

The University of Zambia  
Department of Mathematics and Statistics  
Mat 3110-Engineering Mathematics II

Tutorial Sheet 5

August, 2018.

Submit questions 1 for assessment: Submit on Monday, August 20 , 2018 during class from 11-13hrs.

1. Evaluate the following integrals

(a) 
$$\int_0^2 \int_{-\sqrt{4-x^2}}^0 \int_{\sqrt{5x^2+5y^2}}^{\sqrt{24-x^2-y^2}} 7x \, dz \, dy \, dx$$

(b) 
$$\int_{-4}^4 \int_0^{\sqrt{16-y^2}} \int_0^{6+x} 6yx^2 \, dz \, dx \, dy$$

(c) 
$$\int_{-\sqrt{5}}^{\sqrt{5}} \int_0^{\sqrt{5-y^2}} \int_{-\sqrt{x^2+y^2}}^{-\sqrt{10-x^2-y^2}} 3xz^2 \, dz \, dx \, dy$$

In each of the above, can you tell over which solid we are integrating?

2. Evaluate the following triple integrals over the given solids.

(a) Evaluate 
$$\int \int \int_E 8z \, dV$$
 where  $E$  is the solid bounded by  $z = 2x^2 + 2y^2 - 4$  and  $z = 5 - x^2 - y^2$

(b) Evaluate 
$$\int \int \int_E 4y^2 \, dV$$
 where  $E$  is the solid enclosed by the sphere  $x^2 + y^2 + z^2 = 9$ .

(c) Evaluate 
$$\int \int \int_E 9yz^3 \, dV$$
 where  $E$  is the solid between  $x = -\sqrt{9y^2 + 9z^2}$  and  $x = \sqrt{y^2 + z^2}$  inside the cylinder  $y^2 + z^2 = 1$ .

(d) Evaluate 
$$\int \int \int_E 2yz \, dV$$
 where  $E$  is the solid below  $x^2 + y^2 + z^2 = 16$  and inside  $z = \sqrt{3x^2 + 3y^2}$  that is in the 1<sup>st</sup> octant.

(e) Evaluate 
$$\int \int \int_E x + 2 \, dV$$
 where  $E$  is the solid between the two planes  $2x + y + z = 6$  and  $6x + 3y + 3z = 12$  inside the cylinder  $x^2 + z^2 = 16$ .

(f) Evaluate 
$$\int \int \int_E 5y^2 \, dV$$
 where  $E$  is the portion of the solid sphere  $x^2 + y^2 + z^2 = 4$  with  $x \leq 0$ .

3. Use triple integrals to find the volumes of the following solids.

(a) The solid that is below the plane  $x = z + 3$ , above  $x = -z - 6$  and inside the cylinder  $y^2 + z^2 = 4$ .

(b) The solid bounded by  $y = \sqrt{x^2 + z^2}$ , and  $x^2 + y^2 + z^2 = 2$  in the 1<sup>st</sup> octant.

4. Use triple integrals in cylindrical coordinates to derive the volume of a cylinder of height  $h$  and radius  $a$ .

5. Use a triple integral in spherical coordinates to derive the volume of a sphere with radius  $a$ .