems should be provided for activities involving chemical and radioactive effluents. Ventilation pipes are required to maintain a balanced air pressure throughout the soils and waste system. All access locations for rodding should be reviewed in design and located to enable easy maintenance. Ground-floor fittings should be discharged direct to drains and separate from upper-floor fittings. Consideration should be given to draining basement levels via pumps to reduce the risk of flooding in the event of sewer back-up. In selecting pipework materials, consideration should be given to such items as noise, fixings, condensation and material damage in addition to the general material performance criteria.

All sanitary appliances need to be trapped to prevent sewer and drain smells entering the building. Precautions are required to prevent the seals being broken by siphonic action or plug pressure generated within an adjoining stack. Traps can be protected against these dangers by design or by the incorporation of secondary venting immediately behind the trap. Generally, the provision of sanitary appliances should accord with BS 6465:1984, Part 1.

21.11.5 Public health

The importance of providing a wholesome drinking water

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21.11.5 Public health

The importance of providing a wholesome drinking water supply and an efficient system of sanitation within buildings cannot be stressed strongly enough and improvements in the standards of installation and design must continually be sought to avoid the risk of infection and the creation of health hazards. The inter-relationship of these aspects with the other building services, particularly air-conditioning, is of growing importance as more is understood of the nature and transmission of diseases such as legionnaires. Specialist advice is available from the Department of Health and others on the precautions required.

21.12 Lifts, escalators and passenger conveyors

Many modern buildings are dependent upon lifts and thus demand high standards of performance and reliability from the drive and control systems. The advent of microprocessor controls has meant greater flexibility and quicker response to changing traffic conditions since a wide variety of inputs, such as car positions and car loading, even system failures or the number of people waiting at each landing, can be scanned many times a second. This continuous updating is used to secure the optimum lift performance. Equally, lift-drive systems are benefiting from the use of electronic speed-control techniques which will enable the robust and simple a.c. motor to replace the d.c. motor with its higher maintenance costs. Bulky worm gearing has been used traditionally to reduce the rotational speed of the traction motor but helical gearing with its superior mechanical efficiency and compact dimensions is now being considered.

With the issue of the various parts of BS 5655 for lifts and BS 5656 for escalators, the UK lift industry is now closely aligned with European standards. Only small national differences remain. Although not mandatory, these British Standards define standards of safe and practicable transport for buildings.

High-rise buildings may call for special solutions to the transport needs. One approach is the provision of 'shuttle' lifts where there are common liftshafts shared by two cars. One car covers the zone from ground level to an interchange floor or 'sky lobby', rising nonstop. The second car covers the zone from the sky lobby to the top floor served. A variation is where the shuttle cars are built as double-deckers, serving two levels at a time. Here, of course, two sky lobbies are required.

Undoubtedly, the type of lift attracting the most interest at present is the 'wallclimber' and its close relative, the panorama lift. The wallclimber lifts move on guides attached to the exterior elevation of a building and generally are found only in congenial climates. Panorama lifts resemble more closely a conventional lift but with the car projected through the shaft wall opposite the lift entrance. In both cases there is an emphasis on concealing mechanism and providing the largest practicable area of glass in the car construction. The current popularity of the atrium has added further impetus to the use of such lifts.

A lift pit is required at the bottom of every lift well of depth determined by lift speed; no occupied space is permitted beneath unless special provisions are incorporated to strengthen the pit bottom and lift safety gear. Lift motor rooms should be restricted to lift machinery and associated equipment. The lift well enclosure, pit and motor room form part of the building construction and may require particular construction as a 'protected shaft' passing between fire compartments.

Escalators have a much greater carrying capacity, but can only be used between two floors. Their use is mainly in high-flow areas with a limited number of floors. Capacity is varied by width and speed and can exceed 10 000 persons per hour.

Passenger conveyors are used basically for horizontal movement but increasing use is being made of them on shallow inclines to replace pedestrian ramps. They are used in transport terminals and interchanges.

21.13 Energy

The ready availability of cheap energy and the technology to control the internal environment meant that, for a long time, energy aspects were not a primary consideration in the design of buildings. Generally more effort was put into saving initial cost than into saving energy. All this changed radically when energy costs escalated in the mid 1970s. Today, energy aspects are a fundamental consideration in the design of buildings.

In the UK, analysis showed that buildings use half the nation's energy and that potentially 30% of this could be saved. In new building the potential saving is even greater. For example, a study of the energy used in hospitals[9] showed that savings of 50% and more could be achieved without any major change in hospital standards or building techniques.

Given a reasonable payback period, investment to reduce energy is sound economics. The problem is deciding what is an appropriate payback period. Various energy accounting methods exist. One approach is to compare the primary energy saved with the added primary energy needed to effect the saving. Alternative methods use a traditional financial approach, but these were sensitive to future fuel prices, inflation and interest rates. In the end, economic analysis is seen as a tool to sharpen judgement, particularly when comparing options within the resources available for investment. Other things being equal, priority should be given to those options which would be more difficult to introduce when the building is in occupation. Figure 21.7 is an interesting way of illustrating the combined impact of an energy-saving measure on cost of construction and cost in use. The most attractive measures reduce both initial and running costs of the building as a whole.

Heat loss considerations are important components of building regulations in most countries. In some cases these rely on specific thermal properties for the building fabric. More advanced regulations call for examination of the thermal performance of the building as a whole, thus encouraging the innovative skills of architects and engineers.

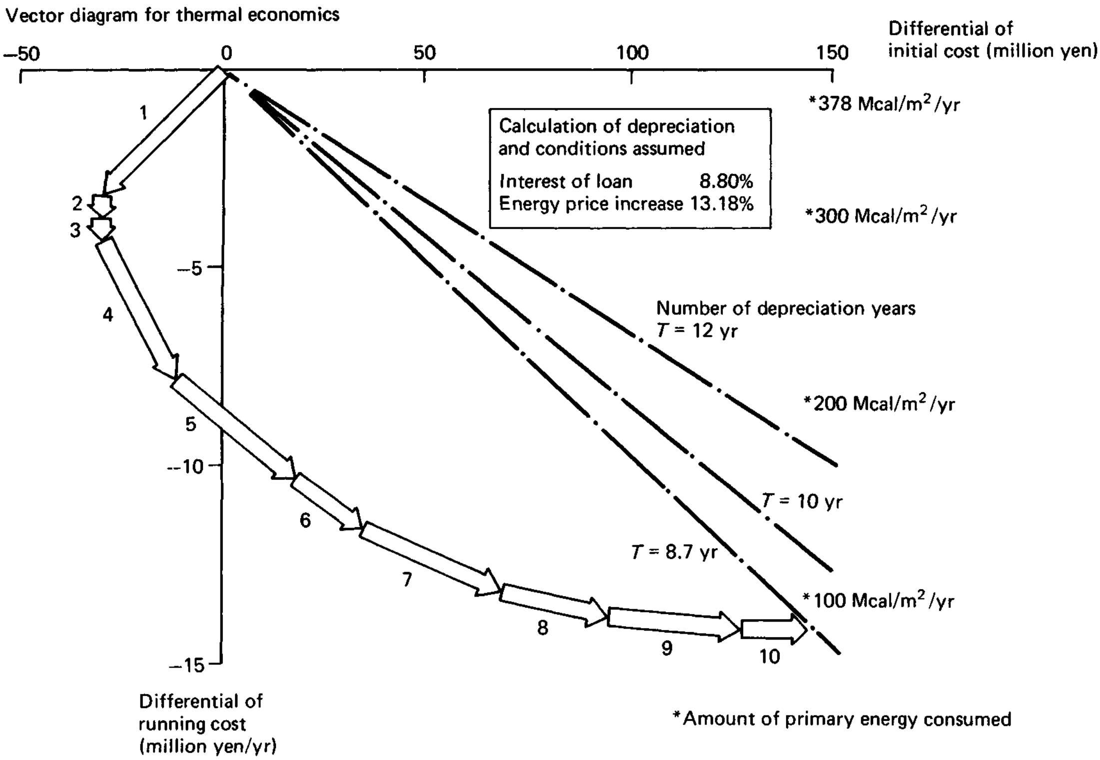
Existing buildings are being adapted to the new energy situation by energy conservation through insulation, the introduction of more sophisticated control systems or by the introduction of new plant.

In new construction, energy-conscious design is now the norm through building form and fabric and through the installations and controls provided. It is not enough simply to provide more thermal insulation; a whole set of measures is required to obtain the optimum solution.

The principal steps in producing an energy-efficient design are as follows:

(1) Computer analysis of local daily and seasonal weather conditions to optimize peak and long-term energy requirements.
(2) Analysis of building orientation, shape, height and construction to control heat gain or loss and the use of internal thermal capacity to reduce peak and total energy demands.
(3) Incorporation of heat conservation and recovery by transfer from points of surplus to points of need and reclamation of waste heat.
(4) Incorporating sophisticated control systems sensitive to variable building use and external weather conditions, to ensure that energy is injected only when needed.
(5) Where applicable using combined heat and power plants so that waste heat may be put to good use.

To save energy buildings are designed so that air-conditioning is unnecessary unless some special factor predominates such as wind, noise or fumes, or client requirement. Walls and roofs are used as the primary climatic modifiers with the environmental



(1) Architectural planning:
 (a) optimum building direction;
 (b) partial underground;
 (c) twin core;
 (d) reduction in floor height;
 (e) cubical structure;
 (f) 6 other methods.

(2) Reduction in power for plumbing facilities:
 (a) water-saving toilet;
 (b) local domestic hot water supply;
 (c) 7 other methods.

(3) Reduction in ventilation:
 (a) local ventilation;
 (b) double-staged use of conditioned air;
 (c) 2 other methods.

(4) Reduction in thermal load:
 (a) use of thermal wheel exchanger;
 (b) control on natural free cooling;
 (c) volume control on minimum outside air intake;
 (d) adoption of outside-air intake through underground pipes;
 (e) use of non-leaky damper;
 (f) 7 other methods.

(5) Insulation, shading device, and ventilation of building:
 (a) reduction in glazing areas;
 (b) use of insulated window shutters;
 (c) use of external louvre blinds;
 (d) adoption of double skin;
 (e) tilting outer glass of double skin;
 (f) 10 other methods.

(6) Reduction in power for fans and pumps:
 (a) adoption of VAV system;
 (b) adoption of large temperature differential system;
 (c) control on number of operating pumps;
 (d) 7 other methods.

(7) Reduction in lighting power:
 (a) task/ambient lighting;
 (b) dimming control on lighting in perimeter zone;
 (c) turning off light during lunch hour;
 (d) 8 other methods.

(8) Upgrade of efficiency:
 (a) adoption of heat reclaim system;
 (b) adoption of thermally-stratified heat storage tank;
 (c) optimal control on starting operation;
 (d) upgraded insulation around mechanical system;
 (e) 9 other methods.

(9) Active solar:
 (a) direct utilization of solar energy for air-conditioning and heating;
 (b) earth heat storage of solar energy;
 (c) 3 other methods.

(10) Reduction in electric power:
 (a) improvement of power factor;
 (b) control on number of operating transformers;
 (c) solar photovolatic power generation system without battery unit;
 (d) 5 other methods.

Figure 21.7 Economics of energy-saving devices in Ohbayashi's energy conservation building. (*Courtesy*: IABSE PERIODICA, IABSE, CH-8093, Zurich) engineering systems providing fine-tuning to match local need. Elevational treatment is directed to the beneficial control of solar gain, the windows being set back to provide shading from the high Sun in summer, but permitting warmth to enter from the low Sun in winter. Glass remains an attractive material but is now less extensively used and is double- or triple-glazed. Passive solar heating can be an integral part of the heating in major buildings. There is now less general lighting and more local or task lighting with automatic systems of control sensitive to external daylight conditions and programmed for planned internal use. There is greater emphasis on variable air systems which use less energy and are more tailored to provide local ventilation needs. Heat reclamation techniques are now widely applied, particularly those based on the use of heat pumps and exhaust-air-inlet heat exchange. Heat from lights and office equipment is reclaimed, and 'run-around' systems are used to transfer heat from the hot, to the cool, side of a building. Energy-consciousness has extended from individual buildings to groups of buildings incorporating complementary energy requirements.

The impact on the structure arises mainly from changes in building form which result from the above considerations, coupled with the possible use of the thermal inertia of the structure to stabilize temperatures or as a heat store. Figure 21.8 illustrates the range of energy-saving measures adopted for the Tokyo Electrical Power Company (TEPCO), Ohtsuka Branch, Tokyo, while Figure 21.9 compares the achieved savings against those estimated.

12.14 Building Regulations

Building control in England and Wales is governed by the Building Act 1984 and the Building Regulations 1985. Scotland and Northern Ireland have separate systems of control.

Building work is defined as: (1) the erection or extension of a building; (2) the material alteration of a building; (3) the provision, extension or material alteration of a controlled service or fitting; and (4) work required on a material change in use. Certain small buildings and extensions and buildings for certain purposes are exempt from the regulations.

The regulations are silent on when repair work becomes subject to control. However, there comes a point in some cases when so much has to be done to repair or replace that the local authority could reasonably require the regulations to be applied.

21.14.1 Procedures

The new regulations contain an important innovation in that the proposed building work may be supervised either by the local authority or by an 'approved inspector'. Under local authority supervision, a further choice is available of depositing either full plans or a 'building notice' which contains much less information.

Full plans may be accompanied by a certificate of compliance with regulation requirements relating to structural stability and/or energy conservation. Such certificates can be given only by an 'approved person' and must be accompanied by a declaration that an approved insurance scheme applies. The details of who will qualify as an 'approved person' have yet to be resolved but it is expected that the relevant professional institutions will become the approving bodies for individuals wishing to undertake this work. When full plans are deposited, the local authority must pass or reject them within 5 weeks, or 2 months if the developer agrees. The 'building notice' option is simpler but is not applicable to shops or offices or any building work subject to the requirement for means of escape in the event of fire. The 'building notice' contains a short description of the work, a block plan and proposals for drainage. The local authority does not issue any approval but may wish to check work in progress.

The Building (Approved Inspectors) Regulations 1985 set out detailed procedures for private certification by approved inspectors as an alternative to local authority supervision. Under private certification, the developer and approved inspector jointly serve an 'initial notice' on the local authority. This describes the proposed works and can be rejected by the local authority only on certain prescribed grounds specified in the Approved Inspectors Regulations. The 'initial notice' must be accompanied by a declaration that an approved scheme of insurance applies to the work. If the local authority does not reject the notice within 10 days it is presumed to have accepted it without conditions. On acceptance, the local authority's powers to enforce the regulations are suspended and the approved inspector becomes responsible for inspecting the plans and building work and, on completion, issuing a final certificate of compliance.

21.14.2 Appeals procedure

When building work is alleged to contravene the regulations, the local authority can require its removal or alteration by serving a Section 36 notice on the building owner who has a right of appeal to a magistrates' court. The building owner may elect to obtain an expert's report from a 'suitably qualified person'. If accepted by the local authority, the Section 36 notice would be withdrawn and the building owner reimbursed his costs. If rejected, the report may be produced to the court and if the appeal is successful the building owner will recover his costs.

21.14.3 Approved Documents and mandatory requirements

The new regulations are much shorter and simpler since the technical details are now contained in a set of nonstatutory Approved Documents which give practical guidance on ways of meeting the regulation requirements. The obligation is to meet the requirement. The Approved Document may be used in whole or in part, or some other arrangement may be adopted provided the basic requirement is met. Use of the Approved Document would tend to be regarded as evidence of compliance with the regulations. If some other method is used, the reverse would apply and demonstration of compliance will be required.

Means of escape from fire, however, are covered by mandatory requirements although the local authority may agree to relax them in particular circumstances. The requirements are covered by the *Rules*.[10] A fire certificate is required for certain uses of building. Such buildings, in addition to means of escape, must include provision for fire alarms and fire-fighting equipment. Currently included are certain factories, offices, shops, railway premises, hotels and boarding houses. The Health and Safety Executive have similar responsibilities for special risk premises such as nuclear installations, buildings at the surfaces of mines and large chemical or petrochemical plants where the processes carried out affect general fire precautions.

Heat losses from certain classes of building are subject to minimum mandatory requirements. Four procedures for meeting the requirements are allowed:

(1) Specified insulation thickness.
(2) Specified U values to take account of full construction features.
(3) Calculated trade-off within glazed and solid areas and in the case of dwellings between solid areas and windows.
(4) Calculated energy use in buildings other than dwellings, allowing heat gains to be set off against heat losses.

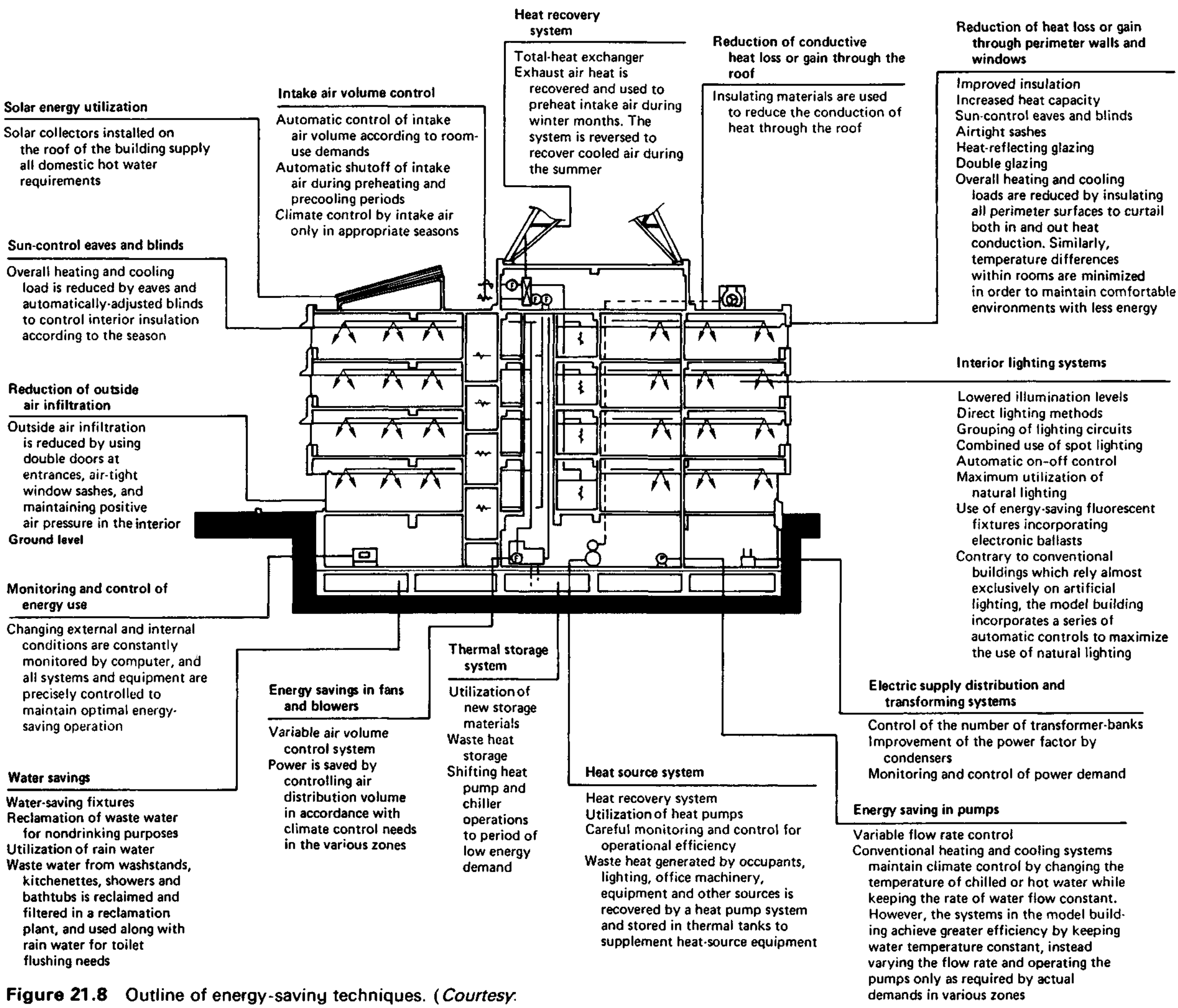


Figure 21.8 Outline of energy-saving techniques. (*Courtesy*: IABSE PERIODICA, IABSE, CH-8093 Zurich)

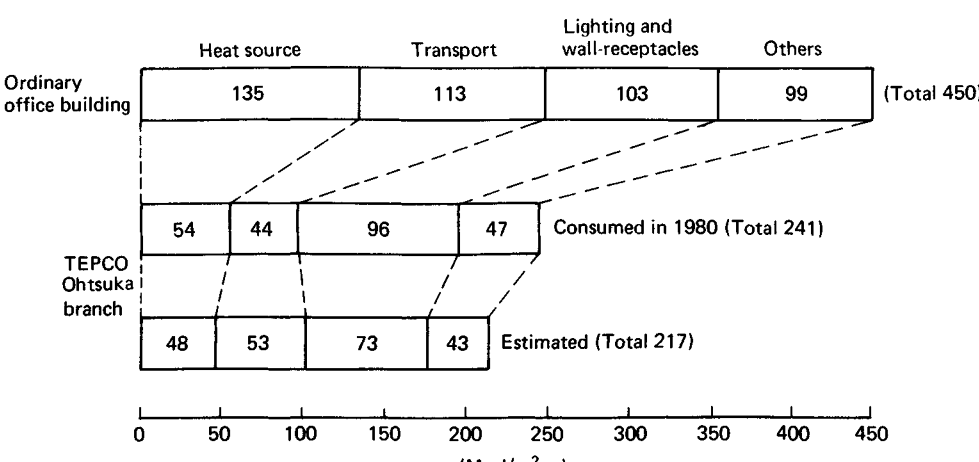


Figure 21.9 Energy consumption in the Tokyo Electrical Power Co., Ohtsuka Branch, Tokyo. (*Courtesy* IABSE PERIODICA, IABSE, CH-8093 Zurich)

21.14.4 Structure

There are three requirements: A1 concerned with loading; A2 with ground movement; and A3 with disproportionate collapse.

A1: Loading.

(1) The building shall be so constructed that the combined dead, imposed and wind loads are sustained and transmitted to the ground:
  (a) safely; and
  (b) without causing such deflection or deformation of any part of the building, or such movement of the ground, as will impair the stability of any part of another building.
(2) In assessing whether a building complies with (1) regard shall be had to the imposed and wind loads to which it is likely to be subjected in the ordinary course of its use for the purpose for which it is intended.

A2: Ground movement

The building shall be so constructed that movements of the subsoil caused by swelling, shrinkage or freezing will not impair the stability of any part of the building.

A3: Disproportionate collapse.

The building shall be so constructed that in the event of an accident the structure will not be damaged to an extent disproportionate to the cause of the damage. This requirement applies only to:

(1) A building having five or more storeys (each basement level being counted as one storey).
(2) A public building the structure of which incorporates a clear span exceeding 9 m between supports.

The Approved Document's guidance is in three sections: (1) Section 1 gives sizes for certain structural elements for houses and other small buildings, covering walls, floors, roofs, chimneys and strip foundations: (2) Section 2 lists codes and standards which are appropriate for all buildings and which may be used to satisfy the requirements of regulations A1 and A2; and (3) the final part deals with disproportionate collapse and quotes codes and standards which may be used in designing to meet the requirement of A3 for a building having five or more storeys, provided the recommendations on ties, and the effect of misuse or accident, are followed. Structural work of concrete is covered by BS 8110:1985, Parts 1 and 2. For steel it is BS 5950:1985, Part 1, and the accident loading referred to in clause 2.4.5.5 should be chosen having particular regard to the importance of the key element and the consequences of its failure; the key element should always be capable of withstanding a load of at least 34 kN/m^2 applied from any direction. British Standard 5628:1978 is quoted as the relevant standard for structural work of masonry.

By way of 'additional information', the Approved Document states that the structural failure of any member not designed as a protected key element or member, in any one storey, should not result in failure of the structure beyond the immediately adjacent storeys or beyond an area within those storeys of: (1) 70 m^2; or (2) 15% of the area of the storey, whichever is less.

Protected key elements or members are single structural elements on which large parts of the structure rely, i.e. supporting a floor or roof area of more than 70 m^2 or 15% of the area of the storey, whichever is less, and their design should take their importance into account. The least loadings they have to withstand are described in the codes and standards listed.

The Approved Document offers no specific guidance on wide-span public buildings of less than five storeys. Clearly, such buildings do require special care in design and construction to reduce risks to human safety and the designer should consider carefully structural behaviour and consequence in the event of an unforeseen hazard or accident.

Structural matters are referred to additionally in other Approved Documents, e.g. in AD7 ('Materials and workmanship') and AD B2/3/4 ('Fire spread').

21.14.5 Fire spread

The requirements cover internal fire spread (surfaces) in B2, internal fire spread (structure) in B3, and external fire spread in B4. Approved Document B/2/3/4 describes how the requirements can be met by controlling aspects of the construction and materials used in the building, e.g. fire resistance and surface spread of flame characteristics.

21.14.5.1 Internal fire spread (surfaces)

The spread of fire over a surface is restricted by provisions for the surface material to have low rates of surface spread of flame, and in some cases to restrict the rate of heat produced.

The provisions are made for walls and ceilings and vary according to the use of the building or compartment, the location of the room or space concerned, and in some cases whether the surface is a wall or ceiling.

21.14.5.2 Internal fire spread (structure)

Premature failure of the structure is prevented and the spread of fire within a building restricted by specifying minimum periods of fire resistance for the elements of structure. Fire resistance includes requirements for one or more of the following:

(1) Resistance to collapse (stability – applicable to load-bearing elements).
(2) Resistance to fire penetration (integrity – applicable to fire-separating elements).
(3) Resistance to the transfer of excess heat (insulation – applicable to fire-separating elements).

Structural stability under fire conditions necessitates consideration of the following aspects of design:

(1) The coincident effects of dead, imposed and wind loads.
(2) The effect of heat on the structural elements.
(3) The provision of structural integrity to resist the effect of fire-induced movements.

The minimum period of fire resistance is set out relevant to the building use and depends upon the height of the building and on the size of the building or compartment. In basements, the provisions are generally more onerous in view of the greater difficulty of dealing with a fire.

21.14.5.3 Compartmentation

The spread of fire can also be restricted by subdividing the building into compartments of restricted floor area and cubic capacity, by means of compartment walls and compartment floors. The degree of subdivision depends upon the use of the building and in some cases its height. In single-storey buildings, the life risk from fire involving the whole building is generally less than that for a multistorey building. Thus, compartmentation in single-storey buildings applies only to those with a significant sleeping risk. There are, however, provisions for compartmentation of most multistorey buildings. Other forms of compartmentation apply between adjoining buildings and between a small garage and a house to which it is attached or forms part. As the minimum period of fire resistance increases with size of building or compartment it may be advantageous to provide compartments of smaller size than specified. Similarly, where a building is used for more than one purpose, it may be desirable to separate by compartmentation the use which has the more onerous fire-resistance requirement.

In order for compartments to be effective, junctions between the different elements enclosing the compartment must be protected. Similarly, any openings connecting one compartment with another should not present a weakness. Any spaces connecting the compartments need to be protected to restrict fire spread. These are termed 'protected shafts'.

21.14.5.4 Concealed spaces and fire stopping

Hidden voids provide a ready route for smoke and flame spread, e.g. above a suspended ceiling or in a roof space. As any spread is concealed, it presents a greater danger than would a more obvious fire weakness in the construction. Provision is therefore made to restrict the hidden spread of fire in concealed spaces by closing the edges of cavities, interrupting cavities which could form a pathway around a fire barrier and subdividing extensive cavities. Similarly, pipe or cable penetrations through a fire protection building element should be sealed appropriately.

21.14.5.5 External fire spread

The construction of external walls and the separation between buildings to prevent external fire spread are closely related, and many of the provisions specified are related to the distance of the wall from the boundary.

Whether a fire will spread across an open space between buildings, and the consequences if it does depend on: (1) the size of the fire; (2) the risk it presents to people in the other buildings; (3) the distance between the buildings; and (4) the fire protection given by their facing sides.

There are provisions limiting the extent of openings in external walls to reduce the risk of fire spread by radiation. Less onerous provisions for separation apply where compartmentation exists. Provisions for roof construction and coverings vary with the size and use of the building and the proximity of the roof to the site boundary.

21.14.5.6 Structural integrity of the building as a whole

The required level of performance will be met if the guidance in AD A1 and AD A2 and the *Guidelines*[11] are followed.

21.14.5.7 Varying the provisions

Where the fire provisions are thought to be unduly restrictive, the fire safety of the building as a whole may be considered and guidance is given about varying the provisions. Factors to be taken into account include: (1) whether the building is new or existing; (2) the construction and fire properties of the materials; (3) fire hazard and fire load; (4) space separation from boundaries and other buildings; (5) means of escape; (6) ease of access for fire-fighting; and (7) the provision of any compensatory features such as sprinklers or other automatic fire-detection systems. For example, the maximum compartment size in shops may be doubled when a sprinkler installation is fitted.

Shopping centres pose special problems and alternative measures and additional compensatory features to those set out in the Approved Document may be appropriate. These include:

(1) Unified ownership and control of fire prevention measures.
(2) Adequate means of escape and smoke control.
(3) Sprinkler protection of all areas of fire load.
(4) Automatic fire alarm system.
(5) Good access for fire fighting.
(6) Materials of limited combustibility and fire spread.
(7) Generally 2 h fire resistance of structure (4 h in basement).
(8) Floors generally of compartment construction.
(9) Walls between shop units of compartment construction.
(10) Compartmentation between large shop units (over 3700 m^2) and a mall, and between opposing shop units (over 2000 m^2) and a mall. Fire shutters may be used for this purpose.

The above items are not exhaustive but draw attention to the need to consider proposals as a comprehensive fire-safety package.

21.14.6 Other Approved Documents

Other Approved Documents are as follows.

C1/2/3	'Site preparation and contaminants'.
C4/	'Resistance to weather and ground moisture'.
D1–	'Cavity insulation'.
E1/2/3	'Airborne and impact sound'.
F1	'Means of ventilation'.
F2	'Condensation'.
H1/2/3/4	'Drainage and waste disposal'.
J1/2/3	'Heat-producing appliances'.
K1/2/3	'Stairways, ramps and guards'.
L2/3/4/5	'Conservation of fuel and power'.
Part 7	'Materials and workmanship'.

21.15 Building security and control

With the increased size and complexity of buildings, systems designed to monitor and control the mechanical and electrical installation, fire protection and escape, burglary, assault and emergency communication have become very important.

In tall buildings, and other major complexes, the most important security requirement is the fire-safety system. In addition to the structural precautions of fire protection and compartmentation, special systems are required to monitor and control: (1) fire detection and suppression; (2) movement and protection of people; (3) smoke control including pressurization and barriers; (4) safe places of refuge; and (5) emergency arrangements and communication.

In major buildings, these arrangements are integrated with those required to monitor and control the heating, ventilation and air-conditioning systems and other aspects of security within a single electronic system. The computer monitors all significant local conditions and appropriate action is taken. Such measures for security and control could, for example, bring in the use of: (1) heating, ventilation and air-conditioning plant and equipment to suit internal and external conditions or programmed requirements; (2) data collection for maintenance and resource management, particularly energy use and analysis, programmed responses to suit anticipated emergencies, e.g. defining smoke-free zones and escape routes in the event of fire; and (3) security interlocks, surveillance and access control.

The terms energy management system (EMS), building automation system (BAS) and building management system (BMS) are used to describe these systems. The EMS controls the environmental functions, the BAS the technical automatic controls, and the BMS includes such matters as status reports on environmental conditions, lifts and the location of people for security purposes. All these aspects influence and are influenced by the overall building designs.

The problems of security are by no means limited to major building complexes. Studies of housing estates with very different crime rates but with comparable density, size and tenant income, demonstrate clear relationships with specific building design characteristics, the nature of the surrounding areas and whether these are under the ready surveillance of the inhabitants. This has given rise to the design concept of 'defensible space' whereby tenants can act as their own 'policemen' simply because they identify with a particular space and easily monitor what is going on.

21.16 Materials

The principal materials used in buildings are concrete, steel, brick and masonry, and timber: each has its own developing technology. Other materials include aluminium, various alloys, glass, plastics and rubber. When used structurally, the essential properties concern strength, rigidity, durability and fire resistance. Relevant general properties concern hardness, thermal characteristics of insulation and expansion, weight, uniformity, appearance and workability. All these may be affected by changing temperature, humidity and weather. The choice of material for a particular building element will be determined by suitability for the intended purpose, cost and availability and compatibility with other materials.

21.16.1 Concrete

As a building and structural material, concrete is durable and relatively impermeable. It is readily available, and with the use of different cements, aggregates, forms and surface treatments it can provide a wide range of strengths, densities and finishes. Its mass automatically provides good airborne sound insulation and high thermal capacity, and with appropriate constituents it is resistant to chemical attack.

Lightweight concrete, in both structural and nonstructural elements, is used when weight is at a premium or when its better fire and thermal resistance is required. Precast applications include building blocks, fire casings, and floor, roof and cladding units. It is used *in situ* for floor slabs, screeds and generally for reinforced or prestressed structures. Its reduced stiffness should be allowed for in design.

Specially dense concrete is used as kentledge or for radiation shielding.

Normal-weight concrete is used unreinforced in mass foundations, gravity retaining walls, screeds and blinding, and precast as paving flags, kerbs and building blocks. For structural work it is normally reinforced with bars of high-tensile or mild steel, fibre-reinforced (with steel, glass or plastic), or prestressed. Fibre reinforcement gives improved impact resistance and can allow thinner sections; current applications are precast cladding, pipes and pile shells. Prestressing is used in precast floor units and in long-span beams as a means of controlling deflection or reducing member depth, and has special application in hanging and transfer structures.

Concrete cladding to buildings is generally precast, and may be nonstructural or structural. Panels may incorporate windows or doors, and may be of sandwich construction with a layer of thermal insulation. Many surface finishes are available including exposed aggregate, mosaic or tiles, and profiles may be plain or sculptured. Great care is required in detailing the profile and surface finish to control staining and to ensure satisfactory weathering.

The choice between precast and *in situ* construction will depend on cost, speed, access and availability of labour. Precasting minimizes the on-site work, but requires good accuracy and suitable lifting equipment. *In situ* work requires more on-site labour, but can be speeded by using prefabricated formwork systems and concrete pumps.

Where reinforced concrete is exposed to the weather, to water or to the ground, durability must be considered. Protecting the reinforcement from corrosion requires an adequate cover of densely compacted concrete with sufficient cement in the mix to maintain the steel in an alkaline environment. Where this cannot be achieved, other measures such as coating the concrete surface, or using galvanized or stainless steel reinforcement, may be needed. Chlorides in concrete must be restricted as they promote corrosion of the reinforcement; they may arise from the use of marine aggregates or from the unwise use of certain admixtures.

At the time of writing (1986), several cases of alkali–silica reaction (ASR) are being reported. This is a reaction which occurs between the alkali in the cement and certain types of aggregate, leading to expansion and loss of strength. It appears to be restricted to high-cement mixes, but there are as yet no reliable tests for aggregates. Although this problem is rare, advice on the latest knowledge should be sought before committing the mix design on large or important concrete elements, particularly if the concrete will be exposed to weather or water.

Glass-reinforced cement (GRC) is a relatively new material which utilizes alkali-resistant glasses developed in the 1970s. A typical mix will contain cement, sand, glass fibre and water, with cement:sand-ratio of 2:1, a water:cement ratio of 0.3:1, and 5% by weight of glass fibre. Careful detailing is needed to accommodate initial drying shrinkage, which is higher than for normal concrete. When young, GRC is a tough material which can deform without breaking. As it ages and weathers, the toughness reduces to a stable level after 2 to 5 yr. Several methods of casting GRC are used, but the normal method is to spray the constituents on a mould to form a layer 6 to 10 mm thick. This is demoulded and carefully cured. Simple shapes can be formed by folding flat sheets before they harden; complex shapes require purpose-made moulds. Glass-reinforced cement shares many applications with glass-reinforced plastic (GRP) but the GRC is heavier, stiffer and has better fire resistance. The main applications are drainage pipes for use underground, permanent formwork for concrete, roof tiles, street furniture and cladding panels. Cladding panels can be formed as a single skin of GRC, or as a sandwich of two skins with insulation between.

21.16.1.1 Fire protection

Fire protection of concrete structures is based upon the provision of adequate thickness of construction and adequate cover to the reinforcement or prestressing tendons. Lightweight concrete has an improved fire resistance because of its better thermal resistance and with some artificial lightweight aggregates is virtually free of spalling during a fire.

21.16.2 Steel

In one form or another steel is extensively used in practically all buildings. As a structural element it is available in a wide range of section and composition to suit the particular requirements of stress, deflection, corrosion or jointing technique: (1) it is of high strength; (2) in itself, occupies little space and is prefabricated for easy and rapid erection on site; and (3) it readily lends itself to extension or alteration. Sections can be cold-curved to form arches or rings. It has two disadvantages: (1) fire; and (2) corrosion. Several methods exist to overcome its fire sensitivity – various coatings are applied to resist corrosion and some steels (Corten) can be left exposed without treatment. Castellated beams are useful to reduce deflection and to provide holes for the passage of services. Hollow sections, of various wall thicknesses, are used in tubular structures, columns, trusses, space frames. Combined sections are commonly used and composite action, via shear connectors, with concrete floor construction can be advantageous. Sheet steel applications include roof and wall cladding, ducting and in floor slab construction, acting compositely with *in situ* concrete topping.

21.16.2.1 Fire protection

Fire protection of structural steel has moved towards the use of boarded lightweight encasements, sprayed surface materials and intumescent coatings. *In situ* concrete casing tends to slow down the construction process, although this can be overcome by precasting the concrete surround leaving only the junctions to be dealt with on site.

Boarded systems. A variety of boards are available in thicknesses from 6 to 80 mm giving fire periods up to 4 h. They are generally manufactured from vermiculite or mica using cement and/or silicate binders and are particularly suitable for column protection and for 'all dry' construction.

Spray systems. A wide variety of lightweight materials are available generally based on vermiculite plus a binder (often cement) or mineral fibres. It is generally applied direct to the steel surface, although in some situations it is applied to an expanded steel lathing to form a hollow box protection. Fire periods of up to 4 h can be achieved; mesh reinforcement may be required for the longer periods.

Intumescent coatings. These thin film coatings or mastics swell under the influence of heat and flames producing an insulating layer 50 times thicker than the original. Fire periods up to 2 h can be achieved. A variety of products are available; not all are suitable for damp environments and only a limited number are approved for the fire protection of columns.

Fire-protection thickness. The thickness required for most steel members given in manufacturers' literature has been given by the Association of Fire Protection Contractors and Constrado.[12] This generally relates the thickness of protection to both the fire resistance period and the H_p/A value of the steel section, where H_p is the perimeter exposed to the fire and A the cross-sectional area. The lower this value, the lower the heating rate.

Structural fire engineering. Whilst it is convenient in most situations to adopt the regulatory approach to fire resistance, in special cases a more fundamental approach may be appropriate and acceptable to the authorities concerned. Fire engineering is directed towards a more accurate assessment of the fire protection required by considering the significance and severity of a real fire in the building and the response of the structure as a totality to it. The process involves the consideration of:

(1) The heating rate and maximum temperature in the compartment – related to the fire load, ventilation and insulation of lining materials.
(2) The temperature rise in the structural member – related to location, weight per metre and perimeter of the structural member exposed along with any fire protection applied and, in the case of beams, their height above the fire.
(3) The stability of the structure – related to the load applied, the grade of steel and the effects of any composite action, restraint, continuity and movement.

Such considerations are particularly applicable to buildings where the function and fire load are unlikely to change. Examples are: schools, offices, hospitals, sports stadia, public assembly buildings and transport terminals. The concept is most cost effective when the analysis leads to the acceptance of the bare structure without fire protection.

The fire load is a measure of the amount of material available to burn and is calculated as weight × calorific value of the contents and building materials used in each compartment. The resulting figure is often converted to the equivalent quantity of wood having the same total heat content. In calculating the heating rate inside the compartment, the available ventilation and insulating properties of the compartment envelope are taken into account. Well-ventilated fires are shorter and hotter and an insulated envelope retains the heat. On the basis of this information, it is possible to predict the likely heating cycle within the compartment which can then be related to the heating strength/deformation curves for the steel.

21.16.3 Brick and masonry

Brick and masonry have the advantages of a long heritage of experience and simple construction based on traditional skills; building plant costs are low, but the labour content is high and not always in sufficient supply. The common materials are bricks and concrete blocks of various types, finishes and strength, and natural or reconstituted stone. The main applications occur in load-bearing walls and piers, particularly in low- and medium-rise buildings, and as cladding. Internally they are used as the inner skin of cavity construction or as partitions and, within limits, may be used to brace framed construction. Reinforcement can be added in the horizontal joints to produce beam action or vertically, in piers, for tying purposes.

In general, the massive nature of these materials provides good sound and thermal insulation together with good compressive strength and durability. Movement due to shrinkage in the case of concrete bricks/blocks and expansion in the case of fired clay bricks, temperature and moisture change, and chemical action must be allowed for and provision made in design against progressive collapse. The design of brickwork has become more sophisticated both structurally and architecturally in keeping with the swing back to more traditional forms of construction.

21.16.4 Timber

The main advantages are: (1) it is readily available in a wide variety of types and section; (2) it is light in weight; and (3) it is easily worked, employing traditional skills. Typical applications are in floors, roofs, framing to light buildings, cladding, wall and ceiling construction and in surface finishings. It is often used in temporary buildings and for temporary works and formwork.

Size, form and consistency limitations have been reduced by the introduction of glued laminates and special fastening systems have made larger-scale structures possible. Combustibility, rot and insect infestation can be retarded by chemical impregnation while treatment with steam or ammonia gas introduces flexibility. Temperature and moisture movements remain problems. Fire resistance of exposed timber members can be assessed from the rate of charring. This varies with different kinds of timber but is generally about 0.6 mm/min on each exposed face.

21.17 Walls, roofs and finishes

21.17.1 External finishes, materials and weathering

The design of the external fabric requires a knowledge of the behaviour of the materials and elements of construction and includes consideration of weathering and water-shedding characteristics. External materials must be durable under the influence of climatic extremes and the local environmental conditions including wind velocity and prevailing direction, and whether coastal, urban, industrial or rural. The cost of maintenance, and accessibility for maintenance are also important considerations. The major functional requirements for the external wall include heat and sound insulation and damp-proofing.

Design elements range from screws to complete assemblies. Deterioration due to weathering may be aesthetic or functional and may be visible or concealed, and design details should be such as to avoid structural deterioration, especially in concealed situations, and should be designed bearing the possible colour change or staining effects of weathering in mind.

The weathering characteristics of the main materials used externally are described in detail elsewhere.[13]

Concrete finishes depend upon the moulds used, the material properties and surface treatment. Blemishes such as surface airholes cannot always be avoided and untreated smooth surfaces generally weather badly, although when, with hard concretes of plain or white mixes, the surface laitance is ground off and sealed, good results can be obtained. Patterning and texture can provide interesting finishes, as with rough board markings, or ribbed surfaces which may be hammered or tooled in various ways. Deep patterning can also be very effective. Exposed aggregate finishes are available from a wide variety of processes and aggregates, and these generally weather well.

Brickwork is traditionally used as external walling and a variety of colours and textures are available in facing bricks. Attention must be given to weathering performance, porosity and freezing effects, efflorescence, sulphate attack, etc. When associated problems are recognized, solutions are available. Details must be designed to accommodate relative movement of the brickwork and other building elements.

Timber and timber products may be used as cladding, but colour changes usually occur on exposure to light and water, which can also lead to damage such as splitting, warping and dirt penetration. Various preservation treatments are available, including the use of varnishes, synthetic resinous clear finishes, opaque paints and applied film overlays.

With the use of metals externally, their particular properties as to electrochemical corrosion in relation to weathering and the problem of bimetallic electrolytic action need to be understood. Aluminium, bronze and copper weather well and are used in roofing, cladding, window framing and flashing applications. Lead may be used in sheet form for special roof covering and more frequently for flashings. Zinc provides useful coatings and flashings. Outstanding durability can be obtained with the use of stainless steel, and low-alloy steels of good weathering properties are available.

Curtain walling can provide an economic form of cladding and glazing, with advantages of lightness, thinness (as affecting usable floor area), flexibility of fenestration and speed of erection without external scaffolding. It is provided in two main types: (1) unit assemblies in which self-supporting panels are prefabricated including glazing and solid infilling with interlock or lap joints; and (2) part assemblies in which frame members are erected and the glass and solid sheets added. Weather resistance and connection to the structure must be adequate to meet high local wind pressures and conditions of driving rain. Sealing methods include the use of mastics, gaskets, cover tapes and spring strips. Thermal movements may be very large since the panels have low heat capacity and respond rapidly to changes in temperature, giving rise to differential movements between one part of the curtain walling and another, and between the curtain walling and the structure.

Different types of glass are available including: (1) clear, coloured, or opaque; (2) heat absorbing, filtering or reflecting; (3) toughened, single or bonded into insulated sandwich construction.

A wide range of plastics for external use is available, with different resistances to ultra-violet light, temperature, water, oxygen, micro-organisms, atmospheric pollution and loading. These include PVC, used for rainwater goods, glassfibre-reinforced polyester resins forming sheet or shell products, polymethyl methacrylate providing transparent sheets of high strength and durability and the phenolic and amino resins for laminated sheets. Polymer films may be applied to other materials such as boards or metals to improve durability.

21.17.2 Floor, ceiling and wall finishes (internal)

Such finishes may be integral with the structure or applied. Type of usage, cost, chemical resistance, aesthetic requirements, fire resistance or 'spread of flame' requirements, maintenance, are some of the factors influencing selection.

Floor finishes integral with concrete rely on good workmanship: power-float finish, use of hardeners, dust inhibitors, waterproofers, or the application of granolithic concrete finishes to 'green' concrete. Timber and metal decking can also be in the 'integral' category. Applied floor finishes can vary from simple sheet or tile materials stuck (or laid loose in some cases) to the structural slab, with or without levelling screeds. Damp-proof membranes or vapour barriers may be required for slabs on the ground, depending on the type of finish to be applied and/or the groundwater conditions. Screeds may need to be of adequate thickness to allow for the running of service conduits, or thickened, reinforced and isolated by insulation in the case of floors to be heated or used for impact sound insulation. Raised floors to accommodate electrical or other services have become increasingly popular in modern office buildings.

Integral wall finishes result from the use of controlled shuttering on concrete work, fair-faced brick or blockwork or self-finished plane or profiled sheet materials. Applied wall finishes can be basically divided into wet and dry applications, plastering – by hand or spray – being typical of the former, and dry-lining such as plasterboards and proprietary insulation boards, acoustic finishes, being among the final finishes that may be required.

Integral ceiling finishes result from the use of untreated structural soffits such as concrete, metal or timber. Applied finishes may be divided into direct and suspended, the former, as indicated, being the application of wet or dry 'lining' or finishing direct to the structural soffit, such as paint, plaster, sprayed finishes, plasterboards, acoustic insulation boards or tiles. Suspended ceilings can be used to conceal structural members, to provide space for services, to reduce room heights for functional or aesthetic reasons, to provide a grid for flexible layout of partitioning. Such ceilings may be partly or wholly demountable for access to services or may be 'monolithic', e.g. plaster on expanded metal.

Building regulations must be referred to when considering internal finishes, requirements for resistance to fire being particularly stringent in areas such as staircases and circulation spaces, but also applicable to other areas and varying according to building use, area and volume.

21.17.3 Roofs

Roofs must keep out the weather, be durable and structurally stable, provide heat insulation in most cases and in certain others provide light and ventilation. The choice of roof structure will generally be determined by the general form of the building and the activities for which it is designed. Unlike floors, there is not usually any restriction on the depth of a roof and this gives a wide flexibility for economical and appropriate solutions. Sometimes the roof structure will be outside the main building.

A roof must carry its own weight together with imposed loads of roof finish and usually insulation, snow, the effects of wind, normal maintenance and often plant. It must resist excessive deflection or distortion which, though not leading to collapse, may damage decorations and services and if visible lead to lack of confidence and anxiety for the occupants. In accommodation for sedentary work or living, heat insulation, lighting, ventilation and sound insulation are important.

In general, the spacing of supports should be as close as possible consistent with present or possible future use.

Short-span roofs below 7.6 m are generally used for houses, blocks of flats, many multistorey buildings and some warehouses. On houses, roofs are often traditional in design. Sheet materials allow a lower pitch but the uplift effect of wind is important. Flats and multistorey buildings are normally roofed with a concrete slab similar to the floor construction.

Medium-span roofs 7.6 to 24 m are generally used for industrial buildings, warehouses, transit buildings, etc. Here, intermediate supports are often a nuisance. Appropriate systems are precast or prestressed concrete beams or steel trusses, lattice girders and portal frames.

Long-span roofs over 24 m are for exhibition halls, industrial buildings, leisure buildings, sports stadia and transport buildings. Many of these buildings require roofs which only keep the elements off the occupants. Systems would include steel lattice girders, space frames, roofs supported by suspended cables, prestressed concrete, arched construction, concrete folded plates and hyperbolic paraboloids.

Roof coverings include slates and tiles for houses, sheet materials flat or profiled, asphalt, felt, new materials based on synthetic rubber, plastics, sprayed-on materials and glass. It must always be remembered that provision must be made for roof drainage.

Fire spread is important in relation to roof coverings and is covered by BS 476:1975, Part 3.

Thermal insulation is often required to conserve heat in the buildings and is covered by the Building Regulations, but it is also important to reduce solar gain and avoid excessive expansion in the structure of the roof, which sometimes distorts the structural frame and outside walls. A reflective external finish to the roof also assists.

Condensation is a serious problem in roofs. Where thermal insulation is provided below the roof deck at ceiling level, cross-ventilation should be provided above the insulation. In some situations, a vapour check is necessary on the warm side of the insulation (the face nearest the inside of the building). Condensation is a subject on which a great deal of research has been carried out in recent years and deserves careful study.

21.17.4 Partitions

Partitions divide large areas into individual spaces for specific purposes such as stores, offices, etc. and separate circulation from working or living areas. The type of partition is determined by requirements of acoustic or thermal insulation, security, privacy, fire resistance and flexibility of planning. When the partitions are structural, brick or blockwork or concrete are commonly used; however, partitioning is generally kept separate from the structures.

Commonly used partitions, in increasing weight, are: (1) light framing with infilling of glass or building board; (2) plasterboard dry partition panels; (3) woodwool and compressed straw building slabs; (4) sandwich composite panels; (5) precast autoclaved concrete panels; (6) light to dense blockwork; and (7) brickwork and concrete.

21.18 Interior design and space planning

Space planning and interior design are the link between the design of the building itself and its eventual use by the occupants. In new building, space planning features at the initial briefing stages, and later in the completion and fitting out of the building for use. During the life of the building many changes are likely to occur in the utilization of the space provided and the building design must allow for this. A structural engineering input is important not only in new building design but also in advising on space utilization in existing construction.

In achieving an efficient, flexible and visually attractive environment the space planner has to balance the client's needs against the restrictions imposed by the nature of the building and the requirements of the statutory bodies. Structural aspects can have a profound effect on the success of the design and the following factors should be considered:

(1) Structural grid related to floor size and shape and intended use.
(2) Actual location of columns and beams.
(3) Form of the structure and its integration into the building fabric including services.
(4) Floor to ceiling heights.
(5) Ability to make satisfactory fixings to the structure.
(6) Floor loadings.

Modular co-ordination in building has not been completely successful. Planning grids have never entirely resolved the conflict between, for example, the sizes of workstations and cellular enclosures on the one hand and the sizes of building boards, ceiling tiles and partitioning systems on the other. Tartan grids are more economic for the internal fitting-out process as there is less need to cut partitions for ceiling tiles. Open-plan arrangements within most buildings do not produce much difficulty but modular co-ordination can become a problem in cellular office accommodation. Offices, for instance, are nearly always 10, 15 or 20 m^2 in area according to the status of the occupier. Building components are 1200, 900 or 300 mm. The most widely used structural grids are 6.0 and 7.2 m; 6.0 m can be subdivided to provide 4×1.5 m or 5×1.2 m window bays while 7.2-m grids give rise only to 1.2 m window modules. Although 1.2 m is very suitable for building boards and partitions it does not lend itself to the provision of individual offices – two window bays gives an office only 2.44 m wide or less. The 1.5 m module is superior in this respect and can accommodate smaller components such as ceiling tiles of 300 mm width; larger components of 1.2 m or 900 mm cannot be installed economically. Perimeter details can be critical: a continuous flush surface allows easy partition connection, avoids cutting around spandrel profiles and service runs, which is both costly and unsightly, and maintains continuity of acoustic insulation. It is more expensive to produce a flush window wall and it is not attempted in speculative buildings. The form of the building often dictates the space-planning principles. Few buildings today are of the traditional narrow form providing two rows of rooms divided by a corridor; most are wider and the most effective use of the space uses a mixture of cellular and open-plan accommodation.

For reasons of cost, buildings often have restricted floor-to-ceiling heights on, or slightly above, the statutory minimum. Alterations to the air-conditioning system are frequently necessary to service individual rooms or to deal with the 'wild heat' produced by the new technologies. Such alterations cause difficulties if the ceiling height is too low or if deep beams have to be traversed; the formation of bulkheads causes problems of another kind. The major problem for the interior designer today lies with the distribution of power, telecommunications and data cables. Speculative offices and older building stock rarely have sufficient capacity to deal with the plethora of wires and the new local area networks. The best solution appears to be the installation of suspended floors fed by extra vertical risers but again the floor-to-ceiling height must be sufficient to allow for this. If it proves impossible then the floor construction should allow for extensive trunking runs in the screed.

Fixings to an existing structure can sometimes prove difficult and in some cases it has not been possible to fix the supports for a plasterboard ceiling to a slab. The strength of the slab gives rise to quite a different set of problems. Even with the growth of information technology there still seems to be more and more paper produced and the filing and storage of bulk paper is dependent upon the floor loading. The mandatory minimum of 2.5 kN/m^2 is rarely enough – especially when the weight of a suspended floor forms part of the live load. Even normal filing units require 5 kN/m^2 and compactors, mechanical and power files, safes, etc. can create serious difficulties. It is often possible to position the storage on beam on lines or directly adjacent to the core walls where maximum strength occurs, but these limitations are detrimental to the functional efficiency of the layout. A value of 4 kN/m^2 live load plus 1 kN/m^2 for partitions, etc. should be achieved if possible.

Similar cautions should be voiced concerning the strength of roofs. It is common to site additional service plants in such locations and the strength of the roof construction is of prime importance.

21.19 Structure

The design of building structures is an iterative process by which the type, shape, dimensions, materials and location of the various structural elements are initially chosen as a first approximation; loads are then determined and the design developed by a process of adjustment and verification that structural performance will be satisfactory. The structure must also satisfy the functional needs of the building, site factors and the many technical requirements concerned with the safety, health, comfort and convenience of the occupants.

21.19.1 Structural behaviour

Assessment of structural behaviour must cover: (1) 'serviceability limit states'; and (2)'ultimate limit states'.

21.19.1.1 'Serviceability limit states'

These are concerned with acceptable vibration, horizontal and vertical deflections and structural cracking and the compatibility of these with the secondary elements supported by the structure, such as partitions, cladding, finishes.

21.19.1.2 'Ultimate limit states'

These are concerned with the provision of adequate reserves of strength to cater for variations in materials, structural behaviour, loading and consequences of failure. Partial factors are used for this purpose as follows:

γ_m allows for variations in strength and is the product of:
- γ_{m_1} to take into account the reduction in strength of materials in the structure as a whole, as compared with the control test specimen; and
- γ_{m_2} to take account of local variations in strength due to other causes, e.g. the construction process.

γ_f allows for variability of loads and load effects and is the product of:
- γ_{f_1} to take account of variability of loads above the characteristic values used in design;
- γ_{f_2} to allow for the reduced probability of combinations of loads; and
- γ_{f_3} to allow for the adverse effects of inaccuracies in design assumptions, constructional tolerances such as dimensions of cross-section, position of steel and eccentricities of loading.

γ_c takes into account the particular behaviour of the structure and its importance in terms of consequential damage, should failure occur. It is the product of γ_{c_1} and γ_{c_2} where:
- γ_{c_1} takes account of the nature of the structure and its behaviour at or near collapse (whether brittle and sudden or ductile and preceded by warning) and the extent of collapse resulting from the failure of a particular member (whether partial or complete); and
- γ_{c_2} takes account of the seriousness of a collapse in terms of its economic consequence and dangers to life and the community.

Relevant structural codes do not give values for the subcomponents (γ_{m_1}, γ_{m_2}, etc.) quoting only global values for γ_m and γ_f, which vary with the circumstances and load combinations being considered. However, the subcomponent definitions are useful reminders of the variables that need to be taken into account.

21.19.1.3 Hazards

Building structures may be subjected to such hazards as: (1) impact from aircraft or vehicular traffic; (2) internal or external explosion caused by, for example, gas or petrol vapour or by sabotage; (3) fire; (4) settlement; (5) coarse errors in design, detailing or construction; and (6) special sensitivities, e.g. as to acceptance of movement or differential movement or as to conditions of elastic instability, not appreciated or allowed for in design. Hazards involving risk of collapse or damage may also be introduced during design, construction or service. They derive from mistakes, ignorance or omission, inadequate communication or organizational weakness.

These hazards cannot be quantified except in special circumstances. However, for buildings of five or more storeys, the Building Regulations requirements concerning progressive collapse provide a general level of protection whereby the stability of a building is not put excessively at risk as a result of local structural damage arising from whatever cause. In cases of known risk the special requirements should be included in the design brief.

Many methods are available for confining the effects of accidental damage to the immediate locality of the incident. These include designing to accept the forces involved, the provision of alternative paths for the loadings, 'fail-safe' and 'back-up' structures. Research has been carried out on partial-stability conditions, whereby the remaining components of the building framework are capable of bridging or stringing over an area of total local damage by beam, catenary or membrane action.

Statutory requirements as to fire resistance and means of escape are devised to ensure continued stability for sufficient time to permit evacuation of the building and fire-fighting to protect adjoining property.

The introduction of new methods and materials requires careful consideration of the structural response to all the events that may occur during manufacture, construction and life of the lies with the distribution of power, telecommunications and data cables. Speculative offices and older building stock rarely have sufficient capacity to deal with the plethora of wires and the new local area networks. The best solution appears to be the installation of suspended floors fed by extra vertical risers but again the floor-to-ceiling height must be sufficient to allow for this. If it proves impossible then the floor construction should allow for extensive trunking runs in the screed.

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The introduction of new methods and materials requires careful consideration of the structural response to all the events that may occur during manufacture, construction and life of the structure, not just those idealized in design procedures, codes or standards.

It is very important to recognize that hazards exist outside the range of conditions normally considered in design; they must be eliminated or the structure designed so that their consequence is acceptable. The alternative is to accept that a particular hazard is so remote a risk that it can be ignored. This conclusion involves not just the statistical assessment of the hazard risk itself but careful examination of the consequence should it occur, since, though the hazard risk itself may be constant, the consequence in one type of building or structure compared with another may be catastrophically different. This applies not only to the protection of human life but also to particular functions, the continuation of which may be of paramount importance to the building user.

Four basic philosophies exist, aimed at reducing hazards or their consequence:

(1) The probabilistic approach.
(2) Discernment of proneness to hazard.
(3) Alternative paths and partial stability considerations.
(4) The hazard–consequence relationship.

These philosophies can be applied individually or in combination as a means of risk control or as a tool for risk comparison between alternative courses of action.

The probabilistic approach (1) recognizes the statistical hazard of adverse combinations of high load and low strength and provides a method of comparative measurement of safety. It is the basis of current limit-state codes. Proneness to hazard (2) recognizes the 'climate' in which errors, misjudgements or accidents occur and is an aid to discerning potential hazard, e.g. a new form of construction being hurriedly adopted under political and/or commercial pressure. Considerations of alternative path/partial stability aspects (3) of a proposed structure, encourages recognition and, hence, elimination of excessive sensitivity to local damage from unforeseen hazards. The hazard–consequence approach (4) seeks to identify the potentially serious consequence and directs resources and attention to the most vulnerable aspects determining future structural performance.

21.19.1.4 Structural tests

The behaviour of the structure is normally assessed by analytical methods but may also be estimated by tests on prototypes or models or by a combination of analysis and experiment. Prototype testing is sometimes used, for example, in precast concrete construction where the accuracy of the design assumptions may be in question or, in cases of repetitive application, where a better or more reliable understanding of structural behaviour may lead to economy. Model testing may be used to determine internal forces or stresses, and in special cases photoelastic analysis may be used to check complex local stress conditions, e.g. around service openings in major structural members.

21.19.2 Robustness

Recent experience has demonstrated the vital importance of the quality of robustness in determining the long-term performance and adaptability of buildings and their structures. These qualities can often be incorporated in the building structure with little, if any, extra cost if appropriate consideration is given in the early stages of design. These desirable qualities include:

(1) An ability to cope with hazards in an acceptable way, i.e. the building and its structure have been consciously designed so that damage would not be disproportionate to the cause.
(2) There is provision to eliminate, or reduce to acceptable levels, the risk or consequence of hazards not allowed for in (1) above.
(3) The structure is not sensitive to:
  (a) marginal departures from the design assumptions;
  (b) local defects or movement;
  (c) environmental change.
(4) The structure does not deflect or vibrate to an extent that alarms the occupants or disturbs intended function.
(5) There is an inbuilt ability to cope with remodelling or increased loading to suit changing use.
(6) The structure is readily buildable and not unduly dependent upon perfect compliance with the specification for workmanship and materials or future maintenance.
(7) The structure is such that early warning would occur before serious defect or collapse.

A very important aspect of robustness is the ability of the structure to cope with change. Very high demand and limited resources in the 1960s coupled with the Modern movement approach to functional design led to 'tailor-made' buildings with very little thought or scope for future change. It is now realized that functional requirements are changing rapidly in many forms of building. Ample spaces for services and the facility to change them are, for the building owner and user, welcome additions to the general robustness of the structure, as is the ability to accept higher loading and some cutting or rearrangement to accommodate additional lifts or service runs. Loading is particularly significant. In many buildings the cost of the structure is small in relation to the value of the finished building. The relative cost of providing for additional loading may be slight and a sound investment for an uncertain future. In the same way, ample plant room space and extra storey height can prove invaluable for the incorporation of additional building services.

A robust structure, however well designed, also needs a robust management and communication system for its production. Analysis of past failures confirms that they result primarily from lack of perception and poor communication rather than from insufficient knowledge of behaviour or circumstance. There are many links in the tortuous chain required to produce a robust building. One of the most significant is that between the designer and builder including those responsible for site inspection and supervision. Two particular aspects emerge from these considerations: (1) buildability; and (2) quality assurance. A readily buildable structure is an essential start to ensuring good quality. Designers should bear in mind the problems of construction and seek advice from a contractor during the early stages of design whenever possible. Conversely, supervisory staff should understand the structural behaviour assumed in design, not only of the structure as a totality on completion but also of parts of the structure during construction of the whole. They should direct particular effort to the more important aspects of quality control and seek to create the right climate on site – to do a good job even when not supervised and to get it right first time. All this is helped by the production of an 'inspection brief' prepared by the design team to ensure the effective deployment of the supervisory staff. Such a document should consolidate all the work leading up to the site start, define responsibilities, confirm and clarify relevant documentation and lines of communication, alert people to critical aspects and co-ordinate specialist inputs, all as a formal handover from those responsible for design to those responsible for construction.

21.19.3 Wind effects on buildings

The airflow around a building is affected by the adjacent land

and building complex and by the shape and size of the building itself, the roof type, the position and size of overhangs, the area and location of openings and the direction of the wind. Account needs to be taken of the loading effect of wind turbulence on the building as a whole (and perhaps during construction) and on components, the local wind environment in the vicinity of buildings, the general weather-tightness, natural ventilation and the air pollution around buildings.

Code of Practice 3:1967. Part 2, Chapter V gives the method to be used for assessing wind loads on individual buildings and takes account of the location of the building, the topography, the ground roughness, the building size, shape and height and a statistical factor related to the probability of given wind speeds occurring during the specified life of the building. For groups of buildings, particularly those including tall buildings, the environmental effects are frequently studied by means of model tests in wind tunnels.

21.19.4 Movement

The problem of movement in buildings is not so much the determination of its absolute value, but more that of achieving a compatibility of movement between parts. Without this, cracking and other disturbances are inevitable and are likely to recur even after repair; in such cases, the accurate assessment of movement serves little purpose. On the other hand, the simple recognition that relative movement will occur, coupled with a broad assessment of its significance, are essential first steps in establishing a compatible design in which the actual amount of movement is relatively unimportant.

When significant parts of a building tend to move appreciably relative to each other, they can be separated into independent blocks. Within each block differential movement will occur between elements, e.g. between the structure and partitions, and also between different parts or different materials forming a particular element.

Factors affecting the division of a building into blocks include: (1) differential foundation settlement due to load or soil variations, changes in foundation type or major intervals in construction; (2) longitudinal movements due to changes in temperature, shrinkage or prestress (immediate and long-term); (3) abrupt changes in building plan or floor or roof level; and (4) abrupt differences in structural stress. Within each block, the most common movement problems are: (1) partitions damaged by floor deflection or relative longitudinal movement; (2) crushing or buckling of cladding and partitions due to relative vertical movement between them and the structure; and (3) separation of surfacings due to movement relative to the backing material.

Structural joints include: (1) hinge details, to permit rotation; (2) expansion and contraction joints, commonly used with flexible seals; and (3) complete separation, e.g. where double columns are introduced. Joints between nonstructural elements allow for expansion or contraction or lozenging by providing appropriate horizontal and vertical gaps between the elements which are sealed with a flexible material or by cover strips secured to one side of the joint. Joints exposed to the weather require special attention in detailing and manufacture and in the choice of sealing materials. The open drain joint coupled with an effective air and water back-seal has proved a successful and reliable joint.

Wherever possible, restraints to longitudinal movement should be avoided by appropriate design and location of wind stiffening cores or bracing walls. For example, in rectangular buildings it is frequently appropriate to locate a service core at one end, providing rigidity in two directions, and a cross-wall at the other which provides the necessary torsional stiffness but permits free movement in the longitudinal direction. Temperature movements are minimized by keeping the structure within the insulation envelope.

21.19.5 Structural arrangement

A great variety of structural arrangement is used in practice depending upon the planning, functional, aesthetic and economic requirements of the building and site. Even for similar buildings, the relative priorities attached to the individual factors affecting structural decisions will vary depending upon the particular circumstances and the views of the client. General rules governing structural arrangement cannot be given, but in most cases the following principles are valid: (1) vertical loads should be transmitted along the shortest and most direct path to the supporting ground; (2) when minimum structural sections are dictated by nonstructural requirements, e.g. sound insulation, they should be deployed to gather extra load; (3) vertical load-bearing elements should be stacked directly over each other; (4) transfer structures, including vertical ties, should be used only when justified; and (5) structural layout should be regular to increase repetition of identical building components and improve construction rhythm.

The structural arrangement, construction method and material may be chosen on the basis of some overriding consideration, such as the provision for future alteration or extension, passage for services, speed of construction, availability of labour and materials or difficulty of access. Where alternatives are equally appropriate, the choice is generally made by cost comparison, but this must take into account all aspects affecting the total cost of the building including, where appropriate, running and maintenance costs.

The primary structural systems available for spanning vertical loads across space are; (1) the catenary, acting in tension; (2) the arch, in compression; and (3) the beam, in bending, item (3) being of most importance in buildings. Columns and walls are the commonly used members for vertical load support, and on occasion tension members are used to suspend lower work from high-level beam or cantilever construction. Frame structures are of two basic types, those in which horizontal forces are taken by shear walls or bracing and those in which the frames, comprising columns rigidly jointed to beams or slabs, are designed to accept horizontal as well as vertical loading.

Structures relying solely on frame action for stability become increasingly inefficient with height and reach a normal practical limit of about 15 to 18 storeys. For tall framed buildings, sway limitations (which must take account of increased lateral displacement due to the action of vertical loads on frames deflected by wind loading) are such that much larger and stiffer members than necessary for vertical loads alone would need to be used.

21.19.6 Resistance to vertical load

21.19.6.1 Columns and walls

Column and load-bearing wall positions are determined mainly by the building use. Where large clear spans are not necessary and regular and permanent space division is required as, for example, in multistorey flats, load-bearing walls are commonly adopted. Column spacing, in conjunction with the floor construction, will be affected by the available structural depth and the necessary provisions for the passage of mechanical and electrical services. Concrete columns may be of any reasonable shape; in tall buildings the section may be kept constant for construction convenience or reduced at upper levels to save usable floor area. When floor space is particularly valuable, spiral reinforcement or solid steel sections may be used or tension supports may be provided; the latter may be prestressed, in stages, to minimize the required sectional area and eliminate cracking.

21.19.6.2 Floors

In addition to their structural function, floors may need to provide impact and airborne sound insulation, thermal insulation and appropriate fire resistance depending upon their location in the building, and the building type. For longer items, deflection and vibration will also be important considerations.

In situ *concrete floors.* Reinforced and prestressed concrete flat-slab construction has the advantages of: (1) minimum structural depth; (2) adaptability to irregular arrangements of columns or walls; and (3) not requiring a suspended ceiling (but if one is provided gives complete freedom for the passage of services). Solid reinforced concrete construction, 150 to 250 mm thick, may be used for spans up to about 7 m, or greater if post-tensioned. Punching shear at the columns, midspan deflection and the size and location of openings require special attention. When larger spans or weight reductions are required, the slab may be coffered to provide one- or two-way spanning: standard or purpose-made plastic, steel or fibreglass moulds are available and can produce a visually attractive self-finished ceiling. Alternatively, permanent cavity formers may be left in position. Beam and slab construction, at the expense of greater floor depth and slower construction, has the advantages of: (1) longer spans; (2) ready provision for large openings, e.g. for stairs and lifts; (3) adaptability to varying size and building shape; and (4) relatively light weight. It is most economic when large repetitive areas, or a heavy loading is required. *In situ* beam and slab construction may be the only valid method of construction for complex shapes and areas.

Precast floors. These have the advantages of speed of erection and quality and accuracy of manufacture; they are economic for medium to large spans particularly where layouts are straightforward and repetitive. They may be used in conjunction with steel or concrete frames in addition to load-bearing wall construction. The largest use of precast flooring is in the form of hollow or solid slabs, reinforced or prestressed, with widths varying normally from 300 mm to 2.7 m and up to 7 or 8 m in the case of large-panel construction where such slabs may incorporate openings, ducting and floating screeds. Precast slabs may be designed to act compositely with the supporting concrete or steel beams. For longer spans, single or double T-beams, which combine floor slab and beam are available; they are connected by welding, bolting or *in situ* jointing to provide secondary load distribution and equalize deflection.

Composite floors. These rely on the composite action between an *in situ* concrete topping and precast concrete soffit elements, which may take the form of precast concrete ribs, planks or slabs incorporating the tension element of the composite slab and having projecting reinforcement or other appropriate interface to ensure composite action. Alternatively, permanent steel shuttering may form the soffit. These floors are easily erected, do not need shuttering and provide the shallow depth of *in situ* slabs. A further example of composite action is that between concrete floor slabs, *in situ* or precast, and steel beam or frame construction.

21.19.6.3 Transfer structures

It is frequently found, in multistorey construction, that the special functions of the lower storeys require an arrangement of columns or bearing walls very different from that required for the efficient support of the superstructure above. A transfer structure is then required to transmit the typical floor column loads to the fewer but larger supports beneath. Often a very substantial structure involving storey height beams is required – this should be taken into account in the early stages of design since it may provide a suitable location for plant. An alternative is to place the transfer structure at roof or intermediate upper-floor level and to suspend the lower structure by means of steel or prestressed concrete hangers.

21.19.7 Resistance to horizontal load

In low- to medium-rise buildings, the structural system is designed primarily to resist the vertical loads and is then checked for lateral forces which may be taken by moment-resisting frame action, braced frames or shear walls conveniently located around lift shafts or stair wells. Simple shear walls provide the necessary horizontal restraint for 'pin-jointed' frameworks which are often convenient and economic. A minimum of three bracing walls are required so disposed as to provide: (1) resistance in each of two directions at right angles; (2) an overall torsional resistance; and (3) minimum restraint to thermal or similar movement.

21.19.8 Multistorey construction

Steel and concrete are the primary structural materials and both can be used in a variety of forms. The choice of material and form will be dependent upon many factors particularly relevant to the building project, such as:

(1) Required speed of construction.
(2) Integration of services.
(3) Adaptability for future change of use.
(4) Lead times for delivery or construction.
(5) Fire protection.
(6) Impact on foundation feasibility and cost.
(7) Buildability and dependence upon workmanship and materials.
(8) Stability during construction.
(9) Road access for delivery and erection.
(10) Off site/on site labour availability.
(11) Dependence upon supplier.
(12) Impact on other trades.
(13) Securing early watertightness.
(14) Cost.

A key aspect is the provision for services either by complete separation from, or integration within, the structural elements. Integration involves deep, perforated structural elements and requires very careful planning and co-ordination. Separation requires zoning outside the structural elements, the configuration of primary and secondary beams to provide these obstruction-free zones is then a very important part of the structural concept.

The common forms of concrete and steel multistorey framed construction are given in Figures 21.10 and 21.11.

Structural continuity is easily effected and commonly adopted in *in situ* reinforced concrete construction, increasing both stiffness and economy. In precast or steel construction, continuity is more difficult to achieve and the theoretical savings in structural material are offset by the complications in construction and reduced future adaptability. Repetition, simplicity and continuity of erection are more important in achieving economy than marginal savings in material obtained by a too-tailored approach in design. However, with modern methods of fabrication, 'specials' can be introduced economically provided the needs of quantity, repetition and simplicity of erection can still be met. The optimum structure will also take into account its impact on other building elements and on the construction process as a whole.

A number of buildings require a special structural response outside the commonly used systems referred to above. Architec-

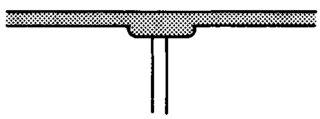
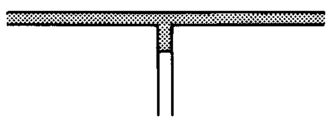
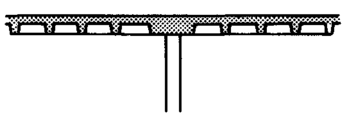
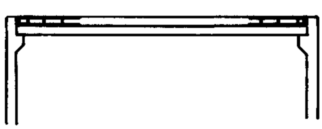
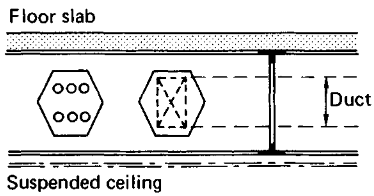
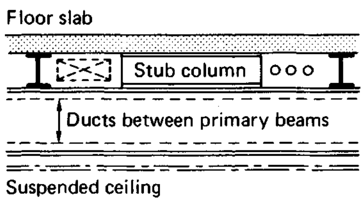
 (a)	Limited spans. Deflections need watching. Uninterrupted service space above suspended ceiling or exposed flat soffit for uplighting and heat sink
 (b)	As for (a) but increased spans. Dead load deflection can be compensated by cable uplift
 (c)	Increased spans Uninterrupted duct space in direction of beams
 (d)	Increased spans. Increased overall floor depth unless services integrated. Generally stiffer construction
 (e)	Increased spans with minimum depth and weight. Decorative soffit if left exposed. Some services can be incorporated in waffle depth
 (f)	As for (d) but generally faster construction
 (g)	Very long spans possible. Clear service runs in direction of span

Figure 21.10 Concrete construction. (a) *In situ* reinforced concrete slab; (b) *in situ* prestressed concrete slab; (c) flat slab with shallow beams, reinforced or prestressed concrete; (d) *in situ* reinforced or prestressed concrete beam or slab; (e) waffle reinforced or prestressed concrete slab; (f) precast beam and slab; (g) precast T on double T



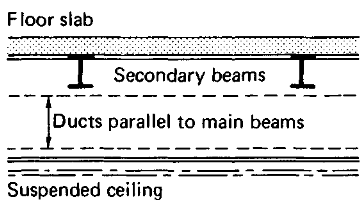
(a)

Optimizes floor depth structurally.
Fitting-in of services is limited and can be difficult. Detailed co-ordination essential



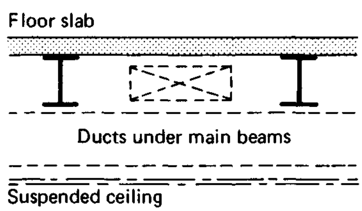
(b)

Composite action with floor slab effected through stub columns increases structural depth while providing space for transverse services at the expense of heavier steel



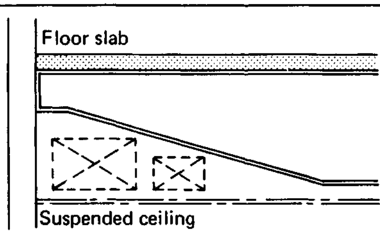
(c)

Main air ducts run parallel to primary beams.
Increased floor zone depth but easier to accommodate and later modify services



(d)

Easiest for services but spans reduced or steel cost increased.
Ideal if floor zone depth not critical



(e)

Large spans possible utilizing full beam depths at span centre.
Transverse service runs accommodated at beam ends or through holes in beam webs near centre of span

Figure 21.11 Floor slabs may be *in situ* or precast concrete and are frequently designed to act compositely with the beams. Profiled steel decking with through-welded stud shear connectors supporting an *in situ* lightweight or normal concrete topping can provide a fast and practical composite floor construction. (a) Castellated deep beams; (b) shallow beams with stub columns; (c) deep primary beams with shallow secondaries supporting slab; (d) shallow primary beams at close centres; (e) tapered beams

tural form, large spans or great height, or special physical criteria, e.g. seismic, may dictate a totally different approach.

21.20 Tall buildings

With increasing height, resistance to horizontal forces begins to dominate the design and may add substantially to the total cost. In addition to structural safety, sway limitations must be satisfied in terms of horizontal accelerations as well as actual movement. In principle, the lateral resistance may be provided by frames in bending, braced frames or by shear walls, as in lower structures, but the greater magnitude of the forces and movements necessitates a more sophisticated approach. The various systems of resistance to horizontal forces in tall buildings are illustrated in Figure 21.12.

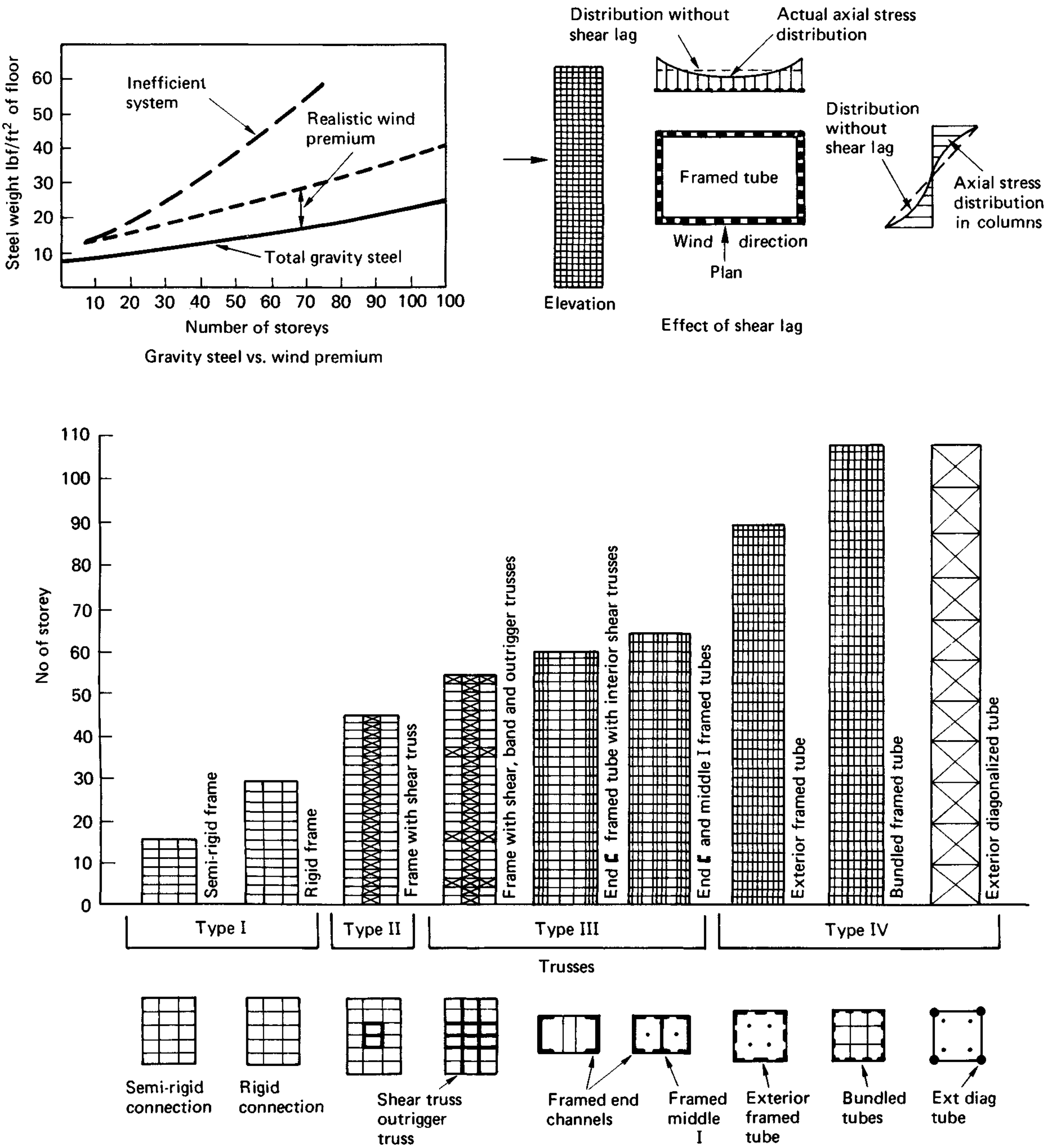


Figure 21.12 Systems of resistance to horizontal forces in tall buildings. (After Iyengar (1972) 'Preliminary design and optimization of steel building systems', *Conference on planning and design of tall buildings.* ASCE/IABLE)

21.20.1 Frames in bending

Internal frames are comparatively inefficient and flexible as a result of the planning necessity for a wide spacing of internal columns and limited floor beam depth. On the other hand, exterior frames, formed in the plane of the external wall, may have closely spaced columns connected by deep spandrels. In this way, the entire perimeter of the building may be developed as a major lateral load-resisting system referred to as the 'boxed frame' or 'framed tube'. Subject to 'shear lag' considerations, the building walls act respectively as the webs or flanges of a box section cantilevering from the foundation. To allow for 'shear lag', two channel sections may be considered operative in place of the complete box. Deep spandrels, although advantageous, are not essential and apartment buildings of up to 46 storeys have been constructed in the US using part of the adjacent flat slab floor as the beam continuous with the closely spaced external columns.

21.20.2 Braced frames

Braced frames usually incorporate single- or double-diagonal braces or K-bracing within the beam and column framework and may be used internally, around service cores or in the external wall. When used externally, intermediate transfer structures may be incorporated to transmit a major proportion of the total vertical load to the corner columns. Such transfer structures may also support and, hence, separate, different configurations of internal supports required when the function varies between different vertical zones of the building. Such arrangements have a major impact on the internal planning and external appearance, and have important relevance to the planning and construction of very tall buildings using steelwork. Similar external truss action may be achieved in concrete by blocking out windows to form solid and continuous diagonal members or, for limited height, by using precast cladding forming a multiple-diagonal system. When they can be accommodated, internal trusses can bring into action lengths of external wall otherwise rendered ineffective by 'shear lag'. By alternating the plan position of such internal trusses, from storey to storey, the structural span of the floors may be reduced to half of the architectural planning bay by providing additional hanging supports from the trusses in the storey above.

21.20.3 Shear walls and cores

Shear walls may be internal or external or may surround internal service areas to form cores; their location and dimensioning are major design elements since they seriously impinge on internal planning and may affect external appearance. In the early formative stages of design, quick structural appraisal of alternative shear walls will be required followed by careful design and analysis of the final arrangement.

In office buildings, the service core – which includes lifts, stairs, ducts and toilets – can occupy 20% or more of the total floor area while fire and sound insulation require this area to be bounded by heavy wall construction. These conditions naturally lead to the use of the service core as a major vertical wind brace. However, away from the core area, open office space is generally required and even if partitions are used they would be demountable to allow for future alteration; internal bracing walls are therefore a planning impediment in offices and are generally avoided. External bracing walls, however, are often used, generally in conjunction with internal cores. In housing or hotels, partitions are normally fixed, need to be heavy for sound insulation and are regularly spaced; they therefore provide many convenient locations for internal bracing walls.

When shear walls alone are used, the general structural requirements are: (1) at least three must be provided of which at least two must be parallel and widely spaced, to provide torsional resistance, with the third at right angles; (2) the centroid of the shear walls should be close to the centre of gravity of the loading; and (3) walls likely to need very large openings should be avoided if alternatives are available, since their stiffness and, hence, load-resisting contribution will be diminished substantially.

Walls with openings produce a stiffness intermediate between that of the total combined length acting as a monolithic wall and the sum of the stiffness of the parts acting separately, depending upon the relative size and location of the openings. Normal analysis assumes that all the shear walls or cores act from a completely rigid foundation such that relative rotation or vertical movement does not occur. Since even small relative movements could seriously invalidate the design, it is important that this assumption is realized in the foundation design or, if this is not practical or economic, the shear wall system should be designed to suit. In general, the total horizontal load is distributed between the shear walls in proportion to their relative stiffnesses taking into account any eccentricity of the applied load. The floor system then acts as a horizontal diaphragm equalizing horizontal displacement and rotation at each floor level.

Shear walls and cores are almost invariably constructed in concrete and are often slipformed. Precast large panels have been successfully used, particularly in high-rise blocks of flats, with the combined functions of load-bearing walls and vertical wind braces. The joints between the panels and the lintels over openings require particular consideration in the design of such structures.

The use of shear walls or cores is an economic and efficient method for resisting large horizontal forces but, in most cases, deflection limits their use to below 30 to 40 storeys. However, if the shear wall, and building, are shaped in plan along their length, a vertical shell or folded plate action could be developed permitting greater heights; otherwise, a 'boxed frame' or one of the combined systems described below is required to control deflection. Another limitation of shear walls, particularly if lightly reinforced, arises from the possibility of brittle failure which could make them unsuitable for seismic structures. However, by suitable framing around the shear wall, the necessary ductile behaviour can be obtained to absorb the considerable strain energy arising from an earthquake.

21.20.4 Combined systems

Internal cores may be used in conjunction with external moment-resisting frames so that the substantial overturning resistance of the façade frame is combined with the excellent shear resistance of the core to form a highly efficient total system known as 'tube in tube'. This arrangement still relies upon closely spaced external columns; when widely spaced external columns are required, a beneficial interaction between the core and the external columns can still be obtained by connecting the two with deep stiff beams rigidly connected to the core and located at convenient levels (roof and service floors). In this arrangement, the core continues to take the shear but the overturning resistance of the full building depth is called into play and deflection is reduced. Another advantage of this system is that it can help control the effects of differential expansion or contraction of the external columns.

Figure 21.13 illustrates the application of the bundled tube principle to the 110-storey Sears building in Chicago. The faces of each tube are stiffened by deep beams and columns in vierendeel action. The tubes are bundled to their maximum effect at ground level but are dropped off with increasing height to suit both structural and internal planning requirements. The internal framing at the junctions between tubes reduces shear lag in the long faces of the multiple tubes as shown in Figure 21.13(b).[14] Maximum effect would be achieved with diagonal bracing incorporated in the tube faces.

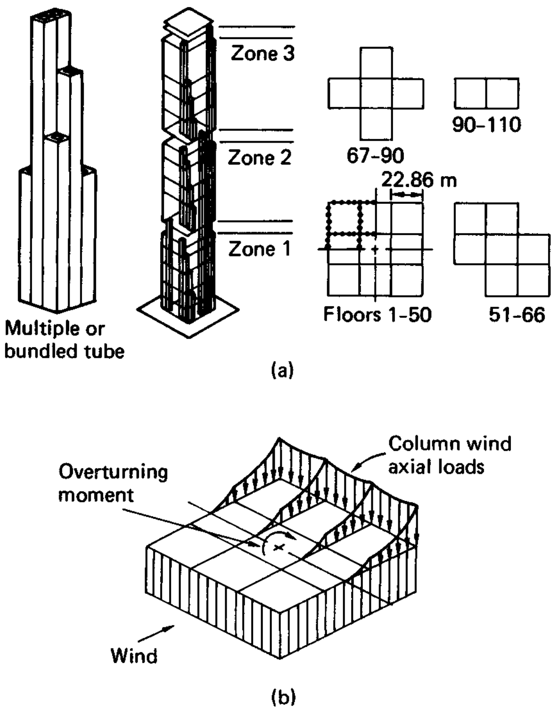


Figure 21.13 The Sears building in Chicago. (After Fischer (ed.) (1980) *Engineering for architecture*. McGraw-Hill, New York)

21.20.5 Vertical movement

Another aspect distinguishing tall buildings is the need to consider the possibility of differential vertical movements due to temperature or stress and, in the case of concrete structures, those due to creep and shrinkage. The movement is most marked between the internal structure and the external columns, particularly if the latter are totally or partly outside the external wall and glazing. It affects most the external cladding and partitions located at right angles to the external wall, as well as any linking structural element.

The effects are best controlled by attempting to achieve uniformity of stress and exposure (including insulation where necessary) and uniform surface:volume ratios for the concrete elements to reduce differential shrinkage or creep. When the problem is particularly severe, the building can be divided into two or more sections by incorporating intermediate transfer structures, in effect producing horizontal expansion or contraction joints. Alternatively, the movement can be restrained by stiff beams connecting the external columns to the core. Another approach is to freely permit the movement and incorporate appropriate movement details in the structure, partitions and finishings.

21.20.6 Lateral movement and dynamic effects

Wind loading on tall buildings is not simply a matter of statics. As with long-span bridges, aerodynamic effects and the dynamic response of the structure under variable wind loading are important considerations. The primary considerations are: (1) vortex shedding, arising from the plan shape of the building; (2) the extent of horizontal movement and related accelerations that can be tolerated; and (3) the means of dissipating the wind energy imparted to the building, i.e. damping.

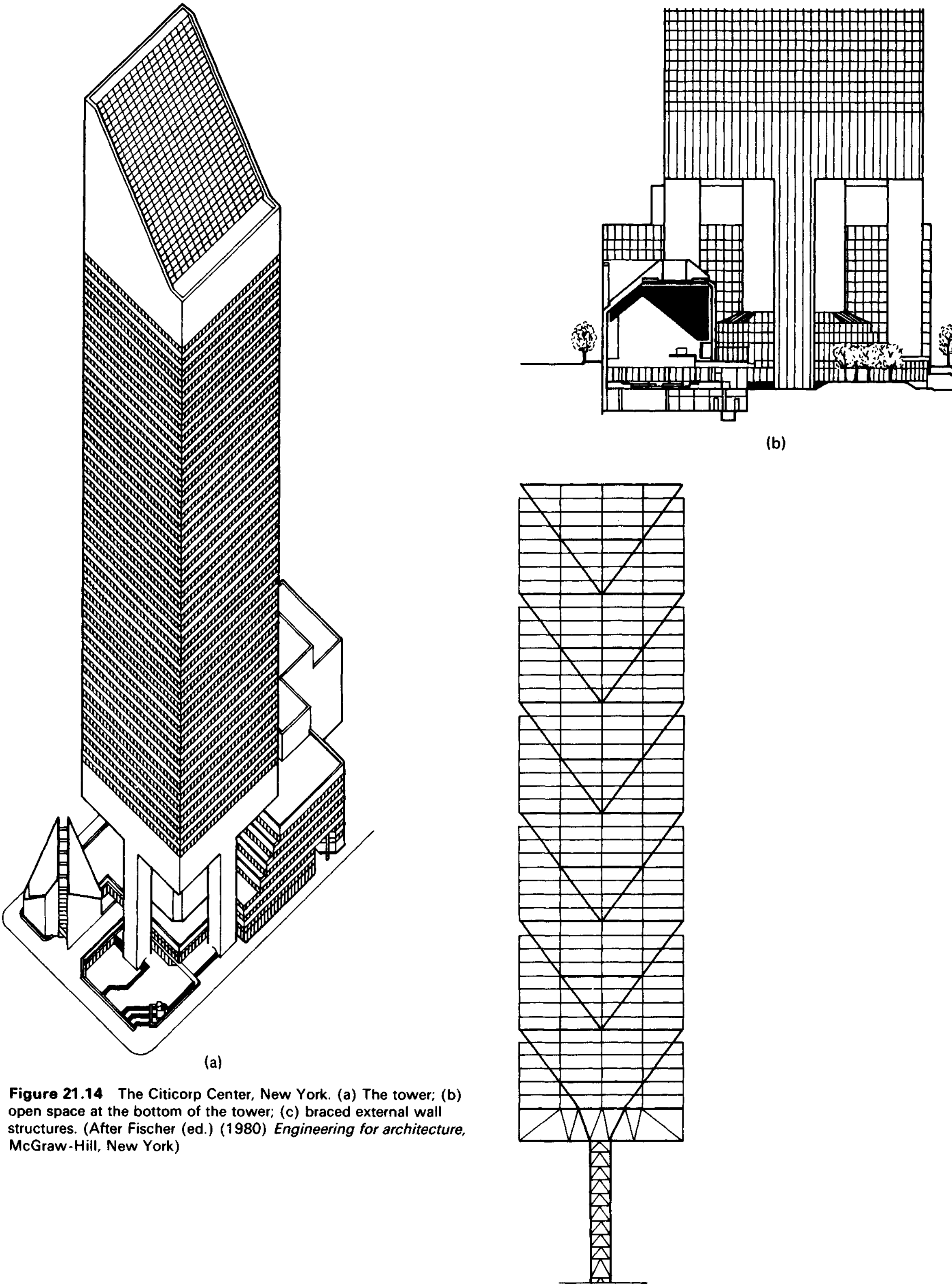
The damping characteristics of the early tall buildings of traditional construction were good. The exterior walls and internal partitions helped to dissipate the wind energy imparted to the building by friction between their parts as they moved relative to each other under wind action. The façades were generally textured, creating turbulence, and sculptured, thus reducing the risk of transverse oscillation from vortex shedding. With the development of simpler shapes and taller, more flexible buildings, these damping qualities have been diminished. Damping through the structure itself is small since, in this context, building structures are very elastic. Damping through the nonstructural elements is difficult to control and can be expensive in maintenance and repair. This leaves aerodynamic damping, through form and cladding texture, or specially designed mechanical damping as the means of absorbing energy and controlling movement to within acceptable limits. The alternative is to increase mass but this can be very expensive.

Acceptable lateral movement is not simply a matter of deflection but more of people's perception of, and psychological response to, movement. Deflection is a product of stiffness and can be controlled, at a cost, through structural quantity and configuration. However, people's reaction to sway is related more to acceleration than actual amount of movement and, more particularly, to the rate of change of acceleration. Increasing the structural stiffness reduces the deflection but does not affect acceleration since the frequency is increased. Indeed, the more critical rate of change of acceleration is also increased. Reducing stiffness increases sway and the relative movement between building elements as well as visual disturbance. Thus, in some cases the only two practical methods of dealing with this problem are to increase the mass or improve the damping characteristics of the building. Increasing the mass is expensive. Aerodynamic damping is possible if suitable building forms are acceptable. Where this is not possible, or in cases of extreme height, additional damping can be provided by incorporating inertia elements strategically located within the building, or energy-absorbing devices at points of movement within the structure. Such damping has the effect of reducing both the amount of movement and the acceleration.

An example incorporating inertia elements is the Citycorp Center in New York which has a tuned mass damper to slow down and reduce movement due to wind. The system is housed in the roof structure and consists of a 400-t concrete inertia block mounted on a 'frictionless' bearing which is free to remain still when the building starts to move under wind action. The mass is connected to a system of pneumatic springs and dashpots in which the energy is absorbed by oil. The dynamics of the damping system are adjustable to suit the building's actual dynamic characteristics. Fail-safe devices are incorporated to ensure that the movement of the mass relative to the building is kept within predetermined limits. Analysis and wind-tunnel testing indicated that this device would reduce the acceleration under wind loading by 38%.

The other methods of damping were adopted at the World Trade Center, also in New York. The simple geometric shape of the twin 110-storey towers was such that the vibration due to vortex shedding could not be discounted. To combat this danger, the building corners were chamfered, so modifying vortex generation, and a viscoelastic damping system introduced. Some 10 000 such dampers were built into each tower, consisting of steel connections incorporating viscoelastic material between the floor trusses and columns.

Figure 21.15 illustrates diagrammatically the dampers incorporated in the 110-storey World Trade Center in New York.



(a) (b) (c)

Figure 21.14 The Citicorp Center, New York. (a) The tower; (b) open space at the bottom of the tower; (c) braced external wall structures. (After Fischer (ed.) (1980) *Engineering for architecture*, McGraw-Hill, New York)

The damping unit comprises two Ts bonded to a central plate by viscoelastic material. The plate is attached to the bottom chord of the beams and the Ts to the columns. Wind energy is absorbed by shear displacement of the viscoelastic material. Figure 21.16 illustrates the general arrangement of a tuned mass damper while Figure 21.17 illustrates graphically the beneficial impact of damping on building oscillation. Similar principles can be used to control the vibration of light structures.

21.20.7 Additional considerations

In addition to the wind loading on buildings, the wind effect between buildings and, particularly, that produced by the presence of a tall building, has proved of considerable concern. Wind-tunnel tests on models are an essential element of design, to check the environmental impact created by a new building within an existing complex.

Special arrangements in the lower levels of tall buildings are encouraged in some locations by city planning policy. In New York, for example, a 20% additional floor area is given as a bonus for providing a public open space or plaza in front of the building; this has now been extended to encourage covered and secure pedestrian freeways through the buildings. Incentives are also given to encourage a richer mixed use of tall buildings, e.g. by incorporating a theatre when the building is in a theatre district, and shops within a shopping district. The intention is to produce a lively combination of activities and more demand for transport and services over a longer period of time.

These aspects have given rise to another important consideration in the design of both high- and low-rise buildings, i.e. their treatment at ground level. For example, the Citycorp Center in New York incorporates at its base a minicentre for culture and commerce including a church, a theatre, a room for jazz performances and a complex of international food boutiques. To accommodate these, the bottom of the structure consists only of the centre core and massive columns placed not at the corners but at the mid-point of each side. In most cases the bridging requirements to achieve these open spaces at ground level are met by a straightforward transfer structure in steel or concrete. This is sometimes placed at roof or upper level and the building beneath hung from it. In other cases, more subtle bridging means have been employed to make use of the whole superstructure, acting as a total bridging entity within itself.

Figures 21.14(a) and (b) illustrate how open space was created at the bottom of the Citicorp Center tower and utilized for religious and cultural purposes. Figure 21.14(c) illustrates the braced external wall structures. Each eight-storey tier is structurally independent with loads from each tier gathered to the four exterior mast columns via the diagonal truss members. Wind shear in each eight-storey tier is taken by a central core structure but the overturning moment is then transferred to the external masts.

In the urban context, the placing and form of tall buildings is of great importance. The architect and engineer have learned to solve the technical and aesthetic problems of tall buildings but generally as isolated elements. As one example which recognizes the very great positive or negative impact that a building of exceptional height may have, the City of San Francisco has adopted an urban plan which sets down fundamental principles and critical urban design relationships to govern future major development. This not only protects community interests but also gives reliable guidance to design.

21.21 Special structures

Special structures include means of covering or enclosing large areas without internal supports (by means of beam, membrane, tension, skeletal or pneumatic structural action), space frames and tall buildings, each of which has its own particular field of application and specialized technology. They also include cases where the normal assumptions regarding structural action may not apply. For example, deep beams, in which the span:depth ratio is small (less than 5:1) behave differently from shallower beams and the normal theory of flexure does not apply. In portal frames, in which the beam spans are large in relation to the column lengths, the distribution of bending moments is highly sensitive to the relative stiffnesses of the members, and the normal assumptions of stiffness of reinforced concrete sections (whether cracked or uncracked) or of steel sections at yield stresses are insufficiently accurate. Similarly, the geometry of the system may exaggerate the effects of movement.

Membrane structures involve the use of thin surfaces geometrically arranged to support vertical loading mainly by forces parallel to the surface. They may be folded plates or singly curved, as in barrel vaults and arches, or doubly curved as in domes, hyperbolic paraboloids and other special forms of curved surface. While secondary bending effects are present at right angles to the surface, the main internal forces are parallel to it and it is essential that the surface and boundary conditions are geometrically correct for resisting the loading. The simple example is the dome, in which compressive forces occur within the dome, and ring tensions or external restraints are required at the perimeter. Concrete, timber and sometimes steel are used in forming long-span membrane structures; they use materials efficiently, but their economic viability depends mainly on the workmanship and labour required. Folded plates provide functional and aesthetically interesting roofs, in which normal flexural action occurs; additional considerations of edge support, end shear, buckling and distribution of out-of-balance loading also apply.

Tension structures involving cable-supported sheeting have been used in exhibition buildings and in sports stadia; the structural system involves steel cables acting in catenary from which decking or tenting is supported.

Skeletal space-frame structures are used in the form of plane grids of rectangular, diagonal ('diagrid'), triangular or hexagonal pattern, arches, domes and other structures analogous to membrane structures, in that the geometric shape of the surface is such that the principal resisting forces act parallel to the surface. These structures have been applied to sports stadia, exhibition buildings, aircraft hangars, terminals and other places requiring very large column-free areas.

Pneumatic construction uses air pressure in various ways to stabilize the membrane of the building. Such structures are light, economical, easy to erect, dismantle and transport, and have proved to be practically successful in application to housing temporary exhibitions, warehousing and covering sports halls and other specialist buildings. The basic engineering principle employed is that the membrane can accept tensile stresses and will fold when not in tension. Internal air pressure is used to maintain membrane tensions when dead and other loads are imposed. Various types of pneumatic structure have been developed, including the basic air-supported membranes and inflated dual-walled and ribbed structures, and various hybrid types. The larger spans are achieved by the use of arched or domed forms, and cables, cable nets and internal membrane walls are used to control shapes and improve stiffness.

21.22 Foundations

Foundations must safely transfer loading from the superstructure to the ground without excessive absolute or differential settlement. The foundation type will depend upon the nature of both the underlying soil and the superstructure since the two must be compatible as far as the settlement characteristics are concerned. In some cases, to economize in foundations, the

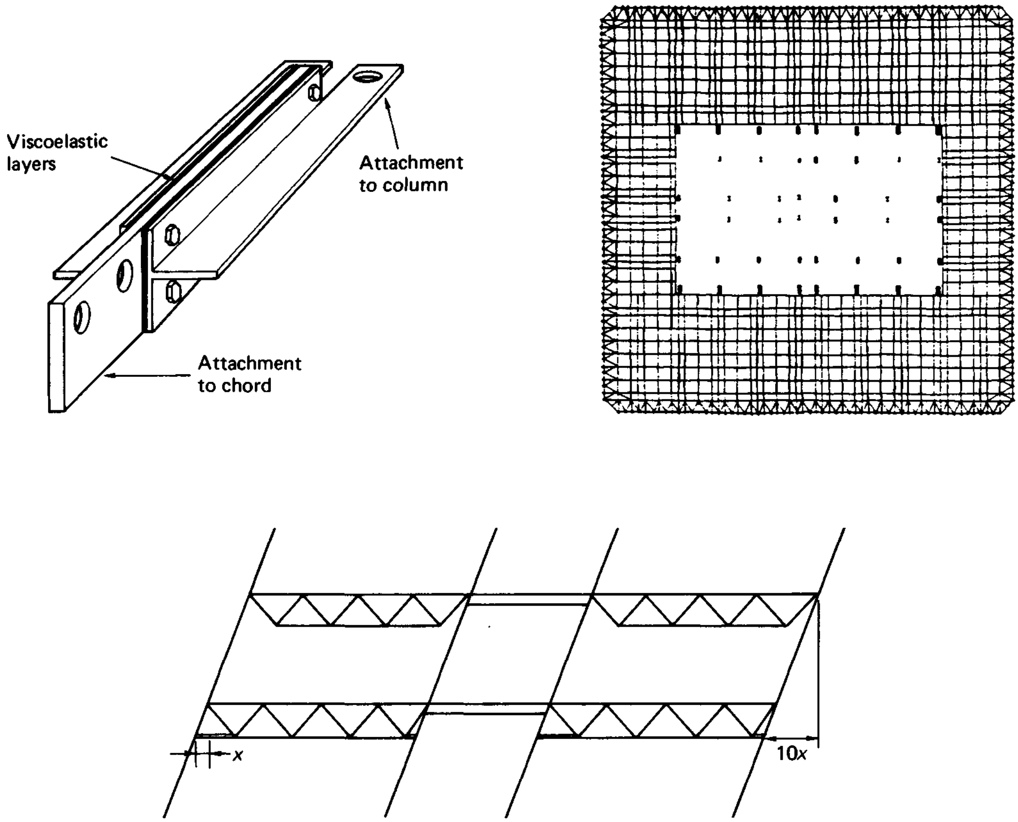


Figure 21.15 Dampers used in the World Trade Center, New York. (After Fischer (ed.) (1980) *Engineerng for architecture.* McGraw-Hill, New York)

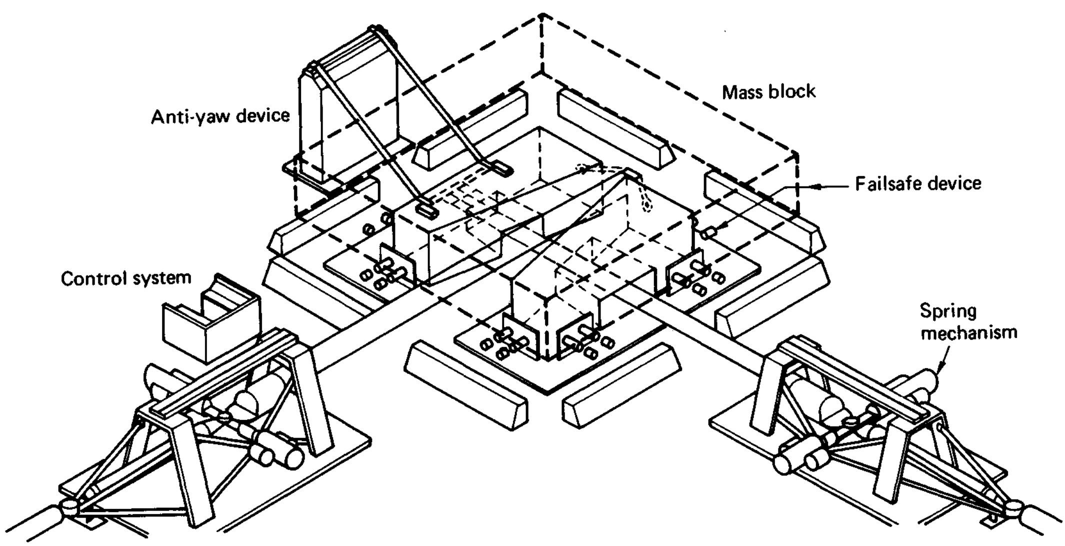


Figure 21.16 General arrangement of a tuned mass damper. (After Fischer (ed.) (1980) *Engineering for architecture,* McGraw-Hill, New York)

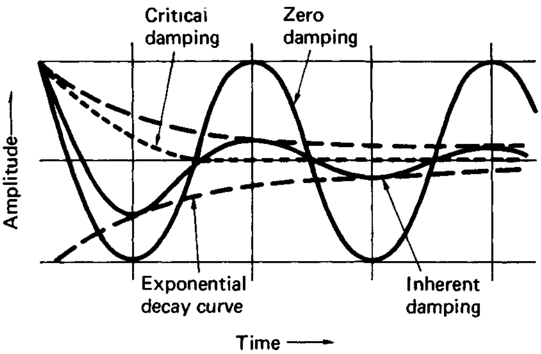


Figure 21.17 The impact of damping on building oscillation. (After Fischer (ed.) (1980) *Engineering for architecture,* McGraw-Hill, New York)

superstructure will be designed to allow for differential settlement by the incorporation of, for example, jointed construction in the structure, cladding and finishings. In other cases, a type of foundation will be adopted which limits settlements to amounts acceptable to the previously determined superstructure. In special instances, the superstructure may be designed to act compositely with the foundation.

In all cases the designs of the superstructure and of the foundations are interdependent; knowledge of the soil conditions is thus essential at all stages of the design process, beginning with at least broad local knowledge prior to land acquisition. Such information may also influence the location of particular buildings on the site.

The risk of mining subsidence must be assessed at an early stage and the appropriate course of action decided, e.g. whether to seal and grout the workings, pile through the workings, design the superstructure to accommodate subsidence, or to accept the risk without special action.

The main types of building foundation are individual pad footings, strip footings, rafts, piers and deep footings, deep-spread foundations and piles. Basement construction is useful in reducing settlements and in controlling differential settlements between parts of buildings of different height.

Ground floor slabs may be suspended, but normally bear directly on the soil; they are required to span local weaknesses or voids due to settlement and to transfer local loading to an appropriate area of the ground. Their design is largely an empirical process; specification, workmanship, joints and surface treatments are important.

Basement construction and ground floor slabs must resist penetration of water or water vapour to a degree determined by the building use; provisions may include: (1) land drainage; (2) good-quality concrete of suitable thickness and with appropriate workmanship and details particularly at the joints as to be adequately waterproof; and (3) the provision of internal or external waterproof membranes. In some buildings it may be more economic to accept some leakage and provide for discharging it than to attempt completely watertight construction. The presence of water may give rise to serious uplift problems in basements during or after construction unless adequately provided for in design.

The construction of deep basements through ground material other than rock is bound to cause ground movement which could affect both the new construction and adjacent property. The magnitude and extent of such movement will be affected by the method of excavation, the sequence of basement construction and any dewatering that may be required.

Top-down methods of basement construction incorporating diaphragm walls or secant piling can be effective in controlling movement. Where speed of construction is important, such methods also allow simultaneous erection of the superstructure. While mathematical modelling exists to predict ground movements, experience has shown that the most reliable indications are obtained by a combination of modelling and reference to basement construction through similar ground conditions. Frequent monitoring of ground movements during construction is necessary to ensure that no excessive movements are developing and to check the reliability of the mathematical predictions. The extent of movement that can be tolerated by existing adjacent property will depend upon its construction and foundation system.

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22 Hydraulic Structures

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Contents

22.1 Open channel structures

22.1.1 Basic concepts

22.1.1.1 The Bernoulli theorem and critical flow

Two important concepts in the hydraulics of flow through structures are the Bernoulli and pressure–momentum theorems. The former (see page **5**/8) expresses conservation of energy, and when applied to straight-line flow in an open channel, taking bed level as reference level, may be expressed as:

$$H = d + \alpha V^2/2g \qquad (22.1)$$

where H is the specific energy head, d the depth of flow above the bed, α coefficient, V the mean velocity and g the gravitational constant

Where the flow is curvilinear, depth will vary across the channel and d is a mean value. Under normal conditions of flow in wide uniform channels, $\alpha = 1.02$ for smooth boundaries but higher for rough boundaries. For example, if $n/d^{1/6} = 0.0225$ (where n = Manning's roughness factor) $\alpha = 1.12$. In order to simplify calculations where velocity head is relatively small, α is often assumed to be unity. Head loss must be allowed for in the value of H. For channels of rectangular cross-section, Equation (22.1) can also be expressed as:

$$H = d + \alpha q^2/2gd^2 \qquad (22.2)$$

where q is the discharge per unit width of channel Q/B where Q is the total discharge and B the width

To derive d from known H and q, with $\alpha = 1$ Figure 22.1 may be used.

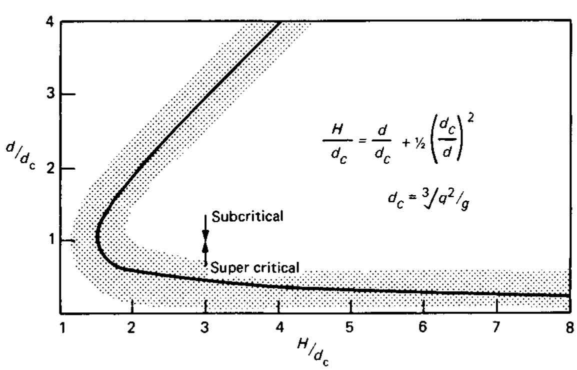


Figure 22.1 Specific energy of flow in open channels. Depth of flow d may be determined from specific energy head H and discharge per unit width q

In a channel of rectangular cross-section with horizontal bed and $\alpha = 1$ (as, for example, immediately downstream of a contraction)

$$\text{critical velocity } V_c = (gd)^{1/2} \qquad (22.3a)$$

and critical depth

$$d_c = V_c^2/g = (q^2/g)^{1/3} \qquad (22.3b)$$

In the more general case, applicable to non-rectangular channels of slope angle θ:

$$\text{critical velocity } V_c = \left(\frac{gd_m \cos\theta}{\alpha}\right)^{1/2} \qquad (22.3c)$$

For critical depth in circular and horseshoe-shaped channels see Figure 22.19 (page **22**/17).

22.1.1.2 Froude number

$F = V/(gd)^{1/2}$ is a useful indicator of the stability of free surface flow. When $F < 1$, the flow is subcritical; when $F = 1$ it is critical and when $F > 1$, supercritical. As F approaches unity from either direction, the flow becomes unstable and surface waves may develop. Surface undulations may occur in subcritical flow when F exceeds 0.5.

22.1.1.3 The pressure–momentum theorem

Unlike the Bernoulli theorem, this applies whether there is head loss or not. It follows from Newton's second law and can be expressed as:

$$P = M_2 - M_1 = \frac{w}{g} Q(V_2 - V_1) \qquad (22.4)$$

where P is the resultant force on a mass of fluid over a specified length, M_1 and M_2 represent momentum at entry and exit, w is the specific weight of fluid, Q the constant discharge and V_1 and V_2 are the flow velocities at entry and exit

P usually is the resultant of fluid pressures and boundary pressures in the direction of flow.

22.1.1.4 Hydraulic jump

This is a relatively abrupt change in flow depth when the flow changes from supercritical to subcritical as described on pages **5**/17 to **5**/19 and illustrated in Figure 22.2. Except at the limiting condition when both depths are critical, it involves a head loss, dissipated in extra turbulence. In Figure 22.1 it can be represented by a transfer from a point on the supercritical curve to a lower point on the subcritical curve. It may be stationary or moving. Its character and movement can be determined by application of the pressure–momentum equation (Equation (22.4)). In a rectangular channel of width B and horizontal bed, $P_1 = \frac{1}{2}Bd_1^2$ at entry and $P_2 = \frac{1}{2}Bd_2^2$ at exit, where d_1 and d_2 are depths; no other pressures have components in the direction of flow. If pressure plus momentum of the supercritical flow $(P_1 + M_1)$ exceeds the pressure plus momentum of the subcritical flow $(P_2 + M_2)$, the jump will move downstream, if they are equal the jump will be stationary and if $(P_2 + M_2)$ exceeds $(P_1 + M_1)$ the jump will move upstream.

For a stationary jump in a horizontal rectangular channel, the relationship between upstream and downstream depths is:

$$\frac{d_2}{d_1} = \sqrt{(0.25 + 2F_1^2)} - 0.5 \qquad (22.5)$$

where d_1 and d_2 are the conjugate depths, i.e. the depths of flow upstream and downstream of the jump, respectively, and F_1 is the Froude number upstream of the jump

A number of laboratory tests have shown close conformity to this relationship.

The jump height, $d_j = (d_2 - d_1)$, on a horizontal floor may be determined from Figure 22.2, which may be extended by use of Equations (22.1) and (22.5). The length of a jump cannot be precisely defined but is approximately 5 to 8 × d_j, the greater factor applying to lower Froude numbers.[1]

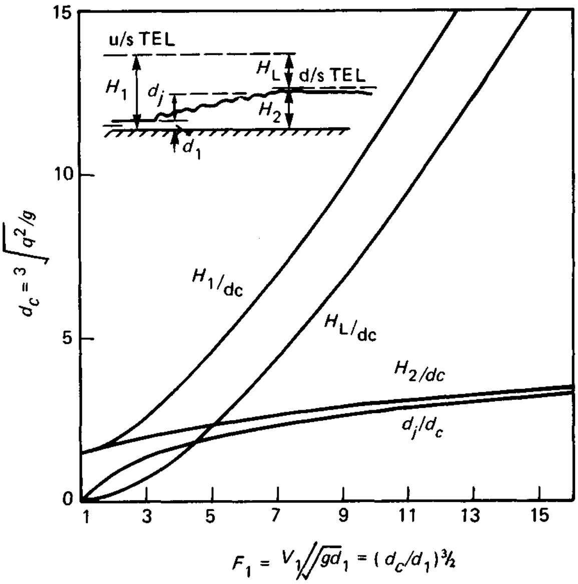


Figure 22.2 Hydraulic jump relationships for horizontal or gently sloping beds. (After Thomas (1958) Discussion on Bradley and Peterka (1957) op. cit. *Proc. Am. Soc. Civ. Engrs,* **84,** HY2, Paper 1616)

Equation (22.5) and Figure 22.2 give results with little error in channels with beds sloping at 10% or less, but with steeper slopes the components of vertical pressures have significant effect.

In channels which are not of rectangular section the jump may be distorted in plan, but the pressure–momentum equation (22.4) can be applied to the whole cross-section. Several methods for calculating the conjugate depths in channels of various shapes are available.[2–5]

22.1.2 Transitions

22.1.2.1 Subcritical flow

In channels of variable cross-section, Equation (22.1) or Figure 22.1 may be used to determine depth of flow, provided changes are sufficiently gradual to avoid significant head loss. In converging flow, q and hence d_c increase with the reduction in width. Therefore with subcritical flow and constant specific energy H, it is evident from Figure 22.1 that d reduces. As examples, a channel may be contracted at a bridge and allowed to expand downstream, or a gated regulator may have a raised sill. In both cases the surface is depressed in the contraction. Provided the flow remains subcritical the process is reversible in a downstream expansion. If, however, a contraction reduces the depth to the critical value, any further contraction has the effect of raising the upstream head, because critical depth is the mini-

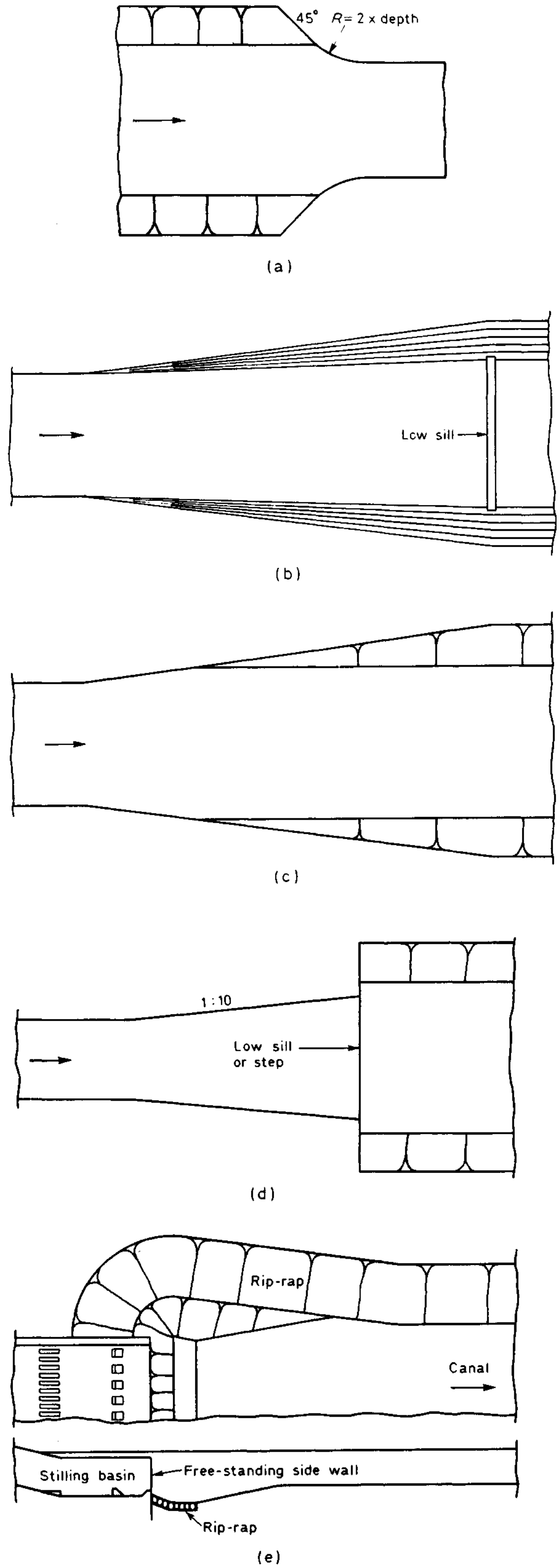


Figure 22.3 Typical transitions for subcritical flow. (a) Contraction from sloping to vertical sides; (b) warped expansion; (c) expansion with vertical sides; (d) short expansion; (e) example of transition from stilling basin to canal in erodible material

mum depth possible for any given specific head (see Figure 22.1). The result is a rise in upstream water level, the excess head generates supercritical flow downstream of the throat, or section of maximum contraction, and is lost in a hydraulic jump where the flow changes back to subcritical. The throat is then acting as a 'control'. If head loss is to be avoided, the Froude number should not be allowed to approach close to unity.

Convergences for subcritical flow may be rapid but external angles in the side walls should be avoided by the use of large-radius curves, as shown in Figure 22.3a. Diverging channels in subcritical flow are liable to result in separation of flow from one or both side walls unless expansion is gradual. Side expansions of 1:10 are usually satisfactory. Sharper divergences may be followed in some conditions;[6] expansion is assisted by a rising floor, baffle blocks or a raised sill downstream and by a hydraulic jump. The expansion ratio is also a factor – see page **22**/17 – where expansions in enclosed flow are discussed. Some examples of diverging transitions are shown in Figure 22.3b to 22.3e. Figure 22.3e illustrates a transition from a drop structure to the canal beyond.

Changes of direction cause head loss because of the secondary flow which distorts the flow pattern; the flow near the bed is deflected more sharply than the surface flow. If the bend is very sharp there may be complete separation at the inner boundary. These effects may be minimized by adopting a large radius for the bend. In rectangular channels with depth: width ratio of 0.6 to 1.2, Shukry[7] found that head loss became minimal with radius 3 × width. In channels with erodible boundaries, unless bank protection is provided, the minimum radius depends on the velocity and erodibility of bank material. On irrigation canals in India the radius is generally 20 to 30 × surface width.

22.1.2.2 Transitions – supercritical flow

The problems here are different from those discussed so far. Whereas in subcritical flow, pressure changes can be transmitted laterally from the side walls to the whole flow, inducing change of depth or direction, in supercritical flow the velocity of transmission of a small disturbance or wave is less than the flow velocity. The result is that a change in direction of a side wall creates an oblique shock wave which is reflected from side to side downstream.

Convergences and divergences should be very gradual. Figure 22.4 shows the shock waves created by a convergence. A sharp convergence may cause high-velocity flow to ride up and overtop the wall. It is therefore preferable, if possible, to locate convergences and other changes in wall direction where the velocity is low, e.g. at the upstream end of a chute, and maintain a straight chute where velocity is high. It may, however, be possible to use lateral inclination of the bed, e.g. superelevation, to assist in convergence or divergence. Where shock waves are unavoidable, they will occur in a zigzag pattern for some distance downstream owing to reflection from side walls. The side walls should therefore be high enough to contain them at points of reflection. Sloping side walls, as in trapezoidal channels, are particularly vulnerable. Methods are available for the calculation of pattern and height of shock waves in simple cases and for minimizing their effects.[9] Scale models may also be used.

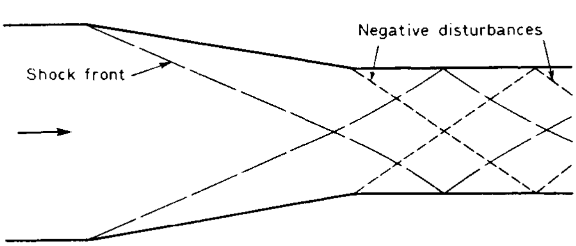


Figure 22.4 Example of shock waves at convergence in supercritical flow. (After Ippen *et al.*, (1951) 'High-velocity flow in open channels.' *Trans. Am. Soc. Civ. Engrs,* **116**, Paper 2434)

Long-radius bends are preferable to short radius, especially where overtopping is a danger. Knapp[9] recommends compound curves for the side walls of bends, with radius $2r$ in the approach and exit over a length of $B/\tan\beta$ in each case, where r is the radius of the centreline of the main curve, B is the channel width and $\sin\beta = F$, the Froude number. This arrangement creates counter waves which tend to neutralize the shock waves generated by the main curve, so reducing disturbances downstream.

22.1.3 Weirs and flumes

22.1.3.1 General

Weirs are used to control flow or water levels, or to measure flow. They range from low walls across streams to the spillway crests of high dams.

The basic equation for free flow over weirs is:

$$q = c(2g)^{\frac{1}{2}}H^{3/2} \tag{22.6}$$

where q is the discharge per unit width, c is a discharge coefficient, g the gravitational acceleration and H the total head level upstream above weir crest, normally taken as $h_1 + V_1^2/2g$, where h_1 is the upstream depth of flow above weir crest level and V_1 is the mean velocity of approach

Equation (22.6) can be derived from Equations (22.2) and (22.3) assuming critical flow and applying a coefficient c to take account of departure from flow on a horizontal bed. The coefficient depends on the shape of the weir and, in general, it varies with head over the weir; only in a few special cases is it constant. There are many weir profiles, each with different characteristics in relation to discharge coefficient and modularity. Weir flow is said to be 'modular' or 'free flow' when it is unaffected by tailwater level. The point at which a rising tailwater begins to affect the upstream head or flow is termed the 'modular limit', expressed as the ratio of downstream to upstream depth above crest level. Values of the coefficients of weirs of many different profiles have been published, e.g. by King and Brater[9] (see also section 22.5). In this section, some types in general use are considered as follows.

Sharp-crested weirs. These are formed of metal plates and are used for precise measurements of flow. Flow over weirs with narrow crests having rectangular upstream corners is effectively sharp crested, with a coefficient c approximately 0.406, provided the nappe springs clear and is fully vented.

Triangular profile weirs. These have sensibly constant coefficients throughout their modular range; no venting is required and the coefficient is greater than that of a sharp-crested weir. For example, the Crump weir (Figure 22.7), with 1:2 upstream and 1:5 downstream slope, has a free-flow coefficient c of 0.442 and a modular limit (within 1% of discharge) of 0.74. Weirs of this type are widely used for measurement of stream flows.

Trapezoidal profile weirs. Trapezoidal profile weirs have flat upstream and downstream slopes and narrow horizontal crests, formed by the gate sill, are often used in gated controls and barrages (see, for example, Figure 22.10). They have a free-flow coefficient which is variable but generally exceeds 0.383 and under drowned conditions the afflux is small.

Broad-crested weirs. These have horizontal crests wide enough for parallel flow effectively to develop. Control is then at the point of critical depth so that $c=1.70$. To ensure that this condition applies and c is constant, the upstream edge should be rounded to avoid the formation of a roller above crest level. In practice, the value of c is 1 to 3% lower due to friction loss. If the downstream floor falls at a gentle slope, say 1:10, the modular limit is between 0.7 and 0.8. Broad-crested weirs have been extensively used for flow measurement and for proportional distribution of flow at dividing points in irrigation systems.

Free-nappe profile weirs. Free-nappe profile weirs with profile according to the shape of an undernappe of flow over a sharp-crested weir (Figure 22.5) have been widely used for overflow spillway crests. The standard profile is one with vertical upstream face and weir height P large compared with head over crest, H. The profile varies with smaller values of P/H and sloping upstream faces. This profile has the advantages that c is comparatively high for the profile discharge (i.e. the discharge corresponding to the nappe profile used), subatmospheric pressures do not develop within the range up to profile discharge, no venting is required and the flow characteristics are well documented and predictable.

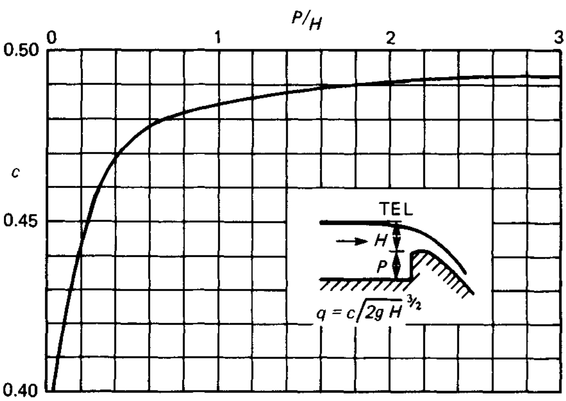


Figure 22.5 Discharge coefficient of free-nappe weirs at design discharge. (Based on USBR data; US Department of the Interior (1960) *Design of small dams.* Denver, Colorado)

The coefficient c of the standard weir at profile discharge is shown in Figure 22.5. By adopting a profile discharge lower than the maximum discharge, a higher coefficient is obtained at flows exceeding profile discharge.[10] Discharge in excess of the profile discharge causes pressures on the face of the weir to fall below atmospheric in the vicinity of the crest where the curvature is sharp.[11] This is usually acceptable provided that the structure is safe against uplift, and a reasonable margin of pressure is allowed above cavitation level to allow for fluctuations.

Profile coordinates have been published[12] from which weirs of standard profile and some variations can be designed.

Sharp side contractions at the abutments of weirs reduce the discharge capacity locally. They should be curved as in Figure 25.3a. Piers have a similar effect, to avoid which spillway piers are often extended upstream, so that the contraction at the pier noses occurs in a region of lower velocity.

22.1.3.2 Submerged weirs

The effect of a tailwater level above the modular limit is to raise the upstream water level for a given discharge. The degree to which the upstream head or discharge is affected depends on the weir profile: moreover in certain ranges of submergence the flow

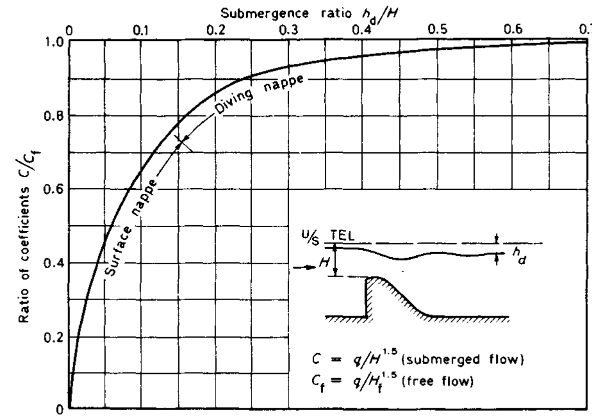


Figure 22.6 Free-nappe profile weirs. Effect of tailwater level on discharge coefficient. (Based on USBR data; US Department of the Interior (1960) *Design of small dams.* Denver, Colorado)

pattern is uncertain and may change from diving nappe, which follows the downstream weir face, to surface nappe, which separates near the weir crest, a roller developing beneath. Observations of discharge related to upstream and downstream heads or water levels therefore cannot be regarded as of general application. Nevertheless, good indications can be obtained. Figure 22.6 shows the effect of submergence on standard free-nappe profile weirs[12] and Figure 22.7 the effect on Crump triangular profile weirs.[13]

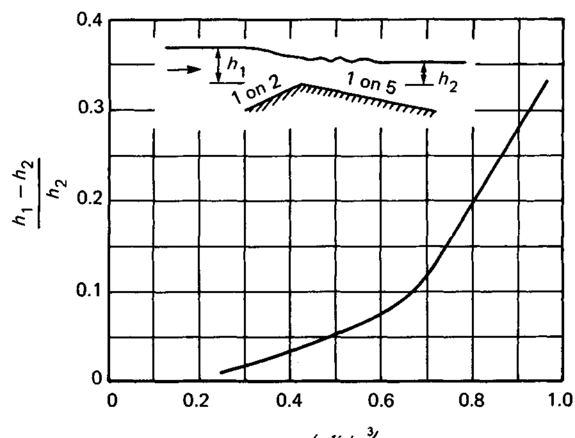


Figure 22.7 Afflux at submerged Crump weirs. (Based on data of White (1971) 'The performance of two-dimensional and flat-vee triangular profile weirs.' *Proc. Instn Civ. Engrs,* Paper 7350S.)

22.1.3.3 Measuring weirs and flumes

For laboratory and other small-scale measurements, sharp-crested weirs consisting of thin plates in the form of rectangular or Vee-notch weirs are found convenient. Standard formulae or tables of discharge for these are available.[9,14] For measurement of larger flows in the field, however, sharp-crested weirs have drawbacks, particularly the need to vent the nappe, the head difference required to ensure modular conditions and the effect of accretion of upstream bed level following the erection of a gauging weir.

Weirs of several other types have been thoroughly investigated and are subjects of national and international standards. A comprehensive account of the performance and use of the main types of weir and flumes is given by Ackers *et al.*[14] Bos[15] has reviewed a wide range of devices capable of use for flow measurement. Those most useful in a civil engineering context depend on the creation of critical flow. This can be induced by providing a sill or weir, or by contracting the width, or by a combination.

The critical flow formula in a rectangular cross-section channel, Equation (22.6), is the most basic formula for flow measurement by weirs and flumes, and applies to free flow, i.e. when the critical flow at the crest of a weir or in the throat of a flume is not drowned by the tailwater level exceeding the modular limit (see para 22.1.3.1). The cross-section of flow may also be made non-rectangular – V-shape, U-shape or trapezoidal – for particular applications, but then adjustment has to be made to the flow formula of Equation (22.6). For precise measurement with weirs and flumes, allowance for boundary layer development and other secondary effects has to be made.[14]

Unless there is already a local drop in level, the introduction of a measuring device will result in a rise in upstream level, though this may be quite small if a device with high modular limit is chosen, or a Crump weir with crest tapping which can be used when drowned by high tailwater level.[13] Where the range of discharge is large and it is desired to obtain an accurate measurement of low flows, a stepped weir may be used, consisting of a short weir at low level for the low flows flanked by longer weirs at a higher level. Alternatively, a flat Vee-weir may be used with crest tapping for submerged conditions.[13]

In the UK, broad-crested weirs with a round nose and Crump weirs have been accepted as standard.[16] In the US, Parshall measuring flumes have been widely used.[17] These were designed with plane surfaces so that they might be easily constructed of wood or concrete. c is not constant but calibration formulae and tables are available. Where the stream to be gauged carries appreciable bed load, a critical-depth flume with a flat or nearly flat bed at the channel bed level is desirable. The bed load can then pass through without excessive accretion upstream, though there may be some at the sides. A measuring flume of this type is shown in Figure 22.8. The degree of contraction sufficient to ensure modular flow can be checked by comparing calculated upstream water levels (using $c = 1.66$) with existing tailwater levels. The broad-crested weir coefficient is applicable, adjusted for head loss upstream of the location of critical depth.[14]

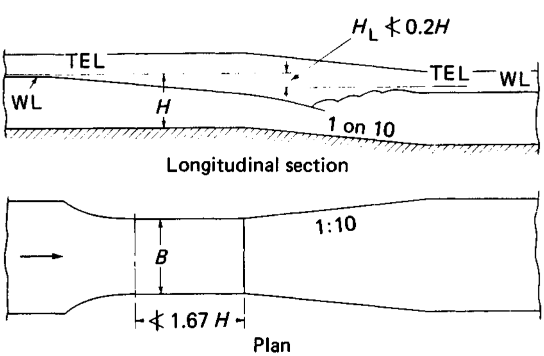


Figure 22.8 Measuring flume with flat floor for debris-laden flow

Structures of many other types are used for flow measurement, mostly depending on the critical depth principle or on orifice control as, for example, devices on irrigation canal outlets.[18]

22.1.4 Control weirs and barrages

22.1.4.1 Gated weirs

Weirs are used to control the water levels of a river or canal, for such purposes as diversion of flow into canals, extraction of water by pumping, creating head for hydro-electric power or maintaining a required depth of water for navigation. A fixed weir also raises flood levels, which may not be acceptable. A gated weir, or barrage, however, does not have this drawback if the gate sill is level with the river bed, or on a low weir crest. The gates are kept closed during low flows, maintaining the required upstream water level, but opened as may be necessary to pass floods. The range of water level is thus much less than with a simple weir, and the gates can be operated to maintain constant water level over a wide range of flow. Types of gates are described on pages **22**/11 to **22**/28.

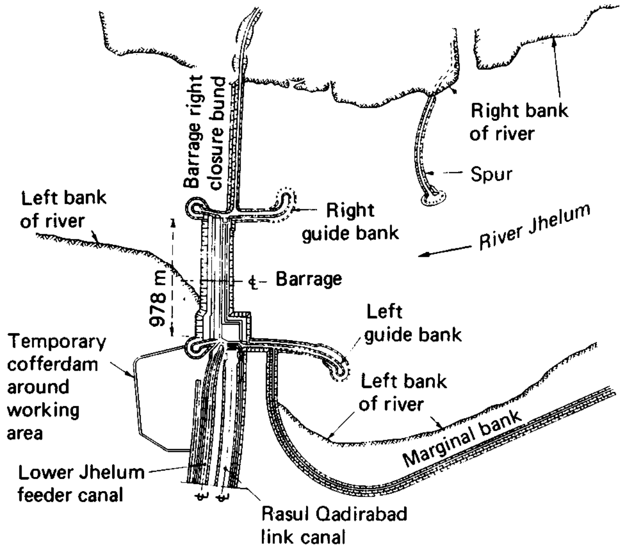
The choice of crest profile depends on the circumstances. For example, a weir with a free-nappe profile is suitable where the crest is to be above the upstream channel bed and there is considerable head difference from upstream to downstream. On the other hand, a low crest with flat triangular profile is better suited where, at high rates of flow, the afflux or rise of upstream water level due to the weir must be kept to a minimum.

22.1.4.2 Control structures in alluvial rivers

Whereas structures in rivers with rocky beds and banks can often be of simple design, with an upstream cutoff wall into the rock and a basin or bucket energy dissipator downstream, the design of control structures in alluvial rivers requires consideration of many other factors.

Firstly, the site and orientation of the structure in relation to the river channel pattern is most important and generally should take priority over other considerations. Alluvial rivers without constraint by structures, training works or outcrops of rock or clay, may change course over a period of years, forming new patterns of river channels. The history of a river course is a good guide to such tendencies. The site for a control structure should be a stable one in the long term, i.e. it should remain operative despite changes in the channel pattern over a number of years, maintained if necessary with the aid of training works. Where a weir or barrage is used for diversion or abstraction of water it is usually desirable to ensure that the quantity of sediment in the water abstracted is a minimum. The best location for the offtake with this in view is generally on the outside of a bend, and the training works should be located to maintain the approach channel accordingly. This consideration applies even where special arrangements are made for sediment exclusion.

A typical barrage forming the headworks of an irrigation canal system on a large river in Pakistan is shown in Figure 22.9. A weir or barrage may occupy only a small part of the width of river channel and floodplain. For example, in India and Pakistan it is general practice to make the width of waterways between abutments equal to or rather greater than the width of Lacey regime channel $4.8Q^{1/2}$ where Q is the maximum design discharge in cubic metres per second.[19] Flanking bunds or embankments are then required extending from the abutments to high ground on either side. Where flood levels are being raised by the control, marginal bunds or flood embankments are often provided extending upstream on each bank. To prevent oblique approach, protect the bunds and avoid outflanking; guide banks are required extending upstream from the abutments (see Figure 22.9). In stable rivers these may be quite short, but where there may be wide swings in the river course they are generally approximately equal in length to the width of waterway between them. In addition, in rivers of this type, spurs or groynes may be provided upstream, but these may cause further



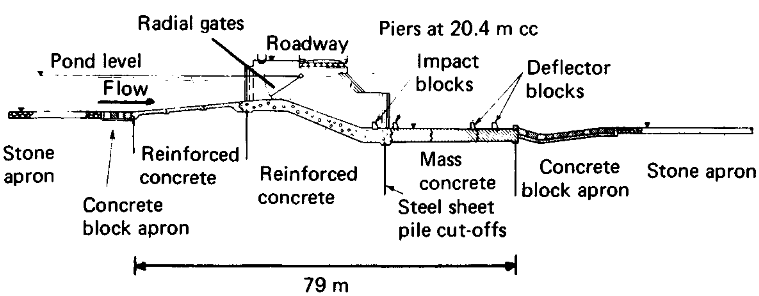


Figure 22.9 Rasul barrage on the river Jhelum, Pakistan. General layout (top), longitudinal section (above). Note the flow from right in layout (top) and from left in section. (Consulting Engineers: Coode and Partners)

trouble unless correctly located. Model tests are desirable before construction. Similar measures are used to train alluvial rivers at bridges. The guide banks and spur heads are protected against scour, by rip-rap or concrete slabs (see pages **22**/15).

Low-level sluices provided in the weir or barrage, generally adjoining the canal regulator, have three functions: (1) they discharge river flow during construction at a low level; (2) during operation of the works they enable the approach to the regulator to be sluiced at intervals to remove deposit of sediment deposit; and (3) if kept open during a flood they draw the main stream towards the canal regulator, thus maintaining a deep channel for water to gain access to the intake during the dry season. To fulfil these functions the sill should be well below the canal regulator sill level and the sluices should have sufficient capacity to influence flood flow distribution. A divide wall is often provided normal to the weir between undersluices and weir to enable the canal to draw supplies from a pocket of low-velocity water, the undersluices being kept closed. A divide wall also facilitates the sluicing operation. If the canal must operate continuously, control of coarse sediment can be provided by tunnels beneath the level of the canal regulator sill, which draw off the bed load and discharge it downstream.[20]

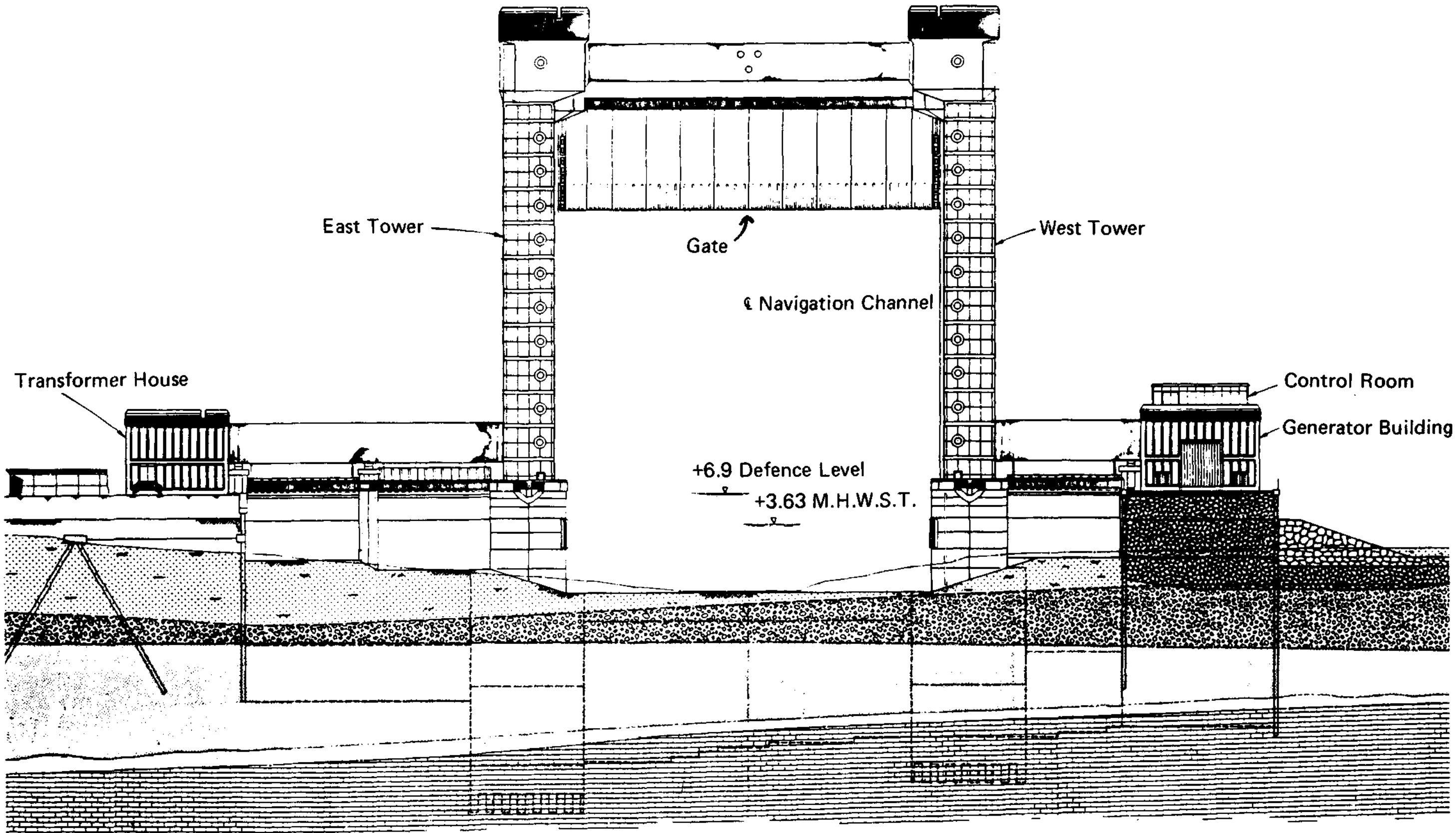
Downstream of the weir and undersluices, a floor is provided to protect the foundations against scour (Figure 22.9). The drop in water level across the weir or undersluices is accompanied by the formation of a hydraulic jump, except possibly at high flood flows when it may be drowned. A flexible apron of loose stone or concrete blocks is beneficial as an extension to the floor.

For design of floor and apron see page **22**/15. To allow for nonuniform discharge distribution, the design discharge per unit width of floor should exceed the mean by an allowance depending on the approach conditions. In India and Pakistan a factor of 20% has generally been added for alluvial rivers but in extreme conditions it should be higher, e.g. where curvature of approach could cause a high concentration.

22.1.4.3 Irrigation canal structures

Canal head regulators are usually located immediately upstream of a weir or barrage (see Figure 22.9). On alluvial rivers the intake should be well above the sill of the undersluices. A stilling basin of sufficient depth, to ensure that the hydraulic jump is retained within it, is essential where the canal bed is erodible, and is also generally provided where the canal is lined.

Where the general ground slope exceeds the design slope of a canal, falls or drop structures are required at intervals to dissipate the excess head and lower the canal to conform to the ground level. Falls are designed in a similar way to weirs, with ungated crest and stilling basin. To reduce cost, the width of waterway is often made less than the width of canal. The upstream contraction presents little difficulty, but the downstream expansion must be gentle to avoid asymmetrical flow downstream (see page **22**/4).



NORTH ELEVATION
(LOOKING DOWNCREEK)

Figure 22.10 Tidal barrier, Barking Creek. (Consulting Engineers: Binnie and Partners)

22.1.4.4 Tidal barriers

The risk of serious flooding from tidal surges penetrating inland via estuaries and tidal inlets has led to several major schemes for tide-excluding barriers. These are in the form of gates, perhaps single gates for schemes of modest size but multiple gates for major estuaries. Navigation is often the controlling feature determining the necessary span, the elevation of the sill and the clearance under any structure spanning over the waterway.

Very large vertical lift gates have been used as tidal barriers and one example, at Barking Creek in the Thames Estuary,[21] is illustrated in Figure 22.10. The gate normally rests at the top of its support towers, thus providing clearance for navigation by medium-sized ships.

Rising sector gates do not require a high supporting structure because they normally rest below the bed of the navigation channel. The main gates of the Thames Barrier are of this type and their operating mechanism is such that they can be rotated through 180° from their normal position in their sills on the estuary bed to raise them above water level for maintenance. They turn through 90° to close the barrier against the tide (Figure 22.11). The rising sector gates in the four main spans of the Thames Barrier have a span of 61 m and effectively they form box-girders between their end wheels. There are six subsidiary gates with spans of 31.5 m.[22]

22.1.5 Permeable foundations

Special consideration is required if a hydraulic gradient will exist across a structure founded on permeable materials. Examples include weirs, regulators across canals, barrages and tidal barriers. Two important requirements are that every part of the structure must be safe against uplift pressures beneath and that underflow or seepage through the permeable materials should be controlled so that there is no failure by 'piping'. Where a continuous impermeable stratum is within reach, underflow can be prevented by a line of sheet piles or a curtain wall intersecting it, or possibly by grouting, but the sealing must be perfect. If, however, the permeable materials are too deep for this treatment, the floor must be safe against uplift pressures exceeding the tailwater level acting on the underside of the structure throughout.

Uplift depends on the hydraulic gradient of flow through the material beneath the work, reducing from the upstream water level to the downstream water level. Its distribution may be affected considerably by the nonuniformity of the materials so a prior investigation of the character of the material, its uniformity and the existence of strata of different permeability is necessary. The floor upstream of a weir or gates is safe against uplift because of the water load above but the downstream floor is particularly vulnerable at times of high upstream and low downstream water levels. Measures to reduce uplift pressures on the downstream floor include the lengthening of the upstream floor and provision of transverse lines of sheet piling upstream or beneath the weir, both serving to lengthen the effective seepage path, and provision of relief drains. Typical protective measures beneath a gated structure are shown in Figure 22.9. 'Piping' consists of the removal of foundation material by the flow of seepage water. It can occur at the tail end of a structure where the underflow emerges and is a potential cause of undermining and ultimate failure of the structure. It is caused by excessive exit gradient. Information on flow nets to determine uplift pressures and exit gradient is given in Chapter 9.

It is usual to protect against piping, where the foundation material is granular, by providing coarser filter material to intercept the seepage over its exit area. This is generally covered by loose stone or other protection against scour, but in case this should fail, other measures are needed to reduce the exit gradient. Such measures include the lengthening of the structure and the provision of transverse lines of sheet piling to reduce the overall hydraulic gradient, provision of relief drains and the provision of a curtain wall or line of sheet piling at the tail end of the floor. The last is most important to avoid a locally steep gradient and protect the floor from undermining by scour, but it should not be too deep because it increases uplift beneath the floor. The upstream or central sheet piling should extend laterally into the flanking embankments, and lines of piling are carried around as may be necessary to intersect seepage paths and box in the foundations. For general design procedures, reference may be made to Haigh,[20] Leliavsky[23] and Foy and Green.[24]

22.1.6 Energy dissipation

22.1.6.1 Stilling basins

At weirs, barrages, sluices, spillways, tunnel outfalls, canal falls and in general where a sharp fall occurs in total energy level, a stilling basin is needed to contain the flow in the region of energy dissipation. This is especially important where the channel bed is erodible. The surplus energy may be dissipated by water spilling into a pool, which may be in bed rock, or lined with rip-rap or concrete.

In most cases the energy head to be dissipated is sufficient to create supercritical flow, defined on page **22**/3. A hydraulic jump is then generally the most effective and economical way of dissipating the surplus energy. The object is to provide a stilling basin lined with nonerodible material, usually concrete, deep enough to retain the jump over the whole range of flow conditions and long enough for the eddies generated in the jump to be reduced to an acceptable intensity before reaching the channel downstream. The minimum depth is thus related to the characteristics of the jump while the minimum length is related also to the degree of stilling required. Where the channel bed is erodible, a greater length of basin is generally required than where it is in rock or is concrete-lined. In the basin, chute blocks, baffle blocks or piers are often provided to help to stabilize the jump and reduce the length of basin required.

As shown earlier, the stability of a hydraulic jump is expressed by the pressure–momentum equation (Equation (22.5)) representing the condition at which the jump is at its limit of stability, i.e. any increase in discharge or upstream head would cause 'sweep-out' or movement of the jump downstream and possibly out of the basin. In the design of stilling basins, however, the quantities which are known are usually the discharge, head drop and tailwater level and it is required to determine the basin floor level. Equation (22.5) therefore cannot be applied directly, but the maximum acceptable floor level can be easily found with the aid of Figure 22.2. The procedure is to compute upstream and downstream total energy levels (water level + velocity head), compute $H_L = H_1 - H_2$ (see Figure 22.2), compute critical depth d_c by Equation (22.3a), compute H_L/d_c, read off H_2/d_c directly beneath H_L/d_c, i.e. for same F_1, and compute H_2. This gives the minimum depth of basin floor beneath tailwater total energy level. It applies to a plain floor and may be reduced by 10 to 20% if chute blocks and/or baffle blocks and end sill are provided. However, it is often the practice to provide the full depth and consider the blocks to provide a safety margin in addition. It is usually necessary to determine minimum basin depth for several discharges throughout the range, because the most severe case is not always with the maximum discharge. When determining q in cases of nonuniform distribution across the basin it may be necessary to use a value rather higher than mean $q = Q/B$, where Q is the total

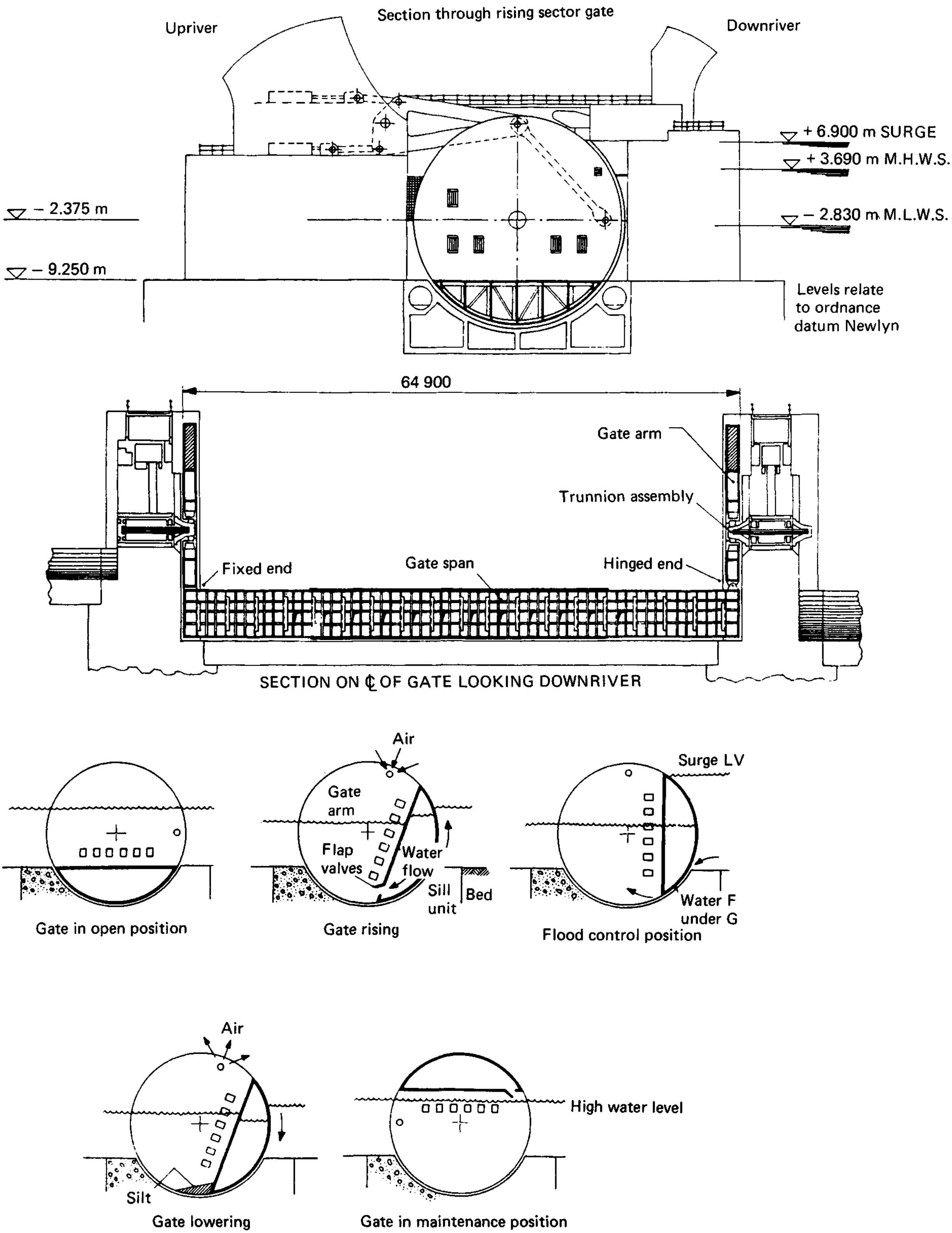


Figure 22.11 Thames Barrier, 61 m span rising sector gate. (Consulting Engineers: Rendel, Palmer and Tritton)

discharge and B the width. Tailwater level is clearly of critical importance for the stability of the jump and it is necessary to have a reliable stage discharge curve, with allowance for future changes as, for example, due to channel bed degradation downstream. The lowest probable levels should be used. In the case of basins for gated spillway releases, where discharge may be increased rapidly over a short period, allowance should be made for low tailwater levels due to time lag.

The length of basin required cannot be defined so precisely. On a plain floor the length of a jump may be 4 or 5 times the depth d_2 in the basin. If residual eddies can be tolerated downstream because the bed is not erodible or is protected by a flexible apron, as in Figure 22.12, a length of $4d_2$ may suffice. Where chute blocks and baffle blocks are provided in such cases, a length of $2.5d_2$ is sometimes considered adequate (but see below).

Many standard designs of hydraulic jump stilling basins have been developed from model tests, one of the most comprehensive being that of Bradley and Peterka.[25,26] Four types of jump were defined according to the Froude number F_1, each with somewhat different characteristics, namely:

F_1 from 1.7 to 2.5	Pre-jump, low energy loss
F_1 from 2.5 to 4.5	Transition, rough pulsating water surface
F_1 from 4.5 to 9.0	Range of good jumps least affected by tailwater variations
F_1 exceeding 9.0	Effective but rough

If F_1 is in the range 2.5 to 4.5 the pulsations are likely to produce surface waves which are propagated downstream. The Froude number is generally determined by other factors, but if there is any choice it is clearly desirable for it to be within the range 4.5 to 9.0. Bradley and Peterka's basin III for F_1 between 4.5 and 9 is shown in Figure 22.12. The dimensions of the chute blocks are made equal to the depth d_1 and those of the baffle blocks range from $1.3d_1$ for $F_1 = 4$ to $3d_1$ for $F_1 = 14$. The height of end sill ranges from $1.2d_1$ for $F_1 = 4$ to $2d_1$ for $F_1 = 14$.

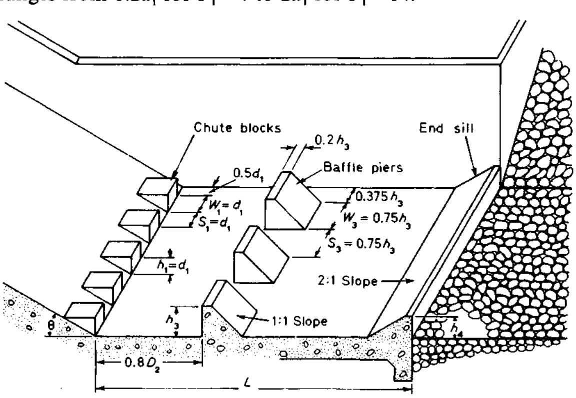


Figure 22.12 US Bureau of Reclamation stilling basin, type III. (After Beichley (1978) 'Hydraulic design of stilling basin for pipe or channel outlets.' USBR Water Resources Research Report No. 24)

Where F_1 is between 2.5 and 4.5 (basin IV) the chute block height is $2d_1$ and the baffle blocks are omitted or, according to Bhowmik,[27] a special arrangement of blocks and deflector may be provided to give improved jump stability. Where F_1 exceeds 9 (basin II) the baffle blocks are omitted and a dentated end sill is recommended. Basins II and IV, having no baffle blocks, are required to be longer than basin III, with floor lengths of approximately $4d_2$. In the case of high head structures, if the velocity much exceeds 15 m/s, chute blocks and baffle blocks are liable to be damaged by cavitation. They can be omitted or protected by steel cladding, as at Mangla Spillway.[28]

Erosion of bed and banks immediately downstream of the stilling basin can be a serious problem, whether the head drop through the structure is great or small – see remarks on transitions, page 22/4.

A normal cause of erosion is the residual turbulence from the hydraulic jump. This may scour the bed beneath the level of the basin floor, so a flexible protection such as rip-rap is needed which will adjust its level to the scoured bed downstream of it (see page 22/8). When the banks are formed of erodible material they need slope protection to guard against local velocities and wave wash. In the case of weirs and barrages on alluvial rivers the banks are carried downstream a short distance – perhaps equal to a quarter of the width of river channel (see Figure 22.9). A loose stone apron is provided at the toe. In the case of canals where the banks are erodible, the slope protection is continued for a distance in which the surface waves will be reduced and velocity distribution will become normal.

A layout of stilling basin and canal banks which has been found satisfactory is shown in Figure 22.3e. The gently diverging side walls are free-standing at their downstream ends, where they consequently do not have to serve as high earth-retaining walls; the channel downstream is widened to accommodate the side rollers which will develop and the banks are protected by rip-rap.

In the case of small flows, shorter and simpler structures have been used, e.g. the straight-drop spillway basin of the US Department of Agriculture.[29]

For large flows and high heads, experience has shown that hydraulic jump basins are generally satisfactory. Damage which has occurred has been due mainly to the basin being of inadequate depth, to cavitation where baffle blocks have been exposed to high velocity flow and to abrasion due to loose materials in the basin.[30,31] In some cases these materials may have remained from river diversion operations but in other cases bed material and even rip-rap has been carried into the basins by backwash. There have also been instances of vibration and shock due to flow instability. In large-scale basins it is especially necessary to guard against flow separation at the side walls, which can be a cause of both these last effects and of backwash.

22.1.6.2 Bucket energy dissipators

The hydraulic jump stilling basins described above are effective but costly, especially for high-discharge concentrations. Where the foundations of the structure are in rock, even an erodible rock, a much higher degree of residual turbulence may be acceptable.

A submerged roller bucket (see Figure 22.13) is suitable over a wide range of Froude numbers. The bucket is placed well below the tailwater level so that a submerged roller forms in the bucket and exit velocities are not excessive. Compared with a hydraulic jump basin, it is deeper but shorter and generally less costly; but

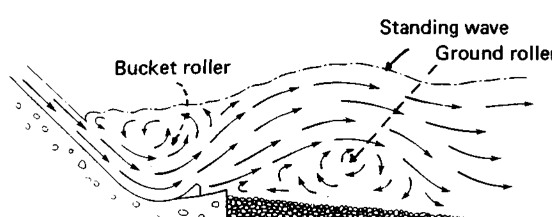


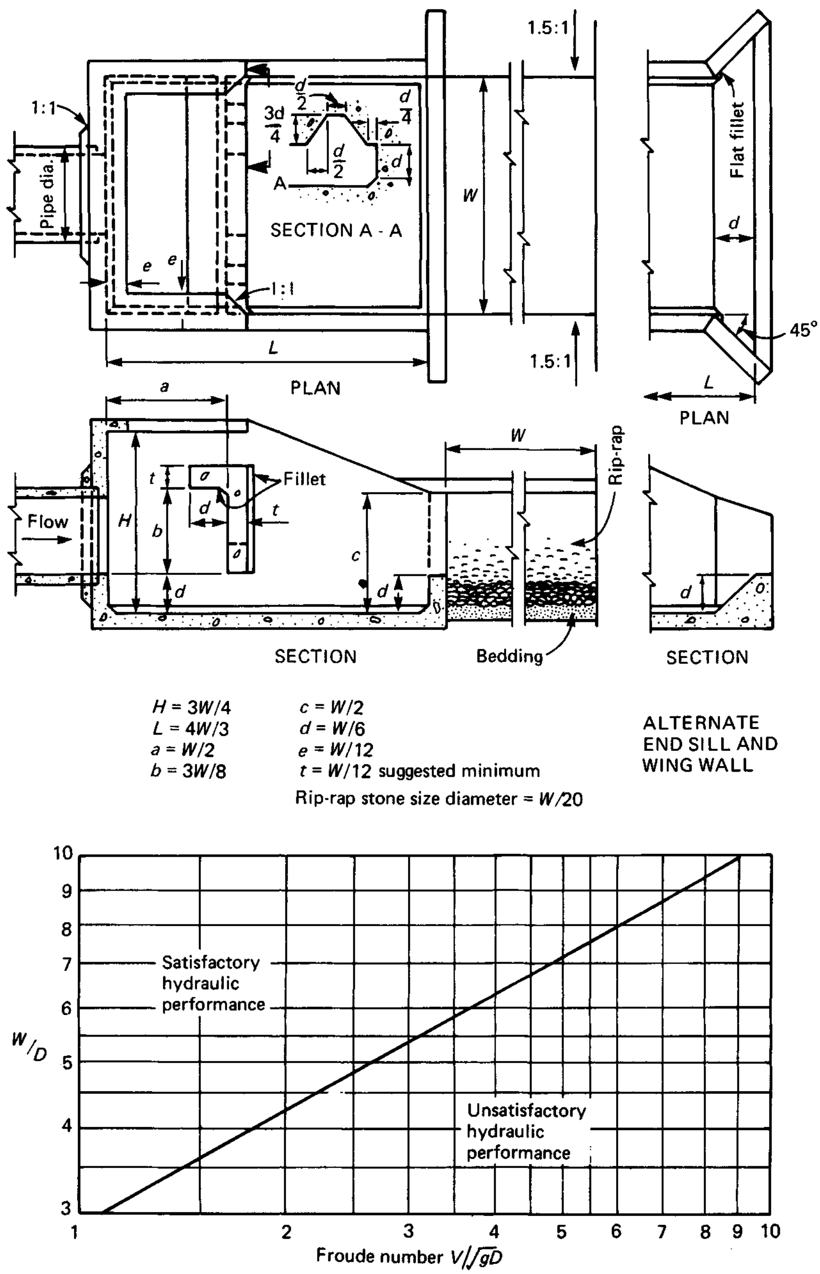
Figure 22.13 Submerged roller bucket – Angostura-type slotted bucket. (After Beichley and Peterka (1959) 'The hydraulic design of slotted spillway buckets.' *Proc. Am. Soc. Civ. Engrs,* **85**, HY10)

the range of tailwater level for satisfactory operation is limited, which precludes its use in some cases. Rules for design have been given by McPherson and Karr[32] and by Bleichley and Peterka[33] who found that slotted buckets were superior to plain buckets.

22.1.6.3 Terminal structures for pipes and valves

High-velocity jets from pipes and terminal valves have considerable erosive power, even on hard rock. Means of protection include the use of valves which disperse the jet in the air, e.g. the cone valve, or valves which project the jet some distance, where a plunge pool can be provided, or structures devised to contain the jet and allow most of the energy to be dissipated before discharge into an erodible channel.

Figure 22.14 shows an impact stilling basin developed by the US Bureau of Reclamation (USBR)[5] for pipe and open-channel outlets with discharges up to 10 m^3/s and velocities up to 9 m/s. It may also be considered for terminal valves within the limits



W = inside width of basin
D = square root of area of flow entering basin
V = velocity of flow entering basin
Tailwater depth uncontrolled

Figure 22.14 US Bureau of Reclamation impact-type energy dissipator – basin VI. (After Beichley (1978) 'Hydraulic design of stilling basin for pipe or channel outlets.' USBR Water Resources Research Report No. 24) stated. The required dimensions may be obtained from Figure 22.14.

A special basin has been developed by the USBR for hollow jet valves.[5,34] Basins have also been used for cone valves. A very effective energy dissipator for pipe outlets is a vertical well in which the pipe outlet is deeply submerged at a short distance above the bottom. Figure 22.15 shows a USBR type of well.[35]

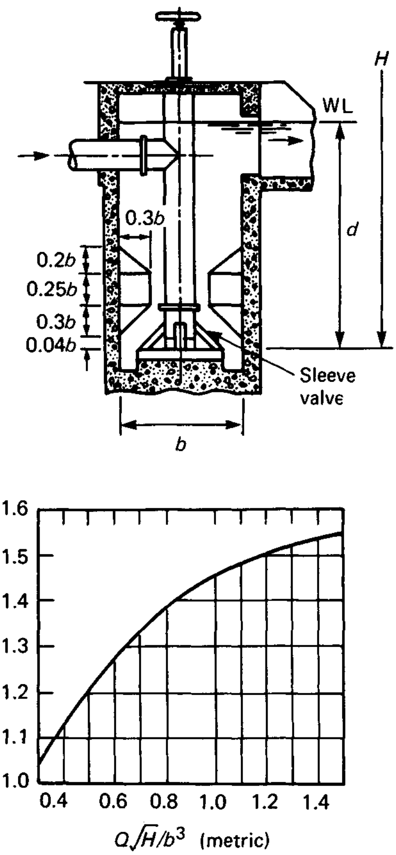


Figure 22.15 Vertical stilling well with sleeve valve (USBR design). Well is of square section in plan with corner fillets as shown. Q is discharge, H is head above pedestal. (After Beichley (1978) 'Hydraulic design for pipe or channel outlets.' USBR Water Resources Research Report No. 24)

Regulation is provided by the sleeve valve at the pipe outlet, operated from above (see page **22**/31).

22.1.7 Scour and erosion

22.1.7.1 Depth of scour at structures

Apart from scour downstream of stilling basins, structures such as bridges, jetties, groynes and constrictions forming obstructions to flow in rivers and channels with erodible beds can give rise to scour due to disturbance of the normal flow pattern. Scour can also be caused by oblique flow at the upstream of control structures such as barrages and regulators. It is generally required to estimate the depth of scour so that adequate protection can be provided or so that the foundations can be located at sufficient depth to avoid the possibility of undermining.

Local scour results from the deflection and, hence, concentration of flow caused by an obstruction. The depth of scour depends on the shape of the obstruction, its orientation to the flow, the channel cross-section and discharge, the character of the erodible bed material, the sediment in transport and the time history of the flow. With so many variables it is not surprising that there is no single formula available for calculation of scour. In the case of important works it is usual to carry out model tests.

It is also possible to determine the order of magnitude and probably the upper limit of scour depth by comparison with depths observed in actual cases, providing a useful check on model results or a fair indication in other cases. Local experience is a guide but may not embrace the highest discharges. To apply historical data from elsewhere it is necessary to adjust for scale. In the cases of rivers in alluvial bed materials, the Lacey regime formulae[19,36] can be used, the depth of scour being related to the normal depth of channel of the same discharge. The relevant formulae in the present context are:

$$B=4.8Q^{1/2} \tag{22.7}$$

and

$$d=0.47(Q/f_L)^{1/3} \tag{22.8}$$

from which can be derived

$$d=1.34q^{2/3}/f_L^{1/3} \tag{22.9}$$

where B is the surface width, d the mean depth, Q the discharge, q the discharge per metre width Q/B, and f_L is a sediment factor, all in metric units relating to stable channels of constant discharge. f_L may be taken as unity for fine sand

Width calculated by Equation (22.7) with Q = design discharge is a useful indicator of the maximum bridge length required for an alluvial river with floodplain, but if the banks are of cohesive materials, the river channel width may be less; Nixon[37] found the average widths of British rivers to be approximately $3Q^{1/2}$, where Q is bank-full discharge. Lacey proposed that the maximum depths of scour at sharp bends in alluvial rivers could be taken as approximately $2d$, where d is calculated from Equation (22.8). Inglis[38] collated data of deep scour observed at structures and training works in alluvial rivers at thirty different locations in India and Pakistan, compared them with the normal depths indicated by Equation (22.8) and reached the following conclusions for maximum depth of scour below water level:

(1) At bridge piers, $2d$.
(2) At large radius guide banks, $2.75d$.
(3) At spurs along river banks, $1.7d$ to $3.8d$, depending on length of spur projection, sharpness of curvature of flow, position and orientation.

Here d is Lacey's normal mean depth calculated from Equation (22.8) using estimated peak discharge. It will be appreciated that large flood flows cannot be measured but are estimated, while maximum depths of scour are transient and may, in fact, have been greater than observed. The scour depths are related to the total rather than the local flow on the grounds that the scour results from the concentration of the whole flow. In the case of braided rivers, allowance could be made for the division of total flow into several channels. Scour depth in rivers in gravel and boulders would be less than indicated but the difference may be small. Scour depths in cohesive materials could be less because of the time required to reach maximum scour.

For the purpose of design of aprons upstream and downstream of barrages in northern India, maximum depth of scour below water level was taken as $1.5d$ at the upstream end of the hard floor and $2d$ at the downstream end of the basin. Here d was calculated from Equation (22.9) using mean q.

Scour at bridge piers has been studied in some detail in models, scour depth being related to discharge per unit width and sometimes expressed as depth below upstream bed level.[39–41] To apply such relationships, the discharge concentration, which can in the worst case greatly exceed the average, has to be estimated, and the upstream depth has then to be calculated for the corresponding flood condition. The latter can be done using Equation (22.9) which is likely to give a conservative value because of time lag and sediment load. The calculation should be checked by use of the appropriate Inglis factor above.

22.1.7.2 Protection against scour

This generally consists of one of the following materials.

Boulders. Boulders from the river bed which are generally rounded and therefore less stable than quarried stone of similar weight.

Rip-Rap. Rip-rap, or pitching of quarried stone, is widely used. In some cases it is hand-packed, especially on side slopes which are expected to remain as placed without settlement, but with increasing use of mechanical equipment it is more often placed in a random manner. This is also preferable in locations where it is expected to settle or move down due to scour. On side slopes the thickness of rip-rap should be sufficient to accommodate the biggest stones without large gaps – at least 1.5 × median stone diameter – and an underlayer or filter of smaller stone is generally provided to prevent the base material from being washed out by wave action. In the case of bed protection, surfaces not subject to scour may be treated in the same way, but at transitions from stilling basins and in general where the channel bed may scour beneath the apron level, the volume of rip-rap should be sufficient to protect a slope at the angle of repose of the rip-rap on the bed material (for a sand bed generally 1:2) extending from the apron level to the level of anticipated deepest scour. For this purpose the rip-rap may be laid on a prepared slope or it may be laid in a horizontal apron which it is assumed will settle to a slope when scour occurs. A margin should be allowed for uneven settlement.

The size of rip-rap which will remain stable may be estimated from Figure 22.16. Sixty per cent by weight of the material should be equal to or larger than the size shown. In the case of rigid structures it may be dangerous to rely on loose stone protection; it is generally best to provide foundations at low levels beneath possible scour. If stone or concrete blocks are used to protect existing structures they should be placed as low as possible beneath normal bed level.

Derrick stone. Derrick stone is stone in blocks too heavy to be placed by bulk handling and which therefore requires individual placing. It is usually placed on an underlayer of graded rip-rap.

Concrete blocks, slabs or units of various shapes. As the density of concrete is less than that of stone, larger and heavier blocks are required than the corresponding stone sizes. Concrete blocks are used in locations where stone of suitable quality and weight is not available or is too costly. Concrete blocks or slabs on edge, e.g. 2 m wide × 0.5 m long × 1 m deep, have been used successfully for flexible aprons downstream of barrages in rivers with sand beds. Concrete slabs are used in slope protection but require good compaction of fill beneath to avoid uneven settlement. Concrete units of special shapes have been developed

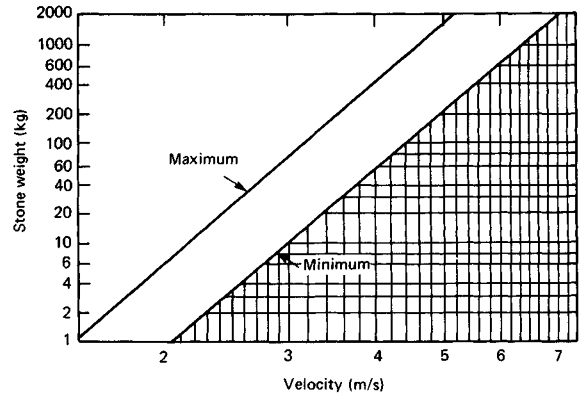


Figure 22.16 Stability of loose rock in flowing water. Graph relates to rock of specific gravity 2.65. For other specific gravity rock weight = weight shown × $1.7s/(s-1)^3$ where s = specific gravity. Use minimum weight graph in normal flow and maximum weight graph for very turbulent flow. (After US Army Corps of Engineers (1952–70) Hydraulic design criteria. US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi)

which require less concrete than do concrete cubes for the same duty; some of these are extensively used in coastal protection and can also be used in river channel works.

Gabions. Gabions, consisting of wire crates containing boulders or broken stone, generally wired together to form an apron, form an economical temporary protection against erosion and have been used in permanent works, though the wire crates may be subject to corrosion. The standard metric size is 2 × 1 × 1 m but thinner mattresses are available. They offer great resistance to removal by flow and a gabion apron has considerable flexibility in adjusting to scour, though less than an apron of rip-rap.

Asphalt. Asphalt provides a smooth impervious cover but does not have much flexibility.

Sheets of polypropylene. These, and other synthetic materials, woven to a fine mesh provide an effective filter layer over sand and have been used to form thin mattresses, with pockets filled with cement grout, for side slope protection.

Brushwood fascine mattresses. These form a traditional protection consisting generally of willow twigs bound in bundles and formed into longitudinal and lateral layers bound together before it is launched by weighting with stone and sinking into place, which is still used and has a considerable life under water.

Vegetation. Vegetation, in particular certain grasses and shrubs, when established above normal water level, can protect a bank against occasional high level wave wash or even shallow overtopping.

For protection of formed banks in cut or fill, any of the above materials would be suitable (see also Chapter 18) subject to adequate protection against scour of the toe of the bank or the channel bed near it. This may be provided by a line of sheet piling at the toe or by a flexible apron laid horizontally which will subside and protect the underwater slope when scour occurs. Quarried stone rip-rap is usually used for the apron where available.

22.2 Enclosed flow

22.2.1 Head loss in large conduits and tunnels

Head loss in pipes is dealt with on pages 5/8 to 5/11. Head loss in large conduits and tunnels may similarly be estimated by the Darcy formula:

$$i = \lambda V^2/2gD = \lambda V^2/8gm \tag{22.10}$$

where i is the hydraulic gradient, λ the friction factor, V the mean velocity, D the diameter of circular conduit flowing full, or m the hydraulic radius to be used for part full and noncircular conduits. λ and Manning's n are related by:

$$n = \lambda^{1/2}D^{1/6}/10.8 = \lambda^{1/2}m^{1/6}/13.6 \tag{22.11}$$

In nearly all actual cases of large conduits the boundary cannot be classed as smooth or rough but falls within the transition region. λ therefore depends on the effective roughness and on the Reynolds number VD/v or $4Vm/v$, where v is the kinematic viscosity (for values of v see pages 5/8 to 5/10). Although many types of roughness are composite, e.g. smooth concrete with projections due to formwork joints, and therefore the equivalent sand roughness concept is not completely representative, it does provide a method of predicting the friction factor, based on recorded experience. In the case of new works this depends on the type of forms used, quality of workmanship and degree to which projections are ground down. Deterioration occurs with age and use. A steel lining may corrode and be roughened by tuberculation, as for pipes. Concrete inverts may be roughened by abrasion during river diversion. There may be deposits due to leaching through joints and cracks in a concrete lining, even vegetation and animal growths, while the deposit of slime by untreated water is commonplace.

Typical values of equivalent sand roughness k for new surfaces, based mainly on Ackers[42] and USBR experience,[43] are given in Table 22.1.

It is more difficult to predict the friction factor in a tunnel after many years of use; the best guide is often obtained from actual measurements in similar tunnels under similar conditions. Observations in many tunnels have been published.[43–45] The effect of slime has been studied by Colebrook.[45]

Table 22.1

Surface	*k range* (mm)
Asbestos	0.012 to 0.015
Spun bitumen lined	0.0 to 0.030
Spun concrete lined pipes	0.0 to 0.030
Uncoated steel	0.015 to 0.060
Coated steel	0.03 to 0.15
Rivetted steel	0.3 to 6.000
Wood stave, planed planks	0.2 to 1.5
Concrete:	
against oiled steel forms with no surface irregularities	0.04 to 0.25
against steel forms, wet mix or spun precast pipes	0.3 to 1.5
against rough forms, rough precast pipes or cement gun	0.6 to 2.0
smooth trowelled surface	0.3 to 1.5
Glazed brickwork	0.6 to 3.0
Brick in cement mortar	1.5 to 6.0
Ashlar and well laid brickwork	1.5
Rough brickwork	3.0

When a suitable k value has been determined, the relative roughness k/D or $k/4m$ can be calculated and a value of λ determined from Figure 22.17 which is based on the Karman–Nikuradse–Prandtl formulae with Colebrook–White transitions described on page 5/9). Some examples of large-conduit observations are shown in Figure 22.17.[43,44] Figures 22.18 and 22.19 show characteristics of circular and horseshoe conduits flowing part full.

The surface of conduits for high-velocity flows should be to a very high standard of finish to avoid damage by cavitation – see page 22/7.

22.2.2 Unlined and lined-invert tunnels in rock

Excavated rock surfaces are very rough and the hydraulics are complicated by 'overbreak', i.e. excavation beyond the minimum required by the specification. Rahm found a relation between the variation in cross-sectional area and the friction

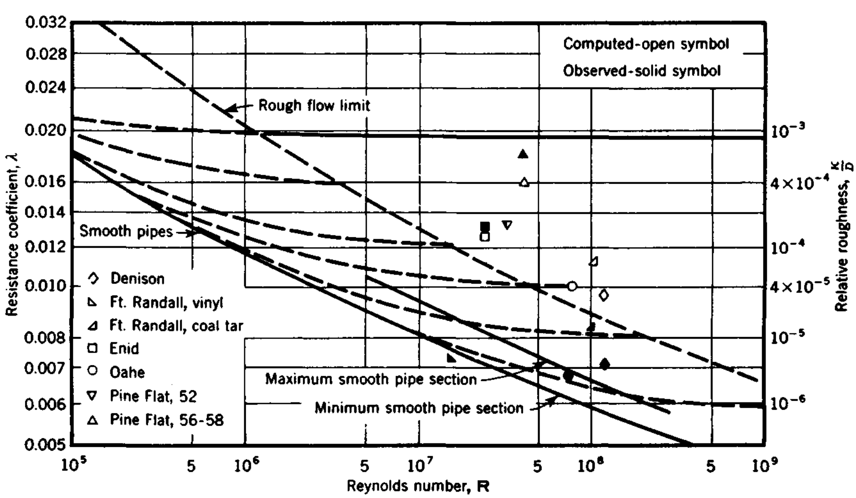


Figure 22.17 Head loss in uniform conduits. Open symbols, computed; solid symbols, observed. (After US Army Corps of Engineers (1952–70) *Hydraulic design criteria*. US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi)

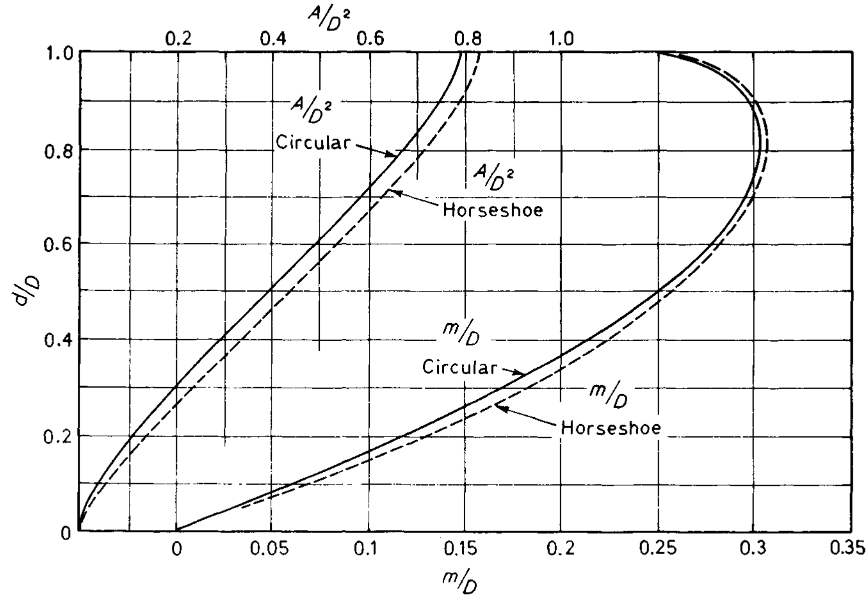


Figure 22.18 Area and hydraulic radius of conduits, part full. For key, see Figure 22.19

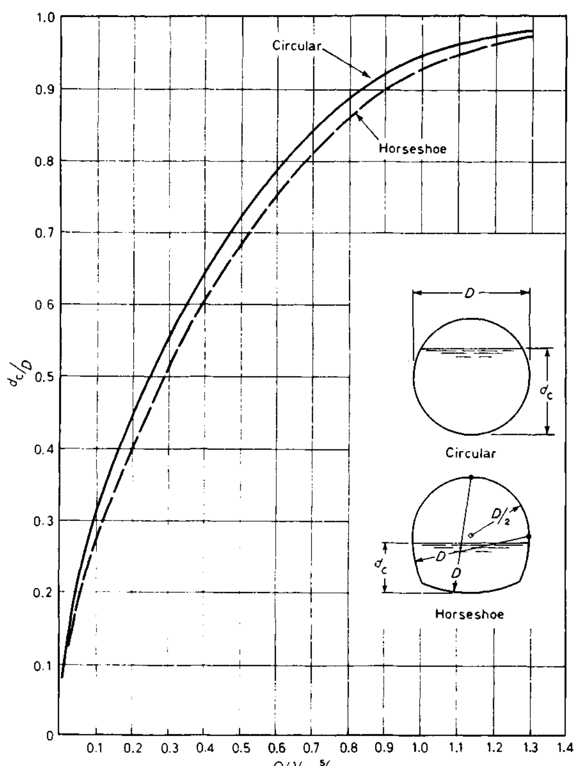


Figure 22.19 Critical depth in circular and horseshoe conduits

factor. The subject was further developed by Colebrook[45] and again by Wright,[46] who showed that the resistance of unlined tunnels can be reduced considerably by providing a concrete invert.

22.2.3 Transitions and bends

Transitions may be from circular to noncircular sections or vice versa, or from one circular section to another of different diameter. In conduits for high-velocity flows, transitions are generally gradual to avoid flow separation and possibly cavitation. It is also necessary to adopt moderate rates of expansion if head is to be conserved and instability of flow downstream avoided. Circular sections can be merged into rectangular or horseshoe sections without double curvature and avoiding sharp local divergences. Figure 22.20 shows head loss in diffusers of circular section; curves of similar pattern but slightly differing values apply to diffusers of rectangular section.[47] It will be seen that for expansion ratios of 2 or more the head loss may be considerable unless the angle of divergence is small. Where rapid expansion is required, divide walls may be used so that the

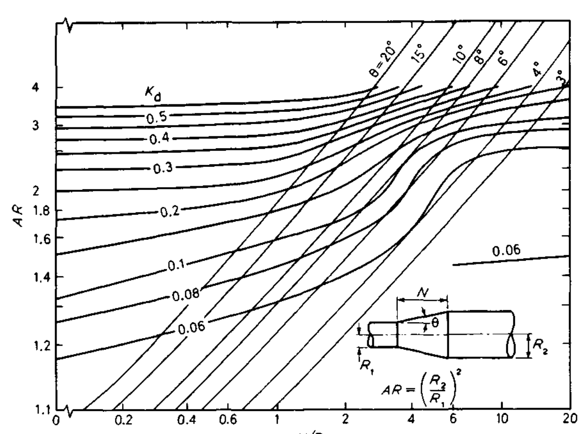


Figure 22.20 Head loss coefficient K_d of conical diffusers with tailpipe. (After Miller (1971) *Internal flow. A guide to losses in pipe and duct systems.* British Hydro-mechanics Research Association, Cranfield)

flow is carried in a number of ducts, each of which is a reasonably efficient diffuser while the overall angle of expansion may be as much as 90°. For head loss in a sudden enlargement, see Chapter 5, pages 5/10 and 5/11; expansions of this type are sometimes used for energy dissipation in closed conduits. Contractions may be more rapid than diffusers, but to avoid head loss due to the formation of a *vena contracta* in the downstream conduit, it is desirable to provide a rounded external angle between the transition and conduit at a radius of at least one-sixth diameter. In high-velocity flow this is an area of potential cavitation and the radius should be greater.

Head losses at bends in large conduits are similar to those in pipe bends (see pages 5/11). A compromise then has often to be reached between the greater head loss of a short-radius bend and the greater cost of long-radius bend; bend radii of between 1.5 and 3 diameters are often adopted. The flow instability induced by a bend persists for a distance of many diameters downstream and may affect the performance of turbines or pumps.

Bifurcations and manifolds, dividing the flow, for example, for two or more machines, are generally designed with great care to achieve a smooth change of velocity, absence of swirl and minimum head loss. Model tests with air are useful to indicate flow pattern and pressure drop in closed conduit transitions; relatively low pressures are used to avoid compressibility effects.

Transitions leading from subcritical flow in open channels to closed conduit flow may, where the approach velocity is low, be designed on the same principles as apply to intakes from reservoirs (see page 22/25). Sharp corners lead to separation and the formation of a *vena contracta* with head loss; this may be avoided by providing a rounded or bellmouth entry. With higher approach velocity the transition should be more gradual, with curves of larger radius. To avoid the formation of a hydraulic jump with resulting air entrainment the design should be such that the contact between free surface and roof occurs where the flow is subcritical, preferably with Froude number well below unity.

22.2.4 Exits

If the exit of a conduit is fully or partially submerged, head loss can be reduced by providing a gradual expansion, which can often be extended in the tail channel. If the conduit exit is not submerged, a free surface may develop some distance upstream of the exit portal, even when the conduit is flowing under pressure. The depth of the exit depends on downstream conditions but where tailwater level is low the flow becomes supercritical and the conduit exit acts as a control. The end depth still depends to some extent on the tail channel, particularly whether the flow is supported at the sides and bed, but the end depth may be estimated from Figure 22.21. If the emerging flow is supercritical and downstream flow subcritical, a hydraulic jump will occur and a stilling basin may be needed.

22.2.5 Flow routing

Flow through conduits can be routed and energy gradient plotted by use of the Bernoulli equation (see page 5/8). Allowance should be made for head loss due to friction, bends, transitions and hydraulic jumps. Subcritical flow is routed in an upstream direction starting from the tail channel or a control, and supercritical flow in a downstream direction, using step methods if necessary. Computer programs exist which ease the burden of calculation. To locate a hydraulic jump the pressure–momentum theorem can be used, taking account of the slope of the conduit and the pressure against the conduit roof if submerged downstream. The method is described by Kalinske and Robertson.[48] Critical depths in circular and horseshoe conduits

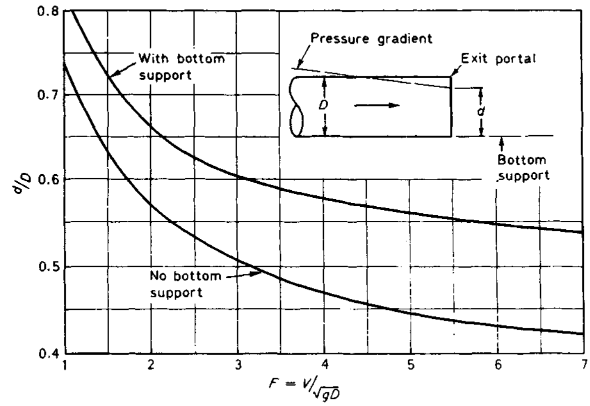


Figure 22.21 Exit depth in circular conduits. *V* is the mean velocity in the conduit flowing full (based on USWES data). (After US Army Corps of Engineers (1952–70) *Hydraulic design criteria*. US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi)

may be determined from Figure 22.19, and diagrams facilitating computation of jumps in conduits of circular and other cross-sections have been published.[2-4] In cases where hydraulic jumps might occur in closed conduits with undesirable results, due to additional head loss or air (see below), it is recommended that the routing be repeated for several discharges using both high and low values of head loss coefficients and upper and lower limits of tailwater rating curve, to obtain a complete account of the flow.

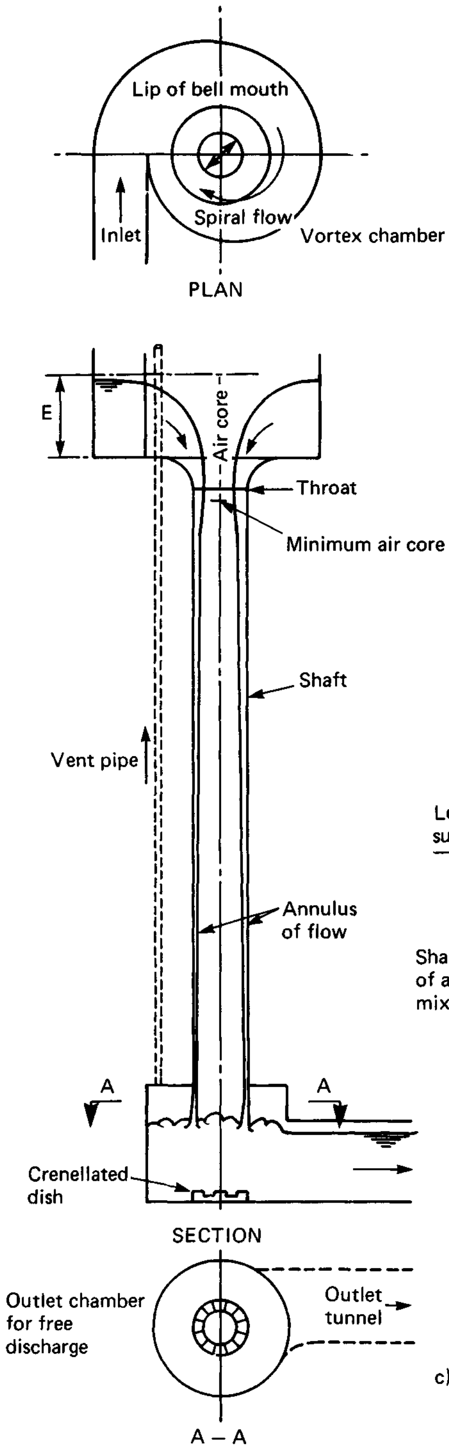
22.2.6 Drop shafts

Sometimes flow has to be dropped from a high-level system to a low-level system, e.g. from shallow sewers to deep interceptors, from river intakes in mountains to a water-transfer tunnel, from the drainage of an open-pit mine to an adit from an adjacent valley. An economic solution is to use a shaft, but there are problems associated with air entrainment and release in any shaft system, especially if the base of the shaft is submerged by the hydraulic conditions in the low-level system. These problems can be minimized by generating a vortex at the top of the shaft, either by a scroll-shaped inlet chamber (Figure 22.22a) or by a tangential vertical slot (Figure 22.22b).

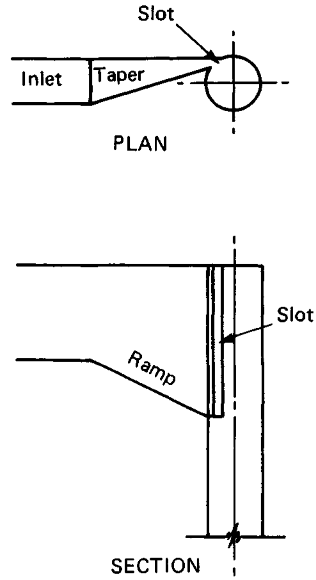
The vortex action ensures that the flow down the shaft will cling to the walls. This has the advantage of minimizing air entrainment and encouraging the return of air back up the centre to the head of the shaft, and at the same time maximizes head dissipation in the shaft by wall friction. The vortex motion is persistent: it will continue for the full length of fairly deep shafts provided the entry is well designed. The theory of the scroll inlet is given by Ackers and Crump,[49] and the slot inlet has been investigated by Eppema, Jain and Kennedy.[47]

If the bottom of the shaft is submerged, as in Figure 22.22c, it will be necessary to provide an air-release chamber. If the full-bore shaft velocity exceeds about 0.5 m/s, bubbles will be carried down with the flow. Problems – perhaps serious ones – could arise if this entrained air was allowed to travel along the tunnel system (due to potentially explosive blowout further downstream) and hence a stilling chamber should be provided to allow the entrained air to separate and rise to the crown of the chamber where the bubbles will coalesce to return via the vent pipe.

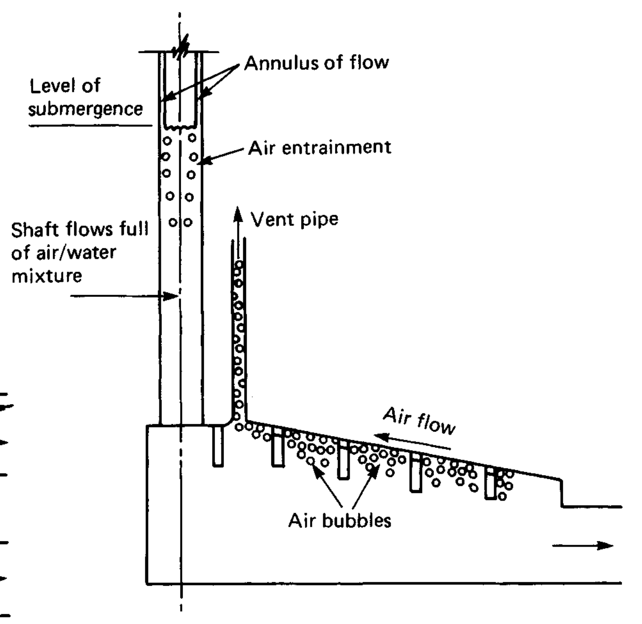
For unsubmerged conditions, Figure 22.22a illustrates a type of collecting chamber at the base of the shaft found suitable for



a) Normal sewage structure with vortex chamber inlet



b) Alternative deep slot inlet



c) Outlet chamber for submerged discharge with air-release provision

Figure 22.22 Vortex drop. Alternative forms: (a) normal sewage structure; (b) alternative deep slot inlet; (c) outlet chamber for submerged discharge with air-release provision sewerage systems. With deep shafts, the annulus of flow may reach terminal conditions where the gravitational component is equalled by the friction at the shaft walls, and so there is a limit to the amount of energy to be dissipated at the base of the shaft.

22.2.7 Air problems in conduits

Air entrained at high velocity releases through gates and valves into conduits, e.g. at outlets from reservoirs, air entering from drop shafts or junctions and air entrained at hydraulic jumps, can lead to dangerous air and cavitation problems unless the conduits are adequately vented. Air can also collect and restrict the flow of water. Air release valves, often combined with vacuum relief to admit air if pressure falls below atmospheric, are therefore provided at high points. Vents are often provided in horizontal tunnels downstream of junctions where entrained air may enter. Air which has collected beneath the soffit tends to be carried forward by the flow, even against a small gradient, but with a variable flow may move upstream and downstream at different times. At vertical shafts in pressure conduits and at deeply submerged exits the intermittent escape of air produces shock waves due to slap on the soffit as water replaces the air. This effect can be minimized by vents for controlled air release.

Hydraulic jumps entrain air and when a jump in a conduit is in contact with the soffit much of the air is released downstream. Following model tests in a conduit with various slopes by Kalinske and Robertson[48] and others, and several observations at full scale, the US Army Corps of Engineers[11] use the formula:

$$\beta = 0.03(F_1 - 1)^{1.6} \qquad (22.12)$$

which gives higher values than found in the model tests to allow for scale effect. Here β is the air:water ratio Q_B/Q_W, $F_1 = V_1/\sqrt{(gd_e)}$, V_1 is the upstream velocity and d_e the effective upstream depth (= water area:surface width). A particular application of these formulae is the estimation of air demand downstream of gates or valves located in closed conduits, where high-velocity flow at part openings is transformed to full-conduit flow through a jump (see Figure 22.23). Full-scale observations in three different cases showed that with rectangular gate openings peak demand occurred at 60 to 85% opening, with a secondary peak at about 5%. Further analysis has been provided by Sharma.[50]

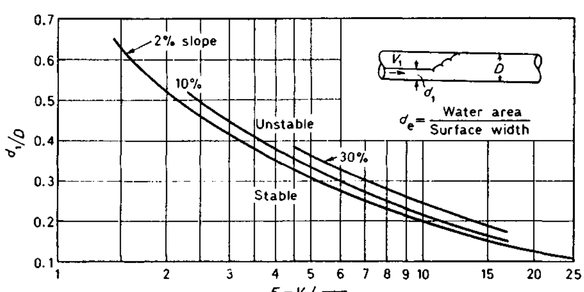


Figure 22.23 Stability of entrained air downstream of hydraulic jumps in circular conduits. (After Kalinske and Robertson (1943) 'Closed conduit flow. Symposium on entrainments of air in flowing water.' *Trans. Am. Soc. Civ. Engrs,* **108,** Paper 2205, 1435)

The air pumped by the jump may be carried downstream by the full-bore flow but, if the velocity is insufficient for this, air will collect immediately downstream of the jump and when a quantity of air has accumulated it will 'blow back' through the jump. Figure 22.23 shows the limiting conditions for the air just carried by the flow, as found by Kalinske and Robertson.[48] Sailer[51] compared these curves with conditions in a number of full-scale inverted siphons and found verification in that five cases where blowback had occurred were represented by higher values of $(F_1 - 1)$ than shown by the curves, while others giving no trouble were on or below the curves. With large flows, blowback through the jump is, like 'blowout' at the exit, explosive and potentially dangerous.

22.3 Spillways

22.3.1 Purpose and types

A spillway is provided to remove surplus water from a reservoir and thus protect the dam and flanking embankments against damage by overtopping.

The best type and location of a spillway depends very much on the topography and geology of the dam site and adjoining area, and on the type of dam. Where the dam is of concrete or masonry founded on hard rock, the spillway may be within the dam, consisting either of a high-level overflow or of submerged orifices, discharging into the river bed beneath. In the case of an earth or rockfill dam, it is usual to site the spillway away from the deepest part of the dam; high flanking ground or a saddle away from the dam site can be suitable locations where a spillway channel may be excavated and control structure provided (see, for example, Figure 22.24). Where the dam is built in a narrow gorge and there is no suitable separate site for the spillway, a side-channel spillway is often adopted (Figure 22.25).

If control is by a fixed ungated weir, the maximum retention level of the reservoir is the weir crest level; at times of spill the reservoir level rises and sufficient freeboard has to be allowed above maximum water level, which is the level at which the design maximum flood discharge is released. In the case of gate-regulated spillways, on the other hand, flood flows can be discharged with reservoir at retention level, which need never be exceeded. For a given dam height, retention storage can thus be greater but, because there is less flood storage, the spillway capacity also may have to be greater. The gates, however, enable the reservoir to be drawn down in advance of a flood peak, given adequate forecasting. Low-level orifices, having greater capacity than required for purposes of normal supply, have greater capability than has a gated crest overflow in drawing down a reservoir in the event of damage to the dam, an important aspect in areas where earthquake risk is present. But crest overflow weirs have a greater rate of increase of capacity as a reservoir level rises above normal, thus providing additional safety margin.

Cost is a major consideration in the choice between a regulated and an unregulated spillway, but spillways without gates have advantages in respect of reliability, absence of mechanical maintenance problems and no power requirements. They are therefore often adopted at remote sites and for small dams where the cost of gates would not be justified.

Siphon spillways carry some of the advantages of both gated and ungated spillways. They can be designed to prime and operate to maximum discharge within a small range of reservoir level and they are automatic, with no moving parts.

Another type of spillway, particularly suited for use with earth or rockfill dams is the bellmouth or 'morning glory' spillway, which can be built quite independently from the dam, and which is described further in section 22.3.5. If the reservoir is for water supply, the bellmouth and shaft are often combined in the same structure as a drawoff tower and the low-level tunnel can be used for river diversion during construction, as discussed and illustrated in section 22.4.1.

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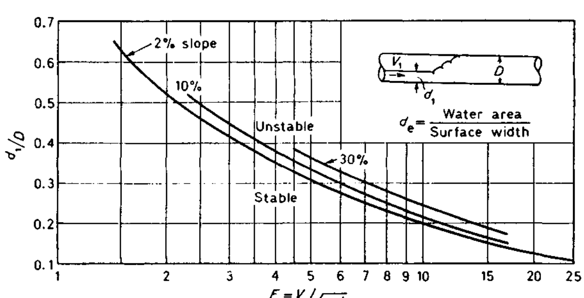

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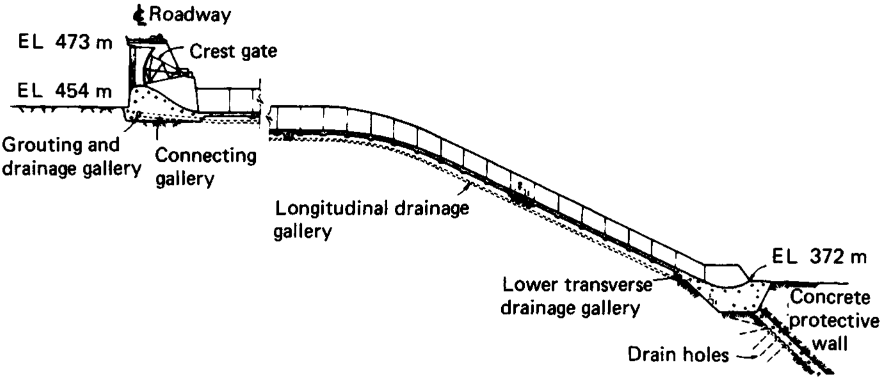


Figure 22.24 Chute spillway, Tarbela Dam, Pakistan. (Consulting engineers: Tippetts–Abbott–McCarthy–Stratton)

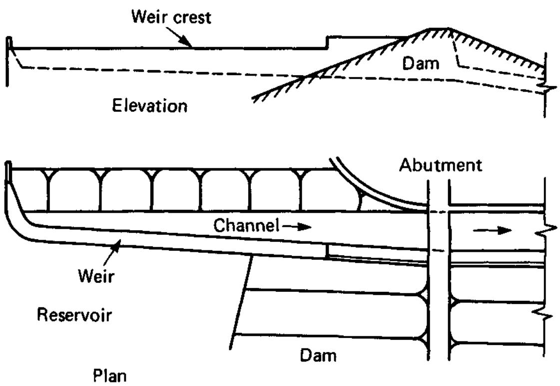


Figure 22.25 Side-channel spillway

floods up to catastrophic, it may be safer and more economical to provide a main or service spillway, fully regulated and capable of controlling all floods up to perhaps a 20- or 50-yr return period, and a secondary or emergency spillway which will bring the combined capacity to the catastrophic level. The capacity of the emergency spillway should be adequate to control normal inflows alone for a reasonable period should the main spillway be damaged.

22.3.2 Channel spillways

The simplest form of spillway, consisting of a channel excavated in rock, is often used for small reservoirs and for emergency spillways. The control is usually provided by a hard sill or weir at the entrance. The channel downstream should be given sufficient slope to ensure that the weir will not be drowned by backwater effect. If the rock is erodible, a curtain wall with bed protection or stilling basin is needed to avoid erosion undermining the downstream end of the weir.

In the case of emergency spillways, a 'fuse plug' is often provided, consisting of an erodible bank across the channel. Its crest is below the dam crest level but above normal operating level. When overtopped it quickly erodes down to the level of the hard sill, bringing the full discharge capacity of the channel into operation. To avoid excessive draw-down of the reservoir, emergency spillways of this type are preferably wide and shallow. Bed and bank protection are usually minimal but it is essential that there is no risk of erosion progressing upstream and breaching the reservoir rim beneath the level of the hard sill.

22.3.3 Weirs

These may be any of the types described earlier on page **22**/5 but are generally of the free-nappe profile type. It is usual for gates to close on to the weir face slightly downstream of the highest point of the crest, so that the jet at small openings will be projected downwards. Even so, sub-atmospheric pressures can develop, and for this and other reasons it is often desirable to avoid prolonged releases under high heads with small gate openings. Provision of separate sluices or valves for small releases is preferable.

Piers affect the rating curve, as described earlier (page **22**/6). A typical rating curve is shown in Figure 22.26 (AB).

A side-channel spillway consists of a weir discharging into a parallel channel, as in Figure 22.25 where the weir is aligned on a ground contour normal to the axis of the dam. The channel must be of sufficient width and depth to allow for energy dissipation and the generation of exit flow without drowning the weir. Calculation is based on the momentum principle.[12,52] Side-channel weirs have been investigated by el-Khashab and Smith.[53]

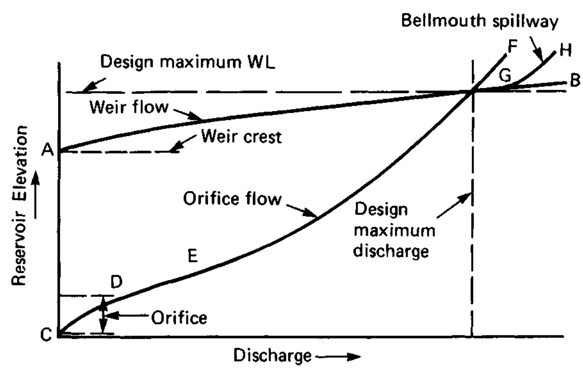


Figure 22.26 Typical rating curves for high-level weir, deep orifice and bellmouth spillways

22.3.4 Low-level outlets

These may discharge through a dam or original ground into conduits, chutes or stilling basins. They are generally of rectangular section and regulated by radial gates (see, for example, Figure 22.27). When the reservoir is drawn-down so that the orifices are not fully submerged, flow is of the free-surface weir

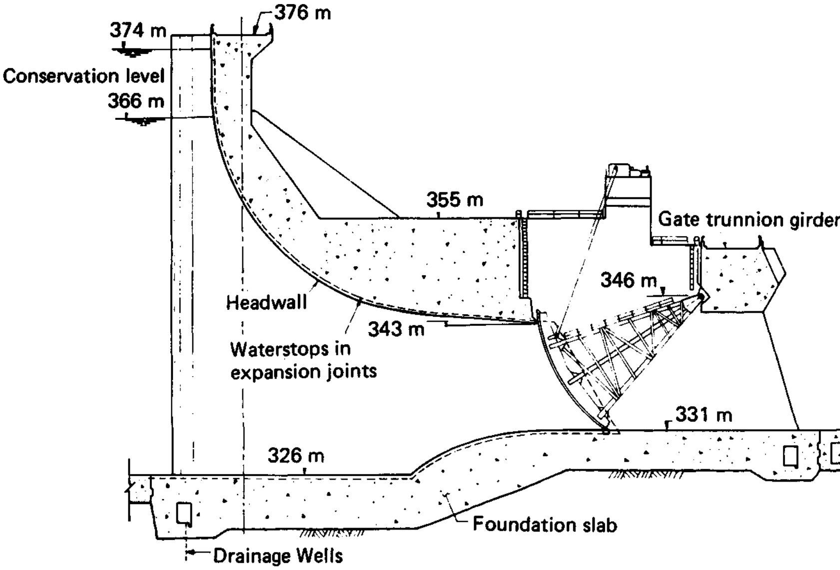


Figure 22.27 Main spillway, Mangla Dam, Pakistan. Section through gate structure. (Consulting Engineers: Binnie and Partners in association with Harza Engineering Co., and Preece, Cardew and Rider)

type. The rating curve of an orifice spillway with gates fully open therefore consists of two main parts, as may be seen from the example in Figure 22.26. The lower part CD relates to weir flow and the upper part EF to submerged orifice flow. Between the two is a transition DE. The discharge for weir flow is $Q=c_1(2g)^{\frac{1}{2}}BH_1^{1.5}$ where Q is the discharge, c_1 is the weir coefficient, B is the width and H_1 is the upstream head above the sill. c_1 is generally a variable.

In the range of orifice flow, $Q=c_2B\sqrt{(2gH_2)}$ where c_2 is the coefficient of discharge and H_2 is the effective head below reservoir level. If the jet springs clear, so that atmospheric pressure obtains around the whole periphery, H_2 is best measured from the centre of the jet at its exit from the orifice. If the jet emerges on a horizontal floor confined within vertical side walls, H_2 is more correctly measured from the soffit level as representing the effective elevation plus pressure head over the jet at the point of separation from the soffit. If there is back pressure H_2 is the differential head.

The requirements in design are those of high-pressure outlets, described on page 22/27. If these are met the coefficient can approach unity. In the case of Mangla spillway[54] (Figure 22.27) it was approximately 0.95. Model tests are used to indicate pressures on the boundary surfaces and provide rating curves for full and partial gate openings.

22.3.5 Bellmouth, shaft and closed-conduit spillways

A bellmouth or 'morning glory' spillway normally consists of an overflow weir, circular in plan, but in some cases multisided, and a vertical shaft discharging into a tunnel or culvert carried through high ground with outfall into the downstream river. The weir can be provided with gates or siphons – Figure 22.28 shows an example of the latter.

The hydraulics of bellmouth and closed-conduit spillways are complicated by the number of potential controls and the entrainment and release of air. At low flows the bellmouth crest provides weir control; at a higher stage the throat at the foot of the bellmouth can exert orifice control; the bend at the foot of the shaft leading into the tunnel can also exert orifice control

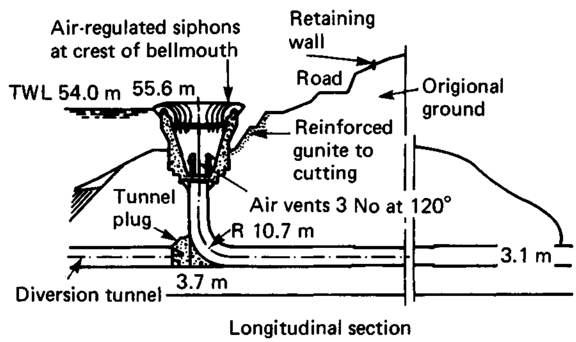


Figure 22.28 Siphon bellmouth spillway, Shek Pik Reservoir, Hong Kong. (Consulting Engineers: Binnie and Partners)

and if the tunnel flows full this may well control the discharge. To avoid instability due to controls operating intermittently, the range of each control should be clearly defined with stable transitions from one to another. It is best to reduce the number of potential controls to one or at most two. The weir (or siphons) provide the primary control and it is normal practice for the design maximum discharge to be reached in this range, with an adequate margin, before the weir is drowned by 'gorging' in the shaft and bellmouth due to controls in the system downstream. Similar considerations apply to spillways which are not of the bellmouth type but which have weir, gate or siphon as a primary control, discharging into shaft and tunnel. If, however, use is to be made of flood storage in the reservoir at higher levels, a bellmouth spillway may be allowed to become completely submerged.

As in the case of straight weirs, sharper curvature raises the coefficient of discharge. Profiles based on the shape of under-nappe of a jet have been designed for weirs circular in plan.[12,55] In several cases of bellmouth spillways measures were necessary to reduce swirl and prevent vortex formation particularly at the highest flows, when it could greatly reduce discharge capacity. Vortex flow is induced by asymmetrical approach in the reservoir; anti-vortex measures usually consist of crest piers or vanes, or a single divide wall on a diameter extending from below crest level to above maximum reservoir level.[28,56] When the jets from opposite sides of the shaft intersect, the entrainment of air can result in negative pressures on the walls. Vents may be necessary to avoid instability and vibrations.

A typical rating curve is shown in Figure 22.26. Weir control is represented by the curve AG. At low flows there is a free surface flow in the bend and tunnel but with rising discharge and downstream conduit not flowing full the bend begins to act as an orifice with water level rapidly rising in the shaft. When it reaches crest level it begins to drown the weir flow. The bellmouth is then said to be 'gorged'. This is represented by the intersection of the two curves at G, above which the bend assumes control of the rate of flow. The rating curve of the spillway is therefore AGH with a short transition at G representing drowned weir control and bend orifice control at H.

If the bend is too sharp, flow from the bend to the conduit is very disturbed; if it is too easy the downstream culvert may flow full; a bend radius of 1.5 to 2 times the diameter is generally satisfactory.

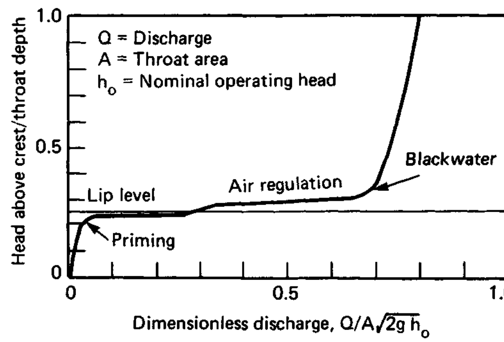
For proper control of flow at the bend and smoother flow in the conduit a deflector may be placed on the inside wall at the upstream of the bend.[57] Where the conduit is used for river diversion during construction, a properly shaped bend can later be formed when the diversion intake is plugged. Unless the conduit is very short, flow with a free surface is desirable with sufficient air space above for entrained air to be released without trouble. Sufficient slope should be provided to ensure that the depth does not exceed the desired limit. As the result of a model study, Mussalli and Carstens[57] recommend upper limits for the proportion of water flow in such conduits, ranging from 97% of the area, when the Froude number is 2, to 50% when it is 8.5. Where the velocity is high enough to entrain air it is desirable to provide an air vent at or near the bend. With high velocities it is also best to avoid bends and other conditions downstream which could cause a hydraulic jump to form in the conduit.

It is evident that the concrete surface at the base of the shafts of bellmouth spillways can be subjected to high impact loads by water, possibly with ice and logs, spilling from a great height. In some cases steel or cast-iron lining has been provided in this area, but from a survey of sixteen bellmouth spillways, of which eight with unlined concrete inverts had undergone a fair test, Bradley[56] found no erosion of a serious nature. Dense concrete with smooth surface finish is called for here and in the conduit.

22.3.6 Siphon spillways

Compared with a free-surface weir, flow through a siphon reaches a high rate of discharge per unit width with only a small rise of reservoir level needed to prime the siphon. This permits a higher retention level for a given maximum water level, or alternatively a higher concentration of flow in a restricted width.

Reservoir retention level is equal to siphon crest level. As the reservoir level rises, the action of a siphon passes through the following successive phases: (1) weir flow, when water spills at low depth over the crest; (2) priming phase, when air is being extracted from the crown of the siphon; and (3) fully primed siphonic flow. When the reservoir falls, (3) gives way to (4), a depriming stage, when air is admitted in sufficient volume to break the siphonic action and the action vellerts to (1), weir flow. In recent years, many air-regulated (or partialized) siphons have been built. In these, phase (3) consists of two parts: in (a) when priming has occurred the entry of air continues so that the flow consists of an air–water mixture. The air intake is so designed that the volume of air admitted is insufficient to break the siphon (except at low flows) but is controlled by very small variations in reservoir level. As the reservoir rises further the volume of air is reduced until stage (b) is reached when the siphon flows 'blackwater', i.e. with no entrained air. This performance is illustrated in the typical stage: discharge function in Figure 22.29a. The advantage of air regulation is that, whereas without it the siphon on priming runs directly to a high blackwater discharge which if in excess of inflow will draw the reservoir down and lead to intermittent priming and depriming, an air-regulated siphon will remain in the fully primed phase over a wide range of flow, with continuous discharge matching the rate of inflow. Examples of air-regulated siphons are Eye Brook,[58] Shek Pik[59] (Figure 22.28) and Plover Cove[60] (Figure 22.29b).



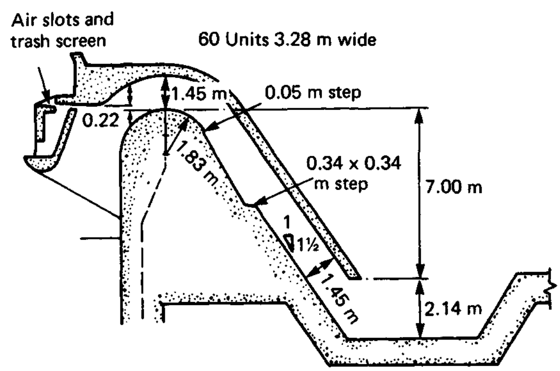


Figure 22.29 Air-regulated siphon spillway. (a) Typical stage: discharge curve; (b) Section through Plover Cove siphon spillway, Hong Kong
(Consulting Engineers: Binnie and Partners)

Spillway siphons are generally designed to prime automatically when the reservoir has risen to a level such that: (1) an upstream air inlet is submerged; (2) the siphon outlet is sealed by a deflected jet or a downstream weir; and (3) the flow is sufficient to entrain and remove air from the crown of the siphon. Various priming devices have been used[61] and the priming depth above crest level is in some cases as little as one-sixth of the throat diameter.

The blackwater discharge capacity of a siphon can be expressed as:

$$Q = cA\sqrt{(2gH)} \tag{22.13}$$

where c is a coefficient allowing for head losses, A is the cross-sectional area of the flow at exit and H is the head from upstream reservoir to effective exit level – usually the downstream lip of the hood

Ackers and Thomas[62] reviewed the design and operation of siphon spillways. The value of c obtained in the Plover Cove siphon model (Figure 22.29b)[60] was 0.68. That of the model of Shek Pik bellmouth siphon (Figure 22.28), where the shape was radial and no sealing weir was provided,[59] was 0.66.

Surface waves in the reservoir are an important factor in siphon design. Model tests showed that despite provision of baffles, waves caused surging in the siphon but the air intakes could be designed to counterbalance the effects of surging and wave wash. The head in siphons is usually limited to about 7 m to avoid cavitation at the crest. Tests on the Plover Cove siphons showed that wave action resulted in transient pressures below average pressures, but a total head of 7.3 m was still feasible. Each case should, however, be examined in the light of the particular conditions obtaining.

22.3.7 Chutes

Chutes may be built into the downstream faces of concrete dams; longer chutes are often provided to convey the flow from side channel or other flanking spillways to the river bed downstream (see Figure 22.24). In general, high head spillways with chutes should not be used for routine releases of water for supply, because of the risk of cavitation damage with small gate openings, also because the chute may have to be taken out of service in the dry season for repairs, and because low flows can create problems of erosion downstream. The gradient is likely to be steep enough to generate high-velocity flow. As lateral changes of direction could result in overtopping of the side walls, any essential changes in the alignment should be made near the control structure where the velocity is relatively low and thereafter the chute should be straight. There should also preferably be no changes of alignment in the side walls where flow is supercritical because diagonal shock waves would be created which might cause overtopping downstream.

High-velocity flow can give rise to high pressure and the most careful precautions are necessary to avoid uplift pressures developing beneath the chute slabs. A high standard of surface finish is called for and the profile should contain only very gradual curvature. Joints between slabs should be keyed and bridged by flexible water stops sealed at intersections. Projections at the joints facing upstream should not be allowed but offsets facing downstream up to 12 mm are often accepted or even specified. Drains are generally provided beneath and parallel to all joints, so that in the event of leakage, uplift pressure cannot build up. For additional protection, chute slabs are often anchored to the foundation rock. Special care is needed where the chute rests on jointed or fissured rock because pressure can be transmitted from leakage at a higher level despite underslab drainage. Chutes at a steep slope are especially vulnerable and may call for deep anchors. Stability should be checked for all possible modes of failure.

The problems of cavitation caused by high-velocity flow are reviewed on page 22/17. Severe damage has occurred on some chute spillways which have been operated at high velocity and hence a careful assessment of the cavitation risk is needed for any chute spillway, especially if the total fall exceeds 50 m. Cavitation damage can be avoided if natural aeration of the flow from its free surface is high enough to provide several per cent of volumetric air concentration at the bed of the chute in the region of high velocity.

The development of flow down the spillway is illustrated in Figure 22.30. A layer of slower-moving fluid influenced by friction at the solid boundary grows in thickness beyond the crest until it occupies the full depth of flow. This defines the 'point of inception' of air entrainment by turbulence at the surface. The entrained air diffuses down within the partially aerated region of flow to occupy the full depth of flow, usually some considerable distance down the spillway. It is only in this fully aerated region that the natural aeration at the solid boundary can be sufficient to prevent cavitation damage if the cavitation index (see page 22/32) drops below the safe limit. The likelihood of cavitation damage therefore depends on whether velocities rise too high as the flow accelerates down the chute before there is sufficient aeration at the bed. This is most likely at high discharge intensities. A method of calculation of the

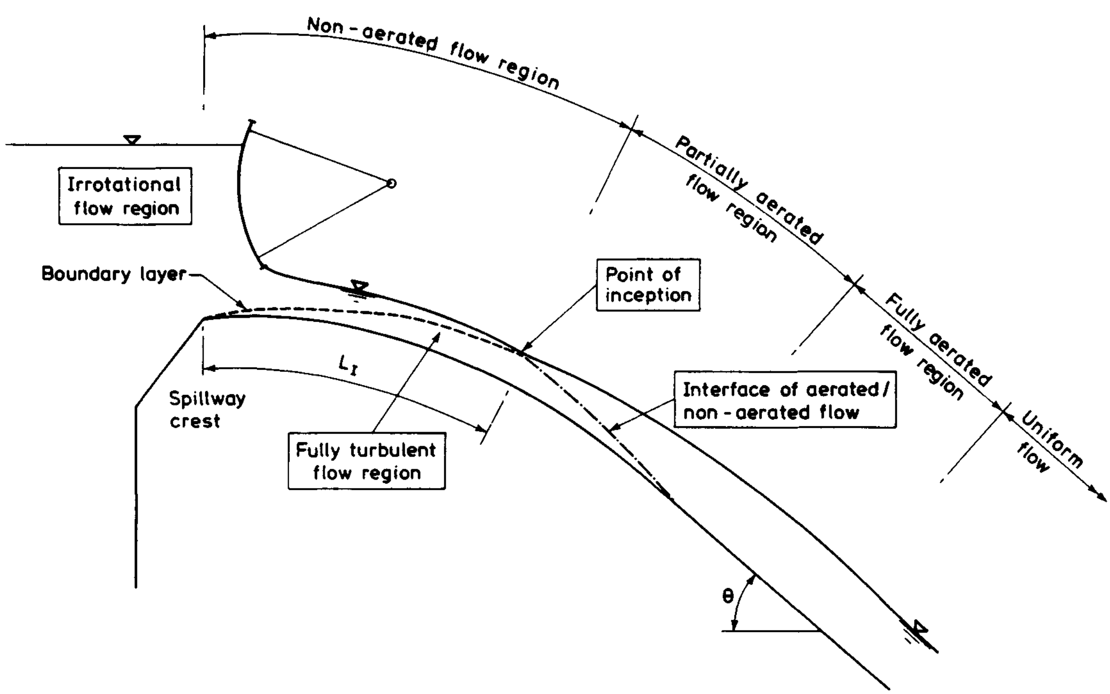


Figure 22.30 Development of flow down a typical spillway

growth of boundary layer is given by Wood, Ackers and Loveless.[63] If natural aeration is not sufficient, purpose-built air-entraining slots should be provided, fed from ducts at either side.[64,65]

Calculation of depth of flow in chutes may be done by a step method beginning at the top, using curves showing energy against depth for the required discharge per unit width, similar to that of Figure 22.1. The calculation should be performed with two roughness coefficients, one representing a maximum, for use in determining side-wall height, and the other representing a minimum, for use in the design of energy-dissipating works. The height of side walls should include an allowance for bulking due to air entrainment. The US Corps of Engineers[11] provide a design curve based on observed data, with the equation:

$$c = 0.436 \log_{10}(S/q^{\frac{1}{5}}) + 0.971 \qquad (22.14)$$

where c is the ratio of air volume to air-plus-water volume, q the discharge per foot width in square metres per second, and S the sine of the angle of chute inclination

22.3.8 Energy dissipation

The energy to be dissipated at the outfall from a spillway is very considerable. The means of protection of the dam and other structures from its erosive action depend largely on the rate of discharge and its head, the erodibility of the materials of the river bed and surrounding ground and on the proximity of the dam.

In the case of rivers in alluvium or other easily erodible ground, a stilling basin designed to contain a hydraulic jump is often provided. This may be of the rectangular type or a submerged roller bucket (see sections 22.1.6.1 and 22.1.6.2). The former is generally more efficient but more costly, especially in the case of large structures where deep retaining walls would be required. Where the river bed material is rock a 'ski jump', trajectory or 'flip' bucket is generally provided. This is elevated above maximum tailwater level, so that the jet trajectory carries the water into a plunge pool some distance from the bucket. Dissipators of this type are generally less costly and are suitable where the bedrock is resistant enough so that erosion does not progress back and endanger the foundations of the dam or other structures. Ski jumps have also been used where the river bed is of alluvium, and the structures are suitably protected against erosion.

The radius of flip buckets is not critical provided it is large relative to the depth of flow. The exit angle is important as it determines the throw distance; the angle is generally between 20 and 40° and the theoretical throw distance is given by x in the formula:

$$\frac{x}{h_v} = \sin 2\theta + 2\cos\theta \left(\sin^2\theta + \frac{y}{h_v} \right)^{1/2} \qquad (22.15)$$

where h_v is the velocity head at the bucket lip, θ the bucket exit angle, measured from the horizontal, y the vertical height of bucket lip above tailwater[11]

However, because of air resistance and internal shear the jet tends to break up and diffuse so that the actual trajectory distance may be 10 to 30% less than indicated by the formula. Elevatorski[5] quotes examples of models and full-scale spillways.

Flip buckets are not necessarily of circular profile, nor axisymmetrical. There are several instances of buckets composed of flat deflecting surfaces, some designed to deflect to one side to suit downstream requirements.[66]

In the design of the side walls, allowance must be made for the additional lateral pressure due to centrifugal force. This may be calculated by methods of Gumensky,[67] Balloffet[68] or the US Army Corps of Engineers.[11] To avoid cavitation damage it is usual to avoid the use of teeth, and in some cases the lip edge is protected by stainless steel.

The size and depth of the plunge pool depend primarily on the discharge concentration and the characteristics of the materials which are eroded. In the formation of a deep pool the eroded material is lifted out by the flow. Incoherent alluvial materials are readily removed and form flat side slopes. Rock disintegrates into fragments by transient pressures in the joints and the fragments are reduced by abrasion until small enough to be removed.[73,74] A plunge pool in rock has relatively steep side slopes and is less extensive in plan than one in alluvium. The erosion is not always confined to the plunge pool because the action of the jet creates large horizontal eddies which can extend back to the chute. Small flows and flows at low heads have shorter trajectories or may not be sufficient to sweep out of the bucket but spill over the lip, causing erosion beneath. In such cases special protection is needed (see, for example, Figure 22.23). Mason[75] has reviewed experience of energy dissipation works at dam outlets.

22.4 Reservoir outlet works

22.4.1 Intakes

The type of intake for drawing water from a reservoir depends on the type of dam and on the purpose of the supply. The velocity may be low and against a back pressure as, for example, in intakes for domestic water supply, and into penstocks for power generation, or it may be high, e.g. in spillways and into diversion tunnels during construction. With high velocity, special problems concerned with head loss and cavitation arise. If the dam is of concrete, the intakes may be located in the dam. If the dam is of earth or rockfill, a separate intake structure may be built (see Figure 22.31), leading into a tunnel, or a free-standing drawoff tower may be provided, sometimes combined with a shaft spillway as in Figure 22.32. A free-standing tower is particularly suitable where drawoff is required at several levels, as where the water is for domestic supply. In such cases a bottom drawoff or 'scour' sluice is generally provided; this is opened at intervals to prevent sediment from building up a deposit in the immediate vicinity of the lowest drawoff to supply.

Deep intakes have advantages in that they will remain submerged at low reservoir levels, are less affected by vortices and are less susceptible to obstruction by ice and floating trash. Against these, the gate structure is more costly and access to the screens for cleaning more difficult.

A square edge or small radius edge to an orifice would result in flow separation and a *vena contracta*, so orifices and sluice entrances, whether circular or rectangular in section, are usually shaped to a bellmouth. The head loss associated with the formation of a *vena contracta* at a circular orifice can be greatly reduced by providing a simple bellmouth, as shown in Figure 22.33a, but for high velocities the curvature should be less to avoid low pressures which might result in cavitation damage. Compound curves of two or more radii and elliptical curves are often suitable profiles. A typical example of an elliptical profile is shown in Figure 22.33b. With this profile the minimum pressure at the boundary is approximately $0.1V^2/2g$ below the corresponding pressure in parallel flow in the orifice downstream where V is mean velocity. As this is a mean pressure, lower pressures may occur owing to fluctuations. For very high velocities this may not be acceptable and the profile may be

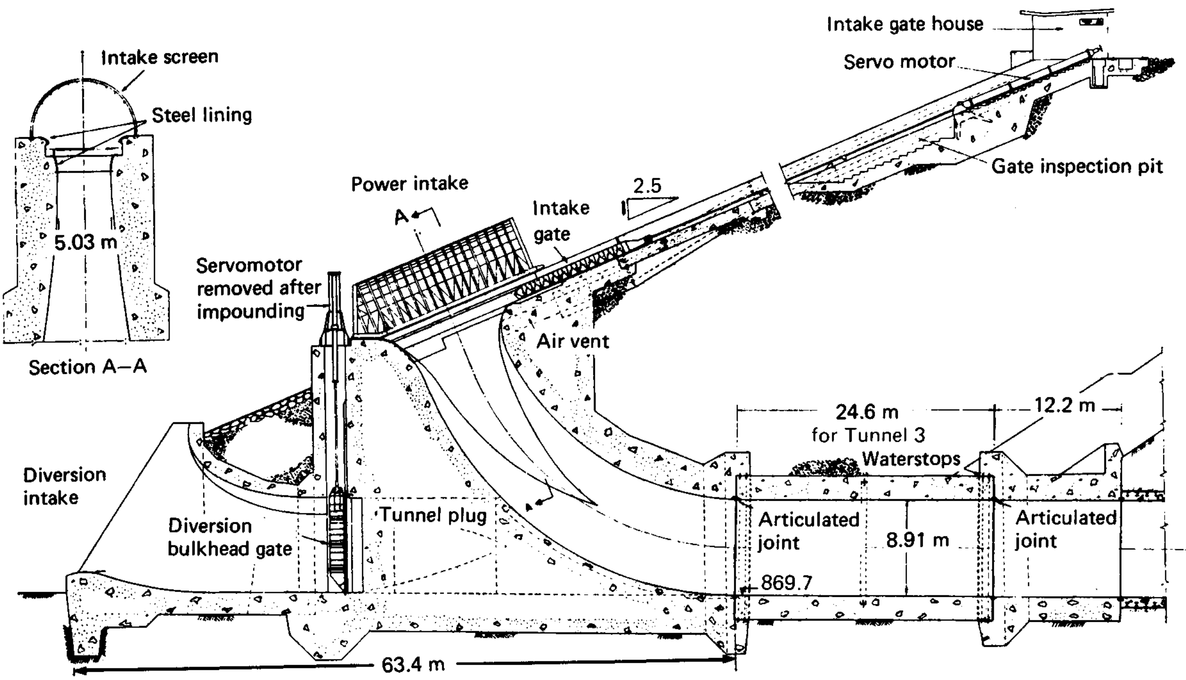


Figure 22.31 Intakes at Mangla Dam, Pakistan. Longitudinal section. (Consulting Engineers: Binnie and Partners in association with Harza Engineering Co., and Preece, Cardew and Rider)

based on the profile of a jet springing from a sharp-edged orifice[72,73] or may be compound elliptical.[11] In the case of important works, especially with high velocities, intake entry curves are usually tested in hydraulic models. Control may be by gate or valve, located at the intake or in a pressure conduit. Radial gates in rectangular orifices, as seen in Figure 22.27, are particularly suited for flood releases. If the gates are to be used for regulation at part openings under heads exceeding 10 m, the inverts immediately downstream are generally lined with steel as protection against cavitation. A second gate is often provided upstream of each service gate for emergency closure and to allow maintenance work on the service gate to proceed when the reservoir is at a higher level. This is generally a vertical-lift gate closing on to a steel sill but requiring side slots. The latter contribute to head loss and, where cavitation is a danger, require special design as, for example, is illustrated in Figure 22.34.

Where outlets fill a vital role in the safety of works, the possibility of failure or malfunction of a gate or valve should be considered and alternative measures provided for an emergency.

22.4.2 Vortices

Though a slight surface swirl may be of no consequence, a vortex with an air core extending to an intake can be harmful in reducing the discharge capacity of the intake, causing gate vibration or resulting in admission of air to pumps or turbines. Any tendency for a vortex to form in a model test should be carefully investigated because vortices form more readily and develop further at full scale.

A free vortex tends to form in accelerating flow towards a region of low pressure, as at a submerged intake. It is facilitated by boundary geometry consistent with vortex shape, and by an initial circulation in the reservoir, and is more marked the greater the pressure drop to the outlet relative to the depth below surface. Vortex action is reduced by deeper submergence of the intake, by reduced velocity at the intake and by obstructions to the rotation, such as horizontal grids and projecting walls.[74,75]

Vortices are also a problem in pump sumps where even a slight swirl may affect pump efficiency and more refined measures are needed. Velocity of approach is generally limited to 1 m/s, but eddies can still form at points of separation. Sharp wall angles and regions of dead water should be avoided and expansions should be gradual. General guidelines are available[76–78] but model tests are often needed to determine optimum pump sump geometry.

There are situations where vortices may be used to advantage, as in the vortex drop structures described earlier.

22.4.3 Screens

Screens or trash racks are provided at intakes to hydro-electric plants, pumps and water-treatment works. Log booms are often placed upstream, but it is generally required to intercept small debris and possibly fish. The spacing of the bars may be 2 to 20 cm depending on the duty. The main requirements in design are that the bars are stiff enough not to vibrate and are arranged for easy cleaning. As a general guide for screen area the mean velocity is usually limited to 0.6 m/s or less. Vibration is avoided if the dimensions of the bars are such that the natural frequency of the bars is higher than the forcing frequency. Screens may be fixed and cleaned by raking or lifted above water for cleaning. For ease of cleaning from above, the vertical bars generally project upstream of the lateral bars.[79,80]

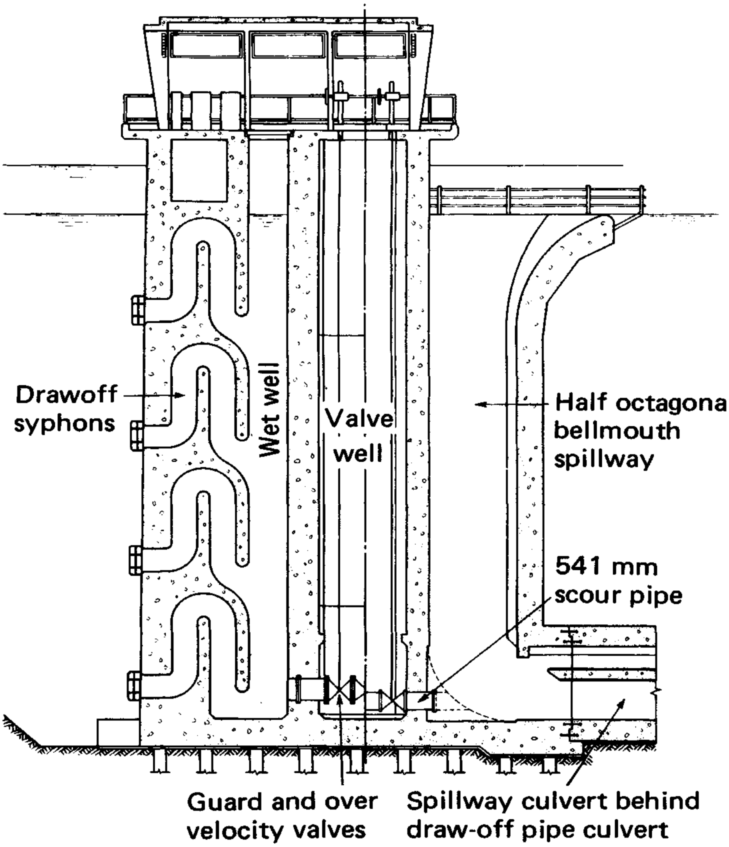


Figure 22.32 Combined drawoff tower and spillway, Seletar Reservoir, Singapore. Air-controlled siphons are used instead of valves for drawoff purposes. (Consulting Engineers: Binnie and Partners, Malaysia)

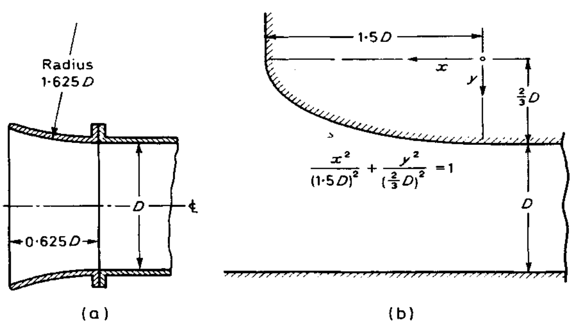


Figure 22.33 (a) Simple bellmouth; (b) elliptical roof profile for conduit intake with parallel sides and horizontal floor. (After US Army Corps of Engineers (1952–70) *Hydraulic design criteria*. US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi)

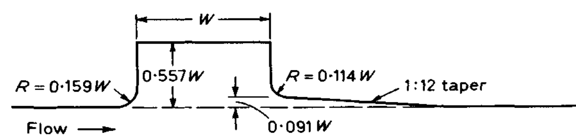


Figure 22.34 Typical gate slot with downstream offset to minimize cavitation. (After US Army Corps of Engineers (1952–70) *Hydraulic design criteria*. US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi)

22.5 Gates and valves

22.5.1 Gates

22.5.1.1 Uses and types

Gates are used to control flow in open channels or closed conduits by restricting or closing the waterway. They may be required:

(1) For regulating the flow, when they must be capable of operating at any required degree of opening.
(2) For emergency or guard purposes, when they must be capable of closing under any condition of runaway flow which could occur.
(3) As bulkhead gates for closing a conduit for inspection, maintenance or construction works. When they are permanent installations, such gates are generally designed to open and close only under balanced pressures, but when used to close diversion tunnels during construction works, closure may be against a considerable flow. Stop logs are similar in function to bulkhead gates, but are in smaller units handled individually and placed above one another.

The types generally used in outlets from reservoirs and in spillways, barrages and canals are as follows.

Vertical lift gates. Vertical lift gates are supported by guides in slots at the side walls of the conduit. They may open by raising or by lowering; in some cases they are in two or even three parts, each operating independently. They may have sliding contact with the guides or may have wheels (fixed-wheel gates) or a moving train of rollers (Stoney gates). They have seals at the sides and (in orifices or closed conduits) also at the top and generally close on to a steel sill with an inset compressible seal if required. Advantages of vertical lift gates are their simplicity and ease of maintenance; disadvantages are the requirement of slots in the side walls and limitations of loading on axles at very high heads. Sliding gates can be used for high heads but require correspondingly powerful actuators. However, sliding gates have been installed for heads as high as 200 m, carrying a water load exceeding 1000 t.[81] A disadvantage in the use of vertical lifting crest gates for spillways, barrages and canals is the requirement of a high superstructure. In the case of conduits the hydraulic disadvantage of side slots in high-velocity flow has been overcome by the introduction of 'jet flow' gates; in these, narrow side-slots are provided but an upstream contraction causes the flow to spring clear of the slots, re-attaching to the side walls downstream. Caterpillar-track-mounted gates are sometimes used for emergency closure, operating on flat bearing faces on the upstream face of a dam. One gate is generally sufficient for a number of orifices, controlled by a mobile gantry from the top.

Inclined lift gates. These are similar in many respects to vertical lift gates but operate on inclined tracks (see Figure 22.31). They are sometimes used for guard or emergency purposes at intakes in earth or rockfill dams, the track being laid on the upstream face of the dam. An advantage of this gate is its low cost compared with alternatives of a vertical lift gate in a tower in the reservoir or in a gate shaft within the dam. Disadvantages may include the remoteness and inaccessibility of the gate in case of emergency and the long-vent shaft.

Radial (or tainter) gates. An example of these may be seen in Figure 22.27. The gate skin is of cylindrical shape and is supported on cross-members spanning two radial arms which rotate on short axles extending from the side walls or piers. The resultant of the water pressures passes through the centreline of the axles creating no moment opposing gate operation; therefore powerful actuators are not required and in the event of power failure quite large gates can be operated manually. Other advantages are simplicity, reliability and low cost. They do not require side slots; side seals are of the sliding type and the gates close on to a steel sill. Where it is required to allow passage of floating debris, or sensitive control of reservoir level, the top of the gate may consist of a hinged flap opening downwards. They are widely used for both weir and orifice control in spillways. They are also sometimes used in pressure conduits but need more space than vertical lift gates and problems of access and removal for maintenance have to be considered.

Hinged leaf, bascule or flap gates. These are sometimes used for crest control where water depth is not great. They are hinged at the bottom and may be used for regulation with water spilling over them; they need venting. They have the advantage of allowing floating debris to pass at small openings but, although they can be of curved profile, the weir crest has to be rather wide to provide the recess. On many European rivers, bascule gates are used for regulating upstream water level, operated by hydraulic actuators located below the weir crest. Hinged gates can be made to open automatically by a simple mechanical device when the upstream water level rises to a given height. Gates hinged at the top are also used where the whole assembly is retractable to allow the passage of ships.

Drum and sector gates. Examples of these can be seen in Figure 22.35. These are crest gates which open downwards, retracting into a recess in the crest. They may be hinged on the upstream (drum) or downstream (sector). The upper surface can be shaped to suit the weir profile when fully open. In the examples shown, the gate consists of a watertight vessel con-

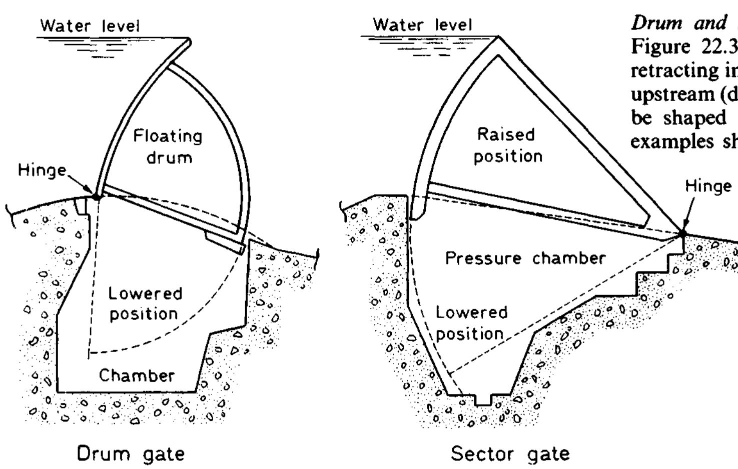


Figure 22.35 Drum and sector gates

trolled by application of headwater pressure beneath; it is sealed at the hinge and gate seat. These gates can be arranged to operate automatically by the upstream water level. (The sector gates of the Thames Barrier are illustrated in Figure 22.11, page **22**/11.)

Bear-trap gates. When raised, bear-trap gates are in the form of a flat 'A', with upstream and downstream leaves forming the two legs, hinged at the bottom, with seals at the hinges and the apex. The gate is raised by admitting water under pressure from the headwater. When lowered the upstream leaf overlaps the downstream leaf and both fold flat. Bear-trap gates have for long been used in Europe and the US for river regulation.

Rolling gates. Rolling gates consist of a roller with toothed-wheel meshing with an inclined toothed rack at each end. The gate is rotated by a chain and accordingly moves up and down the racks. Roller gates have been used for river regulation.

Cylinder or ring gates. Cylinder or ring gates moving on vertical axes have been used as crest gates on bellmouth spillways and for bottom outlets.[81] Some of the former open by being lowered vertically into a recess in the weir crest, controlled by water pressure, others and the bottom outlet gates by being lifted from above. Lateral control of the gate motion is provided by guides.

22.5.1.2 Partial gate openings

These are orifices of which three sides are the fixed boundaries and the fourth is the gate lip. When unsubmerged, the jet springs clear from the gate lip forming a *vena contracta*, the dimensions of which depend on the shape of the gate skin and lip and on the upstream profile of the sill. It is therefore usual to calibrate gates by model tests. Details of calibration of various gates are available.[81–84] Where gates are partially submerged downstream, calibration is complicated by the additional variable, and is also less reliable because of possible variation of flow pattern. Submergence may also lead to vibration problems.

22.5.1.3 Vibration

In general, vibration is a result of resonance where the frequency of a pulsating force is equal or nearly equal to the natural frequency of a flexible part of the structure. Gates are liable to vibrate when: (1) overtopped and not adequately vented; (2) when significant flow occurs both over and under a gate; (3) when the gate is partially or fully submerged; (4) when the location of flow separation is unstable; or (5) there is flow re-attachment. Vibration by the two latter causes may occur at the bottom edge of a lifting or radial gate which should therefore be designed so that the flow separates at a sharp edge and cannot become re-attached by contact elsewhere. This is not always possible at small openings, so that vibration may occur in a limited range of opening. Flexible seals are a potential cause of instability and are often omitted from the bottom edges of gates for this reason. Many cases of gate vibration and remedial methods have been described.[85,86]

22.5.1.4 Downpull and upthrust forces

These forces can act on the upper and lower edges of a gate as well as on the face. They affect the operating forces required and gates are often designed so that the resultant force is of assistance. In particular, if the top edge of a lifting gate is subjected to static pressure whereas the bottom edge is at atmospheric pressure because the lip is on the upstream edge, the resultant downpull assists in gate closure, a safety measure in case of power failure. Pressures measured on bottom edges of various shapes are available.[87]

22.5.1.5 Gate seals

Where a small amount of leakage can be tolerated, as in most works in the open, the seal at the bottom of a lifting or radial gate is usually metal to metal, between the gate edge and a steel sill set flush in the floor. A bottom slot would fill with debris and a projecting rubber seal may vibrate, but if leakage is to be minimal a rubber seal may be inset flush in the floor. Side and top seals can be metal to metal but with close tolerances these may be costly. Flexible rubber seals are therefore frequently used, in the form of strip tightly clamped with small projection, or moulded into bulbous shapes (e.g. music-note type) and arranged to be held in contact by the water pressure. As frictional resistance between metal and rubber seal increases with pressure, brass cladding is often used for sliding seals where the head exceeds 60 m. For very high heads, metal-to-metal contact may be required.

22.5.2 Valves

22.5.2.1 Uses and types

Valves are used to regulate flow or pressure in pipes and conduits or to close them against flow, often at high pressure. A service valve, whether for regulation or closure, is generally protected by an upstream gate or guard valve which can be closed against flow to isolate the regulating valve for maintenance or repair, and to prevent leakage if the regulator is not adequately sealed. Valves may be 'in-line', i.e. with pipeline upstream and downstream, or 'terminal' at the discharge end of a pipeline. Variations in valve design are numerous; only a few types which are normally used in reservoir outlet and hydro-electric power systems are described below. Of these, gate, spherical, butterfly, needle and tube valves are generally used in-line, while needle, tube, hollow jet and sleeve valves are used as terminal regulators.

The discharge capacity of a valve may be expressed as:

$$Q = CA\sqrt{(2gH_L)} \tag{22.16}$$

where Q is the discharge, C the coefficient, A the cross-sectional area of the valve at entry and H_L the head loss across the valve

Gate or sluice valves. In their simplest form (Figure 22.36) these consist of a sliding leaf in a valve body with side slots, thus resembling a vertical lift gate with operating rod sealed to contain the pressure. The sealing contact is metal to metal and the leaf is usually wedge shaped to provide tight sealing when fully closed. Parallel guides prevent vibration at part openings, but gate valves are not suited for regulation except at low or moderate pressures. A bypass is usually provided to balance pressures but a valve for guard or emergency duty may have to close under unbalanced pressure. Advantages of gate valves include the simplicity of design and low head loss when fully open. Disadvantages are the considerable power required to operate under unbalanced heads, cavitation damage to the slots at high velocity and damage by abrasion of the sealing faces if sediment is carried in the flow. The main disadvantages of slots are overcome in the 'ring-follower' valve in which the gate when raised is followed by a cylindrical ring which effectively covers the slots. It requires a cavity for the ring beneath the conduit and is suitable only for guard purposes.

For regulation under high heads a special type of gate valve, termed the 'jet-flow' gate valve, has been devised by the USBR to operate free of cavitation. The flow is expanded, then sharply

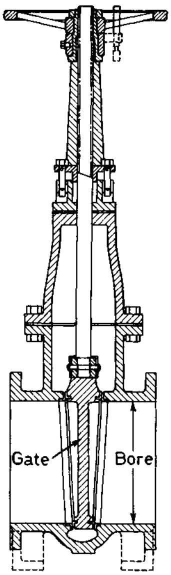


Figure 22.36 Typical gate valve, wedge type. (*Courtesy*: J. Blakeborough and Sons Ltd)

contracted, at the boundaries upstream of the gate slot and springs clear into a vented surround downstream. Valves of this type have been successfully used in sizes up to 2.44 m diameter and under heads to 120 m.[58]

Spherical or rotary plug valves. These are used for guard and on–off duties. They generally consist of a short length of tube of the same diameter as the conduit and length about the same dimension, which is in line with the conduit in the open position and is rotated through 90° to effect a closure. This is enclosed in a body of roughly spherical shape. The great advantage of this valve is that it offers no obstruction to the flow when in the fully open position. Hydraulic characteristics are given by Guins.[89]

Butterfly valves. Butterfly valves (Figure 22.37a) are widely used as guard or isolating valves but under low- or medium-pressure differentials they can be used for regulation. The blade or disc is mounted on a shaft, or on two stub axles; rotation by hydraulic piston and crank is simple and direct. The obstruction caused by the blade in the fully open position inevitably results in head loss and downstream turbulence. The former is reduced by adopting a slim blade of greater diameter than that of the pipe. However, a slim blade may not have the strength to resist high-pressure loading, so to meet this need, blades in some valves consist of two thin parallel discs rigidly connected by structural members parallel to the flow; these have great strength while offering less resistance to the flow than corresponding solid blades. Where the valve is in a terminal position, or is guarding a terminal valve, the pressure of the surrounding fluid may be near atmospheric; this also calls for slim blades to avoid cavitation at high velocities. The resultant torque due to fluid pressure is always acting to close the valve, but for safety when closed it is best for the lower half of the disc (if the axis is horizontal) to close in the direction of flow. Guard valves are often designed to be opened under balanced pressure, for which a bypass valve is provided, but to close against full flow in case of emergency.

Butterfly valves present special sealing problems because of the axles but this problem has been overcome and very low leakage rates have been achieved[88] with rubber seals mounted on disc or body. In some valves the axles are offset to facilitate replacement of the sealing ring. For very high heads, rubber is not suitable and metal-to-metal seals are required.

Hydraulic and torque characteristics of valves with blades of various shapes are available.[89–91]

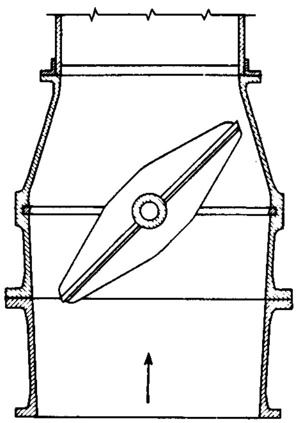
Needle valves. Needle valves have long been used for precise flow regulation in terminal locations. They consist of a needle or tapered plunger which moves axially within an orifice forming part of the valve body (see Figure 22.37b). The plunger is located by guides and operated by screw or hydraulic pressure. In the Larner–Johnson valve, actuation is by the pressure difference across the valve controlled by a pilot valve in the nose of the plunger. The plunger and body are precisely shaped so that the flow accelerates through the valve, to assist actuation, while avoiding cavitation under operating conditions. The discharge coefficient for a fully open valve is from 0.4 to 0.72 depending on the throat area ratio.

Tube valves. Tube valves which were used on some USBR reservoir outlets resemble needle valves but the part of the needle downstream of the sealing ring is omitted. This reduced cavitation problems but vibration was experienced with some valves. This valve has been satisfactorily operated fully submerged. The discharge coefficient, when fully open, based on valve outlet diameter is about 0.6. An in-line regulator of similar general form is shown in Figure 22.37c.[91] This has tubular ports at its discharge end which direct jets towards the centre of the downstream pipe where excess head is dissipated, thus avoiding cavitation damage. The port openings are regulated by an axial movement of the plunger.

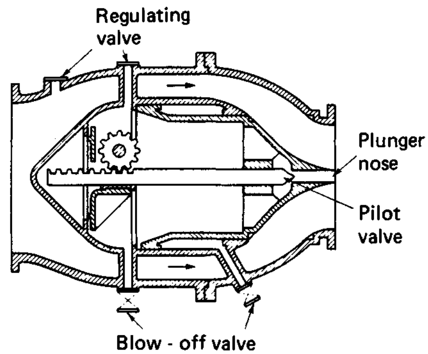
Hollow jet valve. This was developed by the USBR as successor to needle and tube valves and has been generally satisfactory as a high-pressure terminal regulator. It resembles a needle valve with the downstream half of the needle omitted, while the remaining half advances upstream to seal against a ring on the body (Figure 22.37d). The flow is deflected by the surrounding tubular body and is trajected in the form of a jet with nearly parallel sides and hollow centre. Pressure is admitted to the plunger to assist operation, which is mechanical. When fully open the valve normally has a coefficient of discharge of 0.7 based on the outlet diameter.[92,93] Hollow jet valves by the USBR are stated to operate satisfactorily when partially submerged up to centre level, but should not be operated fully submerged.[88] The trajectory can be calculated approximately by the mechanics of a projectile. Though some aeration and dispersion of the jet occurs the jet fallout is concentrated in a relatively small area and in some cases erosion is a problem. A stilling basin has been developed for this valve.[5,34]

Fixed cone-sleeve valves. Also known as Howell–Bunger valves (Figure 22.38), these are widely used in free-discharge terminal applications, including pressure relief for turbines. They have a tubular body on the outside of which is a cylindrical sleeve. This is operated by screws or hydraulic servomotors and retracts to form an opening through which the discharge occurs, deflected outwards by the fixed cone. Discharge coefficient at normal maximum opening is approximately 0.85.[94]

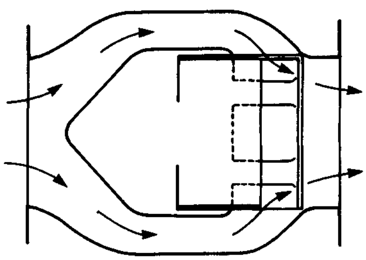
Asymmetry of approach flow may lead to vibration; the valve should not be located too close to a bend in the conduit. A tapered contraction from conduit to valve assists to stabilize the flow. The sharply increasing diameter as the water leaves the valve forces the jet to break up, which is excellent for energy dissipation but sometimes creates problems due to fallout from drifting spray. The limits of the trajectory can be calculated by assuming projectiles at the upper and lower points and two sides of the jet, leaving the valve at velocity equal to $(2gH)^{1/2}$ where H



(a)

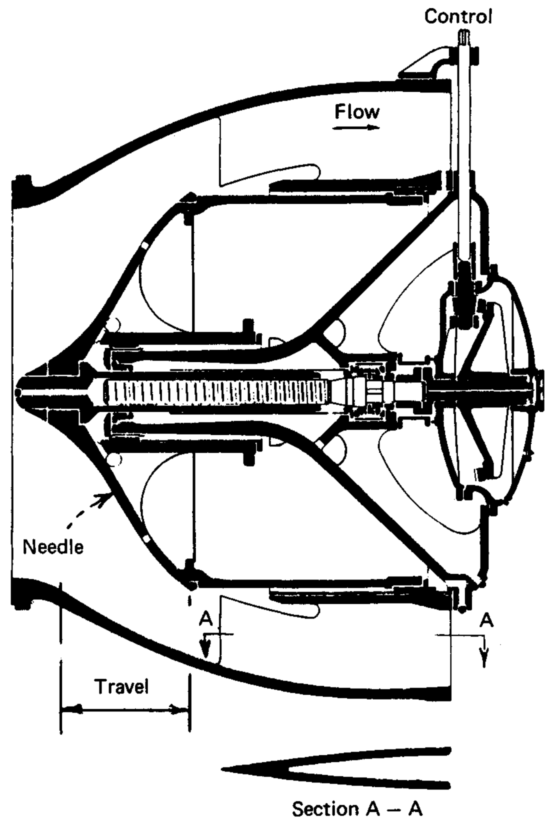


(b)



(c)

Figure 22.37 (a) Example of butterfly valve. The valve diameter exceeds the conduit size to allow for the obstruction of waterway by the blade; (b) Larner–Johnson needle valve, with internal pilot valve control; (c) in-line regulating valve (*Courtesy*: Glenfield and Kennedy Ltd); (d) hollow-jet valve (USBR design)



(d)

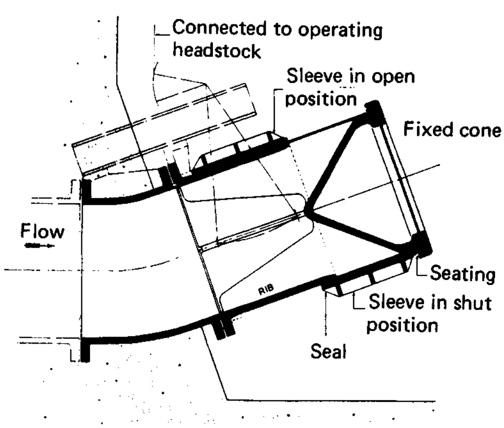


Figure 22.38 Fixed-clone sleeve valve. (*Courtesy*: Glenfield and Kennedy Ltd)

is the pressure plus velocity head in the valve body, and initial trajectory according to the angle of the cone, normally 45° to the axis. Fallout distance is, however, appreciably reduced by air resistance and affected by wind. In some installations a cylindrical hood is provided to restrict the dispersion of the jet, which then becomes tubular in form. To avoid vibration and failure by fatigue this should be rigid and is often of steel with concrete surround. Because of the large air demand, free access of air is essential.

The advantages of sleeve valves are simplicity and relatively low cost, low actuating power and the energy-dissipating characteristics of the jet. They have been made in sizes up to 3.850 m diameter and (smaller valves) for heads up to 280 m. Several cases of damage due to vibration have been recorded, particularly fatigue failure of the ribs attaching the fixed cone to the body, but this weakness has been overcome by increasing rigidity and avoiding causes of instability, in particular by fairing the leading edges of the ribs. Valves of this type have been operated fully submerged but trouble has occurred with partial submergence.

Submerged sleeve valves. Located in a stilling well, a valve of this type is an excellent terminal regulator. As developed by the National Engineering Laboratory, the valve has an internal sleeve sliding in a perforated cylinder. The perforations result in numerous small jets, which can be stilled in a small chamber, and the perforations can be graded to obtain any desired discharge/stroke characteristic. In some valves large ports are provided with the latter object but causing less obstruction and the full opening of the sleeve can be utilized, without perforations or ports. In this case the discharge coefficient is greater but a larger stilling well is needed. Dimensions of stilling wells providing adequate energy dissipation may be determined from Figure 22.15, but shallower, wider wells of about equal volume also have been found satisfactory.

Multijet sleeve valves have also been developed as energy dissipators in pipelines.[95] Energy is dissipated by small jets and, over a wide range of pressure differential, the valve is generally free from vibration and cavitation problems.

22.5.3 Air demand

Air vents are provided downstream of gates and valves in conduits, to relieve low pressures which develop due to regulation and to avoid cavitation or column separation following valve closure. The rate of air demand of a hydraulic jump is discussed on page **22**/20. During closure of the valve, downstream pressure falls and the water standing in the vent pipe is gradually drained. At the same time the water in the conduit, if flowing full, is decelerated, but if the conduit is long and the valve closes before the vent is admitting air, pressure might fall to cavitation level. In such cases it is necessary to limit the speed of valve closure, especially when approaching final closure. Pressure can be estimated by a step calculation.

22.6 Cavitation

In high-velocity flow and regions of low pressure, local pressure may approach the level of vapour pressure causing cavitation bubbles to form and become entrained in the flow. When these reach regions of higher pressure they collapse, producing extremely high local transient pressures which can damage solid boundaries. Examples of cavitation damage have occurred in high-velocity flow past rough surfaces and joints in concrete, in stilling basins below high spillways, downstream of submerged orifices and in valves and rotary pumps. Metal becomes damaged by pitting; concrete begins to disintegrate.

Cavitation in valves, pipe fittings and pumps is characterized by a high-pitched tapping sound and the discharge coefficient or pump efficiency may be affected. Cavitation damage usually occurs immediately downstream of the point of lowest pressure but if the cavities are formed away from solid boundaries as, for example, by fluid shear downstream of a submerged orifice, they can be carried some distance in the flow and in some cases collapse harmlessly in the fluid, but if the collapse occurs near or on a solid boundary this can be damaged. The collapse of a diversion tunnel at Tarbela dam was attributed to this cause.[96]

An index of cavitation potential is the cavitation number:

$$\sigma = \frac{H_2 - h_v}{H_T - H_2} \quad \text{or} \quad \frac{p_2 - p_v}{\rho V^2/2} \qquad (22.17)$$

where H_2 is the pressure head and p_2 the pressure at the point concerned, h_v the vapour pressure head, H_T the total (static + velocity) head, ρ the density of water, p_v the vapour pressure and V_2 the mean or relevant velocity

The heads and pressures appear as differences so should be related to the same datum. If the datum is atmospheric pressure, h_v will be negative, usually taken as − 10 m at sea-level. For the second expression the pressures are usually absolute. The cavitation number represents the ratio of pressure drop required to initiate cavitation to the velocity head available and therefore indicates the potential for cavitation. The number at which cavitation occurs depends on the boundary geometry and flow pattern. If the number for the onset of cavitation in a given situation is known through research or experience it can be used to test whether cavitation will occur over a range of velocities and pressures and (subject to small variations due to scale effect) in situations of geometrical similarity but different absolute dimensions, as in scale models.

The risk of cavitation damage on concrete surfaces in contact with high-velocity flow increases rapidly with increasing velocity, velocities in the range 20 to 40 m/s being of particular concern. It is necessary to provide a very smooth finish; specifications often call for offsets to be ground to a flat slope such as 1:30.[95] In stilling basins below high spillways, concrete baffle blocks may be protected by steel cladding. Use of special additives in the concrete can increase its resistance to damage. Damage can also be alleviated or prevented by aeration as, for example, provided by suction of air at offsets through special ducts.[64,65] In the case of vertical and radial gates and gate valves, mild cavitation may occur with $\sigma = 2.0$ and more severe cavitation with $\sigma = 1.0$. In valves of other types the critical value of σ differs in different designs and with the amount of opening. Butterfly valves were found to have incipient cavitation characteristics with $\sigma = 1.5$ for 30° opening but 3.9 for 80° opening.[97]

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37 Concrete Construction

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Contents

37.1 Introduction

Concrete in its simplest basic form consists of a mixture of cement, sand, stone (aggregate) and water. Its rapid development in this century has made it one of the principal construction materials in use throughout the world.

The Romans have left evidence of their skill in making an earlier form of concrete in the remains of buildings such as the Pantheon in Rome, although evidence of the use of concrete goes back to well before Roman times.

The production of the world's first Portland cement dates back to 1824 when Joseph Aspdin of Leeds took out a patent on the cement he had produced. This cement was formed by heating a mixture of finely divided clay and limestone or chalk in a furnace to a temperature sufficiently high to drive off all the carbon dioxide. He called it Portland cement because he thought it resembled Portland stone in colour.

Since then, there have been many developments in the manufacture of cement and modern Portland cement is now made to high standards in most parts of the world.

Although concrete usually has a reasonably high compressive strength, it always has a relatively low tensile strength. Steel reinforcing bars are therefore generally provided in structural concrete to take the tensile stresses which occur. Hence the term 'reinforced concrete' which is in general use.

37.2 Concrete production

37.2.1 Storage of aggregates

Except in those cases where the components of concrete are readily available at short notice from stockpiles at pits and quarries, ground or bin storage of at least several hours' supply of all materials is called for. In some cases, where concreting operations are concentrated into a relatively small proportion of the total contract time as, for instance, in concrete road construction, the building of large stockpiles adjacent to the batching plant will normally be required.

The equipment required for building and drawing from these large ground stockpiles is expensive and the planning of site operations must be directed towards achieving a sound economic balance between the rate of consumption and the rate of supply. It is essential to avoid ground storage of excessive amounts of materials at batching plants and to rely, as far as is possible, on current production from a number of pits and quarries.

For concretes where the standards of control required are high, provision must be made in the vicinity of batching plants for storing at least two sizes of coarse aggregates and, generally, two days' supply of fine aggregate. This latter requirement stems from the fact that fine aggregates, which are normally washed, can contain up to 15% water which will drain away at a fairly rapid rate, allowing the moisture content to stabilize at a much lower figure over a period of perhaps 24 h.

Large stockpiles should be so constructed that segregation of the larger fractions from graded materials, which could cause embarrassing variations in the overall grading of the concrete aggregate, is avoided to the greatest extent possible. Where large stockpiles of coarse aggregates are to be built, consideration should be given to the use of controlled tipping of lorry loads.

Segregation of the coarser fractions of sands from the finer is not normally a major problem since the moisture content of the whole will be sufficient to prevent particle separation. It is always desirable and will often prove economic to provide adequate paved areas, laid to falls, round the aggregate stockpiles so that access to and drawing from them does not lead to contamination by dirt from the site. It could happen that the loss of aggregate into the ground beneath the stockpile exceeds the cost of providing sufficient hardstanding for aggregate storage.

This paved area will preferably extend well outside the range of any stockpile reclaiming devices and give clean access to, and room for, manoeuvre round the stockpile. However, it will not normally be necessary to pave right up to the batching plant since with most reclaiming units there will be a certain amount of dead storage which is not removed until concreting operations are virtually complete.

A requirement of any binning arrangements made round batching plants is that their walls should be built high enough to avoid overspilling of one grade of material into another; it is also necessary to ensure that overfilling, which leads to spillage and mixing of different grades of aggregate, does not take place.

It is quite often required that concretes be made from aggregates with special properties, e.g. lightweight (for low-density concretes used for better thermal insulation and lower structural weight) or heavyweight (used for such purposes as radiation shielding to nuclear reactors). Rather than complicate binning arrangements round batching plants used for the major part of the concrete it may be found advisable to use a separate batching/mixing plant of sufficient capacity to meet the requirements of the special concrete.

The sizes of stockpiles and the rates at which materials will need to be taken from them are matters which will need to be given proper consideration by job planners and site management. Among the types of equipment most frequently seen operating in Britain are:

(1) Ground hoppers fed with materials drawn from stockpiles by, for example, forward-loading shovels and transferred thence by conveyor belts to short-term storage bins above the batching plants.
(2) Drag shovels, draglines or grabs mounted atop stockpiles located at the batching plant, feeding into short-term storage bins or pulling materials into live storage areas whence they feed by gravity into weighing equipment.

With high-capacity plants these storage bins will, of necessity, hold materials for only a small number of batches, hence the need for adequate ground storage.

37.2.2 Storage of cement

It is generally required that cement be stored after grinding in high-capacity silos at the works. This is done so that the quality of the cement can be assessed for compliance with the appropriate British Standard before delivery and also to ensure, so far as is possible, that the high temperatures attained during the grinding process can be to some extent dissipated before delivery to the site.

The consequences of using physically hot cement as opposed to cooler cement are not thought to be serious, but nevertheless provision must sometimes be made for ensuring that cement temperatures are limited. In some instances where large masses of concrete are to be placed and where overheating due to the exothermic reaction of the cement must be reduced so far as possible, it may be required that the cement be circulated through coolers or kept in circulation through a number of storage units.

Storage of large quantities of cement might be called for to smooth out irregularities in delivery when construction sites are remote from cement works. The movement of bulk supplies of cement from mills to works by special cement trains rather than by road delivery offers the opportunity for short-term site bulk storage in special high-capacity wagons. However, it will generally be necessary for at least part of the journey to be made by road and facilities for pneumatic transfer from rail direct to road vehicles and then into short-term storage at the batching plant, will have to be provided.

On site, overhead cement storage silos of capacities up to about 150 t, either singly or in interconnected groups, are quite widely used. These are generally charged pneumatically by dry-air blowers capable of lifting cement to a height of 30 m or so above the ground, mounted on transport vehicles.

Some cement companies are willing to provide storage facilities on sites and their availability should be investigated.

Although bulk storage of cement in silos is generally preferable to the use of cement in bags, nevertheless there are many small construction jobs, particularly work by small builders, where bagged cement is used. Although the bags are made of strong 3-ply paper, they are not waterproof and it is therefore important that they are protected from the weather. This should preferably be done by storage in dry, well-ventilated sheds or in the case of very small amounts, the bags may be stored outside on a dry platform raised above the ground and covered with plastic sheeting, tarpaulins or similar covers.

Whether cement is stored in bulk in silos or in bags, it should be used in the order in which it has been delivered to the site. Failure to follow this elementary precaution can result in some cement being stored for too long and consequently becoming lumpy, i.e. 'air-set' and unsuitable for use.

When a cement replacement material such as pulverized fuel ash (PFA) or ground blast-furnace slag is included in the concrete being produced, it will usually be necessary to provide another silo to store it. To avoid confusion and the possible misdirection of PFA or ground blast-furnace slag into the cement silo, it is important where bulk deliveries are made, either by road or rail, that the connections at the silos are clearly marked. Where 'split' silos are used, inspection of the diaphragm to check that it is not perforated or damaged in any way is a necessary precaution to ensure that the cement and the cement replacement material stay in their respective compartments.

37.2.3 Water storage

On average, each cubic metre of concrete produced will require between 100 and 140 l of water. In addition, large quantities will be used round the plant at the end of a concreting session for the thorough cleaning down of the whole of the mixing plant and the lorries, skips, pumps or other devices used for the transport of the wet concrete.

These large quantities of water can sometimes be drawn from a water authority's mains, but quite often some local source, such as a stream, will have to be used. Permission to extract water from a stream will generally have to be sought from the local river or water authority.

Overhead storage of water in steel tanks to give a sufficient head to supply a concrete plant can be quite expensive, particularly when drawing from mains is restricted to night-time only and adequate storage has to be provided for all day-time operations. In view of this high cost of water storage a practice sometimes adopted is to excavate a hole of sufficient capacity close to the batching plant and to provide a waterproof lining, generally 1000-gauge polythene sheeting, held round the top edge by embedment in concrete or some other means. From this, stored water can be drawn by pump to supply a small header tank above the batching plant. When this mode of storage is used every precaution must be taken to avoid damage to the lining since repair will be difficult.

The large amounts of water used for cleaning down cannot be discharged into any local water course or sewer without first allowing the cement and aggregates to settle out to such an extent that the effluent is acceptable to the authority. This calls for the building and keeping clean of a comprehensive system of settling tanks from which clear water can be decanted at the end of the line. Provision can be made to recirculate this water into the supply system, but it is doubtful, except where water is very expensive, if recovery is a practical proposition.

37.2.4 Admixtures

The use of admixtures to modify properties of unhardened and hardened concrete in one way or another is becoming increasingly common practice in the construction industry. Generally, these materials are added in very small amounts in relation to the size of a batch and it is usual to measure them by volume and feed them into the mixing water supply line from gauge tanks to the mixer.

When using liquid admixtures it is essential to maintain an adequate supply and to ensure that each batch of concrete has its proper dosage added at the correct time. This calls for the full interlocking of water and admixture supplies so that underdosing or overdosing cannot take place.

37.2.5 Batching concrete

The gauging of concrete to give mixes either of specified proportions or to meet strengths or other requirements is carried out in batching plants using weight as the unit of measurement. In almost all cases, batching plants incorporate a facility for mixing the concrete. However, many ready-mixed concrete plants have a batching facility only, mixing being carried out in truck mixers.

Although there is evidence that given proper control over operations, particularly with regard to the measurement of cement, volume batching of concrete can give high standards of quality control, this system has been almost completely superseded by weight batching. In these plants it is usual to weigh the various aggregates cumulatively in one hopper whilst the cement, any bulky additive such as PFA and water will be measured separately.

Water can be batched into a concrete mix either by weight or volume, but with the increasing tendency to use fully automatic batching/mixing plants in which the moisture content of the fine aggregate is monitored continuously and its batch weight adjusted accordingly, there is rather more emphasis on weight batching.

A small header tank is generally provided and this in turn is kept continuously charged direct from the supply mains, or by pump when the mains pressure is inadequate or supplies have to be drawn from ground storage.

Computer controls are becoming available on modern batching and mixing plants and they are designed to facilitate accurate batching of the materials and the production of concrete mixers of uniform workability. The use of computer control systems also provides a ready means of keeping accurate records of the quantities of materials used and of the weights of the constituents in each batch of concrete.

37.2.6 Mixing concrete

To achieve the full potential strength of a concrete mix it is most important that there should be a proper dispersal of the various constituents within each element of concrete. The speed at which this dispersal takes place will depend upon a number of factors amongst which are:

(1) Type of mixer and its speed of rotation.
(2) Size of charge put into the mixer in relation to the volume of the mixer drum.
(3) Degree of wear on paddles and blades.
(4) Order of charging materials.

Figure 37.1 Production of concrete for motorway base. *Note*: large aggregate stockpiles; ground storage of cement in wheeled bulk silos; small overhead cement storage; groundwater storage (in background); continuous proportioning and mixing of 4 No. aggregates and cement

Various types and sizes of mixer are available and the following are commonly used in British practice:

(1) Rotating-drum mixers:
 (a) tilting drum;
 (b) nontilting drum, including reversing drum.
(2) Split-drum mixers.
(3) Pan and annular-ring mixers.
(4) Trough mixers.
(5) Continuous mixers.

Figure 37.2 shows these mixers in general outline.

Types (1), (2) and (3) are commonly described as 'free fall' mixers since their action is derived from the falling within the drum of elements of concrete materials lifted from the bottom towards the top by a series of blades. In types (4) and (5) the mixing action is more vigorous and it is claimed that this both improves the efficiency of mixing and increases the speed at which a sufficiently high degree of uniformity can be attained.

The largest-capacity batch mixer of any type used to date in British practice has been a 6 m^3 tilting-drum type with an hourly throughput of about 200 m^3. Sizes of the various types available range from a few litres to about 3 m^3 but there are, as noted, some exceptionally large mixers.

37.2.6.1 Rotating-drum mixers

In type (1) mixers which are normally rotated at speeds up to about 20 rpm, mixing is achieved by carrying the ingredients from the bottom of the mixer to the top by a series of paddles of differing form mounted inside the drum. As the paddles approach the top of the mixer, materials are spilled from them and fall to the bottom of the mixer whence they are again lifted towards the top. The speed of the mixer is important in that a slow rotation extends the mixing time while too fast a rate will reduce the efficiency by tending to carry materials over.

A type (1a) tilting-drum mixer is charged at the open end with the axis of rotation of the mixer inclined upwards at an angle of about 45°. Mixing takes place whilst the drum is in this attitude. Discharge is accomplished by moving the axis of rotation through an angle of about 180°. Depression of the axis below the horizontal is carefully controlled to avoid too high a rate of discharge.

With nontilting drum mixers of type (1b), charging is via a retractable or nonretractable chute at one side of the mixer, depending on the loading arrangements, whilst discharge is brought about by inserting an inclined retractable chute at the opposite side. This chute intercepts the 'free-falling' materials within the drum and causes them to be discharged into a receiving hopper or other device. An alternative design is the reversing-drum mixer. In this the concrete is discharged after mixing by reversing the drum; thus, no chutes are needed.

37.2.6.2 Split-drum mixers

This type of mixer had its origin in Belgium but has found a good deal of favour in Britain, largely because of its simplicity, its ability to mix efficiently all types of concrete and its rapid clean discharge. The mixer drum, which rotates on a horizontal axis, is split vertically into two approximately equal volume sections. These sections are closed together during charging and mixing and retracted one from the other for discharging. Mixes are carried from the bottom to the top of the drum by cohesion and a small number of cleats secured to the inside of the drum. It has no blades or paddles of the form usually seen in mixers. Because of the large area of the gap between the two sections,

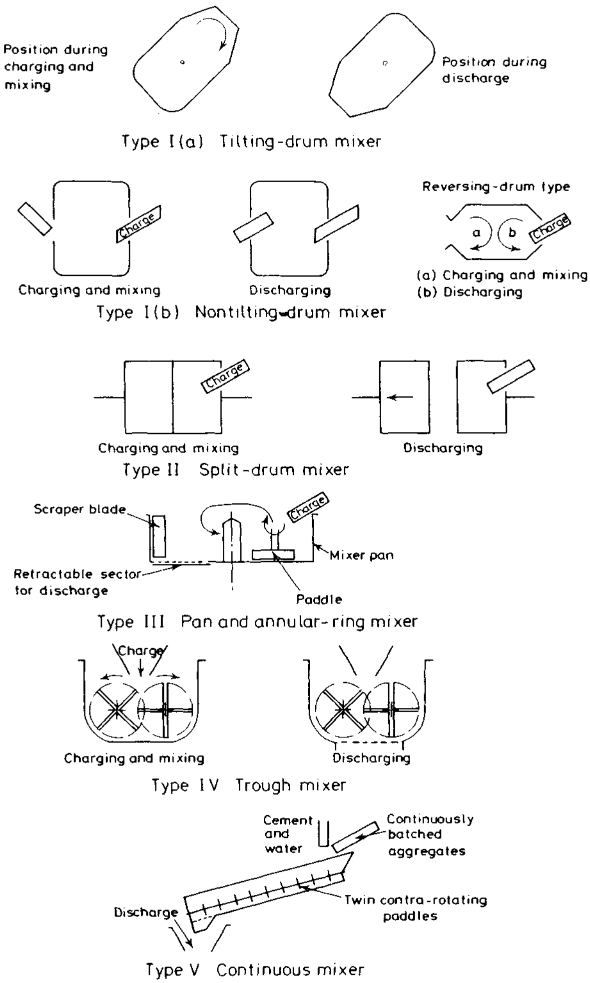

Figure 37.2 Concrete mixers. (*Courtesy*: John Laing and Sons Ltd)

Figure 37.3 Small reversing-drum mixer (Winget 400R). *Note*: drag shovel loading: charge and discharge at opposite ends of the mixer drum; means of controlling point of discharge

when retracted, discharge is very rapid. Mixing cycles are relatively short, in particular because of this rapid discharge, and their efficiency is said to be quite high.

37.2.6.3 Pan and annular-ring mixers

To speed up the mixing cycles and at the same time achieve a higher degree of uniformity of the mixed concrete a series of different types of pan mixer have come to be much more widely used in recent years. Mixers of this type, generally in the smaller sizes, have been used in precast concrete works for many years and have proved very efficient. Because a satisfactory degree of uniformity of the mixed concrete can be achieved with this type of mixer in a good deal shorter time than with type (1) their use has now extended to heavy civil engineering work and there has been a steady increase in the sizes of batches which can be mixed in them.

Since the concrete contained in the pans of these mixers is moved round the pan by a series of paddles whose action and speed varies with the design adopted, they have come to be known as 'forced-action' mixers as opposed to the 'free-falling' type described at (1), (2) and (3) above. The rotating paddles which mix the materials in the pan have varying forms and action and are driven from either above or below the pan. In some types the paddles rotate on their own axis as well as round the mixer pan and, with these particularly, a high degree of uniformity of the mixed concrete is claimed after only a very short period of mixing.

Because of the large diameter of these mixers, which for efficient mixing will have an average of about 150 to 250 mm depth of concrete over the area of the pan, they are seldom made with capacities greater than 2 m^3. A mixer of this capacity will have a pan diameter of about 3 m.

All mixers of this type are quickly discharged by retracting a section of the bottom of the pan to allow concrete to be swept from the pan into a receiver. This feature results in their being mounted quite high in a batching/mixing plant. Each different make of this general type of mixer is claimed to have advantages over its rivals, but it is probable that the final choice will depend more upon such matters as the service afforded by the maker and delivery times. Wear and tear in pan mixers will generally prove to be higher than in a 'free fall' type but wear-resistant metals are coming to be more widely used.

Power consumption is somewhat higher than for 'free-fall' mixers, but this is compensated for by the higher hourly output in relation to the size of the mixer and its accompanying batching arrangements.

37.2.6.4 Trough mixers

This very heavy and robust type of forced-action mixer is more widely used in the production of asphalt and coated materials than for concrete, though a number of them have been built into batching/mixing plants in recent years. They can be used for mixing in much larger batches than can pan mixers since the effective depth of the concrete in them can be greater.

Mixers of this type may have a single or two contra-rotating shafts carrying blades which are so shaped and disposed as to move the constituents of the mix longitudinally along the axis of the mixer as well as round it. Mixing efficiency for short mixing cycles and hourly outputs in relation to the batch sizes are high for all types of concrete.

37.2.6.5 Continuous mixers

Recent developments in machines of this type include proportioning of all the individual materials over a series of weight feeders together with the interlocks which close down propor-

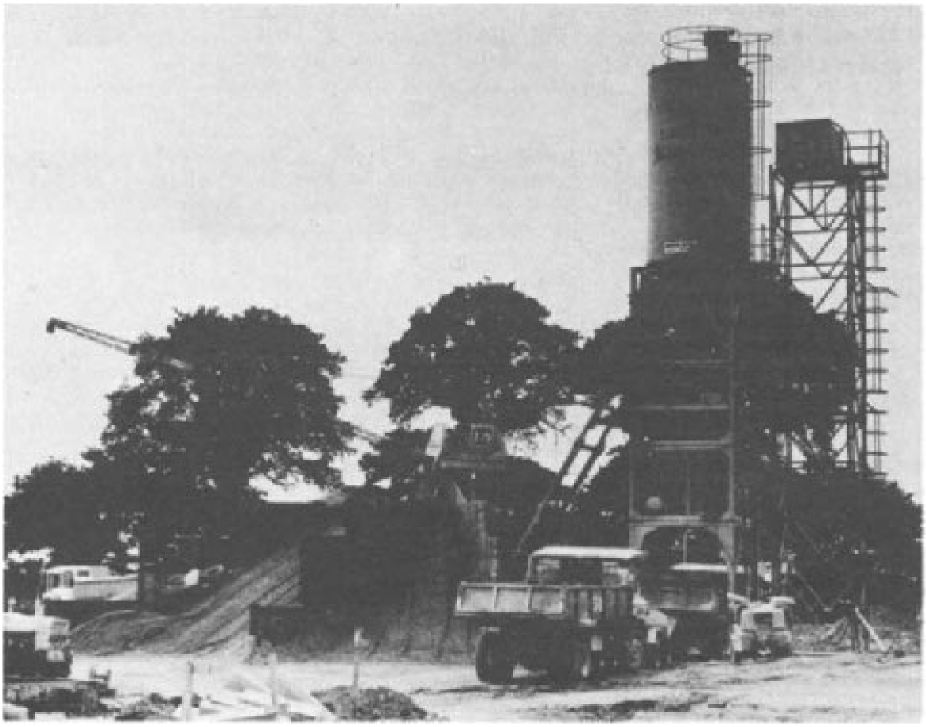

Figure 37.4 Medium-size (30m^3/h) plant (Benford PB 40). *Note*: aggregate stockpiles; overhead cement storage; ground batching; annular-ring mixer.

tioning and mixing operations on the malfunctioning of any one of the feeders. These developments have resulted in there now being available batching/mixing plants which are capable of outputs approaching 300 m^3/h. The continuous flow of accurately proportioned materials through an inclined-trough type of mixer with high-speed contra-rotating paddles ensures very effective mixing with a relatively low power consumption in both proportioning and mixing units of the plant. The rate of feed of the materials can be varied over wide ranges and the retention time in the mixer can be varied by changing the angle of tilt. All types of concrete are quite effectively mixed since the feeders batch constituent materials in their correct proportions on to a continuous moving collecting belt. This ensures that each element of concrete passed through the mixer is correctly proportioned.

37.2.7 Sizes of batching/mixing plants

A decision which will have to be made by a contractor at a very early stage in a contract is that concerning the size of plant to be installed. No doubt early planning of a job whilst preparing a tender will have given a clear indication of what the concreting programme is likely to be and to have set guidelines as to the number, capacity and siting of the batching/mixing plant needed to meet it.

In general it is considered preferable to concentrate concrete production at one central location in the area of maximum demand rather than at a number of dispersed points. This simplifies the problem of supply of materials and of storage. However, over sites to which access can be gained at a number of points it might be thought preferable to locate a number of smaller plants in strategic areas. The early provision of a network of substantially built temporary and permanent site roads will thus be of the greatest value. A further alternative is to provide more than one concrete production unit of smaller size at a central point. Although it will inevitably cost a good deal more to provide a number of small plants with a combined capacity sufficient to meet peak demands than it will one large one, the ability to continue operations when one section is out of commission is particularly attractive. A further point to be considered is that high-capacity single plants producing concrete in large batches need means for carrying these heavy loads around a site and the provision of adequate means for handling the concrete into position.

In many applications of batching and mixing plants – concrete road construction is one – a very obvious requirement of such a plant is that the time occupied in erecting, dismantling, transporting and re-erecting in a new location, and any heavy cranage required for this, should be reduced to a minimum. For this reason a number of manufacturers are building into their plants either self-erecting facilities or features which will result in low costs and short downtimes for these operations. The use of harnessed electrical wiring assembled by no more involved a process than inserting heavy-duty plugs into sockets is a further aid to cost reduction.

For ease of transport it is sought to proportion units of the plant so that they are within acceptable loading gauges on public roads and can be moved without being broken down into smaller units.

37.2.8 Mixing efficiency

In order that concrete should meet the requirements of a specification and be in accordance with mix design developed in the laboratory it is necessary that mixing should continue for such length of time that all the materials are uniformly dispersed throughout a batch. This time will vary with the intensity of the mixing action. Thus it is considered that the 'forced-action' types of mixer are able to achieve a higher degree of uniformity in a shorter time than can 'free-fall' mixers. However, for the reasons given, the capacity of 'forced-action' mixers is limited; hence the largest mixers are usually of the 'free-fall' type.

The time of a mixing cycle is governed by the speed at which the operations of charging, mixing and discharging can be carried out.

Of these three components of the mixing cycle the lengthiest is likely to be the second, though mixing is, to some extent, being carried out throughout the three.

Some specifications require that there should be a minimum lapsed time between completion of charging and discharging, this time being dependent upon the measured time required to achieve the degree of uniformity within the mix suggested in BS 3963:1974 *Method for testing the mixing performance of concrete mixers.*

37.3 Ready-mixed concrete

Over 80% of site-placed concrete in the UK is produced by the ready-mixed concrete industry and this accounts for about 45% of the total cement sales for all purposes. Similar figures apply to other industrialized countries and they demonstrate the increasing tendency of contractors to rely on the supply of a concrete from a ready-mixed concrete plant as opposed to the use of site mixers.

'Building' sites are a major use of ready-mixed concrete since their demand for concrete is usually intermittent and therefore the provision of site mixing facilities is probably not economic. Also there may be a problem of finding space for site mixers and stockpiles of aggregates on many building projects, particularly in congested urban areas.

On 'civil engineering' sites, the choice between the use of ready-mixed concrete or site-mixed concrete is dependent on a number of factors. They include costs (which means a detailed study of the whole concreting operation, not just material costs), continuity of supply, rate of concrete production, site access, availability of concrete with special cements or admixtures and control of quality.

A big advantage arising from the use of ready-mixed concrete on civil engineering works is that deliveries can be brought to locations at, or very close to, the point of use. The unit price quoted will normally be to site and, provided that a reasonable standard of access road to various points is available, this will include anywhere on the site. Thus, the means of transporting large quantities of mixed concrete from a central plant to any point on the site are provided by the ready-mixed concrete operator.

There is often justification for using both ready-mixed concrete and site-mixed concrete on a site. For example, in the early stage of a contract, concrete may be required before the site mixer has been installed. It will therefore be appropriate to use ready-mixed concrete for the early work before the site-mixed concrete can be produced. There is also the situation when the quantities of concrete required in a given time are beyond the capacity of the site mixing plant. In such cases, ready-mixed concrete can be used with advantage to supplement the site production.

There are certain situations where the use of ready-mixed concrete becomes an obvious choice. The placing of large pours of concrete for large foundations in one continuous operation is an example of a construction requirement which demands large-scale production facilities for a short period. The full resources of one or possibly two large ready-mixed concrete plants may be required for a period of up to about 24 h in such a situation. There are other examples of high placing rates which call for the use of ready-mixed concrete such as the concreting of large-bored piles.

On certain road construction sites both in the UK and abroad, ready-mixed concrete operators have entered into subcontracts with main contractors to supply the aggregates and the cement, mix the concrete and distribute it over the site in agitator trucks or other vehicles. The merits of this arrangement depend on a number of factors, not least being the question of whether or not the main contractor can himself find the necessary amounts of aggregates in reasonable proximity to the site.

37.3.1 Plant

Ready-mixed concrete is supplied to jobs in a number of different ways depending largely on the preference of the operators, but perhaps on the terms of the specification in use.

The most widely used method in the UK, since it dispenses with the need for an expensive central mixing plant, is to dry-batch all the materials in two or three operations into the truck mixer whilst the latter is turning at 10 to 15 rpm below the batcher. On completion of the charging of the materials, water is added as required and mixing continues at the plant for at least 100 revolutions of the drum (10 to 15 min). Mixing then continues at about 1 to 2 rpm while the truck mixer travels to the site.

A second 'dry' method is to batch in the same way, but to charge the water required into a tank carried on the mixer. Only on arrival at the site is the calculated amount of water added to the materials in the mixer drum and mixing carried out for a minimum of 100 revolutions (10 to 15 min) before discharge takes place.

Most batching plants now in current use are the two-stage type of plants (i.e. a first stage where the aggregates are weighed and then taken by belt conveyor to the second stage where the cement is weighed and added to the aggregates). Compared with the earlier plants, they have the advantage of reduced height, flexibility of layout, smaller individual structures and ribbon feeding of the batched materials on conveyor belts to the truck mixers. A typical plant will provide for two cement compartments with a total of 60 to 80 t capacity and five aggregates compartments with a total capacity of 250 to 500 t.

Plant mixers are used for a small proportion of central plants supplying ready-mixed concrete; in the UK, probably 10 to 20% of suppliers' depots have central plant mixers. However, in several European countries, especially Germany and France,

Figure 37.5 Truck mixers awaiting loading at ready-mixed concrete depot. *Note*: conveyor-belt attachment on the truck mixer to the left. (*Courtesy*: British Ready Mixed Concrete Association)

the approach is different and the majority of plants are provided with central mixers.

The main advantage of a central mixing plant is that it facilitates better control over the quality of the concrete produced. However, this method of concrete production involves higher capital costs for the plant since a high-capacity mixer capable of completely charging a truck mixer or agitator truck in one, two or three batches is necessary (see Figure 37.5). The concrete is fully mixed at the plant and charged into the truck mixer whilst the latter is rotating at high speed. After complete charging, the concrete is 'kept alive' and prevented from settling and compacting in the drum by rotating at 1 to 2 rpm en route.

Most types of mixer have been used for producing ready-mixed concrete, but the type which is now used most widely in the UK is the tilting drum mixer of 3 m^3 and 6 m^3 capacity. This mixer can be charged and discharged quickly, it occupies low headroom, and because it revolves at relatively low speeds, it uses less power and involves less maintenance than most other types.

37.3.2 Semi-mobile plant

Construction projects of relatively short-term duration are sometimes supplied with concrete by semi-mobile plant. Whereas a mobile plant is one that may easily be moved from place to place, a semi-mobile plant may be defined as one which can only be moved from place to place with a little difficulty.

This type of plant is a compromise between the requirements of total mobility on the one hand and the efficient and economic production of concrete on the other. It is desirable to aim for a 6 m^3 batch size and reasonable quantities of cement and aggregate storage. As a general guide, 60 t of cement and 150 t of overhead aggregate storage should be provided as a minimum requirement.

37.3.3 Truck mixers

Truck mixers are basically free-fall mixers mounted on a truck chassis. They can be used either for mixing and transporting the dry-batched concrete, with water added at the supplier's depot or site, or they can be used for transporting centrally mixed concrete, i.e. as agitators.

The size of drum is largely governed by the official weight restrictions on public roads which results in a nominal capacity of 6 m^3 for the majority of truck mixers used in the UK.

Truck mixers fall broadly into two main types namely: (1) the drum is driven by a separate donkey engine; and (2) the drum is hydraulically driven by power take-off from the truck engine. The use of the separate donkey engine is, however, decreasing because it adds weight to the vehicle and is consequently a handicap to the need to optimize the payload.

On arrival at site, truck mixers generally discharge directly into the required location (for ground and foundation works) or into hoppers (for elevation to a structure) (see Figure 37.6).

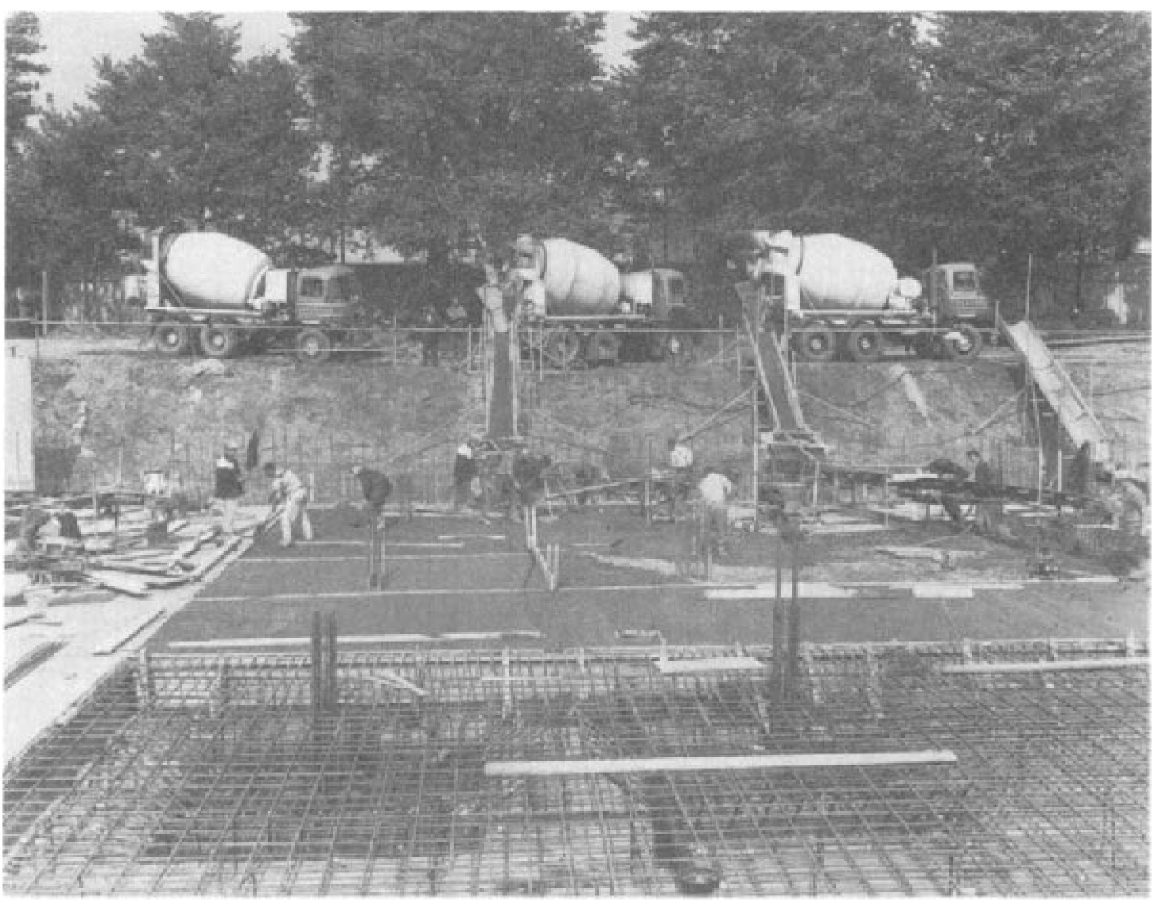

Figure 37.6 Foundation slab being constructed with concrete fed from truck mixers. The concrete is directed down chutes at the side of the excavation. (*Courtesy*: Cement and Concrete Association)

There is an increasing tendency for truck mixers to work in conjunction with mobile pumps, and in this case the concrete is discharged into the receiving hopper of the pump. The use of a pump means that specific attention must be given to the mix design of the concrete to ensure that it is pumpable, and resulting changes to the cement or sand content can result in the imposition of a cost surcharge by the supplier.

Articulated belt conveyors for mounting beneath the discharge chute of a truck mixer are now available and these provide added height and distance when placing concrete. However, since these conveyors are mounted on the truck mixer, there is a loss of payload and consequently their use is very limited.

Whichever method of placing is adopted, safe and adequate site access must be provided for the truck mixers and the method of placing carefully organized to ensure the best use of labour. The supplier normally aims to discharge the concrete at a rate of not more than 5 min/m^3 and therefore expects a truck to spend about 30 min on site. Excessive delay on site can incur an additional charge by the supplier.

37.3.4 Quality control

The principles of quality control for ready-mixed concrete are the same as those for site-mixed concrete and one of the main requirements is that the concrete should meet the specific compressive strength requirement. The strength achieved depends largely on the cement content which is the most expensive ingredient of the concrete. In the face of strong competition, each supplier aims to keep cement content to the minimum possible and there is consequently a risk that the more unscrupulous suppliers will provide a cement content which is too low. Hence the need for tight control by the customer and the regular casting of concrete cubes for testing to check that the required strength has been achieved. In cases where the customer is concerned about the durability of the concrete, he should specify a minimum cement content.

Another primary factor is the water content of the concrete which affects both the strength and workability. The required workability is normally quoted at the ordering stage and the customer is entitled to expect this degree of workability, within certain tolerances, at the time of discharge on site. If the workability at this stage is too low, the onus is on the supplier to add more water to the concrete to bring it to the specified workability. On the other hand, if the workability is correct, but the customer's site staff request the addition of more water to increase the workability, then the supplier will expect a signature to this effect since he cannot be held responsible if the required strength is not then achieved.

In the UK, quality assurance procedures for ready-mixed concrete are controlled by a national certification organization called the Quality Scheme for Ready Mixed Concrete (QSRMC). The scheme establishes technical requirements, including standards of production and quality control by manufacturers of ready-mixed concrete and provides a system for the accreditation of ready-mixed concrete plants.

The main objectives of the QSRMC can be defined as:

(1) Defining technical requirements for the quality systems to be operated by member companies in accordance with BS 5750 'Quality systems' and BS 5328 *Methods for specifying concrete including ready-mixed concrete*.
(2) Providing independent assessments of plants and their associated quality records by qualified specialists in concrete production and technology.
(3) Issuing certificates of accreditation of plants and their associated quality records.
(4) Assuring, by continuing surveillance, that the standards of the scheme are being maintained.
(5) Providing information to the construction industry on the status of accredited plants.

The QSRMC is very similar to other schemes which are now well established in Europe, particularly in Belgium, France, Germany, Holland and Sweden.

37.4 Distribution of concrete

37.4.1 General observations

Whatever the scale of the work, the problem of distributing concrete around a site will arise and a contractor has a wide range of equipment available from which to make his selection.

Where concrete is delivered from a mixing plant to the work by one mode only – e.g. truck mixer – and chuted direct into the work, this can be regarded as primary distribution only. Where it is conveyed for part of the distance by one means and for the rest by another as, for instance, an agitator truck, transferring concrete into a pump hopper, thence into the work, this can be regarded as primary and secondary distribution.

Generally, primary distribution will be by means of wheeled transport of one kind or another, but other methods for moving concrete in bulk over fairly long distances – as, for instance, pumps and conveyor belts – are coming to be used more widely as their design, versatility and standards of reliability are improved.

One problem which sometimes arises, particularly in warm weather or in hot climates, is that changes in workability can take place during transfer and occasionally cause difficulties in placing. However, the use is becoming widespread of plasticizers and set retarders in mixes as means by which a consistency suited to proper placing can be maintained for quite long periods.

In the following sections all the commonly used methods for distribution and handling are briefly described. However, as is so often the case, the personal choice of job planners based on their own experience and evidence from previous works, and, of course, availability of suitable equipment, will play a major part in determining the precise form that primary and secondary distribution are to take.

Amongst the methods of distribution which will be considered here are:

(1) Wheeled transport – for mainly horizontal movement (H).
(2) Hoists – for vertical movement (V) only.
(3) Cranes of different types for H and V.
(4) Concrete pumps for H and V.
(5) Pneumatic methods for H and V.
(6) Conveyors mainly H, but also V.
(7) Cableways for H and V.

37.4.2 Wheeled transport

37.4.2.1 Lorries

The cheapest mode of transport for concrete is undoubtedly the tipping lorry but in general the volume carried and the way in which discharge takes place makes them unsuitable for most purposes. However, they are widely used in paving operations and as flat-bottom transporters for concrete hoppers.

Widely used forms of wheeled transport used for both primary and secondary distribution are dumpers of a range of different types and sizes, flat-bottom tipping lorries (mainly for

Figure 37.7 1-t turntable dumper capable of depositing concrete over arc of 180°. (*Courtesy*: Benford Ltd)

paving operations) and lorry- or trailer-mounted concrete hoppers.

37.4.2.2 Dumpers

Small, hand-propelled dumpers – wheelbarrows and prams – still have their occasional use on even major works. Mechanization of a simple form improves the rate at which concrete can be distributed by this means, but it should be noted that the increase in volume carried, which mechanization allows, does call for rather more sophisticated means of access than just a series of planks serving for a barrow-run.

Mechanical dumpers are supplied in a range of different sizes, generally geared to multiples of the batch size of the mixer they are serving. They are often plagued by the fact that the more workable concretes which are readily discharged from them are very easily spilled unless the haul roads along which they are used are of a reasonably good standard and straight. Clean discharge of the less workable concretes, which are not so readily spilled, is more difficult to achieve and may entail a good deal of stripping out. They also have the disadvantage that concrete is literally dumped from them in one mass; this can pose a number of problems such as the effect of sudden heavy impact loading on formwork and the displacement of reinforcing steel.

To overcome these problems a range of dumpers whose rate of discharge can be controlled hydraulically by the dumper driver adjusting the angle of tilt of the body, has been introduced. A further variant of the simple end-tipping dumper is that which allows rotation of the body to feed concrete over an arc of about 180° (see Figure 37.7).

When used for loading a device for secondary distribution, such as crane skips of various forms, a high discharge level for the concrete is a marked advantage. Equipment of this type is available with a discharge height of up to 2 m and can be used for filling a large range of skips and hoist buckets.

37.4.2.3 Lorry- and trailer-mounted transporters

For the distribution of concrete from the larger sizes of batching plant – say upwards of 25 m^3/h – it is necessary to consider units which are capable of carrying several cubic metres at one time. This implies the use of a chassis of not less than 5 t carrying capacity. High discharge trucks from which discharge is assisted and controlled by the angle of tilt of the body and by a hydraulically driven paddle which propels the concrete towards the outlet, are widely used on sites.

A good deal of concrete is now moved around sites from central mixing plants by truck mixers. By this means, mixing of the concrete is continued whilst in transit, in just the same way as ready-mixed concrete would be.

In those cases where concrete is moved into its final position via crane skips of varying capacity such as, for instance, in dam construction, it is common practice to load the skips themselves at the mixer and to transport these on towed trailer bodies to the pick-up point closest to the concrete's final position. These trailer bodies will often be fitted with cradles into which the crane skips can be easily located, since fully loaded skips will have a high centre of gravity and might well prove unstable on the trailers if difficult ground conditions have to be negotiated.

37.4.3 Hoists

Hoists of various types are used solely for vertical movement of concrete and, despite the competition of a vast range of different crane types, they still play an important role in building and civil engineering construction. They range from the rudimentary platform hoist capable of lifting two or three barrows or a pram full of concrete over a short distance to an automatic hoist of up to about 2 t capacity with skip discharge into receiving hoppers at pre-set levels up to about 200 m. Where concrete in substantial volumes has to be lifted to a considerable height, over which travelling times are likely to be extended, the use of twin hoists is sometimes called for. Apart from giving better continuity of flow of concrete such an arrangement can ensure that work will not cease in the event of breakdown of a single unit.

Whilst most hoists currently in use call for the erection of a substantial tower which, for adequate stability may need to be tied-in to the structure being erected at frequent intervals, there is for some applications, such as concrete chimney construction, a trend towards the use of rope-guided hoisting systems. Here the bottom works including hoist winches, etc. and the headgear are connected only by tensioned guide ropes. Since these guide ropes pass through eyelets on the outside of the hoist cage they serve to constrain the hoist cage, within close limits, to a vertical path.

Building operations generally call for the vertical transportation of personnel as well as the materials of construction. Major movement of personnel will generally be confined to fixed periods but materials of all types will be required throughout the day. For this reason some hoist systems have the facility for on and off loading of special wheeled concrete skips as required.

The design and operation of all loading devices are subject to rigorous regulations to ensure safety in operation and it is incumbent on site management to ensure that these are enforced and that detailed inspection of the whole of the mechanism is carried out at frequent intervals.

37.4.4 Cranes

Handling materials by means of crane and skips is probably one of the oldest construction techniques known to the industry. It would be unwise for any contractor to price work on the basis of using recently developed handling methods without giving full consideration to the use of well-tried equipment such as the crane and skip.

The range of types of crane available to the construction industry is a wide one, consisting as it does of: (1) derricks; (2) tower cranes; (3) cranes mounted on crawler base machines; (4) lorry-mounted cranes and wheeled cranes; and (5) hydraulic cranes.

The lifting capacities of each type also cover a wide range.

Categories (4) and (5) have probably seen the greatest number of very recent advances but since their introduction some years ago both (2) and (3) have seen considerable changes in design and stepping-up of capacity. Advances have been made, too, in derrick design but basically, apart from changes mainly concerned with prime movers, they have remained unchanged for decades. The steam crane is now all but a museum piece; however, its ruggedness, simplicity and general freedom from breakdown still assures its place where electric power cannot be provided at reasonable cost; nor is the standard of maintenance available likely to be adequate for diesel power.

37.4.4.1 Derricks

These fall into three basic types: (1) stiff leg derricks; (2) guyed derricks; and (3) mono-tower derricks. Type (2) are mainly used for special work such as the handling of very heavy, indivisible steelwork loads. They would rarely, if ever, be used for handling concrete.

Stiff leg derricks with carrying capacities up to about 15 t and jib lengths to 45 m are, like all other lifting devices, restricted to loads much below their maxima when their radius of action approaches the maximum. For example, a derrick as above will only handle the maximum load at a radius of less than 30 m; above this figure it will be much reduced. The stays restrict the effective working arc to rather less than 270°, but this restriction rarely rules out their use, particularly when their range can be increased by mounting them on three bogie trucks set on two parallel pairs of rails.

The range of working height can be increased by mounting the lying legs, with their sole plates, on gabbards, which can again be either fixed or on rail-mounted bogies. Ground conditions are always an important factor in considering the stability of derrick cranes and they become of even greater significance when travelling derricks, mounted on tall gabbards, are called for. The need for soundly constructed rail tracks, laid to close tolerances, cannot be overstressed.

Where full-circle operation of a derrick is needed, then the mono-tower crane can be the answer, but such a machine will be restricted in working area swept by the jib from a fixed point. Mono-towers can be made to travel, but problems of the stability of a single moving tower makes this version much less attractive.

37.4.4.2 Tower cranes

Tower cranes were not widely used as an aid to building and civil engineering work in Britain until 1953 but they were coming to be widely used on the continent, particularly in France, for many years before this. Tower cranes, which can be mounted on fixed or movable towers are of two basic types: (1) luffing jib; and (2) saddle jib. Both of them can be mounted on tracked base machines, which can be self-stabilizing, or on lorries fitted with hydraulically actuated outriggers whereby the verticality of the tower can be ensured.

With both the luffing-jib and saddle-jib types of machines, load-carrying capacity at increasing radius is greatly restricted and manufacturers' specifications should be studied before making a selection. Where very tall towers are to be used, these must be tied-in to the erected structure at intervals of about 20 m. When the radius of action of a tower crane is increased by mounting the tower on rail-borne bogies, the same attention to accuracy in track laying – and of course its maintenance – (as for derricks on gabbards) is essential.

Lorry- and crawler-base-mounted tower cranes have generally lower lifting capacities than have the static and rail-mounted types, but they fulfil a useful purpose. However, their high capital cost may restrict their use to those sites where a high degree of manoeuvrability from one working place to another, coupled with a reasonable speed of movement, is of greater significance than is high load-carrying capacity.

Restrictions need to be placed on the operation of any lifting device during high winds and this is particularly the case with tower cranes; this might be considered a serious handicap to their use. However, examination of meteorological and downtime records for the majority of sites shows availability to be generally upwards of 90%. The location of the work will of course have some bearing on this availability factor.

37.4.4.3 Crawler-base-mounted cranes

The attractiveness of this type of machine is that cranage is only one of a range of functions which the base machine can perform. Within a short space of time and with the appropriate equipment, it can be re-rigged as a face shovel or a dragline. However, to ensure stability when ground conditions are difficult, it may be necessary to use a machine with wider and longer tracks than would be needed for excavating functions alone.

The range of base machine available ensures their versatility as means of handling a range of capacities of concrete buckets, and their fast rate of slewing gives good output figures in this activity. The versatility of cranes of this type is much increased when the main jib is supplemented by a fly jib since their working range when operating close into a structure is thereby greatly increased.

The higher-capacity mobile cranes are normally those available on crawler bases but there seems to be a certain amount of rivalry between makers of both crawler-mounted and lorry-mounted cranes to achieve the accolade of highest capacity.

37.4.4.4 Lorry-mounted and wheel-mounted cranes

The former is an independent self-contained crane unit mounted on a multi-wheeled, multi-axle chassis of appropriate size which can be moved under its own power along public roads. The motive power and controls of the crane are completely separated from those of the lorry chassis on which it is mounted.

A wheel-mounted crane is one in which both the travelling of the crane over the ground and the various motions of the crane unit are controlled from one central point. Because of their rather low ground speed these units are normally to be found working in such sections of construction sites as pre-casting

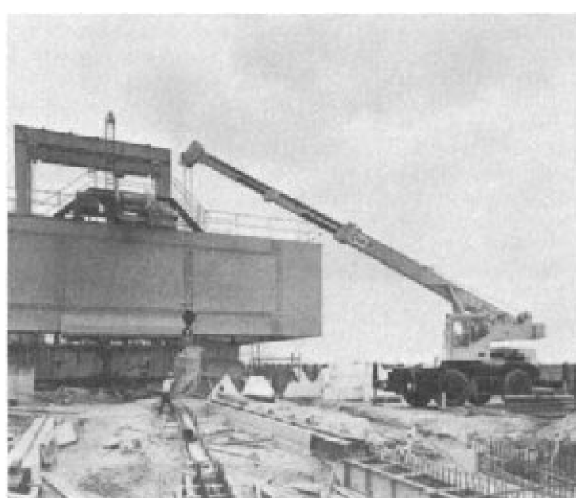

Figure 37.8 Hydraulic crane handling roll-over skip. (*Courtesy*): Coles Cranes Ltd)

yards, plant yards, etc.; they are moved from site to site on low-loading lorries.

The larger sizes of lorry-mounted crane with extended or heavy-duty jibs will normally be moved in at least two units, one carrying the crane mechanism and the other, sections of the jib and fly jib to make up the required mast length.

With all but the smallest loads for which these cranes are used, it is necessary to use the in-built hydraulically actuated outriggers to achieve the necessary degree of stability and to relieve the lorry axles of the crane burden.

37.4.4.5 Hydraulic cranes

The rapid advances which have taken place in the design of hydraulically actuated cranes with their multi-section telescopic jibs, have possibly outstripped those of almost any other type of lifting device during recent years. Their adaptability seems to be endless and new versions, and the uses to which they can be put, are encountered with astonishing frequency.

In this type of crane the only nonhydraulically actuated function is that of hoisting, but even here hydraulic motors can be used for the cable-drum drive. Slewing, luffing, jib extension and outrigger control are all performed by hydraulic cylinders and rams alone or in combination with a form of cable drive. The speed at which the requirements for a particular type of operation can be met make hydraulic cranes one of the most favoured items of equipment at the commencement of work on many sites.

37.4.5 Concrete skips and buckets

Whilst cranes of the various types already described can be and are used for a multitude of purposes on all construction sites, their role in the handling of concrete is that of moving containers charged in one way or another from point to point on a site. These containers are known variously as skips or buckets and there are several different designs to meet differing loading and placing conditions. In size, they range from capacities of about 400 l to 9 m^3 which latter size has been used abroad in concrete dam construction, mainly in conjunction with cableways. Skips are of two basic types: (1) roll-over; and (2) constant attitude. The first (see figure 37.8) is normally charged whilst lying on the ground in a horizontal position, close to the mixer or concrete transporter; it assumes a vertical position when hoisted by a lifting device. Concrete is released through the skip discharge in a controlled flow by means of a simple flap, actuated by a lever which is locked in position during transit on the crane hook to the point of deposit (see Figure 37.9). These skips can be fitted with variously shaped outlets and deflectors which permit their being used to good advantage in filling columns and walls.

The design of a skip which will give clean discharge of concrete of a range of consistencies without recourse to hammering and rodding has been the objective of all manufacturers of this type of equipment but it is probable that the answer to the problem of clean discharge lies just as much with the user as with the designer of the equipment. Cleanliness and freedom from build up of concrete in either its wet or hardened state are essential if discharge problems are to be avoided.

Constant-attitude skips are charged with concrete whilst in the same attitude as they will be during transport and discharge. Generally they are of larger capacity than the roll-over skips, but there is no well-defined range of sizes used for one type or the other.

In the larger sizes, where the weight of concrete above the outlet might be quite substantial, some form of mechanically operated device will be needed to open the gate for discharge. Simple geared clamshell gates are sometimes used at the lower end of the size range, but they tend to be rather slow in operation. For a faster and more positive gate action, pneumatically or hydraulically actuated rams are built into the structure of the bucket. Such mechanisms add substantially to skip weights.

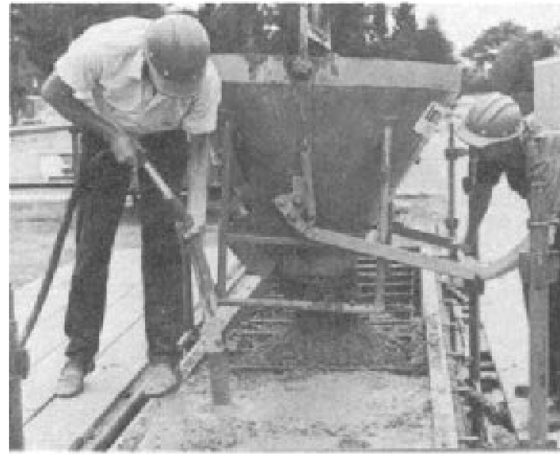

Figure 37.9 Concrete being placed by skip and compacted by poker vibrator. (*Courtesy*: Cement and Concrete Association)

For air operation, it is essential to have a pressure airline available at the point of discharge and, of course, time is spent in coupling up and building up sufficient pressure to actuate the mechanism. Recently developed built-in hydraulically actuated gate-opening devices whose sources of power are hydraulic accumulators charged during hoisting, are said to give very effective control over the discharge of concrete, whether or not the whole batch is to be deposited in one place.

It is clear that the self-weight of a concrete skip or bucket is a factor of much importance to the user since the effective rate of operation of a lifting device will be controlled, in part at least, by this. For this reason, manufacturers have from time to time experimented with different materials for skip construction. Very light weight has been achieved by fabricating in glass-reinforced plastic but because of inability to withstand the inevitable rough usage on sites, these skips have been far from successful. Significantly better concrete weight to total weight ratios are achieved by using magnesium in the manufacture of buckets, but only at very considerable additional capital cost. It should be noted that because of the chemical reaction between concrete and aluminium neither this metal nor its alloys are suitable for this purpose.

37.4.6 Concrete pumping

Production of concrete pumps in the UK by the Concrete Pump Company started in 1932 and by 1939 upwards of seventy machines had been built, many for export.

In these first pumps, the concrete was moved down the pipeline by a piston, actuated mechanically by a diesel engine or electric motor. The energy-consuming process of accelerating a column of concrete in the pipeline, allowing it to come to rest whilst a new charge was fed into the pump cylinder and then re-accelerating the whole, soon came to be recognized as a marked disadvantage of the process. Better continuity of movement along the line was achieved by the German device, developed independently by Torkret and Schwing, of using twin cylinders, one charging whilst the other was discharging. Torkret machines used water as the operating medium, bleeding a small proportion away at each stroke to lubricate the pump cylinder.

Schwing adopted oil as the operating medium, but of necessity used water as cylinder lubrication.

There is now a wide choice of equipment available from both the longer established manufacturers and from newcomers.

The basic differences between various makes of pump are in the actuating medium – oil or water – and in the type of valve used – gate or flapper. So far as can be seen, all types of pump, when operated in the correct manner with suitable concretes, are capable of satisfactory performance but some of the plant servicing departments associated with contractors appear to have their own preferences.

Most of the early single-cylinder ram pumps were of 150 mm bore and suited to pumping concretes with aggregate up to about 38 mm maximum size, at rates approaching 12 to 15 m^3/h, dependent upon the extent to which the pump cylinder was fully charged at each stroke. However, smaller, 75 and 100 mm bore pumps were also available for pumping concrete with aggregate up to about 19 mm at rates of around 6 to 8 m^3/h.

Concurrently with the development of hydraulically actuated pumps in Germany and elsewhere, Challenge–Cook Brothers in America introduced an entirely new concept of a concrete pump which they designated the Squeez-Crete pump. In this machine a short length of flexible but highly abrasion-resistant 100 mm diameter circular tube in the form of a U is charged with concrete from a hopper, this charging being assisted by maintaining a high vacuum in the surrounding chamber. Concrete is expressed from the flexible tube by rollers which, as they rotate on an axis and round the U from inlet to outlet, depress the tube, pushing the contained concrete forward towards the outlet. Since the rotating of the rollers along the U length of flexible tube is continuous, the very desirable objective of continuous flow along the flexible pipe and hence the delivery line, is achieved.

Although 150, 200 mm and even larger diameter pumps are in use, by far the majority of concrete is pumped through pipelines of 75 and 100 mm diameter. One of the main reasons for the use of the smaller sizes is that lengths of large-diameter pipe charged with concrete are very heavy and difficult to move on site. Thus, where large-diameter lines are used they will generally be associated with a semi-permanent pump and pipeline installation so that the advantage of the favourable area:wetted perimeter ratio of the large diameter line can be exploited fully. For flexibility of movement around construction sites, the smaller lorry-mounted and thus fully mobile equipment with pumping mains mounted on hydraulically actuated articulated booms is much favoured (Figure 37.10).

Generally, small-bore pumping equipment is used to handle concrete supplied either from ready-mix plants via truck mixers or concrete mixed on site and distributed in agitator trucks. By this means, the flow of concrete from the carrying unit into the re-mixer and feed hopper of the pump can be accurately controlled so as to keep the head of concrete approximately constant.

Where bigger pumping units are used they often form an

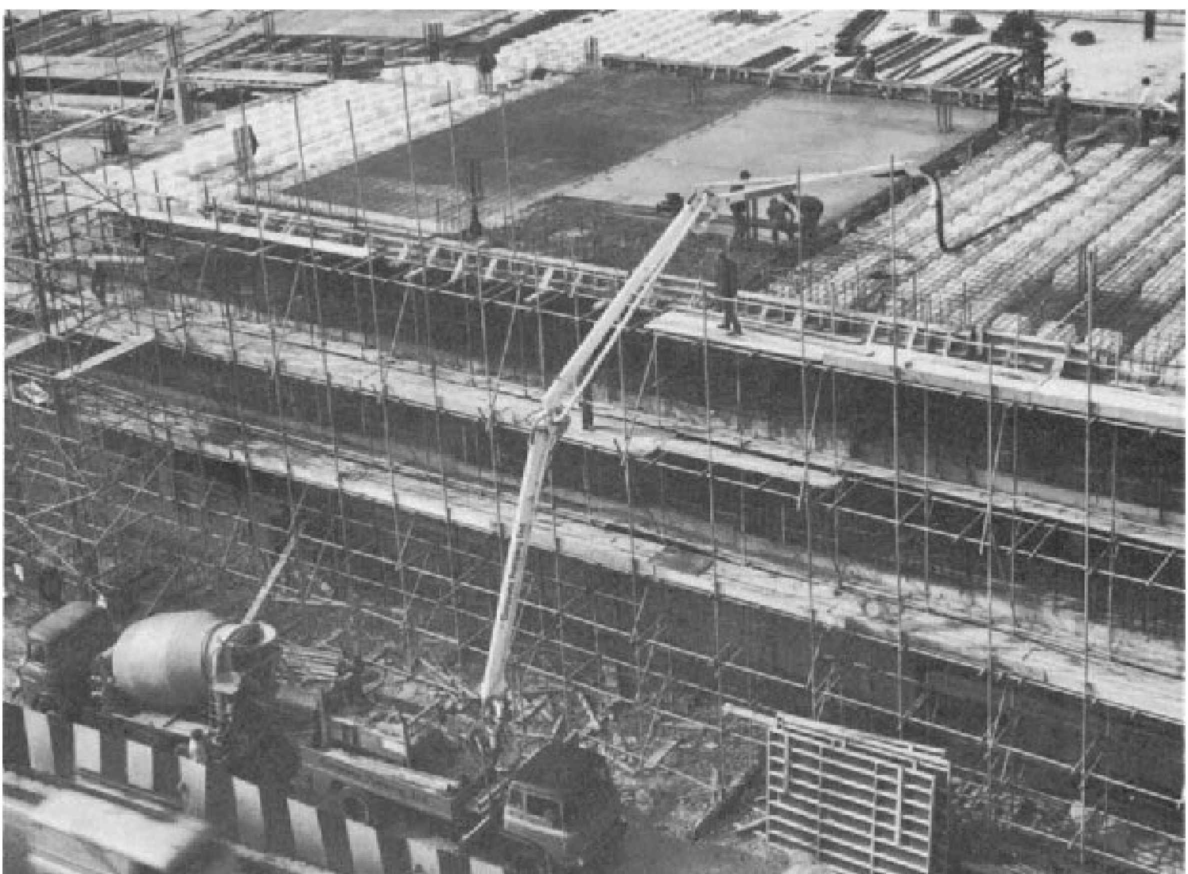

Figure 37.10 Third-floor slab being concreted using mobile pump fitted with folding boom carrying the pipeline. The concrete is supplied to the pump by truck mixer. (*Courtesy*: Cement and Concrete Association)

integral part of a combined batching/mixing/distribution unit where facilities may be provided for alternative methods of distribution as desired.

As would be expected, the effort required to pump concrete vertically is a good deal greater than that for horizontal movement – in ratios varying between 10:1 and 6:1 according to the make. In addition, extra effort is required to negotiate bends, which, when they are unavoidably incorporated into a pipeline, should be of large radius.

When pumping at their maximum range, which is usually claimed as being from 250 to 500 m horizontally and up to 80 m vertically, the output from all pumps tends to fall away, sometimes quite seriously.

It is generally preferred to use small-bore lines for vertical and larger ones for horizontal transport. However, it is not considered advisable to change diameter along a length of pipe. Where a long horizontal movement of concrete is to be followed by considerable vertical movement it might be thought advisable to transfer into a smaller-bore pipeline, through a supplementary pump.

In view of these limitations in the scope of pumping operations, the scale of the work involved should be carefully studied in advance to determine what is likely to be the most appropriate equipment. The layout of pumping points in relation to reception points should also be arranged to keep distances as short as possible.

When concreting operations include the use of expensive pumping equipment, adequate planning to ensure that utilization is as high as possible is essential. This implies that an aim should always be to have a sufficiency of work available to fully use semi-permanent installations on construction sites or to exploit the capabilities of hired-in pumps to the maximum.

37.4.7 Pneumatic placing of concrete

The use of compressed air to convey concrete from a container vessel – generally referred to as a pneumatic placer – along a pipeline to the desired location is little used now. It was, however, quite a popular technique some years ago, being used mainly for such work as tunnel linings for which it may still have some limited application. In the case of the latter, the end of the pipeline is inserted into the space between the tunnel form and the excavation rock face, and the concrete is blown in. The force with which the concrete is blown out of the pipe ensures that it is adequately compacted and this is an advantage where the restricted space prevents compaction by other means.

A particular problem with using pneumatic placers is the tendency of the concrete to segregate at the discharge point. The costs and dangers of using compressed air also tend to distract from their use. In view of these limitations and the availability of more effective modern types of concrete pumps, the use of pneumatic placers has declined.

37.4.8 Conveyor belts

Conveyor belts are very widely used for moving a vast range of materials cheaply and effectively and so it is natural that there should be interest in using them for moving concrete. The

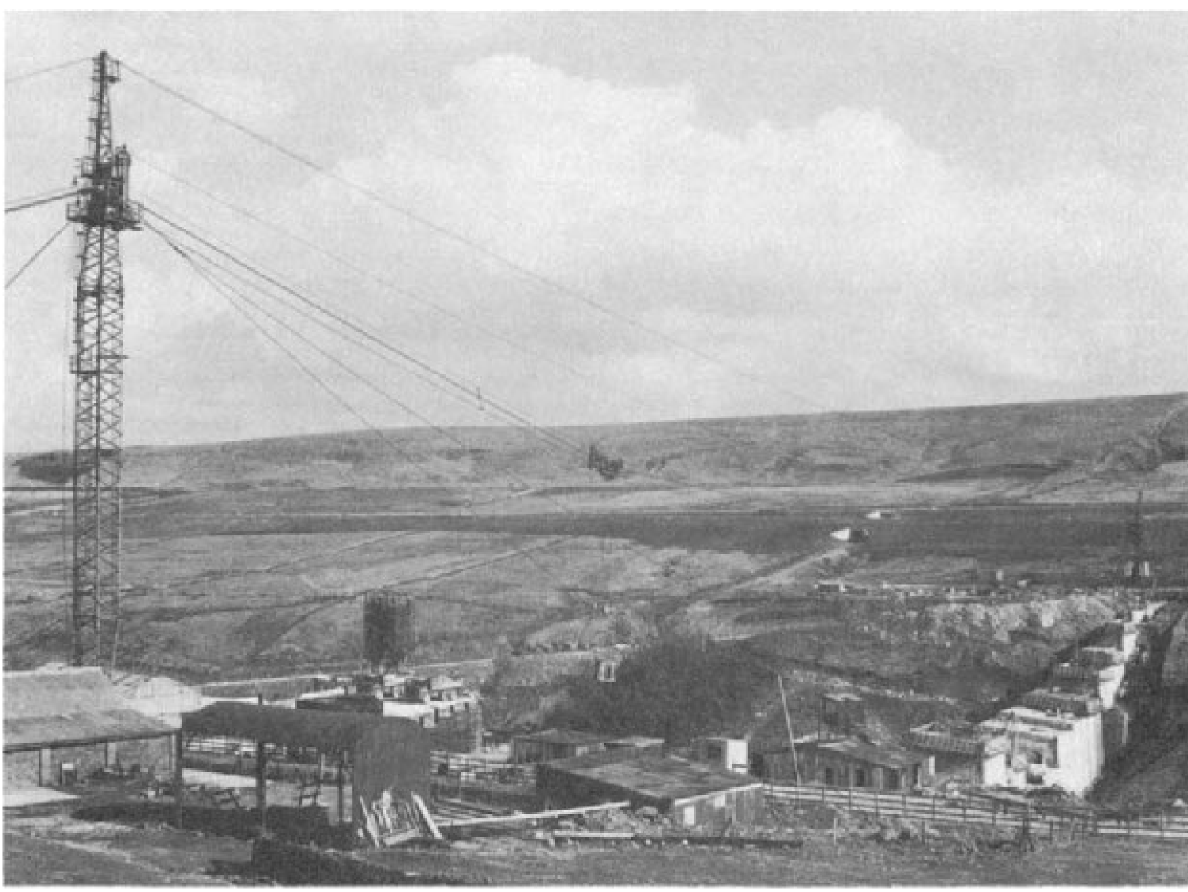

Figure 37.11 Cableway with constant-attitude concrete bucket used in dam construction. (*Courtesy*: J. M. Henderson & Co. Ltd)

throughput of a conveyor system in relation to the power used in moving it is probably more favourable than with any other method of distribution. This is probably as true with concrete as with any other materials, but there are problems associated with conveying concrete which do not apply with other materials, e.g. the tendency for coarse aggregates in the wetter mixes to separate out from the matrix and the need for elaborate and effectively maintained belt-cleaning equipment. There is, in practical terms, no limit to the speed at which a belt carrying concrete can be run so that quite narrow ones achieve high rates of delivery.

Concrete transported on a conveyor belt can have a wide range of slump values, though it is likely that the wetter mixes could cause more problems than the drier ones. There are examples of concrete having been conveyed over long distances, but it should be borne in mind that they are generally open to the weather and that rain and strong sunshine even over a short time can affect the properties of the wet concrete.

By far the widest application of conveyor belts has been in America. There, the fundamental requirements of a conveyor system – a high degree of flexibility and lightness of individual sections – seem to have been met and a high standard of acceptability achieved.

It is an essential feature of effective conveyor-belt operation that arrangements for belt scraping should ensure complete emptying of the belt, down to the rubber, at each discharge point, whether from one belt to another or into the work. This implies the use of vulcanized rubber joints only and the immediate replacement or repair of any section of belting which is damaged for any reason. As mentioned previously, high-speed movement of wet concrete can bring about a tendency to segregation of the coarse aggregate from the matrix; to avoid this, each discharge point should be fitted with a hooded funnel within which the separated elements of the mix can be recombined before discharge on to the next section of belt or into the work.

Evenness of flow on to the conveyor is an essential to effective operation and a form of belt feeder to give this uniformity is a worthwhile investment, even though it is another piece of equipment to be maintained.

In some classes of work the final length of belting is a short one to give discharge over a fairly limited area without of necessity moving the whole system. In others, scraper blades are used to sweep a belt clean of concrete at any point along its length and then to direct it into the work. Yet another method is to use a pivoted conveyor on to which concrete can be deposited at any point along its length. This equipment gives a very wide range of placing facilities by the movement of one belt only of a system.

37.4.9 Cableways

Cableways, working singly or in pairs, have been one of the principal methods used for placing concrete in many of the world's major dams across valleys of various profiles. To give good coverage of the plan area of a dam, cableways are often set out with a head mast fixed in one position but capable of being pivoted to an angle of about 10° from the vertical. The tail mast is normally mounted on a rail-borne carriage which moves over an arc. The stability of both head and tail masts need special consideration in the light of the load to be carried – concrete and containing skip.

To ensure a steady discharge of concrete from the skip and thus gradual return of the cableway to its unloaded bucket condition, special devices for controlling the rate of discharge are essential. These have been discussed.

Cableways will normally be controlled by an operator from the head-mast position, acting under radio guidance from loading and unloading points; discharge of concrete will be controlled locally at deposit points.

37.5 Placing and compacting concrete

37.5.1 Placing

Wet concrete is set into the position in which it is to harden with the aid of crane skips, pump lines, conveyor belts, etc., and the primary objective of placing techniques should be to avoid the need for extensive subsequent movement from the point of deposit to its final position. This will normally be within shutters of one form or another.

In the majority of work, the first pours of concrete will need to be set on or against excavated or filled ground and, in order to avoid contamination of the structural concrete, a thin veneer – up to 100 mm thick – of rather lower-quality blinding concrete is laid over the formation. Besides preventing contamination, the blinding concrete, which can be set to reasonably accurate levels, can be used as a working platform for the erection of reinforcement where required. When the risk of contamination is low, for instance where concrete is to be built up on a rock formation, reasonably effective cleaning of the rock surface should be carried out but over-insistence on a high degree of cleanliness should be avoided. It is most unlikely that any structural design will include the need for a substantial degree of bond between the formation and the structure.

Wherever it is allowable, the cheapest way of filling concrete into deep excavations is via inclined chutes so positioned as to take concrete direct from a transporter into the work. Purpose-made chuting of lightweight but adequate stiffness, achieved by having a narrow but deep section, and supported as necessary, should always be used and generally it should be at an angle greater than about 30° to the horizontal. The range over which concrete can be chuted is sometimes increased by raising the transporter on a specially built movable platform, if necessary, approached by a ramp.

Frequent resiting of the chutes to avoid too great a build-up of concrete at the discharge point should be aimed at – hence the need for light weight. Alternatively, a number of chutes and loading points can be arranged around the work and used in suitable sequence.

Where heavily laden concrete transporters are brought close-in to the sides of an excavation to give maximum range for chuting operations the ability of the excavation support to sustain the heavy surcharge should be checked.

When concrete is to be filled into excavations in which water is rising, it will often be necessary to conduct this water through channels around the periphery of the work to low points or sumps from which it can be raised clear of the work.

Concrete carried out in series of lifts should have the top surface of each prepared by removing laitance whilst the concrete is still relatively unhardened yet not at risk from the action of water jets or other devices used in surface preparation.

With reinforced work, it is particularly important to remove all scraps of tying wire and other debris from joint planes by means of compressed air/water lances or other devices such as suction pipes prior to concreting lest unsightly joints additionally marred by rust stains should develop.

Concrete should be fed into shutters by a method which will give uniform distribution along a section in layers some 350 to 500 mm thick, each layer being placed and compacted before a succeeding layer is placed. By this means, the forming of unsightly segregation planes in the concrete as coarse aggregate separates from the matrix will be avoided. There will also be a reduced risk of displaced reinforcement resulting from unbalanced local concrete pressures. As successive layers are placed, throughput of a conveyor system in relation to the power used in moving it is probably more favourable than with any other method of distribution. This is probably as true with concrete as with any other materials, but there are problems associated with conveying concrete which do not apply with other materials, e.g. the tendency for coarse aggregates in the wetter mixes to separate out from the matrix and the need for elaborate and effectively maintained belt-cleaning equipment. There is, in practical terms, no limit to the speed at which a belt carrying concrete can be run so that quite narrow ones achieve high rates of delivery.

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37.5.2 Placing in deep lifts

At one time, the placing of concrete in lifts of more than 2 m was regarded as unsatisfactory mainly because of segregation troubles with the concrete and the difficulty of obtaining good compaction. However, the continuous placing of concrete in relatively thin sections of concrete walling and columns in heights of up to 10 to 12 m has become an accepted construction technique in recent years.

When pours of this height are adopted, in thin sections it is not usually practicable to use often recommended devices such as full-depth trunking through which to place the concrete, but it may be found advantageous to use special deflector plates, either on the discharge from the concrete skip or at the top of the shutter to direct the bulk of the concrete vertically downwards. There is always a risk that chuting concrete directly into walls between reinforcement will result in a certain amount of aggregate separation. This should be of little consequence provided the concrete is designed to be as cohesive as possible and to contain a slight excess of sand.

When placing concrete in deep lifts, it may be necessary in the case of thin sections to provide openings in the formwork at one side to permit better control over placing and compacting, especially at the bottom of the lift. These openings, which are sometimes referred to as access doors, usually range in size from about 0.3 m square to about 1 m long by 0.5 m deep and they are usually provided at about one-third to one-half the way up and at 2 to 3 m centres laterally. By feeding concrete through these access doors, any problems which arise by dropping the concrete through the full height of the lift are avoided or, at least, substantially reduced. More important, however, the access doors enable the concrete to be seen more clearly and the poker vibrators to be controlled more easily.

Access doors are especially useful for thin walls and columns where the concentration of steel reinforcement is high and where it would not be possible to see the bottom of the lift from the top. They are indispensable for sloping columns and walls because it is very difficult to pass the concrete down from the top satisfactorily with such members.

An alternative to the provision of access doors is to leave out a number of panels of formwork on one side and secure these in place as the concrete rises.

The surface appearance of the concrete may be marred by access doors since there is invariably a clear indication of their positions in the concrete face when the formwork is stripped. With care, a reasonably neat job may be obtained but, whenever possible, it is preferable to position access doors in a side of the member which will not be readily seen.

As far as appearance is concerned, an advantage of deep lift construction is the absence of horizontal construction joints. At best, it is not easy to disguise these joints and sometimes they can be very unsightly. It is for this reason that many columns and walls are now concreted in one lift.

The absence of horizontal construction joints is not the only factor influencing the improved appearance of deep lifts. For example, blowholes on the surface of concrete are generally much more numerous towards the top of a lift regardless of whether the lift is only 2 or 3 m high or whether it is very deep. Consequently, a deep concrete member which is cast in a number of successive lifts may exhibit a band of blowholes at the top of each of these lifts; if only one deep lift is used, the band of blowholes will occur only once.

When walls are poured in deep lifts there is always a risk of excessive water gain at the top surface due to the rising of water from concrete in the lower levels under the high pressures existing there. This water gain, which can often result in a lower-quality concrete marred by less completely closed surfaces and perhaps colour change, is best countered by using somewhat drier batches of concrete as work approaches the top of the lift.

37.5.3 Joints in concrete structures

Joggles set into the top of a concrete lift ostensibly to give a key for succeeding layers are a potent cause of trouble at horizontal construction joints since they are difficult to clean properly and hold water in excess; they should therefore not be used. Instead, joint planes should be slightly crowned to shed any water used in washing down prior to concreting.

Whether or not to use a cement grout or a cement/sand mortar of creamy consistency at joint planes has long been a subject of controversy. Some engineers prefer to use a thin layer of mortar over the area of a joint prior to concreting; others prefer to have the surface dampened but to use no mortar. If mortar is used it is difficult to ensure that it is only thinly spread and worked well into the top of the previous lift; the excessive amounts which may accumulate can give rise to an undesirable amount of shrinkage at the joint plane and perhaps slight colour differences. It is probably wiser to omit mortar – except where a thin (6 to 12 mm) layer can be properly brushed into the hardened concrete of the previous lift – and to rely on complete compaction of the lowest layer of concrete to give the desired quality of construction joint with as high a degree of bond between the two as it is possible to obtain.

The extravagant use of water bars of different types in concrete construction below ground is perhaps indicative of engineers' and architects' lack of confidence that joints can be made to resist the passage of water without their use. They may be desirable and perhaps necessary where water pressures are high but in many instances it has been found that joints incorporating water bars have tended to show traces of water seepage, sometimes severe, whereas those without such a device are much less prone to trouble.

When such faults are investigated it is generally found that the mode of installation has been faulty rather than that the water bar has been inadequate. They must be installed in properly designed movement joints built precisely in accordance with the drawings; construction methods must be such as to ensure that this is done. Horizontal joints in particular, call for a method which will ensure that correct positioning of the bar is maintained; in vertical joints they must be so secured in position that they remain sensibly normal to the joint plane.

Where joints are built in accordance with known good practice it will rarely be necessary to install water bars in horizontal joints when the effective head is less than 5 to 6 m; they may, however, be needed in vertical joints as indicated. Joints should be detailed as shown in Figure 37.12. To avoid the need for water bars in the vertical joints of, say, a basement perimeter wall it will be preferable either to cast *in situ* or precast sections some 5 to 6 m long leaving 1 m gaps between sections. These gaps will be concrete with as dry a mix as can be effectively placed after an interval of more than about 3 weeks. An effective flexible seal built on to the pressure side of the wall should then ensure water tightness.

37.5.4 Underwater concreting

In many civil engineering works it will be necessary to place concrete in situations where it will be under water either all or most of the time. Offshore work between tidal limits is an instance in which concrete may have to be placed during a short period of slack water and protected from the effects of scour very soon after placing and whilst still unhardened. Other work

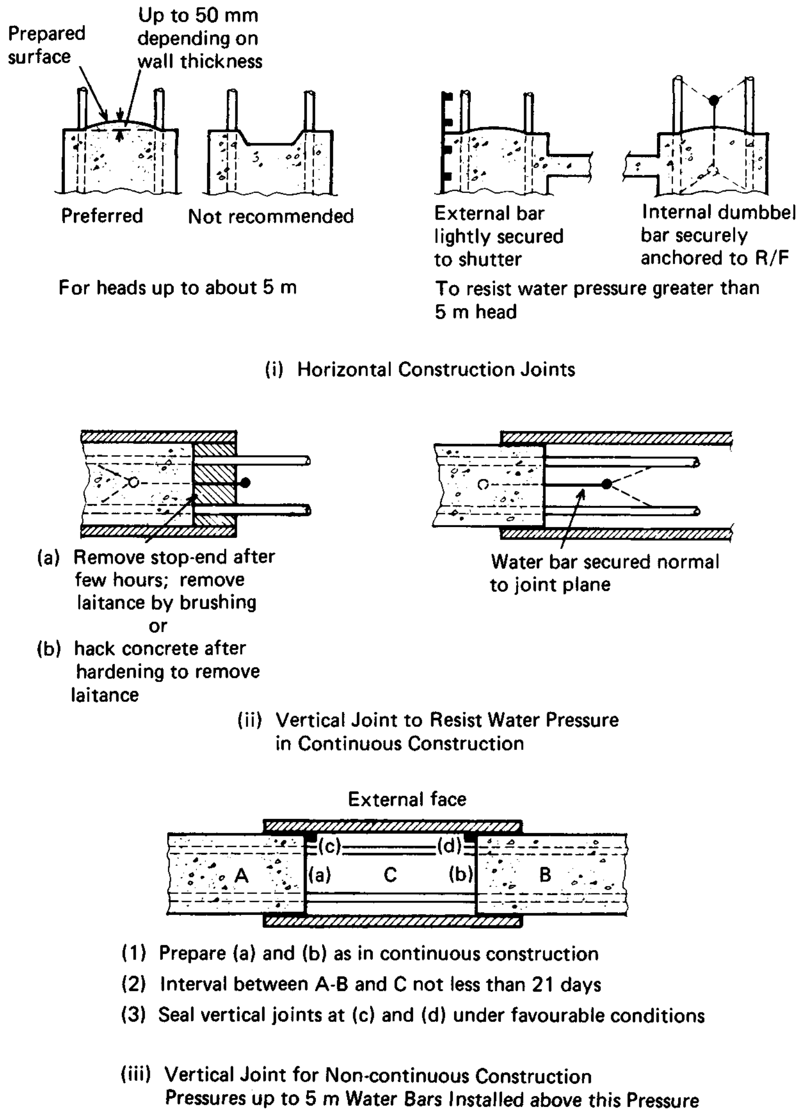

Figure 37.12 Details of construction joints

carried out in tidal and nontidal waters will call for the placing of concrete in parts of structures which are permanently under water. This section concerns itself with this type of work.

Since the hardening of concrete is a purely chemical reaction which can take place only in the presence of water, its setting under water is in no way inhibited. What can happen, however, is that improper methods for placing the concrete into position can cause a serious reduction in quality by virtue of the leaching-out of some of the cement. The objective of underwater concreting techniques is therefore to protect the concrete both during placing and whilst still unhardened, against undue cement loss.

Concrete cannot be placed in fast-running water without recourse to devices such as cofferdamming, but where current velocities are low then it can be placed satisfactorily provided certain precautions are taken both in the proportioning of the mix and in its placing. Some protection against scour or too wide spreading of the concrete mass can be provided by setting the concrete into steel shutters or perhaps walls built up with bagged concrete. Shutters will normally be used if reasonably accurate concrete levels and shapes are to be achieved by rough screeding carried out under water by divers. Where the shape of the mass of concrete is of no particular concern, its only function being to provide a firm base from which the structure proper can be erected, then shuttering is often dispensed with and the concrete mass is allowed to adopt its own angle of repose.

As an alternative to using concrete, it might sometimes be preferred to set single-size coarse stone into a heap of the approximate shape required and then to bind the mass together with a sand–cement mortar fed into the interstices between stones.

This particular technique is generally referred to as the grouted aggregate process and various patented methods, for which the proprietors put forward their various claims, are available. Whichever method is selected it is important that the work be carried out by competent staff, well trained in the art of producing a solid mass of 'concrete' by this apparently simple process.

Whether the choice of an underwater concreting technique falls on the use of conventional concrete or on a grouting method, the clearing of unsuitable material from beneath the structure will be essential. It can be carried out by means of a diver-directed grab, suctionpipe or airlift pump. In those situations where resilting will rapidly take place, concreting should be started as soon as possible after the base has been cleared.

37.5.4.1 Methods for placing concrete

Concrete may be placed under water through a steel tremie tube which will have a diameter some 5 or 6 times the maximum size of the aggregate in it. It will preferably be very workable so that it will flow easily down the tremie pipe and over a large area – up to a radius of about 2.5 m – once it leaves the bottom of the pipe. Where bigger areas than those are required it is normally considered better practice to use more than one pipe or to repeat the whole tremie concreting process at a number of points rather than attempt to cover a large area by moving a single pipe from place to place during one operation.

In operation, the tremie tube, which will often be made of a number of sections to allow easy adjustment of its outlet height above the base of the work, will have a receiving hopper at the top. This hopper, together with the tremie tube, will preferably be slung from a crane or overhead structure and fed with concrete from either a crane skip or a pump.

At the start of operations, the tremie pipe is set hard down on the base of the work, water rising up the pipe. A travelling plug of one of a variety of types is set into the outlet from the concrete hopper. As concrete is poured into the hopper it forces the plug down, displacing water from the tremie and preventing the concrete from falling directly through the water. When the plug has been driven down to the bottom of the tube, the tremie is lifted slightly whereupon the weight of concrete finally forces the plug from the pipe, allowing concrete to well out in all directions, including upwards. Further concrete passing down the tremie is encouraged to flow outwards by slightly raising the pipe, but without allowing its end to lift clear of the mass of concrete.

When an area has been brought to the required height, the tremie should be cleared and moved to a new location, where the sequence of operations will be repeated.

For concreting in deep underwater lifts, which will require reducing tremie lengths, provision must be made for supporting the lower lengths of tremie tube whilst adjustments are made.

An alternative to the use of a tremie tube is a special type of bottom-opening skip. This will be fitted with a canvas cover to protect the concrete during lowering through water and with an enveloping metal shroud which will restrain a sudden surge of wet concrete as the flap-type doors are unlatched, only allowing it to flow as the skip is raised slowly off the bottom or away from previously laid concrete.

Generally speaking, skip placing, because of its intermittent nature, will be technically less satisfactory than tremie placing. Also because of the high cost of such skips and the relatively slower rate of placing, the economics of the method may be less favourable than for tremie work. However, where tolerably accurate surface levels without recourse to heavy underwater screeding are required, work may well be easier with skips. Whatever method is adopted, a consistent supply of concrete which has a sufficiently high cement content to cater for the inevitable loss through leaching is essential to the successful completion of this type of work.

Grouted aggregate method. A useful material for this class of work will be graded 75 to 40 mm and have not more than 10% fines; this is often available as rejects from concrete aggregate processing screens. It should be free from any clay and dusty coatings.

Setting stone into position on a prepared base and grouting should be carried out as quickly as possible, lest silt be deposited over the mass of aggregate or other forms of contamination, such as algae growth which would prevent adequate bond between matrix and stone, should develop. An advantage of using this technique in flowing water is that in passing through the stone mass its velocity will be lowered; this will reduce the likelihood of serious cement teaching.

When using this method of underwater construction the base of the structure should be cleared of any deposits of silt, with an airlift, immediately prior to laying a 75 mm thick bed of sand or pea gravel. Grout pipes are set vertically into the area to be concreted at intervals before stone is tipped round them. These pipes are subsequently coupled through flexible hosing to the grout pumps.

As soon as stone setting is completed – no compaction of any kind will be required – a cement grout of creamy consistency is pumped in turn into the grout pipes and followed with a 1:1.5 cement–sand mortar, again of an easy-flowing creamy consistency. Grouting will normally start at the lowest point in the mass and as this area is seen to be filled, then pipes can be moved further into the mass of stone until such time as the stone and matrix become a solid mass. When grout tubes are lifted in the work to ease pump pressure, care should be taken to ensure that they are always kept at a level slightly below the grout plane. Underwater work carried out by this method alone is not capable of producing a fair surface, but in combination with normal tremie- or skip-placed concrete it will make a first-class base.

37.5.5 Compacting concrete

The relationship between the degree of compaction of concrete and its compressive strength has been indicated in a previous section and the need for achieving a densely compacted mix will be apparent. What is not sufficiently appreciated is that even highly workable concrete mixes need to have some work done on them in order that they should have an adequate standard of compaction.

The effort required to achieve a high degree of compaction with highly workable concrete is minimal, and can be obtained satisfactorily by hand-punning. However, for mixes of medium and low workability, vibration is needed to attain good compaction quickly and the use of vibrators has therefore replaced the earlier hand methods.

Internal vibrators, or poker vibrators as they are more usually called, are the most common type of vibrator in use. They consist of a vibrating tube at the end of a flexible drive and the usual sizes vary from 25 to 75 mm in diameter. The power for the flexible drive can be provided by small petrol engines or electric motors. Although vibrators should not be used for moving concrete (other than the downward movement during compaction), some movement horizontally will inevitably result from their use.

Vibrators fixed to the outside of the formwork are sometimes used, especially where there is a heavy congestion of reinforcement in a wall or deep beam web. In such cases it is not possible to insert a poker vibrator. However, the application of external vibrators is limited by the need to provide heavy formwork which can resist the stresses and shaking produced.

Concrete having been set in the shutters as described in continuous layers about 350 to 500 mm thick along the length of a section, it should be compacted by feeding the vibrator vertically down through the depth of the layer and not more than about 70 mm into the next lower layer. When it is clear that all air has been sensibly expelled from the area being compacted, the vibrator should be slowly withdrawn and plunged into the next section of the work, this process being repeated along the length of the section.

The proper compaction of concrete into highly reinforced zones such as the ends of prestressed concrete beams has always been a problem. The concentration of reinforcing bars and hoop steel round cable anchorages is sometimes such that it is virtually impossible to insert even the smallest-diameter poker vibrator. In such cases external vibrators securely fixed to stiff shutters should be used and moved up the work as concreting proceeds. Honeycombed concrete in the areas of highest steel amounts will be avoided if concrete of a suitable workability is filled into the shutter at a very slow rate only, so that visual inspection will be able to reveal areas of inadequate compaction.

In some situations where suitable lifting devices can be made available, it might be considered to precast the anchorage block on end so that immersion vibrators can be used in the direction of the main reinforcing steel and ducting and not across them. Proper treatment of the top surface of these end blocks, as described, should ensure their proper bonding into the beams with no risk of loss of structural strength of the whole.

37.5.6 Curing concrete

In order to achieve full potential strength and to reduce the amount of drying shrinkage and moisture movement to the lowest possible levels it is necessary to carry out the 'curing' process. This connotes a method whereby the amount of water in the newly placed concrete, which is always higher than that required to fully hydrate the cement present, will be retained over a period of at least several days.

Curing large flat areas such as road slabs and monoliths in plain and reinforced concrete work presents no great problem since, as is indicated in later section, the former can be effectively cured with an impermeable membrane sprayed on to the surface after finishing and the latter with hessian or similar material maintained in a damp condition by means of water sprinklers or occasional drenching with water.

Vertical construction, however, presents a problem which is more difficult to solve. With shuttered vertical faces, the evaporation of moisture from the concrete is effectively barred by the shutters themselves but the economics of construction will generally require that these be used as frequently as possible; this involves removal for re-erection as soon as the concrete has achieved a strength adequate for self-support. At this stage, the concrete is liable to lose moisture to an extent which can result in a good deal of surface crazing and shrinkage cracking. Particularly is this the case if the concrete attains a temperature within the mass which is a good deal higher than the ambient.

Thus for vertical construction, curing of some form should be undertaken as soon as possible after the shutters have been removed. This will preferably consist in draping sections with hessian which can be kept damp by frequent spraying with water. Alternatively a spray bar of plastic tubing perforated at intervals with small-diameter holes can be laid along the top of the concrete and connected to a supply point. It is unnecessary to use an excessive amount of water in this process and, in fact, it is not advisable, since conditions underfoot can thereby be made more unsatisfactory. Polythene sheeting is often used as a barrier to the movement of moisture from concrete but unless it is properly secured to the concrete members with adequate ties it may well be less effective than making no attempt at curing whatsoever, since it encourages the more rapid circulation of air around the member.

In some instances, vertical surfaces are cured by spraying a membrane-forming curing compound on to the concrete surface. The curing compound should be applied, using a hand spray, as soon as the forms have been removed. If the surface has dried out, it should first be sprayed with water and allowed to assume a uniformly damp appearance before the curing membrane is applied.

The application of curing membranes to vertical surfaces suffers from the disadvantage that they stain the concrete surface and do not always disintegrate and fall away after a short time, as claimed by the manufacturers. It must also be remembered that curing membranes should not be used on surfaces that are to receive applied finishes such as plaster or cement renderings, paint, tiles, etc. that require a positive bond with the concrete.

When it is considered desirable for the sake of expedience to cure concrete surfaces which are to receive finishings with a membranous compound, then all traces should be removed by heavy wire brushing or, as has sometimes proved necessary, a more drastic treatment of sand blasting or bush hammering.

37.6 Construction of concrete roads and airfields

37.6.1 General observations

In the last 15 years or so, about 20% of all major roads constructed in the UK have been built in concrete. This figure indicates an increase over the proportion previously constructed (about 6% before 1969) and is attributed to changes in pavement design, the effects of government policy and the rising price of bituminous materials for alternative asphalt-surfaced roads. However, the proportion of concrete roads is still considerably less than in the US where about 50% of major roads are built of concrete.

Important design changes were introduced in the UK in 1969 which permitted the use of unreinforced concrete paving for major roads. Although the omission of reinforcement fabric from the concrete means that many more contraction joints are required, the overall result is a considerable reduction in cost relative to reinforced concrete paving. This cost reduction is one of the main reasons for the increase in the proportion of roads built in concrete.

Machine-laid concrete roads, which have all but replaced hand-laid work throughout the world, are constructed by two different methods. The first uses forms both to contain the unhardened concrete slab and to support rails on which the road-building machines – spreaders, compactors and finishers – are mounted. In Britain, this is sometimes referred to as conventional construction; it is often favoured where reinforced slabs are required by the specification. The second method is slipform paving and, as the term implies, the formwork in which the unhardened concrete assumes the required shape of the road slab moves forward as an integral part of the paving machines, leaving the concrete unsupported after only a very short period of time.

In Britain and Europe, conventional construction has held sway for many years, but in recent years about 30% of major concrete roads in Britain have been constructed by slipform pavers. In America, slipform paving is relentlessly taking over from conventional work and it seems likely that before long there will be little other than this type of concrete road paving work there.

A high proportion of all the runways, taxiways and loading and parking areas for heavy transport aircraft throughout the world are built of concrete of varying thicknesses, reinforced and unreinforced as the design requires. The same methods as are used for building roads can be used for runways, etc. but as thicknesses are often a good deal greater, construction problems may be somewhat different. For example, slipform paving which is successfully used in road construction may not be so attractive a method for building thicker runway slabs because of the greater risk of serious edge slumping. The greater thicknesses involved may also require that compaction from the surface be supplemented by immersion vibrators.

Records of surface profiles of roads laid in recent years indicate that there is now little to choose between the best of the bituminous work and the best of the concrete.

37.6.2 Conventional construction

This class of work is described under the following headings:

(1) Forms and form setting.
(2) Trimming of base and laying sliding membrane.
(3) Setting of expansion and dummy joint assemblies.
(4) Concrete production and transport.
(5) Spreading concrete.
(6) Laying reinforcement – where required.
(7) Compacting and finishing concrete.
(8) Joint forming and sealing.
(9) Texturing of road surfaces.
(10) Curing of road surfaces.

37.6.2.1 Forms and form setting

All formwork for machine-laid concrete roads should be made from steel plate of at least 5 mm gauge and have an adequate number of stiffeners. They should be fitted with heavy-gauge rails on which construction machines, weighing up to about 12 t when loaded, can be run and firmly fixed to a concrete base about 100 mm thick. This base is preferably laid with a wire-guided datum laying machine or similar, capable of laying a strip of good-quality concrete as wide as the form base, to which the formwork can be secured. Normally, laying these base strips to high standards of accuracy should ensure that the forms and rails on which the machines are to run are accurately positioned. Since finishing machines are required to be of the articulated floating beam type (see section 37.6.2.7) the accuracy of the top edge of the formwork is not of great importance, but it is often an advantage to have a square, rather than a rounded, edge to the formwork. When the forms available are shallower than the slab thickness they can still be used since the concrete base on which they are laid can be any thickness greater than about 100 mm. To achieve a balance between adequate rigidity and lowest cost, form depths will generally be about 175 to 200 mm.

In those instances where the design of the road calls for the use of a lean concrete base it might be considered preferable to dispense with the concrete form base and use full-depth forms.

It should be borne in mind that very high loads are imposed on formwork when spreaders are loaded by side-discharge trucks. If these are used care should be taken to ensure adequate strength of the form–base combination so that there is no deflection under these impact loads.

37.6.2.2 Trimming of base and laying sliding membrane

Bases to concrete slabs should be laid accurately to ensure that slab thicknesses are correct. Where the bases are of a granular nature they can be set slightly low and trimmed upwards with fine material to be compacted by roller. If the stronger lean concrete bases are required then they should be laid as accurately as possible with no positive tolerance. An accurately laid base will result in lower shrinkage and temperature stresses developing in the slab than would otherwise be the case. These stresses are sometimes required to be reduced by laying a membrane of polythene or heavy gauge kraft paper. A membrane is perhaps not quite so important when a granular base, trimmed with fines, is used.

37.6.2.3 Setting of expansion and dummy joint assemblies

Under British conditions, expansion joints are not required in concrete roads laid between 21 April and 21 October. It is a natural assumption that contractors will hope to avoid laying concrete in the period October to April, and so expansion joints will not normally be required. However, a full complement of contraction and warping joints is required in the designs. It is essential to have these extensively prefabricated so that setting up at the correct spacing can be carried out expeditiously.

In recent years a dowel bar setting device has become available. This machine provides the means for accurately positioning bars at contraction joints and forcing them down to their correct position within the road slab by vibratory means. The performance of this machine is said to be highly satisfactory as regards both the accuracy with which bars are placed in position and in cost comparisons with methods used hitherto.

37.6.2.4 Concrete production and transport

Concrete for road construction can be produced by any of the methods previously described. The method of construction will dictate the means of transport of concrete but it should be noted that high-capacity end-tipping lorries are likely to result in the lowest overall transport costs. Where it is elected to use box spreaders it has generally been considered necessary to use high-discharge side-tipping lorries whose capacity will be no higher than that of the spreader they are loading. However, various devices have been used by contractors to enable them to use the more economical end-tipper lorries for this purpose.

A matter of some importance when operating high-discharge side-tipping lorries is that, in order to achieve rapid and clean discharge, the load must be quite high above the ground. Concrete laden lorries might be rather unstable at speed unless haul roads are well maintained.

The exposed surface of a lorry load of concrete can be noticeably dried out or wetted by exposure to weather during long hauls and a cover should be provided with each lorry so that it is available for use as required.

The overall rate of paving will depend upon a number of factors, including the programmed time for paving with due allowance for unfavourable weather, and the rate at which supplies of aggregate and cement can be made available, supplemented as necessary by materials drawn from stockpiles accumulated during nonpaving work.

The size of batching and mixing plant required will be governed by the expected rate of forward progress, slab thickness and width of road to be built.

Various alternative combinations of lane widths are allowable and often the overall design, on motorways at least, is simplified to the extent that the surfacing of hard shoulders can be concrete rather than a contrasting bituminous material. Where the design and specification of the road structure allows, it will often be found that construction costs can be reduced by paving in equal widths so that batching, mixing and transporting concrete is carried out at the same tempo throughout the work, with constant width placing and finishing equipment.

The prime requisite for high-quality paving work has been found to be a steady rather than a high rate of progress, but there may be some advantages, so far as quality is concerned, in aiming at a higher rather than lower overall speed of construction. What has been found from past experience is that the irregularity index (q) for the road surface profile is closely linked with the average rate of construction, low q values being associated with high average rates of uninterrupted progress and high values with sporadic working. This implies that the quality of work will be improved if preventive maintenance is carried out diligently to reduce breakdown time of plant to the minimum; the effect of this on the economics of concrete laying will be clear.

37.6.2.5 Spreading concrete

The importance of proper spreading of concrete as a factor contributing to the production of a good riding quality of a concrete road with the minimum effort in finishing cannot be overstated. Ideally the equipment used should be capable of so spreading concrete that its loose density as spread over the formation does not vary by more than about 35 kg/m^3 from point to point.

Concrete is almost universally spread by means of hopper spreaders fed from side-tipping lorries of various designs. The outlet from the hopper may be controlled by a helmet gate or by contraction to a narrow opening through which concrete is passed by vibration. With some models, concrete is tipped into the hopper whilst this is in a roughly horizontal position with the outlet remote from the point of loading. For spreading, the hopper is brought into the erect position and only when this is done does concrete flow from the outlet.

Certain hopper spreaders can be made to function with the hoppers mounted square to the line of the road as opposed to along the line of the road. They can then be loaded by means of end-tipping lorries backed up to the formation; concrete is spread in lanes along the line of the road rather than in bands transverse to it.

This latter system involves the use of a joint assembly which can be very readily and securely set into position on the formation without interfering with the movement of concrete lorries to and from the spreader. It should also be noted that these lorries are liable to cause damage to the formation and any sliding membrane laid on it.

37.6.2.6 Laying reinforcement

When reinforcement is used in road slabs it usually takes the form of welded mesh and is set at about 60 mm below the running surface. Three methods for laying are commonly adopted:

(1) Concrete to the level of the reinforcement is laid and compacted. This operation is normally carried out by the first of two spreaders and compacting beams. After reinforcement has been laid as required, surface concrete is laid and compacted by the second pair of machines. Note that the second spreader will have only about one-third to one-quarter of the throughput of the first.

    As an alternative to this method the reinforcing mesh is laid on the uncompacted lower concrete and depressed into its surface by the beam of the compacting machine before laying the surfacing.

(2) The reinforcement is laid in advance of concreting operations, supported at the correct level on closely spaced stirrups set on the formation, the rate of stirrupping being about 1/m^2. When this method is adopted, the spacing of the main longitudinal bars should be adequate to allow the free passage of concrete through them – a minimum of 100 mm is suggested. Various proprietary systems are available for this purpose and they can be used for either mesh or bar reinforcement.

(3) Loose concrete is spread to the depth required for the full slab thickness. Reinforcement is then laid over the area. Prior to compaction in depth the mesh is depressed into the loose concrete to the required depth by vibrating tines. Besides forcing the mesh into the concrete, these vibrating tines make a contribution to the compaction of the concrete but compaction in depth is carried out from the surface only by compacting beams. This method is fairly widely used in America.

Figure 37.13 Conventional reinforced concrete road construction. Photograph shows from foreground: concrete edge strip carrying rails; sliding membrane and joint assembly; spreading base concrete; laying reinforcement; compacting base concrete through reinforcement; spreading surface concrete; finishing surface with diagonal finishing; texturing and curing (tenting in background)

It should be noted that with method (1) it is allowable to use any quality of concrete in the base (provided of course that it complies with the specification) irrespective of the nature of the aggregate; it need not be air-entrained. The surfacing concrete, which in Britain and many other countries must be air-entrained, can then be laid about 60 mm thick and be made of such materials as will cause the concrete to comply with skid-resistance requirements. Often this smaller volume of concrete for top layer, with air entrainment and a selected aggregate, will be produced in a smaller mixing plant than the rest of the concrete.

With methods (2) and (3) all concrete must be of the same type complying with requirements regarding strength, air entrainment and skid resistance of abraded samples.

In methods (1) and (2) it is usual to lay out the reinforcing mats along the line of the road against the position they will occupy in the slab. As an alternative they can be off-loaded from the supplier's vehicle in bulk on to a wheeled mesh cart which straddles the slab and is either self-propelled or can be towed by one of the leading road-building machines. Mesh is dragged from the pile on the cart as it moves forward with the road slab.

It is claimed that method (1), which requires the final spreading and separate compaction of only 60 mm of surfacing concrete above the base concrete and reinforcement, can lead to easier final finishing to the required surface tolerances. Some of the best work has been carried out by this method. On the other hand good work has been carried out when the whole depth of the slab has been compacted and finished as one layer.

37.6.2.7 Compacting and finishing concrete

Compacting and surface finishing of concrete were at one time carried out by the same machine but, currently, compaction and partial finishing is undertaken by one or two machines of the same type, depending upon whether compaction is carried out to the full depth of the slab or in two distinct layers below and above reinforcement as described. Final finishing is carried out by a further machine which imparts very little compactive effort to the concrete but strikes the surface to a true profile by a to-and-fro screeding action across the slab.

Where two-layer work is adopted, the leading compacting machine will be used to impart a beam finish to the base layer, prior to laying the reinforcement. The surface layer will be compacted and partially finished by the second compacting machine which may have a floating oscillating finishing beam carried on an articulated chassis. Final truing up of the surface will be done with an articulated finishing machine with an angled oscillating single- or double-acting beam. The beam is mounted at an angle of about 50 to 60° to the line of the road and is of particular value in that it can encourage the quick removal of any excess of 'fat' gathered in front of the beam during final screeding.

In some reinforced road construction work, transverse joints are formed by vibratory or other means in the wet concrete immediately behind the second finisher and kept open prior to sealing by a removable insert of one form or another. Here, an angled beam has a marked advantage over a square beam in that it advances steadily from one end of the joint to the other and does not exert a disturbing pressure on any but a short length of the insert at any one time; this ensures that the joint insert remains in its correct position. Furthermore, the angling of the beam extends the wheelbase of the machine considerably and it has been found that this tends to much reduce the 'yaw' of the machine when the resistance to forward movement is greater on one side than the other.

For greatest effectiveness a smoothing beam should be so heavy that there is no tendency to ride over rather than plane off high spots in the concrete. It is considered that for maximum efficiency a beam should weigh about 150 kg/m.

37.6.2.8 Joint forming and sealing

Transverse joints. As noted previously, expansion joints are unlikely to be used in concrete roads except where, inadvertently, construction has been delayed or advanced so that work is in progress during the period mid October to mid April. However, contraction and/or warping joints are necessary and these will be provided with the specified number and sizes of load-transfer bars. Weakening of the slab at these contracting joints to ensure cracking there rather than elsewhere within the slab is provided by a wood or plastic fillet secured to the base and by a groove sawn or formed from the surface directly above the fillet. The reduction in slab thickness by fillet and groove should amount to at least 20% and preferably 33% to ensure adequate weakening.

Joint grooves can be formed in the unhardened concrete by means of vibrating blades or wobbly wheels, either of which will displace sufficient concrete to allow a temporary strip of adequate depth and width to be inserted and held secure in the concrete whilst the disturbed surface is being retrued. This refinishing of the surface is most effectively carried out by the angled finishing machine as described.

As an alternative to forming grooves in the unhardened concrete, they may be sawn when the concrete is sufficiently hard to allow this being done without disruption of the concrete along the joint line.

Concrete saws are power driven and the cutting blades are tipped with various grades of silicon carbide or diamonds. The ease or otherwise of cutting concrete depends much more on the kind of aggregate used than on its strength. Limestone concretes are by far the easiest to cut, next in order are other types of crushed rock whilst hardest are quartzites and flints. The last-named are particularly difficult and expensive to cut.

A difficulty that often arises in cutting partially hardened concrete by saw is that cracking may be induced at the surface before the concrete is hard enough to be sawn. Limestone concretes crack less readily than other types and can be sawn fairly soon after hardening; they are therefore much to be preferred, provided the skid resistance of the concrete can be made to meet specified requirements.

Longitudinal joints. Longitudinal joints may be either full depth at a slab edge or part depth formed by a bottom fillet and a surface groove in the centre of a slab. These surface grooves can be built into the unhardened concrete behind a surface-finishing machine by means of an attachment thereto which displaces concrete and, at the same time, inserts a preformed sealing strip. As with the forming of transverse joints, there is some disturbance of the surface when this insertion is carried out; this is best corrected by using the angled finisher.

Since concrete inevitably shrinks away from the preformed sealer its efficiency is somewhat in doubt. But also in doubt is the real need for a completely efficient longitudinal joint seal.

Full-depth longitudinal joints are made against the formwork and will need tie bars across them, as specified. The ties are most readily positioned by cranking to 90° and laying one arm against the form for later recovery and bending into the adjacent lane. These joints are best sealed by sticking a preformed sealer on to the top of the hardened slab just prior to concreting an adjoining slab.

The adequate sealing of joints in concrete paving is a problem which does not appear to have been solved. It is open to question whether or not sealing against water penetration is necessary when the base, as is generally the case, has been built in such a way as to make it much less susceptible to damage from seeping water. However, there is a need to avoid spalling at joints caused by intrusion of stones and perhaps grit in sealing grooves. If joint edges are left completely unsupported, there is also a risk that traffic will, in time, break away some section of joint edge.

37.6.2.9 Texturing of road surfaces

Recent research into the high-speed skid resistance of various types and textures of concrete surfaces has indicated quite clearly that a deeply textured one, which causes discontinuities in the surface water film in wet weather, markedly improves this characteristic.

It is now the practice on all concrete roads which will be used by high-speed traffic to score the surface, after final finishing, with mechanically operated brushes which impart this texture in the thin layer of surface mortar found on all of them. The result of this scoring is often to produce a drumming similar to that caused by running over heavily surface-dressed black-top roads.

Also available is a grooving machine which consists of a series of vibrating blades at irregular spacings. These make incisions into the unhardened concrete. It is claimed that the wholly random spacing of the grooves formed by this machine does much to reduce the noise nuisance which is sometimes induced where too regular and heavy texturing is a feature of the surface.

37.6.2.10 Curing of road surfaces

Adequate curing of concrete road slabs is essential, particularly in periods of low relative humidity, if damaging cracking and other surface defects such as widespread crazing are to be avoided.

Currently, practice throughout the world is to spray the concrete immediately after finishing with a solution of resin in a volatile carrier, or some other liquid, which will retain a high proportion of the water in the concrete for the vital first few days yet will not remain to reduce the skid resistance of the surface, once it is used by traffic. Where hot sunshine prevails for much of the day, a metallic or white reflecting pigment added to the membrane will reduce heat absorption into the slab.

British requirements in respect of curing are currently that the first 2 or 3 h production of concrete slabs should be protected by tenting which supplements the curing effect of the membrane. This does not pose any problems where rail-mounted equipment is used; but it becomes impracticable when slabs are laid at high speed by slipform paving techniques – in fact, it is most uncommon to see any form of tenting used when slipform paving methods are adopted, except a small amount carried on or close to the machine, for emergency use only.

37.6.3 Slipform paving

A method of building concrete roads at high speed without the use of prefixed formwork – slipform paving – has made a major impact on construction techniques, not only in America but over the rest of the world where concrete roads find favour. Slipform paving is essentially concerned with laying, on a prepared base, a section of road slab in plastic concrete, within a moving form. The sides of the section are defined by formwork which is rigidly mounted on the machine itself at the limit of width of the slab, whilst its upper surface is formed either by a truly flat 'conforming' plate extending to the full width of the slab over a length of several feet or by oscillating screeds which strike off the surface of the concrete at a controlled level. Some machines use both.

The side forms may vary in length between about 4 and 9 m with different machines. Since they generally operate at forward speeds in excess of 1.5 m/min, the concrete slab is supported at its vertical edges for a matter of a few minutes only, in contrast to the normal practice of maintaining support by formwork for several hours before striking.

Slipform pavers are normally positioned as to line and level by means of electrohydraulic equipment. This in turn is controlled by sensors which detect the line and level of wires or cords supported well outside but parallel to and above one or both sides of the slab. Where one wire only is used it is accurately set up to control the position of the nearest slab edge, that of the remote edge being governed by a cross-levelling device on the machine. The locating of this is dictated by the position of the machine in the road, which governs the sense and degree of cross-fall.

Figure 37.14 Slipform paving with G and Z machine. Unreinforced concrete slab on stabilized base. Transverse joints sawn in hardened concrete of left-hand carriageway after 10 to 24 h

The concrete for slipform pavers may be deposited on the formation from end-tipping trucks or discharged into a spreading device in front of the machine. It is then struck off approximately level across the slab by various means, e.g. shuttle spreaders, auger screws and transverse paddles. Compaction to its maximum density by a battery of immersion vibrators, supplemented as necessary by surface vibrators, follows. As the machine moves forward, a conforming plate or other levelling device is passed over the highly fluidized concrete and causes it to take up the shape of the slab. When a conforming plate is used, the amount of concrete passed below the plate will depend to some extent on the head of concrete in front; thus the level at its rear may be subject to slight variations. The action of oscillating screeds, on the other hand, is to strike off excess concrete or to make good deficiencies with concrete carried in front of them. For this reason there may be some technical advantage in using a machine with oscillating screeds rather than a long conforming plate.

It is often preferred to lay concrete road slabs on a very accurately prepared base of cement- or lime-stabilized material, asphalt, sand – asphalt, lean concrete, etc., which will be adequately strong to carry the weight of the paving equipment. When this is done, the paver works at a fixed height above the base tending, by virtue of its long track base, to smooth out irregularities thereon. Direction has to be controlled by means of a line sensor on the machine but apart from this all levels of the finished road slab are dictated by those of the base on which it is being built.

Slipform pavers are best suited to laying unreinforced concrete slabs built without joints but subdivided – when the concrete has been laid a few hours and hardened sufficiently to resist damage – into short lengths by gang saws. However, by deploying other machines such as spreaders and mesh depressors, it is possible to build slabs which are, to all intents and purposes, the equivalent of our reinforced concrete slabs. When this type of road is built, the basic simplicity of slipform paving is lost and the train of equipment becomes not unlike that used for rail-mounted work as previously described. The capital cost of the machines employed is much higher than for rail-mounted equipment but its potential throughput is also much higher.

In America the final running surface behind a slipform paver is often trued-up to a very regular profile by a tube finisher. This simple device consists of a 200 mm diameter tube of length about 1.5 times the slab width, mounted on a wheeled chassis which straddles the newly laid concrete slab.

The profiles of concrete slabs finished by either a slipform paver or a combination of slipform paver and tube finisher are generally very uniform and hence should comply adequately with specification requirements.

Although there do not appear to be any insuperable technical problems associated with the successful operation of slipform pavers, there are those of logistics. These are mainly that the machines operate most economically with high throughputs of concrete. Under normal conditions in Britain this necessarily involves stockpiling large amounts of concrete aggregates and perhaps special arrangements for the supply of corresponding amounts of cement. Building and drawing from stockpiles are expensive operations and significant reductions in construction costs have to be achieved to effect the desired savings in overall costs.

37.7 Concrete floors

37.7.1 Construction procedures

There are many types of toppings and surface finishes that are applied to concrete ground-floors and the choice will depend on the specific requirements in each case. However, in the majority of cases, good finishing techniques on the basic concrete itself will be perfectly adequate.

Floors can be laid in the same way as concrete paving for roads and airfields, although it is customary to see manual or semi-manual methods being used as opposed to machine methods. The latter are limited for economic reasons to the construction of large floors where uninterrupted lengths of at least 80 m are required.

A traditional way of laying concrete floors has been by the 'chequer-board' method of construction in which individual bays are cast alternatively within stop-ends forming the joints. Infill bays are usually specified to be placed no earlier than 7 days afterwards, the basis of this requirement being to allow a considerable proportion of the shrinkage movement of the earlier bays to occur. Since, however, shrinkage of concrete takes place over a period of several months, it is now recognized that this latter requirement is of dubious value. Moreover, this method of working is not efficient and access for constructing the infill bays is poor.

A more modern method of floor construction which has found favour is the so-called 'long-strip' procedure which is basically the same approach as that adapted for concrete road construction. Alternate strips – usually not more than 4.5 m wide – are first laid continuously the full length of the floor area and divided into bays by means of induced joints. These joints may be formed either in the plastic concrete or by sawing shallow slots in the surface 2 or 3 days after the concrete has been laid. The infill strips of concrete are placed a few days later when the first strips of concrete have hardened sufficiently to withstand the effects of the compacting beam without damage to the edges.

Again, as in the case of roads, reinforcement is often provided in the slab near the top surface. This entails laying the concrete in two courses.

37.7.2 Finishing techniques

For many industrial applications, the concrete used for the floor slabs can be directly finished to provide a suitable wearing surface. A 28-day strength of 30 N/mm^2 or more should be specified for the quality of the concrete and it is also desirable that the concrete should have a cement content of at least 330 kg/m^3 in order to provide good durability and resistance to abrasion.

Traditionally, concrete floors are finished by hand trowelling using steel trowels. Each trowelling operation follows the previous one after an interval of about 1 h (during which time further moisture will have evaporated from the concrete surface).

This trowelling process has become mechanized in recent years and it is now usual to see the concrete finished by mechanical means using power floats and power trowels. The former uses a rotating solid circular disc whereas the latter is provided with three or four rotating blades. However, the terminology for these processes varies and the term 'power float' is often taken to cover both power floats and power trowels.

Power floating and power trowelling can, with care, produce very good finishes. In comparison with hand trowelling they enable the work to be done up to 6 times more quickly and their use permits a slightly drier concrete to be used (with consequent advantages in enhanced strength and resistance to wear). However, it is still necessary to resort to hand trowelling in small confined areas, such as floors of domestic dwellings, where it is not practicable to use a power float efficiently.

The power float does not, as is sometimes thought, provide compaction of the concrete. The concrete must have been compacted and screeded to level before the power float is used.

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Traditionally, concrete floors are finished by hand trowelling using steel trowels. Each trowelling operation follows the previous one after an interval of about 1 h (during which time further moisture will have evaporated from the concrete surface).

This trowelling process has become mechanized in recent years and it is now usual to see the concrete finished by mechanical means using power floats and power trowels. The former uses a rotating solid circular disc whereas the latter is provided with three or four rotating blades. However, the terminology for these processes varies and the term 'power float' is often taken to cover both power floats and power trowels.

Power floating and power trowelling can, with care, produce very good finishes. In comparison with hand trowelling they enable the work to be done up to 6 times more quickly and their use permits a slightly drier concrete to be used (with consequent advantages in enhanced strength and resistance to wear). However, it is still necessary to resort to hand trowelling in small confined areas, such as floors of domestic dwellings, where it is not practicable to use a power float efficiently.

The power float does not, as is sometimes thought, provide compaction of the concrete. The concrete must have been compacted and screeded to level before the power float is used.

The time at which the power float is brought into use on the floor surface depends on various factors, notably the ambient temperature and the workability of the concrete. It is recommended that under average conditions it should be used about 1 h after the concrete has been laid. A good guide which can be used to determine the time at which to use the power float is the depth of the impression left on the surface when a man stands on it; if the footprints are about 2 mm deep, the concrete is ready for treatment.

In cool conditions, there may be a considerable delay before the power float can be brought on to the concrete and this may entail overtime working by the operators. To avoid this, a vacuum dewatering process is sometimes applied to the concrete surface soon after it has been compacted and has received its initial surface finish. The process involves the use of a flexible vacuum mat, provided with a fine filter sheet, which is connected to a vacuum generator. When the mat is laid on the concrete surface, a vacuum is created underneath the mat and this causes water to be drawn out of the concrete. The vacuum is usually applied for about 20 min after which the concrete will be sufficiently stiff to receive the first surface treatment with the power float.

37.7.3 Surface hardeners

The use of surface hardeners on concrete floors is rather a controversial subject. On the one hand, a properly laid concrete floor should give a satisfactory performance without any further treatment. On the other hand, the quality of many concrete floors, when laid, is far from perfect and a surface hardening treatment can be beneficial in these cases. However, in the case of weak concrete floors, a surface hardening treatment will not be effective at all.

It follows that the use of a surface hardener should not be regarded as a substitute for producing a good-quality concrete in the first place.

The most commonly used surface hardener is sodium silicate which combines with the lime in the concrete to form a hard glassy substance. Magnesium silicofluoride and certain other salts are also used as surface hardeners.

37.8 Other forms of concrete construction

37.8.1 Dam construction

Circumstances arise from time to time in the construction of reservoirs for either water supply or hydro-electric schemes when it is advantageous to build a concrete dam rather than an earthen or rock-fill embankment.

The design and specifications for the dam will give details of the lengths of each section of the dam, the depth of each lift and the interval which must lapse between the concreting of successive lifts; it will also place a limitation on the depth of concrete which can be poured in the lifts and time interval between adjacent monoliths.

Section lengths are generally of the order of 15 to 18 m and joint planes will normally be at 1.5 to 2 m.

Where it is intended that several monoliths in the same area be brought up together, this can be done provided an adequate gap is left between each for subsequent infilling. Gaps of about 2 m have been used and filled some time after the main lengths of the wall, when major drying and thermal shrinkage of the main blocks has taken place. Proper provision for water bars must be left in each side of this gap so that the method involves somewhat heavier expenditure on joint preparation and sealing.

A number of factors have to be taken into account in deciding the method of construction to be adopted.

First will be the duration of the contract and the likely time to be spent on preparation of the foundations before dam construction can begin. Another major factor will be the profile of the valley across which it will be built. In Britain, the sites on which long and high dams can be built are very few so that a contractor is usually faced with building either a small number of high monoliths or a larger number of low ones.

The required number and dimensions of the monoliths will determine the method by which concrete will best be handled into place. Where a long, low dam is required to be built across a shallow valley, it might be considered to carry the concrete from a central mixing plant in crane skips mounted on flat-bottomed lorries or trailer units, thence into the work via a crawler or other type of crane. Flat-bottomed bogies carried on a narrow-gauge rail track along a low-level gantry might also be considered. To build a shorter but higher dam across a narrow steep-sided valley it might be thought preferable to handle concrete in crane skips via a series of derrick cranes mounted on temporary concrete pillars. These could well be sited on the upstream side of the dam, since the greater volume of concrete will be there.

All possible ways of handling concrete are discussed in a previous chapter and the contractor, in preparing his scheme for the work, will make decisions on the rate of concreting and the means by which it is to be placed in position.

It is common practice to build the main mass of a dam wall with a low cement content and hence fairly low-strength hearting concrete, but to use a higher quality having greater durability at the upstream and downstream faces. This facing must be placed within a short period of placing the hearting concrete so that there shall be a complete bond between the two.

Probably the easiest method is to build up the hearting in a 400 to 500 mm lift over the whole area to within about 400 to 500 mm of the dam faces – or whatever thickness surfacing concrete is called for in the design. The facing is then filled in to the same depth between hearting and the shuttered upstream and downstream faces. The richer concrete is not normally required to be poured against the shuttering to transverse joints.

Rock suitable for good-quality concrete aggregates is often available at the site of work and is generally quarried as required to make concrete on site. Since sections are normally large and there is a need to keep cement contents low, aggregate up to 150 mm maximum is often used. The production of lean and sufficiently workable mixes for low-strength hearting concrete is facilitated by using large-size aggregate. However, the use of plums or displacers is not economic – nor is it good practice.

In some instances where only poorer qualities of rock – as regards their suitability as aggregate – are to be found at site it might be necessary to import the better-quality material for exposed concrete but to use the inferior aggregate for the mass of low-strength concrete in the hearting. Proper mix design and placing techniques are quite capable of producing concretes satisfactory for this work from the most unlikely materials.

A disfiguring feature of many dams in the past has been the tendency to seep water through transverse joints and, more rarely, along horizontal joint planes. The former faults can be avoided by proper detailing of joints to facilitate their building according to the intended design. With horizontal joints it is essential to avoid the presence of laitence, downward joggles and deep indentations which will hold water. Laitence can be removed by air–water blast when the concrete is hardened sufficiently to avoid damage. Treatment of concrete with water sprayed on to hessian strips will ensure its proper curing and avoid any shrinkage cracks which might contribute to failure; it will also keep the whole area clean for subsequent operations.

A recent development is the use of 'rolled concrete' for dam construction (referred to as 'Rollcrete' in the US). The concrete in this case has a low cement content and has a relatively dry consistency which permits compaction by rolling. It may be compared with the dry lean concrete which has been used successfully for the construction of road bases, but the concrete mix used for dams benefits from the inclusion of a fairly high proportion of PFA, the proportion of which has generally ranged from 30 to 75% by volume of cementitious material. The use of rolled concrete for dams has the advantage that conventional earthmoving plant may be used for handling and compacting the concrete and that therefore large outputs can be quickly and economically achieved. Construction can proceed in layers, about 200 to 300 mm thick, laid continuously from one side of the dam to the other.

37.8.2 Tunnel linings

In situ concrete linings may be called-for in hard-ground tunnels to give support to rock which will deteriorate in the course of time or to improve hydraulic characteristics.

The dimensions of the tunnel – length, diameter, number of access points and other factors – will need to be taken into consideration in deciding the method to be used and the order in which work is to be carried out. Mention has already been made of the use of pneumatic placers but concrete pumping is coming to be more widely favoured. A variety of methods for getting the concrete to the working face have been used in the past and new ideas coming from time to time are adequately described in technical literature.

37.8.3 Mass plain and reinforced concrete sections

Concrete will be placed into sections of this type, which will most frequently be found in power stations, foundations to large buildings and other heavy work, by various combinations of methods already described. The sizes of bay to be concreted will be dictated by the output of the batching plant available for the particular operation in hand. Owing to the complexity of shuttering work in reinforced concrete it might be found advantageous to restrict the depth of pour so as to increase the area to be concreted at any one time – shuttering costs are then likely to be rather lower.

In heavily reinforced concrete foundations there is a strong financial incentive to dispense as far as possible with costly construction joints and to place the concrete continuously in large pours. Placing of volumes of concrete of the order of 200 to 300 m^3 is quite common for such foundations and very much larger pours of the order of 3000 m^3 have been placed where the plant facilities and access have permitted such a large-scale approach.

In unreinforced concrete foundations, however, where there is no steel reinforcement to restrain the subsequent shrinkage of the concrete, it is necessary to restrict the areas of concrete cast in one pour in order to avoid cracking resulting from the concrete shrinkage. Due to the absence of reinforcement, the provision of shuttering for the construction joints will be relatively straightforward in these circumstances.

The tendency to crack when the concrete contracts is related to the value of the maximum temperature reached by the concrete soon after it has been placed. The cracking is, of course, caused by the subsequent cooling down to ambient temperature. In large foundation slabs, where cooling is generally in a vertical direction, the maximum temperature rise attained is related to the lift height. However, due to a combination of self-insulating effects and the reduction in the adiabatic rate of heat generation after 24 h, increase in lift height above 2 m causes very little further increase in the maximum temperature reached.

37.8.4 Vertical construction with sliding formwork

For many years, certain types of vertical construction in concrete, e.g. materials storage silos and the service cores of tall buildings, both of which have either a constant cross-section throughout their height or only a small number of variations in wall thickness, have been carried out by using formwork which was moved continuously upwards as the concrete was placed within the forms. The infrequent changes in plan have been accomplished by altering the dimensions of the moving shutter as required; this has involved completion of a section and re-erection of one or both faces of the shutter before work recommenced.

More recent developments have seen modifications to the original conception of moving formwork to allow slight and gradual changes in cross-section as the work proceeded upwards. The construction of reinforced concrete chimneys and the erection of tall towers serving as supports for TV aerials and amenity buildings are examples of work in which both external dimensions and wall thickness have decreased as the height above ground increased.

Sliding formwork, though now widely used, particularly by companies who have become specialists through mastering the techniques involved, is not a new art, there being reference to such work as long as 60 years ago. However, as the use of the method has become more widespread so have improvements been made in the mode of operation.

Basically the method consists in continuously raising formwork of the correct plan dimensions into which concrete with the designed amount of reinforcement is placed in a series of narrow bands up to 150 to 200 mm depth, proceeding over the whole plan area. The formwork to the inner and outer faces is connected at close intervals by a series of straddling yokes, each having a device by means of which the yoke and the attached shutter can be moved upwards in relation to jacking rods at each point. These jacking rods are carried from bottom to top of the structure and are located in circular cavities formed within the walls by tubes which surround the rods below the jacking points.

Apart from the external and internal shutters and the jacking systems and their control, it is necessary to provide a working platform which will generally cover the whole plan area of the structure. From this men can operate and on it can be stored such materials as reinforcement and blocking-out pieces for door openings, etc.

Concrete will normally be raised by a hoist or tower crane and distributed around the outside of the structure by hand barrows, light skips, monorail or other devices considered appropriate.

An essential feature of a sliding form is a platform connected with and below the inner and outer faces of the main shutter, from which operatives can carry out work to impart a sufficiently high standard of surface to the concrete emerging from the shutter. Occasionally, when faults have developed in the work, these will be corrected from the same platforms.

In the earliest examples of sliding formwork, raising of the shutter with reference to the jacking rods was carried out by hand-actuated screw jacks. However, the specially designed jacks used now are almost universally hydraulically operated from a central control panel. The mode of operation is basically that the jaws on the jacks grip the jacking rod firmly whilst the body of the jacks, attached to the yoke, are moved upwards in about 12 mm intervals, the rate of travel varying between 150 and 500 mm/h according to circumstances.

The forming of the narrow cavity round the jacking rods ensures that whilst adequate support is given against buckling under vertical loading as they are lengthened, the rods can be recovered for re-use after completion of the slide.

To ensure success of sliding operations it is essential to

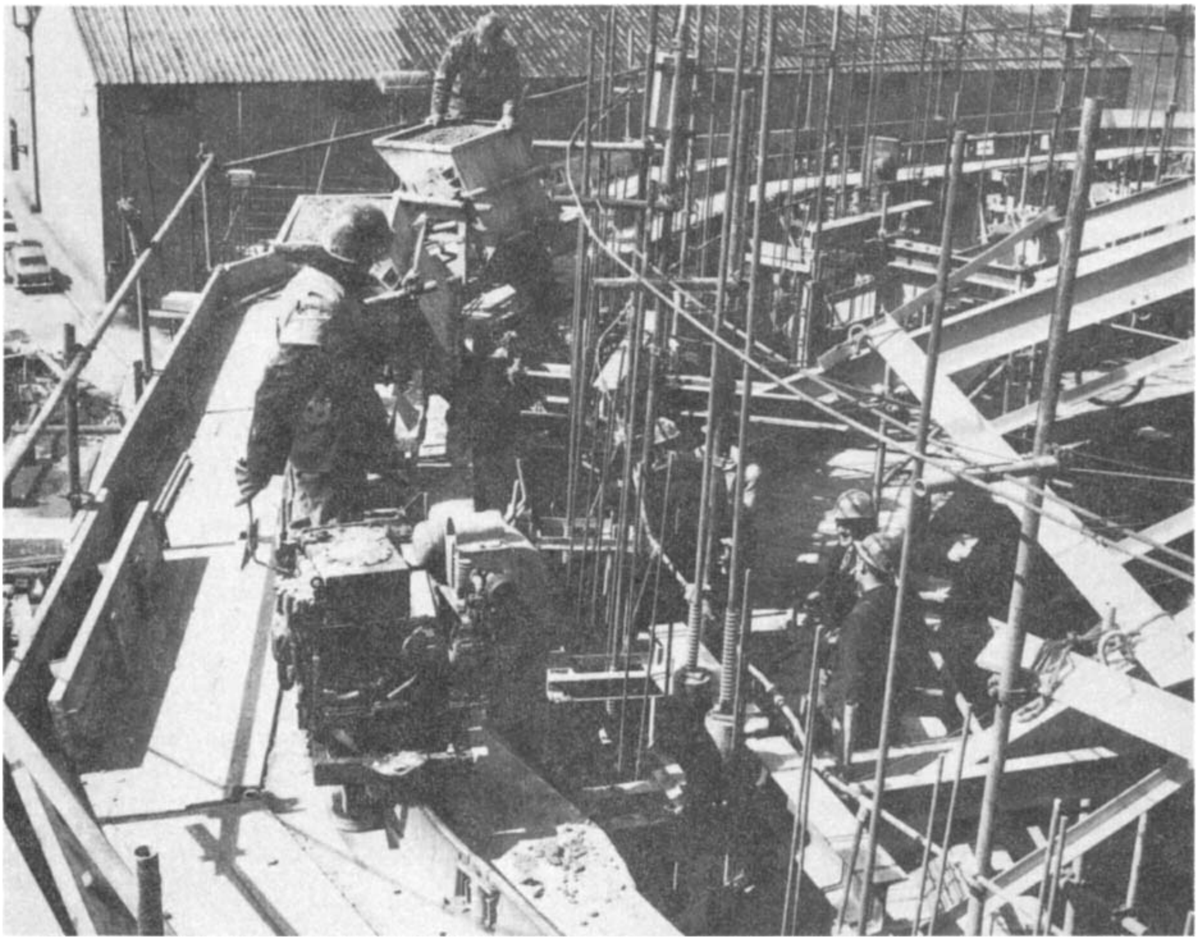

Figure 37.15 Continuous vertical construction of circular silos with sliding formwork. *Note*: sliding form; jacking points; working platform; monorail transporter distributing hoisted concrete. (*Courtesy*: John Laing and Sons Ltd)

provide a high standard of control over the quality of the concrete used and the rate of sliding so that when it emerges from the shutter the concrete is capable of selfsupport without slumping. The concrete must also be amenable to surface floating, etc., to remove minor blemishes.

Sliding operations involve 24-h working, often under very variable weather conditions, and a high standard of job organization is necessary to ensure that there is continuity of all operations involved. Breakdown of equipment which could result in long delays and perhaps in the extreme, the abandonment of a slide, is best guarded against by either duplication of vital items or constant survey to reduce the risk of untimely failure.

37.8.5 Gunite (shotcrete)

Gunite consists essentially of a mixture of cement, sand and water which is sprayed from a nozzle into the required position. In some cases, coarse aggregate of about 10 mm size may also be added. The first step in the process is the mixing of cement and sand in the required proportions. The mixture is then fed into a piece of plant called the 'gun' which consists of one or more chambers connected to a compressed-air supply. This gun feeds the material in a continuous flow into a pipeline, along which it is conveyed pneumatically, until it reaches the nozzle at the placing end. At the nozzle, a spray of water is introduced, under pressure, into the passing material and the resulting mix emerges from the nozzle at high velocity on to the required surface. The optimum distance between the nozzle and the surface is 1 to 1.5 m (Figure 37.16).

The condition of the sand, before mixing with cement, should be damp (a moisture content of 3 to 5% is generally considered desirable) in order that the particles can retain a coating of cement. Sand which is too wet, however, may cause a blockage to develop in the system.

The amount of water added at the nozzle is controlled by a valve operated by the 'nozzleman'. The amount is critical since too wet a mix will result in slumping off the surface, whilst a mix which is too dry will lack cohesion and will result in a considerable loss of material due to excessive rebound off the surface.

This basic guniting procedure is sometimes referred to as the 'dry process' to distinguish it from the so-called 'wet process'.

In the 'wet process', the materials are mixed initially, with the required amount of water (as in the case of normal concrete) before they are fed into the pipeline. The mix is forced along the pipeline by the positive displacement action of a concrete pump or, alternatively, by pneumatic means. At the nozzle end, compressed air is introduced to provide momentum to force the material out in the form of a spray.

Figure 37.16 Gunite being sprayed from a nozzle to increase the thickness of existing concrete on a cooling tower

The amount of water added in the 'wet process' is predetermined to a controlled amount and is not dependent on the judgement of the nozzleman, as in the case of gunite. However, the nature of the wet-mix process requires a mix with a higher water content than that produced in the gunite process, and accordingly, the strength and allied properties of the material will be inferior. There is also more difficulty in clearing any blockages which may occur in the pipeline.

In the US, both the 'dry process' and the 'wet process' are generally referred to as 'shotcrete'.

Gunite has been in use for about 50 years and its main application has been in the repair of deteriorated concrete structures where a layer of good-quality mortar is needed to reinstate the surface. More extensive use of the process has been limited by economics since normal concreting procedures are cheaper. It has, however, been used successfully in constructing swimming pools, culverts, retaining walls, tunnel linings, and intricate curved structures. It has also been used to strengthen existing structures by increasing the thickness of the concrete.

37.8.6 No-fines concrete

As the name implies, no-fines concrete contains no sand or fine aggregate. It is therefore characterized by uniformly distributed voids throughout the mass, which give it a relatively low density. No-fines concrete may therefore be considered to be a particular form of lightweight concrete.

The main applications of no-fines concrete are in the construction of loadbearing walls for low- and medium-rise housing. It has also been used extensively for the infilling panels on high-rise framed structures. Other uses include the provision of drainage layers in civil engineering works and the paving of free-draining parking areas.

When used for building purposes, the optimum size of the aggregate is 10 to 20 mm. It is usual to specify an aggregate of which not more than 5% is retained on a 20 mm mesh sieve and not more than 10% passes the 10 mm sieve.

In order to achieve a satisfactory cellular structure with adequate strength, it is found that mix proportions with a cement:aggregate ratio of 1:8 by volume give the optimum result. In cases where strengths higher than the normal requirements are called for as, for example, on loadbearing no-fines concrete used for four- and five-storey buildings, cement:aggregate ratios of 1:7 by volume, or even slightly richer, may be required.

Cube strengths obtained with 1:8 mixes vary from about 4 to 9 N/mm^2 at 28 days, the corresponding densities being about 1600 to 1850 kg/m^3.

Where strength is less important, as in the case of drainage layers, cement:aggregate ratios can be reduced to 1:10 by volume or leaner.

The water content of no-fines concrete should be the minimum necessary to ensure that each particle of aggregate is coated with a shining film of cement paste. If insufficient water is used, there is a lack of cohesion between the particles giving a friable appearance and loss of strength. Too much water causes the cement paste to run and separate from the aggregate. A water:cement ratio of about 0.40 is usually satisfactory for 1:8 mixes when using dense aggregate.

The main advantages of no-fines concrete, in comparison with normal dense concrete, when used for building construction are:

(1) Lightness in weight.
(2) Low thermal conductivity.
(3) Capillary absorption of water is virtually eliminated.
(4) Light formwork can be used.
(5) The open texture provides an excellent surface for the application of a rendered finish.

As a building process, the no-fines concrete construction technique has the particular benefits of being simple, economical and fast.

A further benefit of using no-fines concrete is that it will not segregate and it can therefore be readily placed in deep lifts of up to three storeys high in one operation, if required. It is important to maintain a level head of no-fines concrete in the formwork along the wall under construction, since localized full-height pouring may cause inclined planes of weakness (pour planes).

Unlike normal concrete it is not necessary to compact no-fines concrete, but some rodding should be given to it to ensure that the formwork is evenly filled. Also careful rodding should be carried out whenever obstacles such as window openings and lintel bearings occur.

No-fines concrete presents some difficulty in the fixing of various fittings and it is necessary to embed nailing blocks of timber which are attached to the formwork prior to pouring. Provision should also be made for suitable openings and chases before pouring since it is difficult to cleanly cut away the no-fines concrete for services.

37.8.7 Concrete diaphragm walls

During the last 15 years or so, there has been a spectacular growth in the construction of concrete diaphragm walls in the UK.

The technique, which was initially developed in Italy to prevent the seepage of water below dams, has subsequently been extended to the construction of retaining walls and loadbearing elements at the sides of deep basements, underpasses, etc. Diaphragm wall construction is invariably carried out by specialist contractors.

The basic process consists of excavating a trench in the ground which is filled with a slurry of bentonite mud to stabilize the sides of the trench as the excavation proceeds. A reinforcement cage is lowered through the bentonite into the trench and concrete is then placed by tremie pipe, gradually displacing the bentonite as it fills the trench.

The key to the process is the use of bentonite which is a thixotropic clay. When it is mixed with water to form a slurry, it has the useful property of forming a membrane of low permeability at the sides of the excavation. The face stabilization is improved as the density of the bentonite slurry is increased but in general the aim should be the achievement of a density of between 1.02 and 1.04 g/cm^3. This may be achieved with a slurry containing 4 to 6% bentonite and 1% fine sand. The gel membrane which is formed along the sides of the trench allows the bentonite slurry to exert a hydrostatic head in excess of the *in situ* head of groundwater and the lateral earth pressures. Additional stabilization is obtained by limited penetration of the bentonite slurry into the adjacent soil.

In the initial stages of diaphragm wall construction, it is necessary to construct concrete guide walls which are usually about 1 m deep and about 300 mm wide. These walls serve the purpose of fixing the line of the wall, controlling the direction of the trenching tool and retaining the soil near the surface.

Percussive, rotary or excavating tools may be used for forming the trench, the first two types being necessary where excavation in rock is required. Excavating tools may be of the auger, bucket, shovel or clamshell grab type which cut the soil in bulk and bring it up above ground level for discharge.

As the excavation proceeds, the bentonite slurry is simultaneously pumped into the trench. Since it is not possible to avoid the mixing of the bentonite with detritus arising from the excavation process, there is inevitably a settling of this sludge to the bottom of the trench and this must be removed before the reinforcement cage is positioned in the trench. If this is not done, it is likely that the concrete which is poured into the bottom of the trench will flow over the sludge and not displace it. Clearly, the presence of such a soft layer would seriously reduce the loadbearing properties of the wall.

After the bottom of the trench has been cleaned out the reinforcement cages are lifted into position and supported at the right level. Concrete is then tremied into the trench and the bentonite slurry is gradually displaced as the level of concrete rises. The displaced slurry is pumped into settling tanks for re-use or else removed from site.

The difference in density between the bentonite slurry and the concrete is generally enough to prevent intermixing except for a layer of about 300 to 600 mm in the interface zone. The concrete should be very workable in order that it can flow readily and the aim should therefore be a concrete having a slump of 150 to 250 mm so that it behaves like a heavy viscous fluid. The coarse aggregate should preferably be rounded gravel of 20 mm maximum size to enhance the flow properties and plasticizing admixtures are normally recommended. A cement content of not less than 400 kg/m^3 is necessary to provide adequate strength in the concrete.

The degree of compaction achieved by gravity in a very workable concrete placed by tremie pipe is generally adequate. Vibration is not required and in any case would be undesirable since it would cause segregation in a very workable concrete.

Diaphragm walls are normally constructed in panels ranging from 700 mm to 1 m wide, 3 to 6 m long and 6 to 30 m deep. The length of a panel is mainly determined by the soil stability and this leads to a practicable maximum length of 6 m in practice. The distance of lateral flow of concrete from the bottom of a tremie pipe should not exceed 3 m in order to ensure a uniform flow. For long panels, therefore, two or more tremies should be used.

Continuity of concrete placing is essential and a continuous rate of at least 20 m^3 concrete per hour is desirable. If serious delays occur in the supply of concrete, difficulties will arise with achieving a correct tremie technique and this can lead to undesirable trapping of bentonite slurry in the body of the concrete wall.

Since diaphragm walls are cast in a series of panels, it is necessary to ensure that the resulting joints between panels are watertight and that they provide an effective key between the panels. This is generally achieved by means of a steel tube installed vertically as a stop end at the end of a panel. This steel tube is removed after the concrete has set to leave a semi-circular-shaped joint at the end of the panel.

One of the problems is the complete removal of the bentonite slurry from the surfaces of the reinforcement bars. Although the tremie technique enables the concrete to displace the bentonite slurry as a mass movement, it cannot be expected to remove completely the coating of bentonite around the reinforcement bars. Since such a coating will adversely affect the bond between the concrete and the steel, it is necessary to use deformed bar reinforcement.

In the case of diaphragm walls constructed round the perimeter of deep basements, the excavation of the ground will follow the completion of the diaphragm wall. Some form of support system will then be necessary to resist the lateral pressure of the earth behind and this may be achieved by props or by ground anchors. The technique whereby the wall is tied back with ground anchors has the particular merit of allowing a clear space free of obstructing props and struts in the excavated area.

37.9 Precast concrete

Precasting of concrete is widely practised in all branches of civil engineering but perhaps the most spectacular is in maritime work. For example, units weighing thousands of tons to be linked together to form submerged vehicle tunnels across narrow waters are frequently built in docks and made temporarily buoyant by adding bulkheads. They are then floated out to their permanent location where a number are strung together on a prepared base below the sea-bed, to make a complete tunnel.

37.9.1 Bridges

Many concrete structures can be built either *in situ* or by using a number of precast units which when assembled together, often by *in situ* work, will form an equivalent structure; here, the emphasis will be on bridge work.

Design studies carried out by the engineer and influenced in large measure by his past experience will indicate which method is the more likely to result in lower cost, simplicity and speed of building. The findings of these studies will be incorporated in the contract designs.

When prestressed concrete beams of various types are a feature of the design then the contractor may have to consider either buying-in or making within his own organization. He will rarely consider setting up equipment to produce long-span box-section beams designed for production by the fully bonded (long-line) system because of the high capital cost involved; but where the beams incorporate their own inbuilt anchorages for prestressing tendons it is often open to consideration whether the beams should be factory made and hauled to site by road and rail or built on the job.

In some instances, for instance where very heavy long-span beams, which could not be brought to the site because of road or rail restrictions, are required, there will be no alternative to site casting. For the smaller, readily transportable, handleable units, such factors as the cost of preparation of suitable casting beds, the cost of concrete production and of providing the high degree of supervision over production need to be considered. Quite often, bridges are built in locations where access is difficult even for small units. Here the alternatives of building on site or waiting until better access can be provided for brought-in beams and the cranes for handling them need to be considered.

Precast concrete units weighing up to about 130 t have formed a substantial part of several overhead urban road works.

37.9.2 Tunnel works

Not perhaps so much in the public eye have been schemes in which the roadways in bored two-lane vehicular tunnels have been precast and set in position somewhat below the axis of the tunnel. In this arrangement the large area below the road is used primarily for ventilation but also for services of all kinds.

Tunnel road deck units will normally be formed in lengths of up to 6 to 8 m in precasting yards at one or both ends of the tunnel, according to the number required and the time available between the completion of driving and lining and opening to traffic.

A high standard of dimensional accuracy and surface finishes is called for, particularly if the upper surface is to form the running surface of the road without recourse to an applied bituminous or further concrete finish; additionally, accurate and sufficient bedding of the units is called for to ensure good performance.

Where ground conditions are appropriate, precast concrete units may be used for lining tunnels. These units may be either solid sections which are stressed into contact with the ground (often manhandled units in the smaller-diameter tunnels), or ribbed sections, both of which are put into position with mechanical erectors. Precast tunnel linings are used as widely as possible because of their low cost as compared with that of cast-iron tubbing.

37.9.3 Cladding panels

Precast concrete cladding panels have been used extensively for the façades of buildings for many years and a great variety of shapes, sizes and surface finishes may be seen throughout the world.

The size of a cladding panel is often determined by the site cranage and the aim should be to limit the weight of a cladding unit to a value which is no more than the other site loads that the crane will have to lift. If this aim is disregarded, it will be necessary to hire-in a heavy mobile crane for lifting the cladding into place.

Although large cladding panels are more expensive to transport and handle, the merits of using them should be considered in each case. The advantages of large panels include faster construction, a reduction in the number of fixings and fewer joints. The latter is a pertinent point as far as leakage problems are concerned.

The weight of the panels will also be influenced by the density of the concrete and there is an obvious benefit in using lightweight concrete. The latter may take the form of concrete made with lightweight aggregate or it may be made from concrete having a cellular structure. An alternative approach to minimizing the weight is to use normal concrete mixes in thin panels, but this introduces problems of providing adequate cover to the steel reinforcement. The consequences of inadequate cover can be seen in the unsightly cracking and corrosion staining that has occurred in those cases where this basic requirement has been given scant attention. For very thin panels where it is not possible to obtain the necessary cover, stainless steel reinforcement should be used.

The development of glass reinforced cement (GRC) in recent years had led to this material being successfuly used as a cladding material. Glass reinforced cement is basically formed in thin sheets which not only gives it the advantage of light weight but also enables it to be provided in panels to a wide variety of shapes. One particular application is the production of panels having a light insulating material (such as polystyrene) sandwiched between two sheets of GRC.

Whatever the material used for cladding panels, dimensional accuracy is an essential requirement. Normal standards of accuracy are covered by the tolerances given in CP 297:1972, 'Precast concrete cladding'. If tighter tolerances are sought, it is necessary to bear in mind that costs increase dramatically as the degree of accuracy is increased.

There are various means of fixing precast concrete cladding panels to the supporting structure and the choice of a suitable fixing arrangement must, in particular, take account of adequate supporting strength, the need for adjustment to accommodate any inaccuracies and the long-term corrosion resistance of the metal used.

37.10 Concrete construction in hot arid countries

37.10.1 Introduction

Concreting in hot arid countries presents difficulties not usually encountered in areas with temperate climates. The prime difficulty is dealing with the adverse effect of the high temperatures and solar radiation not only on the concrete in the handling stages but also on the concrete in the early hardening state.

However, another factor which can be equally difficult in hot arid areas, if not more so, is the presence of aggressive salts in the ground and in sources of fine and coarse aggregates. Sources of water can also contain undesirable levels of salts and even the atmosphere contains wind-borne salts which can cause trouble.

When no allowance is made for these factors in areas of rapid development, such as in parts of the Middle East, the result has been poor-quality concrete work lacking in durability.

37.10.2 Mixing and handling concrete in hot weather

The effects of high ambient temperatures, intense solar radiation and variable humidity pose several problems when planning and carrying out concreting operations. In the Middle East, the annual temperature range in the shade generally varies from about 10 to about 50°C, the latter being experienced during the period June to September. Moreover, the problems caused by high temperatures during this period are increased by the rapid changes in relative humidity which can vary from 25 to 100% within a daily cycle.

This harsh environment influences the behaviour of concrete in both the plastic and the hardened state. In the former, the rapid loss of water by evaporation results in a corresponding loss of workability, and concrete which has been carefully designed to give the right workability at the mixer may well be too stiff for proper compaction by the time it reaches the point of placing. Allowance must therefore be made for this effect by increasing the water content at the mixer above that required. This approach calls for careful judgement since there are several factors which influence the loss of workability, not least being the method of transporting, and the distance to the placing point. It is generally considered preferable to convey the concrete in truck mixers which provide protection from the direct heat of the sun; they may be kept relatively cool by spraying with water periodically. If open transport is used, the concrete should be covered with damp canvas or similar material.

It is beneficial to paint all concreting plant white since the exposed surfaces will more readily reflect the solar radiation. Temperature reductions on former dark surfaces can amount to 10 to 17°C by this straightforward expedient.

Care is required to ensure that the water adjustment at the mixer is not excessive since segregation of the over-wet concrete may otherwise occur during transporting. To avoid this situation, the use of admixtures is recommended. Water-reducing and retarding admixtures have become very popular for concrete work in hot countries since they permit the amount of water in the concrete to be reduced and, moreover, they extend the restricted period of time during which the concrete remains sufficiently workable for handling purposes.

The rate of loss of workability increases as the temperature of the concrete increases and therefore it is advantageous to start with the concrete at as low a temperature as possible. Measures to keep the stockpiles of aggregates cool are therefore of benefit. These measures include shading the aggregates from the sun and spraying with cool water. The latter should be used with discretion because there is a danger that the water content of the aggregates will become very variable and consequently cause difficulties in controlling the water content of the concrete. A fine spray of water uniformly applied two or three times during the day is probably the best approach.

The temperature of the concrete when it is first produced at the mixer will depend on the temperatures, the specific heats, and the proportions of the constituent materials. The temperature of the fresh concrete can be estimated from the following formula:

$$T_c = \frac{(t_c + AT_a + 5Wt_w)}{(1 + A + 5W)}$$

where T_c, t_c, t_a and t_w are the temperatures of the concrete, cement, aggregate and water respectively, A is the aggregate:cement ratio, and W is the water:cement ratio.

This formula is based on a specific heat for water of unity and on a specific heat for cement and aggregates of 0.2 (this figure being used for simplicity instead of the more correct figure of 0.22).

This formula indicates that the initial water temperature may be more important than the aggregate temperature since water has a high specific heat. It is therefore important to keep the water cool and consequently it should preferably be stored in tanks below ground; if this is not done, the tanks should be shaded and painted white. All supply pipes should be buried in the ground and insulated.

In severe conditions, the use of ice will cool the water very significantly. However, ice is frequently not available in the quantities required and also it can be expensive. Ice-making plant or refrigeration equipment is sometimes installed at concrete-mixing plants which provide large quantities of concrete. Generally, it is not considered that ice is really necessary, although it may be desirable in some cases to add blocks of ice to the tank(s) of water early in the morning well before concreting starts.

37.10.3 Maximum temperature of the concrete

To ensure that good-quality concrete is produced, many specifications impose a maximum temperature on the concrete produced. Based on experience in the hot arid zones of the US this maximum temperature is often stated at 32°C.

The practical limitations of this value in the Middle East have led to a more realistic approach there as more experience has been obtained. Depending on circumstances, the maximum temperature allowed in the concrete is now usually between 35 and 38°C. Certainly, a very considerable amount of good-quality concrete has been placed with a maximum temperature requirement of 38°C. However, it must be added that it would not be prudent to relax the temperature requirement to more than 40°C since loss of workability, handling difficulties and the tendency of the concrete to crack after placing increase very significantly above this temperature level.

Clearly, there is no one unique value for a maximum concrete temperature since the behaviour of the concrete will be influenced by the relative humidity and amount of wind as well as the temperature. In some of the coastal regions of the Middle East, the high relative humidity will tend to alleviate the drying effect of the high temperatures. Conversely, where relative humidity is low, it is preferable to be cautious and limit the concrete temperature to a relatively low value, especially where drying winds are prevalent.

The concrete temperature can, of course, be influenced by the time of day during which the construction work takes place. Ideally, it is best to start work either in the early morning or early evening and avoid concreting during the high midday temperatures. Apart from the effect on the concrete, there is the important benefit that the operatives will be able to work more efficiently during the cooler parts of the day. Working at night is sometimes a way of avoiding high daytime temperatures, although it may not be so popular and, moreover, supervision is generally not as good as that in daylight.

37.10.4 Protection and curing of the concrete

Once the concrete has been placed, compacted and finished, adequate protection from the sun and adequate curing is essential. This aspect of the work certainly requires far more attention in hot climates than in temperate ones.

If exposed concrete surfaces are not protected from the sun directly following the finishing operations, continuing evaporation of water is likely to lead to plastic shrinkage cracking. This type of cracking on paved areas is characterized by a series of cracks parallel to each other, often running at about 45° to the line of greatest slope. Although they are usually relatively shallow, they may well penetrate to the steel reinforcement and provide a path for corrosive salts to reach the reinforcement with a risk of consequent steel corrosion.

More severe cracking can occur subsequently in the first 2 or 3 days if the concrete is not protected and is allowed to reach a high temperature before it eventually cools and tries to contract.

The risk of cracking is related to the initial concrete temperature and this adverse effect is one of the main reasons for specifying a maximum concrete temperature at the time of placing, as mentioned above.

The Portland Cement Association[1] has produced a chart (Figure 37.17) which enables the rate of evaporation of water from the concrete to be determined from known values of air

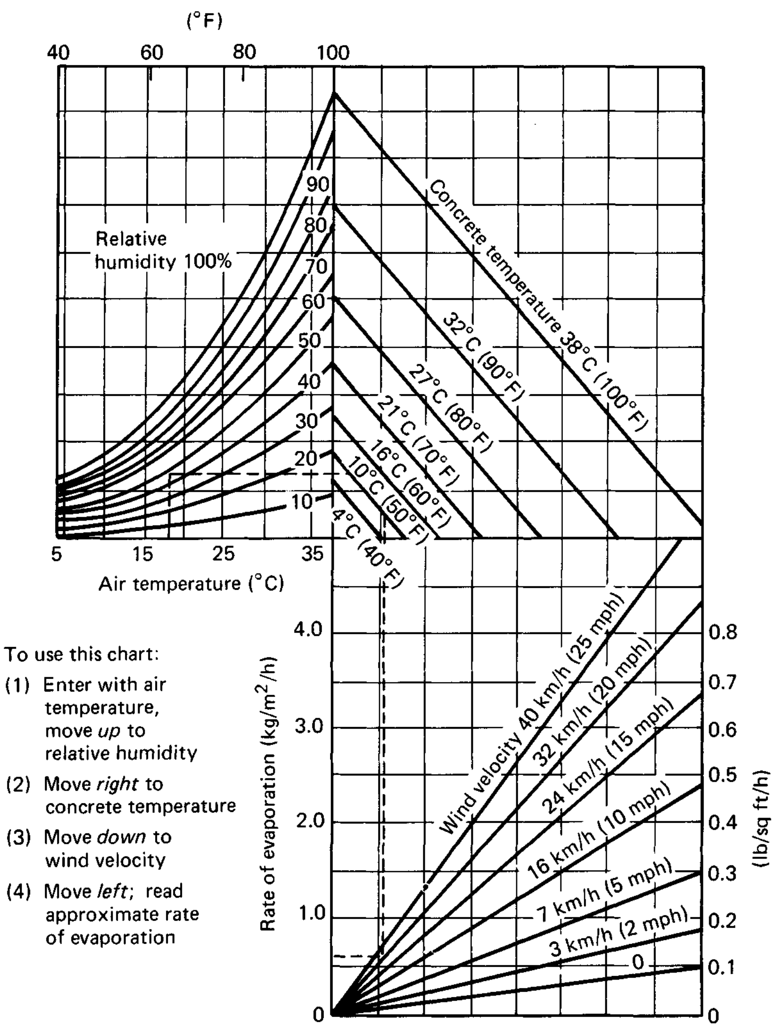

Figure 37.17 Effect of concrete and air temperature, relative humidity, and wind velocity on the rate of evaporation of surface moisture from concrete

temperature, relative humidity, concrete temperature and wind speed. If it is found that the rate of evaporation approaches 1 kg/m^2/h, precautions against plastic shrinkage working are deemed necessary for paving work. These precautions invariably will be required when working in areas with hot arid climates.

Curing procedures in hot conditions are the same as those already described in earlier sections for concrete work in general. However, for hot climates, curing should be started earlier and be applied more diligently. Exposed surfaces, such as paving, should not be left exposed for more than 20 min and preferably less. Curing membranes which are sprayed on to the fresh concrete surfaces should contain a pigment to reflect solar radiation, and two coats should be applied in very hot conditions. It may also be necessary to provide covers or suitable shading over the concrete for several hours to protect it from the excessive heat of the sun. It is important that provision is made for adequate ventilation under these covers.

Where damp hessian is used to cover concrete for several days after placing, the water which is sprayed periodically on to the hessian should be relatively free from salts. If it is not, the repeated application will build up chlorides in the concrete which may reach a level where corrosion of the reinforcement will be initiated. This is an important point to watch in hot arid regions where suitable water supplies are limited. In coastal areas, the temptation to use seawater for curing reinforced concrete must be firmly resisted.

37.10.5 Strength development in hot weather

It is well known that concrete which is placed and cured at high temperatures will achieve higher early strengths than concrete at standard laboratory conditions of 20°C. At an age of 28 days, however, the situation is reversed, and concrete which is maintained at a temperature of 40°C will have a 28-day strength which is nearly 20% less than concrete maintained at 20°C.

Research has shown that concrete made in the laboratory at 38°C produced cube results at 28 days which were about 15% lower than concrete made at 18°C. This reduction occurred in spite of the fact that after 1 day, all the cubes in the investigation were stored in water at 14 to 19°C before testing at 28 days.

37.10.6 Concreting materials

Rigorous quality control is the key to producing trouble-free concrete in hot arid areas, particularly the Middle East, and this applies to the choice and handling of materials as well as the construction process. On major projects, extensive testing is required both before and during construction and even on small projects it is advisable to ensure that the materials have been checked by some basic tests.

Aggregates are probably the main cause for concern and they should be carefully assessed by a full programme of testing before they are approved for use. Natural sands in hot desert countries, especially beach sands, can have very high salt (i.e. sodium chloride) contents and are clearly undesirable for concrete unless the chloride is removed by washing. Many cases of corroded reinforcement and cracked concrete in structures have been due primarily to the use of a sand containing too much salt.

Coarse aggregates may also contain undesirable levels of chloride, particularly some crushed limestones quarried from near the ground surface.

Sulphates must also be checked in both fine and coarse aggregates since undesirable levels can cause expansion within the concrete.

The following recommendations apply to the permissible chloride and sulphate contents of fine and coarse aggregates.

Table 37.1 Limits of chloride content for aggregate used in reinforced concrete

Aggregate	*Maximum chloride content (as Cl) (by weight of aggregate)*	
	Using ordinary Portland cement (%)	*Using sulphate-resisting Portland cement (%)*
Fine aggregate (sand)	0.06	0.04
Coarse aggregate	0.03	0.02

Chlorides (for reinforced concrete)
Small adjustments may be made to these limits, if necessary, subject to the overriding requirement that the acid-soluble chloride present in the concrete does not exceed 0.30% by weight of ordinary Portland cement or 0.20% weight of sulphate-resisting Portland cement.

Sulphates (for all classes of concrete)
The acid-soluble sulphate (as SO_3) content of all aggregates, both coarse and fine, should not exceed a maximum limit of 0.40% by weight of aggregate.

This limit applies when using either ordinary or sulphate-resisting Portland cement. In the case of mixes with relatively low cement contents, it may be necessary to reduce this limit of 0.40% in order to comply with the overriding requirement that the total acid-soluble sulphate present in the concrete (including that present in the cement) does not exceed 4% by weight of cement.

Grading of aggregates should also be carefully checked. Dune sands and beach sands are often too fine to be used on their own as fine aggregate and therefore it may be beneficial to blend them with the fine material from crushed rock and gravel which is usually rather coarse when used on its own. The blending of sands to produce an acceptable grading is often desirable when the concrete is to be placed by pumping.

Dust content is another consideration and in some areas where water is expensive or in short supply it may be necessary to accept dust contents in the processed aggregates which are slightly in excess of BS or ASTM requirements. Although this increases the water demand of the concrete made with such aggregates, the effect is not serious.

The importance of other aggregate tests will depend on the source and type of aggregate and information on these can be obtained from specialist papers.

Apart from the tests, it is recommended that periodic visits are made to the source of the aggregate supplies to ensure that sand is being dug from the area which has been approved or that work is being quarried from suitable working faces. In the case of sandpits where excavation is taken down to water-table level, it is essential to ensure that sand is not taken from just above the water table since it is likely to be contaminated with chlorides that have been absorbed due to upward capillary movement of the water.

In some areas, the quality of cement can be very variable since supplies can be obtained from many different parts of the world. Although consignments may sometimes be accompanied by manufacturers' test certificates, it is nevertheless advisable to check the quality of the cement by tests on samples, especially if the cement has been in transit or store for some months and is no longer fresh. Some form of testing is essential whenever a

Table 37.2 Local criteria for reinforced concrete in the Gulf region

Exposure Conditions	*Range of Specification Limits*		
	Minimum cement content for 20 mm aggregates (kg/m^3)	*Maximum water : cement ratio*	*Minimum cover for reinforcement (mm)*
(1) Superstructures, inland with no risk of wind-borne salts and ground level well above capillary rise zone	300–320	0.52–0.50	30
(2) Superstructures, in areas of saltflats exposed to wind-borne salts. Ground level within capillary rise zone	320	0.50	40
(3) Parts of structures in contact with the soil, well above capillary rise zone and with no risk of water introduced at the surface	320–350	0.50–0.45	40–50
(4) Parts of structures in contact with soil within the capillary rise zone, below groundwater level, or where water may be introduced at the surface:			
(a) soil and groundwater free from significant contamination	300–320	0.50	40–50
(b) soil and groundwater contaminated with sulphates and/or chlorides	320–400*	0.50–0.42*	40–50
(5) Marine structures	370–400	0.45–0.42	75–100
(6) Water-retaining structures	400	0.50	40

Note:
* The wide range of these requirements reflects the range of sulphate concentrations in the soil or groundwater, but takes no account of chloride concentrations. (The five levels of significant sulphate concentration adopted in BRE Digest 250 and BS 8110 are used in local specifications.) A tanking membrane is required for the more severe conditions.

change of source of cement occurs, otherwise there is a risk of low concrete strengths being obtained if no allowance is made for a lower-quality cement.

The importance of good concreting materials and the quality of the resulting concrete has been well covered in the CIRIA *Guide to concrete construction in the Gulf region.*[2]

Table 37.2, taken from information in the CIRIA guide, gives the basic requirements of the concrete as based on specifications in use in the Gulf region. These requirements are related to material exposure conditions which can be very severe in some cases. Particular attention needs to be given to ensuring that the minimum cover to the reinforcement is achieved since lack of adequate cover has been responsible for early corrosion and concrete cracking in many cases.

The recommended type of cement in these various exposure conditions depends on the amount of sulphates and chlorides present. Where resistance is needed against sulphate attack and there is no risk of chloride-induced corrosion, sulphate-resisting cement to BS 4027 or ASTM type V should be used. Where there is no significant exposure to sulphates but there is a risk of chloride-induced corrosion, cement with a medium to high C_3A content is preferred (as found with ordinary Portland cement or ASTM type I). Where resistance is needed against both sulphates and chlorides, a compromise has to be made on the type of cement used. Generally a cement containing at least 3.5, but not more than 9%, C_3A is preferred.

Steel reinforcing bars can also be supplied from a wide variety of sources and therefore checks on quality are advisable. In a saline atmosphere, the reinforcement must be stored under covers to prevent the deposition of salts which will cause corrosion. On important projects it is sometimes considered advisable to clean the steel by grit-blasting or rotary wire-brushing to ensure that it is free from salts and corrosion before being fixed. When reinforcement bars are left projecting from the initial lifts of concrete, they should be covered with polythene sheeting as protection from atmospheric salts if any delay in placing subsequent concrete lifts is expected.

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38

Heavy–welded Structural Fabrication

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Contents

The vast majority of fabrications in steel are now welded and it is rare to see a new fabrication that is joined by any other method. The steel used is generally to BS 4360 *Specification for weldable structural steels*; the revised 1986 edition includes the weathering steels and the whole range covers steels with yield stresses ranging from 230 to 450 N/mm^2.

38.1 Welding processes

Figure 38.1 shows the welding processes most commonly used in steel fabrications; in all cases an arc is struck between the electrode or electrode wire and the workpiece resulting in a high arc temperature which melts off the electrode and deposits it in the joint which has to be made. The manual metal arc (MMA)[1] is the most common process and the electrode is deposited manually with the operator controlling the direction of the weld and the build-up of the weld metal. The flux extruded around the core wire when melted by the heat of the arc provides a gaseous shroud which protects the molten pool and arc from atmospheric contamination and controls the weld metal reactions; it can also be the vehicle for supplying certain alloying constituents to the weld metal. The fused slag around the deposited weld metal also helps to form the weld bead shape. There are several types of electrode coverings which function in different capacities and are classified in BS 639 'Covered electrodes for the manual metal-arc welding of carbon and carbon manganese steels'.

Gas-shielded arcs with bare wire or cored wire, can be of the semi-automatic or automatic type.[1–3] The semi-automatic process utilizes a power source, a wire drive unit, incorporating the necessary control units, and a 'gun' which is held by the operator and manipulated manually; the wire is driven through a flexible tube to the gun and a suitable designed nozzle concentric to the gun orifice supplies the gas to the arc. The automatic process usually has a heavier 'gun' or head with the wire (also known as electrode wire or feed wire) fed directly through the gun without the intervening flexible tube; the whole apparatus travels automatically for longitudinal welds or may be stationary for circular fabrications. Higher welding currents and deposition rates are generally used with subsequent water cooling of the head being necessary. The weld metal and arc is protected from the atmosphere by the shroud but a bare wire must contain deoxidizers such as silicon, manganese and sometimes aluminium; these are necessary to prevent some oxidizing processes which occur within the arc atmosphere. A cored wire has the flux enfolded within the electrode wire as typified in the cross-section shown in Figure 38.1; it may be used in semi- or fully-automatic processes. The necessary deoxidants are carried in the flux which may also be the vehicle for additional alloys to be added to the weld; the flux allows for higher welding currents than that in solid or bare wire shielded welds, with the slag allowing better bead shapes to be obtained, and is generally more tolerant to rusty plate conditions which could otherwise lead to porosity.

Shielding gases used for structural steels are carbon dioxide or argon with addition of oxygen or with carbon dioxide and oxygen, the cheapest being carbon dioxide.

Another type of semi-automatic welding popular in the US and now being used in the UK is the self-shielded arc where the continuous electrode contains in its core flux ingredients which vaporize in the arc, shielding the arc and forming a thin slag around the metal droplets as they transfer across the arc gap; deoxiding materials also form part of the flux. The process requires no gas shield and is therefore better-suited for outdoor operations when windy conditions prevail.

The submerged arc (SA) is a process which feeds a bare wire into the arc and the arc is covered by a granulated flux which is

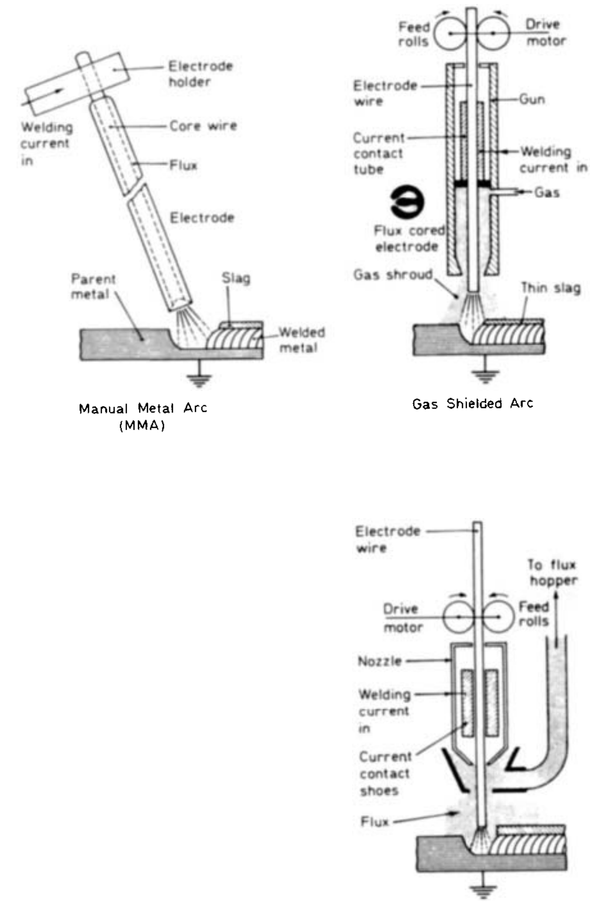


Figure 38.1 Common welding processes

also automatically fed; some of the flux is melted to cover the weld pool as slag and to provide the arc with a gaseous shield. Again alloys can be added to the weld either via the arc or the flux; very high currents can be used in this process[1] and very smooth bead-contour shapes can be obtained. The arc is completely shrouded by the flux and thus it cannot be seen; this gives a total absence of arc glare but, correspondingly, guiding is that much more difficult. This process is more susceptible to rusty or dirty plate conditions than MMA but less susceptible than metal arc inert gas (MIG) and for very heavy weld metal depositions on thick plate, multiple electrodes may be used in the same weld. A semi-automatic form using a small diameter wire can also be obtained.

Figure 38.2 shows schematically two other processes, electro-slag and electro-gas welding; both are completely automatic. In electro-slag welding, the plates to be welded are mounted vertically with the edges of the plate square or unprepared; watercooled copper shoes are mounted either side of the weld seam to contain the molten metal. An arc is struck on the starting block with a little granulated flux added to the weld pool; as the wire or electrode burns off, the temperature of the slag bath increases and the slag becomes electrically conducting; from then on the electrode protrudes into the bath, the arc extinguishes, and the wire metals off due to the I^2R heating of the current. It is thus not an arc process but a continuous cast process used on plate thicknesses over 25 mm and certainly up to 450 mm; for the greater thicknesses three electrodes are fed

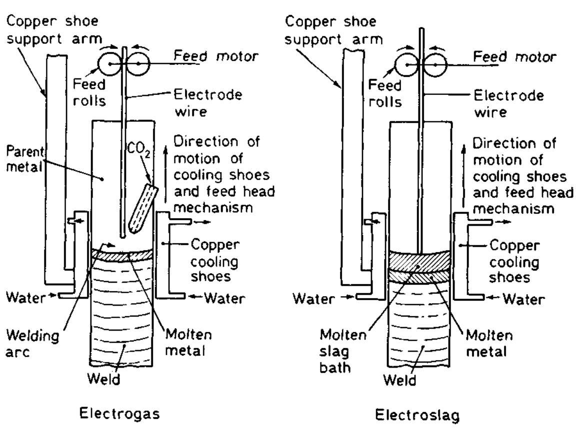


Figure 38.2 Electro-gas and electro-slag processes

simultaneously into the slag bath with, in one application, the whole assembly oscillating across the width of the joint.

There are two methods of applying this process known as electrode or consumable guide (see Figure 38.3); in the electrode method the feed head is at the side of the plate being welded and moves up with the copper cooling shoes as the weld is made. In the consumable guide method the feed head is stationary at the top of the joint to be welded and the wire(s) are fed down to the slag bath by a consumable guide which is insulated from the workpiece by fusible spacers.

As the wire(s) melt so does the bottom of the consumable guide and the copper shoes can be stationary of a length equal to the length of the welded joint; this method requires less sophisticated machinery than the former and is therefore, where it can be applied, cheaper. It cannot obviously be oscillated across the width of the joint.

The electro-gas process is similar to that of electro-slag welding in that the weld metal is contained by watercooled shoes but different in that the weld metal is deposited by a true arc with a thin slag from the flux in the cored electrode; the weld metal and arc is protected by a stream of CO_2.

Since both processes can evolve large heat inputs to the heat-affected zone of the weld and to the weld itself with resultant large grain microstructure, poor fracture toughness may result. Some improvement may be obtained by postweld normalizing treatment, but where fracture toughness may be a problem, expert opinion is advisable.

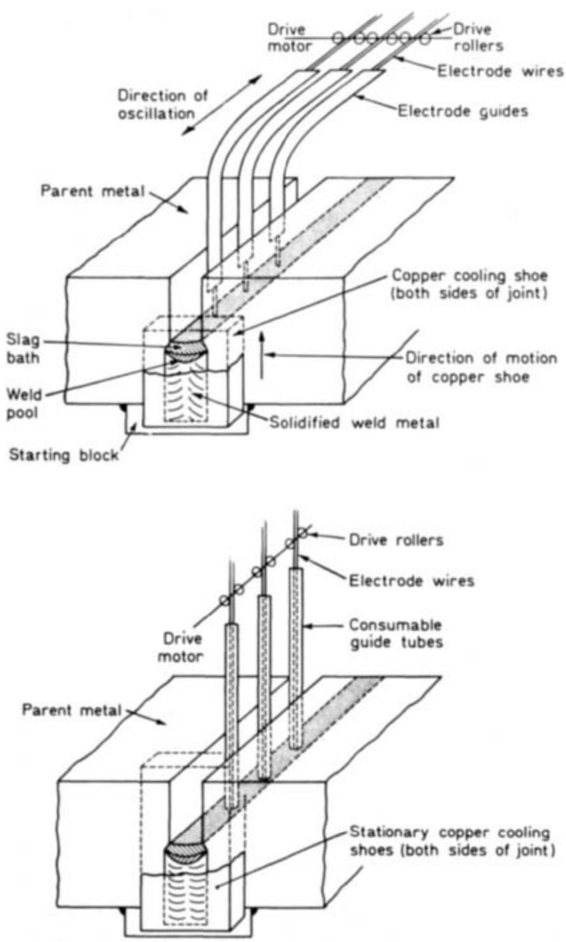


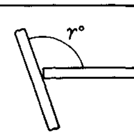
Figure 38.3 Electro-slag welding process

38.2 Weld details

The two main types of welds used in fabrication are the fillet and butt welds. Fillet welds are shown in Figure 38.4; BS 5400[4] and 449[5] lay down that the allowable stress in a fillet weld is based on the throat thickness, t, or '$0.70 \times L$' where L is the leg length, because t for a stated leg length L will vary according to the included angle, $\gamma°$, between the fusion faces. Table 38.1 gives the values of t for varying angle γ.

Table 38.1

Angle between fusion faces	60 to 90°	91 to 100°	101 to 106°	107 to 113°	114 to 120°
Factor by which fillet size is multiplied to give throat thickness	0.70	0.65	0.60	0.55	0.50



Some typical butt welds are shown in Figure 38.5; these are generally for manual welding.[4] The root-run is usually back-grooved (except where a backing strip is used) so that clean weld metal from the previous root is obtained (Figure 38.6); this ensures homogeneity of weld metal at the root area. These same preparations may be used for semi-automatic welding with no root gaps where root gaps are shown, or with a root run of manual weld to seal the root before applying any semi-automatic process for the rest of the weld. The root run on the second side does not generally require back-grooving since the penetration is enough to ensure weld metal homogeneity.

Submerged arc welding is a high-deposition welding process with deep-penetration characteristics although with a direct current electrode negative power source the burn-off or deposition rate increases with a large diminution in penetration; multiple wires or electrodes may be used in the same weld with

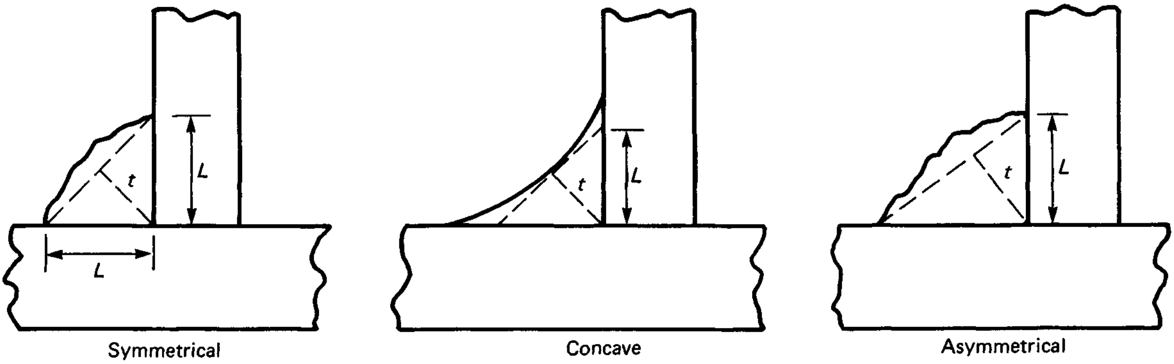


Figure 38.4 Fillet welds. *Note*: minimum length of both legs to be measured for *L*; for concave weld $t \neq 0.7\ L$.

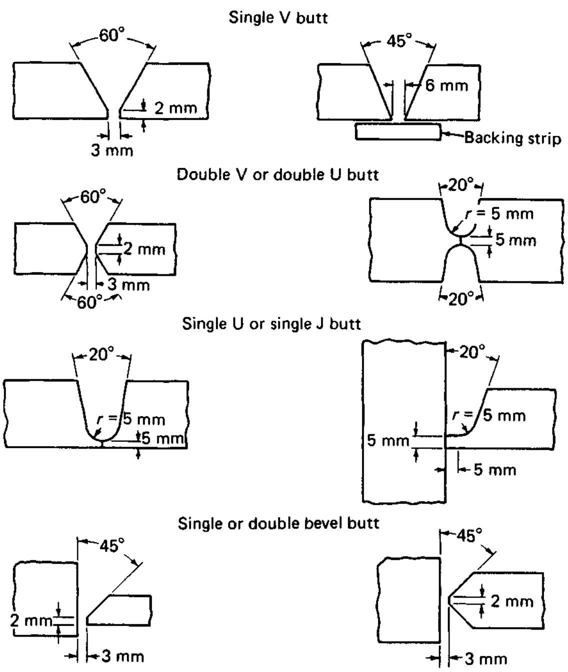


Figure 38.5 Typical butt welds. *Note*: angles and dimensions of root gaps and root faces may be altered to suit welding technique and position of weld, the above being suitable for flat-position welding. Welding is carried out from both sides of all preparations except where a backing strip is employed. To achieve complete penetration, back-gouging (back-grooving) may be employed

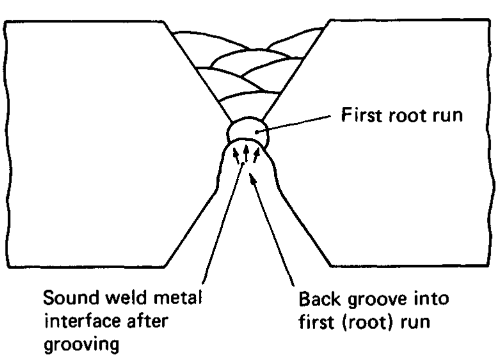


Figure 38.6 Butt weld back groove

the electrodes sharing, in parallel, the same power source or each electrode connected to a separate power source. Weld preparations for such a process are infinite and reference should be made to the suppliers of electrodes and fluxes for their advice; for notch ductile materials, basic fluxes and an increase in the number of runs may be necessary.

38.3 Weld defects

Some typical weld defects are shown in Figure 38.7.

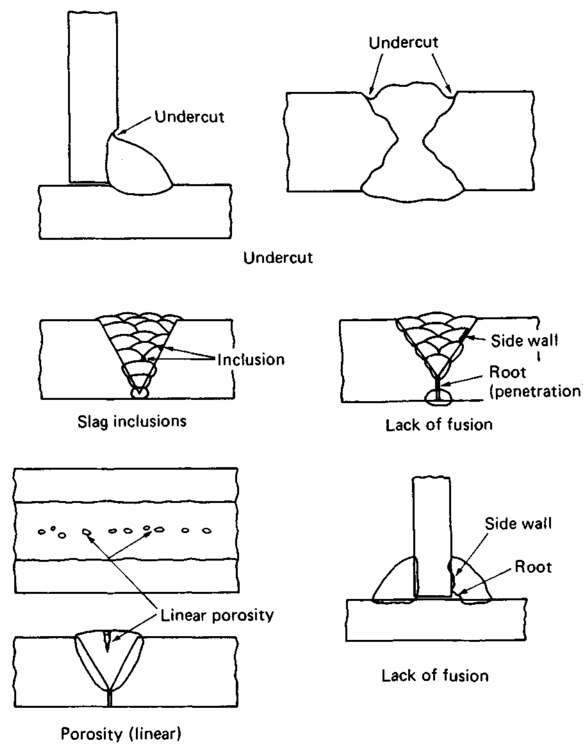


Figure 38.7 Some weld defects

'Undercut.' A groove melted into the parent metal at the toe of a fillet weld or root of a butt-weld – produced by the arc but left unfilled by the filler metal. Undercut may be due to incorrect angle between the electrode and the workpiece, too high an arc voltage or travel speed or scaled parent material.

'Porosity.' Due to dirty or rusty parent material surface, damp consumables, arc instability (as evidenced by the stop or starting of the arc when using MMA), gas entrapment from air due to inefficient shielding gas, grease on filler wires. When linear can denote 'lack of penetration'.

'Lack of penetration' for butt welds. Inclusive angle of prepared faces too small to allow the electrode to get at the root, insufficient current to penetrate the landing or landing too small for the set arc parameters or root gap too small to allow more penetration.

'Lack of fusion.' Incorrect manipulation and angle of electrode to ensure side-wall fusion, or root fusion.

'Slag inclusion.' Nonmetallic solid material entrapped between runs of weld metal or between weld metal and parent metal. Due to inefficient clearing of the slag between each run which in turn may be due to wrong weld parameters giving the wrong-shaped interpass bead and positional requirements.

'Spatter.' Small metallic particles ejected from the weld area and forming on the parent material adjacent to the weld. Spatter varies with the arc process and within that particular process may increase or decrease with differing arc parameters.

'Hot-cracking' (solidification cracking). Cracks appearing in the central region of the weld (Figure 38.8) where segregation of sulphur and phosphorus, the lowest melting-point constituents of the weld metal, occurs; thus at temperatures in the region of the solidus thin films of the liquid segregates occur along the grain boundaries (intergranular). The weld metal may thus become susceptible to cracking because of the high shrinkage stresses generated during the cooling of the weld metal. The effect of sulphur may be reduced by obtaining a higher manganese: sulphur ratio in the weld metal whilst at the same sulphur and phosphorus levels an increased carbon content may cause cracking; likewise silicon. Weld metal on its own is low in all the above elements but high 'pick-up' or dilution may be derived from the parent metal; thus any deep-penetration welding process could lead to hot-cracking. A weld bead or nugget whose depth is greater than its width can, in such deep-penetration processes such as the submerged arc or gas-shielded arcs, promote such cracks when the above metallurgical conditions are marginally operative. In this case a wider preparation or the use of more than one run of weld with lower current values would reduce the dilution factor and the depth: width ratio, to decrease the risk of cracking. Guidance is given in BS 1535 on such cracking.

'Cold-cracking' (underbead or HAZ or hydrogen-induced cracking). The heat-affected zone (HAZ) of a weld is that (generally) narrow zone in the parent metal adjacent to the weld bead affected by the heat input of the weld and whose microstructure and physical properties might be affected by that heat. This zone is rapidly cooled by the mass of the surrounding parent metal and if this cooling rate is high enough a hardened (martensitic) microstructure may be formed. Cracking may develop in this hardened structure (see Figure 38.9) owing to: (1) the alloy content of the parent material increasing; (2) high cooling rate; (3) restraint and therefore higher residual stresses resulting from the weld contraction; (4) stresses within the microstructure due to the transformation to a hardened structure; (5) the presence of absorbed hydrogen from the weld diffusing into the HAZ when that weld cools and contributing to the creation of micro fissures; and (6) for fillet welds, where the fit-up is bad with root gaps.

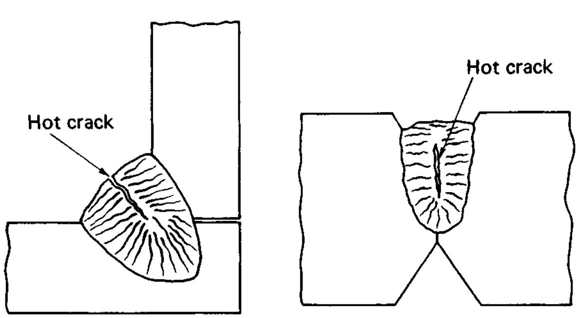


Figure 38.8 Hot-cracking in SA welds

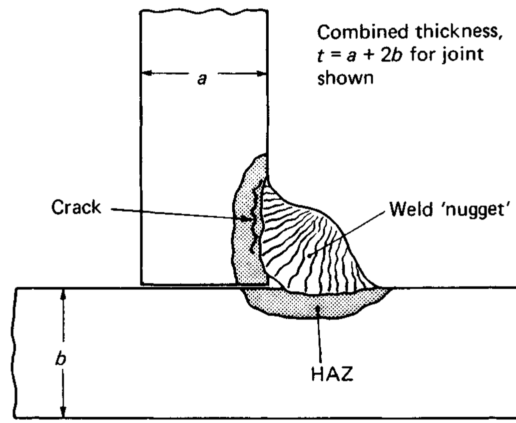


Figure 38.9 HAZ crack

In (1) the presence of alloys in increasing amounts increases the hardenability in the HAZ and their effect can be related to that of carbon by the following carbon equivalent formula:

$$\text{Carbon equivalent\%} = \text{C\%} + \frac{\text{Mn}}{6}\% + \frac{\text{Cr}+\text{Mo}+\text{V}}{5}\% + \frac{\text{Ni}+\text{Cu}}{15}\%$$

Thus any increase of carbon equivalent due to the increase in any of the above alloys will increase the hardenability of the steel. This formula only applies to those steels in BS 4360.

In (2) the cooling rate is assessed partly by the combined thickness t of the joint being welded (Figure 38.9) and partly by the heat input from the weld and any given preheat. The total heat input from any arc may be expressed as:

$$H\ (\text{joules/mm}) = \frac{\text{arc voltage} \times \text{current (amps)}\ XT}{L}$$

where T is the time in seconds to deposit L mm of weld.

Lower t and higher H lead to a lower cooling rate in the HAZ with a less hardenable microstructure.

In (3) the restraint increases with the stiffness of the components making the joint. In (5) the hydrogen content can be reduced by using a low-hydrogen process, e.g. hydrogen-controlled MMA electrodes. The MMA electrode may have to be baked to reduce its hydrogen content to the lowest level possible; in SA welding the flux would have to be dry and preferably of the agglomerate rather than fused flux. With all automatic wires or electrodes no wire drawing compound contaminates should be present; gas-shielded arc processes with solid wire could prove to give weld metals with the lowest hydrogen content.

Preheat curves necessary for combined thicknesses and size of weld deposit (and thus heat input) are given in BS 5135.

Preheat, when applied, reduces the rate of the cooling of the weld and allows more hydrogen to be evolved by the weld metal to the surrounding atmosphere; therefore to be effective it must be applied to the correct temperature and over a sufficient width of the plate. British Standard 5135 indicates that the width of the preheat zone on each side of the weld should be at least 75 mm in any direction from the joint preparation. In practice the temperature is measured by using thermo indicating crayons or paints, the former melting and the latter changing colour when the correct temperature is achieved, and to make certain that the heat has penetrated the full thickness it is customary to heat the far side of the plate with the temperature indicator on the near side, or by heating the near side until the required temperature is indicated on the same side for 2 min for each 25 mm of steel thickness after the heat source has been removed. Although the heats applied are generally low (on the average 100°C), the wide area over which they are used can lead to more distortion than that of the weld itself; it is therefore preferable to use a higher heat input weld source to reduce the preheat required. It is also more economical.

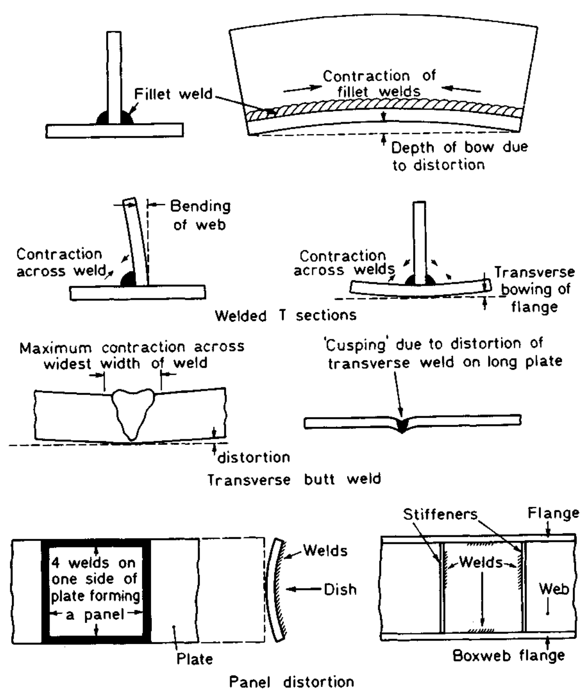


Figure 38.10

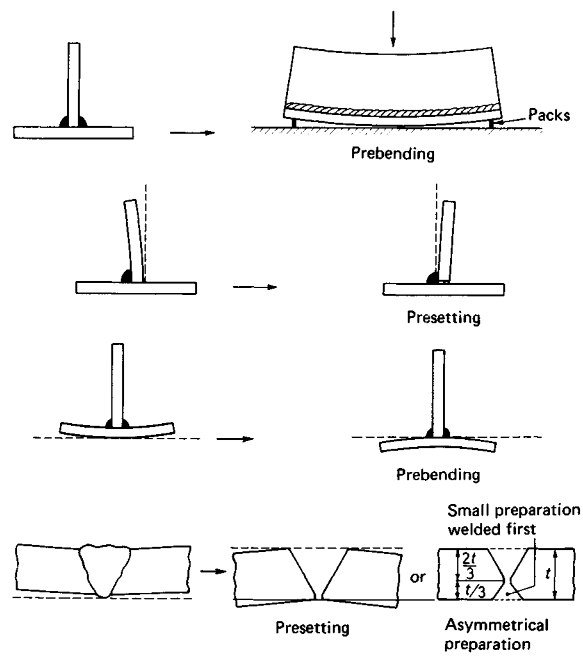


Figure 38.11 Methods to reduce distortion

38.4 Distortion

Distortion due to welding is dependent on the heat input from the weld; such heat is concentrated in a narrow zone around the weld area. The subsequent contraction of the heated weld metal and parent metal produce undue stresses in the fabricated part; if unrestrained the fabrication will distort and, if restrained against distortion, residual stresses up to the yield point of the material may occur. The parts being welded may in themselves have residual stresses due to their shape and size and thus their manufacture; these stresses or some of these stresses may be relieved or increased by the local welding heat and thus their distortion due to welding may be difficult to predict. Metals with differing expansion coefficients, thermal conductivities and physical properties will produce different distortion levels with the same weld heat input.

Figure 38.10 shows distortions for typical welds. Figure 38.11 shows joint preparations, welding procedures and some typical plate presetting to compensate for weld distortions. For the heavier type of fabrication it is generally better to fabricate all subsections prior to incorporating them into the main structure but to control the increased distortions for thin-walled constructions it may be preferable to assemble and tack the whole assembly to give a much stiffer structure more able to withstand distortion.

38.4.1 Correction of distortion

For a dished plate (e.g. a dish resulting from an area of plate welded all round the periphery of that area on one side of the plate only) the amount of dishing resulting from such a weld depends on the heat input of the weld and the thickness of the plate. To flatten such an area, spot heat from a heating torch can be applied in several places within the dished area on the outside (convex side) of the bulge; this will increase the amount of dishing on heating but on contracting that side will shrink and reduce the dish. Heat can be up to red heat (600 to 650°C) but does depend on the thickness of the plate; for very thin plate the applied heat may heat both sides to an equal temperature resulting in equal contractions on both sides of the plate with no decrease in the dish.

Triangular heating on the web and bar heating on the flange of a plate girder or section (Figure 38.12) will increase the camber and can also be applied, within certain limits, to box sections. It is important to note that heating the flange and not

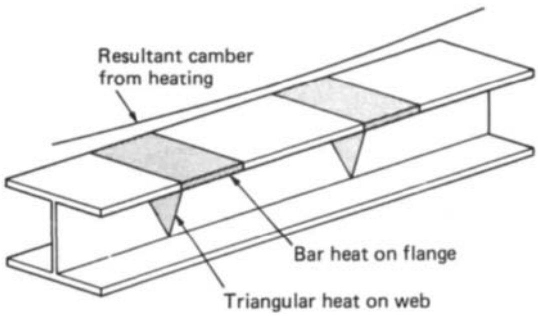


Figure 38.12 Correction of camber

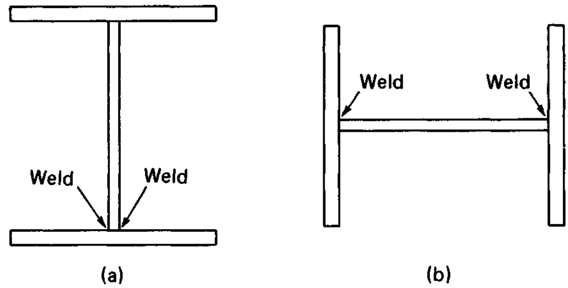


Figure 38.14 Plate girder welds

the web may shrink the flange and increase the camber of the girder but the web, not being heated, cannot shrink to accommodate the increased camber and may therefore buckle.

Angular bending of a flange plate due to the two fillet welds attaching the web to the flange may be corrected by heating in a straight line (Figure 38.13). The effect of introducing heat to shrink areas and to introduce distortions to reduce others, must of a necessity induce stresses into the fabrication; the effect of these stresses and the subsequent increased load on some welds must be watched carefully and if necessary those welds increased in size to accommodate the increased load. All welds or any other form of localized high heats give high residual tensile stresses local to that heat; these stresses in turn generate compressive stresses outside those tensile stress areas. Stress relieving (at about 600 to 650°C) may relieve the structure of any induced stress but in turn must lead to increased or different distortions to accommodate the subsequent movement of the structure.

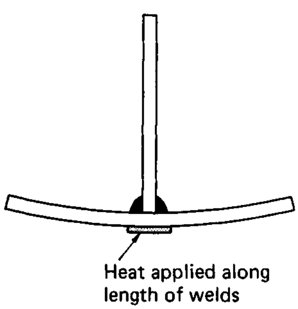


Figure 38.13 Correction for transverse flange distortion

38.5 Assembly

'Plate girders.' These may be welded as in Figure 38.14(a) or (b) by MMA or automatic welding. Tack welding to hold the assembly together must conform to the requirements of BS 5135, with minimum root gaps; large root gaps may lead to HAZ cracks as described previously or 'burn through' when using high current density automatic welds. For girders with top and bottom flanges of differing thicknesses or with top-flange-to-web and bottom-flange-to-web welds of different sizes, different shrinkages may occur in each flange and hence alter the camber of the girder; in such cases it may be necessary to induce triangular heating as described above or to increase deliberately the camber if the web plate is cut to camber in the preparatory stage. For thin flanges it may be found necessary to prebend the plates as shown in Figure 38.11.

For crane girders it may be necessary to make full penetration welds (Figure 38.15) for the web-to-top-flange welds. When using automatic welds such as SA, care must be taken that hot-cracking does not occur; this can happen when trying to achieve penetration and the dilution of the weld metal by the parent metal is high. To reduce dilution back-grooving may be used (but is difficult in this situation) or the web preparation made wider; the increase in the ratio p:w in Figure 38.15 indicates a bigger dilution of the weld metal by the patent metal and the possibility of hot-cracking increases. Hot-cracking invariably occurs on the second side of the joint to be welded since the first weld has made the web–flange assembly rigid or constrained, and it should be noted that hot-cracking may be contained below the surface of the weld and thus not be visible (Figure 38.8).

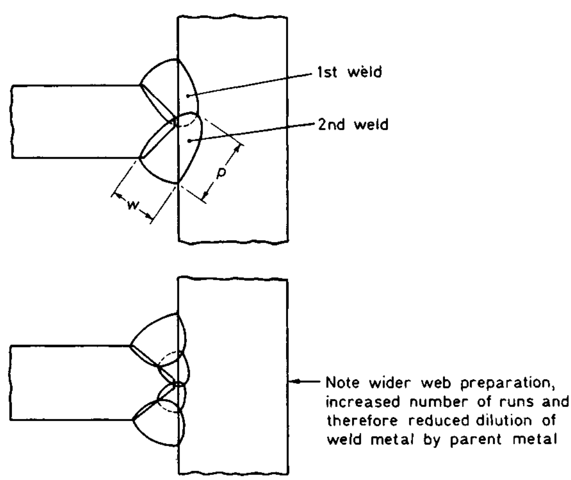


Figure 38.15 Full penetration butt weld, submerged arc

'Box girders.' These are invariably assembled on one flange as the base fabrication plate and must lie perfectly flat on the assembly stallage or a twist in the box may result; all diaphragms are then placed in position after being subassembled and the two webs then tack-welded to diaphragms and flange. As much internal welding as possible is then made before the fourth closing flange plate is placed in position and tack-welded; the four longitudinal web-to-flange welds are then made.

The same comments about differing flange thicknesses or web-to-flange welds in plate girders can apply to box girders.

The choice of flange-to-web longitudinal weld detail may be dictated by the camber and or curvature required in the box. For a large box where it may be difficult to rotate during fabrication (a) in Figure 38.16 may be preferable; where there is camber, (b) is easier to assemble with the flanges outside the webs than (a) where the box-closing flange coming inside the webs can only sit on the diaphragms unless backing strips on the webs are installed between the diaphragms to maintain the closing flange profile.

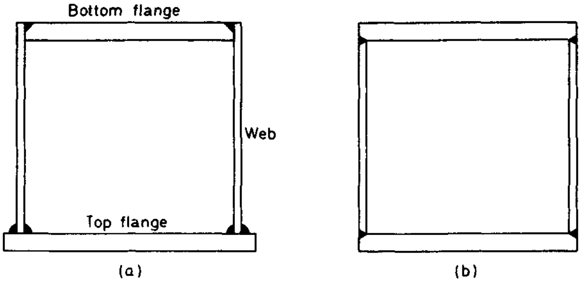


Figure 38.16 Box-girder assembly

Where boxes are to be jointed on site the open ends of adjacent boxes should be stiffened if there are no diaphragms close to the open ends to hold those ends to the required square or trapezoidal profile; for all such ends they would tend, without such stiffening, to have inward bows on all flanges and webs although the four corners are dimensionally correct (Figure 38.17).

In boxes all stiffeners are subassembled on the webs and flanges before the main assembly is completed; to keep all resultant distortion, shrinkage, etc to a minimum it is better that intermittent welding be used on such items if the design requirements can be met with such welds. Stiffeners may be of the bulb, angle or T-type; the latter two may present difficulties for blast or other type of cleaning after welding and also for the subsequent painting.

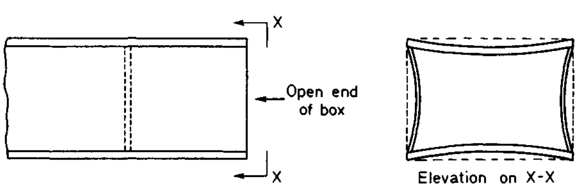


Figure 38.17 Box open-end distortion

38.6 Stud welding

Stud welding shear connectors on the top flange for bridge girders for composite concrete decks is now a widespread practice; the diameters of the studs usually range from 12 to 25 mm and generally vary from 100 to 150 mm in length although 250 mm studs have been welded. The form of the stud is shown in Figure 38.18 and the head of the stud fixes into a gripping chuck in the operator's gun which in turn is placed vertical over the spot to be welded and which rests firmly on a three-point support on the steel surface. When the trigger is pulled an electronic timing device controls the following sequence: The chuck is lifted about 3 mm by an electromagnet and

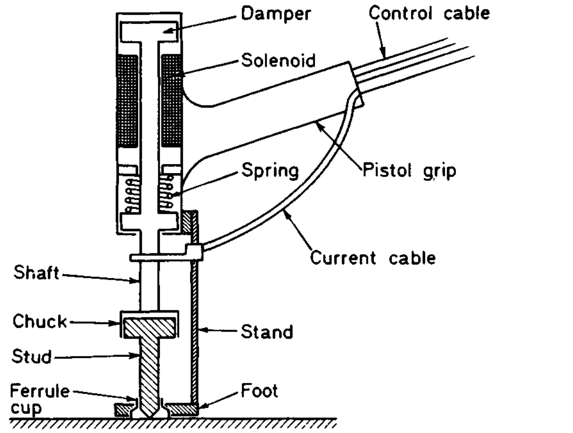


Figure 38.18 Stud-welding gun

a pilot arc is formed about the tapered point (Figure 38.19) which then develops into the main arc, the main arc current being drawn usually from a drooping characteristic transformer–rectifier power source. This arc melts the end of the stud with a resultant melted area on the workpiece and after a preset time the solenoid is de-energized and the stud is plunged by a return spring on to the workpiece while the arc current is still flowing.

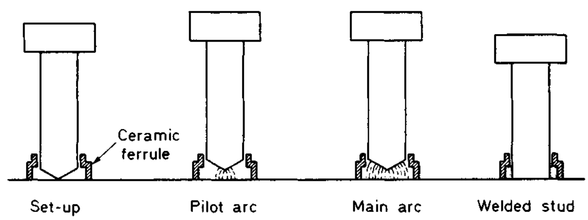


Figure 38.19 Sheer-connector stud

The stud when correctly welded should be of a correct length after welding with a formed upset fillet around it with no undercut; such undercut may be due to incorrect welding parameters or arc blow and when present can lead to easy fracture of the stud from the plate surface. Arc blow because of the high, though transient, currents used may be prevalent when the studs are placed near to the edge of the plate; in such cases an edge plate to extend the magnetic field of the current in the main plate may be utilized (see Figure 38.20).

Studs greater than 22 mm φ are difficult to weld, leading to erratic arcs and sometimes unsound welds; the plate surface on which the studs are being welded should be free of all oily

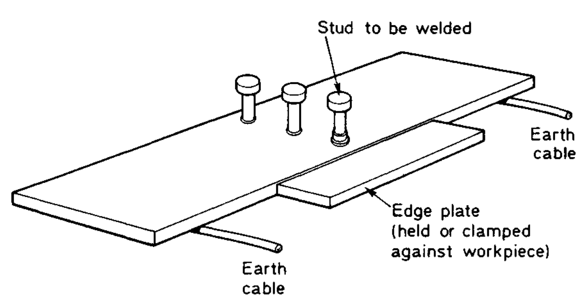


Figure 38.20 Magnetic field edge plate

contaminants, millscale and deep rust. A light surface grinding in the stud area is recommended. The tip of the stud is either sprayed with aluminium or holds a 'slug' of aluminium which acts as a deoxidant when vaporized in the arc; it is important that this deoxidant is not damaged.

One test sometimes employed to ensure the soundness of the steel weld is to bend some to an angle of 30° and to hit or 'ring' the others with a hammer.

38.7 Testing

'Methods.' These may include nondestructive testing methods such as X or gamma radiography, ultrasonics, dye penetrant or magnetic particle testing. Radiography is used almost exclusively on butt welds and ultrasonics on butt and some fillet welds; site welds are invariably tested by ultrasonics and/or gamma radiography employing in general iridium as the source of gamma rays. The standard and scope of testing required is usually determined by the customer and should be ascertained at the enquiry stage.

Before most contracts are commenced some welding procedures may have to be approved by the customer, i.e. a weld joint simulating the thickness, preparation, etc. of an actual weld configuration used in the fabrication must be welded to prove that the proposed welding consumable and method of welding is satisfactory. Such welds may be subsequently tested by nondestructive methods and then physically tested by means of transverse tensile and bend tests, Charpy impact tests, nick-break tests (for fillet welds) and cross-section macrostructures (see BS 709).

The skill of the welders may be approved by the customer's own specific test or by BS 4872 *'Approval testing of welders when welding procedure approval is not required'* or any other subsequent standard with any appropriate nondestructive test requirements.

'Laminations.' Where a plate is laminated or where a plate must be tested for laminations before being incorporated in a fabrication, ultrasonics is the only method by which any such lamination may be detected and the extent measured. Material may be supplied to a standard of lamination testing by the steelmakers; the details of such standards and the appropriate costs may be obtained from the steel supplier. The effect of any lamination on the stability of the structure must be referred to the designer, e.g. the effect of a lamination in a compression member is generally more severe than one in a tension member. It is probably true to say that most structures can tolerate a fairly large degree of lamination in a member before repair is required; the repair of such a lamination is shown in Figure 38.21.

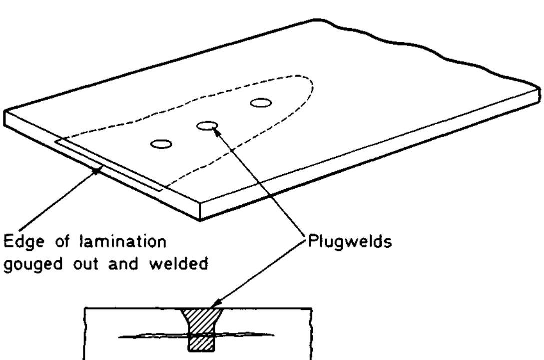


Figure 38.21 Repair to lamination

'Lamellar tearing.' This is a result of nonmetallic inclusions in the steel in the plane of rolling merging into a tear due to the stress imparted by the weld (Figure 38.22) or other derived stresses normal to the plane of inclusions. Because these inclusions are very small and scattered throughout the thickness of the material they are not detected radiographically, and, up to now, although detectable by ultrasonic inspection, they cannot be quantified to assess potential cracking. The tear, when it occurs, is of a ductile nature, fibrous and stepped or ragged as shown schematically in Figure 38.22, the steps resulting from the inclusions in different planes being joined to form the tear; the presence of such inclusions decreases the ability of the material to withstand extension under a load applied across the thickness of the plate. One destructive method of assessing the material for tearing is to machine a small transverse tensile test piece and to measure its reduction in area at failure of applied tensile load.

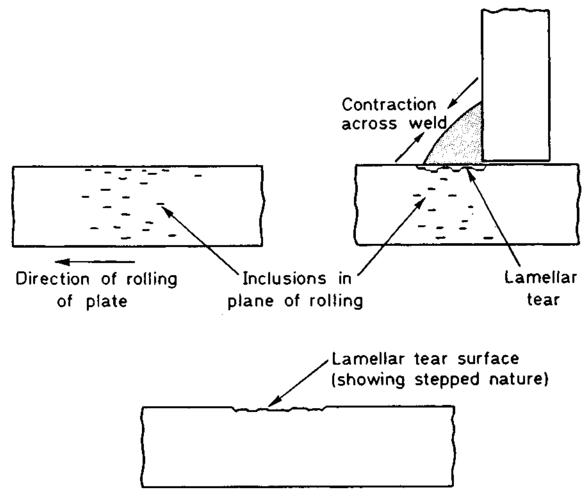


Figure 38.22 Lamellar tearing

Joint details that can promote tearing are shown in Figure 38.23; generally the welds must be large to cause contraction across the welds on cooling to create sufficient tensile stresses across the plate to produce a tear in a susceptible material. For fillet welds to generate such a tear the weld in almost every occasion would have to be greater than 12 mm, although many materials can tolerate much bigger welds, and any details which have both large weld and imposed load stresses across their thickness should be avoided; some preferential details are shown in Figure 38.23. On some suspect material it may be helpful to reduce the risk of tearing by buttering the weld fusion face with MMA or SA welds as also shown.

38.8 Significance of defects

Sudden and catastrophic failures in some steel fabrications has led to the further development of Griffiths'[6] classical work on linear elastic fracture mechanics (LEFM) by Wells[7] and Cottrell,[8] who independently proposed the crack-opening displacement test for determining the fracture toughness of engineering materials where crack propagation ahead of a crack was accompanied by a large plastic deformation at the tip of the crack.

Fracture toughness gives a measure of the material's resistance to failure by cracking due to the stress intensity around the

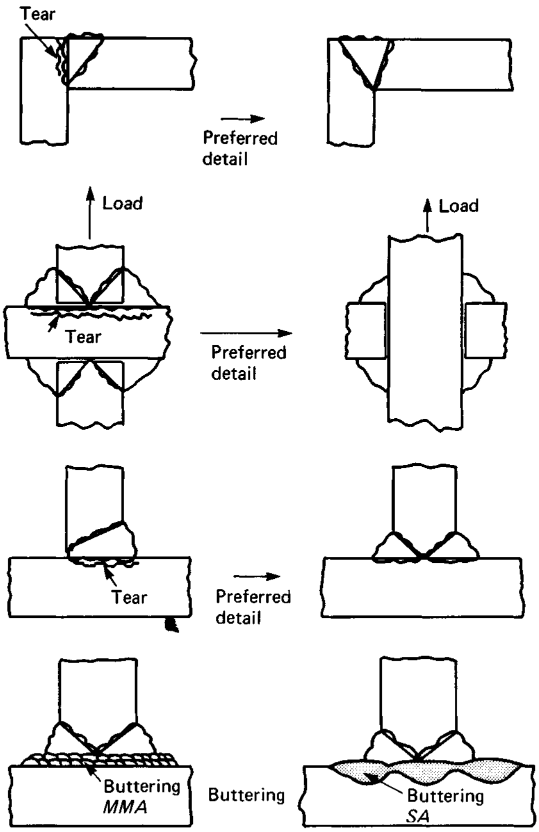


Figure 38.23 Lamellar tearing. Preferred details and buttering

tip of the crack in the presence of an applied and/or residual stress in the material. The measurement of fracture toughness has led to the publication of the BS document PD 6493[9] which gives guidance on the methods for determining acceptance levels of weld defects.

Fracture mechanics can also be applied to the growth of a fatigue crack under load-cycle conditions and the prediction of fatigue failure is largely due to Maddox[10] and Gurney.[11]

Factors which affect fracture toughness of steels are material thickness, service temperature, residual stress due to welding, applied stress, material type and strain rate. Lower service temperatures, thicker material, higher-yield material will adversely affect the fracture toughness; strain rate increase will not permit the normal mechanism of fracturing by slip along the atomic planes in time, and the material behaves elastically. The total stress across a weld defect may be the sum of both applied and residual, the latter due to the stresses set up by the contraction of the weld when cold; it is not unknown for brittle fracture to occur under residual stress only. Once the material has been selected for welding, the correct electrode must be used for yield, ultimate and required impact values; ultimately the level of defects in the weld will govern the service performance of the structure. Crack-like defects normal to principal stresses are most at risk, such as heat-affected zone cracks and lack of penetration; cracks in the outer quarters of a weld are more significant than those in the middle half of the weld. Rounded defects such as porosity and slag inclusions can be ignored in many cases but they themselves may, if profuse or long, indicate possible cracking of some form; such profuse defects may shadow more serious defects when ultrasonic or radiographic testing is applied. However, they indicate bad welding practice and a limit should be applied to the extent of their occurrence.

The fitness for purpose concept, i.e. defects are acceptable in structures if their size, orientation and type do not affect the integrity of the structure, indicate that where stress and design criteria require it an acceptable level of defects must be tabled. A typical statement that no weld should contain defects is inadmissible since all welds contain some defects, however small; with the advent of ultrasonic techniques with improved discriminating powers many small, structurally insignificant defects can be found. Repairs of such defects are not necessary and are expensive, with the possibility of reintroducing more significant defects upon repairing.

Burdekin[12] has proposed acceptance criteria for welded joints in crane girders and proposals are in hand for bridge structures to BS 5400.

Some limited guidance is given in BS5135 related to welders' tests but to relate these tests to a structure's service requirements would be unrealistic. Design detail concepts to minimize the risk of brittle fracture include the obvious step of avoiding welded joints in areas of high stress concentration or high tensile stress and, where this is impossible, to lay down acceptable weld-defect levels. Keep all weld sizes to a minimum commensurate with design criteria to reduce the incidence of weld defects and to reduce weld residual stress. Recognize that a higher-yield material will reduce the acceptable weld defect size and will not be superior to lower strength steel in fatigue life.

To select the correct notch-ductile material related to material thickness and service temperature. A welded joint that has to be tested should be fully accessible to the testing method chosen and to the arc process used to make that joint; poor welding access will inevitably lead to more significant defects. To achieve defect size, location, orientation and type, the use of ultrasonics is mandatory; it lacks a permanent record (compared to radiography) and hence the skill and integrity of the operator is paramount.

Generally, building frames in steel are at little risk to brittle fracture but bridges, pressure vessels and oil platforms all have their areas where care in design, correct steel selection, proven fabrication skill and erection procedures will reduce the possibility of such failures.

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39 Steelwork Erection

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Contents

39.1 Introduction

It is not entirely by chance that the French refer to a bridge as a work of art. For it is indeed a very close liaison between science, and the art of its application, that makes for success in bridge building. This is particularly true for bridges, but is also applicable – though perhaps to a lesser degree – in other types of structures.

A structure, of any type, is essentially designed for its completed condition, and the act of achieving its successful completion is that of coping with logistics of its components, with the partially erected frame, and with the people who will be involved in its erection.

Many variable factors affect the choice of erection method which will be used in any particular case, but on the other hand there are a number of common elements connected with any erection plan which do not vary from job to job.

For any project to be successful it is essential that the designer of the structure has clearly in mind how he visualizes the job will be built. It may well be that the final scheme may vary from that which was originally envisaged due to modifications and improvements, but at least the original conception would have been based on a completely thought out, fully integrated design.

The problems posed by the logistics of a job, the control of materials flow, and also of the organization of the manpower who will build the structure, are common.

Two examples of design points which the original conception should be able to accommodate, immediately spring to mind. The effects of temporary points of support and possible reversal of stresses during bridge building, and secondly the orientation of column webs and flanges to enable the closing beams to be erected in the final stages of completion of a multi-storey frame.

Figure 39.1 This view of the deck of the Forth Road Bridge during erection shows the changes in profile which can occur during the erection process. Arrangements must be made to accommodate and follow such changes in the crane support arrangements. The connections in the structure itself must also accommodate such changes and be designed accordingly

39.2 Effects of the site on a project

The shape and location of the site on which a project is to be built will have had its effect on the basic design of the structure to be erected.

For a tall building, for example, the size of the site, as well as the eventual use of the building will have determined the column centres; the height or number of storeys will have determined the section of the column and, therefore, the weight. In a bridge the configuration of the area to be spanned together with the ground conditions will have determined the spans. Only in uncomplicated cases will the economics of the actual design of the spans themselves be the sole determining factor in the choice of span – or indeed of the method of construction.

Thus, the site through its bearing capacity, or through existing structures or services, has a profound effect on the design of the structure to be erected upon it. That the design of the structure is affected by site conditions is thus clear, but the effect those same conditions have on the erection method to be adopted is even more profound.

A restriction on access for the materials, or the presence of underground services can each affect the placing of cranes and thus the whole approach to the erection plan.

Time on a site costs money and a good scheme should, therefore, aim to reduce as far as is practicable the number of manhours required on that site by the maximum use of prefabrication and subassembly off site. If this is taken to extremes then the components requiring to be lifted could become too large to transport or too heavy to lift. There is thus an optimum balance for any one job, and in major construction projects it is not often that what is right for one job is also right for another.

The ground conditions will have been fully evaluated before the foundations of the structure are designed. It is of equal importance that the ground conditions under temporary foundations are properly evaluated. In a bridge erection scheme, for example, the whole stability of the project can depend upon the adequacy of a temporary pier and, therefore, on the foundations under that pier.

All temporary loads – even of such a transient nature as those under the outriggers of a mobile crane – are important. Only by full attention to such matters can the safety of the men working on the job be safeguarded, and the safety of the structure itself be assured against collapse.

39.3 The effect of plant on design

Everyone is familiar with the contractor's plant yard. Items go there from one job, to be refurbished, tested as required, often given a coat of paint, and there they lie ready to be sent out on to the next job.

The choice of erection plant which is to be used on the next job will, therefore, mean that, for economic reasons, existing tackle will be used where possible. There is, therefore, a need for plant which is as universally useful as possible. Mobile cranes, either mounted on wheels or tracks are one example. At the other end of the scale are the special 'one-off' erection devices built to suit the peculiarities of a particular job. The chances of all the peculiarities repeating exactly on another job are remote, and, therefore, the 'one-off' device is only used on the really big jobs where the very high cost can be written off against that job.

Whatever the type of plant which is being used to erect the steel the economics of that job will be vastly affected by the

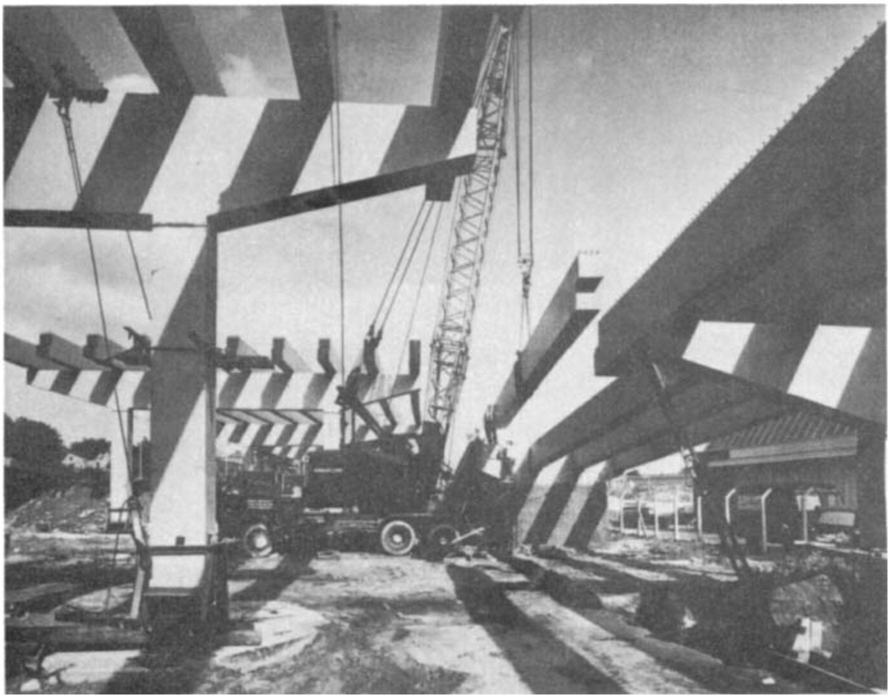

Figure 39.2 The mobile crane is a standard piece of equipment for bridge erection. Points to note are the proper provision and blocking of the outriggers and the use of the minimum radius possible for handling the lifts

variance between the actual average weight of each piece to be lifted and the capacity of the crane at each relevant radius. Thus, in the ideal situation the weight of every piece should equal the capacities given by the safe-load indicator on the crane. Clearly, this is an ideal unlikely to be experienced, but a job on which a conscious effort has been made in that direction will be a more economic job to build.

Where a contractor's plant yard and operational area is remote from access to plant hire companies, greater emphasis must be placed on a careful choice of universally useful pieces of equipment. Little emphasis can be placed on being able to hire a major piece of equipment which may be required for a particular project. An approach often used by international contractors, building major projects in remote areas, is to buy all the equipment needed for a job and then to sell it on the local market at the end of the contract. This option is, generally speaking, not available to a local contractor working his local market by carrying out a large number of smaller contracts. He is in the same position as a small local contractor anywhere, but without the option of hiring the major plant he may need.

When making a choice between one mobile crane and another, consideration must be given to the road conditions and the distances to the likely locations on which the plant will be used. A truck-mounted crane can be driven on the roads, but a crane mounted on caterpillar tracks requires a low-loader and special equipment to move it from one location to another. The existence of suitable maintenance and testing facilities may affect the choice between a hydraulically operated crane or one operated by winches.

If the operational area is large and the likely workload is predominantly at the light end of the market, conditions may be such that the planning emphasis should move towards a more labour-intensive method using easily transportable equipment. A light lattice pole, guyed to the ground at suitable anchorage points can be used to erect most single-storey frames and girders.

Special techniques are needed where the height of the frame being erected requires that the pole – or the crane – be supported on the frame itself as it grows upwards. The winch providing the power to work the crane can either be arranged at ground level, or to climb with the frame.

The choice of plant and the location of that plant during the various stages of building a complex frame will have a significant effect on the stress generated in the structure during erection. Very careful thought must, therefore, be given to the need for temporary material in additional supports and to the need for additional permanent material to transmit the transient additional erection loads, stresses and deflections.

The use of models can be an invaluable help in the positioning of plant for the best use to be made of the capacity available. The increase in the availability of computer facilities is enabling the optimization of erection methods, and the resultant requirements in terms of additional material, manpower and plant, to be achieved more easily. In addition, a large number of alternative possibilities can be considered. A scale model – made of balsa or of cardboard – can also be invaluable as a method of explaining to the erection squad exactly what they have to do, and why.

The temporary supports and the erection cranes are arranged to safeguard the safety of the structure during erection. It is of equal importance that full consideration is given to the many working places where men will have to work during that erection. The provision of safe means of access to these working places will have the dual benefit of improving the safety of the individual, and at the same time of speeding the flow of the work. Careful thought must be given to the use and location of

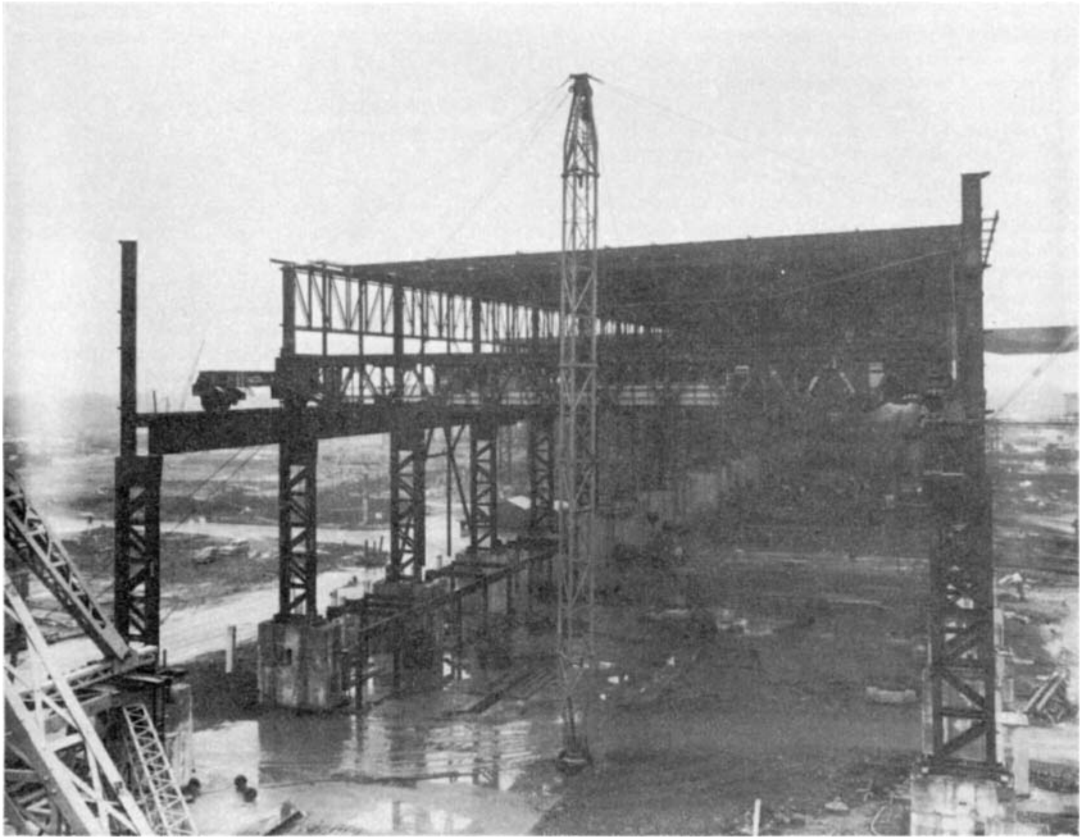

Figure 39.3 An erection pole is much lighter and is more easily broken down into small sections for transport than is a mobile crane with an equivalent lifting capacity. This pole is being used to erect the heavy girders of an overhead electric travelling crane. An erection pole is guyed at the top and bottom and should always be used in as vertical position as possible. By the use of suitable tackles in the overhead and ground guys it is possible to 'walk' the pole from one erection to the next

safety nets since a badly used net can be a hindrance to the job and can indeed be a safety hazard rather than providing the protection intended.

39.4 Tolerances

There are four main areas in which the question of tolerances can affect the engineer. These are: (1) the accuracy of the geometry of the structure in the horizontal and (2) the vertical planes; (3) the accuracy with which the components match at joints and splices; and (4) lastly – but by no means least – the control of deformations in the structure, particularly where welding is involved.

There is only one way of achieving a specified tolerance, and that is to control the operation right from the start. Begin with accurate setting out, follow with accurate fabrication and finish with careful erection. Work which is allowed to slip below standard can seldom be improved at a later stage.

Whilst coarser tolerances can be accepted at the time the ground is being excavated, careful centrelines should be marked on the foundations of columns or piers to ensure that when the steelwork is positioned on them it will be accurate. Levels of both concrete and steelwork can be controlled by the careful use of a dumpy level. It is far easier to place a column on to levelled pads than it is to pull the whole frame into position after it has been erected.

The specification for each job will determine the tolerances which can be accepted on that job, and these will reflect the purposes for which it is to be used. There are a number of standards which give quantitative guidance on acceptable tolerances and these may be quoted in a specification for any particular job.

39.4.1 Camber and vertical curvature of a bridge

Where a bridge is being built using large prefabricated components there is always a possibility that local dimensional variations and twists can occur across the section. Measures must be adopted to accommodate these effects so that force to bring the bridge into contact with its bearings is not necessary. In some cases it may be necessary to provide a temporary support on the centreline of the bridge to allow the structure to take up its natural shape before setting the bearings to suit. If this is necessary, care has to be taken to ensure that the required vertical curvature and camber are maintained as closely as possible. It is important to ensure that controllable errors are not allowed to remain uncorrected and that the adjustment to the bearings is not used merely as a means of compensation for careless work.

The use of machine levels and adjusting screws for levelling the bearings will enable extreme accuracy to be attained in setting the upper bearing surfaces of the bearings. There should be a small air gap between the upper and lower halves of the bearing during final setting to ensure an even distribution of load across the bearing before the load is finally transferred.

Great care must be taken to ensure that the bearing is correctly placed below any diaphragm or stiffener in the structure above.

There should be no need to emphasize the effects which can be caused by loads being applied at points not designed to receive them, or of loads eccentric to their designed position.

The specification of the bridge will lay down the precise tolerances permitted for position and level of the bearings in relation to the structure they are supporting. In multi-span continuous girders the tolerances are tighter than those for simply supported members so far as level is concerned. The position of bearings should be such that they are aligned accurately with the centreline of diaphragms and stiffeners shown on the drawing.

The procedures in the fabricating shop, and the shop assembly of adjacent sections which the specification may have called for, will ensure that these components will recreate the camber called for when they are erected on the bearings we have been discussing.

It is essential to ensure that the methods used for handling, stacking and subsequent erection do not cause permanent distortion of the components. Means of protection should be provided not only to avoid damage to the slings used, but also to prevent damage to the component being lifted or stacked. Incorrect handling can cause deformations and it can also damage any protective treatment which has been previously applied. Many of the points made in this section apply equally to the erection of a building frame.

39.4.2 Tolerances across site butts or joints

The jigs and shop erection procedures used to ensure accuracy of shop fabrication will have minimized the errors in positioning of stiffeners, etc. and will have kept distortions within practicable limits. The responsibility of those on site is then to reassemble the components and erect them into place.

The tolerances allowable in the butt joints between the components are dictated by the need to limit the eccentricity of adjoining members and the secondary stresses those eccentricities would produce. The nature of the stresses, compression or tension, to which the members will be subjected in service, affects the acceptable limits. The compression condition requires a more severe limitation than does a connection loaded in tension. A temporary or transient loading condition, say, during erection, can of course reverse the permanent condition and this must be taken into account when the connections are being designed and fabricated.

If a butt in the flange plate of a girder is to be made by welding then care must be taken to limit the amount of 'cusping' that can occur when the butt is completed. It may be necessary to arrange to have the material preset in order to be able to compensate for the effects of shrinkage and distortion as the welded area cools.

The tolerances both of alignment at butts, and of straightness, will be based on measurement taken over a gauge length dependent on the spacing of stiffeners, and the amounts of allowable deviations will in general be based on the thickness of the plate or flange being measured. Reference has already been made to the relaxation that is possible in connections subject only to tension loading.

There is clearly a balance between the maintenance of practical limits on the accuracy of workmanship and the achievement of very high standards approaching perfection. The fabrication industry has always been keenly aware of the need to maintain a high standard of workmanship in its products whilst at the same time ensuring that it remains competitive in the market places of the world. Careful site planning and control can ensure that these high standards are maintained in the completed structure.

39.5 The interaction of design and erection factors

In section 39.3 consideration was given to the choice of plant which would be used in a particular project. The weight of the pieces to be lifted is clearly of paramount importance in this consideration.

The weight of a component is a function of its size. Piece weights, therefore, affect the location of the splices connecting the components. In addition there are local limitations, and associated regulations, which limit the size of component that can be transported. Thus, the site and any limitations imposed by its access limitations can affect the design just as much as the capacity of the available crane on the job. All these points must be borne in mind in addition to the purely design considerations.

The 'nesting' of components must be considered during the design phase in order to minimize the volume required to be occupied by a given weight of material, particularly where transportation by sea is involved. The cost penalties for shipping a bulky lightweight component are very heavy.

The restrictions referred to, where size must be limited, can result in components which are uneconomical to erect. In these cases, consideration should be given to ways of enabling subassembly or prefabrication to be carried out at site before finally lifting the component into place. An example of this in the tall-building field is shown in Figure 39.4. In this example the floor beam stubs have been welded to the column shaft after arrival on site, but before erection. There were a number of advantages in this procedure. The plain components would 'nest' well for transport, thus reducing the cost of transportation. The depth of the floor girder sections could be increased at the columns in order to be able better to transmit the wind loads, and lastly the splices remaining to be made up in the air are in a much more accessible location. Splices should be located just above a floor level to give safe access for bolting up. The location of the splices between sections of columns is limited both by the capacity of the crane available for the erection, and also by the climbing cycle which has been decided upon. Three floor levels in one piece is normal, with a reduction towards the lower sections where heavier scantlings will be used.

39.6 Foundations and temporary supports

The design of the permanent foundations for a structure will have been considered in very great detail when the structure was in the design office. From the point of view of the erector, however, the temporary condition must be examined. This applies more particularly to bridge building, but the principle applies in any erection plan. Not only can transient loadings and moments which will be applied to foundations during erection exceed very considerably those which can occur after the structure is complete, but temporary foundations may also be required.

Where temporary foundations are required, as much consideration should be given to their design and construction as their importance to the safety of the erection scheme dictates. The main foundations will not have been designed without reference being made to the results of test borings. Where loads are significant or the effects of settlement on the safety of the

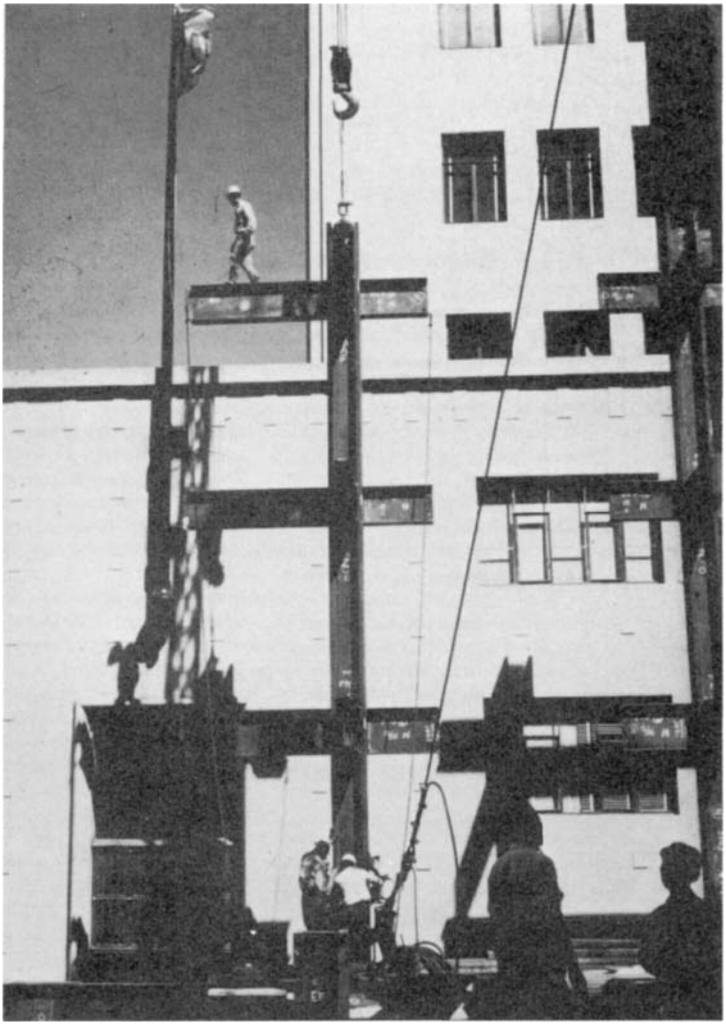

Figure 39.4 A site-subassembled column section for a high-rise building being erected. Site-subassembly enables more easily transportable components to be built up into economic units for erection. In this case also moment connections could be more easily welded up in optimum conditions and the stub-beam connection reduced to an easily jigged and readily accessible connection

structure could be dangerous, then a trial boring should be put down local to the temporary foundation to ensure that adequate precautions are taken.

The additional loads that must be considered when designing temporary foundations should include those that arise from the incompleteness of the structure that they are supporting. The effects of wind loading and aerodynamic instability must be considered. The weights of items of plant, plus the loads they are carrying and any resultant uplift and dynamic loads due to movement, can affect the design.

Particularly in bridgework there will be temporary piers to place on the foundations discussed earlier, and also temporary extensions to the permanent piers. These are commonly constructed of standard components which can be stored and re-used on future work. In the event that specially designed and fabricated temporary supports are not being used, a check calculation should be made to ensure the adequacy of the standard components planned for use.

The points at which it is required to make connections between the permanent structure and the temporary erection structures should be given careful consideration. They must be incorporated in the original design conception. They must be adequate to carry the various loads and stresses to which they will be subjected during the progress of the work. It must be remembered that temperature changes will result in the movement of the structure, and that these movements have to be accommodated by the design of the temporary works. Remember too, that the temporary structure will deflect when the load comes on to it. There must be provision for relieving this load before the structure can be removed.

In buildings, the temporary structures most commonly used are, again, props or columns used to support girder structures at intermediate points during the construction, for example, large lattice girders spanning distances too great to permit erection of the component in one subassembly. It is important here to check the adequacy of the beam carrying the prop, or of the lower column length being temporarily extended. It is not uncommon where the temporary erection loads have to be accommodated for permanent additional material to be incorporated to provide the additional carrying capacity required.

39.7 The partially completed structure

We have considered the need for temporary supports during the erection of a structure, and these considerations have dealt principally with the need to shorten cantilevers during bridge erection or to prop long girders during assembly in a building.

However, all structures are designed to have a minimum amount of redundancy and it follows, therefore, that until an erection procedure is complete the structure is at risk. It is to eliminate this risk and to take due account of the changing stress patterns that will arise during an erection procedure that the engineer must be concerned. It has already been stressed that an erection scheme should be borne in mind when the original design is being made, since only by that means can a feasible, safe and well regulated design be produced.

If a lattice girder structure is to be built out by cantilevering from a pier it is clear that the stresses in the members will be reversed until the next pier is reached and the girder spans from pier to pier. Similarly, only when all the spans are completed will any continuity assumed by the designer be achieved.

Where the lattice girder is supported by piers or, in the case of an arch rib, may be supported from above by temporary cables (see Figure 39.5), very special stress patterns can develop. It may be necessary to carry out a number of case studies at progressive stages of erection in order to ensure that the most critical condition for each member and for each connection has been considered and adequately dealt with.

A building designed to derive its rigidity from shear walls or from the composite action of the floor slab will be unstable until these features are completed. Temporary means of providing this rigidity must be arranged.

The location of these temporary members, and their stiffness relative to adjacent members of the partially completed frame must be carefully considered. It must be possible to construct the permanent members without removing the temporary load-carrying members, and to be able to remove the temporary members afterwards.

The permanent structure must be capable of absorbing the loads induced by the temporary members. These can in many cases frame into locations not intended to carry those loads when the frame is completed. Cases of eccentricity need special attention in view of the secondary stresses which can be induced.

39.8 Stockyards and transport

We have seen that prefabrication enables manhours to be expended on the ground rather than at a height, and thus reduces to the absolute minimum the time that a site is occupied. It is this factor that enables a steel frame to be put up so quickly, and to provide an instant support for the follow-up trades.

Inevitably, components must be transported from the factory to the site. Nothing can ever be perfectly controlled, and a stockyard is therefore necessary to absorb the differences between materials arriving on site, and those actually required from day to day for erection. The effects of delays in the transport system must be cushioned, otherwise expensive erection equipment and manpower would be kept idle.

It has been very truly said that a good stockyard control can make a successful job. The reverse is certainly true: if components and fasteners are not sent out to the erection front as required there will be delays.

Incoming material must be recorded correctly, and located in order that it can be relocated and transported out when required without resorting to double handling. It is often convenient to colour-code the material to conform either to types or area of

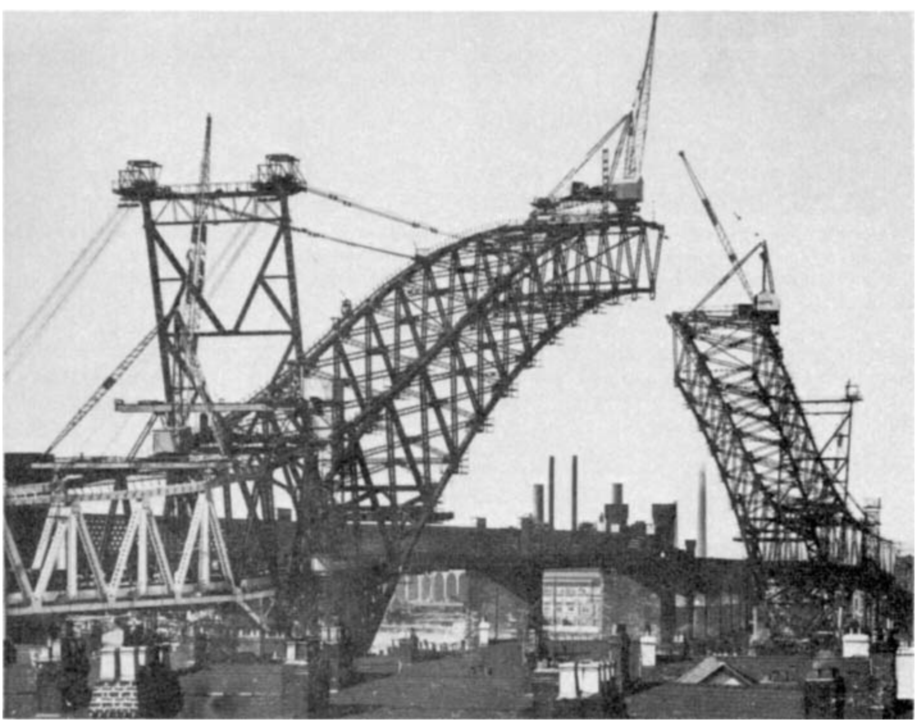

Figure 39.5 In the erection of an arch bridge, temporary supports are required to assist in cantilevering the two halves of the arch out from the abutments. Transient loads, deformations and stress reversals which occur during the erection process must all be calculated carefully for all erection stages and constant checks made of the actual behaviour of the structure as it grows out of the crown

the project. Cranage will be provided to handle the weights of components involved, with the same rules applying as were discussed in section 39.2. A light crane with large coverage will be required to handle the light components, and all the heavy components will be placed under the heavy crane. A Goliath-type crane is often used for stockyard duties since it occupies little ground area for its tracks and can easily cover a large area without reduction of capacity.

Adequate roads – and rail tracks where required – must be provided to give all-weather service. Axle loads of the vehicles using a stockyard are heavy and maintenance time on broken roads is time lost for erection.

Vehicles bringing material into a stockyard will be the normal form of fixed axle or semi-trailer type of transport common on the public roads. Only in exceptional cases will it be necessary to bring components on to site on special transport vehicles requiring police escort. There are limitations imposed by law on both the weight and dimensions of loads that can be moved on the public highway and a knowledge of those restrictions is essential. Additionally, many sites are located in places where the road layout, or low bridges, impose restrictions and a study of these must be undertaken before the component gets stuck and the site is disrupted.

On-site transport is not restricted in the same way, and the size of load is similarly subject to different controls. Site subassembly can lead to awkward loads, and the safe loading and fixing of these on the site transport is important.

The proper control and distribution of fasteners is important. Bolts should be rebagged into 'sets' required for particular connections or areas. This saves time at the splice, and also cuts down the waste of bolts when an excessive number are issued by the store. The proper storage of welding electrodes, their issue and conditioning while being held at the point of use are also functions often delegated to the stockyard stores control function.

39.9 Manpower and safety

None of the operations discussed in this chapter can be carried out without an adequate and sufficiently skilled number of men. The proper use of subassembly techniques can reduce the number of manhours which have to be worked at a height, but work has still to be done up on the open steel, and this must be properly planned.

Steel erection is a task that can only be learned by experience. Formal training can teach the basic skills of rigging, scaffolding, slinging, crane driving, burning and welding, but only experience can enable these skills to be applied on the job. It is the individual erector who brings his expertise and skill; it is the employer who must provide the planned erection method and adequate equipment to enable the erector to safely apply his skill. He needs not only the crane and the spanners, but also a safe place in which to work. Figure 39.6 shows a man working on an unsafe platform. Not only is it dangerous for a person working in such conditions, but the safety of the whole job could be put at risk. The provision of boxes to contain loose tools, and good housekeeping in the handling of small components in general can each help to ensure a smooth running, and a safe, job.

In the UK, a large number of regulations control the manner of the ordering of work on a construction site, and these, together with summaries of them are available from HMSO and from RoSPA. These cover such major items as the testing and retesting of lifting equipment and the dimensional requirements for working platforms and access ways. Many accidents, though, are caused not by major problems, but by insufficient attention being given to the small items.

Thought given to the design and detailing of the steelwork itself can result in a real reduction in the risk of an accident occurring during erection.

Accidents will always happen, but careful planning from the

Figure 39.6 An example of an unsafe working platform. To ask a man to work under such conditions endangers not only the man on the platform but also the safety of the other men with whom he is working, and the safety of the whole job could be endangered

start will prevent a simple accident from escalating into a major calamity.

39.10 Construction management

The construction manager is a man whose function is becoming increasingly important. The basic object of management on a construction site is to have the right component at the right place at the right time with adequate equipment and men to handle it, and to repeat this over and again to complete the structure on time and within a cost target.

To this basic requirement must be added the co-ordination of a number of contractors, each with his own problems and targets, together with an ability to overcome problems created by other influences, often outside his direct control.

There are a number of contractual forms within which construction work can be carried out, but one important feature of all these is that the roles of the parties to the contract are clearly defined. It is essential that responsibilities are clearly defined in this way since we have seen how closely the original design affects the erection process and vice versa. These interconnecting factors must be appreciated by all the parties or disputes, and perhaps tragedies, can result.

There is a variety of ways in which a prospective owner can have the management of the construction of his project organized. One factor common to all methods, however, is that the earlier in the time-scale all the parties who are to be involved can add their particular expertise to the planning of that project, then the sooner the project can be started, and the sooner it can be completed. The attainment of this ideal situation is made easier with some forms of contract and management organization than with others.

In what is being presented here as the ideal situation, a project team is brought together at the inception of the planning of the project. This team will comprise those who will be directly responsible for each of the phases of the project, from the design, not only of the frame, but of all the associated services, right through to the site erection functions, at the other end of the planning and construction time-scale.

The other end of the spectrum is the job where all the functions are carried out in watertight compartments, the designs being prepared in a number of engineers' offices, and the contractors tendering in isolation. With this system no meaningful discussions can take place between those who either are, or will be, most intimately involved with the project until so late a time in the construction process that their contribution can only be limited.

39.11 Summary

The more complex a project becomes and the more services are involved – a tall building is an excellent example – the greater the need for early consultation and planning. In a tall building where the lower floors can be approaching occupancy while those above are having services installed and those at the top are still being framed, the whole complex construction pattern is seen in microcosm.

Accesses for foundation construction must integrate with the demolition of what was there before. Material arriving on site requires close control and schedules if traffic chaos and a hopelessly congested site are to be avoided.

The positioning of cranes and the stability of the partially completed frame and of any temporary works must be very carefully thought through.

The safety of those working on the structure and of those working, and perhaps also living, in the vicinity of the new structure must be constantly considered.

Only by the co-operation of everybody who will be involved in the project, be it bridge or building, can the best design be made, the best construction method adopted and the earliest completion date obtained.

All these factors – design, method of construction and construction time – must be optimized if the owner is to be able to make use of his new facility at the earliest possible time and so be able to generate revenue from his investment.

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40

Buried Pipelines and Sewer Construction

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Contents

Buried pipelines are used to transport fluids which may be gases, liquids or slurries. The pipes may operate at high or low pressure. The pipelines may form part of extensive structures such as large national grids or be limited in extent, such as culverts under roads. Consequently, a wide range of skill and engineering expertise is needed to construct and maintain the structures which represent a considerable capital investment and form a significant proportion of any developed country's infrastructure.

40.1 Routing

The choice of a pipeline route is usually governed by economic considerations but practical and legal factors will have a considerable influence which varies upon the substance being conveyed. A useful checklist of matters to be considered is provided in Part 1 of *Pipelines in land*.[1]

The development of pipelines in the UK is regulated by a number of Acts of Parliament which must be strictly followed by the pipeline promoter. The principal legislation comprises:

Water Acts 1945 and 1948
Gas Acts 1948 and 1965
Public Health Acts 1936 and 1961
Requisitioned Land and Waterworks Acts 1945 and 1948
Land Powers Act 1958

Other legislation which is relevant is:

Public Utilities Street Works Act 1950
(for pipes laid along or across streets)

Coast Protection Act 1949
(for parts of pipelines laid below high water)

Land Drainage Act 1961
(for pipelines crossing rivers and streams)

The legal situation is frequently complex and it is essential to obtain expert advice. The Pipelines Inspectorate, the Health and Safety Executive and the relevant local and statutory authorities should be approached at an early stage in the development of any large project. It should be noted that a planning consent or pipeline authorization does not confer any rights to enter or carry out works in land and it is up to the pipeline promoter to obtain the necessary rights from the owners or occupiers. If these cannot be obtained voluntarily a Compulsory Rights Order may be required.

The greatest problems, both practical and legal, will be encountered in congested urban areas. Pressure pipes for oil, gas and water can be laid at relatively shallow depths and within limits may follow the ground surface profile. Sewers, however, are generally laid to falls so that the contents flow under gravity and this may result in pipelines being constructed at greater depths with potentially greater problems for adjacent owners.

40.2 Materials

The material for a pipeline must be selected to suit its type and purpose (see Table 40.1). The manufacture and use of all materials listed are covered by British Standards which should be consulted to ensure that both the pipe material and the jointing system will comply with the desired service conditions. For work outside the UK, other relevant national standards may apply and should be consulted.

Most pipe manufacturers have devised their own pattern of joint and modern practice uses flexible joints rather than rigid. These are easier to fix and allow a limited degree of relative settlement between pipes without the risk of leakage or fracture. 'Push in' joints using rubber sealing rings are used for low-pressure work. At higher pressures the seal may be compressed by loose flanges and bolts or screwed fittings. Steel and some plastic pipes can also be jointed by welding and this process is usual where 100% watertightness is required for safety reasons. Heat-shrink plastic and other proprietary joints are gaining favour particularly for repair work in the smaller diameters of pipe.

It should be noted that most flexible joints do not resist longitudinal forces. Therefore, pressure mains utilizing such joints should be provided with anchors at bends, tees and end caps to resist thrusts arising from the internal pressure.

40.3 Pipes in trench

40.3.1 Structural design

A buried pipe and the soil surrounding it are interactive structures. The extent of the interaction and hence the magnitude of the pipe loads arising depends on the relative stiffnesses. As a result two separate traditions of design have been evolved; one for 'rigid', and one for 'flexible', pipes.

40.3.2 Rigid and flexible pipes

A rigid pipe may be defined as likely to fracture under very small deformation. The pipe wall resists the external load by circumferential bending and the tensile strength of the pipe material is the usual limiting factor. The tensile bending stress due to the applied loads will be affected by the circumferential tensile stress resulting from any internal pressure caused by the pipe contents.

A flexible pipe will not necessarily crack under slow deformation even when this becomes large. It will fail under vertical loading by buckling or flattening if it is not adequately supported laterally. A circular flexible pipe reacts to external loading above it by deflecting downwards at the crown and outwards at the sides. The latter movement induces a passive resistance in the adjacent soil. The pipe should be capable of sustaining a crown deflection of at least 10% of the original diameter without damage, although it is usual to limit the working deflection to 5% or even less if protective linings are used.[2]

It should be noted that in reality the behaviour of any soil-pipe system varies with the pipe diameter : wall thickness ratio, its stiffness and the modulus of the soil. Consequently, there is an intermediate category of 'semi-flexible' pipes which for convenience are normally designed as rigid pipes.[2]

40.3.3 Longitudinal bending

This is also known as axial bending or beam effect and arises due to differential settlement along the pipeline (which is likely in most soils) or from local concentration of support. The use of flexible joints and good workmanship will assist in reducing the effects of longitudinal bending but special attention should be paid to small rigid pipes laid at shallow depths under heavy wheel loads, and where pipes intersect with structures such as buildings and manholes. A flexible joint as close to the face as possible and a short 'rocker' pipe should be used to accommodate differential movement at structures. If concrete beddings are used for the pipe it is essential to retain flexibility at pipe joints by forming flexible joints through the bedding.

In poor ground it may be necessary to use a piled foundation with a continuous reinforced concrete capping beam to support the pipeline at the required line and grade.

Table 40.1 General pipeline applications (based on BS CP 2010)

Pipe material	Design method	*Application*: Crude oil and petroleum products	Liquefied petroleum gases	Natural gas	Town gas	Industrial gases	Chemicals	Brine	Sludges and slurries	Water	Sewage and trade effluent	Remarks
Asbestos	R							A	A	A	A	
Clay	I						A	A		A	A	
Concrete	G							A	A	A	A	Larger diameters are reinforced
Grey cast iron	I			A^{Δ}	A^{Δ}		A	A	A	A*	A	
	D											
Ductile cast iron				A^{Δ}	A^{Δ}		A	A	A	A*	A	
Pitch fibre	F										A	
Steel with butt-	L											
welded joints	E	A	A	A	A	A	A*	A*	A	A*	A*	
Steel with other	X											
than butt-welded	I											
joints	B			A^{Δ}	A^{Δ}		A*	A*	A	A*	A*	
	L											
Plastics	E			A	A		A	A	A	A	A	Lightweight; some types can be chemically or heat welded

Notes:
(1) *Indicates pipeline may need special lining to prevent corrosion.
(2) $^{\Delta}$Used at lower operating pressures.
(3) It is necessary to ensure that any jointing compounds are resistant to the fluid being carried.

40.4 Imposed loads on pipes in trench

The most widely used method of estimating external loads on a buried pipeline was pioneered by Marston, Spangler and Schlick in the US. It was further developed and extended for UK practice by Clark and Young and is generally termed the 'Marston' or 'computed load' method (Table 40.2).

The method is to some extent empirical and uses a soil model based on Rankine's theories of soil behaviour. This has been the cause of some criticism but considerable testing and successful practical experience has demonstrated its merits, and it remains the standard method in the UK, Europe and the US.

40.4.1 Installation conditions considered for design

40.4.1.1 Narrow trench condition

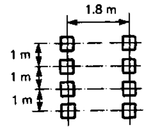
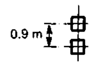
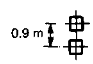
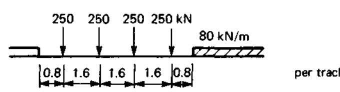
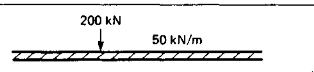
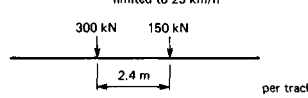
This is the case when the trench is narrow and deep compared to the width of the pipe. The upper limit of narrow trench width for a given combination of trench depth and pipe diameter is known as the 'transition width' (see below). The narrow trench condition results in smaller soil loads on pipes than other conditions but there are practical construction difficulties which can prevent the design assumptions being realized.

The theory is based on an analogy with the Jansen theory of

Table 40.2 Notes on imposed loads on pipes in trench

Source of load	*Comment*
(1) Soil overburden	The magnitude of the vertical load on the pipe is estimated using the Marston model and is influenced by: (a) the depth of the fill and its nature (b) the width of the trench (c) whether negative or positive projection (d) when the trench sheeting is removed (e) level of water table
(2) Superimposed loads on surface	
(a) uniformly distributed load of large extent, e.g. temporary filling	(a) the load is expressed as an equivalent additional depth of fill in the Marston model but ignores any shear forces induced in the surcharge due to differential settlement (b) for narrow trench conditions an appreciable error may result when the notional increase in depth approaches the same magnitude as the original depth
(b) uniformly distributed loads of limited extent (permanent), e.g. foundations of structures, stacking of construction materials, ground loads from caterpillar tracks	(a) the load on the pipe is estimated using Newmark's integration of the Boussinesq equation (b) the soil stress at the pipe crown is calculated and assumed to be constant over 1 m run of pipe and across the diameter (c) the calculated load is added to the soil loads
(c) concentrated loads e.g. vehicle loads	(a) the load on the pipe is estimated by the Boussinesq method for the distribution of stress in a semi-infinite homogeneous elastic medium due to a point load at the surface (b) the method results in a peaked load along the pipe. This is converted to an average load over a length of 0.9 m or less if appropriate (c) the load thus calculated is added to the soil loads (d) the loading conditions are shown in Table 40.3 together with appropriate impact factors (e) all pipes should be checked for the worst case arising during construction as well as the permanent works service condition
(3) Fluid load, i.e. load of pipe contents	(a) weight of pipe contents causes circumferential bending in the pipe wall. (Note self-weight of pipe usually neglected) (b) the magnitude of the bending moment depends on the manner in which the pipe is bedded and whether the pipe is running full (c) the effect is allowed by adding an 'equivalent water load' to the other loads on the pipe. The value can be obtained from charts or can be estimated as 0.75 times water load
(4) Internal pressure	(a) pipes should be designed for the worst internal surge or test pressure which is likely to arise (b) in gravity pipelines the maximum static head (ignoring surge) occurs when the velocity of flow is zero (c) in pumping mains the maximum head will be either: (i) the sum of the maximum static head plus friction head at maximum flow plus any other loss of head, or (ii) maximum surge pressure due to sudden stoppage of the pumps and closing of non-return valves particularly if no surge suppression is provided in the system (d) partial vacuum conditions arising from inefficient air valves, etc. can be treated as an additional temporary external water pressure

Table 40.3 Concentrated vehicle loads on pipelines

Condition	*Reference*		*Loading*
Main roads	BS 5400: 1975 type HB		Eight wheel loads of 112.5 kN each (including impact factor of 1.25) distributed over circular or square contact area at effective pressure of 1100 kN/m^2
Light roads	Ministry of Housing and Local Government (1967) *Working party on the design and construction of underground pipe sewers*, 2nd Report, HMSO, London, as quoted in: Young and O'Reilly (1983) *A guide to design loadings for buried rigid pipes.* Transport and Road Research Laboratory, HMSO, London		Two wheel loads of 105 kN each (i.e. 70 kN static weight with impact factor of 1.5) and contact pressure of 700 kN/m^2
Field loading	Ministry of Housing and Local Government (1967) *Working party on the design and construction of underground pipe sewers*, 2nd Report, HMSO, London, as quoted in: Young and O'Reilly (1983) *A guide to design loadings for buried rigid pipes.* Transport and Road Research Laboratory, HMSO, London		Two wheel loads of 60 kN each (i.e. 30 kN static weight with impact factor of 2.0) and contact pressure of 400 kN/m^2
Railway loading RU (mainline railways of 1.4 m gauge and above)	BS 5400: 1978	 Loads are static and must be increased by impact factor of 2.0 for pipes up to 3 m diameter	
Railway loading RL (reduced loading for passenger rapid transit systems where main line locomotives and rolling stock do not operate	BS 5400: 1978	 Impact factors: 1.20 for ballasted track; 1.40 for unballasted track; 1.00 for temporary works with rail traffic limited to 25 km/h OR  Impact included	
Construction vehicles	Trott and Gaunt (1976) *Experimental pipelines under a major road: performance during and after road construction.* Report LR 692, Transport and Road Research Laboratory, Crowthorne	Manufacturers' loading data for actual plant should be used wherever possible. Alternatively, loads can be estimated from following:	

Plant	*Total mass (t)*	*Static wheel load (t)*	*Tyre inflation pressure (KN/m^2)*
Small scraper	23.2	6	200–400
Large scraper	110.3	28	500–600
Small dump truck	24.3	4	350–700
Large dump truck	80.4	20	up to 650
Ready mix truck (6 m^3 capacity)	24.0	5.5	up to 750

pressures within silos (see also Table 40.3). In a vertically sided trench (Figure 40.1) the load on the pipe is the weight of the prism of fill at level X–X minus the friction of the fill on the adjacent soil. The theory ignores the effect of cohesion on the shear surface and, when rigid pipes are used, any support provided by the fill below level X–X.

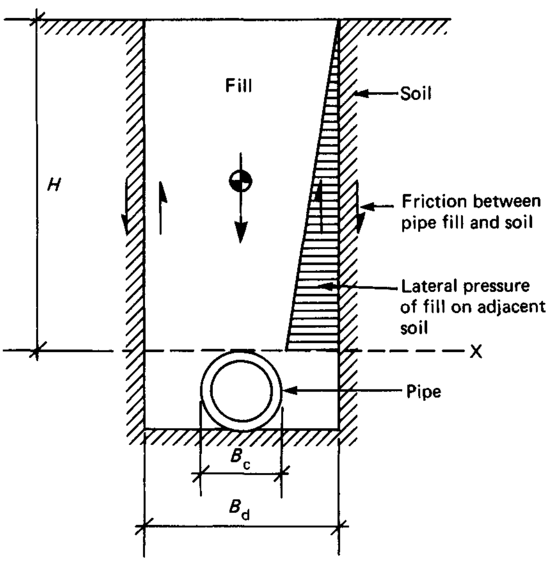


Figure 40.1 Narrow trench conditions: diagram of forces

The load on the pipe is given by the expression:

$$W_c = \gamma B_d^2 \left[\frac{1-e^{-2K\mu'}}{2K\mu'} \times (H/B_d) \right]$$

$$= \gamma B_d^2 \, C_d$$

where W_c = fill load on pipe in narrow conditions in kilonewtons per metre; γ = unit weight of soil in kilonewtons per cubic metre; B_d = effective width of trench in metres; e = base of Napierian logarithms; H = height of ground surface above top of pipe in metres; K = Rankine active earth pressure coefficient; and μ' = coefficient of sliding friction of backfill against trench sides

$K\mu'$ is a semi-empirical constant for particular soil types and it is common to use design charts to determine values of C_d for different soil types; μ' is used to distinguish the coefficient of sliding friction from the coefficient of internal friction for backfill used elsewhere in Young's and O'Reilly's calculations.[3]

It has been shown that the narrow trench condition can exist in battered and stepped trenches (Figure 40.2) and the value of $K\mu'$ appropriate to the material in which shear occurs should be used for design.

40.4.1.2 Late removal of trench support sheeting in the narrow trench condition

The narrow trench loading condition assumes that the pipe load will be reduced by friction at interface between the fill and the soil. However, in close-sheeted trenches it is frequently not practicable for reasons of safety to remove the sheets before placing and compacting the backfill. In such cases it should be

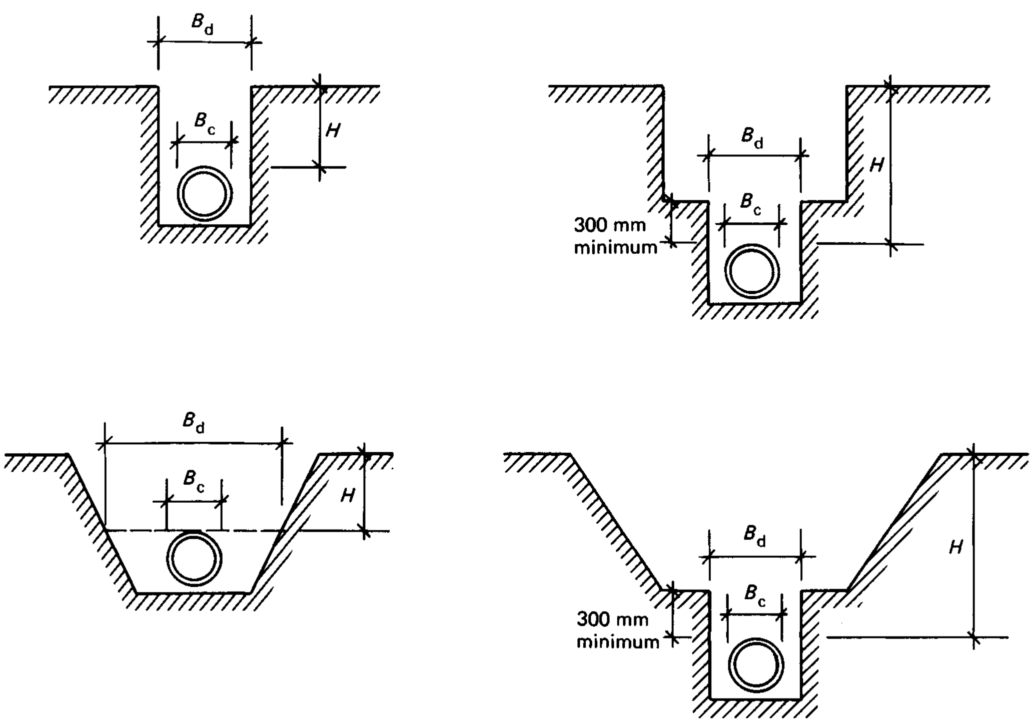
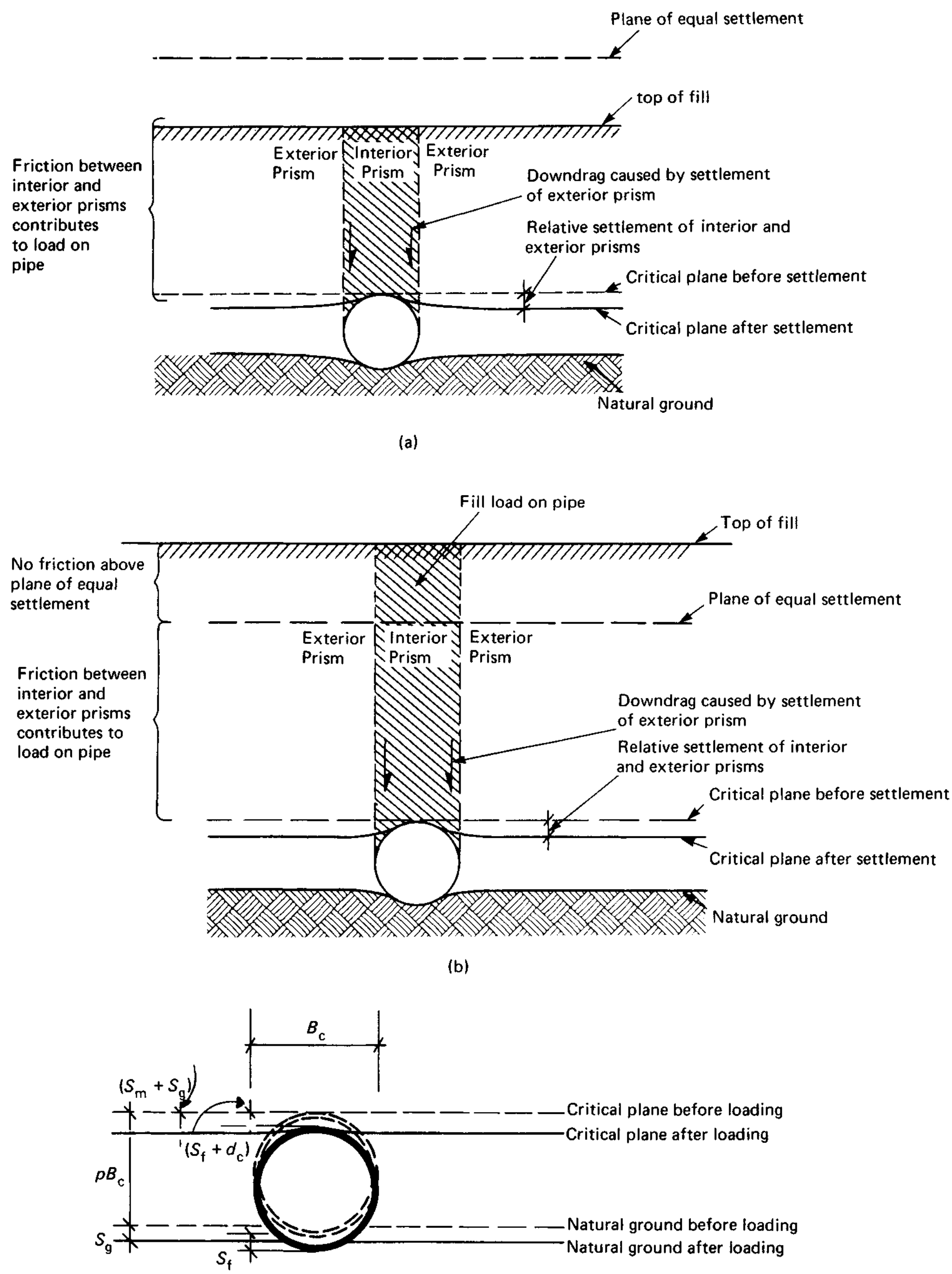


Figure 40.2 Effective trench widths in battered and stepped trenches



Settlement ratio r_{sd}

$$= \frac{(S_m + S_g) - (S_f + d_c)}{S_m}$$

p = projection ratio
pB_c = projection height
S_m = compression of soil in projection height due to overlying fill
S_f = total settlement of pipe invert
S_g = settlement of natural ground surface
d_c = vertical deflection (shortening) of the pipe

(Note: It is not practical to predetermine the settlement ratio and it is always assessed from experimental load measurements and practical experience: see Young and O' Reilly (1983) *A guide to design loadings for buried rigid pipes.* Transport and Road Research Laboratory, HMSO, London.)

(c)

Figure 40.3 Positive projection condition under valley fill or embankment. (a) Complete projection, i.e. shear stresses between interior and exterior prisms extend to top of fill; (b) incomplete projection, i.e. shear stresses between interior and exterior prisms do not extend to top of fill; (c) settlement ratio assumed that the total weight of backfill will be exerted on the pipe.

40.4.1.3 Embankment or valley fill with positive projection condition

As the trench width increases, the prism of soil over the pipe will be bounded on either side by deeper prisms of fill down to formation level (Figure 40.3). The latter are likely to settle relative to the pipe and, hence, will exert downward forces on the sides of the centre prism and increase the load on the pipe.

In a low embankment the shear stresses on the centre prism will extend up to the surface of the fill. This is known as the 'complete projection condition'.

In a sufficiently high embankment, the shear stresses do not extend to the surface but cease at an intermediate level known as the 'plane of equal settlement'. This is the 'incomplete projection condition' and no frictional forces are exerted on the centre prism above the plane of equal settlement.

It should be noted that the amount of deformation of the critical plane in Figures 40.3(a) and (b) depends on, among other things, the 'projection ratio', i.e. the extent to which the pipe projects above the natural ground.

The total deformation of the critical plane is made up of:

(1) The settlement of the fill in the projection height pB_c.
(2) The total settlement of the pipe invert.
(3) The settlement of the natural ground surface.
(4) The vertical shortening of the pipe.
(5) The 'settlement ratio' defined in Figure 40.3(c). (Actual values of settlement ratio have been derived semi-empirically.[3])

The total load on the pipe is given by the expression:

$$W_c' = C_c \gamma B_c^2$$

where W_c' = fill load on pipe in kilonewtons per metre; C_c = fill load coefficient for positive projection case; and B_c = external diameter of pipe

C_c is derived from complex expressions depending on soil and fill characteristics and geometry, and it is usual to obtain design values from standard charts.[3]

40.4.1.4 Wide-trench condition

It has been shown that if a trench is progressively widened and other conditions do not change then the narrow trench loading does not continue to increase but reaches a limiting value given by the appropriate positive projection equation. The trench width at which this limit is reached is known as the 'transition width'.

The same logic applies to a narrow trench being reduced in depth. The depth at which the narrow trench load equals the positive projection load is known as the 'transition depth'. For depths deeper than the transition depth the narrow trench condition applies.

40.4.1.5 Embankment or valley fill with negative projection condition

This case differs from positive projection in that the pipe is laid in a subtrench below the natural ground level (Figure 40.4). The middle prism of soil will tend to settle relative to the adjacent prisms on either side thus reducing the potential fill load on the pipe.

The concepts of the plane of equal settlement and settlemert ratios are also used in this case.

The total load on the pipe is given by:

$$W_c' = C_n \gamma B_d^2$$

where W_c' = fill load on pipe in kilonewtons per metre; C_n = fill load coefficient for negative projection case; and B_d = width of trench

As in the case of positive projection, the fill load coefficient C_n is dependent on soil and fill characteristics and geometry, and values may be obtained from standard charts.[3]

40.4.1.6 Loads on flexible pipes

The calculation of fill loads on flexible pipes can be simplified by

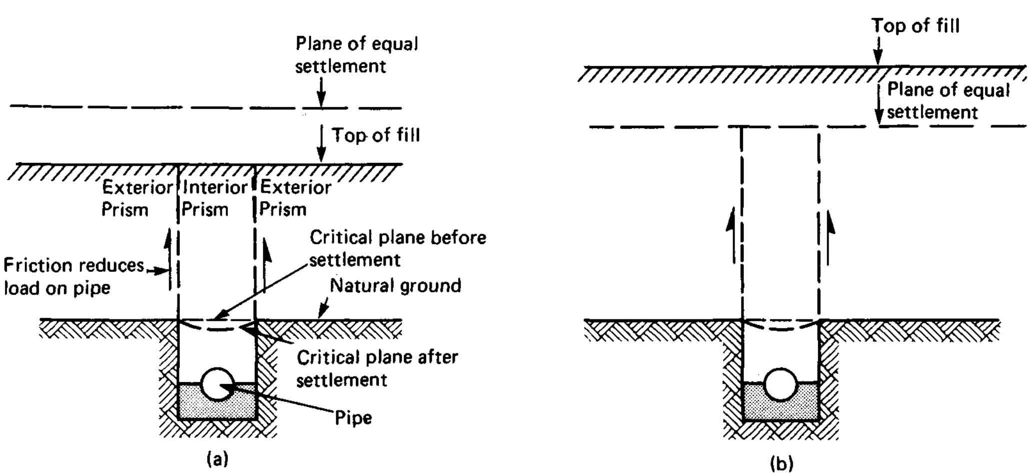


Figure 40.4 Negative projection condition under embankment or valley fill. (a) Complete projection, i.e. shear stresses between interior and exterior prisms extend to top of fill;(b) incomplete projection, i.e. shear stresses do not extend to top of fill

taking the total effective weight of the prism above the pipe of width equal to the pipe horizontal diameter[4] and extending up to the ground surface. To this may be added the self-weight of the pipe above the midplane and the buoyancy caused by any groundwater above the level of the midplane although these loads will be minimal in the case of small pipes.

The effect of concentrated vehicle loads may be calculated using a Boussinesq distribution or by reference to design curves in Compston *et al.*[4] and Nath.[5] The pressures due to vehicle loads will tend to be greatest at the crown and those from external water pressure greatest at the invert. The maximum radial pressure is selected for design and is assumed to be distributed as ring compression around the pipe wall[4] in addition to the fill load.

For the design of corrugated buried steel structures under roadways in the UK, the Department of Transport[6] should be consulted.

40.5 Pipe strength

40.5.1 Rigid pipes

The design of a rigid pipe wall is based on its resistance to the circumferential bending moments induced by external loads plus any circumferential tensile stress caused by internal pressure.

The effect of external loads on a pipe can be calculated by mathematical theory. However, due to the difficulties in accurately modelling the conditions which apply in practice, it is usual to design rigid pipes using an empirical method linked to standard crushing values in the British Standards Specification.

The empirical design approach takes account of the benefits gained by using specific bedding methods immediately around the pipe (Figure 40.5). These distribute the foundation reaction around the lower pipe periphery thus reducing the circumferential bending moment in the pipe wall.

The 'bedding factor' is the ratio by which the ultimate strength of the pipe in the ground is increased compared to its strength in the standard 'three-edge bearing test'. Standard tables of bedding factors are published[2,3] and they normally ignore the effects of lateral restraint. When pipes are laid under embankments with positive projection, the soil will exert an active soil pressure acting horizontally to the pipe. Since this pressure tends to produce a bending moment in the pipe ring which opposes that produced by the vertical load, the net effect is to produce an apparent increase in the bedding factor. The enhanced value of bedding factor can be calculated by methods presented by Young and Trott.[7] The supporting strength of a pipe must be equal to or greater than the total external load (W_e). Therefore, the required strength of pipe W_T is:

$$W_T = \frac{\text{External load } (W_e) \times \text{factor of safety}}{\text{Bedding factor}}$$

The factor of safety can be chosen at the discretion of the designer. It is usual practice to use a factor of safety of 1.25 in conjunction with the maximum crushing test load of a pipe, or factor of safety = 1.0 in conjunction with the proof test load for an unreinforced concrete pipe. (All pipes must withstand the specified maximum load and reinforced concrete pipes should not crack by more than a specified amount under proof load.)

Tables are published for structural design of different types of pipe under various load conditions.[8-10]

40.5.1.1 Internal pressure in rigid pipes

As previously noted, internal pressure produces circumferential tension which reduces the available strength for resisting circumferential bending tension due to the external load. In these cases, Young and O'Reilly[3] and Young and Trott[7] should be used for design.

40.5.2 Flexible pipes

The design of flexible pipes is based on the yield strength of the pipe material in compression and on its resistance to circumferential buckling and distortion while being restrained by the surrounding ground. In pressure pipes the yield strength in tension must also be considered.

A significant element of the strength of a flexible pipe is derived from the adjacent soil and consequently the adequacy of the surrounding material and the workmanship used will have a significant effect on the performance of the pipe.

The pipe wall thickness for external loading is derived from ring compression theory.[4,11] It should be noted that a minimum ratio of pipe diameter to wall thickness must be provided to avoid damage to the pipe during handling.

40.5.3 Practical pipe design

The design and construction of pipes for most uses are covered by appropriate national standards. These usually give preferred methods of analysis and allowable stresses and deal with practical matters. Minimum cover depths (usually not less than 0.9 m) and the need to provide anchorages, air valves and other venting arrangements may have significant effects on design loads.

Nevertheless, the designer will be required to make a number of engineering judgements and it is essential that he should understand the basic theories involved and their limitations. Variability in soil, pipe strength and site workmanship means that extreme precision in design is not appropriate. Design tables are available[8-10] which reduce tedious calculation in a large number of situations. In situations where site conditions vary and the tables do not apply, the works by Young and O'Reilly[3] and Young and Trott[7] should be used.

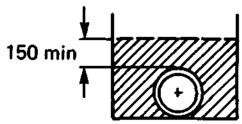
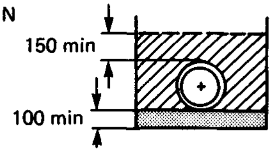
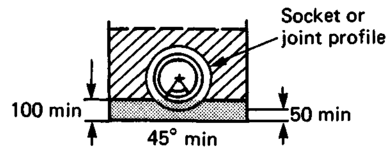
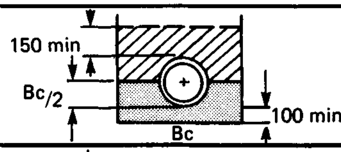
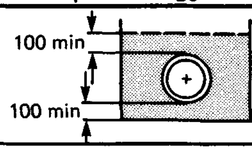
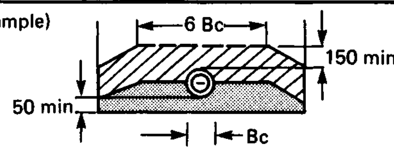
The worst conditions for a length of pipeline should be used for design. Sources of extreme stress should be avoided rather than strengthening the pipe to resist them. The designer's assumptions should not demand excessively high qualities of workmanship or supervision on site. In particular, the designer must recognize that trenches need to be wide enough for access and support during construction. Experience shows that in most circumstances it is cheaper and more satisfactory in service to use strong pipes with cheap beddings (Class B for sewer pipes,[12]) and to employ the least expensive method of installation appropriate to the site conditions.

40.6 Construction of pipes in trench

40.6.1 Site investigation

Both the pipeline designer and the constructor require knowledge of the site conditions and the ground. A proper site investigation should be carried out at an early stage in the development of a project and it should include all the relevant topographical information and a soil profile extending to at least 1.5 times the trench depth.

The results of the investigation should be reported formally.[13] It is unlikely that extensive soil testing will be required since visual descriptions will be sufficient for most methods of design for pipe strength and trench support. Particular attention should be given to groundwater tables and to potentially corrosive conditions.

Figure	Bedding class	Bedding	Comments
5a	D 	1.1	Pipe laid on trimmed trench bottom
5b	N 	1.1	Pipe laid on a flat layer of granular material with CF[1] not greater than 0.3
5c	F 	1.5	Pipe laid on flat layer of granular material with CF[1] not greater than 0.2. Illustrated after settlement
5d	B 	1.9	Pipe laid on granular material to half diameter with CF[1] not greater than 0.2
5e	S 	2.2	Pipe fully surrounded by granular material with CF[1] not greater than 0.2
5f	B (example) 	1.9 (example)	Construction as in Figures 5b, 5c, 5d and 5e in this table except that when the trench width exceeds 4 times the outside diameter of the pipe barrel, the granular material may be sloped down from that width to the trench formation

All dimensions are in millimetres

Key: Selected fill; Granular material

Notes: (1) CF = Compaction fraction (see Appendix D, BS 8005: Sewerage, Part 1)

(2) Recommendations for granular pill and selected fill: are given in clauses 5.5.1 and 5.5.2 of BS 8005.

(3) Where pipes with sockets are used these sockets should not be less than 50 mm above the floor of the trench.

Figure 40.5 Beddings for rigid pipes

40.6.2 Ground movement

All ground has existing *in situ* stresses and the balance of these stresses can be disturbed by the construction of the pipeline. In a trench excavation the sides and bottom lose the restraint of the excavated spoil and deform until either sufficient shear strength is mobilized in the ground to restore equilibrium or failure occurs. The insertion of the trench support assists in reducing this potential movement and preventing collapse. However, it should be recognized that ground movement cannot be eliminated.

The stress–strain relationships of soils are very complex. The degree of strain is dependent not only on the magnitude of the original stress and the properties of the soil but also on time and other factors. In weak soils, e.g. soft clay, light loading can cause large deformations. Wide trenches in soft clay can heave and fail at the base even when side support has been installed. In strong soils, e.g. cemented sands, the same loads will cause only small deformations with no failure.

The deformations are shear deflections which commence immediately excavation starts. There is a time lag before the maximum potential movement is achieved at each level of dig. For this reason soil can continue to creep after the trench support has been installed since it may have gaps between it and the soil and will in any case deflect under the load. Other movements may occur due to:

(1) Inadequately compacted backfill.
(2) Volume changes due to moisture variations particularly in fine grained soils.
(3) Short-term consolidation resulting from increased effective stresses caused by groundwater lowering during construction.
(4) Long-term consolidation caused by structures and geological processes.
(5) Ground heave due to groundwater pressures beneath clay.
(6) Frost heave.
(7) Traffic loading.

Symons[14] and Attewell and Taylor[15] give useful information on the possible components of total movement and their magnitude.

Soil movement is a cause of concern since buried pipelines will move with the soil and hence must accommodate the movement without distress. Pipelines with flexible joints are better able to resist such movements than pipes with rigid joints.

The water and gas industries in the UK have recognized the risk caused by movements arising from deep excavation to adjacent buried pipes, particularly those manufactured of grey cast-iron. For this reason a consultative procedure has been set up between them to investigate construction works so that potential problems are avoided.[16]

40.6.3 Groundwater control

This can be required during trench construction either to cope with groundwater flowing into the trench or to reduce uplift pressures on the bottom of the trench. Water flows may be controlled either by driving impermeable sheets to obtain a cutoff or by dewatering.

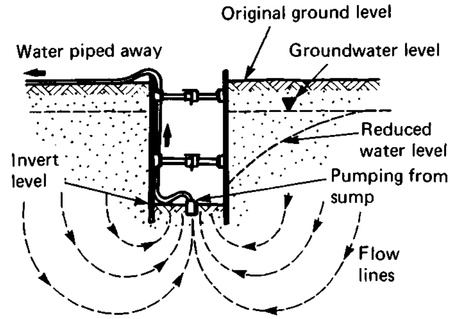
Dewatering of trenches down to 6 m is most commonly done by sump pumping and well pointing (see Figure 40.6). The effectiveness depends on the nature of the soil, the trench geometry and the degree and rate of lowering required. Sump pumping is the cheapest and simplest method but draws water inwards towards the base of the trench which may cause instability due to seepage forces and erosion of fine particles.

Well points have the advantage of drawing water away from the trench bottom which tends to stabilize the ground by increasing its shear resistance. Well points are most effective in sands; clays and fine silts are usually too impermeable.

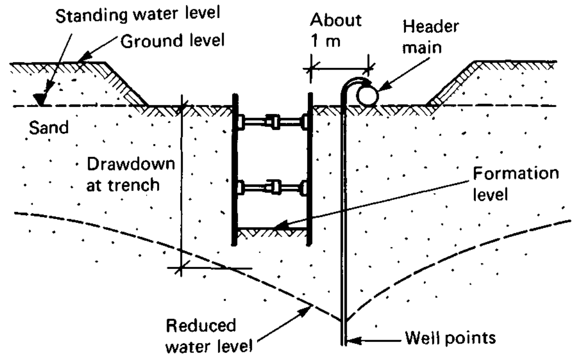
In these cases exclusion methods are generally employed, although ground freezing or electro-osmosis has been used successfully at great cost.[17]

40.6.4 Battered trenches

Where sufficient space is available, battered trenches are often the quickest and cheapest method of construction. Large excavators can be employed and the need for side support is eliminated by cutting the sides of the excavation at a safe slope.[13] Slips due to subsequent deterioration of the exposed faces must be prevented by suitable measures where they are likely to cause problems.



(a)



(b)

Figure 40.6 (a) Sump pumping; (b) single-sided well point system. (This figure is reproduced by courtesy of the Construction Industry Research and Information Association, Report No. 97 *Trenching practice* by permission of the Director of CIRIA)

40.6.5 Trench support

Trench support has two functions:

(1) To provide a safe place of work for operatives working within the trench.
(2) To maintain the stability of adjacent ground and hence the integrity of structures supported by it.

Trench support systems can be considered in two groups: (1) those based on traditional timbering methods but now frequently employing steel sheets and adjustable struts; and (2) those methods which use proprietary components or complete proprietary systems.[13,18,19]

In the traditional system, the sheets may be either predriven or inserted as excavation proceeds, and then supported by walings and adjustable struts (Figure 40.7).

The sizes of the components are estimated by simple calculation.[13] The use of highly mathematical theories is not justified since soil characteristics are generally not accurately known and the reuse of simple components keeps material costs low. The system can be adopted readily to suit changes in conditions or geometry but it should be installed only by men experienced in its use.

40.6.6 Proprietary systems

These may be described[13,18] as:

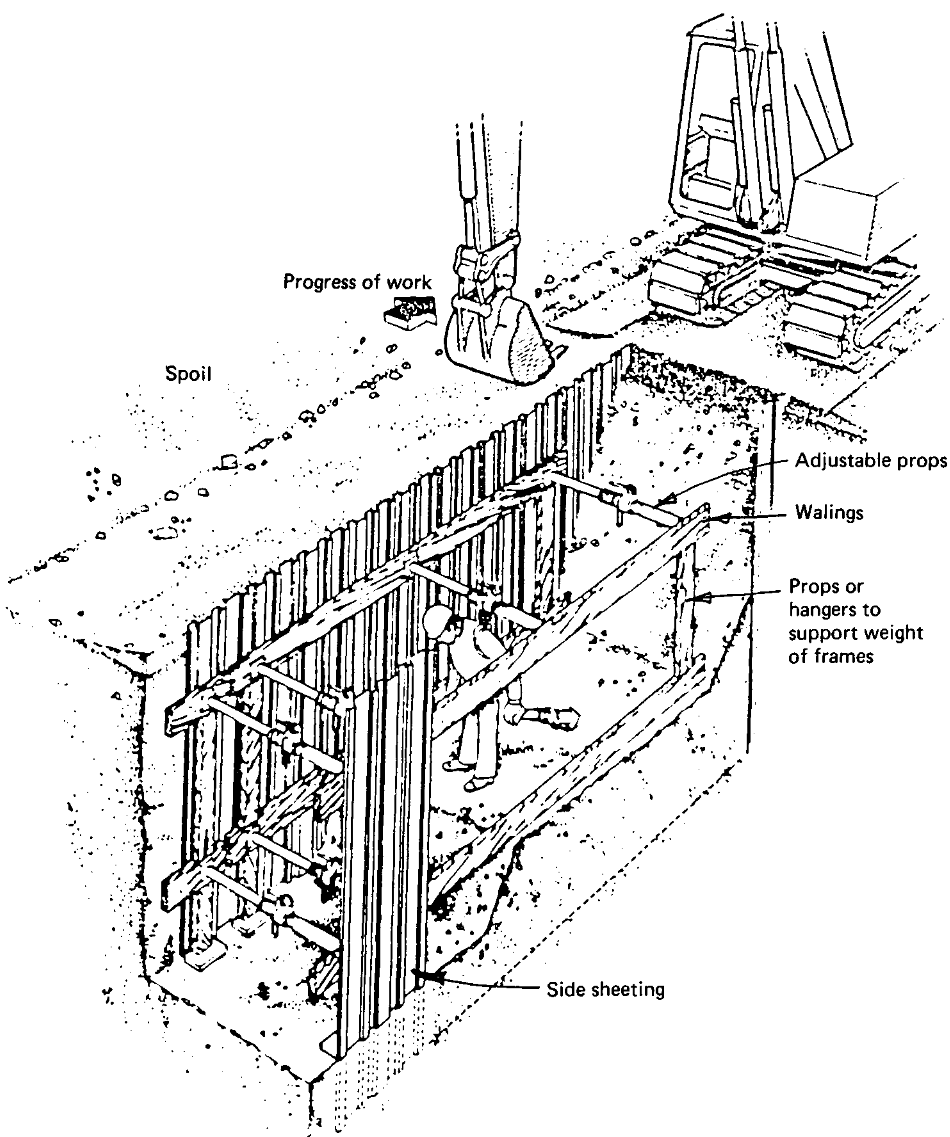


Figure 40.7 Typical trench support system with struts, walings and side sheeting. (This figure is reproduced by courtesy of the Construction Industry Research and Information Association, Report No. 97 *Trenching practice* by permission of the Director of CIRIA)

(1) Hydraulic frames and shores.
(2) Boxes.
(3) Slide rail systems.
(4) Shields.
(5) Piling frames.

Hydraulic frames (Figure 40.8) have adjustable struts permanently fixed to walings and may be used in place of conventional screw struts and separate walings. The advantage of the hydraulic system is that in some conditions it can be placed and tightened without the need for operatives to enter the trench. The vertical shores are of similar construction and can be used as pinchers where full face sheeting is not required.

Boxes are used to form modular strutted support walls (Figure 40.8(b)). They are either lowered into a predug trench to provide a protective box for operatives or are buried using the dig-and-push technique. In this case, the box is progressively pushed down and the spoil dug out between the walls. Where efforts are made to limit overbreak, the latter method will result in the box being a tighter fit in the ground thus giving more positive support to the trench sides. It is normal practice when laying pipes to use three or four boxes in line.

Slide rail systems have vertical slide rails or soldiers strutted apart with horizontal walls spanning between them (Figure 40.8(c)). The dig-and-push installation method is used and the wall panels slide vertically in the rails.

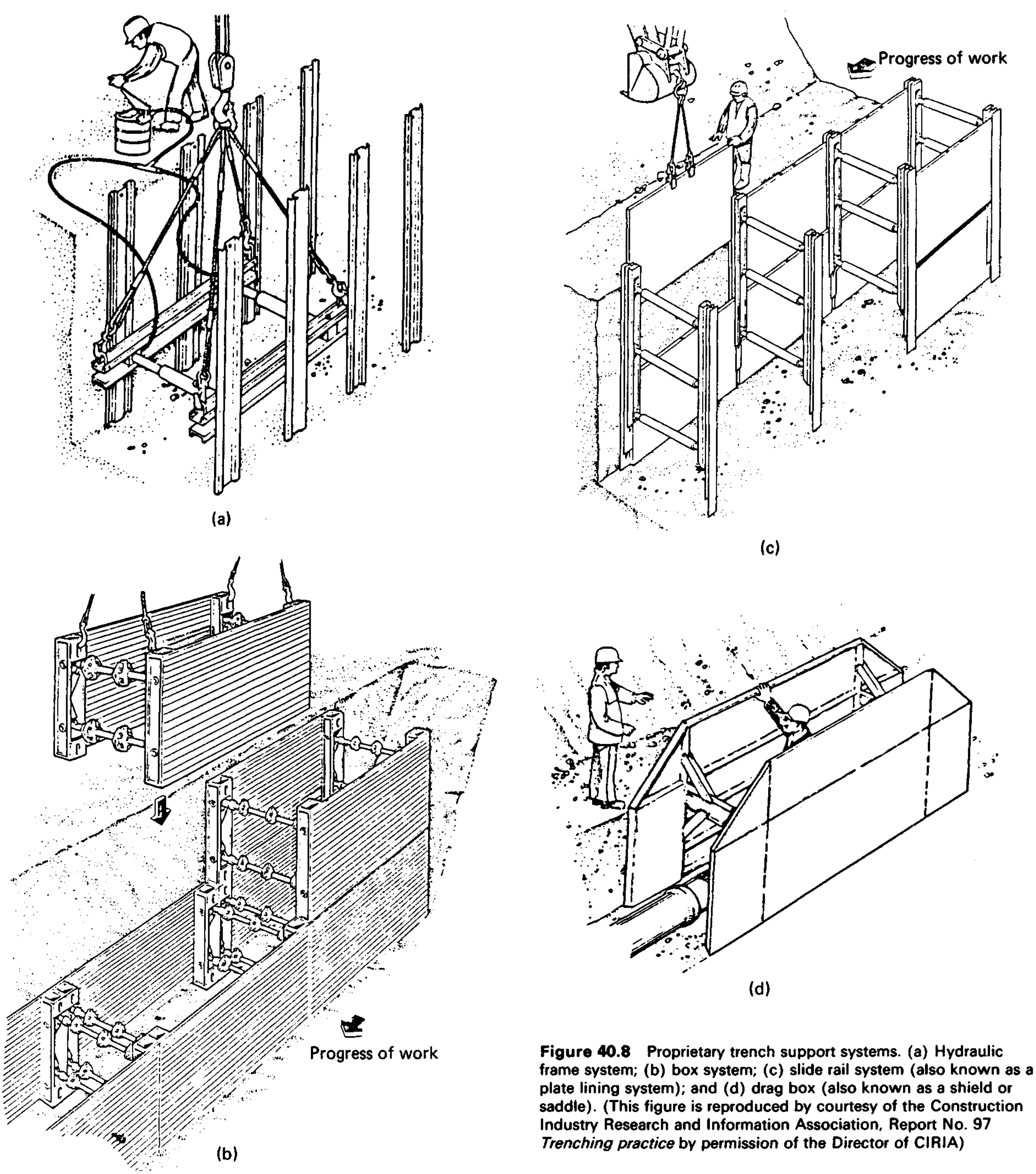


Figure 40.8 Proprietary trench support systems. (a) Hydraulic frame system; (b) box system; (c) slide rail system (also known as a plate lining system); and (d) drag box (also known as a shield or saddle). (This figure is reproduced by courtesy of the Construction Industry Research and Information Association, Report No. 97 *Trenching practice* by permission of the Director of CIRIA)

Shields, also called drag boxes or saddles (Figure 40.8(d)), are vertical support walls permanently strutted apart. The shield is lowered into the excavation and dragged forward by the excavator as the trench is extended. It is a loose fit in the ground and provides a protected place of work.

The presence of existing crossing services causes practical difficulties for boxes, shields and slide rail systems. Consequently, short lengths of traditional trench support may be needed in these areas.

Proprietary piling frames can be used to install vertical sheeting in the ground (Figure 40.9). The non-powered type consists of two sets of parallel walings forming a gate through which the sheets are inserted. The walings are permanently strutted apart and dig-and-push methods are used to force the sheets into the ground. In the powered version, the waling gates are equipped with hydraulic rams which enable the machine to push individual sheets up or down. The sheets are specially made for the machine and can be used to depths greater than 6 m if intermediate waling frames are used.

A further development for installing small-diameter pipes is

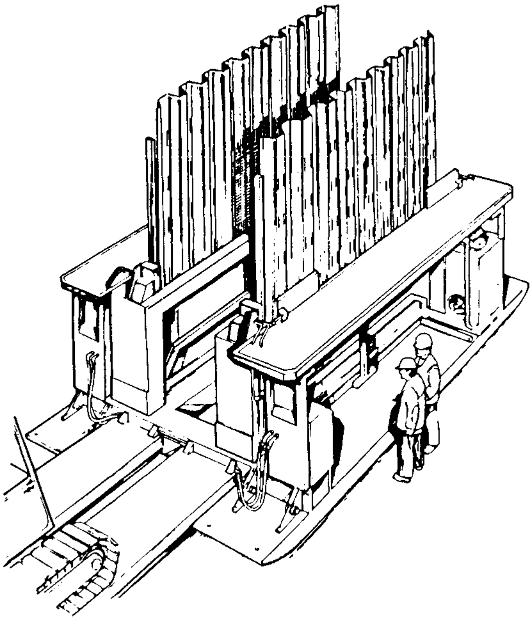


Figure 40.9 Proprietary rail-mounted piling frame. (This figure is reproduced by courtesy of the Construction Industry Research and Information Association, Technical Note 95 *Proprietary trench support systems*)

to use a self-propelled machine which cuts a continuous slot in the ground. It carries horizontal shields to keep the slot open so that the pipes can be jointed continuously at the surface and installed down a ladder guide into the bottom of the trench.

40.6.7 Submarine crossings

Pipes to be laid under water can be installed by several methods depending on circumstances:

(1) In cofferdam.
(2) Floating and sinking.
(3) Bottom pulling.
(4) Lay barge.

Cofferdams are costly and not used for pipelines of great length. Floating and sinking requires the pipeline to be fabricated on shore and fitted with buoyancy tanks before it is towed into position and sunk. Care must be taken not to bend the pipe and its protective coating beyond their allowable stresses.

Bottom pulling is more frequently used since it is easier to control stresses. The pipe is fabricated on shore in line with the trench and then pulled into the trench using a winch. In the lay barge method the pipe lengths are jointed on the barge as it is winched forward. The pipe spans between the barge and the trench bottom and frequently a ladder is used to give intermediate support.

In all cases, sufficient weight must be provided to anchor the pipe in place when it is empty.

40.6.8 Trench backfilling and reinstatement

The backfill forms part of the load-bearing system of a pipeline structure and must be constructed with care. The fill around the pipe needs particular attention to achieve an adequate bedding factor while ensuring no damage is caused by over-compaction. The materials used for backfill should be capable of densification to the required standard without undue effort. The reuse of poor material from the excavation can be a false economy particularly under trafficked areas. The permanent reinstatement of road surfacing should be done as soon as practicable. The use of temporary surfacing may cause more problems than it is intended to cure, particularly where it does not give adequate support to the adjacent pavement or protect the subgrade.

40.6.9 Protection

Pipelines may require protection both internally and externally against a wide range of aggressive conditions. Steel and ductile iron pipes will tend to corrode in the presence of moisture and air. Consequently, they may require protection both internally and externally using a wide range of materials such as reinforced bitumen, coal tars, plastics, epoxy resin-based coatings and Portland cement mortars. Factory-applied protection is to be preferred but, since accidental damage is likely to occur during installation, ease of repair must be an important consideration.

The corrosion of ferrous materials is an electro-chemical process and cathodic protection can be used to provide external protection in addition to coatings. This may be particularly necessary in aggressive ground conditions and where clay soil could promote bacteriological corrosion. The pipeline is made cathodic by using either an impressed direct current or by connecting the pipeline to sacrificial anodes which corrode in preference.[20] It is important that cathodic protection systems are properly engineered since corrosion may be accelerated by an inadequate design.

Concrete pipes can be damaged due to sulphate in the groundwater and septicity in sewage. The latter generates hydrogen sulphide which can be reduced to sulphuric acid by certain types of bacteria. Septicity can to some degree be prevented by careful design of the sewer system to ensure an adequate oxygen supply.

40.6.10 Testing

Pipelines should be inspected for line level and material defects as they are constructed. Welded joints in steel pipelines can be checked using radiographic or other non-destructive tests. Internal proof pressure tests should be applied, preferably before joints have been backfilled. Air pressure is usually quicker and more convenient than water pressure; smoke tests can be used to discover leaks. However, air-pressure tests can be dangerous if failure should occur on large-diameter pipes. Water testing is to be preferred in these cases. Special arrangements may be necessary to anchor unbalanced forces particularly when testing large pipes. Also, the practical problems of obtaining sufficient water and its subsequent disposal can be considerable.

40.6.11 Safety

The principal legislation dealing with construction is the Health and Safety at Work Act 1974[21] which defines duties and responsibilities of employers, controllers of premises, manufacturers and suppliers of equipment and employees. This Act is supported by the Construction Regulations[22,23] together with other legislation. Practical advice on safety can be found elsewhere.[13,19,24,25]

The object of the legislation is to provide a safe place of work for operatives with safe means of entry and egress and a safe system of work. At the same time the safety of persons outside the site must also be ensured.

By law, trenches must be supported properly and inspected regularly.[22] Excavators used in the work can be used as cranes

Table 40.4 Safety checks for confined spaces

Before work starts:
(1) Check ground conditions for hazards and sources of gas such as organic strata, refuse, sewers, gas mains, industrial pipelines and the interaction between carbonate and acid (particularly in chalk, limestone and greensands)
(2) Ensure personnel are fit and properly trained
(3) Ensure breathing apparatus, lifelines and safety equipment is available. Define the procedures for contact with the emergency services
(4) Check gas monitoring equipment is available and working

Before entering:
(5) Check atmosphere for oxygen deficiency and explosive or toxic gases
(6) Check arrangements for ventilation
(7) Use breathing apparatus if the atmosphere is dangerous
(8) Check arrangements for entry and egress and lighting are adequate

While working:
(9) Continuously monitor the atmosphere
(10) Ensure space is properly ventilated
(11) Do not smoke
(12) Ensure direct communication is maintained between everyone involved in the work
(13) Ensure all equipment is maintained in good order and properly used

In emergencies:
(14) Do not enter the space without proper equipment or without an attendant at the entrance. A lifeline and harness should be used when entering and pulling out victims.
(15) Do not attempt to purge dangerous atmospheres with pure oxygen which will cause an explosive hazard

only if special exemption is obtained[26] and should be inspected weekly.

It should be noted that trenches may in some circumstances constitute a confined space and appropriate safety measures should be applied (see Table 40.4).

The legal responsibilities for safety cannot easily be subcontracted and the Swan Hunter case[27] clarified further duties for providing information. Contractors working alongside other contractors (whether subcontractors, tertiary contractors or independent contractors) have a duty to provide sufficient information so that each employer is aware how his operations affect others and how the operations of others affect his own employees. This duty applies not only downwards from main contractor to subcontractor but upwards also.

40.7 Trenchless pipelaying

In urban areas the use of trenchless techniques can reduce to a large extent the disruption caused by trench construction and also allow obstructions such as rivers, railways, major roads, etc. to be negotiated conveniently. The techniques currently used may be roughly divided into microtunnelling (formation of a new bore smaller than man-entry size) and conventional tunnelling (larger than man-entry size). The lower limit of man-entry size is generally accepted to be about 900 mm diameter. This is the smallest diameter in which a man can work effectively (albeit with difficulty). Where pipes are to be inspected only, men can enter down to 600 mm diameter if proper precautions are taken.

Since 95% of the UK sewer system is 900 mm diameter or smaller, microtunnelling is frequently used for sewer refurbishment as well as new construction.

40.7.1 Microtunnelling design

In recent years there has been a considerable emphasis on the development of microtunnelling techniques, and methods are available for producing bores from 50 to 900 mm diameter. Figure 40.10 indicates the current techniques, which are based mainly on pipe-jacking methods.

Little research has been done on the vertical loads exerted by the ground on pipes installed by jacking. It is usually assumed that a slight overbreak of soil around the pipe may occur which will create a soil stress system similar to a Marston narrow trench condition. Therefore, soil load can be estimated by the equation by Young and O'Reilly (page 40/7) where B_d = effective width of trench. This assumption is likely to be conservative in stable soil.[7] Other soil theories may also be used; in particular Terzaghi's theory for buried pipes[28] which is based on assumptions similar to those used by Marston.

The loads on the pipe due to ground surcharge may also be calculated using the methods derived for pipes in trench.

The pipes must be strong enough to cope with the axial loads produced by the jacking operation as well as the external and service loads. Experience shows that pipes which are designed to cope with the jacking forces are generally adequate for the ground loads. Once the ground forces have been established, the pipe strength can be checked either using the bedding factor method (a value of 1.9 is generally accepted for pipes installed by jacking) or by using elastic theories of ring compression.[29]

The jacking loads arise mainly from friction. This varies with different ground conditions from 5 to 25 kN/m^2 although more extreme figures have been known. Sands and gravels cause higher values than cohesive soils and friction can be reduced by lubricating the outside of the pipe using bentonite slurry or other methods.

Jacking pipes of different materials and joint design vary in the load accepted in end bearing, the different materials available including steel, concrete, clay and GRP. Angular variations arising between pipes during installation tend to reduce the failure load and many types of joint have been tried to reduce this danger.[28] The jacking forces must be resisted at the thrust pit by a thrust wall or a heavy foundation. These are designed by conventional soil mechanics methods.

40.7.2 Site investigation

A proper site investigation is an indispensable requirement. It is

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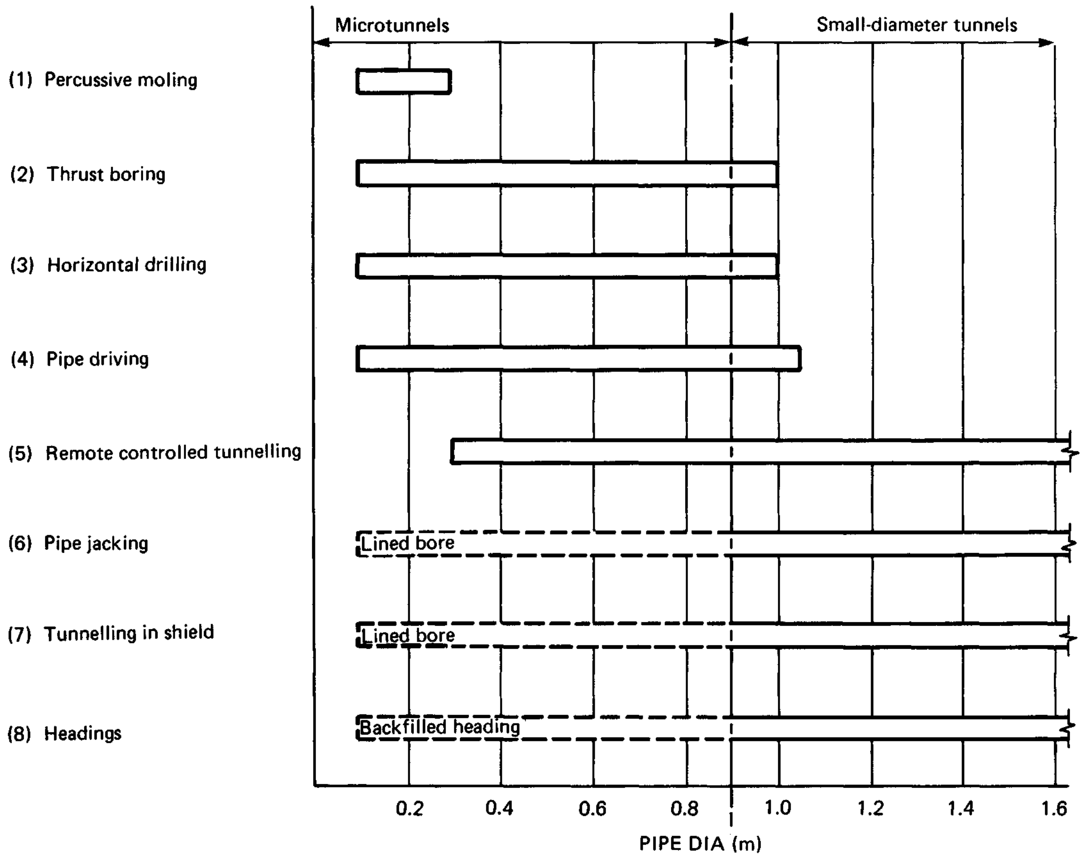


Figure 40.10 Trenchless pipelaying techniques for new pipelines

essential to choose the right construction method for the conditions, since there is no access to the end of the bore and the tunnelling machine may be lost if unacceptable ground conditions are encountered. If this happens it will probably require a shaft or second tunnel to be dug to recover the machine. In extreme cases the machine and the original tunnel must be abandoned.

A full topographical survey is required including the accurate location of all existing buried structures. The soil investigation should extend to a generous depth below the invert of the pipe. Boreholes and trial pits should not be located exactly on the centreline of the pipe to avoid weakening the ground and they should be properly grouted up and backfilled for the same reason. Soil testing is required to ascertain the description, density and strength of the ground. Cobbles and boulders are a particular hazard and small-diameter boreholes may be inadequate for identifying such conditions.

Information on groundwater levels and permeabilities must be provided. Poor conditions may need treatment before the tunnel excavation starts. Suitable techniques might include groundwater lowering with well points or deep wells, grouting or ground freezing.

The detection of existing services may be done by a variety of methods. Magnetic field location is commonly used. Passive locators are used to locate buried conductors by detecting the natural magnetic field surrounding it. The method is quick and simple but only locates. It is not appropriate to trace or identify a buried service. If this is required, an active locator should be used. This comprises a transmitter, which applies a signal of known frequency to the pipe or cable and a receiver which is tuned to the signal and used to trace the service. The method can be time-consuming and requires a skilled operator but gives greater accuracy than the passive technique.

Non-metallic pipes can be traced by the use of small self-contained transmitters known as sondes. These are moved through the pipes using rods or high-pressure water or towing cables, and the signal is traced from ground level.

Other location methods are available including ground-probing radar, seismic reflection techniques and the detection of variations in static magnetic fields. Also dowsing methods have been used with success.

A common fault of existing service maps is lack of accuracy. The digitization of records and production of digital maps based on Ordnance Survey maps is more accurate and is gaining favour. This should improve the situation in the future and ease the exchange of information between utility companies and others.

40.7.3 Ground movement

The installation techniques for microtunnels may be classified as 'convergent', where the volume of excavation exceeds the volume of the pipe installed, or 'expansive' where the opposite is the case.

In the convergent case, ground may be lost due to face and peripheral encroachment (similar to overdig in larger tunnels),

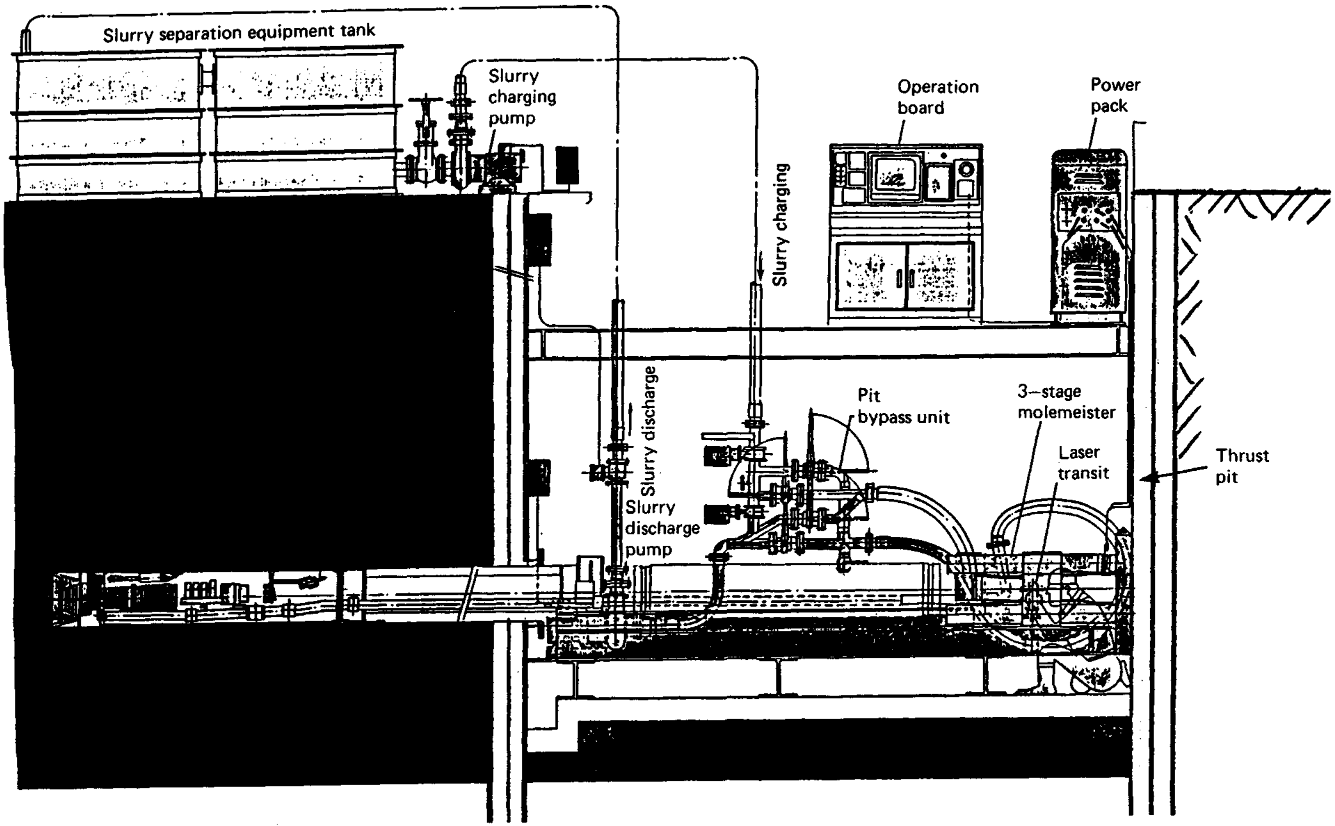


Figure 40.11 Microtunnelling with a bentonite slurry shield. (Courtesy: Iseti Poly-Tech Inc.)

consolidation due to changes in drainage patterns, and local instabilities during driving. Because of the small diameter of the hole the absolute volume lost tends to be small and, hence, the potential movement is usually limited compared to man-accessible tunnels.

Expansive techniques include percussive moling, pipe driving and on-line pipe replacement and may disturb adjacent buried pipes or cause ground heave at shallow depths. This problem is discussed by Howe and Hunter[30] and O'Rouke.[31]

40.7.4 Pipe jacking with slurry shields

These machines are essentially miniaturized versions of shields used for conventional tunnelling and employ a full-face cutting head within a shield (Figure 40.11). The cutter can be moved relative to the shield, allowing it to exert a constant pressure on the ground irrespective of the rate of advance of the pipe. The groundwater pressure is balanced by flooding the cutting face with water or a bentonite slurry. In many machines the slurry is used to transport the spoil back to the surface.

The ability to balance soil and water pressure gives good control of ground movement at the face. The alignment of the machine is monitored by using a laser beam focused on a target on the shield. The target is observed by an operator at ground level via closed-circuit TV and he can adjust the alignment by remote control using steering jacks within the shield.

The shield is jacked forward from the jacking pit leading a pipe string through which its power and slurry lines run. As new pipes are added to the pipe string the power and other lines must be broken and reconnected. One manufacturer avoids this by using a temporary lining behind the shield which has a recess to accommodate the lines. The temporary lining is replaced by the permanent pipe in a second-stage operation.

Slurry shield machines can operate in most soft ground conditions although the cutter type and the slurry separation systems may need adjustment for different materials which can cause practical difficulties in variable ground conditions. The size of cobble which can be excavated depends on the size and type of machine used. One manufacturer has machines which can crush stones measuring up to about one-third of the diameter of the shield. In ground containing large stones it will be necessary to install a larger machine which can cope with the expected conditions.

40.7.5 Pipe jacking with steerable borers

This category covers a wide range of machines. The steering may be by means of jacks on the shield or by rotating a special cam behind the articulated cutting collar. Alignment is monitored by visual surveying from the launch pit or by using a laser system observed by closed-circuit TV. Spoil is usually removed back to the launch pit by screw conveyor which also forms the drive shaft. Water or slurry may be used to cool the cutting head and assist in spoil transport. Other systems form a pilot hole which is then reamed out by passing spoil forward to the receiving pit.

Steerable borers can deal with a wide range of ground conditions by selecting an appropriate cutting head. Auger bits

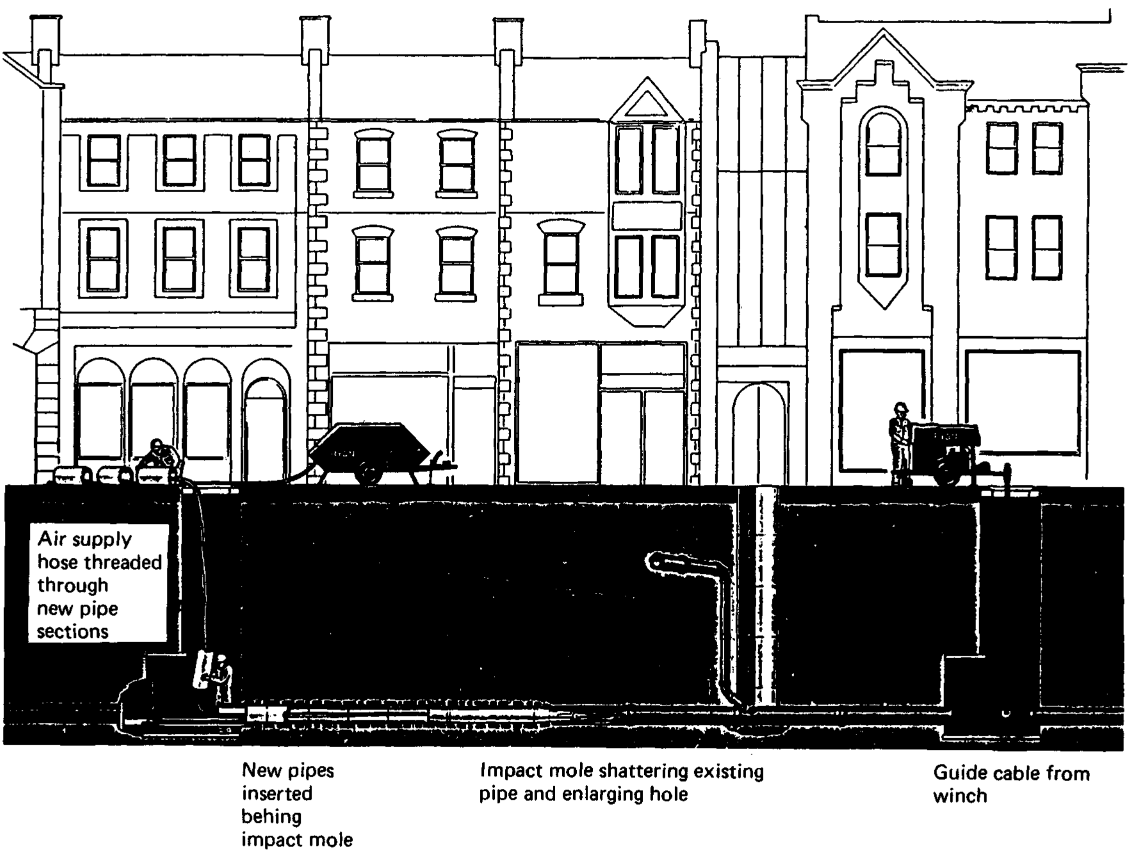


Figure 40.12 Impact mole used for upsizing an existing sewer

can be used in soft strata while rock roller bits can be used in harder materials. Various methods are employed for dealing with water pressure including short pitch augers or pressurizing the face with water or compressed air.

40.7.6 Pipe jacking with non-steerable augers

There are less complicated tools than those described above although they do incorporate similar features. The non-steering auger uses a simple cutting head rotated by an auger screw. The rotational power and thrust for the auger is provided by a drive unit at the launch pit. The cutting head is difficult to control accurately once boring has progressed a few metres and often alignment is not monitored between the launch and receiving pits during excavation. Large deviations may occur if the bore obliquely crosses hard strata. In long drives steady bearings have been installed in intermediate pits and reasonable accuracy achieved. A wide range of ground conditions can be excavated but problems may arise in water-bearing sands and gravels.

40.7.7 Pipe ramming

This technique, also known as pipe driving or pipe pushing, is analagous to pile driving. It normally utilizes a steel pipe which is driven into the ground from a launch pit using percussive loading at the trailing end. Small-diameter pipes are driven closed-ended but larger diameters are left open and cleaned out using augers or water jets.

Pipe ramming can be carried out in most types of ground and the percussive action tends to break up cobbles and other obstructions. There is little control of accuracy once the drive is under way and obstructions can cause deviations. The final accuracy depends on ground conditions and lengths of drive do not usually exceed 30 m.

40.7.8 Impact moling

This technique relies on displacing and compacting soil to form a void in the ground. The mole is usually powered by compressed air which causes a hammer piston to strike a chisel-headed anvil. This pierces the ground and the tool moves forward. The machines are generally reversible to aid recovery if refusal conditions are encountered.

Moles can be used to make pilot holes for subsequent enlargement or alternatively they can tow in pipes while forming the bore if polyethelene or plastic pipes are used (Figure 40.12).

Impact moles can operate in most materials and are able to break up isolated stones or boulders although these may cause some deviation in line. In very soft conditions the mole tends to sink under its own weight and follows a downward curving trajectory. In shallow bores the mole may curve upwards. Therefore to prevent ground heave, and to control accuracy, a minimum cover of 9 times the diameter should be used. Accuracy depends on ground conditions and is typically about 1%.

Impact moles are also used widely in replacing and up-sizing gas and sewer pipes (see below).

40.7.9 Directional drilling

This method employs techniques used in drilling oil- and gaswells, and is particularly suited to the construction of river crossings, etc. A surface-mounted rig is used to drill a pilot hole from one side of the river to the other using a down-the-hole motor at the end of drill rods. The motor is powered by bentonite slurry pumped through the drill rods. The returning slurry cools the drill bit and pushes the cuttings back to the surface. The curved profile is achieved by steering the drill rods using a special fitting mounted behind the motor.

The pilot hole is cased with a steel pipe called a washover pipe which is then used as a draw string for a reaming tool when the pilot hole is complete. The reaming tool is a circular cutter which is attached to the leading end of the washover pipe. The rig then rotates the pipe and pulls it back through the pilot hole (Figure 40.13). A further washover pipe is towed behind the reaming tool. Bentonite slurry is used together with the second washover pipe to stabilize the enlarged hole. The permanent pipe is then pulled into place using the second washover pipe. Further reaming and smoothing tools may be used in this operation to ensure a proper fit for the permanent pipe.

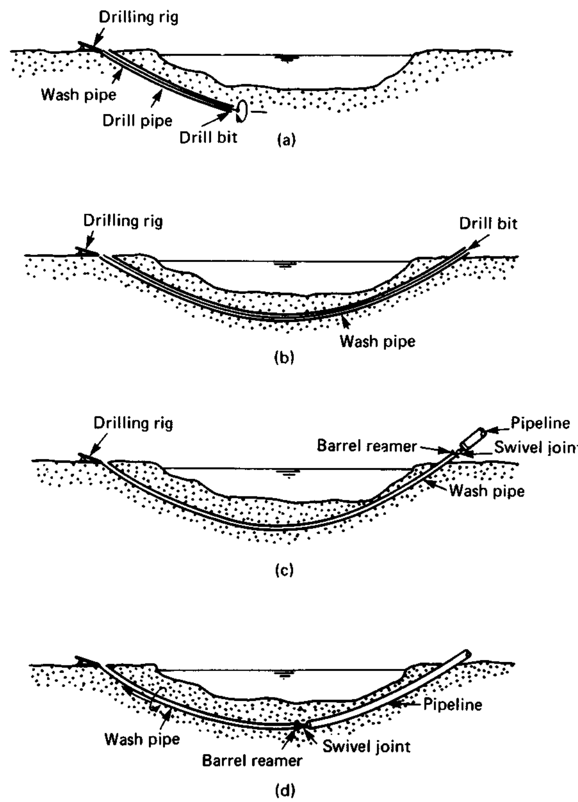


Figure 40.13 Directional drilling. (a) Stage 1: pilot hole drilled by advancing the drill string and overdrilling with the washover pipe in stages of approximately 80 m; (b) stage 2: pilot hole completed when both drill string and washover pipe exit on opposite bank; (c) stage 3: drill pipe is removed, barrel reamer connected to washover pipe which is in turn connected to the pipeline pulling head by a swivel joint; (d) stage 4: the barrel reamer is pulled back and rotated by the drill rig positioning the non-rotating pipeline into the formed hole

40.8 Man-accessible tunnels

Man-accessible tunnels are defined as having diameters greater than 900 mm which is the smallest in which a man can effectively work.

A wide range of excavation and lining methods are available depending on ground conditions and operational requirements.[32] In soft ground timbered headings, segmental linings and pipe jacking are the principal construction methods used.

Excavation is usually done by hand in timber heading. In segmental tunnels and pipe jacking hand excavation may be used but more usually a variety of mechanized methods are employed, including backhoe excavators within a shield, and full face tunnelling machines. The latter include bentonite slurry shields and mechanical earth pressure machines intended to minimize ground loss and movement at the excavation face.

Soft ground tunnels are invariably lined and the choice of lining depends on a number of factors including ground conditions, operational requirements and the construction method to be used. The relationship of these factors is complex and the final choice should be based on mutual agreement between the employer and the contractor.

In hard rock the traditional method of excavation is drill and blast which has the advantage of coping with a wide range of ground conditions. Machine excavation can be much quicker and hence cheaper, but it is more sensitive to changing ground conditions which may be outside its range.

Linings for bores in hard competent rock may be unnecessary for structural reasons but could be desirable to provide a smooth hydraulically efficient surface. In this case an *in situ* sprayed or poured concrete lining may be used. In less competent ground the rock may be stabilized with rock bolts prior to lining with *in situ* concrete or a bolted precast concrete lining may be used.

In many cases the primary lining supporting the excavation may be too large for efficient hydraulic design and the tunnel must be lined with a small carrier pipe or a channel invert. Timbered headings are invariably backfilled around the carrier pipe which must be designed for the loads arising.[3]

40.8.1 Site investigation

The main factor affecting construction cost of a tunnel is the nature of the ground. The site investigation will be similar in nature to that for microtunnelling but usually will be more extensive due to the larger works being constructed. When assessing suitable construction methods and the loads on the lining, more extensive *in situ* and laboratory testing will be required. Full descriptions must be given for the strata together with a geotechnical interpretive report since the ground structure has increasing significance as the tunnel size increases. When large tunnels are contemplated, consideration should be given to the possibility of constructing a pilot tunnel as a separate contract. This will yield valuable information for the design and construction of the larger tunnel.

A tunnel lining is interactive with the surrounding soils in a manner which depends on their relative stiffnesses. Measurements in existing tunnels show that frequently the lining does not carry the whole overburden load.

The fundamental design problem is predicting the behaviour of the ground and the way it is varied by the construction process. The current trend is towards designing flexible linings which will deflect to produce virtual equality of radial stress in the ground, usually by the horizontal axis extending as the vertical axis becomes shorter. The process of design must consider the construction method and the operational requirements of the lining, and then check that it is adequate for the ground loads expected. Handling stresses in the lining elements often give the worst design conditions particularly in tunnels for shallow depths.

A variety of methods of designing lining thickness are available.[32,33] Empirical techniques based on field observations allow the designer to estimate the likely ground loads and deformation of the lining when similar construction methods are used in similar ground conditions. Other design procedures are based on mathematical analysis and use closed form elastic solutions or numerical methods. These require the soil behaviour to be modelled in terms of its elastic, plastic and time-dependent characteristics to predict the changes in the *in situ* ground stresses. It is difficult to determine representative values for these factors and to assess the *in situ* state of stress of the ground. Hence, the methods are frequently used to model a range of possibilities to assess the sensitivity of each parameter and allow an engineering judgement to be made on likely stable or unstable configurations.

Pipes to be installed by jacking methods are designed as rigid structures as previously described for microtunnelling. Particular attention must be paid to the leading pipe and the trailing pipes at intermediate jacking stations since these are repeatedly subject to the full ram load during the jacking operation.

40.8.2 Headings

Headings are rectangular or square in section and usually dug by hand. The ground is supported by timber frames which are erected as the excavation progresses (Figure 40.14). The timber sizes are either estimated by rule of thumb or designed as temporary works with appropriate factors of safety.[19] The dimensions of the heading must allow sufficient working room for the construction of the carrier pipe and the compaction of the backfilling around it. The smallest practical size is about 1.15×0.7 m clear between supports.

The technique requires experienced and skilled operatives and is relatively expensive. As a result its use is restricted to special circumstances.

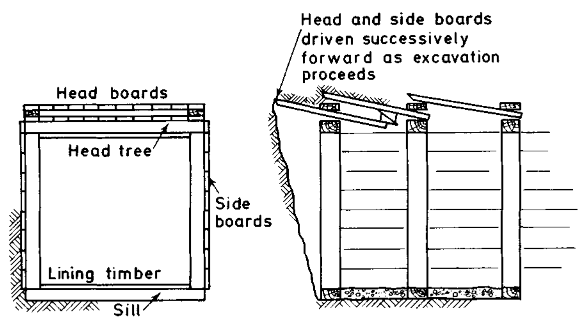


Figure 40.14 Timbered heading

40.8.3 Steel liner plates

Steel liner plates may be used for lining small-diameter tunnels in firm ground. Typically, four plates are used to construct the ring and the plates are bolted on the longitudinal and circumferential joints. The plates are back-grouted to ensure even bearing of the ground. Joints can be welded if necessary to prevent water ingress.

The finished lining behaves as a flexible pipe and is generally given a secondary lining.

40.8.4 Precast concrete segmental linings

These are available for either bolted or boltless construction. Bolted segments have flanges on all four sides and tend to be used in poor ground and where there are problems with water ingress (Figure 40.15). The segments are erected within the protection of a shield and are back-grouted every ring or every shift. The method is particularly useful in conditions requiring compressed air and for hydraulic tunnels with a moderate pressure.

Where a smooth internal finish is required it is necessary to use a secondary lining. To avoid this, smooth bore, bolted

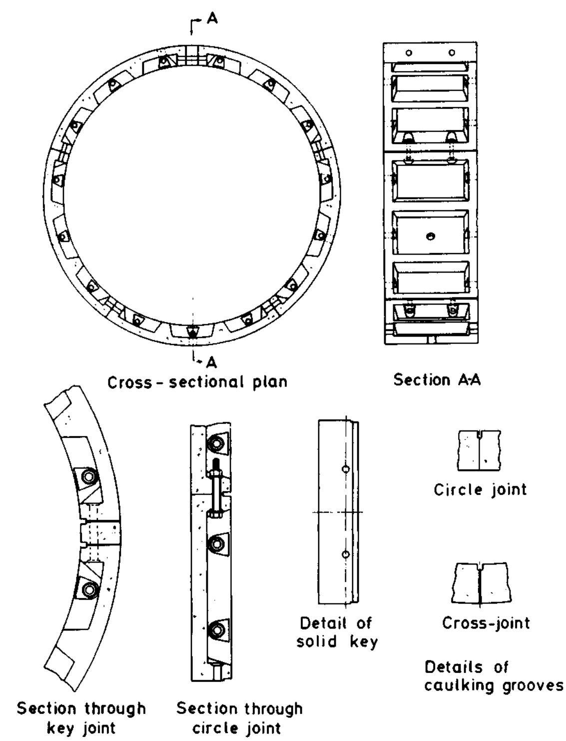


Figure 40.15 Bolted concrete segmental ring

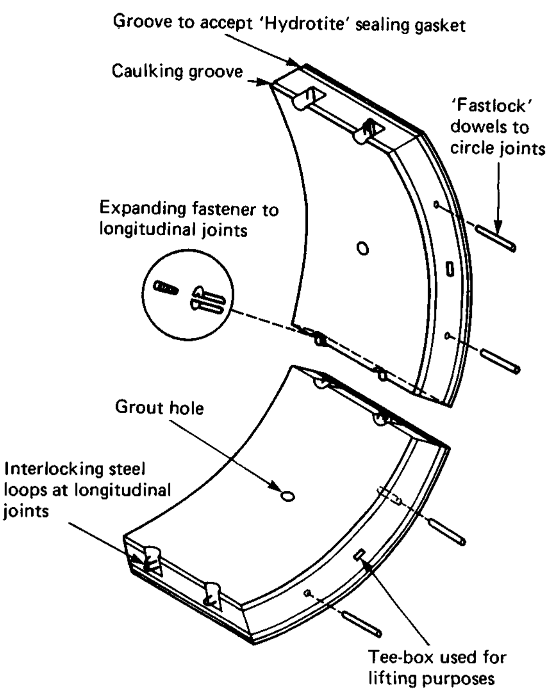


Figure 40.16 Bolted smooth bore concrete segmental ring. (*Courtesy*: Charcon Tunnels Ltd)

segments have been developed and their use is generally known as one pass construction (Figure 40.16).

Boltless segments also produce a smooth bore tunnel and are used in ground which has a reasonable stand-up time. The segments are erected on a temporary steel former ring using bolted connections. After the annulus behind the concrete ring is grouted the steel former is demounted and the process repeated (Figure 40.17).

Other smooth bore linings are of the expanded type intended for use in clay. The linings are erected on an erector arm or bars and expanded by using a wedge-shaped block or by cirumferential jacks which form spaces to be filled with dry pack or special blocks.

Grey cast iron has great strength and traditionally was used where heavy ground loads were expected. More recently it has been replaced by spheroidal cast iron which has a better tensile strength. Cast iron has been used for both bolted and expanded systems. The segments can be manufactured to a better tolerance than concrete and, hence, produce a better seal against water ingress in bad conditions.

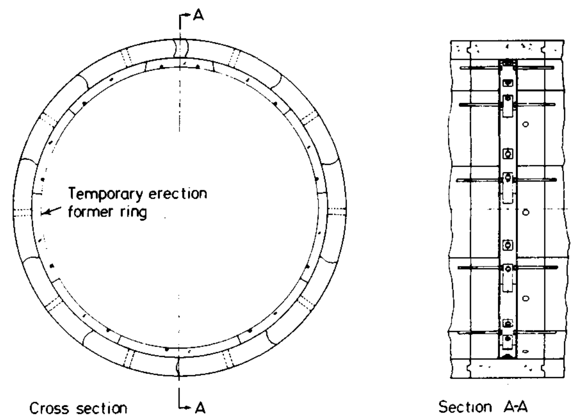


Figure 40.17 Boltless smooth bore concrete segmental ring. (*Courtesy*: Charcon Tunnels Ltd)

40.8.5 Triple segmental block (minitunnel system)

This proprietary system has been developed to provide smooth-bore tunnels in the range 1.0 to 1.3 m diameter. The lining consists of three segments which can be erected without the use of a former ring (Figure 40.18). Each segment has longitudinal V-shaped grooves inside and outside which function as stress raisers to ensure that the lining deforms and acts with forces in compression.

The segments are erected within the protection of a shield which is used to dig an oversize hole. The overbreak is filled with gravel injected through an orifice in the rear of the shield tail. The gravel packs around the completed segment ring and supports the ground. The main longitudinal and circumferential joints are sealed with a rubber bitumen strip compound during erection and the gravel annulus may be grouted up to provide additional waterproofing.

40.8.6 Pipe jacking

As with the microtunnelling technique, the pipes are forced into the ground using hydraulic jacks at a working shaft (Figure 40.19). A steel or concrete cutting edge is used at the leading end of the pipe string and miners working within the pipe excavate

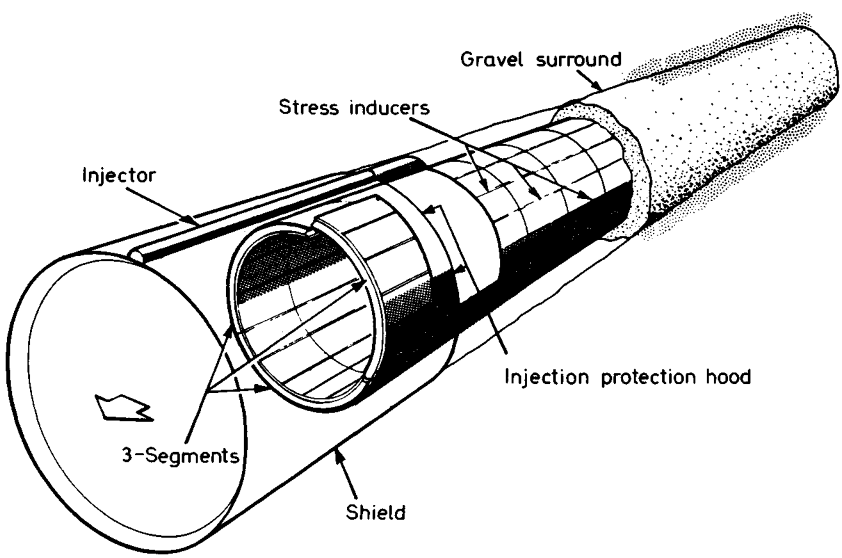


Figure 40.18 Mini tunnel system. (*Courtesy*: William F. Rees Ltd)

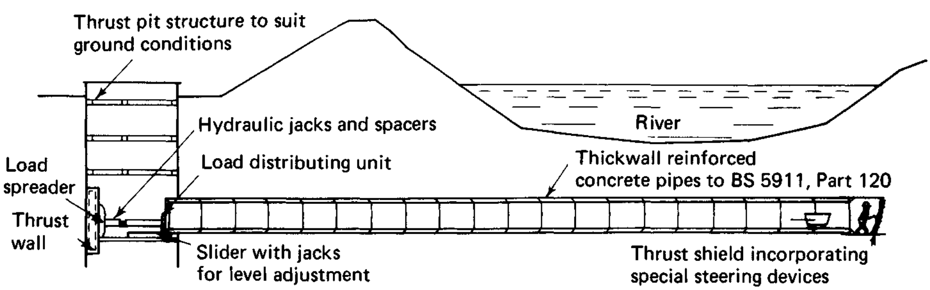


Figure 40.19 Section through a typical pipe jacking operation

material as in a normal tunnel shield operation. The accuracy of the drive can be improved by using trimming jacks or steering fins to adjust the attitude of the cutting shield. These also allow pipes to be jacked round bends.

The maximum length of drive can be increased by injecting bentonite or other lubricants to reduce the friction on the outside of the pipe. Also intermediate jacking stations can be built into the pipe string to increase overall jacking capacity and to restrict the length of pipe to be pushed by each set of jacks.

40.8.7 Ground movement

This is caused by ground loss during tunnel excavation and by additional consolidation as the porewater pressure is reduced due to drainage induced by the presence of the tunnel. Observations of various tunnels in the past have provided a basis for estimating the magnitude of potential ground movements caused by tunnelling.[15,34] Efficient management and good workmanship are critical factors in reducing movement to a practicable minimum.

In soft, loose and very permeable ground some pretreatment may be necessary to limit the ground loss and to improve the safety of the construction phase. Typical methods include dewatering, grouting and freezing.

The use of compressed air may be considered to overcome technical difficulties during excavation in soft clays or water-bearing ground. It acts in three ways, by: (1) balancing the water head; (2) providing a direct reaction to field forces; and (3) drying the skin of the soil, thus increasing its effective stress. However, it also has significant physiological risks for operatives and must be regarded as a last resort.

40.9 Working in confined spaces

A confined space may be defined as a workplace which does not have the benefit of natural ventilation. Consequently, the danger exists that the atmosphere in such places may become either deficient in oxygen due to the build up of gases or vapours, or hazardous due to concentrations of toxic or flammable gases or vapours.

The construction and maintenance of buried pipelines involves working in a variety of confined spaces. Examples are manholes, shafts, tunnels, pits, boreholes and, in some situations, trenches. Precautions must be taken in these cases to ensure that safe working conditions exist and the work being carried out does not give rise to hazards. Much specialized advice is available[24–27,35–37] and tunnels and boreholes have Bri-

Table 40.5 Summary of the most commonly encountered dangerous gases in confined spaces

Gas		*Specific gravity*	*Hazard*	*TLV** *(p.p.m.)*	*Explosive limits* *Lower* *(%)*	*Upper* *(%)*	*Principal sources*	*Prevention*	*In case of fire*
Carbon monoxide	CO	0.97	Toxic flammable explosive	50	—	—	Explosives/engines	Ventilation control	Carbon dioxide dry powder
Carbon dioxide	CO_2	1.53	Asphyxiant	5000	—	—	Natural/engines	Ventilation control	—
Nitrogen oxides	NO	1.04	Toxic	25	—	—	Explosives/engines	Ventilation control	—
	NO_2	1.60	Extremely toxic	5	—	—			
Nitrogen	N_2	0.80	Asphyxiant liquid causes burns	—	—	—	Freezing processes	Ventilation control	—
Methane	CH_4	0.60	Explosive and asphyxiant	—	5.3	14	Natural	Ventilation control	Carbon dioxide dry powder
Hydrogen sulphide	H_2S	1.70	Toxic and explosive	10	4.3	46	Natural	Ventilation control	Carbon dioxide dry powder
Sulphur dioxide	SO_2	2.30	Toxic	5	—	—	Natural	Ventilation control	—
Propane		1.55	Explosive and asphyxiant	1000	2.2	9.5	Leakages	Ventilation control Cylinder care	Carbon dioxide dry powder
Butane		2.10		600	1.5	8.5	Leakages		
Acetylene		0.91		—	2.5	81.0	Leakages		
Deoxygenated air	N_2		Asphyxiant	—	—	—	Natural/induced	Ventilation control	—
Petrol/diesel vapour			Explosive	—	1.3	7.5.	Spillage	Ventilation control	Foam carbon dioxide dry powder vaporizing liquid

*Threshold limit value.

tish Standard requirements.[38,39] The Construction Regulations and Factory Acts also give guidance.

Work in confined spaces should be undertaken only by persons properly trained for the job. Under the Health and Safety at Work Act 1974 the employer has a duty to provide such information, instruction, training and supervision as is necessary to ensure, so far as is reasonably practicable, the health and safety at work of his employees. The employer should also check that employees who are expected to work in confined spaces are physically and mentally suitable.

Safe systems of work must be laid down and strictly observed. The minimum requirements will be:

(1) To test the atmosphere before entering the confined space.
(2) To monitor the atmosphere while people are working.
(3) To maintain contact between the operatives and an attendant in free air. The attendant must be trained to carry out emergency procedures.

Adequate means of access and egress must be provided. Rectangular or oval holes of 458 × 407 mm and circular holes of 407 mm diameter are the minimum sizes laid down by the Factories Act.

Larger access should be provided where feasible so that rescue equipment can be used more easily. Permit to Enter and Permit to Work systems should be established.[24] These must be administered by a responsible person who will keep records of the safety measures taken and will sign a certificate accordingly.

Gas-testing of the atmosphere may be done using simple, rugged equipment. Potential hazards can often be anticipated by an intelligent assessment of local conditions. Works near waste tips are likely to encounter methane or hydrogen sulphide generated by the decomposition of organic material as well as other industrial compounds. Carbon dioxide, which is heavier than air, may be produced by the action of acidic water on limestone or other calcareous rock. It may also arise from the exhausts of nearby internal combustion engines and will be mixed with carbon monoxide. Table 40.5 summarizes the commonly encountered dangerous gases. Oxygen deficiency can arise due to rusting processes or a fall in atmospheric pressure causing deoxygenated gas to seep out of the surrounding ground.

In sewers additional hazards arise due to the danger of being swept away or drowned by fast-flowing streams. Dangerous conditions can arise quickly and may be noticed by increasing movement of air or water through the sewer or the noise of approaching water. Attention should be paid to weather forecasts as a preliminary precaution.

Sewers can also give rise to infection from a number of causes and all persons entering should be inoculated. Personal hygiene must be meticulous and any injury or abrasion should receive proper medical attention.

Confined spaces can rapidly become contaminated from the work processes being used. Often these processes are themselves intended to avoid other hazards. For instance, ground freezing and chemical grouting for excluding water, and pipe and cable freezing to isolate sections of pipe for maintenance or repair. Burning and thermal welding processes use up oxygen and give rise to toxic fumes as do many solvents and paints. Pipes which have contained flammable materials must be purged thoroughly before any work is carried out.

Dust and noise must also be regarded as dangerous contaminants in confined spaces. Lighting must be efficient and provide illumination to at least 20 lx using flameproof equipment and no more than 24 V. An emergency lighting system must be available for immediate use in the event of failure of the main system.

Rescue from confined spaces should not be attempted without proper equipment. The rescue team must be a minimum of two so that one person can attend at the entrance while the other enters. Non-observance of these requirements is likely to make the situation worse.

40.10 Rehabilitation of pipes and sewers

Trenchless methods for the rehabilitation of buried pipes offer the advantages of reduced disruption and cost. The gas, water supply and water disposal industries in particular have developed new methods of construction, particularly for non-man-entry pipes, and further innovation can be expected.

The methods are divided broadly into those incorporating the original pipe structure, e.g. relining, and those removing or destroying the existing structure, e.g. pipe bursting with percussive moles. The choice of method is heavily influenced by the size of new pipe required and the access available. In non-man-entry sizes work must be carried out either by remote methods or by open trench.

40.10.1 Inspection and basic strategy

Before repair and refurbishing operations can be undertaken the pipe must be inspected. Manual inspection is generally preferred where possible but in dangerous atmospheres and small-diameter pipes closed-circuit television is used routinely. It should be recognized that in such cases breathing apparatus or forced ventilation may be required when inserting and recovering the camera.

The results of the inspection should be recorded in a standard manner to reduce the subjective nature of assessing defects. In sewers the recommendations of the National Water Council[40] should be used.

After inspection, a repair strategy must be decided. In sewers,

Table 40.6 Definitions of remedial works in sewers

Rehabilitation:	All aspects of upgrading performance of existing sewers. Structural rehabilitation includes repair, renovation and renewal. Hydraulic rehabilitation includes replacement, reinforcement, flow reduction or attenuation and occasionally renovation
Maintenance:	Minor repairs not involving reconstruction of main sewer or alteration of dimensions
Repair:	Rectification of damage to the structural fabric of the sewer and reconstruction of short lengths but not reconstruction of the whole pipeline.
Renovation:	Improvement of performance of sewer by incorporating the original sewer fabric
Reinforcement:	Provision of an additional pipeline which, in conjunction with the existing sewer, increases overall flow capacity
Renewal:	Construction of a new sewer on or off the line of an existing sewer. The function and capacity of the new sewer is similar to the old
Replacement:	Construction of a new sewer on or off the line of an existing sewer. The function of the new sewer will incorporate that of the old but may also include improvement or development work.

Table 40.7 Pipe stabilization and relining methods

Technique	*Brief description*	*Original effective pipe diameter*	*Advantage*	*Disadvantage*
Cleaning				
(1) Scaling	High-pressure water jetting to remove scale and encrustation	All	Can be used in all diameters of pipe. In small diameter pipes the work can be done remotely and monitored by closed-circuit TV	High-pressure equipment is expensive and requires trained operators
(2) Removal of protrusions	In small-diameter pipes cutting is done by remotely controlled mechanical or high-pressure cutters operated under observation by closed-circuit TV	All		
Stabilization				
(3) Repointing	Hand- or pressure-pointing used to renew joints in masonry or brick sewers where these have not closed up	0.9 m upwards	Minimal disruption to existing service. Materials and equipment inexpensive	Man-access necessary and existing sewer must be structurally sound. Infiltration must be controlled and flow must be diverted when working in the invert. Quality control difficult due to poor working conditions
(4) Cement grouting	Cement grout injected to seal leaks, fill voids and strengthen ground	All	Materials and equipment inexpensive. PFA and other additives can be added to grout to improve properties. Can be injected from within pipe or from ground surface	May not be suitable where infiltration is actually flowing
(5) Chemical grouting	Low-viscosity chemicals injected into ground generally to seal leaks. Some increase in soil strength may be achieved	All	Can be used for man-entry and non-man-entry pipes. Fast gel time allows method to be used where infiltration is occurring. Low flows tolerated within pipe. Remote method for small pipes allows joints to be air tested as work proceeds	Expensive particularly where large voids exist. High flows must be diverted. Restricted number of specialists for non-man-entry system. Difficult to seal packers where pipe surface is irregular
Pipe lining				
(6) Pipelining generally	New pipes are inserted in existing pipes		Can improve structural stability, hydraulic capacity and chemical resistance. Structural design will require annulus between old and new pipes to be grouted up. Existing pipe may then form part of the structure	Reduces cross-sectional area but may improve hydraulic performance to compensate. Problems may arise where laterals occur
(7) Sliplining	After cleaning and proving the diameter of the existing pipe the new pipe (usually polyolefin) is jointed at ground level using butt fusion welding and then towed or pushed into the existing pipe via a lead in trench	0.1–0.9 m	Quick and large-diameter bends may be accommodated	Lateral connections must be re-made quickly. This could take a lot of time or considerable excavation in non-man-access pipes and services could be disrupted. Pipe may tend to float during grouting. Lead in trench disruptive. Circular cross-section only available

(8) Sectional lining pipes	Lining pipes are jointed within a working pit and then jacked or towed into the existing pipe	All	Various materials may be used (e.g. μPVC, reinforced plastic mortar, GRP, GRC). Quick large-diameter bends may be accommodated. Joints can be screwed, socket and spigot or welded depending on material. Cost effective for short deep lengths. May be possible to vary shape of cross-section with some materials	Generally needs working pit and large number of joints. Relatively high loss of cross-sectional area. Pipe may tend to float during grouting. Lateral connections may cause a problem in non-man-accessible diameters
(9) Inversion: polyester lining cured in place	After cleaning the existing pipe a special polyester felt liner impregnated with polyester thermosetting resin is inserted into the pipe through an existing manhole. The liner is initially inside out and unrolled into the pipe by flooding with water. The liner is specially tailored to suit the diameter and length of the pipe. The resin is then cured by circulating hot water within the lined pipe	0.1–1.2 m	Rapid installation using existing manholes. Bends and minor deformations in the existing pipe can be accommodated. Thickness is approximately 3 mm minimum but can be increased to 19 mm. Smooth finish improves flow characteristics. Resins can be specially formulated to give chemical-resistant finish	Full over-pumping required during installation. Site set-up cost is relatively high on small jobs. Monopoly supplier
(10) Segmental relining	These methods are frequently used for non-circular man-accessible pipes			
(a) GRC segments	Segments are usually produced in two sections with lap joints. Connections are made using bolts, pop rivets, self-tapping screws, etc. The invert is laid first and bedded on mortar or wooden blocks. The crown is then installed and centralized with wedges. The annulus is grouted up and the laterals reconnected as work proceeds	0.6 m upwards	Variety of cross-sections available. Material easily cut to form connections	Jointing is labour-intensive. Strutting often required during grouting operation
(b) GRP segments	Segments can be either one piece with a single longitudinal joint or two pieces with lap joints	0.6 m upwards	Deforms easily to suit cross-section. High strength : weight ratio	Jointing is labour-intensive. Support usually required during grouting. Care must be taken not to overstrain or otherwise damage the barrier layer of resin
(c) Preshot gunite segments	Segments are installed by laying invert and then jacking crown into position. Reinforcement is tied and *in situ* gunite used to make joints. Annulus then pressure grouted	See note	Variety of cross-sections available. No strutting required. Segments can support earth load	Segments are heavy. Jointing is labour-intensive. Not suitable for pipe clearances less than 1070 × 760 mm
(d) Precast resin concrete segments	Similar to (c) but lap jointed using epoxy-modified mortar seals	See note	Similar to (c)	Similar to (c). Not suitable for pipe clearances less than 900 × 600 mm

Table 40.7 Pipe stabilization and relining methods (*continued*)

In situ *coatings*				
	All methods need thorough cleaning of the pipe and dewatering of the pipe to prevent infiltration			
(11) *In situ* gunite	Standard gunite process used in man-accessible pipes to rebuild inside of pipe. Small-diameter rod or mesh required for structural applications	0.9 m upwards	Cheap material and simple process produces a jointless lining. Connections and changing cross-section easily accommodated. Access through existing manholes	Dusty and difficult to supervise. Very dependent on operator skill. Must remove rebound material from invert. Overpumping required
(12) Centrifugal cement mortar lining	The wet mortar mix is pumped through a revolving mortar dispenser on to the walls of the pipe. The lining machine is towed through the pipe at a specified rate followed by rotary or drag trowels which give a smooth finish	All	Existing manholes can be used for access. Suitable for a wide range of pipe diameters. Cheap material	Non-structural. Laterals difficult to deal with. Flow must be diverted
(13) Polyurethane or cold-cured resin	Similar to (12) and currently undergoing development			

the pipeline is likely to form only one part of a larger reticulation and changes in its flow characteristics may have adverse effects elsewhere. The *Sewerage rehabilitation manual*[41] sets out the requirements of a proper management strategy for sewer rehabilitation and gives definitions of the various terms to be used when describing the works (Table 40.6).

40.10.2 Methods of rehabilitation

Some methods of rehabilitation are based on microtunnelling techniques while others are based on more traditional procedures (Table 40.7).

The practical requirements common to gas, water and sewer pipes are: (1) the need to cope with existing flows; (2) the need to work through existing manholes wherever possible; and (3) the ability to cope with lateral connections.

When relining gas and sewer pipes it is sometimes possible to use the annulus between the old and new pipes to maintain existing service connections. The annulus is grouted up once the service connections have been remade and the new pipe is on stream. In other cases bypasses and overpumping must be used with the possibility of interrupted services for the consumer.

Lateral connections present a considerable problem in sewers. They must be located accurately prior to any work being carried out and all live connections identified. The position and angle of entry must be recorded with great accuracy. If the connection protrudes into the sewer then it may be necessary to cut the projection back even before the inspection phase can be completed.

Various remote methods of reconnection have been invented,[42,43] but local excavation is often the only practical solution. On occasions, relined sewers can be reconnected using a 'down the drain' remote-controlled cutter observed by closed-circuit TV within the new sewer or using a cutter working within the new sewer. Accurate location of the cutting tool is required to ensure coincidence within the existing branch and several specialized devices are available to achieve this. Once the new pipe has been holed through, special sleeves or grouting techniques are used to make a watertight joint usually working from within the pipe.

In cases where excavation is unavoidable for reconnection special 'keyhole' techniques have been evolved which require a minimum of spoil to be removed so that special pipe-fitting tools can be used from ground level.

40.11 Costs

The total cost of constructing any buried pipeline is made up of the costs incurred by its promoter and the social costs borne by the community at large. The social costs can arise in a variety of ways. Examples are traffic delays, additional wear on unsuitable diversion roads, business losses, etc. which are not direct charges to the promoter. There is increasing concern in some critical urban situations that the social cost may be greater than the initial construction cost of the pipeline, particularly where trenched construction is used.[44] Thus, concern has given added impetus to the development of trenchless techniques which may offer significant benefits in congested areas by reducing disruption.

40.11.1 Project cost appraisal

The prudent promoter of a new pipeline will consider its total lifetime costs which simply stated are:

$$\text{Initial construction cost} + \text{Running cost} + \text{Maintenance cost} + \text{Replacement cost}$$

It is likely that several different routes and types of construction appear feasible and the promoter will undertake cost/benefit studies to ascertain which combination offers the best advantage within given financial limits. The studies will require a number of estimates to be made of future events. Consequently, the results are unlikely to be accurate in absolute terms but will offer a relative assessment of the choices available. A further factor to be considered in cost/benefit analysis is the possibility of maximizing benefit for a small increase in construction cost. An example of this is the enlargement of ducts and tunnels to permit them to be used by two or more separate services.

The degree to which social costs are considered in such studies will depend on legal requirements and the nature of the promoter. The law gives some third parties entitlement to recompense for financial damage suffered as a result of the works. In other cases, for instance, where diverted traffic uses more fuel, the promoter is unlikely to have to foot the bill and will not consider such costs in the feasibility study unless political requirements or public policy dictate otherwise. Table 40.8

Table 40.8 Typical direct and indirect costs associated with sewer construction in trench

	Direct costs	*Indirect costs*
Engineering costs		
Planning and design	Yes	No
Site investigations	Yes	No
Construction	Yes	No
Supervision	Yes	No
Service diversions	Yes	No
Reinstatement and maintenance	Initial reinstatement	Increased long-term maintenance (damage due to trenching)
Social costs		
Other service replacements	For example, gas mains, water mains, etc.	No
Traffic diversions and delays	Traffic lights, signs	Cost to road users of delay and extra fuel, additional policing, wear and tear on alternative routes
Loss of business	Compensation	Any costs not met by compensation
Environmental	Include in permanent works	Yes
Dislocation of supply	Temporary works	Consumer inconvenience

indicates typical direct and indirect costs which may arise in the construction of a sewer in trench.

40.11.2 Construction costs

The estimation of costs for civil engineering construction is a skilled procedure. If prices are required to be within a reasonable order of accuracy then it is necessary to build them up from first principles. The variation of conditions for design and construction will tend to make the use of historical overall unit costs unreliable except on the simplest jobs.

Ground conditions have a major influence on both the design and the risks arising during construction and in any type of construction the cost of a proper site investigation should be regarded as money well spent. The report should consider the problems likely to arise in both the permanent and temporary conditions and the report should be made available to the tenderers.

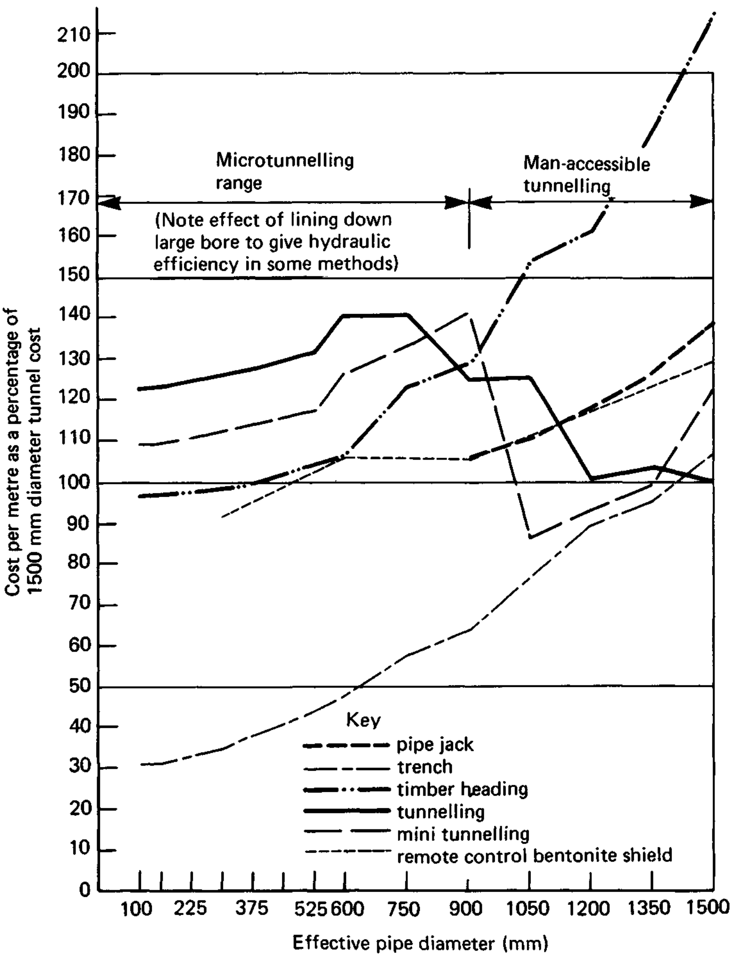
The promoter must decide at an early stage what the contractual basis will be for the design and the construction of the works. The contracts with the designer and the constructor must place the obligations and responsibilities for the risks in an equitable manner. The promoter must recognize that by bearing some part of the potential risks himself, e.g. unexpected ground conditions, then the prices tendered are likely to be more realistic.

Decisions made during the design phase may result in some methods of construction being precluded and the effect of this must be assessed. Also different construction methods impress their own individual characteristics on the serviceability and life of the completed works which must be recognized by the designer.

The total construction cost of a pipeline is made up of:

(1) Labour cost.
(2) Plant cost.
(3) Materials cost.
(4) Constructor's on-costs including temporary works.
(5) Constructor's profit.
(6) Design cost.
(7) Promoter's supervision cost.

In most cases, the cost of the pipe and other permanent materials is not affected significantly by the depth of construc-



Note: Based on 600-m long sewer including manholes

Figure 40.20 Comparison of direct engineering costs of various tunnelling methods in good ground tion. For a given diameter of pipe constructed in trench the cost of labour and plant increases as a power function of the depth. The cost of pipelines laid by non-disruptive methods is largely unaffected by depth.

The comparison of costs for different types of construction must be made on the basis of the same ground and site conditions. Where conditions change, then the relativities and ranking of the various methods are also likely to change. Some construction methods can cope readily with unexpected ground conditions, whereas others may need to be supplemented with expensive secondary operations such as extensive grouting or ground freezing. The potential cost of such risks must be assessed at the feasibility stage of a project.

The total job size will also have an influence on the unit costs of construction especially when using methods with a high initial set-up cost. Figure 40.20 illustrates the relative costs for installing 600-m long sewers by different methods in good ground.

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