

## PROBABILITIES, RANDOM VARIABLES AND THEIR DISTRIBUTIONS.

### 3.4.2 Standard Normal Distribution

- This is a special type of normal distribution which has a mean of 0 and variance is 1.
- The random variable is denoted by capital letter Z. It is therefore represented as:

$$\mathbf{Z \sim N(0, 1)}$$

The above is read as “the random variable Z is normally distributed with mean 0 and variance 1.

- Values from normal distributions can be mapped directly onto standard normal distributions through a transformation as follows:

$$\mathbf{Z = (x - \mu) / \sigma}$$

- Standard normal tables are then used to find the probabilities. Tables are normally used to find the probability of obtaining values Z that are less than some specified value z (denoted by small letter). The argument is that either  $Z < z$  or  $Z \geq z$  because together, these two account for all the distribution. Remember that probability under the normal curve adds up to 1 therefore:

$$P(Z < z) + P(Z \geq z) = 1$$

Examples (Use standard statistical table I will provide)

- 1)  $P(Z < 1.5) = 0.9332$
- 2)  $P(Z \geq 1.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668$
- 3)  $P(Z < -1.5) = P(Z \geq 1.5)$
- 4)  $P(Z > -0.75) = P(Z < 0.75)$
- 5) In a course that is modelled by a normal distribution, what is the probability of a student getting a total score of less than 180 marks if the average is 161 and the variance is 132?

$$P(X < 180)$$

- Convert normal to standard normal

$$\begin{aligned} Z &= (x - \mu) / \sigma \\ &= (180 - 161) / \sqrt{132} \\ &= 1.65 \end{aligned}$$

- Use standard normal table

$$P(X < 180) = P(Z < 1.65) = \mathbf{0.9505}$$

The chance of a student getting less than 180 marks in a test is 95.05%

- 6) A certain bacteria has a lifetime which is normally distributed with a mean of 62 hours and standard deviation of 3 hours. What is the probability of this bacteria lasting less than 68 hours?  
*This will be solved in the video. Please try it on your own as well.*