QUESTION 1

$$x\sqrt{x+y} = 8 - xy$$

$$x(x+y)^{\frac{1}{2}} = 8 - xy$$

$$x\left(\tfrac{1}{2}\right)(x+y)^{-\frac{1}{2}}(1+y') + (x+y)^{\frac{1}{2}} = -xy' - y$$

$$\frac{x(1+y')}{2(x+y)^{\frac{1}{2}}} + (x+y)^{\frac{1}{2}} = -xy' - y$$

$$\frac{x + xy'}{2(x+y)^{\frac{1}{2}}} + (x+y)^{\frac{1}{2}} = -xy' - y$$

$$\frac{x}{2(x+y)^{\frac{1}{2}}} + \frac{xy'}{2(x+y)^{\frac{1}{2}}} + (x+y)^{\frac{1}{2}} = -xy' - y$$

$$\frac{xy'}{2(x+y)^{\frac{1}{2}}} + xy' = -y - \frac{\cancel{x}}{2(x+y)^{\frac{1}{2}}} - (x+y)^{\frac{1}{2}}$$

$$xy' + 2xy'(x+y)^{\frac{1}{2}} = -2y(x+y)^{\frac{1}{2}} - x - 2(x+y)$$

$$y'\left(x + 2x(x+y)^{\frac{1}{2}}\right) = -2y(x+y)^{\frac{1}{2}} - 3x + 2y$$

$$\therefore\ y' = \frac{-2y(x+y)^{\frac{1}{2}} - 3x - 2y}{x + 2x(x+y)^{\frac{1}{2}}}$$

QUESTION 2

$\cancel{\sin 33^0}$

$f(x) = \sin 33^0$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

i) $L(x) = f(a) + f'(a)(x-a)$

$= \sin 30 + \cos 30\,(\sin 33 - \sin 30)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{11\pi}{60} - \frac{\pi}{6}\right)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{60}\right)$

$= \underline{\underline{0.545344984}}$

ii) $f(x) = -\sin a < 0$, error is negative

$|f''(a)| = |-\sin a| = \sin a = \sin 30 = 0.5 = K$

$E = \frac{K}{2}(x-a)^2$

$= \frac{0.5}{2}\left(\frac{\pi}{60}\right)^2$

$= \underline{\underline{0.000685389}}$

~~iii) $f''(x) = -\sin a < 0$~~

iii) Negative error mean $L(x) > f(x)$

$\therefore$ Range of $\sin$ ~~33~~ lies in, $\underline{(0.54466,\ 0.545345)}$

QUESTION 3

Let $f(x) = \frac{1}{2+x}$, about $x=1$ order n

$$f(x) = (2+x)^{-1}$$
$$= \frac{1}{3}$$
$$f'(x) = (2+x)^{-1}$$
$$= -(2+x)^{-2}$$
$$= -\frac{1}{9}$$

$$f''(x) = 2(2+x)^{-3}$$
$$= \frac{2}{27}$$

$$\therefore P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots + \frac{f^n(a)(x-a)^n}{n!}$$

$$P_n(x) = \frac{1}{3} - \frac{1}{9}(x-a) + \frac{1}{27}(x-1)^2 - \frac{1}{81}(x-1)^3 + \dots + \frac{f^n(x-a)}{n!}$$
$$= \frac{1}{3} - \frac{(x-1)}{9} + \frac{(x-1)^2}{27} - \frac{(x-1)^3}{81} + \dots + \frac{f^n(a)(x-a)^n}{n!}$$

QUESTION 4

$= \lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right)$

$= \lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right) \cdot \frac{\left(x+\sqrt{x^2+x}\right)}{\left(x+\sqrt{x^2+x}\right)}$

$= \lim_{x\to\infty} \frac{x^2 + x\sqrt{x^2+x} - x\sqrt{x^2+x} - x^2 + x}{x+\sqrt{x^2+x}}$

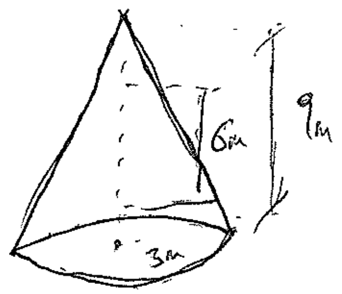
$= \lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+\cancel{x}/x}}$

$= \lim_{x\to\infty} \frac{-1}{1+\sqrt{1+1/\infty}}$

$= \lim_{x\to\infty} = \underline{\frac{-1}{2}}$

QUESTION 5

$\frac{dv}{dt_n} = \frac{dv}{dt_{in}} + \frac{dv}{dt_{out}}$



$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{3}{9}$

$r = \frac{1}{3}h$

$\therefore v = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi h^3}{27}$

$\frac{dv}{dt} = \frac{3\pi h^2}{27}\cdot\frac{dh}{\cancel{dt}}$

$\frac{dv}{dt} = \frac{3\pi(6)^2}{27}\cdot 0.2 m/h$

$= \cancel{0.05\pi/h}$ ~~0.05...~~

$= \pi/0.8 \; m^3/h$

$\therefore$ ~~$\pi/0.8$~~

$2.51 = 10 + \frac{dv}{dt_{in}}$

$\therefore \frac{dv}{dt_{in}} = 7.487 m^3/h$

QUESTION 5

$\Rightarrow \$40 \rightarrow 80$ rooms

$\$(40+x) \Rightarrow (80-2x)$

Income $= (40+x)(80\cancel{+2x})$

$= 3200 \cancel{+2}x \cdots (i)$

Cost $= \$10(80-2x) + 2(2x)$

$= 800 - 20x + 4x$

$= 800 - 16x$

$\therefore$ ~~Profit = Income~~

Profit = Income − Cost

$= 3200 - 2x^2 - 800 + 16x$

$$\frac{d\,\text{profit}}{dx} = -4x^{\cancel{2}} + 16$$

$-4x^{\cancel{2}} + 16 = 0,$

$4x = 16$

$x = \$4$

$\therefore = (40+x) =$

$= (40+4)$

$= \underline{\underline{\$44 \text{ to maximise}}}$

QUESTION 6

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$= y^2 = \left(1 - \frac{x^2}{a^2}\right) b^2$ and $x^2 = a^2 - \frac{a^2y^2}{b^2}$ ... (i)

$= y^2 = b^2 - \frac{b^2x^2}{a^2}$

$= \frac{dy}{dx} \cdot 2y = \frac{-2b^2x}{a^2}$

$= \frac{dy}{dx} = \frac{-b^2x}{ya^2}$ ... (ii)

$A = 2x \cdot 2y$ (Area of rectangle)

$= \quad = 4xy$

$= \therefore \frac{dA}{dx} = 4y + 4x \cdot \frac{dy}{dx}$ ... (iii)

$= \therefore 4y + 4x\left(\frac{-b^2x}{a^2y}\right) = 0$

$= 4y - \frac{4b^2x^2}{a^2y} = 0$

$= 4a^2y^2 = 4b^2x^2$

$= y^2 = \frac{b^2x^2}{a^2}$ and $= x^2 = \frac{a^2y^2}{b^2}$

$= b^2 - \frac{b^2x^2}{a^2} = \frac{b^2x^2}{a^2}$

$= a^2b^2 - b^2x^2 = b^2x^2$

$a^2b^2 = 2b^2x^2$

$2x^2 = a^2$

$= x = \frac{a}{\sqrt{2}}$

and

$= a^2 - \frac{a^2y^2}{b^2} = \frac{a^2y^2}{b^2}$

$= a^2b^2 - a^2y^2 = a^2y^2$

$2a^2y^2 = a^2b^2$

$2y^2 = b^2$

$= y = \frac{b}{\sqrt{2}}$

$\therefore A = 4xy$

$= 4\left(\frac{a}{\sqrt{2}}\right)\left(\frac{b}{\sqrt{2}}\right)$

$= 4\left(\frac{ab}{2}\right)$

$A = 2ab$

QUESTION 7

$I_n = \int_0^{\pi/2} x^n \sin x \, dx$ Let $u = x^n$

$dv = \sin x \, dx$

$\Rightarrow I_n = -x^n \cos x - \int v \, du$ $\frac{du}{dx} = n x^{n-1}$

$= -x^n \cos x + \int \cos x \, n x^{n-1} dx$ $du = n x^{n-1} dx, \quad V = -\cos x$

$= x^n \cos x + n \int x^{n-1} \cos x \, dx$ $u_1 = x^{n-1}$

$\Rightarrow I_n = -x^n \cos x + n [x^{n-1} \sin x] - \int \sin x (n-1) x^{n-1} dx$ $\frac{du_1}{dx} = (n-1) x^{n-1}$

$= -x^n \cos x + n x^{n-1} \sin x - n(n-1) \int x^{n-2} \sin x \, dx$

$= -x^n \cos x + n x^{n-1} \sin x - n(n-1) I_{n-2}$ $du_1 = (n-1) x^{n-1} dx$

$dv = \cos x \, dx$

$\Rightarrow I_6 = x^6 \cos x + 6x^5 \sin x - 30 I_4 + C$ $\int dv = \sin x$

$I_4 = -x^4 \cos x + 4x^3 \sin x - 12 I_2 + C$

$\Rightarrow I_2 = -x^2 \cos x + 2 \sin x + 2 \cos x + C$

$\Rightarrow I_6 = -x^6 \cos x + 6x^5 \sin x - 30 [-x^4 \cos x + 4x^3 \sin x - 12(-x \cos x + 2 \sin x + 2 \cos x)]$

$I_6 = x^6 \cos x + 6x^5 \sin x + 30x^4 \cos x + 120 x^5 \sin x - 360 x^2 \cos x + 120 \sin x + 720 \cos x$

$\Rightarrow I^6 \int_0^{\pi/2} = (6x^5 - 120 x^5 + 720 x) - (-x^6 + 30x^4 - 360 x^2 + 720)$

$\therefore I_6 \Big|_0^{\pi/2} = x^6 + 6x^5 - 30x^4 - 120 x^3 + 360 x^2 + 720 x + 720$

QUESTION 8

$I = \int_{-\frac{1}{3}}^{2} \frac{x}{\sqrt[3]{3x+2}} \, dx$

let $u = 3x+2$

$\frac{du}{dx} = 3$

~~$dx = \frac{u^3 - 2}{3}$~~

$dx = \frac{u^3-2}{3}$ --- (1)

$= \int \frac{\frac{u-2}{3}}{\sqrt[3]{u}} \cdot \frac{du}{3}$

$= \int \frac{u-2}{9\sqrt[3]{u}} \, du$

$= \frac{1}{9} \int \left[\frac{u-2}{\sqrt[3]{u}} \right] du$

$= \frac{1}{9} \left[\frac{3}{5} u^{\frac{5}{3}} - \frac{9}{2} u^{\frac{2}{3}} \right]$

$= \frac{1}{5} u^{\frac{5}{3}} - \frac{1}{2} u^{\frac{2}{3}}$

$= \frac{(3x+2)^{\frac{5}{3}}}{15} - \frac{(3x+2)^{\frac{2}{3}}}{2}$

$\Rightarrow \left[\frac{(3x+2)^{\frac{5}{3}}}{15} - \frac{(3x+2)^{\frac{2}{3}}}{2} \right]_{-\frac{1}{3}}^{2}$

$= \left[\frac{(3(2)+2)^{\frac{5}{3}}}{15} - \frac{(3(2)+2)^{\frac{2}{3}}}{2} \right] - \left[\frac{(3(-\frac{1}{3})+2)^{\frac{5}{3}}}{15} - \frac{(3(-\frac{1}{3})+2)^{\frac{2}{3}}}{2} \right]$

$= \left[\frac{8^{\frac{5}{3}}}{15} - \frac{8^{\frac{2}{3}}}{3} \right] - \left[\frac{1}{15} - \frac{1}{3} \right]$

$= \left[\frac{32}{15} - \frac{4}{3} \right] - \left[-\frac{4}{15} \right]$

$= \frac{16}{15} \approx 1.0667$