Univesity of Zambia
Department of Mathematics and Statistics
Mat 3110 - Engineering Mathematics II

Tutorial Sheet 1 **April, 2018.**

Submit the following questions for assessment: Q2, Q4(h), Q6(c), Q9 and Q10(h). Submit on 13th April during class 11-13hrs.

1. Apply the power series method to solve the following differential equations.
   (a) $y' = 2y$
   (b) $y' + y = 0$
   (c) $y' = ky$ (k is a real constant)
   (d) $(1-x)y' = y$
   (e) $(x+1)y' = 3y$
   (f) $(1+x)y' + y = 0$
   (g) $y' + 2xy = 0$
   (h) $y' = 3x^2y$
   (i) $y'' - y = 0$
   (j) $y'' + 2xy = 0$
   (k) $y'' - y' = 0$
   (l) $y'' - 9y = 0$

2. Show that
$$\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} = \sum_{j=1}^{\infty}(j+a)ja_{j+1}x^{j-1} = \sum_{s=0}^{\infty}(s+2)(s=1)a_{s+2}x^s$$

3. For each of the series below, shift eh index so the the power under the summation sign is x^m.
   (a)
$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{3n}x^{n+2}$$
   (b)
$$\sum_{s=2}^{\infty}\frac{s(s+1)}{s^2+1}x^{s-1}$$
   (c)
$$\sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{6^k}x^{k-3}$$

4. Find a general solution of the following differential equations.
   (a) $xy' = 3y + 3$
   (b) $(x-3)y' - xy = 0$
   (c) $y' = 2xy$
   (d) $(1-x^4)y' = 4x^3y$
   (e) $(x+1)y' - (2x+3)y = 0$
   (f) $y'' - y = x$
   (g) $y'' - 3y' + 2y = 0$
   (h) $y'' - 4xy' + (4x^2 - 2)y = 0$
   (i) $(1-x^2)y'' - 2xy' + 2y = 0$
   (j) $y'' - xy' + y = 0$

5. Find a general solution of the following differential equations.
   (a) $x^2y'' - 6y = 0$.
   (b) $xy'' + 4y' = 0$
   (c) $x^2y'' - 2xy' + 2y = 0$
   (d) $x^2y'' + 9xy' + 16y = 0$
   (e) $x^2y'' + xy' - y = 0$
   (f) $x^2y'' + 3xy' + y = 0$
   (g) $x^2y'' + 3xy' + 5y = 0$
   (h) $x^2y'' + xy' + y = 0$

6. Solve the following initial value problems.
   (a) $x^2y'' - 4xy' + 4y = 0$, $y(1) = 4, \quad y'(1) = 13.$
   (b) $4x^2y'' - 4xy' - y = 0$ $y(4) = 2, \quad y'(4) = -0.25.$
   (c) $x^2y'' - 5xy' + 8y = 0$ $y(1) = 5, \quad y'(1) = 18.$

7. Convert each linear ode into a system of first order linear odes.
   (a) $y'' - 4y + 5y = 0.$
   (b) $y''' - 5y'' + 9y = t\cos 2t$
   (c) $y^(4) + 3y''' - \pi y'' + 2\pi y' - 6y = 11$

8. Rewrite each system you found in the question above into a matrix-vector equation.

9. Convert the third order linear equation below into a system of 3 first order linear odes using
   (a) the usual substitutions
   (b) substitutions in the reverse order: $x_1 = y'', x_2 = y', x_3 = y$.

   Deduce the fact that there are multiple ways to rewrite each n-th order linear equation into a linear system of n equations.

$$y''' + 6y'' + y' - 2y = 0.$$

10. Find the eigenvalues and corresponding eigenvectors of the following matrices.
    (a) $\begin{pmatrix} 5 & -2 \\ 9 & -6 \end{pmatrix}$
    (b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
    (c) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$
    (d) $\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$
    (e) $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$
    (f) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
    (g) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$
    (h) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

11. Rewrite the following second linear equation into a system of two equations.

$$y'' + 5y - 6y = 0.$$

    Then find the system's general solution.

12. Find the general solution of each system below.
    (a) $\mathbf{x}' = \begin{pmatrix} 2 & 7 \\ -5 & -10 \end{pmatrix}\mathbf{x}$
    (b) $\mathbf{x}' = \begin{pmatrix} -3 & 6 \\ -3 & 3 \end{pmatrix}\mathbf{x}$
    (c) $\mathbf{x}' = \begin{pmatrix} 8 & -4 \\ 1 & 4 \end{pmatrix}\mathbf{x}$
    (d) $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -5 \end{pmatrix}\mathbf{x}$
    (e) $\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\mathbf{x}$
    (f) $\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -4 \end{pmatrix}\mathbf{x}$