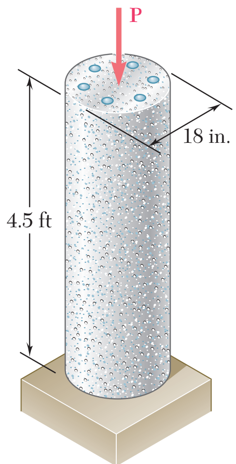


PROBLEM 2.35

The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete.

P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(1.125\text{ in.})^2 = 5.9641\text{ in}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(18\text{ in.})^2 - 5.9641\text{ in}^2$$

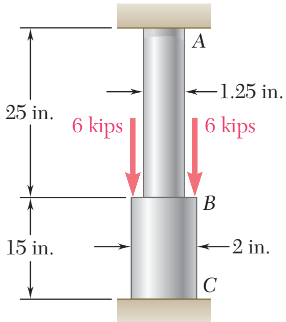
$$= 248.51\text{ in}^2$$

$$L = 4.5\text{ ft} = 54\text{ in.}$$

$$\varepsilon = \frac{-350 \times 10^3\text{ lb}}{(4.2 \times 10^6\text{ psi})(248.51\text{ in}^2) + (29 \times 10^6\text{ psi})(5.9641\text{ in}^2)} = -2.8767 \times 10^{-4}$$

$$\sigma_s = E_s\varepsilon = (29 \times 10^6\text{ psi})(-2.8767 \times 10^{-4}) = -8.3424 \times 10^3\text{ psi}$$ $\sigma_s = -8.34$ ksi ◀

$$\sigma_c = E_c\varepsilon = (4.2 \times 10^6\text{ psi})(-2.8767 \times 10^{-4}) = 1.20821 \times 10^3\text{ psi}$$ $\sigma_c = -1.208$ ksi ◀

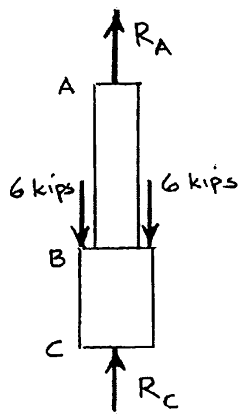


PROBLEM 2.39

A polystyrene rod consisting of two cylindrical portions AB and BC is restrained at both ends and supports two 6-kip loads as shown. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the reactions at A and C, (*b*) the normal stress in each portion of the rod.

SOLUTION

(*a*) We express that the elongation of the rod is zero.



$$\delta = \frac{P_{AB}L_{AB}}{\frac{\pi}{4}d_{AB}^2 E} + \frac{P_{BC}L_{BC}}{\frac{\pi}{4}d_{BC}^2 E} = 0$$

But $\quad P_{AB} = +R_A \qquad P_{BC} = -R_C$

Substituting and simplifying,

$$\frac{R_A L_{AB}}{d_{AB}^2} - \frac{R_C L_{BC}}{d_{BC}^2} = 0$$

$$R_C = \frac{L_{AB}}{L_{BC}}\left(\frac{d_{BC}}{d_{AB}}\right)^2 R_A = \frac{25}{15}\left(\frac{2}{1.25}\right)^2 R_A$$

$$R_C = 4.2667 R_A \qquad (1)$$

From the free body diagram, $\quad R_A + R_C = 12$ kips $\qquad (2)$

Substituting (1) into (2), $\quad 5.2667 R_A = 12$

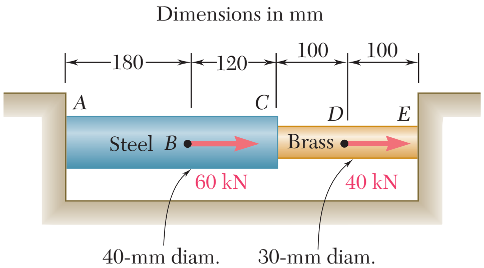
$$R_A = 2.2785 \text{ kips} \qquad R_A = 2.28 \text{ kips} \uparrow \blacktriangleleft$$

From (1), $\quad R_C = 4.2667(2.2785) = 9.7217$ kips

$$R_C = 9.72 \text{ kips} \uparrow \blacktriangleleft$$

(*b*) $$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{+R_A}{A_{AB}} = \frac{2.2785}{\frac{\pi}{4}(1.25)^2} \qquad \sigma_{AB} = +1.857 \text{ ksi} \blacktriangleleft$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{-R_C}{A_{BC}} = \frac{-9.7217}{\frac{\pi}{4}(2)^2} \qquad \sigma_{BC} = -3.09 \text{ ksi} \blacktriangleleft$$



PROBLEM 2.41

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (a) the reactions at A and E, (b) the deflection of point C.

SOLUTION

<u>A to C</u>: $E = 200\times 10^9$ Pa

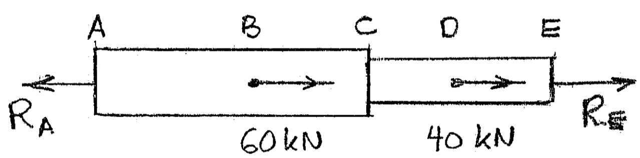
$$A = \frac{\pi}{4}(40)^2 = 1.25664\times 10^3\ \text{mm}^2 = 1.25664\times 10^{-3}\ \text{m}^2$$

$$EA = 251.327\times 10^6\ \text{N}$$

<u>C to E</u>: $E = 105\times 10^9$ Pa

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times 10^{-6}\ \text{m}^2$$

$$EA = 74.220\times 10^6\ \text{N}$$



<u>A to B</u>: $P = R_A$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327\times 10^6}$$

$$= 716.20\times 10^{-12} R_A$$

<u>B to C</u>: $P = R_A - 60\times 10^3$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times 10^3)(0.120)}{251.327\times 10^6}$$

$$= 447.47\times 10^{-12} R_A - 26.848\times 10^{-6}$$

PROBLEM 2.41 *(Continued)*

<u>C to D</u>: $P = R_A - 60\times 10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times 10^3)(0.100)}{74.220\times 10^6}$$

$$= 1.34735\times 10^{-9} R_A - 80.841\times 10^{-6}$$

<u>D to E</u>: $P = R_A - 100\times 10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times 10^3)(0.100)}{74.220\times 10^6}$$

$$= 1.34735\times 10^{-9} R_A - 134.735\times 10^{-6}$$

<u>A to E</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times 10^{-9} R_A - 242.424\times 10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(*a*) $3.85837\times 10^{-9} R_A - 242.424\times 10^{-6} = 0 \quad R_A = 62.831\times 10^3 \text{ N}$ $R_A = 62.8 \text{ kN} \leftarrow$ ◀

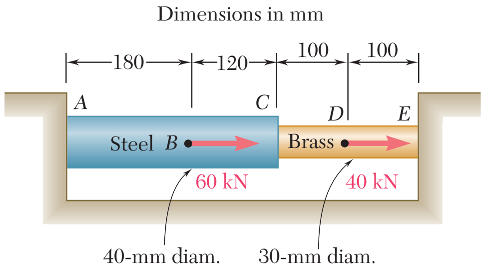
$R_E = R_A - 100\times 10^3 = 62.8\times 10^3 - 100\times 10^3 = -37.2\times 10^3 \text{ N}$ $R_E = 37.2 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times 10^{-9} R_A - 26.848\times 10^{-6}$

$$= (1.16369\times 10^{-9})(62.831\times 10^3) - 26.848\times 10^{-6}$$

$$= 46.3\times 10^{-6} \text{ m}$$

$\delta_C = 46.3 \ \mu\text{m} \rightarrow$ ◀



PROBLEM 2.42

Solve Prob. 2.41, assuming that rod *AC* is made of brass and rod *CE* is made of steel.

PROBLEM 2.41 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.

SOLUTION

<u>*A* to *C*</u>: $E = 105 \times 10^9 \text{ Pa}$

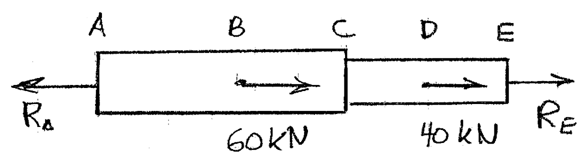
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$EA = 131.947 \times 10^6 \text{ N}$$

<u>*C* to *E*</u>: $E = 200 \times 10^9 \text{ Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$EA = 141.372 \times 10^6 \text{ N}$$



<u>*A* to *B*</u>: $P = R_A$

$$L = 180 \text{ mm} = 0.180 \text{ m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947 \times 10^6} = 1.36418 \times 10^{-9} R_A$$

<u>*B* to *C*</u>: $P = R_A - 60 \times 10^3$

$$L = 120 \text{ mm} = 0.120 \text{ m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{131.947 \times 10^6} = 909.456 \times 10^{-12} R_A - 54.567 \times 10^{-6}$$

<u>*C* to *D*</u>: $P = R_A - 60 \times 10^3$

$$L = 100 \text{ mm} = 0.100 \text{ m}$$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 42.441 \times 10^{-6}$$

PROBLEM 2.42 *(Continued)*

<u>*D* to *E*</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{141.372\times10^6}$$

$$= 707.354\times10^{-12} R_A - 70.735\times10^{-6}$$

<u>*A* to *E*</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.68834\times10^{-9} R_A - 167.743\times10^{-6}$$

Since point *E* cannot move relative to *A*, $\delta_{AE} = 0$

(*a*) $3.68834\times10^{-9} R_A - 167.743\times10^{-6} = 0 \quad R_A = 45.479\times10^3 \text{ N}$ $R_A = 45.5 \text{ kN} \leftarrow$ ◀

$R_E = R_A - 100\times10^3 = 45.479\times10^3 - 100\times10^3 = -54.521\times10^3$ $R_E = 54.5 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364\times10^{-9} R_A - 54.567\times10^{-6}$

$$= (2.27364\times10^{-9})(45.479\times10^3) - 54.567\times10^{-6}$$

$$= 48.8\times10^{-6} \text{ m}$$

$\delta_C = 48.8\,\mu\text{m} \rightarrow$ ◀