The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Tutorial Sheet 3 **June, 2018.**

Submit questions 8, 9 and 10(a), 10(b) and 10(c) for assessment: Submit on Friday, June 22, 2018 during class from 11-13hrs.

1. Give the equation of the line in vector form of the following.
   (a) The line through the points $(2, -4, 1)$ and $(0, 4, -10)$.
   (b) The line through the point $(7, 2, 4)$ and parallel to the line given by $r(t) = (5 - 8t)\ \mathbf{i} + (6 + t)\ \mathbf{j} - 12t\ \mathbf{k}$.

2. Write down the equation of the plane for each of the following.
   (a) The plane that contains the points $(4, -3, 1)$, $(-3, -1, 1)$ and $(4, -2, 8)$.
   (b) The plane containing the piint $(3, 0, -4)$ and orthogonal to the line given by $r(t) = (12 - t)\ \mathbf{i} + 8t\ \mathbf{j} + (4 + 6t)\ \mathbf{k}$.
   (c) The plane containing the point $(-8, 3, 7)$ and parallel to the plane given by $4x + 8y - 2z = 45$.

3. Sketch each of the following quadratic surfaces.
   (a) $\frac{y^2}{9} + z^2 = 1$
   (b) $\frac{x^2}{4}\frac{y^2}{9} + \frac{z^2}{6} = 1$
   (c) $z = \frac{x^2}{4} + \frac{y^2}{4} - 6$
   (d) $y^2 = 4x^2 + 16z^2$
   (e) $x = 4 - 5y^2 - 9z^2$
   (f) $z^2 = x^2 + \frac{y^2}{2}$
   (g) $x = 2 + 4x^2 + 6y^2$
   (h) $4x^2 + y^2 + 3z^2 = 1$
   (i) $\frac{x^2}{9} + \frac{y^2}{16} = 1$
   (j) $y = \frac{x^2}{9} + \frac{z^2}{3} - 7$
   (k) $6x^2 + 2z^2 = 1$
   (l) $x = 12 - \frac{y^2}{4} - 3z^2$
   (m) $x^2 = 4y^2 + 9z^2$
   (n) $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$
   (o) $-x^2 - 4y^2 + z^2 = 4$

4. Sketch the graph of the given vector functions.
   (a) $r(t) = (t + 1)\ \mathbf{i} + (\frac{1}{4}t^2 + 3)\ \mathbf{j}$
   (b) $r(t) = -2\cos t\ \mathbf{i} + 5\sin t\ \mathbf{j}$
   (c) $r(t) = (2t + 1)\ \mathbf{i} + (t^2 - 1)\ \mathbf{j}$
   (d) $r(t) = (t^2 + 4)\ \mathbf{i} + (6 - t^2)\ \mathbf{j}$

5. Identify the graph of the vector function without sketching the graph.
   (a) $r(t) = 3\cos(6t)\ \mathbf{i} - 4\ \mathbf{j} + \sin(6t)\ \mathbf{k}$
   (b) $r(t) = (2 - t)\ \mathbf{i} + (4 + 7t)\ \mathbf{j} + (-1 - 3t)\ \mathbf{k}$
   (c) $r(t) = 6\ \mathbf{i} + (2 + 8t)\ \mathbf{j} + (-1 + 10t)\ \mathbf{k}$
   (d) $r(t) = -2t\ \mathbf{i} + 6\cos t\ \mathbf{j} + 6\sin t\ \mathbf{k}$

6. Write down the vector function of the line segment between the two points.
   (a) The line segment starting at $(1, 3)$ and ending at $(-4, 6)$.
   (b) The line segment starting at $(0, 2, -1)$ and ending at $(7, -9, 2)$.

7. Compute the given double integrals over the indicted regions.
   (a) $\int\int_D 3xy^2 - 2\ \mathrm{d}A$, where D is the unit circle centered at the origin.
   (b) $\int\int_D 4x - 2y\ \mathrm{d}A$, where D is the top half region between $x^2 + y^2 = 4$ and $x^2 + y^2 = 25$.
   (c) $\int\int_D 6xy + 4x^2\ \mathrm{d}A$, where D is the portion of the disc $x^2 + y^2 = 9$ in the 2^{nd} quadrant.
   (d) $\int\int_D x^5\sin(y^4)\ \mathrm{d}A$, where D is the region in the 2^{nd} quadrant bounded by $y = 3x^2$, $y = 12$ and the y-axis.
   (e) $\int\int_D 7y^3e^{x^2+1}\ \mathrm{d}A$, where D is the region bounded by $y = 2\sqrt[4]{x}$, $x = 9$ and the x-axis.

(f) $\int\int_D e^{y^2+1}\,\mathrm{d}A$, where D is the triangle with vertices $(0,0)$, $(-2,4)$ and $(8,4)$.

(g) $\int\int_D 6y(x+6)^2\,\mathrm{d}A$, where D is the region bounded by $x=-y^2$ and $x=y-6$.

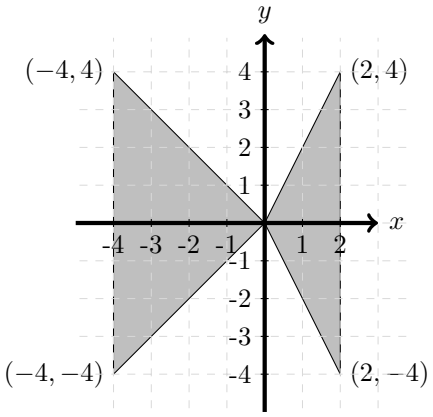
(h) $\int\int_D 15x^2-6y\,\mathrm{d}A$, where D is the region bounded by $x=\frac{1}{2}y^2$ and $x=4\sqrt{y}$.

(i) $\int\int_D 9-\frac{6y^2}{x^2}\,\mathrm{d}A$, where D is the region in the 1st quadrant bounded by $y=x^3$ and $y=4x$.

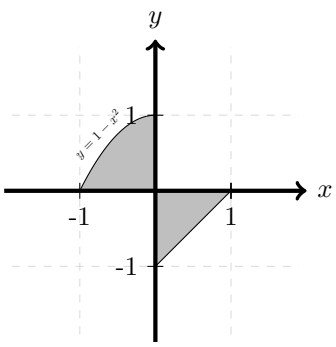
(j) $\int\int_D 8yx^3\,\mathrm{d}A$, where $D=\{(x,y)\mid -1\le y\le 2,\ -1\le x\le 1+y^2\}$.

(k) $\int\int_D xy\cos(yx^2)\,\mathrm{d}A$, where $D=\{(x,y)\mid -2\le x\le 3,\ -1\le y\le 1\}$.

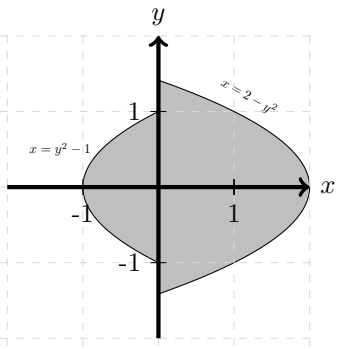
(l) $\int\int_D xy-y^2\,\mathrm{d}A$ where D is shown below.



(m) $\int\int_D 12x^3-3\,\mathrm{d}A$ where D is shown below.



(n) $\int\int_D 6y^2+10yx^4\,\mathrm{d}A$ where D is shown below.



8. Evaluate the following double integrals.

(a) $\int_{-2}^{0}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} x^2\,\mathrm{d}x\mathrm{d}y$ (b) $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\sqrt{x^2+y^2}\,\mathrm{d}y\mathrm{d}x$ (c) $\int_{0}^{8}\int_{y^{\frac{1}{3}}}^{2}\frac{y}{x^7+1}\,\mathrm{d}x\mathrm{d}y$ (d) $\int_{-4}^{0}\int_{\sqrt{-x}}^{2}x^{-\frac{2}{3}}\sqrt{y^{\frac{5}{3}}+1}\,\mathrm{d}y\mathrm{d}x$

9. Use double integrals to determine the area of the following regions in the xy-plane.
   (a) The region bounded by$y=x^2+1$and$y=\frac{1}{2}x^2+3$.
   (b) The region bounded by$x=-y^2$and$x=y-6$.
   (c) The region bounded by$y=x^2+1$and$y=\frac{1}{2}x^2+3$.

10. Use a double integral to determine the volume of the following solids.
   (a) The solid bounded by the planes $z=4-2x-2y$, $y=2x$, $x=0$ and $z=0$.
   (b) The solid that is inside both the cylinder $x^2+y^2=9$ and $x^2+y^2+z^2=16$.
   (c) The solid that is bounded by $z=12-3x^2-3y^2$ and $z=x^2+y^2-8$.
   (d) The solid that is below $z=9-4x^2-4y^2$ and above the xy-plane.
   (e) The solid the in first octant that is below the plane given by $2x+6y+4z=8$.
   (f) The solid that is bounded by the plane $z=3-2y$, the surface $y=1-x^2$, the plane $x=0$ and the plane $z=0$.
   (g) The solid that is bounded by $z=12-3x^2-3y^2$ and $z=x^2+y^2-8$.
   (h) The solid that is inside both the cylinder $x^2+y^2=9$ and the sphere $x^2+y^2+z^2=16$.
   (i) The solid bounded by the planes$z=4-2x-2y$, $y=2x$, $x=0$and$z=0$.
   (j) The solid bounded by $z=3-2y$, the surface $y=1-x^2$ and the planes $x=0$ and $z=0$.
   (k) The solid in the first octant that is below the plane given by $2x+6y+4z=8$.
   (l) The solid that lies under the surface $f(x,y)=9x^2+4xy+4$ and above the rectangle $R=\{(x,y)\mid -1\le x\le 1, 0\le y\le 2\}$.