THE UNVIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

ENGINEERING MATHEMATICS

EM212

By Kenneth Mukosha

The University of Zambia

Department of Mathematics & Statistics

Engineering Mathematics II - EM212

Quiz 1 **Duration 50mins** **February 2012**

1. Let C be the curve given by $\underline{r}(t) = (\cos^2 t, \sin t, \sin^2 t)$, $\frac{-\pi}{2} \leq t \leq \frac{\pi}{2}$. Find two surfaces on which C lies and use them to sketch C.

2. Let C be the curve of intersection of the surfaces $z = x^2 + y^2$ and $2y - z = 0$.
   (a) Sketch the two surfaces on the same set of axes.
   (b) Find the cartesian equation of a cylinder on which C lies.

3. Find the limit, if it exists, or show that the limit does not exist:-

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2 + y^2}$ b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

4. Let $g(x, y) = y - \ln x$. Draw the family of level curves for the function $g(x, y)$ (labelling a few level curves).

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Masiye David Mbindawina

D.O.D

Never Back Down.

The University of Zambia

Department of Mathematics & Statistics

Engineering Mathematics II - EM212

Tutorial Sheet 1

February 2012

1. In each of the following cases sketch the curve given by $\mathbf{r}(t)$:

   (a) $\mathbf{r}(t) = (\cos t, \sin t), \quad 0 \leq t \leq \frac{\pi}{2}$

   (b) $\mathbf{r}(t) = (\cos 2t, \sin 2t), \quad 0 \leq t \leq \frac{\pi}{4}$

   (c) $\mathbf{r}(t) = (t, \sqrt{1-t^2}), \quad 0 \leq t \leq 1$

   (d) $\mathbf{r}(t) = (\ln t, \sqrt{1-(\ln t)^2}), \quad 1 \leq t \leq e$.

   What did you observe? Could this curve also have been given parametrically by $\mathbf{r}(t) = (e^t, \sqrt{1-e^{2t}})$? Explain.

2. Let $\mathbf{r}(t) = (2\sin t, 2\cos t, 5t), \; t \geq 0$.

   (i) Sketch the curve given by $\mathbf{r}(t)$.

   (ii) Does the curve intersect the cylinder $x^2 + y^2 = 4$? If so, at which points?

   (iii) Does this curve intersect the upper half cone $z = \sqrt{x^2+y^2}$? If so, at which points?

3. In each of the following, find the domain and range of the function:-

   (a) $z = f(x,y) = \frac{\sqrt{x^2+y^2-9}}{x-y}$

   (b) $z = f(x,y) = 3/\sqrt{x^2-y}$

   (c) $z = f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$

   (d) $z = f(x,y) = \ln(1+x-y^2)$

   (e) $z = f(x,y) = \sqrt{\frac{x-y}{x+y}}$

   Sketch the domain and/or range where possible.

4. Let

$$f(x,y) = \begin{cases} x^2 & \text{if } x \neq y, \\ y^2 & \text{if } x = y. \end{cases}$$

   Find $f_x(2,2)$ and $f_y(2,2)$.

5. Let

$$g(x,y) = \begin{cases} 4 & \text{if } x = 0 \text{ or } y = 0, \\ x^2 + y^2 & \text{otheriwse.} \end{cases}$$

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Mathematics & Statistics Dept.
EM 212 Tutorial sheet

1. Solve the following equations

   (i) $(D^2 - 4)y = 4\sinh^2 x$

   (ii) $(D^2 + 2D + 5)y = 34\cos 2x$

   (iii) $(D^2 + 4D)y = e^x + \sin x$

   (iv) $(D^2 + 1)y = 3x^4$

   (v) $(2D + 1)y = xe^{-x}$

   (vi) $(D_2 + 2D + 1)y = x\sin x$

2. Find the general solution for the given equations.

   (i) $y'' + y = \cos 2x$

   (ii) $y'' + 4y' + 5y = e^{-x}$

   (iii) $y''' + y = x + \sin x + \cos x$

   (iv) $y'' - 2y' - 3y = 2e^{2x} + 3\sin x$

3. Find the general solution of the following equations by variation of parameters

   (i) $y'' + y = x^3$

   (ii) $y'' + y' = 2\sin x$, $y(0) = 0, y'(0) = -3$

   (iii) $y''' - y'' = 4x^2$

   (iv) $y'' - 9y = 5e^{3x}$

   (v) $y'' - y' = \sin^2 x$

4. Use power series to solve the following equations and find the interval of convergence.

   (i) $y' + 3xy = 0$

   (ii) $y'' - xy = 0$

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EM 212 Tutorial sheet

1. Show that the following are linear Transformation

   (a) $T:\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 3x \\ -2y \end{pmatrix}$ (b) $T:\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x+y+z \\ x+y+z \\ x+y+z \end{pmatrix}$

2. Find T^{-1} where possible in the following

   (a) $T^{-1}:\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x+y \\ x+y+z \\ 2y+z \end{pmatrix}$ (b) $T:\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x+2y \\ y+3x \end{pmatrix}$

3. Given that $T:\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2x+z \\ y \\ -y+z \end{pmatrix}$ and $U:\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -x+2y-3z \\ 2x-y+4z \\ 3x+4y+z \end{pmatrix}$

   Find (a) T^{-1} , (b) U^{-1} (c) $(TU)^{-1}$, (d) $(UT)^{-1}$

4. Find the eigenvalues of matrix A where

   $$A = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 0 & -2 \\ 0 & -2 & 1 \end{pmatrix}$$

   Given that the matrix P is such $P^{-1}AP$ is a diagonal matrix B, write down a possible form of B.

5. Find an orthogonal matrix P such that $P^{T}AP$ diagonalises the symmetric matrix A, where A is ;

   (a) $\begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & 7 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -2 & -4 \\ -2 & -2 & -6 \\ -4 & -6 & -1 \end{pmatrix}$

6. Find the eigenvalues and the corresponding eigenvectors of the following matrices;

   (a) $\begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$

$y = Ax^2 - Bx + C.$

$y' = 2Ax - B$

$y'' = 2A$

Daniel Masuse

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Mathematics & Statistics Dept.

EM 212 Quiz 2

Time Allowed: 1hour 30min.

$2A + 8Ax - 4B + 3Ax^2 - 9Bx + 3C = x^2 - 3x$

$8Ax - 9Bx = -3 \quad 2A + 3C - 4B = 0, \quad 3A = 1 \quad A = \frac{1}{3}$

$\frac{8}{3} - 9B = -3$

$-9B = -3 - \frac{8}{3}$

$-\frac{17}{3} = -9B$

$-\frac{17}{27}$

1. Solve the following equations:

   (i) $(x \ln x)y' + y = 6x^3$

   (ii) $2xydx + (y^2 - 3x^2)dy = 0$

   (iii) $xdy + ydx = xy^2dx$

2. Show that $y_1(x) = e^x$ and $y_2(x) = e^{2x}$ are linearly independent solutions of $y'' - 3y' + 2y = 0$

3. Solve the equations

   (i) $D^2(D-3)y = 2 + x$

   (ii) $y'' - 3y' + 2y = 4e^{3x}$

   (iii) $(D^2 - 2D + 2)y = x + e^x$

4. Find the particular solution of

   (i) $y'' - 3y' + 2y = \sin 2x$

   (ii) $(D+1)(D+3)y = x^2 - 3x$

END of Quiz

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Department of Mathematics & Statistics

Engineering Mathematics II - EM212

Tutorial Sheet 1

February 2012

1. In each of the following cases sketch the curve given by $\mathbf{r}(t)$:
   (a) $\mathbf{r}(t) = (\cos t, \sin t), \quad 0 \le t \le \frac{\pi}{2}$
   (b) $\mathbf{r}(t) = (\cos 2t, \sin 2t), \quad 0 \le t \le \frac{\pi}{4}$
   (c) $\mathbf{r}(t) = (t, \sqrt{1-t^2}), \quad 0 \le t \le 1$
   (d) $\mathbf{r}(t) = (\ln t, \sqrt{1-(\ln t)^2}), \quad 1 \le t \le e$.

   What did you observe? Could this curve also have been given parametrically by $\mathbf{r}(t) = (e^t, \sqrt{1-e^{2t}})$? Explain.

2. Let $\mathbf{r}(t) = (2\sin t, 2\cos t, 5t), \ t \ge 0$.
   (i) Sketch the curve given by $\mathbf{r}(t)$.
   (ii) Does the curve intersect the cylinder $x^2+y^2=4$? If so, at which points?
   (iii) Does this curve intersect the upper half cone $z = \sqrt{x^2+y^2}$? If so, at which points?

3. In each of the following, find the domain and range of the function:-
   (a) $z = f(x,y) = \frac{\sqrt{x^2+y^2-9}}{x-y}$
   (b) $z = f(x,y) = 3/\sqrt{x^2-y}$
   (c) $z = f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$
   (d) $z = f(x,y) = \ln(1+x-y^2)$?
   (e) $z = f(x,y) = \sqrt{\frac{x-y}{x+y}}$

   Sketch the domain and/or range where possible.

4. Let

$$f(x,y) = \begin{cases} x^2 & \text{if } x \ne y, \\ y^2 & \text{if } x = y. \end{cases}$$

   Find $f_x(2,2)$ and $f_y(2,2)$.

5. Let

$$g(x,y) = \begin{cases} 4 & \text{if } x=0 \text{ or } y=0, \\ x^2+y^2 & \text{otheriwse.} \end{cases}$$

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INGRID

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

Second Semester Examinations - May 2012

EM212 - Engineering Mathematics II

Time allowed : Three (3) hours Full marks : 100

Instructions:
- Attempt **any (5) five** questions. All questions carry equal marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.
- Calculators are **not** allowed.

This paper consists of 3 pages of questions.

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1. a) Given the function

$$z = (x + 5y\sin x)^{\frac{4}{3}},$$

find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

b) Let $z = f(u, v)$ have continuous second order partial derivatives and suppose that $u(x, y) = xy$, $v(x, y) = \frac{y}{x}$.

(i) Find $\frac{\partial z}{\partial x}$ in terms of $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

(ii) Find an expression for $x^2 \frac{\partial^2 z}{\partial x^2}$ in terms of u, v and the partial derivatives of z with respect to u and v.

c) Let

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

(i) Show that f is not continuous at $(0, 0)$.

(ii) Show that f_x and f_y exist at $(0, 0)$.

2. a) (i) Let $\mathbf{s}(t) = (t, t+1, t^2)$, $t \in \mathbb{R}$. Sketch the parametric curve by finding two surfaces on which it lies.

(ii) Find and classify the stationary point(s) of the function

$$f(x, y) = (4 - x - y)xy\ .$$

b) Find the points on the sphere $x^2 + y^2 + z^2 = 1$ closest and farthest from the point $(1, 2, 3)$.

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c) Use the total differential to estimate $\sqrt{2.98^2 + 4.03^2}$.

3. a) Given the symmetric matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

(i) Find an orthogonal matrix P such that $P^T AP$ is a diagonal matrix.

(ii) Evaluate $P^T AP$. 15

b) Let

$$B = \begin{pmatrix} 0 & -1 & 1 \\ 2 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}, \qquad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Show that $B^3 = I$, and hence, find B^{-1}.

4. a) A is a 3×3 matrix of the form

$$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix},$$

where a, b, c, d, e, f are real. The transformation represented by A maps the points $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ onto the points $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ respectively.

Given that $\det A = 8$, find A.

60

b) Solve the following equations :-

(i) $\cos x \dfrac{dy}{dx} + (\cos x + \sin x)y = 2 + \sin 2x, \quad y(0) = 2$

(ii) $(D^2 + 2D + 5)y = e^{-x} \sin^2 x$

2

5. a) (i) Find the eigenvalues and corresponding eigenvectors of

$$\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}.$$

(ii) Normalize the eigenvectors in (i).

b) Solve the following equations :-

(i) $(y^5 - xy)\dfrac{dy}{dx} = 1$

(ii) $(D^2 - 4)y = 4\sinh^2 x$

5

6. a) Let

$$M = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 4 & 3 & 1 \end{pmatrix}$$

be a 3×3 matrix. A transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix M. Find a cartesian equation of the image of the line with equation

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}.$$

b) Solve the following equations :-

(i) $y'' + y = \tan x$, by variation of parameters. 10

(ii) $y^3 y' + xy^4 = xe^{-x^2}$.

END!

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The University of Zambia

Department of Mathematics & Statistics

Engineering Mathematics II - EM212

Tutorial Sheet 1 February 2012

1. In each of the following cases sketch the curve given by $\mathbf{r}(t)$:

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   (c) $\mathbf{r}(t) = (t, \sqrt{1-t^2}), \quad 0 \leq t \leq 1$

   (d) $\mathbf{r}(t) = (\ln t, \sqrt{1-(\ln t)^2}), \quad 1 \leq t \leq e$.

   What did you observe? Could this curve also have been given parametrically by $\mathbf{r}(t) = (e^t, \sqrt{1-e^{2t}})$? Explain.

2. Let $\mathbf{r}(t) = (2\sin t, 2\cos t, 5t), \ t \geq 0$.

   (i) Sketch the curve given by $\mathbf{r}(t)$.

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3. In each of the following, find the domain and range of the function:-

   (a) $z = f(x,y) = \frac{\sqrt{x^2+y^2-9}}{x-y}$

   (b) $z = f(x,y) = 3/\sqrt{x^2-y}$

   (c) $z = f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$

   (d) $z = f(x,y) = \ln(1+x-y^2)$

   (e) $z = f(x,y) = \sqrt{\frac{x-y}{x+y}}$

   Sketch the domain and/or range where possible.

4. Let

$$f(x,y) = \begin{cases} x^2 & \text{if } x \neq y, \\ y^2 & \text{if } x = y. \end{cases}$$

   Find $f_x(2,2)$ and $f_y(2,2)$.

5. Let

$$g(x,y) = \begin{cases} 4 & \text{if } x = 0 \text{ or } y = 0, \\ x^2 + y^2 & \text{otheriwse.} \end{cases}$$

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Department of Mathematics & Statistics

Engineering Mathematics II - EM212

Tutorial Sheet 1 **February 2012**

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   What did you observe? Could this curve also have been given parametrically by $\mathbf{r}(t) = (e^t, \sqrt{1-e^{2t}})$? Explain.

2. Let $\mathbf{r}(t) = (2\sin t, 2\cos t, 5t), \ t \ge 0$.

   (i) Sketch the curve given by $\mathbf{r}(t)$.

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3. In each of the following, find the domain and range of the function:-

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   Sketch the domain and/or range where possible.

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   Find $f_x(2,2)$ and $f_y(2,2)$.

5. Let

$$g(x,y) = \begin{cases} 4 & \text{if } x = 0 \text{ or } y = 0, \\ x^2+y^2 & \text{otheriwse.} \end{cases}$$

Ziqui Dennk

The University of Zambia

Department of Mathematics & Statistics

Second Semester Examinations - February 2008

EM212 - Engineering Mathematics II

Time allowed : Three (3) hrs

Full marks : 100

Instructions: Attempt any four (4) questions. All questions carry equal marks.

Full credit will only be given when necessary work is shown.

Indicate your computer number on all answer booklets.

This paper consists of 4 pages of questions.

1. a) Find the domain and range of the function

$$f(x,y) = \sqrt{\frac{x-y}{x+y}}\ .$$

b) Let $f(x,y) = 8 - 3(x-2)^2 - 4(y-1)^2$.

(i) Sketch a few level curves of $f(x,y)$.

(ii) Sketch the surface $z = f(x,y)$.

(iii) Sketch and describe the curve of intersection C of the surface $z = f(x,y)$ with the plane $x = 3$.

c) Find and classify the stationary points of $g(x,y) = 3xy - x^2y - xy^2$.

d) Let $z = \ln(x^2 + y^2)^{\frac{1}{2}}$.

(i) Find and simplify the expression

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\ .$$

(ii) Prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0\ .$$

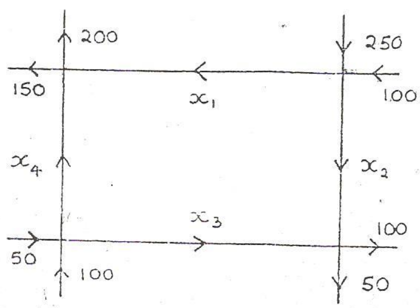
$-4 = 4(y-1)^2$

$350 - x_1$

2\. a) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \\ 3y \end{pmatrix}$.

Is T a linear map? Explain.

b) The following diagram shows a road network where all the streets are one-way. The flow of traffic in and out of the network is measured in vehicles per hour, and is indicated on the diagram. Let x_1, x_2, x_3 and x_4 denote the number of vehicles flowing along the various branches per hour.



(i) Construct a system of linear equations in the unknowns x_1, x_2, x_3 and x_4 that describes the traffic flow in this road network.

(ii) Express the system of equations in matrix form.

(iii) Solve the system of equations you constructed in (i).

(iv) Use your solution to deduce the maximum and minimum values of x_2.

Pr babs c) Find conditions on a, b, and c, so that $(a, b, c) \in \mathbb{R}^3$ belongs to the space generated by $u = (2, 1, 0)$, $v = (1, -1, 2)$ and $w = (0, 3, -4)$.

d) Determine whether the set

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \right\}$$

forms a basis for $\mathbb{R}^3$.

Form a augmented matrix. Row reduced. $c_1 = c_2 = c_3 = 0$. [illegible] [illegible] basis.

3. a) Show that the differential equation

$$\frac{\ln y}{x}\,dx - \left(\frac{[illegible]}{y} + \sin y\right) dy = 0$$

is exact, and find the general solution.

b) Solve the differential equation $y'' - 2y' - 3y = \cos x$.

c) Solve the following differential equations :

(i)

$$\frac{dy}{dx} + \frac{1+y^3}{xy^2(1+x^2)} = 0$$

(ii)

$$\frac{dy}{dx} - xy = x$$

d) A weight attached to a spring moves up and down so that the equation of motion is

$$\frac{d^2s}{dt^2} + 16s = 0 .$$

where s is the stretch of the spring at time t. If $s = 2$, and $ds/dt = 1$ when $t = 0$, find s in terms of t.

4. a) The following functions are continuous everywhere except at the origin $(0,0)$, where they are not defined. Can they be made continuous there? Explain.

(i) $z = \dfrac{\sin(x+y)}{x+y}$ (ii) $z = \dfrac{xy}{x^2+y^2}$

b) Let
Or others

$$A = \begin{pmatrix} 1 & 2 & k \\ 1 & 1 & -1 \\ -1 & 3 & k^2 \end{pmatrix} \qquad k \in \mathbb{R} .$$

(i) Find all values of k for which A does not have an inverse.

(ii) Find the inverse of A in the case when $k = 0$.

c) Find the directional derivative of the function at the given point in the direction of the vector v.

$$f(x,y) = 1 + 2x\sqrt{y} , \qquad (3,4) , \qquad \mathrm{v} = \langle 4, -3 \rangle .$$

d) Solve the differential equation $9y'' - 24y' + 16y = 0$

3

5. a) Let

$$f(x,y) = \begin{cases} xy/(x^2+y^2) & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Show that f_x and f_y exist at $(0,0)$ but that f is not continuous there.

b) Let $V(x,y,z) = xyz$ for $x \geq 0$, $y \geq 0$ and $z \geq 0$. Find values for x, y and z that maximize the value of V subject to the constraint $2x + 2y + z = 108$.

c) Let

$$B = \begin{pmatrix} 1-\lambda & 7 & 1 & -1 \\ 0 & 3-\lambda & 2 & -2 \\ 1 & 0 & 1-\lambda & -1 \\ 1 & 0 & 1 & -1-\lambda \end{pmatrix}$$

Solve the equation $\det B = 0$.

d) Use the total differential to approximate the change in the hypotenuse of a right angled triangle of legs 6 and 8 inches, when the shorter leg is lengthened by $\frac{1}{4}$ in. and the longer leg is shortened by $\frac{1}{8}$ in.

$\boxed{a_{11}(C_{ofac})}$ +

END!

$$\begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$$

$$1-\lambda \begin{vmatrix} 3-\lambda & 2 & -2 \\ 0 & 1-\lambda & -1 \\ 0 & 1 & -1-\lambda \end{vmatrix}$$

$$-7 \begin{vmatrix} 0 & 2 & -2 \\ 1 & 1-\lambda & -1 \\ 1 & 1 & -1-\lambda \end{vmatrix} + (3-\lambda) \begin{vmatrix} 1-\lambda & 1 & -1 \\ 1 & 1-\lambda & -1 \\ 1 & 1 & -1-\lambda \end{vmatrix}$$

$$7\left(-2\begin{vmatrix} 1 & -1 \\ 1 & -1-\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & 1-\lambda \\ 1 & 1 \end{vmatrix}\right) + (3-\lambda)\left[1-\lambda \begin{vmatrix} 1-\lambda & -1 \\ 1 & -1-\lambda \end{vmatrix} - \begin{vmatrix} 1 & -1 \\ 1 & -1-\lambda \end{vmatrix}\right]$$

D.O.D
Masuzu David Mbindawike.

The University of Zambia

Department of Mathematics & Statistics

Engineering Mathematics II - EM212

Tutorial Sheet 2 **February 2012**

1. (a) By finding two different paths along which the function approaches different values, show that

$$\lim_{(x,y)\to(0,0)} \frac{y^2 - x^2}{x^2 + y^2}$$

does not exist.

(b) Show that the function

$$\frac{4x^2y^3}{x^3 + y^9}$$

approaches 0 as (x, y) tends to $(0, 0)$ along all straight lines through the origin.

(c) Show that

$$\lim_{(x,y)\to(0,0)} \frac{4x^2y^3}{x^3 + y^9}$$

does not exist.

2. Calculate the value of the given partial derivative at the given point.

(a) $f(x, y) = \sinh(x - y)$; $f_x(3, 3)$.

(b) $f(x, y) = \sqrt[3]{x^2y - y^2x^5}$; $f_y(-2, 4)$.

(c) $f(x, y) = \ln(x^2 + y^4)$; $f_y(3, 1)$.

3. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

$$f(x, y) = 1 + 2x\sqrt{y}\ , \qquad (3, 4)\ , \qquad \mathbf{v} = \langle 4, -3 \rangle\ .$$

4. Find an approximate value for the change in z, on the surface $z = 2x^2 - 3y^2$ when x changes form 4 to 4.3 and y changes from 5 to 4.8.

5. Use the total differential to approximate the change in the hypotenuse of a right angled triangle of legs 6 and 8 inches, when the shorter leg is lengthened by $\frac{1}{4}$ in. and the longer leg is shortened by $\frac{1}{8}$ in.

6. Find dw/dt using the chain rule and then expressing everything in terms of t.

(a) $w = 2xy/(x^2 + y^2)$, $x = 2t$, $y = t^2$.

(b) $w = \ln(x^2 + 3xy^2 + 4y^4)$, $x = 2t^2$, $y = 3t$.

1

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7. Let $z = f(u, v)$ have continuous second order partial derivatives, and suppose $u(x,$ $xy, v(x, y) = y/x$.
   (i) Find $\partial z/\partial x$ in terms of z with respect to u and v.
   (ii) Find an expression for $x^2 \frac{\partial^2 z}{\partial x^2}$ in terms of u, v and the partial derivatives of z w.r.t. respect to u and v.

8. Find and classify the stationary point(s) of

   a) $f(x, y) = 2x^3 - x^2y + y^2$ b) $f(x, y) = x^2 - 2xy + \frac{1}{3}y^3 - 3y$ c) $f(x, y) = 4xy - x$ -

9. Let $g(x, y) = 2(x^2 + y^2)e^{-(x^2+y^2)}$.
   (a) Find all the stationary points of g. DO NOT try to classify them using the sec... derivative test.
   (b) For which (x, y) is $g(x, y) < 0$?
   (c) What happens to $g(x, y)$ as x and y both tend to infinity? Explain.
   (d) Use your results from (b) and (c) to classify the stationary points of g.
   (e) Try to sketch (and describe) the graph of the surface $z = g(x, y)$.

10. (a) Use Lagrange multipliers to find the points on the curve of intersection of the surf... $x - y - 1 = 0$, $y^2 - z^2 = 1$, that are nearest to and furthest from the origin.
    (b) Draw a sketch to ensure that you have the correct absolute maximum and minim... in (a)

11. Let $f(x, y) = x^3 + y^2 - 3xy + y$.
    (a) Locate and classify the stationary points of f.
    (b) Let R be the region $\{(x, y) : 0 \le x \le 2, 0 \le y \le 2\}$. Find the (global) maximum and minimum of f over R.

12. A rectangular box without a lid is to be made from $12\,\mathrm{m}^2$ of cardboard. Find the maxim... volume of such a box.

13. Let $V(x, y, z) = xyz$ for $x \ge 0$, $y \ge 0$ and $z \ge 0$. Find values for x, y and z that maximize the value of V subject to the constraint $2x + 2y + z = 108$.

14. Use Lagrange multipliers to find the maximum and minimum of the function subject to the given constraint(s).
    (a) $f(x, y) = x^2y$; $x^2 + 2y^2 = 6$
    (b) $f(x, y) = (x - 2)^2 + y^2$; $x^2 - y^2 = 1$
    (c) $f(x, y) = x + y$; $x^2 + y^2 = 1$
    Loui 4 (d) $f(x, y, z) = 2x + 6y + 10z$; $x^2 + y^2 + z^2 = 35$
    Bio 4 (e) $f(x, y, z) = x + 2y$; $x + y + z = 1$, $y^2 + z^2 = 4$

Eng - 3
Math - 2
Crick - 2
Science - 2
Acc - 2

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13 in 6
11 in 5

2

Be Peaceful Bye Sena.

D.O.D
Maswe David
Mbindawina

The University of Zambia

Department of Mathematics & Statistics

EM212 : Eng. Mathematical

Assignment No. 3

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1. Solve each of the following equation

   (a) $\frac{dx}{dy} - \frac{2x}{y} = x^4$ (b) $y' + y = \sin x$

   (c $xy' + y = 3x^3y^2$ (d) $y' - ay = e^{ax}$

   (e) done $xdy + ydx = xy^2dx$ (f) $y' - y = 2e^x$

   (g) $y(1+xy)dx + x(1-xy)dy = 0$

   (h) $y' + (y-1)\cos x = 0$

2. Solve the equation $(y - 2xy - x^2)dx + x^2dy = 0$, for which y = 0 for x = 1.

3. Determine the solution of $y'' - y' - 12y = 0$, for which y(0) = 3, y'(0) = 5

4. In each of the following , find the G.S. of each equation.

   (i) $y'' + y' = 0$

   (ii) $2y'' + y' - y = 0$

   (iii) $y''' - 6y' + 11y = 0$

   (iv) $y^{(4)} - y = 0$ $y^4 - y = 0$

   (v) $y_{''} + 6y_{'} + 9y = 0$

5. Show that sin2x and cos2x are linearly independent solution of $y'' + 4y = 0$.

   Show that $y_1(x) = 3\sin 2x$ and $y_2(x) = \cos 2x - \sin 2x$ are also linearly independent solution of $y''' + 4y' = 0$. Determine constants C_1 and C_2 such that $6\sin 2x + 4\cos 2x = C_1y_1(x) + C_2y_2(x)$.

6. Show that $y_2(x) = xe^x$ and $y_2(x) = e^x$ are linearly independent solution of $y'' - 2y' + y = 0$. Determine the solution y(x) of this equation for which y(0) = 1, $y'(0) = 0$

7. Find the particular solution of the linear differential equations

   (i) $y'' - y' - 30y = 0$, with conditions: y(0) = 0, y'(0) = -4

   (ii) $y'' + 16y = 0$, with conditions: y(0) = 0, y'(0) = 2.

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DOD

KALENGA

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2010 Academic Year Second Semester Final Examinations

EM212 - Engineering Mathematics II

Time allowed : **Three (3) hours** Full marks : 100

Instructions:
- There are six (6) questions in this paper.
- Attempt any **five (5)** questions. **All** questions carry equal marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.
- **Calculators** are **not** allowed.

This paper consists of 3 pages of questions.

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1. a) Use the total differential to estimate the value of $\sqrt{(2.98)^2 + (4.03)^2}$.

   b) Find the absolute maximum and minimum of the function $f(x, y) = 3x^2 + 2y^2 - 4y$ over the region bounded by $y = x^2$ and $y = 4$.

   c) Is $A = \begin{pmatrix} 3 & 3 & 6 \\ 0 & 1 & 2 \\ -2 & 0 & 0 \end{pmatrix}$ invertible? Justify your answer.

2. a) Determine whether or not the following sets of vectors are linearly independent

   i) $\{(1, 0, 3), (1, 0, 1), (2, 0, 0), (1, 1, 0)\}$;

   ii) $\left\{ \begin{pmatrix} 8 & -2 \\ 10 & 0 \end{pmatrix}, \begin{pmatrix} -12 & 3 \\ -15 & 0 \end{pmatrix} \right\}$.

   b) Use the Lagrange multiplier technique to find the maximum and minimum values of $f(x, y, z) = x^2y + z$ subject to $x^2 + y^2 = 1$ and $z = y$.

   c) Let $w = x\ln(x^2 + y^2)$; $x = s + t$ and $y = s - t$. Show that, in terms of s and t,

$$\frac{\partial w}{\partial s} = \ln\left(2(s^2 + t^2)\right) + \frac{2s(s+t)}{s^2 + t^2}.$$

1

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5. a) Given that $A = \begin{pmatrix} 8 & 2 & 2 \\ 2 & 5 & -4 \\ 2 & -4 & 5 \end{pmatrix}$, find

i) the eigenvalues of A;

ii) the eigenspaces corresponding to each eigenvalue found in (i);

b) Describe and draw the level curves for the given c-values of

$$f(x,y) = \frac{3x^2 + 4y^2}{12}; \quad c = 1, 2, 3$$

c) Find the general solution of $xy' - 2y = x^2$.

6. a) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x,y,z) = (x+y, y-z)$. Is T linear? Justify

b) Use Gauss-Jordan elimination method to solve the system

$$\begin{aligned} -2x + y - z &= 4 \\ x + 2y + 3z &= 13 \\ 3x + 7y + z &= -1. \end{aligned}$$

c) Solve the initial value problem

$$y'' + 2y' - 8y = e^{3x}; \ y(0) = 1, \ y'(0) = \frac{3}{7}.$$

END OF EXAMINATION!

"Failure without attempt is Laziness"

LIMGEE © 2012

The University of Zambia

Department of Mathematics & Statistics

Second Semester Examinations - April 2009

EM212 - Engineering Mathematics II

Time allowed : Three (3) hrs   Full marks : 100

Instructions:
- Attempt **any five (5)** questions. All questions carry equal marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.
- **Calculators** are **not** allowed.

This paper consists of 3 pages of questions.

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1. a) Let $f(x,y) = e^{x^2-y}$. Draw the family of level curves for the function (labelling a few level curves).

b) Given the linear system of equations

$$\begin{aligned} x + y - 4z &= 0 \\ 2x + 3y + z &= 1 \\ 4x + 7y + \lambda z &= \mu , \end{aligned}$$

find the conditions which λ and μ must satisfy for the system of equations to have

(i) no solution?

(ii) a unique solution?

(iii) more than one solution?

Find all the solutions (in terms of λ and μ for the unique solution) whenever possible.

c) Show that the differential equation

$$(2x + y^3)dx + (3xy^2 - e^{-2y})dy = 0$$

is exact, and find the particular solution such that $y(-1) = 0$.

2. a) Prove that $\{(1,0,3),(5,2,1),(0,1,6)\}$ is a basis for $\mathbb{R}^3$.

b) Let

$$f(x,y)=\begin{cases}(x^3y-xy^3)/(x^2+y^2) & \text{if } (x,y)\neq(0,0)\ ,\\ 0 & \text{if } (x,y)=(0,0)\ .\end{cases}$$

(i) Show that $f_x(0,0)=f_y(0,0)=0$.

(ii) Hence show that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

c) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$$

for which $y(0)=0\ ,\ \ y'(0)=2$.

3. a) Find and sketch the domain of the function

$$f(x,y)=\ln(1+x-y^2)\ .$$

What is the range of f?

b) Solve the following differential equations

(i) $x^2y'-yx^2=y$

(ii) $xy'+(2+3x)y=xe^{-3x}$

c) Let $T:\mathbb{R}^2\to\mathbb{R}^3$ be a linear transformation such that

$$T\begin{pmatrix}2\\-3\end{pmatrix}=\begin{pmatrix}-1\\5\\-9\end{pmatrix}\qquad\text{and}\qquad T\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}2\\0\\3\end{pmatrix}.$$

Find

(i) $T\begin{pmatrix}2\\1\end{pmatrix}$ (ii) $T\begin{pmatrix}-3\\4\end{pmatrix}$

4. a) Let $\mathbf{r}(t)=(2\cos t, 2\sin t, 2t),\ t\geq 0$. Sketch the curve given by $\mathbf{r}(t)$.

b) Let

$$z=\frac{4}{xy}-\frac{x}{y}\ ,\qquad x=u^2\ ,\qquad y=uv\ .$$

Find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$.

c) Find the general solution of the differential equation

$$y''-2y'+y=\frac{e^x}{x}\ .$$

5. a) Let $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x - y \\ y + z \\ 3y + z \end{pmatrix}$.

(i) Show that $\mathbf{T}$ is a linear transformation.

(ii) Find a cartesian equation for the image under $\mathbf{T}$ of the plane $x + 2y + 3z = 0$.

b) Use Lagrange multipliers to find the maximum and minimum of the function $f(x, y) = x + y$, subject to the constraint $x^2 + y^2 = 1$.

6. a) Given the function

$$f(x, y) = e^{xy}\sqrt{x^2 + y^2},$$

find the first partial derivatives with respect to x and y.

b) Find and classify the stationary point(s) of the function

$$f(x, y) = x^2 - 2xy + \frac{1}{3}y^3 - 3y;$$

c) Find all eigenvalues and a basis of each eigenspace of the operator $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $\mathbf{T}(x, y, z) = (2x + y, y - z, 2y + 4z)$.

END!

3

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2009/2010 ACADEMIC YEAR
SECOND SEMESTER FINAL EXAMINATIONS

EM212 - ENGINEERING MATHEMATICS II

TIME ALLOWED: **THREE (3) HRS**

FULL MARKS: **100**

INSTRUCTIONS:

- There are **six (6)** questions in this paper. Attempt **any five (5)** questions. All questions carry **equal** marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on **all** answer books.
- **Calculators** are **not** allowed.

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1. a) Locate and classify the stationary points of $f(x, y) = x^3 - 3x + xy^2$.

   b) What does it mean that two square matrices A and B are similar.

   c) Given that $A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$; find

   λ

   i) the eigenvalues of A
   ii) the eigenvetors and the eigenspaces corresponding to each eigenvalue found in (i)
   iii) the algebraic and geometric multiplicities of each eigenvalue.
   iv) Is A diagonalizable? Justify.

2. a) Given $A = \begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{pmatrix}$, find

   f_x
   f_y

   i) the determinant of A
   ii) the adjoint of A

$b = x - 0$

$f_{xx}(x_0, y_0)$
$f_{yy}(x_0, y_0)$
$f_{xy}(x_0, y_0)$

1 $f_{xx}, f_{yy}, f_{xy} =$

$g(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

iii) Hence or otherwise, find the solution to the system

$$3x_1 + 2x_2 - x_3 = 4$$
$$2x_1 - x_2 + 2x_3 = 10$$
$$x_1 - 3x_2 - 4x_3 = 5$$

b) Find the general solution of $\frac{dy}{dx} = \frac{3y-1}{x}$.

c) Find the linearization $L(x, y)$ of $f(x, y) = \frac{1}{1+x-y}$ near the point $(2, \ 1)$.

3. a) Use total differential to approximate the value of $\sqrt{35.6}\ \sqrt[3]{64.08}$ correct to two (2) decimal places.

b) For what value(s) of k are the vectors $(1, \ 2, \ 3), (2, \ -1, \ 4), (3, \ k, \ 4)$ linearly independent?

c) Show that $f(x, y) = \frac{x^2 + y^2}{xy}$ has no limit.

d) Find the general solution of the differential equation $(1 - \sin x \tan y)dx + (\cos x \sec^2 y)dy = 0$. EXAMPLE YAMU CLASS

4. a) i) Show that $T : \mathbf{R}^3 \to \mathbf{R}^3$ defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-y \\ x+2y-z \\ 2x+y+z \end{pmatrix}$ is a linear transformation.

ii) Find the matrix A_T representing the linear transformation T in (i) relative to the basis $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ of $\mathbf{R}^3$.

b) Solve the initial value problem $y'' + y' = 2\sin x; \quad y(0) = 0, \ y'(0) = -3$.

c) Discuss the continuity of $f(x, y) = \frac{\sin z}{e^x + e^y}$.

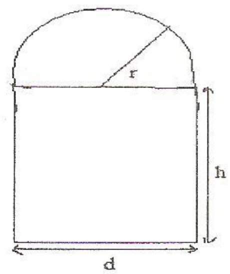
5. a) Find the general solution of $y' + \frac{y}{x} - \sqrt{y} = 0$.

b) Find the condition(s) for k so that the system

$$2x - 3y + z = -1$$
$$-x + 2y - z = 0$$
$$y - z = k$$

has infinite solutions

c) A hot water storage tank is a vertical cylinder surmounted by a hemispherical top of the same diameter. See the figure below. The tank is designed to hold $400m^3$ of liquid.



Determine the total height and the total diameter of the tank if the surface heat loss is to be minimum. (Volume of a sphere $= \frac{4}{3}\pi r^3$, Surface area of a sphere $= 4\pi r^2$).

6. a) Find the series solution of the differential equation $(x^2+1)y''-4xy'-6y=0$.

b) The base radius of a right circular cone is increasing at the rate of $1.5mm/s$ while the perpendicular height is decreasing at $6mm/s$. Determine the rate at which the volume V is changing when $r=1.2cm$ and $h=2.4cm$ (Volume of a cone$=\frac{1}{3}\pi r^2 h$).

c) Find a basis for the set of vectors lying on the plane $x+2y-3z=0$.

END OF EXAMINATION

3

The University of Zambia

Department of Mathematics & Statistics

EM212-Engineering Mathematics II

Test 2 - 20th April 2011

Duration : Two (2) hrs

Instructions: Attempt all questions. Show all essential working.

1. Given that $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -4 \end{pmatrix}$. Find

   a) the determinant of A;

   b) the adjoint of A; and

   c) hence, the inverse of A.

2. a) Show that $V = \{(x, y) : x, y \in \mathbb{R}\}$ is not a vetor space over $\mathbb{R}$, where addition and sclar multiplication is defined by $(x, y) + (z, w) = (x + 2z, y + w)$ and $\alpha(x, y) = (\alpha x, \alpha y)$, respectively.

   b) Show that $W = \{(0, x, y) | x, y \in \mathbb{R}\}$ is a vector subspace of $\mathbb{R}^3$.

3. a) What does it mean that a set of vectors $\{v_1, v_2, \ldots, v_n\}$ is linearly dependent?

   b) Determine whether or not the set of vectors $\{x^3 + 4, x^2 - 2, x^3 + x\}$ is linearly dependent.

   c) Show that $\{(2, -1, 4), (4, 1, 6)\}$ is a basis for the plane $5x - 2y - 3z = 0$.

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (2x + y, y - z, 2y + 4z)$. Then

   a) show that T is a linear transformation;

   b) find the eigenvalues of the matrix A_T representing T;

   c) find the eigenvector(s) corresponding to each eigenvalue found in (b).

THE END!

"Failure without attempt is laziness"